1

UNITS, PHYSICAL QUANTITIES AND VECTORS

1.1. **IDENTIFY:** Convert units from mi to km and from km to ft.

SET UP: $1\text{ in.} = 2.54\text{ cm}$, $1\text{ km} = 1000\text{ m}$, $12\text{ in.} = 1\text{ ft}$, $1\text{ mi} = 5280\text{ ft}$.

EXECUTE: **(a)** $1.00\text{ mi} = (1.00\text{ mi})\left(\dfrac{5280\text{ ft}}{1\text{ mi}}\right)\left(\dfrac{12\text{ in.}}{1\text{ ft}}\right)\left(\dfrac{2.54\text{ cm}}{1\text{ in.}}\right)\left(\dfrac{1\text{ m}}{10^2\text{ cm}}\right)\left(\dfrac{1\text{ km}}{10^3\text{ m}}\right) = 1.61\text{ km}$

(b) $1.00\text{ km} = (1.00\text{ km})\left(\dfrac{10^3\text{ m}}{1\text{ km}}\right)\left(\dfrac{10^2\text{ cm}}{1\text{ m}}\right)\left(\dfrac{1\text{ in.}}{2.54\text{ cm}}\right)\left(\dfrac{1\text{ ft}}{12\text{ in.}}\right) = 3.28\times10^3\text{ ft}$

EVALUATE: A mile is a greater distance than a kilometer. There are 5280 ft in a mile but only 3280 ft in a km.

1.2. **IDENTIFY:** Convert volume units from L to in.^3.

SET UP: $1\text{ L} = 1000\text{ cm}^3$. $1\text{ in.} = 2.54\text{ cm}$

EXECUTE: $0.473\text{ L}\times\left(\dfrac{1000\text{ cm}^3}{1\text{ L}}\right)\times\left(\dfrac{1\text{ in.}}{2.54\text{ cm}}\right)^3 = 28.9\text{ in.}^3$.

EVALUATE: 1 in.^3 is greater than 1 cm^3, so the volume in in.^3 is a smaller number than the volume in cm^3, which is 473 cm^3.

1.3. **IDENTIFY:** We know the speed of light in m/s. $t = d/v$. Convert 1.00 ft to m and t from s to ns.

SET UP: The speed of light is $v = 3.00\times10^8\text{ m/s}$. $1\text{ ft} = 0.3048\text{ m}$. $1\text{ s} = 10^9\text{ ns}$.

EXECUTE: $t = \dfrac{0.3048\text{ m}}{3.00\times10^8\text{ m/s}} = 1.02\times10^{-9}\text{ s} = 1.02\text{ ns}$

EVALUATE: In 1.00 s light travels $3.00\times10^8\text{ m} = 3.00\times10^5\text{ km} = 1.86\times10^5\text{ mi}$.

1.4. **IDENTIFY:** Convert the units from g to kg and from cm^3 to m^3.

SET UP: $1\text{ kg} = 1000\text{ g}$. $1\text{ m} = 1000\text{ cm}$.

EXECUTE: $11.3\ \dfrac{\text{g}}{\text{cm}^3}\times\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)\times\left(\dfrac{100\text{ cm}}{1\text{ m}}\right)^3 = 1.13\times10^4\ \dfrac{\text{kg}}{\text{m}^3}$

EVALUATE: The ratio that converts cm to m is cubed, because we need to convert cm^3 to m^3.

1.5. **IDENTIFY:** Convert volume units from in.^3 to L.

SET UP: $1\text{ L} = 1000\text{ cm}^3$. $1\text{ in.} = 2.54\text{ cm}$.

EXECUTE: $(327\text{ in.}^3)\times(2.54\text{ cm/in.})^3\times(1\text{ L}/1000\text{ cm}^3) = 5.36\text{ L}$

EVALUATE: The volume is 5360 cm^3. 1 cm^3 is less than 1 in.^3, so the volume in cm^3 is a larger number than the volume in in.^3.

1.6. **IDENTIFY:** Convert ft^2 to m^2 and then to hectares.

SET UP: $1.00\text{ hectare} = 1.00\times10^4\text{ m}^2$. $1\text{ ft} = 0.3048\text{ m}$.

EXECUTE: The area is $(12.0\text{ acres})\left(\dfrac{43{,}600\text{ ft}^2}{1\text{ acre}}\right)\left(\dfrac{0.3048\text{ m}}{1.00\text{ ft}}\right)^2\left(\dfrac{1.00\text{ hectare}}{1.00\times10^4\text{ m}^2}\right) = 4.86\text{ hectares}$.

EVALUATE: Since $1\text{ ft} = 0.3048\text{ m}$, $1\text{ ft}^2 = (0.3048)^2\text{ m}^2$.

1.7. **IDENTIFY:** Convert seconds to years.

SET UP: $1\text{ billion seconds} = 1\times10^9\text{ s}$. $1\text{ day} = 24\text{ h}$. $1\text{ h} = 3600\text{ s}$.

EXECUTE: $1.00\text{ billion seconds} = (1.00\times10^9\text{ s})\left(\dfrac{1\text{ h}}{3600\text{ s}}\right)\left(\dfrac{1\text{ day}}{24\text{ h}}\right)\left(\dfrac{1\text{ y}}{365\text{ days}}\right) = 31.7\text{ y}$.

EVALUATE: The conversion $1\text{ y} = 3.156\times 10^7\text{ s}$ assumes $1\text{ y} = 365.24\text{ d}$, which is the average for one extra day every four years, in leap years. The problem says instead to assume a 365-day year.

1.8. **IDENTIFY:** Apply the given conversion factors.

SET UP: $1\text{ furlong} = 0.1250\text{ mi}$ and $1\text{ fortnight} = 14\text{ days}$. $1\text{ day} = 24\text{ h}$.

EXECUTE: $$(180{,}000\text{ furlongs/fortnight})\left(\frac{0.125\text{ mi}}{1\text{ furlong}}\right)\left(\frac{1\text{ fortnight}}{14\text{ days}}\right)\left(\frac{1\text{ day}}{24\text{ h}}\right) = 67\text{ mi/h}$$

EVALUATE: A furlong is less than a mile and a fortnight is many hours, so the speed limit in mph is a much smaller number.

1.9. **IDENTIFY:** Convert miles/gallon to km/L.

SET UP: $1\text{ mi} = 1.609\text{ km}$. $1\text{ gallon} = 3.788\text{ L}$.

EXECUTE: **(a)** $$55.0\text{ miles/gallon} = (55.0\text{ miles/gallon})\left(\frac{1.609\text{ km}}{1\text{ mi}}\right)\left(\frac{1\text{ gallon}}{3.788\text{ L}}\right) = 23.4\text{ km/L}.$$

(b) The volume of gas required is $\dfrac{1500\text{ km}}{23.4\text{ km/L}} = 64.1\text{ L}$. $\dfrac{64.1\text{ L}}{45\text{ L/tank}} = 1.4\text{ tanks}$.

EVALUATE: $1\text{ mi/gal} = 0.425\text{ km/L}$. A km is very roughly half a mile and there are roughly 4 liters in a gallon, so $1\text{ mi/gal} \sim \frac{2}{4}\text{ km/L}$, which is roughly our result.

1.10. **IDENTIFY:** Convert units.

SET UP: Use the unit conversions given in the problem. Also, $100\text{ cm} = 1\text{ m}$ and $1000\text{ g} = 1\text{ kg}$.

EXECUTE: **(a)** $$\left(60\frac{\text{mi}}{\text{h}}\right)\left(\frac{1\text{h}}{3600\text{s}}\right)\left(\frac{5280\text{ ft}}{1\text{mi}}\right) = 88\frac{\text{ft}}{\text{s}}$$

(b) $$\left(32\frac{\text{ft}}{\text{s}^2}\right)\left(\frac{30.48\text{cm}}{1\text{ft}}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right) = 9.8\frac{\text{m}}{\text{s}^2}$$

(c) $$\left(1.0\frac{\text{g}}{\text{cm}^3}\right)\left(\frac{100\text{ cm}}{1\text{ m}}\right)^3\left(\frac{1\text{ kg}}{1000\text{ g}}\right) = 10^3\frac{\text{kg}}{\text{m}^3}$$

EVALUATE: The relations $60\text{ mi/h} = 88\text{ ft/s}$ and $1\text{ g/cm}^3 = 10^3\text{ kg/m}^3$ are exact. The relation $32\text{ ft/s}^2 = 9.8\text{ m/s}^2$ is accurate to only two significant figures.

1.11. **IDENTIFY:** We know the density and mass; thus we can find the volume using the relation $\text{density} = \text{mass/volume} = m/V$. The radius is then found from the volume equation for a sphere and the result for the volume.

SET UP: $\text{Density} = 19.5\text{ g/cm}^3$ and $m_{\text{critical}} = 60.0\text{ kg}$. For a sphere $V = \frac{4}{3}\pi r^3$.

EXECUTE: $$V = m_{\text{critical}}/\text{density} = \left(\frac{60.0\text{ kg}}{19.5\text{ g/cm}^3}\right)\left(\frac{1000\text{ g}}{1.0\text{ kg}}\right) = 3080\text{ cm}^3.$$

$$r = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3}{4\pi}(3080\text{ cm}^3)} = 9.0\text{ cm}.$$

EVALUATE: The density is very large, so the 130 pound sphere is small in size.

1.12. **IDENTIFY:** Use your calculator to display $\pi\times 10^7$. Compare that number to the number of seconds in a year.

SET UP: $1\text{ yr} = 365.24\text{ days}$, $1\text{ day} = 24\text{ h}$, and $1\text{ h} = 3600\text{ s}$.

EXECUTE: $$(365.24\text{ days/1 yr})\left(\frac{24\text{ h}}{1\text{ day}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 3.15567\ldots\times 10^7\text{ s};\ \pi\times 10^7\text{ s} = 3.14159\ldots\times 10^7\text{ s}$$

The approximate expression is accurate to two significant figures.

EVALUATE: The close agreement is a numerical accident.

1.13. **IDENTIFY:** The percent error is the error divided by the quantity.

SET UP: The distance from Berlin to Paris is given to the nearest 10 km.

EXECUTE: **(a)** $$\frac{10\text{ m}}{890\times 10^3\text{ m}} = 1.1\times 10^{-3}\%.$$

(b) Since the distance was given as 890 km, the total distance should be 890,000 meters. We know the total distance to only three significant figures.

EVALUATE: In this case a very small percentage error has disastrous consequences.

1.14. **IDENTIFY:** When numbers are multiplied or divided, the number of significant figures in the result can be no greater than in the factor with the fewest significant figures. When we add or subtract numbers it is the location of the decimal that matters.

SET UP: 12 mm has two significant figures and 5.98 mm has three significant figures.

EXECUTE: **(a)** $(12\text{ mm})\times(5.98\text{ mm}) = 72\text{ mm}^2$ (two significant figures)

(b) $\dfrac{5.98\text{ mm}}{12\text{ mm}} = 0.50$ (also two significant figures)

(c) 36 mm (to the nearest millimeter)

(d) 6 mm

(e) 2.0 (two significant figures)

EVALUATE: The length of the rectangle is known only to the nearest mm, so the answers in parts (c) and (d) are known only to the nearest mm.

1.15. **IDENTIFY** and **SET UP:** In each case, estimate the precision of the measurement.

EXECUTE: **(a)** If a meter stick can measure to the nearest millimeter, the error will be about 0.13%.

(b) If the chemical balance can measure to the nearest milligram, the error will be about $8.3\times10^{-3}\%$.

(c) If a handheld stopwatch (as opposed to electric timing devices) can measure to the nearest tenth of a second, the error will be about $2.8\times10^{-2}\%$.

EVALUATE: The percent errors are those due only to the limit of precision of the measurement.

1.16. **IDENTIFY:** Use the extreme values in the piece's length and width to find the uncertainty in the area.

SET UP: The length could be as large as 5.11 cm and the width could be as large as 1.91 cm.

EXECUTE: The area is $9.69 \pm 0.07\text{ cm}^2$. The fractional uncertainty in the area is $\dfrac{0.07\text{ cm}^2}{9.69\text{ cm}^2} = 0.72\%,$ and the fractional uncertainties in the length and width are $\dfrac{0.01\text{ cm}}{5.10\text{ cm}} = 0.20\%$ and $\dfrac{0.01\text{ cm}}{1.9\text{ cm}} = 0.53\%.$ The sum of these fractional uncertainties is $0.20\% + 0.53\% = 0.73\%$, in agreement with the fractional uncertainty in the area.

EVALUATE: The fractional uncertainty in a product of numbers is greater than the fractional uncertainty in any of the individual numbers.

1.17. **IDENTIFY:** Calculate the average volume and diameter and the uncertainty in these quantities.

SET UP: Using the extreme values of the input data gives us the largest and smallest values of the target variables and from these we get the uncertainty.

EXECUTE: **(a)** The volume of a disk of diameter *d* and thickness *t* is $V = \pi(d/2)^2 t.$

The average volume is $V = \pi(8.50\text{ cm}/2)^2(0.50\text{ cm}) = 2.837\text{ cm}^3.$ But *t* is given to only two significant figures so the answer should be expressed to two significant figures: $V = 2.8\text{ cm}^3.$

We can find the uncertainty in the volume as follows. The volume could be as large as $V = \pi(8.52\text{ cm}/2)^2(0.055\text{ cm}) = 3.1\text{ cm}^3,$ which is 0.3 cm^3 larger than the average value. The volume could be as small as $V = \pi(8.52\text{ cm}/2)^2(0.045\text{ cm}) = 2.5\text{ cm}^3,$ which is 0.3 cm^3 smaller than the average value. The uncertainty is $\pm0.3\text{ cm}^3,$ and we express the volume as $V = 2.8 \pm 0.3\text{ cm}^3.$

(b) The ratio of the average diameter to the average thickness is $8.50\text{ cm}/0.050\text{ cm} = 170.$ By taking the largest possible value of the diameter and the smallest possible thickness we get the largest possible value for this ratio: $8.52\text{ cm}/0.045\text{ cm} = 190.$ The smallest possible value of the ratio is $8.48/0.055 = 150.$ Thus the uncertainty is ±20 and we write the ratio as $170 \pm 20.$

EVALUATE: The thickness is uncertain by 10% and the percentage uncertainty in the diameter is much less, so the percentage uncertainty in the volume and in the ratio should be about 10%.

1.18. **IDENTIFY:** Estimate the number of people and then use the estimates given in the problem to calculate the number of gallons.

SET UP: Estimate 3×10^8 people, so 2×10^8 cars.

EXECUTE: $(\text{Number of cars} \times \text{miles/car day})/(\text{mi/gal}) = \text{gallons/day}$

$$(2\times10^8\text{ cars}\times 10000\text{ mi/yr/car}\times 1\text{ yr/365 days})/(20\text{ mi/gal}) = 3\times10^8\text{ gal/day}$$

EVALUATE: The number of gallons of gas used each day approximately equals the population of the U.S.

1.19. **IDENTIFY:** Express 200 kg in pounds. Express each of 200 m, 200 cm and 200 mm in inches. Express 200 months in years.

SET UP: A mass of 1 kg is equivalent to a weight of about 2.2 lbs. $1\text{ in.} = 2.54\text{ cm}$. $1\text{ y} = 12\text{ months}$.

EXECUTE: **(a)** 200 kg is a weight of 440 lb. This is much larger than the typical weight of a man.

(b) $200\text{ m} = (2.00\times10^4\text{ cm})\left(\dfrac{1\text{ in.}}{2.54\text{ cm}}\right) = 7.9\times10^3\text{ inches}$. This is much greater than the height of a person.

(c) $200\text{ cm} = 2.00\text{ m} = 79\text{ inches} = 6.6\text{ ft}$. Some people are this tall, but not an ordinary man.

(d) 200 mm = 0.200 m = 7.9 inches . This is much too short.

(e) $200 \text{ months} = (200 \text{ mon})\left(\frac{1 \text{ y}}{12 \text{ mon}}\right) = 17 \text{ y}$. This is the age of a teenager; a middle-aged man is much older than this.

EVALUATE: None are plausible. When specifying the value of a measured quantity it is essential to give the units in which it is being expressed.

1.20. **IDENTIFY:** The number of kernels can be calculated as $N = V_{\text{bottle}} / V_{\text{kernel}}$.

SET UP: Based on an Internet search, Iowan corn farmers use a sieve having a hole size of 0.3125 in. ≅ 8 mm to remove kernel fragments. Therefore estimate the average kernel length as 10 mm, the width as 6 mm and the depth as 3 mm. We must also apply the conversion factors $1 \text{ L} = 1000 \text{ cm}^3$ and 1 cm = 10 mm.

EXECUTE: The volume of the kernel is: $V_{\text{kernel}} = (10 \text{ mm})(6 \text{ mm})(3 \text{ mm}) = 180 \text{ mm}^3$. The bottle's volume is: $V_{\text{bottle}} = (2.0 \text{ L})\left[(1000 \text{ cm}^3)/(1.0 \text{ L})\right]\left[(10 \text{ mm})^3/(1.0 \text{ cm})^3\right] = 2.0 \times 10^6 \text{ mm}^3$. The number of kernels is then $N_{\text{kernels}} = V_{\text{bottle}} / V_{\text{kernels}} \approx (2.0 \times 10^6 \text{ mm}^3)/(180 \text{ mm}^3) = 11{,}000 \text{ kernels}$.

EVALUATE: This estimate is highly dependent upon your estimate of the kernel dimensions. And since these dimensions vary amongst the different available types of corn, acceptable answers could range from 6,500 to 20,000.

1.21. **IDENTIFY:** Estimate the number of pages and the number of words per page.

SET UP: Assuming the two-volume edition, there are approximately a thousand pages, and each page has between 500 and a thousand words (counting captions and the smaller print, such as the end-of-chapter exercises and problems).

EXECUTE: An estimate for the number of words is about 10^6 .

EVALUATE: We can expect that this estimate is accurate to within a factor of 10.

1.22. **IDENTIFY:** Approximate the number of breaths per minute. Convert minutes to years and cm^3 to m^3 to find the volume in m^3 breathed in a year.

SET UP: Assume 10 breaths/min . $1 \text{ y} = (365 \text{ d})\left(\frac{24 \text{ h}}{1 \text{ d}}\right)\left(\frac{60 \text{ min}}{1 \text{ h}}\right) = 5.3 \times 10^5 \text{ min}$. $10^2 \text{ cm} = 1 \text{ m}$ so $10^6 \text{ cm}^3 = 1 \text{ m}^3$. The volume of a sphere is $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$, where *r* is the radius and *d* is the diameter. Don't forget to account for four astronauts.

EXECUTE: **(a)** The volume is $(4)(10 \text{ breaths/min})(500 \times 10^{-6} \text{ m}^3)\left(\frac{5.3 \times 10^5 \text{ min}}{1 \text{ y}}\right) = 1 \times 10^4 \text{ m}^3/\text{yr}$.

(b) $d = \left(\frac{6V}{\pi}\right)^{1/3} = \left(\frac{6[1 \times 10^4 \text{ m}^3]}{\pi}\right)^{1/3} = 27 \text{ m}$

EVALUATE: Our estimate assumes that each cm^3 of air is breathed in only once, where in reality not all the oxygen is absorbed from the air in each breath. Therefore, a somewhat smaller volume would actually be required.

1.23. **IDENTIFY:** Estimate the number of blinks per minute. Convert minutes to years. Estimate the typical lifetime in years.

SET UP: Estimate that we blink 10 times per minute. 1 y = 365 days . 1 day = 24 h , 1 h = 60 min . Use 80 years for the lifetime.

EXECUTE: The number of blinks is $(10 \text{ per min})\left(\frac{60 \text{ min}}{1 \text{ h}}\right)\left(\frac{24 \text{ h}}{1 \text{ day}}\right)\left(\frac{365 \text{ days}}{1 \text{ y}}\right)(80 \text{ y/lifetime}) = 4 \times 10^8$

EVALUATE: Our estimate of the number of blinks per minute can be off by a factor of two but our calculation is surely accurate to a power of 10.

1.24. **IDENTIFY:** Estimate the number of beats per minute and the duration of a lifetime. The volume of blood pumped during this interval is then the volume per beat multiplied by the total beats.

SET UP: An average middle-aged (40 year-old) adult at rest has a heart rate of roughly 75 beats per minute. To calculate the number of beats in a lifetime, use the current average lifespan of 80 years.

EXECUTE: $N_{\text{beats}} = (75 \text{ beats/min})\left(\frac{60 \text{ min}}{1 \text{ h}}\right)\left(\frac{24 \text{ h}}{1 \text{ day}}\right)\left(\frac{365 \text{ days}}{\text{yr}}\right)\left(\frac{80 \text{ yr}}{\text{lifespan}}\right) = 3 \times 10^9 \text{ beats/lifespan}$

$V_{\text{blood}} = (50 \text{ cm}^3/\text{beat})\left(\frac{1 \text{ L}}{1000 \text{ cm}^3}\right)\left(\frac{1 \text{ gal}}{3.788 \text{ L}}\right)\left(\frac{3 \times 10^9 \text{ beats}}{\text{lifespan}}\right) = 4 \times 10^7 \text{ gal/lifespan}$

EVALUATE: This is a very large volume.

1.25. **IDENTIFY:** Estimation problem

SET UP: Estimate that the pile is $18\text{ in.}\times 18\text{ in.}\times 5\text{ ft } 8\text{ in.}$. Use the density of gold to calculate the mass of gold in the pile and from this calculate the dollar value.

EXECUTE: The volume of gold in the pile is $V = 18\text{ in.}\times 18\text{ in.}\times 68\text{ in.} = 22{,}000\text{ in.}^3$. Convert to cm^3:

$$V = 22{,}000\text{ in.}^3(1000\text{ cm}^3/61.02\text{ in.}^3) = 3.6\times 10^5\text{ cm}^3.$$

The density of gold is 19.3 g/cm^3, so the mass of this volume of gold is

$$m = (19.3\text{ g/cm}^3)(3.6\times 10^5\text{ cm}^3) = 7\times 10^6\text{ g}.$$

The monetary value of one gram is \$10, so the gold has a value of $(\$10/\text{gram})(7\times 10^6\text{ grams}) = \$7\times 10^7$, or about $\$100\times 10^6$ (one hundred million dollars).

EVALUATE: This is quite a large pile of gold, so such a large monetary value is reasonable.

1.26. **IDENTIFY:** Estimate the diameter of a drop and from that calculate the volume of a drop, in m^3. Convert m^3 to L.

SET UP: Estimate the diameter of a drop to be $d = 2\text{ mm}$. The volume of a spherical drop is $V = \frac{4}{3}\pi r^3 = \frac{1}{6}\pi d^3$. $10^3\text{ cm}^3 = 1\text{ L}$.

EXECUTE: $V = \frac{1}{6}\pi(0.2\text{ cm})^3 = 4\times 10^{-3}\text{ cm}^3$. The number of drops in 1.0 L is $\dfrac{1000\text{ cm}^3}{4\times 10^{-3}\text{ cm}^3} = 2\times 10^5$

EVALUATE: Since $V \sim d^3$, if our estimate of the diameter of a drop is off by a factor of 2 then our estimate of the number of drops is off by a factor of 8.

1.27. **IDENTIFY:** Estimate the number of students and the average number of pizzas eaten by each student in a school year.

SET UP: Assume a school of thousand students, each of whom averages ten pizzas a year (perhaps an underestimate)

EXECUTE: They eat a total of 10^4 pizzas.

EVALUATE: The same answer applies to a school of 250 students averaging 40 pizzas a year each.

1.28. **IDENTIFY:** The number of bills is the distance to the moon divided by the thickness of one bill.

SET UP: Estimate the thickness of a dollar bills by measuring a short stack, say ten, and dividing the measurement by the total number of bills. I obtain a thickness of roughly 1 mm. From Appendix F, the distance from the earth to the moon is $3.8\times 10^8\text{ m}$.

EXECUTE: $N_{\text{bills}} = \left(\dfrac{3.8\times 10^8\text{ m}}{0.1\text{ mm/bill}}\right)\left(\dfrac{10^3\text{ mm}}{1\text{ m}}\right) = 3.8\times 10^{12}\text{ bills} \approx 4\times 10^{12}\text{ bills}$

EVALUATE: This answer represents 4 trillion dollars! The cost of a single space shuttle mission in 2005 is significantly less – roughly 1 billion dollars.

1.29. **IDENTIFY:** The cost would equal the number of dollar bills required; the surface area of the U.S. divided by the surface area of a single dollar bill.

SET UP: By drawing a rectangle on a map of the U.S., the approximate area is 2600 mi by 1300 mi or $3{,}380{,}000\text{ mi}^2$. This estimate is within 10 percent of the actual area, $3{,}794{,}083\text{ mi}^2$. The population is roughly 3.0×10^8 while the area of a dollar bill, as measured with a ruler, is approximately $6\frac{1}{8}$ in. by $2\frac{5}{8}$ in.

EXECUTE: $A_{\text{U.S.}} = (3{,}380{,}000\text{ mi}^2)[(5280\text{ ft})/(1\text{ mi})]^2[(12\text{ in.})/(1\text{ ft})]^2 = 1.4\times 10^{16}\text{ in.}^2$

$A_{\text{bill}} = (6.125\text{ in.})(2.625\text{ in.}) = 16.1\text{ in.}^2$

Total cost $= N_{\text{bills}} = A_{\text{U.S.}}/A_{\text{bill}} = (1.4\times 10^{16}\text{ in.}^2)/(16.1\text{ in.}^2/\text{bill}) = 9\times 10^{14}\text{ bills}$

Cost per person $= (9\times 10^{14}\text{ dollars})/(3.0\times 10^8\text{ persons}) = 3\times 10^6$ dollars/person

EVALUATE: The actual cost would be somewhat larger, because the land isn't flat.

1.30. **IDENTIFY:** The displacements must be added as vectors and the magnitude of the sum depends on the relative orientation of the two displacements.

SET UP: The sum with the largest magnitude is when the two displacements are parallel and the sum with the smallest magnitude is when the two displacements are antiparallel.

EXECUTE: The orientations of the displacements that give the desired sum are shown in Figure 1.30.

EVALUATE: The orientations of the two displacements can be chosen such that the sum has any value between 0.6 m and 4.2 m.

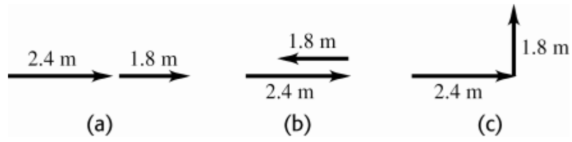


Figure 1.30

1.31. **IDENTIFY:** Draw each subsequent displacement tail to head with the previous displacement. The resultant displacement is the single vector that points from the starting point to the stopping point.

SET UP: Call the three displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$. The resultant displacement $\vec{R}$ is given by $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.

EXECUTE: The vector addition diagram is given in Figure 1.31. Careful measurement gives that $\vec{R}$ is $7.8 \text{ km}, 38^\circ$ north of east.

EVALUATE: The magnitude of the resultant displacement, 7.8 km, is less than the sum of the magnitudes of the individual displacements, $2.6 \text{ km} + 4.0 \text{ km} + 3.1 \text{ km}$.

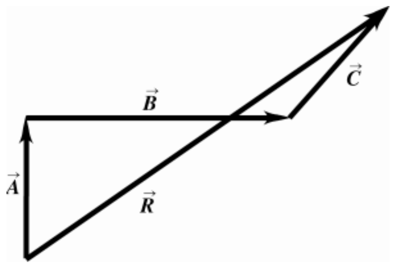


Figure 1.31

1.32. **IDENTIFY:** Draw the vector addition diagram, so scale.

SET UP: The two vectors $\vec{A}$ and $\vec{B}$ are specified in the figure that accompanies the problem.

EXECUTE: **(a)** The diagram for $\vec{C} = \vec{A} + \vec{B}$ is given in Figure 1.32a. Measuring the length and angle of $\vec{C}$ gives $C = 9.0$ m and an angle of $\theta = 34^\circ$.

(b) The diagram for $\vec{D} = \vec{A} - \vec{B}$ is given in Figure 1.32b. Measuring the length and angle of $\vec{D}$ gives $D = 22$ m and an angle of $\theta = 250^\circ$.

(c) $-\vec{A} - \vec{B} = -(A + B)$, so $-\vec{A} - \vec{B}$ has a magnitude of 9.0 m (the same as $\vec{A} + \vec{B}$) and an angle with the $+x$ axis of 214° (opposite to the direction of $\vec{A} + \vec{B}$).

(d) $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{B} - \vec{A}$ has a magnitude of 22 m and an angle with the $+x$ axis of 70° (opposite to the direction of $\vec{A} - \vec{B}$).

EVALUATE: The vector $-\vec{A}$ is equal in magnitude and opposite in direction to the vector $\vec{A}$.

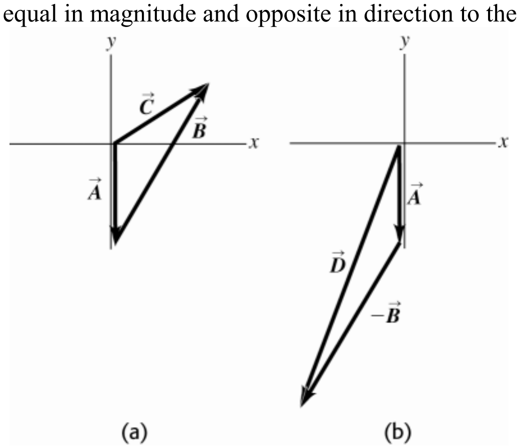


Figure 1.32

1.33. **IDENTIFY:** Since she returns to the starting point, the vectors sum of the four displacements must be zero.

SET UP: Call the three given displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$, and call the fourth displacement $\vec{D}$. $\vec{A} + \vec{B} + \vec{C} + \vec{D} = 0$.

EXECUTE: The vector addition diagram is sketched in Figure 1.33. Careful measurement gives that $\vec{D}$ is $144 \text{ m}, 41^\circ$ south of west.

EVALUATE: $\vec{D}$ is equal in magnitude and opposite in direction to the sum $\vec{A}+\vec{B}+\vec{C}$.

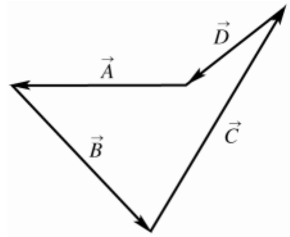


Figure 1.33

1.34. **IDENTIFY** and **SET UP:** Use a ruler and protractor to draw the vectors described. Then draw the corresponding horizontal and vertical components.

EXECUTE: **(a)** Figure 1.34 gives components 4.7 m, 8.1 m.

(b) Figure 1.34 gives components -15.6 km, 15.6 km.

(c) Figure 1.34 gives components 3.82 cm, -5.07 cm.

EVALUATE: The signs of the components depend on the quadrant in which the vector lies.

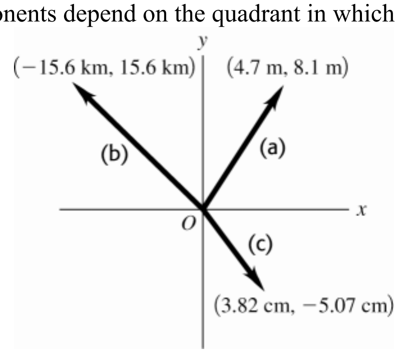


Figure 1.34

1.35. **IDENTIFY:** For each vector $\vec{V}$, use that $V_x = V\cos\theta$ and $V_y = V\sin\theta$, when θ is the angle $\vec{V}$ makes with the $+x$ axis, measured counterclockwise from the axis.

SET UP: For $\vec{A}$, $\theta = 270.0°$. For $\vec{B}$, $\theta = 60.0°$. For $\vec{C}$, $\theta = 205.0°$. For $\vec{D}$, $\theta = 143.0°$.

EXECUTE: $A_x = 0$, $A_y = -8.00$ m. $B_x = 7.50$ m, $B_y = 13.0$ m. $C_x = -10.9$ m, $C_y = -5.07$ m. $D_x = -7.99$ m, $D_y = 6.02$ m.

EVALUATE: The signs of the components correspond to the quadrant in which the vector lies.

1.36. **IDENTIFY:** $\tan\theta = \dfrac{A_y}{A_x}$, for θ measured counterclockwise from the $+x$-axis.

SET UP: A sketch of A_x, A_y and $\vec{A}$ tells us the quadrant in which $\vec{A}$ lies.

EXECUTE:

(a) $\tan\theta = \dfrac{A_y}{A_X} = \dfrac{-1.00\text{ m}}{2.00\text{ m}} = -0.500$. $\theta = \tan^{-1}(-0.500) = 360° - 26.6° = 333°$.

(b) $\tan\theta = \dfrac{A_y}{A_x} = \dfrac{1.00\text{ m}}{2.00\text{ m}} = 0.500$. $\theta = \tan^{-1}(0.500) = 26.6°$.

(c) $\tan\theta = \dfrac{A_y}{A_x} = \dfrac{1.00\text{ m}}{-2.00\text{ m}} = -0.500$. $\theta = \tan^{-1}(-0.500) = 180° - 26.6° = 153°$.

(d) $\tan\theta = \dfrac{A_y}{A_x} = \dfrac{-1.00\text{ m}}{-2.00\text{ m}} = 0.500$. $\theta = \tan^{-1}(0.500) = 180° + 26.6° = 207°$

EVALUATE: The angles 26.6° and 207° have the same tangent. Our sketch tells us which is the correct value of θ.

1.37. **IDENTIFY:** Find the vector sum of the two forces.

SET UP: Use components to add the two forces. Take the $+x$-direction to be forward and the $+y$-direction to be upward.

EXECUTE: The second force has components $F_{2x} = F_2 \cos 32.4° = 433$ N and $F_{2y} = F_2 \sin 32.4° = 275$ N. The first force has components $F_{1x} = 725$ N and $F_{1y} = 0$.

$F_x = F_{1x} + F_{2x} = 1158$ N and $F_y = F_{1y} + F_{2y} = 275$ N

The resultant force is 1190 N in the direction $13.4°$ above the forward direction.

EVALUATE: Since the two forces are not in the same direction the magnitude of their vector sum is less than the sum of their magnitudes.

1.38. **IDENTIFY:** Find the vector sum of the three given displacements.

SET UP: Use coordinates for which $+x$ is east and $+y$ is north. The driver's vector displacements are:

$\vec{A} = 2.6$ km, 0° of north; $\vec{B} = 4.0$ km, 0° of east; $\vec{C} = 3.1$ km, 45° north of east .

EXECUTE: $R_x = A_x + B_x + C_x = 0 + 4.0\text{ km} + (3.1\text{ km})\cos(45°) = 6.2\text{ km}$; $R_y = A_y + B_y + C_y =$ $2.6\text{ km} + 0 + (3.1\text{ km})(\sin 45°) = 4.8\text{ km}$; $R = \sqrt{R_x^2 + R_y^2} = 7.8\text{ km}$; $\theta = \tan^{-1}[(4.8\text{ km})/(6.2\text{ km})] = 38°$;

$\vec{R} = 7.8$ km, 38° north of east. This result is confirmed by the sketch in Figure 1.38.

EVALUATE: Both R_x and R_y are positive and $\vec{R}$ is in the first quadrant.

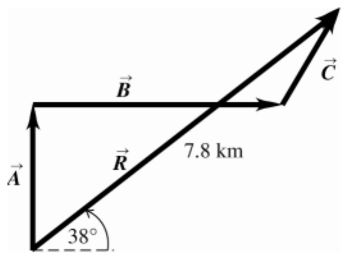


Figure 1.38

1.39. **IDENTIFY:** If $\vec{C} = \vec{A} + \vec{B}$, then $C_x = A_x + B_x$ and $C_y = A_y + B_y$. Use C_x and C_y to find the magnitude and direction of $\vec{C}$.

SET UP: From Figure 1.34 in the textbook, $A_x = 0$, $A_y = -8.00$ m and $B_x = +B \sin 30.0° = 7.50$ m , $B_y = +B \cos 30.0° = 13.0$ m .

EXECUTE: **(a)** $\vec{C} = \vec{A} + \vec{B}$ so $C_x = A_x + B_x = 7.50$ m and $C_y = A_y + B_y = +5.00$ m . $C = 9.01$ m .

$\tan\theta = \dfrac{C_y}{C_x} = \dfrac{5.00\text{ m}}{7.50\text{ m}}$ and $\theta = 33.7°$.

(b) $\vec{B} + \vec{A} = \vec{A} + \vec{B}$, so $\vec{B} + \vec{A}$ has magnitude 9.01 m and direction specified by $33.7°$.

(c) $\vec{D} = \vec{A} - \vec{B}$ so $D_x = A_x - B_x = -7.50$ m and $D_y = A_y - B_y = -21.0$ m . $D = 22.3$ m . $\tan\phi = \dfrac{D_y}{D_x} = \dfrac{-21.0\text{ m}}{-7.50\text{ m}}$ and $\phi = 70.3°$. $\vec{D}$ is in the 3rd quadrant and the angle θ counterclockwise from the $+x$ axis is $180° + 70.3° = 250.3°$.

(d) $\vec{B} - \vec{A} = -(\vec{A} - \vec{B})$, so $\vec{B} - \vec{A}$ has magnitude 22.3 m and direction specified by $\theta = 70.3°$.

EVALUATE: These results agree with those calculated from a scale drawing in Problem 1.32.

1.40. **IDENTIFY:** Use Equations (1.7) and (1.8) to calculate the magnitude and direction of each of the given vectors.

SET UP: A sketch of A_x , A_y and $\vec{A}$ tells us the quadrant in which $\vec{A}$ lies.

EXECUTE: **(a)** $\sqrt{(-8.60\text{ cm})^2 + (5.20\text{ cm})^2} = 10.0$ cm, $\arctan\left(\dfrac{5.20}{-8.60}\right) = 148.8°$ (which is $180° - 31.2°$).

(b) $\sqrt{(-9.7\text{ m})^2 + (-2.45\text{ m})^2} = 10.0$ m, $\arctan\left(\dfrac{-2.45}{-9.7}\right) = 14° + 180° = 194°$.

(c) $\sqrt{(7.75\text{ km})^2 + (-2.70\text{ km})^2} = 8.21$ km, $\arctan\left(\dfrac{-2.7}{7.75}\right) = 340.8°$ (which is $360° - 19.2°$).

EVALUATE: In each case the angle is measured counterclockwise from the $+x$ axis. Our results for θ agree with our sketches.

1.41. **IDENTIFY:** Vector addition problem. We are given the magnitude and direction of three vectors and are asked to find their sum.

SET UP:

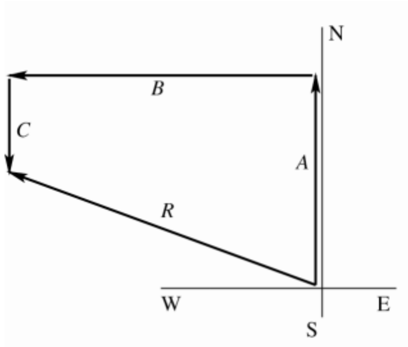


Figure 1.41a

$A = 3.25$ km
$B = 4.75$ km
$C = 1.50$ km

Select a coordinate system where $+x$ is east and $+y$ is north. Let $\vec{A}$, $\vec{B}$ and $\vec{C}$ be the three displacements of the professor. Then the resultant displacement $\vec{R}$ is given by $\vec{R} = \vec{A} + \vec{B} + \vec{C}$. By the method of components, $R_x = A_x + B_x + C_x$ and $R_y = A_y + B_y + C_y$. Find the *x* and *y* components of each vector; add them to find the components of the resultant. Then the magnitude and direction of the resultant can be found from its *x* and *y* components that we have calculated. As always it is essential to draw a sketch.

EXECUTE:

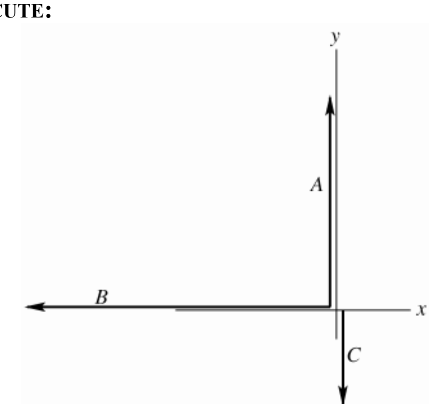


Figure 1.41b

$A_x = 0,\quad A_y = +3.25 \text{ km}$

$B_x = -4.75 \text{ km},\quad B_y = 0$

$C_x = 0,\qquad C_y = -1.50 \text{ km}$

$R_x = A_x + B_x + C_x$

$R_x = 0 - 4.75 \text{ km} + 0 = -4.75 \text{ km}$

$R_y = A_y + B_y + C_y$

$R_y = 3.25 \text{ km} + 0 - 1.50 \text{ km} = 1.75 \text{ km}$

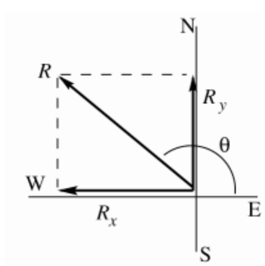


Figure 1.41c

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-4.75 \text{ km})^2 + (1.75 \text{ km})^2}$$

$$R = 5.06 \text{ km}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{1.75 \text{ km}}{-4.75 \text{ km}} = -0.3684$$

$$\theta = 159.8^\circ$$

The angle θ measured counterclockwise from the $+x$-axis. In terms of compass directions, the resultant displacement is 20.2° N of W.

EVALUATE: $R_x < 0$ and $R_y > 0$, so $\vec{R}$ is in 2nd quadrant. This agrees with the vector addition diagram.

1.42. **IDENTIFY:** Add the vectors using components. $\vec{B} - \vec{A} = \vec{B} + (-\vec{A})$.

SET UP: If $\vec{C} = \vec{A} + \vec{B}$ then $C_x = A_x + B_x$ and $C_y = A_y + B_y$. If $\vec{D} = \vec{B} - \vec{A}$ then $D_x = B_x - A_x$ and $D_y = B_y - A_y$.

EXECUTE: **(a)** The *x*- and *y*-components of the sum are $1.30 \text{ cm} + 4.10 \text{ cm} = 5.40 \text{ cm}$, $2.25 \text{ cm} + (-3.75 \text{ cm}) = -1.50 \text{ cm}$.

(b) Using Equations (1.7) and (1.8), $\sqrt{(5.40\text{cm})^2 (-1.50 \text{ cm})^2} = 5.60 \text{ cm}$, $\arctan\left(\dfrac{-1.50}{+5.40}\right) = 344.5^\circ$ ccw.

(c) Similarly, $4.10\text{ cm} - (1.30\text{ cm}) = 2.80\text{ cm}$, $-3.75\text{ cm} - (2.25\text{ cm}) = -6.00\text{ cm}$.

(d) $\sqrt{(2.80\text{cm})^2 + (-6.00\text{cm})^2} = 6.62\text{ cm}$, $\arctan\left(\dfrac{-6.00}{2.80}\right) = 295°$ (which is $360° - 65°$).

EVALUATE: We can draw the vector addition diagram in each case and verify that our results are qualitatively correct.

1.43. **IDENTIFY:** Vector addition problem. $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$.

SET UP: Find the *x*- and *y*-components of $\vec{A}$ and $\vec{B}$. Then the *x*- and *y*-components of the vector sum are calculated from the *x*- and *y*-components of $\vec{A}$ and $\vec{B}$.

EXECUTE:

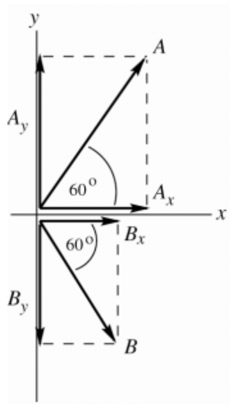


Figure 1.43a

$A_x = A\cos(60.0°)$

$A_x = (2.80\text{ cm})\cos(60.0°) = +1.40\text{ cm}$

$A_y = A\sin(60.0°)$

$A_y = (2.80\text{ cm})\sin(60.0°) = +2.425\text{ cm}$

$B_x = B\cos(-60.0°)$

$B_x = (1.90\text{ cm})\cos(-60.0°) = +0.95\text{ cm}$

$B_y = B\sin(-60.0°)$

$B_y = (1.90\text{ cm})\sin(-60.0°) = -1.645\text{ cm}$

Note that the signs of the components correspond to the directions of the component vectors.

(a) Now let $\vec{R} = \vec{A} + \vec{B}$.

$R_x = A_x + B_x = +1.40\text{ cm} + 0.95\text{ cm} = +2.35\text{ cm}$.

$R_y = A_y + B_y = +2.425\text{ cm} - 1.645\text{ cm} = +0.78\text{ cm}$.

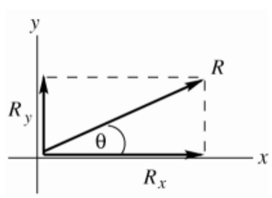


Figure 1.43b

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(2.35\text{ cm})^2 + (0.78\text{ cm})^2}$

$R = 2.48\text{ cm}$

$\tan\theta = \dfrac{R_y}{R_x} = \dfrac{+0.78\text{ cm}}{+2.35\text{ cm}} = +0.3319$

$\theta = 18.4°$

EVALUATE: The vector addition diagram for $\vec{R} = \vec{A} + \vec{B}$ is

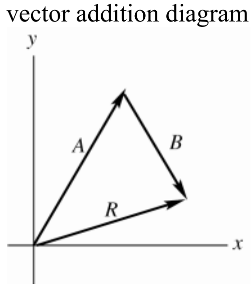


Figure 1.43c

$\vec{R}$ is in the 1st quadrant, with $|R_y| < |R_x|$, in agreement with our calculation.

(b) EXECUTE: Now let $\vec{R} = \vec{A} - \vec{B}$.

$R_x = A_x - B_x = +1.40\text{ cm} - 0.95\text{ cm} = +0.45\text{ cm}.$

$R_y = A_y - B_y = +2.425\text{ cm} + 1.645\text{ cm} = +4.070\text{ cm}.$

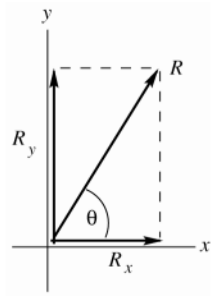


Figure 1.43d

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(0.45\text{ cm})^2 + (4.070\text{ cm})^2}$$

$$R = 4.09\text{ cm}$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{4.070\text{ cm}}{0.45\text{ cm}} = +9.044$$

$$\theta = 83.7°$$

EVALUATE: The vector addition diagram for $\vec{R} = \vec{A} + \left(-\vec{B}\right)$ is

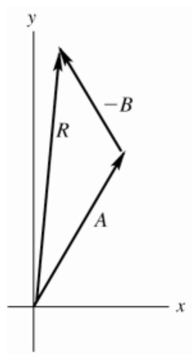


Figure 1.43e

$\vec{R}$ is in the 1st quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

(c) EXECUTE:

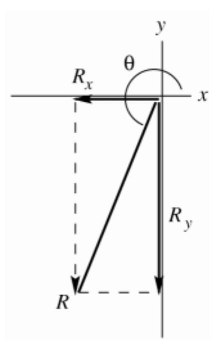


Figure 1.43f

$\vec{B} - \vec{A} = -\left(\vec{A} - \vec{B}\right)$

$\vec{B} - \vec{A}$ and $\vec{A} - \vec{B}$ are equal in magnitude and opposite in direction.

$R = 4.09\text{ cm}$ and

$\theta = 83.7° + 180° = 264°$

EVALUATE: The vector addition diagram for $\vec{R} = \vec{B} + \left(-\vec{A}\right)$ is

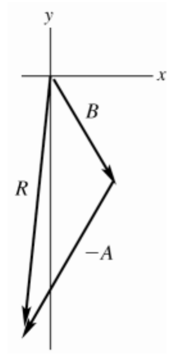


Figure 1.43g

$\vec{R}$ is in the 3rd quadrant, with $|R_x| < |R_y|$, in agreement with our calculation.

1.44. IDENTIFY: The velocity of the boat relative to the earth, $\vec{v}_{\text{B/E}}$, the velocity of the water relative to the earth, $\vec{v}_{\text{W/E}}$, and the velocity of the boat relative to the water, $\vec{v}_{\text{B/W}}$, are related by $\vec{v}_{\text{B/E}} = \vec{v}_{\text{B/W}} + \vec{v}_{\text{W/E}}$.

SET UP: $\vec{v}_{\text{W/E}} = 5.0\text{ km/h}$, north and $\vec{v}_{\text{B/W}} = 7.0\text{ km/h}$, west. The vector addition diagram is sketched in Figure 1.44.

EXECUTE: $v_{\text{B/E}}^2 = v_{\text{W/E}}^2 + v_{\text{B/W}}^2$ and $v_{\text{B/E}} = \sqrt{(5.0\text{ km/h})^2 + (7.0\text{ km/h})^2} = 8.6\text{ km/h}$. $\tan\phi = \dfrac{v_{\text{W/E}}}{v_{\text{B/W}}} = \dfrac{5.0\text{ km/h}}{7.0\text{ km/h}}$ and $\phi = 36°$, north of west.

EVALUATE: Since the two vectors we are adding are perpendicular we can use the Pythagorean theorem directly to find the magnitude of their vector sum.

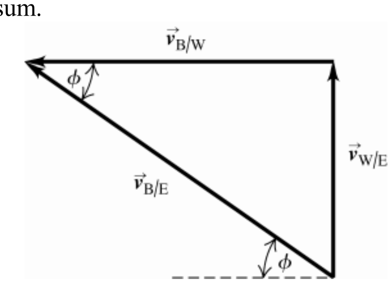


Figure 1.44

1.45. IDENTIFY: Let $A = 625\text{ N}$ and $B = 875\text{ N}$. We are asked to find the vector $\vec{C}$ such that $\vec{A} + \vec{B} = \vec{C} = 0$.

SET UP: $A_x = 0$, $A_y = -625\text{ N}$. $B_x = (875\text{ N})\cos 30° = 758\text{ N}$, $B_y = (875\text{ N})\sin 30° = 438\text{ N}$.

EXECUTE: $C_x = -(A_x + B_x) = -(0 + 758\text{ N}) = -758\text{ N}$. $C_y = -(A_y + B_y) = -(-625\text{ N} + 438\text{ N}) = +187\text{ N}$. Vector $\vec{C}$ and its components are sketched in Figure 1.45. $C = \sqrt{C_x^2 + C_y^2} = 781\text{ N}$. $\tan\phi = \dfrac{|C_y|}{|C_x|} = \dfrac{187\text{ N}}{758\text{ N}}$ and $\phi = 13.9°$.

$\vec{C}$ is at an angle of $13.9°$ above the $-x$-axis and therefore at an angle $180° - 13.9° = 166.1°$ counterclockwise from the $+x$-axis.

EVALUATE: A vector addition diagram for $\vec{A} + \vec{B} + \vec{C}$ verifies that their sum is zero.

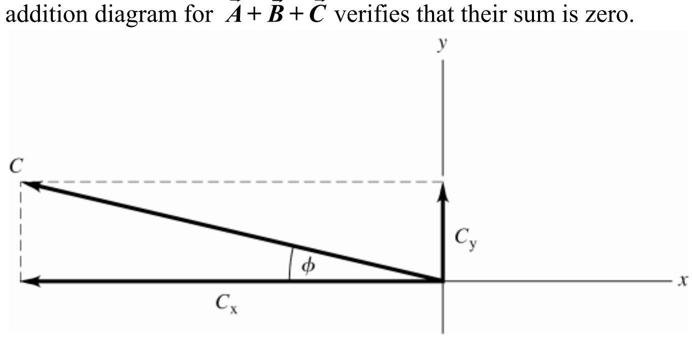


Figure 1.45

1.46. **IDENTIFY:** We know the vector sum and want to find the magnitude of the vectors. Use the method of components.

SET UP: The two vectors $\vec{A}$ and $\vec{B}$ and their resultant $\vec{C}$ are shown in Figure 1.46. Let $+y$ be in the direction of the resultant. $A = B$.

EXECUTE: $C_y = A_y + B_y$. $372\text{ N} = 2A\cos 43.0°$ and $A = 254\text{ N}$.

EVALUATE: The sum of the magnitudes of the two forces exceeds the magnitude of the resultant force because only a component of each force is upward.

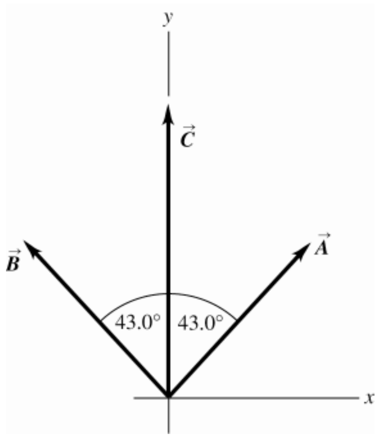


Figure 1.46

1.47. **IDENTIFY:** Find the components of each vector and then use Eq.(1.14).

SET UP: $A_x = 0$, $A_y = -8.00\text{ m}$. $B_x = 7.50\text{ m}$, $B_y = 13.0\text{ m}$. $C_x = -10.9\text{ m}$, $C_y = -5.07\text{ m}$. $D_x = -7.99\text{ m}$, $D_y = 6.02\text{ m}$.

EXECUTE: $\vec{A} = (-8.00\text{ m})\hat{j}$; $\vec{B} = (7.50\text{ m})\hat{i} + (13.0\text{ m})\hat{j}$; $\vec{C} = (-10.9\text{ m})\hat{i} + (-5.07\text{ m})\hat{j}$;

$\vec{D} = (-7.99\text{ m})\hat{i} + (6.02\text{ m})\hat{j}$.

EVALUATE: All these vectors lie in the *xy*-plane and have no *z*-component.

1.48. **IDENTIFY:** The general expression for a vector written in terms of components and unit vectors is $\vec{A} = A_x\hat{i} + A_y\hat{j}$

SET UP: $5.0\vec{B} = 5.0(4\hat{i} - 6\hat{j}) = 20\hat{i} - 30\hat{j}$

EXECUTE: **(a)** $A_x = 5.0$, $A_y = -6.3$ **(b)** $A_x = 11.2$, $A_y = -9.91$ **(c)** $A_x = -15.0$, $A_y = 22.4$

(d) $A_x = 20$, $A_y = -30$

EVALUATE: The components are signed scalars.

1.49. **IDENTIFY:** Use trig to find the components of each vector. Use Eq.(1.11) to find the components of the vector sum. Eq.(1.14) expresses a vector in terms of its components.

SET UP: Use the coordinates in the figure that accompanies the problem.

EXECUTE: **(a)** $\vec{A} = (3.60\text{ m})\cos 70.0°\hat{i} + (3.60\text{ m})\sin 70.0°\hat{j} = (1.23\text{ m})\hat{i} + (3.38\text{ m})\hat{j}$

$\vec{B} = -(2.40\text{ m})\cos 30.0°\hat{i} - (2.40\text{ m})\sin 30.0°\hat{j} = (-2.08\text{ m})\hat{i} + (-1.20\text{ m})\hat{j}$

(b) $\vec{C} = (3.00)\vec{A} - (4.00)\vec{B} = (3.00)(1.23\text{ m})\hat{i} + (3.00)(3.38\text{ m})\hat{j} - (4.00)(-2.08\text{ m})\hat{i} - (4.00)(-1.20\text{ m})\hat{j}$

$= (12.01\text{ m})\hat{i} + (14.94)\hat{j}$

(c) From Equations (1.7) and (1.8),

$$C = \sqrt{(12.01\text{ m})^2 + (14.94\text{ m})^2} = 19.17\text{ m, arctan}\left(\frac{14.94\text{ m}}{12.01\text{ m}}\right) = 51.2°$$

EVALUATE: C_x and C_y are both positive, so θ is in the first quadrant.

1.50. **IDENTIFY:** Find *A* and *B*. Find the vector difference using components.

SET UP: Deduce the *x*- and *y*-components and use Eq.(1.8).

EXECUTE: **(a)** $\vec{A} = 4.00\hat{i} + 3.00\hat{j}$; $A_x = +4.00$; $A_y = +3.00$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(4.00)^2 + (3.00)^2} = 5.00$$

$\vec{B} = 5.00\hat{i} - 2.00\hat{j}$; $B_x = +5.00$; $B_y = -2.00$

$B = \sqrt{B_x^2 + B_y^2} = \sqrt{(5.00)^2 + (-2.00)^2} = 5.39$

EVALUATE: Note that the magnitudes of $\vec{A}$ and $\vec{B}$ are each larger than either of their components.

EXECUTE: **(b)** $\vec{A} - \vec{B} = 4.00\hat{i} + 3.00\hat{j} - \left(5.00\hat{i} - 2.00\hat{j}\right) = (4.00 - 5.00)\hat{i} + (3.00 + 2.00)\hat{j}$

$\vec{A} - \vec{B} = -1.00\hat{i} + 5.00\hat{j}$

(c) Let $\vec{R} = \vec{A} - \vec{B} = -1.00\hat{i} + 5.00\hat{j}$. Then $R_x = -1.00$, $R_y = 5.00$.

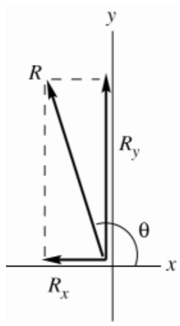


Figure 1.50

$R = \sqrt{R_x^2 + R_y^2}$

$R = \sqrt{(-1.00)^2 + (5.00)^2} = 5.10.$

$\tan\theta = \dfrac{R_y}{R_x} = \dfrac{5.00}{-1.00} = -5.00$

$\theta = -78.7° + 180° = 101.3°.$

EVALUATE: $R_x < 0$ and $R_y > 0$, so $\vec{R}$ is in the 2nd quadrant.

1.51. **IDENTIFY:** A unit vector has magnitude equal to 1.

SET UP: The magnitude of a vector is given in terms of its components by Eq.(1.12).

EXECUTE: **(a)** $\left|\hat{i} + \hat{j} + \hat{k}\right| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3} \neq 1$ so it is not a unit vector.

(b) $\left|\vec{A}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2}$. If any component is greater than $+1$ or less than -1, $\left|\vec{A}\right| > 1$, so it cannot be a unit vector. $\vec{A}$ can have negative components since the minus sign goes away when the component is squared.

(c) $\left|\vec{A}\right| = 1$ gives $\sqrt{a^2(3.0)^2 + a^2(4.0)^2} = 1$ and $\sqrt{a^2}\sqrt{25} = 1$. $a = \pm\dfrac{1}{5.0} = \pm 0.20$.

EVALUATE: The magnitude of a vector is greater than the magnitude of any of its components.

1.52. **IDENTIFY:** If vectors $\vec{A}$ and $\vec{B}$ commute for addition, $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. If they commute for the scalar product, $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$.

SET UP: Express the sum and scalar product in terms of the components of $\vec{A}$ and $\vec{B}$.

EXECUTE: **(a)** Let $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$. $\vec{A} + \vec{B} = \left(A_x + B_x\right)\hat{i} + \left(A_y + B_y\right)\hat{j}$.

$\vec{B} + \vec{A} = \left(B_x + A_x\right)\hat{i} + \left(B_y + A_y\right)\hat{j}$. Scalar addition is commutative, so $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y$ and $\vec{B}\cdot\vec{A} = B_xA_x + B_yA_y$. Scalar multiplication is commutative, so $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$.

(b) $\vec{A}\times\vec{B} = \left(A_yB_z - A_zB_y\right)\hat{i} + \left(A_zB_x - A_xB_z\right)\hat{j} + \left(A_xB_y - A_yB_x\right)\hat{k}$.

$\vec{B}\times\vec{A} = \left(B_yA_z - B_zA_y\right)\hat{i} + \left(B_zA_x - B_xA_z\right)\hat{j} + \left(B_xA_y - B_yA_x\right)\hat{k}$. Comparison of each component in each vector product shows that one is the negative of the other.

EVALUATE: The result in part (b) means that $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ have the same magnitude and opposite direction.

1.53. **IDENTIFY:** $\vec{A}\cdot\vec{B} = AB\cos\phi$

SET UP: For $\vec{A}$ and $\vec{B}$, $\phi = 150.0°$. For $\vec{B}$ and $\vec{C}$, $\phi = 145.0°$. For $\vec{A}$ and $\vec{C}$, $\phi = 65.0°$.

EXECUTE: **(a)** $\vec{A}\cdot\vec{B} = (8.00\text{ m})(15.0\text{ m})\cos 150.0° = -104\text{ m}^2$

(b) $\vec{B}\cdot\vec{C} = (15.0\text{ m})(12.0\text{ m})\cos 145.0° = -148\text{ m}^2$

(c) $\vec{A}\cdot\vec{C} = (8.00\text{ m})(12.0\text{ m})\cos 65.0° = 40.6\text{ m}^2$

EVALUATE: When $\phi < 90°$ the scalar product is positive and when $\phi > 90°$ the scalar product is negative.

1.54. **IDENTIFY:** Target variables are $\vec{A}\cdot\vec{B}$ and the angle ϕ between the two vectors.

SET UP: We are given $\vec{A}$ and $\vec{B}$ in unit vector form and can take the scalar product using Eq.(1.19). The angle ϕ can then be found from Eq.(1.18).

EXECUTE:

(a) $\vec{A} = 4.00\hat{i} + 3.00\hat{j}$, $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$; $A = 5.00$, $B = 5.39$

$\vec{A}\cdot\vec{B} = (4.00\hat{i} + 3.00\hat{j})\cdot(5.00\hat{i} - 2.00\hat{j}) = (4.00)(5.00) + (3.00)(-2.00) = 20.0 - 6.0 = +14.0.$

(b) $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{14.0}{(5.00)(5.39)} = 0.519;\ \ \phi = 58.7°.$

EVALUATE: The component of $\vec{B}$ along $\vec{A}$ is in the same direction as $\vec{A}$, so the scalar product is positive and the angle ϕ is less than $90°$.

1.55. **IDENTIFY:** For all of these pairs of vectors, the angle is found from combining Equations (1.18) and (1.21), to give the angle ϕ as $\phi = \arccos\left(\dfrac{\vec{A}\cdot\vec{B}}{AB}\right) = \arccos\left(\dfrac{A_x B_x + A_y B_y}{AB}\right)$.

SET UP: Eq.(1.14) shows how to obtain the components for a vector written in terms of unit vectors.

EXECUTE: **(a)** $\vec{A}\cdot\vec{B} = -22$, $A = \sqrt{40}$, $B = \sqrt{13}$, and so $\phi = \arccos\left(\dfrac{-22}{\sqrt{40}\,\sqrt{13}}\right) = 165°$.

(b) $\vec{A}\cdot\vec{B} = 60$, $A = \sqrt{34}$, $B = \sqrt{136}$, $\quad \phi = \arccos\left(\dfrac{60}{\sqrt{34}\sqrt{136}}\right) = 28°$.

(c) $\vec{A}\cdot\vec{B} = 0$ and $\phi = 90°$.

EVALUATE: If $\vec{A}\cdot\vec{B} > 0$, $0 \le \phi < 90°$. If $\vec{A}\cdot\vec{B} < 0$, $90° < \phi \le 180°$. If $\vec{A}\cdot\vec{B} = 0$, $\phi = 90°$ and the two vectors are perpendicular.

1.56. **IDENTIFY:** $\vec{A}\cdot\vec{B} = AB\cos\phi$ and $\left|\vec{A}\times\vec{B}\right| = AB\sin\phi$, where ϕ is the angle between $\vec{A}$ and $\vec{B}$.

SET UP: Figure 1.56 shows $\vec{A}$ and $\vec{B}$. The components $A_{\|}$ of $\vec{A}$ along $\vec{B}$ and $A_{\perp}$ of $\vec{A}$ perpendicular to $\vec{B}$ are shown in Figure 1.56a. The components of $B_{\|}$ of $\vec{B}$ along $\vec{A}$ and $B_{\perp}$ of $\vec{B}$ perpendicular to $\vec{A}$ are shown in Figure 1.56b.

EXECUTE: **(a)** From Figures 1.56a and b, $A_{\|} = A\cos\phi$ and $B_{\|} = B\cos\phi$. $\vec{A}\cdot\vec{B} = AB\cos\phi = BA_{\|} = AB_{\|}$.

(b) $A_{\perp} = A\sin\phi$ and $B_{\perp} = B\sin\phi$. $\left|\vec{A}\times\vec{B}\right| = AB\sin\phi = BA_{\perp} = AB_{\perp}$.

EVALUATE: When $\vec{A}$ and $\vec{B}$ are perpendicular, $\vec{A}$ has no component along $\vec{B}$ and $\vec{B}$ has no component along $\vec{A}$ and $\vec{A}\cdot\vec{B} = 0$. When $\vec{A}$ and $\vec{B}$ are parallel, $\vec{A}$ has no component perpendicular to $\vec{B}$ and $\vec{B}$ has no component perpendicular to $\vec{A}$ and $\left|\vec{A}\times\vec{B}\right| = 0$.

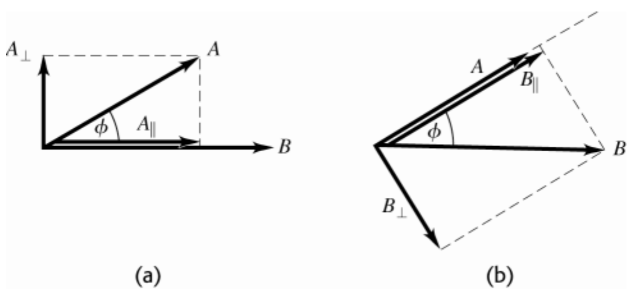


Figure 1.56

1.57. **IDENTIFY:** $\vec{A}\times\vec{D}$ has magnitude $AD\sin\phi$. Its direction is given by the right-hand rule.

SET UP: $\phi = 180° - 53° = 127°$

EXECUTE: $\left|\vec{A}\times\vec{D}\right| = (8.00\text{ m})(10.0\text{ m})\sin 127° = 63.9\text{ m}^2$. The right-hand rule says $\vec{A}\times\vec{D}$ is in the $-z$-direction (into the page).

EVALUATE: The component of $\vec{D}$ perpendicular to $\vec{A}$ is $D_{\perp} = D\sin 53.0° = 7.00\text{ m}$. $\left|\vec{A}\times\vec{D}\right| = AD_{\perp} = 63.9\text{ m}^2$, which agrees with our previous result.

1.58. **IDENTIFY:** Target variable is the vector $\vec{A}\times\vec{B}$, expressed in terms of unit vectors.

SET UP: We are given $\vec{A}$ and $\vec{B}$ in unit vector form and can take the vector product using Eq.(1.24).

EXECUTE: $\vec{A} = 4.00\hat{i} + 3.00\hat{j}$, $\vec{B} = 5.00\hat{i} - 2.00\hat{j}$

$\vec{A}\times\vec{B}=\left(4.00\hat{\boldsymbol{i}}+3.00\hat{\boldsymbol{j}}\right)\times\left(5.00\hat{\boldsymbol{i}}-2.00\hat{\boldsymbol{j}}\right)=\ 20.0\hat{\boldsymbol{i}}\times\hat{\boldsymbol{i}}-8.00\hat{\boldsymbol{i}}\times\hat{\boldsymbol{j}}+15.0\hat{\boldsymbol{j}}\times\hat{\boldsymbol{i}}-6.00\hat{\boldsymbol{j}}\times\hat{\boldsymbol{j}}$

But $\hat{\boldsymbol{i}}\times\hat{\boldsymbol{i}}=\hat{\boldsymbol{j}}\times\hat{\boldsymbol{j}}=0$ and $\hat{\boldsymbol{i}}\times\hat{\boldsymbol{j}}=\hat{\boldsymbol{k}},\ \ \hat{\boldsymbol{j}}\times\hat{\boldsymbol{i}}=-\hat{\boldsymbol{k}},$ so $\vec{A}\times\vec{B}=-8.00\hat{\boldsymbol{k}}+15.0\left(-\hat{\boldsymbol{k}}\right)=-23.0\hat{\boldsymbol{k}}.$

The magnitude of $\vec{A}\times\vec{B}$ is 23.0.

EVALUATE: Sketch the vectors $\vec{A}$ and $\vec{B}$ in a coordinate system where the *xy*-plane is in the plane of the paper and the *z*-axis is directed out toward you.

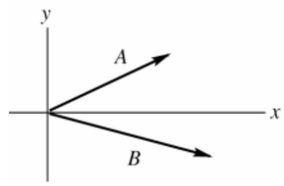


Figure 1.58

By the right-hand rule $\vec{A}\times\vec{B}$ is directed into the plane of the paper, in the $-z$-direction. This agrees with the above calculation that used unit vectors.

1.59. **IDENTIFY:** The right-hand rule gives the direction and Eq.(1.22) gives the magnitude.

SET UP: $\phi=120.0°$.

EXECUTE: **(a)** The direction of $\vec{A}\times\vec{B}$ is into the page (the $-z$-direction). The magnitude of the vector product is $AB\sin\phi=\left(2.80\text{ cm}\right)\left(1.90\text{ cm}\right)\sin 120^\circ=4.61\text{ cm}^2$.

(b) Rather than repeat the calculations, Eq. (1.23) may be used to see that $\vec{B}\times\vec{A}$ has magnitude 4.61 cm^2 and is in the $+z$-direction (out of the page).

EVALUATE: For part (a) we could use Eq. (1.27) and note that the only non-vanishing component is

$$C_z=A_x B_y-A_y B_x=\left(2.80\text{ cm}\right)\cos 60.0°\left(-1.90\text{ cm}\right)\sin 60°$$
$$-\left(2.80\text{ cm}\right)\sin 60.0°\left(1.90\text{ cm}\right)\cos 60.0°=-4.61\text{ cm}^2.$$

This gives the same result.

1.60. **IDENTIFY:** Area is length times width. Do unit conversions.

SET UP: $1\text{ mi}=5280\text{ ft}$. $1\text{ ft}^3=7.477\text{ gal}$.

EXECUTE: **(a)** The area of one acre is $\frac{1}{8}\text{ mi}\times\frac{1}{80}\text{ mi}=\frac{1}{640}\text{ mi}^2$, so there are 640 acres to a square mile.

(b) $\left(1\text{ acre}\right)\times\left(\dfrac{1\text{ mi}^2}{640\text{ acre}}\right)\times\left(\dfrac{5280\text{ ft}}{1\text{ mi}}\right)^2=43{,}560\text{ ft}^2$

(all of the above conversions are exact).

(c) $(1\text{ acre-foot})=\left(43{,}560\text{ ft}^3\right)\times\left(\dfrac{7.477\text{ gal}}{1\text{ ft}^3}\right)=3.26\times10^5\text{ gal},$ which is rounded to three significant figures.

EVALUATE: An acre is much larger than a square foot but less than a square mile. A volume of 1 acre-foot is much larger than a gallon.

1.61. **IDENTIFY:** The density relates mass and volume. Use the given mass and density to find the volume and from this the radius.

SET UP: The earth has mass $m_{\text{E}}=5.97\times10^{24}\text{ kg}$ and radius $r_{\text{E}}=6.38\times10^6\text{ m}$. The volume of a sphere is $V=\frac{4}{3}\pi r^3$. $\rho=1.76\text{ g/cm}^3=1760\text{ km/m}^3$.

EXECUTE: **(a)** The planet has mass $m=5.5m_{\text{E}}=3.28\times10^{25}\text{ kg}$. $V=\dfrac{m}{\rho}=\dfrac{3.28\times10^{25}\text{ kg}}{1760\text{ kg/m}^3}=1.86\times10^{22}\text{ m}^3$.

$$r=\left(\frac{3V}{4\pi}\right)^{1/3}=\left(\frac{3[1.86\times10^{22}\text{ m}^3]}{4\pi}\right)^{1/3}=1.64\times10^7\text{ m}=1.64\times10^4\text{ km}$$

(b) $r=2.57r_{\text{E}}$

EVALUATE: Volume *V* is proportional to mass and radius *r* is proportional to $V^{1/3}$, so *r* is proportional to $m^{1/3}$. If the planet and earth had the same density its radius would be $(5.5)^{1/3}r_{\text{E}}=1.8r_{\text{E}}$. The radius of the planet is greater than this, so its density must be less than that of the earth.

1.62. **IDENTIFY** and **SET UP:** Unit conversion.

EXECUTE: **(a)** $f = 1.420\times10^9$ cycles/s, so $\dfrac{1}{1.420\times10^9}$ s $= 7.04\times10^{-10}$ s for one cycle.

(b) $\dfrac{3600\ \text{s/h}}{7.04\times10^{-10}\ \text{s/cycle}} = 5.11\times10^{12}$ cycles/h

(c) Calculate the number of seconds in 4600 million years $= 4.6\times10^9$ y and divide by the time for 1 cycle:

$$\frac{(4.6\times10^9\ \text{y})(3.156\times10^7\ \text{s/y})}{7.04\times10^{-10}\ \text{s/cycle}} = 2.1\times10^{26}\ \text{cycles}$$

(d) The clock is off by 1 s in $100{,}000\ \text{y} = 1\times10^5$ y, so in 4.60×10^9 y it is off by $(1\ \text{s})\left(\dfrac{4.60\times10^9}{1\times10^5}\right) = 4.6\times10^4$ s (about 13 h).

EVALUATE: In each case the units in the calculation combine algebraically to give the correct units for the answer.

1.63. **IDENTIFY:** The number of atoms is your mass divided by the mass of one atom.

SET UP: Assume a 70-kg person and that the human body is mostly water. Use Appendix D to find the mass of one H_2O molecule: $18.015\ \text{u}\times1.661\times10^{-27}\ \text{kg/u} = 2.992\times10^{-26}$ kg/molecule.

EXECUTE: $(70\ \text{kg})/(2.992\times10^{-26}\ \text{kg/molecule}) = 2.34\times10^{27}$ molecules. Each H_2O molecule has 3 atoms, so there are about 6×10^{27} atoms.

EVALUATE: Assuming carbon to be the most common atom gives 3×10^{27} molecules, which is a result of the same order of magnitude.

1.64. **IDENTIFY:** Estimate the volume of each object. The mass *m* is the density times the volume.

SET UP: The volume of a sphere of radius *r* is $V = \frac{4}{3}\pi r^3$. The volume of a cylinder of radius *r* and length *l* is $V = \pi r^2 l$. The density of water is $1000\ \text{kg/m}^3$.

EXECUTE: **(a)** Estimate the volume as that of a sphere of diameter 10 cm: $V = 5.2\times10^{-4}\,\text{m}^3$.
$m = (0.98)(1000\ \text{kg/m}^3)(5.2\times10^{-4}\,\text{m}^3) = 0.5\ \text{kg}$.

(b) Approximate as a sphere of radius $r = 0.25\mu\text{m}$ (probably an over estimate): $V = 6.5\times10^{-20}\,\text{m}^3$.
$m = (0.98)(1000\ \text{kg/m}^3)(6.5\times10^{-20}\ \text{m}^3) = 6\times10^{-17}\ \text{kg} = 6\times10^{-14}\ \text{g}$.

(c) Estimate the volume as that of a cylinder of length 1 cm and radius 3 mm: $V = \pi r^2 l = 2.8\times10^{-7}\,\text{m}^3$.
$m = (0.98)(1000\ \text{kg/m}^3)(2.8\times10^{-7}\ \text{m}^3) = 3\times10^{-4}\ \text{kg} = 0.3\ \text{g}$.

EVALUATE: The mass is directly proportional to the volume.

1.65. **IDENTIFY:** Use the volume *V* and density ρ to calculate the mass *M*: $\rho = \dfrac{M}{V}$, so $V = \dfrac{M}{\rho}$.

SET UP: The volume of a cube with sides of length *x* is x^3. The volume of a sphere with radius *R* is $\frac{4}{3}\pi R^3$.

EXECUTE: **(a)** $x^3 = \dfrac{0.200\ \text{kg}}{7.86\times10^3\ \text{kg/m}^3} = 2.54\times10^{-5}\,\text{m}^3$. $x = 2.94\times10^{-2}\,\text{m} = 2.94\ \text{cm}$.

(b) $\dfrac{4}{3}\pi R^3 = 2.54\times10^{-5}\,\text{m}^3$. $R = 1.82\times10^{-2}\,\text{m} = 1.82\ \text{cm}$.

EVALUATE: $\frac{4}{3}\pi = 4.2$, so a sphere with radius *R* has a greater volume than a cube whose sides have length *R*.

1.66. **IDENTIFY:** Estimate the volume of sand in all the beaches on the earth. The diameter of a grain of sand determines its volume. From the volume of one grain and the total volume of sand we can calculate the number of grains.

SET UP: The volume of a sphere of diameter *d* is $V = \frac{1}{6}\pi d^3$. Consulting an atlas, we estimate that the continents have about 1.45×10^5 km of coastline. Add another 25% of this for rivers and lakes, giving 1.82×10^5 km of coastline. Assume that a beach extends 50 m beyond the water and that the sand is 2 m deep. 1 billion $= 1\times10^9$.

EXECUTE: **(a)** The volume of sand is $(1.82\times10^8\ \text{m})(50\ \text{m})(2\ \text{m}) = 2\times10^{10}\ \text{m}^3$. The volume of a grain is $V = \frac{1}{6}\pi(0.2\times10^{-3}\ \text{m})^3 = 4\times10^{-12}\ \text{m}^3$. The number of grains is $\dfrac{2\times10^{10}\ \text{m}^3}{4\times10^{-12}\ \text{m}^3} = 5\times10^{21}$. The number of grains of sand is about 10^{22}.

(b) The number of stars is $(100\times10^9)(100\times10^9) = 10^{22}$. The two estimates result in comparable numbers for these two quantities.

EVALUATE: Both numbers are crude estimates but are probably accurate to a few powers of 10.

1.67. **IDENTIFY:** The number of particles is the total mass divided by the mass of one particle.

SET UP: $1\text{ mol} = 6.0\times10^{23}$ atoms. The mass of the earth is 6.0×10^{24} kg. The mass of the sun is 2.0×10^{30} kg. The distance from the earth to the sun is 1.5×10^{11} m. The volume of a sphere of radius R is $\frac{4}{3}\pi R^3$. Protons and neutrons each have a mass of 1.7×10^{-27} kg and the mass of an electron is much less.

EXECUTE: **(a)** $(6.0\times10^{24}\text{ kg})\times\left(\dfrac{6.0\times10^{23}\ \frac{\text{atoms}}{\text{mole}}}{14\times10^{-3}\ \frac{\text{kg}}{\text{mole}}}\right) = 2.6\times10^{50}$ atoms.

(b) The number of neutrons is the mass of the neutron star divided by the mass of a neutron:

$$\frac{(2)(2.0\times10^{30}\text{ kg})}{(1.7\times10^{-27}\text{ kg/neutron})} = 2.4\times10^{57}\text{ neutrons.}$$

(c) The average mass of a particle is essentially $\frac{2}{3}$ the mass of either the proton or the neutron, 1.7×10^{-27} kg. The total number of particles is the total mass divided by this average, and the total mass is the volume times the average density. Denoting the density by ρ,

$$\frac{M}{m_{\text{ave}}} = \frac{\frac{4}{3}\pi R^3\rho}{\frac{2}{3}m_{\text{p}}} = \frac{(2\pi)(1.5\times10^{11}\text{ m})^3\,(10^{18}\text{ kg/m}^3)}{1.7\times10^{-27}\text{ kg}} = 1.2\times10^{79}.$$

Note the conversion from g/cm^3 to kg/m^3.

EVALUATE: These numbers of particles are each very, very large but are still much less than a googol.

1.68. **IDENTIFY:** Let $\vec{D}$ be the fourth force. Find $\vec{D}$ such that $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$, so $\vec{D}=-\left(\vec{A}+\vec{B}+\vec{C}\right)$.

SET UP: Use components and solve for the components D_x and D_y of $\vec{D}$.

EXECUTE: $A_x = +A\cos30.0° = +86.6\text{ N}$, $A_y = +A\cos30.0° = +50.00\text{ N}$.

$B_x = -B\sin30.0° = -40.00\text{ N}$, $B_y = +B\cos30.0° = +69.28\text{ N}$.

$C_x = +C\cos53.0° = -24.07\text{ N}$, $C_y = -C\sin53.0° = -31.90\text{ N}$.

Then $D_x = -22.53$ N, $D_y = -87.34$ N and $D = \sqrt{D_x^2 + D_y^2} = 90.2$ N. $\tan\alpha = \left|D_y / D_x\right| = 87.34/22.53$. $\alpha = 75.54°$. $\phi = 180° + \alpha = 256°$, counterclockwise from the $+x$-axis.

EVALUATE: As shown in Figure 1.68, since D_x and D_y are both negative, $\vec{D}$ must lie in the third quadrant.

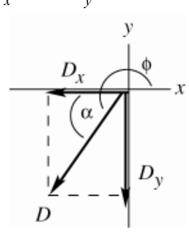


Figure 1.68

1.69. **IDENTIFY:** We know the magnitude and direction of the sum of the two vector pulls and the direction of one pull. We also know that one pull has twice the magnitude of the other. There are two unknowns, the magnitude of the smaller pull and its direction. $A_x + B_x = C_x$ and $A_y + B_y = C_y$ give two equations for these two unknowns.

SET UP: Let the smaller pull be $\vec{A}$ and the larger pull be $\vec{B}$. $B = 2A$. $\vec{C} = \vec{A} + \vec{B}$ has magnitude 350.0 N and is northward. Let $+x$ be east and $+y$ be north. $B_x = -B\sin25.0°$ and $B_y = B\cos25.0°$. $C_x = 0$, $C_y = 350.0$ N.

$\vec{A}$ must have an eastward component to cancel the westward component of $\vec{B}$. There are then two possibilities, as sketched in Figures 1.69 a and b. $\vec{A}$ can have a northward component or $\vec{A}$ can have a southward component.

EXECUTE: In either Figure 1.69 a or b, $A_x + B_x = C_x$ and $B = 2A$ gives $(2A)\sin25.0° = A\sin\phi$ and $\phi = 57.7°$. In Figure 1.69a, $A_y + B_y = C_y$ gives $2A\cos25.0° + A\cos57.7° = 350.0$ N and $A = 149$ N. In Figure 1.69b, $2A\cos25.0° - A\cos57.7° = 350.0$ N and $A = 274$ N. One solution is for the smaller pull to be $57.7°$ east of north. In this case, the smaller pull is 149 N and the larger pull is 298 N. The other solution is for the smaller pull to be $57.7°$ east of south. In this case the smaller pull is 274 N and the larger pull is 548 N.

EVALUATE: For the first solution, with $\vec{A}$ east of north, each worker has to exert less force to produce the given resultant force and this is the sensible direction for the worker to pull.

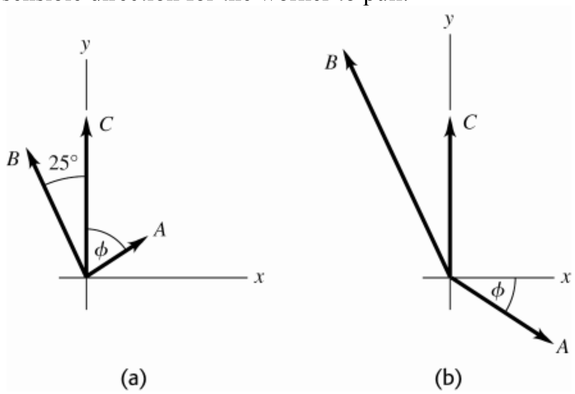


Figure 1.69

1.70. **IDENTIFY:** Find the vector sum of the two displacements.

SET UP: Call the two displacements $\vec{A}$ and $\vec{B}$, where $A = 170$ km and $B = 230$ km. $\vec{A} + \vec{B} = \vec{R}$. $\vec{A}$ and $\vec{B}$ are as shown in Figure 1.70.

EXECUTE: $R_x = A_x + B_x = (170 \text{ km}) \sin 68° + (230 \text{ km}) \cos 48° = 311.5 \text{ km}$.

$R_y = A_y + B_y = (170 \text{ km}) \cos 68° - (230 \text{ km}) \sin 48° = -107.2 \text{ km}$.

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(311.5 \text{ km})^2 + (-107.2 \text{ km})^2} = 330 \text{ km}$. $\tan\theta_R = \left|\frac{R_y}{R_x}\right| = \frac{107.2 \text{ km}}{311.5 \text{ km}} = 0.344$.

$\theta_R = 19°$ south of east.

EVALUATE: Our calculation using components agrees with $\vec{R}$ shown in the vector addition diagram, Figure 1.70.

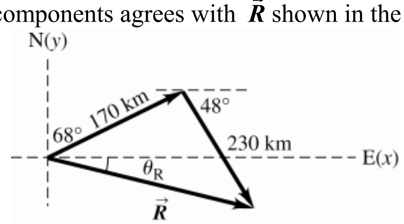


Figure 1.70

1.71. **IDENTIFY:** $\vec{A} + \vec{B} = \vec{C}$ (or $\vec{B} + \vec{A} = \vec{C}$). The target variable is vector $\vec{A}$.

SET UP: Use components and Eq.(1.10) to solve for the components of $\vec{A}$. Find the magnitude and direction of $\vec{A}$ from its components.

EXECUTE: **(a)**

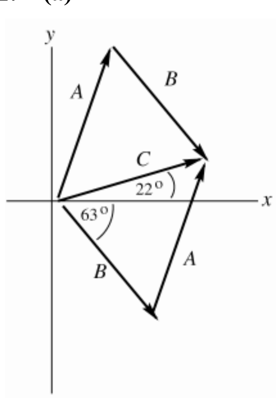


Figure 1.71a

$C_x = A_x + B_x$, so $A_x = C_x - B_x$

$C_y = A_y + B_y$, so $A_y = C_y - B_y$

$C_x = C\cos 22.0° = (6.40 \text{ cm})\cos 22.0°$

$C_x = +5.934$ cm

$C_y = C\sin 22.0° = (6.40 \text{ cm})\sin 22.0°$

$C_y = +2.397$ cm

$B_x = B\cos(360° - 63.0°) = (6.40 \text{ cm})\cos 297.0°$

$B_x = +2.906$ cm

$B_y = B\sin 297.0° = (6.40 \text{ cm})\sin 297.0°$

$B_y = -5.702$ cm

(b) $A_x = C_x - B_x = +5.934 \text{ cm} - 2.906 \text{ cm} = +3.03 \text{ cm}$

$A_y = C_y - B_y = +2.397 \text{ cm} - (-5.702) \text{ cm} = +8.10 \text{ cm}$

(c)

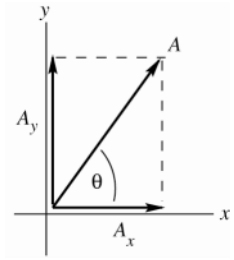


Figure 1.71b

$$A = \sqrt{A_x^2 + A_y^2}$$
$$A = \sqrt{(3.03\text{ cm})^2 + (8.10\text{ cm})^2} = 8.65\text{ cm}$$
$$\tan\theta = \frac{A_y}{A_x} = \frac{8.10\text{ cm}}{3.03\text{ cm}} = 2.67$$
$$\theta = 69.5^\circ$$

EVALUATE: The $\vec{A}$ we calculated agrees qualitatively with vector $\vec{A}$ in the vector addition diagram in part (a).

1.72. **IDENTIFY:** Add the vectors using the method of components.

SET UP: $A_x = 0$, $A_y = -8.00\text{ m}$. $B_x = 7.50\text{ m}$, $B_y = 13.0\text{ m}$. $C_x = -10.9\text{ m}$, $C_y = -5.07\text{ m}$.

EXECUTE: **(a)** $R_x = A_x + B_x + C_x = -3.4\text{ m}$. $R_y = A_y + B_y + C_y = -0.07\text{ m}$. $R = 3.4\text{ m}$. $\tan\theta = \dfrac{-0.07\text{ m}}{-3.4\text{ m}}$.

$\theta = 1.2^\circ$ below the $-x$-axis.

(b) $S_x = C_x - A_x - B_x = -18.4\text{ m}$. $S_y = C_y - A_y - B_y = -10.1\text{ m}$. $S = 21.0\text{ m}$. $\tan\theta = \dfrac{S_y}{S_x} = \dfrac{-10.1\text{ m}}{-18.4\text{ m}}$. $\theta = 28.8^\circ$ below the $-x$-axis.

EVALUATE: The magnitude and direction we calculated for $\vec{R}$ and $\vec{S}$ agree with our vector diagrams.

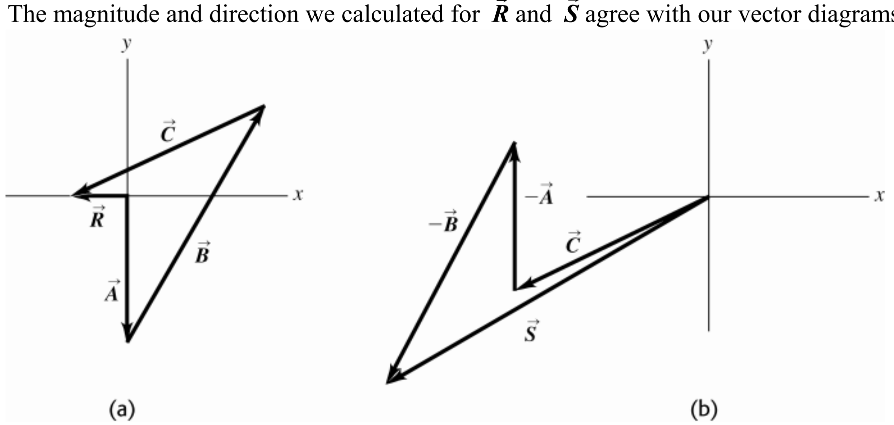


Figure 1.72

1.73. **IDENTIFY:** Vector addition. Target variable is the 4th displacement.

SET UP: Use a coordinate system where east is in the $+x$-direction and north is in the $+y$-direction.

Let $\vec{A}$, $\vec{B}$, and $\vec{C}$ be the three displacements that are given and let $\vec{D}$ be the fourth unmeasured displacement. Then the resultant displacement is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. And since she ends up back where she started, $\vec{R} = 0$.

$0 = \vec{A} + \vec{B} + \vec{C} + \vec{D}$, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$

$D_x = -(A_x + B_x + C_x)$ and $D_y = -(A_y + B_y + C_y)$

EXECUTE:

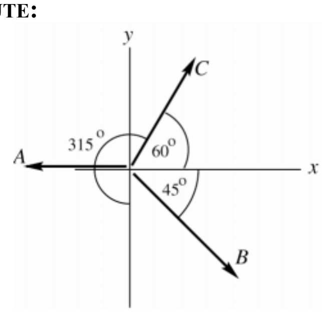


Figure 1.73a

$A_x = -180\text{ m}$, $A_y = 0$

$B_x = B\cos 315^\circ = (210\text{ m})\cos 315^\circ = +148.5\text{ m}$

$B_y = B\sin 315^\circ = (210\text{ m})\sin 315^\circ = -148.5\text{ m}$

$C_x = C\cos 60^\circ = (280\text{ m})\cos 60^\circ = +140\text{ m}$

$C_y = C\sin 60^\circ = (280\text{ m})\sin 60^\circ = +242.5\text{ m}$

$D_x = -(A_x + B_x + C_x) = -(-180\text{ m} + 148.5\text{ m} + 140\text{ m}) = -108.5\text{ m}$

$D_y = -(A_y + B_y + C_y) = -(0 - 148.5\text{ m} + 242.5\text{ m}) = -94.0\text{ m}$

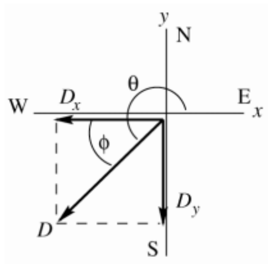


Figure 1.73b

$D = \sqrt{D_x^2 + D_y^2}$

$D = \sqrt{(-108.5\text{ m})^2 + (-94.0\text{ m})^2} = 144\text{ m}$

$\tan\theta = \dfrac{D_y}{D_x} = \dfrac{-94.0\text{ m}}{-108.5\text{ m}} = 0.8664$

$\theta = 180° + 40.9° = 220.9°$

($\vec{D}$ is in the third quadrant since both D_x and D_y are negative.)

The direction of $\vec{D}$ can also be specified in terms of $\phi = \theta - 180° = 40.9°$; $\vec{D}$ is $41°$ south of west.

EVALUATE: The vector addition diagram, approximately to scale, is

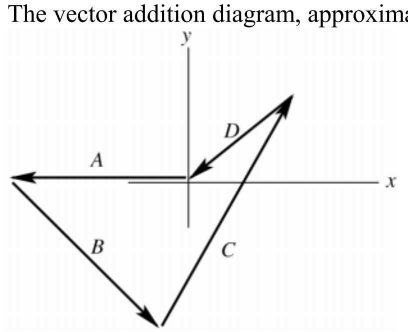


Figure 1.73c

Vector $\vec{D}$ in this diagram agrees qualitatively with our calculation using components.

1.74. **IDENTIFY:** Solve for one of the vectors in the vector sum. Use components.

SET UP: Use coordinates for which $+x$ is east and $+y$ is north. The vector displacements are:

$\vec{A} = 2.00$ km, 0°of east; $\vec{B} = 3.50$ m, 45° south of east; and $\vec{R} = 5.80$ m, 0° east

EXECUTE: $C_x = R_x - A_x - B_x = 5.80\text{ km} - (2.00\text{ km}) - (3.50\text{ km})(\cos 45°) = 1.33\text{ km}$; $C_y = R_y - A_y - B_y$ $= 0\text{ km} - 0\text{ km} - (-3.50\text{ km})(\sin 45°) = 2.47\text{ km}$; $C = \sqrt{(1.33\text{ km})^2 + (2.47\text{ km})^2} = 2.81\text{ km}$; $\theta = \tan^{-1}\left[(2.47\text{ km})/(1.33\text{ km})\right] = 61.7°$ north of east. The vector addition diagram in Figure 1.74 shows good qualitative agreement with these values.

EVALUATE: The third leg lies in the first quadrant since its *x* and *y* components are both positive.

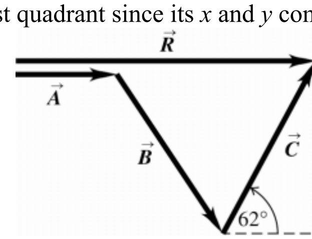


Figure 1.74

1.75. **IDENTIFY:** The sum of the vector forces on the beam sum to zero, so their *x* components and their *y* components sum to zero. Solve for the components of $\vec{F}$.

SET UP: The forces on the beam are sketched in Figure 1.75a. Choose coordinates as shown in the sketch. The 100-N pull makes an angle of $30.0° + 40.0° = 70.0°$ with the horizontal. $\vec{F}$ and the 100-N pull have been replaced by their *x* and *y* components.

EXECUTE: **(a)** The sum of the *x*-components is equal to zero gives $F_x + (100\text{ N})\cos 70.0° = 0$ and $F_x = -34.2\text{ N}$. The sum of the *y*-components is equal to zero gives $F_y + (100\text{ N})\sin 70.0° - 124\text{ N} = 0$ and $F_y = +30.0\text{ N}$. $\vec{F}$ and its components are sketched in Figure 1.75b. $F = \sqrt{F_x^2 + F_y^2} = 45.5\text{ N}$. $\tan\phi = \dfrac{|F_y|}{|F_x|} = \dfrac{30.0\text{ N}}{34.2\text{ N}}$ and $\phi = 41.3°$. $\vec{F}$ is directed at $41.3°$ above the $-x$ -axis in Figure 1.75a.

(b) The vector addition diagram is given in Figure 1.75c. $\vec{F}$ determined from the diagram agrees with $\vec{F}$ calculated in part (a) using components.

EVALUATE: The vertical component of the 100 N pull is less than the 124 N weight so $\vec{F}$ must have an upward component if all three forces balance.

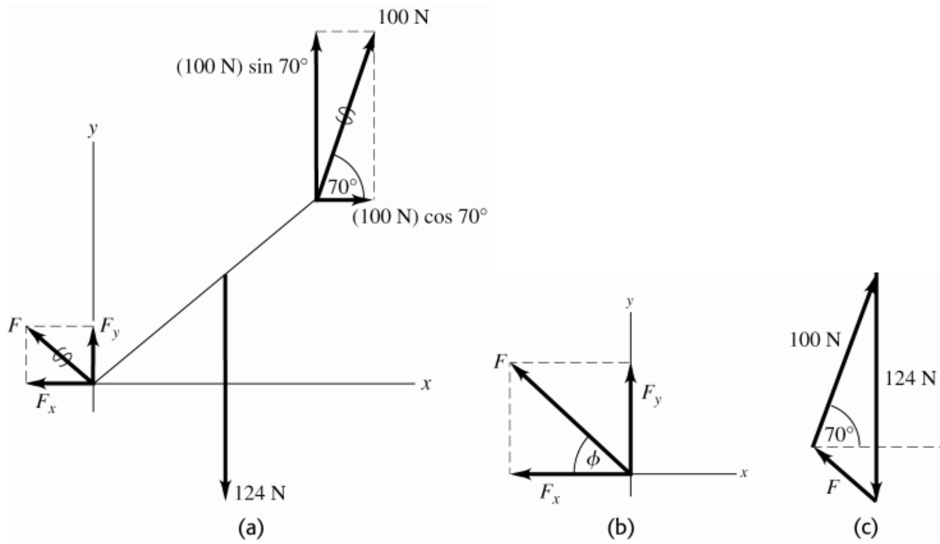


Figure 1.75

1.76. **IDENTIFY:** The four displacements return her to her starting point, so $\vec{D} = -(\vec{A} + \vec{B} + \vec{C})$, where $\vec{A}$, $\vec{B}$ and $\vec{C}$ are in the three given displacements and $\vec{D}$ is the displacement for her return.

START UP: Let $+x$ be east and $+y$ be north.

EXECUTE: **(a)** $D_x = -[(147\text{ km})\sin 85° + (106\text{ km})\sin 167° + (166\text{ km})\sin 235°] = -34.3\text{ km}$.

$D_y = -[(147\text{ km})\cos 85° + (106\text{ km})\cos 167° + (166\text{ km})\cos 235°] = +185.7\text{ km}$.

$D = \sqrt{(-34.3\text{ km})^2 + (185.7\text{ km})^2} = 189\text{ km}$.

(b) The direction relative to north is $\phi = \arctan\left(\dfrac{34.3\text{ km}}{185.7\text{ km}}\right) = 10.5°$. Since $D_x < 0$ and $D_y > 0$, the direction of $\vec{D}$ is $10.5°$ west of north.

EVALUATE: The four displacements add to zero.

1.77. **IDENTIFY** and **SET UP:** The vector $\vec{A}$ that connects points (x_1, y_1) and (x_2, y_2) has components $A_x = x_2 - x_1$ and $A_y = y_2 - y_1$.

EXECUTE: **(a)** Angle of first line is $\theta = \tan^{-1}\left(\dfrac{200-20}{210-10}\right) = 42°$. Angle of second line is $42° + 30° = 72°$.

Therefore $X = 10 + 250\cos 72° = 87$, $Y = 20 + 250\sin 72° = 258$ for a final point of (87,258).

(b) The computer screen now looks something like Figure 1.77. The length of the bottom line is

$\sqrt{(210-87)^2 + (200-258)^2} = 136$ and its direction is $\tan^{-1}\left(\dfrac{258-200}{210-87}\right) = 25°$ below straight left.

EVALUATE: Figure 1.77 is a vector addition diagram. The vector first line plus the vector arrow gives the vector for the second line.

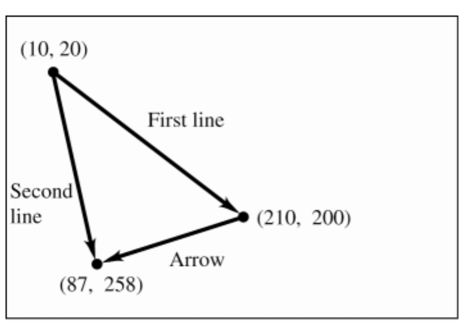


Figure 1.77

1.78. **IDENTIFY:** Let the three given displacements be $\vec{A}$, $\vec{B}$ and $\vec{C}$, where $A = 40$ steps, $B = 80$ steps and $C = 50$ steps. $\vec{R} = \vec{A} + \vec{B} + \vec{C}$. The displacement $\vec{C}$ that will return him to his hut is $-\vec{R}$.

SET UP: Let the east direction be the $+x$-direction and the north direction be the $+y$-direction.

EXECUTE: **(a)** The three displacements and their resultant are sketched in Figure 1.78.

(b) $R_x = (40)\cos 45° - (80)\cos\ 60° = -11.7$ and $R_y = (40)\sin 45° + (80)\sin 60° - 50 = 47.6$.

The magnitude and direction of the resultant are $\sqrt{(-11.7)^2 + (47.6)^2} = 49$, $\arctan\left(\frac{47.6}{11.7}\right) = 76°$, north of west.

We know that $\vec{R}$ is in the second quadrant because $R_x < 0$, $R_y > 0$. To return to the hut, the explorer must take 49 steps in a direction $76°$ south of east, which is $14°$ east of south.

EVALUATE: It is useful to show R_x, R_y and $\vec{R}$ on a sketch, so we can specify what angle we are computing.

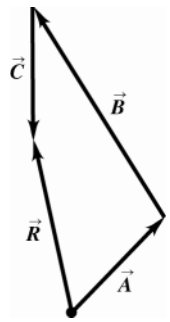


Figure 1.78

1.79. **IDENTIFY:** Vector addition. One vector and the sum are given; find the second vector (magnitude and direction).

SET UP: Let $+x$ be east and $+y$ be north. Let $\vec{A}$ be the displacement 285 km at $40.0°$ north of west and let $\vec{B}$ be the unknown displacement.

$\vec{A} + \vec{B} = \vec{R}$ where $\vec{R} = 115$ km, east

$\vec{B} = \vec{R} - \vec{A}$

$B_x = R_x - A_x$, $B_y = R_y - A_y$

EXECUTE: $A_x = -A\cos 40.0° = -218.3$ km, $A_y = +A\sin 40.0° = +183.2$ km

$R_x = 115$ km, $R_y = 0$

Then $B_x = 333.3$ km, $B_y = -183.2$ km. $B = \sqrt{B_x^2 + B_y^2} = 380$ km;

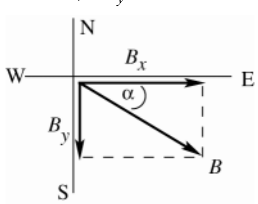


Figure 1.79

$\tan\alpha = |B_y / B_x| = (183.2\text{ km})/(333.3\text{ km})$

$\alpha = 28.8°$, south of east

EVALUATE: The southward component of $\vec{B}$ cancels the northward component of $\vec{A}$. The eastward component of $\vec{B}$ must be 115 km larger than the magnitude of the westward component of $\vec{A}$.

1.80. **IDENTIFY:** Find the components of the weight force, using the specified coordinate directions.

SET UP: For parts (a) and (b), take $+x$ direction along the hillside and the $+y$ direction in the downward direction and perpendicular to the hillside. For part (c), $\alpha = 35.0°$ and $w = 550$ N.

EXECUTE: **(a)** $w_x = w\sin\alpha$

(b) $w_y = w\cos\alpha$

(c) The maximum allowable weight is $w = w_x/(\sin\alpha) = (550\text{ N})/(\sin 35.0°) = 959$ N.

EVALUATE: The component parallel to the hill increases as α increases and the component perpendicular to the hill increases as α decreases.

1.81. **IDENTIFY:** Vector addition. One force and the vector sum are given; find the second force.

SET UP: Use components. Let $+y$ be upward.

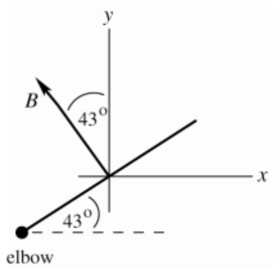


Figure 1.81a

$\vec{B}$ is the force the biceps exerts.

$\vec{E}$ is the force the elbow exerts. $\vec{E}+\vec{B}=\vec{R}$, where $R=132.5\text{ N}$ and is upward.

$E_x = R_x - B_x$, $\qquad E_y = R_y - B_y$

EXECUTE: $B_x = -B\sin 43° = -158.2\text{ N}$, $B_y = +B\cos 43° = +169.7\text{ N}$, $R_x = 0$, $R_y = +132.5\text{ N}$

Then $E_x = +158.2\text{ N}$, $E_y = -37.2\text{ N}$

$E = \sqrt{E_x^2 + E_y^2} = 160\text{ N};$

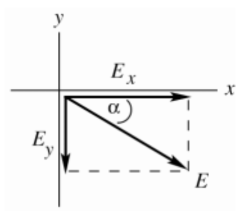


Figure 1.81b

$\tan\alpha = |E_y / E_x| = 37.2/158.2$

$\alpha = 13°$, below horizontal

EVALUATE: The *x*-component of $\vec{E}$ cancels the *x*-component of $\vec{B}$. The resultant upward force is less than the upward component of $\vec{B}$, so E_y must be downward.

1.82. **IDENTIFY:** Find the vector sum of the four displacements.

SET UP: Take the beginning of the journey as the origin, with north being the *y*-direction, east the *x*-direction, and the *z*-axis vertical. The first displacement is then $(-30\text{ m})\hat{k}$, the second is $(-15\text{ m})\hat{j}$, the third is $(200\text{ m})\hat{i}$, and the fourth is $(100\text{ m})\hat{j}$.

EXECUTE: **(a)** Adding the four displacements gives

$(-30\text{ m})\hat{k} + (-15\text{ m})\hat{j} + (200\text{ m})\hat{i} + (100\text{ m})\hat{j} = (200\text{ m})\hat{i} + (85\text{ m})\hat{j} - (30\text{ m})\hat{k}.$

(b) The total distance traveled is the sum of the distances of the individual segments: $30\text{ m} + 15\text{ m} + 200\text{ m} + 100\text{ m} = 345\text{ m}$. The magnitude of the total displacement is:

$$D = \sqrt{D_x^2 + D_y^2 + D_z^2} = \sqrt{(200\text{ m})^2 + (85\text{ m})^2 + \left(-30\text{ m}\right)^2} = 219\text{ m}.$$

EVALUATE: The magnitude of the displacement is much less than the distance traveled along the path.

1.83. **IDENTIFY:** The sum of the force displacements must be zero. Use components.

SET UP: Call the displacements $\vec{A}$, $\vec{B}$, $\vec{C}$ and $\vec{D}$, where $\vec{D}$ is the final unknown displacement for the return from the treasure to the oak tree. Vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ are sketched in Figure 1.83a. $\vec{A}+\vec{B}+\vec{C}+\vec{D}=0$ says $A_x + B_x + C_x + D_x = 0$ and $A_y + B_y + C_y + D_y = 0$. $A = 825\text{ m}$, $B = 1250\text{ m}$, and $C = 1000\text{ m}$. Let $+x$ be eastward and $+y$ be north.

EXECUTE: **(a)** $A_x + B_x + C_x + D_x = 0$ gives $D_x = -(A_x + B_x + C_x) = -(0 - [1250\text{ m}]\sin 30.0° + [1000\text{ m}]\cos 40.0°) = -141\text{ m}$. $A_y + B_y + C_y + D_y = 0$ gives $D_y = -(A_y + B_y + C_y) = -(-825\text{ m} + [1250\text{ m}]\cos 30.0° + [1000\text{ m}]\sin 40.0°) = -900\text{ m}$.

The fourth displacement $\vec{D}$ and its components are sketched in Figure 1.83b. $D = \sqrt{D_x^2 + D_y^2} = 911\text{ m}$.

$\tan\phi = \dfrac{|D_x|}{|D_y|} = \dfrac{141\text{ m}}{900\text{ m}}$ and $\phi = 8.9°$. You should head $8.9°$ west of south and must walk 911 m.

(b) The vector diagram is sketched in Figure 1.83c. The final displacement $\vec{D}$ from this diagram agrees with the vector $\vec{D}$ calculated in part (a) using components.

EVALUATE: Note that $\vec{D}$ is the negative of the sum of $\vec{A}$, $\vec{B}$, and $\vec{C}$.

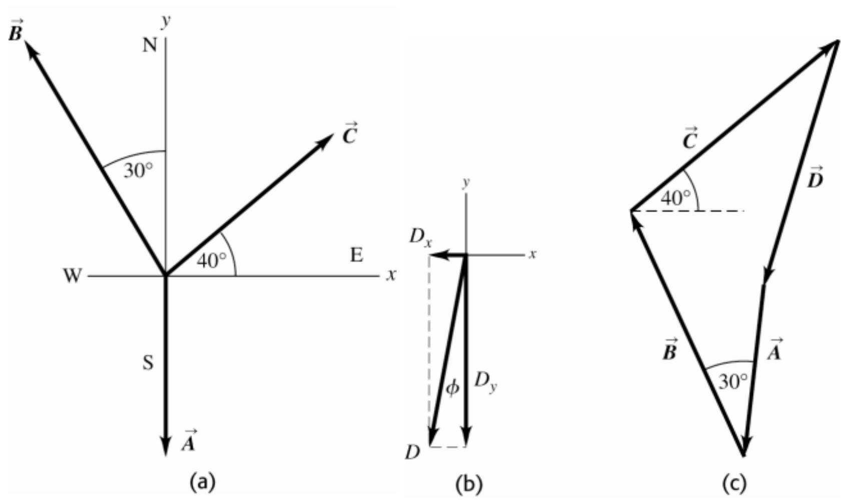


Figure 1.83

1.84. IDENTIFY: If the vector from your tent to Joe's is $\vec{A}$ and from your tent to Karl's is $\vec{B}$, then the vector from Joe's tent to Karl's is $\vec{B}-\vec{A}$.

SET UP: Take your tent's position as the origin. Let $+x$ be east and $+y$ be north.

EXECUTE: The position vector for Joe's tent is

$([21.0\text{ m}]\cos 23°)\hat{i}-([21.0\text{ m}]\sin 23°)\hat{j}=(19.33\text{ m})\hat{i}-(8.205\text{ m})\hat{j}.$

The position vector for Karl's tent is $([32.0\text{ m}]\cos 37°)\hat{i}+([32.0\text{ m}]\sin 37°)\hat{j}=(25.56\text{ m})\hat{i}+(19.26\text{ m})\hat{j}.$

The difference between the two positions is

$(19.33\text{ m}-25.56\text{ m})\hat{i}+(-8.205\text{ m}-19.25\text{ m})\hat{j}=-(6.23\text{ m})\hat{i}-(27.46\text{ m})\hat{j}.$ The magnitude of this vector is the distance between the two tents: $D=\sqrt{(-6.23\text{ m})^2+(-27.46\text{ m})^2}=28.2\text{ m}$

EVALUATE: If both tents were due east of yours, the distance between them would be $32.0\text{ m}-21.0\text{ m}=17.0\text{ m}$. If Joe's was due north of yours and Karl's was due south of yours, then the distance between them would be $32.0\text{ m}+21.0\text{ m}=53.0\text{ m}$. The actual distance between them lies between these limiting values.

1.85. IDENTIFY: In Eqs.(1.21) and (1.27) write the components of $\vec{A}$ and $\vec{B}$ in terms of A, B, θ_A and θ_B.

SET UP: From Appendix B, $\cos(a-b)=\cos a\cos b+\sin a\sin b$ and $\sin(a-b)=\sin a\cos b-\cos a\sin b$.

EXECUTE: **(a)** With $A_z=B_z=0$, Eq.(1.21) becomes

$A_xB_x+A_yB_y=(A\cos\theta_A)(B\cos\theta_B)+(A\sin\theta_A)(B\sin\theta_B)$

$A_xB_x+A_yB_y=AB(\cos\theta_A\cos\theta_B+\sin\theta_A\sin\theta_B)=AB\cos(\theta_A-\theta_B)=AB\cos\phi$, where the expression for the cosine of the difference between two angles has been used.

(b) With $A_z=B_z=0$, $\vec{C}=C_z\hat{k}$ and $C=|C_z|$. From Eq.(1.27),

$|C|=|A_xB_y-A_yB_x|=|(A\cos\theta_A)(B\sin\theta_B)-(A\sin\theta_A)(B\cos\theta_A)|$

$|C|=AB|\cos\theta_A\sin\theta_B-\sin\theta_A\cos\theta_B|=AB|\sin(\theta_B-\theta_A)|=AB\sin\phi$, where the expression for the sine of the difference between two angles has been used.

EVALUATE: Since they are equivalent, we may use either Eq.(1.18) or (1.21) for the scalar product and either (1.22) or (1.27) for the vector product, depending on which is the more convenient in a given application.

1.86. IDENTIFY: Apply Eqs.(1.18) and (1.22).

SET UP: The angle between the vectors is $20°+90°+30°=140°$.

EXECUTE: **(a)** Eq. (1.18) gives $\vec{A}\cdot\vec{B}=(3.60\text{ m})(2.40\text{ m})\cos 140°=-6.62\text{ m}^2$.

(b) From Eq.(1.22), the magnitude of the cross product is $(3.60\text{ m})(2.40\text{ m})\sin 140°=5.55\text{ m}^2$ and the direction, from the right-hand rule, is out of the page (the $+z$-direction).

EVALUATE: We could also use Eqs.(1.21) and (1.27), with the components of $\vec{A}$ and $\vec{B}$.

1.87. IDENTIFY: Compare the magnitude of the cross product, $AB\sin\phi$, to the area of the parallelogram.

SET UP: The two sides of the parallelogram have lengths A and B. ϕ is the angle between $\vec{A}$ and $\vec{B}$.

EXECUTE: **(a)** The length of the base is B and the height of the parallelogram is $A\sin\phi$, so the area is $AB\sin\phi$. This equals the magnitude of the cross product.

(b) The cross product $\vec{A}\times\vec{B}$ is perpendicular to the plane formed by $\vec{A}$ and $\vec{B}$, so the angle is 90°.

EVALUATE: It is useful to consider the special cases $\phi = 0^\circ$, where the area is zero, and $\phi = 90^\circ$, where the parallelogram becomes a rectangle and the area is AB.

1.88. IDENTIFY: Use Eq.(1.27) for the components of the vector product.

SET UP: Use coordinates with the $+x$-axis to the right, $+y$-axis toward the top of the page, and $+z$-axis out of the page. $A_x = 0$, $A_y = 0$ and $A_z = -3.50\text{ cm}$. The page is 20 cm by 35 cm, so $B_x = 20\text{ cm}$ and $B_y = 35\text{ cm}$.

EXECUTE: $\left(\vec{A}\times\vec{B}\right)_x = 122\text{ cm}^2$, $\left(\vec{A}\times\vec{B}\right)_y = -70\text{ cm}^2$, $\left(\vec{A}\times\vec{B}\right)_z = 0.$

EVALUATE: From the components we calculated the magnitude of the vector product is 141 cm^2. $B = 40.3\text{ cm}$ and $\phi = 90^\circ$, so $AB\sin\phi = 141\text{ cm}^2$, which agrees.

1.89. IDENTIFY: $\vec{A}$ and $\vec{B}$ are given in unit vector form. Find A, B and the vector difference $\vec{A}-\vec{B}$.

SET UP: $\vec{A} = -2.00\vec{i} + 3.00\vec{j} + 4.00\vec{k}$, $\vec{B} = 3.00\vec{i} + 1.00\vec{j} - 3.00\vec{k}$

Use Eq.(1.8) to find the magnitudes of the vectors.

EXECUTE: **(a)** $A = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(-2.00)^2 + (3.00)^2 + (4.00)^2} = 5.38$

$B = \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(3.00)^2 + (1.00)^2 + (-3.00)^2} = 4.36$

(b) $\vec{A}-\vec{B} = (-2.00\hat{i} + 3.00\hat{j} + 4.00\hat{k}) - (3.00\hat{i} + 1.00\hat{j} - 3.00\hat{k})$

$\vec{A}-\vec{B} = (-2.00-3.00)\hat{i} + (3.00-1.00)\hat{j} + (4.00-(-3.00))\hat{k} = -5.00\hat{i} + 2.00\hat{j} + 7.00\hat{k}.$

(c) Let $\vec{C} = \vec{A}-\vec{B}$, so $C_x = -5.00,\ C_y = +2.00,\ C_z = +7.00$

$$C = \sqrt{C_x^2 + C_y^2 + C_z^2} = \sqrt{(-5.00)^2 + (2.00)^2 + (7.00)^2} = 8.83$$

$\vec{B}-\vec{A} = -(\vec{A}-\vec{B})$, so $\vec{A}-\vec{B}$ and $\vec{B}-\vec{A}$ have the same magnitude but opposite directions.

EVALUATE: A, B and C are each larger than any of their components.

1.90. IDENTIFY: Calculate the scalar product and use Eq.(1.18) to determine ϕ.

SET UP: The unit vectors are perpendicular to each other.

EXECUTE: The direction vectors each have magnitude $\sqrt{3}$, and their scalar product is $(1)(1)+(1)(-1)+(1)(-1) = -1$, so from Eq. (1.18) the angle between the bonds is

$$\arccos\left(\frac{-1}{\sqrt{3}\sqrt{3}}\right) = \arccos\left(-\frac{1}{3}\right) = 109^\circ.$$

EVALUATE: The angle between the two vectors in the bond directions is greater than 90°.

1.91. IDENTIFY: Use the relation derived in part (a) of Problem 1.92: $C^2 = A^2 + B^2 + 2AB\cos\phi$, where ϕ is the angle between $\vec{A}$ and $\vec{B}$.

SET UP: $\cos\phi = 0$ for $\phi = 90^\circ$. $\cos\phi < 0$ for $90^\circ < \phi < 180^\circ$ and $\cos\phi > 0$ for $0^\circ < \phi < 90^\circ$.

EXECUTE: **(a)** If $C^2 = A^2 + B^2$, $\cos\phi = 0$, and the angle between $\vec{A}$ and $\vec{B}$ is 90° (the vectors are perpendicular).

(b) If $C^2 < A^2 + B^2$, $\cos\phi < 0$, and the angle between $\vec{A}$ and $\vec{B}$ is greater than 90°.

(c) If $C^2 > A^2 + B^2$, $\cos\phi > 0$, and the angle between $\vec{A}$ and $\vec{B}$ is less than 90°.

EVALUATE: It is easy to verify the expression from Problem 1.92 for the special cases $\phi = 0$, where $C = A + B$, and for $\phi = 180^\circ$, where $C = A - B$.

1.92. IDENTIFY: Let $\vec{C} = \vec{A} + \vec{B}$ and calculate the scalar product $\vec{C}\cdot\vec{C}$.

SET UP: For any vector $\vec{V}$, $\vec{V}\cdot\vec{V} = V^2$. $\vec{A}\cdot\vec{B} = AB\cos\phi$.

EXECUTE: **(a)** Use the linearity of the dot product to show that the square of the magnitude of the sum $\vec{A}+\vec{B}$ is

$$\left(\vec{A}+\vec{B}\right)\cdot\left(\vec{A}+\vec{B}\right) = \vec{A}\cdot\vec{A} + \vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{A} + \vec{B}\cdot\vec{B} = \vec{A}\cdot\vec{A} + \vec{B}\cdot\vec{B} + 2\vec{A}\cdot\vec{B} = A^2 + B^2 + 2\vec{A}\cdot\vec{B}$$
$$= A^2 + B^2 + 2AB\cos\phi$$

(b) Using the result of part (a), with $A = B$, the condition is that $A^2 = A^2 + A^2 + 2A^2\cos\phi$, which solves for $1 = 2 + 2\cos\phi$, $\cos\phi = -\frac{1}{2}$, and $\phi = 120°$.

EVALUATE: The expression $C^2 = A^2 + B^2 + 2AB\cos\phi$ is called the law of cosines.

1.93. **IDENTIFY:** Find the angle between specified pairs of vectors.

SET UP: Use $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

EXECUTE: **(a)** $\vec{A} = \hat{k}$ (along line ab)

$\vec{B} = \hat{i} + \hat{j} + \hat{k}$ (along line ad)

$A = 1,\ B = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$

$\vec{A}\cdot\vec{B} = \hat{k}\cdot\left(\hat{i} + \hat{j} + \hat{k}\right) = 1$

So $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB} = 1/\sqrt{3};\ \ \phi = 54.7°$

(b) $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ (along line ad)

$\vec{B} = \hat{j} + \hat{k}$ (along line ac)

$A = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3};\ \ B = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\vec{A}\cdot\vec{B} = \left(\hat{i} + \hat{j} + \hat{k}\right)\cdot\left(\hat{i} + \hat{j}\right) = 1 + 1 = 2$

So $\cos\phi = \dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{2}{\sqrt{3}\sqrt{2}} = \dfrac{2}{\sqrt{6}};\ \ \phi = 35.3°$

EVALUATE: Each angle is computed to be less than $90°$, in agreement with what is deduced from Fig. 1.43 in the textbook.

1.94. **IDENTIFY:** The cross product $\vec{A}\times\vec{B}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

SET UP: Use Eq.(1.27) to calculate the components of $\vec{A}\times\vec{B}$.

EXECUTE: The cross product is

$(-13.00)\hat{i} + (6.00)\hat{j} + (-11.00)\hat{k} = 13\left[-(1.00)\hat{i} + \left(\dfrac{6.00}{13.00}\right)\hat{j} - \dfrac{11.00}{13.00}\hat{k}\right]$. The magnitude of the vector in square brackets is $\sqrt{1.93}$, and so a unit vector in this direction is

$$\left[\frac{-(1.00)\hat{i} + (6.00/13.00)\hat{j} - (11.00/13.00)\hat{k}}{\sqrt{1.93}}\right].$$

The negative of this vector,

$$\left[\frac{(1.00)\hat{i} - (6.00/13.00)\hat{j} + (11.00/13.00)\hat{k}}{\sqrt{1.93}}\right],$$

is also a unit vector perpendicular to $\vec{A}$ and $\vec{B}$.

EVALUATE: Any two vectors that are not parallel or antiparallel form a plane and a vector perpendicular to both vectors is perpendicular to this plane.

1.95. **IDENTIFY** and **SET UP:** The target variables are the components of $\vec{C}$. We are given $\vec{A}$ and $\vec{B}$. We also know $\vec{A}\cdot\vec{C}$ and $\vec{B}\cdot\vec{C}$, and this gives us two equations in the two unknowns C_x and C_y.

EXECUTE: $\vec{A}$ and $\vec{C}$ are perpendicular, so $\vec{A}\cdot\vec{C} = 0$. $A_xC_x + A_yC_y = 0$, which gives $5.0C_x - 6.5C_y = 0$.

$\vec{B}\cdot\vec{C} = 15.0$, so $-3.5C_x + 7.0C_y = 15.0$

We have two equations in two unknowns C_x and C_y. Solving gives $C_x = 8.0$ and $C_y = 6.1$

EVALUATE: We can check that our result does give us a vector $\vec{C}$ that satisfies the two equations $\vec{A}\cdot\vec{C} = 0$ and $\vec{B}\cdot\vec{C} = 15.0$.

1.96. **IDENTIFY:** Calculate the magnitude of the vector product and then use Eq.(1.22).

SET UP: The magnitude of a vector is related to its components by Eq.(1.12).

EXECUTE: $\left|\vec{A}\times\vec{B}\right| = AB\sin\theta$. $\sin\theta = \dfrac{\left|\vec{A}\times\vec{B}\right|}{AB} = \dfrac{\sqrt{(-5.00)^2+(2.00)^2}}{(3.00)(3.00)} = 0.5984$ and $\theta = \sin^{-1}(0.5984) = 36.8°$.

EVALUATE: We haven't found $\vec{A}$ and $\vec{B}$, just the angle between them.

1.97. **(a) IDENTIFY:** Prove that $\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C}$.

SET UP: Express the scalar and vector products in terms of components.

EXECUTE:

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = A_x(\vec{B}\times\vec{C})_x + A_y(\vec{B}\times\vec{C})_y + A_z(\vec{B}\times\vec{C})_z$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = A_x(B_yC_z - B_zC_y) + A_y(B_zC_x - B_xC_z) + A_z(B_xC_y - B_yC_x)$$

$$(\vec{A}\times\vec{B})\cdot\vec{C} = (\vec{A}\times\vec{B})_x C_x + (\vec{A}\times\vec{B})_y C_y + (\vec{A}\times\vec{B})_z C_z$$

$$(\vec{A}\times\vec{B})\cdot\vec{C} = (A_yB_z - A_zB_y)C_x + (A_zB_x - A_xB_z)C_y + (A_xB_y - A_yB_x)C_z$$

Comparison of the expressions for $\vec{A}\cdot(\vec{B}\times\vec{C})$ and $(\vec{A}\times\vec{B})\cdot\vec{C}$ shows they contain the same terms, so $\vec{A}\cdot(\vec{B}\times\vec{C}) = (\vec{A}\times\vec{B})\cdot\vec{C}$.

(b) IDENTIFY: Calculate $(\vec{A}\times\vec{B})\cdot\vec{C}$, given the magnitude and direction of $\vec{A}$, $\vec{B}$, and $\vec{C}$.

SET UP: Use Eq.(1.22) to find the magnitude and direction of $\vec{A}\times\vec{B}$. Then we know the components of $\vec{A}\times\vec{B}$ and of $\vec{C}$ and can use an expression like Eq.(1.21) to find the scalar product in terms of components.

EXECUTE: $A = 5.00$; $\theta_A = 26.0°$; $B = 4.00$, $\theta_B = 63.0°$

$\left|\vec{A}\times\vec{B}\right| = AB\sin\phi$.

The angle ϕ between $\vec{A}$ and $\vec{B}$ is equal to $\phi = \theta_B - \theta_A = 63.0° - 26.0° = 37.0°$. So

$\left|\vec{A}\times\vec{B}\right| = (5.00)(4.00)\sin 37.0° = 12.04$, and by the right hand-rule $\vec{A}\times\vec{B}$ is in the $+z$-direction. Thus

$(\vec{A}\times\vec{B})\cdot\vec{C} = (12.04)(6.00) = 72.2$

EVALUATE: $\vec{A}\times\vec{B}$ is a vector, so taking its scalar product with $\vec{C}$ is a legitimate vector operation. $(\vec{A}\times\vec{B})\cdot\vec{C}$ is a scalar product between two vectors so the result is a scalar.

1.98. **IDENTIFY:** Use the maximum and minimum values of the dimensions to find the maximum and minimum areas and volumes.

SET UP: For a rectangle of width W and length L the area is LW. For a rectangular solid with dimensions L, W and H the volume is LWH.

EXECUTE: **(a)** The maximum and minimum areas are $(L+l)(W+w) = LW + lW + Lw$, $(L-l)(W-w) = LW - lW - Lw$, where the common terms wl have been omitted. The area and its uncertainty are then $WL \pm (lW + Lw)$, so the uncertainty in the area is $a = lW + Lw$.

(b) The fractional uncertainty in the area is $\dfrac{a}{A} = \dfrac{lW + Wl}{WL} = \dfrac{l}{L} + \dfrac{w}{W}$, the sum of the fractional uncertainties in the length and width.

(c) The similar calculation to find the uncertainty v in the volume will involve neglecting the terms lwH, lWh and Lwh as well as lwh; the uncertainty in the volume is $v = lWH + LwH + LWh$, and the fractional uncertainty in the volume is $\dfrac{v}{V} = \dfrac{lWH + LwH + LWh}{LWH} = \dfrac{l}{L} + \dfrac{w}{W} + \dfrac{h}{H}$, the sum of the fractional uncertainties in the length, width and height.

EVALUATE: The calculation assumes the uncertainties are small, so that terms involving products of two or more uncertainties can be neglected.

1.99. **IDENTIFY:** Add the vector displacements of the receiver and then find the vector from the quarterback to the receiver.

SET UP: Add the x-components and the y-components.

EXECUTE: The receiver's position is

$[(+1.0+9.0-6.0+12.0)\ \text{yd}]\hat{\boldsymbol{i}}+[(-5.0+11.0+4.0+18.0)\ \text{yd}]\hat{\boldsymbol{j}}=(16.0\ \text{yd})\hat{\boldsymbol{i}}+(28.0\ \text{yd})\hat{\boldsymbol{j}}$.

The vector from the quarterback to the receiver is the receiver's position minus the quarterback's position, or $(16.0\ \text{yd})\hat{\boldsymbol{i}}+(35.0\ \text{yd})\hat{\boldsymbol{j}}$, a vector with magnitude $\sqrt{(16.0\ \text{yd})^2+(35.0\ \text{yd})^2}=38.5\ \text{yd}$. The angle is

$\arctan\left(\dfrac{16.0}{35.0}\right)=24.6°$ to the right of downfield.

EVALUATE: The vector from the quarterback to receiver has positive *x*-component and positive *y*-component.

1.100. IDENTIFY: Use the *x* and *y* coordinates for each object to find the vector from one object to the other; the distance between two objects is the magnitude of this vector. Use the scalar product to find the angle between two vectors.

SET UP: If object *A* has coordinates (x_A, y_A) and object *B* has coordinates (x_B, y_B), the vector $\vec{\boldsymbol{r}}_{AB}$ from *A* to *B* has *x*-component $x_B - x_A$ and *y*-component $y_B - y_A$.

EXECUTE: **(a)** The diagram is sketched in Figure 1.100.

(b) (i) In AU, $\sqrt{(0.3182)^2+(0.9329)^2}=0.9857$.

(ii) In AU, $\sqrt{(1.3087)^2+(-0.4423)^2+(-0.0414)^2}=1.3820$.

(iii) In AU $\sqrt{(0.3182-1.3087)^2+(0.9329-(-0.4423))^2+(0.0414)^2}=1.695$.

(c) The angle between the directions from the Earth to the Sun and to Mars is obtained from the dot product. Combining Equations (1.18) and (1.21),

$$\phi=\arccos\left(\frac{(-0.3182)(1.3087-0.3182)+(-0.9329)(-0.4423-0.9329)+(0)}{(0.9857)(1.695)}\right)=54.6°.$$

(d) Mars could not have been visible at midnight, because the Sun-Mars angle is less than 90°.

EVALUATE: Our calculations correctly give that Mars is farther from the Sun than the earth is. Note that on this date Mars was farther from the earth than it is from the Sun.

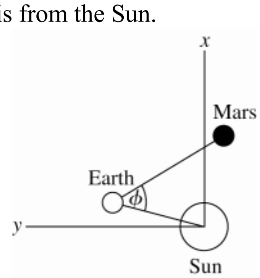


Figure 1.100

1.101. IDENTIFY: Draw the vector addition diagram for the position vectors.

SET UP: Use coordinates in which the Sun to Merak line lies along the *x*-axis. Let $\vec{\boldsymbol{A}}$ be the position vector of Alkaid relative to the Sun, $\vec{\boldsymbol{M}}$ is the position vector of Merak relative to the Sun, and $\vec{\boldsymbol{R}}$ is the position vector for Alkaid relative to Merak. $A=138$ ly and $M=77$ ly.

EXECUTE: The relative positions are shown in Figure 1.101. $\vec{\boldsymbol{M}}+\vec{\boldsymbol{R}}=\vec{\boldsymbol{A}}$. $A_x=M_x+R_x$ so $R_x=A_x-M_x=(138\ \text{ly})\cos 25.6°-77\ \text{ly}=47.5\ \text{ly}$. $R_y=A_y-M_y=(138\ \text{ly})\sin 25.6°-0=59.6\ \text{ly}$. $R=76.2$ ly is the distance between Alkaid and Merak.

(b) The angle is angle ϕ in Figure 1.101. $\cos\theta=\dfrac{R_x}{R}=\dfrac{47.5\ \text{ly}}{76.2\ \text{ly}}$ and $\theta=51.4°$. Then $\phi=180°-\theta=129°$.

EVALUATE: The concepts of vector addition and components make these calculations very simple.

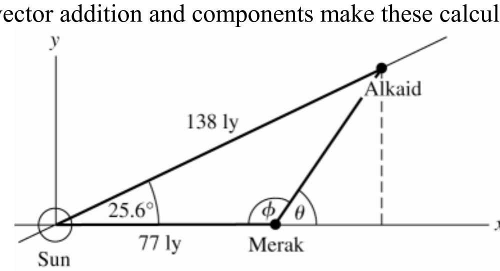


Figure 1.101

1.102. **IDENTIFY:** Define $\vec{S} = A\hat{i} + B\hat{j} + C\hat{k}$. Show that $\vec{r} \cdot \vec{S} = 0$ if $Ax + By + Cz = 0$.

SET UP: Use Eq.(1.21) to calculate the scalar product.

EXECUTE: $\vec{r} \cdot \vec{S} = (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (A\hat{i} + B\hat{j} + C\hat{k}) = Ax + By + Cz$

If the points satisfy $Ax + By + Cz = 0$, then $\vec{r} \cdot \vec{S} = 0$ and all points $\vec{r}$ are perpendicular to $\vec{S}$. The vector and plane are sketched in Figure 1.102.

EVALUATE: If two vectors are perpendicular their scalar product is zero.

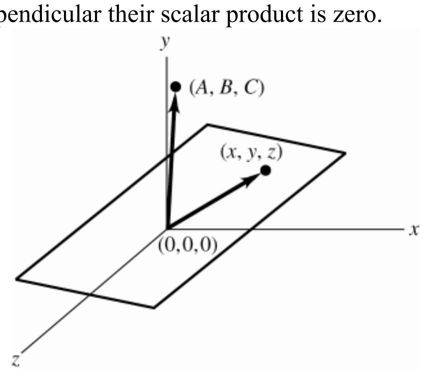


Figure 1.102

2

MOTION ALONG A STRAIGHT LINE

2.1. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$.

SET UP: Let $+x$ be upward.

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{1000\text{ m} - 63\text{ m}}{4.75\text{ s}} = 197\text{ m/s}$

(b) $v_{\text{av-}x} = \dfrac{1000\text{ m} - 0}{5.90\text{ s}} = 169\text{ m/s}$

EVALUATE: For the first 1.15 s of the flight, $v_{\text{av-}x} = \dfrac{63\text{ m} - 0}{1.15\text{ s}} = 54.8\text{ m/s}$. When the velocity isn't constant the average velocity depends on the time interval chosen. In this motion the velocity is increasing.

2.2. **IDENTIFY:** $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$

SET UP: $13.5\text{ days} = 1.166 \times 10^5\text{ s}$. At the release point, $x = +5.150 \times 10^6\text{ m}$.

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{x_2 - x_1}{\Delta t} = \dfrac{5.150 \times 10^6\text{ m}}{1.166 \times 10^6\text{ s}} = -4.42\text{ m/s}$

(b) For the round trip, $x_2 = x_1$ and $\Delta x = 0$. The average velocity is zero.

EVALUATE: The average velocity for the trip from the nest to the release point is positive.

2.3. **IDENTIFY:** Target variable is the time Δt it takes to make the trip in heavy traffic. Use Eq.(2.2) that relates the average velocity to the displacement and average time.

SET UP: $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ so $\Delta x = v_{\text{av-}x}\Delta t$ and $\Delta t = \dfrac{\Delta x}{v_{\text{av-}x}}$.

EXECUTE: Use the information given for normal driving conditions to calculate the distance between the two cities:

$$\Delta x = v_{\text{av-}x}\Delta t = (105\text{ km/h})(1\text{ h/60 min})(140\text{ min}) = 245\text{ km}.$$

Now use $v_{\text{av-}x}$ for heavy traffic to calculate Δt; Δx is the same as before:

$$\Delta t = \frac{\Delta x}{v_{\text{av-}x}} = \frac{245\text{ km}}{70\text{ km/h}} = 3.50\text{ h} = 3\text{ h and 30 min.}$$

The trip takes an additional 1 hour and 10 minutes.

EVALUATE: The time is inversely proportional to the average speed, so the time in traffic is $(105/70)(140\text{ m}) = 210\text{ min}$.

2.4. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. Use the average speed for each segment to find the time traveled in that segment. The average speed is the distance traveled by the time.

SET UP: The post is 80 m west of the pillar. The total distance traveled is $200\text{ m} + 280\text{ m} = 480\text{ m}$.

EXECUTE: **(a)** The eastward run takes time $\dfrac{200\text{ m}}{5.0\text{ m/s}} = 40.0\text{ s}$ and the westward run takes $\dfrac{280\text{ m}}{4.0\text{ m/s}} = 70.0\text{ s}$. The average speed for the entire trip is $\dfrac{480\text{ m}}{110.0\text{ s}} = 4.4\text{ m/s}$.

(b) $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{-80\text{ m}}{110.0\text{ s}} = -0.73\text{ m/s}$. The average velocity is directed westward.

EVALUATE: The displacement is much less than the distance traveled and the magnitude of the average velocity is much less than the average speed. The average speed for the entire trip has a value that lies between the average speed for the two segments.

2.5. **IDENTIFY:** When they first meet the sum of the distances they have run is 200 m.

SET UP: Each runs with constant speed and continues around the track in the same direction, so the distance each runs is given by $d = vt$. Let the two runners be objects A and B.

EXECUTE: **(a)** $d_A + d_B = 200\text{ m}$, so $(6.20\text{ m/s})t + (5.50\text{ m/s})t = 200\text{ m}$ and $t = \dfrac{200\text{ m}}{11.70\text{ m/s}} = 17.1\text{ s}$.

(b) $d_A = v_A t = (6.20\text{ m/s})(17.1\text{ s}) = 106\text{ m}$. $d_B = v_B t = (5.50\text{ m/s})(17.1\text{ s}) = 94\text{ m}$. The faster runner will be 106 m from the starting point and the slower runner will be 94 m from the starting point. These distances are measured around the circular track and are not straight-line distances.

EVALUATE: The faster runner runs farther.

2.6. **IDENTIFY:** To overtake the slower runner the first time the fast runner must run 200 m farther. To overtake the slower runner the second time the faster runner must run 400 m farther.

SET UP: t and x_0 are the same for the two runners.

EXECUTE: **(a)** Apply $x - x_0 = v_{0x}t$ to each runner: $(x - x_0)_\text{f} = (6.20\text{ m/s})t$ and $(x - x_0)_\text{s} = (5.50\text{ m/s})t$.

$(x - x_0)_\text{f} = (x - x_0)_\text{s} + 200\text{ m}$ gives $(6.20\text{ m/s})t = (5.50\text{ m/s})t + 200\text{ m}$ and $t = \dfrac{200\text{ m}}{6.20\text{ m/s} - 5.50\text{ m/s}} = 286\text{ s}$.

$(x - x_0)_\text{f} = 1770\text{ m}$ and $(x - x_0)_\text{s} = 1570\text{ m}$.

(b) Repeat the calculation but now $(x - x_0)_\text{f} = (x - x_0)_\text{s} + 400\text{ m}$. $t = 572\text{ s}$. The fast runner has traveled 3540 m. He has made 17 full laps for 3400 m and 140 m past the starting line in this 18$^\text{th}$ lap.

EVALUATE: In part (a) the fast runner will have run 8 laps for 1600 m and will be 170 m past the starting line in his 9$^\text{th}$ lap.

2.7. **IDENTIFY:** In time t_S the S-waves travel a distance $d = v_\text{S} t_\text{S}$ and in time t_P the P-waves travel a distance $d = v_\text{P} t_\text{P}$.

SET UP: $t_\text{S} = t_\text{P} + 33\text{ s}$

EXECUTE: $\dfrac{d}{v_\text{S}} = \dfrac{d}{v_\text{P}} + 33\text{ s}$. $d\left(\dfrac{1}{3.5\text{ km/s}} - \dfrac{1}{6.5\text{ km/s}}\right) = 33\text{ s}$ and $d = 250\text{ km}$.

EVALUATE: The times of travel for each wave are $t_\text{S} = 71\text{ s}$ and $t_\text{P} = 38\text{ s}$.

2.8. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$. Use $x(t)$ to find x for each t.

SET UP: $x(0) = 0$, $x(2.00\text{ s}) = 5.60\text{ m}$, and $x(4.00\text{ s}) = 20.8\text{ m}$

EXECUTE: **(a)** $v_{\text{av-}x} = \dfrac{5.60\text{ m} - 0}{2.00\text{ s}} = +2.80\text{ m/s}$

(b) $v_{\text{av-}x} = \dfrac{20.8\text{ m} - 0}{4.00\text{ s}} = +5.20\text{ m/s}$

(c) $v_{\text{av-}x} = \dfrac{20.8\text{ m} - 5.60\text{ m}}{2.00\text{ s}} = +7.60\text{ m/s}$

EVALUATE: The average velocity depends on the time interval being considered.

2.9. **(a) IDENTIFY:** Calculate the average velocity using Eq.(2.2).

SET UP: $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$ so use $x(t)$ to find the displacement Δx for this time interval.

EXECUTE: $t = 0$: $x = 0$

$t = 10.0\text{ s}$: $x = (2.40\text{ m/s}^2)(10.0\text{ s})^2 - (0.120\text{ m/s}^3)(10.0\text{ s})^3 = 240\text{ m} - 120\text{ m} = 120\text{ m}$.

Then $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{120\text{ m}}{10.0\text{ s}} = 12.0\text{ m/s}$.

(b) IDENTIFY: Use Eq.(2.3) to calculate $v_x(t)$ and evaluate this expression at each specified t.

SET UP: $v_x = \dfrac{dx}{dt} = 2bt - 3ct^2$.

EXECUTE: (i) $t = 0$: $v_x = 0$

(ii) $t = 5.0\text{ s}$: $v_x = 2(2.40\text{ m/s}^2)(5.0\text{ s}) - 3(0.120\text{ m/s}^3)(5.0\text{ s})^2 = 24.0\text{ m/s} - 9.0\text{ m/s} = 15.0\text{ m/s}$.

(iii) $t = 10.0\text{ s}$: $v_x = 2(2.40\text{ m/s}^2)(10.0\text{ s}) - 3(0.120\text{ m/s}^3)(10.0\text{ s})^2 = 48.0\text{ m/s} - 36.0\text{ m/s} = 12.0\text{ m/s}$.

(c) IDENTIFY: Find the value of t when $v_x(t)$ from part (b) is zero.

SET UP: $v_x = 2bt - 3ct^2$

$v_x = 0$ at $t = 0$.

$v_x = 0$ next when $2bt - 3ct^2 = 0$

EXECUTE: $2b = 3ct$ so $t = \frac{2b}{3c} = \frac{2(2.40\text{ m/s}^2)}{30(.120\text{ m/s}^3)} = 13.3\text{ s}$

EVALUATE: $v_x(t)$ for this motion says the car starts from rest, speeds up, and then slows down again.

2.10. **IDENTIFY** and **SET UP:** The instantaneous velocity is the slope of the tangent to the x versus t graph.
EXECUTE: **(a)** The velocity is zero where the graph is horizontal; point IV.
(b) The velocity is constant and positive where the graph is a straight line with positive slope; point I.
(c) The velocity is constant and negative where the graph is a straight line with negative slope; point V.
(d) The slope is positive and increasing at point II.
(e) The slope is positive and decreasing at point III.
EVALUATE: The sign of the velocity indicates its direction.

2.11. **IDENTIFY:** The average velocity is given by $v_{\text{av-}x} = \frac{\Delta x}{\Delta t}$. We can find the displacement Δt for each constant velocity time interval. The average speed is the distance traveled divided by the time.
SET UP: For $t = 0$ to $t = 2.0\text{ s}$, $v_x = 2.0\text{ m/s}$. For $t = 2.0\text{ s}$ to $t = 3.0\text{ s}$, $v_x = 3.0\text{ m/s}$. In part (b), $v_x = -3.0\text{ m/s}$ for $t = 2.0\text{ s}$ to $t = 3.0\text{ s}$. When the velocity is constant, $\Delta x = v_x \Delta t$.
EXECUTE: **(a)** For $t = 0$ to $t = 2.0\text{ s}$, $\Delta x = (2.0\text{ m/s})(2.0\text{ s}) = 4.0\text{ m}$. For $t = 2.0\text{ s}$ to $t = 3.0\text{ s}$, $\Delta x = (3.0\text{ m/s})(1.0\text{ s}) = 3.0\text{ m}$. For the first 3.0 s, $\Delta x = 4.0\text{ m} + 3.0\text{ m} = 7.0\text{ m}$. The distance traveled is also 7.0 m.

The average velocity is $v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{7.0\text{ m}}{3.0\text{ s}} = 2.33\text{ m/s}$. The average speed is also 2.33 m/s.

(b) For $t = 2.0$ s to 3.0 s, $\Delta x = (-3.0\text{ m/s})(1.0\text{ s}) = -3.0\text{ m}$. For the first 3.0 s, $\Delta x = 4.0\text{ m} + (-3.0\text{ m}) = +1.0\text{ m}$. The dog runs 4.0 m in the $+x$-direction and then 3.0 m in the $-x$-direction, so the distance traveled is still 7.0 m. $v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{1.0\text{ m}}{3.0\text{ s}} = 0.33\text{ m/s}$. The average speed is $\frac{7.00\text{ m}}{3.00\text{ s}} = 2.33\text{ m/s}$.
EVALUATE: When the motion is always in the same direction, the displacement and the distance traveled are equal and the average velocity has the same magnitude as the average speed. When the motion changes direction during the time interval, those quantities are different.

2.12. **IDENTIFY** and **SET UP:** $a_{\text{av},x} = \frac{\Delta v_x}{\Delta t}$. The instantaneous acceleration is the slope of the tangent to the v_x versus t graph.
EXECUTE: **(a)** 0 s to 2 s: $a_{\text{av},x} = 0$; 2 s to 4 s: $a_{\text{av},x} = 1.0\text{ m/s}^2$; 4 s to 6 s: $a_{\text{av},x} = 1.5\text{ m/s}^2$; 6 s to 8 s: $a_{\text{av},x} = 2.5\text{ m/s}^2$; 8 s to 10 s: $a_{\text{av},x} = 2.5\text{ m/s}^2$; 10 s to 12 s: $a_{\text{av},x} = 2.5\text{ m/s}^2$; 12 s to 14 s: $a_{\text{av},x} = 1.0\text{ m/s}^2$; 14 s to 16 s: $a_{\text{av},x} = 0$. The acceleration is not constant over the entire 16 s time interval. The acceleration is constant between 6 s and 12 s.
(b) The graph of v_x versus t is given in Fig. 2.12. $t = 9\text{ s}$: $a_x = 2.5\text{ m/s}^2$; $t = 13\text{ s}$: $a_x = 1.0\text{ m/s}^2$; $t = 15\text{ s}$: $a_x = 0$.

EVALUATE: The acceleration is constant when the velocity changes at a constant rate. When the velocity is constant, the acceleration is zero.

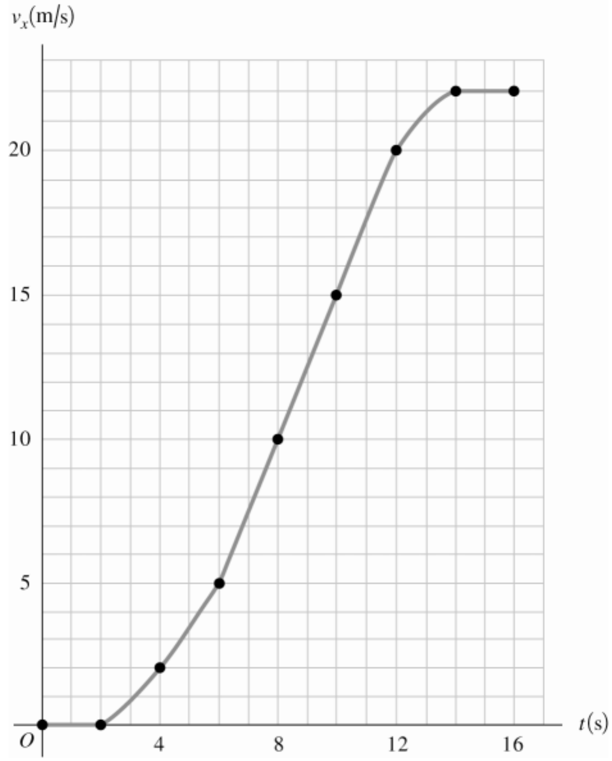


Figure 2.12

2.13. **IDENTIFY:** The average acceleration for a time interval Δt is given by $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$.

SET UP: Assume the car is moving in the $+x$ direction. $1\text{ mi/h} = 0.447\text{ m/s}$, so $60\text{ mi/h} = 26.82\text{ m/s}$, $200\text{ mi/h} = 89.40\text{ m/s}$ and $253\text{ mi/h} = 113.1\text{ m/s}$.

EXECUTE: **(a)** The graph of v_x versus t is sketched in Figure 2.13. The graph is not a straight line, so the acceleration is not constant.

(b) (i) $a_{\text{av-}x} = \dfrac{26.82\text{ m/s} - 0}{2.1\text{ s}} = 12.8\text{ m/s}^2$ (ii) $a_{\text{av-}x} = \dfrac{89.40\text{ m/s} - 26.82\text{ m/s}}{20.0\text{ s} - 2.1\text{ s}} = 3.50\text{ m/s}^2$ (iii) $a_{\text{av-}x} = \dfrac{113.1\text{ m/s} - 89.40\text{ m/s}}{53\text{ s} - 20.0\text{ s}} = 0.718\text{ m/s}^2$. The slope of the graph of v_x versus t decreases as t increases. This is consistent with an average acceleration that decreases in magnitude during each successive time interval.

EVALUATE: The average acceleration depends on the chosen time interval. For the interval between 0 and 53 s, $a_{\text{av-}x} = \dfrac{113.1\text{ m/s} - 0}{53\text{ s}} = 2.13\text{ m/s}^2$.

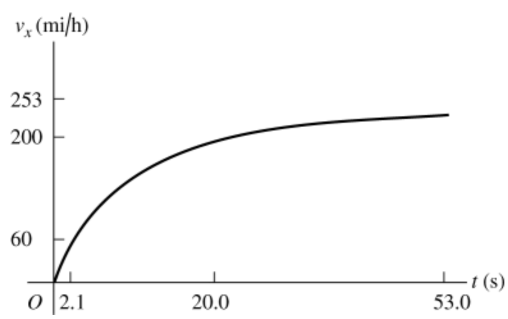


Figure 2.13

2.14. **IDENTIFY:** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$. $a_x(t)$ is the slope of the v_x versus t graph.

SET UP: 60 km/h $= 16.7$ m/s

EXECUTE: **(a)** (i) $a_{\text{av-}x} = \dfrac{16.7\text{ m/s} - 0}{10\text{ s}} = 1.7\text{ m/s}^2$. (ii) $a_{\text{av-}x} = \dfrac{0 - 16.7\text{ m/s}}{10\text{ s}} = -1.7\text{ m/s}^2$.

(iii) $\Delta v_x = 0$ and $a_{\text{av-}x} = 0$. (iv) $\Delta v_x = 0$ and $a_{\text{av-}x} = 0$.

(b) At $t = 20$ s, v_x is constant and $a_x = 0$. At $t = 35$ s, the graph of v_x versus t is a straight line and $a_x = a_{\text{av-}x} = -1.7\text{ m/s}^2$.

EVALUATE: When $a_{\text{av-}x}$ and v_x have the same sign the speed is increasing. When they have opposite sign the speed is decreasing.

2.15. **IDENTIFY** and **SET UP:** Use $v_x = \dfrac{dx}{dt}$ and $a_x = \dfrac{dv_x}{dt}$ to calculate $v_x(t)$ and $a_x(t)$.

EXECUTE: $v_x = \dfrac{dx}{dt} = 2.00\text{ cm/s} - (0.125\text{ cm/s}^2)t$

$a_x = \dfrac{dv_x}{dt} = -0.125\text{ cm/s}^2$

(a) At $t = 0$, $x = 50.0$ cm, $v_x = 2.00$ cm/s, $a_x = -0.125\text{ cm/s}^2$.

(b) Set $v_x = 0$ and solve for t: $t = 16.0$ s.

(c) Set $x = 50.0$ cm and solve for t. This gives $t = 0$ and $t = 32.0$ s. The turtle returns to the starting point after 32.0 s.

(d) Turtle is 10.0 cm from starting point when $x = 60.0$ cm or $x = 40.0$ cm.
Set $x = 60.0$ cm and solve for t: $t = 6.20$ s and $t = 25.8$ s.
At $t = 6.20$ s, $v_x = +1.23$ cm/s.
At $t = 25.8$ s, $v_x = -1.23$ cm/s.
Set $x = 40.0$ cm and solve for t: $t = 36.4$ s (other root to the quadratic equation is negative and hence nonphysical).
At $t = 36.4$ s, $v_x = -2.55$ cm/s.

(e) The graphs are sketched in Figure 2.15.

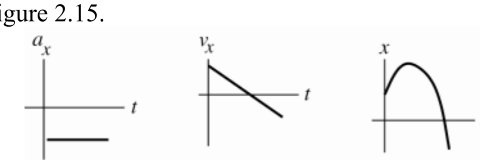


Figure 2.15

EVALUATE: The acceleration is constant and negative. v_x is linear in time. It is initially positive, decreases to zero, and then becomes negative with increasing magnitude. The turtle initially moves farther away from the origin but then stops and moves in the $-x$-direction.

2.16. **IDENTIFY:** Use Eq.(2.4), with $\Delta t = 10$ s in all cases.

SET UP: v_x is negative if the motion is to the right.

EXECUTE: **(a)** $\left((5.0\text{ m/s}) - (15.0\text{ m/s})\right)/(10\text{ s}) = -1.0\text{ m/s}^2$

(b) $\left((-15.0\text{ m/s}) - (-5.0\text{ m/s})\right)/(10\text{ s}) = -1.0\text{ m/s}^2$

(c) $\left((-15.0\text{ m/s}) - (+15.0\text{ m/s})\right)/(10\text{ s}) = -3.0\text{ m/s}^2$

EVALUATE: In all cases, the negative acceleration indicates an acceleration to the left.

2.17. **IDENTIFY:** The average acceleration is $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$

SET UP: Assume the car goes from rest to 65 mi/h (29 m/s) in 10 s. In braking, assume the car goes from 65 mi/h to zero in 4.0 s. Let $+x$ be in the direction the car is traveling.

EXECUTE: **(a)** $a_{\text{av-}x} = \dfrac{29\text{ m/s} - 0}{10\text{ s}} = 2.9\text{ m/s}^2$

(b) $a_{\text{av-}x} = \dfrac{0 - 29\text{ m/s}}{4.0\text{ s}} = -7.2\text{ m/s}^2$

(c) In part (a) the speed increases so the acceleration is in the same direction as the velocity. If the velocity direction is positive, then the acceleration is positive. In part (b) the speed decreases so the acceleration is in the direction opposite to the direction of the velocity. If the velocity direction is positive then the acceleration is negative, and if the velocity direction is negative then the acceleration direction is positive.

EVALUATE: The sign of the velocity and of the acceleration indicate their direction.

2.18. **IDENTIFY:** The average acceleration is $a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t}$. Use $v_x(t)$ to find v_x at each *t*. The instantaneous acceleration is $a_x = \frac{dv_x}{dt}$.

SET UP: $v_x(0) = 3.00$ m/s and $v_x(5.00\text{ s}) = 5.50$ m/s.

EXECUTE: **(a)** $a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{5.50\text{ m/s} - 3.00\text{ m/s}}{5.00\text{ s}} = 0.500\text{ m/s}^2$

(b) $a_x = \frac{dv_x}{dt} = (0.100\text{ m/s}^3)(2t) = (0.200\text{ m/s}^3)t$. At $t = 0$, $a_x = 0$. At $t = 5.00$ s, $a_x = 1.00\text{ m/s}^2$.

(c) Graphs of $v_x(t)$ and $a_x(t)$ are given in Figure 2.18.

EVALUATE: $a_x(t)$ is the slope of $v_x(t)$ and increases at *t* increases. The average acceleration for $t = 0$ to $t = 5.00$ s equals the instantaneous acceleration at the midpoint of the time interval, $t = 2.50$ s, since $a_x(t)$ is a linear function of *t*.

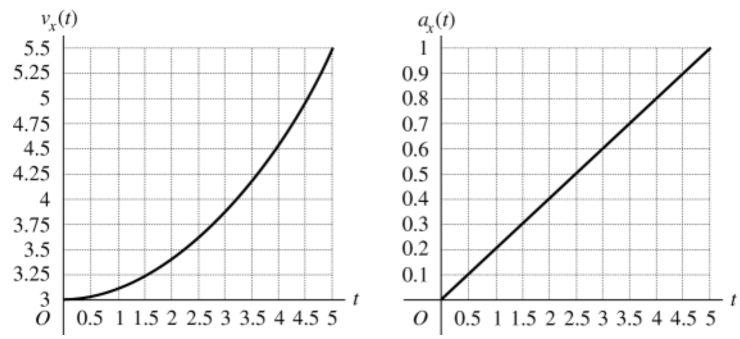


Figure 2.18

2.19. **(a) IDENTIFY** and **SET UP:** v_x is the slope of the *x* versus *t* curve and a_x is the slope of the v_x versus *t* curve.

EXECUTE: $t = 0$ to $t = 5$ s: *x* versus *t* is a parabola so a_x is a constant. The curvature is positive so a_x is positive. v_x versus *t* is a straight line with positive slope. $v_{0x} = 0$.

$t = 5$ s to $t = 15$ s: *x* versus *t* is a straight line so v_x is constant and $a_x = 0$. The slope of *x* versus *t* is positive so v_x is positive.

$t = 15$ s to $t = 25$ s: *x* versus *t* is a parabola with negative curvature, so a_x is constant and negative. v_x versus *t* is a straight line with negative slope. The velocity is zero at 20 s, positive for 15 s to 20 s, and negative for 20 s to 25 s.

$t = 25$ s to $t = 35$ s: *x* versus *t* is a straight line so v_x is constant and $a_x = 0$. The slope of *x* versus *t* is negative so v_x is negative.

$t = 35$ s to $t = 40$ s: *x* versus *t* is a parabola with positive curvature, so a_x is constant and positive. v_x versus *t* is a straight line with positive slope. The velocity reaches zero at $t = 40$ s.

The graphs of $v_x(t)$ and $a_x(t)$ are sketched in Figure 2.19a.

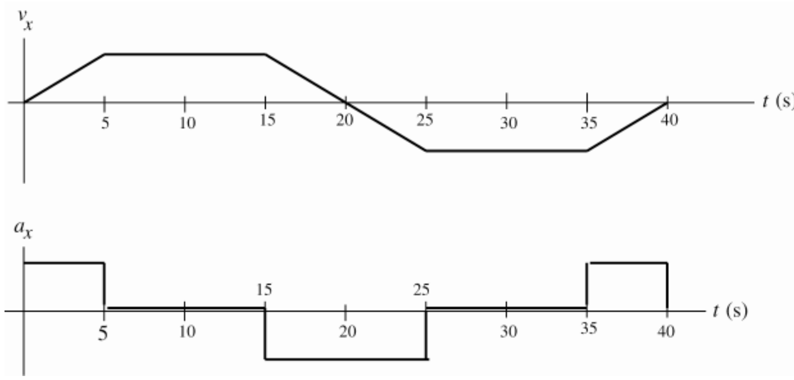


Figure 2.19a

(b) The motions diagrams are sketched in Figure 2.19b.

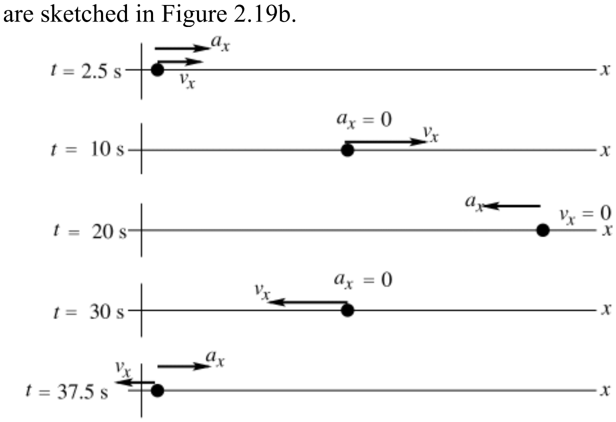


Figure 2.19b

EVALUATE: The spider speeds up for the first 5 s, since v_x and a_x are both positive. Starting at $t = 15$ s the spider starts to slow down, stops momentarily at $t = 20$ s, and then moves in the opposite direction. At $t = 35$ s the spider starts to slow down again and stops at $t = 40$ s.

2.20. IDENTIFY: $v_x(t) = \dfrac{dx}{dt}$ and $a_x(t) = \dfrac{dv_x}{dt}$

SET UP: $\dfrac{d}{dt}(t^n) = nt^{n-1}$ for $n \geq 1$.

EXECUTE: **(a)** $v_x(t) = (9.60\text{ m/s}^2)t - (0.600\text{ m/s}^6)t^5$ and $a_x(t) = 9.60\text{ m/s}^2 - (3.00\text{ m/s}^6)t^4$. Setting $v_x = 0$ gives $t = 0$ and $t = 2.00$ s. At $t = 0$, $x = 2.17$ m and $a_x = 9.60\text{ m/s}^2$. At $t = 2.00$ s, $x = 15.0$ m and $a_x = -38.4\text{ m/s}^2$.

(b) The graphs are given in Figure 2.20.

EVALUATE: For the entire time interval from $t = 0$ to $t = 2.00$ s, the velocity v_x is positive and x increases. While a_x is also positive the speed increases and while a_x is negative the speed decreases.

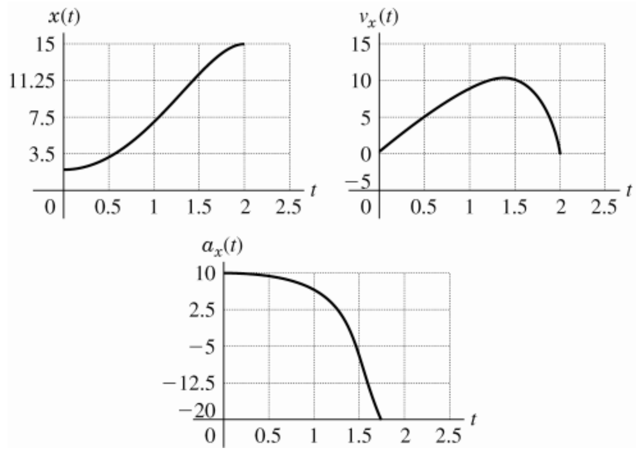
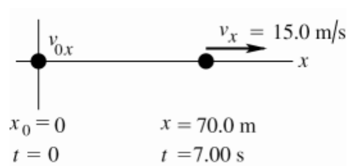


Figure 2.20

2.21. **IDENTIFY:** Use the constant acceleration equations to find v_{0x} and a_x.

(a) SET UP: The situation is sketched in Figure 2.21.



$x - x_0 = 70.0$ m
$t = 7.00$ s
$v_x = 15.0$ m/s
$v_{0x} = ?$

Figure 2.21

EXECUTE: Use $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$, so $v_{0x} = \frac{2(x - x_0)}{t} - v_x = \frac{2(70.0\text{ m})}{7.00\text{ s}} - 15.0\text{ m/s} = 5.0\text{ m/s}.$

(b) Use $v_x = v_{0x} + a_x t$, so $a_x = \frac{v_x - v_{0x}}{t} = \frac{15.0\text{ m/s} - 5.0\text{ m/s}}{7.00\text{ s}} = 1.43\text{ m/s}^2.$

EVALUATE: The average velocity is $(70.0\text{ m})/(7.00\text{ s}) = 10.0\text{ m/s}$. The final velocity is larger than this, so the antelope must be speeding up during the time interval; $v_{0x} < v_x$ and $a_x > 0$.

2.22. **IDENTIFY:** Apply the constant acceleration kinematic equations.

SET UP: Let $+x$ be in the direction of the motion of the plane. $173\text{ mi/h} = 77.33\text{ m/s}$. $307\text{ ft} = 93.57\text{ m}$.

EXECUTE: **(a)** $v_{0x} = 0$, $v_x = 77.33$ m/s and $x - x_0 = 93.57$ m. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(77.33\text{ m/s})^2 - 0}{2(93.57\text{ m})} = 32.0\text{ m/s}^2.$$

(b) $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$ gives $t = \frac{2(x - x_0)}{v_{0x} + v_x} = \frac{2(93.57\text{ m})}{0 + 77.33\text{ m/s}} = 2.42\text{ s}$

EVALUATE: Either $v_x = v_{0x} + a_x t$ or $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ could also be used to find t and would give the same result as in part (b).

2.23. **IDENTIFY:** For constant acceleration, Eqs. (2.8), (2.12), (2.13) and (2.14) apply.

SET UP: Assume the ball starts from rest and moves in the $+x$-direction.

EXECUTE: **(a)** $x - x_0 = 1.50$ m, $v_x = 45.0$ m/s and $v_{0x} = 0$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(45.0\text{ m/s})^2}{2(1.50\text{ m})} = 675\text{ m/s}^2.$$

(b) $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t$ gives $t = \frac{2(x - x_0)}{v_{0x} + v_x} = \frac{2(1.50\text{ m})}{45.0\text{ m/s}} = 0.0667\text{ s}$

EVALUATE: We could also use $v_x = v_{0x} + a_x t$ to find $t = \frac{v_x}{a_x} = \frac{45.0\text{ m/s}}{675\text{ m/s}^2} = 0.0667$ s which agrees with our previous result. The acceleration of the ball is very large.

2.24. **IDENTIFY:** For constant acceleration, Eqs. (2.8), (2.12), (2.13) and (2.14) apply.

SET UP: Assume the ball moves in the $+x$ direction.

EXECUTE: **(a)** $v_x = 73.14\text{ m/s}$, $v_{0x} = 0$ and $t = 30.0\text{ ms}$. $v_x = v_{0x} + a_x t$ gives

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{73.14\text{ m/s} - 0}{30.0\times 10^{-3}\text{ s}} = 2440\text{ m/s}^2.$$

(b) $x - x_0 = \left(\dfrac{v_{0x}+v_x}{2}\right)t = \left(\dfrac{0+73.14\text{ m/s}}{2}\right)(30.0\times 10^{-3}\text{ s}) = 1.10\text{ m}$

EVALUATE: We could also use $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to calculate $x - x_0$:

$x - x_0 = \frac{1}{2}(2440\text{ m/s}^2)(30.0\times 10^{-3}\text{ s})^2 = 1.10\text{ m}$, which agrees with our previous result. The acceleration of the ball is very large.

2.25. **IDENTIFY:** Assume that the acceleration is constant and apply the constant acceleration kinematic equations. Set $|a_x|$ equal to its maximum allowed value.

SET UP: Let $+x$ be the direction of the initial velocity of the car. $a_x = -250\text{ m/s}^2$. $105\text{ km/h} = 29.17\text{ m/s}$.

EXECUTE: $v_{0x} = +29.17\text{ m/s}$. $v_x = 0$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = \dfrac{0-(29.17\text{ m/s})^2}{2(-250\text{ m/s}^2)} = 1.70\text{ m}$.

EVALUATE: The car frame stops over a shorter distance and has a larger magnitude of acceleration. Part of your 1.70 m stopping distance is the stopping distance of the car and part is how far you move relative to the car while stopping.

2.26. **IDENTIFY:** Apply constant acceleration equations to the motion of the car.

SET UP: Let $+x$ be the direction the car is moving.

EXECUTE: **(a)** From Eq. (2.13), with $v_{0x} = 0$, $a_x = \dfrac{v_x^2}{2(x-x_0)} = \dfrac{(20\text{ m/s})^2}{2(120\text{ m})} = 1.67\text{ m/s}^2$.

(b) Using Eq. (2.14), $t = 2(x - x_0)/v_x = 2(120\text{ m})/(20\text{ m/s}) = 12\text{ s}$.

(c) $(12\text{ s})(20\text{ m/s}) = 240\text{ m}$.

EVALUATE: The average velocity of the car is half the constant speed of the traffic, so the traffic travels twice as far.

2.27. **IDENTIFY:** The average acceleration is $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$. For constant acceleration, Eqs. (2.8), (2.12), (2.13) and (2.14) apply.

SET UP: Assume the shuttle travels in the $+x$ direction. $161\text{ km/h} = 44.72\text{ m/s}$ and $1610\text{ km/h} = 447.2\text{ m/s}$. $1.00\text{ min} = 60.0\text{ s}$

EXECUTE: **(a)** (i) $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{44.72\text{ m/s} - 0}{8.00\text{ s}} = 5.59\text{ m/s}^2$

(ii) $a_{\text{av-}x} = \dfrac{447.2\text{ m/s} - 44.72\text{ m/s}}{60.0\text{ s} - 8.00\text{ s}} = 7.74\text{ m/s}^2$

(b) (i) $t = 8.00\text{ s}$, $v_{0x} = 0$, and $v_x = 44.72\text{ m/s}$. $x - x_0 = \left(\dfrac{v_{0x}+v_x}{2}\right)t = \left(\dfrac{0+44.72\text{ m/s}}{2}\right)(8.00\text{ s}) = 179\text{ m}$.

(ii) $\Delta t = 60.0\text{ s} - 8.00\text{ s} = 52.0\text{ s}$, $v_{0x} = 44.72\text{ m/s}$, and $v_x = 447.2\text{ m/s}$.

$$x - x_0 = \left(\frac{v_{0x}+v_x}{2}\right)t = \left(\frac{44.72\text{ m/s} + 447.2\text{ m/s}}{2}\right)(52.0\text{ s}) = 1.28\times 10^4\text{ m}.$$

EVALUATE: When the acceleration is constant the instantaneous acceleration throughout the time interval equals the average acceleration for that time interval. We could have calculated the distance in part (a) as $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = \frac{1}{2}(5.59\text{ m/s}^2)(8.00\text{ s})^2 = 179\text{ m}$, which agrees with our previous calculation.

2.28. **IDENTIFY:** Apply the constant acceleration kinematic equations to the motion of the car.

SET UP: $0.250\text{ mi} = 1320\text{ ft}$. $60.0\text{ mph} = 88.0\text{ ft/s}$. Let $+x$ be the direction the car is traveling.

EXECUTE: **(a)** braking: $v_{0x} = 88.0\text{ ft/s}$, $x - x_0 = 146\text{ ft}$, $v_x = 0$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{0 - (88.0\text{ ft/s})^2}{2(146\text{ ft})} = -26.5\text{ ft/s}^2$$

Speeding up: $v_{0x} = 0$, $x - x_0 = 1320\text{ ft}$, $t = 19.9\text{ s}$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$$a_x = \frac{2(x - x_0)}{t^2} = \frac{2(1320\text{ ft})}{(19.9\text{ s})^2} = 6.67\text{ ft/s}^2$$

(b) $v_x = v_{0x} + a_x t = 0 + (6.67\ \text{ft/s}^2)(19.9\ \text{s}) = 133\ \text{ft/s} = 90.5\ \text{mph}$

(c) $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{0 - 88.0\ \text{ft/s}}{-26.5\ \text{ft/s}^2} = 3.32\ \text{s}$

EVALUATE: The magnitude of the acceleration while braking is much larger than when speeding up. That is why it takes much longer to go from 0 to 60 mph than to go from 60 mph to 0.

2.29. **IDENTIFY:** The acceleration a_x is the slope of the graph of v_x versus *t*.

SET UP: The signs of v_x and of a_x indicate their directions.

EXECUTE: **(a)** Reading from the graph, at $t = 4.0\ \text{s}$, $v_x = 2.7\ \text{cm/s}$, to the right and at $t = 7.0\ \text{s}$, $v_x = 1.3\ \text{cm/s}$, to the left.

(b) v_x versus *t* is a straight line with slope $-\dfrac{8.0\ \text{cm/s}}{6.0\ \text{s}} = -1.3\ \text{cm/s}^2$. The acceleration is constant and equal to $1.3\ \text{cm/s}^2$, to the left. It has this value at all times.

(c) Since the acceleration is constant, $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$. For $t = 0$ to 4.5 s,

$x - x_0 = (8.0\ \text{cm/s})(4.5\ \text{s}) + \frac{1}{2}(-1.3\ \text{cm/s}^2)(4.5\ \text{s})^2 = 22.8\ \text{cm}$. For $t = 0$ to 7.5 s,

$x - x_0 = (8.0\ \text{cm/s})(7.5\ \text{s}) + \frac{1}{2}(-1.3\ \text{cm/s}^2)(7.5\ \text{s})^2 = 23.4\ \text{cm}$

(d) The graphs of a_x and *x* versus *t* are given in Fig. 2.29.

EVALUATE: In part (c) we could have instead used $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t$.

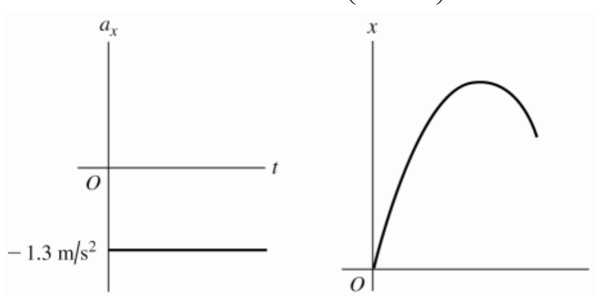


Figure 2.29

2.30. **IDENTIFY:** Use the constant acceleration equations to find *x*, v_{0x}, v_x and a_x for each constant-acceleration segment of the motion.

SET UP: Let $+x$ be the direction of motion of the car and let $x = 0$ at the first traffic light.

EXECUTE: **(a)** For $t = 0$ to $t = 8\ \text{s}$: $x = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{0 + 20\ \text{m/s}}{2}\right)(8\ \text{s}) = 80\ \text{m}$.

$a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{20\ \text{m/s}}{8\ \text{s}} = +2.50\ \text{m/s}^2$. The car moves from $x = 0$ to $x = 80\ \text{m}$. The velocity v_x increases linearly from zero to 20 m/s. The acceleration is a constant $2.50\ \text{m/s}^2$.

Constant speed for 60 m: The car moves from $x = 80\ \text{m}$ to $x = 140\ \text{m}$. v_x is a constant 20 m/s. $a_x = 0$. This interval starts at $t = 8\ \text{s}$ and continues until $t = \dfrac{60\ \text{m}}{20\ \text{m/s}} + 8\ \text{s} = 11\ \text{s}$.

Slowing from 20 m/s until stopped: The car moves from $x = 140\ \text{m}$ to $x = 180\ \text{m}$. The velocity decreases linearly from 20 m/s to zero. $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t$ gives $t = \dfrac{2(40\ \text{m})}{20\ \text{m/s} + 0} = 4\ \text{s}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$a_x = \dfrac{-(20.0\ \text{m/s})^2}{2(40\ \text{m})} = -5.00\ \text{m/s}^2$ This segment is from $t = 11\ \text{s}$ to $t = 15\ \text{s}$. The acceleration is a constant $-5.00\ \text{m/s}^2$.

The graphs are drawn in Figure 2.30a.

(b) The motion diagram is sketched in Figure 2.30b.

EVALUATE: When $\vec{a}$ and $\vec{v}$ are in the same direction, the speed increases ($t = 0$ to $t = 8$ s). When $\vec{a}$ and $\vec{v}$ are in opposite directions, the speed decreases ($t = 11$ s to $t = 15$ s). When $a = 0$ the speed is constant $t = 8$ s to $t = 11$ s.

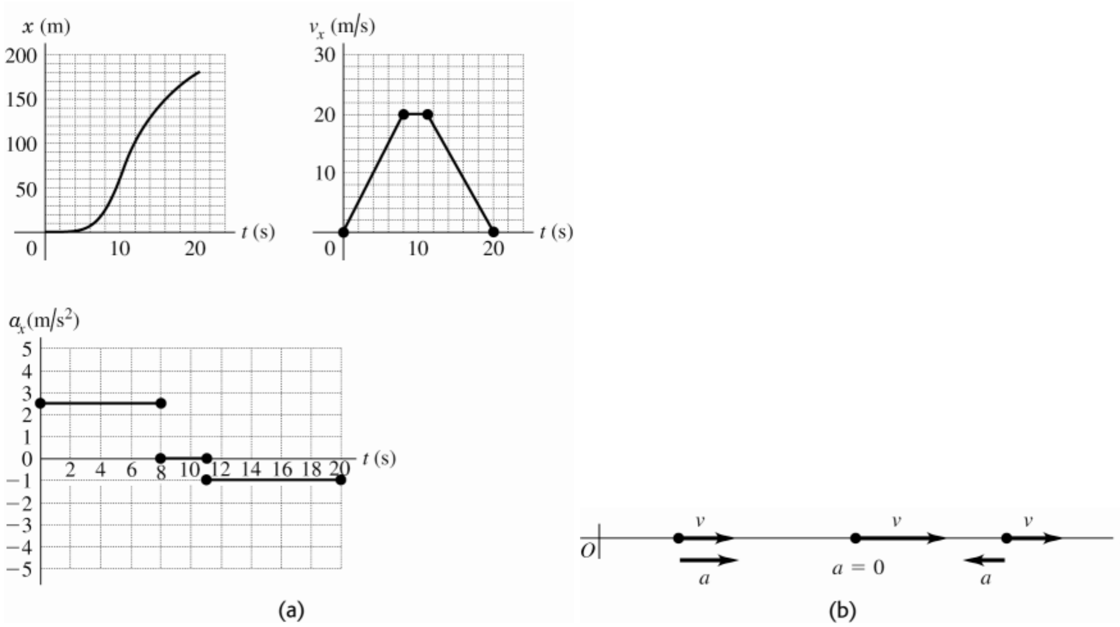


Figure 2.30a-b

2.31. **(a) IDENTIFY** and **SET UP:** The acceleration a_x at time t is the slope of the tangent to the v_x versus t curve at time t.

EXECUTE: At $t = 3$ s, the v_x versus t curve is a horizontal straight line, with zero slope. Thus $a_x = 0$.

At $t = 7$ s, the v_x versus t curve is a straight-line segment with slope $\dfrac{45\text{ m/s} - 20\text{ m/s}}{9\text{ s} - 5\text{ s}} = 6.3\text{ m/s}^2$.

Thus $a_x = 6.3\text{ m/s}^2$.

At $t = 11$ s the curve is again a straight-line segment, now with slope $\dfrac{-0 - 45\text{ m/s}}{13\text{ s} - 9\text{ s}} = -11.2\text{ m/s}^2$.

Thus $a_x = -11.2\text{ m/s}^2$.

EVALUATE: $a_x = 0$ when v_x is constant, $a_x > 0$ when v_x is positive and the speed is increasing, and $a_x < 0$ when v_x is positive and the speed is decreasing.

(b) IDENTIFY: Calculate the displacement during the specified time interval.

SET UP: We can use the constant acceleration equations only for time intervals during which the acceleration is constant. If necessary, break the motion up into constant acceleration segments and apply the constant acceleration equations for each segment. For the time interval $t = 0$ to $t = 5$ s the acceleration is constant and equal to zero. For the time interval $t = 5$ s to $t = 9$ s the acceleration is constant and equal to 6.25 m/s^2. For the interval $t = 9$ s to $t = 13$ s the acceleration is constant and equal to -11.2 m/s^2.

EXECUTE: During the first 5 seconds the acceleration is constant, so the constant acceleration kinematic formulas can be used.

$v_{0x} = 20\text{ m/s}$ $\quad a_x = 0$ $\quad t = 5\text{ s}$ $\quad x - x_0 = ?$

$x - x_0 = v_{0x}t$ $\quad (a_x = 0$ so no $\tfrac{1}{2}a_xt^2$ term)

$x - x_0 = (20\text{ m/s})(5\text{ s}) = 100\text{ m}$; this is the distance the officer travels in the first 5 seconds.

During the interval $t = 5$ s to 9 s the acceleration is again constant. The constant acceleration formulas can be applied to this 4 second interval. It is convenient to restart our clock so the interval starts at time $t = 0$ and ends at time $t = 5$ s. (Note that the acceleration is *not* constant over the entire $t = 0$ to $t = 9$ s interval.)

$v_{0x} = 20\text{ m/s}$ $\quad a_x = 6.25\text{ m/s}^2$ $\quad t = 4\text{ s}$ $\quad x_0 = 100\text{ m}$ $\quad x - x_0 = ?$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_xt^2$

$x - x_0 = (20\text{ m/s})(4\text{ s}) + \tfrac{1}{2}(6.25\text{ m/s}^2)(4\text{ s})^2 = 80\text{ m} + 50\text{ m} = 130\text{ m}.$

Thus $x - x_0 + 130\text{ m} = 100\text{ m} + 130\text{ m} = 230\text{ m}.$

At $t=9$ s the officer is at $x=230$ m, so she has traveled 230 m in the first 9 seconds.

During the interval $t=9$ s to $t=13$ s the acceleration is again constant. The constant acceleration formulas can be applied for this 4 second interval but *not* for the whole $t=0$ to $t=13$ s interval. To use the equations restart our clock so this interval begins at time $t=0$ and ends at time $t=4$ s.

$v_{0x}=45$ m/s (at the start of this time interval)

$a_x=-11.2\text{ m/s}^2$ $t=4$ s $x_0=230$ m $x-x_0=?$

$x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$

$x-x_0=(45\text{ m/s})(4\text{ s})+\frac{1}{2}(-11.2\text{ m/s}^2)(4\text{ s})^2=180\text{ m}-89.6\text{ m}=90.4\text{ m}.$

Thus $x=x_0+90.4\text{ m}=230\text{ m}+90.4\text{ m}=320\text{ m}.$

At $t=13$ s the officer is at $x=320$ m, so she has traveled 320 m in the first 13 seconds.

EVALUATE: The velocity v_x is always positive so the displacement is always positive and displacement and distance traveled are the same. The average velocity for time interval Δt is $v_{\text{av-}x}=\Delta x/\Delta t.$ For $t=0$ to 5 s, $v_{\text{av-}x}=20$ m/s. For $t=0$ to 9 s, $v_{\text{av-}x}=26$ m/s. For $t=0$ to 13 s, $v_{\text{av-}x}=25$ m/s. These results are consistent with Fig. 2.33.

2.32. **IDENTIFY:** In each constant acceleration interval, the constant acceleration equations apply.

SET UP: When a_x is constant, the graph of v_x versus t is a straight line and the graph of x versus t is a parabola. When $a_x=0$, v_x is constant and x versus t is a straight line.

EXECUTE: The graphs are given in Figure 2.32.

EVALUATE: The slope of the x versus t graph is $v_x(t)$ and the slope of the v_x versus t graph is $a_x(t)$.

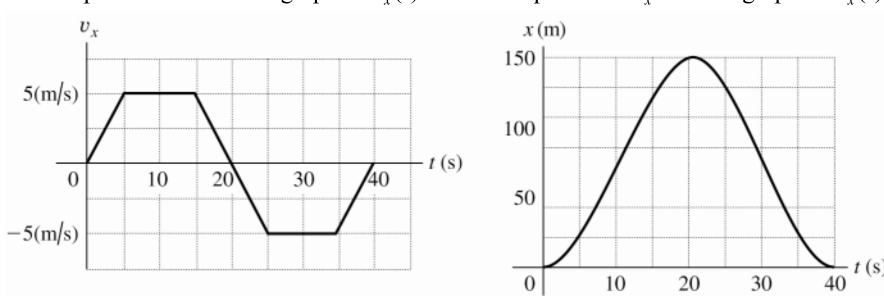


Figure 2.32

2.33. **(a) IDENTIFY:** The maximum speed occurs at the end of the initial acceleration period.

SET UP: $a_x=20.0\text{ m/s}^2$ $t=15.0\text{ min}=900$ s $v_{0x}=0$ $v_x=?$

$v_x=v_{0x}+a_xt$

EXECUTE: $v_x=0+(20.0\text{ m/s}^2)(900\text{ s})=1.80\times10^4\text{ m/s}$

(b) IDENTIFY: Use constant acceleration formulas to find the displacement Δx. The motion consists of three constant acceleration intervals. In the middle segment of the trip $a_x=0$ and $v_x=1.80\times10^4$ m/s, but we can't directly find the distance traveled during this part of the trip because we don't know the time. Instead, find the distance traveled in the first part of the trip (where $a_x=+20.0\text{ m/s}^2$) and in the last part of the trip (where $a_x=-20.0\text{ m/s}^2$). Subtract these two distances from the total distance of 3.84×10^8 m to find the distance traveled in the middle part of the trip (where $a_x=0$).

<u>first segment</u>

SET UP: $x-x_0=?$ $t=15.0\text{ min}=900$ s $a_x=+20.0\text{ m/s}^2$ $v_{0x}=0$

$x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$

EXECUTE: $x-x_0=0+\frac{1}{2}(20.0\text{ m/s}^2)(900\text{ s})^2=8.10\times10^6\text{ m}=8.10\times10^3\text{ km}$

<u>second segment</u>

SET UP: $x-x_0=?$ $t=15.0\text{ min}=900$ s $a_x=-20.0\text{ m/s}^2$

$v_{0x}=1.80\times10^4$ m/s

$x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$

EXECUTE: $x-x_0=(1.80\times10^4\text{ s})(900\text{ s})+\frac{1}{2}(-20.0\text{ m/s}^2)(900\text{ s})^2=8.10\times10^6\text{ m}=8.10\times10^3\text{ km}$ (The same distance as traveled as in the first segment.)

Therefore, the distance traveled at constant speed is
$3.84\times10^8\ \text{m} - 8.10\times10^6\ \text{m} - 8.10\times10^6\ \text{m} = 3.678\times10^8\ \text{m} = 3.678\times10^5\ \text{km}$.

The fraction this is of the total distance is $\dfrac{3.678\times10^8\ \text{m}}{3.84\times10^8\ \text{m}} = 0.958$.

(c) IDENTIFY: We know the time for each acceleration period, so find the time for the constant speed segment.

SET UP: $x - x_0 = 3.678\times10^8\ \text{m}$ $v_x = 1.80\times10^4\ \text{m/s}$ $a_x = 0$ $t = ?$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$

EXECUTE: $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{3.678\times10^8\ \text{m}}{1.80\times10^4\ \text{m/s}} = 2.043\times10^4\ \text{s} = 340.5\ \text{min}$.

The total time for the whole trip is thus $15.0\ \text{min} + 340.5\ \text{min} + 15.0\ \text{min} = 370\,\text{min}$.

EVALUATE: If the speed was a constant $1.80\times10^4\ \text{m/s}$ for the entire trip, the trip would take $(3.84\times10^8\ \text{m})/(1.80\times10^4\ \text{m/s}) = 356\ \text{min}$. The trip actually takes a bit longer than this since the average velocity is less than $1.80\times10^8\ \text{m/s}$ during the relatively brief acceleration phases.

2.34. **IDENTIFY:** Use constant acceleration equations to find $x - x_0$ for each segment of the motion.

SET UP: Let $+x$ be the direction the train is traveling.

EXECUTE: $t = 0$ to 14.0 s: $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}(1.60\ \text{m/s}^2)(14.0\ \text{s})^2 = 157\ \text{m}$.

At $t = 14.0\ \text{s}$, the speed is $v_x = v_{0x} + a_x t = (1.60\ \text{m/s}^2)(14.0\ \text{s}) = 22.4\ \text{m/s}$. In the next 70.0 s, $a_x = 0$ and $x - x_0 = v_{0x}t = (22.4\ \text{m/s})(70.0\ \text{s}) = 1568\ \text{m}$.

For the interval during which the train is slowing down, $v_{0x} = 22.4\ \text{m/s}$, $a_x = -3.50\ \text{m/s}^2$ and $v_x = 0$.

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = \dfrac{0 - (22.4\ \text{m/s})^2}{2(-3.50\ \text{m/s}^2)} = 72\ \text{m}$.

The total distance traveled is $157\ \text{m} + 1568\ \text{m} + 72\ \text{m} = 1800\ \text{m}$.

EVALUATE: The acceleration is not constant for the entire motion but it does consist of constant acceleration segments and we can use constant acceleration equations for each segment.

2.35 **IDENTIFY:** $v_x(t)$ is the slope of the *x* versus *t* graph. Car *B* moves with constant speed and zero acceleration. Car *A* moves with positive acceleration; assume the acceleration is constant.

SET UP: For car *B*, v_x is positive and $a_x = 0$. For car *A*, a_x is positive and v_x increases with *t*.

EXECUTE: **(a)** The motion diagrams for the cars are given in Figure 2.35a.

(b) The two cars have the same position at times when their *x*-*t* graphs cross. The figure in the problem shows this occurs at approximately $t = 1\ \text{s}$ and $t = 3\ \text{s}$.

(c) The graphs of v_x versus *t* for each car are sketched in Figure 2.35b.

(d) The cars have the same velocity when their *x*-*t* graphs have the same slope. This occurs at approximately $t = 2\ \text{s}$.

(e) Car *A* passes car *B* when x_A moves above x_B in the *x*-*t* graph. This happens at $t = 3\ \text{s}$.

(f) Car *B* passes car *A* when x_B moves above x_A in the *x*-*t* graph. This happens at $t = 1\ \text{s}$.

EVALUATE: When $a_x = 0$, the graph of v_x versus *t* is a horizontal line. When a_x is positive, the graph of v_x versus *t* is a straight line with positive slope.

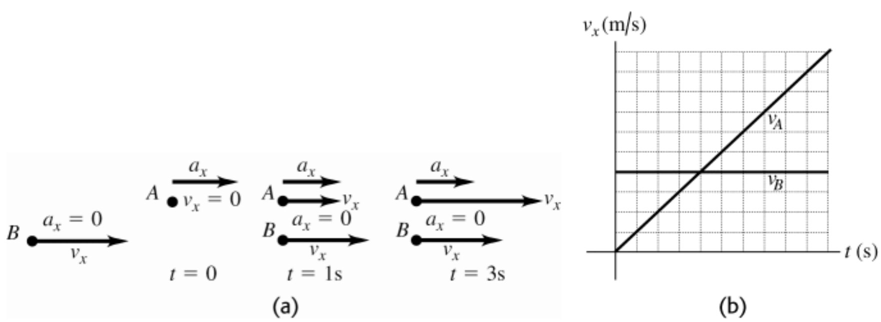


Figure 2.35a-b

2.36. **IDENTIFY:** Apply the constant acceleration equations to the motion of each vehicle. The truck passes the car when they are at the same *x* at the same $t > 0$.

SET UP: The truck has $a_x = 0$. The car has $v_{0x} = 0$. Let $+x$ be in the direction of motion of the vehicles. Both vehicles start at $x_0 = 0$. The car has $a_C = 3.20 \text{ m/s}^2$. The truck has $v_x = 20.0 \text{ m/s}$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x_T = v_{0T}t$ and $x_C = \frac{1}{2}a_C t^2$. Setting $x_T = x_C$ gives $t = 0$ and $v_{0T} = \frac{1}{2}a_C t$, so $t = \dfrac{2v_{0T}}{a_C} = \dfrac{2(20.0 \text{ m/s})}{3.20 \text{ m/s}^2} = 12.5 \text{ s}$. At this t, $x_T = (20.0 \text{ m/s})(12.5 \text{ s}) = 250 \text{ m}$ and $x = \frac{1}{2}(3.20 \text{ m/s}^2)(12.5 \text{ s})^2 = 250 \text{ m}$.

The car and truck have each traveled 250 m.

(b) At $t = 12.5 \text{ s}$, the car has $v_x = v_{0x} + a_x t = (3.20 \text{ m/s}^2)(12.5 \text{ s}) = 40 \text{ m/s}$.

(c) $x_T = v_{0T}t$ and $x_C = \frac{1}{2}a_C t^2$. The *x*-*t* graph of the motion for each vehicle is sketched in Figure 2.36a.

(d) $v_T = v_{0T}$. $v_C = a_C t$. The v_x-*t* graph for each vehicle is sketched in Figure 2.36b.

EVALUATE: When the car overtakes the truck its speed is twice that of the truck.

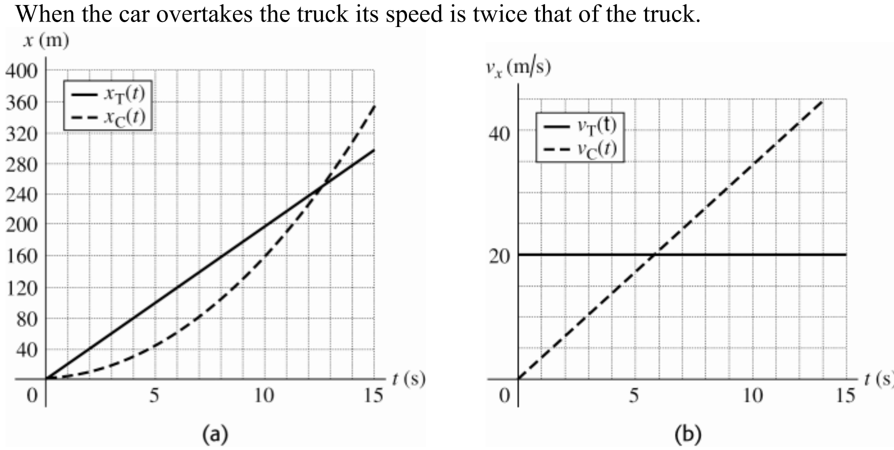


Figure 2.36a-b

2.37. IDENTIFY: For constant acceleration, Eqs. (2.8), (2.12), (2.13) and (2.14) apply.

SET UP: Take $+y$ to be downward, so the motion is in the $+y$ direction. $19{,}300 \text{ km/h} = 5361 \text{ m/s}$, $1600 \text{ km/h} = 444.4 \text{ m/s}$, and $321 \text{ km/h} = 89.2 \text{ m/s}$. $4.0 \text{ min} = 240 \text{ s}$.

EXECUTE: **(a)** Stage *A*: $t = 240 \text{ s}$, $v_{0y} = 5361 \text{ m/s}$, $v_y = 444.4 \text{ m/s}$. $v_y = v_{0y} + a_y t$ gives

$$a_y = \frac{v_y - v_{0y}}{t} = \frac{444.4 \text{ m/s} - 5361 \text{ m/s}}{240 \text{ s}} = -20.5 \text{ m/s}^2.$$

Stage *B*: $t = 94 \text{ s}$, $v_{0y} = 444.4 \text{ m/s}$, $v_y = 89.2 \text{ m/s}$. $v_y = v_{0y} + a_y t$ gives

$$a_y = \frac{v_y - v_{0y}}{t} = \frac{89.2 \text{ m/s} - 444.4 \text{ m/s}}{94 \text{ s}} = -3.8 \text{ m/s}^2.$$

Stage *C*: $y - y_0 = 75 \text{ m}$, $v_{0y} = 89.2 \text{ m/s}$, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$a_y = \dfrac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \dfrac{0 - (89.2 \text{ m/s})^2}{2(75 \text{ m})} = -53.0 \text{ m/s}^2$. In each case the negative sign means that the acceleration is upward.

(b) Stage *A*: $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t = \left(\dfrac{5361 \text{ m/s} + 444.4 \text{ m/s}}{2}\right)(240 \text{ s}) = 697 \text{ km}$.

Stage *B*: $y - y_0 = \left(\dfrac{444.4 \text{ m/s} + 89.2 \text{ m/s}}{2}\right)(94 \text{ s}) = 25 \text{ km}$.

Stage *C*: The problem states that $y - y_0 = 75 \text{ m} = 0.075 \text{ km}$.

The total distance traveled during all three stages is $697 \text{ km} + 25 \text{ km} + 0.075 \text{ km} = 722 \text{ km}$.

EVALUATE: The upward acceleration produced by friction in stage *A* is calculated to be greater than the upward acceleration due to the parachute in stage *B*. The effects of air resistance increase with increasing speed and in reality the acceleration was probably not constant during stages *A* and *B*.

2.38. IDENTIFY: Assume an initial height of 200 m and a constant acceleration of 9.80 m/s^2.

SET UP: Let $+y$ be downward. $1 \text{ km/h} = 0.2778 \text{ m/s}$ and $1 \text{ mi/h} = 0.4470 \text{ m/s}$.

EXECUTE: **(a)** $y - y_0 = 200\text{ m}$, $a_y = 9.80\text{ m/s}^2$, $v_{0y} = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = \sqrt{2(9.80\text{ m/s}^2)(200\text{ m})} = 60\text{ m/s} = 200\text{ km/h} = 140\text{ mi/h}$.

(b) Raindrops actually have a speed of about 1 m/s as they strike the ground.

(c) The actual speed at the ground is much less than the speed calculated assuming free-fall, so neglect of air resistance is a very poor approximation for falling raindrops.

EVALUATE: In the absence of air resistance raindrops would land with speeds that would make them very dangerous.

2.39. **IDENTIFY:** Apply the constant acceleration equations to the motion of the flea. After the flea leaves the ground, $a_y = g$, downward. Take the origin at the ground and the positive direction to be upward.

(a) SET UP: At the maximum height $v_y = 0$.

$v_y = 0 \quad y - y_0 = 0.440\text{ m} \quad a_y = -9.80\text{ m/s}^2 \quad v_{0y} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\text{ m/s}^2)(0.440\text{ m})} = 2.94\text{ m/s}$

(b) SET UP: When the flea has returned to the ground $y - y_0 = 0$.

$y - y_0 = 0 \quad v_{0y} = +2.94\text{ m/s} \quad a_y = -9.80\text{ m/s}^2 \quad t = ?$

$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$

EXECUTE: With $y - y_0 = 0$ this gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(2.94\text{ m/s})}{-9.80\text{ m/s}^2} = 0.600\text{ s}$.

EVALUATE: We can use $v_y = v_{0y} + a_y t$ to show that with $v_{0y} = 2.94\text{ m/s}$, $v_y = 0$ after 0.300 s.

2.40. **IDENTIFY:** Apply constant acceleration equations to the motion of the lander.

SET UP: Let $+y$ be positive. Since the lander is in free-fall, $a_y = +1.6\text{ m/s}^2$.

EXECUTE: $v_{0y} = 0.8\text{ m/s}$, $y - y_0 = 5.0\text{ m}$, $a_y = +1.6\text{ m/s}^2$ in $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = \sqrt{v_{0y}^2 + 2a_y(y - y_0)} = \sqrt{(0.8\text{ m/s})^2 + 2(1.6\text{ m/s}^2)(5.0\text{ m})} = 4.1\text{ m/s}$.

EVALUATE: The same descent on earth would result in a final speed of 9.9 m/s, since the acceleration due to gravity on earth is much larger than on the moon.

2.41. **IDENTIFY:** Apply constant acceleration equations to the motion of the meterstick. The time the meterstick falls is your reaction time.

SET UP: Let $+y$ be downward. The meter stick has $v_{0y} = 0$ and $a_y = 9.80\text{ m/s}^2$. Let d be the distance the meterstick falls.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $d = (4.90\text{ m/s}^2)t^2$ and $t = \sqrt{\dfrac{d}{4.90\text{ m/s}^2}}$.

(b) $t = \sqrt{\dfrac{0.176\text{ m}}{4.90\text{ m/s}^2}} = 0.190\text{ s}$

EVALUATE: The reaction time is proportional to the square of the distance the stick falls.

2.42. **IDENTIFY:** Apply constant acceleration equations to the vertical motion of the brick.

SET UP: Let $+y$ be downward. $a_y = 9.80\text{ m/s}^2$

EXECUTE: **(a)** $v_{0y} = 0$, $t = 2.50\text{ s}$, $a_y = 9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}(9.80\text{ m/s}^2)(2.50\text{ s})^2 = 30.6\text{ m}$. The building is 30.6 m tall.

(b) $v_y = v_{0y} + a_y t = 0 + (9.80\text{ m/s}^2)(2.50\text{ s}) = 24.5\text{ m/s}$

(c) The graphs of a_y, v_y and y versus t are given in Fig. 2.42. Take $y = 0$ at the ground.

EVALUATE: We could use either $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$ or $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ to check our results.

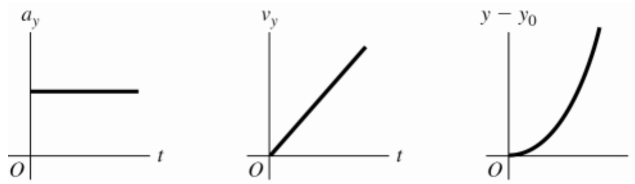


Figure 2.42

2.43. **IDENTIFY:** When the only force is gravity the acceleration is $9.80\ \text{m/s}^2$, downward. There are two intervals of constant acceleration and the constant acceleration equations apply during each of these intervals.

SET UP: Let $+y$ be upward. Let $y = 0$ at the launch pad. The final velocity for the first phase of the motion is the initial velocity for the free-fall phase.

EXECUTE: **(a)** Find the velocity when the engines cut off. $y - y_0 = 525\ \text{m}$, $a_y = +2.25\ \text{m/s}^2$, $v_{0y} = 0$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(2.25\ \text{m/s}^2)(525\ \text{m})} = 48.6\ \text{m/s}$.

Now consider the motion from engine cut off to maximum height: $y_0 = 525\ \text{m}$, $v_{0y} = +48.6\ \text{m/s}$, $v_y = 0$ (at the maximum height), $a_y = -9.80\ \text{m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (48.6\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 121\ \text{m}$ and

$y = 121\ \text{m} + 525\ \text{m} = 646\ \text{m}$.

(b) Consider the motion from engine failure until just before the rocket strikes the ground: $y - y_0 = -525\ \text{m}$, $a_y = -9.80\ \text{m/s}^2$, $v_{0y} = +48.6\ \text{m/s}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$v_y = -\sqrt{(48.6\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(-525\ \text{m})} = -112\ \text{m/s}$. Then $v_y = v_{0y} + a_y t$ gives

$t = \frac{v_y - v_{0y}}{a_y} = \frac{-112\ \text{m/s} - 48.6\ \text{m/s}}{-9.80\ \text{m/s}^2} = 16.4\ \text{s}$.

(c) Find the time from blast-off until engine failure: $y - y_0 = 525\ \text{m}$, $v_{0y} = 0$, $a_y = +2.25\ \text{m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y - y_0)}{a_y}} = \sqrt{\frac{2(525\ \text{m})}{2.25\ \text{m/s}^2}} = 21.6\ \text{s}$. The rocket strikes the launch pad $21.6\ \text{s} + 16.4\ \text{s} = 38.0\ \text{s}$ after blast off. The acceleration a_y is $+2.25\ \text{m/s}^2$ from $t = 0$ to $t = 21.6\ \text{s}$. It is $-9.80\ \text{m/s}^2$ from $t = 21.6\ \text{s}$ to $38.0\ \text{s}$. $v_y = v_{0y} + a_y t$ applies during each constant acceleration segment, so the graph of v_y versus t is a straight line with positive slope of $2.25\ \text{m/s}^2$ during the blast-off phase and with negative slope of $-9.80\ \text{m/s}^2$ after engine failure. During each phase $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. The sign of a_y determines the curvature of $y(t)$. At $t = 38.0\ \text{s}$ the rocket has returned to $y = 0$. The graphs are sketched in Figure 2.43.

EVALUATE: In part (b) we could have found the time from $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$, finding v_y first allows us to avoid solving for t from a quadratic equation.

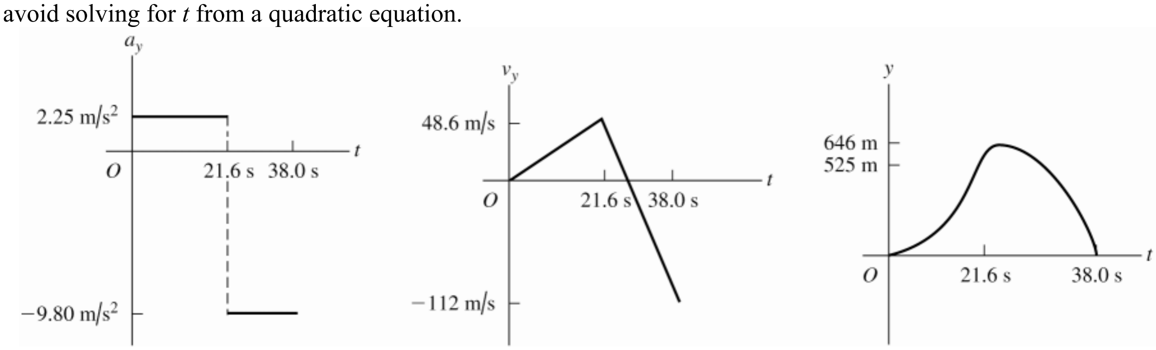


Figure 2.43

2.44. **IDENTIFY:** Apply constant acceleration equations to the vertical motion of the sandbag.

SET UP: Take $+y$ upward. $a_y = -9.80 \text{ m/s}^2$. The initial velocity of the sandbag equals the velocity of the balloon, so $v_{0y} = +5.00 \text{ m/s}$. When the balloon reaches the ground, $y - y_0 = -40.0 \text{ m}$. At its maximum height the sandbag has $v_y = 0$.

EXECUTE: **(a)** $t = 0.250 \text{ s}$: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (5.00 \text{ m/s})(0.250 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.250 \text{ s})^2 = 0.94 \text{ m}$. The sandbag is 40.9 m above the ground. $v_y = v_{0y} + a_y t = +5.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(0.250 \text{ s}) = 2.55 \text{ m/s}$.

$t = 1.00 \text{ s}$: $y - y_0 = (5.00 \text{ m/s})(1.00 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(1.00 \text{ s})^2 = 0.10 \text{ m}$. The sandbag is 40.1 m above the ground. $v_y = v_{0y} + a_y t = +5.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(1.00 \text{ s}) = -4.80 \text{ m/s}$.

(b) $y - y_0 = -40.0 \text{ m}$, $v_{0y} = 5.00 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $-40.0 \text{ m} = (5.00 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$. $(4.90 \text{ m/s}^2)t^2 - (5.00 \text{ m/s})t - 40.0 \text{ m} = 0$ and

$t = \frac{1}{9.80}\left(5.00 \pm \sqrt{(-5.00)^2 - 4(4.90)(-40.0)}\right) \text{ s} = (0.51 \pm 2.90) \text{ s}$. t must be positive, so $t = 3.41 \text{ s}$.

(c) $v_y = v_{0y} + a_y t = +5.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(3.41 \text{ s}) = -28.4 \text{ m/s}$

(d) $v_{0y} = 5.00 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (5.00 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 1.28 \text{ m}$. The maximum height is 41.3 m above the ground.

(e) The graphs of a_y, v_y, and y versus t are given in Fig. 2.44. Take $y = 0$ at the ground.

EVALUATE: The sandbag initially travels upward with decreasing velocity and then moves downward with increasing speed.

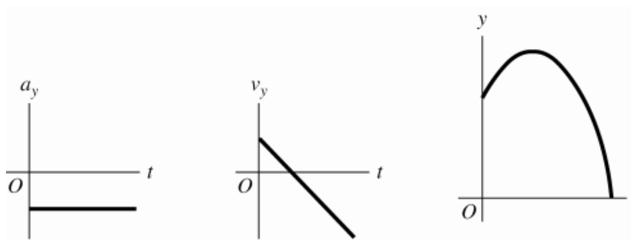


Figure 2.44

2.45. **IDENTIFY:** The balloon has constant acceleration $a_y = g$, downward.

(a) SET UP: Take the $+y$ direction to be upward.

$t = 2.00 \text{ s}$, $v_{0y} = -6.00 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$, $v_y = ?$

EXECUTE: $v_y = v_{0y} + a_y t = -6.00 \text{ m/s} + (-9.80 \text{ m/s}^2)(2.00 \text{ s}) = -25.5 \text{ m/s}$

(b) SET UP: $y - y_0 = ?$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (-6.00 \text{ m/s})(2.00 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(2.00 \text{ s})^2 = -31.6 \text{ m}$

(c) SET UP: $y - y_0 = -10.0 \text{ m}$, $v_{0y} = -6.00 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$, $v_y = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)} = -\sqrt{(-6.00 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-10.0 \text{ m})} = -15.2 \text{ m/s}$

(d) The graphs are sketched in Figure 2.45.

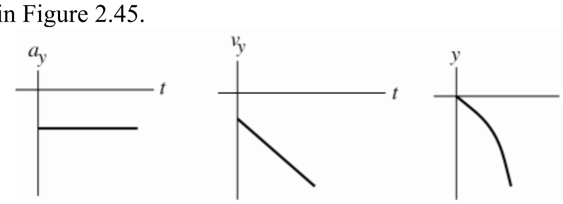


Figure 2.45

EVALUATE: The speed of the balloon increases steadily since the acceleration and velocity are in the same direction. $|v_y| = 25.5 \text{ m/s}$ when $|y - y_0| = 31.6 \text{ m}$, so $|v_y|$ is less than this (15.2 m/s) when $|y - y_0|$ is less (10.0 m).

2.46. **IDENTIFY:** Since air resistance is ignored, the egg is in free-fall and has a constant downward acceleration of magnitude 9.80 m/s^2. Apply the constant acceleration equations to the motion of the egg.

SET UP: Take $+y$ to be upward. At the maximum height, $v_y = 0$.

EXECUTE: **(a)** $y - y_0 = -50.0 \text{ m}$, $t = 5.00 \text{ s}$, $a_y = -9.80 \text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$v_{0y} = \frac{y - y_0}{t} - \tfrac{1}{2}a_y t = \frac{-50.0 \text{ m}}{5.00 \text{ s}} - \tfrac{1}{2}(-9.80 \text{ m/s}^2)(5.00 \text{ s}) = +14.5 \text{ m/s}.$$

(b) $v_{0y} = +14.5 \text{ m/s}$, $v_y = 0$ (at the maximum height), $a_y = -9.80 \text{ m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (14.5 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 10.7 \text{ m}.$$

(c) At the maximum height $v_y = 0$.

(d) The acceleration is constant and equal to 9.80 m/s^2, downward, at all points in the motion, including at the maximum height.

(e) The graphs are sketched in Figure 2.46.

EVALUATE: The time for the egg to reach its maximum height is $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-14.5 \text{ m/s}}{-9.8 \text{ m/s}^2} = 1.48 \text{ s}$. The egg has returned to the level of the cornice after 2.96 s and after 5.00 s it has traveled downward from the cornice for 2.04 s.

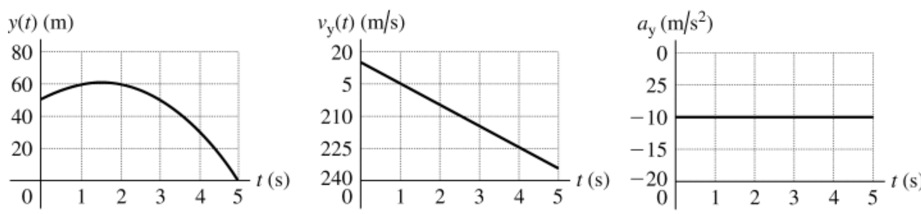


Figure 2.46

2.47. **IDENTIFY:** Use the constant acceleration equations to calculate a_x and $x - x_0$.

(a) SET UP: $v_x = 224 \text{ m/s}$, $v_{0x} = 0$, $t = 0.900 \text{ s}$, $a_x = ?$

$v_x = v_{0x} + a_x t$

EXECUTE: $a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{224 \text{ m/s} - 0}{0.900 \text{ s}} = 249 \text{ m/s}^2$

(b) $a_x / g = (249 \text{ m/s}^2)/(9.80 \text{ m/s}^2) = 25.4$

(c) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(249 \text{ m/s}^2)(0.900 \text{ s})^2 = 101 \text{ m}$

(d) SET UP: Calculate the acceleration, assuming it is constant:

$t = 1.40 \text{ s}$, $v_{0x} = 283 \text{ m/s}$, $v_x = 0$ (stops), $a_x = ?$

$v_x = v_{0x} + a_x t$

EXECUTE: $a_x = \dfrac{v_x - v_{0x}}{t} = \dfrac{0 - 283 \text{ m/s}}{1.40 \text{ s}} = -202 \text{ m/s}^2$

$a_x / g = (-202 \text{ m/s}^2)/(9.80 \text{ m/s}^2) = -20.6$; $a_x = -20.6g$

If the acceleration while the sled is stopping is constant then the magnitude of the acceleration is only 20.6*g*. But if the acceleration is not constant it is certainly possible that at some point the instantaneous acceleration could be as large as 40*g*.

EVALUATE: It is reasonable that for this motion the acceleration is much larger than *g*.

2.48. **IDENTIFY:** Since air resistance is ignored, the boulder is in free-fall and has a constant downward acceleration of magnitude 9.80 m/s^2. Apply the constant acceleration equations to the motion of the boulder.

SET UP: Take $+y$ to be upward.

EXECUTE: **(a)** $v_{0y} = +40.0 \text{ m/s}$, $v_y = +20.0 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$. $v_y = v_{0y} + a_y t$ gives

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{20.0 \text{ m/s} - 40.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = +2.04 \text{ s}.$$

(b) $v_y = -20.0\text{ m/s}$. $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-20.0\text{ m/s} - 40.0\text{ m/s}}{-9.80\text{ m/s}^2} = +6.12\text{ s}$.

(c) $y - y_0 = 0$, $v_{0y} = +40.0\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 0$ and $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(40.0\text{ m/s})}{-9.80\text{ m/s}^2} = +8.16\text{ s}$.

(d) $v_y = 0$, $v_{0y} = +40.0\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$. $v_y = v_{0y} + a_y t$ gives $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{0 - 40.0\text{ m/s}}{-9.80\text{ m/s}^2} = 4.08\text{ s}$.

(e) The acceleration is 9.80 m/s^2, downward, at all points in the motion.

(f) The graphs are sketched in Figure 2.48.

EVALUATE: $v_y = 0$ at the maximum height. The time to reach the maximum height is half the total time in the air, so the answer in part (d) is half the answer in part (c). Also note that $2.04\text{ s} < 4.08\text{ s} < 6.12\text{ s}$. The boulder is going upward until it reaches its maximum height and after the maximum height it is traveling downward.

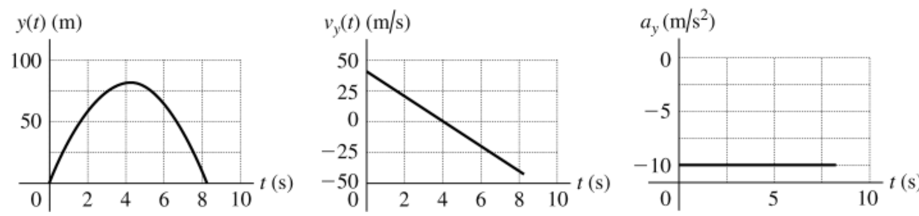


Figure 2.48

2.49. **IDENTIFY:** We can avoid solving for the common height by considering the relation between height, time of fall and acceleration due to gravity and setting up a ratio involving time of fall and acceleration due to gravity.

SET UP: Let g_{En} be the acceleration due to gravity on Enceladus and let g be this quantity on earth. Let h be the common height from which the object is dropped. Let $+y$ be downward, so $y - y_0 = h$. $v_{0y} = 0$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $h = \frac{1}{2}gt_{\text{E}}^2$ and $h = \frac{1}{2}g_{\text{En}}t_{\text{En}}^2$. Combining these two equations gives $gt_{\text{E}}^2 = g_{\text{En}}t_{\text{En}}^2$ and $g_{\text{En}} = g\left(\dfrac{t_{\text{E}}}{t_{\text{En}}}\right)^2 = (9.80\text{ m/s}^2)\left(\dfrac{1.75\text{ s}}{18.6\text{ s}}\right)^2 = 0.0868\text{ m/s}^2$.

EVALUATE: The acceleration due to gravity is inversely proportional to the square of the time of fall.

2.50. **IDENTIFY:** The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use Eqs.(2.17) and (2.18). Use the values of v_x and of x at $t = 1.0\text{ s}$ to evaluate v_{0x} and x_0.

SET UP: $\int t^n dt = \dfrac{1}{n+1}t^{n+1}$, for $n \ge 0$.

EXECUTE: **(a)** $v_x = v_{0x} + \int_0^t \alpha t dt = v_{0x} + \frac{1}{2}\alpha t^2 = v_{0x} + (0.60\text{ m/s}^3)t^2$. $v_x = 5.0\text{ m/s}$ when $t = 1.0\text{ s}$ gives $v_{0x} = 4.4\text{ m/s}$. Then, at $t = 2.0\text{ s}$, $v_x = 4.4\text{ m/s} + (0.60\text{ m/s}^3)(2.0\text{ s})^2 = 6.8\text{ m/s}$.

(b) $x = x_0 + \int_0^t (v_{0x} + \frac{1}{2}\alpha t^2)dt = x_0 + v_{0x}t + \dfrac{1}{6}\alpha t^3$. $x = 6.0\text{ m}$ at $t = 1.0\text{ s}$ gives $x_0 = 1.4\text{ m}$. Then, at $t = 2.0\text{ s}$, $x = 1.4\text{ m} + (4.4\text{ m/s})(2.0\text{ s}) + \dfrac{1}{6}(1.24\text{ m/s}^3)(2.0\text{ s})^3 = 11.8\text{ m}$.

(c) $x(t) = 1.4\text{ m} + (4.4\text{ m/s})t + (0.20\text{ m/s}^3)t^3$. $v_x(t) = 4.4\text{ m/s} + (0.60\text{ m/s}^3)t^2$. $a_x(t) = (1.20\text{m/s}^3)t$. The graphs are sketched in Figure 2.50.

EVALUATE: We can verify that $a_x = \dfrac{dv_x}{dt}$ and $v_x = \dfrac{dx}{dt}$.

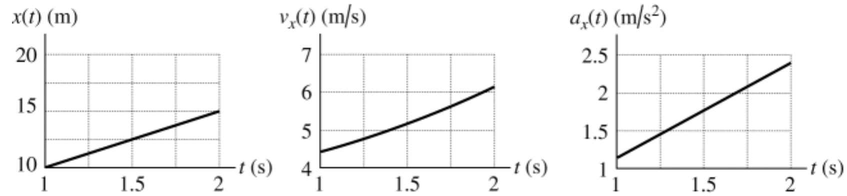


Figure 2.50

2.51. $a_x = At - Bt^2$ with $A = 1.50\text{ m/s}^3$ and $B = 0.120\text{ m/s}^4$

(a) IDENTIFY: Integrate $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t)$.

SET UP: $v_x = v_{0x} + \int_0^t a_x\, dt$

EXECUTE: $v_x = v_{0x} + \int_0^t (At - Bt^2)\, dt = v_{0x} + \frac{1}{2}At^2 - \frac{1}{3}Bt^3$

At rest at $t = 0$ says that $v_{0x} = 0$, so

$v_x = \frac{1}{2}At^2 - \frac{1}{3}Bt^3 = \frac{1}{2}(1.50\text{ m/s}^3)t^2 - \frac{1}{3}(0.120\text{ m/s}^4)t^3$

$v_x = (0.75\text{ m/s}^3)t^2 - (0.040\text{ m/s}^4)t^3$

SET UP: $x - x_0 + \int_0^t v_x\, dt$

EXECUTE: $x = x_0 + \int_0^t \left(\frac{1}{2}At^2 - \frac{1}{3}Bt^3\right) dt = x_0 + \frac{1}{6}At^3 - \frac{1}{12}Bt^4$

At the origin at $t = 0$ says that $x_0 = 0$, so

$x = \frac{1}{6}At^3 - \frac{1}{12}Bt^4 = \frac{1}{6}(1.50\text{ m/s}^3)t^3 - \frac{1}{12}(0.120\text{ m/s}^4)t^4$

$x = (0.25\text{ m/s}^3)t^3 - (0.010\text{ m/s}^4)t^4$

EVALUATE: We can check our results by using them to verify that $v_x(t) = \dfrac{dx}{dt}$ and $a_x(t) = \dfrac{dv_x}{dt}$.

(b) IDENTIFY and SET UP: At time *t*, when v_x is a maximum, $\dfrac{dv_x}{dt} = 0$. (Since $a_x = \dfrac{dv_x}{dt}$, the maximum velocity is when $a_x = 0$. For earlier times a_x is positive so v_x is still increasing. For later times a_x is negative and v_x is decreasing.)

EXECUTE: $a_x = \dfrac{dv_x}{dt} = 0$ so $At - Bt^2 = 0$

One root is $t = 0$, but at this time $v_x = 0$ and not a maximum.

The other root is $t = \dfrac{A}{B} = \dfrac{1.50\text{ m/s}^3}{0.120\text{ m/s}^4} = 12.5\text{ s}$

At this time $v_x = (0.75\text{ m/s}^3)t^2 - (0.040\text{ m/s}^4)t^3$ gives

$v_x = (0.75\text{ m/s}^3)(12.5\text{ s})^2 - (0.040\text{ m/s}^4)(12.5\text{ s})^3 = 117.2\text{ m/s} - 78.1\text{ m/s} = 39.1\text{ m/s}.$

EVALUATE: For $t < 12.5$ s, $a_x > 0$ and v_x is increasing. For $t > 12.5$ s, $a_x < 0$ and v_x is decreasing.

2.52. **IDENTIFY:** $a(t)$ is the slope of the *v* versus *t* graph and the distance traveled is the area under the *v* versus *t* graph.

SET UP: The *v* versus *t* graph can be approximated by the graph sketched in Figure 2.52.

EXECUTE: **(a)** Slope $= a = 0$ for $t \ge 1.3$ ms.

(b)

$h_{\text{max}} = \text{Area under } v\text{-}t \text{ graph} \approx A_{\text{Triangle}} + A_{\text{Rectangle}} \approx \frac{1}{2}(1.3\text{ ms})(133\text{ cm/s}) + (2.5\text{ ms} - 1.3\text{ ms})(133\text{ cm/s}) \approx 0.25\text{ cm}$

(c) $a = \text{slope of } v\text{-}t$ graph. $a(0.5\text{ ms}) \approx a(1.0\text{ ms}) \approx \dfrac{133\text{cm/s}}{1.3\text{ms}} = 1.0\times10^5\text{ cm/s}^2$.

$a(1.5\text{ ms}) = 0$ because the slope is zero.

(d) $h = \text{area}$ under *v-t* graph. $h(0.5\text{ ms}) \approx A_{\text{Triangle}} = \frac{1}{2}(0.5\text{ ms})(33\text{ cm/s}) = 8.3\times10^{-3}\text{ cm}$.

$h(1.0\text{ ms}) \approx A_{\text{Triangle}} = \frac{1}{2}(1.0\text{ ms})(100\text{ cm/s}) = 5.0\times10^{-2}\text{ cm}$.

$h(1.5\text{ ms}) \approx A_{\text{Triangle}} + A_{\text{Rectangle}} = \frac{1}{2}(1.3\text{ ms})(133\text{ cm/s})(0.2\text{ ms})(1.33) = 0.11\text{ cm}$

EVALUATE: The acceleration is constant until $t = 1.3$ ms, and then it is zero. $g = 980\text{ cm/s}^2$. The acceleration during the first 1.3 ms is much larger than this and gravity can be neglected for the portion of the jump that we are considering.

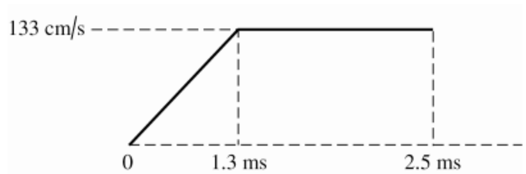


Figure 2.52

2.53. **(a) IDENTIFY** and **SET UP:** The change in speed is the area under the a_x versus *t* curve between vertical lines at $t = 2.5$ s and $t = 7.5$ s.

EXECUTE: This area is $\frac{1}{2}(4.00\text{ cm/s}^2 + 8.00\text{ cm/s}^2)(7.5\text{ s} - 2.5\text{ s}) = 30.0\text{ cm/s}$

This acceleration is positive so the change in velocity is positive.

(b) Slope of v_x versus *t* is positive and increasing with *t*. The graph is sketched in Figure 2.53.

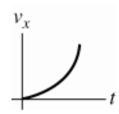


Figure 2.53

EVALUATE: The calculation in part (a) is equivalent to $\Delta v_x = (a_{\text{av-}x})\Delta t$. Since a_x is linear in *t*, $a_{\text{av-}x} = (a_{0x} + a_x)/2$. Thus $a_{\text{av-}x} = \frac{1}{2}(4.00\text{ cm/s}^2 + 8.00\text{ cm/s}^2)$ for the time interval $t = 2.5$ s to $t = 7.5$ s.

2.54. **IDENTIFY:** The average speed is the total distance traveled divided by the total time. The elapsed time is the distance traveled divided by the average speed.

SET UP: The total distance traveled is 20 mi. With an average speed of 8 mi/h for 10 mi, the time for that first 10 miles is $\frac{10\text{ mi}}{8\text{ mi/h}} = 1.25\text{ h}$.

EXECUTE: **(a)** An average speed of 4 mi/h for 20 mi gives a total time of $\frac{20\text{ mi}}{4\text{ mi/h}} = 5.0\text{ h}$. The second 10 mi must be covered in $5.0\text{ h} - 1.25\text{ h} = 3.75\text{ h}$. This corresponds to an average speed of $\frac{10\text{ mi}}{3.75\text{ h}} = 2.7\text{ mi/h}$.

(b) An average speed of 12 mi/h for 20 mi gives a total time of $\frac{20\text{ mi}}{12\text{ mi/h}} = 1.67\text{ h}$. The second 10 mi must be covered in $1.67\text{ h} - 1.25\text{ h} = 0.42\text{ h}$. This corresponds to an average speed of $\frac{10\text{ mi}}{0.42\text{ h}} = 24\text{ mi/h}$.

(c) An average speed of 16 mi/h for 20 mi gives a total time of $\frac{20\text{ mi}}{16\text{ mi/h}} = 1.25\text{ h}$. But 1.25 h was already spent during the first 10 miles and the second 10 miles would have to be covered in zero time. This is not possible and an average speed of 16 mi/h for the 20-mile ride is not possible.

EVALUATE: The average speed for the total trip is not the average of the average speeds for each 10-mile segment. The rider spends a different amount of time traveling at each of the two average speeds.

2.55. **IDENTIFY:** $v_x(t) = \frac{dx}{dt}$ and $a_x = \frac{dv_x}{dt}$.

SET UP: $\frac{d}{dt}(t^n) = nt^{n-1}$, for $n \ge 1$.

EXECUTE: **(a)** $v_x(t) = (9.00\text{ m/s}^3)t^2 - (20.0\text{ m/s}^2)t + 9.00\text{ m/s}$. $a_x(t) = (18.0\text{ m/s}^3)t - 20.0\text{ m/s}^2$. The graphs are sketched in Figure 2.55.

(b) The particle is instantaneously at rest when $v_x(t) = 0$. $v_{0x} = 0$ and the quadratic formula gives

$t = \frac{1}{18.0}(20.0 \pm \sqrt{(20.0)^2 - 4(9.00)(9.00)})$ s $= 1.11$ s ± 0.48 s. $t = 0.63$ s and $t = 1.59$ s. These results agree with the v_x-t graphs in part (a).

(c) For $t = 0.63$ s, $a_x = (18.0\text{ m/s}^3)(0.63\text{ s}) - 20.0\text{ m/s}^2 = -8.7\text{ m/s}^2$. For $t = 1.59$ s, $a_x = +8.6\text{ m/s}^2$. At $t = 0.63$ s the slope of the v_x-t graph is negative and at $t = 1.59$ s it is positive, so the same answer is deduced from the $v_x(t)$ graph as from the expression for $a_x(t)$.

(d) $v_x(t)$ is instantaneously not changing when $a_x = 0$. This occurs at $t = \frac{20.0\text{ m/s}^2}{18.0\text{m/s}^3} = 1.11$ s.

(e) When the particle is at its greatest distance from the origin, $v_x = 0$ and $a_x < 0$ (so the particle is starting to move back toward the origin). This is the case for $t = 0.63$ s, which agrees with the *x*-*t* graph in part (a). At $t = 0.63$ s, $x = 2.45$ m.

(f) The particle's speed is changing at its greatest rate when a_x has its maximum magnitude. The a_x-t graph in part (a) shows this occurs at $t = 0$ and at $t = 2.00$ s. Since v_x is always positive in this time interval, the particle is speeding up at its greatest rate when a_x is positive, and this is for $t = 2.00$ s.

The particle is slowing down at its greatest rate when a_x is negative and this is for $t = 0$.

EVALUATE: Since $a_x(t)$ is linear in *t*, $v_x(t)$ is a parabola and is symmetric around the point where $|v_x(t)|$ has its minimum value ($t = 1.11$ s). For this reason, the answer to part (d) is midway between the two times in part (c).

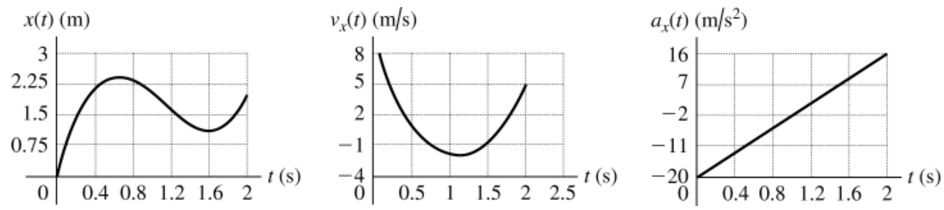


Figure 2.55

2.56. **IDENTIFY:** The average velocity is $v_{\text{av-}x} = \frac{\Delta x}{\Delta t}$. The average speed is the distance traveled divided by the elapsed time.

SET UP: Let $+x$ be in the direction of the first leg of the race. For the round trip, $\Delta x \geq 0$ and the total distance traveled is 50.0 m. For each leg of the race both the magnitude of the displacement and the distance traveled are 25.0 m.

EXECUTE: **(a)** $|v_{\text{av-}x}| = \left|\frac{\Delta x}{\Delta t}\right| = \frac{25.0\text{ m}}{20.0\text{ s}} = 1.25\text{ m/s}$. This is the same as the average speed for this leg of the race.

(b) $|v_{\text{av-}x}| = \left|\frac{\Delta x}{\Delta t}\right| = \frac{25.0\text{ m}}{15.0\text{ s}} = 1.67\text{ m/s}$. This is the same as the average speed for this leg of the race.

(c) $\Delta x = 0$ so $v_{\text{av-}x} = 0$.

(d) The average speed is $\frac{50.0\text{ m}}{35.0\text{ s}} = 1.43\text{ m/s}$.

EVALUATE: Note that the average speed for the round trip is not equal to the arithmetic average of the average speeds for each leg.

2.57. **IDENTIFY:** Use information about displacement and time to calculate average speed and average velocity. Take the origin to be at Seward and the positive direction to be west.

(a) SET UP: average speed $= \frac{\text{distance traveled}}{\text{time}}$

EXECUTE: The distance traveled (different from the net displacement $(x - x_0)$) is $76\text{ km} + 34\text{ km} = 110\text{ km}$.

Find the total elapsed time by using $v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t}$ to find *t* for each leg of the journey.

Seward to Auora: $t = \frac{x - x_0}{v_{\text{av-}x}} = \frac{76\text{ km}}{88\text{ km/h}} = 0.8636\text{ h}$

Auora to York: $t = \dfrac{x - x_0}{v_{\text{av-}x}} = \dfrac{-34\ \text{km}}{-72\ \text{km/h}} = 0.4722\ \text{h}$

Total $t = 0.8636\ \text{h} + 0.4722\ \text{h} = 1.336\ \text{h}.$

Then average speed $= \dfrac{110\ \text{km}}{1.336\ \text{h}} = 82\ \text{km/h}.$

(b) SET UP: $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t},$ where Δx is the displacement, not the total distance traveled.

For the whole trip he ends up $76\ \text{km} - 34\ \text{km} = 42\ \text{km}$ west of his starting point. $v_{\text{av-}x} = \dfrac{42\ \text{km}}{1.336\ \text{h}} = 31\ \text{km/h}.$

EVALUATE: The motion is not uniformly in the same direction so the displacement is less than the distance traveled and the magnitude of the average velocity is less than the average speed.

2.58. **IDENTIFY:** The vehicles are assumed to move at constant speed. The speed (mi/h) divided by the frequency with which vehicles pass a given point (vehicles/h) is the total space per vehicle (the length of the vehicle plus space to the next vehicle).

SET UP: $96\ \text{km/h} = 96 \times 10^3\ \text{m/h}$

EXECUTE: **(a)** The total space per vehicle is $\dfrac{96 \times 10^3\ \text{m/h}}{2400\ \text{vehicles/h}} = 40\ \text{m/vehicle}$. Since the average length of a vehicle is 4.6 m, the average space between vehicles is $40\ \text{m} - 4.6\ \text{m} = 35\ \text{m}$.

(b) The frequency of vehicles (vehicles/h) is $\dfrac{96 \times 10^3\ \text{m/h}}{(4.6 + 9.2)\ \text{m/vehicle}} = 7000\ \text{vehicles/h}$.

EVALUATE: The traffic flow rate per lane would nearly triple. Note that the traffic flow rate is directly proportional to the traffic speed.

2.59. **(a) IDENTIFY:** Calculate the average acceleration using $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t} = \dfrac{v_x - v_{0x}}{t}$ Use the information about the time and total distance to find his maximum speed.

SET UP: $v_{0x} = 0$ since the runner starts from rest.

$t = 4.0\ \text{s},$ but we need to calculate $v_x,$ the speed of the runner at the end of the acceleration period.

EXECUTE: For the last $9.1\ \text{s} - 4.0\ \text{s} = 5.1\ \text{s}$ the acceleration is zero and the runner travels a distance of $d_1 = (5.1\ \text{s})v_x$ (obtained using $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$)

During the acceleration phase of 4.0 s, where the velocity goes from 0 to $v_x,$ the runner travels a distance

$$d_2 = \left(\frac{v_{0x} + v_x}{2}\right)t = \frac{v_x}{2}(4.0\ \text{s}) = (2.0\ \text{s})v_x$$

The total distance traveled is 100 m, so $d_1 + d_2 = 100\ \text{m}.$ This gives $(5.1\ \text{s})v_x + (2.0\ \text{s})v_x = 100\ \text{m}.$

$v_x = \dfrac{100\ \text{m}}{7.1\ \text{s}} = 14.08\ \text{m/s}.$

Now we can calculate $a_{\text{av-}x}$: $a_{\text{av-}x} = \dfrac{v_x - v_{0x}}{t} = \dfrac{14.08\ \text{s} - 0}{4.0\ \text{s}} = 3.5\ \text{m/s}^2.$

(b) For this time interval the velocity is constant, so $a_{\text{av}-x} = 0.$

EVALUATE: Now that we have v_x we can calculate $d_1 = (5.1\ \text{s})(14.08\ \text{m/s}) = 71.9\ \text{m}$ and $d_2 = (2.0\ \text{s})(14.08\ \text{m/s}) = 28.2\ \text{m}.$ So, $d_1 + d_2 = 100\ \text{m},$ which checks.

(c) IDENTIFY and **SET UP:** $a_{\text{av-}x} = \dfrac{v_x - v_{0x}}{t},$ where now the time interval is the full 9.1 s of the race.

We have calculated the final speed to be $14.08\ \text{m/s},$ so

$$a_{\text{av-}x} = \frac{14.08\ \text{m/s}}{9.1\ \text{s}} = 1.5\ \text{m/s}^2.$$

EVALUATE: The acceleration is zero for the last 5.1 s, so it makes sense for the answer in part (c) to be less than half the answer in part (a).

(d) The runner spends different times moving with the average accelerations of parts (a) and (b).

2.60. **IDENTIFY:** Apply the constant acceleration equations to the motion of the sled. The average velocity for a time interval Δt is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$.

SET UP: Let $+x$ be parallel to the incline and directed down the incline. The problem doesn't state how much time it takes the sled to go from the top to 14.4 m from the top.

EXECUTE: **(a)** 14.4 m to 25.6 m: $v_{\text{av-}x} = \dfrac{25.6\text{ m} - 14.4\text{ m}}{2.00\text{ s}} = 5.60\text{ m/s}$. 25.6 to 40.0 m:

$v_{\text{av-}x} = \dfrac{40.0\text{ m} - 25.6\text{ m}}{2.00\text{ s}} = 7.20\text{ m/s}$. 40.0 m to 57.6 m: $v_{\text{av-}x} = \dfrac{57.6\text{ m} - 40.0\text{ m}}{2.00\text{ s}} = 8.80\text{ m/s}$.

(b) For each segment we know $x - x_0$ and t but we don't know v_{0x} or v_x. Let $x_1 = 14.4\text{ m}$ and $x_2 = 25.6\text{ m}$. For this interval $\left(\dfrac{v_1 + v_2}{2}\right) = \dfrac{x_2 - x_1}{t}$ and $at = v_2 - v_1$. Solving for v_2 gives $v_2 = \frac{1}{2}at + \dfrac{x_2 - x_1}{t}$. Let $x_2 = 25.6\text{ m}$ and $x_3 = 40.0\text{ m}$. For this second interval, $\left(\dfrac{v_2 + v_3}{2}\right) = \dfrac{x_3 - x_2}{t}$ and $at = v_3 - v_2$. Solving for v_2 gives $v_2 = -\frac{1}{2}at + \dfrac{x_3 - x_2}{t}$. Setting these two expressions for v_2 equal to each other and solving for a gives

$a = \dfrac{1}{t^2}[(x_3 - x_2) - (x_2 - x_1)] = \dfrac{1}{(2.00\text{ s})^2}[(40.0\text{ m} - 25.6\text{ m}) - (25.6\text{ m} - 14.4\text{ m})] = 0.80\text{ m/s}^2$.

Note that this expression for a says $a = \dfrac{v_{\text{av-23}} - v_{\text{av-12}}}{t}$, where $v_{\text{av-12}}$ and $v_{\text{av-23}}$ are the average speeds for successive 2.00 s intervals.

(c) For the motion from $x = 14.4\text{ m}$ to $x = 25.6\text{ m}$, $x - x_0 = 11.2\text{ m}$, $a_x = 0.80\text{ m/s}^2$ and $t = 2.00\text{ s}$.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_{0x} = \dfrac{x - x_0}{t} - \frac{1}{2}a_x t = \dfrac{11.2\text{ m}}{2.00\text{ s}} - \dfrac{1}{2}(0.80\text{ m/s}^2)(2.00\text{ s}) = 4.80\text{ m/s}$.

(d) For the motion from $x = 0$ to $x = 14.4\text{ m}$, $x - x_0 = 14.4\text{ m}$, $v_{0x} = 0$, and $v_x = 4.8\text{ m/s}$.

$x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t$ gives $t = \dfrac{2(x - x_0)}{v_{0x} + v_x} = \dfrac{2(14.4\text{ m})}{4.8\text{ m/s}} = 6.0\text{ s}$.

(e) For this 1.00 s time interval, $t = 1.00\text{ s}$, $v_{0x} = 4.8\text{ m/s}$, $a_x = 0.80\text{ m/s}^2$.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (4.8\text{ m/s})(1.00\text{ s}) + \frac{1}{2}(0.80\text{ m/s}^2)(1.00\text{ s})^2 = 5.2\text{ m}$.

EVALUATE: With $x = 0$ at the top of the hill, $x(t) = v_{0x}t + \frac{1}{2}a_x t^2 = (0.40\text{ m/s}^2)t^2$. We can verify that $t = 6.0\text{ s}$ gives $x = 14.4\text{ m}$, $t = 8.0\text{ s}$ gives 25.6 m, $t = 10.0\text{ s}$ gives 40.0 m, and $t = 12.0\text{ s}$ gives 57.6 m.

2.61. **IDENTIFY:** When the graph of v_x versus t is a straight line the acceleration is constant, so this motion consists of two constant acceleration segments and the constant acceleration equations can be used for each segment. Since v_x is always positive the motion is always in the $+x$ direction and the total distance moved equals the magnitude of the displacement. The acceleration a_x is the slope of the v_x versus t graph.

SET UP: For the $t = 0$ to $t = 10.0\text{ s}$ segment, $v_{0x} = 4.00\text{ m/s}$ and $v_x = 12.0\text{ m/s}$. For the $t = 10.0\text{ s}$ to 12.0 s segment, $v_{0x} = 12.0\text{ m/s}$ and $v_x = 0$.

EXECUTE: **(a)** For $t = 0$ to $t = 10.0\text{ s}$, $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = \left(\dfrac{4.00\text{ m/s} + 12.0\text{ m/s}}{2}\right)(10.0\text{ s}) = 80.0\text{ m}$. For $t = 10.0\text{ s}$ to $t = 12.0\text{ s}$, $x - x_0 = \left(\dfrac{12.0\text{ m/s} + 0}{2}\right)(2.00\text{ s}) = 12.0\text{ m}$. The total distance traveled is 92.0 m.

(b) $x - x_0 = 80.0\text{ m} + 12.0\text{ m} = 92.0\text{ m}$

(c) For $t = 0$ to 10.0 s, $a_x = \dfrac{12.0\text{ m/s} - 4.00\text{ m/s}}{10.0\text{ s}} = 0.800\text{ m/s}^2$. For $t = 10.0\text{ s}$ to 10.2 s,

$a_x = \dfrac{0 - 12.0\text{ m/s}}{2.00\text{ s}} = -6.00\text{ m/s}^2$. The graph of a_x versus t is given in Figure 2.61.

EVALUATE: When v_x and a_x are both positive, the speed increases. When v_x is positive and a_x is negative, the speed decreases.

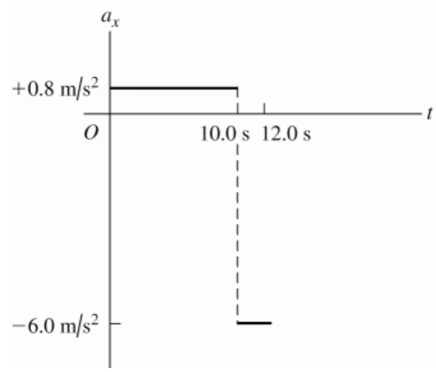


Figure 2.61

2.62. **IDENTIFY:** Since light travels at constant speed, $d = ct$

SET UP: The distance from the earth to the sun is 1.50×10^{11} m. The distance from the earth to the moon is 3.84×10^{8} m. $c = 186{,}000$ mi/s.

EXECUTE: **(a)** $d = ct = (3.0\times10^{8}\ \text{m/s})(1\ \text{y})\left(\dfrac{365\frac{1}{4}\ \text{d}}{1\ \text{y}}\right)\left(\dfrac{24\ \text{h}}{1\ \text{d}}\right)\left(\dfrac{3600\ \text{s}}{1\ \text{h}}\right) = 9.5\times10^{15}\ \text{m}$

(b) $d = ct = (3.0\times10^{8}\ \text{m/s})(10^{-9}\ \text{s}) = 0.30\ \text{m}$

(c) $t = \dfrac{d}{c} = \dfrac{1.5\times10^{11}\ \text{m}}{3.0\times10^{8}\ \text{m/s}} = 500\ \text{s} = 8.33\ \text{min}$

(d) $t = \dfrac{d}{c} = \dfrac{2(3.84\times10^{8}\ \text{m})}{3.0\times10^{8}\ \text{m/s}} = 2.6\ \text{s}$

(e) $t = \dfrac{d}{c} = \dfrac{3\times10^{9}\ \text{mi}}{186{,}000\ \text{mi/s}} = 16{,}100\ \text{s} = 4.5\ \text{h}$

EVALUATE: The speed of light is very large but it still takes light a measurable length of time to travel a large distance.

2.63. **IDENTIFY:** Speed is distance *d* divided by time *t*. The distance around a circular path is $d = 2\pi R$, where *R* is the radius of the circular path.

SET UP: The radius of the earth is $R_{\text{E}} = 6.38\times10^{6}$ m. The earth rotates once in $1\ \text{day} = 86{,}400$ s. The radius of the earth's orbit around the sun is 1.50×10^{11} m and the earth completes this orbit in $1\ \text{year} = 3.156\times10^{7}$ s. The speed of light in vacuum is $c = 3.00\times10^{8}$ m/s.

EXECUTE: **(a)** $v = \dfrac{d}{t} = \dfrac{2\pi R_{\text{E}}}{t} = \dfrac{2\pi(6.38\times10^{6}\ \text{m})}{86{,}400\ \text{s}} = 464\ \text{m/s}$.

(b) $v = \dfrac{2\pi R}{t} = \dfrac{2\pi(1.50\times10^{11}\ \text{m})}{3.156\times10^{7}\ \text{s}} = 2.99\times10^{4}\ \text{m/s}$.

(c) The time for light to go around once is $t = \dfrac{d}{c} = \dfrac{2\pi R_{\text{E}}}{\text{c}} = \dfrac{2\pi(6.38\times10^{6}\ \text{m})}{3.00\times10^{8}\ \text{m/s}} = 0.1336\ \text{s}$. In 1.00 s light would go around the earth $\dfrac{1.00\ \text{s}}{0.1336\ \text{s}} = 7.49\ \text{times}$.

EVALUATE: All these speeds are large compared to speeds of objects in our everyday experience.

2.64. **IDENTIFY:** When the graph of v_x versus *t* is a straight line the acceleration is constant, so this motion consists of two constant acceleration segments and the constant acceleration equations can be used for each segment. For $t = 0$ to 5.0 s, v_x is positive and the ball moves in the $+x$ direction. For $t = 5.0$ s to 20.0 s, v_x is negative and the ball moves in the $-x$ direction. The acceleration a_x is the slope of the v_x versus *t* graph.

SET UP: For the $t = 0$ to $t = 5.0$ s segment, $v_{0x} = 0$ and $v_x = 30.0$ m/s. For the $t = 5.0$ s to $t = 20.0$ s segment, $v_{0x} = -20.0$ m/s and $v_x = 0$.

EXECUTE: **(a)** For $t = 0$ to 5.0 s, $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = \left(\frac{0 + 30.0\text{ m/s}}{2}\right)(5.0\text{ m/s}) = 75.0\text{ m}$. The ball travels a distance of 75.0 m. For $t = 5.0$ s to 20.0 s, $x - x_0 = \left(\frac{-20.0\text{ m/s} + 0}{2}\right)(15.0\text{ m/s}) = -150.0\text{ m}$. The total distance traveled is $75.0\text{ m} + 150.0\text{ m} = 225.0\text{ m}$.

(b) The total displacement is $x - x_0 = 75.0\text{ m} + (-150.0\text{ m}) = -75.0\text{ m}$. The ball ends up 75.0 m in the negative x-direction from where it started.

(c) For $t = 0$ to 5.0 s, $a_x = \frac{30.0\text{ m/s} - 0}{5.0\text{ s}} = 6.00\text{ m/s}^2$. For $t = 5.0$ s to 20.0 s, $a_x = \frac{0 - (-20.0\text{ m/s})}{15.0\text{ s}} = +1.33\text{ m/s}^2$. The graph of a_x versus t is given in Figure 2.64.

(d) The ball is in contact with the floor for a small but nonzero period of time and the direction of the velocity doesn't change instantaneously. So, no, the actual graph of $v_x(t)$ is not really vertical at 5.00 s.

EVALUATE: For $t = 0$ to 5.0 s, both v_x and a_x are positive and the speed increases. For $t = 5.0$ s to 20.0 s, v_x is negative and a_x is positive and the speed decreases. Since the direction of motion is not the same throughout, the displacement is not equal to the distance traveled.

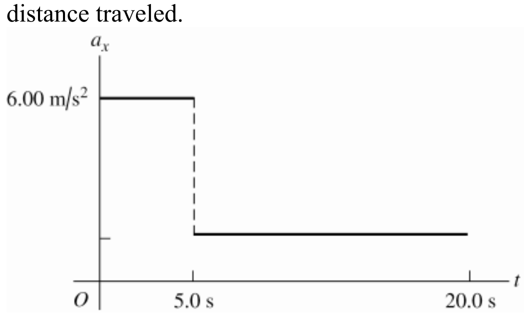


Figure 2.64

2.65. **IDENTIFY** and **SET UP:** Apply constant acceleration equations.
Find the velocity at the start of the second 5.0 s; this is the velocity at the end of the first 5.0 s. Then find $x - x_0$ for the first 5.0 s.

EXECUTE: For the first 5.0 s of the motion, $v_{0x} = 0$, $t = 5.0$ s.

$v_x = v_{0x} + a_x t$ gives $v_x = a_x(5.0\text{ s})$.

This is the initial speed for the second 5.0 s of the motion. For the second 5.0 s:

$v_{0x} = a_x(5.0\text{ s})$, $t = 5.0$ s, $x - x_0 = 150$ m.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $150\text{ m} = (25\text{ s}^2)a_x + (12.5\text{ s}^2)a_x$ and $a_x = 4.0\text{ m/s}^2$

Use this a_x and consider the first 5.0 s of the motion:

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(4.0\text{ m/s}^2)(5.0\text{ s})^2 = 50.0\text{ m}$.

EVALUATE: The ball is speeding up so it travels farther in the second 5.0 s interval than in the first. In fact, $x - x_0$ is proportional to t^2 since it starts from rest. If it goes 50.0 m in 5.0 s, in twice the time (10.0 s) it should go four times as far. In 10.0 s we calculated it went $50\text{ m} + 150\text{ m} = 200\text{ m}$, which is four times 50 m.

2.66. **IDENTIFY:** Apply $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to the motion of each train. A collision means the front of the passenger train is at the same location as the caboose of the freight train at some common time.

SET UP: Let P be the passenger train and F be the freight train. For the front of the passenger train $x_0 = 0$ and for the caboose of the freight train $x_0 = 200$ m. For the freight train $v_F = 15.0$ m/s and $a_F = 0$. For the passenger train $v_P = 25.0$ m/s and $a_P = -0.100\text{ m/s}^2$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ for each object gives $x_P = v_P t + \frac{1}{2}a_P t^2$ and $x_F = 200\text{ m} + v_F t$. Setting $x_P = x_F$ gives $v_P t + \frac{1}{2}a_P t^2 = 200\text{ m} + v_F t$. $(0.0500\text{ m/s}^2)t^2 - (10.0\text{ m/s})t + 200\text{ m} = 0$. The quadratic formula gives $t = \frac{1}{0.100}\left(+10.0 \pm \sqrt{(10.0)^2 - 4(0.0500)(200)}\right)\text{ s} = (100 \pm 77.5)\text{ s}$. The collision occurs at $t = 100\text{ s} - 77.5\text{ s} = 22.5\text{ s}$. The equations that specify a collision have a physical solution (real, positive t), so a collision does occur.

(b) $x_P = (25.0\ \text{m/s})(22.5\ \text{s}) + \frac{1}{2}(-0.100\ \text{m/s}^2)(22.5\ \text{s})^2 = 537\ \text{m}$. The passenger train moves 537 m before the collision. The freight train moves $(15.0\ \text{m/s})(22.5\ \text{s}) = 337\ \text{m}$.

(c) The graphs of x_F and x_P versus t are sketched in Figure 2.66.

EVALUATE: The second root for the equation for t, $t = 177.5$ s is the time the trains would meet again if they were on parallel tracks and continued their motion after the first meeting.

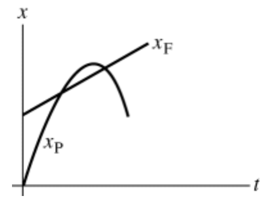


Figure 2.66

2.67. **IDENTIFY:** Apply constant acceleration equations to the motion of the two objects, you and the cockroach. You catch up with the roach when both objects are at the same place at the same time. Let T be the time when you catch up with the cockroach.

SET UP: Take $x = 0$ to be at the $t = 0$ location of the roach and positive x to be in the direction of motion of the two objects.

<u>roach</u>:

$v_{0x} = 1.50\ \text{m/s},\ a_x = 0,\ x_0 = 0,\ x = 1.20\ \text{m},\ t = T$

<u>you</u>:

$v_{0x} = 0.80\ \text{m/s},\ x_0 = -0.90\ \text{m},\ x = 1.20\ \text{m},\ t = T,\ a_x = ?$

Apply $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ to both objects:

EXECUTE: <u>roach</u>: $1.20\ \text{m} = (1.50\ \text{m/s})T$, so $T = 0.800$ s.

<u>you</u>: $1.20\ \text{m} - (-0.90\ \text{m}) = (0.80\ \text{m/s})T + \frac{1}{2}a_x T^2$

$2.10\ \text{m} = (0.80\ \text{m/s})(0.800\ \text{s}) + \frac{1}{2}a_x(0.800\ \text{s})^2$

$2.10\ \text{m} = 0.64\ \text{m} + (0.320\ \text{s}^2)a_x$

$a_x = 4.6\ \text{m/s}^2.$

EVALUATE: Your final velocity is $v_x = v_{0x} + a_x t = 4.48$ m/s. Then $x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right)t = 2.10\ \text{m}$, which checks.

You have to accelerate to a speed greater than that of the roach so you will travel the extra 0.90 m you are initially behind.

2.68. **IDENTIFY:** The insect has constant speed 15 m/s during the time it takes the cars to come together.

SET UP: Each car has moved 100 m when they hit.

EXECUTE: The time until the cars hit is $\frac{100\ \text{m}}{10\ \text{m/s}} = 10\ \text{s}$. During this time the grasshopper travels a distance of $(15\ \text{m/s})(10\ \text{s}) = 150\ \text{m}$.

EVALUATE: The grasshopper ends up 100 m from where it started, so the magnitude of his final displacement is 100 m. This is less than the total distance he travels since he spends part of the time moving in the opposite direction.

2.69. **IDENTIFY:** Apply constant acceleration equations to each object.

Take the origin of coordinates to be at the initial position of the truck, as shown in Figure 2.69a

Let d be the distance that the auto initially is behind the truck, so $x_0(\text{auto}) = -d$ and $x_0(\text{truck}) = 0$. Let T be the time it takes the auto to catch the truck. Thus at time T the truck has undergone a displacement $x - x_0 = 40.0$ m, so is at $x = x_0 + 40.0\ \text{m} = 40.0$ m. The auto has caught the truck so at time T is also at $x = 40.0$ m.

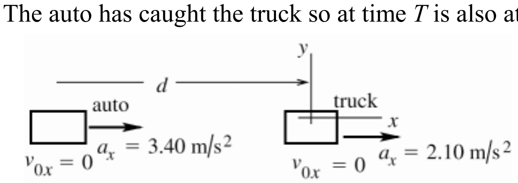


Figure 2.69a

(a) SET UP: Use the motion of the truck to calculate T:

$x - x_0 = 40.0\text{ m}$, $v_{0x} = 0$ (starts from rest), $a_x = 2.10\text{ m/s}^2$, $t = T$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$

Since $v_{0x} = 0$, this gives $t = \sqrt{\dfrac{2(x - x_0)}{a_x}}$

EXECUTE: $T = \sqrt{\dfrac{2(40.0\text{ m})}{2.10\text{ m/s}^2}} = 6.17\text{ s}$

(b) SET UP: Use the motion of the auto to calculate d:

$x - x_0 = 40.0\text{ m} + d$, $v_{0x} = 0$, $a_x = 3.40\text{ m/s}^2$, $t = 6.17\text{ s}$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$

EXECUTE: $d + 40.0\text{ m} = \tfrac{1}{2}(3.40\text{ m/s}^2)(6.17\text{ s})^2$

$d = 64.8\text{ m} - 40.0\text{ m} = 24.8\text{ m}$

(c) auto: $v_x = v_{0x} + a_x t = 0 + (3.40\text{ m/s}^2)(6.17\text{ s}) = 21.0\text{ m/s}$

truck: $v_x = v_{0x} + a_x t = 0 + (2.10\text{ m/s}^2)(6.17\text{ s}) = 13.0\text{ m/s}$

(d) The graph is sketched in Figure 2.69b.

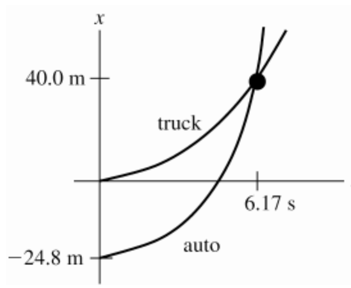


Figure 2.69b

EVALUATE: In part (c) we found that the auto was traveling faster than the truck when they come abreast. The graph in part (d) agrees with this: at the intersection of the two curves the slope of the *x-t* curve for the auto is greater than that of the truck. The auto must have an average velocity greater than that of the truck since it must travel farther in the same time interval.

2.70. **IDENTIFY:** Apply the constant acceleration equations to the motion of each car. The collision occurs when the cars are at the same place at the same time.

SET UP: Let $+x$ be to the right. Let $x = 0$ at the initial location of car 1, so $x_{01} = 0$ and $x_{02} = D$. The cars collide when $x_1 = x_2$. $v_{0x1} = 0$, $a_{x1} = a_x$, $v_{0x2} = -v_0$ and $a_{x2} = 0$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$ gives $x_1 = \tfrac{1}{2}a_x t^2$ and $x_2 = D - v_0 t$. $x_1 = x_2$ gives $\tfrac{1}{2}a_x t^2 = D - v_0 t$.

$\tfrac{1}{2}a_x t^2 + v_0 t - D = 0$. The quadratic formula gives $t = \dfrac{1}{a_x}\left(-v_0 \pm \sqrt{v_0^2 + 2a_x D}\right)$. Only the positive root is physical, so $t = \dfrac{1}{a_x}\left(-v_0 + \sqrt{v_0^2 + 2a_x D}\right)$.

(b) $v_1 = a_x t = \sqrt{v_0^2 + 2a_x D} - v_0$

(c) The *x-t* and v_x-*t* graphs for the two cars are sketched in Figure 2.70.

EVALUATE: In the limit that $a_x = 0$, $D - v_0 t = 0$ and $t = D/v_0$, the time it takes car 2 to travel distance *D*. In the limit that $v_0 = 0$, $t = \sqrt{\dfrac{2D}{a_x}}$, the time it takes car 1 to travel distance *D*.

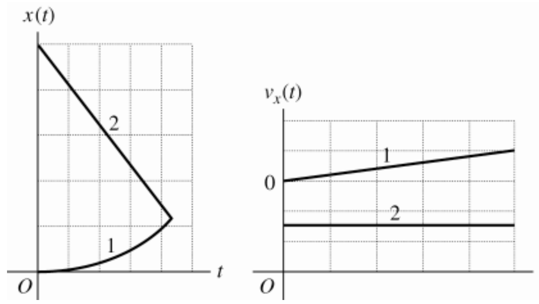


Figure 2.70

2.71. IDENTIFY: The average speed is the distance traveled divided by the time. The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t}$.

SET UP: The distance the ball travels is half the circumference of a circle of diameter 50.0 cm so is $\frac{1}{2}\pi d = \frac{1}{2}\pi(50.0\text{ cm}) = 78.5\text{ cm}$. Let $+x$ be horizontally from the starting point toward the ending point, so Δx equals the diameter of the bowl.

EXECUTE: **(a)** The average speed is $\dfrac{\frac{1}{2}\pi d}{t} = \dfrac{78.5\text{ cm}}{10.0\text{ s}} = 7.85\text{ cm/s}$.

(b) The average velocity is $v_{\text{av-}x} = \dfrac{\Delta x}{\Delta t} = \dfrac{50.0\text{ cm}}{10.0\text{ s}} = 5.00\text{ cm/s}$.

EVALUATE: The average speed is greater than the magnitude of the average velocity, since the distance traveled is greater than the magnitude of the displacement.

2.72. IDENTIFY: a_x is the slope of the v_x versus *t* graph. *x* is the area under the v_x versus *t* graph.

SET UP: The slope of v_x is positive and decreasing in magnitude. As v_x increases, the displacement in a given amount of time increases.

EXECUTE: The a_x-t and *x*-*t* graphs are sketched in Figure 2.72.

EVALUATE: v_x is the slope of the *x* versus *t* graph. The $x(t)$ graph we sketch has zero slope at $t = 0$, the slope is always positive, and the slope initially increases and then approaches a constant. This behavior agrees with the $v_x(t)$ that is given in the graph in the problem.

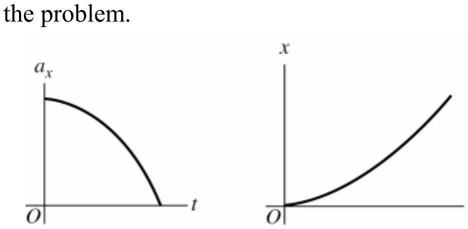


Figure 2.72

2.73. IDENTIFY: Apply constant acceleration equations to each vehicle.
SET UP: **(a)** It is very convenient to work in coordinates attached to the truck.
Note that these coordinates move at constant velocity relative to the earth. In these coordinates the truck is at rest, and the initial velocity of the car is $v_{0x} = 0$. Also, the car's acceleration in these coordinates is the same as in coordinates fixed to the earth.
EXECUTE: First, let's calculate how far the car must travel relative to the truck: The situation is sketched in Figure 2.73.

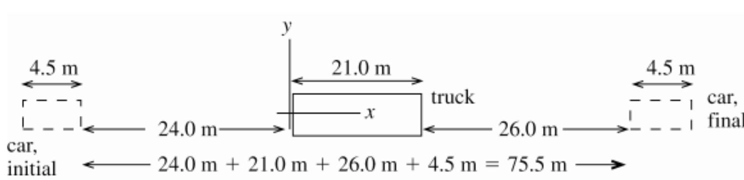


Figure 2.73

The car goes from $x_0 = -24.0$ m to $x = 51.5$ m. So $x - x_0 = 75.5$ m for the car.

Calculate the time it takes the car to travel this distance:

$a_x = 0.600\text{ m/s}^2$, $v_{0x} = 0$, $x - x_0 = 75.5$ m, $t = ?$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$

$t = \sqrt{\dfrac{2(x - x_0)}{a_x}} = \sqrt{\dfrac{2(75.5\text{ m})}{0.600\text{ m/s}^2}} = 15.86\text{ s}$

It takes the car 15.9 s to pass the truck.

(b) Need how far the car travels relative to the earth, so go now to coordinates fixed to the earth. In these coordinates $v_{0x} = 20.0$ m/s for the car. Take the origin to be at the initial position of the car.

$v_{0x} = 20.0$ m/s, $a_x = 0.600\text{ m/s}^2$, $t = 15.86$ s, $x - x_0 = ?$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 = (20.0\text{ m/s})(15.86\text{ s}) + \tfrac{1}{2}(0.600\text{ m/s}^2)(15.86\text{ s})^2$

$x - x_0 = 317.2\text{ m} + 75.5\text{ m} = 393\text{ m}.$

(c) In coordinates fixed to the earth:

$v_x = v_{0x} + a_x t = 20.0\text{ m/s} + (0.600\text{ m/s}^2)(15.86\text{ s}) = 29.5\text{ m/s}$

EVALUATE: In 15.9 s the truck travels $x - x_0 = (20.0\text{ m/s})(15.86\text{ s}) = 317.2$ m. The car travels $392.7\text{ m} - 317.2\text{ m} = 75$ m farther than the truck, which checks with part (a). In coordinates attached to the truck, for the car $v_{0x} = 0$, $v_x = 9.5$ m/s and in 15.86 s the car travels $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t = 75$ m, which checks with part (a).

2.74. **IDENTIFY:** The acceleration is not constant so the constant acceleration equations cannot be used. Instead, use $a_x(t) = \dfrac{dv_x}{dt}$ and $x = x_0 + \int_0^t v_x(t)dt$.

SET UP: $\int t^n dt = \dfrac{1}{n+1}t^{n+1}$ for $n \ge 0$.

EXECUTE: **(a)** $x(t) = x_0 + \int_0^t [\alpha - \beta t^2]dt = x_0 + \alpha t - \tfrac{1}{3}\beta t^3$. $x = 0$ at $t = 0$ gives $x_0 = 0$ and

$x(t) = \alpha t - \tfrac{1}{3}\beta t^3 = (4.00\text{ m/s})t - (0.667\text{ m/s}^3)t^3$. $a_x(t) = \dfrac{dv_x}{dt} = -2\beta t = -(4.00\text{ m/s}^3)t$.

(b) The maximum positive *x* is when $v_x = 0$ and $a_x < 0$. $v_x = 0$ gives $\alpha - \beta t^2 = 0$ and

$t = \sqrt{\dfrac{\alpha}{\beta}} = \sqrt{\dfrac{4.00\text{ m/s}}{2.00\text{ m/s}^3}} = 1.41\text{ s}$. At this *t*, a_x is negative. For $t = 1.41$ s ,

$x = (4.00\text{ m/s})(1.41\text{ s}) - (0.667\text{ m/s}^3)(1.41\text{ s})^3 = 3.77\text{ m}$.

EVALUATE: After $t = 1.41$ s the object starts to move in the $-x$ direction and goes to $x = -\infty$ as $t \to \infty$.

2.75. $a(t) = \alpha + \beta t$, with $\alpha = -2.00\text{ m/s}^2$ and $\beta = 3.00\text{ m/s}^3$

(a) IDENTIFY and **SET UP:** Integrage $a_x(t)$ to find $v_x(t)$ and then integrate $v_x(t)$ to find $x(t)$.

EXECUTE: $v_x = v_{0x} + \int_0^t a_x\, dt = v_{0x} + \int_0^t (\alpha + \beta)\, dt = v_{0x} + \alpha t + \tfrac{1}{2}\beta t^2$

$x = x_0 + \int_0^t v_x\, dt = x_0 + \int_0^t (v_{0x} + \alpha t + \tfrac{1}{2}\beta t^2)\, dt = x_0 + v_{0x}t + \tfrac{1}{2}\alpha t^2 + \tfrac{1}{6}\beta t^3$

At $t = 0$, $x = x_0$.

To have $x = x_0$ at $t_1 = 4.00$ s requires that $v_{0x}t_1 + \tfrac{1}{2}\alpha t_1^2 + \tfrac{1}{6}\beta t_1^3 = 0.$

Thus $v_{0x} = -\tfrac{1}{6}\beta t_1^2 - \tfrac{1}{2}\alpha t_1 = -\tfrac{1}{6}(3.00\text{ m/s}^3)(4.00\text{ s})^2 - \tfrac{1}{2}(-2.00\text{ m/s}^2)(4.00\text{ s}) = -4.00\text{ m/s}.$

(b) With v_{0x} as calculated in part (a) and $t = 4.00$ s,

$v_0 = v_{0x} + \alpha t + \tfrac{1}{2}\beta t^2 = -4.00\text{ s} + (-2.00\text{ m/s}^2)(4.00\text{ s}) + \tfrac{1}{2}(3.00\text{ m/s}^3)(4.00\text{ s})^2 = +12.0\text{ m/s}.$

EVALUATE: $a_x = 0$ at $t = 0.67$ s. For $t > 0.67$ s, $a_x > 0.$ At $t = 0,$ the particle is moving in the $-x$-direction and is speeding up. After $t = 0.67$ s, when the acceleration is positive, the object slows down and then starts to move in the $+x$-direction with increasing speed.

2.76. **IDENTIFY:** Find the distance the professor walks during the time t it takes the egg to fall to the height of his head.

SET UP: Let $+y$ be downward. The egg has $v_{0y}=0$ and $a_y=9.80\text{ m/s}^2$. At the height of the professor's head, the egg has $y-y_0=44.2\text{ m}$.

EXECUTE: $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $t=\sqrt{\dfrac{2(y-y_0)}{a_y}}=\sqrt{\dfrac{2(44.2\text{ m})}{9.80\text{ m/s}^2}}=3.00\text{ s}$. The professor walks a distance $x-x_0=v_{0x}t=(1.20\text{ m/s})(3.00\text{ s})=3.60\text{ m}$. Release the egg when your professor is 3.60 m from the point directly below you.

EVALUATE: Just before the egg lands its speed is $(9.80\text{ m/s}^2)(3.00\text{s})=29.4\text{ m/s}$. It is traveling much faster than the professor.

2.77. **IDENTIFY:** Use the constant acceleration equations to establish a relationship between maximum height and acceleration due to gravity and between time in the air and acceleration due to gravity.

SET UP: Let $+y$ be upward. At the maximum height, $v_y=0$. When the rock returns to the surface, $y-y_0=0$.

EXECUTE: **(a)** $v_y^2=v_{0y}^2+2a_y(y-y_0)$ gives $a_yH=-\frac{1}{2}v_{0y}^2$, which is constant, so $a_\text{E}H_\text{E}=a_\text{M}H_\text{M}$.

$$H_\text{M}=H_\text{E}\left(\frac{a_\text{E}}{a_\text{M}}\right)=H\left(\frac{9.80\text{ m/s}^2}{3.71\text{ m/s}^2}\right)=2.64H\ .$$

(b) $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ with $y-y_0=0$ gives $a_yt=-2v_{0y}$, which is constant, so $a_\text{E}T_\text{E}=a_\text{M}T_\text{M}$.

$$T_\text{M}=T_\text{E}\left[\frac{a_\text{E}}{a_\text{M}}\right]=2.64T\ .$$

EVALUATE: On Mars, where the acceleration due to gravity is smaller, the rocks reach a greater height and are in the air for a longer time.

2.78. **IDENTIFY:** Calculate the time it takes her to run to the table and return. This is the time in the air for the thrown ball. The thrown ball is in free-fall after it is thrown. Assume air resistance can be neglected.

SET UP: For the thrown ball, let $+y$ be upward. $a_y=-9.80\text{ m/s}^2$. $y-y_0=0$ when the ball returns to its original position.

EXECUTE: **(a)** It takes her $\dfrac{5.50\text{ m}}{2.50\text{ m/s}}=2.20\text{ s}$ to reach the table and an equal time to return. For the ball, $y-y_0=0$, $t=4.40\text{ s}$ and $a_y=-9.80\text{ m/s}^2$. $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $v_{0y}=-\frac{1}{2}a_yt=-\frac{1}{2}(-9.80\text{ m/s}^2)(4.40\text{ s})=21.6\text{ m/s}$.

(b) Find $y-y_0$ when $t=2.20\text{ s}$. $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2=(21.6\text{ m/s})(2.20\text{ s})+\frac{1}{2}(-9.80\text{ m/s}^2)(2.20\text{ s})^2=23.8\text{ m}$

EVALUATE: It takes the ball the same amount of time to reach its maximum height as to return from its maximum height, so when she is at the table the ball is at its maximum height. Note that this large maximum height requires that the act either be done outdoors, or in a building with a very high ceiling.

2.79. **(a) IDENTIFY:** Use constant acceleration equations, with $a_y=g$, downward, to calculate the speed of the diver when she reaches the water.

SET UP: Take the origin of coordinates to be at the platform, and take the $+y$-direction to be downward.

$y-y_0=+21.3\text{ m}$, $a_y=+9.80\text{ m/s}^2$, $v_{0y}=0$ (since diver just steps off), $v_y=?$

$v_y^2=v_{0y}^2+2a_y(y-y_0)$

EXECUTE: $v_y=+\sqrt{2a_y(y-y_0)}=+\sqrt{2(9.80\text{ m/s}^2)(31.3\text{ m})}=+20.4\text{ m/s}.$

We know that v_y is positive because the diver is traveling downward when she reaches the water.

The announcer has exaggerated the speed of the diver.

EVALUATE: We could also use $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ to find $t=2.085\text{ s}$. The diver gains 9.80 m/s of speed each second, so has $v_y=(9.80\text{ m/s}^2)(2.085\text{ s})=20.4\text{ m/s}$ when she reaches the water, which checks.

(b) IDENTIFY: Calculate the initial upward velocity needed to give the diver a speed of 25.0 m/s when she reaches the water. Use the same coordinates as in part (a).

SET UP: $v_{0y}=?$, $v_y=+25.0\text{ m/s}$, $a_y=+9.80\text{ m/s}^2$, $y-y_0=+21.3\text{ m}$

$v_y^2=v_{0y}^2+2a_y(y-y_0)$

EXECUTE: $v_{0y} = -\sqrt{v_y^2 - 2a_y(y - y_0)} = -\sqrt{(25.0\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(21.3\text{ m})} = -14.4\text{ m/s}$

(v_{0y} is negative since the direction of the initial velocity is upward.)

EVALUATE: One way to decide if this speed is reasonable is to calculate the maximum height above the platform it would produce:

$v_{0y} = -14.4\text{ m/s}$, $v_y = 0$ (at maximum height), $a_y = +9.80\text{ m/s}^2$, $y - y_0 = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (-14.4\text{ s})^2}{2(+9.80\text{ m/s})} = -10.6\text{ m}$$

This is not physically attainable; a vertical leap of 10.6 m upward is not possible.

2.80. IDENTIFY: The flowerpot is in free-fall. Apply the constant acceleration equations. Use the motion past the window to find the speed of the flowerpot as it reaches the top of the window. Then consider the motion from the windowsill to the top of the window.

SET UP: Let $+y$ be downward. Throughout the motion $a_y = +9.80\text{ m/s}^2$.

EXECUTE: Motion past the window: $y - y_0 = 1.90\text{ m}$, $t = 0.420\text{ s}$, $a_y = +9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$v_{0y} = \dfrac{y - y_0}{t} - \frac{1}{2}a_y t = \dfrac{1.90\text{ m}}{0.420\text{ s}} - \frac{1}{2}(9.80\text{ m/s}^2)(0.420\text{ s}) = 2.466\text{ m/s}$. This is the velocity of the flowerpot when it is at the top of the window.

Motion from the windowsill to the top of the window: $v_{0y} = 0$, $v_y = 2.466\text{ m/s}$, $a_y = +9.80\text{ m/s}^2$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{(2.466\text{ m/s})^2 - 0}{2(9.80\text{ m/s}^2)} = 0.310\text{ m}$. The top of the window is 0.310 m below the windowsill.

EVALUATE: It takes the flowerpot $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{2.466\text{ m/s}}{9.80\text{ m/s}^2} = 0.252\text{ s}$ to fall from the sill to the top of the window. Our result says that from the windowsill the pot falls $0.310\text{ m} + 1.90\text{ m} = 2.21\text{ m}$ in $0.252\text{ s} + 0.420\text{ s} = 0.672\text{ s}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\text{ m/s}^2)(0.672\text{ s})^2 = 2.21\text{ m}$, which checks.

2.81. IDENTIFY: For parts (a) and (b) apply the constant acceleration equations to the motion of the bullet. In part (c) neglect air resistance, so the bullet is free-fall. Use the constant acceleration equations to establish a relation between initial speed v_0 and maximum height *H*.

SET UP: For parts (a) and (b) let $+x$ be in the direction of motion of the bullet. For part (c) let $+y$ be upward, so $a_y = -g$. At the maximum height, $v_y = 0$.

EXECUTE: **(a)** $x - x_0 = 0.700\text{ m}$, $v_{0x} = 0$, $v_x = 965\text{ m/s}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$a_x = \dfrac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \dfrac{(965\text{ m/s})^2 - 0}{2(0.700\text{ m})} = 6.65\times10^5\text{ m/s}^2$. $\dfrac{a_x}{g} = 6.79\times10^4$, so $a_x = (6.79\times10^4)g$.

(b) $x - x_0 = \left(\dfrac{v_{0x} + v_x}{2}\right)t$ gives $t = \dfrac{2(x - x_0)}{v_{0x} + v_x} = \dfrac{2(0.700\text{ m})}{0 + 965\text{ m/s}} = 1.45\text{ ms}$.

(c) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ and $v_y = 0$ gives $\dfrac{v_{0y}^2}{y - y_0} = -2a_y$, which is constant. $\dfrac{v_{01}^2}{H_1} = \dfrac{v_{02}^2}{H_2}$.

$$H_2 = H_1\left(\frac{v_{02}^2}{v_{01}^2}\right) = H\left(\frac{\frac{1}{2}v_{01}}{v_{01}}\right)^2 = H/4.$$

EVALUATE: $H = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{-(965\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 47.5\text{ km}$. Rifle bullets fired vertically don't actually reach such a large height; it is not an accurate approximation to ignore air resistance.

2.82. IDENTIFY: Assume the firing of the second stage lasts a very short time, so the rocket is in free-fall after 25.0 s. The motion consists of two constant acceleration segments.

SET UP: Let $+y$ be upward. After $t = 25.0\text{ s}$, $a_y = -9.80\text{ m/s}^2$.

EXECUTE: **(a)** Find the height of the rocket at $t = 25.0\text{ s}$: $v_{0y} = 0$, $a_y = +3.50\text{ m/s}^2$, $t = 25.0\text{ s}$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(3.50\text{ m/s})(25.0\text{ s})^2 = 1.0938\times10^3\text{ m}$. Find the displacement of the rocket from firing of the

second stage until the maximum height is reached: $v_{0y} = 132.5$ m/s, $v_y = 0$ (at maximum height), $a_y = -9.80$ m/s^2.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (132.5\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 896\text{ m}$. The total height is

$1094\text{ m} + 896\text{ m} = 1990\text{ m}$.

(b) $v_{0y} = +132.5$ m/s, $a_y = -9.80$ m/s^2, $y - y_0 = -1094$ m. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$-1093.8\text{ m} = (132.5\text{ m/s})t - (4.90\text{ m/s}^2)t^2$. The quadratic formula gives $t = 33.7$ s as the positive root. The rocket returns to the launch pad 33.7 s after the second stage fires.

(c) $v_y = v_{0y} + a_y t = +132.5\text{ m/s} + (-9.80\text{ m/s}^2)(33.7\text{ s}) = -198\text{ m/s}$. The rocket has speed 198 m/s as it reaches the launch pad.

EVALUATE: The speed when the rocket returns to the launch pad is greater than 132.5 m/s. When the rocket returns to the height where the second stage fired, its velocity is 132.5 m/s downward and it continues to speed up during the rest of the descent.

2.83. Take positive y to be upward.

(a) IDENTIFY: Consider the motion from when he applies the acceleration to when the shot leaves his hand.

SET UP: $v_{0y} = 0$, $v_y = ?$, $a_y = 45.0$ m/s^2, $y - y_0 = 0.640$ m

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(45.0\text{ m/s}^2)(0.640\text{ m})} = 7.59\text{ m/s}$

(b) IDENTIFY: Consider the motion of the shot from the point where he releases it to its maximum height, where $v = 0$. Take $y = 0$ at the ground.

SET UP: $y_0 = 2.20$ m, $y = ?$, $a_y = -9.80$ m/s^2 (free fall), $v_{0y} = 7.59$ m/s

(from part (a), $v_y = 0$ at maximum height)

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (7.59\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 2.94\text{ m}$

$y = 2.20\text{ m} + 2.94\text{ m} = 5.14\text{ m}.$

(c) IDENTIFY: Consider the motion of the shot from the point where he releases it to when it returns to the height of his head. Take $y = 0$ at the ground.

SET UP: $y_0 = 2.20$ m, $y = 1.83$ m, $a_y = -9.80$ m/s^2, $v_{0y} = +7.59$ m/s, $t = ?$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $1.83\text{ m} - 2.20\text{ m} = (7.59\text{ m/s})t + \frac{1}{2}(-9.80\text{ m/s}^2)t^2$

$= (7.59\text{ m/s})t - (4.90\text{ m/s}^2)t^2$

$4.90t^2 - 7.59t - 0.37 = 0$, with t in seconds.

Use the quadratic formula to solve for t:

$t = \dfrac{1}{9.80}\left(7.59 \pm \sqrt{(7.59)^2 - 4(4.90)(-0.37)}\right) = 0.774 \pm 0.822$

t must be positive, so $t = 0.774\text{ s} + 0.822\text{ s} = 1.60\text{ s}$

EVALUATE: Calculate the time to the maximum height: $v_y = v_{0y} + a_y t$, so

$t = (v_y - v_{0y})/a_y = -(7.59\text{ m/s})/(-9.80\text{ m/s}^2) = 0.77\text{ s}$. It also takes 0.77 s to return to 2.2 m above the ground, for a total time of 1.54 s. His head is a little lower than 2.20 m, so it is reasonable for the shot to reach the level of his head a little later than 1.54 s after being thrown; the answer of 1.60 s in part (c) makes sense.

2.84. IDENTIFY: The teacher is in free-fall and falls with constant acceleration 9.80 m/s^2, downward. The sound from her shout travels at constant speed. The sound travels from the top of the cliff, reflects from the ground and then travels upward to her present location. If the height of the cliff is h and she falls a distance y in 3.0 s, the sound must travel a distance $h + (h - y)$ in 3.0 s.

SET UP: Let $+y$ be downward, so for the teacher $a_y = 9.80$ m/s^2 and $v_{0y} = 0$. Let $y = 0$ at the top of the cliff.

EXECUTE: **(a)** For the teacher, $y = \frac{1}{2}(9.80\text{ m/s}^2)(3.0\text{ s})^2 = 44.1\text{ m}$. For the sound, $h + (h - y) = v_s t$.

$h = \frac{1}{2}(v_s t + y) = \frac{1}{2}([340\text{ m/s}][3.0\text{ s}] + 44.1\text{ m}) = 532\text{ m}$, which rounds to 530 m.

(b) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(9.80\text{ m/s}^2)(532\text{ m})} = 102\text{ m/s}$.

EVALUATE: She is in the air for $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{102 \text{ m/s}}{9.80 \text{ m/s}^2} = 10.4 \text{ s}$ and strikes the ground at high speed.

2.85. **IDENTIFY** and **SET UP:** Let $+y$ be upward. Each ball moves with constant acceleration $a_y = -9.80 \text{ m/s}^2$. In parts (c) and (d) require that the two balls be at the same height at the same time.

EXECUTE: **(a)** At ceiling, $v_y = 0$, $y - y_0 = 3.0 \text{ m}$, $a_y = -9.80 \text{ m/s}^2$. Solve for v_{0y}.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_{0y} = 7.7 \text{ m/s}$.

(b) $v_y = v_{0y} + a_y t$ with the information from part (a) gives $t = 0.78 \text{ s}$.

(c) Let the first ball travel downward a distance *d* in time *t*. It starts from its maximum height, so $v_{0y} = 0$.

$$y - y_0 = v_{0_y} t = \tfrac{1}{2} a_y t^2 \text{ gives } d = (4.9 \text{ m/s}^2)t^2$$

The second ball has $v_{0y} = \frac{2}{3}(7.7 \text{ m/s}) = 5.1 \text{ m/s}$. In time *t* it must travel upward $3.0 \text{ m} - d$ to be at the same place as the first ball.

$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$ gives $3.0 \text{ m} - d = (5.1 \text{ m/s})t - (4.9 \text{ m/s}^2)t^2$.

We have two equations in two unknowns, *d* and *t*. Solving gives $t = 0.59 \text{ s}$ and $d = 1.7 \text{ m}$.

(d) $3.0 \text{ m} - d = 1.3 \text{ m}$

EVALUATE: In 0.59 s the first ball falls $d = (4.9 \text{ m/s}^2)(0.59 \text{ s})^2 = 1.7 \text{ m}$, so is at the same height as the second ball.

2.86. **IDENTIFY:** The helicopter has two segments of motion with constant acceleration: upward acceleration for 10.0 s and then free-fall until it returns to the ground. Powers has three segments of motion with constant acceleration: upward acceleration for 10.0 s, free-fall for 7.0 s and then downward acceleration of 2.0 m/s^2.

SET UP: Let $+y$ be upward. Let $y = 0$ at the ground.

EXECUTE: **(a)** When the engine shuts off both objects have upward velocity $v_y = v_{0y} + a_y t = (5.0 \text{ m/s}^2)(10.0 \text{ s}) = 50.0 \text{ m/s}$ and are at $y = v_{0y} t + \frac{1}{2} a_y t^2 = \frac{1}{2}(5.0 \text{ m/s}^2)(10.0 \text{ s})^2 = 250 \text{ m}$. For the helicopter, $v_y = 0$ (at the maximum height), $v_{0y} = +50.0 \text{ m/s}$, $y_0 = 250 \text{ m}$, and $a_y = -9.80 \text{ m/s}^2$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y = \dfrac{v_y^2 - v_{0y}^2}{2a_y} + y_0 = \dfrac{0 - (50.0 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} + 250 \text{ m} = 378 \text{ m}$, which rounds to 380 m.

(b) The time for the helicopter to crash from the height of 250 m where the engines shut off can be found using $v_{0y} = +50.0 \text{ m/s}$, $a_y = -9.80 \text{ m/s}^2$, and $y - y_0 = -250 \text{ m}$. $y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$ gives $-250 \text{ m} = (50.0 \text{ m/s})t - (4.90 \text{ m/s}^2)t^2$. $(4.90 \text{ m/s}^2)t^2 - (50.0 \text{ m/s})t - 250 \text{ m} = 0$. The quadratic formula gives $t = \dfrac{1}{9.80}\left(50.0 \pm \sqrt{(50.0)^2 + 4(4.90)(250)}\right) \text{ s}$. Only the positive solution is physical, so $t = 13.9 \text{ s}$. Powers therefore has free-fall for 7.0 s and then downward acceleration of 2.0 m/s^2 for $13.9 \text{ s} - 7.0 \text{ s} = 6.9 \text{ s}$. After 7.0 s of free-fall he is at $y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 = 250 \text{ m} + (50.0 \text{ m/s})(7.0 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(7.0 \text{ s})^2 = 360 \text{ m}$ and has velocity $v_x = v_{0x} + a_x t = 50.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(7.0 \text{ s}) = -18.6 \text{ m/s}$. After the next 6.9 s he is at $y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2 = 360 \text{ m} + (-18.6 \text{ m/s})(6.9 \text{ s}) + \frac{1}{2}(-2.00 \text{ m/s}^2)(6.9 \text{ s})^2 = 184 \text{ m}$. Powers is 184 m above the ground when the helicopter crashes.

EVALUATE: When Powers steps out of the helicopter he retains the initial velocity he had in the helicopter but his acceleration changes abruptly from 5.0 m/s^2 upward to 9.80 m/s^2 downward. Without the jet pack he would have crashed into the ground at the same time as the helicopter. The jet pack slows his descent so he is above the ground when the helicopter crashes.

2.87. **IDENTIFY:** Apply the constant acceleration equations to his motion. Consider two segments of the motion: the last 1.0 s and the motion prior to that. The final velocity for the first segment is the initial velocity for the second segment.

SET UP: Let $+y$ be downward, so $a_y = +9.80 \text{ m/s}^2$.

EXECUTE: Motion from the roof to a height of $h/4$ above ground: $y - y_0 = 3h/4$, $a_y = +9.80 \text{ m/s}^2$, $v_{0y} = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2a_y(y - y_0)} = 3.834\sqrt{h}\ \sqrt{\text{m}}/\text{s}$. Motion from height of $h/4$ to the ground: $y - y_0 = h/4$, $a_y = +9.80 \text{ m/s}^2$, $v_{0y} = 3.834\sqrt{h}\ \sqrt{\text{m}}/\text{s}$, $t = 1.00 \text{ s}$. $y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$ gives

$\dfrac{h}{4} = 3.834\sqrt{h}\ \sqrt{\text{m}} + 4.90\ \text{m}$. Let $h = u^2$ and solve for *u*. $\frac{1}{4}u^2 - 3.834u\ \sqrt{\text{m}} - 4.90\ \text{m} = 0$.

$u = 2\left(3.834 \pm \sqrt{(-3.834)^2 + 4.90}\right)\ \sqrt{\text{m}}$. Only the positive root is physical, so $u = 16.52\ \sqrt{\text{m}}$ and $h = u^2 = 273\ \text{m}$, which rounds to 270 m. The building is 270 m tall.

EVALUATE: With $h = 273\ \text{m}$ the total time of fall is $t = \sqrt{\dfrac{2h}{a_y}} = 7.46\ \text{s}$. In $7.47\ \text{s} - 1.00\ \text{s} = 6.46\ \text{s}$ Spider-Man falls a distance $y - y_0 = \frac{1}{2}(9.80\ \text{m/s}^2)(6.46\ \text{s})^2 = 204\ \text{m}$. This leaves 69 m for the last 1.0 s of fall, which is $h/4$.

2.88. **IDENTIFY:** Apply constant acceleration equations to the motion of the rock. Sound travels at constant speed.

SET UP: Let t_{fall} be the time for the rock to fall to the ground and let t_{s} be the time it takes the sound to travel from the impact point back to you. $t_{\text{fall}} + t_{\text{s}} = 10.0\ \text{s}$. Both the rock and sound travel a distance *d* that is equal to the height of the cliff. Take $+y$ downward for the motion of the rock. The rock has $v_{0y} = 0$ and $a_y = 9.80\ \text{m/s}^2$.

EXECUTE: **(a)** For the rock, $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $t_{\text{fall}} = \sqrt{\dfrac{2d}{9.80\ \text{m/s}^2}}$.

For the sound, $t_{\text{s}} = \dfrac{d}{330\ \text{m/s}} = 10.0\ \text{s}$. Let $\alpha^2 = d$. $0.00303\alpha^2 + 0.4518\alpha - 10.0 = 0$. $\alpha = 19.6$ and $d = 384\ \text{m}$.

(b) You would have calculated $d = \frac{1}{2}(9.80\ \text{m/s}^2)(10.0\ \text{s})^2 = 490\ \text{m}$. You would have overestimated the height of the cliff. It actually takes the rock less time than 10.0 s to fall to the ground.

EVALUATE: Once we know *d* we can calculate that $t_{\text{fall}} = 8.8\ \text{s}$ and $t_{\text{s}} = 1.2\ \text{s}$. The time for the sound of impact to travel back to you is 12% of the total time and cannot be neglected. The rock has speed 86 m/s just before it strikes the ground.

2.89. **(a) IDENTIFY:** Let $+y$ be upward. The can has constant acceleration $a_y = -g$. The initial upward velocity of the can equals the upward velocity of the scaffolding; first find this speed.

SET UP: $y - y_0 = -15.0\ \text{m}$, $t = 3.25\ \text{s}$, $a_y = -9.80\ \text{m/s}^2$, $v_{0y} = ?$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $v_{0y} = 11.31\ \text{m/s}$

Use this v_{0y} in $v_y = v_{0y} + a_yt$ to solve for v_y: $v_y = -20.5\ \text{m/s}$

(b) IDENTIFY: Find the maximum height of the can, above the point where it falls from the scaffolding:

SET UP: $v_y = 0$, $v_{0y} = +11.31\ \text{m/s}$, $a_y = -9.80\ \text{m/s}^2$, $y - y_0 = ?$

EXECUTE: $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = 6.53\ \text{m}$

The can will pass the location of the other painter. Yes, he gets a chance.

EVALUATE: Relative to the ground the can is initially traveling upward, so it moves upward before stopping momentarily and starting to fall back down.

2.90. **IDENTIFY:** Both objects are in free-fall. Apply the constant acceleration equations to the motion of each person.

SET UP: Let $+y$ be downward, so $a_y = +9.80\ \text{m/s}^2$ for each object.

EXECUTE: **(a)** Find the time it takes the student to reach the ground: $y - y_0 = 180\ \text{m}$, $v_{0y} = 0$, $a_y = 9.80\ \text{m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(180\ \text{m})}{9.80\ \text{m/s}^2}} = 6.06\ \text{s}$. Superman must reach the ground in $6.06\ \text{s} - 5.00\ \text{s} = 1.06\ \text{s}$: $t = 1.06\ \text{s}$, $y - y_0 = 180\ \text{m}$, $a_y = +9.80\ \text{m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives

$v_{0y} = \dfrac{y - y_0}{t} - \frac{1}{2}a_yt = \dfrac{180\ \text{m}}{1.06\ \text{s}} - \frac{1}{2}(9.80\ \text{m/s}^2)(1.06\ \text{s}) = 165\ \text{m/s}$. Superman must have initial speed $v_0 = 165\ \text{m/s}$.

(b) The graphs of *y-t* for Superman and for the student are sketched in Figure 2.90.

(c) The minimum height of the building is the height for which the student reaches the ground in 5.00 s, before Superman jumps. $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 = \frac{1}{2}(9.80\ \text{m/s}^2)(5.00\ \text{s})^2 = 122\ \text{m}$. The skyscraper must be at least 122 m high.

EVALUATE: $165\text{ m/s} = 369\text{ mi/h}$, so only Superman could jump downward with this initial speed.

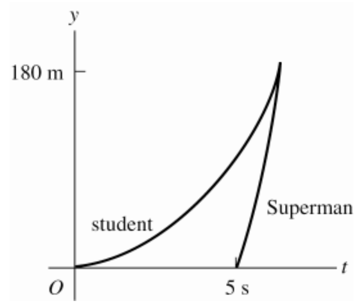


Figure 2.90

2.91. IDENTIFY: Apply constant acceleration equations to the motion of the rocket and to the motion of the canister after it is released. Find the time it takes the canister to reach the ground after it is released and find the height of the rocket after this time has elapsed. The canister travels up to its maximum height and then returns to the ground.

SET UP: Let $+y$ be upward. At the instant that the canister is released, it has the same velocity as the rocket. After it is released, the canister has $a_y = -9.80\text{ m/s}^2$. At its maximum height the canister has $v_y = 0$.

EXECUTE: **(a)** Find the speed of the rocket when the canister is released: $v_{0y} = 0$, $a_y = 3.30\text{ m/s}^2$, $y - y_0 = 235\text{ m}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(3.30\text{ m/s}^2)(235\text{ m})} = 39.4\text{ m/s}$. For the motion of the canister after it is released, $v_{0y} = +39.4\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$, $y - y_0 = -235\text{ m}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $-235\text{ m} = (39.4\text{ m/s})t - (4.90\text{ m/s}^2)t^2$. The quadratic formula gives $t = 12.0\text{ s}$ as the positive solution. Then for the motion of the rocket during this 12.0 s, $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 = 235\text{ m} + (39.4\text{ m/s})(12.0\text{ s}) + \frac{1}{2}(3.30\text{ m/s}^2)(12.0\text{ s})^2 = 945\text{ m}$.

(b) Find the maximum height of the canister above its release point: $v_{0y} = +39.4\text{ m/s}$, $v_y = 0$, $a_y = -9.80\text{ m/s}^2$.

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (39.4\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 79.2\text{ m}$. After its release the canister travels upward 79.2 m to its maximum height and then back down $79.2\text{ m} + 235\text{ m}$ to the ground. The total distance it travels is 393 m.

EVALUATE: The speed of the rocket at the instant that the canister returns to the launch pad is $v_y = v_{0y} + a_yt = 39.4\text{ m/s} + (3.30\text{ m/s}^2)(12.0\text{ s}) = 79.0\text{ m/s}$. We can calculate its height at this instant by $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $v_{0y} = 0$ and $v_y = 79.0\text{ m/s}$. $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{(79.0\text{ m/s})^2}{2(3.30\text{ m/s}^2)} = 946\text{ m}$, which agrees with our previous calculation.

2.92. IDENTIFY: Both objects are in free-fall and move with constant acceleration 9.80 m/s^2, downward. The two balls collide when they are at the same height at the same time.

SET UP: Let $+y$ be upward, so $a_y = -9.80\text{ m/s}^2$ for each ball. Let $y = 0$ at the ground. Let ball *A* be the one thrown straight up and ball *B* be the one dropped from rest at height *H*. $y_{0A} = 0$, $y_{0B} = H$.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ applied to each ball gives $y_A = v_0t - \frac{1}{2}gt^2$ and $y_B = H - \frac{1}{2}gt^2$. $y_A = y_B$ gives $v_0t - \frac{1}{2}gt^2 = H - \frac{1}{2}gt^2$ and $t = \dfrac{H}{v_0}$.

(b) For ball *A* at its highest point, $v_{yA} = 0$ and $v_y = v_{0y} + a_yt$ gives $t = \dfrac{v_0}{g}$. Setting this equal to the time in part (a) gives $\dfrac{H}{v_0} = \dfrac{v_0}{g}$ and $H = \dfrac{v_0^2}{g}$.

EVALUATE: In part (a), using $t = \dfrac{H}{v_0}$ in the expressions for y_A and y_B gives $y_A = y_B = H\left(1 - \dfrac{gH}{2v_0^2}\right)$. *H* must be less than $\dfrac{2v_0^2}{g}$ in order for the balls to collide before ball *A* returns to the ground. This is because it takes ball *A*

time $t = \dfrac{2v_0}{g}$ to return to the ground and ball B falls a distance $\frac{1}{2}gt^2 = \dfrac{2v_0^2}{g}$ during this time. When $H = \dfrac{2v_0^2}{g}$ the two balls collide just as ball A reaches the ground and for H greater than this ball A reaches the ground before they collide.

2.93. **IDENTIFY** and **SET UP:** Use $v_x = dx/dt$ and $a_x = dv_x/dt$ to calculate $v_x(t)$ and $a_x(t)$ for each car. Use these equations to answer the questions about the motion.

EXECUTE: $x_A = \alpha t + \beta t^2$, $v_{Ax} = \dfrac{dx_A}{dt} = \alpha + 2\beta t$, $a_{Ax} = \dfrac{dv_{Ax}}{dt} = 2\beta$

$x_B = \gamma t^2 - \delta t^3$, $v_{Bx} = \dfrac{dx_B}{dt} = 2\gamma t - 3\delta t^2$, $a_{Bx} = \dfrac{dv_{Bx}}{dt} - 2\gamma - 6\delta t$

(a) IDENTIFY and **SET UP:** The car that initially moves ahead is the one that has the larger v_{0x}.

EXECUTE: At $t = 0$, $v_{Ax} = \alpha$ and $v_{Bx} = 0$. So initially car A moves ahead.

(b) IDENTIFY and **SET UP:** Cars at the same point implies $x_A = x_B$.

$\alpha t + \beta t^2 = \gamma t^2 - \delta t^3$

EXECUTE: One solution is $t = 0$, which says that they start from the same point. To find the other solutions, divide by t: $\alpha + \beta t = \gamma t - \delta t^2$

$\delta t^2 + (\beta - \gamma)t + \alpha = 0$

$$t = \frac{1}{2\delta}\left(-(\beta-\gamma) \pm \sqrt{(\beta-\gamma)^2 - 4\delta\alpha}\right) = \frac{1}{0.40}\left(+1.60 \pm \sqrt{(1.60)^2 - 4(0.20)(2.60)}\right) = 4.00\text{ s} \pm 1.73\text{ s}$$

So $x_A = x_B$ for $t = 0$, $t = 2.27$ s and $t = 5.73$ s.

EVALUATE: Car A has constant, positive a_x. Its v_x is positive and increasing. Car B has $v_{0x} = 0$ and a_x that is initially positive but then becomes negative. Car B initially moves in the $+x$-direction but then slows down and finally reverses direction. At $t = 2.27$ s car B has overtaken car A and then passes it. At $t = 5.73$ s, car B is moving in the $-x$-direction as it passes car A again.

(c) IDENTIFY: The distance from A to B is $x_B - x_A$. The rate of change of this distance is $\dfrac{d(x_B - x_A)}{dt}$. If this distance is not changing, $\dfrac{d(x_B - x_A)}{dt} = 0$. But this says $v_{Bx} - v_{Ax} = 0$. (The distance between A and B is neither decreasing nor increasing at the instant when they have the same velocity.)

SET UP: $v_{Ax} = v_{Bx}$ requires $\alpha + 2\beta t = 2\gamma t - 3\delta t^2$

EXECUTE: $3\delta t^2 + 2(\beta - \gamma)t + \alpha = 0$

$$t = \frac{1}{6\delta}\left(-2(\beta-\gamma) \pm \sqrt{4(\beta-\gamma)^2 - 12\delta\alpha}\right) = \frac{1}{1.20}\left(3.20 \pm \sqrt{4(-1.60)^2 - 12(0.20)(2.60)}\right)$$

$t = 2.667\text{ s} \pm 1.667$ s, so $v_{Ax} = v_{Bx}$ for $t = 1.00$ s and $t = 4.33$ s.

EVALUATE: At $t = 1.00$ s, $v_{Ax} = v_{Bx} = 5.00$ m/s. At $t = 4.33$ s, $v_{Ax} = v_{Bx} = 13.0$ m/s. Now car B is slowing down while A continues to speed up, so their velocities aren't ever equal again.

(d) IDENTIFY and **SET UP:** $a_{Ax} = a_{Bx}$ requires $2\beta = 2\gamma - 6\delta t$

EXECUTE: $t = \dfrac{\gamma - \beta}{3\delta} = \dfrac{2.80\text{ m/s}^2 - 1.20\text{ m/s}^2}{3(0.20\text{ m/s}^3)} = 2.67$ s.

EVALUATE: At $t = 0$, $a_{Bx} > a_{Ax}$, but a_{Bx} is decreasing while a_{Ax} is constant. They are equal at $t = 2.67$ s but for all times after that $a_{Bx} < a_{Ax}$.

2.94. **IDENTIFY:** The apple has two segments of motion with constant acceleration. For the motion from the tree to the top of the grass the acceleration is g, downward and the apple falls a distance $H - h$. For the motion from the top of the grass to the ground the acceleration is a, upward, the apple travels downward a distance h, and the final speed is zero.

SET UP: Let $+y$ be upward and let $y = 0$ at the ground. The apple is initially a height $H + h$ above the ground.

EXECUTE: **(a)** Motion from $y_0 = H + h$ to $y = H$: $y - y_0 = -H$, $v_{0y} = 0$, $a_y = -g$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = -\sqrt{2gH}$. The speed of the apple is $\sqrt{2gH}$ as it enters the grass.

(b) Motion from $y_0 = h$ to $y = 0$: $y - y_0 = -h$, $v_{0y} = -\sqrt{2gH}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$a_y = \dfrac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \dfrac{-2gH}{2(-h)} = \dfrac{gH}{h}$. The acceleration of the apple while it is in the grass is gH/h, upward.

(c) Graphs of y-t, v_y-t and a_y-t are sketched in Figure 2.94.

EVALUATE: The acceleration a produced by the grass increases when H increases and decreases when h increases.

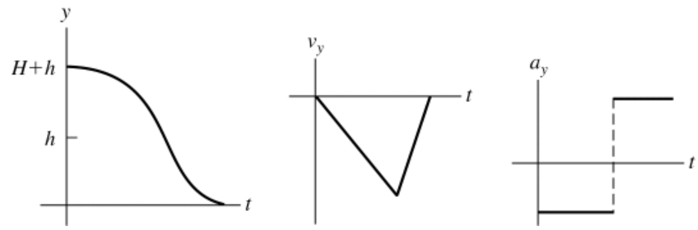


Figure 2.94

2.95. IDENTIFY: Apply constant acceleration equations to the motion of the two objects, the student and the bus.

SET UP: For convenience, let the student's (constant) speed be v_0 and the bus's initial position be x_0. Note that these quantities are for separate objects, the student and the bus. The initial position of the student is taken to be zero, and the initial velocity of the bus is taken to be zero. The positions of the student x_1 and the bus x_2 as functions of time are then $x_1 = v_0 t$ and $x_2 = x_0 + (1/2)at^2$.

EXECUTE: **(a)** Setting $x_1 = x_2$ and solving for the times t gives $t = \dfrac{1}{a}\left(v_0 \pm \sqrt{v_0^2 - 2ax_0}\right)$.

$$t = \frac{1}{(0.170\ \text{m/s}^2)}\left((5.0\ \text{m/s}) \pm \sqrt{(5.0\ \text{m/s})^2 - 2(0.170\ \text{m/s}^2)(40.0\ \text{m})}\right) = 9.55\ \text{s and } 49.3\ \text{s}.$$

The student will be likely to hop on the bus the first time she passes it (see part (d) for a discussion of the later time). During this time, the student has run a distance $v_0 t = (5\ \text{m/s})(9.55\ \text{s}) = 47.8\ \text{m}$.

(b) The speed of the bus is $(0.170\ \text{m/s}^2)(9.55\ \text{s}) = 1.62\ \text{m/s}$.

(c) The results can be verified by noting that the x lines for the student and the bus intersect at two points, as shown in Figure 2.95a.

(d) At the later time, the student has passed the bus, maintaining her constant speed, but the accelerating bus then catches up to her. At this later time the bus's velocity is $\left(0.170\ \text{m/s}^2\right)\left(49.3\ \text{s}\right) = 8.38\ \text{m/s}$.

(e) No; $v_0^2 < 2ax_0$, and the roots of the quadratic are imaginary. When the student runs at $3.5\ \text{m/s}$, Figure 2.95b shows that the two lines do *not* intersect:

(f) For the student to catch the bus, $v_0^2 > 2ax_0$. and so the minimum speed is

$\sqrt{2\left(0.170\ \text{m/s}^2\right)\left(40\ \text{m/s}\right)} = 3.688\ \text{m/s}$. She would be running for a time $\dfrac{3.69\ \text{m/s}}{0.170\ \text{m/s}^2} = 21.7\ \text{s}$, and covers a distance $\left(3.688\ \text{m/s}\right)\ \left(21.7\ \text{s}\right) = 80.0\ \text{m}$.

However, when the student runs at $3.688\ \text{m/s}$, the lines intersect at *one* point, at $x = 80\ \text{m}$, as shown in Figure 2.95c.

EVALUATE: The graph in part (c) shows that the student is traveling faster than the bus the first time they meet but at the second time they meet the bus is traveling faster.

$t_2 = t_{\text{tot}} - t_1$

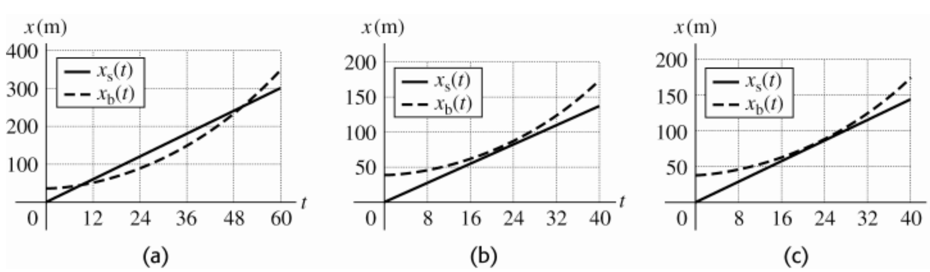


Figure 2.95

2.96. **IDENTIFY:** Apply $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ to the motion from the maximum height, where $v_{0y} = 0$. The time spent above $y_{\max}/2$ on the way down equals the time spent above $y_{\max}/2$ on the way up.

SET UP: Let $+y$ be downward. $a_y = g$. $y - y_0 = y_{\max}/2$ when he is a distance $y_{\max}/2$ above the floor.

EXECUTE: The time from the maximum height to $y_{\max}/2$ above the floor is given by $y_{\max}/2 = \frac{1}{2}gt_1^2$. The time from the maximum height to the floor is given by $y_{\max} = \frac{1}{2}gt_{\text{tot}}^2$ and the time from a height of $y_{\max}/2$ to the floor is .

$$\frac{t_1}{t_2} = \frac{\sqrt{y_{\max}/2}}{\sqrt{y_{\max}} - \sqrt{y_{\max}/2}} = \frac{1}{\sqrt{2} - 1} = 2.4\,.$$

EVALUATE: The person spends over twice as long above $y_{\max}/2$ as below $y_{\max}/2$. His average speed is less above $y_{\max}/2$ than it is when he is below this height.

2.97. **IDENTIFY:** Apply constant acceleration equations to both objects.

SET UP: Let $+y$ be upward, so each ball has $a_y = -g$. For the purpose of doing all four parts with the least repetition of algebra, quantities will be denoted symbolically. That is, let $y_1 = h + v_0 t - \dfrac{1}{2}gt^2$, $y_2 = h - \dfrac{1}{2}g(t - t_0)^2$. In this case, $t_0 = 1.00\text{ s}$.

EXECUTE: **(a)** Setting $y_1 = y_2 = 0$, expanding the binomial $(t - t_0)^2$ and eliminating the common term $\frac{1}{2}gt^2$ yields $v_0 t = gt_0 t - \frac{1}{2}gt_0^2$. Solving for *t*: $t = \dfrac{\frac{1}{2}gt_0^2}{gt_0 - v_0} = \dfrac{t_0}{2}\left(\dfrac{1}{1 - v_0/(gt_0)}\right)$.

Substitution of this into the expression for y_1 and setting $y_1 = 0$ and solving for *h* as a function of v_0 yields, after some algebra, $h = \frac{1}{2}gt_0^2\dfrac{\left(\frac{1}{2}gt_0 - v_0\right)^2}{\left(gt_0 - v_0\right)^2}$. Using the given value $t_0 = 1.00\text{ s}$ and $g = 9.80\text{ m/s}^2$,

$$h = 20.0\text{ m} = (4.9\text{ m})\left(\frac{4.9\text{ m/s} - v_0}{9.8\text{ m/s} - v_0}\right)^2.$$

This has two solutions, one of which is unphysical (the first ball is still going up when the second is released; see part (c)). The physical solution involves taking the negative square root before solving for v_0, and yields 8.2 m/s. The graph of *y* versus *t* for each ball is given in Figure 2.97.

(b) The above expression gives for (i), 0.411 m and for (ii) 1.15 km.

(c) As v_0 approaches 9.8 m/s, the height *h* becomes infinite, corresponding to a relative velocity at the time the second ball is thrown that approaches zero. If $v_0 > 9.8\text{ m/s}$, the first ball can never catch the second ball.

(d) As v_0 approaches 4.9 m/s, the height approaches zero. This corresponds to the first ball being closer and closer (on its way down) to the top of the roof when the second ball is released. If $v_0 < 4.9\text{ m/s}$, the first ball will already have passed the roof on the way down before the second ball is released, and the second ball can never catch up.

EVALUATE: Note that the values of v_0 in parts (a) and (b) are all greater than $v_{\min}$ and less than $v_{\max}$.

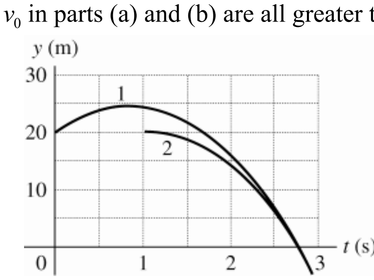


Figure 2.97

2.98. **IDENTIFY:** Apply constant acceleration equations to the motion of the boulder.

SET UP: Let $+y$ be downward, so $a_y = +g$.

EXECUTE: (a) Let the height be h and denote the 1.30-s interval as Δt; the simultaneous equations $h = \frac{1}{2}gt^2, \frac{2}{3}h = \frac{1}{2}g(t - \Delta t)^2$ can be solved for t. Eliminating h and taking the square root, $\frac{t}{t - \Delta t} = \sqrt{\frac{3}{2}}$, and $t = \frac{\Delta t}{1 - \sqrt{2/3}}$, and substitution into $h = \frac{1}{2}gt^2$ gives $h = 246$ m.

(b) The above method assumed that $t > 0$ when the square root was taken. The negative root (with $\Delta t = 0$) gives an answer of 2.51 m, clearly not a "cliff". This would correspond to an object that was initially near the bottom of this "cliff" being thrown upward and taking 1.30 s to rise to the top and fall to the bottom. Although physically possible, the conditions of the problem preclude this answer.

EVALUATE: For the first two-thirds of the distance, $y - y_0 = 164$ m, $v_{0y} = 0$, and $a_y = 9.80$ m/s^2. $v_y = \sqrt{2a_y(y - y_0)} = 56.7$ m/s. Then for the last third of the distance, $y - y_0 = 82.0$ m, $v_{0y} = 56.7$ m/s and $a_y = 9.80$ m/s^2. $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $(4.90\text{ m/s}^2)t^2 + (56.7\text{ m/s})t - 82.0\text{ m} = 0$.

$t = \frac{1}{9.8}\left(-56.7 + \sqrt{(56.7)^2 + 4(4.9)(82.0)}\right)$ s $= 1.30$ s, as required.

MOTION IN TWO OR THREE DIMENSIONS

3

3.1. **IDENTIFY** and **SET UP:** Use Eq.(3.2), in component form.

EXECUTE: $\left(v_{av}\right)_x = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{5.3\text{ m} - 1.1\text{ m}}{3.0\text{ s} - 0} = 1.4\text{ m/s}$

$\left(v_{av}\right)_y = \dfrac{\Delta y}{\Delta t} = \dfrac{y_2 - y_1}{t_2 - t_1} = \dfrac{-0.5\text{ m} - 3.4\text{ m}}{3.0\text{ s} - 0} = -1.3\text{ m/s}$

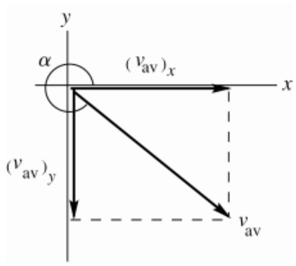


Figure 3.1

$\tan\alpha = \dfrac{\left(v_{av}\right)_y}{\left(v_{av}\right)_x} = \dfrac{-1.3\text{ m/s}}{1.4\text{ m/s}} = -0.9286$

$\alpha = 360° - 42.9° = 317°$

$v_{av} = \sqrt{\left(v_{av}\right)_x^2 + \left(v_{av}\right)_y^2}$

$v_{av} = \sqrt{(1.4\text{ m/s})^2 + (-1.3\text{ m/s})^2} = 1.9\text{ m/s}$

EVALUATE: Our calculation gives that $\vec{v}_{av}$ is in the 4th quadrant. This corresponds to increasing x and decreasing y.

3.2. **IDENTIFY:** Use Eq.(3.2), written in component form. The distance from the origin is the magnitude of $\vec{r}$.

SET UP: At time t_1, $x_1 = y_1 = 0$.

EXECUTE: **(a)** $x = (v_{av\text{-}x})\Delta t = (-3.8\text{ m/s})(12.0\text{ s}) = -45.6\text{ m}$ and $y = (v_{av\text{-}y})\Delta t = (4.9\text{ m/s})(12.0\text{ s}) = 58.8\text{ m}$.

(b) $r = \sqrt{x^2 + y^2} = \sqrt{(-45.6\text{ m})^2 + (58.8\text{ m})^2} = 74.4\text{ m}$.

EVALUATE: $\Delta\vec{r}$ is in the direction of $\vec{v}_{av}$. Therefore, Δx is negative since $v_{av\text{-}x}$ is negative and Δy is positive since $v_{av\text{-}y}$ is positive.

3.3. **(a) IDENTIFY** and **SET UP:** From $\vec{r}$ we can calculate x and y for any t.
Then use Eq.(3.2), in component form.

EXECUTE: $\vec{r} = \left[4.0\text{ cm} + \left(2.5\text{ cm/s}^2\right)t^2\right]\hat{i} + \left(5.0\text{ cm/s}\right)t\hat{j}$

At $t = 0$, $\vec{r} = \left(4.0\text{ cm}\right)\hat{i}$.

At $t = 2.0\text{ s}$, $\vec{r} = \left(14.0\text{ cm}\right)\hat{i} + \left(10.0\text{ cm}\right)\hat{j}$.

$\left(v_{av}\right)_x = \dfrac{\Delta x}{\Delta t} = \dfrac{10.0\text{ cm}}{2.0\text{ s}} = 5.0\text{ cm/s}.$

$\left(v_{av}\right)_y = \dfrac{\Delta y}{\Delta t} = \dfrac{10.0\text{ cm}}{2.0\text{ s}} = 5.0\text{ cm/s}.$

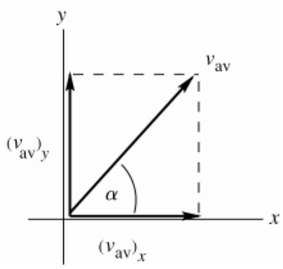


Figure 3.3a

$$v_{av} = \sqrt{(v_{av})_x^2 + (v_{av})_y^2} = 7.1 \text{ cm/s}$$

$$\tan\alpha = \frac{(v_{av})_y}{(v_{av})_x} = 1.00$$

$\theta = 45°.$

EVALUATE: Both x and y increase, so $\vec{v}_{av}$ is in the 1st quadrant.

(b) IDENTIFY and **SET UP:** Calculate $\vec{r}$ by taking the time derivative of $\vec{r}(t)$.

EXECUTE: $\vec{v} = \frac{d\vec{r}}{dt} = \left(\left[5.0 \text{ cm/s}^2\right]t\right)\hat{i} + (5.0 \text{ cm/s})\hat{j}$

$\underline{t = 0}$: $v_x = 0$, $v_y = 5.0$ cm/s; $v = 5.0$ cm/s and $\theta = 90°$

$\underline{t = 1.0 \text{ s}}$: $v_x = 5.0$ cm/s, $v_y = 5.0$ cm/s; $v = 7.1$ cm/s and $\theta = 45°$

$\underline{t = 2.0 \text{ s}}$: $v_x = 10.0$ cm/s, $v_y = 5.0$ cm/s; $v = 11$ cm/s and $\theta = 27°$

(c) The trajectory is a graph of y versus x.

$x = 4.0 \text{ cm} + (2.5 \text{ cm/s}^2)t^2$, $y = (5.0 \text{ cm/s})t$

For values of t between 0 and 2.0 s, calculate x and y and plot y versus x.

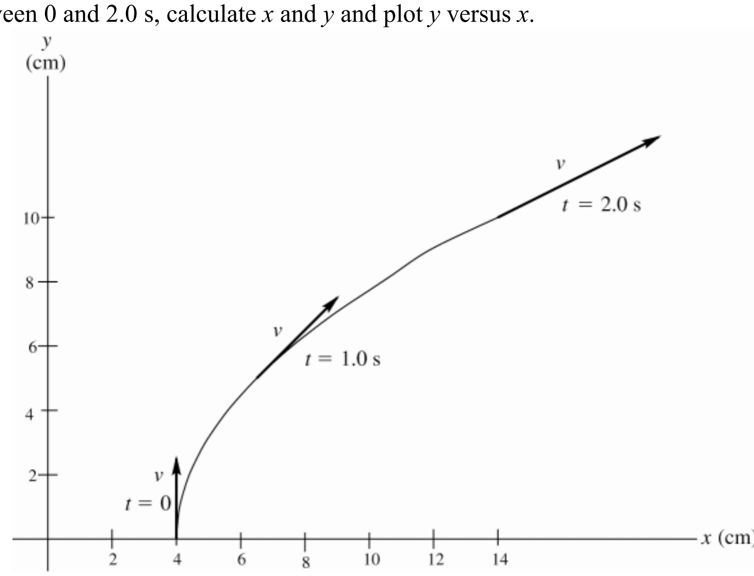


Figure 3.3b

EVALUATE: The sketch shows that the instantaneous velocity at any t is tangent to the trajectory.

3.4. **IDENTIFY:** $\vec{v} = d\vec{r}/dt$. This vector will make a $45°$-angle with both axes when its x- and y-components are equal.

SET UP: $\frac{d(t^n)}{dt} = nt^{n-1}$.

EXECUTE: $\vec{v} = 2bt\hat{i} + 3ct^2\hat{j}$. $v_x = v_y$ gives $t = 2b/3c$.

EVALUATE: Both components of $\vec{v}$ change with t.

3.5. **IDENTIFY** and **SET UP:** Use Eq.(3.8) in component form to calculate $(a_{av})_x$ and $(a_{av})_y$.

EXECUTE: **(a)** The velocity vectors at $t_1 = 0$ and $t_2 = 30.0\text{ s}$ are shown in Figure 3.5a.

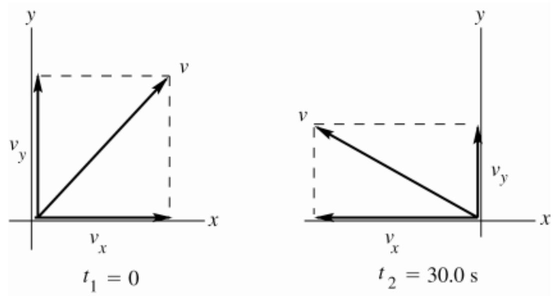


Figure 3.5a

(b) $(a_{\text{av}})_x = \dfrac{\Delta v_x}{\Delta t} = \dfrac{v_{2x} - v_{1x}}{t_2 - t_1} = \dfrac{-170\text{ m/s} - 90\text{ m/s}}{30.0\text{ s}} = -8.67\text{ m/s}^2$

$(a_{\text{av}})_y = \dfrac{\Delta v_y}{\Delta t} = \dfrac{v_{2y} - v_{1y}}{t_2 - t_1} = \dfrac{40\text{ m/s} - 110\text{ m/s}}{30.0\text{ s}} = -2.33\text{ m/s}^2$

(c)

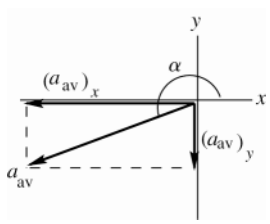


Figure 3.5b

$$a = \sqrt{(a_{\text{av}})_x^2 + (a_{\text{av}})_y^2} = 8.98\text{ m/s}^2$$

$$\tan\alpha = \frac{(a_{\text{av}})_y}{(a_{\text{av}})_x} = \frac{-2.33\text{ m/s}^2}{-8.67\text{ m/s}^2} = 0.269$$

$$\alpha = 15^\circ + 180^\circ = 195^\circ$$

EVALUATE: The changes in v_x and v_y are both in the negative x or y direction, so both components of $\vec{a}_{\text{av}}$ are in the 3rd quadrant.

3.6. **IDENTIFY:** Use Eq.(3.8), written in component form.

SET UP: $a_x = (0.45\text{m/s}^2)\cos 31.0^\circ = 0.39\text{m/s}^2$, $a_y = (0.45\text{m/s}^2)\sin 31.0^\circ = 0.23\text{m/s}^2$

EXECUTE: **(a)** $a_{\text{av-}x} = \dfrac{\Delta v_x}{\Delta t}$ and $v_x = 2.6\text{ m/s} + (0.39\text{ m/s}^2)(10.0\text{ s}) = 6.5\text{ m/s}$. $a_{\text{av-}y} = \dfrac{\Delta v_y}{\Delta t}$ and

$v_y = -1.8\text{ m/s} + (0.23\text{ m/s}^2)(10.0\text{ s}) = 0.52\text{ m/s}$.

(b) $v = \sqrt{(6.5\text{m/s})^2 + (0.52\text{m/s})^2} = 6.48\text{m/s}$, at an angle of $\arctan\left(\dfrac{0.52}{6.5}\right) = 4.6^\circ$ above the horizontal.

(c) The velocity vectors $\vec{v}_1$ and $\vec{v}_2$ are sketched in Figure 3.6. The two velocity vectors differ in magnitude and direction.

EVALUATE: $\vec{v}_1$ is at an angle of 35° below the $+x$-axis and has magnitude $v_1 = 3.2\text{ m/s}$, so $v_2 > v_1$ and the direction of $\vec{v}_2$ is rotated counterclockwise from the direction of $\vec{v}_1$.

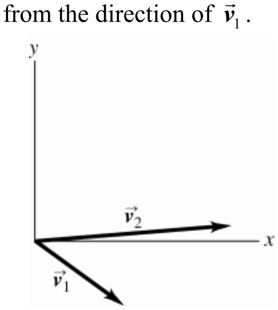


Figure 3.6

3.7. **IDENTIFY** and **SET UP:** Use Eqs.(3.4) and (3.12) to find v_x, v_y, a_x, and a_y as functions of time. The magnitude and direction of $\vec{r}$ and $\vec{a}$ can be found once we know their components.

EXECUTE: **(a)** Calculate *x* and *y* for *t* values in the range 0 to 2.0 s and plot *y* versus *x*. The results are given in Figure 3.7a.

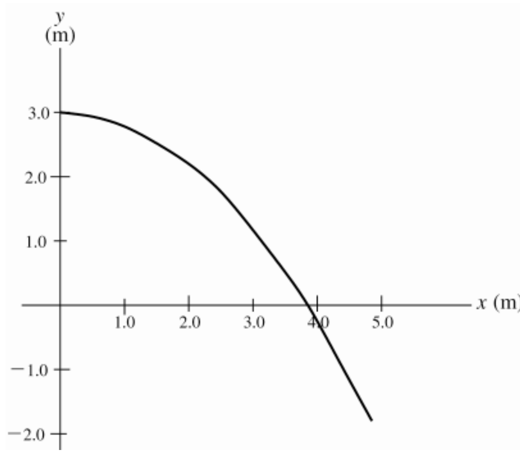


Figure 3.7a

(b) $v_x = \dfrac{dx}{dt} = \alpha \quad v_y = \dfrac{dy}{dt} = -2\beta t$

$a_y = \dfrac{dv_x}{dt} = 0 \quad a_y = \dfrac{dv_y}{dt} = -2\beta$

Thus $\vec{\boldsymbol{v}} = a\hat{\boldsymbol{i}} - 2\beta t\hat{\boldsymbol{j}} \quad \vec{\boldsymbol{a}} = -2\beta\hat{\boldsymbol{j}}$

(c) velocity: At $t = 2.0\ \text{s}, \quad v_x = 2.4\ \text{m/s}, \quad v_y = -2(1.2\ \text{m/s}^2)(2.0\ \text{s}) = -4.8\ \text{m/s}$

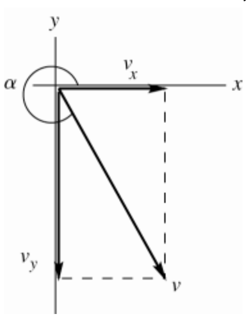


Figure 3.7b

$$v = \sqrt{v_x^2 + v_y^2} = 5.4\ \text{m/s}$$

$$\tan\alpha = \frac{v_y}{v_x} = \frac{-4.8\ \text{m/s}}{2.4\ \text{m/s}} = -2.00$$

$$\alpha = -63.4^\circ + 360^\circ = 297^\circ$$

acceleration: At $t = 2.0\ \text{s}, \quad a_x = 0, \quad a_y = -2(1.2\ \text{m/s}^2) = -2.4\ \text{m/s}^2$

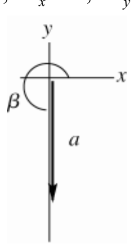


Figure 3.7c

$$a = \sqrt{a_x^2 + a_y^2} = 2.4\ \text{m/s}^2$$

$$\tan\beta = \frac{a_y}{a_x} = \frac{-2.4\ \text{m/s}^2}{0} = -\infty$$

$$\beta = 270^\circ$$

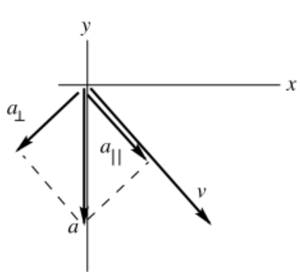


Figure 3.7d

EVALUATE: **(d)** $\vec{\boldsymbol{a}}$ has a component $a_\parallel$ in the same direction as $\vec{\boldsymbol{v}}$, so we know that *v* is increasing (the bird is speeding up.) $\vec{\boldsymbol{a}}$ also has a component $a_\perp$ perpendicular to $\vec{\boldsymbol{v}}$, so that the direction of $\vec{\boldsymbol{v}}$ is changing; the bird is turning toward the $-y$-direction (toward the right)

$\vec{v}$ is always tangent to the path; $\vec{v}$ at $t = 2.0$ s shown in part (c) is tangent to the path at this t, conforming to this general rule. $\vec{a}$ is constant and in the $-y$-direction; the direction of $\vec{v}$ is turning toward the $-y$-direction.

3.8. **IDENTIFY:** The component $\vec{a}_\perp$ of $\vec{a}$ perpendicular to the path is related to the change in direction of $\vec{v}$ and the component $\vec{a}_\parallel$ of $\vec{a}$ parallel to the path is related to the change in the magnitude of $\vec{v}$.

SET UP: When the speed is increasing, $\vec{a}_\parallel$ is in the direction of $\vec{v}$ and when the speed is decreasing, $\vec{a}_\parallel$ is opposite to the direction of $\vec{v}$. When v is constant, $a_\parallel$ is zero and when the path is a straight line, $a_\perp$ is zero.

EXECUTE: The acceleration vectors in each case are sketched in Figure 3.8a-c.

EVALUATE: $\vec{a}_\perp$ is toward the center of curvature of the path.

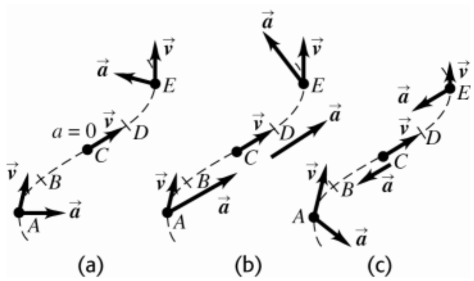


Figure 3.8a-c

3.9. **IDENTIFY:** The book moves in projectile motion once it leaves the table top. Its initial velocity is horizontal.
SET UP: Take the positive y-direction to be upward. Take the origin of coordinates at the initial position of the book, at the point where it leaves the table top.

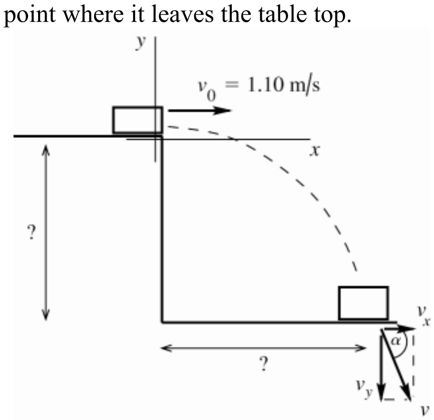


Figure 3.9a

<u>x-component</u>:
$a_x = 0,\ v_{0x} = 1.10$ m/s,
$t = 0.350$ s
<u>y-component</u>:
$a_y = -9.80\ \text{m/s}^2$,
$v_{0y} = 0$,
$t = 0.350$ s

Use constant acceleration equations for the x and y components of the motion, with $a_x = 0$ and $a_y = -g$.

EXECUTE: **(a)** $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(-9.80\ \text{m/s}^2)(0.350\ \text{s})^2 = -0.600$ m. The table top is 0.600 m above the floor.

(b) $x - x_0 = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (1.10\ \text{m/s})(0.350\ \text{s}) + 0 = 0.358$ m.

(c) $v_x = v_{0x} + a_x t = 1.10$ m/s (The x-component of the velocity is constant, since $a_x = 0$.)

$v_y = v_{0y} + a_y t = 0 + (-9.80\ \text{m/s}^2)(0.350\ \text{s}) = -3.43$ m/s

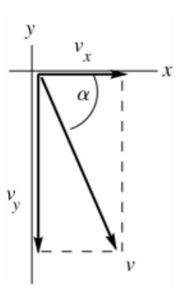


Figure 3.9b

$v = \sqrt{v_x^2 + v_y^2} = 3.60$ m/s

$\tan\alpha = \dfrac{v_y}{v_x} = \dfrac{-3.43\ \text{m/s}}{1.10\ \text{m/s}} = -3.118$

$\alpha = -72.2°$

Direction of $\vec{v}$ is $72.2°$ below the horizontal

(d) The graphs are given in Figure 3.9c

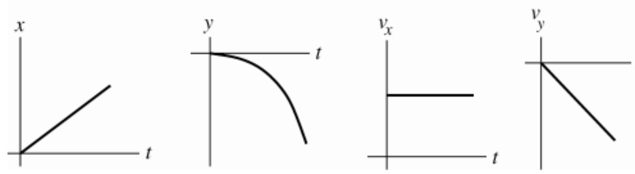


Figure 3.9c

EVALUATE: In the *x*-direction, $a_x = 0$ and v_x is constant. In the *y*-direction, $a_y = -9.80\text{ m/s}^2$ and v_y is downward and increasing in magnitude since a_y and v_y are in the same directions. The *x* and *y* motions occur independently, connected only by the time. The time it takes the book to fall 0.600 m is the time it travels horizontally.

3.10. IDENTIFY: The bomb moves in projectile motion. Treat the horizontal and vertical components of the motion separately. The vertical motion determines the time in the air.

SET UP: The initial velocity of the bomb is the same as that of the helicopter. Take $+y$ downward, so $a_x = 0$, $a_y = +9.80\text{ m/s}^2$, $v_{0x} = 60.0\text{ m/s}$ and $v_{0y} = 0$.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 300\text{ m}$ gives $t = \sqrt{\frac{2(y-y_0)}{a_y}} = \sqrt{\frac{2(300\text{ m})}{9.80\text{ m/s}^2}} = 7.82\text{ s}$.

(b) The bomb travels a horizontal distance $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (60.0\text{ m/s})(7.82\text{ s}) = 470\text{ m}$.

(c) $v_x = v_{0x} = 60.0\text{ m/s}$. $v_y = v_{0y} + a_y t = (9.80\text{ m/s}^2)(7.82\text{ s}) = 76.6\text{ m/s}$.

(d) The graphs are given in Figure 3.10.

(e) Because the airplane and the bomb always have the same *x*-component of velocity and position, the plane will be 300 m directly above the bomb at impact.

EVALUATE: The initial horizontal velocity of the bomb doesn't affect its vertical motion.

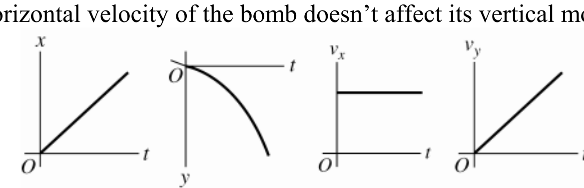


Figure 3.10

3.11. IDENTIFY: Each object moves in projectile motion.

SET UP: Take $+y$ to be downward. For each cricket, $a_x = 0$ and $a_y = +9.80\text{ m/s}^2$. For Chirpy, $v_{0x} = v_{0y} = 0$. For Milada, $v_{0x} = 0.950\text{ m/s}$, $v_{0y} = 0$

EXECUTE: Milada's horizontal component of velocity has no effect on her vertical motion. She also reaches the ground in 3.50 s. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (0.950\text{ m/s})(3.50\text{ s}) = 3.32\text{ m}$

EVALUATE: The *x* and *y* components of motion are totally separate and are connected only by the fact that the time is the same for both.

3.12. IDENTIFY: The person moves in projectile motion. She must travel 1.75 m horizontally during the time she falls 9.00 m vertically.

SET UP: Take $+y$ downward. $a_x = 0$, $a_y = +9.80\text{ m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

EXECUTE: Time to fall 9.00 m: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\frac{2(y-y_0)}{a_y}} = \sqrt{\frac{2(9.00\text{ m})}{9.80\text{ m/s}^2}} = 1.36\text{ s}$.

Speed needed to travel 1.75 m horizontally during this time: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$$v_0 = v_{0x} = \frac{x - x_0}{t} = \frac{1.75\text{ m}}{1.36\text{ s}} = 1.29\text{ m/s}.$$

EVALUATE: If she increases her initial speed she still takes 1.36 s to reach the level of the ledge, but has traveled horizontally farther than 1.75 m.

3.13. IDENTIFY: The car moves in projectile motion. The car travels $21.3\text{ m} - 1.80\text{ m} = 19.5\text{ m}$ downward during the time it travels 61.0 m horizontally.

SET UP: Take $+y$ to be downward. $a_x = 0$, $a_y = +9.80\text{ m/s}^2$. $v_{0x} = v_0$, $v_{0y} = 0$.

EXECUTE: Use the vertical motion to find the time in the air:

$y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $t=\sqrt{\dfrac{2(y-y_0)}{a_y}}=\sqrt{\dfrac{2(19.5\text{ m})}{9.80\text{ m/s}^2}}=1.995\text{ s}$

Then $x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$ gives $v_0=v_{0x}=\dfrac{x-x_0}{t}=\dfrac{61.0\text{ m}}{1.995\text{ s}}=30.6\text{ m/s}$.

(b) $v_x=30.6\,\text{m/s}$ since $a_x=0$. $v_y=v_{0y}+a_yt=-19.6\,\text{m/s}$. $v=\sqrt{v_x^2+v_y^2}=36.3\,\text{m/s}$.

EVALUATE: We calculate the final velocity by calculating its *x* and *y* components.

3.14. **IDENTIFY:** The marble moves with projectile motion, with initial velocity that is horizontal and has magnitude v_0. Treat the horizontal and vertical motions separately. If v_0 is too small the marble will land to the left of the hole and if v_0 is too large the marble will land to the right of the hole.

SET UP: Let $+x$ be horizontal to the right and let $+y$ be upward. $v_{0x}=v_0$, $v_{0y}=0$, $a_x=0$, $a_y=-9.80\text{ m/s}^2$

EXECUTE: Use the vertical motion to find the time it takes the marble to reach the height of the level ground; $y-y_0=-2.75\text{ m}$. $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $t=\sqrt{\dfrac{2(y-y_0)}{a_y}}=\sqrt{\dfrac{2(-2.75\text{ m})}{-9.80\text{ m/s}^2}}=0.749\text{ s}$. The time does not depend on v_0.

Minimum v_0: $x-x_0=2.00\text{ m}$, $t=0.749\text{ s}$. $x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$ gives $v_0=\dfrac{x-x_0}{t}=\dfrac{2.00\text{ m}}{0.749\text{ s}}=2.67\text{ m/s}$.

Maximum v_0: $x-x_0=3.50\text{ m}$ and $v_0=\dfrac{3.50\text{ m}}{0.749\text{ s}}=4.67\text{ m/s}$.

EVALUATE: The horizontal and vertical motions are independent and are treated separately. Their only connection is that the time is the same for both.

3.15. **IDENTIFY:** The ball moves with projectile motion with an initial velocity that is horizontal and has magnitude v_0. The height *h* of the table and v_0 are the same; the acceleration due to gravity changes from $g_{\text{E}}=9.80\text{ m/s}^2$ on earth to g_{X} on planet X.

SET UP: Let $+x$ be horizontal and in the direction of the initial velocity of the marble and let $+y$ be upward. $v_{0x}=v_0$, $v_{0y}=0$, $a_x=0$, $a_y=-g$, where *g* is either g_{E} or g_{X}.

EXECUTE: Use the vertical motion to find the time in the air: $y-y_0=-h$. $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $t=\sqrt{\dfrac{2h}{g}}$.

Then $x-x_0=v_{0x}t+\frac{1}{2}a_xt^2$ gives $x-x_0=v_{0x}t=v_0\sqrt{\dfrac{2h}{g}}$. $x-x_0=D$ on earth and 2.76*D* on Planet X.

$(x-x_0)\sqrt{g}=v_0\sqrt{2h}$, which is constant, so $D\sqrt{g_{\text{E}}}=2.76D\sqrt{g_{\text{X}}}$. $g_{\text{X}}=\dfrac{g_{\text{E}}}{(2.76)^2}=0.131g_{\text{E}}=1.28\text{ m/s}^2$.

EVALUATE: On Planet X the acceleration due to gravity is less, it takes the ball longer to reach the floor, and it travels farther horizontally.

3.16. **IDENTIFY:** The football moves in projectile motion.

SET UP: Let $+y$ be upward. $a_x=0$, $a_y=-g$. At the highest point in the trajectory, $v_y=0$.

EXECUTE: **(a)** $v_y=v_{0y}+a_yt$. The time *t* is $\dfrac{v_{0y}}{g}=\dfrac{16.0\,\text{m/s}}{9.80\,\text{m/s}^2}=1.63\,\text{s}$.

(b) Different constant acceleration equations give different expressions but the same numerical result:

$\frac{1}{2}gt^2=\frac{1}{2}v_{y0}t=\dfrac{v_{0y}^2}{2g}=13.1\text{ m}$.

(c) Regardless of how the algebra is done, the time will be twice that found in part (a), or 3.27 s

(d) $a_x=0$, so $x-x_0=v_{0x}t=(20.0\text{ m/s})(3.27\text{ s})=65.3\text{ m}$.

(e) The graphs are sketched in Figure 3.16.

EVALUATE: When the football returns to its original level, $v_x = 20.0$ m/s and $v_y = -16.0$ m/s .

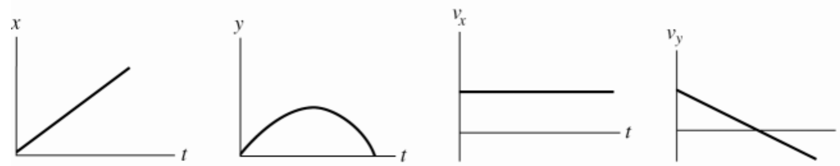


Figure 3.16

3.17. **IDENTIFY:** The shell moves in projectile motion.

SET UP: Let $+x$ be horizontal, along the direction of the shell's motion, and let $+y$ be upward. $a_x = 0$, $a_y = -9.80\ \text{m/s}^2$.

EXECUTE: **(a)** $v_{0x} = v_0 \cos\alpha_0 = (80.0\ \text{m/s})\cos 60.0° = 40.0$ m/s , $v_{0y} = v_0 \sin\alpha_0 = (80.0\ \text{m/s})\sin 60.0° = 69.3$ m/s .

(b) At the maximum height $v_y = 0$. $v_y = v_{0y} + a_y t$ gives $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{0 - 69.3\ \text{m/s}}{-9.80\ \text{m/s}^2} = 7.07$ s .

(c) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (69.3\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = 245$ m .

(d) The total time in the air is twice the time to the maximum height, so $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (40.0\ \text{m/s})(14.14\ \text{s}) = 566$ m .

(e) At the maximum height, $v_x = v_{0x} = 40.0$ m/s and $v_y = 0$. At all points in the motion, $a_x = 0$ and $a_y = -9.80\ \text{m/s}^2$.

EVALUATE: The equation for the horizontal range *R* derived in Example 3.8 is $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$. This gives $R = \dfrac{(80.0\ \text{m/s})^2 \sin(120.0°)}{9.80\ \text{m/s}^2} = 566$ m , which agrees with our result in part (d).

3.18. **IDENTIFY:** The flare moves with projectile motion. The equations derived in Example 3.8 can be used to find the maximum height *h* and range *R*.

SET UP: From Example 3.8, $h = \dfrac{v_0^2 \sin^2\alpha_0}{2g}$ and $R = \dfrac{v_0^2 \sin 2\alpha_0}{g}$.

EXECUTE: **(a)** $h = \dfrac{(125\ \text{m/s})^2(\sin 55.0°)^2}{2(9.80\ \text{m/s}^2)} = 535$ m . $R = \dfrac{(125\ \text{m/s})^2(\sin 110.0°)}{9.80\ \text{m/s}^2} = 1500$ m .

(b) *h* and *R* are proportional to $1/g$, so on the Moon, $h = \left(\dfrac{9.80\ \text{m/s}^2}{1.67\ \text{m/s}^2}\right)(535\ \text{m}) = 3140$ m and $R = \left(\dfrac{9.80\ \text{m/s}^2}{1.67\ \text{m/s}^2}\right)(1500\ \text{m}) = 8800$ m .

EVALUATE: The projectile travels on a parabolic trajectory. It is incorrect to say that $h = (R/2)\tan\alpha_0$.

3.19. **IDENTIFY:** The baseball moves in projectile motion. In part (c) first calculate the components of the velocity at this point and then get the resultant velocity from its components.

SET UP: First find the *x*- and *y*-components of the initial velocity. Use coordinates where the $+y$-direction is upward, the $+x$-direction is to the right and the origin is at the point where the baseball leaves the bat.

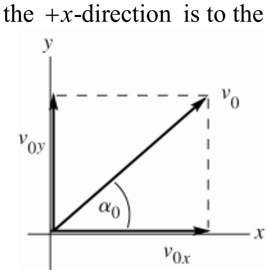


Figure 3.19a

$$v_{0x} = v_0 \cos\alpha_0 = (30.0\ \text{m/s})\cos 36.9° = 24.0\ \text{m/s}$$
$$v_{0y} = v_0 \sin\alpha_0 = (30.0\ \text{m/s})\sin 36.9° = 18.0\ \text{m/s}$$

Use constant acceleration equations for the *x* and *y* motions, with $a_x = 0$ and $a_y = -g$.

EXECUTE: **(a)** y-component (vertical motion):

$y - y_0 = +10.0\ \text{m/s},\ \ v_{0y} = 18.0\ \text{m/s},\ \ a_y = -9.80\ \text{m/s}^2,\ \ t = ?$

$y - y_0 = v_{0y} + \frac{1}{2}a_y t^2$

$10.0\ \text{m} = (18.0\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$

$(4.90\ \text{m/s}^2)t^2 - (18.0\ \text{m/s})t + 10.0\ \text{m} = 0$

Apply the quadratic formula: $t = \frac{1}{9.80}\left[18.0 \pm \sqrt{(-18.0)^2 - 4(4.90)(10.0)}\right]\ \text{s} = (1.837 \pm 1.154)\ \text{s}$

The ball is at a height of 10.0 above the point where it left the bat at $t_1 = 0.683\ \text{s}$ and at $t_2 = 2.99\ \text{s}$. At the earlier time the ball passes through a height of 10.0 m as its way up and at the later time it passes through 10.0 m on its way down.

(b) $v_x = v_{0x} = +24.0\ \text{m/s}$, at all times since $a_x = 0$.

$v_y = v_{0y} + a_y t$

$t_1 = 0.683\ \text{s}$: $v_y = +18.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(0.683\ \text{s}) = +11.3\ \text{m/s}$. ($v_y$ is positive means that the ball is traveling upward at this point.

$t_2 = 2.99\ \text{s}$: $v_y = +18.0\ \text{m/s} + (-9.80\ \text{m/s}^2)(2.99\ \text{s}) = -11.3\ \text{m/s}$. ($v_y$ is negative means that the ball is traveling downward at this point.)

(c) $v_x = v_{0x} = 24.0\ \text{m/s}$

Solve for v_y:

$v_y = ?,\ \ y - y_0 = 0$ (when ball returns to height where motion started),

$a_y = -9.80\ \text{m/s}^2,\ \ v_{0y} = +18.0\ \text{m/s}$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y = -v_{0y} = -18.0\ \text{m/s}$ (negative, since the baseball must be traveling downward at this point)

Now that have the components can solve for the magnitude and direction of $\vec{v}$.

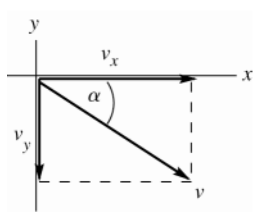


Figure 3.19b

$v = \sqrt{v_x^2 + v_y^2}$

$v = \sqrt{(24.0\ \text{m/s})^2 + (-18.0\ \text{m/s})^2} = 30.0\ \text{m/s}$

$\tan\alpha = \frac{v_y}{v_x} = \frac{-18.0\ \text{m/s}}{24.0\ \text{m/s}}$

$\alpha = -36.9°$, $36.9°$ below the horizontal

The velocity of the ball when it returns to the level where it left the bat has magnitude 30.0 m/s and is directed at an angle of $36.9°$ below the horizontal.

EVALUATE: The discussion in parts (a) and (b) explains the significance of two values of t for which $y - y_0 = +10.0\ \text{m}$. When the ball returns to its initial height, our results give that its speed is the same as its initial speed and the angle of its velocity below the horizontal is equal to the angle of its initial velocity above the horizontal; both of these are general results.

3.20. **IDENTIFY:** The shot moves in projectile motion.

SET UP: Let $+y$ be upward.

EXECUTE: **(a)** If air resistance is to be ignored, the components of acceleration are 0 horizontally and $-g = -9.80\ \text{m/s}^2$ vertically downward.

(b) The x-component of velocity is constant at $v_x = (12.0\ \text{m/s})\cos 51.0° = 7.55\ \text{m/s}$. The y-component is $v_{0y} = (12.0\ \text{m/s})\sin 51.0° = 9.32\ \text{m/s}$ at release and $v_y = v_{0y} - gt = (10.57\ \text{m/s}) - (9.80\ \text{m/s}^2)(2.08\ \text{s}) = -11.06\ \text{m/s}$ when the shot hits.

(c) $x - x_0 = v_{0x}t = (7.55\ \text{m/s})(2.08\ \text{s}) = 15.7\ \text{m}$.

(d) The initial and final heights are not the same.

(e) With $y = 0$ and v_{0y} as found above, Eq.(3.18) gives $y_0 = 1.81\text{m}$.

(f) The graphs are sketched in Figure 3.20.

EVALUATE: When the shot returns to its initial height, $v_y = -9.32$ m/s . The shot continues to accelerate downward as it travels downward 1.81 m to the ground and the magnitude of v_y at the ground is larger than 9.32 m/s.

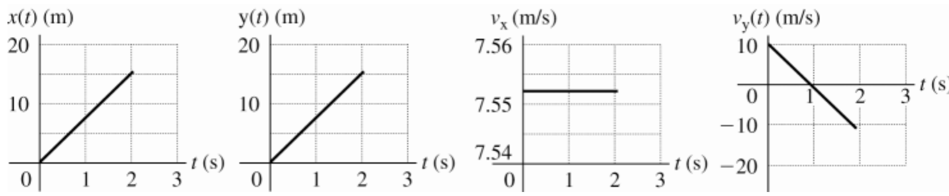
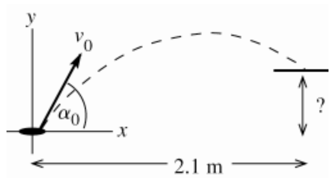


Figure 3.20

3.21. **IDENTIFY:** Take the origin of coordinates at the point where the quarter leaves your hand and take positive *y* to be upward. The quarter moves in projectile motion, with $a_x = 0$, and $a_y = -g$. It travels vertically for the time it takes it to travel horizontally 2.1 m.



$$v_{0x} = v_0 \cos\alpha_0 = (6.4 \text{ m/s})\cos 60°$$
$$v_{0x} = 3.20 \text{ m/s}$$
$$v_{0y} = v_0 \sin\alpha_0 = (6.4 \text{ m/s})\sin 60°$$
$$v_{0y} = 5.54 \text{ m/s}$$

Figure 3.21

(a) SET UP: Use the horizontal (*x*-component) of motion to solve for *t*, the time the quarter travels through the air:

$t = ?$, $x - x_0 = 2.1$ m, $v_{0x} = 3.2$ m/s, $a_x = 0$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = v_{0x}t$, since $a_x = 0$

EXECUTE: $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{2.1 \text{ m}}{3.2 \text{ m/s}} = 0.656$ s

SET UP: Now find the vertical displacement of the quarter after this time:

$y - y_0 = ?$, $a_y = -9.80 \text{ m/s}^2$, $v_{0y} = +5.54$ m/s, $t = 0.656$ s

$y - y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $y - y_0 = (5.54 \text{ m/s})(0.656 \text{ s}) + \frac{1}{2}(-9.80 \text{ m/s}^2)(0.656 \text{ s})^2 = 3.63 \text{ m} - 2.11 \text{ m} = 1.5 \text{ m}.$

(b) SET UP: $v_y = ?$, $t = 0.656$ s, $a_y = -9.80 \text{ m/s}^2$, $v_{0y} = +5.54$ m/s

$v_y = v_{0y} + a_y t$

EXECUTE: $v_y = 5.54 \text{ m/s} + (-9.80 \text{ m/s}^2)(0.656 \text{ s}) = -0.89 \text{ m/s}.$

EVALUATE: The minus sign for v_y indicates that the *y*-component of $\vec{v}$ is downward. At this point the quarter has passed through the highest point in its path and is on its way down. The horizontal range if it returned to its original height (it doesn't!) would be 3.6 m. It reaches its maximum height after traveling horizontally 1.8 m, so at $x - x_0 = 2.1$ m it is on its way down.

3.22. **IDENTIFY:** Use the analysis of Example 3.10.

SET UP: From Example 3.10, $t = \dfrac{d}{v_0 \cos\alpha_0}$ and $y_{\text{dart}} = (v_0 \sin\alpha_0)t - \frac{1}{2}gt^2$.

EXECUTE: Substituting for *t* in terms of *d* in the expression for y_{dart} gives

$$y_{\text{dart}} = d\left(\tan\alpha_0 - \frac{gd}{2v_0^2 \cos^2\alpha_0}\right).$$

Using the given values for *d* and α_0 to express this as a function of v_0 ,

$$y = (3.00 \text{ m})\left(0.90 - \frac{26.62 \text{ m}^2/\text{s}^2}{v_0^2}\right).$$

(a) $v_0 = 12.0$ m/s gives $y = 2.14$ m .

(b) $v_0 = 8.0$ m/s gives $y = 1.45$ m .

(c) $v_0 = 4.0$ m/s gives $y = -2.29$ m . In this case, the dart was fired with so slow a speed that it hit the ground before traveling the 3-meter horizontal distance.

EVALUATE: For (a) and (d) the trajectory of the dart has the shape shown in Figure 3.26 in the textbook. For (c) the dart moves in a parabola and returns to the ground before it reaches the *x*-coordinate of the monkey.

3.23. **IDENTIFY:** Take the origin of coordinates at the roof and let the $+y$-direction be upward. The rock moves in projectile motion, with $a_x = 0$ and $a_y = -g$. Apply constant acceleration equations for the *x* and *y* components of the motion.

SET UP:

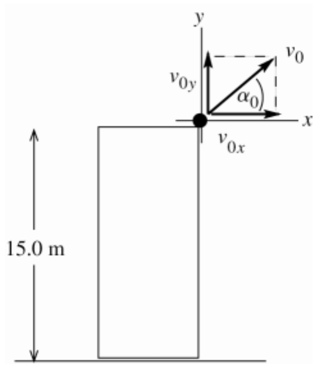


Figure 3.23a

$v_{0x} = v_0 \cos\alpha_0 = 25.2$ m/s

$v_{0y} = v_0 \sin\alpha_0 = 16.3$ m/s

(a) At the maximum height $v_y = 0$.

$a_y = -9.80 \text{ m/s}^2, \quad v_y = 0, \quad v_{0y} = +16.3 \text{ m/s}, \quad y - y_0 = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (16.3 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = +13.6 \text{ m}$

(b) SET UP: Find the velocity by solving for its *x* and *y* components.

$v_x = v_{0x} = 25.2$ m/s (since $a_x = 0$)

$v_y = ?, \quad a_y = -9.80 \text{ m/s}^2, \quad y - y_0 = -15.0 \text{ m}$ (negative because at the ground the rock is below its initial position),

$v_{0y} = 16.3$ m/s

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$v_y = -\sqrt{v_{0y}^2 + 2a_y(y - y_0)}$ (v_y is negative because at the ground the rock is traveling downward.)

EXECUTE: $v_y = -\sqrt{(16.3 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(-15.0 \text{ m})} = -23.7 \text{ m/s}$

Then $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(25.2 \text{ m/s})^2 + (-23.7 \text{ m/s})^2} = 34.6 \text{ m/s}.$

(c) SET UP: Use the vertical motion (*y*-component) to find the time the rock is in the air:

$t = ?, \quad v_y = -23.7$ m/s (from part (b)), $a_y = -9.80 \text{ m/s}^2, \quad v_{0y} = +16.3 \text{ m/s}$

EXECUTE: $t = \dfrac{v_y - v_{0y}}{a_y} = \dfrac{-23.7 \text{ m/s} - 16.3 \text{ m/s}}{-9.80 \text{ m/s}^2} = +4.08 \text{ s}$

SET UP: Can use this *t* to calculate the horizontal range:

$t = 4.08 \text{ s}, \quad v_{0x} = 25.2 \text{ m/s}, \quad a_x = 0, \quad x - x_0 = ?$

EXECUTE: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (25.2 \text{ m/s})(4.08 \text{ s}) + 0 = 103 \text{ m}$

(d) Graphs of *x* versus *t*, *y* versus *t*, v_x versus *t*, and v_y versus *t*:

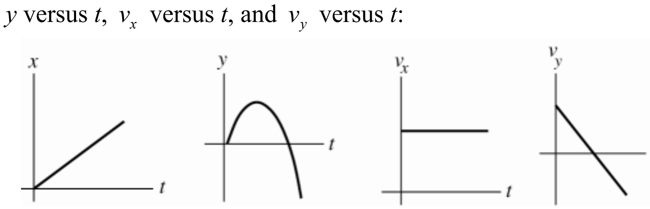


Figure 3.23b

EVALUATE: The time it takes the rock to travel vertically to the ground is the time it has to travel horizontally. With $v_{0y} = +16.3\text{ m/s}$ the time it takes the rock to return to the level of the roof $(y = 0)$ is $t = 2v_{0y}/g = 3.33\text{ s}$. The time in the air is greater than this because the rock travels an additional 15.0 m to the ground.

3.24. **IDENTIFY:** Consider the horizontal and vertical components of the projectile motion. The water travels 45.0 m horizontally in 3.00 s.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. $v_{0x} = v_0\cos\theta_0$, $v_{0y} = v_0\sin\theta_0$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$ gives $x - x_0 = v_0(\cos\theta_0)t$ and $\cos\theta_0 = \dfrac{45.0\text{ m}}{(25.0\text{ m/s})(3.00\text{ s})} = 0.600$; $\theta_0 = 53.1°$

(b) At the highest point $v_x = v_{0x} = (25.0\text{ m/s})\cos 53.1° = 15.0\text{ m/s}$, $v_y = 0$ and $v = \sqrt{v_x^2 + v_y^2} = 15.0\text{ m/s}$. At all points in the motion, $a = 9.80\text{ m/s}^2$ downward.

(c) Find $y - y_0$ when $t = 3.00\text{s}$:

$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2 = (25.0\text{ m/s})(\sin 53.1°)(3.00\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(3.00\text{ s})^2 = 15.9\text{ m}$

$v_x = v_{0x} = 15.0\text{ m/s}$, $v_y = v_{0y} + a_yt = (25.0\text{ m/s})(\sin 53.1°) - (9.80\text{m/s}^2)(3.00\text{ s}) = -9.41\text{ m/s}$, and

$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{(15.0\text{ m/s})^2 + (-9.41\text{ m/s})^2} = 17.7\text{ m/s}$

EVALUATE: The acceleration is the same at all points of the motion. It takes the water $t = -\dfrac{v_{0y}}{a_y} = -\dfrac{20.0\text{ m/s}}{-9.80\text{ m/s}^2} = 2.04\text{ s}$ to reach its maximum height. When the water reaches the building it has passed its maximum height and its vertical component of velocity is downward.

3.25. **IDENTIFY** and **SET UP:** The stone moves in projectile motion. Its initial velocity is the same as that of the balloon. Use constant acceleration equations for the *x* and *y* components of its motion. Take $+y$ to be upward.

EXECUTE: **(a)** Use the vertical motion of the rock to find the initial height.

$t = 6.00\text{ s}$, $v_{0y} = +20.0\text{ m/s}$, $a_y = +9.80\text{ m/s}^2$, $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $y - y_0 = 296\text{ m}$

(b) In 6.00 s the balloon travels downward a distance $y - y_0 = (20.0\text{ s})(6.00\text{ s}) = 120\text{ m}$. So, its height above ground when the rock hits is $296\text{ m} - 120\text{ m} = 176\text{ m}$.

(c) The horizontal distance the rock travels in 6.00 s is 90.0 m. The vertical component of the distance between the rock and the basket is 176 m, so the rock is $\sqrt{(176\text{ m})^2 + (90\text{ m})^2} = 198\text{ m}$ from the basket when it hits the ground.

(d) (i) The basket has no horizontal velocity, so the rock has horizontal velocity 15.0 m/s relative to the basket. Just before the rock hits the ground, its vertical component of velocity is $v_y = v_{0y} + a_yt =$ $20.0\text{ m/s} + (9.80\text{ m/s}^2)(6.00\text{ s}) = 78.8\text{ m/s}$, downward, relative to the ground. The basket is moving downward at 20.0 m/s, so relative to the basket the rock has downward component of velocity 58.8 m/s.

(e) horizontal: 15.0 m/s; vertical: 78.8 m/s

EVALUATE: The rock has a constant horizontal velocity and accelerates downward

3.26. **IDENTIFY:** The shell moves as a projectile. To just clear the top of the cliff, the shell must have $y - y_0 = 25.0\text{ m}$ when it has $x - x_0 = 60.0\text{ m}$.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -g$. $v_{0x} = v_0\cos 43°$, $v_{0y} = v_0\sin 43°$.

EXECUTE: **(a)** horizontal motion: $x - x_0 = v_{0x}t$ so $t = \dfrac{60.0\text{ m}}{(v_0\cos 43°)t}$.

vertical motion: $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $25.0\text{m} = (v_0\ \sin 43.0°)t + \frac{1}{2}(-9.80\text{m/s}^2)t^2$.

Solving these two simultaneous equations for v_0 and *t* gives $v_0 = 3.26\text{ m/s}$ and $t = 2.51\text{ s}$.

(b) v_y when shell reaches cliff:

$$v_y = v_{0y} + a_yt = (32.6\text{ m/s})\sin 43.0° - (9.80\text{ m/s}^2)(2.51\text{ s}) = -2.4\text{ m/s}$$

The shell is traveling downward when it reaches the cliff, so it lands right at the edge of the cliff.

EVALUATE: The shell reaches its maximum height at $t = -\dfrac{v_{0y}}{a_y} = 2.27\text{ s}$, which confirms that at $t = 2.51\text{ s}$ it has passed its maximum height and is on its way down when it strikes the edge of the cliff.

3.27. **IDENTIFY:** The suitcase moves in projectile motion. The initial velocity of the suitcase equals the velocity of the airplane.

SET UP: Take $+y$ to be upward. $a_x = 0$, $a_y = -g$.

EXECUTE: Use the vertical motion to find the time it takes the suitcase to reach the ground:
$v_{0y} = v_0 \sin 23°$, $a_y = -9.80 \text{ m/s}^2$, $y - y_0 = -114 \text{ m}$, $t = ?$ $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 9.60 \text{ s}$.
The distance the suitcase travels horizontally is $x - x_0 = v_{0x} = (v_0 \cos 23.0°)t = 795 \text{ m}$.

EVALUATE: An object released from rest at a height of 114 m strikes the ground at $t = \sqrt{\dfrac{2(y - y_0)}{-g}} = 4.82 \text{ s}$. The suitcase is in the air much longer than this since it initially has an upward component of velocity.

3.28. **IDENTIFY:** Determine how a_{rad} depends on the rotational period T.

SET UP: $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$.

EXECUTE: For any item in the washer, the centripetal acceleration will be inversely proportional to the square of the rotational period; tripling the centripetal acceleration involves decreasing the period by a factor of $\sqrt{3}$, so that the new period T' is given in terms of the previous period T by $T' = T/\sqrt{3}$.
EVALUATE: The rotational period must be decreased in order to increase the rate of rotation and therefore increase the centripetal acceleration.

3.29. **IDENTIFY:** Apply Eq. (3.30).
SET UP: $T = 24 \text{ h}$.

EXECUTE: **(a)** $a_{\text{rad}} = \dfrac{4\pi^2(6.38\times10^6 \text{ m})}{((24 \text{ h})(3600 \text{ s/h}))^2} = 0.034 \text{ m/s}^2 = 3.4\times10^{-3} g.$

(b) Solving Eq. (3.30) for the period T with $a_{\text{rad}} = g$, $T = \sqrt{\dfrac{4\pi^2(6.38\times10^6 \text{m})}{9.80 \text{ m/s}^2}} = 5070 \text{ s} = 1.4 \text{ h}.$

EVALUATE: a_{rad} is proportional to $1/T^2$, so to increase a_{rad} by a factor of $\dfrac{1}{3.4\times10^{-3}} = 294$ requires that T be multiplied by a factor of $\dfrac{1}{\sqrt{294}}$. $\dfrac{24 \text{ h}}{\sqrt{294}} = 1.4 \text{ h}$.

3.30. **IDENTIFY:** Each blade tip moves in a circle of radius $R = 3.40 \text{ m}$ and therefore has radial acceleration $a_{\text{rad}} = v^2/R$.

SET UP: 550 rev/min = 9.17 rev/s , corresponding to a period of $T = \dfrac{1}{9.17 \text{ rev/s}} = 0.109 \text{ s}$.

EXECUTE: **(a)** $v = \dfrac{2\pi R}{T} = 196 \text{ m/s}$.

(b) $a_{\text{rad}} = \dfrac{v^2}{R} = 1.13\times10^4 \text{ m/s}^2 = 1.15\times10^3 g$.

EVALUATE: $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$ gives the same results for a_{rad} as in part (b).

3.31. **IDENTIFY:** Apply Eq.(3.30).
SET UP: $R = 7.0 \text{ m}$. $g = 9.80 \text{ m/s}^2$.

EXECUTE: **(a)** Solving Eq. (3.30) for T in terms of R and a_{rad} ,
$T = \sqrt{4\pi^2 R/a_{\text{rad}}} = \sqrt{4\pi^2(7.0 \text{ m})/(3.0)(9.80 \text{ m/s}^2)} = 3.07 \text{ s}$.

(b) $a_{\text{rad}} = 10g$ gives $T = 1.68 \text{ s}$.

EVALUATE: When a_{rad} increases, T decreases.

3.32. **IDENTIFY:** Each planet moves in a circular orbit and therefore has acceleration $a_{\text{rad}} = v^2/R$.

SET UP: The radius of the earth's orbit is $r = 1.50\times10^{11} \text{ m}$ and its orbital period is $T = 365 \text{ days} = 3.16\times10^7 \text{ s}$. For Mercury, $r = 5.79\times10^{10} \text{ m}$ and $T = 88.0 \text{ days} = 7.60\times10^6 \text{ s}$.

EXECUTE: **(a)** $v = \dfrac{2\pi r}{T} = 2.98\times10^4 \text{ m/s}$

(b) $a_{\text{rad}} = \dfrac{v^2}{r} = 5.91\times10^{-3} \text{ m/s}^2$.

(c) $v = 4.79\times10^4 \text{ m/s}$, and $a_{\text{rad}} = 3.96\times10^{-2} \text{ m/s}^2$.

EVALUATE: Mercury has a larger orbital velocity and a larger radial acceleration than earth.

3.33. **IDENTIFY:** Uniform circular motion.

SET UP: Since the magnitude of $\vec{v}$ is constant. $v_{\tan} = \dfrac{d|\vec{v}|}{dt} = 0$ and the resultant acceleration is equal to the radial component. At each point in the motion the radial component of the acceleration is directed in toward the center of the circular path and its magnitude is given by v^2/R.

EXECUTE: **(a)** $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(7.00\text{ m/s})^2}{14.0\text{ m}} = 3.50\text{ m/s}^2$, upward.

(b) The radial acceleration has the same magnitude as in part (a), but now the direction toward the center of the circle is downward. The acceleration at this point in the motion is 3.50 m/s^2, downward.

(c) SET UP: The time to make one rotation is the period *T*, and the speed *v* is the distance for one revolution divided by *T*.

EXECUTE: $v = \dfrac{2\pi R}{T}$ so $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(14.0\text{ m})}{7.00\text{ m/s}} = 12.6\text{ s}$

EVALUATE: The radial acceleration is constant in magnitude since *v* is constant and is at every point in the motion directed toward the center of the circular path. The acceleration is perpendicular to $\vec{v}$ and is nonzero because the direction of $\vec{v}$ changes.

3.34. **IDENTIFY:** The acceleration is the vector sum of the two perpendicular components, a_{rad} and $a_{\tan}$.

SET UP: $a_{\tan}$ is parallel to $\vec{v}$ and hence is associated with the change in speed; $a_{\tan} = 0.500\text{ m/s}^2$.

EXECUTE: **(a)** $a_{\text{rad}} = v^2/R = (3\text{ m/s})^2/(14\text{ m}) = 0.643\text{ m/s}^2$.

$a = ((0.643\text{ m/s}^2)^2 + (0.5\text{ m/s}^2)^2)^{1/2} = 0.814\text{ m/s}^2,\ 37.9°$ to the right of vertical.

(b) The sketch is given in Figure 3.34.

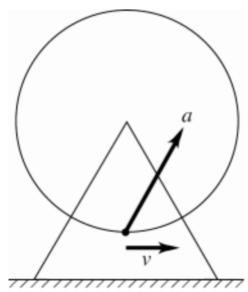


Figure 3.34

3.35. **IDENTIFY:** Each part of his body moves in uniform circular motion, with $a_{\text{rad}} = \dfrac{v^2}{R}$. The speed in rev/s is $1/T$, where *T* is the period in seconds (time for 1 revolution). The speed *v* increases with *R* along the length of his body but all of him rotates with the same period *T*.

SET UP: For his head $R = 8.84\text{ m}$ and for his feet $R = 6.84\text{ m}$.

EXECUTE: **(a)** $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(8.84\text{ m})(12.5)(9.80\text{ m/s}^2)} = 32.9\text{ m/s}$

(b) Use $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$. Since his head has $a_{\text{rad}} = 12.5g$ and $R = 8.84\text{ m}$,

$T = 2\pi\sqrt{\dfrac{R}{a_{\text{rad}}}} = 2\pi\sqrt{\dfrac{8.84\text{ m}}{12.5(9.80\text{ m/s}^2)}} = 1.688\text{ s}$. Then his feet have $a_{\text{rad}} = \dfrac{R}{T^2} = \dfrac{4\pi^2(6.84\text{ m})}{(1.688\text{ s})^2} = 94.8\text{ m/s}^2 = 9.67g$.

The difference between the acceleration of his head and his feet is $12.5g - 9.67g = 2.83g = 27.7\text{ m/s}^2$.

(c) $\dfrac{1}{T} = \dfrac{1}{1.69\text{ s}} = 0.592\text{ rev/s} = 35.5\text{ rpm}$

EVALUATE: His feet have speed $v = \sqrt{Ra_{\text{rad}}} = \sqrt{(6.84\text{ m})(94.8\text{ m/s}^2)} = 25.5\text{ m/s}$

3.36. **IDENTIFY:** The relative velocities are $\vec{v}_{\text{S/F}}$, the velocity of the scooter relative to the flatcar, $\vec{v}_{\text{S/G}}$, the scooter relative to the ground and $\vec{v}_{\text{F/G}}$, the flatcar relative to the ground. $\vec{v}_{\text{S/G}} = \vec{v}_{\text{S/F}} + \vec{v}_{\text{F/G}}$. Carry out the vector addition by drawing a vector addition diagram.

SET UP: $\vec{v}_{\text{S/F}} = \vec{v}_{\text{S/G}} - \vec{v}_{\text{F/G}}$. $\vec{v}_{\text{F/G}}$ is to the right, so $-\vec{v}_{\text{F/G}}$ is to the left.

EXECUTE: In each case the vector addition diagram gives

(a) 5.0 m/s to the right
(b) 16.0 m/s to the left
(c) 13.0 m/s to the left.

EVALUATE: The scooter has the largest speed relative to the ground when it is moving to the right relative to the flatcar, since in that case the two velocities $\vec{v}_{S/F}$ and $\vec{v}_{F/G}$ are in the same direction and their magnitudes add.

3.37. IDENTIFY: Relative velocity problem. The time to walk the length of the moving sidewalk is the length divided by the velocity of the woman relative to the ground.

SET UP: Let W stand for the woman, G for the ground, and S for the sidewalk. Take the positive direction to be the direction in which the sidewalk is moving.

The velocities are $v_{W/G}$ (woman relative to the ground), $v_{W/S}$ (woman relative to the sidewalk), and $v_{S/G}$ (sidewalk relative to the ground).

Eq.(3.33) becomes $v_{W/G} = v_{W/S} + v_{S/G}$.

The time to reach the other end is given by $t = \dfrac{\text{distance traveled relative to ground}}{v_{W/G}}$

EXECUTE: (a) $v_{S/G} = 1.0\text{ m/s}$

$v_{W/S} = +1.5\text{ m/s}$

$v_{W/G} = v_{W/S} + v_{S/G} = 1.5\text{ m/s} + 1.0\text{ m/s} = 2.5\text{ m/s}.$

$t = \dfrac{35.0\text{ m}}{v_{W/G}} = \dfrac{35.0\text{ m}}{2.5\text{ m/s}} = 14\text{ s}.$

(b) $v_{S/G} = 1.0\text{ m/s}$

$v_{W/S} = -1.5\text{ m/s}$

$v_{W/G} = v_{W/S} + v_{S/G} = -1.5\text{ m/s} + 1.0\text{ m/s} = -0.5\text{ m/s}.$ (Since $v_{W/G}$ now is negative, she must get on the moving sidewalk at the opposite end from in part (a).)

$t = \dfrac{-35.0\text{ m}}{v_{W/G}} = \dfrac{-35.0\text{ m}}{-0.5\text{ m/s}} = 70\text{ s}.$

EVALUATE: Her speed relative to the ground is much greater in part (a) when she walks with the motion of the sidewalk.

3.38. IDENTIFY: Calculate the rower's speed relative to the shore for each segment of the round trip.

SET UP: The boat's speed relative to the shore is 6.8 km/h downstream and 1.2 km/h upstream.

EXECUTE: The walker moves a total distance of 3.0 km at a speed of 4.0 km/h, and takes a time of three fourths of an hour (45.0 min).

The total time the rower takes is $\dfrac{1.5\text{ km}}{6.8\text{ km/h}} + \dfrac{1.5\text{ km}}{1.2\text{ km/h}} = 1.47\text{ h} = 88.2\text{ min}.$

EVALUATE: It takes the rower longer, even though for half the distance his speed is greater than 4.0 km/h. The rower spends more time at the slower speed.

3.39. IDENTIFY: Apply the relative velocity relation.

SET UP: The relative velocities are $\vec{v}_{C/E}$, the canoe relative to the earth, $\vec{v}_{R/E}$, the velocity of the river relative to the earth and $\vec{v}_{C/R}$, the velocity of the canoe relative to the river.

EXECUTE: $\vec{v}_{C/E} = \vec{v}_{C/R} + \vec{v}_{R/E}$ and therefore $\vec{v}_{C/R} = \vec{v}_{C/E} - \vec{v}_{R/E}$. The velocity components of $\vec{v}_{C/R}$ are $-0.50\text{ m/s} + (0.40\text{ m/s})/\sqrt{2},$ east and $(0.40\text{ m/s})/\sqrt{2},$ south, for a velocity relative to the river of 0.36 m/s, at $52.5°$ south of west.

EVALUATE: The velocity of the canoe relative to the river has a smaller magnitude than the velocity of the canoe relative to the earth.

3.40. IDENTIFY: Use the relation that relates the relative velocities.

SET UP: The relative velocities are the velocity of the plane relative to the ground, $\vec{v}_{P/G}$, the velocity of the plane relative to the air, $\vec{v}_{P/A}$, and the velocity of the air relative to the ground, $\vec{v}_{A/G}$. $\vec{v}_{P/G}$ must due west and $\vec{v}_{A/G}$ must be south. $v_{A/G} = 80\text{ km/h}$ and $v_{P/A} = 320\text{ km/h}$. $\vec{v}_{P/G} = \vec{v}_{P/A} + \vec{v}_{A/G}$. The relative velocity addition diagram is given in Figure 3.40.

EXECUTE: (a) $\sin\theta = \dfrac{v_{A/G}}{v_{P/A}} = \dfrac{80\text{ km/h}}{320\text{ km/h}}$ and $\theta = 14°$, north of west.

(b) $v_{P/G} = \sqrt{v_{P/A}^2 - v_{A/G}^2} = \sqrt{(320\text{ km/h})^2 - (80.0\text{ km/h})^2} = 310\text{ km/h}$.

EVALUATE: To travel due west the velocity of the plane relative to the air must have a westward component and also a component that is northward, opposite to the wind direction.

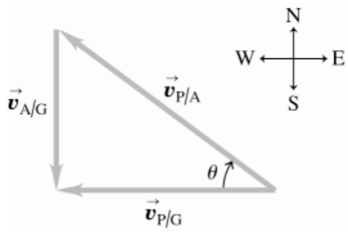


Figure 3.40

3.41. IDENTIFY: Relative velocity problem in two dimensions. His motion relative to the earth (time displacement) depends on his velocity relative to the earth so we must solve for this velocity.

(a) SET UP: View the motion from above.

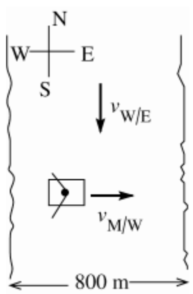


Figure 3.41a

The velocity vectors in the problem are:

$\vec{v}_{M/E}$, the velocity of the man relative to the earth

$\vec{v}_{W/E}$, the velocity of the water relative to the earth

$\vec{v}_{M/W}$, the velocity of the man relative to the water

The rule for adding these velocities is

$\vec{v}_{M/E} = \vec{v}_{M/W} + \vec{v}_{W/E}$

The problem tells us that $\vec{v}_{W/E}$ has magnitude 2.0 m/s and direction due south. It also tells us that $\vec{v}_{M/W}$ has magnitude 4.2 m/s and direction due east. The vector addition diagram is then as shown in Figure 3.41b

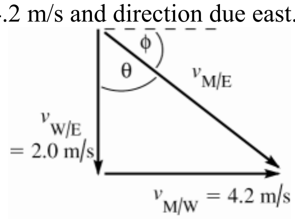


Figure 3.41b

This diagram shows the vector addition $\vec{v}_{M/E} = \vec{v}_{M/W} + \vec{v}_{W/E}$ and also has $\vec{v}_{M/W}$ and $\vec{v}_{W/E}$ in their specified directions. Note that the vector diagram forms a right triangle.

The Pythagorean theorem applied to the vector addition diagram gives $v_{M/E}^2 = v_{M/W}^2 + v_{W/E}^2$.

EXECUTE: $v_{M/E} = \sqrt{v_{M/W}^2 + v_{W/E}^2} = \sqrt{(4.2\text{ m/s})^2 + (2.0\text{ m/s})^2} = 4.7\text{ m/s}$ $\tan\theta = \dfrac{v_{M/W}}{v_{W/E}} = \dfrac{4.2\text{ m/s}}{2.0\text{ m/s}} = 2.10$; $\theta = 65°$; or $\phi = 90° - \theta = 25°$. The velocity of the man relative to the earth has magnitude 4.7 m/s and direction 25° S of E.

(b) This requires careful thought. To cross the river the man must travel 800 m due east relative to the earth. The man's velocity relative to the earth is $\vec{v}_{M/E}$. But, from the vector addition diagram the eastward component of $v_{M/E}$ equals $v_{M/W} = 4.2$ m/s.

Thus $t = \dfrac{x - x_0}{v_x} = \dfrac{800\text{ m}}{4.2\text{ m/s}} = 190$ s.

(c) The southward component of $\vec{v}_{M/E}$ equals $v_{W/E} = 2.0$ m/s. Therefore, in the 190 s it takes him to cross the river the distance south the man travels relative to the earth is

$$y - y_0 = v_y t = (2.0\text{ m/s})(190\text{ s}) = 380\text{ m}.$$

EVALUATE: If there were no current he would cross in the same time, $(800\text{ m})/(4.2\text{ m/s}) = 190$ s. The current carries him downstream but doesn't affect his motion in the perpendicular direction, from bank to bank.

3.42. IDENTIFY: Use the relation that relates the relative velocities.

SET UP: The relative velocities are the water relative to the earth, $\vec{v}_{W/E}$, the boat relative to the water, $\vec{v}_{B/W}$, and the boat relative to the earth, $\vec{v}_{B/E}$. $\vec{v}_{B/E}$ is due east, $\vec{v}_{W/E}$ is due south and has magnitude 2.0 m/s. $v_{B/W} = 4.2$ m/s. $\vec{v}_{B/E} = \vec{v}_{B/W} + \vec{v}_{W/E}$. The velocity addition diagram is given in Figure 3.42.

EXECUTE: **(a)** Find the direction of $\vec{v}_{B/W}$. $\sin\theta = \frac{v_{W/E}}{v_{B/W}} = \frac{2.0 \text{ m/s}}{4.2 \text{ m/s}}$. $\theta = 28.4°$, north of east.

(b) $v_{B/E} = \sqrt{v_{B/W}^2 - v_{W/E}^2} = \sqrt{(4.2 \text{ m/s})^2 - (2.0 \text{ m/s})^2} = 3.7 \text{ m/s}$

(c) $t = \frac{800 \text{ m}}{v_{B/E}} = \frac{800 \text{ m}}{3.7 \text{ m/s}} = 216 \text{ s}$.

EVALUATE: It takes longer to cross the river in this problem than it did in Problem 3.41. In the direction straight across the river (east) the component of his velocity relative to the earth is lass than 4.2 m/s.

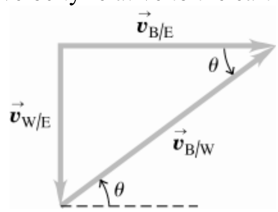


Figure 3.42

3.43. **IDENTIFY:** Relative velocity problem in two dimensions.

(a) SET UP: $\vec{v}_{P/A}$ is the velocity of the plane relative to the air. The problem states that $\vec{v}_{P/A}$ has magnitude 35 m/s and direction south.

$\vec{v}_{A/E}$ is the velocity of the air relative to the earth. The problem states that $\vec{v}_{A/E}$ is to the southwest ($45°$ S of W) and has magnitude 10 m/s.

The relative velocity equation is $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$.

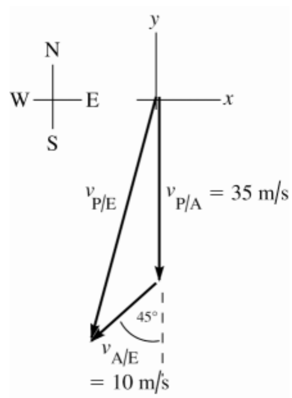


Figure 3.43a

EXECUTE: **(b)** $(v_{P/A})_x = 0$, $(v_{P/A})_y = -35 \text{ m/s}$

$(v_{A/E})_x = -(10 \text{ m/s})\cos 45° = -7.07 \text{ m/s}$,

$(v_{A/E})_y = -(10 \text{ m/s})\sin 45° = -7.07 \text{ m/s}$

$(v_{P/E})_x = (v_{P/A})_x + (v_{A/E})_x = 0 - 7.07 \text{ m/s} = -7.1 \text{ m/s}$

$(v_{P/E})_y = (v_{P/A})_y + (v_{A/E})_y = -35 \text{ m/s} - 7.07 \text{ m/s} = -42 \text{ m/s}$

(c)

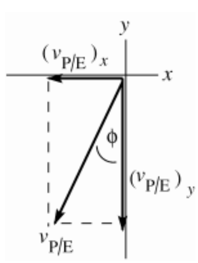


Figure 3.43b

$$v_{P/E} = \sqrt{(v_{P/E})_x^2 + (v_{P/E})_y^2}$$

$$v_{P/E} = \sqrt{(-7.1 \text{ m/s})^2 + (-42 \text{ m/s})^2} = 43 \text{ m/s}$$

$$\tan\phi = \frac{(v_{P/E})_x}{(v_{P/E})_y} = \frac{-7.1}{-42} = 0.169$$

$\phi = 9.6°$; ($9.6°$ west of south)

EVALUATE: The relative velocity addition diagram does not form a right triangle so the vector addition must be done using components. The wind adds both southward and westward components to the velocity of the plane relative to the ground.

3.44. **IDENTIFY:** Use Eqs.(2.17) and (2.18).

SET UP: At the maximum height $v_y = 0$.

EXECUTE: **(a)** $v_x = v_{0x} + \frac{\alpha}{3}t^3$, $v_y = v_{0y} + \beta t - \frac{\gamma}{2}t^2$, and $x = v_{0x}t + \frac{\alpha}{12}t^4$, $y = v_{0y}t + \frac{\beta}{2}t^2 - \frac{\gamma}{6}t^3$.

(b) Setting $v_y = 0$ yields a quadratic in t, $0 = v_{0y} + \beta t - \frac{\gamma}{2}t^2$, which has as the positive solution

$t = \frac{1}{\gamma}\left[\beta + \sqrt{\beta^2 + 2v_0\gamma}\right] = 13.59\text{ s}$. Using this time in the expression for $y(t)$ gives a maximum height of 341 m.

(c) The path of the rocket is sketched in Figure 3.44.

(d) $y = 0$ gives $0 = v_{0y}t + \frac{\beta}{2}t^2 - \frac{\gamma}{6}t^3$ and $\frac{\gamma}{6}t^2 - \frac{\beta}{2}t - v_{0y} = 0$. The positive solution is $t = 20.73\text{ s}$. For this t, $x = 3.85\times10^4\text{ m}$.

EVALUATE: The graph in part (c) shows the path is not symmetric about the highest point and the time to return to the ground is less than twice the time to the maximum height.

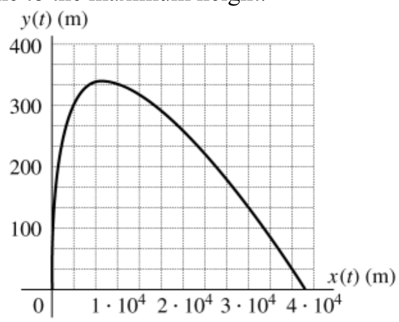


Figure 3.44

3.45. **IDENTIFY:** $\vec{v} = \frac{d\vec{r}}{dt}$ and $\vec{a} = \frac{d\vec{v}}{dt}$

SET UP: $\frac{d}{dt}(t^n) = nt^{n-1}$. At $t = 1.00\text{ s}$, $a_x = 4.00\text{ m/s}^2$ and $a_y = 3.00\text{ m/s}^2$. At $t = 0$, $x = 0$ and $y = 50.0\text{ m}$.

EXECUTE: **(a)** $v_x = \frac{dx}{dt} = 2Bt$. $a_x = \frac{dv_x}{dt} = 2B$, which is independent of t. $a_x = 4.00\text{ m/s}^2$ gives $B = 2.00\text{ m/s}^2$.

$v_y = \frac{dy}{dt} = 3Dt^2$. $a_y = \frac{dv_y}{dt} = 6Dt$. $a_y = 3.00\text{ m/s}^2$ gives $D = 0.500\text{ m/s}^2$. $x = 0$ at $t = 0$ gives $A = 0$. $y = 50.0\text{ m}$ at $t = 0$ gives $C = 50.0\text{ m}$.

(b) At $t = 0$, $v_x = 0$ and $v_y = 0$, so $\vec{v} = 0$. At $t = 0$, $a_x = 2B = 4.00\text{ m/s}^2$ and $a_y = 0$, so $\vec{a} = (4.00\text{ m/s}^2)\hat{i}$.

(c) At $t = 10.0\text{ s}$, $v_x = 2(2.00\text{ m/s}^2)(10.0\text{ s}) = 40.0\text{ m/s}$ and $v_y = 3(0.500\text{ m/s}^3)(10.0\text{ s})^2 = 150\text{ m/s}$.

$v = \sqrt{v_x^2 + v_y^2} = 155\text{ m/s}$.

(d) $x = (2.00\text{ m/s}^2)(10.0\text{ s})^2 = 200\text{ m}$, $y = 50.0\text{ m} + (0.500\text{ m/s}^3)(10.0\text{ s})^3 = 550\text{ m}$. $\vec{r} = (200\text{ m})\hat{i} + (550\text{ m})\hat{j}$.

EVALUATE: The velocity and acceleration vectors as functions of time are $\vec{v}(t) = (2Bt)\hat{i} + (3Dt^2)\hat{j}$ and $\vec{a}(t) = (2B)\hat{i} + (6Dt)\hat{j}$. The acceleration is not constant.

3.46. **IDENTIFY:** $\vec{r} = \vec{r}_0 + \int_0^t \vec{v}(t)dt$ and $\boldsymbol{a} = \frac{d\vec{v}}{dt}$.

SET UP: At $t = 0$, $x_0 = 0$ and $y_0 = 0$.

EXECUTE: **(a)** Integrating, $\vec{r} = (\alpha t - \frac{\beta}{3}t^3)\hat{i} + (\frac{\gamma}{2}t^2)\hat{j}$. Differentiating, $\vec{a} = (-2\beta t)\hat{i} + \gamma\,\hat{j}$.

(b) The positive time at which $x = 0$ is given by $t^2 = 3\alpha/\beta$. At this time, the y-coordinate is

$$y = \frac{\gamma}{2}t^2 = \frac{3\alpha\gamma}{2\beta} = \frac{3(2.4\text{ m/s})(4.0\text{ m/s}^2)}{2(1.6\text{ m/s}^3)} = 9.0\text{ m}.$$

EVALUATE: The acceleration is not constant.

3.47. **IDENTIFY:** Once the rocket leaves the incline it moves in projectile motion. The acceleration along the incline determines the initial velocity and initial position for the projectile motion.

SET UP: For motion along the incline let $+x$ be directed up the incline. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = \sqrt{2(1.25\text{ m/s}^2)(200\text{ m})} = 22.36\text{ m/s}$. When the projectile motion begins the rocket has $v_0 = 22.36\text{ m/s}$ at $35.0°$ above the horizontal and is at a vertical height of $(200.0\text{ m})\sin 35.0° = 114.7\text{ m}$. For the projectile motion let $+x$ be horizontal to the right and let $+y$ be upward. Let $y = 0$ at the ground. Then $y_0 = 114.7\text{ m}$, $v_{0x} = v_0\cos 35.0° = 18.32\text{ m/s}$, $v_{0y} = v_0 \sin 35.0° = 12.83\text{ m/s}$, $a_x = 0$, $a_y = -9.80\text{ m/s}^2$. Let $x = 0$ at point *A*, so $x_0 = (200.0\text{ m})\cos 35.0° = 163.8\text{ m}$.

EXECUTE: **(a)** At the maximum height $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$y - y_0 = \dfrac{v_y^2 - v_{0y}^2}{2a_y} = \dfrac{0 - (12.83\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = 8.40\text{ m}$ and $y = 114.7\text{ m} + 8.40\text{ m} = 123\text{ m}$. The maximum height above ground is 123 m.

(b) The time in the air can be calculated from the vertical component of the projectile motion: $y - y_0 = -114.7\text{ m}$, $v_{0y} = 12.83\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $(4.90\text{ m/s}^2)t^2 - (12.83\text{ m/s})t - 114.7\text{ m}$. The quadratic formula gives $t = \dfrac{1}{9.80}\left(12.83 \pm \sqrt{(12.83)^2 + 4(4.90)(114.7)}\right)\text{ s}$. The positive root is $t = 6.32\text{ s}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (18.32\text{ m/s})(6.32\text{ s}) = 115.8\text{ m}$ and $x = 163.8\text{ m} + 115.8\text{ m} = 280\text{ m}$. The horizontal range of the rocket is 280 m.

EVALUATE: The expressions for *h* and *R* derived in Example 3.8 do not apply here. They are only for a projectile fired on level ground.

3.48. **IDENTIFY:** The person moves in projectile motion. Use the results in Example 3.8 to determine how *T*, *h* and *D* depend on *g* and set up a ratio.

SET UP: From Example 3.8, the time in the air is $t = \dfrac{2v_0 \sin\alpha_0}{g}$, the maximum height is $h = \dfrac{v_0^2 \sin^2\alpha_0}{2g}$ and the horizontal range (called *D* in the problem) is $D = \dfrac{v_0^2 \sin 2\alpha_0}{g}$. The person has the same v_0 and α_0 on Mars as on the earth.

EXECUTE: $tg = 2v_0\sin\alpha_0$, which is constant, so $t_E g_E = t_M g_M$. $t_M = \left(\dfrac{g_E}{g_M}\right)t_E = \left(\dfrac{g_E}{0.379 g_E}\right)t_E = 2.64 t_E$.

$hg = \dfrac{v_0^2 \sin^2\alpha_0}{2}$, which is constant, so $h_E g_E = h_M g_M$. $h_M = \left(\dfrac{g_E}{g_M}\right)h_E = 2.64 h_E$. $Dg = v_0^2 \sin 2\alpha_0$, which is constant, so $D_E g_E = D_M g_M$. $D_M = \left(\dfrac{g_E}{g_M}\right)D_E = 2.64 D_E$.

EVALUATE: All three quantities are proportional to $1/g$ so all increase by the same factor of $g_E/g_M = 2.64$.

3.49. **IDENTIFY:** The range for a projectile that lands at the same height from which it was launched is $R = \dfrac{v_0^2 \sin 2\alpha}{g}$.

SET UP: The maximum range is for $\alpha = 45°$.

EXECUTE: Assuming $\alpha = 45°$, and $R = 50\text{ m}$, $v_0 = \sqrt{gR} = 22\text{ m/s}$.

EVALUATE: We have assumed that debris was launched at all angles, including the angle of $45°$ that gives maximum range.

3.50. **IDENTIFY:** The velocity has a horizontal tangential component and a vertical component. The vertical component of acceleration is zero and the horizontal component is $a_{rad} = \dfrac{v_x^2}{R}$

SET UP: Let $+y$ be upward and $+x$ be in the direction of the tangential velocity at the instant we are considering.

EXECUTE: **(a)** The bird's tangential velocity can be found from

$$v_x = \frac{\text{circumference}}{\text{time of rotation}} = \frac{2\pi(8.00\text{ m})}{5.00\text{ s}} = \frac{50.27\text{ m}}{5.00\text{ s}} = 10.05\text{ m/s}$$

Thus its velocity consists of the components $v_x = 10.05$ m/s and $v_y = 3.00$ m/s. The speed relative to the ground is then $v = \sqrt{v_x^2 + v_y^2} = 10.5$ m/s **.**

(b) The bird's speed is constant, so its acceleration is strictly centripetal–entirely in the horizontal direction, toward the center of its spiral path–and has magnitude $a_{\text{rad}} = \frac{v_x^2}{r} = \frac{(10.05\text{ m/s})^2}{8.00\text{ m}} = 12.6\text{ m/s}^2$ **.**

(c) Using the vertical and horizontal velocity components $\theta = \tan^{-1}\frac{3.00\text{ m/s}}{10.05\text{ m/s}} = 16.6°$ **.**

EVALUATE: The angle between the bird's velocity and the horizontal remains constant as the bird rises.

3.51. **IDENTIFY:** Take $+y$ to be downward. Both objects have the same vertical motion, with v_{0y} and $a_y = +g$. Use constant acceleration equations for the *x* and *y* components of the motion.

SET UP: Use the vertical motion to find the time in the air:

$v_{0y} = 0,\ \ a_y = 9.80\text{ m/s}^2,\ \ y - y_0 = 25\text{ m},\ \ t = ?$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 2.259$ s

During this time the dart must travel 90 m, so the horizontal component of its velocity must be

$$v_{0x} = \frac{x - x_0}{t} = \frac{90\text{ m}}{2.25\text{ s}} = 40\text{ m/s}$$

EVALUATE: Both objects hit the ground at the same time. The dart hits the monkey for any muzzle velocity greater than 40 m/s.

3.52. **IDENTIFY:** The person moves in projectile motion. Her vertical motion determines her time in the air.

SET UP: Take $+y$ upward. $v_{0x} = 15.0$ m/s, $v_{0y} = +10.0$ m/s, $a_x = 0$, $a_y = -9.80\text{ m/s}^2$.

EXECUTE: **(a)** Use the vertical motion to find the time in the air: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = -30.0$ m gives $-30.0\text{ m} = (10.0\text{ m/s})t - (4.90\text{ m/s}^2)t^2$. The quadratic formula gives

$t = \frac{1}{2(4.9)}\left(+10.0 \pm \sqrt{(-10.0)^2 - 4(4.9)(-30)}\right)$ s. The positive solution is $t = 3.70$ s. During this time she travels a horizontal distance $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (15.0\text{ m/s})(3.70\text{ s}) = 55.5$ m. She will land 55.5 m south of the point where she drops from the helicopter and this is where the mats should have been placed.

(b) The *x*-*t*, *y*-*t*, v_x-*t* and v_y-*t* graphs are sketched in Figure 3.52.

EVALUATE: If she had dropped from rest at a height of 30.0 m it would have taken her $t = \sqrt{\frac{2(30.0\text{ m})}{9.80\text{ m/s}^2}} = 2.47$ s. She is in the air longer than this because she has an initial vertical component of velocity that is upward.

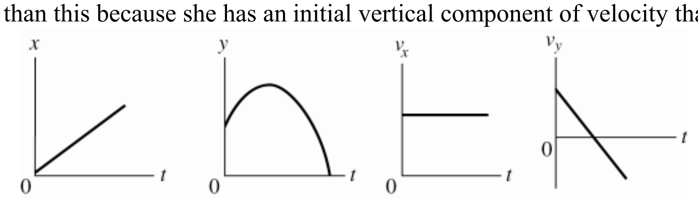


Figure 3.52

3.53. **IDENTIFY:** The cannister moves in projectile motion. Its initial velocity is horizontal. Apply constant acceleration equations for the *x* and *y* components of motion.

SET UP:

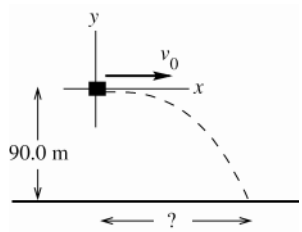


Figure 3.53

Take the origin of coordinates at the point where the canister is released. Take $+y$ to be upward. The initial velocity of the canister is the velocity of the plane, 64.0 m/s in the $+x$-direction.

Use the vertical motion to find the time of fall:
$t = ?$, $v_{0y} = 0$, $a_y = -9.80\text{ m/s}^2$, $y - y_0 = -90.0\text{ m}$ (When the canister reaches the ground it is 90.0 m below the origin.)

$$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$$

EXECUTE: Since $v_{0y} = 0$, $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(-90.0\text{ m})}{-9.80\text{ m/s}^2}} = 4.286\text{ s}.$

SET UP: Then use the horizontal component of the motion to calculate how far the canister falls in this time:
$x - x_0 = ?$, $a_x - 0$, $v_{0x} = 64.0\text{ m/s}$,

EXECUTE: $x - x_0 = v_0 t + \tfrac{1}{2}at^2 = (64.0\text{ m/s})(4.286\text{ s}) + 0 = 274\text{ m}.$

EVALUATE: The time it takes the cannister to fall 90.0 m, starting from rest, is the time it travels horizontally at constant speed.

3.54. **IDENTIFY:** The equipment moves in projectile motion. The distance *D* is the horizontal range of the equipment plus the distance the ship moves while the equipment is in the air.

SET UP: For the motion of the equipment take $+x$ to be to the right and $+y$ to be upwards. Then $a_x = 0$, $a_y = -9.80\text{ m/s}^2$, $v_{0x} = v_0\cos\alpha_0 = 7.50\text{ m/s}$ and $v_{0y} = v_0\sin\alpha_0 = 13.0\text{ m/s}$. When the equipment lands in the front of the ship, $y - y_0 = -8.75\text{ m}$.

EXECUTE: Use the vertical motion of the equipment to find its time in the air: $y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $t = \dfrac{1}{9.80}\left(13.0 \pm \sqrt{(-13.0)^2 + 4(4.90)(8.75)}\right)\text{ s}$. The positive root is $t = 3.21\text{ s}$. The horizontal range of the equipment is $x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 = (7.50\text{ m/s})(3.21\text{ s}) = 24.1\text{ m}$. In 3.21 s the ship moves a horizontal distance $(0.450\text{ m/s})(3.21\text{ s}) = 1.44\text{ m}$, so $D = 24.1\text{ m} + 1.44\text{ m} = 25.5\text{ m}$.

EVALUATE: The equation $R = \dfrac{v_0^2\sin 2\alpha_0}{g}$ from Example 3.8 can't be used because the starting and ending points of the projectile motion are at different heights.

3.55. **IDENTIFY:** Projectile motion problem.

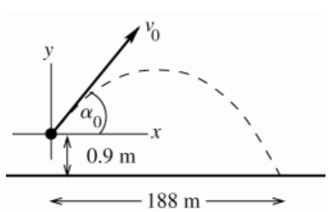


Figure 3.55

Take the origin of coordinates at the point where the ball leaves the bat, and take $+y$ to be upward.
$v_{0x} = v_0\cos\alpha_0$
$v_{0y} = v_0\sin\alpha_0,$
but we don't know v_0.

Write down the equation for the horizontal displacement when the ball hits the ground and the corresponding equation for the vertical displacement. The time *t* is the same for both components, so this will give us two equations in two unknowns (v_0 and *t*).

(a) SET UP: <u>y-component</u>:

$a_y = -9.80\text{ m/s}^2,\quad y - y_0 = -0.9\text{ m},\quad v_{0y} = v_0 \sin 45°$

$y - y_0 = v_{0y} + t\tfrac{1}{2}a_y t^2$

EXECUTE: $-0.9\text{ m} = (v_0 \sin 45°)t + \tfrac{1}{2}(-9.80\text{ m/s}^2)t^2$

SET UP: <u>x-component</u>:

$a_x = 0,\quad x - x_0 = 188\text{ m},\quad v_{0x} = v_0 \cos 45°$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$

EXECUTE: $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{188\text{ m}}{v_0 \cos 45°}$

Put the expression for t from the x-component motion into the y-component equation and solve for v_0. (Note that $\sin 45° = \cos 45°$.)

$$-0.9\text{ m} = (v_0 \sin 45°)\left(\frac{188\text{ m}}{v_0 \cos 45°}\right) - (4.90\text{ m/s}^2)\left(\frac{188\text{ m}}{v_0 \cos 45°}\right)^2$$

$$4.90\text{ m/s}^2\left(\frac{188\text{ m}}{v_0 \cos 45°}\right)^2 = 188\text{ m} + 0.9\text{ m} = 188.9\text{ m}$$

$$\left(\frac{v_0 \cos 45°}{188\text{ m}}\right)^2 = \frac{4.90\text{ m/s}^2}{188.9\text{ m}},\quad v_0 = \left(\frac{188\text{ m}}{\cos 45°}\right)\sqrt{\frac{4.90\text{ m/s}^2}{188.9\text{ m}}} = 42.8\text{ m/s}$$

(b) Use the horizontal motion to find the time it takes the ball to reach the fence:

SET UP: <u>x-component</u>:

$x - x_0 = 116\text{ m},\quad a_x = 0,\quad v_{0x} = v_0 \cos 45° = (42.8\text{ m/s})\cos 45° = 30.3\text{ m/s},\quad t = ?$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$

EXECUTE: $t = \dfrac{x - x_0}{v_{0x}} = \dfrac{116\text{ m}}{30.3\text{ m/s}} = 3.83\text{ s}$

SET UP: Find the vertical displacement of the ball at this t:

<u>y-component</u>:

$y - y_0 = ?,\quad a_y = -9.80\text{ m/s}^2,\quad v_{0y} = v_0 \sin 45° = 30.3\text{ m/s},\quad t = 3.83\text{ s}$

$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$

EXECUTE: $y - y_0 = (30.3\text{ s})(3.83\text{ s}) + \tfrac{1}{2}(-9.80\text{ m/s}^2)(3.83\text{ s})^2$

$y - y_0 = 116.0\text{ m} - 71.9\text{ m} = +44.1\text{ m}$, above the point where the ball was hit. The height of the ball above the ground is $44.1\text{ m} + 0.90\text{ m} = 45.0\text{ m}$. It's height then above the top of the fence is $45.0\text{ m} - 3.0\text{ m} = 42.0\text{ m}$.

EVALUATE: With $v_0 = 42.8\text{ m/s}$, $v_{0y} = 30.3\text{ m/s}$ and it takes the ball 6.18 s to return to the height where it was hit and only slightly longer to reach a point 0.9 m below this height. $t = (188\text{ m})/(v_0 \cos 45°)$ gives $t = 6.21\text{ s}$, which agrees with this estimate. The ball reaches its maximum height approximately $(188\text{ m})/2 = 94\text{ m}$ from home plate, so at the fence the ball is not far past its maximum height of 47.6 m, so a height of 45.0 m at the fence is reasonable.

3.56. IDENTIFY: The water moves in projectile motion.

SET UP: Let $x_0 = y_0 = 0$ and take $+y$ to be positive. $a_x = 0$, $a_y = -g$.

EXECUTE: The equations of motions are $y = (v_0 \sin \alpha)t - \tfrac{1}{2}gt^2$ and $x = (v_0 \cos \alpha)t$. When the water goes in the tank for the *minimum* velocity, $y = 2D$ and $x = 6D$. When the water goes in the tank for the *maximum* velocity, $y = 2D$ and $x = 7D$. In both cases, $\sin \alpha = \cos \alpha = \sqrt{2}/2$.

To reach the *minimum* distance: $6D = \dfrac{\sqrt{2}}{2}v_0 t$, and $2D = \dfrac{\sqrt{2}}{2}v_0 t - \tfrac{1}{2}gt^2$. Solving the first equation for t gives $t = \dfrac{6D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 6D - \tfrac{1}{2}g\left(\dfrac{6D\sqrt{2}}{v_0}\right)^2$. Solving this for v_0 gives $v_0 = 3\sqrt{gD}$.

To reach the *maximum* distance: $7D = \frac{\sqrt{2}}{2}v_0 t$, and $2D = \frac{\sqrt{2}}{2}v_0 t - \frac{1}{2}gt^2$. Solving the first equation for *t* gives $t = \frac{7D\sqrt{2}}{v_0}$. Substituting this into the second equation gives $2D = 7D - \frac{1}{2}g\left(\frac{7D\sqrt{2}}{v_0}\right)^2$. Solving this for v_0 gives $v_0 = \sqrt{49gD/5} = 3.13\sqrt{gD}$, which, as expected, is larger than the previous result.

EVALUATE: A launch speed of $v_0 = \sqrt{6}\sqrt{gD} = 2.45\sqrt{gD}$ is required for a horizontal range of 6*D*. The minimum speed required is greater than this, because the water must be at a height of at least 2*D* when it reaches the front of the tank.

3.57. **IDENTIFY:** The equations for *h* and *R* from Example 3.8 can be used.

SET UP: $h = \frac{v_0^2 \sin^2 \alpha_0}{2g}$ and $R = \frac{v_0^2 \sin 2\alpha_0}{g}$. If the projectile is launched straight up, $\alpha_0 = 90°$.

EXECUTE: **(a)** $h = \frac{v_0^2}{2g}$ and $v_0 = \sqrt{2gh}$.

(b) Calculate α_0 that gives a maximum height of *h* when $v_0 = 2\sqrt{2gh}$. $h = \frac{8gh\sin^2 \alpha_0}{2g} = 4h\sin^2 \alpha_0$. $\sin\alpha_0 = \frac{1}{2}$ and $\alpha_0 = 30.0°$.

(c) $R = \frac{\left(2\sqrt{2gh}\right)^2 \sin 60.0°}{g} = 6.93h$.

EVALUATE: $\frac{v_0^2}{g} = \frac{2h}{\sin^2 \alpha_0}$ so $R = \frac{2h\sin(2\alpha_0)}{\sin^2 \alpha_0}$. For a given α_0 , *R* increases when *h* increases. For $\alpha_0 = 90°$, $R = 0$ and for $\alpha_0 = 0°$, $h = 0$ and $R = 0$. For $\alpha_0 = 45°$, $R = 4h$.

3.58. **IDENTIFY:** To clear the bar the ball must have a height of 10.0 ft when it has a horizontal displacement of 36.0 ft. The ball moves as a projectile. When v_0 is very large, the ball reaches the goal posts in a very short time and the acceleration due to gravity causes negligible downward displacement.

SET UP: 36.0 ft = 10.97 m ; 10.0 ft = 3.048 m . Let $+x$ be to the right and $+y$ be upward, so $a_x = 0$, $a_y = -g$, $v_{0x} = v_0 \cos\alpha_0$ and $v_{0y} = v_0 \sin\alpha_0$

EXECUTE: **(a)** The ball cannot be aimed lower than directly at the bar. $\tan\alpha_0 = \frac{10.0 \text{ ft}}{36.0 \text{ ft}}$ and $\alpha_0 = 15.5°$.

(b) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \frac{x - x_0}{v_{0x}} = \frac{x - x_0}{v_0 \cos\alpha_0}$. Then $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$$y - y_0 = (v_0 \sin\alpha_0)\left(\frac{x - x_0}{v_0 \cos\alpha_0}\right) - \frac{1}{2}g\frac{(x - x_0)^2}{v_0^2 \cos^2 \alpha_0} = (x - x_0)\tan\alpha_0 - \frac{1}{2}g\frac{(x - x_0)^2}{v_0^2 \cos^2 \alpha_0} .$$

$$v_0 = \frac{(x - x_0)}{\cos\alpha_0}\sqrt{\frac{g}{2[(x - x_0)\tan\alpha_0 - (y - y_0)]}} = \frac{10.97 \text{ m}}{\cos 45.0°}\sqrt{\frac{9.80 \text{ m/s}^2}{2[10.97 \text{ m} - 3.048 \text{ m}]}} = 12.2 \text{ m/s}$$

EVALUATE: With the v_0 in part (b) the horizontal range of the ball is $R = \frac{v_0^2 \sin 2\alpha_0}{g} = 15.2 \text{ m} = 49.9 \text{ ft}$. The ball reaches the highest point in its trajectory when $x - x_0 = R/2$, so when it reaches the goal posts it is on its way down.

3.59. **IDENTIFY:** Apply Eq.(3.27) and solve for *x*.

SET UP: The change in height is $y = -h$.

EXECUTE: **(a)** We get a quadratic equation in *x*, the solution to which is

$$x = \frac{v_0^2 \cos\alpha_0}{g}\left[\tan^2 \alpha_0 + \frac{2gh}{v_0^2 \cos\alpha_0}\right] = \frac{v_0 \cos\alpha_0}{g}\left[v_0 \sin\alpha_0 + \sqrt{v_0^2 \sin^2 \alpha_0 + 2gh}\right].$$

If $h = 0$, the square root reduces to $v_0 \sin \alpha_0$, and $x = R$.

(b) The expression for x becomes $x = (10.2\text{ m})\cos\alpha_0 + [\sin^2\alpha_0 + \sqrt{\sin^2\alpha_0 + 0.98}]$. The graph of x as a function of α_0 is sketched in Figure 3.59. The angle $\alpha_0 = 90°$ corresponds to the projectile being launched straight up, and there is no horizontal motion. If $\alpha_0 = 0$, the projectile moves horizontally until it has fallen the distance h.

(d) The graph shows that the maximum horizontal distance is for an angle less than $45°$.

EVALUATE: For $\alpha_0 = 45°$ the x and y components of the initial velocity are equal. For $\alpha_0 < 45°$ the x component of the initial velocity is less than the y component. Height comes from the initial position and less vertical component of initial velocity is needed for the maximum range.

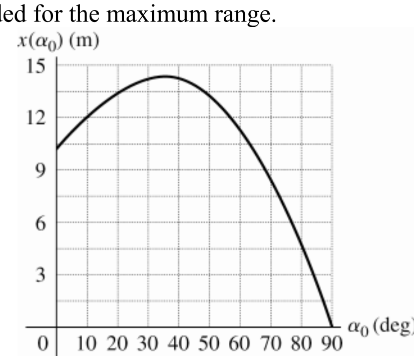


Figure 3.59

3.60. **IDENTIFY:** The snowball moves in projectile motion. In part (a) the vertical motion determines the time in the air. In part (c), find the height of the snowball above the ground after it has traveled horizontally 4.0 m.

SET UP: Let $+y$ be downward. $a_x = 0$, $a_y = +9.80\text{ m/s}^2$. $v_{0x} = v_0\cos\theta_0 = 5.36\text{ m/s}$, $v_{0y} = v_0\sin\theta_0 = 4.50\text{ m/s}$.

EXECUTE: **(a)** Use the vertical motion to find the time in the air: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 14.0\text{ m}$ gives $14.0\text{ m} = (4.50\text{ m/s})t + (4.9\text{ m/s}^2)t^2$. The quadratic formula gives $t = \frac{1}{2(4.9)}\left(-4.50 \pm \sqrt{(4.50)^2 - 4(4.9)(-14.0)}\right)\text{ s}$.

The positive root is $t = 1.29\text{ s}$. Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (5.36\text{ m/s})(1.29\text{ s}) = 6.91\text{ m}$.

(b) The x-t, y-t, v_x-t and v_y-t graphs are sketched in Figure 3.60.

(c) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \frac{x - x_0}{v_{0x}} = \frac{4.0\text{ m}}{5.36\text{ m/s}} = 0.746\text{ s}$. In this time the snowball travels downward a distance $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = 6.08\text{ m}$ and is therefore $14.0\text{ m} - 6.08\text{ m} = 7.9\text{ m}$ above the ground. The snowball passes well above the man and doesn't hit him.

EVALUATE: If the snowball had been released from rest at a height of 14.0 m it would have reached the ground in $t = \sqrt{\frac{2(14.0\text{ m})}{9.80\text{ m/s}^2}} = 1.69\text{ s}$. The snowball reaches the ground in a shorter time than this because of its initial downward component of velocity.

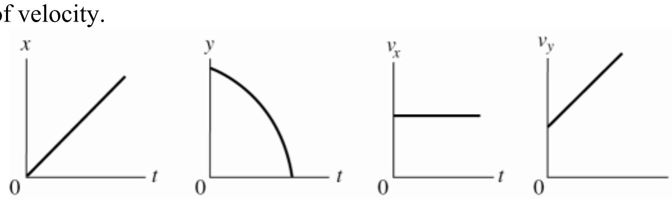


Figure 3.60

3.61. **(a) IDENTIFY** and **SET UP:** Use the equation derived in Example 3.8:

$$R = (v_0\cos\alpha_0)\left(\frac{2v_0\sin\alpha_0}{g}\right)$$

Call the range R_1 when the angle is α_0 and R_2 when the angle is $90° - \alpha$.

$$R_1 = (v_0 \cos\alpha_0)\left(\frac{2v_0 \sin\alpha_0}{g}\right)$$

$$R_2 = (v_0 \cos(90° - \alpha_0))\left(\frac{2v_0 \sin(90° - \alpha_0)}{g}\right)$$

The problem asks us to show that $R_1 = R_2$.

EXECUTE: We can use the trig identities in Appendix B to show:

$\cos(90° - \alpha_0) = \cos(\alpha_0 - 90°) = \sin\alpha_0$

$\sin(90° - \alpha_0) = -\sin(\alpha_0 - 90°) = -(-\cos\alpha_0) = +\cos\alpha_0$

Thus $R_2 = (v_0 \sin\alpha_0)\left(\frac{2v_0 \cos\alpha_0}{g}\right) = (v_0 \cos\alpha_0)\left(\frac{2v_0 \sin\alpha_0}{g}\right) = R_1.$

(b) $R = \frac{v_0^2 \sin 2\alpha_0}{g}$ so $\sin 2\alpha_0 = \frac{Rg}{v_0^2} = \frac{(0.25\text{ m})(9.80\text{ m/s}^2)}{(2.2\text{ m/s})^2}.$

This gives $\alpha = 15°$ or $75°$.

EVALUATE: $R = (v_0^2 \sin 2\alpha_0)/g$, so the result in part (a) requires that $\sin^2(2\alpha_0) = \sin^2(180° - 2\alpha_0)$, which is true. (Try some values of α_0 and see!)

3.62. **IDENTIFY:** Mary Belle moves in projectile motion.

SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -g$.

EXECUTE: **(a)** Eq.(3.27) with $x = 8.2$ m, $y = 6.1$ m and $\alpha_0 = 53°$ gives $v_0 = 13.8$ m/s.

(b) When she reached Joe Bob, $t = \frac{8.2\text{ m}}{v_0 \cos 53°} = 0.9874$ s. $v_x = v_{0x} = 8.31$ m/s and $v_y = v_{0y} + a_y t = +1.34$ m/s.

$v = 8.4$ m/s, at an angle of $9.16°$.

(c) The graph of $v_x(t)$ is a horizontal line. The other graphs are sketched in Figure 3.62.

(d) Use Eq. (3.27), which becomes $y = (1.327)x - (0.071115\ \text{m}^{-1})x^2$. Setting $y = -8.6$ m gives $x = 23.8$ m as the positive solution.

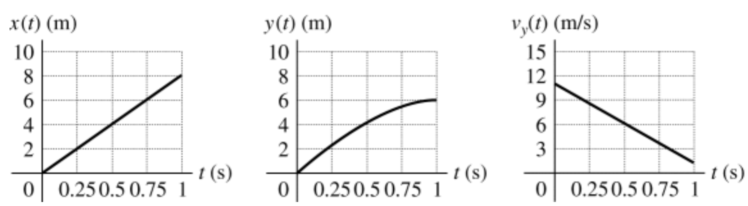


Figure 3.62

3.63. **(a) IDENTIFY:** Projectile motion.

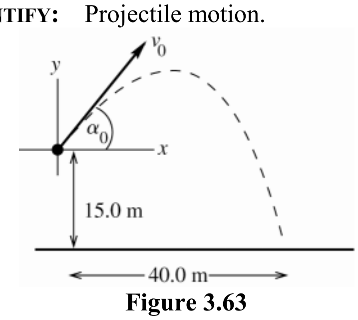


Figure 3.63

Take the origin of coordinates at the top of the ramp and take $+y$ to be upward. The problem specifies that the object is displaced 40.0 m to the right when it is 15.0 m below the origin.

We don't know *t*, the time in the air, and we don't know v_0. Write down the equations for the horizontal and vertical displacements. Combine these two equations to eliminate one unknown.

SET UP: *y*-component:

$y - y_0 = -15.0$ m, $a_y = -9.80\text{ m/s}^2$, $v_{0y} = v_0 \sin 53.0°$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

EXECUTE: $-15.0\text{ m} = (v_0 \sin 53.0°)t - (4.90\text{ m/s}^2)t^2$

SET UP: <u>x-component</u>:
$x - x_0 = 40.0\text{ m},\ \ a_x = 0,\ \ v_{0x} = v_0\cos 53.0°$
$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$
EXECUTE: $40.0\text{ m} = (v_0 t)\cos 53.0°$

The second equation says $v_0 t = \dfrac{40.0\text{ m}}{\cos 53.0°} = 66.47\text{ m}.$

Use this to replace $v_0 t$ in the first equation:

$$-15.0\text{ m} = (66.47\text{ m})\sin 53° - (4.90\text{ m/s}^2)t^2$$

$$t = \sqrt{\frac{(66.46\text{ m})\sin 53° + 15.0\text{ m}}{4.90\text{ m/s}^2}} = \sqrt{\frac{68.08\text{ m}}{4.90\text{ m/s}^2}} = 3.727\text{ s}.$$

Now that we have t we can use the x-component equation to solve for v_0:

$$v_0 = \frac{40.0\text{ m}}{t\cos 53.0°} = \frac{40.0\text{ m}}{(3.727\text{ s})\cos 53.0°} = 17.8\text{ m/s}.$$

EVALUATE: Using these values of v_0 and t in the $y = y_0 = v_{0y} + \tfrac{1}{2}a_y t^2$ equation verifies that $y - y_0 = -15.0\text{ m}$.

(b) IDENTIFY: $v_0 = (17.8\text{ m/s})/2 = 8.9\text{ m/s}$
This is less than the speed required to make it to the other side, so he lands in the river.
Use the vertical motion to find the time it takes him to reach the water:
SET UP: $y - y_0 = -100\text{ m};\ \ v_{0y} = +v_0\sin 53.0° = 7.11\text{ m/s};\ \ a_y = -9.80\text{ m/s}^2$
$y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$ gives $-100 = 7.11t - 4.90t^2$
EXECUTE: $4.90t^2 - 7.11t - 100 = 0$ and $t = \tfrac{1}{9.80}\left(7.11 \pm \sqrt{(7.11)^2 - 4(4.90)(-100)}\right)$
$t = 0.726\text{ s} \pm 4.57\text{ s}$ so $t = 5.30\text{ s}.$
The horizontal distance he travels in this time is

$$x - x_0 = v_{0x}t = (v_0\cos 53.0°)t = (5.36\text{ m/s})(5.30\text{ s}) = 28.4\text{ m}.$$

He lands in the river a horizontal distance of 28.4 m from his launch point.
EVALUATE: He has half the minimum speed and makes it only about halfway across.

3.64. **IDENTIFY:** The rock moves in projectile motion.
SET UP: Let $+y$ be upward. $a_x = 0$, $a_y = -g$. Eqs.(3.22) and (3.23) give v_x and v_y.
EXECUTE: Combining equations 3.25, 3.22 and 3.23 gives
$v^2 = v_0^2\cos^2\alpha_0 + (v_0\sin\alpha_0 - gt)^2 = v_0^2(\sin^2\alpha_0 + \cos^2\alpha_0) - 2v_0\sin\alpha_0 gt + (gt)^2$.
$v^2 = v_0^2 - 2g(v_0\sin\alpha_0 t - \dfrac{1}{2}gt^2) = v_0^2 - 2gy$, where Eq.(3.21) has been used to eliminate t in favor of y. For the case of a rock thrown from the roof of a building of height h, the speed at the ground is found by substituting $y = -h$ into the above expression, yielding $v = \sqrt{v_0^2 + 2gh}$, which is independent of α_0.
EVALUATE: This result, as will be seen in the chapter dealing with conservation of energy (Chapter 7), is valid for any y, positive, negative or zero, as long as $v_0^2 - 2gy > 0$.

3.65. **IDENTIFY and SET UP:** Take $+y$ to be upward. The rocket moves with projectile motion, with $v_{0y} = +40.0\text{ m/s}$ and $v_{0x} = 30.0\text{ m/s}$ relative to the ground. The vertical motion of the rocket is unaffected by its horizontal velocity.
EXECUTE: **(a)** $v_y = 0$ (at maximum height), $v_{0y} = +40.0\text{ m/s},\ \ a_y = -9.80\text{ m/s}^2,\ \ y - y_0 = ?$
$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $y - y_0 = 81.6\text{ m}$
(b) Both the cart and the rocket have the same constant horizontal velocity, so both travel the same horizontal distance while the rocket is in the air and the rocket lands in the cart.
(c) Use the vertical motion of the rocket to find the time it is in the air.
$v_{0y} = 40\text{ m/s},\ \ a_y = -9.80\text{ m/s}^2,\ \ v_y = -40\text{ m/s},\ \ t = ?$
$v_y = v_{0y} + a_y t$ gives $t = 8.164\text{ s}$
Then $x - x_0 = v_{0x}t = (30.0\text{ m/s})(8.164\text{ s}) = 245\text{ m}.$

(d) Relative to the ground the rocket has initial velocity components $v_{0x} = 30.0$ m/s and $v_{0y} = 40.0$ m/s, so it is traveling at $53.1°$ above the horizontal.
(e) (i)

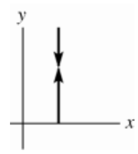


Figure 3.65a

Relative to the cart, the rocket travels straight up and then straight down
(ii)

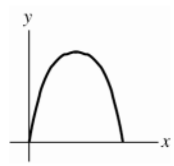


Figure 3.65b

Relative to the ground the rocket travels in a parabola.
EVALUATE: Both the cart and rocket have the same constant horizontal velocity. The rocket lands in the cart.

3.66. **IDENTIFY:** The ball moves in projectile motion.
SET UP: The woman and ball travel for the same time and must travel the same horizontal distance, so for the ball $v_{0x} = 6.00\text{ m/s}$.
EXECUTE: **(a)** $v_{0x} = v_0 \cos\theta_0$. $\cos\theta_0 = \dfrac{v_{0x}}{v_0} = \dfrac{6.00\text{ m/s}}{20.0\text{ m/s}}$ and $\theta_0 = 72.5°$.
(b) Relative to the ground the ball moves in a parabola. The ball and the runner have the same horizontal component of velocity, so relative to the runner the ball has only vertical motion. The trajectories as seen by each observer are sketched in Figure 3.66.
EVALUATE: The ball could be thrown with a different speed, so long as the angle at which it was thrown was adjusted to keep $v_{0x} = 6.00\text{ m/s}$.

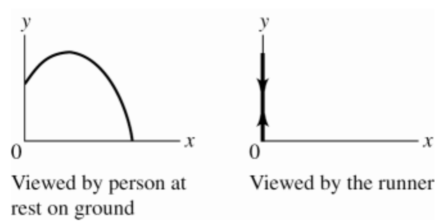


Figure 3.66

3.67. **IDENTIFY:** The boulder moves in projectile motion.
SET UP: Take $+y$ downward. $v_{0x} = v_0$, $v_{0y} = 0$. $a_x = 0$, $a_y = +9.80\text{ m/s}^2$.
EXECUTE: **(a)** Use the vertical motion to find the time for the boulder to reach the level of the lake: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = +20$ m gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(20\text{ m})}{9.80\text{ m/s}^2}} = 2.02$ s. The rock must travel horizontally 100 m during this time. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $v_0 = v_{0x} = \dfrac{x - x_0}{t} = \dfrac{100\text{ m}}{2.02\text{ s}} = 49.5\text{ m/s}$
(b) In going from the edge of the cliff to the plain, the boulder travels downward a distance of $y - y_0 = 45$ m.
$t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(45\text{ m})}{9.80\text{ m/s}^2}} = 3.03$ s and $x - x_0 = v_{0x}t = (49.5\text{ m/s})(3.03\text{ s}) = 150$ m. The rock lands $150\text{ m} - 100\text{ m} = 50\text{ m}$ beyond the foot of the dam.
EVALUATE: The boulder passes over the dam 2.02 s after it leaves the cliff and then travels an additional 1.01 s before landing on the plain. If the boulder has an initial speed that is less than 49 m/s, then it lands in the lake.

3.68. **IDENTIFY:** The bagels move in projectile motion. Find Henrietta's location when the bagels reach the ground, and require the bagels to have this horizontal range.

SET UP: Let $+y$ be downward and let $x_0 = y_0 = 0$. $a_x = 0$, $a_y = +g$. When the bagels reach the ground, $y = 43.9\text{ m}$.

EXECUTE: **(a)** When she catches the bagels, Henrietta has been jogging for 9.00 s plus the time for the bagels to fall 43.9 m from rest. Get the time to fall: $y = \frac{1}{2}gt^2$, $43.9\text{ m} = \frac{1}{2}(9.80\text{ m/s}^2)t^2$ and $t = 2.99\text{ s}$. So, she has been jogging for $9.00\text{ s} + 2.99\text{ s} = 12.0\text{ s}$. During this time she has gone $x = vt = (3.05\text{ m/s})(12.0\text{ s}) = 36.6\text{ m}$. Bruce must throw the bagels so they travel 36.6 m horizontally in 2.99 s. This gives $x = vt$. $36.6\text{ m} = v(2.99\text{ s})$ and $v = 12.2\text{ m/s}$.

(b) 36.6 m from the building.

EVALUATE: If $v > 12.2\text{ m/s}$ the bagels land in front of her and if $v < 12.2\text{ m/s}$ they land behind her. There is a range of velocities greater than 12.2 m/s for which she would catch the bagels in the air, at some height above the sidewalk.

3.69. **IDENTIFY:** The shell moves in projectile motion. To find the horizontal distance between the tanks we must find the horizontal velocity of one tank relative to the other. Take $+y$ to be upward.

(a) SET UP: The vertical motion of the shell is unaffected by the horizontal motion of the tank. Use the vertical motion of the shell to find the time the shell is in the air:

$v_{0y} = v_0 \sin\alpha = 43.4\text{ m/s}$, $a_y = -9.80\text{ m/s}^2$, $y - y_0 = 0$ (returns to initial height), $t = ?$

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = 8.86\text{ s}$

SET UP: Consider the motion of one tank relative to the other.

EXECUTE: Relative to tank #1 the shell has a constant horizontal velocity $v_0\cos\alpha = 246.2\text{ m/s}$. Relative to the ground the horizontal velocity component is $246.2\text{ m/s} + 15.0\text{ m/s} = 261.2\text{ m/s}$. Relative to tank #2 the shell has horizontal velocity component $261.2\text{ m/s} - 35.0\text{ m/s} = 226.2\text{ m/s}$. The distance between the tanks when the shell was fired is the $(226.2\text{ m/s})(8.86\text{ s}) = 2000\text{ m}$ that the shell travels relative to tank #2 during the 8.86 s that the shell is in the air.

(b) The tanks are initially 2000 m apart. In 8.86 s tank #1 travels 133 m and tank #2 travels 310 m, in the same direction. Therefore, their separation increases by $310\text{ m} - 133\text{ m} = 177\text{ m}$. So, the separation becomes 2180 m (rounding to 3 significant figures).

EVALUATE: The retreating tank has greater speed than the approaching tank, so they move farther apart while the shell is in the air. We can also calculate the separation in part (b) as the relative speed of the tanks times the time the shell is in the air: $(35.0\text{ m/s} - 15.0\text{ m/s})(8.86\text{ s}) = 177\text{ m}$.

3.70. **IDENTIFY:** The object moves with constant acceleration in both the horizontal and vertical directions.

SET UP: Let $+y$ be downward and let $+x$ be the direction in which the firecracker is thrown.

EXECUTE: The firecracker's falling time can be found from the vertical motion: $t = \sqrt{\frac{2h}{g}}$.

The firecracker's horizontal position at any time *t* (taking the student's position as $x = 0$) is $x = vt - \frac{1}{2}at^2$. $x = 0$ when cracker hits the ground, so $t = 2v/a$. Combining this with the expression for the falling time gives $\frac{2v}{a} = \sqrt{\frac{2h}{g}}$ and $h = \frac{2v^2 g}{a^2}$.

EVALUATE: When *h* is smaller, the time in the air is smaller and either *v* must be smaller or *a* must be larger.

3.71. **IDENTIFY:** The velocity $\vec{v}_{\text{T/G}}$ of the tank relative to the ground is related to the velocity $\vec{v}_{\text{R/G}}$ of the rocket relative to the ground and the velocity $\vec{v}_{\text{T/R}}$ of the tank relative to the rocket by $\vec{v}_{\text{T/G}} = \vec{v}_{\text{T/R}} + \vec{v}_{\text{R/G}}$.

SET UP: Let $+y$ be upward and take $y = 0$ at the ground. Let $+x$ be in the direction of the horizontal component of the tank's motion. Once the tank is released it has $a_x = 0$, $a_y = -9.80\text{ m/s}^2$, relative to the ground.

EXECUTE: **(a)** For the rocket $v_y = v_{0y} + a_y t = (1.75\text{m/s}^2)(22.0\text{ s}) = 38.5\text{ m/s}$ and $v_x = 0$. The rocket has speed 38.5 m/s at the instant when the fuel tank is released.

(b) (i) The rocket's path is vertical, so relative to the crew member $v_{\text{T/R-}x} = +25.0\text{ m/s}$ and $v_{\text{T/R-}y} = 0$. (ii) $\vec{v}_{\text{R/G}}$ is vertical and $\vec{v}_{\text{T/R}}$ is horizontal, so $v_{\text{T/G-}x} = +25.0\text{ m/s}$ and $v_{\text{T/G}-y} = +38.5\text{ m/s}$.

(c) (i) The tank initially moves horizontally, at an angle of zero. (ii) $\tan\alpha_0 = \frac{v_{\text{T/G-}y}}{v_{\text{T/G-}x}} = \frac{38.5\text{ m/s}}{25.0\text{ m/s}}$ and $\alpha_0 = 57.0°$.

(d) Consider the motion of the tank, in the reference frame of the technician on the ground. At the instant the tank is released the rocket at a height $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2=\frac{1}{2}(1.75\text{ m/s}^2)(22.0\text{ s})^2=423.5\text{ m}$. So, for the tank $y_0=423.5\text{ m}$, $v_{0y}=38.5\text{ m/s}$ and $a_y=-9.80\text{ m/s}^2$. $v_y=0$ at the maximum height. $v_y^2=v_{0y}^2+2a_y(y-y_0)$ gives $y-y_0=\dfrac{v_y^2-v_{0y}^2}{2a_y}=\dfrac{0-(38.5\text{ m/s})^2}{2(9.80\text{ m/s}^2)}=75.6\text{ m}$. $y=423.5\text{ m}+75.6\text{ m}=499\text{ m}$. The tank reaches a height of 499 m above the launch pad.

EVALUATE: Relative to the crew member in the rocket the jettisoned tank has an acceleration of $1.75\text{ m/s}^2+9.80\text{ m/s}^2=11.5\text{ m/s}^2$, downward. Relative to the rocket the tank follows a parabolic path, but with zero initial vertical velocity and with a downward acceleration that has magnitude greater than *g*.

3.72. **IDENTIFY:** The velocity $\vec{v}_{\text{R/G}}$ of the rocket relative to the ground is related to the velocity $\vec{v}_{\text{S/G}}$ of the secondary rocket relative to the ground and the velocity $\vec{v}_{\text{S/R}}$ of the secondary rocket relative to the rocket by $\vec{v}_{\text{S/G}}=\vec{v}_{\text{S/R}}+\vec{v}_{\text{R/G}}$.

SET UP: Let $+y$ be upward and let $y=0$ at the ground. Let $+x$ be in the direction of the horizontal component of the secondary rocket's motion. After it is launched the secondary rocket has $a_x=0$ and $a_y=-9.80\text{ m/s}^2$, relative to the ground.

EXECUTE: **(a)** (i) $v_{\text{S/R-}x}=(12.0\text{ m/s})\cos53.0°=7.22\text{ m/s}$ and $v_{\text{S/R-y}}=(12.0\text{ m/s})\sin53.0°=9.58\text{ m/s}$.

(ii) $v_{\text{R/G-}x}=0$ and $v_{\text{R/G-}y}=8.50\text{ m/s}$. $v_{\text{S/G-}x}=v_{\text{S/R-}x}+v_{\text{R/G-}x}=7.22\text{ m/s}$ and $v_{\text{S/G-}y}=v_{\text{S/R-}y}+v_{\text{R/G-}y}=$ $9.58\text{ m/s}+8.50\text{ m/s}=18.1\text{ m/s}$.

(b) $v_{\text{S/G}}=\sqrt{(v_{\text{S/G-}x})^2+(v_{\text{S/G-}y})^2}=19.5\text{ m/s}$. $\tan\alpha_0=\dfrac{v_{\text{S/G-}y}}{v_{\text{S/G-}x}}=\dfrac{18.1\text{ m/s}}{7.22\text{ m/s}}$ and $\alpha_0=68.3°$.

(c) Relative to the ground the secondary rocket has $y_0=145\text{ m}$, $v_{0y}=+18.1\text{ m/s}$, $a_y=-9.80\text{ m/s}^2$ and $v_y=0$ (at the maximum height). $v_y^2=v_{0y}^2+2a_y(y-y_0)$ gives $y-y_0=\dfrac{v_y^2-v_{0y}^2}{2a_y}=\dfrac{0-(18.1\text{ m/s})^2}{2(-9.80\text{ m/s}^2)}=16.7\text{ m}$.

$y=145\text{ m}+16.7\text{ m}=162\text{ m}$.

EVALUATE: The secondary rocket reaches its maximum height in time $t=\dfrac{v_y-v_{0y}}{a_y}=\dfrac{-18.1\text{ m/s}}{-9.80\text{ m/s}^2}=1.85\text{ s}$ after it is launched. At this time the primary rocket has height $145\text{ m}+(8.50\text{ m/s})(1.85\text{ s})=161\text{ m}$, so is at nearly the same height as the secondary rocket. The secondary rocket first moves upward from the primary rocket but then loses vertical velocity due to the acceleration of gravity.

3.73. **IDENTIFY:** The original firecracker moves as a projectile. At its maximum height it's velocity is horizontal. The velocity $\vec{v}_{\text{A/G}}$ of fragment *A* relative to the ground is related to the velocity $\vec{v}_{\text{F/G}}$ of the original firecracker relative to the ground and the velocity $\vec{v}_{\text{A/F}}$ of the fragment relative to the original firecracker by $\vec{v}_{\text{A/G}}=\vec{v}_{\text{A/F}}+\vec{v}_{\text{F/G}}$. Fragment *B* obeys a similar equation.

SET UP: Let $+x$ be along the direction of the horizontal motion of the firecracker before it explodes and let $+y$ be upward. Fragment *A* moves at $53.0°$ above the $+x$ direction and fragment *B* moves at $53.0°$ below the $+x$ direction. Before it explodes the firecracker has $a_x=0$ and $a_y=-9.80\text{ m/s}^2$

EXECUTE: The horizontal component of the firecracker's velocity relative to the ground is constant (since $a_x=0$), so $v_{\text{F/G-}x}=(25.0\text{ m/s})\cos30.0°=21.65\text{ m/s}$. At the time of the explosion, $v_{\text{F/G-}y}=0$. For fragment *A*, $v_{\text{A/F-}x}=(20.0\text{ m/s})\cos53.0°=12.0\text{ m/s}$ and $v_{\text{A/F-}y}=(20.0\text{ m/s})\sin53.0°=16.0\text{ m/s}$.

$v_{\text{A/G-}x}=v_{\text{A/F-}x}+v_{\text{F/G-}x}=12.0\text{ m/s}+21.65\text{ m/s}=33.7\text{ m/s}$. $v_{\text{A/G-}y}=v_{\text{A/F-}y}+v_{\text{F/G-}y}=16.0\text{ m/s}$.

$\tan\alpha_0=\dfrac{v_{\text{A/G-}y}}{v_{\text{A/G-}x}}=\dfrac{16.0\text{ m/s}}{33.7\text{ m/s}}$ and $\alpha_0=25.4°$. The calculation for fragment *B* is the same, except $v_{\text{A/F-}y}=-16.0\text{ m/s}$.

The fragments move at $25.4°$ above and $25.4°$ below the horizontal.

EVALUATE: As the initial velocity of the firecracker increases the angle with the horizontal for the fragments, as measured from the ground, decreases.

3.74. **IDENTIFY:** The grenade moves in projectile motion. $110\text{ km/h}=30.6\text{ m/s}$. The horizontal range *R* of the grenade must be 15.8 m plus the distance *d* that the enemy's car travels while the grenade is in the air.

SET UP: For the grenade take $+y$ upward, so $a_x = 0$, $a_y = -g$. Let v_0 be the magnitude of the velocity of the grenade relative to the hero. $v_{0x} = v_0 \cos 45°$, $v_{0y} = v_0 \sin 45°$. 90 km/h = 25 m/s ; The enemy's car is traveling away from the hero's car with a relative velocity of $v_{\text{rel}} = 30.6 \text{ m/s} - 25 \text{ m/s} = 5.6 \text{ m/s}$.

EXECUTE: $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ with $y - y_0 = 0$ gives $t = -\dfrac{2v_{0y}}{a_y} = \dfrac{2v_0 \sin 45°}{g}$. $d = v_{\text{rel}}t = \dfrac{\sqrt{2}v_0 v_{\text{rel}}}{g}$.

$R = v_{0x}t = v_0(\cos 45°)t = \dfrac{2v_0^2 \sin 45° \cos 45°}{g} = \dfrac{v_0^2}{g}$. $R = d + 15.8$ m gives that $\dfrac{v_0^2}{g} = \dfrac{\sqrt{2}v_{\text{rel}}}{g}v_0 + 15.8$ m.

$v_0^2 - \sqrt{2}v_{\text{rel}}v_0 - (15.8 \text{ m})g = 0$. $v_0^2 - 7.92v_0 - 154.8 = 0$. The quadratic formula gives $v_0 = 17.0 \text{ m/s} = 61.2 \text{ km/h}$. The grenade has velocity of magnitude 61.2 km/h relative to the hero. Relative to the hero the velocity of the grenade has components $v_{0x} = v_0 \cos 45° = 43.3$ km/h and $v_{0y} = v_0 \sin 45° = 43.3$ km/h. Relative to the earth the velocity of the grenade has components $v_{\text{E}x} = 43.3 \text{ km/h} + 90 \text{ km/h} = 133.3 \text{ km/h}$ and $v_{\text{E}y} = 43.3$ km/h. The magnitude of the velocity relative to the earth is $v_{\text{E}} = \sqrt{v_{\text{E}x}^2 + v_{\text{E}y}^2} = 140$ km/h.

EVALUATE: The time the grenade is in the air is $t = \dfrac{2v_0 \sin 45°}{g} = \dfrac{2(17.0 \text{ m/s})\sin 45°}{9.80 \text{ m/s}^2} = 2.45$ s. During this time the grenade travels a horizontal distance $x - x_0 = (133.3 \text{ km/h})(2.45 \text{ s})(1 \text{ h}/3600 \text{ s}) = 90.7$ m, relative to the earth, and the enemy's car travels a horizontal distance $x - x_0 = (110 \text{ km/h})(2.45 \text{ s})(1 \text{ h}/3600 \text{ s}) = 74.9$ m, relative to the earth. The grenade has traveled 15.8 m farther.

3.75. IDENTIFY and SET UP: Use Eqs. (3.4) and (3.12) to get the velocity and acceleration components from the position components.

EXECUTE: $x = R\cos\omega t,\ \ y = R\sin\omega t$

(a) $r = \sqrt{x^2 + y^2} = \sqrt{R^2\cos^2\omega t + R^2\sin^2\omega t} = \sqrt{R^2(\sin^2\omega t + \cos^2\omega t)} = \sqrt{R^2} = R,$

since $\sin^2\omega t + \cos^2\omega t = 1$.

(b) $v_x = \dfrac{dx}{dt} = -R\omega\sin\omega t,\ \ v_y = \dfrac{dy}{dt} = R\omega\cos\omega t$

$\vec{v}\cdot\vec{r} = v_x x + v_y y = (-R\omega\sin\omega t)(R\cos\omega t) + (R\omega\cos\omega t)(R\sin\omega t)$

$\vec{v}\cdot\vec{r} = R^2\omega(-\sin\omega t\cos\omega t + \sin\omega t\cos\omega t) = 0,$ so $\vec{v}$ is perpendicular to $\vec{r}$.

(c) $a_x = \dfrac{dv_x}{dt} = -R\omega^2\cos\omega t = -\omega^2 x$

$a_y = \dfrac{dv_y}{dt} = -R\omega^2\sin\omega t = -\omega^2 y$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\omega^4 x^2 + \omega^4 y^2} = \omega^2\sqrt{x^2 + y^2} = R\omega^2.$

$\vec{a} = a_x\hat{i} + a_y\hat{j} = -\omega^2(x\hat{i} + y\hat{j}) = -\omega^2\vec{r}.$

Since ω^2 is positive this means that the direction of $\vec{a}$ is opposite to the direction of $\vec{r}$.

(d) $v = \sqrt{v_x^2 + v_y^2} = \sqrt{R^2\omega^2\sin^2\omega t + R^2\omega^2\cos^2\omega t} = \sqrt{R^2\omega^2(\sin^2\omega t + \cos^2\omega t)}.$ $v = \sqrt{R^2\omega^2} = R\omega.$

(e) $a = R\omega^2,\ \ \omega = v/R,$ so $a = R(v^2/R^2) = v^2/R.$

EVALUATE: The rock moves in uniform circular motion. The position vector is radial, the velocity is tangential, and the acceleration is radially inward.

3.76. IDENTIFY: All velocities are constant, so the distance traveled is $d = v_{\text{B/E}}t$, where $v_{\text{B/E}}$ is the magnitude of the velocity of the boat relative to the earth. The relative velocities $\vec{v}_{\text{B/E}}$, $\vec{v}_{\text{S/W}}$ (boat relative to the water) and $\vec{v}_{\text{W/E}}$ (water relative to the earth) are related by $\vec{v}_{\text{B/E}} = \vec{v}_{\text{B/W}} + \vec{v}_{\text{W/E}}$.

SET UP: Let $+x$ be east and let $+y$ be north. $v_{\text{W/E-}x} = +30.0$ m/min and $v_{\text{W/E-}y} = 0$. $v_{\text{B/W}} = 100.0$ m/min. The direction of $\vec{v}_{\text{B/W}}$ is the direction in which the boat is pointed or aimed.

EXECUTE: **(a)** $v_{\text{B/W-}y} = +100.0$ m/min and $v_{\text{B/W-}x} = 0$. $v_{\text{B/E-}x} = v_{\text{B/W-}x} + v_{\text{W/E-}x} = 30.0$ m/min and $v_{\text{B/E-}y} = v_{\text{B/W-}y} + v_{\text{W/E-}y} = 100.0$ m/min. The time to cross the river is $t = \dfrac{y - y_0}{v_{\text{B/E-}y}} = \dfrac{400.0 \text{ m}}{100.0 \text{ m/min}} = 4.00$ min.

$x - x_0 = (30.0 \text{ m/min})(4.00 \text{ min}) = 120.0 \text{ m}$. You will land 120.0 m east of point *B*, which is 45.0 m east of point *C*. The distance you will have traveled is $\sqrt{(400.0 \text{ m})^2 + (120.0 \text{ m})^2} = 418 \text{ m}$.

(b) $\vec{v}_{\text{B/W}}$ is directed at angle ϕ east of north, where $\tan\phi = \dfrac{75.0 \text{ m}}{400.0 \text{ m}}$ and $\phi = 10.6°$.

$v_{\text{B/W-}x} = (100.0 \text{ m/min})\sin 10.6° = 18.4 \text{ m/min}$ and $v_{\text{B/W-}y} = (100.0 \text{ m/min})\cos 10.6° = 98.3 \text{ m/min}$.

$v_{\text{B/E-}x} = v_{\text{B/W-}x} + v_{\text{W/E-}x} = 18.4 \text{ m/min} + 30.0 \text{ m/min} = 48.4 \text{ m/min}$. $v_{\text{B/E-}y} = v_{\text{B/W-}y} + v_{\text{W/E-}y} = 98.3 \text{ m/min}$.

$t = \dfrac{y - y_0}{v_{\text{B/E-}y}} = \dfrac{400.0 \text{ m}}{98.3 \text{ m/min}} = 4.07 \text{ min}$. $x - x_0 = (48.4 \text{ m/min})(4.07 \text{ min}) = 197 \text{ m}$. You will land 197 m downstream from *B*, so 122 m downstream from *C*.

(c) (i) If you reach point *C*, then $\vec{v}_{\text{B/E}}$ is directed at $10.6°$ east of north, which is $79.4°$ north of east. We don't know the magnitude of $\vec{v}_{\text{B/E}}$ and the direction of $\vec{v}_{\text{B/W}}$. In part (a) we found that if we aim the boat due north we will land east of *C*, so to land at *C* we must aim the boat west of north. Let $\vec{v}_{\text{B/W}}$ be at an angle ϕ of north of west. The relative velocity addition diagram is sketched in Figure 3.76. The law of sines says $\dfrac{\sin\theta}{v_{\text{W/E}}} = \dfrac{\sin 79.4°}{v_{\text{B/W}}}$.

$\sin\theta = \left(\dfrac{30.0 \text{ m/min}}{100.0 \text{ m/min}}\right)\sin 79.4°$ and $\theta = 17.15°$. Then $\phi = 180° - 79.4° - 17.15° = 83.5°$. The boat will head $83.5°$ north of west, so $6.5°$ west of north.

$v_{\text{B/E-}x} = v_{\text{B/W-}x} + v_{\text{W/E-}x} = -(100.0 \text{ m/min})\cos 83.5° + 30.0 \text{ m/min} = 18.7 \text{ m/min}$.

$v_{\text{B/E-}y} = v_{\text{B/W-}y} + v_{\text{W/E-}y} = -(100.0 \text{ m/min})\sin 83.5° = 99.4 \text{ m/min}$. Note that these two components do give the direction of $\vec{v}_{\text{B/E}}$ to be $79.4°$ north of east, as required. (ii) The time to cross the river is

$t = \dfrac{y - y_0}{v_{\text{B/E-}y}} = \dfrac{400.0 \text{ m}}{99.4 \text{ m/min}} = 4.02 \text{ min}$. (iii) You travel from *A* to *C*, a distance of $\sqrt{(400.0 \text{ m})^2 + (75.0 \text{ m})^2} = 407 \text{ m}$.

(iv) $v_{\text{B/E}} = \sqrt{(v_{\text{B/E-}x})^2 + (v_{\text{B/E-}y})^2} = 101 \text{ m/min}$. Note that $v_{\text{B/E}}t = 406 \text{ m}$, the distance traveled (apart from a small difference due to rounding).

EVALUATE: You cross the river in the shortest time when you head toward point *B*, as in part (a), even though you travel farther than in part (c).

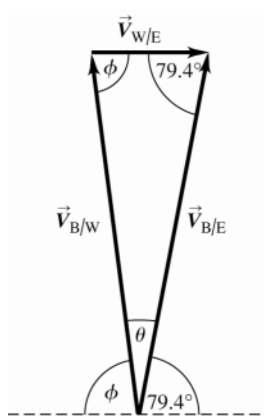


Figure 3.76

3.77. IDENTIFY: $v_x = dx/dt$, $v_y = dy/dt$, $a_x = dv_x/dt$ and $a_y = dv_y/dt$.

SET UP: $\dfrac{d(\sin\omega t)}{dt} = \omega\cos(\omega t)$ and $\dfrac{d(\cos\omega t)}{dt} = -\omega\sin(\omega t)$.

EXECUTE: **(a)** The path is sketched in Figure 3.77.

(b) To find the velocity components, take the derivative of *x* and *y* with respect to time: $v_x = R\omega(1 - \cos\omega t)$, and $v_y = R\omega\sin\omega t$. To find the acceleration components, take the derivative of v_x and v_y with respect to time: $a_x = R\omega^2 \sin\omega t$, and $a_y = R\omega^2\cos\omega t$.

(c) The particle is at rest $(v_y = v_x = 0)$ every period, namely at $t = 0,\ 2\pi/\omega,\ 4\pi/\omega, \ldots$. At that time, $x = 0,\ 2\pi R,\ 4\pi R, \ldots$; and $y = 0$. The acceleration is $a = R\omega^2$ in the $+y$-direction.

(d) No, since $a = \left[\left(R\omega^2 \sin\omega t\right)^2 + \left(R\omega^2 \cos\omega t\right)^2\right]^{1/2} = R\omega^2$. The magnitude of the acceleration is the same as for uniform circular motion.

EVALUATE: The velocity is tangent to the path. v_{0x} is always positive; v_y changes sign during the motion.

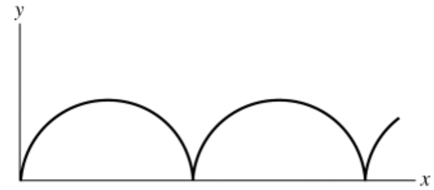


Figure 3.77

3.78. **IDENTIFY:** At the highest point in the trajectory the velocity of the projectile relative to the earth is horizontal. The velocity $\vec{v}_{\text{P/E}}$ of the projectile relative to the earth, the velocity $\vec{v}_{\text{F/P}}$ of a fragment relative to the projectile, and the velocity $\vec{v}_{\text{F/E}}$ of a fragment relative to the earth are related by $\vec{v}_{\text{F/E}} = \vec{v}_{\text{F/P}} + \vec{v}_{\text{P/E}}$.

SET UP: Let $+x$ be along the horizontal component of the projectile motion. Let the speed of each fragment relative to the projectile be v. Call the fragments 1 and 2, where fragment 1 travels in the $+x$ direction and fragment 2 is in the $-x$-direction, and let the speeds just after the explosion of the two fragments relative to the earth be v_1 and v_2. Let v_{p} be the speed of the projectile just before the explosion.

EXECUTE: $v_{\text{F/E-}x} = v_{\text{F/P-}x} + v_{\text{P/E-}x}$ gives $v_1 = v_{\text{p}} + v$ and $-v_2 = v_{\text{p}} - v$. Both fragments start from the same height with zero vertical component of velocity relative to the earth, so they both fall for the same time t, and this is also the same time as it took for the projectile to travel a horizontal distance D, so $v_{\text{p}}t = D$. Since fragment 2 lands at A it travels a horizontal distance D as it falls and $v_2 t = D$. $-v_2 = +v_{\text{p}} - v$ gives $v = v_{\text{p}} + v_2$ and $vt = v_{\text{p}}t + v_2 t = 2D$. Then $v_1 t = v_{\text{p}}t + vt = 3D$. This fragment lands a horizontal distance $3D$ from the point of explosion and hence $4D$ from A.

EVALUATE: Fragment 1, that is ejected in the direction of the motion of the projectile travels with greater speed relative to the earth than the fragment that travels in the opposite direction.

3.79. **IDENTIFY:** $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{4\pi^2 R}{T^2}$. All points on the centrifuge have the same period T.

SET UP: The period T in seconds is related to n, the number of revolutions per minute, by $n = \dfrac{60\text{ s/min}}{T}$.

EXECUTE: **(a)** $\dfrac{a_{\text{rad}}}{R} = \dfrac{4\pi^2}{T^2}$, which is constant. $\dfrac{a_{\text{rad,1}}}{R_1} = \dfrac{a_{\text{rad,2}}}{R_2}$. Let $R_1 = R$, so $a_{\text{rad,1}} = 5.00g$ and let $R_2 = R/2$.

$a_{\text{rad,2}} = a_{\text{rad,1}}\left(\dfrac{R_2}{R_1}\right) = (5.00g)(1/2) = 2.50g$.

(b) $T = \left(\dfrac{60\text{ s/min}}{n}\right)$ and $a_{\text{rad}} = \dfrac{4\pi^2 R}{T^2}$ gives $a_{\text{rad}} = 4\pi^2 R n^2/(60\text{ s/min})^2$. $\dfrac{a_{\text{rad}}}{n^2} = \dfrac{4\pi^2 R}{(60\text{ s/min})^2}$, which is constant.

$\dfrac{a_{\text{rad,1}}}{n_1^2} = \dfrac{a_{\text{rad,2}}}{n_2^2}$. Let $a_{\text{rad,1}} = 5.00g$, so $n_1 = n$ and $a_{\text{rad,2}} = 5g_{\text{Mercury}} = 5(0.378)g$. Then

$n_2 = n_1\sqrt{\dfrac{a_{\text{rad,2}}}{a_{\text{rad,1}}}} = n\sqrt{\dfrac{5(0.378)g}{5.00g}} = 0.615n$.

EVALUATE: The radial acceleration is less for points closer to the rotation axis. Since $g_{\text{Mercury}} < g$, a smaller rotation rate is required to produce $5g_{\text{Mercury}}$ than to produce $5g$.

3.80. **IDENTIFY:** Use the relation that relates the relative velocities.

SET UP: The relative velocities are the raindrop relative to the earth, $\vec{v}_{\text{R/E}}$, the raindrop relative to the train, $\vec{v}_{\text{R/T}}$, and the train relative to the earth, $\vec{v}_{\text{T/E}}$. $\vec{v}_{\text{R/E}} = \vec{v}_{\text{R/T}} + \vec{v}_{\text{T/E}}$. $\vec{v}_{\text{T/E}}$ is due east and has magnitude 12.0 m/s. $\vec{v}_{\text{R/T}}$ is 30.0° west of vertical. $\vec{v}_{\text{R/E}}$ is vertical. The relative velocity addition diagram is given in Figure 3.80.

EXECUTE: **(a)** $\vec{v}_{\text{R/E}}$ is vertical and has zero horizontal component. The horizontal component of $\vec{v}_{\text{R/T}}$ is $-\vec{v}_{\text{T/E}}$, so is 12.0 m/s westward.

(b) $v_{\text{R/E}} = \dfrac{v_{\text{T/E}}}{\tan 30.0^\circ} = \dfrac{12.0\text{ m/s}}{\tan 30.0^\circ} = 20.8\text{ m/s}$. $v_{\text{R/T}} = \dfrac{v_{\text{T/E}}}{\sin 30.0^\circ} = \dfrac{12.0\text{ m/s}}{\sin 30.0^\circ} = 24.0\text{ m/s}$.

EVALUATE: The speed of the raindrop relative to the train is greater than its speed relative to the earth, because of the motion of the train.

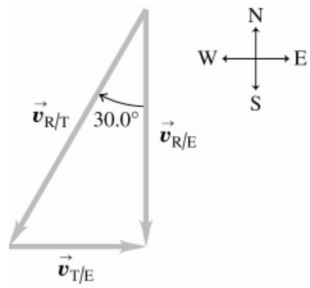


Figure 3.80

3.81. **IDENTIFY:** Relative velocity problem. The plane's motion relative to the earth is determined by its velocity relative to the earth.

SET UP: Select a coordinate system where $+y$ is north and $+x$ is east.

The velocity vectors in the problem are:

$\vec{v}_{P/E}$, the velocity of the plane relative to the earth.

$\vec{v}_{P/A}$, the velocity of the plane relative to the air (the magnitude $v_{P/A}$ is the air speed of the plane and the direction of $\vec{v}_{P/A}$ is the compass course set by the pilot).

$\vec{v}_{A/E}$, the velocity of the air relative to the earth (the wind velocity).

The rule for combining relative velocities gives $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$.

(a) We are given the following information about the relative velocities:

$\vec{v}_{P/A}$ has magnitude 220 km/h and its direction is west. In our coordinates is has components $(v_{P/A})_x = -220$ km/h and $(v_{P/A})_y = 0$.

From the displacement of the plane relative to the earth after 0.500 h, we find that $\vec{v}_{P/E}$ has components in our coordinate system of

$$(v_{P/E})_x = -\frac{120\text{ km}}{0.500\text{ h}} = -240\text{ km/h (west)}$$

$$(v_{P/E})_y = -\frac{20\text{ km}}{0.500\text{ h}} = -40\text{ km/h (south)}$$

With this information the diagram corresponding to the velocity addition equation is shown in Figure 3.81a.

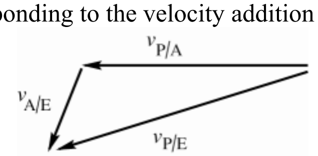


Figure 3.81a

We are asked to find $\vec{v}_{A/E}$, so solve for this vector:

$\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$ gives $\vec{v}_{A/E} = \vec{v}_{P/E} - \vec{v}_{P/A}$.

EXECUTE: The x-component of this equation gives

$(v_{A/E})_x = (v_{P/E})_x - (v_{P/A})_x = -240\text{ km/h} - (-220\text{ km/h}) = -20\text{ km/h}$.

The y-component of this equation gives

$(v_{A/E})_y = (v_{P/E})_y - (v_{P/A})_y = -40\text{ km/h}$.

Now that we have the components of $\vec{v}_{A/E}$ we can find its magnitude and direction.

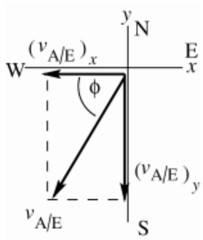


Figure 3.81b

$$v_{A/E} = \sqrt{(v_{A/E})_x^2 + (v_{A/E})_y^2}$$

$$v_{A/E} = \sqrt{(-20\text{ km/h})^2 + (-40\text{ km/h})^2} = 44.7\text{ km/h}$$

$$\tan\phi = \frac{40\text{ km/h}}{20\text{ km/h}} = 2.00;\quad \phi = 63.4°$$

The direction of the wind velocity is 63.4° S of W, or 26.6° W of S.

EVALUATE: The plane heads west. It goes farther west than it would without wind and also travels south, so the wind velocity has components west and south.

(b) SET UP: The rule for combining the relative velocities is still $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$, but some of these velocities have different values than in part (a).

$\vec{v}_{P/A}$ has magnitude 220 km/h but its direction is to be found.

$\vec{v}_{A/E}$ has magnitude 40 km/h and its direction is due south.

The direction of $\vec{v}_{P/E}$ is west; its magnitude is not given.

The vector diagram for $\vec{v}_{P/E} = \vec{v}_{P/A} + \vec{v}_{A/E}$ and the specified directions for the vectors is shown in Figure 3.81c.

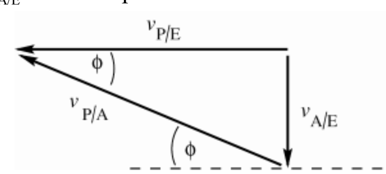


Figure 3.81c

The vector addition diagram forms a right triangle.

EXECUTE: $\sin\phi = \dfrac{v_{A/E}}{v_{P/A}} = \dfrac{40\text{ km/h}}{220\text{ km/h}} = 0.1818;\quad \phi = 10.5°.$

The pilot should set her course 10.5° north of west.

EVALUATE: The velocity of the plane relative to the air must have a northward component to counteract the wind and a westward component in order to travel west.

3.82. IDENTIFY: Both the bolt and the elevator move vertically with constant acceleration.

SET UP: Let $+y$ be upward and let $y = 0$ at the initial position of the floor of the elevator, so y_0 for the bolt is 3.00 m.

EXECUTE: **(a)** The position of the bolt is $3.00\text{ m} + (2.50\text{ m/s})t - (1/2)(9.80\text{ m/s}^2)t^2$ and the position of the floor is (2.50 m/s)t. Equating the two, $3.00\text{ m} = (4.90\text{ m/s}^2)t^2$. Therefore, $t = 0.782\text{ s}$.

(b) The velocity of the bolt is $2.50\text{ m/s} - (9.80\text{ m/s}^2)(0.782\text{ s}) = -5.17\text{ m/s}$ relative to Earth, therefore, relative to an observer in the elevator $v = -5.17\text{ m/s} - 2.50\text{ m/s} = -7.67\text{ m/s}$.

(c) As calculated in part (b), the speed relative to Earth is 5.17 m/s.

(d) Relative to Earth, the distance the bolt traveled is

$(2.50\text{ m/s})t - (1/2)(9.80\text{ m/s}^2)t^2 = (2.50\text{ m/s})(0.782\text{ s}) - (4.90\text{ m/s}^2)(0.782\text{ s})^2 = -1.04\text{ m}$.

EVALUATE: As viewed by an observer in the elevator, the bolt has $v_{0y} = 0$ and $a_y = -9.80\text{ m/s}^2$, so in 0.782 s it falls $-\frac{1}{2}(9.80\text{ m/s}^2)(0.782\text{ s})^2 = -3.00\text{ m}$.

3.83. IDENTIFY: In an earth frame the elevator accelerates upward at 4.00 m/s^2 and the bolt accelerates downward at 9.80 m/s^2. Relative to the elevator the bolt has a downward acceleration of $4.00\text{ m/s}^2 + 9.80\text{ m/s}^2 = 13.80\text{ m/s}^2$. In either frame, that of the earth or that of the elevator, the bolt has constant acceleration and the constant acceleration equations can be used.

SET UP: Let $+y$ be upward. The bolt travels 3.00 m downward relative to the elevator.

EXECUTE: **(a)** In the frame of the elevator, $v_{0y} = 0$, $y - y_0 = -3.00\text{ m}$, $a_y = -13.8\text{ m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(-3.00\text{ m})}{-13.8\text{ m/s}^2}} = 0.659\text{ s}$.

(b) $v_y = v_{0y} + a_y t$. $v_{0y} = 0$ and $t = 0.659\ \text{s}$. (i) $a_y = -13.8\ \text{m/s}^2$ and $v_y = -9.09\ \text{m/s}$. The bolt has speed 9.09 m/s when it reaches the floor of the elevator. (ii) $a_y = -9.80\ \text{m/s}^2$ and $v_y = -6.46\ \text{m/s}$. In this frame the bolt has speed 6.46 m/s when it reaches the floor of the elevator.

(c) $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. $v_{0y} = 0$ and $t = 0.659\ \text{s}$. (i) $a_y = -13.8\ \text{m/s}^2$ and $y - y_0 = \frac{1}{2}(-13.8\ \text{m/s}^2)(0.659\ \text{s})^2 = -3.00\ \text{m}$. The bolt falls 3.00 m, which is correctly the distance between the floor and roof of the elevator. (ii) $a_y = -9.80\ \text{m/s}^2$ and $y - y_0 = \frac{1}{2}(-9.80\ \text{m/s}^2)(0.659\ \text{s})^2 = -2.13\ \text{m}$. The bolt falls 2.13 m.

EVALUATE: In the earth's frame the bolt falls 2.13 m and the elevator rises $\frac{1}{2}(4.00\ \text{m/s}^2)(0.659\ \text{s})^2 = 0.87\ \text{m}$ during the time that the bolt travels from the ceiling to the floor of the elevator.

3.84. **IDENTIFY:** The velocity $\vec{v}_{\text{P/E}}$ of the plane relative to the earth is related to the velocity $\vec{v}_{\text{P/A}}$ of the plane relative to the air and the velocity $\vec{v}_{\text{A/E}}$ of the air relative to the earth (the wind velocity) by $\vec{v}_{\text{P/E}} = \vec{v}_{\text{P/A}} + \vec{v}_{\text{A/E}}$.

SET UP: Let $+x$ be to the east. With no wind $v_{\text{P/A}} = v_{\text{P/E}} = \dfrac{5550\ \text{km}}{6.60\ \text{h}} = 840.9\ \text{km/h}$. $v_{\text{A/E-}x} = +225\ \text{km/h}$. The distance between *A* and *B* is 2775 km.

EXECUTE: $v_{\text{P/E-}x} = v_{\text{P/A-}x} + v_{\text{A/E-}x}$. For the trip *A* to *B*, $v_{\text{P/A-}x} = +840.9\ \text{km/h}$ and $v_{\text{P/E-}x} = 840.9\ \text{km/h} + 225\ \text{km/h} = 1065.9\ \text{km/h}$ and the travel time is $t_{AB} = \dfrac{2775\ \text{km}}{1065.9\ \text{km/h}} = 2.60\ \text{h}$. For the trip *B* to *A*, $v_{\text{P/A-}x} = -840.9\ \text{km/h}$ and $v_{\text{P/E-}x} = -840.9\ \text{km/h} + 225\ \text{km/h} = -615.9\ \text{km/h}$ and the travel time is $t_{BA} = \dfrac{-2775\ \text{km}}{-615.9\ \text{km/h}} = 4.51\ \text{h}$. The total time for the round trip will be $t = t_{AB} + t_{BA} = 7.11\ \text{h}$.

EVALUATE: The round trip takes longer when the wind blows, even though the plane travels with the wind for one leg of the trip. The arithmetic average of the speeds for each leg is $\dfrac{1065.9\ \text{km/h} + 615.9\ \text{km/h}}{2} = 840.9\ \text{km/h}$, the same speed when there is no wind. But the plane spends more time traveling at the slower speed relative to the ground and the average speed is less than the arithmetic average of the speeds for each half of the trip.

3.85. **IDENTIFY:** Relative velocity problem.

SET UP: The three relative velocities are:

$\vec{v}_{\text{J/G}}$, Juan relative to the ground. This velocity is due north and has magnitude $v_{\text{J/G}} = 8.00$ m/s.

$\vec{v}_{\text{B/G}}$, the ball relative to the ground. This vector is $37.0°$ east of north and has magnitude $v_{\text{B/G}} = 12.00$ m/s.

$\vec{v}_{\text{B/J}}$, the ball relative to Juan. We are asked to find the magnitude and direction of this vector.

The relative velocity addition equation is $\vec{v}_{\text{B/G}} = \vec{v}_{\text{B/J}} + \vec{v}_{\text{J/G}}$, so $\vec{v}_{\text{B/J}} = \vec{v}_{\text{B/G}} - \vec{v}_{\text{J/G}}$.

The relative velocity addition diagram does not form a right triangle so we must do the vector addition using components.

Take $+y$ to be north and $+x$ to be east.

EXECUTE: $v_{\text{B/J}x} = +v_{\text{B/G}} \sin 37.0° = 7.222\ \text{m/s}$

$v_{\text{B/J}y} = +v_{\text{B/G}} \cos 37.0° - v_{\text{J/G}} = 1.584\ \text{m/s}$

These two components give $v_{\text{B/J}} = 7.39\ \text{m/s}$ at $12.4°$ north of east.

EVALUATE: Since Juan is running due north, the ball's eastward component of velocity relative to him is the same as its eastward component relative to the earth. The northward component of velocity for Juan and the ball are in the same direction, so the component for the ball relative to Juan is the difference in their components of velocity relative to the ground.

3.86. **IDENTIFY:** **(a)** The ball moves in projectile motion. When it is moving horizontally, $v_y = 0$.

SET UP: Let $+x$ be to the right and let $+y$ be upward. $a_x = 0$, $a_y = -g$.

EXECUTE: **(a)** $v_{0y} = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(4.90\ \text{m})} = 9.80\ \text{m/s}$.

(b) $v_{0y}/g = 1.00\ \text{s}$.

(c) The horizontal component of the velocity of the ball relative to the man is $\sqrt{(10.8\ \text{m/s})^2 - (9.80\ \text{m/s})^2} = 4.54\ \text{m/s}$, the horizontal component of the velocity relative to the hoop is $4.54\ \text{m/s} + 9.10\ \text{m/s} = 13.6\ \text{m/s}$, and the man must be 13.6 m in front of the hoop at release.

(d) Relative to the flat car, the ball is projected at an angle $\theta = \tan^{-1}\left(\dfrac{9.80\text{ m/s}}{4.54\text{ m/s}}\right) = 65°$. Relative to the ground the angle is $\theta = \tan^{-1}\left(\dfrac{9.80\text{ m/s}}{4.54\text{ m/s} + 9.10\text{ m/s}}\right) = 35.7°$.

EVALUATE: In both frames of reference the ball moves in a parabolic path with $a_x = 0$ and $a_y = -g$. The only difference between the description of the motion in the two frames is the horizontal component of the ball's velocity.

3.87. IDENTIFY: The pellets move in projectile motion. The vertical motion determines their time in the air.

SET UP: $v_{0x} = v_0 \cos 1.0°$, $v_{0y} = v_0 \sin 1.0°$.

EXECUTE: (a) $t = \dfrac{2v_{0y}}{g}$. $x - x_0 = v_{0x}t$ gives $x - x_0 = (v_0 \cos 1.0°)\left(\dfrac{2v_0 \sin 1.0°}{g}\right) = 80\text{ m}$.

(b) The probability is 1000 times the ratio of the area of the top of the person's head to the area of the circle in which the pellets land. $(1000)\left(\dfrac{\pi(10\times10^{-2}\text{ m})^2}{\pi(80\text{ m})^2}\right) = 1.6\times10^{-3}$.

(c) The slower rise will tend to reduce the time in the air and hence reduce the radius. The slower horizontal velocity will also reduce the radius. The lower speed would tend to increase the time of descent, hence increasing the radius. As the bullets fall, the friction effect is smaller than when they were rising, and the overall effect is to decrease the radius.

EVALUATE: The small angle of deviation from the vertical still causes the pellets to spread over a large area because their time in the air is large.

3.88. IDENTIFY: Write an expression for the square of the distance (D^2) from the origin to the particle, expressed as a function of time. Then take the derivative of D^2 with respect to *t*, and solve for the value of *t* when this derivative is zero. If the discriminant is zero or negative, the distance *D* will never decrease.

SET UP: $D^2 = x^2 + y^2$, with $x(t)$ and $y(t)$ given by Eqs.(3.20) and (3.21).

EXECUTE: Following this process, $\sin^{-1}\sqrt{8/9} = 70.5°$.

EVALUATE: We know that if the object is thrown straight up it moves away from *P* and then returns, so we are not surprised that the projectile angle must be less than some maximum value for the distance to always increase with time.

3.89. IDENTIFY: The baseball moves in projectile motion.

SET UP: Use coordinates where the *x*-axis is horizontal and the *y*-axis is vertical.

EXECUTE: (a) The trajectory of the projectile is given by Eq. (3.27), with $\alpha_0 = \theta + \varphi,$ and the equation describing the incline is $y = x\tan\theta$. Setting these equal and factoring out the $x = 0$ root (where the projectile is on the incline) gives a value for x_0; the range measured along the incline is

$$x/\cos\theta = \left[\frac{2v_0^2}{g}\right][\tan(\theta+\varphi) - \tan\theta]\left[\frac{\cos^2(\theta+\varphi)}{\cos\theta}\right].$$

(b) Of the many ways to approach this problem, a convenient way is to use the same sort of substitution, involving double angles, as was used to derive the expression for the range along a horizontal incline. Specifically, write the above in terms of $\alpha = \theta + \varphi,$ as

$$R = \left[\frac{2v_0^2}{g\cos^2\theta}\right][\sin\alpha\cos\alpha\cos\theta - \cos^2\alpha\sin\theta].$$

The dependence on α and hence φ is in the second term. Using the identities

$\sin\alpha\cos\alpha = (1/2)\sin 2\alpha$ and $\cos^2\alpha = (1/2)(1+\cos 2\alpha)$, this term becomes

$$(1/2)[\cos\theta\sin 2\alpha - \sin\theta\cos 2\alpha - \sin\theta] = (1/2)[\sin(2\alpha-\theta) - \sin\theta].$$

This will be a maximum when $\sin(2\alpha - \theta)$ is a maximum, at $2\alpha - \theta = 2\varphi + \theta = 90°,$ or $\varphi = 45° - \theta/2$.

EVALUATE: Note that the result reduces to the expected forms when $\theta = 0$ (a flat incline, $\varphi = 45°$ and when $\theta = -90°$ (a vertical cliff), when a horizontal launch gives the greatest distance).

3.90. IDENTIFY: The arrow moves in projectile motion.

SET UP: Use coordinates that for which the axes are horizontal and vertical. Let θ be the angle of the slope and let ϕ be the angle of projection relative to the sloping ground.

EXECUTE: The horizontal distance x in terms of the angles is

$$\tan\theta = \tan(\theta+\phi) - \left(\frac{gx}{2v_0^2}\right)\frac{1}{\cos^2(\theta+\phi)}.$$

Denote the dimensionless quantity $gx/2v_0^2$ by β; in this case

$$\beta = \frac{(9.80\ \text{m/s}^2)(60.0\ \text{m})\cos 30.0°}{2(32.0\ \text{m/s})^2} = 0.2486.$$

The above relation can then be written, on multiplying both sides by the product $\cos\theta\cos(\theta+\phi)$,

$$\sin\theta\cos(\theta+\phi) = \sin(\theta+\phi)\cos\theta - \frac{\beta\cos\theta}{\cos(\theta+\phi)},$$

and so $\sin(\theta+\phi)\cos\theta - \cos(\theta+\phi)\sin\theta = \dfrac{\beta\cos\theta}{\cos(\theta+\phi)}$. The term on the left is $\sin((\theta+\phi)-\theta) = \sin\phi$, so the result of this combination is $\sin\phi\cos(\theta+\phi) = \beta\cos\theta$.

Although this can be done numerically (by iteration, trial-and-error, or other methods), the expansion $\sin a\cos b = \frac{1}{2}(\sin(a+b)+\sin(a-b))$ allows the angle ϕ to be isolated; specifically, then

$\dfrac{1}{2}(\sin(2\phi+\theta)+\sin(-\theta)) = \beta\cos\theta$, with the net result that $\sin(2\phi+\theta) = 2\beta\cos\theta + \sin\theta$.

(a) For $\theta = 30°$, and β as found above, $\phi = 19.3°$ and the angle above the horizontal is $\theta+\phi = 49.3°$. For level ground, using $\beta = 0.2871$, gives $\phi = 17.5°$.

(b) For $\theta = -30°$, the same β as with $\theta = 30°$ may be used $(\cos 30° = \cos(-30°))$, giving $\phi = 13.0°$ and $\phi+\theta = -17.0°$.

EVALUATE: For $\theta = 0$ the result becomes $\sin(2\phi) = 2\beta = gx/v_0^2$. This is equivalent to the expression $R = \dfrac{v_0^2\sin(2\alpha_0)}{g}$ derived in Example 3.8.

3.91. **IDENTIFY:** Find $\Delta\vec{v}$ and use this to calculate the magnitude and direction of the average acceleration.

SET UP: In a time Δt, the velocity vector has moved through an angle (in radians) $\Delta\phi = \dfrac{v\Delta t}{R}$ (see Figure 3.28 in the textbook). By considering the isosceles triangle formed by the two velocity vectors, the magnitude $|\Delta\vec{\mathbf{v}}|$ is seen to be $2v\sin(\phi/2)$.

EXECUTE: $|\vec{a}_{\text{av}}| = \dfrac{|\Delta\vec{v}|}{\Delta t} = 2\dfrac{v}{\Delta t}\sin\left(\dfrac{v\Delta t}{2R}\right) = \dfrac{10\ \text{m/s}}{\Delta \text{t}}\sin([1.0/\text{s}]\Delta \text{t})$

Using the given values gives magnitudes of $9.59\ \text{m/s}^2, 9.98\ \text{m/s}^2$ and $10.0\ \text{m/s}^2$. The changes in direction of the velocity vectors are given by $\Delta\theta = \dfrac{v\Delta t}{R}$ and are, respectively, 1.0 rad, 0.2 rad, and 0.1 rad. Therefore, the angle of the average acceleration vector with the original velocity vector is $\dfrac{\pi+\Delta\theta}{2} = \pi/2 + 1/2\ \text{rad}(\text{or } 118.6°)$, $\pi/2 + 0.1\ \text{rad}(\text{or } 95.7°)$, and $\pi/2 + 0.05\ \text{rad}(\text{or } 92.9°)$.

EVALUATE: The instantaneous acceleration magnitude, $v^2/R = (5.00\ \text{m/s})^2/(2.50\ \text{m}) = 10.0\ \text{m/s}^2$ is indeed approached in the limit at $\Delta t \to 0$. Also, the direction of $\vec{a}_{\text{av}}$ approaches the radially inward direction as $\Delta t \to 0$.

3.92. **IDENTIFY:** The rocket has two periods of constant acceleration motion.

SET UP: Let $+y$ be upward. During the free-fall phase, $a_x = 0$ and $a_y = -g$. After the engines turn on, $a_x = (3.00g)\cos 30.0°$ and $a_y = (3.00g)\sin 30.0°$. Let t be the total time since the rocket was dropped and let T be the time the rocket falls before the engine starts.

EXECUTE: (i) The diagram is given in Figure 3.92a.

(ii) The x-position of the plane is $(236\ \text{m/s})t$ and the x-position of the rocket is $(236\ \text{m/s})t + (1/2)(3.00)(9.80\ \text{m/s}^2)\cos 30°(t-T)^2$. The graphs of these two equations are sketched in Figure 3.92b.

(iii) If we take $y=0$ to be the altitude of the airliner, then

$y(t)=-1/2gT^2-gT(t-T)+1/2(3.00)(9.80\text{ m/s}^2)(\sin 30°)(t-T)^2$ for the rocket. The airliner has constant y. The graphs are sketched in Figure 3.92b.

In each of the Figures 3.92a-c, the rocket is dropped at $t=0$ and the time T when the motor is turned on is indicated.

By setting $y=0$ for the rocket, we can solve for t in terms of T:

$0=-(4.90\text{ m/s}^2)T^2-(9.80\text{ m/s}^2)T(t-T)+(7.35\text{ m/s}^2)(t-T)^2$. Using the quadratic formula for the variable $x=t-T$ we find $x=t-T=\dfrac{(9.80\text{ m/s}^2)T+\sqrt{(9.80\text{ m/s}^2 T)^2+(4)(7.35\text{ m/s}^2)(4.9)T^2}}{2(7.35\text{ m/s}^2)}$, or $t=2.72\ T$. Now, using the condition that $x_{\text{rocket}}-x_{\text{plane}}=1000\text{ m}$, we find $(236\text{ m/s})t+(12.7\text{ m/s}^2)(t-T)^2-(236\text{ m/s})t=1000\text{ m}$, or $(1.72T)^2=78.6\text{ s}^2$. Therefore $T=5.15\text{ s}$.

EVALUATE: During the free-fall phase the rocket and airliner have the same x coordinate but the rocket moves downward from the airliner. After the engines fire, the rocket starts to move upward and its horizontal component of velocity starts to exceed that of the airliner.

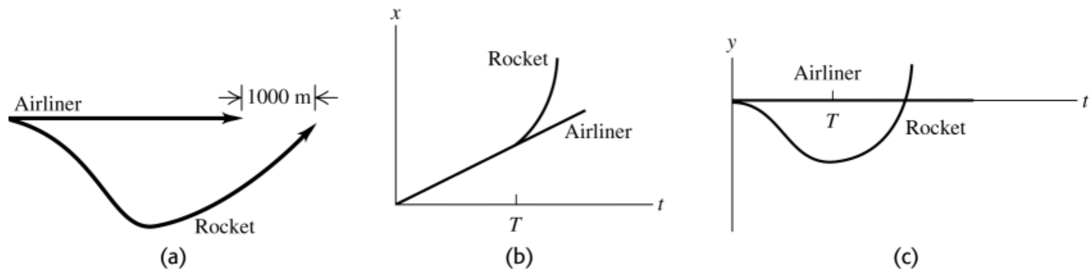


Figure 3.92

3.93. IDENTIFY: Apply the relative velocity relation.

SET UP: Let $v_{\text{C/W}}$ be the speed of the canoe relative to water and $v_{\text{W/G}}$ be the speed of the water relative to the ground.

EXECUTE: **(a)** Taking all units to be in km and h, we have three equations. We know that heading upstream $v_{\text{C/W}}-v_{\text{W/G}}=2$. We know that heading downstream for a time t, $(v_{\text{C/W}}+v_{\text{W/G}})t=5$. We also know that for the bottle $v_{\text{W/G}}(t+1)=3$. Solving these three equations for $v_{\text{W/G}}=x$, $v_{\text{C/W}}=2+x$, therefore $(2+x+x)t=5$ or $(2+2x)t=5$. Also $t=3/x-1$, so $(2+2x)\left(\dfrac{3}{x}-1\right)=5$ or $2x^2+x-6=0$. The positive solution is $x=v_{\text{W/G}}=1.5\text{ km/h}$.

(b) $v_{\text{C/W}}=2\text{ km/h}+v_{\text{W/G}}=3.5\text{ km/h}$.

EVALUATE: When they head upstream, their speed relative to the ground is $3.5\text{ km/h}-1.5\text{ km/h}=2.0\text{ km/h}$. When they head downstream, their speed relative to the ground is $3.5\text{ km/h}+1.5\text{ km/h}=5.0\text{ km/h}$. The bottle is moving downstream at 1.5 km/s relative to the earth, so they are able to overtake it.

NEWTON'S LAWS OF MOTION

4

4.1. **IDENTIFY:** Consider the vector sum in each case.

SET UP: Call the two forces $\vec{F}_1$ and $\vec{F}_2$. Let $\vec{F}_1$ be to the right. In each case select the direction of $\vec{F}_2$ such that $\vec{F} = \vec{F}_1 + \vec{F}_2$ has the desired magnitude.

EXECUTE: **(a)** For the magnitude of the sum to be the sum of the magnitudes, the forces must be parallel, and the angle between them is zero. The two vectors and their sum are sketched in Figure 4.1a.

(b) The forces form the sides of a right isosceles triangle, and the angle between them is $90°$. The two vectors and their sum are sketched in Figure 4.1b.

(c) For the sum to have zero magnitude, the forces must be antiparallel, and the angle between them is $180°$. The two vectors are sketched in Figure 4.1c.

EVALUATE: The maximum magnitude of the sum of the two vectors is 2*F*, as in part (a).

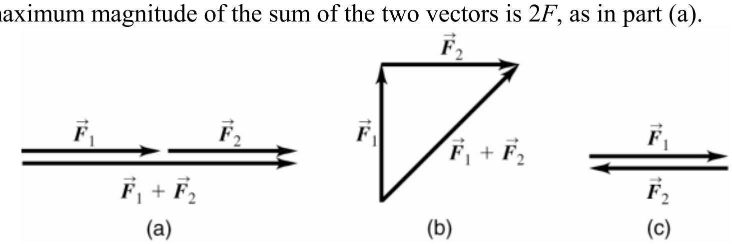


Figure 4.1

4.2. **IDENTIFY:** Add the three forces by adding their components.

SET UP: In the new coordinates, the 120-N force acts at an angle of $53°$ from the $-x$-axis, or $233°$ from the $+x$-axis, and the 50-N force acts at an angle of $323°$ from the $+x$-axis.

EXECUTE: **(a)** The components of the net force are

$$R_x = (120\text{ N})\cos 233° + (50\text{ N})\cos 323° = -32\text{ N}$$

$$R_y = (250\text{ N}) + (120\text{ N})\sin 233° + (50\text{ N})\sin 323° = 124\text{ N}.$$

(b) $R = \sqrt{R_x^2 + R_y^2} = 128\text{ N},\ \arctan\left(\frac{124}{-32}\right) = 104°$. The results have the same magnitude as in Example 4.1, and the angle has been changed by the amount $(37°)$ that the coordinates have been rotated.

EVALUATE: We can use any set of coordinate axes that we wish to and can therefore select axes for which the analysis of the problem is the simplest.

4.3. **IDENTIFY:** Use right-triangle trigonometry to find the components of the force.

SET UP: Let $+x$ be to the right and let $+y$ be downward.

EXECUTE: The horizontal component of the force is $(10\text{ N})\cos 45° = 7.1\text{ N}$ to the right and the vertical component is $(10\text{ N})\sin 45° = 7.1\text{ N}$ down.

EVALUATE: In our coordinates each component is positive; the signs of the components indicate the directions of the component vectors.

4.4. **IDENTIFY:** $F_x = F\cos\theta$, $F_y = F\sin\theta$.

SET UP: Let $+x$ be parallel to the ramp and directed up the ramp. Let $+y$ be perpendicular to the ramp and directed away from it. Then $\theta = 30.0°$.

EXECUTE: **(a)** $F = \dfrac{F_x}{\cos\theta} = \dfrac{60.0\text{ N}}{\cos 30°} = 69.3\text{ N}.$

(b) $F_y = F\sin\theta = F_x\tan\theta = 34.6\text{ N}.$

EVALUATE: We can verify that $F_x^2 + F_y^2 = F^2$. The signs of F_x and F_y show their direction.

4.5. **IDENTIFY:** Vector addition.

SET UP: Use a coordinate system where the $+x$-axis is in the direction of $\vec{F}_A$, the force applied by dog A. The forces are sketched in Figure 4.5.

EXECUTE:

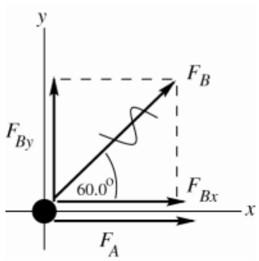


Figure 4.5a

$F_{Ax} = +270\text{ N},\ \ F_{Ay} = 0$

$F_{Bx} = F_B\cos 60.0° = (300\text{ N})\cos 60.0° = +150\text{ N}$

$F_{By} = F_B\sin 60.0° = (300\text{ N})\sin 60.0° = +260\text{ N}$

$\vec{R} = \vec{F}_A + \vec{F}_B$

$R_x = F_{Ax} + F_{Bx} = +270\text{ N} + 150\text{ N} = +420\text{ N}$

$R_y = F_{Ay} + F_{By} = 0 + 260\text{ N} = +260\text{ N}$

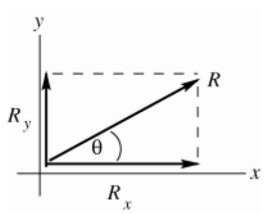


Figure 4.5b

$R = \sqrt{R_x^2 + R_y^2}$

$R = \sqrt{(420\text{ N})^2 + (260\text{ N})^2} = 494\text{ N}$

$\tan\theta = \dfrac{R_y}{R_x} = 0.619$

$\theta = 31.8°$

EVALUATE: The forces must be added as vectors. The magnitude of the resultant force is less than the sum of the magnitudes of the two forces and depends on the angle between the two forces.

4.6. **IDENTIFY:** Add the two forces using components.

SET UP: $F_x = F\cos\theta$, $F_y = F\sin\theta$, where θ is the angle $\vec{F}$ makes with the $+x$ axis.

EXECUTE: **(a)** $F_{1x} + F_{2x} = (9.00\text{ N})\cos 120° + (6.00\text{ N})\cos(233.1°) = -8.10\text{ N}$

$F_{1y} + F_{2y} = (9.00\text{ N})\sin 120° + (6.00\text{ N})\sin(233.1°) = +3.00\text{ N}.$

(b) $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(8.10\text{ N})^2 + (3.00\text{ N})^2} = 8.64\text{ N}.$

EVALUATE: Since $F_x < 0$ and $F_y > 0$, $\vec{F}$ is in the second quadrant.

4.7. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$.

SET UP: Let $+x$ be in the direction of the force.

EXECUTE: $a_x = F_x/m = (132\text{ N})/(60\text{ kg}) = 2.2\text{ m/s}^2$.

EVALUATE: The acceleration is in the direction of the force.

4.8. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$.

SET UP: Let $+x$ be in the direction of the acceleration.

EXECUTE: $F_x = ma_x = (135\text{ kg})(1.40\text{ m/s}^2) = 189\text{ N}.$

EVALUATE: The net force must be in the direction of the acceleration.

4.9. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to the box.

SET UP: Let $+x$ be the direction of the force and acceleration. $\sum F_x = 48.0\text{ N}$.

EXECUTE: $\sum F_x = ma_x$ gives $m = \dfrac{\sum F_x}{a_x} = \dfrac{48.0\text{ N}}{3.00\text{ m/s}^2} = 16.0\text{ kg}$.

EVALUATE: The vertical forces sum to zero and there is no motion in that direction.

4.10. IDENTIFY: Use the information about the motion to find the acceleration and then use $\sum F_x = ma_x$ to calculate *m*.

SET UP: Let $+x$ be the direction of the force. $\sum F_x = 80.0\text{ N}$.

EXECUTE: (a) $x - x_0 = 11.0\text{ m}$, $t = 5.00\text{ s}$, $v_{0x} = 0$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives

$a_x = \dfrac{2(x - x_0)}{t^2} = \dfrac{2(11.0\text{ m})}{(5.00\text{ s})^2} = 0.880\text{ m/s}^2$. $m = \dfrac{\sum F_x}{a_x} = \dfrac{80.0\text{ N}}{0.880\text{ m/s}^2} = 90.9\text{ kg}$.

(b) $a_x = 0$ and v_x is constant. After the first 5.0 s, $v_x = v_{0x} + a_x t = (0.880\text{ m/s}^2)(5.00\text{ s}) = 4.40\text{ m/s}$.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (4.40\text{ m/s})(5.00\text{ s}) = 22.0\text{ m}$.

EVALUATE: The mass determines the amount of acceleration produced by a given force. The block moves farther in the second 5.00 s than in the first 5.00 s.

4.11. IDENTIFY and **SET UP:** Use Newton's second law in component form (Eq.4.8) to calculate the acceleration produced by the force. Use constant acceleration equations to calculate the effect of the acceleration on the motion.

EXECUTE: (a) During this time interval the acceleration is constant and equal to

$$a_x = \frac{F_x}{m} = \frac{0.250\text{ N}}{0.160\text{ kg}} = 1.562\text{ m/s}^2$$

We can use the constant acceleration kinematic equations from Chapter 2.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}(1.562\text{ m/s}^2)(2.00\text{ s})^2$,

so the puck is at $x = 3.12$ m.

$v_x = v_{0x} + a_x t = 0 + (1.562\text{ m/s}^2)(2.00\text{ s}) = 3.13\text{ m/s}$.

(b) In the time interval from $t = 2.00$ s to 5.00 s the force has been removed so the acceleration is zero. The speed stays constant at $v_x = 3.12$ m/s. The distance the puck travels is $x - x_0 = v_{0x}t = (3.12\text{ m/s})(5.00\text{ s} - 2.00\text{ s}) = 9.36\text{ m}$.

At the end of the interval it is at $x = x_0 + 9.36\text{ m} = 12.5\text{ m}$.

In the time interval from $t = 5.00$ s to 7.00 s the acceleration is again $a_x = 1.562\text{ m/s}^2$. At the start of this interval $v_{0x} = 3.12$ m/s and $x_0 = 12.5$ m.

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 = (3.12\text{ m/s})(2.00\text{ s}) + \frac{1}{2}(1.562\text{ m/s}^2)(2.00\text{ s})^2$.

$x - x_0 = 6.24\text{ m} + 3.12\text{ m} = 9.36\text{ m}$.

Therefore, at $t = 7.00$ s the puck is at $x = x_0 + 9.36\text{ m} = 12.5\text{ m} + 9.36\text{ m} = 21.9\text{ m}$.

$$v_x = v_{0x} + a_x t = 3.12\text{ m/s} + (1.562\text{ m/s}^2)(2.00\text{ s}) = 6.24\text{ m/s}$$

EVALUATE: The acceleration says the puck gains 1.56 m/s of velocity for every second the force acts. The force acts a total of 4.00 s so the final velocity is $(1.56\text{ m/s})(4.0\text{ s}) = 6.24\text{ m/s}$.

4.12. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$. Then use a constant acceleration equation to relate the kinematic quantities.

SET UP: Let $+x$ be in the direction of the force.

EXECUTE: (a) $a_x = F_x / m = (140\text{ N})/(32.5\text{ kg}) = 4.31\text{ m/s}^2$.

(b) $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$. With $v_{0x} = 0$, $x = \frac{1}{2}at^2 = 215\text{ m}$.

(c) $v_x = v_{0x} + a_x t$. With $v_{0x} = 0$, $v_x = a_x t = 2x/t = 43.0\text{ m/s}$.

EVALUATE: The acceleration connects the motion to the forces.

4.13. IDENTIFY: The force and acceleration are related by Newton's second law.

SET UP: $\sum F_x = ma_x$, where $\sum F_x$ is the net force. $m = 4.50\text{ kg}$.

EXECUTE: (a) The maximum net force occurs when the acceleration has its maximum value.

$\sum F_x = ma_x = (4.50\text{ kg})(10.0\text{ m/s}^2) = 45.0\text{ N}$. This maximum force occurs between 2.0 s and 4.0 s.

(b) The net force is constant when the acceleration is constant. This is between 2.0 s and 4.0 s.

(c) The net force is zero when the acceleration is zero. This is the case at $t = 0$ and $t = 6.0$ s .

EVALUATE: A graph of $\sum F_x$ versus *t* would have the same shape as the graph of a_x versus *t*.

4.14. **IDENTIFY:** The force and acceleration are related by Newton's second law. $a_x = \dfrac{dv_x}{dt}$, so a_x is the slope of the graph of v_x versus *t*.

SET UP: The graph of v_x versus *t* consists of straight-line segments. For $t = 0$ to $t = 2.00\text{ s}$, $a_x = 4.00\text{ m/s}^2$. For $t = 2.00\text{ s}$ to 6.00 s, $a_x = 0$. For $t = 6.00\text{ s}$ to 10.0 s, $a_x = 1.00\text{ m/s}^2$.

$\sum F_x = ma_x$, with $m = 2.75\text{ kg}$. $\sum F_x$ is the net force.

EXECUTE: **(a)** The maximum net force occurs when the acceleration has its maximum value.

$\sum F_x = ma_x = (2.75\text{ kg})(4.00\text{ m/s}^2) = 11.0\text{ N}$. This maximum occurs in the interval $t = 0$ to $t = 2.00\text{ s}$.

(b) The net force is zero when the acceleration is zero. This is between 2.00 s and 6.00 s.

(c) Between 6.00 s and 10.0 s, $a_x = 1.00\text{ m/s}^2$, so $\sum F_x = (2.75\text{ kg})(1.00\text{ m/s}^2) = 2.75\text{ N}$.

EVALUATE: The net force is largest when the velocity is changing most rapidly.

4.15. **IDENTIFY:** The net force and the acceleration are related by Newton's second law. When the rocket is near the surface of the earth the forces on it are the upward force $\vec{F}$ exerted on it because of the burning fuel and the downward force $\vec{F}_{\text{grav}}$ of gravity. $F_{\text{grav}} = mg$.

SET UP: Let $+y$ be upward. The weight of the rocket is $F_{\text{grav}} = (8.00\text{ kg})(9.80\text{ m/s}^2) = 78.4\text{ N}$.

EXECUTE: **(a)** At $t = 0$, $F = A = 100.0\text{ N}$. At $t = 2.00\text{ s}$, $F = A + (4.00\text{ s}^2)B = 150.0\text{ N}$ and

$B = \dfrac{150.0\text{ N} - 100.0\text{ N}}{4.00\text{ s}^2} = 12.5\text{ N/s}^2$.

(b) (i) At $t = 0$, $F = A = 100.0\text{ N}$. The net force is $\sum F_y = F - F_{\text{grav}} = 100.0\text{ N} - 78.4\text{ N} = 21.6\text{ N}$.

$a_y = \dfrac{\sum F_y}{m} = \dfrac{21.6\text{ N}}{8.00\text{ kg}} = 2.70\text{ m/s}^2$. (ii) At $t = 3.00\text{ s}$, $F = A + B(3.00\text{ s})^2 = 212.5\text{ N}$.

$\sum F_y = 212.5\text{ N} - 78.4\text{ N} = 134.1\text{ N}$. $a_y = \dfrac{\sum F_y}{m} = \dfrac{134.1\text{ N}}{8.00\text{ kg}} = 16.8\text{ m/s}^2$.

(c) Now $F_{\text{grav}} = 0$ and $\sum F_y = F = 212.5\text{ N}$. $a_y = \dfrac{212.5\text{ N}}{8.00\text{ kg}} = 26.6\text{ m/s}^2$.

EVALUATE: The acceleration increases as *F* increases.

4.16. **IDENTIFY:** Use constant acceleration equations to calculate a_x and *t*. Then use $\sum \vec{F} = m\vec{a}$ to calculate the net force.

SET UP: Let $+x$ be in the direction of motion of the electron.

EXECUTE: **(a)** $v_{0x} = 0$, $(x - x_0) = 1.80 \times 10^{-2}\text{ m}$, $v_x = 3.00 \times 10^6\text{ m/s}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(3.00 \times 10^6\text{ m/s})^2 - 0}{2(1.80 \times 10^{-2}\text{ m})} = 2.50 \times 10^{14}\text{ m/s}^2$$

(b) $v_x = v_{0x} + a_x t$ gives $t = \dfrac{v_x - v_{0x}}{a_x} = \dfrac{3.00 \times 10^6\text{ m/s} - 0}{2.50 \times 10^{14}\text{ m/s}^2} = 1.2 \times 10^{-8}\text{ s}$

(c) $\sum F_x = ma_x = (9.11 \times 10^{-31}\text{ kg})(2.50 \times 10^{14}\text{ m/s}^2) = 2.28 \times 10^{-16}\text{ N}$.

EVALUATE: The acceleration is in the direction of motion since the speed is increasing, and the net force is in the direction of the acceleration.

4.17. **IDENTIFY** and **SET UP:** $F = ma$. We must use $w = mg$ to find the mass of the boulder.

EXECUTE: $m = \dfrac{w}{g} = \dfrac{2400\text{ N}}{9.80\text{ m/s}^2} = 244.9\text{ kg}$

Then $F = ma = (244.9\text{ kg})(12.0\text{ m/s}^2) = 2940\text{ N}$.

EVALUATE: We must use mass in Newton's second law. Mass and weight are proportional.

4.18. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$.

SET UP: $m = w/g = (71.2\text{ N})/(9.80\text{ m/s}^2) = 7.27\text{ kg}$.

EXECUTE: $a_x = \dfrac{F_x}{m} = \dfrac{160\text{ N}}{7.27\text{ kg}} = 22.0\text{ m/s}^2$

EVALUATE: The weight of the ball is a vertical force and doesn't affect the horizontal acceleration. However, the weight is used to calculate the mass.

4.19. **IDENTIFY** and **SET UP:** $w = mg$. The mass of the watermelon is constant, independent of its location. Its weight differs on earth and Jupiter's moon. Use the information about the watermelon's weight on earth to calculate its mass:

EXECUTE: $w = mg$ gives that $m = \dfrac{w}{g} = \dfrac{44.0\text{ N}}{9.80\text{ m/s}^2} = 4.49\text{ kg}.$

On Jupiter's moon, $m = 4.49$ kg, the same as on earth. Thus the weight on Jupiter's moon is

$w = mg = (4.49\text{ kg})(1.81\text{ m/s}^2) = 8.13\text{ N}.$

EVALUATE: The weight of the watermelon is less on Io, since g is smaller there.

4.20. **IDENTIFY:** Weight and mass are related by $w = mg$. The mass is constant but g and w depend on location.

SET UP: On earth, $g = 9.80\text{ m/s}^2$.

EXECUTE: **(a)** $\dfrac{w}{g} = m$, which is constant, so $\dfrac{w_E}{g_E} = \dfrac{w_A}{g_A}$. $w_E = 17.5$ N, $g_E = 9.80\text{ m/s}^2$, and $w_A = 3.24$ N.

$$g_A = \left(\frac{w_A}{w_E}\right)g_E = \left(\frac{3.24\text{ N}}{17.5\text{ N}}\right)(9.80\text{ m/s}^2) = 1.81\text{ m/s}^2.$$

(b) $m = \dfrac{w_E}{g_E} = \dfrac{17.5\text{ N}}{9.80\text{ m/s}^2} = 1.79\text{ kg}$.

EVALUATE: The weight at a location and the acceleration due to gravity at that location are directly proportional.

4.21. **IDENTIFY:** Apply $\sum F_x = ma_x$ to find the resultant horizontal force.

SET UP: Let the acceleration be in the $+x$ direction.

EXECUTE: $\sum F_x = ma_x = (55\text{ kg})(15\text{ m/s}^2) = 825\text{ N}$. The force is exerted by the blocks. The blocks push on the sprinter because the sprinter pushes on the blocks.

EVALUATE: The force the blocks exert on the sprinter has the same magnitude as the force the sprinter exerts on the blocks. The harder the sprinter pushes, the greater the force on him.

4.22. **IDENTIFY:** $\sum \vec{F} = m\vec{a}$ refers to forces that all act on one object. The third law refers to forces that a pair of objects exert on each other.

SET UP: An object is in equilibrium if the vector sum of all the forces on it is zero. A third law pair of forces have the same magnitude regardless of the motion of either object.

EXECUTE: **(a)** the earth (gravity)

(b) 4 N; the book

(c) no, these two forces are exerted on the same object

(d) 4 N; the earth; the book; upward

(e) 4 N, the hand; the book; downward

(f) second (The two forces are exerted on the same object and this object has zero acceleration.)

(g) third (The forces are between a pair of objects.)

(h) No. There is a net upward force on the book equal to 1 N.

(i) No. The force exerted on the book by your hand is 5 N, upward. The force exerted on the book by the earth is 4 N, downward.

(j) Yes. These forces form a third-law pair and are equal in magnitude and opposite in direction.

(k) Yes. These forces form a third-law pair and are equal in magnitude and opposite in direction.

(l) One, only the gravity force.

(m) No. There is a net downward force of 5 N exerted on the book.

EVALUATE: Newton's second and third laws give complementary information about the forces that act.

4.23. **IDENTIFY:** Identify the forces on the bottle.

SET UP: Classify forces as contact or noncontact forces. The noncontact force is gravity and the contact forces come from things that touch the object. Gravity is always directed downward toward the center of the earth. Air resistance is always directed opposite to the velocity of the object relative to the air.

EXECUTE: **(a)** The free-body diagram for the bottle is sketched in Figure 4.23a

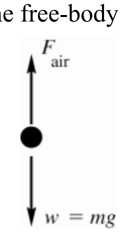


Figure 4.23a

The only forces on the bottle are gravity (downward) and air resistance (upward).

(b)

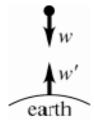


Figure 4.23b

w is the force of gravity that the earth exerts on the bottle. The reaction to this force is w', force that the bottle exerts on the earth

Note that these two equal and opposite forces produce very different accelerations because the bottle and the earth have very different masses.

F_{air} is the force that the air exerts on the bottle and is upward. The reaction to this force is a downward force F'_{air} that the bottle exerts on the air. These two forces have equal magnitudes and opposite directions.

EVALUATE: The only thing in contact with the bottle while it is falling is the air. Newton's third law always deals with forces on two different objects.

4.24. **IDENTIFY:** The reaction forces in Newton's third law are always between a pair of objects. In Newton's second law all the forces act on a single object.

SET UP: Let $+y$ be downward. $m = w/g$.

EXECUTE: The reaction to the upward normal force on the passenger is the downward normal force, also of magnitude 620 N, that the passenger exerts on the floor. The reaction to the passenger's weight is the gravitational force that the passenger exerts on the earth, upward and also of magnitude 650 N. $\dfrac{\sum F_y}{m} = a_y$ gives

$a_y = \dfrac{650\text{ N} - 620\text{ N}}{(650\text{ N})/(9.80\text{ m/s}^2)} = 0.452\text{ m/s}^2$. The passenger's acceleration is 0.452 m/s^2 , downward.

EVALUATE: There is a net downward force on the passenger and the passenger has a downward acceleration.

4.25. **IDENTIFY:** Apply Newton's second law to the earth.

SET UP: The force of gravity that the earth exerts on her is her weight, $w = mg = (45\text{ kg})(9.8\text{ m/s}^2) = 441\text{ N}.$ By Newton's 3rd law, she exerts an equal and opposite force on the earth.

Apply $\sum \vec{F} = m\vec{a}$ to the earth, with $\left|\sum \vec{F}\right| = w = 441\text{ N},$ but must use the mass of the earth for *m*.

EXECUTE: $a = \dfrac{w}{m} = \dfrac{441\text{ N}}{6.0\times 10^{24}\text{ kg}} = 7.4\times 10^{-23}\text{ m/s}^2.$

EVALUATE: This is *much* smaller than her acceleration of $9.8\text{ m/s}^2.$ The force she exerts on the earth equals in magnitude the force the earth exerts on her, but the acceleration the force produces depends on the mass of the object and her mass is *much* less than the mass of the earth.

4.26. **IDENTIFY** and **SET UP:** The only force on the ball is the gravity force, $\vec{F}_{\text{grav}}$. This force is mg , downward and is independent of the motion of the object.

EXECUTE: The free-body diagram is sketched in Figure 4.26. The free-body diagram is the same in all cases.

EVALUATE: Some forces, such as friction, depend on the motion of the object but the gravity force does not.



Figure 4.26

4.27. **IDENTIFY:** Identify the forces on each object.

SET UP: In each case the forces are the noncontact force of gravity (the weight) and the forces applied by objects that are in contact with each crate. Each crate touches the floor and the other crate, and some object applies $\vec{F}$ to crate A.

EXECUTE: **(a)** The free-body diagrams for each crate are given in Figure 4.27.

F_{AB} (the force on m_A due to m_B) and F_{BA} (the force on m_B due to m_A) form an action-reaction pair.

(b) Since there is no horizontal force opposing F, any value of F, no matter how small, will cause the crates to accelerate to the right. The weight of the two crates acts at a right angle to the horizontal, and is in any case balanced by the upward force of the surface on them.

EVALUATE: Crate B is accelerated by F_{BA} and crate A is accelerated by the net force $F - F_{AB}$. The greater the total weight of the two crates, the greater their total mass and the smaller will be their acceleration.

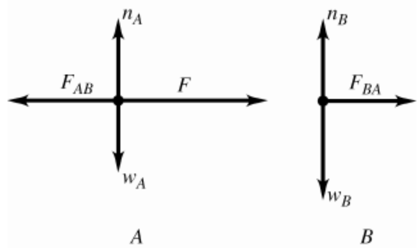


Figure 4.27

4.28. **IDENTIFY:** The surface of block B can exert both a friction force and a normal force on block A. The friction force is directed so as to oppose relative motion between blocks B and A. Gravity exerts a downward force w on block A.
SET UP: The pull is a force on B not on A.
EXECUTE: **(a)** If the table is frictionless there is a net horizontal force on the combined object of the two blocks, and block B accelerates in the direction of the pull. The friction force that B exerts on A is to the right, to try to prevent A from slipping relative to B as B accelerates to the right. The free-body diagram is sketched in Figure 4.28a. f is the friction force that B exerts on A and n is the normal force that B exerts on A.
(b) The pull and the friction force exerted on B by the table cancel and the net force on the system of two blocks is zero. The blocks move with the same constant speed and B exerts no friction force on A. The free-body diagram is sketched in Figure 4.28b.
EVALUATE: If in part (b) the pull force is decreased, block B will slow down, with an acceleration directed to the left. In this case the friction force on A would be to the left, to prevent relative motion between the two blocks by giving A an acceleration equal to that of B.

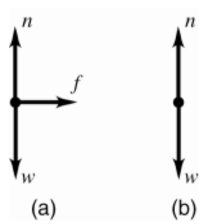


Figure 4.28

4.29. **IDENTIFY:** Since the observer in the train sees the ball hang motionless, the ball must have the same acceleration as the train car. By Newton's second law, there must be a net force on the ball in the same direction as its acceleration.
SET UP: The forces on the ball are gravity, which is w, downward, and the tension $\vec{T}$ in the string, which is directed along the string.
EXECUTE: **(a)** The acceleration of the train is zero, so the acceleration of the ball is zero. There is no net horizontal force on the ball and the string must hang vertically. The free-body diagram is sketched in Figure 4.29a.
(b) The train has a constant acceleration directed east so the ball must have a constant eastward acceleration. There must be a net horizontal force on the ball, directed to the east. This net force must come from an eastward component of $\vec{T}$ and the ball hangs with the string displaced west of vertical. The free-body diagram is sketched in Figure 4.29b.
EVALUATE: When the motion of an object is described in an inertial frame, there must be a net force in the direction of the acceleration.

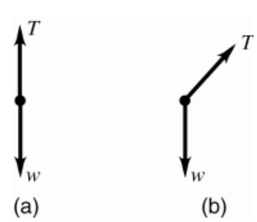


Figure 4.29

4.30. **IDENTIFY:** Identify the forces for each object. Action-reaction pairs of forces act between two objects.
SET UP: Friction is parallel to the surfaces and is directly to oppose relative motion between the surfaces.
EXECUTE: The free-body diagram for the box is given in Figure 4.30a. The free body diagram for the truck is given in Figure 4.30b. The box's friction force on the truck bed and the truck bed's friction force on the box form an action-reaction pair. There would also be some small air-resistance force action to the left, presumably negligible at this speed.

EVALUATE: The friction force on the box, exerted by the bed of the truck, is in the direction of the truck's acceleration. This friction force can't be large enough to give the box the same acceleration that the truck has and the truck acquires a greater speed than the box.

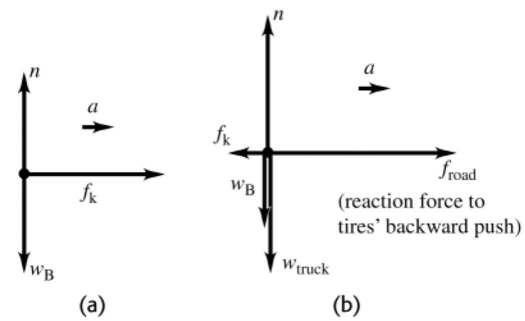


Figure 4.30

4.31. **IDENTIFY:** Identify the forces on the chair. The floor exerts a normal force and a friction force.
SET UP: Let $+y$ be upward and let $+x$ be in the direction of the motion of the chair.
EXECUTE: **(a)** The free-body diagram for the chair is given in Figure 4.31.
(b) For the chair, $a_y = 0$ so $\sum F_y = ma_y$ gives $n - mg - F\sin 37° = 0$ and $n = 142\text{ N}$.
EVALUATE: n is larger than the weight because $\vec{F}$ has a downward component.

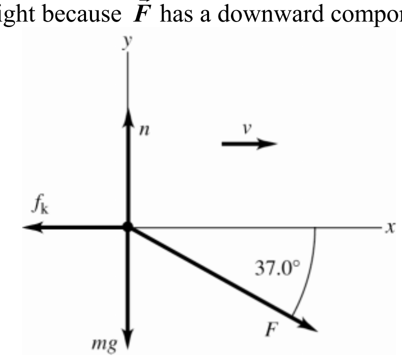


Figure 4.31

4.32. **IDENTIFY:** Identify the forces on the skier and apply $\sum \vec{F} = m\vec{a}$. Constant speed means $a = 0$.
SET UP: Use coordinates that are parallel and perpendicular to the slope.
EXECUTE: **(a)** The free-body diagram for the skier is given in Figure 4.32.
(b) $\sum F_x = ma_x$ with $a_x = 0$ gives $T = mg\sin\theta = (65.0\text{ kg})(9.80\text{ m/s}^2)\sin 26.0° = 279\text{ N}$.
EVALUATE: T is less than the weight of the skier. It is equal to the component of the weight that is parallel to the incline.

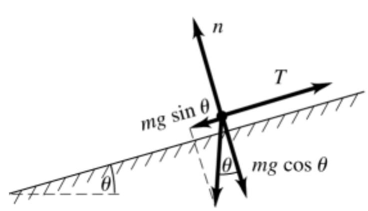


Figure 4.32

4.33. **IDENTIFY:** $\sum \vec{F} = m\vec{a}$ must be satisfied for each object. Newton's third law says that the force $\vec{F}_{\text{C on T}}$ that the car exerts on the truck is equal in magnitude and opposite in direction to the force $\vec{F}_{\text{T on C}}$ that the truck exerts on the car.
SET UP: The only horizontal force on the car is the force $\vec{F}_{\text{T on C}}$ exerted by the truck. The car exerts a force $\vec{F}_{\text{C on T}}$ on the truck. There is also a horizontal friction force $\vec{f}$ that the highway surface exerts on the truck. Assume the system is accelerating to the right in the free-body diagrams.
EXECUTE: **(a)** The free-body diagram for the car is sketched in Figure 4.33a
(b) The free-body diagram for the truck is sketched in Figure 4.33b.

(c) The friction force $\vec{f}$ accelerates the system forward. The tires of the truck push backwards on the highway surface as they rotate, so by Newton's third law the roadway pushes forward on the tires.

EVALUATE: $F_{\text{T on C}}$ and $F_{\text{C on T}}$ each equal the tension T in the rope. Both objects have the same acceleration $\vec{a}$. $T = m_{\text{C}}a$ and $f - T = m_{\text{T}}a$, so $f = (m_{\text{C}} + m_{\text{T}})a$. The acceleration of the two objects is proportional to *f*.

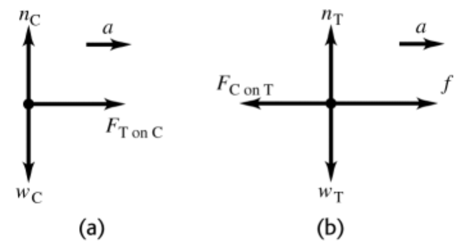


Figure 4.33

4.34. **IDENTIFY:** Use a constant acceleration equation to find the stopping time and acceleration. Then use $\sum \vec{F} = m\vec{a}$ to calculate the force.

SET UP: Let $+x$ be in the direction the bullet is traveling. $\vec{F}$ is the force the wood exerts on the bullet.

EXECUTE: **(a)** $v_{0x} = 350\text{ m/s}$, $v_x = 0$ and $(x - x_0) = 0.130\text{ m}$. $(x - x_0) = \left(\dfrac{v_{0x} + v_x}{2}\right)t$

gives $t = \dfrac{2(x - x_0)}{v_{0x} + v_x} = \dfrac{2(0.130\text{ m})}{350\text{ m/s}} = 7.43 \times 10^{-4}\text{ s}$.

(b) $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $a_x = \dfrac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \dfrac{0 - (350\text{ m/s})^2}{2(0.130\text{ m})} = -4.71 \times 10^5\text{ m/s}^2$

$\sum F_x = ma_x$ gives $-F = ma_x$ and $F = -ma_x = -(1.80 \times 10^{-3}\text{ kg})(-4.71 \times 10^5\text{ m/s}^2) = 848\text{ N}$.

EVALUATE: The acceleration and net force are opposite to the direction of motion of the bullet.

4.35. **IDENTIFY:** Vector addition problem. Write the vector addition equation in component form. We know one vector and its resultant and are asked to solve for the other vector.

SET UP: Use coordinates with the $+x$-axis along $\vec{F}_1$ and the $+y$-axis along $\vec{R}$; as shown in Figure 4.35a.

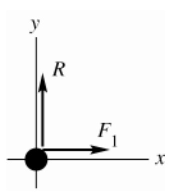


Figure 4.35a

$F_{1x} = +1300\text{ N}$, $F_{1y} = 0$

$R_x = 0$, $R_y = +1300\text{ N}$

$\vec{F}_1 + \vec{F}_2 = \vec{R}$, so $\vec{F}_2 = \vec{R} - \vec{F}_1$

EXECUTE: $F_{2x} = R_x - F_{1x} = 0 - 1300\text{ N} = -1300\text{ N}$

$F_{2y} = R_y - F_{1y} = +1300\text{ N} - 0 = +1300\text{ N}$

The components of $\vec{F}_2$ are sketched in Figure 4.35b.

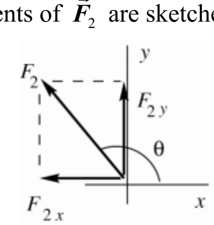


Figure 4.35b

$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2} = \sqrt{(-1300\text{ N})^2 + (1300\text{ N})^2}$

$F = 1840\text{ N}$

$\tan\theta = \dfrac{F_{2y}}{F_{2x}} = \dfrac{+1300\text{ N}}{-1300\text{ N}} = -1.00$

$\theta = 135°$

The magnitude of $\vec{F}_2$ is 1840 N and its direction is $135°$ counterclockwise from the direction of $\vec{F}_1$.

EVALUATE: $\vec{F}_2$ has a negative *x*-component to cancel $\vec{F}_1$ and a *y*-component to equal $\vec{R}$.

4.36. **IDENTIFY:** Use the motion of the ball to calculate *g*, the acceleration of gravity on the planet. Then $w = mg$.

SET UP: Let $+y$ be downward and take $y_0 = 0$. $v_{0y} = 0$ since the ball is released from rest.

EXECUTE: Get g on X: $y = \frac{1}{2}gt^2$ gives $10.0\text{ m} = \frac{1}{2}g(2.2\text{ s})^2$. $g = 4.13\text{ m/s}^2$ and then $w_X = mg_X = (0.100\text{ kg})(4.03\text{ m/s}^2) = 0.41\text{ N}$.

EVALUATE: g on Planet X is smaller than on earth and the object weighs less than it would on earth.

4.37. **IDENTIFY:** If the box moves in the $+x$-direction it must have $a_y = 0$, so $\sum F_y = 0$.

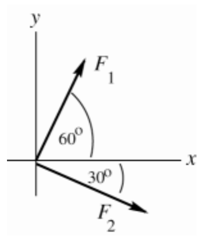


Figure 4.37

The smallest force the child can exert and still produce such motion is a force that makes the y-components of all three forces sum to zero, but that doesn't have any x-component.

SET UP: $\vec{F}_1$ and $\vec{F}_2$ are sketched in Figure 4.37. Let $\vec{F}_3$ be the force exerted by the child.

$\sum F_y = ma_y$ implies $F_{1y} + F_{2y} + F_{3y} = 0$, so $F_{3y} = -(F_{1y} + F_{2y})$.

EXECUTE: $F_{1y} = +F_1 \sin 60° = (100\text{ N})\sin 60° = 86.6\text{ N}$

$F_{2y} = +F_2 \sin(-30°) = -F_2 \sin 30° = -(140\text{ N})\sin 30° = -70.0\text{ N}$

Then $F_{3y} = -(F_{1y} + F_{2y}) = -(86.6\text{ N} - 70.0\text{ N}) = -16.6\text{ N}$; $F_{3x} = 0$

The smallest force the child can exert has magnitude 17 N and is directed at $90°$ clockwise from the $+x$-axis shown in the figure.

(b) IDENTIFY and **SET UP:** Apply $\sum F_x = ma_x$. We know the forces and a_x so can solve for m. The force exerted by the child is in the $-y$-direction and has no x-component.

EXECUTE: $F_{1x} = F_1 \cos 60° = 50\text{ N}$

$F_{2x} = F_2 \cos 30° = 121.2\text{ N}$

$\sum F_x = F_{1x} + F_{2x} = 50\text{ N} + 121.2\text{ N} = 171.2\text{ N}$

$$m = \frac{\sum F_x}{a_x} = \frac{171.2\text{ N}}{2.00\text{ m/s}^2} = 85.6\text{ kg}$$

Then $w = mg = 840\text{ N}$.

EVALUATE: In part (b) we don't need to consider the y-component of Newton's second law. $a_y = 0$ so the mass doesn't appear in the $\sum F_y = ma_y$ equation.

4.38. **IDENTIFY:** Use $\sum \vec{F} = m\vec{a}$ to calculate the acceleration of the tanker and then use constant acceleration kinematic equations.

SET UP: Let $+x$ be the direction the tanker is moving initially. Then $a_x = -F/m$.

EXECUTE: $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ says that if the reef weren't there the ship would stop in a distance of

$$x - x_0 = -\frac{v_{0x}^2}{2a_x} = \frac{v_0^2}{2(F/m)} = \frac{mv_0^2}{2F} = \frac{(3.6\times10^7\text{ kg})(1.5\text{ m/s})^2}{2(8.0\times10^4\text{ N})} = 506\text{ m},$$

so the ship would hit the reef. The speed when the tanker hits the reef is found from $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$, so it is

$$v = \sqrt{v_0^2 - (2Fx/m)} = \sqrt{(1.5\text{ m/s})^2 - \frac{2(8.0\times10^4\text{ N})(500\text{ m})}{(3.6\times10^7\text{ kg})}} = 0.17\text{ m/s},$$

and the oil should be safe.

EVALUATE: The force and acceleration are directed opposite to the initial motion of the tanker and the speed decreases.

4.39. **IDENTIFY:** We can apply constant acceleration equations to relate the kinematic variables and we can use Newton's second law to relate the forces and acceleration.

(a) SET UP: First use the information given about the height of the jump to calculate the speed he has at the instant his feet leave the ground. Use a coordinate system with the $+y$-axis upward and the origin at the position when his feet leave the ground.

$v_y = 0$ (at the maximum height), $v_{0y} = ?$, $a_y = -9.80\ \text{m/s}^2$, $y - y_0 = +1.2\ \text{m}$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_{0y} = \sqrt{-2a_y(y-y_0)} = \sqrt{-2(-9.80\ \text{m/s}^2)(1.2\ \text{m})} = 4.85\ \text{m/s}$

(b) SET UP: Now consider the acceleration phase, from when he starts to jump until when his feet leave the ground. Use a coordinate system where the $+y$-axis is upward and the origin is at his position when he starts his jump.

EXECUTE: Calculate the average acceleration:

$$(a_{\text{av}})_y = \frac{v_y - v_{0y}}{t} = \frac{4.89\ \text{m/s} - 0}{0.300\ \text{s}} = 16.2\ \text{m/s}^2$$

(c) SET UP: Finally, find the average upward force that the ground must exert on him to produce this average upward acceleration. (Don't forget about the downward force of gravity.) The forces are sketched in Figure 4.39.

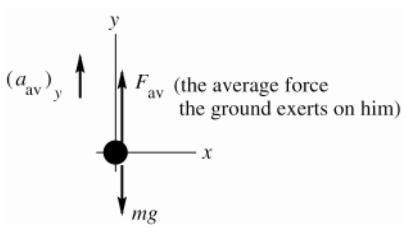


Figure 4.39

EXECUTE:

$m = w/g = \dfrac{890\ \text{N}}{9.80\ \text{m/s}^2} = 90.8\ \text{kg}$

$\sum F_y = ma_y$

$F_{\text{av}} - mg = m(a_{\text{av}})_y$

$F_{\text{av}} = m(g + (a_{\text{av}})_y)$

$F_{\text{av}} = 90.8\ \text{kg}(9.80\ \text{m/s}^2 + 16.2\ \text{m/s}^2)$

$F_{\text{av}} = 2360\ \text{N}$

This is the average force exerted on him by the ground. But by Newton's 3rd law, the average force he exerts on the ground is equal and opposite, so is 2360 N, downward.

EVALUATE: In order for him to accelerate upward, the ground must exert an upward force greater than his weight.

4.40. IDENTIFY: Use constant acceleration equations to calculate the acceleration a_x that would be required. Then use $\sum F_x = ma_x$ to find the necessary force.

SET UP: Let $+x$ be the direction of the initial motion of the auto.

EXECUTE: $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ with $v_x = 0$ gives $a_x = -\dfrac{v_{0x}^2}{2(x - x_0)}$. The force F is directed opposite to the motion and $a_x = -\dfrac{F}{m}$. Equating these two expressions for a_x gives

$$F = m\frac{v_{0x}^2}{2(x - x_0)} = (850\ \text{kg})\frac{(12.5\ \text{m/s})^2}{2(1.8\times10^{-2}\ \text{m})} = 3.7\times10^6\ \text{N}.$$

EVALUATE: A very large force is required to stop such a massive object in such a short distance.

4.41. IDENTIFY: Apply Newton's second law to calculate a.

(a) SET UP: The free-body diagram for the bucket is sketched in Figure 4.41.

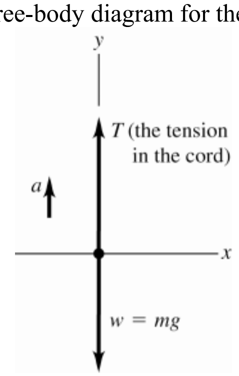


Figure 4.41

The net force on the bucket is $T - mg$, upward.

(b) EXECUTE: $\sum F_y = ma_y$ gives $T - mg = ma$

$$a = \frac{T - mg}{m} = \frac{75.0\text{ N} - (4.80\text{ kg})(9.80\text{ m/s}^2)}{4.80\text{ kg}} = \frac{75.0\text{ N} - 47.04\text{ N}}{4.80\text{ kg}} = 5.82\text{ m/s}^2.$$

EVALUATE: The weight of the bucket is 47.0 N. The upward force exerted by the cord is larger than this, so the bucket accelerates upward.

4.42. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the parachutist.

SET UP: Let $+y$ be upward. $\vec{F}_{\text{air}}$ is the force of air resistance.

EXECUTE: (a) $w = mg = (55.0\text{ kg})(9.80\text{ m/s}^2) = 539\text{ N}$

(b) The free-body diagram is given in Fig. 4.42. $\sum F_y = F_{\text{air}} - w = 620\text{ N} - 539\text{ N} = 81\text{ N}$. The net force is upward.

(c) $a_y = \frac{\sum F_y}{m} = \frac{81\text{ N}}{55.0\text{ kg}} = 1.5\text{ m/s}^2$, upward.

EVALUATE: Both the net force and the acceleration are upward. Since her velocity is downward and her acceleration is upward, her speed decreases.

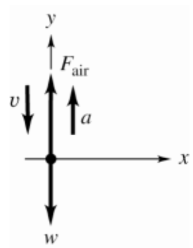


Figure 4.42

4.43. IDENTIFY: Use Newton's 2nd law to relate the acceleration and forces for each crate.

(a) SET UP: Since the crates are connected by a rope, they both have the same acceleration, 2.50 m/s^2.

(b) The forces on the 4.00 kg crate are shown in Figure 4.43a.

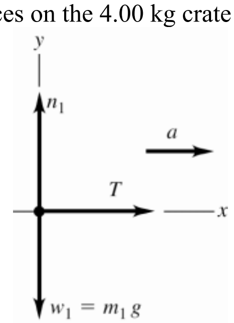


Figure 4.43a

EXECUTE:

$\sum F_x = ma_x$

$T = m_1 a = (4.00\text{ kg})(2.50\text{ m/s}^2) = 10.0\text{ N}.$

(c) SET UP: Forces on the 6.00 kg crate are shown in Figure 4.43b

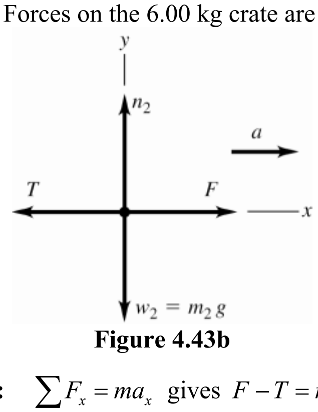


Figure 4.43b

The crate accelerates to the right, so the net force is to the right. *F* must be larger than *T*.

(d) EXECUTE: $\sum F_x = ma_x$ gives $F - T = m_2 a$

$$F = T + m_2 a = 10.0\text{ N} + (6.00\text{ kg})(2.50\text{ m/s}^2) = 10.0\text{ N} + 15.0\text{ N} = 25.0\text{ N}$$

EVALUATE: We can also consider the two crates and the rope connecting them as a single object of mass $m = m_1 + m_2 = 10.0$ kg. The free-body diagram is sketched in Figure 4.43c.

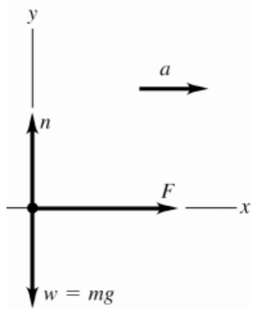


Figure 4.43c

$\sum F_x = ma_x$

$F = ma = (10.0\text{ kg})(2.50\text{ m/s}^2) = 25.0\text{ N}$

This agrees with our answer in part (d).

4.44. **IDENTIFY:** Apply Newton's second and third laws.

SET UP: Action-reaction forces act between a pair of objects. In the second law all the forces act on the same object.

EXECUTE: **(a)** The force the astronaut exerts on the cable and the force that the cable exerts on the astronaut are an action-reaction pair, so the cable exerts a force of 80.0 N on the astronaut.

(b) The cable is under tension.

(c) $a = \dfrac{F}{m} = \dfrac{80.0\text{ N}}{105.0\text{ kg}} = 0.762\text{ m/s}^2$.

(d) There is no net force on the massless cable, so the force that the shuttle exerts on the cable must be 80.0 N (this is *not* an action-reaction pair). Thus, the force that the cable exerts on the shuttle must be 80.0 N.

(e) $a = \dfrac{F}{m} = \dfrac{80.0\text{ N}}{9.05\times 10^4\text{ kg}} = 8.84\times 10^{-4}\text{ m/s}^2$.

EVALUATE: Since the cable is massless the net force on it is zero and the tension is the same at each end.

4.45. **IDENTIFY** and **SET UP:** Take derivatives of $x(t)$ to find v_x and a_x. Use Newton's second law to relate the acceleration to the net force on the object.

EXECUTE:

(a) $x = (9.0\times 10^3\text{ m/s}^2)t^2 - (8.0\times 10^4\text{m/s}^3)t^3$

$x = 0$ at $t = 0$

When $t = 0.025$ s, $x = (9.0\times 10^3\text{ m/s}^2)(0.025\text{ s})^2 - (8.0\times 10^4\text{ m/s}^3)(0.025\text{ s})^3 = 4.4$ m.

The length of the barrel must be 4.4 m.

(b) $v_x = \dfrac{dx}{dt} = (18.0\times 10^3\text{ m/s}^2)t - (24.0\times 10^4\text{ m/s}^3)t^2$

At $t = 0$, $v_x = 0$ (object starts from rest).

At $t = 0.025$ s, when the object reaches the end of the barrel,

$v_x = (18.0\times 10^3\text{ m/s}^2)(0.025\text{ s}) - (24.0\times 10^4\text{ m/s}^3)(0.025\text{ s})^2 = 300\text{ m/s}$

(c) $\sum F_x = ma_x$, so must find a_x.

$a_x = \dfrac{dv_x}{dt} = 18.0\times 10^3\text{ m/s}^2 - (48.0\times 10^4\text{ m/s}^3)t$

(i) At $t = 0$, $a_x = 18.0\times 10^3\text{ m/s}^2$ and $\sum F_x = (1.50\text{ kg})(18.0\times 10^3\text{ m/s}^2) = 2.7\times 10^4$ N.

(ii) At $t = 0.025$ s, $a_x = 18\times 10^3\text{ m/s}^2 - (48.0\times 10^4\text{ m/s}^3)(0.025\text{ s}) = 6.0\times 10^3\text{ m/s}^2$ and $\sum F_x = (1.50\text{ kg})(6.0\times 10^3\text{ m/s}^2) = 9.0\times 10^3$ N.

EVALUATE: The acceleration and net force decrease as the object moves along the barrel.

4.46. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ and solve for the mass m of the spacecraft.

SET UP: $w = mg$. Let $+y$ be upward.

EXECUTE: **(a)** The velocity of the spacecraft is downward. When it is slowing down, the acceleration is upward. When it is speeding up, the acceleration is downward.

(b) In each case the net force is in the direction of the acceleration. Speeding up: $w > F$ and the net force is downward. Slowing down: $w < F$ and the net force is upward.

(c) Denote the y-component of the acceleration when the thrust is F_1 by a_1 and the y-component of the acceleration when the thrust is F_2 by a_1. $a_y = +1.20\ \text{m/s}^2$ and $a_2 = -0.80\ \text{m/s}^2$. The forces and accelerations are then related by $F_1 - w = ma_1$, $F_2 - w = ma_2$. Dividing the first of these by the second to eliminate the mass gives $\dfrac{F_1 - w}{F_2 - w} = \dfrac{a_1}{a_2}$, and solving for the weight w gives

$w = \dfrac{a_1F_2 - a_2F_1}{a_1 - a_2}$. Substituting the given numbers, with $+y$ upward, gives

$$w = \frac{(1.20\ \text{m/s}^2)(10.0\times10^3\ \text{N}) - (-0.80\ \text{m/s}^2)(25.0\times10^3\ \text{N})}{1.20\ \text{m/s}^2 - (-0.80\ \text{m/s}^2)} = 16.0\times10^3\ \text{N}.$$

EVALUATE: The acceleration due to gravity at the surface of Mercury did not need to be found.

4.47. **IDENTIFY:** The ship and instrument have the same acceleration. The forces and acceleration are related by Newton's second law. We can use a constant acceleration equation to calculate the acceleration from the information given about the motion.

SET UP: Let $+y$ be upward. The forces on the instrument are the upward tension $\vec{T}$ exerted by the wire and the downward force $\vec{w}$ of gravity. $w = mg = (6.50\ \text{kg})(9.80\ \text{m/s}^2) = 63.7\ \text{N}$

EXECUTE: **(a)** The free-body diagram is sketched in Figure 4.47. The acceleration is upward, so $T > w$.

$y - y_0 = 276\ \text{m}$, $t = 15.0\ \text{s}$, $v_{0y} = 0$. $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives $a_y = \dfrac{2(y-y_0)}{t^2} = \dfrac{2(276\ \text{m})}{(15.0\ \text{s})^2} = 2.45\ \text{m/s}^2$.

$\sum F_y = ma_y$ gives $T - w = ma$ and $T = w + ma = 63.7\ \text{N} + (6.50\ \text{kg})(2.45\ \text{m/s}^2) = 79.6\ \text{N}$.

EVALUATE: There must be a net force in the direction of the acceleration.

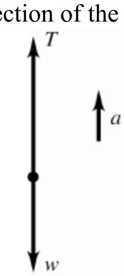


Figure 4.47

4.48. If the rocket is moving downward and its speed is decreasing, its acceleration is upward, just as in Problem 4.47. The solution is identical to that of Problem 4.47.

4.49. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the gymnast.

SET UP: The upward force on the gymnast gives the tension in the rope. The free-body diagram for the gymnast is given in Figure 4.49.

EXECUTE: **(a)** If the gymnast climbs at a constant rate, there is no net force on the gymnast, so the tension must equal the weight; $T = mg$.

(b) No motion is no acceleration, so the tension is again the gymnast's weight.

(c) $T - w = T - mg = ma = m|\mathbf{\vec{a}}|$ (the acceleration is upward, the same direction as the tension), so $T = m(g + |\mathbf{\vec{a}}|)$.

(d) $T - w = T - mg = ma = -m|\mathbf{\vec{a}}|$ (the acceleration is downward, the opposite direction to the tension), so $T = m(g - |\mathbf{\vec{a}}|)$.

EVALUATE: When she accelerates upward the tension is greater than her weight and when she accelerates downward the tension is less than her weight.



Figure 4.49

4.50. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the elevator to relate the forces on it to the acceleration.

(a) SET UP: The free-body diagram for the elevator is sketched in Figure 4.50.

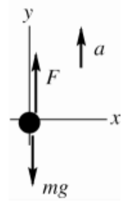


Figure 4.50

The net force is $T - mg$ (upward).

Take the $+y$-direction to be upward since that is the direction of the acceleration. The maximum upward acceleration is obtained from the maximum possible tension in the cables.

EXECUTE: $\sum F_y = ma_y$ gives $T - mg = ma$

$$a = \frac{T - mg}{m} = \frac{28{,}000\ \text{N} - (2200\ \text{kg})(9.80\ \text{m/s}^2)}{2200\ \text{kg}} = 2.93\ \text{m/s}^2.$$

(b) What changes is the weight *mg* of the elevator.

$$a = \frac{T - mg}{m} = \frac{28{,}000\ \text{N} - (2200\ \text{kg})(1.62\ \text{m/s}^2)}{2200\ \text{kg}} = 11.1\ \text{m/s}^2.$$

EVALUATE: The cables can give the elevator a greater acceleration on the moon since the downward force of gravity is less there and the same *T* then gives a greater net force.

4.51. **IDENTIFY:** He is in free-fall until he contacts the ground. Use the constant acceleration equations and apply $\sum \vec{F} = m\vec{a}$.

SET UP: Take $+y$ downward. While he is in the air, before he touches the ground, his acceleration is $a_y = 9.80\ \text{m/s}^2$.

EXECUTE: **(a)** $v_{0y} = 0$, $y - y_0 = 3.10\ \text{m}$, and $a_y = 9.80\ \text{m/s}^2$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$v_y = \sqrt{2a_y(y - y_0)} = \sqrt{2(9.80\ \text{m/s}^2)(3.10\ \text{m})} = 7.79\ \text{m/s}$$

(b) $v_{0y} = 7.79\ \text{m/s}$, $v_y = 0$, $y - y_0 = 0.60\ \text{m}$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{0 - (7.79\ \text{m/s})^2}{2(0.60\ \text{m})} = -50.6\ \text{m/s}^2$$. The acceleration is upward.

(c) The free-body diagram is given in Fig. 4.51. $\vec{F}$ is the force the ground exerts on him.

$\sum F_y = ma_y$ gives $mg - F = -ma$. $F = m(g + a) = (75.0\ \text{kg})(9.80\ \text{m/s}^2 + 50.6\ \text{m/s}^2) = 4.53 \times 10^3\ \text{N}$, upward.

$$\frac{F}{w} = \frac{4.53 \times 10^3\ \text{N}}{(75.0\ \text{kg})(9.80\ \text{m/s}^2)} = 6.16$$, so $F = 6.16w$.

By Newton's third law, the force his feet exert on the ground is $-\vec{F}$.

EVALUATE: The force the ground exerts on him is about six times his weight.

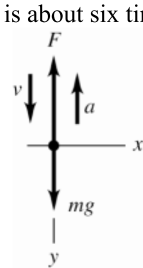


Figure 4.51

4.52. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the hammer head. Use a constant acceleration equation to relate the motion to the acceleration.

SET UP: Let $+y$ be upward.

EXECUTE: **(a)** The free-body diagram for the hammer head is sketched in Figure 4.52.

(b) The acceleration of the hammer head is given by $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $v_y = 0$, $v_{0y} = -3.2\ \text{m/s}^2$ and $y - y_0 = -0.0045\ \text{m}$. $a_y = v_{0y}^2 / 2(y - y_0) = (3.2\ \text{m/s})^2 / 2(0.0045\ \text{cm}) = 1.138 \times 10^3\ \text{m/s}^2$. The mass of the hammer

head is its weight divided by g, $(4.9\ \text{N})/(9.80\ \text{m/s}^2) = 0.50\ \text{kg}$, and so the net force on the hammer head is $(0.50\ \text{kg})(1.138\times 10^3\ \text{m/s}^2) = 570\ \text{N}$. This is the sum of the forces on the hammer head: the upward force that the nail exerts, the downward weight and the downward 15-N force. The force that the nail exerts is then 590 N, and this must be the magnitude of the force that the hammer head exerts on the nail.

(c) The distance the nail moves is 0.12 m, so the acceleration will be $4267\ \text{m/s}^2$, and the net force on the hammer head will be 2133 N. The magnitude of the force that the nail exerts on the hammer head, and hence the magnitude of the force that the hammer head exerts on the nail, is 2153 N, or about 2200 N.

EVALUATE: For the shorter stopping distance the acceleration has a larger magnitude and the force between the nail and hammer head is larger.

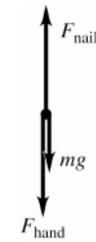


Figure 4.52

4.53. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to some portion of the cable.

SET UP: The free-body diagrams for the whole cable, the top half of the cable and the bottom half are sketched in Figure 4.53. The cable is at rest, so in each diagram the net force is zero.

EXECUTE: **(a)** The net force on a point of the cable at the top is zero; the tension in the cable must be equal to the weight w.

(b) The net force on the cable must be zero; the difference between the tensions at the top and bottom must be equal to the weight w, and with the result of part (a), there is no tension at the bottom.

(c) The net force on the bottom half of the cable must be zero, and so the tension in the cable at the middle must be half the weight, $w/2$. Equivalently, the net force on the upper half of the cable must be zero. From part (a) the tension at the top is w, the weight of the top half is $w/2$ and so the tension in the cable at the middle must be $w - w/2 = w/2$.

(d) A graph of T vs. distance will be a negatively sloped line.

EVALUATE: The tension decreases linearly from a value of w at the top to zero at the bottom of the cable.

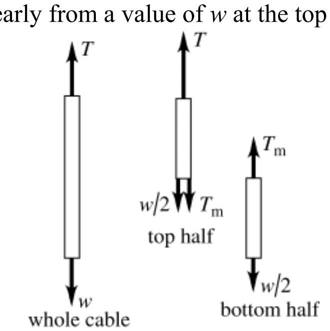


Figure 4.53

4.54. **IDENTIFY:** Note that in this problem the mass of the rope is given, and that it is not negligible compared to the other masses. Apply $\sum \vec{F} = m\vec{a}$ to each object to relate the forces to the acceleration.

(a) SET UP: The free-body diagrams for each block and for the rope are given in Figure 4.54a.

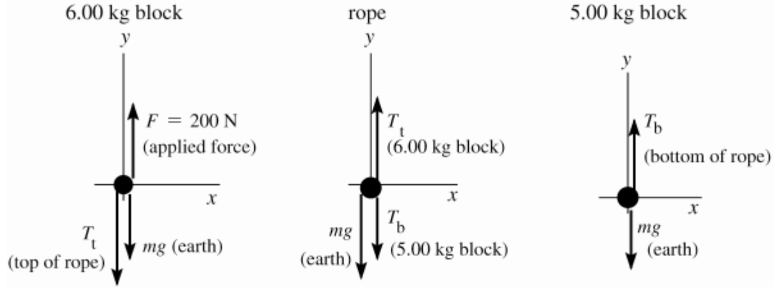


Figure 4.54a

T_t is the tension at the top of the rope and T_b is the tension at the bottom of the rope.

EXECUTE: **(b)** Treat the rope and the two blocks together as a single object, with mass $m = 6.00\text{ kg} + 4.00\text{ kg} + 5.00\text{ kg} = 15.0\text{ kg}$. Take $+y$ upward, since the acceleration is upward. The free-body diagram is given in Figure 4.54b.

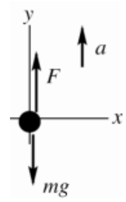


Figure 4.54b

$$\sum F_y = ma_y$$
$$F - mg = ma$$
$$a = \frac{F - mg}{m}$$
$$a = \frac{200\text{ N} - (15.0\text{ kg})(9.80\text{ m/s}^2)}{15.0\text{ kg}} = 3.53\text{ m/s}^2$$

(c) Consider the forces on the top block $(m = 6.00\text{ kg})$, since the tension at the top of the rope (T_t) will be one of these forces.

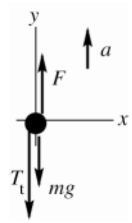


Figure 4.54c

$$\sum F_y = ma_y$$
$$F - mg - T_t = ma$$
$$T_t = F - m(g + a)$$
$$T = 200\text{ N} - (6.00\text{ kg})(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2) = 120\text{ N}$$

Alternatively, can consider the forces on the combined object rope plus bottom block $(m = 9.00\text{ kg})$:

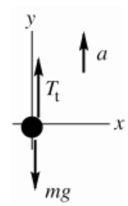


Figure 4.54d

$$\sum F_y = ma_y$$
$$T_t - mg = ma$$
$$T_t = m(g + a) = 9.00\text{ kg}(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2) = 120\text{ N},$$

which checks

(d) One way to do this is to consider the forces on the top half of the rope $(m = 2.00\text{ kg})$. Let T_m be the tension at the midpoint of the rope.

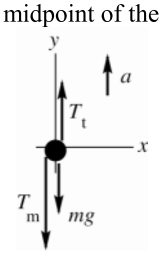


Figure 4.54e

$$\sum F_y = ma_y$$
$$T_t - T_m - mg = ma$$
$$T_m = T_t - m(g + a) = 120\text{ N} - 2.00\text{ kg}(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2) = 93.3\text{ N}$$

To check this answer we can alternatively consider the forces on the bottom half of the rope plus the lower block taken together as a combined object $(m = 2.00\text{ kg} + 5.00\text{ kg} = 7.00\text{ kg})$:

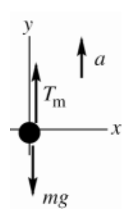


Figure 4.54f

$$\sum F_y = ma_y$$
$$T_m - mg = ma$$
$$T_m = m(g + a) = 7.00\text{ kg}(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2) = 93.3\text{ N},$$

which checks

EVALUATE: The tension in the rope is not constant but increases from the bottom of the rope to the top. The tension at the top of the rope must accelerate the rope as well the 5.00-kg block. The tension at the top of the rope is less than F; there must be a net upward force on the 6.00-kg block.

4.55. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the barbell and to the athlete. Use the motion of the barbell to calculate its acceleration.

SET UP: Let $+y$ be upward.

EXECUTE: **(a)** The free-body diagrams for the baseball and for the athlete are sketched in Figure 4.55.

(b) The athlete's weight is $mg = (90.0\text{ kg})(9.80\text{ m/s}^2) = 882\text{ N}$. The upward acceleration of the barbell is found from $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$. $a_y = \dfrac{2(y-y_0)}{t^2} = \dfrac{2(0.600\text{ m})}{(1.6\text{ s})^2} = 0.469\text{ m/s}^2$. The force needed to lift the barbell is given by $F_{\text{lift}} - w_{\text{barbell}} = ma_y$. The barbell's mass is $(490\text{ N})/(9.80\text{ m/s}^2) = 50.0\text{ kg}$, so

$F_{\text{lift}} = w_{\text{barbell}} + ma = 490\text{ N} + (50.0\text{ kg})(0.469\text{ m/s}^2) = 490\text{ N} + 23\text{ N} = 513\text{ N}$.

The athlete is not accelerating, so $F_{\text{floor}} - F_{\text{lift}} - w_{\text{athlete}} = 0$. $F_{\text{floor}} = F_{\text{lift}} + w_{\text{athlete}} = 513\text{ N} + 882\text{ N} = 1395\text{ N}$.

EVALUATE: Since the athlete pushes upward on the barbell with a force greater than its weight the barbell pushes down on him and the normal force on the athlete is greater than the total weight, 1362 N, of the athlete plus barbell.

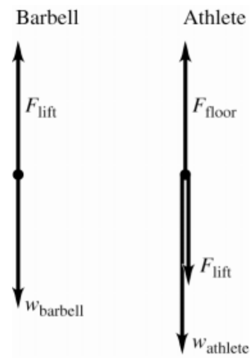


Figure 4.55

4.56. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the balloon and its passengers and cargo, both before and after objects are dropped overboard.

SET UP: When the acceleration is downward take $+y$ to be downward and when the acceleration is upward take $+y$ to be upward.

EXECUTE: **(a)** The free-body diagram for the descending balloon is given in Figure 4.56. L is the lift force.

(b) $\sum F_y = ma_y$ gives $Mg - L = M(g/3)$ and $L = 2Mg/3$.

(c) Now $+y$ is upward, so $L - mg = m(g/2)$, where m is the mass remaining.

$L = 2Mg/3$, so $m = 4M/9$. Mass $5M/9$ must be dropped overboard.

EVALUATE: In part (b) the lift force is greater than the total weight and in part (c) the lift force is less than the total weight.

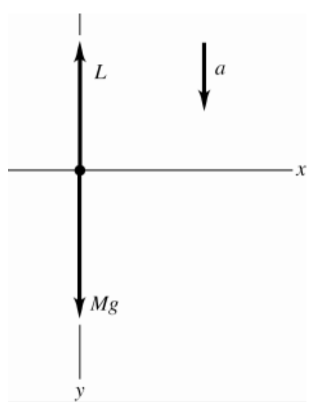


Figure 4.56

4.57. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the entire chain and to each link.

SET UP: $m =$ mass of one link. Let $+y$ be upward.

EXECUTE: **(a)** The free-body diagrams are sketched in Figure 4.57. F_{top} is the force the top and middle links exert on each other. F_{middle} is the force the middle and bottom links exert on each other.

(b) (i) The weight of each link is $mg = (0.300\text{ kg})(9.80\text{ m/s}^2) = 2.94\text{ N}$. Using the free-body diagram for the whole chain:

$$a = \frac{F_{\text{student}} - 3mg}{3m} = \frac{12\text{ N} - 3(2.94\text{ N})}{0.900\text{ kg}} = \frac{3.18\text{ N}}{0.900\text{ kg}} = 3.53\text{ m/s}^2$$

(ii) The top link also accelerates at 3.53 m/s^2, so $F_{\text{student}} - F_{\text{top}} - mg = ma$.

$F_{\text{top}} = F_{\text{student}} - m(g+a) = 12\text{ N} - (0.300\text{ kg})(9.80\text{ m/s}^2 + 3.53\text{ m/s}^2) = 8.0\text{ N}$.

EVALUATE: The force exerted by the middle link on the bottom link is given by $F_{\text{middle}} - mg = ma$ and $F_{\text{middle}} = m(g+a) = 4.0\text{ N}$. We can verify that with our results $\sum F_y = ma_y$ is satisfied for the middle link.

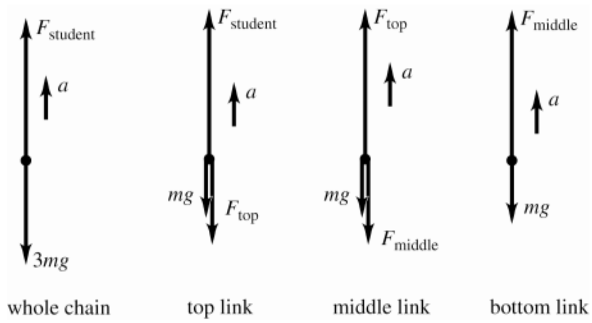


Figure 4.57

4.58. **IDENTIFY:** Calculate $\vec{a}$ from $\vec{a} = d^2\vec{r}/dt^2$. Then $\vec{F}_{\text{net}} = m\vec{a}$.

SET UP: $w = mg$

EXECUTE: Differentiating twice, the acceleration of the helicopter as a function of time is $\vec{a} = (0.120\text{ m/s}^3)t\hat{i} - (0.12\text{ m/s}^2)\hat{k}$ and at $t = 5.0\text{s}$, the acceleration is $\vec{a} = (0.60\text{ m/s}^2)\hat{i} - (0.12\text{ m/s}^2)\hat{k}$. The force is then

$$\vec{F} = m\vec{a} = \frac{w}{g}\vec{a} = \frac{(2.75\times10^5\text{ N})}{(9.80\text{ m/s}^2)}\left[(0.60\text{ m/s}^2)\hat{i} - (0.12\text{ m/s}^2)\hat{k}\right] = (1.7\times10^4\text{ N})\hat{i} - (3.4\times10^3\text{ N})\hat{k}$$

EVALUATE: The force and acceleration are in the same direction. They are both time dependent.

4.59. **IDENTIFY:** $F_x = ma_x$ and $a_x = \dfrac{d^2x}{dt^2}$.

SET UP: $\dfrac{d}{dt}(t^n) = nt^{n-1}$

EXECUTE: The velocity as a function of time is $v_x(t) = A - 3Bt^2$ and the acceleration as a function of time is $a_x(t) = -6Bt$, and so the force as a function of time is $F_x(t) = ma(t) = -6mBt$.

EVALUATE: Since the acceleration is along the *x*-axis, the force is along the *x*-axis.

4.60. **IDENTIFY:** $\vec{a} = \vec{F}/m$. $\vec{v} = \vec{v}_0 + \int_0^t \vec{a}\,dt$.

SET UP: $v_0 = 0$ since the object is initially at rest.

EXECUTE: $\vec{v}(t) = \dfrac{1}{m}\displaystyle\int_0^t \vec{F}\,dt = \dfrac{1}{m}\left(k_1 t\hat{i} + \dfrac{k_2}{4}t^4\hat{j}\right)$.

EVALUATE: $\vec{F}$ has both *x* and *y* components, so $\vec{v}$ develops *x* and *y* components.

4.61. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: The chain rule for differentiating says $\dfrac{dv}{dt} = \dfrac{dv}{dx}\dfrac{dv}{dt} = \dfrac{dv}{dx}v$.

EXECUTE: **(a)** The equation of motion, $-Cv^2 = m\frac{dv}{dt}$ cannot be integrated with respect to time, as the unknown function $v(t)$ is part of the integrand. The equation must be *separated* before integration; that is, $-\frac{C}{m}dt = \frac{dv}{v^2}$ and

$$-\frac{Ct}{m} = -\frac{1}{v} + \frac{1}{v_0},$$

where v_0 is the constant of integration that gives $v = v_0$ at $t = 0$. Note that this form shows that if $v_0 = 0$, there is no motion. This expression may be rewritten as $v = \frac{dx}{dt} = \left(\frac{1}{v_0} + \frac{Ct}{m}\right)^{-1}$,

which may be integrated to obtain $x - x_0 = \frac{m}{C}\ln\left[1 + \frac{Ctv_0}{m}\right].$

To obtain *x* as a function of *v*, the time *t* must be eliminated in favor of *v*; from the expression obtained after the first integration, $\frac{Ctv_0}{m} = \frac{v_0}{v} - 1$, so $x - x_0 = \frac{m}{C}\ln\left(\frac{v_0}{v}\right).$

(b) Applying the chain rule, $\sum F = m\frac{dv}{dt} = mv\frac{dv}{dx}$. Using the given expression for the net force, $-Cv^2 = \left(v\frac{dv}{dx}\right)m$. $-\frac{C}{m}dx = \frac{dv}{v}$. Integrating gives $-\frac{C}{m}(x - x_0) = \ln\left(\frac{v}{v_0}\right)$ and $x - x_0 = \frac{m}{C}\ln\left(\frac{v_0}{v}\right).$

EVALUATE: If *C* is positive, our expression for $v(t)$ shows it decreases from its value of v_0. As *v* decreases, so does the acceleration and therefore the rate of decrease of *v*.

4.62. **IDENTIFY:** $x = \int_0^t v_x dt$ and $v_x = \int_0^t a_x dt$, and similar equations apply to the *y*-component.

SET UP: In this situation, the *x*-component of force depends explicitly on the *y*-component of position. As the *y*-component of force is given as an explicit function of time, v_y and *y* can be found as functions of time and used in the expression for $a_x(t)$.

EXECUTE: $a_y = (k_3/m)t$, so $v_y = (k_3/2m)t^2$ and $y = (k_3/6m)t^3$, where the initial conditions $v_{0y} = 0, y_0 = 0$ have been used. Then, the expressions for a_x, v_x and *x* are obtained as functions of time: $a_x = \frac{k_1}{m} + \frac{k_2k_3}{6m^2}t^3$,

$v_x = \frac{k_1}{m}t + \frac{k_2k_3}{24m^2}t^4$ and $x = \frac{k_1}{2m}t^2 + \frac{k_2k_3}{120m^2}t^5$.

In vector form, $\vec{r} = \left(\frac{k_1}{2m}t^2 + \frac{k_2k_3}{120m^2}t^5\right)\hat{i} + \left(\frac{k_3}{6m}t^3\right)\hat{j}$ and $\vec{v} = \left(\frac{k_1}{m}t + \frac{k_2k_3}{24m^2}t^4\right)\hat{i} + \left(\frac{k_3}{2m}t^2\right)\hat{j}$.

EVALUATE: a_x depends on time because it depends on *y*, and *y* is a function of time.

5

APPLYING NEWTON'S LAWS

5.1. **IDENTIFY:** $a = 0$ for each object. Apply $\sum F_y = ma_y$ to each weight and to the pulley.

SET UP: Take $+y$ upward. The pulley has negligible mass. Let T_r be the tension in the rope and let T_c be the tension in the chain.

EXECUTE: **(a)** The free-body diagram for each weight is the same and is given in Figure 5.1a. $\sum F_y = ma_y$ gives $T_r = w = 25.0\text{ N}$.

(b) The free-body diagram for the pulley is given in Figure 5.1b. $T_c = 2T_r = 50.0\text{ N}$.

EVALUATE: The tension is the same at all points along the rope.

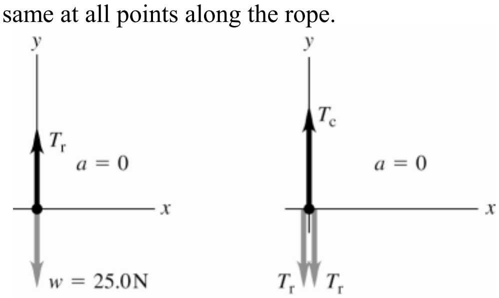


Figure 5.1a, b

5.2. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each weight.

SET UP: Two forces act on each mass**:** w down and $T(= w)$ up.

EXECUTE: In all cases, each string is supporting a weight w against gravity, and the tension in each string is w.

EVALUATE: The tension is the same in all three cases.

5.3. **IDENTIFY:** Both objects are at rest and $a = 0$. Apply Newton's first law to the appropriate object. The maximum tension T_{max} is at the top of the chain and the minimum tension is at the bottom of the chain.

SET UP: Let $+y$ be upward. For the maximum tension take the object to be the chain plus the ball. For the minimum tension take the object to be the ball. For the tension T three-fourths of the way up from the bottom of the chain, take the chain below this point plus the ball to be the object. The free-body diagrams in each of these three cases are sketched in Figures 5.3a, 5.3b and 5.3c. $m_{b+c} = 75.0\text{ kg} + 26.0\text{ kg} = 101.0\text{ kg}$. $m_b = 75.0\text{ kg}$. m is the mass of three-fourths of the chain: $m = \frac{3}{4}(26.0\text{ kg}) = 19.5\text{ kg}$.

EXECUTE: **(a)** From Figure 5.3a, $\sum F_y = 0$ gives $T_{max} - m_{b+c}g = 0$ and $T_{max} = (101.0\text{ kg})(9.80\text{ m/s}^2) = 990\text{ N}$.

From Figure 5.3b, $\sum F_y = 0$ gives $T_{min} - m_b g = 0$ and $T_{min} = (75.0\text{ kg})(9.80\text{ m/s}^2) = 735\text{ N}$.

(b) From Figure 5.3c, $\sum F_y = 0$ gives $T - (m + m_b)g = 0$ and $T = (19.5\text{ kg} + 75.0\text{ kg})(9.80\text{ m/s}^2) = 926\text{ N}$.

EVALUATE: The tension in the chain increases linearly from the bottom to the top of the chain.

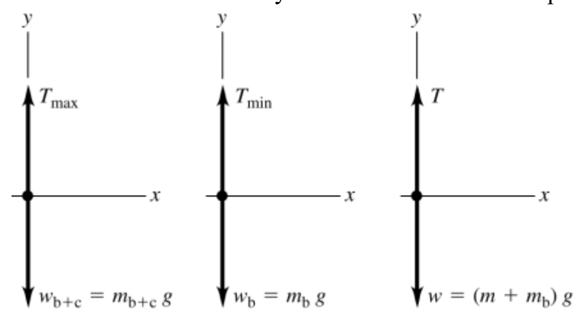
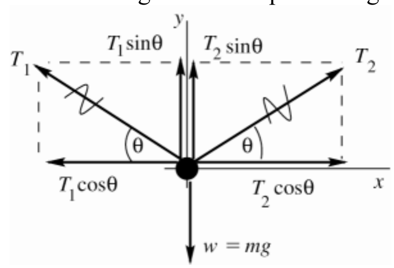


Figure 5.3a–c

5.4. IDENTIFY: Apply Newton's 1st law to the person. Each half of the rope exerts a force on him, directed along the rope and equal to the tension T in the rope.

SET UP: **(a)** The force diagram for the person is given in Figure 5.4



T_1 and T_2 are the tensions in each half of the rope.

Figure 5.4

EXECUTE: $\sum F_x = 0$

$T_2\cos\theta - T_1\cos\theta = 0$

This says that $T_1 = T_2 = T$ (The tension is the same on both sides of the person.)

$\sum F_y = 0$

$T_1\sin\theta + T_2\sin\theta - mg = 0$

But $T_1 = T_2 = T$, so $2T\sin\theta = mg$

$$T = \frac{mg}{2\sin\theta} = \frac{(90.0\text{ kg})(9.80\text{ m/s}^2)}{2\sin 10.0°} = 2540\text{ N}$$

(b) The relation $2T\sin\theta = mg$ still applies but now we are given that $T = 2.50\times10^4$ N (the breaking strength) and are asked to find θ.

$$\sin\theta = \frac{mg}{2T} = \frac{(90.0\text{ kg})(9.80\text{ m/s}^2)}{2(2.50\times10^4\text{ N})} = 0.01764,\quad \theta = 1.01°.$$

EVALUATE: $T = mg/(2\sin\theta)$ says that $T = mg/2$ when $\theta = 90°$ (rope is vertical).

$T \to \infty$ when $\theta \to 0$ since the upward component of the tension becomes a smaller fraction of the tension.

5.5. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the frame.

SET UP: Let w be the weight of the frame. Since the two wires make the same angle with the vertical, the tension is the same in each wire. $T = 0.75w$.

EXECUTE: The vertical component of the force due to the tension in each wire must be half of the weight, and this in turn is the tension multiplied by the cosine of the angle each wire makes with the vertical. $\frac{w}{2} = \frac{3w}{4}\cos\theta$ and $\theta = \arccos\frac{2}{3} = 48°$.

EVALUATE: If $\theta = 0°$, $T = w/2$ and $T \to \infty$ as $\theta \to 90°$. Therefore, there must be an angle where $T = 3w/4$.

5.6. IDENTIFY: Apply Newton's 1st law to the car. The forces are the same as in Example 5.5.

SET UP: The free-body diagram is sketched in Figure 5.6.

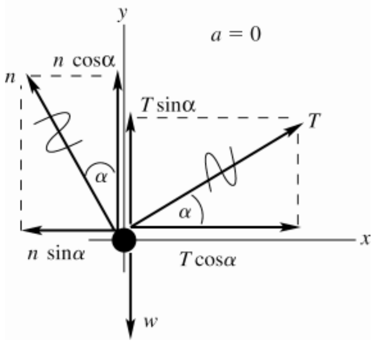


Figure 5.6

EXECUTE:

$\sum F_x = ma_x$

$T\cos\alpha - n\sin\alpha = 0$

$T\cos\alpha = n\sin\alpha$

$\sum F_y = ma_y$

$n\cos\alpha + T\sin\alpha - w = 0$

$n\cos\alpha + T\sin\alpha = w$

The first equation gives $n = T\left(\dfrac{\cos\alpha}{\sin\alpha}\right)$.

Use this in the second equation to eliminate n:

$$\left(T\frac{\cos\alpha}{\sin\alpha}\right)\cos\alpha + T\sin\alpha = w$$

Multiply this equation by $\sin\alpha$:

$T(\cos^2\alpha + \sin^2\alpha) = w\sin\alpha$

$T = w\sin\alpha$ (since $\cos^2\alpha + \sin^2\alpha = 1$).

Then $n = T\left(\dfrac{\cos\alpha}{\sin\alpha}\right) = w\sin\alpha\left(\dfrac{\cos\alpha}{\sin\alpha}\right) = w\cos\alpha.$

EVALUATE: These results are the same as obtained in Example 5.5. The choice of coordinate axes is up to us. Some choices may make the calculation easier, but the results are the same for any choice of axes.

5.7. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the car.

SET UP: Use coordinates with $+x$ parallel to the surface of the street.

EXECUTE: $\sum F_x = 0$ gives $T = w\sin\alpha$. $F = mg\sin\theta = (1390\text{ kg})(9.80\text{ m/s}^2)\sin17.5° = 4.10\times10^3\text{ N}$.

EVALUATE: The force required is less than the weight of the car by the factor $\sin\alpha$.

5.8. IDENTIFY: Apply Newton's 1st law to the wrecking ball. Each cable exerts a force on the ball, directed along the cable.

SET UP: The force diagram for the wrecking ball is sketched in Figure 5.8.

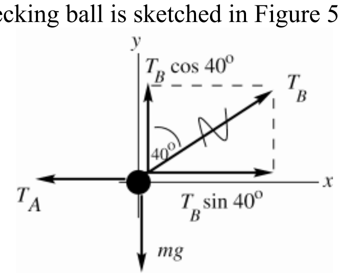


Figure 5.8

EXECUTE:

(a) $\sum F_y = ma_y$

$T_B\cos40° - mg = 0$

$T_B = \dfrac{mg}{\cos40°} = \dfrac{(4090\text{ kg})(9.80\text{ m/s}^2)}{\cos40°} = 5.23\times10^4\text{ N}$

(b) $\sum F_x = ma_x$

$T_B\sin40° - T_A = 0$

$T_A = T_B\sin40° = 3.36\times10^4\text{ N}$

EVALUATE: If the angle $40°$ is replaces by $0°$ (cable B is vertical), then $T_B = mg$ and $T_A = 0$.

5.9. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the object and to the knot where the cords are joined.

SET UP: Let $+y$ be upward and $+x$ be to the right.

EXECUTE: **(a)** $T_C = w$, $T_A \sin 30° + T_B \sin 45° = T_C = w$, and $T_A \cos 30° - T_B \cos 45° = 0$. Since $\sin 45° = \cos 45°$, adding the last two equations gives $T_A(\cos 30° + \sin 30°) = w$, and so $T_A = \dfrac{w}{1.366} = 0.732w$. Then,

$$T_B = T_A \frac{\cos 30°}{\cos 45°} = 0.897w.$$

(b) Similar to part (a), $T_C = w$, $-T_A \cos 60° + T_B \sin 45° = w$, and $T_A \sin 60° - T_B \cos 45° = 0$.

Adding these two equations, $T_A = \dfrac{w}{(\sin 60° - \cos 60°)} = 2.73w$, and $T_B = T_A \dfrac{\sin 60°}{\cos 45°} = 3.35w$.

EVALUATE: In part (a), $T_A + T_B > w$ since only the vertical components of T_A and T_B hold the object against gravity. In part (b), since T_A has a downward component T_B is greater than w.

5.10. **IDENTIFY:** Apply Newton's first law to the car.

SET UP: Use x and y coordinates that are parallel and perpendicular to the ramp.

EXECUTE: **(a)** The free-body diagram for the car is given in Figure 5.10. The vertical weight w and the tension T in the cable have each been replaced by their x and y components.

(b) $\sum F_x = 0$ gives $T\cos 31.0° - w\sin 25.0° = 0$ and $T = w\dfrac{\sin 25.0°}{\cos 31.0°} = (1130 \text{ kg})(9.80 \text{ m/s}^2)\dfrac{\sin 25.0°}{\cos 31.0°} = 5460 \text{ N}$.

(c) $\sum F_y = 0$ gives $n + T\sin 31.0° - w\cos 25.0° = 0$ and

$$n = w\cos 25.0° - T\sin 31.0° = (1130 \text{ kg})(9.80 \text{ m/s}^2)\cos 25.0° - (5460 \text{ N})\sin 31.0° = 7220 \text{ N}$$

EVALUATE: We could also use coordinates that are horizontal and vertical and would obtain the same values of n and T.

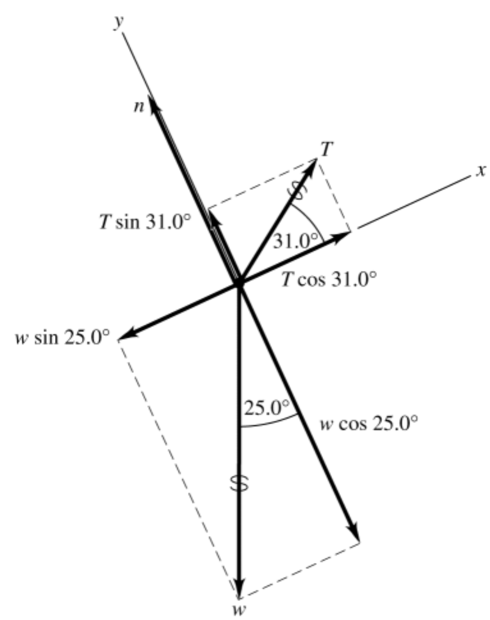


Figure 5.10

5.11. **IDENTIFY:** Since the velocity is constant, apply Newton's first law to the piano. The push applied by the man must oppose the component of gravity down the incline.

SET UP: The free-body diagrams for the two cases are shown in Figures 5.11a and b. $\vec{F}$ is the force applied by the man. Use the coordinates shown in the figure.

EXECUTE: **(a)** $\sum F_x = 0$ gives $F - w\sin 11.0° = 0$ and $F = (180 \text{ kg})(9.80 \text{ m/s}^2)\sin 11.0° = 337 \text{ N}$.

(b) $\sum F_y = 0$ gives $n\cos 11.0° - w = 0$ and $n = \dfrac{w}{\cos 11.0°}$. $\sum F_x = 0$ gives $F - n\sin 11.0° = 0$ and

$$F = \left(\frac{w}{\cos 11.0°}\right)\sin 11.0° = w\tan 11.0° = 343 \text{ N}.$$

EVALUATE: A slightly greater force is required when the man pushes parallel to the floor. If the slope angle of the incline were larger, $\sin\alpha$ and $\tan\alpha$ would differ more and there would be more difference in the force needed in each case.

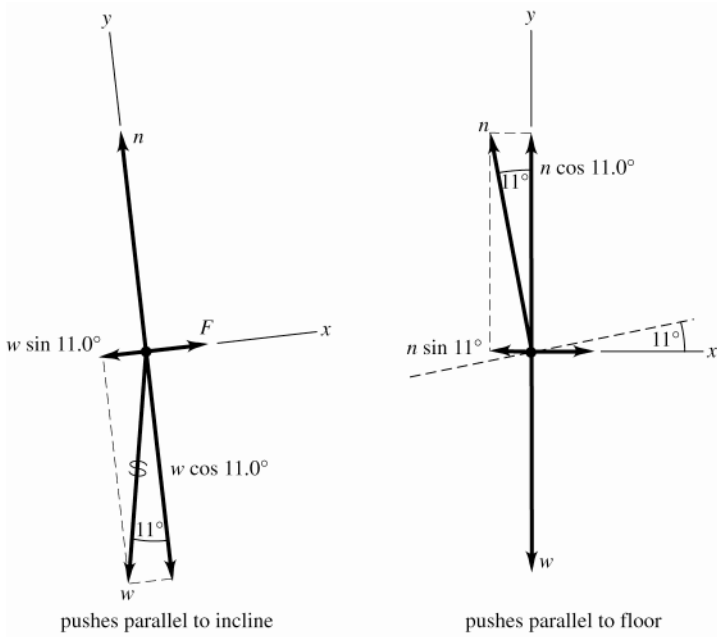


Figure 5.11a, b

5.12. **IDENTIFY:** Apply Newton's 1st law to the hanging weight and to each knot. The tension force at each end of a string is the same.

(a) Let the tensions in the three strings be T, T', and T'', as shown in Figure 5.12a.

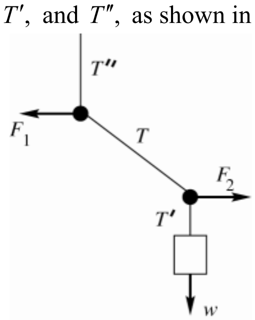


Figure 5.12a

SET UP: The free-body diagram for the block is given in Figure 5.12b.

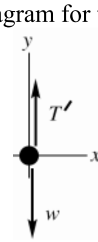


Figure 5.12b

EXECUTE:

$$\sum F_y = 0$$
$$T' - w = 0$$
$$T' = w = 60.0\text{ N}$$

SET UP: The free-body diagram for the lower knot is given in Figure 5.12c.

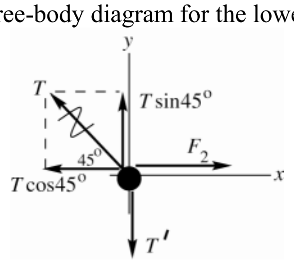


Figure 5.12c

EXECUTE:

$$\sum F_y = 0$$
$$T\sin 45° - T' = 0$$
$$T = \frac{T'}{\sin 45°} = \frac{60.0\text{ N}}{\sin 45°} = 84.9\text{ N}$$

(b) Apply $\sum F_x = 0$ to the force diagram for the lower knot:

$\sum F_x = 0$

$F_2 = T\cos 45° = (84.9\text{ N})\cos 45° = 60.0\text{ N}$

SET UP: The free-body diagram for the upper knot is given in Figure 5.12d.

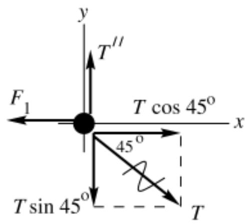


Figure 5.12d

EXECUTE:

$\sum F_x = 0$

$T\cos 45° - F_1 = 0$

$F_1 = (84.9\text{ N})\cos 45°$

$F_1 = 60.0\text{ N}$

Note that $F_1 = F_2$.

EVALUATE: Applying $\sum F_y = 0$ to the upper knot gives $T'' = T\sin 45° = 60.0\text{ N} = w.$ If we treat the whole system as a single object, the force diagram is given in Figure 5.12e.

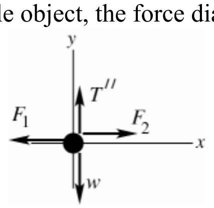


Figure 5.12e

$\sum F_x = 0$ gives $F_2 = F_1,$ which checks

$\sum F_y = 0$ gives $T'' = w,$ which checks

5.13. **IDENTIFY:** Apply Newton's first law to the ball. The force of the wall on the ball and the force of the ball on the wall are related by Newton's third law.

SET UP: The forces on the ball are its weight, the tension in the wire, and the normal force applied by the wall.

To calculate the angle ϕ that the wire makes with the wall, use Figure 5.13a. $\sin\phi = \dfrac{16.0\text{ cm}}{46.0\text{ cm}}$ and $\phi = 20.35°$

EXECUTE: **(a)** The free-body diagram is shown in Figure 5.13b. Use the x and y coordinates shown in the figure.

$\sum F_y = 0$ gives $T\cos\phi - w = 0$ and $T = \dfrac{w}{\cos\phi} = \dfrac{(45.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 20.35°} = 470\text{ N}$

(b) $\sum F_x = 0$ gives $T\sin\phi - n = 0$. $n = (470\text{ N})\sin 20.35° = 163\text{ N}$. By Newton's third law, the force the ball exerts on the wall is 163 N, directed to the right.

EVALUATE: $n = \left(\dfrac{w}{\cos\phi}\right)\sin\phi = w\tan\phi$. As the angle ϕ decreases (by increasing the length of the wire), T decreases and n decreases.

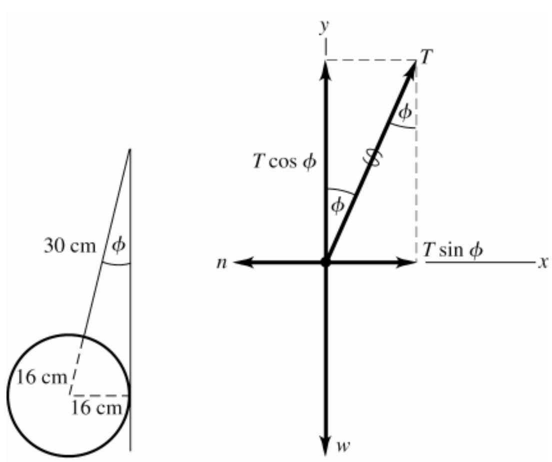


Figure 5.13a, b

5.14. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. $a = 0$.

SET UP: Take $+y$ perpendicular to the incline and $+x$ parallel to the incline.

EXECUTE: The free-body diagrams for each block, A and B, are given in Figure 5.14.

(a) For B, $\sum F_x = ma_x$ gives $T_1 - w\sin\alpha = 0$ and $T_1 = w\sin\alpha$.

(b) For block A, $\sum F_x = ma_x$ gives $T_1 - T_2 - w\sin\alpha = 0$ and $T_2 = 2w\sin\alpha$.

(c) $\sum F_y = ma_y$ for each block gives $n_A = n_B = w\cos\alpha$.

(d) For $\alpha \to 0$, $T_1 = T_2 \to 0$ and $n_A = n_B \to w$. For $\alpha \to 90°$, $T_1 = w$, $T_2 = 2w$ and $n_A = n_B = 0$.

EVALUATE: The two tensions are different but the two normal forces are the same.

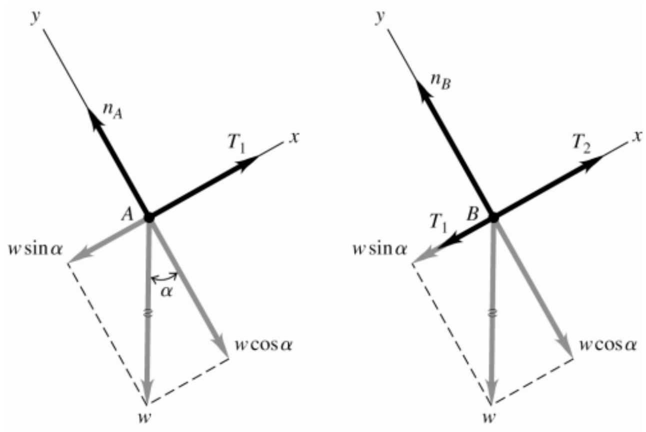


Figure 5.14a, b

5.15. IDENTIFY: Apply Newton's first law to the ball. Treat the ball as a particle.

SET UP: The forces on the ball are gravity, the tension in the wire and the normal force exerted by the surface. The normal force is perpendicular to the surface of the ramp. Use x and y axes that are horizontal and vertical.

EXECUTE: **(a)** The free-body diagram for the ball is given in Figure 5.15. The normal force has been replaced by its x and y components.

(b) $\sum F_y = 0$ gives $n\cos 35.0° - w = 0$ and $n = \dfrac{mg}{\cos 35.0°} = 1.22mg$.

(c) $\sum F_x = 0$ gives $T - n\sin 35.0° = 0$ and $T = (1.22mg)\sin 35.0° = 0.700mg$.

EVALUATE: Note that the normal force is greater than the weight, and increases without limit as the angle of the ramp increases towards $90°$. The tension in the wire is $w\tan\phi$, where ϕ is the angle of the ramp and T also increases without limit as $\phi \to 90°$.

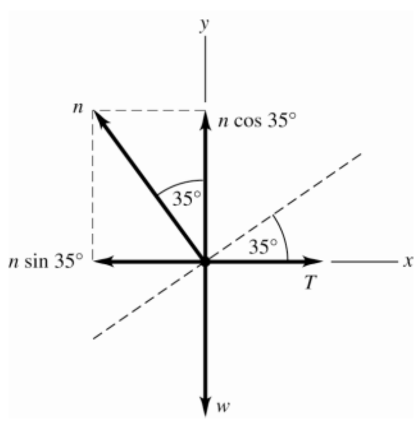


Figure 5.15

5.16. IDENTIFY: Apply Newton's second law to the rocket plus its contents and to the power supply. Both the rocket and the power supply have the same acceleration.

SET UP: The free-body diagrams for the rocket and for the power supply are given in Figures 5.16a and b. Since the highest altitude of the rocket is 120 m, it is near to the surface of the earth and there is a downward gravity force on each object. Let $+y$ be upward, since that is the direction of the acceleration. The power supply has mass $m_{ps} = (15.5\text{ N})/(9.80\text{ m/s}^2) = 1.58\text{ kg}$

EXECUTE: **(a)** $\sum F_y = ma_y$ applied to the rocket gives $F - m_r g = m_r a$.

$$a = \frac{F - m_r g}{m_r} = \frac{1720\text{ N} - (125\text{ kg})(9.80\text{ m/s}^2)}{125\text{ kg}} = 3.96\text{ m/s}^2.$$

(b) $\sum F_y = ma_y$ applied to the power supply gives $n - m_{ps} g = m_{ps} a$.

$$n = m_{ps}(g + a) = (1.58\text{ kg})(9.80\text{ m/s}^2 + 3.96\text{ m/s}^2) = 21.7\text{ N}.$$

EVALUATE: The acceleration is constant while the thrust is constant and the normal force is constant while the acceleration is constant. The altitude of 120 m is not used in the calculation.

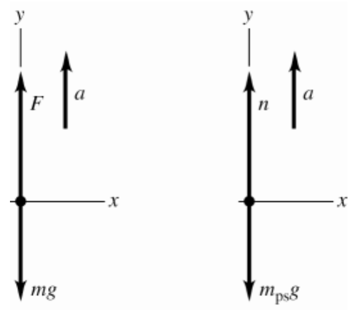


Figure 5.16a, b

5.17. **IDENTIFY:** Use the kinematic information to find the acceleration of the capsule and the stopping time. Use Newton's second law to find the force F that the ground exerted on the capsule during the crash.

SET UP: Let $+y$ be upward. $311\text{ km/h} = 86.4\text{ m/s}$. The free-body diagram for the capsule is given in Figure 15.17.

EXECUTE: $y - y_0 = -0.810\text{ m}$, $v_{0y} = -86.4\text{ m/s}$, $v_y = 0$. $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{0 - (-86.4\text{ m/s})^2}{2(-0.810)\text{ m}} = 4610\text{ m/s}^2 = 470g.$$

(b) $\sum F_y = ma_y$ applied to the capsule gives $F - mg = ma$ and

$$F = m(g + a) = (210\text{ kg})(9.80\text{ m/s}^2 + 4610\text{ m/s}^2) = 9.70 \times 10^5\text{ N} = 471w.$$

(c) $y - y_0 = \left(\frac{v_{0y} + v_y}{2}\right)t$ gives $t = \frac{2(y - y_0)}{v_{0y} + v_y} = \frac{2(-0.810\text{ m})}{-86.4\text{ m/s}^2 + 0} = 0.0187\text{ s}$

EVALUATE: The upward force exerted by the ground is much larger than the weight of the capsule and stops the capsule in a short amount of time. After the capsule has come to rest, the ground still exerts a force mg on the capsule, but the large 9.00×10^5 N force is exerted only for 0.0187 s.

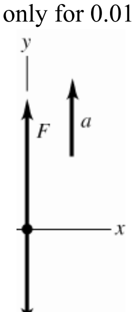


Figure 5.17

5.18. **IDENTIFY:** Apply Newton's second law to the three sleds taken together as a composite object and to each individual sled. All three sleds have the same horizontal acceleration a.

SET UP: The free-body diagram for the three sleds taken as a composite object is given in Figure 5.18a and for each individual sled in Figure 5.18b-d. Let $+x$ be to the right, in the direction of the acceleration. $m_{tot} = 60.0\text{ kg}$.

EXECUTE: **(a)** $\sum F_x = ma_x$ for the three sleds as a composite object gives $P = m_{tot} a$ and

$$a = \frac{P}{m_{tot}} = \frac{125\text{ N}}{60.0\text{ kg}} = 2.08\text{ m/s}^2.$$

(b) $\sum F_x = ma_x$ applied to the 10.0 kg sled gives $P - T_A = m_{10}a$ and $T_A = P - m_{10}a = 125\text{ N} - (10.0\text{ kg})(2.08\text{ m/s}^2) = 104\text{ N}$. $\sum F_x = ma_x$ applied to the 30.0 kg sled gives $T_B = m_{30}a = (30.0\text{ kg})(2.08\text{ m/s}^2) = 62.4\text{ N}$.

EVALUATE: If we apply $\sum F_x = ma_x$ to the 20.0 kg sled and calculate a from T_A and T_B found in part (b), we get $T_A - T_B = m_{20}a$. $a = \dfrac{T_A - T_B}{m_{20}} = \dfrac{104\text{ N} - 62.4\text{ N}}{20.0\text{ kg}} = 2.08\text{ m/s}^2$, which agrees with the value we calculated in part (a).

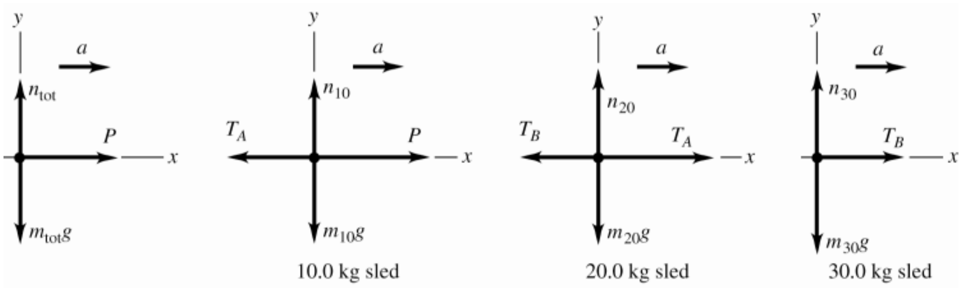


Figure 5.18a–d

5.19. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the load of bricks and to the counterweight. The tension is the same at each end of the rope. The rope pulls up with the same force (T) on the bricks and on the counterweight. The counterweight accelerates downward and the bricks accelerate upward; these accelerations have the same magnitude.

(a) SET UP: The free-body diagrams for the bricks and counterweight are given in Figure 5.19.

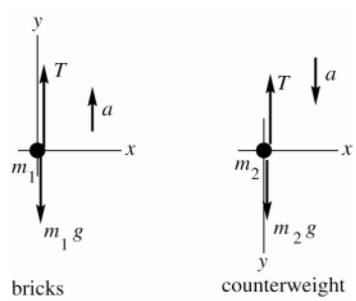


Figure 5.19

(b) EXECUTE: Apply $\sum F_y = ma_y$ to each object. The acceleration magnitude is the same for the two objects. For the bricks take $+y$ to be upward since $\vec{a}$ for the bricks is upward. For the counterweight take $+y$ to be downward since $\vec{a}$ is downward.

bricks: $\sum F_y = ma_y$

$T - m_1 g = m_1 a$

counterweight: $\sum F_y = ma_y$

$m_2 g - T = m_2 a$

Add these two equations to eliminate T:

$$(m_2 - m_1)g = (m_1 + m_2)a$$

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g = \left(\frac{28.0\text{ kg} - 15.0\text{ kg}}{15.0\text{ kg} + 28.0\text{ kg}}\right)(9.80\text{ m/s}^2) = 2.96\text{ m/s}^2$$

(c) $T - m_1 g = m_1 a$ gives $T = m_1(a + g) = (15.0\text{ kg})(2.96\text{ m/s}^2 + 9.80\text{ m/s}^2) = 191\text{ N}$

As a check, calculate T using the other equation.

$m_2 g - T = m_2 a$ gives $T = m_2(g - a) = 28.0\text{ kg}(9.80\text{ m/s}^2 - 2.96\text{ m/s}^2) = 191\text{ N}$, which checks.

EVALUATE: The tension is 1.30 times the weight of the bricks; this causes the bricks to accelerate upward. The tension is 0.696 times the weight of the counterweight; this causes the counterweight to accelerate downward. If $m_1 = m_2$, $a = 0$ and $T = m_1 g = m_2 g$. In this special case the objects don't move. If $m_1 = 0$, $a = g$ and $T = 0$; in this special case the counterweight is in free-fall. Our general result is correct in these two special cases.

5.20. **IDENTIFY:** In part (a) use the kinematic information and the constant acceleration equations to calculate the acceleration of the ice. Then apply $\sum \vec{F} = m\vec{a}$. In part (b) use $\sum \vec{F} = m\vec{a}$ to find the acceleration and use this in the constant acceleration equations to find the final speed.

SET UP: Figures 5.20a and b give the free-body diagrams for the ice both with and without friction. Let $+x$ be directed down the ramp, so $+y$ is perpendicular to the ramp surface. Let ϕ be the angle between the ramp and the horizontal. The gravity force has been replaced by its x and y components.

EXECUTE: **(a)** $x - x_0 = 1.50\text{ m}$, $v_{0x} = 0$, $v_x = 2.50\text{ m/s}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$a_x = \dfrac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \dfrac{(2.50\text{ m/s})^2 - 0}{2(1.50\text{ m})} = 2.08\text{ m/s}^2$. $\sum F_x = ma_x$ gives $mg\sin\phi = ma$ and $\sin\phi = \dfrac{a}{g} = \dfrac{2.08\text{ m/s}^2}{9.80\text{ m/s}^2}$.

$\phi = 12.3°$.

(b) $\sum F_x = ma_x$ gives $mg\sin\phi - f = ma$ and

$a = \dfrac{mg\sin\phi - f}{m} = \dfrac{(8.00\text{ kg})(9.80\text{ m/s}^2)\sin 12.3° - 10.0\text{ N}}{8.00\text{ kg}} = 0.838\text{ m/s}^2$.

Then $x - x_0 = 1.50\text{ m}$, $v_{0x} = 0$, $a_x = 0.838\text{ m/s}^2$ and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives

$$v_x = \sqrt{2a_x(x - x_0)} = \sqrt{2(0.838\text{ m/s}^2)(1.50\text{ m})} = 1.59\text{ m/s}$$

EVALUATE: With friction present the speed at the bottom of the ramp is less.

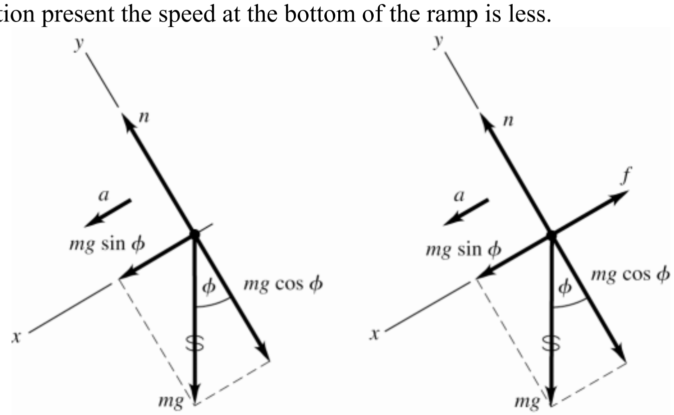


Figure 5.20a, b

5.21. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. Each block has the same magnitude of acceleration a.

SET UP: Assume the pulley is to the right of the 4.00 kg block. There is no friction force on the 4.00 kg block, the only force on it is the tension in the rope. The 4.00 kg block therefore accelerates to the right and the suspended block accelerates downward. Let $+x$ be to the right for the 4.00 kg block, so for it $a_x = a$, and let $+y$ be downward for the suspended block, so for it $a_y = a$.

EXECUTE: **(a)** The free-body diagrams for each block are given in Figures 5.21a and b.

(b) $\sum F_x = ma_x$ applied to the 4.00 kg block gives $T = (4.00\text{ kg})a$ and $a = \dfrac{T}{4.00\text{ kg}} = \dfrac{10.0\text{ N}}{4.00\text{ kg}} = 2.50\text{ m/s}^2$.

(c) $\sum F_y = ma_y$ applied to the suspended block gives $mg - T = ma$ and

$m = \dfrac{T}{g - a} = \dfrac{10.0\text{ N}}{9.80\text{ m/s}^2 - 2.50\text{ m/s}^2} = 1.37\text{ kg}$.

(d) The weight of the hanging block is $mg = (1.37\text{ kg})(9.80\text{ m/s}^2) = 13.4\text{ N}$. This is greater than the tension in the rope; $T = 0.75mg$.

EVALUATE: Since the hanging block accelerates downward, the net force on this block must be downward and the weight of the hanging block must be greater than the tension in the rope. Note that the blocks accelerate no matter how small m is. It is not necessary to have $m > 4.00\text{ kg}$, and in fact in this problem m is less than 4.00 kg.

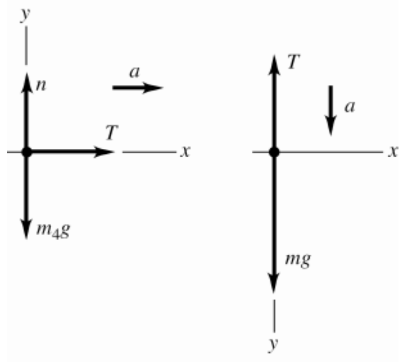


Figure 5.21a, b

5.22. **IDENTIFY:** (a) Consider both gliders together as a single object, apply $\sum \vec{F} = m\vec{a}$, and solve for a. Use a in a constant acceleration equation to find the required runway length.

(b) Apply $\sum \vec{F} = m\vec{a}$ to the second glider and solve for the tension T_g in the towrope that connects the two gliders.

SET UP: In part (a), set the tension T_t in the towrope between the plane and the first glider equal to its maximum value, $T_t = 12{,}000\text{ N}$.

EXECUTE: **(a)** The free-body diagram for both gliders as a single object of mass $2m = 1400\text{ kg}$ is given in Figure 5.22a. $\sum F_x = ma_x$ gives $T_t - 2f = (2m)a$ and $a = \dfrac{T_t - 2f}{2m} = \dfrac{12{,}000\text{ N} - 5000\text{ N}}{1400\text{ kg}} = 5.00\text{ m/s}^2$. Then

$a_x = 5.00\text{ m/s}^2$, $v_{0x} = 0$ and $v_x = 40\text{ m/s}$ in $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $(x - x_0) = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = 160\text{ m}$.

(b) The free-body diagram for the second glider is given in Figure 5.22b.

$\sum F_x = ma_x$ gives $T_g - f = ma$ and $T = f + ma = 2500\text{ N} + (700\text{ kg})(5.00\text{ m/s}^2) = 6000\text{ N}$.

EVALUATE: We can verify that $\sum F_x = ma_x$ is also satisfied for the first glider.

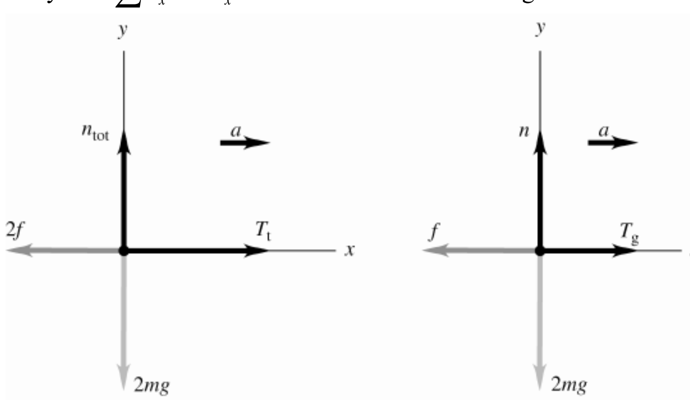


Figure 5.22a, b

5.23. **IDENTIFY:** The maximum tension in the chain is at the top of the chain. Apply $\sum \vec{F} = m\vec{a}$ to the composite object of chain and boulder. Use the constant acceleration kinematic equations to relate the acceleration to the time.

SET UP: Let $+y$ be upward. The free-body diagram for the composite object is given in Figure 5.23.

$T = 2.50 w_{\text{chain}}$. $m_{\text{tot}} = m_{\text{chain}} + m_{\text{boulder}} = 1325\text{ kg}$.

EXECUTE: **(a)** $\sum F_y = ma_y$ gives $T - m_{\text{tot}} g = m_{\text{tot}} a$. $a = \dfrac{T - m_{\text{tot}} g}{m_{\text{tot}}} = \dfrac{2.50 m_{\text{chain}} g - m_{\text{tot}} g}{m_{\text{tot}}} = \left(\dfrac{2.50 m_{\text{chain}}}{m_{\text{tot}}} - 1\right) g$

$a = \left(\dfrac{2.50[575\text{ kg}]}{1325\text{ kg}} - 1\right)(9.80\text{ m/s}^2) = 0.832\text{ m/s}^2$.

(b) Assume the acceleration has its maximum value: $a_y = 0.832\text{ m/s}^2$, $y - y_0 = 125\text{ m}$ and $v_{0y} = 0$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2(y-y_0)}{a_y}} = \sqrt{\dfrac{2(125\text{ m})}{0.832\text{ m/s}^2}} = 17.3\text{ s}$

EVALUATE: The tension in the chain is $T = 1.41\times10^4\text{ N}$ and the total weight is $1.30\times10^4\text{ N}$. The upward force exceeds the downward force and the acceleration is upward.

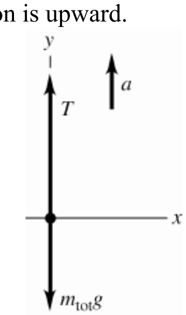


Figure 5.23

5.24. IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to the composite object of elevator plus student ($m_{tot} = 850\text{ kg}$) and also to the student ($w = 550\text{ N}$). The elevator and the student have the same acceleration.

SET UP: Let $+y$ be upward. The free-body diagrams for the composite object and for the student are given in Figure 5.24a and b. T is the tension in the cable and n is the scale reading, the normal force the scale exerts on the student. The mass of the student is $m = w/g = 56.1\text{ kg}$.

EXECUTE: (a) $\sum F_y = ma_y$ applied to the student gives $n - mg = ma_y$.

$a_y = \dfrac{n-mg}{m} = \dfrac{450\text{ N} - 550\text{ N}}{56.1\text{ kg}} = -1.78\text{ m/s}^2$. The elevator has a downward acceleration of 1.78 m/s^2.

(b) $a_y = \dfrac{670\text{ N} - 550\text{ N}}{56.1\text{ kg}} = 2.14\text{ m/s}^2$.

(c) $n = 0$ means $a_y = -g$. The student should worry; the elevator is in free-fall.

(d) $\sum F_y = ma_y$ applied to the composite object gives $T - m_{tot}g = m_{tot}a$. $T = m_{tot}(a_y + g)$. In part (a), $T = (850\text{ kg})(-1.78\text{ m/s}^2 + 9.80\text{ m/s}^2) = 6820\text{ N}$. In part (c), $a_y = -g$ and $T = 0$.

EVALUATE: In part (b), $T = (850\text{ kg})(2.14\text{ m/s}^2 + 9.80\text{ m/s}^2) = 10{,}150\text{ N}$. The weight of the composite object is 8330 N. When the acceleration is upward the tension is greater than the weight and when the acceleration is downward the tension is less than the weight.

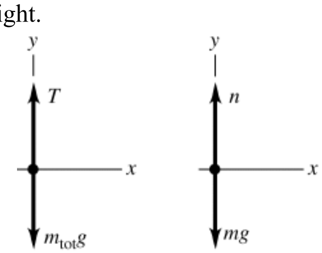


Figure 5.24a, b

5.25. IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to the puck. Use the information about the motion to calculate the acceleration. The table must slope downward to the right.

SET UP: Let α be the angle between the table surface and the horizontal. Let the $+x$-axis be to the right and parallel to the surface of the table.

EXECUTE: $\sum F_x = ma_x$ gives $mg\sin\alpha = ma_x$. The time of travel for the puck is L/v_0, where $L = 1.75\text{ m}$ and $v_0 = 3.80\text{ m/s}$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $a_x = \dfrac{2x}{t^2} = \dfrac{2xv_0^2}{L^2}$, where $x = 0.0250\text{ m}$. $\sin\alpha = \dfrac{a_x}{g} = \dfrac{2xv_0^2}{gL^2}$.

$\alpha = \arcsin\left(\dfrac{2(2.50\times10^{-2}\text{m})(3.80\text{ m/s})^2}{(9.80\text{ m/s}^2)(1.75\text{ m})^2}\right) = 1.38°$.

EVALUATE: The table is level in the direction along its length, since the velocity in that direction is constant. The angle of slope to the right is small, so the acceleration and deflection in that direction are small.

5.26. **IDENTIFY:** Acceleration and velocity are related by $a_y = \dfrac{dv_y}{dt}$. Apply $\sum \vec{F} = m\vec{a}$ to the rocket.

SET UP: Let $+y$ be upward. The free-body diagram for the rocket is sketched in Figure 5.26. $\vec{F}$ is the thrust force.

EXECUTE: **(a)** $v_y = At + Bt^2$. $a_y = A + 2Bt$. At $t = 0$, $a_y = 1.50\ \text{m/s}^2$ so $A = 1.50\ \text{m/s}^2$. Then $v_y = 2.00\ \text{m/s}$ at $t = 1.00\ \text{s}$ gives $2.00\ \text{m/s} = (1.50\ \text{m/s}^2)(1.00\ \text{s}) + B(1.00\ \text{s})^2$ and $B = 0.50\ \text{m/s}^3$.

(b) At $t = 4.00\ \text{s}$, $a_y = 1.50\ \text{m/s}^2 + 2(0.50\ \text{m/s}^3)(4.00\ \text{s}) = 5.50\ \text{m/s}^2$.

(c) $\sum F_y = ma_y$ applied to the rocket gives $T - mg = ma$ and

$T = m(a + g) = (2540\ \text{kg})(9.80\ \text{m/s}^2 + 5.50\ \text{m/s}^2) = 3.89 \times 10^4\ \text{N}$. $T = 1.56w$.

(d) When $a = 1.50\ \text{m/s}^2$, $T = (2540\ \text{kg})(9.80\ \text{m/s}^2 + 1.50\ \text{m/s}^2) = 2.87 \times 10^4\ \text{N}$

EVALUATE: During the time interval when $v(t) = At + Bt^2$ applies the magnitude of the acceleration is increasing, and the thrust is increasing.

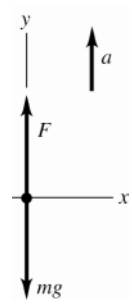


Figure 5.26

5.27. **IDENTIFY:** Consider the forces in each case. There is the force of gravity and the forces from objects that touch the object in question.

SET UP: A surface exerts a normal force perpendicular to the surface, and a friction force, parallel to the surface.

EXECUTE: The free-body diagrams are sketched in Figure 5.27a-c.

EVALUATE: Friction opposes relative motion between the two surfaces. When one surface is stationary the friction force on the other surface is directed opposite to its motion.

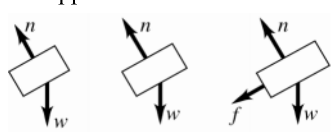


Figure 5.27a–c

5.28. **IDENTIFY:** $f_s \le \mu_s n$ and $f_k = \mu_k n$. The normal force n is determined by applying $\sum \vec{F} = m\vec{a}$ to the block. Normally, $\mu_k \le \mu_s$. f_s is only as large as it needs to be to prevent relative motion between the two surfaces.

SET UP: Since the table is horizontal, with only the block present $n = 135\ \text{N}$. With the brick on the block, $n = 270\ \text{N}$.

EXECUTE: **(a)** The friction is static for $P = 0$ to $P = 75.0\ \text{N}$. The friction is kinetic for $P > 75.0\ \text{N}$.

(b) The maximum value of f_s is $\mu_s n$. From the graph the maximum f_s is $f_s = 75.0\ \text{N}$, so

$\mu_s = \dfrac{\max f_s}{n} = \dfrac{75.0\ \text{N}}{135\ \text{N}} = 0.556$. $f_k = \mu_k n$. From the graph, $f_k = 50.0\ \text{N}$ and $\mu_k = \dfrac{f_k}{n} = \dfrac{50.0\ \text{N}}{135\ \text{N}} = 0.370$.

(c) When the block is moving the friction is kinetic and has the constant value $f_k = \mu_k n$, independent of *P*. This is why the graph is horizontal for $P > 75.0\ \text{N}$. When the block is at rest, $f_s = P$ since this prevents relative motion. This is why the graph for $P < 75.0\ \text{N}$ has slope $+1$.

(d) max f_s and f_k would double. The values of *f* on the vertical axis would double but the shape of the graph would be unchanged.

EVALUATE: The coefficients of friction are independent of the normal force.

5.29. **(a) IDENTIFY:** Constant speed implies $a = 0$. Apply Newton's 1st law to the box. The friction force is directed opposite to the motion of the box.

SET UP: Consider the free-body diagram for the box, given in Figure 5.29a. Let $\vec{F}$ be the horizontal force applied by the worker. The friction is kinetic friction since the box is sliding along the surface.

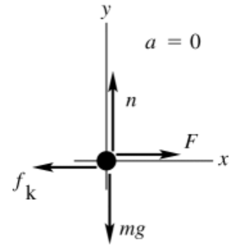


Figure 5.29a

EXECUTE:

$\sum F_y = ma_y$

$n - mg = 0$

$n = mg$

So $f_k = \mu_k n = \mu_k mg$

$\sum F_x = ma_x$

$F - f_k = 0$

$F = f_k = \mu_k mg = (0.20)(11.2\text{ kg})(9.80\text{ m/s}^2) = 22\text{ N}$

(b) IDENTIFY: Now the only horizontal force on the box is the kinetic friction force. Apply Newton's 2nd law to the box to calculate its acceleration. Once we have the acceleration, we can find the distance using a constant acceleration equation. The friction force is $f_k = \mu_k mg$, just as in part (a).

SET UP: The free-body diagram is sketched in Figure 5.29b.

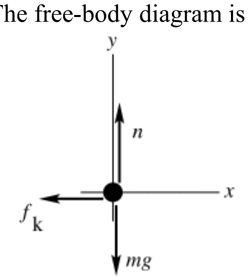


Figure 5.29b

EXECUTE:

$\sum F_x = ma_x$

$-f_k = ma_x$

$-\mu_k mg = ma_x$

$a_x = -\mu_k g = -(0.20)(9.80\text{ m/s}^2) = -1.96\text{ m/s}^2$

Use the constant acceleration equations to find the distance the box travels:

$v_x = 0,\ v_{0x} = 3.50\text{ m/s},\ a_x = -1.96\text{ m/s}^2,\ x - x_0 = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

$$x - x_0 = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - (3.50\text{ m/s})^2}{2(-1.96\text{ m/s}^2)} = 3.1\text{ m}$$

EVALUATE: The normal force is the component of force exerted by a surface perpendicular to the surface. Its magnitude is determined by $\sum \vec{F} = m\vec{a}$. In this case *n* and *mg* are the only vertical forces and $a_y = 0$, so $n = mg$. Also note that f_k and *n* are proportional in magnitude but perpendicular in direction.

5.30. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box.

SET UP: Since the only vertical forces are *n* and *w*, the normal force on the box equals its weight. Static friction is as large as it needs to be to prevent relative motion between the box and the surface, up to its maximum possible value of $f_s^{max} = \mu_s n$. If the box is sliding then the friction force is $f_k = \mu_k n$.

EXECUTE: **(a)** If there is no applied force, no friction force is needed to keep the box at rest.

(b) $f_s^{max} = \mu_s n = (0.40)(40.0\text{ N}) = 16.0\text{ N}$. If a horizontal force of 6.0 N is applied to the box, then $f_s = 6.0\text{ N}$ in the opposite direction.

(c) The monkey must apply a force equal to f_s^{max}, 16.0 N.

(d) Once the box has started moving, a force equal to $f_k = \mu_k n = 8.0\text{ N}$ is required to keep it moving at constant velocity.

EVALUATE: $\mu_k < \mu_s$ and less force must be applied to the box to maintain its motion than to start it moving.

5.31. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the crate. $f_s \le \mu_s n$ and $f_k = \mu_k n$.

SET UP: Let $+y$ be upward and let $+x$ be in the direction of the push. Since the floor is horizontal and the push is horizontal, the normal force equals the weight of the crate: $n = mg = 441$ N. The force it takes to start the crate moving equals max f_s and the force required to keep it moving equals f_k

EXECUTE: max $f_s = 313$ N, so $\mu_s = \dfrac{313\text{ N}}{441\text{ N}} = 0.710$. $f_k = 208$ N, so $\mu_k = \dfrac{208\text{ N}}{441\text{ N}} = 0.472$.

(b) The friction is kinetic. $\sum F_x = ma_x$ gives $F - f_k = ma$ and $F = f_k + ma = 208 + (45.0\text{ kg})(1.10\text{ m/s}^2) = 258$ N.

(c) (i) The normal force now is $mg = 72.9$ N. To cause it to move, $F = \max f_s = \mu_s n = (0.710)(72.9\text{ N}) = 51.8$ N.

(ii) $F = f_k + ma$ and $a = \dfrac{F - f_k}{m} = \dfrac{258\text{ N} - (0.472)(72.9\text{ N})}{45.0\text{ kg}} = 4.97\text{ m/s}^2$

EVALUATE: The kinetic friction force is independent of the speed of the object. On the moon, the mass of the crate is the same as on earth, but the weight and normal force are less.

5.32. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box and calculate the normal and friction forces. The coefficient of kinetic friction is the ratio $\dfrac{f_k}{n}$.

SET UP: Let $+x$ be in the direction of motion. $a_x = -0.90\text{ m/s}^2$. The box has mass 8.67 kg.

EXECUTE: The normal force has magnitude $85\text{ N} + 25\text{ N} = 110$ N. The friction force, from $F_H - f_k = ma$ is

$f_k = F_H - ma = 20\text{ N} - (8.67\text{ kg})(-0.90\text{ m/s}^2) = 28$ N. $\mu_k = \dfrac{28\text{ N}}{110\text{ N}} = 0.25.$

EVALUATE: The normal force is greater than the weight of the box, because of the downward component of the push force.

5.33. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the composite object consisting of the two boxes and to the top box. The friction the ramp exerts on the lower box is kinetic friction. The upper box doesn't slip relative to the lower box, so the friction between the two boxes is static. Since the speed is constant the acceleration is zero.

SET UP: Let $+x$ be up the incline. The free-body diagrams for the composite object and for the upper box are given in Figures 5.33a and b. The slope angle ϕ of the ramp is given by $\tan\phi = \dfrac{2.50\text{ m}}{4.75\text{ m}}$, so $\phi = 27.76°$. Since the boxes move down the ramp, the kinetic friction force exerted on the lower box by the ramp is directed up the incline. To prevent slipping relative to the lower box the static friction force on the upper box is directed up the incline. $m_{tot} = 32.0\text{ kg} + 48.0\text{ kg} = 80.0$ kg.

EXECUTE: **(a)** $\sum F_y = ma_y$ applied to the composite object gives $n_{tot} = m_{tot} g\cos\phi$ and $f_k = \mu_k m_{tot} g\cos\phi$.

$\sum F_x = ma_x$ gives $f_k + T - m_{tot} g\sin\phi = 0$ and

$T = (\sin\phi - \mu_k\cos\phi)m_{tot}g = (\sin 27.76° - [0.444]\cos 27.76°)(80.0\text{ kg})(9.80\text{ m/s}^2) = 57.1$ N.

The person must apply a force of 57.1 N, directed up the ramp.

(b) $\sum F_x = ma_x$ applied to the upper box gives $f_s = mg\sin\phi = (32.0\text{ kg})(9.80\text{ m/s}^2)\sin 27.76° = 146$ N, directed up the ramp.

EVALUATE: For each object the net force is zero.

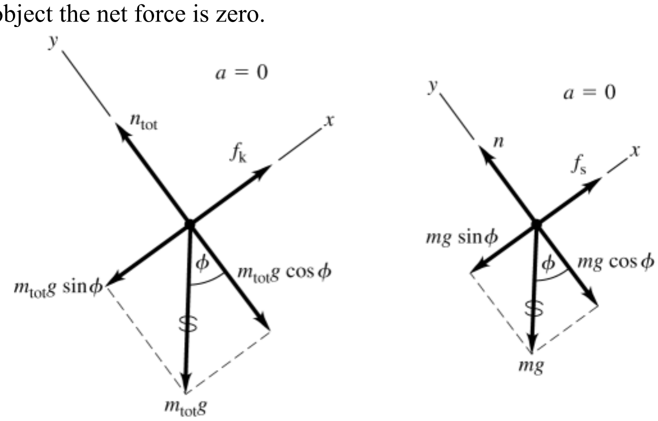


Figure 5.33a, b

5.34. **IDENTIFY:** Use $\sum \vec{F} = m\vec{a}$ to find the acceleration that can be given to the car by the kinetic friction force. Then use a constant acceleration equation.

SET UP: Take $+x$ in the direction the car is moving.

EXECUTE: **(a)** The free-body diagram for the car is shown in Figure 5.34. $\sum F_y = ma_y$ gives $n = mg$. $\sum F_x = ma_x$ gives $-\mu_k n = ma_x$. $-\mu_k mg = ma_x$ and $a_x = -\mu_k g$. Then $v_x = 0$ and $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $(x - x_0) = -\dfrac{v_{0x}^2}{2a_x} = +\dfrac{v_{0x}^2}{2\mu_k g} = \dfrac{(29.1\text{ m/s})^2}{2(0.80)(9.80\text{ m/s}^2)} = 54.0\text{ m}$.

(b) $v_{0x} = \sqrt{2\mu_k g(x - x_0)} = \sqrt{2(0.25)(9.80\text{ m/s}^2)(54.0\text{ m})} = 16.3\text{ m/s}$

EVALUATE: For constant stopping distance $\dfrac{v_{0x}^2}{\mu_k}$ is constant and v_{0x} is proportional to $\sqrt{\mu_k}$. The answer to part (b) can be calculated as $(29.1\text{ m/s})\sqrt{0.25/0.80} = 16.3\text{ m/s}$.

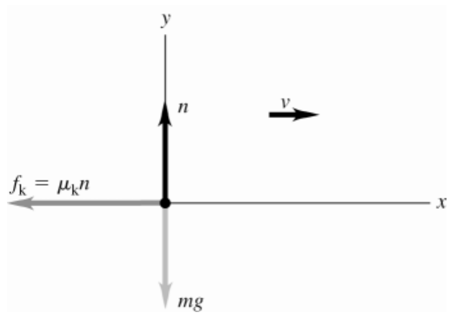


Figure 5.34

5.35. **IDENTIFY:** For a given initial speed, the distance traveled is inversely proportional to the coefficient of kinetic friction.

SET UP: From Table 5.1 the coefficient of kinetic friction is 0.04 for Teflon on steel and 0.44 for brass on steel.

EXECUTE: The ratio of the distances is $\dfrac{0.44}{0.04} = 11$.

EVALUATE: The smaller the coefficient of kinetic friction the smaller the retarding force of friction, and the greater the stopping distance.

5.36. **IDENTIFY:** Constant speed means zero acceleration for each block. If the block is moving the friction force the tabletop exerts on it is kinetic friction. Apply $\sum \vec{F} = m\vec{a}$ to each block.

SET UP: The free-body diagrams and choice of coordinates for each block are given by Figure 5.36. $m_A = 4.59\text{ kg}$ and $m_B = 2.55\text{ kg}$.

EXECUTE: **(a)** $\sum F_y = ma_y$ with $a_y = 0$ applied to block *B* gives $m_B g - T = 0$ and $T = 25.0\text{ N}$. $\sum F_x = ma_x$ with $a_x = 0$ applied to block *A* gives $T - f_k = 0$ and $f_k = 25.0\text{ N}$. $n_A = m_A g = 45.0\text{ N}$ and $\mu_k = \dfrac{f_k}{n_A} = \dfrac{25.0\text{ N}}{45.0\text{ N}} = 0.556$.

(b) Now let *A* be block *A* plus the cat, so $m_A = 9.18\text{ kg}$. $n_A = 90.0\text{ N}$ and $f_k = \mu_k n = (0.556)(90.0\text{ N}) = 50.0\text{ N}$. $\sum F_x = ma_x$ for *A* gives $T - f_k = m_A a_x$. $\sum F_y = ma_y$ for block *B* gives $m_B g - T = m_B a_y$. a_x for *A* equals a_y for *B*, so adding the two equations gives $m_B g - f_k = (m_A + m_B)a_y$ and $a_y = \dfrac{m_B g - f_k}{m_A + m_B} = \dfrac{25.0\text{ N} - 50.0\text{ N}}{9.18\text{ kg} + 2.55\text{ kg}} = -2.13\text{ m/s}^2$.

The acceleration is upward and block *B* slows down.

EVALUATE: The equation $m_B g - f_k = (m_A + m_B)a_y$ has a simple interpretation. If both blocks are considered together then there are two external forces: $m_B g$ that acts to move the system one way and f_k that acts oppositely. The net force of $m_B g - f_k$ must accelerate a total mass of $m_A + m_B$.

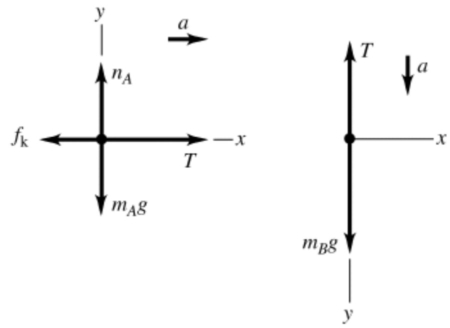


Figure 5.36

5.37. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each crate. The rope exerts force T to the right on crate A and force T to the left on crate B. The target variables are the forces T and F. Constant v implies $a = 0$.

SET UP: The free-body diagram for A is sketched in Figure 5.37a

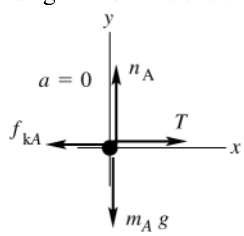


Figure 5.37a

EXECUTE:

$$\sum F_y = ma_y$$
$$n_A - m_A g = 0$$
$$n_A = m_A g$$
$$f_{kA} = \mu_k n_A = \mu_k m_A g$$

$$\sum F_x = ma_x$$
$$T - f_{kA} = 0$$
$$T = \mu_k m_A g$$

SET UP: The free-body diagram for B is sketched in Figure 5.37b.

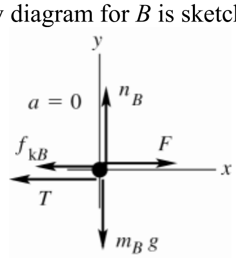


Figure 5.37b

EXECUTE:

$$\sum F_y = ma_y$$
$$n_B - m_B g = 0$$
$$n_B = m_B g$$
$$f_{kB} = \mu_k n_B = \mu_k m_B g$$

$$\sum F_x = ma_x$$
$$F - T - f_{kB} = 0$$
$$F = T + \mu_k m_B g$$

Use the first equation to replace T in the second:

$$F = \mu_k m_A g + \mu_k m_B g.$$

(a) $F = \mu_k (m_A + m_B)g$

(b) $T = \mu_k m_A g$

EVALUATE: We can also consider both crates together as a single object of mass $(m_A + m_B)$. $\sum F_x = ma_x$ for this combined object gives $F = f_k = \mu_k (m_A + m_B)g$, in agreement with our answer in part (a).

5.38. **IDENTIFY:** $f = \mu_r n$. Apply $\sum \vec{F} = m\vec{a}$ to the tire.

SET UP: $n = mg$ and $f = ma$.

EXECUTE: $a_x = \dfrac{v^2 - v_0^2}{L}$, where L is the distance covered before the wheel's speed is reduced to half its original speed and $v = v_0/2$. $\mu_r = \dfrac{a}{g} = \dfrac{v_0^2 - v^2}{2Lg} = \dfrac{v_0^2 - \frac{1}{4}v_0^2}{2Lg} = \dfrac{3}{8}\dfrac{v_0^2}{Lg}$.

Low pressure, $L = 18.1$ m and $\dfrac{3}{8}\dfrac{(3.50\text{ m/s})^2}{(18.1\text{ m})(9.80\text{ m/s}^2)} = 0.0259$.

High pressure, $L = 92.9$ m and $\dfrac{3}{8}\dfrac{(3.50\text{ m/s})^2}{(3.50\text{ m/s})^2} = 0.00505$.

EVALUATE: μ_r is inversely proportional to the distance L, so $\dfrac{\mu_{r1}}{\mu_{r2}} = \dfrac{L_2}{L_1}$.

5.39. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box. Use the information about sliding to calculate the mass of the box.

SET UP: $f_k = \mu_k n$, $f_r = \mu_r n$ and $n = mg$.

EXECUTE: Without the dolly: $n = mg$ and $F - \mu_k n = 0$ ($a_x = 0$ since speed is constant).

$$m = \frac{F}{\mu_k g} = \frac{160\text{ N}}{(0.47)\,(9.80\text{ m/s}^2)} = 34.74\text{ kg}$$

With the dolly: the total mass is $34.7\text{ kg} + 5.3\text{ kg} = 40.04\text{ kg}$ and friction now is rolling friction, $f_r = \mu_r mg$.

$F - \mu_r mg = ma$. $a = \dfrac{F - \mu_r mg}{m} = 3.82\text{ m/s}^2$.

EVALUATE: $f_k = \mu_k mg = 160$ N and $f_r = \mu_r mg = 4.36$ N, or, $\dfrac{f_r}{f_k} = \dfrac{\mu_r}{\mu_k}$. The rolling friction force is much less than the kinetic friction force.

5.40. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the truck. For constant speed, $a = 0$ and $F_{\text{horiz}} = f_r$.

SET UP: $f_r = \mu_r n = \mu_r mg$. Let $m_2 = 1.42 m_1$ and $\mu_{r2} = 0.81\mu_{r1}$.

EXECUTE: Since the speed is constant and we are neglecting air resistance, we can ignore the 2.4 m/s, and F_{net} in the horizontal direction must be zero. Therefore $f_r = \mu_r n = F_{\text{horiz}} = 200$ N before the weight and pressure changes are made. After the changes, $(0.81\mu_r)\,(1.42n) = F_{\text{horiz}}$, because the speed is still constant and $F_{\text{net}} = 0$. We can simply divide the two equations: $\dfrac{(0.81\mu_r)(1.42n)}{\mu_r n} = \dfrac{F_{\text{horiz}}}{200\text{ N}}$ and $(0.81)\,(1.42)\,(200\text{ N}) = F_{\text{horiz}} = 230$ N.

EVALUATE: The increase in weight increases the normal force and hence the friction force, whereas the decrease in μ_r reduces it. The percentage increase in the weight is larger, so the net effect is an increase in the friction force.

5.41. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. The target variables are the tension T in the cord and the acceleration a of the blocks. Then a can be used in a constant acceleration equation to find the speed of each block. The magnitude of the acceleration is the same for both blocks.

SET UP: The system is sketched in Figure 5.41a.

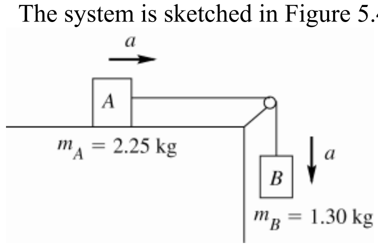


Figure 5.41a

For each block take a positive coordinate direction to be the direction of the block's acceleration.

block on the table: The free-body is sketched in Figure 5.41b.

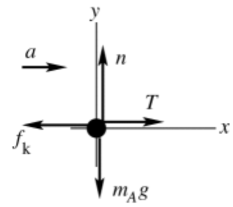


Figure 5.41b

EXECUTE:

$\sum F_y = ma_y$

$n - m_A g = 0$

$n = m_A g$

$f_k = \mu_k n = \mu_k m_A g$

$\sum F_x = ma_x$

$T - f_k = m_A a$

$T - \mu_k m_A g = m_A a$

SET UP: hanging block: The free-body is sketched in Figure 5.41c.

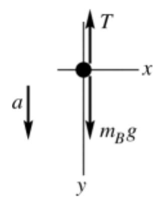


Figure 5.41c

EXECUTE:

$\sum F_y = ma_y$

$m_B g - T = m_B a$

$T = m_B g - m_B a$

(a) Use the second equation in the first

$$m_B g - m_B a - \mu_k m_A g = m_A a$$

$$(m_A + m_B)a = (m_B - \mu_k m_A)g$$

$$a = \frac{(m_B - \mu_k m_A)g}{m_A + m_B} = \frac{(1.30\text{ kg} - (0.45)(2.25\text{ kg}))(9.80\text{ m/s}^2)}{2.25\text{ kg} + 1.30\text{ kg}} = 0.7937\text{ m/s}^2$$

SET UP: Now use the constant acceleration equations to find the final speed. Note that the blocks have the same speeds. $x - x_0 = 0.0300$ m, $a_x = 0.7937\text{ m/s}^2$, $v_{0x} = 0$, $v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

EXECUTE: $v_x = \sqrt{2a_x(x - x_0)} = \sqrt{2(0.7937\text{ m/s}^2)(0.0300\text{ m})} = 0.218\text{ m/s} = 21.8\text{ cm/s}.$

(b) $T = m_B g - m_B a = m_B(g - a) = 1.30\text{ kg}(9.80\text{ m/s}^2 - 0.7937\text{ m/s}^2) = 11.7\text{ N}$

Or, to check, $T - \mu_k m_A g = m_A a$

$T = m_A(a + \mu_k g) = 2.25\text{ kg}(0.7937\text{ m/s}^2 + (0.45)(9.80\text{ m/s}^2)) = 11.7\text{ N}$, which checks.

EVALUATE: The force T exerted by the cord has the same value for each block. $T < m_B g$ since the hanging block accelerates downward. Also, $f_k = \mu_k m_A g = 9.92$ N. $T > f_k$ and the block on the table accelerates in the direction of T.

5.42. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box. When the box is ready to slip the static friction force has its maximum possible value, $f_s = \mu_s n$.

SET UP: Use coordinates parallel and perpendicular to the ramp.

EXECUTE: **(a)** The normal force will be $w\cos\theta$ and the component of the gravitational force along the ramp is $w\sin\theta$. The box begins to slip when $w\sin\theta > \mu_s w\cos\theta$, or $\tan\theta > \mu_s = 0.35$, so slipping occurs at $\theta = \arctan(0.35) = 19.3°$.

(b) When moving, the friction force along the ramp is $\mu_k w\cos\theta$, the component of the gravitational force along the ramp is $w\sin\theta$, so the acceleration is

$$(w\sin\theta - w\mu_k\cos\theta)/m = g(\sin\theta - \mu_k\cos\theta) = 0.92\text{ m/s}^2.$$

(c) Since $v_{0x} = 0$, $2ax = v^2$, so $v = (2ax)^{1/2}$, or $v = [(2)(0.92\text{m/s}^2)(5\text{ m})]^{1/2} = 3\text{ m/s}$.

EVALUATE: When the box starts to move, friction changes from static to kinetic and the friction force becomes smaller.

5.43. **(a) IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the crate. Constant v implies $a = 0$. Crate moving says that the friction is kinetic friction. The target variable is the magnitude of the force applied by the woman.
SET UP: The free-body diagram for the crate is sketched in Figure 5.43.

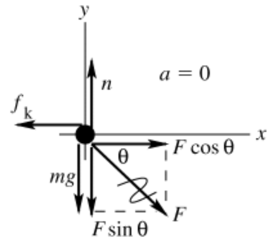


Figure 5.43

EXECUTE:

$\sum F_y = ma_y$

$n - mg - F\sin\theta = 0$

$n = mg + F\sin\theta$

$f_k = \mu_k n = \mu_k mg + \mu_k F\sin\theta$

$\sum F_x = ma_x$

$F\cos\theta - f_k = 0$

$F\cos\theta - \mu_k mg - \mu_k F\sin\theta = 0$

$F(\cos\theta - \mu_k\sin\theta) = \mu_k mg$

$$F = \frac{\mu_k mg}{\cos\theta - \mu_k\sin\theta}$$

(b) IDENTIFY and **SET UP:** "start the crate moving" means the same force diagram as in part (a), except that μ_k is replaced by μ_s. Thus $F = \dfrac{\mu_s mg}{\cos\theta - \mu_s\sin\theta}$.

EXECUTE: $F \to \infty$ if $\cos\theta - \mu_s\sin\theta = 0$. This gives $\mu_s = \dfrac{\cos\theta}{\sin\theta} = \dfrac{1}{\tan\theta}$.

EVALUATE: $\vec{F}$ has a downward component so $n > mg$. If $\theta = 0$ (woman pushes horizontally), $n = mg$ and $F = f_k = \mu_k mg$.

5.44. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box.

SET UP: Let $+y$ be upward and $+x$ be horizontal, in the direction of the acceleration. Constant speed means $a = 0$.

EXECUTE: **(a)** There is no net force in the vertical direction, so $n + F\sin\theta - w = 0$, or $n = w - F\sin\theta = mg - F\sin\theta$. The friction force is $f_k = \mu_k n = \mu_k(mg - F\sin\theta)$. The net horizontal force is $F\cos\theta - f_k = F\cos\theta - \mu_k(mg - F\sin\theta)$, and so at constant speed,

$$F = \frac{\mu_k mg}{\cos\theta + \mu_k\sin\theta}$$

(b) Using the given values, $F = \dfrac{(0.35)(90\text{ kg})(9.80\text{ m/s}^2)}{(\cos 25° + (0.35)\sin 25°)} = 290\text{ N}$.

EVALUATE: If $\theta = 0°$, $F = \mu_k mg$.

5.45. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block.

SET UP: For block B use coordinates parallel and perpendicular to the incline. Since they are connected by ropes, blocks A and B also move with constant speed.

EXECUTE: **(a)** The free-body diagrams are sketched in Figure 5.45.

(b) The blocks move with constant speed, so there is no net force on block A; the tension in the rope connecting A and B must be equal to the frictional force on block A, $\mu_k = (0.35)\ (25.0\text{ N}) = 9\text{ N}$.

(c) The weight of block C will be the tension in the rope connecting B and C; this is found by considering the forces on block B. The components of force along the ramp are the tension in the first rope (9 N, from part (a)), the component of the weight along the ramp, the friction on block B and the tension in the second rope. Thus, the weight of block C is

$$w_C = 9\text{ N} + w_B(\sin 36.9° + \mu_k\cos 36.9°) = 9\text{ N} + (25.0\text{ N})(\sin 36.9° + (0.35)\cos 36.9°) = 31.0\text{ N}$$

The intermediate calculation of the first tension may be avoided to obtain the answer in terms of the common weight w of blocks A and B, $w_C = w(\mu_k + (\sin\theta + \mu_k\cos\theta))$, giving the same result.

(d) Applying Newton's Second Law to the remaining masses (B and C) gives:

$$a = g(w_C - \mu_k w_B\cos\theta - w_B\sin\theta)/(w_B + w_C) = 1.54\text{ m/s}^2.$$

EVALUATE: Before the rope between A and B is cut the net external force on the system is zero. When the rope is cut the friction force on A is removed from the system and there is a net force on the system of blocks B and C.

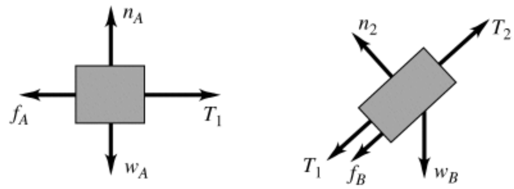


Figure 5.45

5.46. IDENTIFY and **SET UP:** The derivative of v_y gives a_y as a function of time, and the integral of v_y gives y as a function of time.

EXECUTE: Differentiating Eq. (5.10) with respect to time gives the acceleration

$a = v_t\left(\frac{k}{m}\right)e^{-(k/m)t} = ge^{-(k/m)t}$, where Eq. (5.9), $v_t = mg/k$, has been used. Integrating Eq. (5.10) with respect to time with $y_0 = 0$ gives

$$y = \int_0^t v_t[1-e^{-(k/m)t}]\,dt = v_t\left[t+\left(\frac{m}{k}\right)e^{-(k/m)t}\right] - v_t\left(\frac{m}{k}\right) = v_t\left[t-\frac{m}{k}\left(1-e^{-(k/m)t}\right)\right].$$

EVALUATE: We can verify that $dy/dt = v_y$.

5.47. IDENTIFY and **SET UP:** Apply Eq.(5.13).

EXECUTE: **(a)** Solving for D in terms of v_t, $D = \frac{mg}{v_t^2} = \frac{(80\text{ kg})\,(9.80\text{ m/s}^2)}{(42\text{ m/s})^2} = 0.44\text{ kg/m}$.

(b) $v_t = \sqrt{\frac{mg}{D}} = \sqrt{\frac{(45\text{ kg})(9.80\text{ m/s}^2)}{(0.25\text{ kg/m})}} = 42\text{ m/s}$.

EVALUATE: v_t is less for the daughter since her mass is less.

5.48. IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to the ball. At the terminal speed, $f = mg$.

SET UP: The fluid resistance is directed opposite to the velocity of the object. At half the terminal speed, the magnitude of the frictional force is one-fourth the weight.

EXECUTE: **(a)** If the ball is moving up, the frictional force is down, so the magnitude of the net force is (5/4)w and the acceleration is (5/4)g, down.

(b) While moving down, the frictional force is up, and the magnitude of the net force is (3/4)w and the acceleration is (3/4)g, down.

EVALUATE: The frictional force is less than mg in each case and in each case the net force is downward and the acceleration is downward.

5.49. IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to one of the masses. The mass moves in a circular path, so has acceleration $a_{\text{rad}} = \frac{v^2}{R}$, directed toward the center of the path.

SET UP: In each case, $R = 0.200$ m. In part (a), let $+x$ be toward the center of the circle, so $a_x = a_{\text{rad}}$. In part (b) let $+y$ be toward the center of the circle, so $a_y = a_{\text{rad}}$. $+y$ is downward when the mass is at the top of the circle and $+y$ is upward when the mass is at the bottom of the circle. Since a_{rad} has its greatest possible value, $\vec{F}$ is in the direction of $\vec{a}_{\text{rad}}$ at both positions.

EXECUTE: **(a)** $\sum F_x = ma_x$ gives $F = ma_{\text{rad}} = m\frac{v^2}{R}$. $F = 75.0$ N and $v = \sqrt{\frac{FR}{m}} = \sqrt{\frac{(75.0\text{ N})(0.200\text{ m})}{1.15\text{ kg}}} = 3.61$ m/s.

(b) The free-body diagrams for a mass at the top of the path and at the bottom of the path are given in figure 5.49. At the top, $\sum F_y = ma_y$ gives $F = ma_{\text{rad}} - mg$ and at the bottom it gives $F = mg + ma_{\text{rad}}$. For a given rotation rate and hence value of a_{rad}, the value of F required is larger at the bottom of the path.

(c) $F = mg + ma_{\text{rad}}$ so $\frac{v^2}{R} = \frac{F}{m} - g$ and

$$v = \sqrt{R\left(\frac{F}{m} - g\right)} = \sqrt{(0.200\text{ m})\left(\frac{75.0\text{ N}}{1.15\text{ kg}} - 9.80\text{ m/s}^2\right)} = 3.33\text{ m/s}$$

EVALUATE: The maximum speed is less for the vertical circle. At the bottom of the vertical path $\vec{F}$ and the weight are in opposite directions so *F* must exceed ma_{rad} by an amount equal to *mg*. At the top of the vertical path *F* and *mg* are in the same direction and together provide the required net force, so *F* must be larger at the bottom.

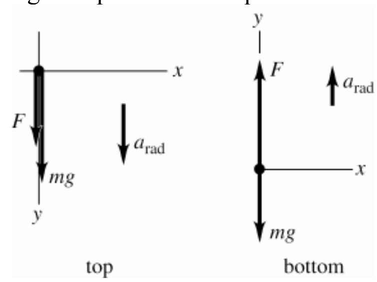


Figure 5.49

5.50. **IDENTIFY:** Since the car travels in an arc of a circle, it has acceleration $a_{\text{rad}} = v^2/R$, directed toward the center of the arc. The only horizontal force on the car is the static friction force exerted by the roadway. To calculate the minimum coefficient of friction that is required, set the static friction force equal to its maximum value, $f_s = \mu_s n$. Friction is static friction because the car is not sliding in the radial direction.

SET UP: The free-body diagram for the car is given in Figure 5.50. The diagram assumes the center of the curve is to the left of the car.

EXECUTE: **(a)** $\sum F_y = ma_y$ gives $n = mg$. $\sum F_x = ma_x$ gives $\mu_s n = m\frac{v^2}{R}$. $\mu_s mg = m\frac{v^2}{R}$ and

$$\mu_s = \frac{v^2}{gR} = \frac{(25.0 \text{ m/s})^2}{(9.80 \text{ m/s}^2)(220 \text{ m})} = 0.290$$

(b) $\frac{v^2}{\mu_s} = Rg = \text{constant}$, so $\frac{v_1^2}{\mu_{s1}} = \frac{v_2^2}{\mu_{s2}}$. $v_2 = v_1\sqrt{\frac{\mu_{s2}}{\mu_{s1}}} = (25.0 \text{ m/s})\sqrt{\frac{\mu_{s1}/3}{\mu_{s1}}} = 14.4 \text{ m/s}$.

EVALUATE: A smaller coefficient of friction means a smaller maximum friction force, a smaller possible acceleration and therefore a smaller speed.

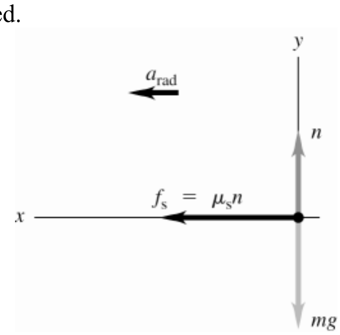


Figure 5.50

5.51. **IDENTIFY:** We can use the analysis done in Example 5.23. As in that example, we assume friction is negligible.

SET UP: From Example 5.23, the banking angle β is given by $\tan\beta = \frac{v^2}{gR}$. Also, $n = mg/\cos\beta$.

$65.0 \text{ mi/h} = 29.1 \text{ m/s}$.

EXECUTE: **(a)** $\tan\beta = \frac{(29.1 \text{ m/s})^2}{(9.80 \text{ m/s}^2)(225 \text{ m})}$ and $\beta = 21.0°$. The expression for $\tan\beta$ does not involve the mass of the vehicle, so the truck and car should travel at the same speed.

(b) For the car, $n_{\text{car}} = \frac{(1125 \text{ kg})(9.80 \text{ m/s}^2)}{\cos 21.0°} = 1.18\times 10^4 \text{ N}$ and $n_{\text{truck}} = 2n_{\text{car}} = 2.36\times 10^4 \text{ N}$, since $m_{\text{truck}} = 2m_{\text{car}}$.

EVALUATE: The vertical component of the normal force must equal the weight of the vehicle, so the normal force is proportional to *m*.

5.52. **IDENTIFY:** The acceleration of the person is $a_{\text{rad}} = v^2/R$, directed horizontally to the left in the figure in the problem. The time for one revolution is the period $T = \frac{2\pi R}{v}$. Apply $\sum \vec{F} = m\vec{a}$ to the person.

SET UP: The person moves in a circle of radius $R = 3.00\text{ m} + (5.00\text{ m})\sin 30.0° = 5.50\text{ m}$. The free-body diagram is given in Figure 5.52. $\vec{F}$ is the force applied to the seat by the rod.

EXECUTE: **(a)** $\sum F_y = ma_y$ gives $F\cos 30.0° = mg$ and $F = \dfrac{mg}{\cos 30.0°}$. $\sum F_x = ma_x$ gives $F\sin 30.0° = m\dfrac{v^2}{R}$.

Combining these two equations gives $v = \sqrt{Rg\tan\theta} = \sqrt{(5.50\text{ m})(9.80\text{ m/s}^2)\tan 30.0°} = 5.58\text{ m/s}$. Then the period is $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(5.50\text{ m})}{5.58\text{ m/s}} = 6.19\text{ s}$.

(b) The net force is proportional to m so in $\sum\vec{F} = m\vec{a}$ the mass divides out and the angle for a given rate of rotation is independent of the mass of the passengers.

EVALUATE: The person moves in a horizontal circle so the acceleration is horizontal. The net inward force required for circular motion is produced by a component of the force exerted on the seat by the rod.

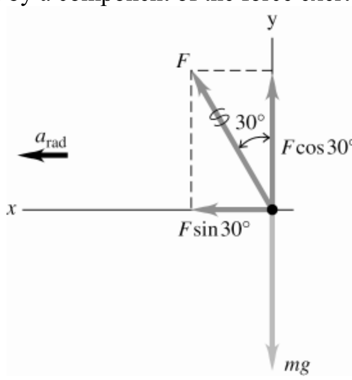


Figure 5.52

5.53. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to the composite object of the person plus seat. This object moves in a horizontal circle and has acceleration a_{rad}, directed toward the center of the circle.

SET UP: The free-body diagram for the composite object is given in Figure 5.53. Let $+x$ be to the right, in the direction of $\vec{a}_{\text{rad}}$. Let $+y$ be upward. The radius of the circular path is $R = 7.50\text{ m}$. The total mass is $(255\text{ N} + 825\text{ N})/(9.80\text{ m/s}^2) = 110.2\text{ kg}$. Since the rotation rate is $32.0\text{ rev/min} = 0.5333\text{ rev/s}$, the period T is $\dfrac{1}{0.5333\text{ rev/s}} = 1.875\text{ s}$.

EXECUTE: $\sum F_y = ma_y$ gives $T_A\cos 40.0° - mg = 0$ and $T_A = \dfrac{mg}{\cos 40.0°} = \dfrac{255\text{ N} + 825\text{ N}}{\cos 40.0°} = 1410\text{ N}$.

$\sum F_x = ma_x$ gives $T_A\sin 40.0° + T_B = ma_{\text{rad}}$ and

$$T_B = m\frac{4\pi^2 R}{T^2} - T_A\sin 40.0° = (110.2\text{ kg})\frac{4\pi^2(7.50\text{ m})}{(1.875\text{ s})^2} - (1410\text{ N})\sin 40.0° = 8370\text{ N}.$$

The tension in the horizontal cable is 8370 N and the tension in the other cable is 1410 N.

EVALUATE: The weight of the composite object is 1080 N. The tension in cable A is larger than this since its vertical component must equal the weight. $ma_{\text{rad}} = 9280\text{ N}$. The tension in cable B is less than this because part of the required inward force comes from a component of the tension in cable A.

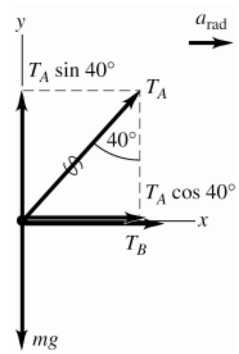


Figure 5.53

5.54. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the button. The button moves in a circle, so it has acceleration a_{rad}.

SET UP: The situation is equivalent to that of Example 5.22.

EXECUTE: **(a)** $\mu_s = \dfrac{v^2}{Rg}$. Expressing v in terms of the period T, $v = \dfrac{2\pi R}{T}$ so $\mu_s = \dfrac{4\pi^2 R}{T^2 g}$. A platform speed of 40.0 rev/min corresponds to a period of 1.50 s, so $\mu_s = \dfrac{4\pi^2(0.150\ \text{m})}{(1.50\ \text{s})^2(9.80\ \text{m/s}^2)} = 0.269.$

(b) For the same coefficient of static friction, the maximum radius is proportional to the square of the period (longer periods mean slower speeds, so the button may be moved further out) and so is inversely proportional to the square of the speed. Thus, at the higher speed, the maximum radius is (0.150 m) $\left(\dfrac{40.0}{60.0}\right)^2 = 0.067\ \text{m}$.

EVALUATE: $a_{rad} = \dfrac{4\pi^2 R}{T^2}$. The maximum radial acceleration that friction can give is $\mu_s mg$. At the faster rotation rate T is smaller so R must be smaller to keep a_{rad} the same.

5.55. **IDENTIFY:** The acceleration due to circular motion is $a_{rad} = \dfrac{4\pi^2 R}{T^2}$.

SET UP: $R = 800\ \text{m}$. $1/T$ is the number of revolutions per second.

EXECUTE: **(a)** Setting $a_{rad} = g$ and solving for the period T gives

$$T = 2\pi\sqrt{\frac{R}{g}} = 2\pi\sqrt{\frac{400\ \text{m}}{9.80\ \text{m/s}^2}} = 40.1\ \text{s},$$

so the number of revolutions per minute is $(60\ \text{s/min})/(40.1\ \text{s}) = 1.5\ \text{rev/min}$.

(b) The lower acceleration corresponds to a longer period, and hence a lower rotation rate, by a factor of the square root of the ratio of the accelerations, $T' = (1.5\ \text{rev/min}) \times \sqrt{3.70/9.8} = 0.92\ \text{rev/min}.$

EVALUATE: In part (a) the tangential speed of a point at the rim is given by $a_{rad} = \dfrac{v^2}{R}$, so $v = \sqrt{Ra_{rad}} = \sqrt{Rg} = 62.6\ \text{m/s}$; the space station is rotating rapidly.

5.56. **IDENTIFY:** $T = \dfrac{2\pi R}{v}$. The apparent weight of a person is the normal force exerted on him by the seat he is sitting on. His acceleration is $a_{rad} = v^2/R$, directed toward the center of the circle.

SET UP: The period is $T = 60.0$ s. The passenger has mass $m = w/g = 90.0\ \text{kg}$.

EXECUTE: **(a)** $v = \dfrac{2\pi R}{T} = \dfrac{2\pi(50.0\ \text{m})}{60.0\ \text{s}} = 5.24\ \text{m/s}$. Note that $a_{rad} = \dfrac{v^2}{R} = \dfrac{(5.24\ \text{m/s})^2}{50.0\ \text{m}} = 0.549\ \text{m/s}^2$.

(b) The free-body diagram for the person at the top of his path is given in Figure 5.56a. The acceleration is downward, so take $+y$ downward. $\sum F_y = ma_y$ gives $mg - n = ma_{rad}$.

$n = m(g - a_{rad}) = (90.0\ \text{kg})(9.80\ \text{m/s}^2 - 0.549\ \text{m/s}^2) = 833\ \text{N}$.

The free-body diagram for the person at the bottom of his path is given in Figure 5.56b. The acceleration is upward, so take $+y$ upward. $\sum F_y = ma_y$ gives $n - mg = ma_{rad}$ and $n = m(g + a_{rad}) = 931\ \text{N}$.

(c) Apparent weight $= 0$ means $n = 0$ and $mg = ma_{rad}$. $g = \dfrac{v^2}{R}$ and $v = \sqrt{gR} = 22.1\ \text{m/s}$. The time for one revolution would be $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(50.0\ \text{m})}{22.1\ \text{m/s}} = 14.2\ \text{s}$. Note that $a_{rad} = g$.

(d) $n = m(g + a_{rad}) = 2mg = 2(882\ \text{N}) = 1760\ \text{N}$, twice his true weight.

EVALUATE: At the top of his path his apparent weight is less than his true weight and at the bottom of his path his apparent weight is greater than his true weight.

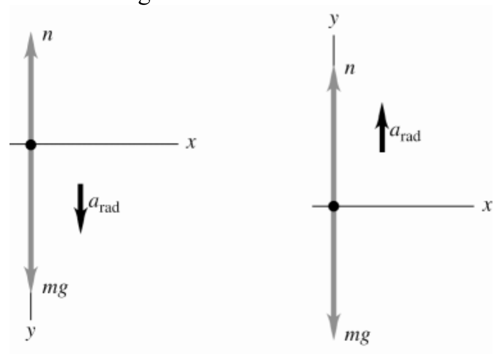


Figure 5.56a, b

5.57. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the motion of the pilot. The pilot moves in a vertical circle. The apparent weight is the normal force exerted on him. At each point $\vec{a}_{rad}$ is directed toward the center of the circular path.

(a) SET UP: "the pilot feels weightless" means that the vertical normal force n exerted on the pilot by the chair on which the pilot sits is zero. The force diagram for the pilot at the top of the path is given in Figure 5.57a.

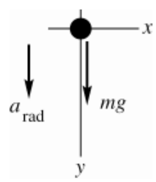


Figure 5.57a

EXECUTE:

$$\sum F_y = ma_y$$

$$mg = ma_{rad}$$

$$g = \frac{v^2}{R}$$

Thus $v = \sqrt{gR} = \sqrt{(9.80\ \text{m/s}^2)(150\ \text{m})} = 38.34\ \text{m/s}$

$$v = (38.34\ \text{m/s})\left(\frac{1\ \text{km}}{10^3\ \text{m}}\right)\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = 138\ \text{km/h}$$

(b) SET UP: The force diagram for the pilot at the bottom of the path is given in Figure 5.57b. Note that the vertical normal force exerted on the pilot by the chair on which the pilot sits is now upward.

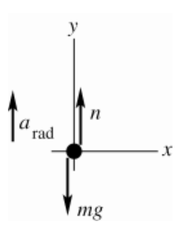


Figure 5.57b

EXECUTE:

$$\sum F_y = ma_y$$

$$n - mg = m\frac{v^2}{R}$$

$$n = mg + m\frac{v^2}{R}$$

This normal force is the pilot's apparent weight.

$w = 700\ \text{N}$, so $m = \dfrac{w}{g} = 71.43\ \text{kg}$

$$v = (280\ \text{km/h})\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right) = 77.78\ \text{m/s}$$

Thus $n = 700\ \text{N} + 71.43\ \text{kg}\dfrac{(77.78\ \text{m/s})^2}{150\ \text{m}} = 3580\ \text{N}.$

EVALUATE: In part (b), $n > mg$ since the acceleration is upward. The pilot feels he is much heavier than when at rest. The speed is not constant, but it is still true that $a_{rad} = v^2/R$ at each point of the motion.

5.58. **IDENTIFY:** $a_{rad} = v^2/R$, directed toward the center of the circular path. At the bottom of the dive, $\vec{a}_{rad}$ is upward. The apparent weight of the pilot is the normal force exerted on her by the seat on which she is sitting.

SET UP: The free-body diagram for the pilot is given in Figure 5.58.

EXECUTE: (a) $a_{\text{rad}} = \dfrac{v^2}{R}$ gives $R = \dfrac{v^2}{a_{\text{rad}}} = \dfrac{(95.0\text{ m/s})^2}{4.00(9.80\text{ m/s}^2)} = 230\text{ m}$.

(b) $\sum F_y = ma_y$ gives $n - mg = ma_{\text{rad}}$.

$n = m(g + a_{\text{rad}}) = m(g + 4.00g) = 5.00mg = (5.00)(50.0\text{ kg})(9.80\text{ m/s}^2) = 2450\text{ N}$

EVALUATE: Her apparent weight is five times her true weight, the force of gravity the earth exerts on her.

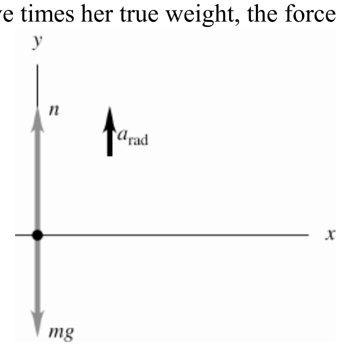


Figure 5.58

5.59. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the water. The water moves in a vertical circle. The target variable is the speed v; we will calculate a_{rad} and then get v from $a_{\text{rad}} = v^2/R$

SET UP: Consider the free-body diagram for the water when the pail is at the top of its circular path, as shown in Figures 5.59a and b.

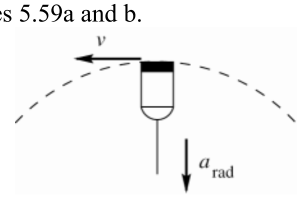


Figure 5.59a

The radial acceleration is in toward the center of the circle so at this point is downward. n is the downward normal force exerted on the water by the bottom of the pail.

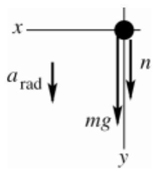


Figure 5.59b

EXECUTE:

$\sum F_y = ma_y$

$n + mg = m\dfrac{v^2}{R}$

At the minimum speed the water is just ready to lose contact with the bottom of the pail, so at this speed, $n \to 0$. (Note that the force n cannot be upward.)

With $n \to 0$ the equation becomes $mg = m\dfrac{v^2}{R}$. $v = \sqrt{gR} = \sqrt{(9.80\text{ m/s}^2)(0.600\text{ m})} = 2.42\text{ m/s}$.

EVALUATE: At the minimum speed $a_{\text{rad}} = g$. If v is less than this minimum speed, gravity pulls the water (and bucket) out of the circular path.

5.60. IDENTIFY: The ball has acceleration $a_{\text{rad}} = v^2/R$, directed toward the center of the circular path. When the ball is at the bottom of the swing, its acceleration is upward.

SET UP: Take $+y$ upward, in the direction of the acceleration. The bowling ball has mass $m = w/g = 7.27\text{ kg}$.

EXECUTE: (a) $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(4.20\text{ m/s})^2}{3.80\text{ m}} = 4.64\text{ m/s}$, upward.

(b) The free-body diagram is given in Figure 5.60. $\sum F_y = ma_y$ gives $T - mg = ma_{\text{rad}}$.

$T = m(g + a_{\text{rad}}) = (7.27\text{ kg})(9.80\text{ m/s}^2 + 4.64\text{ m/s}^2) = 105\text{ N}$

EVALUATE: The acceleration is upward, so the net force is upward and the tension is greater than the weight.

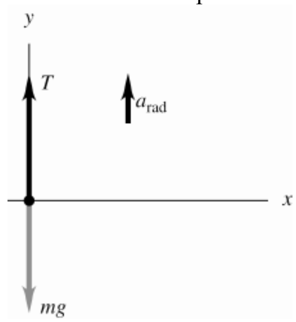


Figure 5.60

5.61. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the knot.

SET UP: $a = 0$. Use coordinates with axes that are horizontal and vertical.

EXECUTE: **(a)** The free-body diagram for the knot is sketched in Figure 5.61.

T_1 is more vertical so supports more of the weight and is larger. You can also see this from $\sum F_x = ma_x$:

$T_2 \cos 40° - T_1 \cos 60° = 0$. $T_2 \cos 40° - T_1 \cos 60° = 0$.

(b) T_1 is larger so set $T_1 = 5000$ N. Then $T_2 = T_1/1.532 = 3263.5$ N. $\sum F_y = ma_y$ gives

$T_1 \sin 60° + T_2 \sin 40° = w$ and $w = 6400$ N.

EVALUATE: The sum of the vertical components of the two tensions equals the weight of the suspended object. The sum of the tensions is greater than the weight.

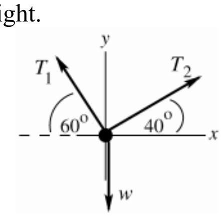


Figure 5.61

5.62. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to each object . Constant speed means $a = 0$.

SET UP: The free-body diagrams are sketched in Figure 5.62. T_1 is the tension in the lower chain, T_2 is the tension in the upper chain and $T = F$ is the tension in the rope.

EXECUTE: The tension in the lower chain balances the weight and so is equal to *w*. The lower pulley must have no net force on it, so twice the tension in the rope must be equal to *w* and the tension in the rope, which equals *F*, is $w/2$. Then, the downward force on the upper pulley due to the rope is also *w*, and so the upper chain exerts a force *w* on the upper pulley, and the tension in the upper chain is also *w*.

EVALUATE: The pulley combination allows the worker to lift a weight *w* by applying a force of only $w/2$.

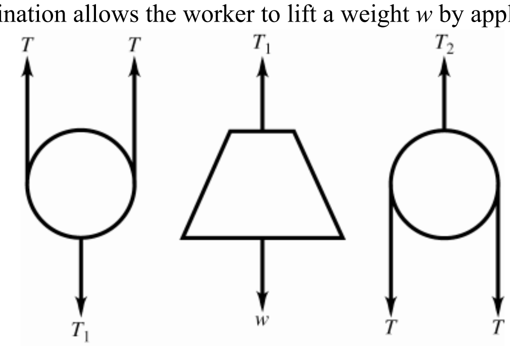


Figure 5.62

5.63. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the rope.

SET UP: The hooks exert forces on the ends of the rope. At each hook, the force that the hook exerts and the force due to the tension in the rope are an action-reaction pair.

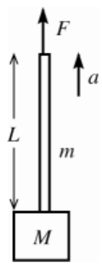
EXECUTE: **(a)** The vertical forces that the hooks exert must balance the weight of the rope, so each hook exerts an upward vertical force of $w/2$ on the rope. Therefore, the downward force that the rope exerts at each end is $T_{\text{end}} \sin\theta = w/2$, so $T_{\text{end}} = w/(2\sin\theta) = Mg/(2\sin\theta)$.

(b) Each half of the rope is itself in equilibrium, so the tension in the middle must balance the horizontal force that each hook exerts, which is the same as the horizontal component of the force due to the tension at the end; $T_{\text{end}}\cos\theta = T_{\text{middle}}$, so $T_{\text{middle}} = Mg\cos\theta/(2\sin\theta) = Mg/(2\tan\theta)$.

(c) Mathematically speaking, $\theta \neq 0$ because this would cause a division by zero in the equation for T_{end} or T_{middle}. Physically speaking, we would need an infinite tension to keep a non-massless rope perfectly straight.

EVALUATE: The tension in the rope is not the same at all points along the rope.

5.64. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to the combined rope plus block to find a. Then apply $\sum\vec{F} = m\vec{a}$ to a section of the rope of length x. First note the limiting values of the tension. The system is sketched in Figure 5.64a.



At the top of the rope $T = F$
At the bottom of the rope $T = M(g+a)$

Figure 5.64a

SET UP: Consider the rope and block as one combined object, in order to calculate the acceleration: The free-body diagram is sketched in Figure 5.64b.

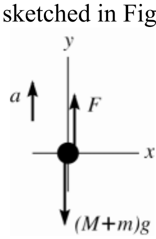


Figure 5.64b

EXECUTE:

$$\sum F_y = ma_y$$

$$F-(M+m)g = (M+m)a$$

$$a = \frac{F}{M+m} - g$$

SET UP: Now consider the forces on a section of the rope that extends a distance $x < L$ below the top. The tension at the bottom of this section is $T(x)$ and the mass of this section is $m(x/L)$. The free-body diagram is sketched in Figure 5.64c.

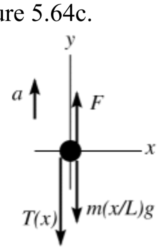


Figure 5.64c

EXECUTE:

$$\sum F_y = ma_y$$

$$F - T(x) - m(x/L)g = m(x/L)a$$

$$T(x) = F - m(x/L)g - m(x/L)a$$

Using our expression for a and simplifying gives

$$T(x) = F\left(1 - \frac{mx}{L(M+m)}\right)$$

EVALUATE: Important to check this result for the limiting cases:
$x = 0$: The expression gives the correct value of $T = F$.
$x = L$: The expression gives $T = F(M/(M+m))$. This should equal $T = M(g+a)$, and when we use the expression for a we see that it does.

5.65. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to each block.

SET UP: Constant speed means $a = 0$. When the blocks are moving, the friction force is f_k and when they are at rest, the friction force is f_s.

EXECUTE: **(a)** The tension in the cord must be $m_2 g$ in order that the hanging block move at constant speed. This tension must overcome friction and the component of the gravitational force along the incline, so $m_2 g = (m_1 g\sin\alpha + \mu_k m_1 g\cos\alpha)$ and $m_2 = m_1(\sin\alpha + \mu_k\cos\alpha)$.

(b) In this case, the friction force acts in the same direction as the tension on the block of mass m_1, so $m_2 g = (m_1 g\sin\alpha - \mu_k m_1 g\cos\alpha)$, or $m_2 = m_1(\sin\alpha - \mu_k\cos\alpha)$.

(c) Similar to the analysis of parts (a) and (b), the largest m_2 could be is $m_1(\sin\alpha + \mu_s \cos\alpha)$ and the smallest m_2 could be is $m_1(\sin\alpha - \mu_s \cos\alpha)$.

EVALUATE: In parts (a) and (b) the friction force changes direction when the direction of the motion of m_1 changes. In part (c), for the largest m_2 the static friction force on m_1 is directed down the incline and for the smallest m_2 the static friction force on m_1 is directed up the incline.

5.66. **IDENTIFY:** The system is in equilibrium. Apply Newton's 1st law to block *A*, to the hanging weight and to the knot where the cords meet. Target variables are the two forces.

(a) SET UP: The free-body diagram for the hanging block is given in Figure 5.66a.

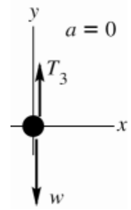


Figure 5.66a

EXECUTE:

$\sum F_y = ma_y$

$T_3 - w = 0$

$T_3 = 12.0\ \text{N}$

SET UP: The free-body diagram for the knot is given in Figure 5.66b.

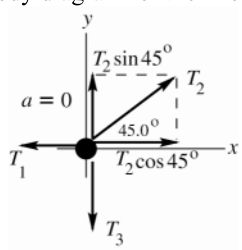


Figure 5.66b

EXECUTE:

$\sum F_y = ma_y$

$T_2 \sin 45.0° - T_3 = 0$

$T_2 = \dfrac{T_3}{\sin 45.0°} = \dfrac{12.0\ \text{N}}{\sin 45.0°}$

$T_2 = 17.0\ \text{N}$

$\sum F_x = ma_x$

$T_2 \cos 45.0° - T_1 = 0$

$T_1 = T_2 \cos 45.0° = 12.0\ \text{N}$

SET UP: The free-body diagram for block *A* is given in Figure 5.66c.

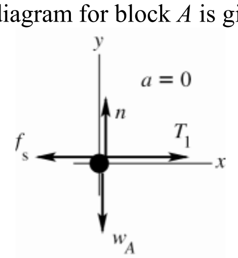


Figure 5.66c

EXECUTE:

$\sum F_x = ma_x$

$T_1 - f_s = 0$

$f_s = T_1 = 12.0\ \text{N}$

EVALUATE: Also can apply $\sum F_y = ma_y$ to this block:

$$n - w_A = 0$$

$$n = w_A = 60.0\ \text{N}$$

Then $\mu_s n = (0.25)(60.0\ \text{N}) = 15.0\ \text{N};$ this is the maximum possible value for the static friction force. We see that $f_s < \mu_s n;$ for this value of *w* the static friction force can hold the blocks in place.

(b) SET UP: We have all the same free-body diagrams and force equations as in part (a) but now the static friction force has its largest possible value, $f_s = \mu_s n = 15.0\ \text{N}.$ Then $T_1 = f_s = 15.0\ \text{N}.$

EXECUTE: From the equations for the forces on the knot

$T_2 \cos 45.0° - T_1 = 0$ implies $T_2 = T_1/\cos 45.0° = \dfrac{15.0\ \text{N}}{\cos 45.0°} = 21.2\ \text{N}$

$T_2 \sin 45.0° - T_3 = 0$ implies $T_3 = T_2 \sin 45.0° = (21.2\ \text{N})\sin 45.0° = 15.0\ \text{N}$

And finally $T_3 - w = 0$ implies $w = T_3 = 15.0\ \text{N}.$

EVALUATE: Compared to part (a), the friction is larger in part (b) by a factor of (15.0/12.0) and *w* is larger by this same ratio.

5.67. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. Use Newton's 3rd law to relate forces on A and on B.

SET UP: Constant speed means $a = 0$.

EXECUTE: **(a)** Treat A and B as a single object of weight $w = w_A + w_B = 4.80\text{ N}$. The free-body diagram for this combined object is given in Figure 5.67a. $\sum F_y = ma_y$ gives $n = w = 4.80\text{ N}$. $f_k = \mu_k n = 1.44\text{ N}$. $\sum F_x = ma_x$ gives $F = f_k = 1.44\text{ N}$

(b) The free-body force diagrams for blocks A and B are given in Figure 5.67b. n and f_k are the normal and friction forces applied to block B by the tabletop and are the same as in part (a). f_{kB} is the friction force that A applies to B. It is to the right because the force from A opposes the motion of B. n_B is the downward force that A exerts on B. f_{kA} is the friction force that B applies to A. It is to the left because block B wants A to move with it. n_A is the normal force that block B exerts on A. By Newton's third law, $f_{kB} = f_{kA}$ and these forces are in opposite directions. Also, $n_A = n_B$ and these forces are in opposite directions.

$\sum F_y = ma_y$ for block A gives $n_A = w_A = 1.20\text{ N}$, so $n_B = 1.20\text{ N}$.

$f_{kA} = \mu_k n_A = (0.300)(1.20\text{ N}) = 0.36\text{ N}$, and $f_{kB} = 0.36\text{ N}$.

$\sum F_x = ma_x$ for block A gives $T = f_{kA} = 0.36\text{ N}$.

$\sum F_x = ma_x$ for block B gives $F = f_{kB} + f_k = 0.36\text{ N} + 1.44\text{ N} = 1.80\text{ N}$

EVALUATE: In part (a) block A is at rest with respect to B and it has zero acceleration. There is no horizontal force on A besides friction, and the friction force on A is zero. A larger force F is needed in part (b), because of the friction force between the two blocks.

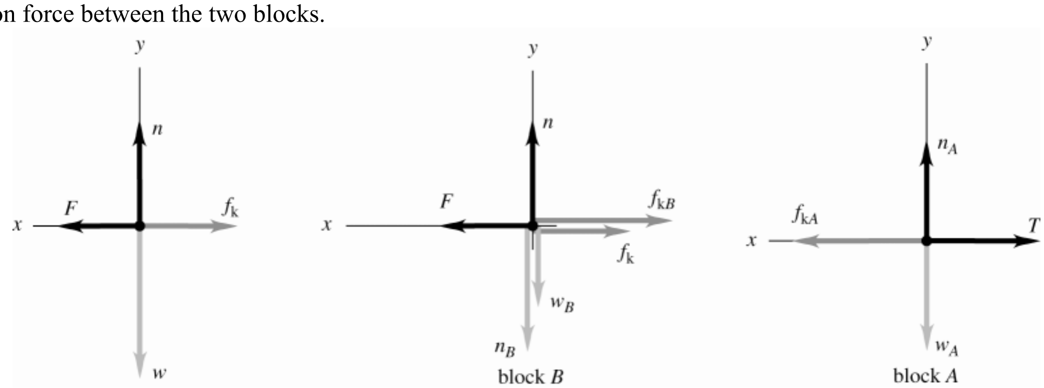


Figure 5.67a–c

5.68. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the brush. Constant speed means $a = 0$. Target variables are two of the forces on the brush.

SET UP: Note that the normal force exerted by the wall is horizontal, since it is perpendicular to the wall. The kinetic friction force exerted by the wall is parallel to the wall and opposes the motion, so it is vertically downward. The free-body diagram is given in Figure 5.68.

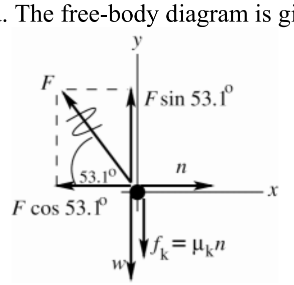


Figure 5.68

EXECUTE:

$\sum F_x = ma_x$

$n - F\cos 53.1° = 0$

$n = F\cos 53.1°$

$f_k = \mu_k n = \mu_k F\cos 53.1°$

$\sum F_y = ma_y$

$F\sin 53.1° - w - f_k = 0$

$F\sin 53.1° - w - \mu_k F\cos 53.1° = 0$

$F(\sin 53.1° - \mu_k \cos 53.1°) = w$

$$F = \frac{w}{\sin 53.1° - \mu_k \cos 53.1°}$$

(a) $F = \dfrac{w}{\sin 53.1° - \mu_k \cos 53.1°} = \dfrac{120 \text{ N}}{\sin 53.1° - (0.15)\cos 53.1°} = 16.9 \text{ N}$

(b) $n = F\cos 53.1° = (16.9 \text{ N})\cos 53.1° = 10.1 \text{ N}$

EVALUATE: In the absence of friction $w = F\sin 53.1°$, which agrees with our expression.

5.69. **IDENTIFY:** The net force at any time is $F_{\text{net}} = ma$.

SET UP: At $t = 0$, $a = 62g$. The maximum acceleration is $140g$ at $t = 1.2$ ms.

EXECUTE: **(a)** $F_{\text{net}} = ma = 62mg = 62(210\times10^{-9} \text{ kg})(9.80 \text{ m/s}^2) = 1.3\times10^{-4} \text{ N}$. This force is 62 times the flea's weight.

(b) $F_{\text{net}} = 140mg = 2.9\times10^{-4} \text{ N}$.

(c) Since the initial speed is zero, the maximum speed is the area under the a_x-t graph. This gives 1.2 m/s.

EVALUATE: a is much larger than g and the net external force is much larger than the flea's weight.

5.70. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the instrument and calculate the acceleration. Then use constant acceleration equations to describe the motion.

SET UP: The free-body diagram for the instrument is given in Figure 5.70. The instrument has mass $m = w/g = 1.531 \text{ kg}$.

EXECUTE: **(a)** For on the instrument, $\sum F_y = ma_y$ gives $T - mg = ma$ and $a = \dfrac{T - mg}{m} = 13.07 \text{ m/s}^2$.

$v_{0y} = 0,\ v_y = 330 \text{ m/s},\ a_y = 13.07 \text{ m/s}^2,\ t = ?$ Then $v_y = v_{0y} + a_y t$ gives $t = 25.3$ s. Consider forces on the rocket; rocket has the same a_y. Let F be the thrust of the rocket engines. $F - mg = ma$ and

$F = m(g + a) = (25{,}000 \text{ kg})(9.80 \text{ m/s}^2 + 13.07 \text{ m/s}^2) = 5.72\times10^5 \text{ N}$.

(b) $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = 4170$ m.

EVALUATE: The rocket and instrument have the same acceleration. The tension in the wire is over twice the weight of the instrument and the upward acceleration is greater than g.

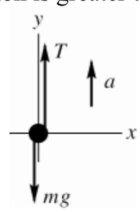


Figure 5.70

5.71. **IDENTIFY:** $a = dv/dt$. Apply $\sum \vec{F} = m\vec{a}$ to yourself.

SET UP: The reading of the scale is equal to the normal force the scale applies to you.

EXECUTE: The elevator's acceleration is

$$a = \frac{dv(t)}{dt} = 3.0 \text{ m/s}^2 + 2(0.20 \text{ m/s}^3)t = 3.0 \text{ m/s}^2 + (0.40 \text{ m/s}^3)t$$

At $t = 4.0$ s, $a = 3.0 \text{ m/s}^2 + (0.40 \text{ m/s}^3)(4.0 \text{ s}) = 4.6 \text{ m/s}^2$. From Newton's Second Law, the net force on you is $F_{\text{net}} = F_{\text{scale}} - w = ma$ and

$$F_{\text{scale}} = w + ma = (72 \text{ kg})(9.8 \text{ m/s}^2) + (72 \text{ kg})(4.6 \text{ m/s}^2) = 1040 \text{ N}$$

EVALUATE: a increases with time, so the scale reading is increasing.

5.72. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the passenger to find the maximum allowed acceleration. Then use a constant acceleration equation to find the maximum speed.

SET UP: The free-body diagram for the passenger is given in Figure 5.72.

EXECUTE: $\sum F_y = ma_y$ gives $n - mg = ma$. $n = 1.6mg$, so $a = 0.60\,g = 5.88 \text{ m/s}^2$.

$y - y_0 = 3.0 \text{ m},\ a_y = 5.88 \text{ m/s}^2,\ v_{0y} = 0$ so $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = 5.0 \text{ m/s}$.

EVALUATE: A larger final speed would require a larger value of a_y, which would mean a larger normal force on the person.

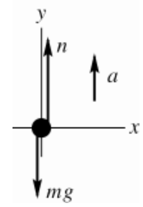


Figure 5.72

5.73. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the package. Calculate *a* and then use a constant acceleration equation to describe the motion.

SET UP: Let $+x$ be directed up the ramp.

EXECUTE: **(a)** $F_{\text{net}} = -mg\sin 37° - f_{\text{k}} = -mg\sin 37° - \mu_{\text{k}} mg\cos 37° = ma$ and

$$a = -(9.8\ \text{m/s}^2)(0.602 + (0.30)(0.799)) = -8.25\,\text{m/s}^2$$

Since we know the length of the slope, we can use $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ with $x_0 = 0$ and $v_x = 0$ at the top.

$v_0^2 = -2ax = -2(-8.25\ \text{m/s}^2)(8.0\ \text{m}) = 132\ \text{m}^2/\text{s}^2$ and $v_0 = \sqrt{132\ \text{m}^2/\text{s}^2} = 11.5\ \text{m/s}$

(b) For the trip back down the slope, gravity and the friction force operate in opposite directions to each other.

$F_{\text{net}} = -mg\sin 37° + \mu_{\text{k}} mg\cos 37° = ma$ and

$a = g(-\sin 37° + 0.30\ \cos 37°) = (9.8\ \text{m/s}^2)((-0.602) + (0.30)(0.799)) = -3.55\ \text{m/s}^2$.

Now we have $v_0 = 0,\ x_0 = -8.0\ \text{m},\ x = 0$ and $v^2 = v_0^2 + 2a(x - x_0) = 0 + 2(-3.55\ \text{m/s}^2)(-8.0\ \text{m}) = 56.8\ \text{m}^2/\text{s}^2$, so

$v = \sqrt{56.8\ \text{m}^2/\text{s}^2} = 7.54\ \text{m/s}$.

EVALUATE: In both cases, moving up the incline and moving down the incline, the acceleration is directed down the incline. The magnitude of *a* is greater when the package is going up the incline, because $mg\sin 37°$ and f_{k} are in the same direction whereas when the package is going down these two forces are in opposite directions.

5.74. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the hammer. Since the hammer is at rest relative to the bus its acceleration equals that of the bus.

SET UP: The free-body diagram for the hammer is given in Figure 5.74.

EXECUTE: $\sum F_y = ma_y$ gives $T\sin 74° - mg = 0$ so $T\sin 74° = mg$. $\sum F_x = ma_x$ gives $T\cos 74° = ma$. Divide the second equation by the first: $\dfrac{a}{g} = \dfrac{1}{\tan 74°}$ and $a = 2.8\ \text{m/s}^2$.

EVALUATE: When the acceleration increases the angle between the rope and the ceiling of the bus decreases, and the angle the rope makes with the vertical increases.

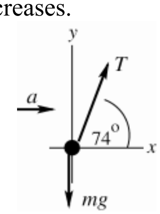


Figure 5.74

5.75. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the washer and to the crate. Since the washer is at rest relative to the crate, these two objects have the same acceleration.

SET UP: The free-body diagram for the washer is given in Figure 5.75.

EXECUTE: It's interesting to look at the string's angle measured from the perpendicular to the top of the crate. This angle is $\theta_{\text{string}} = 90° -$ angle measured from the top of the crate . The free-body diagram for the washer then leads to the following equations, using Newton's Second Law and taking the upslope direction as positive:

$$-m_{\text{w}} g\sin\theta_{\text{slope}} + T\sin\theta_{\text{string}} = m_{\text{w}} a \text{ and } T\sin\theta_{\text{string}} = m_{\text{w}}(a + g\ \sin\theta_{\text{slope}})$$

$$-m_{\text{w}} g\cos\theta_{\text{slope}} + T\cos\theta_{\text{string}} = 0 \text{ and } T\cos\theta_{\text{string}} = m_{\text{w}} g\cos\theta_{\text{slope}}$$

Dividing the two equations: $\tan\theta_{\text{string}} = \dfrac{a + g\sin\theta_{\text{slope}}}{g\cos\theta_{\text{slope}}}$

For the crate, the component of the weight along the slope is $-m_{\text{c}}g\sin\theta_{\text{slope}}$ and the normal force is $m_{\text{c}}g\cos\theta_{\text{slope}}$.

Using Newton's Second Law again: $-m_{\text{c}}g\sin\ \theta_{\text{slope}} + \mu_{\text{k}}m_{\text{c}}g\cos\theta_{\text{slope}} = m_{\text{c}}a$. $\mu_{\text{k}} = \dfrac{a + g\sin\theta_{\text{slope}}}{g\cos\theta_{\text{slope}}}$. This leads to the interesting observation that the string will hang at an angle whose tangent is equal to the coefficient of kinetic friction**:**

$$\mu_{\text{k}} = \tan\theta_{\text{string}} = \tan(90° - 68°) = \tan\ 22° = 0.40\ .$$

EVALUATE: In the limit that $\mu_{\text{k}} \to 0$, $\theta_{\text{string}} \to 0$ and the string is perpendicular to the top of the crate.

As μ_{k} increases, θ_{string} increases.

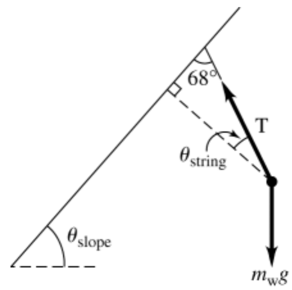


Figure 5.75

5.76. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to yourself and calculate *a*. Then use constant acceleration equations to describe the motion.

SET UP: The free-body diagram is given in Figure 5.76.

EXECUTE: **(a)** $\Sigma F_y = ma_y$ gives $n = mg\cos\alpha$. $\Sigma F_x = ma_x$ gives $mg\sin\alpha - f_{\text{k}} = ma$. Combining these two equations, we have $a = g(\sin\alpha - \mu_{\text{k}}\cos\ \alpha) = -3.094\ \text{m/s}^2$. Find your stopping distance:

$v_x = 0,\ a_x = -3.094\ \text{m/s}^2,\ v_{0x} = 20\ \text{m/s}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $x - x_0 = 64.6$ m, which is greater than 40 m. You don't stop before you reach the hole, so you fall into it.

(b) $a_x = -3.094\ \text{m/s}^2,\ x - x_0 = 40\ \text{m},\ v_x = 0$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_{0x} = 16\ \text{m/s}$.

EVALUATE: Your stopping distance is proportional to the square of your initial speed, so your initial speed is proportional to the square root of your stopping distance. To stop in 40 m instead of 64.6 m your initial speed must be $(20\ \text{m/s})\sqrt{\dfrac{40\ \text{m}}{64.6\ \text{m}}} = 16\ \text{m/s}$.

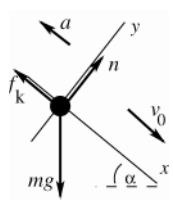


Figure 5.76

5.77. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to each block and to the rope. The key idea in solving this problem is to recognize that if the system is accelerating, the tension that block *A* exerts on the rope is different from the tension that block *B* exerts on the rope. (Otherwise the net force on the rope would be zero, and the rope couldn't accelerate.)

SET UP: Take a positive coordinate direction for each object to be in the direction of the acceleration of that object. All three objects have the same magnitude of acceleration.

EXECUTE: The Second Law equations for the three different parts of the system are**:**

Block *A* (The only horizontal forces on *A* are tension to the right, and friction to the left): $-\mu_{\text{k}}m_Ag + T_A = m_Aa.$

Block *B* (The only vertical forces on *B* are gravity down, and tension up): $m_Bg - T_B = m_Ba.$

Rope (The forces on the rope along the direction of its motion are the tensions at either end and the weight of the portion of the rope that hangs vertically): $m_R\left(\dfrac{d}{L}\right)g + T_B - T_A = m_Ra.$

To solve for a and eliminate the tensions, add the left hand sides and right hand sides of the three equations:

$-\mu_k m_A g + m_B g + m_R\left(\frac{d}{L}\right)g = (m_A + m_B + m_R)a$, or $a = g\frac{m_B + m_R(d/L) - \mu_k m_A}{(m_A + m_B + m_R)}$.

(a) When $\mu_k = 0$, $a = g\frac{m_B + m_R(d/L)}{(m_A + m_B + m_R)}$. As the system moves, d will increase, approaching L as a limit, and thus the acceleration will approach a maximum value of $a = g\frac{m_B + m_R}{(m_A + m_B + m_R)}$.

(b) For the blocks to just begin moving, $a > 0$, so solve $0 = [m_B + m_R(d/L) - \mu_s m_A]$ for d. Note that we must use static friction to find d for when the block will *begin* to move. Solving for d, $d = \frac{L}{m_R}(\mu_s m_A - m_B)$ or

$d = \frac{1.0 \text{ m}}{0.160 \text{ kg}}(0.25(2 \text{ kg}) - 0.4 \text{ kg}) = 0.63 \text{ m}.$

(c) When $m_R = 0.04$ kg, $d = \frac{1.0 \text{ m}}{0.04 \text{ kg}}(0.25(2 \text{ kg}) - 0.4 \text{ kg}) = 2.50 \text{ m}$. This is not a physically possible situation since $d > L$. The blocks won't move, no matter what portion of the rope hangs over the edge.

EVALUATE: For the blocks to move when released, the weight of B plus the weight of the rope that hangs vertically must be greater than the maximum static friction force on A, which is $\mu_s n = 4.9$ N.

5.78. **IDENTIFY:** Apply Newton's 1st law to the rope. Let m_1 be the mass of that part of the rope that is on the table, and let m_2 be the mass of that part of the rope that is hanging over the edge. ($m_1 + m_2 = m$, the total mass of the rope). Since the mass of the rope is not being neglected, the tension in the rope varies along the length of the rope. Let T be the tension in the rope at that point that is at the edge of the table.

SET UP: The free-body diagram for the hanging section of the rope is given in Figure 5.78a

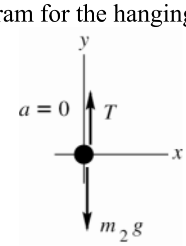


Figure 5.78a

EXECUTE:

$\sum F_y = ma_y$

$T - m_2 g = 0$

$T = m_2 g$

SET UP: The free-body diagram for that part of the rope that is on the table is given in Figure 5.78b.

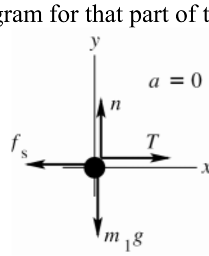


Figure 5.78b

EXECUTE:

$\sum F_y = ma_y$

$n - m_1 g = 0$

$n = m_1 g$

When the maximum amount of rope hangs over the edge the static friction has its maximum value:

$f_s = \mu_s n = \mu_s m_1 g$

$\sum F_x = ma_x$

$T - f_s = 0$

$T = \mu_s m_1 g$

Use the first equation to replace T:

$m_2 g = \mu_s m_1 g$

$m_2 = \mu_s m_1$

The fraction that hangs over is $\frac{m_2}{m} = \frac{\mu_s m_1}{m_1 + \mu_s m_1} = \frac{\mu_s}{1 + \mu_s}$.

EVALUATE: As $\mu_s \to 0$, the fraction goes to zero and as $\mu_s \to \infty$, the fraction goes to unity.

5.79. **IDENTIFY:** First calculate the maximum acceleration that the static friction force can give to the case. Apply $\sum \vec{F} = m\vec{a}$ to the case.

(a) SET UP: The static friction force is to the right in Figure 5.79a (northward) since it tries to make the case move with the truck. The maximum value it can have is $f_s = \mu_s N$.

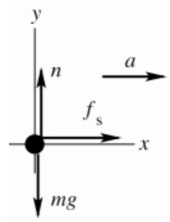


Figure 5.79a

EXECUTE:

$\sum F_y = ma_y$

$n - mg = 0$

$n = mg$

$f_s = \mu_s n = \mu_s mg$

$\sum F_x = ma_x$

$f_s = ma$

$\mu_s mg = ma$

$a = \mu_s g = (0.30)(9.80\ \text{m/s}^2) = 2.94\ \text{m/s}^2$

The truck's acceleration is less than this so the case doesn't slip relative to the truck; the case's acceleration is $a = 2.20\ \text{m/s}^2$ (northward). Then $f_s = ma = (30.0\ \text{kg})(2.20\ \text{m/s}^2) = 66\ \text{N}$, northward.

(b) IDENTIFY: Now the acceleration of the truck is greater than the acceleration that static friction can give the case. Therefore, the case slips relative to the truck and the friction is kinetic friction. The friction force still tries to keep the case moving with the truck, so the acceleration of the case and the friction force are both southward. The free-body diagram is sketched in Figure 5.79b.

SET UP:

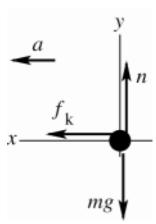


Figure 5.79b

EXECUTE:

$\sum F_y = ma_y$

$n - mg = 0$

$n = mg$

$f_k = \mu_k mg = (0.20)(30.0\ \text{kg})(9.80\ \text{m/s}^2)$

$f_k = 59\ \text{N}$, southward

EVALUATE: $f_k = ma$ implies $a = \dfrac{f_k}{m} = \dfrac{59\ \text{N}}{30.0\ \text{kg}} = 2.0\ \text{m/s}^2$. The magnitude of the acceleration of the case is less than that of the truck and the case slides toward the front of the truck. In both parts (a) and (b) the friction is in the direction of the motion and accelerates the case. Friction opposes *relative* motion between two surfaces in contact.

5.80. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the car to calculate its acceleration. Then use a constant acceleration equation to find the initial speed.

SET UP: Let $+x$ be in the direction of the car's initial velocity. The friction force f_k is then in the $-x$-direction. $192\ \text{ft} = 58.52\ \text{m}$.

EXECUTE: $n = mg$ and $f_k = \mu_k mg$. $\sum F_x = ma_x$ gives $-\mu_k mg = ma_x$ and $a_x = -\mu_k g = -(0.750)(9.80\ \text{m/s}^2) = -7.35\ \text{m/s}^2$. $v_x = 0$ (stops), $x - x_0 = 58.52\ \text{m}$. $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_{0x} = \sqrt{-2a_x(x - x_0)} = \sqrt{-2(-7.35\ \text{m/s}^2)(58.52\ \text{m})} = 29.3\ \text{m/s} = 65.5\ \text{mi/h}$. He was guilty.

EVALUATE: $x - x_0 = \dfrac{v_x^2 - v_{0x}^2}{2a_x} = -\dfrac{v_{0x}^2}{2a_x}$. If his initial speed had been 45 mi/h he would have stopped in $\left(\dfrac{45\ \text{mi/h}}{65.5\ \text{mi/h}}\right)^2 (192\ \text{ft}) = 91\ \text{ft}$.

5.81. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the point where the three wires join and also to one of the balls. By symmetry the tension in each of the 35.0 cm wires is the same.

SET UP: The geometry of the situation is sketched in Figure 5.81a. The angle ϕ that each wire makes with the vertical is given by $\sin\phi = \dfrac{12.5\text{ cm}}{47.5\text{ cm}}$ and $\phi = 15.26°$. Let T_A be the tension in the vertical wire and let T_B be the tension in each of the other two wires. Neglect the weight of the wires. The free-body diagram for the left-hand ball is given in Figure 5.81b and for the point where the wires join in Figure 5.81c. n is the force one ball exerts on the other.

EXECUTE: **(a)** $\sum F_y = ma_y$ applied to the ball gives $T_B\cos\phi - mg = 0$.

$T_B = \dfrac{mg}{\cos\phi} = \dfrac{(15.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 15.26°} = 152\text{ N}$. Then $\sum F_y = ma_y$ applied in Figure 5.81c gives $T_A - 2T_B\cos\phi = 0$ and $T_A = 2(152\text{ N})\cos\phi = 294\text{ N}$.

(b) $\sum F_x = ma_x$ applied to the ball gives $n - T_B\sin\phi = 0$ and $n = (152\text{ N})\sin 15.26° = 40.0\text{ N}$.

EVALUATE: T_A equals the total weight of the two balls.

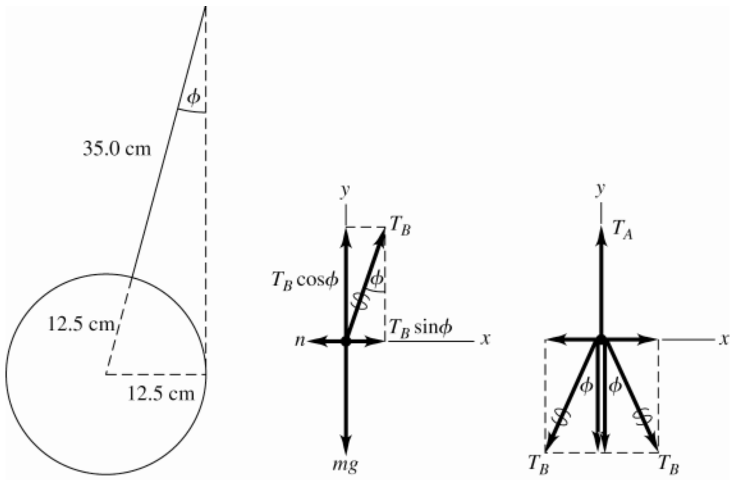


Figure 5.81a–c

5.82. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the box. Compare the acceleration of the box to the acceleration of the truck and use constant acceleration equations to describe the motion.

SET UP: Both objects have acceleration in the same direction; take this to be the $+x$-direction.

EXECUTE: If the block were to remain at rest relative to the truck, the friction force would need to cause an acceleration of 2.20 m/s^2; however, the maximum acceleration possible due to static friction is $(0.19)(9.80\text{ m/s}^2) = 1.86\text{ m/s}^2$, and so the block will move relative to the truck; the acceleration of the box would be $\mu_k g = (0.15)(9.80\text{ m/s}^2) = 1.47\text{ m/s}^2$. The difference between the distance the truck moves and the distance the box moves (*i.e.*, the distance the box moves relative to the truck) will be 1.80 m after a time

$$t = \sqrt{\frac{2\Delta x}{a_{\text{truck}} - a_{\text{box}}}} = \sqrt{\frac{2(1.80\text{ m})}{(2.20\text{ m/s}^2 - 1.47\text{ m/s}^2)}} = 2.221\text{ s}.$$

In this time, the truck moves $\frac{1}{2}a_{\text{truck}}t^2 = \frac{1}{2}(2.20\text{m/s}^2)\,(2.221\text{ s})^2 = 5.43\text{ m}$.

EVALUATE: To prevent the box from sliding off the truck the coefficient of static friction would have to be $\mu_s = (2.20\text{ m/s}^2)/g = 0.224$.

5.83. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. Forces between the blocks are related by Newton's 3rd law. The target variable is the force F. Block B is pulled to the left at constant speed, so block A moves to the right at constant speed and $a = 0$ for each block.

SET UP: The free-body diagram for block A is given in Figure 5.83a. n_{BA} is the normal force that B exerts on A. $f_{BA} = \mu_k n_{BA}$ is the kinetic friction force that B exerts on A. Block A moves to the right relative to B, and f_{BA} opposes this motion, so f_{BA} is to the left.

Note also that F acts just on B, not on A.

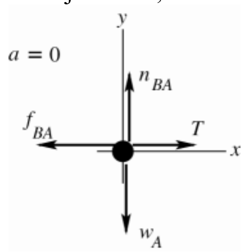


Figure 5.83a

EXECUTE:
$\sum F_y = ma_y$
$n_{BA} - w_A = 0$
$n_{BA} = 1.40\ \text{N}$
$f_{BA} = \mu_k n_{BA} = (0.30)(1.40\ \text{N}) = 0.420\ \text{N}$

$\sum F_x = ma_x$
$T - f_{BA} = 0$
$T = f_{BA} = 0.420\ \text{N}$

SET UP: The free-body diagram for block B is given in Figure 5.83b.

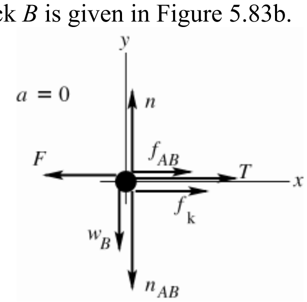


Figure 5.83b

EXECUTE: n_{AB} is the normal force that block A exerts on block B. By Newton's third law n_{AB} and n_{BA} are equal in magnitude and opposite in direction, so $n_{AB} = 1.40\ \text{N}$. f_{AB} is the kinetic friction force that A exerts on B. Block B moves to the left relative to A and f_{AB} opposes this motion, so f_{AB} is to the right.
$f_{AB} = \mu_k n_{AB} = (0.30)(1.40\ \text{N}) = 0.420\ \text{N}.$
n and f_k are the normal and friction force exerted by the floor on block B; $f_k = \mu_k n.$ Note that block B moves to the left relative to the floor and f_k opposes this motion, so f_k is to the right.
$\sum F_y = ma_y$
$n - w_B - n_{AB} = 0$
$n = w_B + n_{AB} = 4.20\ \text{N} + 1.40\ \text{N} = 5.60\ \text{N}$
Then $f_k = \mu_k n = (0.30)(5.60\ \text{N}) = 1.68\ \text{N}.$
$\sum F_x = ma_x$
$f_{AB} + T + f_k - F = 0$
$F = T + f_{AB} + f_k = 0.420\ \text{N} + 0.420\ \text{N} + 1.68\ \text{N} = 2.52\ \text{N}$

EVALUATE: Note that f_{AB} and f_{BA} are a third law action-reaction pair, so they must be equal in magnitude and opposite in direction and this is indeed what our calculation gives.

5.84. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the person to find the acceleration the PAPS unit produces. Apply constant acceleration equations to her free-fall motion and to her motion after the PAPS fires.

SET UP: We take the upward direction as positive.

EXECUTE: The explorer's vertical acceleration is $-3.7\ \text{m/s}^2$ for the first 20 s. Thus at the end of that time her vertical velocity will be $v_y = a_y t = (-3.7\ \text{m/s}^2)(20\ \text{s}) = -74\ \text{m/s}.$ She will have fallen a distance $d = v_{av} t = \left(\frac{-74\ \text{m/s}}{2}\right)(20\ \text{s}) = -740\ \text{m}$ and will thus be $1200\ \text{m} - 740\ \text{m} = 460\ \text{m}$ above the surface. Her vertical velocity must reach zero as she touches the ground; therefore, taking the ignition point of the PAPS as

$y_0 = 0,\ v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $a_y = \dfrac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \dfrac{0 - (-74\ \text{m/s})^2}{-460\ \text{m}} = 5.95\ \text{m/s}^2$, which is the vertical acceleration that must be provided by the PAPS. The time it takes to reach the ground is given by

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - (-74\ \text{m/s})}{5.95\ \text{m/s}^2} = 12.4\ \text{s}$$

Using Newton's Second Law for the vertical direction $F_{\text{PAPSv}} + mg = ma$. This gives

$$F_{\text{PAPSv}} = ma - mg = m(a + g) = (150\ \text{kg})(5.95 - (-3.7))\ \text{m/s}^2 = 1450\ \text{N}\ ,$$

which is the vertical component of the PAPS force. The vehicle must also be brought to a stop horizontally in 12.4 seconds; the acceleration needed to do this is

$$a_y = \frac{v_y - v_{0y}}{t} = \frac{0 - 33\ \text{m/s}^2}{12.4\ \text{s}} = 2.66\ \text{m/s}^2$$

and the force needed is $F_{\text{PAPSh}} = ma = (150\ \text{kg})(2.66\ \text{m/s}^2) = 400\ \text{N}$, since there are no other horizontal forces.

EVALUATE: The acceleration produced by the PAPS must bring to zero both her horizontal and vertical components of velocity.

5.85. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. Parts (a) and (b) will be done together.

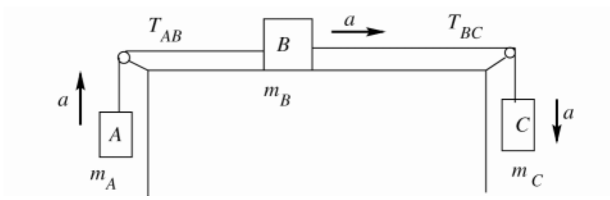


Figure 5.85a

Note that each block has the same magnitude of acceleration, but in different directions. For each block let the direction of $\vec{a}$ be a positive coordinate direction.

SET UP: The free-body diagram for block *A* is given in Figure 5.85b.

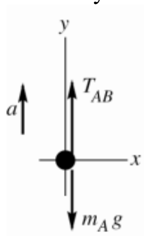


Figure 5.85b

EXECUTE:

$\sum F_y = ma_y$

$T_{AB} - m_A g = m_A a$

$T_{AB} = m_A(a + g)$

$T_{AB} = 4.00\ \text{kg}(2.00\ \text{m/s}^2 + 9.80\ \text{m/s}^2) = 47.2\ \text{N}$

SET UP: The free-body diagram for block *B* is given in Figure 5.85b.

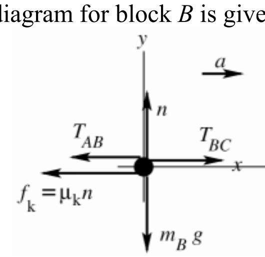


Figure 5.85c

EXECUTE:

$\sum F_y = ma_y$

$n - m_B g = 0$

$n = m_B g$

$f_k = \mu_k n = \mu_k m_B g = (0.25)(12.0\ \text{kg})(9.80\ \text{m/s}^2) = 29.4\ \text{N}$

$\sum F_x = ma_x$

$T_{BC} - T_{AB} - f_k = m_B a$

$T_{BC} = T_{AB} + f_k + m_B a = 47.2\ \text{N} + 29.4\ \text{N} + (12.0\ \text{kg})(2.00\ \text{m/s}^2)$

$T_{BC} = 100.6\ \text{N}$

SET UP: The free-body diagram for block C is sketched in Figure 5.85d.

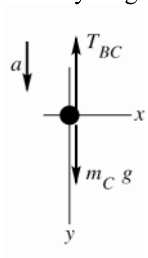


Figure 5.85d

EXECUTE:

$$\sum F_y = ma_y$$

$$m_C g - T_{BC} = m_C a$$

$$m_C(g-a) = T_{BC}$$

$$m_C = \frac{T_{BC}}{g-a} = \frac{100.6\text{ N}}{9.80\text{ m/s}^2 - 2.00\text{ m/s}^2} = 12.9\text{ kg}$$

EVALUATE: If all three blocks are considered together as a single object and $\sum \vec{F} = m\vec{a}$ is applied to this combined object, $m_C g - m_A g - \mu_k m_B g = (m_A + m_B + m_C)a$. Using the values for μ_k, m_A and m_B given in the problem and the mass m_C we calculated, this equation gives $a = 2.00\text{ m/s}^2$, which checks.

5.86. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. They have the same magnitude of acceleration, a.

SET UP: Consider positive accelerations to be to the right (up and to the right for the left-hand block, down and to the right for the right-hand block).

EXECUTE: **(a)** The forces along the inclines and the accelerations are related by $T - (100\text{ kg})g\sin 30° = (100\text{ kg})a$ and $(50\text{ kg})g\sin 53° - T = (50\text{ kg})a$, where T is the tension in the cord and a the mutual magnitude of acceleration. Adding these relations, $(50\text{ kg}\sin 53° - 100\text{ kg}\sin 30°)g = (50\text{ kg} + 100\text{ kg})a$, or $a = -0.067g$. Since a comes out negative, the blocks will slide to the left; the 100-kg block will slide down. Of course, if coordinates had been chosen so that positive accelerations were to the left, a would be $+0.067g$.

(b) $a = 0.067(9.80\text{ m/s}^2) = 0.658\text{ m/s}^2$.

(c) Substituting the value of a (including the proper sign, depending on choice of coordinates) into either of the above relations involving T yields 424 N.

EVALUATE: For part (a) we could have compared $mg\sin\theta$ for each block to determine which direction the system would move.

5.87. **IDENTIFY:** Let the tensions in the ropes be T_1 and T_2.

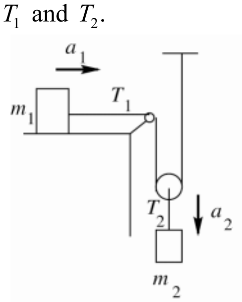


Figure 5.87a

Consider the forces on each block. In each case take a positive coordinate direction in the direction of the acceleration of that block.

SET UP: The free-body diagram for m_1 is given in Figure 5.87b.

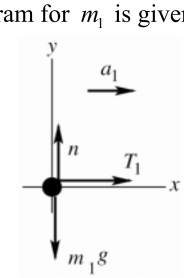


Figure 5.87b

EXECUTE:

$$\sum F_x = ma_x$$

$$T_1 = m_1 a_1$$

SET UP: The free-body diagram for m_2 is given in Figure 5.87c.

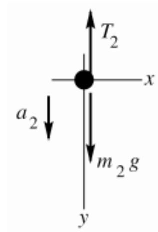


Figure 5.87c

EXECUTE:
$\sum F_y = ma_y$
$m_2g - T_2 = m_2a_2$

This gives us two equations, but there are 4 unknowns (T_1, T_2, a_1, and a_2) so two more equations are required.

SET UP: The free-body diagram for the moveable pulley (mass *m*) is given in Figure 5.87d.

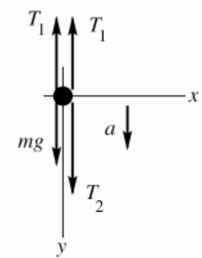


Figure 5.87d

EXECUTE:
$\sum F_y = ma_y$
$mg + T_2 - 2T_1 = ma$

But our pulleys have negligible mass, so $mg = ma = 0$ and $T_2 = 2T_1$. Combine these three equations to eliminate T_1 and T_2: $m_2g - T_2 = m_2a_2$ gives $m_2g - 2T_1 = m_2a_2$. And then with $T_1 = m_1a_1$ we have $m_2g - 2m_1a_1 = m_2a_2$.

SET UP: There are still two unknowns, a_1 and a_2. But the accelerations a_1 and a_2 are related. In any time interval, if m_1 moves to the right a distance *d*, then in the same time m_2 moves downward a distance $d/2$. One of the constant acceleration kinematic equations says $x - x_0 = v_{0x}t + \frac{1}{2}a_xt^2$, so if m_2 moves half the distance it must have half the acceleration of m_1: $a_2 = a_1/2$, or $a_1 = 2a_2$.

EXECUTE: This is the additional equation we need. Use it in the previous equation and get

$m_2g - 2m_1(2a_2) = m_2a_2$.

$a_2(4m_1 + m_2) = m_2g$

$a_2 = \dfrac{m_2g}{4m_1 + m_2}$ and $a_1 = 2a_2 = \dfrac{2m_2g}{4m_1 + m_2}$.

EVALUATE: If $m_2 \to 0$ or $m_1 \to \infty$, $a_1 = a_2 = 0$. If $m_2 >> m_1$, $a_2 = g$ and $a_1 = 2g$.

5.88. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to block *B*, to block *A* and *B* as a composite object and to block *C*. If *A* and *B* slide together all three blocks have the same magnitude of acceleration.

SET UP: If *A* and *B* don't slip the friction between them is static. The free-body diagrams for block *B*, for blocks *A* and *B*, and for *C* are given in Figures 5.88a-c. Block *C* accelerates downward and *A* and *B* accelerate to the right. In each case take a positive coordinate direction to be in the direction of the acceleration. Since block *A* moves to the right, the friction force f_s on block *B* is to the right, to prevent relative motion between the two blocks. When *C* has its largest mass, f_s has its largest value: $f_s = \mu_s n$.

EXECUTE: $\sum F_x = ma_x$ applied to the block *B* gives $f_s = m_Ba$. $n = m_Bg$ and $f_s = \mu_s m_B g$. $\mu_s m_B g = m_B a$ and $a = \mu_s g$. $\sum F_x = ma_x$ applied to blocks $A + B$ gives $T = m_{AB}a = m_{AB}\mu_s g$. $\sum F_y = ma_y$ applied to block *C* gives $m_C g - T = m_C a$. $m_C g - m_{AB}\mu_s g = m_C \mu_s g$. $m_C = \dfrac{m_{AB}\mu_s}{1 - \mu_s} = (5.00\text{ kg} + 8.00\text{ kg})\left(\dfrac{0.750}{1 - 0.750}\right) = 39.0\text{ kg}$.

EVALUATE: With no friction from the tabletop, the system accelerates no matter how small the mass of C is. If m_C is less than 39.0 kg, the friction force that A exerts on B is less than $\mu_s n$. If m_C is greater than 39.0 kg, blocks C and A have a larger acceleration than friction can give to block B and A accelerates out from under B.

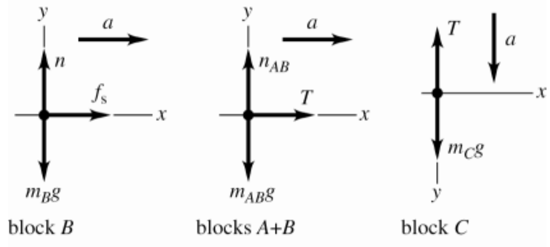


Figure 5.88

5.89. **IDENTIFY:** Apply the method of Exercise 5.19 to calculate the acceleration of each object. Then apply constant acceleration equations to the motion of the 2.00 kg object.

SET UP: After the 5.00 kg object reaches the floor, the 2.00 kg object is in free-fall, with downward acceleration g.

EXECUTE: The 2.00-kg object will accelerate upward at $g\dfrac{5.00\text{ kg}-2.00\text{ kg}}{5.00\text{ kg}+2.00\text{ kg}}=3g/7$, and the 5.00-kg object will accelerate downward at $3g/7$. Let the initial height above the ground be h_0. When the large object hits the ground, the small object will be at a height $2h_0$, and moving upward with a speed given by $v_0^2=2ah_0=6gh_0/7$. The small object will continue to rise a distance $v_0^2/2g=3h_0/7$, and so the maximum height reached will be $2h_0+3h_0/7=17h_0/7=1.46\text{ m}$ above the floor, which is 0.860 m above its initial height.

EVALUATE: The small object is 1.20 m above the floor when the large object strikes the floor, and it rises an additional 0.26 m after that.

5.90. **IDENTIFY:** Apply $\sum\vec{F}=m\vec{a}$ to the box.

SET UP: The box has an upward acceleration of $a=1.90\text{ m/s}^2$.

EXECUTE: The floor exerts an upward force n on the box, obtained from $n-mg=ma$, or $n=m(a+g)$. The friction force that needs to be balanced is

$$\mu_k n=\mu_k m(a+g)\ =(0.32)(28.0\text{ kg})(1.90\text{ m/s}^2+9.80\text{ m/s}^2)=105\text{ N}.$$

EVALUATE: If the elevator wasn't accelerating the normal force would be $n=mg$ and the friction force that would have to be overcome would be 87.8 N. The upward acceleration increases the normal force and that increases the friction force.

5.91. **IDENTIFY:** Apply $\sum\vec{F}=m\vec{a}$ to the block. The cart and the block have the same acceleration. The normal force exerted by the cart on the block is perpendicular to the front of the cart, so is horizontal and to the right. The friction force on the block is directed so as to hold the block up against the downward pull of gravity. We want to calculate the minimum a required, so take static friction to have its maximum value, $f_s=\mu_s n$.

SET UP: The free-body diagram for the block is given in Figure 5.91.

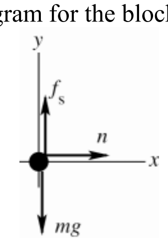


Figure 5.91

EXECUTE:

$\sum F_x=ma_x$

$n=ma$

$f_s=\mu_s n=\mu_s ma$

$\sum F_y=ma_y$

$f_s-mg=0$

$\mu_s ma=mg$

$a=g/\mu_s$

EVALUATE: An observer on the cart sees the block pinned there, with no reason for a horizontal force on it because the block is at rest relative to the cart. Therefore, such an observer concludes that $n=0$ and thus $f_s=0$, and he doesn't understand what holds the block up against the downward force of gravity. The reason for this

difficulty is that $\sum \vec{F} = m\vec{a}$ does not apply in a coordinate frame attached to the cart. This reference frame is accelerated, and hence not inertial. The smaller μ_s is, the larger a must be to keep the block pinned against the front of the cart.

5.92. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block.

SET UP: Use coordinates where $+x$ is directed down the incline.

EXECUTE: **(a)** Since the larger block (the trailing block) has the larger coefficient of friction, it will need to be pulled down the plane; *i.e.*, the larger block will not move faster than the smaller block, and the blocks will have the same acceleration. For the smaller block, $(4.00\text{ kg})g(\sin 30° - (0.25)\cos 30°) - T = (4.00\text{ kg})a$, or $11.11\text{ N} - T = (4.00\text{ kg})a$, and similarly for the larger, $15.44\text{ N} + T = (8.00\text{ kg})a$. Adding these two relations, $26.55\text{ N} = (12.00\text{ kg})a$, $a = 2.21\text{ m/s}^2$.

(b) Substitution into either of the above relations gives $T = 2.27\text{ N}$.

(c) The string will be slack. The 4.00-kg block will have $a = 2.78\text{ m/s}^2$ and the 8.00-kg block will have $a = 1.93\text{ m/s}^2$, until the 4.00-kg block overtakes the 8.00-kg block and collides with it.

EVALUATE: If the string is cut the acceleration of each block will be independent of the mass of that block and will depend only on the slope angle and the coefficient of kinetic friction. The 8.00-kg block would have a smaller acceleration even though it has a larger mass, since it has a larger μ_k.

5.93. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the block and to the plank.

SET UP: Both objects have $a = 0$.

EXECUTE: Let n_B be the normal force between the plank and the block and n_A be the normal force between the block and the incline. Then, $n_B = w\cos\theta$ and $n_A = n_B + 3w\cos\theta = 4w\cos\theta$. The net frictional force on the block is $\mu_k(n_A + n_B) = \mu_k 5w\cos\theta$. To move at constant speed, this must balance the component of the block's weight along the incline, so $3w\sin\theta = \mu_k 5w\cos\theta$, and $\mu_k = \frac{3}{5}\tan\theta = \frac{3}{5}\tan 37° = 0.452$.

EVALUATE: In the absence of the plank the block slides down at constant speed when the slope angle and coefficient of friction are related by $\tan\theta = \mu_k$. For $\theta = 36.9°$, $\mu_k = 0.75$. A smaller μ_k is needed when the plank is present because the plank provides an additional friction force.

5.94. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the ball, to m_1 and to m_2

SET UP: The free-body diagrams for the ball, m_1 and m_2 are given in Figures 5.94a-c. All three objects have the same magnitude of acceleration. In each case take the direction of $\vec{a}$ to be a positive coordinate direction.

EXECUTE: **(a)** $\sum F_y = ma_y$ applied to the ball gives $T\cos\theta = mg$. $\sum F_x = ma_x$ applied to the ball gives $T\sin\theta = ma$. Combining these two equations to eliminate T gives $\tan\theta = a/g$.

(b) $\sum F_x = ma_x$ applied to m_2 gives $T = m_2 a$. $\sum F_y = ma_y$ applied to m_1 gives $m_1 g - T = m_1 a$. Combining these two equations gives $a = \left(\dfrac{m_1}{m_1 + m_2}\right)g$. Then $\tan\theta = \dfrac{m_1}{m_1 + m_2} = \dfrac{250\text{ kg}}{1500\text{ kg}}$ and $\theta = 9.46°$.

(c) As m_1 becomes much larger than m_2, $a \to g$ and $\tan\theta \to 1$, so $\theta \to 45°$.

EVALUATE: The device requires that the ball is at rest relative to the platform; any motion swinging back and forth must be damped out. When $m_1 << m_2$ the system still accelerates, but with small a and $\theta \to 0°$.

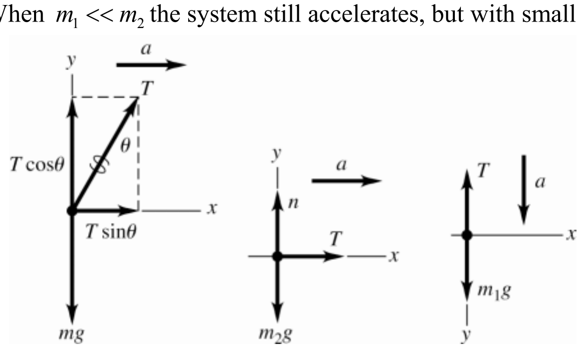


Figure 5.94a–c

5.95. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the automobile.

SET UP: The "correct" banking angle is for zero friction and is given by $\tan\beta = \dfrac{v_0^2}{gR}$, as derived in Example 5.23. Use coordinates that are vertical and horizontal, since the acceleration is horizontal.

EXECUTE: For speeds larger than v_0, a frictional force is needed to keep the car from skidding. In this case, the inward force will consist of a part due to the normal force n and the friction force f; $n\sin\beta + f\cos\beta = ma_{\text{rad}}$. The normal and friction forces both have vertical components; since there is no vertical acceleration, $n\cos\beta - f\sin\beta = mg$. Using $f = \mu_s n$ and $a_{\text{rad}} = \dfrac{v^2}{R} = \dfrac{(1.5v_0)^2}{R} = 2.25\,g\tan\beta$, these two relations become $n\sin\beta + \mu_s n\cos\beta = 2.25\,mg\tan\beta$ and $n\cos\beta - \mu_s n\sin\beta = mg$. Dividing to cancel n gives $\dfrac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta} = 2.25\tan\beta$. Solving for μ_s and simplifying yields $\mu_s = \dfrac{1.25\sin\beta\cos\beta}{1 + 1.25\sin^2\beta}$. Using $\beta = \arctan\left(\dfrac{(20\text{ m/s})^2}{(9.80\text{ m/s}^2)(120\text{ m})}\right) = 18.79°$ gives $\mu_s = 0.34$.

EVALUATE: If μ_s is insufficient, the car skids away from the center of curvature of the roadway, so the friction in inward.

5.96. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the car. The car moves in the arc of a horizontal circle, so $\vec{a} = \vec{a}_{\text{rad}},$ directed toward the center of curvature of the roadway. The target variable is the speed of the car. a_{rad} will be calculated from the forces and then v will be calculated from $a_{\text{rad}} = v^2/R$.

(a) To keep the car from sliding up the banking the static friction force is directed down the incline. At maximum speed the static friction force has its maximum value $f_s = \mu_s n$.

SET UP: The free-body diagram for the car is sketched in Figure 5.96a.

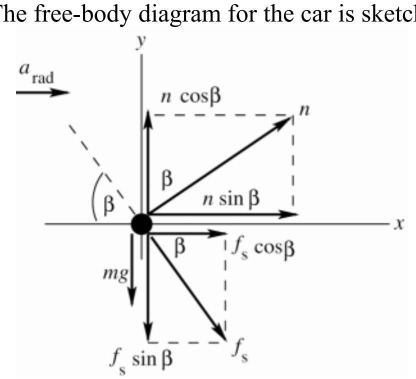


Figure 5.96a

EXECUTE:

$\sum F_y = ma_y$

$n\cos\beta - f_s\sin\beta - mg = 0$

But $f_s = \mu_s n$, so

$n\cos\beta - \mu_s n\sin\beta - mg = 0$

$$n = \frac{mg}{\cos\beta - \mu_s\sin\beta}$$

$\sum F_x = ma_x$

$n\sin\beta + \mu_s n\cos\beta = ma_{\text{rad}}$

$n(\sin\beta + \mu_s\cos\beta) = ma_{\text{rad}}$

Use the $\sum F_y$ equation to replace n:

$$\left(\frac{mg}{\cos\beta - \mu_s\sin\beta}\right)(\sin\beta + \mu_s\cos\beta) = ma_{\text{rad}}$$

$$a_{\text{rad}} = \left(\frac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta}\right)g = \left(\frac{\sin 25° + (0.30)\cos 25°}{\cos 25° - (0.30)\sin 25°}\right)(9.80\text{ m/s}^2) = 8.73\text{ m/s}^2$$

$a_{\text{rad}} = v^2/R$ implies $v = \sqrt{a_{\text{rad}}R} = \sqrt{(8.73\text{ m/s}^2)(50\text{ m})} = 21\text{ m/s}$.

(b) IDENTIFY: To keep the car from sliding *down* the banking the static friction force is directed up the incline. At the minimum speed the static friction force has its maximum value $f_s = \mu_s n$.

SET UP: The free-body diagram for the car is sketched in Figure 5.96b.

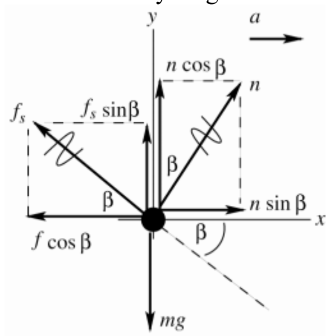


Figure 5.96b

The free-body diagram is identical to that in part (a) except that now the components of f_s have opposite directions. The force equations are all the same except for the opposite sign for terms containing μ_s.

EXECUTE: $a_{rad} = \left(\dfrac{\sin\beta - \mu_s\cos\beta}{\cos\beta + \mu_s\sin\beta}\right)g = \left(\dfrac{\sin 25° - (0.30)\cos 25°}{\cos 25° + (0.30)\sin 25°}\right)(9.80\text{ m/s}^2) = 1.43\text{ m/s}^2$

$v = \sqrt{a_{rad}R} = \sqrt{(1.43\text{ m/s}^2)(50\text{ m})} = 8.5\text{ m/s}.$

EVALUATE: For v between these maximum and minimum values, the car is held on the road at a constant height by a static friction force that is less than $\mu_s n$. When $\mu_s \to 0$, $a_{rad} = g\tan\beta$. Our analysis agrees with the result of Example 5.23 in this special case.

5.97. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to the car.

SET UP: $1\text{ mi/h} = 0.447\text{ m/s}$. The acceleration of the car is $a_{rad} = v^2/r$, directed toward the center of curvature of the roadway.

EXECUTE: **(a)** $80\text{ mi/h} = 35.7\text{ m/s}$. The centripetal force needed to keep the car on the road is provided by friction; thus $\mu_s mg = \dfrac{mv^2}{r}$ and $r = \dfrac{v^2}{\mu_s g} = \dfrac{(35.7\text{ m/s})^2}{(0.76)(9.8\text{ m/s}^2)} = 171\text{ m}$.

(b) If $\mu_s = 0.20$,

$$v = \sqrt{r\mu_s g} = \sqrt{(171\text{ m})(0.20)(9.8\text{ m/s}^2)} = 18.3\text{ m/s or about 41 mi/h}.$$

(c) If $\mu_s = 0.37$,

$$v = \sqrt{(171\text{ m})(0.37)(9.8\text{ m/s}^2)} = 24.9\text{ m/s or about 56 mi/h}$$

The speed limit is evidently designed for these conditions.

EVALUATE: The maximum safe speed is proportional to $\sqrt{\mu_s}$. $\sqrt{0.20/0.76} = 0.51$, so the maximum safe speed for wet-ice conditions is about half what it is for a dry road.

5.98. **IDENTIFY:** The analysis of this problem is the same as that of Example 5.21.

SET UP: From Example 5.21, $\tan\beta = \dfrac{a_{rad}}{g} = \dfrac{v^2}{rg}$.

EXECUTE: Solving for v in terms of β and R, $v = \sqrt{gR\tan\beta} = \sqrt{(9.80\text{ m/s}^2)(50.0)\tan 30.0°} = 16.8\text{ m/s}$, about 60.6 km/h.

EVALUATE: The greater the speed of the bus the larger will be the angle β, so T will have a larger horizontal, inward component.

5.99. **IDENTIFY** and **SET UP:** The monkey and bananas have the same mass and the tension in the rope has the same upward value at the bananas and at the monkey. Therefore, the monkey and bananas will have the same net force and hence the same acceleration, in both magnitude and direction.

EXECUTE: **(a)** For the monkey to move up, $T > mg$. The bananas also move up.

(b) The bananas and monkey move with the same acceleration and the distance between them remains constant.

(c) Both the monkey and bananas are in free fall. They have the same initial velocity and as they fall the distance between them doesn't change.

(d) The bananas will slow down at the same rate as the monkey. If the monkey comes to a stop, so will the bananas.

EVALUATE: None of these actions bring the monkey any closer to the bananas.

5.100. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$, with $f = kv$.

SET UP: Follow the analysis that leads to Eq.(5.10), except now the initial speed is $v_{0y} = 3mg/k = 3v_t$ rather than zero.

EXECUTE: The separated equation of motion has a lower limit of $3v_t$ instead of 0; specifically,

$$\int_{3v_t}^{v} \frac{dv}{v - v_t} = \ln\frac{v_t - v}{-2v_t} = \ln\left(\frac{v}{2v_t} - \frac{1}{2}\right) = -\frac{k}{m}t, \text{ or } v = 2v_t\left[\frac{1}{2} + e^{-(k/m)t}\right].$$

EVALUATE: As $t \to \infty$ the speed approaches v_t. The speed is always greater than v_t and this limit is approached from above.

5.101. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the rock.

SET UP: Equations 5.9 through 5.13 apply, but with a_0 rather than g as the initial acceleration.

EXECUTE: **(a)** The rock is released from rest, and so there is initially no resistive force and $a_0 = (18.0\text{ N})/(3.00\text{ kg}) = 6.00\text{ m/s}^2$.

(b) $(18.0\text{ N} - (2.20\text{ N}\cdot\text{s/m})\ (3.00\text{ m/s}))/(3.00\text{ kg}) = 3.80\text{ m/s}^2$.

(c) The net force must be 1.80 N, so $kv = 16.2\text{ N}$ and $v = (16.2\text{ N})/(2.20\text{ N}\cdot\text{s/m}) = 7.36\text{ m/s}$.

(d) When the net force is equal to zero, and hence the acceleration is zero, $kv_t = 18.0\text{ N}$ and $v_t = (18.0\text{ N})/(2.20\text{ N}\cdot\text{s/m}) = 8.18\text{ m/s}$.

(e) From Eq.(5.12),

$$y = (8.18\text{ m/s})\left[(2.00\text{ s}) - \frac{3.00\text{ kg}}{2.20\text{ N}\cdot\text{s/m}}\left(1 - e^{-((2.20\text{ N}\cdot\text{s/m})/(3.00\text{ kg}))(2.00\text{ s})}\right)\right] = +7.78\text{ m}.$$

From Eq. (5.10), $v = (8.18\text{ m/s})[1 - e^{-((2.20\text{ N}\cdot\text{s/m})/(3.00\text{ kg}))(2.00\text{ s})}] = 6.29\text{ m/s}$.

From Eq.(5.11), but with a_0 instead of g, $a = (6.00\text{ m/s}^2)e^{-((2.20\text{ N}\cdot\text{s/m})/(3.00\text{ kg}))(2.00\text{ s})} = 1.38\text{ m/s}^2$.

(f) $1 - \dfrac{v}{v_t} = 0.1 = e^{-(k/m)t}$ and $t = \dfrac{m}{k}\ln\ (10) = 3.14\text{ s}$.

EVALUATE: The acceleration decreases with time until it becomes zero when $v = v_t$. The speed increases with time and approaches v_t as $t \to \infty$.

5.102. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the rock. $a = \dfrac{dv}{dt}$ and $v = \dfrac{dx}{dt}$ yield differential equations that can be integrated to give $v(t)$ and $x(t)$.

SET UP: The retarding force of the surface is the only horizontal force acting.

EXECUTE: **(a)** Thus $a = \dfrac{F_{\text{net}}}{m} = \dfrac{F_R}{m} = \dfrac{-kv^{1/2}}{m} = \dfrac{dv}{dt}$ and $\dfrac{dv}{v^{1/2}} = -\dfrac{k}{m}dt$. Integrating gives $\int_{v_0}^{v} \dfrac{dv}{v^{1/2}} = -\dfrac{k}{m}\int_0^t dt$ and $2v^{1/2}\Big|_{v_0}^{v} = -\dfrac{kt}{m}$. This gives $v = v_0 - \dfrac{v_0^{1/2}kt}{m} + \dfrac{k^2t^2}{4m^2}$.

For the rock's position: $\dfrac{dx}{dt} = v_0 - \dfrac{v_0^{1/2}kt}{m} + \dfrac{k^2t^2}{4m^2}$ and $dx = v_0dt - \dfrac{v_0^{1/2}ktdt}{m} + \dfrac{k^2t^2dt}{4m^2}$.

Integrating gives $x = v_0t - \dfrac{v_0^{1/2}kt^2}{2m} + \dfrac{k^2t^3}{12m^2}$.

(b) $v = 0 = v_0 - \dfrac{v_0^{1/2}kt}{m} + \dfrac{k^2t^2}{2m^2}$. This is a quadratic equation in *t*; from the quadratic formula we can find the single solution $t = \dfrac{2mv_0^{1/2}}{k}$.

(c) Substituting the expression for *t* into the equation for *x*:

$$x = v_0 \cdot \frac{2mv_0^{1/2}}{k} - \frac{v_0^{1/2}k}{2m}\cdot\frac{4m^2v_0}{k^2} + \frac{k^2}{12m^2}\cdot\frac{8m^3v_0^{3/2}}{k^3} = \frac{2mv_0^{3/2}}{3k}$$

EVALUATE: The magnitude of the average acceleration is $a_{\text{av}} = \left|\dfrac{\Delta v}{\Delta t}\right| - \dfrac{v_0}{(2mv_0^{1/2}/k)} = \dfrac{1}{2}\dfrac{kv_0^{1/2}}{m}$. The average force is $F_{\text{av}} = ma_{\text{av}} = \frac{1}{2}kv_0^{1/2}$, which is $\frac{1}{2}$ times the initial value of the force.

5.103. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the object, with and without including the buoyancy force.

SET UP: At the terminal speed v_t, $a = 0$.

EXECUTE: Without buoyancy, $kv_t = mg$, so $k = \dfrac{mg}{v_t} = \dfrac{mg}{0.36\text{ s}}$. With buoyancy included there is the additional upward buoyancy force B, so $B + kv_t = mg$. $B = mg - kv_t = mg\left(1 - \dfrac{0.24\text{ m/s}}{0.36\text{ m/s}}\right) = mg/3$.

EVALUATE: At the terminal speed, B and $f = kv$ together equal mg. The presence of B reduces the value of f required, so the presence of B reduces the terminal speed.

5.104. IDENTIFY: The block has acceleration $a_{rad} = v^2/r$, directed to the left in the figure in the problem. Apply $\sum \vec{F} = m\vec{a}$ to the block.

SET UP: The block moves in a horizontal circle of radius $r = \sqrt{(1.25\text{ m})^2 - (1.00\text{ m})^2} = 0.75\text{ m}$. Each string makes an angle θ with the vertical. $\cos\theta = \dfrac{1.00\text{ m}}{1.25\text{ m}}$, so $\theta = 36.9°$. The free-body diagram for the block is given in Figure 5.104. Let $+x$ be to the left and let $+y$ be upward.

EXECUTE: (a) $\sum F_y = ma_y$ gives $T_u\cos\theta - T_l\cos\theta - mg = 0$.

$$T_l = T_u - \frac{mg}{\cos\theta} = 80.0\text{ N} - \frac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{\cos 36.9°} = 31.0\text{ N}.$$

(b) $\sum F_x = ma_x$ gives $(T_u + T_l)\sin\theta = m\dfrac{v^2}{r}$.

$v = \sqrt{\dfrac{r(T_u + T_l)\sin\theta}{m}} = \sqrt{\dfrac{(0.75\text{ m})(80.0\text{ N} + 31.0\text{ N})\sin 36.9°}{4.00\text{ kg}}} = 3.53\text{ m/s}$. The number of revolutions per second is

$$\frac{v}{2\pi r} = \frac{3.53\text{ m/s}}{2\pi(0.75\text{ m})} = 0.749\text{ rev/s} = 44.9\text{ rev/min}.$$

(c) If $T_l \to 0$, $T_u\cos\theta = mg$ and $T_u = \dfrac{mg}{\cos\theta} = \dfrac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{\cos 36.9°} = 49.0\text{ N}$. $T_u\sin\theta = m\dfrac{v^2}{r}$.

$v = \sqrt{\dfrac{rT_u\sin\theta}{m}} = \sqrt{\dfrac{(0.75\text{ m})(49.0\text{ N})\sin 36.9°}{4.00\text{ kg}}} = 2.35\text{ m/s}$. The number of revolutions per minute is

$$(44.9\text{ rev/min})\left(\frac{2.35\text{ m/s}}{3.53\text{ m/s}}\right) = 29.9\text{ rev/min}$$

EVALUATE: The tension in the upper string must be greater than the tension in the lower string so that together they produce an upward component of force that balances the weight of the block.

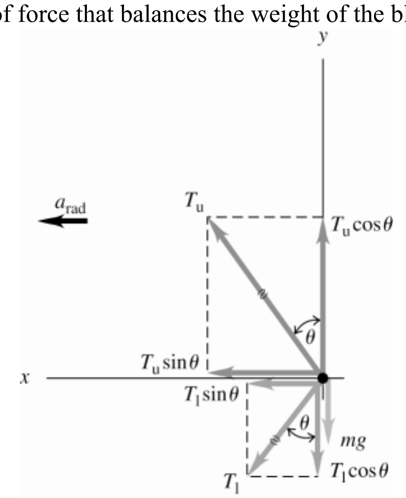


Figure 5.104

5.105. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the falling object.

SET UP: Follow the steps that lead to Eq.(5.10), except now $v_{0y} = v_0$ and is not zero.

EXECUTE: **(a)** Newton's 2nd law gives $m\dfrac{dv_y}{dt} = mg - kv_y$, where $\dfrac{mg}{k} = v_\text{t}$. $\displaystyle\int_{v_0}^{v_y} \frac{dv_y}{v_y - v_\text{t}} = -\frac{k}{m}\int_0^t dt$. This is the same expression used in the derivation of Eq. (5.10), except the lower limit in the velocity integral is the initial speed v_0 instead of zero. Evaluating the integrals and rearranging gives $v = v_0 e^{-kt/m} + v_\text{t}(1 - e^{-kt/m})$. Note that at $t = 0$ this expression says $v_y = v_0$ and at $t \to \alpha$ it says $v_y \to v_\text{t}$.

(b) The downward gravity force is larger than the upward fluid resistance force so the acceleration is downward, until the fluid resistance force equals gravity when the terminal speed is reached. The object speeds up until $v_y = v_\text{t}$. Take $+y$ to be downward. The graph is sketched in Figure 5.105a.

(c) The upward resistance force is larger than the downward gravity force so the acceleration is upward and the object slows down, until the fluid resistance force equals gravity when the terminal speed is reached. Take $+y$ to be downward. The graph is sketched in Figure 5.105b.

(d) When $v_0 = v_\text{t}$ the acceleration at $t = 0$ is zero and remains zero; the velocity is constant and equal to the terminal velocity.

EVALUATE: In all cases the speed becomes v_t as $t \to \infty$.

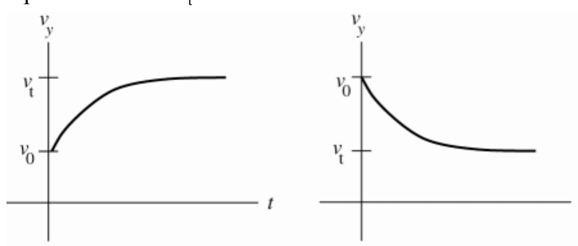


Figure 5.105a, b

5.106. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the rock.

SET UP: At the maximum height, $v_y = 0$. Let $+y$ be upward. Suppress the *y* subscripts on *v* and *a*.

EXECUTE: **(a)** To find the maximum height and time to the top without fluid resistance: $v^2 = v_0^2 + 2a(y - y_0)$ and $y - y_0 = \dfrac{v^2 - v_0^2}{2a} = \dfrac{0 - (6.0\text{ m/s})^2}{2(-9.8\text{ m/s}^2)} = 1.84\text{ m}$. $t = \dfrac{v - v_0}{a} = \dfrac{0 - 6.0\text{ m/s}}{-9.8\text{ m/s}^2} = 0.61\text{ s}$.

(b) Starting from Newton's Second Law for this situation $m\dfrac{dv}{dt} = mg - kv$. We rearrange and integrate, taking downward as positive as in the text and noting that the velocity at the top of the rock's flight is zero:

$\displaystyle\int_v^0 \frac{dv}{v - v_\text{t}} = -\frac{k}{m}t$. $\ln(v - v_\text{t})\Big|_v^0 = \ln\dfrac{-v_\text{t}}{v - v_\text{t}} = \ln\dfrac{-2.0\text{ m/s}}{-6.0\text{ m/s} - 2.0\text{ m/s}} = \ln(0.25) = -1.386$

From Eq.(5.9), $m/k = v_\text{t}/g = (2.0\text{ m/s}^2)/(9.8\text{ m/s}^2) = 0.204\text{ s}$, and $t = -\dfrac{m}{k}(-1.386) = (0.204\text{ s})(1.386) = 0.283\text{ s}$ to the top. Equation 5.10 in the text gives us $\dfrac{dx}{dt} = v_\text{t}(1 - e^{-(k/m)t}) = v_\text{t} - v_\text{t}e^{-(k/m)t}$.

$$x = \int_0^x dx = \int_0^t v_\text{t}dt - \int_0^t v_\text{t}e^{-(k/m)t}dt = v_\text{t}t + \frac{v_\text{t}m}{k}(e^{-(k/m)t} - 1).$$

$$x = (2.0\text{ m/s})(0.283\text{ s}) + (2.0\text{ m/s})(0.204\text{ s})(\text{e}^{-1.387} - 1) = 0.26\text{ m}.$$

EVALUATE: With fluid resistance present the maximum height is much less and the time to reach it is less.

5.107. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the car.

SET UP: The forces on the car are the air drag force $f_\text{D} = Dv^2$ and the rolling friction force $\mu_\text{r}mg$. Take the velocity to be in the $+x$-direction. The forces are opposite in direction to the velocity.

EXECUTE: **(a)** $\sum F_x = ma_x$ gives $-Dv^2 - \mu_\text{r}mg = ma$. We can write this equation twice, once with $v = 32\text{ m/s}$ and $a = -0.42\text{ m/s}^2$ and once with $v = 24\text{ m/s}$ and $a = -0.30\text{ m/s}^2$. Solving these two simultaneous equations in the unknowns *D* and μ_r gives $\mu_\text{r} = 0.015$ and $D = 0.36\text{ N}\cdot\text{s}^2/\text{m}^2$.

(b) $n = mg\cos\beta$ and the component of gravity parallel to the incline is $mg\sin\beta$, where $\beta = 2.2°$. For constant speed, $mg\sin 2.2° - \mu_\text{r}mg\cos 2.2° - Dv^2 = 0$. Solving for v gives $v = 29\text{ m/s}$.

(c) For angle β, $mg\sin\beta - \mu_r mg\cos\beta - Dv^2 = 0$ and $v = \sqrt{\dfrac{mg(\sin\beta - \mu_r\cos\beta)}{D}}$. The terminal speed for a falling object is derived from $Dv_t^2 - mg = 0$, so $v_t = \sqrt{mg/D}$. $v/v_t = \sqrt{\sin\beta - \mu_r\cos\beta}$. And since $\mu_r = 0.015$, $v/v_t = \sqrt{\sin\beta - (0.015)\cos\beta}$.

EVALUATE: In part (c), $v \to v_t$ as $\beta \to 90°$, since in that limit the incline becomes vertical.

5.108. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the person and to the cart.

SET UP: The apparent weight, w_{app}, which is the same as the upward force on the person exerted by the car seat.

EXECUTE: **(a)** The apparent weight is the actual weight of the person minus the centripetal force needed to keep him moving in its circular path:

$$w_{app} = mg - \frac{mv^2}{R} = (70\text{ kg})\left[(9.8\text{ m/s}^2) - \frac{(12\text{ m/s})^2}{40\text{ m}}\right] = 434\text{ N}.$$

(b) The cart will lose contact with the surface when its apparent weight is zero; i.e., when the road no longer has to exert any upward force on it: $mg - \dfrac{mv^2}{R} = 0$. $v = \sqrt{Rg} = \sqrt{(40\text{ m})(9.8\text{ m/s}^2)} = 19.8\text{ m/s}$. The answer doesn't depend on the cart's mass, because the centripetal force needed to hold it on the road is proportional to its mass and so to its weight, which provides the centripetal force in this situation.

EVALUATE: At the speed calculated in part (b), the downward force needed for circular motion is provided by gravity. For speeds greater than this more, downward force is needed and there is no source for it and the cart leaves the circular path. For speeds less than this, less downward force than gravity is needed, so the roadway must exert an upward vertical force.

5.109. **(a) IDENTIFY:** Use the information given about Jena to find the time t for one revolution of the merry-go-round. Her acceleration is a_{rad}, directed in toward the axis. Let $\vec{F}_1$ be the horizontal force that keeps her from sliding off. Let her speed be v_1 and let R_1 be her distance from the axis. Apply $\sum \vec{F} = m\vec{a}$ to Jena, who moves in uniform circular motion.

SET UP: The free-body diagram for Jena is sketched in Figure 5.109a

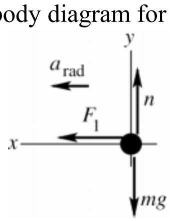


Figure 5.109a

EXECUTE:

$\sum F_x = ma_x$

$F_1 = ma_{rad}$

$F_1 = m\dfrac{v_1^2}{R_1}$, $v_1 = \sqrt{\dfrac{R_1F_1}{m}} = 1.90\text{ m/s}$

The time for one revolution is $t = \dfrac{2\pi R_1}{v_1} = 2\pi R_1\sqrt{\dfrac{m}{R_1F_1}}$. Jackie goes around once in the same time but her speed (v_2) and the radius of her circular path (R_2) are different.

$$v_2 = \frac{2\pi R_2}{t} = 2\pi R_2\left(\frac{1}{2\pi R_1}\right)\sqrt{\frac{R_1F_1}{m}} = \frac{R_2}{R_1}\sqrt{\frac{R_1F_1}{m}}.$$

IDENTIFY: Now apply $\sum \vec{F} = m\vec{a}$ to Jackie. She also moves in uniform circular motion.

SET UP: The free-body diagram for Jackie is sketched in Figure 5.109b.

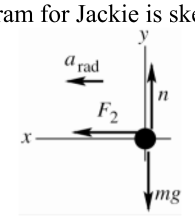


Figure 5.109b

EXECUTE:

$\sum F_x = ma_x$

$F_2 = ma_{rad}$

$$F_2 = m\frac{v_2^2}{R_2} = \left(\frac{m}{R_2}\right)\left(\frac{R_2^2}{R_1^2}\right)\left(\frac{R_1F_1}{m}\right) = \left(\frac{R_2}{R_1}\right)F_1 = \left(\frac{3.60\text{ m}}{1.80\text{ m}}\right)(60.0\text{ N}) = 120.0\text{ N}$$

(b) $F_2 = m\dfrac{v_2^2}{R_2}$, so $v_2 = \sqrt{\dfrac{F_2R_2}{m}} = \sqrt{\dfrac{(120.0\text{ N})(3.60\text{ m})}{30.0\text{ kg}}} = 3.79\text{ m/s}$

EVALUATE: Both girls rotate together so have the same period T. By Eq.(5.16), a_{rad} is larger for Jackie so the force on her is larger. Eq.(5.15) says $R_1/v_1 = R_2/v_2$ so $v_2 = v_1(R_2/R_1)$; this agrees with our result in (a).

5.110. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the passenger. The passenger has acceleration a_{rad}, directed inward toward the center of the circular path.

SET UP: The passenger's velocity is $v = 2\pi R/t = 8.80\ \text{m/s}$. The vertical component of the seat's force must balance the passenger's weight and the horizontal component must provide the centripetal force.

EXECUTE: **(a)** $F_{seat}\sin\theta = mg = 833\ \text{N}$ and $F_{seat}\cos\theta = \dfrac{mv^2}{R} = 188\ \text{N}$. Therefore

$\tan\theta = (833\ \text{N})/(188\ \text{N}) = 4.43$; $\theta = 77.3°$ above the horizontal. The magnitude of the net force exerted by the seat (note that this is not the net force on the passenger) is

$$F_{seat} = \sqrt{(833\ \text{N})^2 + (188\ \text{N})^2} = 854\ \text{N}$$

(b) The magnitude of the force is the same, but the horizontal component is reversed.

EVALUATE: At the highest point in the motion, $F_{seat} = mg - m\dfrac{v^2}{R} = 645\ \text{N}$. At the lowest point in the motion,

$F_{seat} = mg + m\dfrac{v^2}{R} = 1021\ \text{N}$. The result in parts (a) and (b) lies between these extreme values.

5.111. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the person. The person moves in a horizontal circle so his acceleration is $a_{rad} = v^2/R$, directed toward the center of the circle. The target variable is the coefficient of static friction between the person and the surface of the cylinder. $v = (0.60\ \text{rev/s})\left(\dfrac{2\pi R}{1\ \text{rev}}\right) = (0.60\ \text{rev/s})\left(\dfrac{2\pi(2.5\ \text{m})}{1\ \text{rev}}\right) = 9.425\ \text{m/s}$

(a) SET UP: The problem situation is sketched in Figure 5.111a.

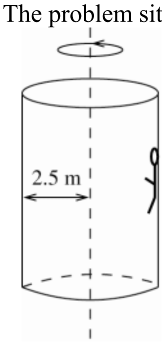


Figure 5.111a

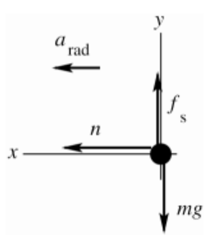


Figure 5.111b

The free-body diagram for the person is sketched in Figure 5.111b. The person is held up against gravity by the static friction force exerted on him by the wall. The acceleration of the person is a_{rad}, directed in towards the axis of rotation.

(b) EXECUTE: To calculate the minimum μ_s required, take f_s to have its maximum value, $f_s = \mu_s n$.

$\sum F_y = ma_y$

$f_s - mg = 0$

$\mu_s n = mg$

$\sum F_x = ma_x$

$n = mv^2/R$

Combine these two equations to eliminate n:

$\mu_s mv^2/R = mg$

$$\mu_s = \frac{Rg}{v^2} = \frac{(2.5\ \text{m})(9.80\ \text{m/s}^2)}{(9.425\ \text{m/s})^2} = 0.28$$

(c) EVALUATE: No, the mass of the person divided out of the equation for μ_s. Also, the smaller μ_s is, the larger v must be to keep the person from sliding down. For smaller μ_s the cylinder must rotate faster to make n larger enough.

5.112. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the combined object of motorcycle plus rider.

SET UP: The object has acceleration $a_{rad} = v^2/r$, directed toward the center of the circular path.

EXECUTE: **(a)** For the tires not to lose contact, there must be a downward force on the tires. Thus, the (downward) acceleration at the top of the sphere must exceed mg, so $m\frac{v^2}{R} > mg$, and

$v > \sqrt{gR} = \sqrt{(9.80\ \text{m/s}^2)\ (13.0\ \text{m})} = 11.3\ \text{m/s}.$

(b) The (upward) acceleration will then be $4g$, so the upward normal force must be $5mg = 5(110\ \text{kg})\ (9.80\ \text{m/s}^2) = 5390\ \text{N}.$

EVALUATE: At any nonzero speed the normal force at the bottom of the path exceeds the weight of the object.

5.113. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to your friend. Your friend moves in the arc of a circle as the car turns.

(a) Turn to the right. The situation is sketched in Figure 5.113a.

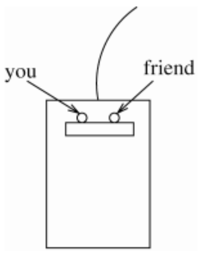


Figure 5.113a

As viewed in an inertial frame, in the absence of sufficient friction your friend doesn't make the turn completely and you move to the right toward your friend.

(b) The maximum radius of the turn is the one that makes a_{rad} just equal to the maximum acceleration that static friction can give to your friend, and for this situation f_s has its maximum value $f_s = \mu_s n$.

SET UP: The free-body diagram for your friend, as viewed by someone standing behind the car, is sketched in Figure 5.113b.

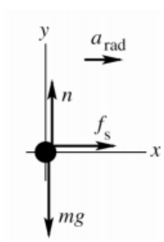


Figure 5.113b

EXECUTE:

$\sum F_y = ma_y$

$n - mg = 0$

$n = mg$

$\sum F_x = ma_x$

$f_s = ma_{rad}$

$\mu_s n = mv^2/R$

$\mu_s mg = mv^2/R$

$$R = \frac{v^2}{\mu_s g} = \frac{(20\ \text{m/s})^2}{(0.35)(9.80\ \text{m/s}^2)} = 120\ \text{m}$$

EVALUATE: The larger μ_s is, the smaller the radius R must be.

5.114. **IDENTIFY:** The tension F in the string must be the same as the weight of the hanging block, and must also provide the resultant force necessary to keep the block on the table in uniform circular motion.

SET UP: The acceleration of the block is $a_{rad} = v^2/r$, directed toward the hole.

EXECUTE: $Mg = F = m\frac{v^2}{r}$, so $v = \sqrt{grM/m}.$

EVALUATE: The larger M is the greater must be the speed v, if r remains the same.

5.115. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the circular motion of the bead. Also use Eq.(5.16) to relate a_{rad} to the period of rotation *T*.

SET UP: The bead and hoop are sketched in Figure 5.115a.

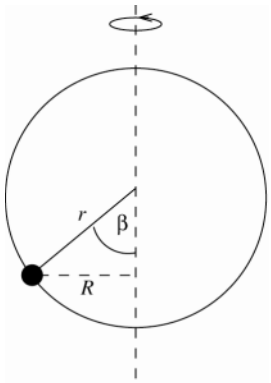


Figure 5.115a

The bead moves in a circle of radius $R = r\sin\beta$.
The normal force exerted on the bead by the hoop is radially inward.

The free-body diagram for the bead is sketched in Figure 5.115b.

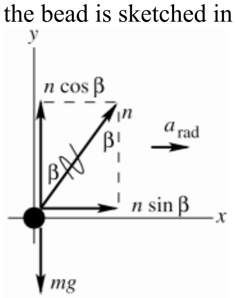


Figure 5.115b

EXECUTE:

$\sum F_y = ma_y$

$n\cos\beta - mg = 0$

$n = mg/\cos\beta$

$\sum F_x = ma_x$

$n\sin\beta = ma_{\text{rad}}$

Combine these two equations to eliminate *n*:

$$\left(\frac{mg}{\cos\beta}\right)\sin\beta = ma_{\text{rad}}$$

$$\frac{\sin\beta}{\cos\beta} = \frac{a_{\text{rad}}}{g}$$

$a_{\text{rad}} = v^2/R$ and $v = 2\pi R/T$, so $a_{\text{rad}} = 4\pi^2 R/T^2$, where *T* is the time for one revolution.

$R = r\sin\beta$, so $a_{\text{rad}} = \dfrac{4\pi^2 r\sin\beta}{T^2}$

Use this in the above equation: $\dfrac{\sin\beta}{\cos\beta} = \dfrac{4\pi^2 r\sin\beta}{T^2 g}$

This equation is satisfied by $\sin\beta = 0$, so $\beta = 0$, or by

$\dfrac{1}{\cos\beta} = \dfrac{4\pi^2 r}{T^2 g}$, which gives $\cos\beta = \dfrac{T^2 g}{4\pi^2 r}$

(a) 4.00 rev/s implies $T = (1/4.00)\text{ s} = 0.250\text{ s}$

Then $\cos\beta = \dfrac{(0.250\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2(0.100\text{ m})}$ and $\beta = 81.1°$.

(b) This would mean $\beta = 90°$. But $\cos 90° = 0$, so this requires $T \to 0$. So β approaches $90°$ as the hoop rotates very fast, but $\beta = 90°$ is not possible.

(c) 1.00 rev/s implies $T = 1.00\text{ s}$

The $\cos\beta = \dfrac{T^2 g}{4\pi^2 r}$ equation then says $\cos\beta = \dfrac{(1.00\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2(0.100\text{ m})} = 2.48$, which is not possible. The only way to have the $\sum \vec{F} = m\vec{a}$ equations satisfied is for $\sin\beta = 0$. This means $\beta = 0$; the bead sits at the bottom of the hoop.

Evaluate: $\beta \to 90°$ as $T \to 0$ (hoop moves faster). The largest value T can have is given by $T^2 g/(4\pi^2 r) = 1$ so $T = 2\pi\sqrt{r/g} = 0.635$ s. This corresponds to a rotation rate of $(1/0.635)$ rev/s $= 1.58$ rev/s. For a rotation rate less than 1.58 rev/s, $\beta = 0$ is the only solution and the bead sits at the bottom of the hoop. Part (c) is an example of this.

5.116. **Identify:** $a_x = \dfrac{d^2x}{dt^2}$ and $a_y = \dfrac{d^2y}{dt^2}$. Then apply $\sum \vec{F} = m\vec{a}$ to calculate the components of the net force.

Set Up: The components of $\vec{F}$ determine its magnitude and direction.

Execute: **(a)** Differentiating twice, $a_x = -6\beta t$ and $a_y = -2\delta$, so

$F_x = ma_x = (2.20\text{ kg})(-0.72\text{ N/s})t = -(1.58\text{ N/s})t$ and $F_y = ma_y = (2.20\text{ kg})(-2.00\text{ m/s}^2) = -4.40\text{ N}$.

(b) The graph is given in Figure 5.116.

(c) At $t = 3.00$ s, $F_x = -4.75$ N and $F_y = -4.40$ N, so $F = \sqrt{(-4.75\text{ N})^2 + (-4.40\text{ N})^2} = 6.48\text{ N}$ at an angle of $\arctan\left(\dfrac{-4.40}{-4.75}\right) = 223°$.

Evaluate: F_y is constant and negative. F_x is zero at $t = 0$ and becomes increasingly more negative as t increases.

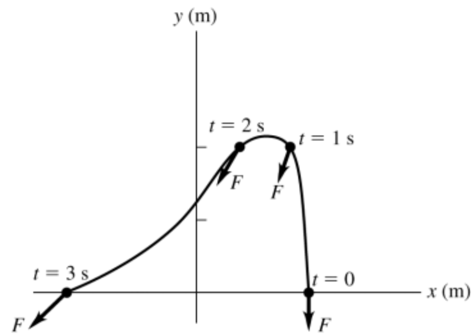


Figure 5.116

5.117. **Identify:** The velocity is tangent to the path. The acceleration has a tangential component when the speed is changing and a radial component when the path is curving.

Set Up: $\vec{a}_{\text{rad}}$ is toward the center of curvature of the path. $\vec{a}_{\text{tan}}$ is parallel to $\vec{v}$ when the speed is increasing and antiparallel to $\vec{v}$ when the speed is decreasing. The net force $\vec{F}$ is proportional to $\vec{a}$.

Execute: The diagram is sketched in Figure 5.117.

Evaluate: $\vec{v}$, $\vec{a}$, and $\vec{F}$ all change during the motion.

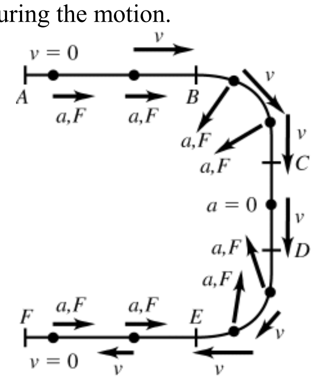


Figure 5.117

5.118. **Identify:** Apply $\sum \vec{F} = m\vec{a}$ to the car. It has acceleration $\vec{a}_{\text{rad}}$, directed toward the center of the circular path.

Set Up: The analysis is the same as in Example 5.24.

Execute: **(a)** $F_A = m\left(g + \dfrac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{ m/s}^2 + \dfrac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = 61.8\text{ N}.$

(b) $F_B = m\left(g - \dfrac{v^2}{R}\right) = (1.60\text{ kg})\left(9.80\text{ m/s}^2 - \dfrac{(12.0\text{ m/s})^2}{5.00\text{ m}}\right) = -30.4\text{ N}.$, where the minus sign indicates that the track pushes *down* on the car. The magnitude of this force is 30.4 N.

Evaluate: $|F_A| > |F_B|$. $|F_A| - 2mg$.

5.119. **IDENTIFY:** The analysis is the same as for Problem 5.96.

SET UP: The speed is related to the period by $v = 2\pi R/T = 2\pi h(\tan\beta)/T$, or $T = 2\pi h(\tan\beta)/v$.

EXECUTE: The maximum and minimum speeds are the same as those found in Problem 5.96,

$$v_{\max} = \sqrt{gh\tan\beta\frac{\cos\beta + \mu_s\sin\beta}{\sin\beta - \mu_s\cos\beta}} \text{ and } v_{\min} = \sqrt{gh\tan\beta\frac{\cos\beta - \mu_s\sin\beta}{\sin\beta + \mu_s\cos\beta}}.$$

The minimum and maximum values of the period T are then

$$T_{\min} = 2\pi\sqrt{\frac{h\tan\beta}{g}\frac{\sin\beta - \mu_s\cos\beta}{\cos\beta + \mu_s\sin\beta}} \text{ and } T_{\max} = 2\pi\sqrt{\frac{h\tan\beta}{g}\frac{\sin\beta + \mu_s\cos\beta}{\cos\beta - \mu_s\sin\beta}}.$$

EVALUATE: For $\mu_s = 0$ the results for the speeds reduce to $v_{\min} = v_{\max} = \sqrt{gh}$. $h = \frac{R}{\tan\beta}$. The result for v then agrees with the result in Example 5.23, if we take into account that in this problem β is measured from the vertical whereas in Example 5.23 it is measured relative to the horizontal.

5.120. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to the block and to the wedge.

SET UP: For both parts, take the x-direction to be horizontal and positive to the right, and the y-direction to be vertical and positive upward. The normal force between the block and the wedge is n; the normal force between the wedge and the horizontal surface will not enter, as the wedge is presumed to have zero vertical acceleration. The horizontal acceleration of the wedge is A, and the components of acceleration of the block are a_x and a_y.

EXECUTE: **(a)** The equations of motion are then $MA = -n\sin\alpha$, $ma_x = n\sin\alpha$ and $ma_y = n\cos\alpha - mg$. Note that the normal force gives the wedge a negative acceleration; the wedge is expected to move to the left. These are three equations in four unknowns, A, a_x, a_y and n. Solution is possible with the imposition of the relation between A, a_x and a_y. An observer on the wedge is not in an inertial frame, and should not apply Newton's laws, but the kinematic relation between the components of acceleration are not so restricted. To such an observer, the vertical acceleration of the block is a_y, but the horizontal acceleration of the block is $a_x - A$. To this observer, the block descends at an angle α, so the relation needed is $\frac{a_y}{a_x - A} = -\tan\alpha$. At this point, algebra is unavoidable. A possible approach is to eliminate a_x by noting that $a_x = -\frac{M}{m}A$, using this in the kinematic constraint to eliminate a_y and then eliminating n. The results are:

$$A = \frac{-gm}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_x = \frac{gM}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

$$a_y = \frac{-g(M+\mathrm{m})\tan\alpha}{(M+m)\tan\alpha + (M/\tan\alpha)}$$

(b) When $M >> m, A \to 0$, as expected (the large block won't move). Also,

$a_x \to \frac{g}{\tan\alpha + (1/\tan\alpha)} = g\frac{\tan\alpha}{\tan^2\alpha + 1} = g\sin\alpha\cos\alpha$ which is the acceleration of the block ($g\sin\alpha$ in this case), with the factor of $\cos\alpha$ giving the horizontal component. Similarly, $a_y \to -g\sin^2\alpha$.

(c) The trajectory is a spiral.

EVALUATE: If $m >> M$, our general results give $a_x = 0$ and $a_y = -g$. The massive block accelerates straight downward, as if it were in free-fall.

5.121. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to the block and to the wedge.

SET UP: From Problem 5.120, $ma_x = n\sin\alpha$ and $ma_y = n\cos\alpha - mg$ for the block. $a_y = 0$ gives $a_x = g\tan\alpha$.

EXECUTE: If the block is not to move vertically, both the block and the wedge have this horizontal acceleration and the applied force must be $F = (M+m)a = (M+m)g\tan\alpha$.

EVALUATE: $F \to 0$ as $\alpha \to 0$ and $F \to \infty$ as $\alpha \to \infty$.

5.122. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$.

SET UP: Let $+x$ be directed up the ramp.

EXECUTE: The normal force that the ramp exerts on the box will be $n = w\cos\alpha - T\sin\alpha$. The rope provides a force of $T\cos\theta$ up the ramp, and the component of the weight down the ramp is $w\sin\alpha$. Thus, the net force up the ramp is

$$F = T\cos\theta - w\sin\alpha - \mu_k(w\cos\alpha - T\sin\theta) = T(\cos\theta + \mu_k\sin\theta) - w(\sin\alpha + \mu_k\cos\alpha)$$

The acceleration will be the greatest when the first term in parentheses is greatest and this occurs when $\tan\theta = \mu_k$.

EVALUATE: Small θ means F is more nearly in the direction of the motion. But $\theta \to 90°$ means F is directed to reduce the normal force and thereby reduce friction. The optimum value of θ is somewhere in between and depends on μ_k. When $\mu_k = 0$, the optimum value of θ is $\theta = 0°$.

5.123. **IDENTIFY:** Use the results of Problem 5.44.

SET UP: $f(x)$ is a minimum when $\dfrac{df}{dx} = 0$ and $\dfrac{d^2 f}{dx^2} > 0$.

EXECUTE: **(a)** $F = \mu_k w/(\cos\theta + \mu_k\sin\theta)$

(b) The graph of F versus θ is given in Figure 5.123.

(c) F is minimized at $\tan\theta = \mu_k$. For $\mu_k = 0.25$, $\theta = 14.0°$.

EVALUATE: Small θ means F is more nearly in the direction of the motion. But $\theta \to 90°$ means F is directed to reduce the normal force and thereby reduce friction. The optimum value of θ is somewhere in between and depends on μ_k.

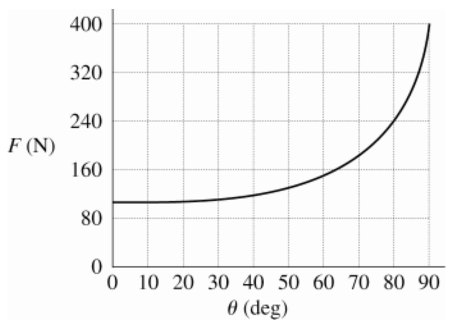


Figure 5.123

5.124. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the ball. At the terminal speed, $a = 0$.

SET UP: For convenience, take the positive direction to be down, so that for the baseball released from rest, the acceleration and velocity will be positive, and the speed of the baseball is the same as its positive component of velocity. Then the resisting force, directed against the velocity, is upward and hence negative.

EXECUTE: **(a)** The free-body diagram for the falling ball is sketched in Figure 5.124.

(b) Newton's Second Law is then $ma = mg - Dv^2$. Initially, when $v = 0$, the acceleration is g, and the speed increases. As the speed increases, the resistive force increases and hence the acceleration decreases. This continues as the speed approaches the terminal speed.

(c) At terminal velocity, $a = 0$, so $v_t = \sqrt{\dfrac{mg}{D}}$ in agreement with Eq. (5.13).

(d) The equation of motion may be rewritten as $\dfrac{dv}{dt} = \dfrac{g}{v_t^2}(v_t^2 - v^2)$. This is a separable equation and may be expressed as $\displaystyle\int \frac{dv}{v_t^2 - v^2} = \frac{g}{v_t^2}\int dt$ or $\dfrac{1}{v_t}\operatorname{arctanh}\left(\dfrac{v}{v_t}\right) = \dfrac{gt}{v_t^2}$. $v = v_t \tanh\left(gt/v_t\right)$.

EVALUATE: $\tanh x = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$. At $t \to 0$, $\tanh(gt/v_t) \to 0$ and $v \to 0$. At $t \to \infty$, $\tanh(gt/v_t) \to \infty$ and $v \to v_t$.

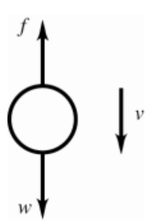


Figure 5.124

5.125. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each of the three masses and to the pulley *B*.

SET UP: Take all accelerations to be positive downward. The equations of motion are straightforward, but the kinematic relations between the accelerations, and the resultant algebra, are not immediately obvious. If the acceleration of pulley *B* is a_B, then $a_B = -a_3$, and a_B is the average of the accelerations of masses 1 and 2, or $a_1 + a_2 = 2a_B = -2a_3$.

EXECUTE: **(a)** There can be no net force on the massless pulley *B*, so $T_C = 2T_A$. The five equations to be solved are then $m_1 g - T_A = m_1 a_1$, $m_2 g - T_A = m_2 a_2$, $m_3 g - T_C = m_3 a_3$, $a_1 + a_2 + 2a_3 = 0$ and $2T_A - T_C = 0$. These are five equations in five unknowns, and may be solved by standard means.

The accelerations a_1 and a_2 may be eliminated by using $2a_3 = -(a_1 + a_2) = -(2g - T_A((1/m_1) + (1/m_2)))$.

The tension T_A may be eliminated by using $T_A = (1/2)T_C = (1/2)m_3(g - a_3)$.

Combining and solving for a_3 gives $a_3 = g\dfrac{-4m_1m_2 + m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$.

(b) The acceleration of the pulley *B* has the same magnitude as a_3 and is in the opposite direction.

(c) $a_1 = g - \dfrac{T_A}{m_1} = g - \dfrac{T_C}{2m_1} = g - \dfrac{m_3}{2m_1}(g - a_3)$. Substituting the above expression for a_3 gives

$$a_1 = g\frac{4m_1m_2 - 3m_2m_3 + m_1m_3}{4m_1m_2 + m_2m_3 + m_1m_3}.$$

(d) A similar analysis (or, interchanging the labels 1 and 2) gives $a_2 = g\dfrac{4m_1m_2 - 3m_1m_3 + m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$.

(e), **(f)** Once the accelerations are known, the tensions may be found by substitution into the appropriate equation of motion, giving $T_A = g\dfrac{4m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$, $T_C = g\dfrac{8m_1m_2m_3}{4m_1m_2 + m_2m_3 + m_1m_3}$.

(g) If $m_1 = m_2 = m$ and $m_3 = 2m$, all of the accelerations are zero, $T_C = 2mg$ and $T_A = mg$. All masses and pulleys are in equilibrium, and the tensions are equal to the weights they support, which is what is expected.

EVALUATE: It is useful to consider special cases. For example, when $m_1 = m_2 >> m_3$ our general result gives $a_1 = a_2 = +g$ and $a_3 = -g$.

5.126. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each block. The tension in the string is the same at both ends. If $T < w$ for a block, that block remains at rest.

SET UP: In all cases, the tension in the string will be half of *F*.

EXECUTE: **(a)** $F/2 = 62$ N, which is insufficient to raise either block; $a_1 = a_2 = 0$.

(b) $F/2 = 62$ N. The larger block (of weight 196 N) will not move, so $a_1 = 0$, but the smaller block, of weight 98 N, has a net upward force of 49 N applied to it, and so will accelerate upwards with $a_2 = \dfrac{49\text{ N}}{10.0\text{ kg}} = 4.9\text{ m/s}^2$.

(c) $F/2 = 212$ N, so the net upward force on block *A* is 16 N and that on block *B* is 114 N, so

$$a_1 = \frac{16\text{ N}}{20.0\text{ kg}} = 0.8\text{ m/s}^2 \text{ and } a_2 = \frac{114\text{ N}}{10.0\text{ kg}} = 11.4\text{ m/s}^2.$$

EVALUATE: The two blocks need not have accelerations with the same magnitudes.

5.127. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the ball at each position.

SET UP: When the ball is at rest, $a = 0$. When the ball is swinging in an arc it has acceleration component $a_{\text{rad}} = \dfrac{v^2}{R}$, directed inward.

EXECUTE: Before the horizontal string is cut, the ball is in equilibrium, and the vertical component of the tension force must balance the weight, so $T_A \cos\beta = w$ or $T_A = w/\cos\beta$. At point *B*, the ball is not in equilibrium; its speed is instantaneously 0, so there is no radial acceleration, and the tension force must balance the radial component of the weight, so $T_B = w\cos\beta$ and the ratio $(T_B/T_A) = \cos^2\beta$.

EVALUATE: At point *B* the net force on the ball is not zero; the ball has a tangential acceleration.

WORK AND KINETIC ENERGY

6

6.1. IDENTIFY: Apply Eq.(6.2).

SET UP: The bucket rises slowly, so the tension in the rope may be taken to be the bucket's weight.

EXECUTE: **(a)** $W = Fs = mgs = (6.75\text{ kg})\,(9.80\text{ m/s}^2)(4.00\text{ m}) = 265\text{ J}.$

(b) Gravity is directed opposite to the direction of the bucket's motion, so Eq.(6.2) gives the negative of the result of part (a), or -265 J.

(c) The total work done on the bucket is zero.

EVALUATE: When the force is in the direction of the displacement, the force does positive work. When the force is directed opposite to the displacement, the force does negative work.

6.2. IDENTIFY: In each case the forces are constant and the displacement is along a straight line, so $W = Fs\cos\phi$.

SET UP: In part (a), when the cable pulls horizontally $\phi = 0°$ and when it pulls at $35.0°$ above the horizontal $\phi = 35.0°$. In part (b), if the cable pulls horizontally $\phi = 180°$. If the cable pulls on the car at $35.0°$ above the horizontal it pulls on the truck at $35.0°$ below the horizontal and $\phi = 145.0°$. For the gravity force $\phi = 90°$, since the force is vertical and the displacement is horizontal.

EXECUTE: **(a)** When the cable is horizontal, $W = (850\text{ N})(5.00\times10^3\text{ m})\cos 0° = 4.25\times10^6\text{ J}$. When the cable is $35.0°$ above the horizontal, $W = (850\text{ N})(5.00\times10^3\text{ m})\cos 35.0° = 3.48\times10^6\text{ J}$.

(b) $\cos180° = -\cos0°$ and $\cos145.0° = -\cos35.0°$, so the answers are $-4.26\times10^6\text{ J}$ and $-3.48\times10^6\text{ J}$.

(c) Since $\cos\phi = \cos90° = 0$, $W = 0$ in both cases.

EVALUATE: If the car and truck are taken together as the system, the tension in the cable does no net work.

6.3. IDENTIFY: Each force can be used in the relation $W = F_{\|}s = (F\cos\phi)s$ for parts (b) through (d). For part (e), apply the net work relation as $W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f$.

SET UP: In order to move the crate at constant velocity, the worker must apply a force that equals the force of friction, $F_{\text{worker}} = f_{\text{k}} = \mu_{\text{k}} n.$

EXECUTE: **(a)** The magnitude of the force the worker must apply is:

$$F_{\text{worker}} = f_{\text{k}} = \mu_{\text{k}} n = \mu_{\text{k}} mg = (0.25)(30.0\text{ kg})(9.80\text{ m/s}^2) = 74\text{ N}$$

(b) Since the force applied by the worker is horizontal and in the direction of the displacement, $\phi = 0°$ and the work is:

$$W_{\text{worker}} = (F_{\text{worker}}\cos\phi)s = [(74\text{ N})(\cos0°)](4.5\text{ m}) = +333\text{ J}$$

(c) Friction acts in the direction opposite of motion, thus $\phi = 180°$ and the work of friction is:

$$W_f = (f_{\text{k}}\cos\phi)s = [(74\text{ N})(\cos180°)](4.5\text{ m}) = -333\text{ J}$$

(d) Both gravity and the normal force act perpendicular to the direction of displacement. Thus, neither force does any work on the crate and $W_{\text{grav}} = W_n = 0.0\text{ J}.$

(e) Substituting into the net work relation, the net work done on the crate is:

$$W_{\text{net}} = W_{\text{worker}} + W_{\text{grav}} + W_n + W_f = +333\text{ J} + 0.0\text{ J} + 0.0\text{ J} - 333\text{ J} = 0.0\text{ J}$$

EVALUATE: The net work done on the crate is zero because the two contributing forces, F_{worker} and F_f, are equal in magnitude and opposite in direction.

6.4. IDENTIFY: The forces are constant so Eq.(6.2) can be used to calculate the work. Constant speed implies $a = 0$. We must use $\sum\vec{F} = m\vec{a}$ applied to the crate to find the forces acting on it.

(a) SET UP: The free-body diagram for the crate is given in Figure 6.4.

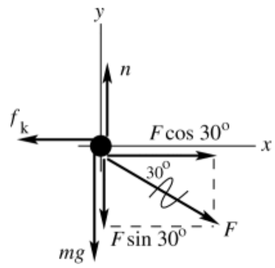


Figure 6.4

EXECUTE: $\sum F_y = ma_y$

$n - mg - F\sin 30° = 0$

$n = mg + F\sin 30°$

$f_k = \mu_k n = \mu_k mg + F\mu_k \sin 30°$

$\sum F_x = ma_x$

$F\cos 30° - f_k = 0$

$F\cos 30° - \mu_k mg - \mu_k \sin 30° F = 0$

$$F = \frac{\mu_k mg}{\cos 30° - \mu_k \sin 30°} = \frac{0.25(30.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 30° - (0.25)\sin 30°} = 99.2\text{ N}$$

(b) $W_F = (F\cos\phi)s = (99.2\text{ N})(\cos 30°)(4.5\text{ m}) = 387\text{ J}$

($F\cos 30°$ is the horizontal component of $\vec{F}$; the work done by $\vec{F}$ is the displacement times the component of $\vec{F}$ in the direction of the displacement.)

(c) We have an expression for f_k from part (a):

$$f_k = \mu_k(mg + F\sin 30°) = (0.250)[(30.0\text{ kg})(9.80\text{ m/s}^2) + (99.2\text{ N})(\sin 30°)] = 85.9\text{ N}$$

$\phi = 180°$ since f_k is opposite to the displacement. Thus $W_f = (f_k\cos\phi)s = (85.9\text{ N})(\cos 180°)(4.5\text{ m}) = -387\text{ J}$

(d) The normal force is perpendicular to the displacement so $\phi = 90°$ and $W_n = 0$. The gravity force (the weight) is perpendicular to the displacement so $\phi = 90°$ and $W_w = 0$

(e) $W_{tot} = W_F + W_f + W_n + W_w = +387\text{ J} + (-387\text{ J}) = 0$

EVALUATE: Forces with a component in the direction of the displacement do positive work, forces opposite to the displacement do negative work and forces perpendicular to the displacement do zero work. The total work, obtained as the sum of the work done by each force, equals the work done by the net force. In this problem, $F_{net} = 0$ since $a = 0$ and $W_{tot} = 0$, which agrees with the sum calculated in part (e).

6.5. IDENTIFY: The gravity force is constant and the displacement is along a straight line, so $W = Fs\cos\phi$.

SET UP: The displacement is upward along the ladder and the gravity force is downward, so $\phi = 180.0° - 30.0° = 150.0°$. $w = mg = 735\text{ N}$.

EXECUTE: **(a)** $W = (735\text{ N})(2.75\text{ m})\cos 150.0° = -1750\text{ J}$.

(b) No, the gravity force is independent of the motion of the painter.

EVALUATE: Gravity is downward and the vertical component of the displacement is upward, so the gravity force does negative work.

6.6. IDENTIFY and SET UP: $W_F = (F\cos\phi)s$, since the forces are constant. We can calculate the total work by summing the work done by each force. The forces are sketched in Figure 6.6.

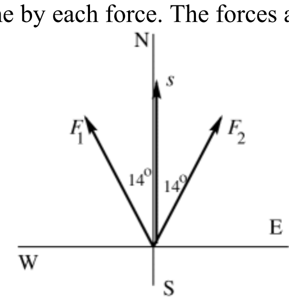


Figure 6.6

EXECUTE: $W_1 = F_1 s\cos\phi_1$

$W_1 = (1.80\times 10^6\text{ N})(0.75\times 10^3\text{ m})\cos 14°$

$W_1 = 1.31\times 10^9\text{ J}$

$W_2 = F_2 s\cos\phi_2 = W_1$

$W_{tot} = W_1 + W_2 = 2(1.31\times 10^9\text{ J}) = 2.62\times 10^9\text{ J}$

EVALUATE: Only the component $F\cos\phi$ of force in the direction of the displacement does work. These components are in the direction of $\vec{s}$ so the forces do positive work.

6.7. **IDENTIFY:** All forces are constant and each block moves in a straight line. so $W = Fs\cos\phi$. The only direction the system can move at constant speed is for the 12.0 N block to descend and the 20.0 N block to move to the right.

SET UP: Since the 12.0 N block moves at constant speed, $a = 0$ for it and the tension T in the string is $T = 12.0\text{ N}$. Since the 20.0 N block moves to the right at constant speed the friction force f_k on it is to the left and $f_k = T = 12.0\text{ N}$.

EXECUTE: **(a)** (i) $\phi = 0°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 0° = 9.00\text{ J}$. (ii) $\phi = 180°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 180° = -9.00\text{ J}$.

(b) (i) $\phi = 90°$ and $W = 0$. (ii) $\phi = 0°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 0° = 9.00\text{ J}$. (iii) $\phi = 180°$ and $W = (12.0\text{ N})(0.750\text{ m})\cos 180° = -9.00\text{ J}$. (iv) $\phi = 90°$ and $W = 0$.

(c) $W_{\text{tot}} = 0$ for each block.

EVALUATE: For each block there are two forces that do work, and for each block the two forces do work of equal magnitude and opposite sign. When the force and displacement are in opposite directions, the work done is negative.

6.8. **IDENTIFY:** Apply Eq.(6.5).

SET UP: $\hat{\boldsymbol{i}} \cdot \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}} \cdot \hat{\boldsymbol{j}} = 1$ and $\hat{\boldsymbol{i}} \cdot \hat{\boldsymbol{j}} = \hat{\boldsymbol{j}} \cdot \hat{\boldsymbol{i}} = 0$

EXECUTE: The work you do is $\vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}} = ((30\text{ N})\hat{\boldsymbol{i}} - (40\text{ N})\hat{\boldsymbol{j}}) \cdot ((-9.0\text{ m})\hat{\boldsymbol{i}} - (3.0\text{ m})\hat{\boldsymbol{j}})$

$\vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{s}} = (30\text{ N})(-9.0\text{ m}) + (-40\text{ N})(-3.0\text{ m}) = -270\text{ N}\cdot\text{m} + 120\text{ N}\cdot\text{m} = -150\text{ J}$.

EVALUATE: The x-component of $\vec{\boldsymbol{F}}$ does negative work and the y-component of $\vec{\boldsymbol{F}}$ does positive work. The total work done by $\vec{\boldsymbol{F}}$ is the sum of the work done by each of its components.

6.9. **IDENTIFY:** Apply Eq.(6.2) or (6.3).

SET UP: The gravity force is in the $-y$-direction , so $\vec{\boldsymbol{F}}_{mg} \cdot \vec{\boldsymbol{s}} = -mg(y_2 - y_1)$

EXECUTE: **(a)** (i) Tension force is always perpendicular to the displacement and does no work.
(ii) Work done by gravity is $-mg(y_2 - y_1)$. When $y_1 = y_2$, $W_{mg} = 0$.

(b) (i) Tension does no work. (ii) Let l be the length of the string. $W_{mg} = -mg(y_2 - y_1) = -mg(2l) = -25.1\text{ J}$

EVALUATE: In part (b) the displacement is upward and the gravity force is downward, so the gravity force does negative work.

6.10. **IDENTIFY:** $K = \frac{1}{2}mv^2$

SET UP: $65\text{ mi/h} = 29.1\text{ m/s}$

EXECUTE: **(a)** $K = \frac{1}{2}(750\text{ kg})(29.1\text{ m/s})^2 = 3.18\times10^5\text{ J}$

(b) $K_1 = \frac{1}{2}mv_1^2$. $K_2 = \frac{1}{2}mv_2^2$, with $v_2 = v_1/2$, so $K_2 = \frac{1}{2}m(v_1/2)^2 = \frac{1}{4}(\frac{1}{2}mv_1^2) = K_1/4$. The change in kinetic energy is a decrease of $\frac{3}{4}K_1$.

(c) $K_2 = \frac{1}{2}K_1$. $\dfrac{K}{v^2} = \dfrac{m}{2} =$ constant , so $\dfrac{K_1}{v_1^2} = \dfrac{K_2}{v_2^2}$. $v_2 = v_1\sqrt{K_2/K_1} = (65\text{ mi/h})\sqrt{\frac{1}{2}K_1/K_1} = 46\text{ mi/h}$.

EVALUATE: Since $K \sim v^2$, to have half the kinetic energy the speed must be less than half of the original speed.

6.11. **IDENTIFY:** $K = \frac{1}{2}mv^2$. Since the meteor comes to rest the energy it delivers to the ground equals its original kinetic energy.

SET UP: $v = 12\text{ km/s} = 1.2\times10^4\text{ m/s}$. A 1.0 megaton bomb releases 4.184×10^{15} J of energy.

EXECUTE: **(a)** $K = \frac{1}{2}(1.4\times10^8\text{ kg})(1.2\times10^4\text{ m/s})^2 = 1.0\times10^{16}\text{ J}$.

(b) $\dfrac{1.0\times10^{16}\text{ J}}{4.184\times10^{15}\text{ J}} = 2.4$. The energy is equivalent to 2.4 one-megaton bombs.

EVALUATE: Part of the energy transferred to the ground lifts soil and rocks into the air and creates a large crater.

6.12. **IDENTIFY:** $K = \frac{1}{2}mv^2$. Use the equations for free-fall to find the speed of the weight when it reaches the ground.

SET UP: Estimate that a person has speed 2 m/s when walking and 6 m/s when running. The mass of an electron is 9.11×10^{-31} kg. In part (c) take $+y$ downward, so $a_y = +9.80\text{ m/s}^2$. Estimate a shoulder height of 1.6 m.

EXECUTE: **(a)** Walking: $K = \frac{1}{2}(75\text{ kg})(2\text{ m/s})^2 = 150\text{ J}$. Running: $K = \frac{1}{2}(75\text{ kg})(6\text{ m/s})^2 = 1400\text{ J}$.

(b) $K = \frac{1}{2}(9.11\times10^{-31}\text{ kg})(2.19\times10^6\text{ m/s})^2 = 2.2\times10^{-18}\text{ J}$.

(c) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(9.80\text{ m/s}^2)(1.6\text{ m})} = 5.6\text{ m/s}$. $K = \frac{1}{2}(1.0\text{ kg})(5.6\text{ m/s})^2 = 16\text{ J}$.

(d) $v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2(100\text{ J})}{30\text{ kg}}}=2.6\text{ m/s}$. Yes, this is reasonable.

EVALUATE: A walking speed of 2 m/s corresponds to walking a mile in about 13 min. A running speed of 6 m/s corresponds to running a 100 m dash in about 17 s.

6.13. **IDENTIFY:** $K=\frac{1}{2}mv^2$. Set up a ratio that relates *K*, *m* and *v*.

SET UP: $m_\text{p}=1836m_\text{e}$

EXECUTE: **(a)** $K_\text{p}=K_\text{e}$ gives $m_\text{e}v_\text{e}^2=m_\text{p}v_\text{p}^2$. $v_\text{e}=v_\text{p}\sqrt{m_\text{p}/m_\text{e}}=V\sqrt{1836}=42.85V$.

(b) $v_\text{p}=v_\text{e}$ gives $\frac{K_\text{p}}{m_\text{p}}=\frac{K_\text{e}}{m_\text{e}}$. $K_\text{p}=K_\text{e}(m_\text{p}/m_\text{e})=1836K$.

EVALUATE: The electron has less mass so must travel faster to have the same kinetic energy. And with equal speeds the proton has more kinetic energy.

6.14. **IDENTIFY:** Only gravity does work on the watermelon, so $W_\text{tot}=W_\text{grav}$. $W_\text{tot}=\Delta K$ and $K=\frac{1}{2}mv^2$.

SET UP: Since the watermelon is dropped from rest, $K_1=0$.

EXECUTE: **(a)** $W_\text{grav}=mgs=(4.80\text{ kg})(9.80\text{ m/s}^2)(25.0\text{ m})=1180\text{ J}$

(b) $W_\text{tot}=K_2-K_1$ so $K_2=1180\text{ J}$. $v=\sqrt{\frac{2K_2}{m}}=\sqrt{\frac{2(1180\text{ J})}{4.80\text{ kg}}}=22.2\text{ m/s}$.

(c) The work done by gravity would be the same. Air resistance would do negative work and W_tot would be less than W_grav . The answer in (a) would be unchanged and both answers in (b) would decrease.

EVALUATE: The gravity force is downward and the displacement is downward, so gravity does positive work.

6.15. **IDENTIFY:** $W_\text{tot}=K_2-K_1$. In each case calculate W_tot from what we know about the force and the displacement.

SET UP: The gravity force is *mg*, downward. The friction force is $f_\text{k}=\mu_\text{k}n=\mu_\text{k}mg$ and is directed opposite to the displacement. The mass of the object isn't given, so we expect that it will divide out in the calculation.

EXECUTE: **(a)** $K_1=0$. $W_\text{tot}=W_\text{grav}=mgs$. $mgs=\frac{1}{2}mv_2^2$ and $v_2=\sqrt{2gs}=\sqrt{2(9.80\text{ m/s}^2)(95.0\text{ m})}=43.2\text{ m/s}$.

(b) $K_2=0$ (at the maximum height). $W_\text{tot}=W_\text{grav}=-mgs$. $-mgs=-\frac{1}{2}mv_1^2$ and

$v_1=\sqrt{2gs}=\sqrt{2(9.80\text{ m/s}^2)(525\text{ m})}=101\text{ m/s}$.

(c) $K_1=\frac{1}{2}mv_1^2$. $K_2=0$. $W_\text{tot}=W_f=-\mu_\text{k}mgs$. $-\mu_\text{k}mgs=-\frac{1}{2}mv_1^2$. $s=\frac{v_1^2}{2\mu_\text{k}g}=\frac{(5.00\text{ m/s})^2}{2(0.220)(9.80\text{ m/s}^2)}=5.80\text{ m}$.

(d) $K_1=\frac{1}{2}mv_1^2$. $K_2=\frac{1}{2}mv_2^2$. $W_\text{tot}=W_f=-\mu_\text{k}mgs$. $K_2=W_\text{tot}+K_1$. $\frac{1}{2}mv_2^2=-\mu_\text{k}mgs+\frac{1}{2}mv_1^2$

$v_2=\sqrt{v_1^2-2\mu_\text{k}gs}=\sqrt{(5.00\text{ m/s})^2-2(0.220)(9.80\text{ m/s}^2)(2.90\text{ m})}=3.53\text{ m/s}$.

(e) $K_1=\frac{1}{2}mv_1^2$. $K_2=0$. $W_\text{grav}=-mgy_2$, where y_2 is the vertical height. $-mgy_2=-\frac{1}{2}mv_1^2$ and

$y_2=\frac{v_1^2}{2g}=\frac{(12.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)}=7.35\text{ m}$.

EVALUATE: In parts (c) and (d), friction does negative work and the kinetic energy is reduced. In part (a), gravity does positive work and the speed increases. In parts (b) and (e), gravity does negative work and the speed decreases. The vertical height in part (e) is independent of the slope angle of the hill.

6.16. **IDENTIFY:** From the work-energy relation, $W=W_\text{grav}=\Delta K_\text{rock}$.

SET UP: As the rock rises, the gravitational force, $F=mg$, does work on the rock. Since this force acts in the direction opposite to the motion and displacement, *s*, the work is negative. Let *h* be the vertical distance the rock travels.

EXECUTE: **(a)** Applying $W_\text{grav}=K_2-K_1$ we obtain $-mgh=\frac{1}{2}mv_2^2-\frac{1}{2}mv_1^2$. Dividing by *m* and solving for v_1 ,

$v_1=\sqrt{v_2^2+2gh}$. Substituting $h=15.0\text{ m}$ and $v_2=25.0\text{ m/s}$,

$$v_1=\sqrt{(25.0\text{ m/s})^2+2(9.80\text{ m/s}^2)(15.0\text{ m})}=30.3\text{ m/s}$$

(b) Solve the same work-energy relation for *h*. At the maximum height $v_2=0$.

$-mgh=\frac{1}{2}mv_2^2-\frac{1}{2}mv_1^2$ and $h=\frac{v_1^2-v_2^2}{2g}=\frac{(30.3\text{ m/s})^2-(0.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)}=46.8\text{ m}$.

EVALUATE: Note that the weight of 20 N was never used in the calculations because both gravitational potential and kinetic energy are proportional to mass, *m*. Thus any object, that attains 25.0 m/s at a height of 15.0 m, must have an initial velocity of 30.3 m/s. As the rock moves upward gravity does negative work and this reduces the kinetic energy of the rock.

6.17. **IDENTIFY** and **SET UP:** Apply Eq.(6.6) to the box. Let point 1 be at the bottom of the incline and let point 2 be at the skier. Work is done by gravity and by friction. Solve for K_1 and from that obtain the required initial speed.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2, \;\; K_2 = 0$

Work is done by gravity and friction, so $W_{\text{tot}} = W_{mg} + W_f$.

$W_{mg} = -mg(y_2 - y_1) = -mgh$

$W_f = -fs$. The normal force is $n = mg\cos\alpha$ and $s = h/\sin\alpha$, where *s* is the distance the box travels along the incline.

$W_f = -(\mu_{\text{k}} mg\cos\alpha)(h/\sin\alpha) = -\mu_{\text{k}} mgh/\tan\alpha$

Substituting these expressions into the work-energy theorem gives

$-mgh - \mu_{\text{k}} mgh/\tan\alpha = -\frac{1}{2}mv_0^2$.

Solving for v_0 then gives $v_0 = \sqrt{2gh(1+\mu_{\text{k}}/\tan\alpha)}$.

EVALUATE: The result is independent of the mass of the box. As $\alpha \to 90°$, $h = s$ and $v_0 = \sqrt{2gh}$, the same as throwing the box straight up into the air. For $\alpha = 90°$ the normal force is zero so there is no friction.

6.18. **IDENTIFY:** Apply $W = Fs\cos\phi$ and $W_{\text{tot}} = \Delta K$.

SET UP: Parallel to incline: force component $W_{\|} = mg\sin\alpha$, down incline; displacement $s = h/\sin\alpha$, down incline. Perpendicular to the incline: $s = 0$.

EXECUTE: **(a)** $W_{\|} = (mg\sin\alpha)(h/\sin\alpha) = mgh$. $W_{\perp} = 0$, since there is no displacement in this direction.

$W_{mg} = W_{\|} + W_{\perp} = mgh$, same as falling height *h*.

(b) $W_{\text{tot}} = K_2 - K_1$ gives $mgh = \frac{1}{2}mv^2$ and $v = \sqrt{2gh}$, same as if had been dropped from height *h*. The work done by gravity depends only on the vertical displacement of the object. When the slope angle is small, there is a small force component in the direction of the displacement but a large displacement in this direction. When the slope angle is large, the force component in the direction of the displacement along the incline is larger but the displacement in this direction is smaller.

(c) $h = 15.0\text{ m}$, so $v = \sqrt{2gh} = 17.1\text{ s}$.

EVALUATE: The acceleration and time of travel are different for an object sliding down an incline and an object in free-fall, but the final velocity is the same in these two cases.

6.19. **IDENTIFY:** $W_{\text{tot}} = K_2 - K_1$ with $W_{\text{tot}} = W_f$. The car stops, so $K_2 = 0$. In each case identify what is constant and set up a ratio.

SET UP: $W_f = -fs$, so $-fs = -\frac{1}{2}mv_0^2$.

EXECUTE: **(a)** $v_{0b} = 3v_{0a}$. $s_a = D$. *f* is constant. $\dfrac{v_0^2}{s} = \dfrac{2f}{m} = \text{constant}$, so $\dfrac{v_{0a}^2}{s_a} = \dfrac{v_{0b}^2}{s_b}$. $s_b = s_a\left(\dfrac{v_{0b}}{v_{0a}}\right)^2 = D(3)^2 = 9D$.

(b) $f_b = 3f_a$. v_0 is constant. $fs = \frac{1}{2}mv_0^2 = \text{constant}$, so $f_a s_a = f_b s_b$. $s_b = s_a\left(\dfrac{f_a}{f_b}\right) = D/3$.

EVALUATE: The stopping distance is proportional to the square of the initial speed. When the friction force increases, the stopping distance decreases.

6.20. **IDENTIFY** and **SET UP:** Apply Eq.(6.6). The relation between the speeds v_1 and v_2 tells us the relation between K_1 and K_2.

EXECUTE: **(a)** $W = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2, \;\; K_2 = \frac{1}{2}mv_2^2$

$v_2 = \frac{1}{4}v_1$ gives that $K_2 = \frac{1}{2}m\left(\frac{1}{4}v_1\right)^2 = \frac{1}{16}\left(\frac{1}{2}mv_1^2\right) = \frac{1}{16}K_1$

$W = K_2 - K_1 = \frac{1}{16}K_1 - K_1 = -\frac{15}{16}K_1$

(b) EVALUATE: *K* depends only on the magnitude of $\vec{v}$ not on its direction, so the answer for *W* in part (a) does *not* depend on the final direction of the electron's motion. The electron slows down, so its kinetic energy decreases and the total work done on it is negative.

6.21. **IDENTIFY:** Apply $W = Fs\cos\phi$ and $W_{\text{tot}} = \Delta K$.

SET UP: $\phi = 0°$

EXECUTE: From Equations (6.1), (6.5) and (6.6), and solving for *F*,

$$F = \frac{\Delta K}{s} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{s} = \frac{\frac{1}{2}(8.00\text{ kg})((6.00\text{ m/s})^2 - (4.00\text{ m/s})^2)}{(2.50\text{ m})} = 32.0\text{ N}.$$

EVALUATE: The force is in the direction of the displacement, so the force does positive work and the kinetic energy of the object increases.

6.22. **IDENTIFY** and **SET UP:** Use Eq.(6.6) to calculate the work done by the foot on the ball. Then use Eq.(6.2) to find the distance over which this force acts.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.420\text{ kg})(2.00\text{ m/s})^2 = 0.84\text{ J}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.420\text{ kg})(6.00\text{ m/s})^2 = 7.56\text{ J}$

$W_{\text{tot}} = K_2 - K_1 = 7.56\text{ J} - 0.84\text{ J} = 6.72\text{ J}$

The 40.0 N force is the only force doing work on the ball, so it must do 6.72 J of work. $W_F = (F\cos\phi)s$ gives that

$$s = \frac{W}{F\cos\phi} = \frac{6.72\text{ J}}{(40.0\text{ N})(\cos 0)} = 0.168\text{ m}$$

EVALUATE: The force is in the direction of the motion so positive work is done and this is consistent with an increase in kinetic energy.

6.23. **IDENTIFY:** Apply $W_{\text{tot}} = \Delta K$.

SET UP: $v_1 = 0$, $v_2 = v$. $f_k = \mu_k mg$ and f_k does negative work. The force $F = 36.0\text{ N}$ is in the direction of the motion and does positive work.

EXECUTE: **(a)** If there is no work done by friction, the final kinetic energy is the work done by the applied force, and solving for the speed,

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2Fs}{m}} = \sqrt{\frac{2(36.0\text{ N})(1.20\text{ m})}{(4.30\text{ kg})}} = 4.48\text{ m/s}.$$

(b) The net work is $Fs - f_k s = (F - \mu_k mg)s$, so

$$v = \sqrt{\frac{2(F - \mu_k mg)s}{m}} = \sqrt{\frac{2(36.0\text{ N} - (0.30)(4.30\text{ kg})(9.80\text{ m/s}^2))(1.20\text{ m})}{(4.30\text{ kg})}} = 3.61\text{ m/s}$$

EVALUATE: The total work done is larger in the absence of friction and the final speed is larger in that case.

6.24. **IDENTIFY:** Apply $W = Fs\cos\phi$ and $W_{\text{tot}} = \Delta K$

SET UP: The gravity force has magnitude *mg* and is directed downward.

EXECUTE: **(a)** On the way up, gravity is opposed to the direction of motion, and so

$W = -mgs = -(0.145\text{ kg})(9.80\text{ m/s}^2)(20.0\text{ m}) = -28.4\text{ J}$.

(b) $v_2 = \sqrt{v_1^2 + 2\frac{W}{m}} = \sqrt{(25.0\text{ m/s})^2 + \frac{2(-28.4\text{ J})}{(0.145\text{ kg})}} = 15.3\text{ m/s}$.

(c) No; in the absence of air resistance, the ball will have the same speed on the way down as on the way up. On the way down, gravity will have done both negative and positive work on the ball, but the net work at this height will be the same.

EVALUATE: As the baseball moves upward, gravity does negative work and the speed of the baseball decreases.

6.25. **(a) IDENTIFY** and **SET UP:** Use Eq.(6.2) to find the work done by the positive force. Then use Eq.(6.6) to find the final kinetic energy, and then $K_2 = \frac{1}{2}mv_2^2$ gives the final speed.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$, so $K_2 = W_{\text{tot}} + K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(7.00\text{ kg})(4.00\text{ m/s})^2 = 56.0\text{ J}$

The only force that does work on the wagon is the 10.0 N force. This force is in the direction of the displacement so $\phi = 0°$ and the force does positive work:

$$W_F = (F\cos\phi)s = (10.0\text{ N})(\cos 0)(3.0\text{ m}) = 30.0\text{ J}$$

Then $K_2 = W_{\text{tot}} + K_1 = 30.0\text{ J} + 56.0\text{ J} = 86.0\text{ J}$.

$$K_2 = \tfrac{1}{2}mv_2^2;\quad v_2 = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(86.0\text{ J})}{7.00\text{ kg}}} = 4.96\text{ m/s}$$

(b) IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the wagon to calculate *a*. Then use a constant acceleration equation to calculate the final speed. The free-body diagram is given in Figure 6.25.

SET UP:

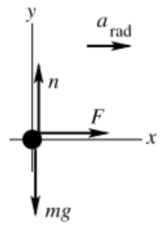


Figure 6.25

EXECUTE: $\sum F_x = ma_x$

$F = ma_x$

$a_x = \frac{F}{m} = \frac{10.0\text{ N}}{7.00\text{ kg}} = 1.43\text{ m/s}^2$

$v_{2x}^2 = v_{1x}^2 + 2a_2(x - x_0)$

$v_{2x} = \sqrt{v_{1x}^2 + 2a_x(x - x_0)} = \sqrt{(4.00\text{ m/s})^2 + 2(1.43\text{ m/s}^2)(3.0\text{ m})} = 4.96\text{ m/s}$

EVALUATE: This agrees with the result calculated in part (a). The force in the direction of the motion does positive work and the kinetic energy and speed increase. In part (b), the equivalent statement is that the force produces an acceleration in the direction of the velocity and this causes the magnitude of the velocity to increase.

6.26. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $K_1 = 0$. The normal force does no work. The work *W* done by gravity is $W = mgh$, where $h = L\sin\theta$ is the vertical distance the block has dropped when it has traveled a distance *L* down the incline and θ is the angle the plane makes with the horizontal.

EXECUTE: The work-energy theorem gives $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2W}{m}} = \sqrt{2gh} = \sqrt{2gL\sin\theta}$. Using the given numbers,

$v = \sqrt{2(9.80\text{ m/s}^2)(0.75\text{ m})\sin 36.9°} = 2.97\text{ m/s}$.

EVALUATE: The final speed of the block is the same as if it had been dropped from a height *h*.

6.27. **IDENTIFY:** $W_{\text{tot}} = K_2 - K_1$. Only friction does work.

SET UP: $W_{\text{tot}} = W_{f_k} = -\mu_k mgs$. $K_2 = 0$ (car stops). $K_1 = \frac{1}{2}mv_0^2$.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$ gives $-\mu_k mgs = -\frac{1}{2}mv_0^2$. $s = \frac{v_0^2}{2\mu_k g}$.

(b) (i) $\mu_{kb} = 2\mu_{ka}$. $s\mu_k = \frac{v_0^2}{2g} = \text{constant}$ so $s_a\mu_{ka} = s_b\mu_{kb}$. $s_b = \left(\frac{\mu_{ka}}{\mu_{kb}}\right)s_a = s_a/2$. The minimum stopping distance would be halved. (ii) $v_{0b} = 2v_{0a}$. $\frac{s}{v_0^2} = \frac{1}{2\mu_k g} = \text{constant}$, so $\frac{s_a}{v_{0a}^2} = \frac{s_b}{v_{0b}^2}$. $s_b = s_a\left(\frac{v_{0b}}{v_{0a}}\right)^2 = 4s_a$. The stopping distance would become 4 times as great. (iii) $v_{0b} = 2v_{0a}$, $\mu_{kb} = 2\mu_{ka}$. $\frac{s\mu_k}{v_0^2} = \frac{1}{2g} = \text{constant}$, so $\frac{s_a\mu_{ka}}{v_{0a}^2} = \frac{s_b\mu_{kb}}{v_{0b}^2}$.

$s_b = s_a\left(\frac{\mu_{ka}}{\mu_{kb}}\right)\left(\frac{v_{0b}}{v_{0a}}\right)^2 = s_a\left(\frac{1}{2}\right)(2)^2 = 2s_a$. The stopping distance would double.

EVALUATE: The stopping distance is directly proportional to the square of the initial speed and indirectly proportional to the coefficient of kinetic friction.

6.28. **IDENTIFY:** The work that must be done to move the end of a spring from x_1 to x_2 is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$. The force required to hold the end of the spring at displacement *x* is $F_x = kx$.

SET UP: When the spring is at its unstretched length, $x = 0$. When the spring is stretched, $x > 0$, and when the spring is compressed, $x < 0$.

EXECUTE: **(a)** $x_1 = 0$ and $W = \frac{1}{2}kx_2^2$. $k = \frac{2W}{x_2^2} = \frac{2(12.0\text{ J})}{(0.0300\text{ m})^2} = 2.67\times10^4\text{ N/m}$.

(b) $F_x = kx = (2.67\times10^4\text{ N/m})(0.0300\text{ m}) = 801\text{ N}$.

(c) $x_1 = 0$, $x_2 = -0.0400\text{ m}$. $W = \frac{1}{2}(2.67\times10^4\text{ N/m})(-0.0400\text{ m})^2 = 21.4\text{ J}$.

$F_x = kx = (2.67\times10^4\text{ N/m})(0.0400\text{ m}) = 1070\text{ N}$.

EVALUATE: When a spring, initially unstretched, is either compressed or stretched, positive work is done by the force that moves the end of the spring.

6.29. **IDENTIFY** and **SET UP:** Use Eq.(6.8) to calculate *k* for the spring. Then Eq.(6.10), with $x_1 = 0$, can be used to calculate the work done to stretch or compress the spring an amount x_2.

EXECUTE: Use the information given to calculate the force constant of the spring.

$F_x = kx$ gives $k = \dfrac{F_x}{x} = \dfrac{160\text{ N}}{0.050\text{ m}} = 3200\text{ N/m}$

(a) $F_x = kx = (3200\text{ N/m})(0.015\text{ m}) = 48\text{ N}$

$F_x = kx = (3200\text{ N/m})(-0.020\text{ m}) = -64\text{ N}$ (magnitude 64 N)

(b) $W = \frac{1}{2}kx^2 = \frac{1}{2}(3200\text{ N/m})(0.015\text{ m})^2 = 0.36\text{ J}$

$W = \frac{1}{2}kx^2 = \frac{1}{2}(3200\text{ N/m})(-0.020\text{ m})^2 = 0.64\text{ J}$

Note that in each case the work done is positive.

EVALUATE: The force is not constant during the displacement so Eq.(6.2) *cannot* be used. A force in the $+x$ direction is required to stretch the spring and a force in the opposite direction to compress it. The force F_x is in the same direction as the displacement, so positive work is done in both cases.

6.30. **IDENTIFY:** The magnitude of the work can be found by finding the area under the graph.

SET UP: The area under each triangle is 1/2 $\text{base} \times \text{height}$. $F_x > 0$, so the work done is positive when *x* increases during the displacement.

EXECUTE: **(a)** $1/2\,(8\text{ m})(10\text{ N}) = 40\text{ J}$.

(b) $1/2\,(4\text{ m})(10\text{ N}) = 20\text{ J}$.

(c) $1/2\,(12\text{ m})(10\text{ N}) = 60\text{ J}$.

EVALUATE: The sum of the answers to parts (a) and (b) equals the answer to part (c).

6.31. **IDENTIFY:** Use the work-energy theorem and the results of Problem 6.30.

SET UP: For $x = 0$ to $x = 8.0\text{ m}$, $W_{\text{tot}} = 40\text{ J}$. For $x = 0$ to $x = 12.0\text{ m}$, $W_{\text{tot}} = 60\text{ J}$.

EXECUTE: **(a)** $v = \sqrt{\dfrac{(2)(40\text{ J})}{10\text{ kg}}} = 2.83\text{ m/s}$

(b) $v = \sqrt{\dfrac{(2)(60\text{ J})}{10\text{ kg}}} = 3.46\text{ m/s}$.

EVALUATE: $\vec{F}$ is always in the $+x$-direction. For this motion $\vec{F}$ does positive work and the speed continually increases during the motion.

6.32. **IDENTIFY:** The force has only an *x*-component and the motion is along the *x*-direction, so $W = \int_{x_1}^{x_2} F_x dx$.

SET UP: $x_1 = 0$ and $x_2 = 6.9\text{ m}$.

EXECUTE: The work you do with your changing force is

$W = \int_{x_1}^{x_2} F(x)dx = \int_{x_1}^{x_2} (-20.0\text{ N})dx - \int_{x_1}^{x_2} (3.0\text{ N/m})xdx = (-20.0\text{ N})x\big|_{x_1}^{x_2} - (3.0\text{ N/m})(x^2/2)\big|_{x_1}^{x_2}$

$W = -138\text{ N}\cdot\text{m} - 71.4\text{ N}\cdot\text{m} = -209\text{ J}$.

EVALUATE: The work is negative because the cow continues to move forward (in the $+x$-direction) as you vainly attempt to push her backward.

6.33. **IDENTIFY:** Apply Eq.(6.6) to the box.

SET UP: Let point 1 be just before the box reaches the end of the spring and let point 2 be where the spring has maximum compression and the box has momentarily come to rest.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2$, $K_2 = 0$

Work is done by the spring force. $W_{\text{tot}} = -\frac{1}{2}kx_2^2$, where x_2 is the amount the spring is compressed.

$-\frac{1}{2}kx_2^2 = -\frac{1}{2}mv_0^2$ and $x_2 = v_0\sqrt{m/k} = (3.0\text{ m/s})\sqrt{(6.0\text{ kg})/(7500\text{ N/m})} = 8.5\text{ cm}$

EVALUATE: The compression of the spring increases when either v_0 or *m* increases and decreases when *k* increases (stiffer spring).

6.34. **IDENTIFY:** The force applied to the springs is $F_x = kx$. The work done on a spring to move its end from x_1 to x_2 is $W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$. Use the information that is given to calculate *k*.

SET UP: When the springs are compressed 0.200 m from their uncompressed length, $x_1 = 0$ and $x_2 = -0.200\text{ m}$. When the platform is moved 0.200 m farther, x_2 becomes -0.400 m.

EVALUATE: **(a)** $k = \dfrac{2W}{x_2^2 - x_1^2} = \dfrac{2(80.0\text{ J})}{(0.200\text{ m})^2 - 0} = 4000\text{ N/m}$. $F_x = kx = (4000\text{ N/m})(-0.200\text{ m}) = -800\text{ N}$. The magnitude of force that is required is 800 N.

(b) To compress the springs from $x_1 = 0$ to $x_2 = -0.400\text{ m}$, the work required is $W = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}(4000\text{ N/m})(-0.400\text{ m})^2 = 320\text{ J}$. The additional work required is $320\text{ J} - 80\text{ J} = 240\text{ J}$. For $x = -0.400\text{ m}$, $F_x = kx = -1600\text{ N}$. The magnitude of force required is 1600 N.

EVALUATE: More work is required to move the end of the spring from $x = -0.200\text{ m}$ to $x = -0.400\text{ m}$ than to move it from $x = 0$ to $x = -0.200\text{ m}$, even though the displacement of the platform is the same in each case. The magnitude of the force increases as the compression of the spring increases.

6.35. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to calculate the μ_s required for the static friction force to equal the spring force.

SET UP: **(a)** The free-body diagram for the glider is given in Figure 6.35.

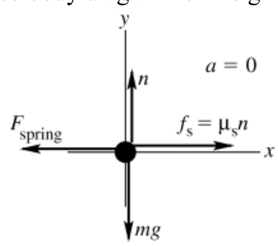


Figure 6.35

EXECUTE: $\sum F_y = ma_y$

$n - mg = 0$

$n = mg$

$f_s = \mu_s mg$

$\sum F_x = ma_x$

$f_s - F_{\text{spring}} = 0$

$\mu_s mg - kd = 0$

$$\mu_s = \frac{kd}{mg} = \frac{(20.0\text{ N/m})(0.086\text{ m})}{(0.100\text{ kg})(9.80\text{ m/s}^2)} = 1.76$$

(b) IDENTIFY and **SET UP:** Apply $\sum \vec{F} = m\vec{a}$ to find the maximum amount the spring can be compressed and still have the spring force balanced by friction. Then use $W_{\text{tot}} = K_2 - K_1$ to find the initial speed that results in this compression of the spring when the glider stops.

EXECUTE: $\mu_s mg = kd$

$$d = \frac{\mu_s mg}{k} = \frac{(0.60)(0.100\text{ kg})(9.80\text{ m/s}^2)}{20.0\text{ N/m}} = 0.0294\text{ m}$$

Now apply the work-energy theorem to the motion of the glider:

$W_{\text{tot}} = K_2 - K_1$

$K_1 = \tfrac{1}{2}mv_1^2,\ \ K_2 = 0$ (instantaneously stops)

$W_{\text{tot}} = W_{\text{spring}} + W_{\text{fric}} = -\tfrac{1}{2}kd^2 - \mu_k mgd$ (as in Example 6.8)

$W_{\text{tot}} = -\tfrac{1}{2}(20.0\text{ N/m})(0.0294\text{ m})^2 - 0.47(0.100\text{ kg})(9.80\text{ m/s}^2)(0.0294\text{ m}) = -0.02218\text{ J}$

Then $W_{\text{tot}} = K_2 - K_1$ gives $-0.02218\text{ J} = -\tfrac{1}{2}mv_1^2$.

$$v_1 = \sqrt{\frac{2(0.02218\text{ J})}{0.100\text{ kg}}} = 0.67\text{ m/s}$$

EVALUATE: In Example 6.8 an initial speed of 1.50 m/s compresses the spring 0.086 m and in part (a) of this problem we found that the glider doesn't stay at rest. In part (b) we found that a smaller displacement of 0.0294 m when the glider stops is required if it is to stay at rest. And we calculate a smaller initial speed (0.67 m/s) to produce this smaller displacement.

6.36. **IDENTIFY:** For the spring, $W = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2$. Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $x_1 = -0.025\text{ m}$ and $x_2 = 0$.

EXECUTE: **(a)** $W = \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}(200\text{ N/m})(-0.025\text{ m})^2 = 0.060\text{ J}$.

(b) The work-energy theorem gives $v_2 = \sqrt{\dfrac{2W}{m}} = \sqrt{\dfrac{2(0.060\text{ J})}{(4.0\text{ kg})}} = 0.18\text{ m/s}$.

EVALUATE: The block moves in the direction of the spring force, the spring does positive work and the kinetic energy of the block increases.

6.37. **IDENTIFY** and **SET UP:** The magnitude of the work done by F_x equals the area under the F_x versus *x* curve. The work is positive when F_x and the displacement are in the same direction; it is negative when they are in opposite directions.

EXECUTE: **(a)** F_x is positive and the displacement Δx is positive, so $W > 0$.

$W = \frac{1}{2}(2.0\text{ N})(2.0\text{ m}) + (2.0\text{ N})(1.0\text{ m}) = +4.0\text{ J}$

(b) During this displacement $F_x = 0$, so $W = 0$.

(c) F_x is negative, Δx is positive, so $W < 0$. $W = -\frac{1}{2}(1.0\text{ N})(2.0\text{ m}) = -1.0\text{ J}$

(d) The work is the sum of the answers to parts (a), (b), and (c), so $W = 4.0\text{ J} + 0 - 1.0\text{ J} = +3.0\text{ J}$

(e) The work done for $x = 7.0\text{ m}$ to $x = 3.0\text{ m}$ is $+1.0$ J. This work is positive since the displacement and the force are both in the $-x$-direction. The magnitude of the work done for $x = 3.0\text{ m}$ to $x = 2.0\text{ m}$ is 2.0 J, the area under F_x versus *x*. This work is negative since the displacement is in the $-x$-direction and the force is in the $+x$-direction. Thus $W = +1.0\text{ J} - 2.0\text{ J} = -1.0\text{ J}$

EVALUATE: The work done when the car moves from $x = 2.0\text{ m}$ to $x = 0$ is $-\frac{1}{2}(2.0\text{ N})(2.0\text{ m}) = -2.0\text{ J}$. Adding this to the work for $x = 7.0\text{ m}$ to $x = 2.0\text{ m}$ gives a total of $W = -3.0\text{ J}$ for $x = 7.0\text{ m}$ to $x = 0$. The work for $x = 7.0\text{ m}$ to $x = 0$ is the negative of the work for $x = 0$ to $x = 7.0\text{ m}$.

6.38. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $K_1 = 0$. From Exercise 6.37, the work for $x = 0$ to $x = 3.0$ m is 4.0 J. *W* for $x = 0$ to $x = 4.0$ m is also 4.0 J. For $x = 0$ to $x = 7.0\text{ m}$, $W = 3.0\text{ J}$.

EXECUTE: **(a)** $K = 4.0\text{ J}$, so $v = \sqrt{2K/m} = \sqrt{2(4.0\text{ J})/(2.0\text{ kg})} = 2.00\text{ m/s}$.

(b) No work is done between $x = 3.0\text{ m}$ and $x = 4.0\text{ m}$, so the speed is the same, 2.00 m/s.

(c) $K = 3.0\text{ J}$, so $v = \sqrt{2K/m} = \sqrt{2(3.0\text{ J})/(2.0\text{ kg})} = 1.73\text{ m/s}$.

EVALUATE: In each case the work done by *F* is positive and the car gains kinetic energy.

6.39. **IDENTIFY** and **SET UP:** Apply Eq.(6.6). Let point 1 be where the sled is released and point 2 be at $x = 0$ for part (a) and at $x = -0.200\text{ m}$ for part (b). Use Eq.(6.10) for the work done by the spring and calculate K_2. Then $K_2 = \frac{1}{2}mv_2^2$ gives v_2.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$ so $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$ (released with no initial velocity), $K_2 = \frac{1}{2}mv_2^2$

The only force doing work is the spring force. Eq.(6.10) gives the work done *on* the spring to move its end from x_1 to x_2. The force the spring exerts on an object attached to it is $F = -kx$, so the work the spring does is

$W_{\text{spr}} = -\left(\frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2\right) = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Here $x_1 = -0.375\text{ m}$ and $x_2 = 0$. Thus $W_{\text{spr}} = \frac{1}{2}(4000\text{ N/m})(-0.375\text{ m})^2 - 0 = 281\text{ J}$.

$K_2 = K_1 + W_{\text{tot}} = 0 + 281\text{ J} = 281\text{ J}$

Then $K_2 = \frac{1}{2}mv_2^2$ implies $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(281\text{ J})}{70.0\text{ kg}}} = 2.83\text{ m/s}$.

(b) $K_2 = K_1 + W_{\text{tot}}$

$K_1 = 0$

$W_{\text{tot}} = W_{\text{spr}} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$. Now $x_2 = 0.200\text{ m}$, so

$W_{\text{spr}} = \frac{1}{2}(4000\text{ N/m})(-0.375\text{ m})^2 - \frac{1}{2}(4000\text{ N/m})(-0.200\text{ m})^2 = 281\text{ J} - 80\text{ J} = 201\text{ J}$

Thus $K_2 = 0 + 201\text{ J} = 201\text{ J}$ and $K_2 = \frac{1}{2}mv_2^2$ gives $v_2 = \sqrt{\dfrac{2K_2}{m}} = \sqrt{\dfrac{2(201\text{ J})}{70.0\text{ kg}}} = 2.40\text{ m/s}$.

EVALUATE: The spring does positive work and the sled gains speed as it returns to $x = 0$. More work is done during the larger displacement in part (a), so the speed there is larger than in part (b).

6.40. **IDENTIFY:** $F_x = kx$

SET UP: When the spring is in equilibrium, the same force is applied to both ends of any segment of the spring.

EXECUTE: **(a)** When a force *F* is applied to each end of the original spring, the end of the spring is displaced a distance *x*. Each half of the spring elongates a distance x_{h}, where $x_{\text{h}} = x/2$. Since *F* is also the force applied to each half of the spring, $F = kx$ and $F = k_{\text{h}}x_{\text{h}}$. $kx = k_{\text{h}}x_{\text{h}}$ and $k_{\text{h}} = k\left(\dfrac{x}{x_{\text{h}}}\right) = 2k$.

(b) The same reasoning as in part (a) gives $k_{seg} = 3k$, where k_{seg} is the force constant of each segment.

EVALUATE: For half of the spring the same force produces less displacement than for the original spring. Since $k = F/x$, smaller *x* for the same *F* means larger *k*.

6.41. **IDENTIFY** and **SET UP:** Apply Eq.(6.6) to the glider. Work is done by the spring and by gravity. Take point 1 to be where the glider is released. In part (a) point 2 is where the glider has traveled 1.80 m and $K_2 = 0$. There two points are shown in Figure 6.41a. In part (b) point 2 is where the glider has traveled 0.80 m.

EXECUTE: **(a)** $W_{tot} = K_2 - K_1 = 0$. Solve for x_1, the amount the spring is initially compressed.

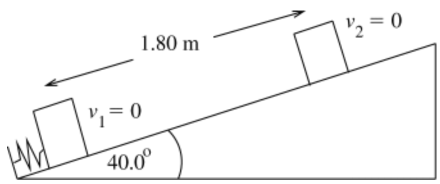


Figure 6.41a

$W_{tot} = W_{spr} + W_w = 0$

So $W_{spr} = -W_w$

(The spring does positive work on the glider since the spring force is directed up the incline, the same as the direction of the displacement.)

The directions of the displacement and of the gravity force are shown in Figure 6.41b.

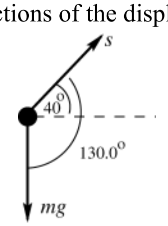


Figure 6.41b

$W_w = (w\cos\phi)s = (mg\cos 130.0°)s$

$W_w = (0.0900\text{ kg})(9.80\text{ m/s}^2)(\cos 130.0°)(1.80\text{ m}) = -1.020\text{ J}$

(The component of *w* parallel to the incline is directed down the incline, opposite to the displacement, so gravity does negative work.)

$W_{spr} = -W_w = +1.020\text{ J}$

$W_{spr} = \frac{1}{2}kx_1^2$ so $x_1 = \sqrt{\frac{2W_{spr}}{k}} = \sqrt{\frac{2(1.020\text{ J})}{640\text{ N/m}}} = 0.0565\text{ m}$

(b) The spring was compressed only 0.0565 m so at this point in the motion the glider is no longer in contact with the spring. Points 1 and 2 are shown in Figure 6.41c.

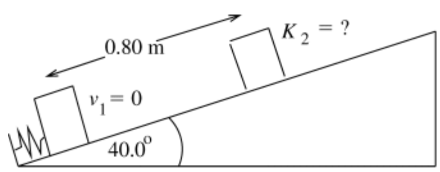


Figure 6.41c

$W_{tot} = K_2 - K_1$

$K_2 = K_1 + W_{tot}$

$K_1 = 0$

$W_{tot} = W_{spr} + W_w$

From part (a), $W_{spr} = 1.020\text{ J}$ and

$W_w = (mg\cos 130.0°)s = (0.0900\text{ kg})(9.80\text{ m/s}^2)(\cos 130.0°)(0.80\text{ m}) = -0.454\text{ J}$

Then $K_2 = W_{spr} + W_w = +1.020\text{ J} - 0.454\text{ J} = +0.57\text{ J}.$

EVALUATE: The kinetic energy in part (b) is positive, as it must be. In part (a), $x_2 = 0$ since the spring force is no longer applied past this point. In computing the work done by gravity we use the full 0.80 m the glider moves.

6.42. **IDENTIFY:** Apply $W_{tot} = K_2 - K_1$ to the brick. Work is done by the spring force and by gravity.

SET UP: At the maximum height. $v = 0$. Gravity does negative work, $W_{grav} = -mgh$. The work done by the spring is $\frac{1}{2}kd^2$, where *d* is the distance the spring is compressed initially.

EXECUTE: The initial and final kinetic energies of the brick are both zero, so the net work done on the brick by the spring and gravity is zero, so $(1/2)kd^2 - mgh = 0$, or

$d = \sqrt{2mgh/k} = \sqrt{2(1.80\text{ kg})(9.80\text{ m/s}^2)(3.6\text{ m})/(450\text{ N/m})} = 0.53\text{ m}.$ The spring will provide an upward force while the spring and the brick are in contact. When this force goes to zero, the spring is at its uncompressed length. But when the spring reaches its uncompressed length the brick has an upward velocity and leaves the spring.

EVALUATE: Gravity does negative work because the gravity force is downward and the brick moves upward. The spring force does positive work on the brick because the spring force is upward and the brick moves upward.

6.43. IDENTIFY: Apply the relation between energy and power.

SET UP: Use $P = \dfrac{W}{\Delta t}$ to solve for W, the energy the bulb uses. Then set this value equal to $\frac{1}{2}mv^2$ and solve for the speed.

EXECUTE: $W = P\Delta t = (100\ \text{W})(3600\ \text{s}) = 3.6\times10^5\ \text{J}$

$$K = 3.6\times10^5\ \text{J so } v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(3.6\times10^5\ \text{J})}{70\ \text{kg}}} = 100\ \text{m/s}$$

EVALUATE: Olympic runners achieve speeds up to approximately 36 m/s, or roughly one third the result calculated.

6.44. IDENTIFY: Energy is power times time.

SET UP: $1\ \text{W} = 1\ \text{J/s}$. $1\ \text{yr} = 3.16\times10^7\ \text{s}$.

EXECUTE: **(a)** $\dfrac{(1.0\times10^{19}\ \text{J/yr})}{(3.16\times10^7\ \text{s/yr})} = 3.2\times10^{11}\ \text{W}$.

(b) $\dfrac{3.2\times10^{11}\ \text{W}}{3.0\times10^8\ \text{folks}} = 1.1\ \text{kW/person}$.

(c) $A = \dfrac{3.2\times10^{11}\ \text{W}}{(0.40)1.0\times10^3\ \text{W/m}^2} = 8.0\times10^8\ \text{m}^2 = 800\ \text{km}^2$.

EVALUATE: The area in part (c) corresponds to a square about 28 km on a side, which is about 18 miles. The space required is not an impediment.

6.45. IDENTIFY: $P_{\text{av}} = \dfrac{\Delta W}{\Delta t}$. ΔW is the energy released.

SET UP: ΔW is to be the same. $1\ \text{y} = 3.156\times10^7\ \text{s}$.

EXECUTE: $P_{\text{av}}\Delta t = \Delta W = \text{constant}$, so $P_{\text{av-sun}}\Delta t_{\text{sun}} = P_{\text{av-m}}\Delta t_{\text{m}}$.

$$P_{\text{av-m}} = P_{\text{av-sun}}\left(\frac{\Delta t_{\text{sun}}}{\Delta t_{\text{m}}}\right) = \left(\frac{[2.5\times10^5\ \text{y}][3.156\times10^7\ \text{s/y}]}{0.20\ \text{s}}\right) = 3.9\times10^{13}P.$$

EVALUATE: Since the power output of the magnetar is so much larger than that of our sun, the mechanism by which it radiates energy must be quite different.

6.46. IDENTIFY: The thermal energy is produced as a result of the force of friction, $F = \mu_{\text{k}}mg$. The average thermal power is thus the average rate of work done by friction or $P = F_{\|}v_{\text{av}}$.

SET UP: $v_{\text{av}} = \dfrac{v_2 + v_1}{2} = \left(\dfrac{8.00\ \text{m/s} + 0}{2}\right) = 4.00\ \text{m/s}$

EXECUTE: $P = Fv_{\text{av}} = \left[(0.200)(20.0\ \text{kg})(9.80\ \text{m/s}^2)\right](4.00\ \text{m/s}) = 157\ \text{W}$

EVALUATE: The power could also be determined as the rate of change of kinetic energy, $\Delta K/t$, where the time is calculated from $v_{\text{f}} = v_{\text{i}} + at$ and a is calculated from a force balance, $\sum F = ma = \mu_{\text{k}}mg$.

6.47. IDENTIFY: Use the relation $P = F_{\|}v$ to relate the given force and velocity to the total power developed.

SET UP: $1\ \text{hp} = 746\ \text{W}$

EXECUTE: The total power is $P = F_{\|}v = (165\ \text{N})(9.00\ \text{m/s}) = 1.49\times10^3\ \text{W}$. Each rider therefore contributes

$P_{\text{each rider}} = (1.49\times10^3\ \text{W})/2 = 745\ \text{W} \approx 1\ \text{hp}$.

EVALUATE: The result of one horsepower is very large; a rider could not sustain this output for long periods of time.

6.48. IDENTIFY and SET UP: Calculate the power used to make the plane climb against gravity. Consider the vertical motion since gravity is vertical.

EXECUTE: The rate at which work is being done against gravity is

$P = Fv = mgv = (700\ \text{kg})(9.80\ \text{m/s}^2)(2.5\ \text{m/s}) = 17.15\ \text{kW}$.

This is the part of the engine power that is being used to make the airplane climb. The fraction this is of the total is $17.15\ \text{kW}/75\ \text{kW} = 0.23$.

EVALUATE: The power we calculate for making the airplane climb is considerably less than the power output of the engine.

6.49. **IDENTIFY:** $P_{av} = \frac{\Delta W}{\Delta t}$. The work you do in lifting mass *m* a height *h* is *mgh*.

SET UP: $1 \text{ hp} = 746 \text{ W}$

EXECUTE: **(a)** The number per minute would be the average power divided by the work (*mgh*) required to lift one box, $\frac{(0.50 \text{ hp})\,(746 \text{ W/hp})}{(30 \text{ kg})\,(9.80 \text{ m/s}^2)\,(0.90 \text{ m})} = 1.41\ /\text{s, or } 84.6\ /\text{min}.$

(b) Similarly, $\frac{(100 \text{ W})}{(30 \text{ kg})\,(9.80 \text{ m/s}^2)\,(0.90 \text{ m})} = 0.378\ /\text{s, or } 22.7\ /\text{min}.$

EVALUATE: A 30-kg crate weighs about 66 lbs. It is not possible for a person to perform work at this rate.

6.50. **IDENTIFY** and **SET UP:** Use Eq.(6.15) to relate the power provided and the amount of work done against gravity in 16.0 s. The work done against gravity depends on the total weight which depends on the number of passengers.

EXECUTE: Find the total mass that can be lifted:

$$P_{av} = \frac{\Delta W}{\Delta t} = \frac{mgh}{t}, \text{ so } m = \frac{P_{av} t}{gh}$$

$$P_{av} = (40 \text{ hp})\left(\frac{746 \text{ W}}{1 \text{ hp}}\right) = 2.984 \times 10^4 \text{ W}$$

$$m = \frac{P_{av} t}{gh} = \frac{(2.984 \times 10^4 \text{ W})(16.0 \text{ s})}{(9.80 \text{ m/s}^2)(20.0 \text{ m})} = 2.436 \times 10^3 \text{ kg}$$

This is the total mass of elevator plus passengers. The mass of the passengers is $2.436 \times 10^3 \text{ kg} - 600 \text{ kg} = 1.836 \times 10^3 \text{ kg}$.

The number of passengers is $\frac{1.836 \times 10^3 \text{ kg}}{65.0 \text{ kg}} = 28.2.$ 28 passengers can ride.

EVALUATE: Typical elevator capacities are about half this, in order to have a margin of safety.

6.51. **IDENTIFY:** Calculate the gallons of gasoline consumed and from that the energy consumed. Find the time Δt for the trip and use $P_{av} = \frac{\Delta W}{\Delta t}$, where ΔW is the energy consumed.

SET UP: $200 \text{ km} = 124 \text{ mi}$

EXECUTE: **(a)** The gallons of gasoline consumed is $\frac{124 \text{ mi}}{30 \text{ mi/gal}} = 4.13 \text{ gal}$. The energy consumed is

$(4.13 \text{ gal})(1.3 \times 10^9 \text{ J/gal}) = 5.4 \times 10^9 \text{ J}$.

(b) The time for the trip is $\frac{124 \text{ mi}}{60 \text{ mi/h}} = 2.07 \text{ h} = 7450 \text{ s}$. $P_{av} = \frac{\Delta W}{\Delta t} = \frac{5.4 \times 10^9 \text{ J}}{7450 \text{ s}} = 7.2 \times 10^5 \text{ W} = 720 \text{ kW}$.

EVALUATE: The rate of energy consumption is $\frac{720 \times 10^3 \text{ W}}{746 \text{ W/hp}} = 970 \text{ hp}$.

6.52. **IDENTIFY:** Apply $P = F_{\parallel} v$. $F_{\parallel}$ is the force *F* of water resistance.

SET UP: $1 \text{ hp} = 746 \text{ W}$. $1 \text{ km/h} = 0.228 \text{ m/s}$

EXECUTE: $F = \frac{(0.70)\,P}{v} = \frac{(0.70)\,(280{,}000 \text{ hp})(746 \text{ W/hp})}{(65 \text{ km/h})\,((0.228 \text{ m/s})/(1 \text{ km/h}))} = 8.1 \times 10^6 \text{ N}.$

EVALUATE: The power required depends on speed, because of the factor of *v* in $P = F_{\parallel} v$ and also because the resistive force increases with speed.

6.53. **IDENTIFY:** To lift the skiers, the rope must do positive work to counteract the negative work developed by the component of the gravitational force acting on the total number of skiers,

$F_{rope} = Nmg \sin\alpha$.

SET UP: $P = F_{\parallel} v = F_{rope} v$

EXECUTE: $P_{rope} = F_{rope} v = \left[+Nmg(\cos\phi)\right]v$.

$$P_{rope} = \left[(50 \text{ riders})(70.0 \text{ kg})(9.80 \text{ m/s}^2)(\cos 75.0^\circ)\right]\left[(12.0 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.60 \text{ km/h}}\right)\right].$$

$P_{rope} = 2.96 \times 10^4 \text{ W} = 29.6 \text{ kW}$.

EVALUATE: Some additional power would be needed to give the riders kinetic energy as they are accelerated from rest.

6.54. **IDENTIFY:** Relate power, work and time.

SET UP: Work done in each stroke is $W = Fs$ and $P_{av} = W/t$.

EXECUTE: 100 strokes per second means $P_{av} = 100Fs/t$ with $t = 1.00$ s, $F = 2mg$ and $s = 0.010$ m. $P_{av} = 0.20$ W.

EVALUATE: For a 70 kg person to apply a force of twice his weight through a distance of 0.50 m for 100 times per second, the average power output would be 7.0×10^5 W. This power output is very far beyond the capability of a person.

6.55. **IDENTIFY:** For mass dm located a distance x from the axis and moving with speed v, the kinetic energy is $K = \frac{1}{2}(dm)v^2$. Follow the procedure specified in the hint.

SET UP: The bar and an infinitesimal mass element along the bar are sketched in Figure 6.55. Let $M =$ total mass and $T =$ time for one revolution. $v = \dfrac{2\pi x}{T}$.

EXECUTE: $K = \int \frac{1}{2}(dm)v^2$. $dm = \dfrac{M}{L}dx$, so

$$K = \int_0^L \frac{1}{2}\left(\frac{M}{L}dx\right)\left(\frac{2\pi x}{T}\right)^2 = \frac{1}{2}\left(\frac{M}{L}\right)\left(\frac{4\pi^2}{T^2}\right)\int_0^L x^2 dx = \frac{1}{2}\left(\frac{M}{L}\right)\left(\frac{4\pi^2}{T^2}\right)\left(\frac{L^3}{3}\right) = \frac{2}{3}\pi^2 ML^2/T^2$$

There are 5 revolutions in 3 seconds, so $T = 3/5 \text{ s} = 0.60$ s

$$K = \frac{2}{3}\pi^2(12.0\text{ kg})\ (2.00\text{ m})^2/(0.60\text{ s})^2 = 877\text{ J}.$$

EVALUATE: If a point mass 12.0 kg is 2.00 m from the axis and rotates at the same rate as the bar, $v = \dfrac{2\pi(2.00\text{ m})}{0.60\text{ s}} = 20.9$ m/s and $K = \frac{1}{2}mv^2 = \frac{1}{2}(12\text{ kg})(20.9\text{ m/s})^2 = 2.62\times10^3$ J. K for the bar is smaller by a factor of 0.33. The speed of a segment of the bar decreases toward the axis.

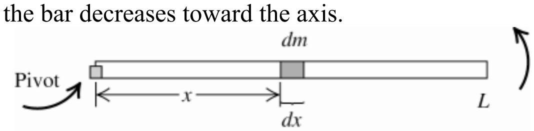


Figure 6.55

6.56. **IDENTIFY:** Density is mass per unit volume, $\rho = m/V$, so we can calculate the mass of the asteroid. $K = \frac{1}{2}mv^2$. Since the asteroid comes to rest, the kinetic energy it delivers equals its initial kinetic energy.

SET UP: The volume of a sphere is related to its diameter by $V = \dfrac{1}{6}\pi d^3$.

EXECUTE: **(a)** $V = \dfrac{\pi}{6}(320\text{ m})^3 = 1.72\times10^7\text{ m}^3$. $m = \rho V = (2600\text{ kg/m}^3)(1.72\times10^7\text{ m}^3) = 4.47\times10^{10}$ kg.

$K = \frac{1}{2}mv^2 = \frac{1}{2}(4.47\times10^{10}\text{ kg})(12.6\times10^3\text{ m/s})^2 = 3.55\times10^{18}$ J.

(b) The yield of a Castle/Bravo device is $(1\text{ s})(4.184\times10^{15}\text{ J}) = 6.28\times10^{16}$ J. $\dfrac{3.55\times10^{18}\text{ J}}{6.28\times10^{16}\text{ J}} = 56.5$ devices.

EVALUATE: If such an asteroid were to hit the earth the effect would be catastrophic.

6.57. **IDENTIFY** and **SET UP:** Since the forces are constant, Eq.(6.2) can be used to calculate the work done by each force. The forces on the suitcase are shown in Figure 6.57a.

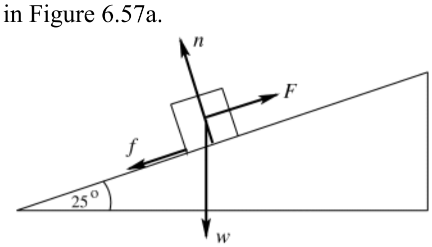


Figure 6.57a

In part (f), Eq.(6.6) is used to relate the total work to the initial and final kinetic energy.

EXECUTE: **(a)** $W_F = (F\cos\phi)s$

Both $\vec{F}$ and $\vec{s}$ are parallel to the incline and in the same direction, so $\phi = 90°$ and $W_F = Fs = (140\text{ N})(3.80\text{ m}) = 532$ J

(b) The directions of the displacement and of the gravity force are shown in Figure 6.57b.

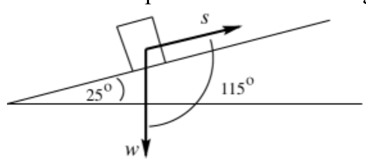


Figure 6.57b

$W_w = (w\cos\phi)s$

$\phi = 115°$, so

$W_w = (196\text{ N})(\cos 115°)(3.80\text{ m})$

$W_w = -315\text{ J}$

Alternatively, the component of w parallel to the incline is $w\sin 25°$. This component is down the incline so its angle with $\vec{s}$ is $\phi = 180°$. $W_{w\sin 25°} = (196\text{ N}\sin 25°)(\cos 180°)(3.80\text{ m}) = -315\text{ J}$. The other component of w, $w\cos 25°$, is perpendicular to $\vec{s}$ and hence does no work. Thus $W_w = W_{w\sin 25°} = -315\text{ J}$, which agrees with the above.

(c) The normal force is perpendicular to the displacement $(\phi = 90°)$, so $W_n = 0$.

(d) $n = w\cos 25°$ so $f_k = \mu_k n = \mu_k w\cos 25° = (0.30)(196\text{ N})\cos 25° = 53.3\text{ N}$

$W_f = (f_k\cos\phi)x = (53.3\text{ N})(\cos 180°)(3.80\text{ m}) = -202\text{ J}$

(e) $W_{tot} = W_F + W_w + W_n + W_f = +532\text{ J} - 315\text{ J} + 0 - 202\text{ J} = 15\text{ J}$

(f) $W_{tot} = K_2 - K_1$, $K_1 = 0$, so $K_2 = W_{tot}$

$\frac{1}{2}mv_2^2 = W_{tot}$ so $v_2 = \sqrt{\frac{2W_{tot}}{m}} = \sqrt{\frac{2(15\text{ J})}{20.0\text{ kg}}} = 1.2\text{ m/s}$

EVALUATE: The total work done is positive and the kinetic energy of the suitcase increases as it moves up the incline.

6.58. **IDENTIFY:** The work he does to lift his body a distance h is $W = mgh$. The work per unit mass is $(W/m) = gh$.

SET UP: The quantity gh has units of N/kg.

EXECUTE: **(a)** The man does work, $(9.8\text{ N/kg})(0.4\text{ m}) = 3.92\text{ J/kg}$.

(b) $(3.92\text{ J/kg})/(70\text{ J/kg})\times 100 = 5.6\%$.

(c) The child does work $(9.8\text{ N/kg})(0.2\text{ m}) = 1.96\text{ J/kg}$. $(1.96\text{ J/kg})/(70\text{ J/kg})\times 100 = 2.8\%$.

(d) If both the man and the child can do work at the rate of 70 J/kg, and if the child only needs to use 1.96 J/kg instead of 3.92 J/kg, the child should be able to do more chin-ups.

EVALUATE: Since the child has arms half the length of his father's arms, the child must lift his body only 0.20 m to do a chin-up.

6.59. **IDENTIFY:** Apply the definitions of IMA and AMA given in the problem.

SET UP: When the object moves a distance L along the ramp, it rises a vertical distance $L\sin\alpha$.

EXECUTE: **(a)** $s_{in} = L$, $s_{out} = L\sin\alpha$, so $IMA = \dfrac{1}{\sin\alpha}$.

(b) If $AMA = IMA$, $(F_{out}/F_{in}) = (s_{in}/s_{out})$ and so $(F_{out})(s_{out}) = (F_{in})(s_{in})$, or $W_{out} = W_{in}$.

(c) The pulley is sketched in Figure 6.59.

(d) $e = \dfrac{W_{out}}{W_{in}} = \dfrac{(F_{out})(s_{out})}{(F_{in})(s_{in})} = \dfrac{F_{out}/F_{in}}{s_{in}/s_{out}} = \dfrac{AMA}{IMA}$.

EVALUATE: $F_{in} = w\sin\alpha$ and $F_{out} = w$. $(F_{in})(s_{in}) = (w\sin\alpha)L$. $(F_{out})(s_{out}) = w(\sin\alpha L)$. Therefore, $(F_{in})(s_{in}) = (F_{out})(s_{out})$. A smaller force acting over a larger distance does the same amount of work as a larger force acting over a smaller distance.

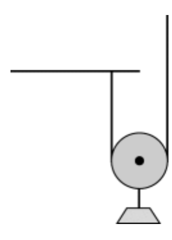


Figure 6.59

6.60. **IDENTIFY:** Apply $\sum\vec{F} = m\vec{a}$ to each block to find the tension in the string. Each force is constant and $W = Fs\cos\phi$.

SET UP: The free-body diagram for each block is given in Figure 6.60. $m_A = \dfrac{20.0\text{ N}}{g} = 2.04\text{ kg}$ and

$m_B = \dfrac{12.0\text{ N}}{g} = 1.22\text{ kg}$.

EXECUTE: $T - f_k = m_A a$. $w_B - T = m_B a$. $w_B - f_k = (m_A + m_B)a$.

$f_k = 0$. $a = \left(\dfrac{w_B}{m_A + m_B}\right)$ and $T = w_B\left(\dfrac{m_A}{m_A + m_B}\right) = w_B\left(\dfrac{w_A}{w_A + w_B}\right) = 7.50\text{ N}$.

20.0 N block: $W_{tot} = Ts = (7.50\text{ N})(0.750\text{ m}) = 5.62\text{ J}$.

12.0 N block: $W_{tot} = (w_B - T)s = (12.0\text{ N} - 7.50\text{ N})(0.750\text{ m}) = 3.38\text{ J}$

(b) $f_k = \mu_k w_A = 6.50\text{ N}$. $a = \dfrac{w_B - \mu_k w_A}{m_A + m_B}$. $T = f_k + (w_B - \mu_k w_A)\left(\dfrac{m_A}{m_A + m_B}\right) = \mu_k w_A + (w_B - \mu_k w_A)\left(\dfrac{w_A}{w_A + w_B}\right)$.

$T = 6.50\text{ N} + (5.50\text{ N})(0.625) = 9.94\text{ N}$.

20.0 N block: $W_{tot} = (T - f_k)s = (9.94\text{ N} - 6.50\text{ N})(0.750\text{ m}) = 2.58\text{ J}$.

12.0 N block: $W_{tot} = (w_B - T)s = (12.0\text{ N} - 9.94\text{ N})(0.750\text{ m}) = 1.54\text{ J}$.

EVALUATE: Since the two blocks move with equal speeds, for each block $W_{tot} = K_2 - K_1$ is proportional to the mass (or weight) of that block. With friction the gain in kinetic energy is less, so the total work on each block is less.

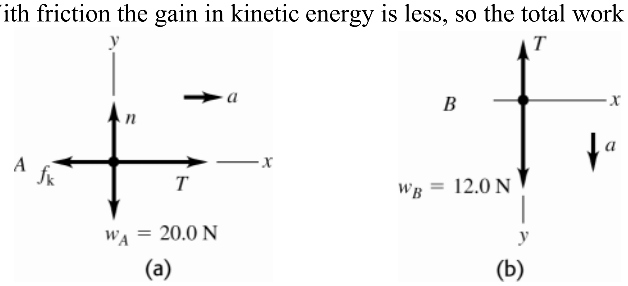


Figure 6.60

6.61. **IDENTIFY:** $K = \frac{1}{2}mv^2$. Find the speed of the shuttle relative to the earth and relative to the satellite.

SET UP: Velocity is distance divided by time. For one orbit the shuttle travels a distance $2\pi R$.

EXECUTE: **(a)** $\dfrac{1}{2}mv^2 = \dfrac{1}{2}m\left(\dfrac{2\pi R}{T}\right)^2 = \dfrac{1}{2}(86{,}400\text{ kg})\left(\dfrac{2\pi(6.66\times10^6\text{ m})}{(90.1\text{ min})(60\text{ s/min})}\right)^2 = 2.59\times10^{12}\text{ J}.$

(b) $(1/2)\,mv^2 = (1/2)\,(86{,}400\text{ kg})\,((1.00\text{ m})/(3.00\text{ s}))^2 = 4.80\times10^3\text{ J}.$

EVALUATE: The kinetic energy of an object depends on the reference frame in which it is measured.

6.62. **IDENTIFY:** $W = Fs\cos\phi$. $W_{tot} = K_2 - K_1$.

SET UP: $f_k = \mu_k n$. The normal force is $n = mg\cos\theta$, with $\theta = 12.0°$. The component of the weight parallel to the incline is $mg\sin\theta$.

EXECUTE: **(a)** $\phi = 180°$ and $W_f = -f_k s = -(\mu_k mg\cos\theta)s = -(0.31)(5.00\text{ kg})(9.80\text{ m/s}^2)(\cos 12.0°)(1.50\text{ m}) = -22.3\text{ J}$

(b) $(5.00\text{ kg})(9.80\text{ m/s}^2)(\sin 12.0°)(1.50\text{ m}) = 15.3\text{ J}.$

(c) The normal force does no work.

(d) $W_{tot} = 15.3\text{ J} - 22.3\text{ J} = -7.0\text{ J}.$

(e) $K_2 = K_1 + W_{tot} = (1/2)(5.00\text{ kg})(2.2\text{ m/s})^2 - 7.0\text{ J} = 5.1\text{ J}$, and so $v_2 = \sqrt{2(5.1\text{ J})/(5.00\text{ kg})} = 1.4\text{ m/s}$.

EVALUATE: Friction does negative work and gravity does positive work. The net work is negative and the kinetic energy of the object decreases.

6.63. **IDENTIFY:** The effective force constant is defined by $k_{eff} = F/x$, where F is the force applied to each end of the spring combination and x is the amount the spring combination is stretched.

SET UP: Consider a force F applied to each end of the combination. Then F_1 and F_2 are the forces applied to each spring and $F = F_1 + F_2$. Each spring stretches the same amount x.

EXECUTE: **(a)** $F = k_{eff}x$. $F = F_1 + F_2 = k_1 x + k_2 x$. Equating the two expressions for F gives $k_{eff} = k_1 + k_2$.

(b) The same procedure as in part (a) gives $k_{eff} = k_1 + k_2 + \cdots + k_N$.

EVALUATE: The effective force constant of the configuration is greater than any of the force constants of the individual springs. More force is required to stretch the parallel combination that is required to stretch each separate spring the same amount.

6.64. **IDENTIFY:** The effective force constant is defined by $k_{\text{eff}} = F/x$, where F is the force applied to each end of the spring combination and x is the amount the spring combination is stretched.

SET UP: Consider a force F applied to each end of the combination. The same force F is applied to each spring. Spring 1 stretches a distance x_1 and spring 2 stretches a distance x_2, where $x_1 = F/k_1$ and $x_2 = F/k_2$. The total distance the combination stretches is $x = x_1 + x_2$.

EXECUTE: **(a)** $x = x_1 + x_2$ gives $\dfrac{F}{k_{\text{eff}}} = \dfrac{F}{k_1} = \dfrac{F}{k_2}$ and $\dfrac{1}{k_{\text{eff}}} = \dfrac{1}{k_1} + \dfrac{1}{k_2}$.

(b) The same procedure as in part (a) gives $\dfrac{1}{k_{\text{eff}}} = \dfrac{1}{k_1} + \dfrac{1}{k_2} + \cdots + \dfrac{1}{k_N}$.

EVALUATE: For two springs the result in part (a) can be written as $k_{\text{eff}} = \dfrac{k_1 k_2}{k_1 + k_2}$. The effective force constant for the two springs in series is less than the force constant for each individual spring. It takes less force to stretch the combination an amount x than to stretch either separate spring an amount x.

6.65. **IDENTIFY:** Apply Eq.(6.7).

SET UP: $\int \dfrac{dx}{x^2} = -\dfrac{1}{x}$.

EXECUTE: **(a)** $W = \int_{x_1}^{x_2} F_x dx = -k\int_{x_1}^{x_2} \dfrac{dx}{x^2} = -k\left[-\dfrac{1}{x}\right]_{x_1}^{x_2} = k\left(\dfrac{1}{x_2} - \dfrac{1}{x_1}\right)$. The force is given to be attractive, so $F_x < 0$, and k must be positive. If $x_2 > x_1$, $\dfrac{1}{x_2} < \dfrac{1}{x_1}$, and $W < 0$.

(b) Taking "slowly" to be constant speed, the net force on the object is zero. The force applied by the hand is opposite F_x, and the work done is negative of that found in part (a), or $k\left(\dfrac{1}{x_1} - \dfrac{1}{x_2}\right)$, which is positive if $x_2 > x_1$.

(c) The answers have the same magnitude but opposite signs; this is to be expected, in that the net work done is zero.

EVALUATE: Your force is directed away from the origin, so when the object moves away from the origin your force does positive work.

6.66. **IDENTIFY:** Apply Eq.(6.6) to the motion of the asteroid.

SET UP: Let point 1 be at a great distance and let point 2 be at the surface of the earth. Assume $K_1 = 0$. From the information given about the gravitational force its magnitude as a function of distance r from the center of the earth must be $F = mg(R_{\text{E}}/r)^2$. This force is directed in the $-\hat{\boldsymbol{r}}$ direction since it is a "pull". F is not constant so Eq.(6.7) must be used to calculate the work it does.

EXECUTE: $W = -\int_1^2 F\, ds = -\int_{\infty}^{R_{\text{E}}} \left(\dfrac{mgR_{\text{E}}^2}{r^2}\right) dr = -mgR_{\text{E}}^2\left(-(1/r)\Big|_{\infty}^{R_{\text{E}}}\right) = mgR_{\text{E}}$

$W_{\text{tot}} = K_2 - K_1,\ \ K_1 = 0$

This gives $K_2 = mgR_{\text{E}} = 1.25 \times 10^{12}\ \text{J}$

$K_2 = \frac{1}{2}mv_2^2$ so $v_2 = \sqrt{2K_2/m} = 11{,}000\ \text{m/s}$

EVALUATE: Note that $v_2 = \sqrt{2gR_{\text{E}}}$, the impact speed is independent of the mass of the asteroid.

6.67. **IDENTIFY:** Calculate the work done by friction and apply $W_{\text{tot}} = K_2 - K_1$. Since the friction force is not constant, use Eq.(6.7) to calculate the work.

SET UP: Let x be the distance past P. Since μ_{k} increases linearly with x, $\mu_{\text{k}} = 0.100 + Ax$. When $x = 12.5\ \text{m}$, $\mu_{\text{k}} = 0.600$, so $A = 0.500/(12.5\ \text{m}) = 0.0400/\text{m}$

EXECUTE: **(a)** $W_{\text{tot}} = \Delta K = K_2 - K_1$ gives $-\int \mu_{\text{k}} mg dx = 0 - \dfrac{1}{2}mv_1^2$. Using the above expression for μ_{k},

$g\int_0^{x_2}(0.100 + Ax)dx = \dfrac{1}{2}v_1^2$ and $g\left[(0.100)x_2 + A\dfrac{x_2^2}{2}\right] = \dfrac{1}{2}v_1^2$. $(9.80\ \text{m/s}^2)\left[(0.100)x_{\text{f}} + (0.0400/\text{m})\dfrac{x_{\text{f}}^2}{2}\right] = \dfrac{1}{2}(4.50\ \text{m/s})^2$.

Solving for x_2 gives $x_2 = 5.11\ \text{m}$.

(b) $\mu_{\text{k}} = 0.100 + (0.0400/\text{m})(5.11\ \text{m}) = 0.304$

(c) $W_{\text{tot}} = K_2 - K_1$ gives $-\mu_k m g x_2 = 0 - \frac{1}{2}mv_1^2$. $x_2 = \frac{v_1^2}{2\mu_k g} = \frac{(4.50\text{ m/s})^2}{2(0.100)(9.80\text{ m/s}^2)} = 10.3\text{ m}$.

EVALUATE: The box goes farther when the friction coefficient doesn't increase.

6.68. IDENTIFY: Use Eq.(6.7) to calculate *W*.

SET UP: $x_1 = 0$. In part (a), $x_2 = 0.050\text{ m}$. In part (b), $x_2 = -0.050\text{ m}$.

EXECUTE: **(a)** $W = \int_0^{x_2} F dx = \int_0^{x_2} (kx - bx^2 + cx^3)dx = \frac{k}{2}x_2^2 - \frac{b}{3}x_2^3 + \frac{c}{4}x_2^4$.

$W = (50.0\text{ N/m})x_2^2 - (233\text{ N/m}^2)x_2^3 + (3000\text{ N/m}^3)x_2^4$. When $x_2 = 0.050\text{ m}$, $W = 0.12\text{ J}$.

(b) When $x_2 = -0.050$ m, $W = 0.17$ J.

(c) It's easier to stretch the spring; the quadratic $-bx^2$ term is always in the $-x$-direction, and so the needed force, and hence the needed work, will be less when $x_2 > 0$.

EVALUATE: When $x = 0.050\text{ m}$, $F_x = 4.75\text{ N}$. When $x = -0.050\text{ m}$, $F_x = 8.25\text{ N}$.

6.69. IDENTIFY and SET UP: Use $\sum \vec{F} = m\vec{a}$ to find the tension force *T*. The block moves in uniform circular motion and $\vec{a} = \vec{a}_{\text{rad}}$.

(a) The free-body diagram for the block is given in Figure 6.69.

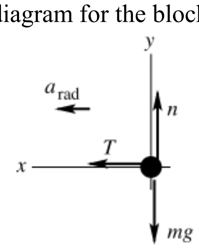


Figure 6.69

EXECUTE: $\sum F_x = ma_x$

$$T = m\frac{v^2}{R}$$

$$T = (0.120\text{ kg})\frac{(0.70\text{ m/s})^2}{0.40\text{ m}} = 0.15\text{ N}$$

(b) $T = m\frac{v^2}{R} = (0.120\text{ kg})\frac{(2.80\text{ m/s})^2}{0.10\text{ m}} = 9.4\text{ N}$

(c) SET UP: The tension changes as the distance of the block from the hole changes. We could use $W = \int_{x_1}^{x_2} F_x\, dx$ to calculate the work. But a much simpler approach is to use $W_{\text{tot}} = K_2 - K_1$.

EXECUTE: The only force doing work on the block is the tension in the cord, so $W_{\text{tot}} = W_T$.

$$K_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(0.120\text{ kg})(0.70\text{ m/s})^2 = 0.0294\text{ J}$$

$$K_2 = \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}(0.120\text{ kg})(2.80\text{ m/s})^2 = 0.470\text{ J}$$

$$W_{\text{tot}} = K_2 - K_1 = 0.470\text{ J} - 0.029\text{ J} = 0.44\text{ J}$$

This is the amount of work done by the person who pulled the cord.

EVALUATE: The block moves inward, in the direction of the tension, so *T* does positive work and the kinetic energy increases.

6.70. IDENTIFY: Use Eq.(6.7) to find the work done by *F*. Then apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: $\int \frac{dx}{x^2} = -\frac{1}{x}$.

EXECUTE: $W = \int_{x_1}^{x_2} \frac{\alpha}{x^2} dx = \alpha\left(\frac{1}{x_1} - \frac{1}{x_2}\right)$. $W = (2.12\times10^{-26}\text{ N}\cdot\text{m}^2)((0.200\text{ m}^{-1}) - (1.25\times10^{9}\text{ m}^{-1})) = -2.65\times10^{-17}\text{ J}$.

Note that x_1 is so large compared to x_2 that the term $1/x_1$ is negligible. Then, using Eq. (6.13)) and solving for v_2,

$$v_2 = \sqrt{v_1^2 + \frac{2W}{m}} = \sqrt{(3.00\times10^5\text{ m/s})^2 + \frac{2(-2.65\times10^{-17}\text{ J})}{(1.67\times10^{-27}\text{ kg})}} = 2.41\times10^5\text{ m/s}.$$

(b) With $K_2 = 0, W = -K_1$. Using $W = -\frac{\alpha}{x_2}$,

$$x_2 = \frac{\alpha}{K_1} = \frac{2\alpha}{mv_1^2} = \frac{2(2.12\times10^{-26}\text{ N}\cdot\text{m}^2)}{(1.67\times10^{-27}\text{ kg})(3.00\times10^5\text{ m/s})^2} = 2.82\times10^{-10}\text{ m}.$$

(c) The repulsive force has done no net work, so the kinetic energy and hence the speed of the proton have their original values, and the speed is 3.00×10^5 m/s .

EVALUATE: As the proton moves toward the uranium nucleus the repulsive force does negative work and the kinetic energy of the proton decreases. As the proton moves away from the uranium nucleus the repulsive force does positive work and the kinetic energy of the proton increases.

6.71. **IDENTIFY** and **SET UP:** Use $v_x = dx/dt$ and $a_x = dv_x/dt$. Use $\sum\vec{F} = m\vec{a}$ to calculate $\vec{F}$ from $\vec{a}$.

EXECUTE: **(a)** $x(t) = \alpha t^2 + \beta t^3,\ \ v_x(t) = \dfrac{dx}{dt} = 2\alpha t + 3\beta t^2$

$t = 4.00$ s: $v_x = 2(0.200\text{ m/s}^2)(4.00\text{ s}) + 3(0.0200\text{ m/s}^3)(4.00\text{ s})^2 = 2.56$ m/s.

(b) $a_x(t) = \dfrac{dv_x}{dt} = 2\alpha + 6\beta t$

$F_x = ma_x = m(2\alpha + 6\beta t)$

$t = 4.00$ s: $F_x = 6.00\text{ kg}(2(0.200\text{ m/s}^2) + 6(0.0200\text{ m/s}^3)(4.00\text{ s})) = 5.28$ N

(c) IDENTIFY and **SET UP:** Use Eq.(6.6) to calculate the work.

EXECUTE: $W_{\text{tot}} = K_2 - K_1$

At $t_1 = 0,\ v_1 = 0$ so $K_1 = 0.$

$W_{\text{tot}} = W_F$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(6.00\text{ kg})(2.56\text{ m/s})^2 = 19.7$ J

Then $W_{\text{tot}} = K_2 - K_1$ gives that $W_F = 19.7$ J

EVALUATE: v increases with t so the kinetic energy increases and the work done is positive. We can also calculate W_F directly from Eq.(6.7), by writing dx as $v_x\ dt$ and performing the integral.

6.72. **IDENTIFY:** Since the capsule comes to rest, the amount of work the capsule does on the ground equals its original kinetic energy. Use constant acceleration kinematic equations to calculate the stopping time t; $\Delta t = t$.

SET UP: 311 km/h = 86.4 m/s . Let $+y$ be the direction the capsule is traveling before the crash.

EXECUTE: $\Delta W = K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(210\text{ kg})(86.4\text{ m/s})^2 = 7.84\times10^5$ J . $y - y_0 = 0.810$ m , $v_{0y} = 86.4$ m/s and $v_y = 0$.

$y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $t = \dfrac{2(y - y_0)}{v_{0y}} = \dfrac{2(0.810\text{ m})}{86.4\text{ m/s}} = 0.01875$ s . $\dfrac{\Delta W}{\Delta t} = \dfrac{7.84\times10^5\text{ J}}{0.01875\text{ s}} = 4.18\times10^7$ W

EVALUATE: A large amount of work is done in a very small amount of time.

6.73. **IDENTIFY** and **SET UP:** Use Eq.(6.6). You do positive work and gravity does negative work. Let point 1 be at the base of the bridge and point 2 be at the top of the bridge.

EXECUTE: **(a)** $W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2 = 1000$ J

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(80.0\text{ kg})(1.50\text{ m/s})^2 = 90$ J

$W_{\text{tot}} = 90\text{ J} - 1000\text{ J} = -910$ J

(b) Neglecting friction, work is done by you (with the force you apply to the pedals) and by gravity: $W_{\text{tot}} = W_{\text{you}} + W_{\text{gravity}}$. The gravity force is $w = mg = (80.0\text{ kg})(9.80\text{ m/s}^2) = 784$ N, downward. The displacement is 5.20 m, upward. Thus $\phi = 180°$ and

$$W_{\text{gravity}} = (F\cos\phi)s = (784\text{ N})(5.20\text{ m})\cos180° = -4077\text{ J}$$

Then $W_{\text{tot}} = W_{\text{you}} + W_{\text{gravity}}$ gives

$$W_{\text{you}} = W_{\text{tot}} - W_{\text{gravity}} = -910\text{ J} - (-4077\text{ J}) = +3170\text{ J}$$

EVALUATE: The total work done is negative and you lose kinetic energy.

6.74. **IDENTIFY:** Use Eq.(6.7) to calculate W.

SET UP: $\int x^{-n}dx = -\dfrac{1}{n-1}x^{-(n-1)}$

EXECUTE: **(a)** $W = \int_{x_0}^{\infty}\dfrac{b}{x^n}dx = \dfrac{b}{(n-1))x^{n-1}}\Bigg|_{x_0}^{\infty} = \dfrac{b}{(n-1)x_0^{n-1}}$. Note that for this part, for $n > 1, x^{1-n} \to 0$ as $x \to \infty$.

(b) When $0 < n < 1$, the improper integral must be used, $W = \lim_{x_2 \to \infty} \left[\frac{b}{(n-1)} (x_2^{n-1} - x_0^{n-1}) \right]$, and because the exponent on the x_2^{n-1} is positive, the limit does not exist, and the integral diverges. This is interpreted as the force F doing an infinite amount of work, even though $F \to 0$ as $x_2 \to \infty$.

EVALUATE: The work-energy theorem says that an object gains an infinite amount of kinetic energy when an infinite amount of work is done on it.

6.75. IDENTIFY: The negative work done by the spring equals the change in kinetic energy of the car.

SET UP: The work done by a spring when it is compressed a distance x from equilibrium is $-\frac{1}{2}kx^2$. $K_2 = 0$.

EXECUTE: $-\frac{1}{2}kx^2 = K_2 - K_1$ gives $\frac{1}{2}kx^2 = \frac{1}{2}mv_1^2$ and

$k = (mv_1^2)/x^2 = [(1200\text{ kg})(0.65\text{ m/s})^2]/(0.070\text{ m})^2 = 1.0 \times 10^5\text{ N/m}$.

EVALUATE: When the spring is compressed, the spring force is directed opposite to the displacement of the object and the work done by the spring is negative.

6.76. IDENTIFY: Apply $W_{\text{tot}} = K_2 - K_1$.

SET UP: Let x_0 be the initial distance the spring is compressed. The work done by the spring is $\frac{1}{2}kx_0^2 - \frac{1}{2}kx^2$, where x is the final distance the spring is compressed.

EXECUTE: **(a)** Equating the work done by the spring to the gain in kinetic energy, $\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2$, so

$v = \sqrt{\frac{k}{m}}x_0 = \sqrt{\frac{400\text{ N/m}}{0.0300\text{ kg}}}(0.060\text{ m}) = 6.93\text{ m/s}.$

(b) W_{tot} must now include friction, so $\frac{1}{2}mv^2 = W_{\text{tot}} = \frac{1}{2}kx_0^2 - fx_0$, where f is the magnitude of the friction force. Then,

$$v = \sqrt{\frac{k}{m}x_0^2 - \frac{2f}{m}x_0} = \sqrt{\frac{400\text{ N/m}}{0.0300\text{ kg}}(0.060\text{ m})^2 - \frac{2(6.00\text{ N})}{(0.0300\text{ kg})}(0.060\text{ m})} = 4.90\text{ m/s}.$$

(c) The greatest speed occurs when the acceleration (and the net force) are zero. Let x be the amount the spring is still compressed, so the distance the ball has moved is $x_0 - x$. $kx = f$, $x = \frac{f}{k} = \frac{6.00\text{ N}}{400\text{ N/m}} = 0.0150\text{ m}$. To find the speed, the net work is $W_{\text{tot}} = \frac{1}{2}k(x_0^2 - x^2) - f(x_0 - x)$, so the maximum speed is $v_{\text{max}} = \sqrt{\frac{k}{m}(x_0^2 - x^2) - \frac{2f}{m}(x_0 - x)}$.

$$v_{\text{max}} = \sqrt{\frac{400\text{ N/m}}{(0.0300\text{ kg})}((0.060\text{ m})^2 - (0.0150\text{ m})^2) - \frac{2(6.00\text{ N})}{(0.0300\text{ kg})}(0.060\text{ m} - 0.0150\text{ m})} = 5.20\text{ m/s}$$

EVALUATE: The maximum speed with friction present (part (c)) is larger than the result of part (b) but smaller than the result of part (a).

6.77. IDENTIFY and SET UP: Use Eq.(6.6). Work is done by the spring and by gravity. Let point 1 be where the textbook is released and point 2 be where it stops sliding. $x_2 = 0$ since at point 2 the spring is neither stretched nor compressed. The situation is sketched in Figure 6.77.

EXECUTE:

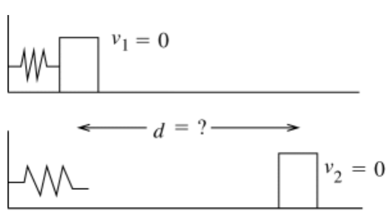


Figure 6.77

$W_{\text{tot}} = K_2 - K_1$

$K_1 = 0,\quad K_2 = 0$

$W_{\text{tot}} = W_{\text{fric}} + W_{\text{spr}}$

$W_{\text{spr}} = \frac{1}{2}kx_1^2$, where $x_1 = 0.250\text{ m}$ (Spring force is in direction of motion of block so it does positive work.)

$W_{\text{fric}} = -\mu_k mgd$

Then $W_{\text{tot}} = K_2 - K_1$ gives $\frac{1}{2}kx_1^2 - \mu_k mgd = 0$

$d = \frac{kx_1^2}{2\mu_k mg} = \frac{(250\text{ N/m})(0.250\text{ m})^2}{2(0.30)(2.50\text{ kg})(9.80\text{ m/s}^2)} = 1.1\text{ m}$, measured from the point where the block was released.

EVALUATE: The positive work done by the spring equals the magnitude of the negative work done by friction. The total work done during the motion between points 1 and 2 is zero and the textbook starts and ends with zero kinetic energy.

6.78. **IDENTIFY:** Apply $W_{tot} = K_2 - K_1$ to the cat.

SET UP: Let point 1 be at the bottom of the ramp and point 2 be at the top of the ramp.

EXECUTE: The work done by gravity is $W_g = -mgL\sin\theta$ (negative since the cat is moving up), and the work done by the applied force is *FL*, where *F* is the magnitude of the applied force. The total work is

$$W_{tot} = (100\text{ N})(2.00\text{ m}) - (7.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})\sin 30° = 131.4\text{ J}.$$

The cat's initial kinetic energy is $\frac{1}{2}mv_1^2 = \frac{1}{2}(7.00\text{ kg})(2.40\text{ m/s})^2 = 20.2\text{ J}$, and

$$v_2 = \sqrt{\frac{2(K_1 + W)}{m}} = \sqrt{\frac{2(20.2\text{ J} + 131.4\text{ J})}{(7.00\text{ kg})}} = 6.58\text{ m/s}.$$

EVALUATE: The net work done on the cat is positive and the cat gains speed. Without your push, $W_{tot} = W_{grav} = -68.6\text{ J}$ and the cat wouldn't have enough initial kinetic energy to reach the top of the ramp.

6.79. **IDENTIFY:** Apply $W_{tot} = K_2 - K_1$ to the vehicle.

SET UP: Call the bumper compression *x* and the initial speed v_0. The work done by the spring is $-\frac{1}{2}kx^2$ and $K_2 = 0$.

EXECUTE: **(a)** The necessary relations are $\frac{1}{2}kx^2 = \frac{1}{2}mv_0^2$, $kx < 5\,mg$. Combining to eliminate *k* and then *x*, the two inequalities are $x > \frac{v^2}{5g}$ and $k < 25\frac{mg^2}{v^2}$. Using the given numerical values, $x > \frac{(20.0\text{ m/s})^2}{5(9.80\text{ m/s}^2)} = 8.16\text{ m}$ and $k < 25\frac{(1700\text{ kg})(9.80\text{ m/s}^2)^2}{(20.0\text{ m/s})^2} = 1.02\times 10^4\text{ N/m}.$

(b) A distance of 8 m is not commonly available as space in which to stop a car. Also, the car stops only momentarily and then returns to its original speed when the spring returns to its original length.

EVALUATE: If *k* were doubled, to $2.04\times 10^4\text{ N/m}$, then $x = 5.77\text{ m}$. The stopping distance is reduced by a factor of $1/\sqrt{2}$, but the maximum acceleration would then be $kx/m = 69.2\text{ m/s}^2$, which is $7.07g$.

6.80. **IDENTIFY:** Apply $W_{tot} = K_2 - K_1$. $W = Fs\cos\phi$.

SET UP: The students do positive work, and the force that they exert makes an angle of $30.0°$ with the direction of motion. Gravity does negative work, and is at an angle of $120.0°$ with the chair's motion,

EXECUTE: The total work done is $W_{tot} = ((600\text{ N})\cos 30.0° + (85.0\text{ kg})(9.80\text{ m/s}^2)\cos 120.0°)(2.50\text{ m}) = 257.8\text{ J}$, and so the speed at the top of the ramp is $v_2 = \sqrt{v_1^2 + \frac{2W_{tot}}{m}} = \sqrt{(2.00\text{ m/s})^2 + \frac{2(257.8\text{ J})}{(85.0\text{ kg})}} = 3.17\text{ m/s}.$

EVALUATE: The component of gravity down the incline is $mg\sin 30° = 417\text{ N}$ and the component of the push up the incline is $(600\text{ N})\cos 30° = 520\text{ N}$. The force component up the incline is greater than the force component down the incline, the net work done is positive and the speed increases.

6.81. **IDENTIFY:** Apply $W_{tot} = K_2 - K_1$ to the blocks.

SET UP: If *X* is the distance the spring is compressed, the work done by the spring is $-\frac{1}{2}kX^2$. At maximum compression, the spring (and hence the block) is not moving, so the block has no kinetic energy and $x_2 = 0$.

EXECUTE: **(a)** The work done *by* the block is equal to its initial kinetic energy, and the maximum compression is found from $\frac{1}{2}kX^2 = \frac{1}{2}mv_0^2$ and $X = \sqrt{\frac{m}{k}}v = \sqrt{\frac{5.00\text{ kg}}{500\text{ N/m}}}(6.00\text{ m/s}) = 0.600\text{ m}.$

(b) Solving for v_0 in terms of a known *X*, $v_0 = \sqrt{\frac{k}{m}}X = \sqrt{\frac{500\text{ N/m}}{5.00\text{ kg}}}(0.150\text{ m}) = 1.50\text{ m/s}.$

EVALUATE: The negative work done by the spring removes the kinetic energy of the block.

6.82. **IDENTIFY:** Apply $W_{tot} = K_2 - K_1$ to the system of the two blocks. The total work done is the sum of that done by gravity (on the hanging block) and that done by friction (on the block on the table).

SET UP: Let *h* be the distance the 6.00 kg block descends. The work done by gravity is (6.00 kg)*gh* and the work done by friction is $-\mu_k(8.00\text{ kg})gh$.

EXECUTE: $W_{\text{tot}} = (6.00\text{ kg} - (0.25)(8.00\text{ kg}))\,(9.80\text{ m/s}^2)\,(1.50\text{ m}) = 58.8\text{ J}$. This work increases the kinetic energy of both blocks: $W_{\text{tot}} = \frac{1}{2}(m_1 + m_2)v^2$, so $v = \sqrt{\frac{2(58.8\text{ J})}{(14.00\text{ kg})}} = 2.90\text{ m/s}$.

EVALUATE: Since the two blocks are connected by the rope, they move the same distance h and have the same speed v.

6.83. **IDENTIFY** and **SET UP:** Apply $W_{\text{tot}} = K_2 - K_1$ to the system consisting of both blocks. Since they are connected by the cord, both blocks have the same speed at every point in the motion. Also, when the 6.00-kg block has moved downward 1.50 m, the 8.00-kg block has moved 1.50 m to the right. The target variable, μ_k, will be a factor in the work done by friction. The forces on each block are shown in Figure 6.83.

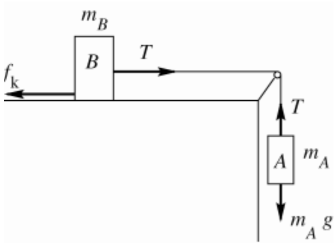


Figure 6.83

EXECUTE: $K_1 = \frac{1}{2}m_A v_1^2 + \frac{1}{2}m_B v_1^2 = \frac{1}{2}(m_A + m_B)v_1^2$

$K_2 = 0$

The tension T in the rope does positive work on block B and the same magnitude of negative work on block A, so T does no net work on the system. Gravity does work $W_{mg} = m_A g d$ on block A, where $d = 2.00\text{ m}$. (Block B moves horizontally, so no work is done on it by gravity.) Friction does work $W_{\text{fric}} = -\mu_k m_B g d$ on block B. Thus $W_{\text{tot}} = W_{mg} + W_{\text{fric}} = m_A g d - \mu_k m_B g d$. Then $W_{\text{tot}} = K_2 - K_1$ gives $m_A g d - \mu_k m_B g d = -\frac{1}{2}(m_A + m_B)v_1^2$ and

$$\mu_k = \frac{m_A}{m_B} + \frac{\frac{1}{2}(m_A + m_B)v_1^2}{m_B g d} = \frac{6.00\text{ kg}}{8.00\text{ kg}} + \frac{(6.00\text{ kg} + 8.00\text{ kg})(0.900\text{ m/s})^2}{2(8.00\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})} = 0.786$$

EVALUATE: The weight of block A does positive work and the friction force on block B does negative work, so the net work is positive and the kinetic energy of the blocks increases as block A descends. Note that K_1 includes the kinetic energy of both blocks. We could have applied the work-energy theorem to block A alone, but then W_{tot} includes the work done on block A by the tension force.

6.84. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$. The work done by the force from the bow is the area under the graph of F_x versus the draw length.

SET UP: One possible way of estimating the work is to approximate the F versus x curve as a parabola which goes to zero at $x = 0$ and $x = x_0$, and has a maximum of F_0 at $x = x_0/2$, so that $F(x) = (4F_0/x_0^2)x(x_0 - x)$. This may seem like a crude approximation to the figure, but it has the advantage of being easy to integrate.

EXECUTE: $\int_0^{x_0} F dx = \frac{4F_0}{x_0^2}\int_0^{x_0}(x_0 x - x^2)\,dx = \frac{4F_0}{x_0^2}\left(x_0\frac{x_0^2}{2} - \frac{x_0^3}{3}\right) = \frac{2}{3}F_0 x_0$. With $F_0 = 200\text{ N}$ and $x_0 = 0.75\text{ m}$, $W = 100\text{ J}$. The speed of the arrow is then $\sqrt{\frac{2W}{m}} = \sqrt{\frac{2(100\text{ J})}{(0.025\text{ kg})}} = 89\text{ m/s}$.

EVALUATE: We could alternatively represent the area as that of a rectangle 180 N by 0.55 m. This gives $W = 99\text{ J}$, in close agreement with our more elaborate estimate.

6.85. **IDENTIFY:** Apply Eq.(6.6) to the skater.

SET UP: Let point 1 be just before she reaches the rough patch and let point 2 be where she exits from the patch. Work is done by friction. We don't know the skater's mass so can't calculate either friction or the initial kinetic energy. Leave her mass m as a variable and expect that it will divide out of the final equation.

EXECUTE: $f_k = 0.25mg$ so $W_f = W_{\text{tot}} = -(0.25mg)s$, where s is the length of the rough patch.

$W_{\text{tot}} = K_2 - K_1$

$K_1 = \frac{1}{2}mv_0^2,\quad K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}m(0.45v_0)^2 = 0.2025\left(\frac{1}{2}mv_0^2\right)$

The work-energy relation gives $-(0.25mg)s = (0.2025 - 1)\frac{1}{2}mv_0^2$

The mass divides out, and solving gives $s = 1.5\text{ m}$.

EVALUATE: Friction does negative work and this reduces her kinetic energy.

6.86. **IDENTIFY:** $P_{av} = F_{\|}v_{av}$. Use $F = ma$ to calculate the force.

SET UP: $v_{av} = \frac{0 + 6.00\text{ m/s}}{2} = 3.00\text{ m/s}$

EXECUTE: Your friend's average acceleration is $a = \frac{v - v_0}{t} = \frac{6.00\text{ m/s}}{3.00\text{ s}} = 2.00\text{ m/s}^2$. Since there are no other horizontal forces acting, the force you exert on her is given by $F_{net} = ma = (65.0\text{ kg})(2.00\text{ m/s}^2) = 130\text{ N}$.

$P_{av} = (130\text{ N})(3.00\text{ m/s}) = 390\text{ W}$.

EVALUATE: We could also use the work-energy theorem: $W = K_2 - K_1 = \frac{1}{2}(65.0\text{ kg})(6.00\text{ m/s})^2 = 1170\text{ J}$.

$P_{av} = \frac{W}{t} = \frac{1170\text{ J}}{3.00\text{ s}} = 390\text{ W}$, the same as obtained by our other approach.

6.87. **IDENTIFY:** To lift a mass m a height h requires work $W = mgh$. To accelerate mass m from rest to speed v requires $W = K_2 - K_1 = \frac{1}{2}mv^2$. $P_{av} = \frac{\Delta W}{\Delta t}$.

SET UP: $t = 60\text{ s}$

EXECUTE: **(a)** $(800\text{ kg})(9.80\text{ m/s}^2)(14.0\text{ m}) = 1.10\times10^5\text{ J}$

(b) $(1/2)(800\text{ kg})(18.0\text{ m/s}^2) = 1.30\times10^5\text{ J}$.

(c) $\frac{1.10\times10^5\text{ J} + 1.30\times10^5\text{ J}}{60\text{ s}} = 3.99\text{ kW}$.

EVALUATE: Approximately the same amount of work is required to lift the water against gravity as to accelerate it to its final speed.

6.88. **IDENTIFY:** $P = F_{\|}v$ and $F_{\|} = ma$.

SET UP: From Problem 6.71, $v = 2\alpha t + 3\beta t^2$ and $a = 2\alpha + 6\beta t$.

EXECUTE: $P = F_{\|}v = mav = m(2\alpha + 6\beta t)(2\alpha t + 3\beta t^2) = m(4\alpha^2 t + 18\alpha\beta t^2 + 18\beta^2 t^3)$.

$P = (0.96\text{ N/s})t + (0.43\text{ N/s}^2)t^2 + (0.043\text{ N/s}^3)t^3$. At $t = 4.00\text{ s}$, the power output is 13.5 W.

EVALUATE: P increases in time because v increase and because a increases.

6.89. **IDENTIFY** and **SET UP:** Energy is $P_{av}t$. The total energy expended in one day is the sum of the energy expended in each type of activity.

EXECUTE: $1\text{ day} = 8.64\times10^4\text{ s}$

Let t_{walk} be the time she spends walking and t_{other} be the time she spends in other activities; $t_{other} = 8.64\times10^4\text{ s} - t_{walk}$. The energy expended in each activity is the power output times the time, so

$E = Pt = (280\text{ W})t_{walk} + (100\text{ W})t_{other} = 1.1\times10^7\text{ J}$

$(280\text{ W})t_{walk} + (100\text{ W})(8.64\times10^4\text{ s} - t_{walk}) = 1.1\times10^7\text{ J}$

$(180\text{ W})t_{walk} = 2.36\times10^6\text{ J}$

$t_{walk} = 1.31\times10^4\text{ s} = 218\text{ min} = 3.6\text{ h}$.

EVALUATE: Her average power for one day is $(1.1\times10^7\text{ J})/([24][3600\text{ s}]) = 127\text{ W}$. This is much closer to her 100 W rate than to her 280 W rate, so most of her day is spent at the 100 W rate.

6.90. **IDENTIFY** and **SET UP:** $W = Pt$

EXECUTE: **(a)** The hummingbird produces energy at a rate of 0.7 J/s to 1.75 J/s. At 10 beats/s, the bird must expend between 0.07 J/beat and 0.175 J/beat.

(b) The steady output of the athlete is (500 W)/(70 kg) = 7 W/kg, which is below the 10 W/kg necessary to stay aloft. Though the athlete can expend 1400 W/70 kg = 20 W/kg for short periods of time, no human-powered aircraft could stay aloft for very long.

EVALUATE: Movies of early attempts at human-powered flight bear out our results.

6.91. **IDENTIFY** and **SET UP:** Use Eq.(6.15). The work done on the water by gravity is mgh, where $h = 170\text{ m}$. Solve for the mass m of water for 1.00 s and then calculate the volume of water that has this mass.

EXECUTE: The power output is $P_{av} = 2000 \text{ MW} = 2.00\times10^9 \text{ W}$. $P_{av} = \dfrac{\Delta W}{\Delta t}$ and 92% of the work done on the water by gravity is converted to electrical power output, so in 1.00 s the amount of work done on the water by gravity is

$$W = \frac{P_{av}\Delta t}{0.92} = \frac{(2.00\times10^9 \text{ W})(1.00 \text{ s})}{0.92} = 2.174\times10^9 \text{ J}$$

$W = mgh$, so the mass of water flowing over the dam in 1.00 s must be

$$m = \frac{W}{gh} = \frac{2.174\times10^9 \text{ J}}{(9.80 \text{ m/s}^2)(170 \text{ m})} = 1.30\times10^6 \text{ kg}$$

$$\text{density} = \frac{m}{V} \text{ so } V = \frac{m}{\text{density}} = \frac{1.30\times10^6 \text{ kg}}{1.00\times10^3 \text{ kg/m}^3} = 1.30\times10^3 \text{ m}^3.$$

EVALUATE: The dam is 1270 m long, so this volume corresponds to about a m^3 flowing over each 1 m length of the dam, a reasonable amount.

6.92. **IDENTIFY:** $P = \dfrac{W}{t}$ and $W = \frac{1}{2}mv^2$, if the object starts from rest. $a = \dfrac{dv}{dt}$ and $x - x_0 = \int v dt$.

SET UP: $\dfrac{d}{dt}t^{1/2} = \frac{1}{2}t^{-1/2}$. $\int t^{1/2}dt = \frac{2}{3}t^{3/2}$.

EXECUTE: **(a)** The power P is related to the speed by $Pt = K = \frac{1}{2}mv^2$, so $v = \sqrt{\dfrac{2Pt}{m}}$.

(b) $a = \dfrac{dv}{dt} = \dfrac{d}{dt}\sqrt{\dfrac{2Pt}{m}} = \sqrt{\dfrac{2P}{m}}\dfrac{d}{dt}\sqrt{t} = \sqrt{\dfrac{2P}{m}}\dfrac{1}{2\sqrt{t}} = \sqrt{\dfrac{P}{2mt}}.$

(c) $x - x_0 = \int v\, dt = \sqrt{\dfrac{2P}{m}}\int t^{\frac{1}{2}}\, dt = \sqrt{\dfrac{2P}{m}}\dfrac{2}{3}t^{\frac{3}{2}} = \sqrt{\dfrac{8P}{9m}}t^{\frac{3}{2}}.$

EVALUATE: v, a, and $x - x_0$ at a particular time are all proportional to $P^{1/2}$. The result in part (b) could also be obtained from $P = Fv$ and $a = F/m$, so $a = \dfrac{P}{vm}$.

6.93. **IDENTIFY and SET UP:** For part (a) calculate m from the volume of blood pumped by the heart in one day. For part (b) use W calculated in part (a) in Eq.(6.15).

EXECUTE: **(a)** $W = mgh$, as in Example 6.11. We need the mass of blood lifted; we are given the volume

$$V = (7500 \text{ L})\left(\frac{1\times10^{-3} \text{ m}^3}{1 \text{ L}}\right) = 7.50 \text{ m}^3.$$

$$m = \text{density}\times\text{volume} = (1.05\times10^3 \text{ kg/m}^3)(7.50 \text{ m}^3) = 7.875\times10^3 \text{ kg}$$

Then $W = mgh = (7.875\times10^3 \text{ kg})(9.80 \text{ m/s}^2)(1.63 \text{ m}) = 1.26\times10^5 \text{ J}$.

(b) $P_{av} = \dfrac{\Delta W}{\Delta t} = \dfrac{1.26\times10^5 \text{ J}}{(24 \text{ h})(3600 \text{ s/h})} = 1.46 \text{ W}.$

EVALUATE: Compared to light bulbs or common electrical devices, the power output of the heart is rather small.

6.94. **IDENTIFY:** $P = F_{\parallel}v = Mav$. To overcome gravity on a slope that is at an angle α above the horizontal, $P = (Mg\sin\alpha)v$.

SET UP: $1 \text{ MW} = 10^6 \text{ W}$. $1 \text{ kN} = 10^3 \text{ N}$. When α is small, $\tan\alpha \approx \sin\alpha$.

EXECUTE: **(a)** The number of cars is the total power available divided by the power needed per car,

$\dfrac{13.4\times10^6 \text{ W}}{(2.8\times10^3 \text{ N})(27 \text{ m/s})} = 177$, rounding down to the nearest integer.

(b) To accelerate a total mass M at an acceleration a and speed v, the extra power needed is Mav. To climb a hill of angle α, the extra power needed is $(Mg\sin\alpha)v$. This will be nearly the same if $a \sim g\sin\alpha$; if $g\sin\alpha \sim g\tan\alpha \sim 0.10 \text{ m/s}^2$, the power is about the same as that needed to accelerate at 0.10 m/s^2.

(c) $P = (Mg\sin\alpha)v$, where M is the total mass of the diesel units.

$$P = (1.10\times10^6 \text{ kg})(9.80 \text{ m/s}^2)(0.010)(27 \text{ m/s}) = 2.9 \text{ MW}.$$

(d) The power available to the cars is 13.4 MW, minus the 2.9 MW needed to maintain the speed of the diesel units on the incline. The total number of cars is then $\dfrac{13.4\times10^6 \text{ W} - 2.9\times10^6 \text{ W}}{(2.8\times10^3 \text{ N} + (8.2\times10^4 \text{ kg})(9.80 \text{ m/s}^2)(0.010))(27 \text{ m/s})} = 36$, rounding to the nearest integer.

EVALUATE: For a single car, $Mg\sin\alpha = (8.2\times10^4\ \text{kg})(9.80\ \text{m/s}^2)(0.010) = 8.0\times10^3\ \text{N}$, which is over twice the 2.8 kN required to pull the car at 27 m/s on level tracks. Even a slope as gradual as 1.0% greatly increases the power requirements, or for constant power greatly decreases the number of cars that can be pulled.

6.95. IDENTIFY: $P = F_{\parallel}v$. The force required to give mass m an acceleration a is $F = ma$. For an incline at an angle α above the horizontal, the component of mg down the incline is $mg\sin\alpha$.

SET UP: For small α, $\sin\alpha \approx \tan\alpha$.

EXECUTE: **(a)** $P_0 = Fv = (53\times10^3\ \text{N})(45\ \text{m/s}) = 2.4\ \text{MW}$.

(b) $P_1 = mav = (9.1\times10^5\ \text{kg})(1.5\ \text{m/s}^2)(45\ \text{m/s}) = 61\ \text{MW}$.

(c) Approximating $\sin\alpha$, by $\tan\alpha$, and using the component of gravity down the incline as $mg\sin\alpha$,

$P_2 = (mg\sin\alpha)v = (9.1\times10^5\ \text{kg})(9.80\ \text{m/s}^2)(0.015)(45\ \text{m/s}) = 6.0\ \text{MW}$.

EVALUATE: From Problem 6.94, we would expect that a $0.15\ \text{m/s}^2$ acceleration and a 1.5% slope would require the same power. We found that a $1.5\ \text{m/s}^2$ acceleration requires ten times more power than a 1.5% slope, which is consistent.

6.96. IDENTIFY: $W = \int_{x_1}^{x_2} F_x dx$, and F_x depends on both x and y.

SET UP: In each case, use the value of y that applies to the specified path. $\int x dx = \frac{1}{2}x^2$. $\int x^2 dx = \frac{1}{3}x^3$

EXECUTE: **(a)** Along this path, y is constant, with the value $y = 3.00\ \text{m}$.

$W = \alpha y\int_{x_1}^{x_2} x dx = (2.50\ \text{N/m}^2)(3.00\ \text{m})\left(\frac{2.00\ \text{m}}{2}\right) = 15.0\ \text{J}$, since $x_1 = 0$ and $x_2 = 2.00\ \text{m}$.

(b) Since the force has no y-component, no work is done moving in the y-direction.

(c) Along this path, y varies with position along the path, given by $y = 1.5x$, so $F_x = \alpha(1.5x)x = 1.5\alpha x^2$, and

$$W = \int_{x_1}^{x_2} F dx = 1.5\alpha\int_{x_1}^{x_2} x^2 dx = 1.5(2.50\ \text{N/m}^2)\frac{(2.00\ \text{m})^3}{3} = 10.0\ \text{J}.$$

EVALUATE: The force depends on the position of the object along its path.

6.97. IDENTIFY and SET UP: Use Eq.(6.18) to relate the forces to the power required. The air resistance force is $F_{\text{air}} = \frac{1}{2}CA\rho v^2$, where C is the drag coefficient.

EXECUTE: **(a)** $P = F_{\text{tot}}v$, with $F_{\text{tot}} = F_{\text{roll}} + F_{\text{air}}$

$F_{\text{air}} = \frac{1}{2}CA\rho v^2 = \frac{1}{2}(1.0)(0.463\ \text{m}^3)(1.2\ \text{kg/m}^3)(12.0\ \text{m/s})^2 = 40.0\ \text{N}$

$F_{\text{roll}} = \mu_r n = \mu_r w = (0.0045)(490\ \text{N} + 118\ \text{N}) = 2.74\ \text{N}$

$P = (F_{\text{roll}} + F_{\text{air}})v = (2.74\ \text{N} + 40.0\ \text{N})(12.0\ \text{s}) = 513\ \text{W}$

(b) $F_{\text{air}} = \frac{1}{2}CA\rho v^2 = \frac{1}{2}(0.88)(0.366\ \text{m}^3)(1.2\ \text{kg/m}^3)(12.0\ \text{m/s})^2 = 27.8\ \text{N}$

$F_{\text{roll}} = \mu_r n = \mu_r w = (0.0030)(490\ \text{N} + 88\ \text{N}) = 1.73\ \text{N}$

$P = (F_{\text{roll}} + F_{\text{air}})v = (1.73\ \text{N} + 27.8\ \text{N})(12.0\ \text{s}) = 354\ \text{W}$

(c) $F_{\text{air}} = \frac{1}{2}CA\rho v^2 = \frac{1}{2}(0.88)(0.366\ \text{m}^3)(1.2\ \text{kg/m}^3)(6.0\ \text{m/s})^2 = 6.96\ \text{N}$

$F_{\text{roll}} = \mu_r n = 1.73\ \text{N}$ (unchanged)

$P = (F_{\text{roll}} + F_{\text{air}})v = (1.73\ \text{N} + 6.96\ \text{N})(6.0\ \text{s}) = 52.1\ \text{W}$

EVALUATE: Since F_{air} is proportional to v^2 and $P = Fv$, reducing the speed greatly reduces the power required.

6.98. IDENTIFY: $P = F_{\parallel}v$

SET UP: $1\ \text{m/s} = 3.6\ \text{km/h}$

EXECUTE: **(a)** $F = \dfrac{P}{v} = \dfrac{28.0\times10^3\ \text{W}}{(60.0\ \text{km/h})((1\ \text{m/s})/(3.6\ \text{km/h}))} = 1.68\times10^3\ \text{N}.$

(b) The speed is lowered by a factor of one-half, and the resisting force is lowered by a factor of $(0.65 + 0.35/4)$, and so the power at the lower speed is $(28.0\ \text{kW})(0.50)(0.65 + 0.35/4) = 10.3\ \text{kW} = 13.8\ \text{hp}$.

(c) Similarly, at the higher speed, $(28.0\ \text{kW})(2.0)(0.65 + 0.35\times4) = 114.8\ \text{kW} = 154\ \text{hp}$.

EVALUATE: At low speeds rolling friction dominates the power requirement but at high speeds air resistance dominates.

6.99. **IDENTIFY** and **SET UP:** Use Eq.(6.18) to relate F and P. In part (a), F is the retarding force. In parts (b) and (c), F includes gravity.

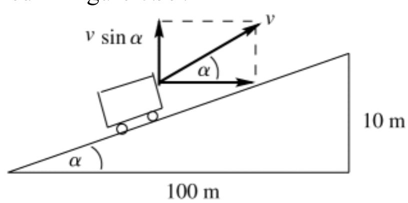
EXECUTE: **(a)** $P = Fv$, so $F = P/v$.

$$P = (8.00 \text{ hp})\left(\frac{746 \text{ W}}{1 \text{ hp}}\right) = 5968 \text{ W}$$

$$v = (60.0 \text{ km/h})\left(\frac{1000 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 16.67 \text{ m/s}$$

$$F = \frac{P}{v} = \frac{5968 \text{ W}}{16.67 \text{ m/s}} = 358 \text{ N}.$$

(b) The power required is the 8.00 hp of part (a) plus the power P_g required to lift the car against gravity. The situation is sketched in Figure 6.99.



$$\tan\alpha = \frac{10 \text{ m}}{100 \text{ m}} = 0.10$$
$$\alpha = 5.71°$$

Figure 6.99

The vertical component of the velocity of the car is $v\sin\alpha = (16.67 \text{ m/s})\sin 5.71° = 1.658 \text{ m/s}$.

Then $P_g = F(v\sin a) = mgv\sin\alpha = (1800 \text{ kg})(9.80 \text{ m/s}^2)(1.658 \text{ m/s}) = 2.92\times 10^4 \text{ W}$

$$P_g = 2.92\times 10^4 \text{ W}\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = 39.1 \text{ hp}$$

The total power required is $8.00 \text{ hp} + 39.1 \text{ hp} = 47.1 \text{ hp}$.

(c) The power required from the engine is *reduced* by the rate at which gravity does positive work. The road incline angle α is given by $\tan\alpha = 0.0100$, so $\alpha = 0.5729°$.

$P_g = mg(v\sin\alpha) = (1800 \text{ kg})(9.80 \text{ m/s}^2)(16.67 \text{ m/s})\sin 0.5729° = 2.94\times 10^3 \text{ W} = 3.94 \text{ hp}$.

The power required from the engine is then $8.00 \text{ hp} - 3.94 \text{ hp} = 4.06 \text{ hp}$.

(d) No power is needed from the engine if gravity does work at the rate of $P_g = 8.00 \text{ hp} = 5968 \text{ W}$

$$P_g = mgv\sin\alpha, \text{ so } \sin\alpha = \frac{P_g}{mgv} = \frac{5968 \text{ W}}{(1800 \text{ kg})(9.80 \text{ m/s}^2)(16.67 \text{ m/s})} = 0.02030$$

$\alpha = 1.163°$ and $\tan\alpha = 0.0203$, a 2.03% grade.

EVALUATE: More power is required when the car goes uphill and less when it goes downhill. In part (d), at this angle the component of gravity down the incline is $mg\sin\alpha = 358 \text{ N}$ and this force cancels the retarding force and no force from the engine is required. The retarding force depends on the speed so it is the same in parts (a), (b), and (c).

6.100. **IDENTIFY:** Apply $W_{\text{tot}} = K_2 - K_1$ to relate the initial speed v_0 to the distance x along the plank that the box moves before coming to rest.

SET UP: The component of weight down the incline is $mg\sin\alpha$, the normal force is $mg\cos\alpha$ and the friction force is $f = \mu mg\cos\alpha$.

EXECUTE: $\Delta K = 0 - \frac{1}{2}mv_0^2$ and $W = \int_0^x (-mg\sin\alpha - \mu mg\cos\alpha)dx$. Then,

$$W = -mg\int_0^x (\sin\alpha + Ax\cos\alpha)dx,\ W = -mg\left[\sin\alpha x + \frac{Ax^2}{2}\cos\alpha\right].$$

Set $W = \Delta K$: $-\frac{1}{2}mv_0^2 = -mg\left[\sin\alpha x + \frac{Ax^2}{2}\cos\alpha\right]$. To eliminate x, note that the box comes to a rest when the force of static friction balances the component of the weight directed down the plane. So, $mg\sin\alpha = Ax\, mg\cos\alpha$. Solve this for x and substitute into the previous equation: $x = \frac{\sin\alpha}{A\cos\alpha}$. Then, $\frac{1}{2}v_0^2 = +g\left[\sin\alpha\frac{\sin\alpha}{A\cos\alpha} + \frac{1}{2}A\left(\frac{\sin\alpha}{A\cos\alpha}\right)^2\cos\alpha\right]$, and upon canceling factors and collecting terms, $v_0^2 = \frac{3g\sin^2\alpha}{A\cos\alpha}$. The box will remain stationary whenever $v_0^2 \geq \frac{3g\sin^2\alpha}{A\cos\alpha}$.

EVALUATE: If v_0 is too small the box stops at a point where the friction force is too small to hold the box in place. $\sin\alpha$ increases and $\cos\alpha$ decreases as α increases, so the v_0 required increases as α increases.

6.101. **IDENTIFY:** In part (a) follow the steps outlined in the problem. For parts (b), (c) and (d) apply the work-energy theorem.

SET UP: $\int x^2 dx = \frac{1}{3}x^3$

EXECUTE: **(a)** Denote the position of a piece of the spring by *l*; $l = 0$ is the fixed point and $l = L$ is the moving end of the spring. Then the velocity of the point corresponding to *l*, denoted *u*, is $u(l) = v(\mathrm{I}/L)$ (when the spring is moving, *l* will be a function of time, and so *u* is an implicit function of time). The mass of a piece of length *dl* is $dm = (M/L)dl$, and so $dK = \frac{1}{2}(dm)u^2 = \frac{1}{2}\frac{Mv^2}{L^3}l^2 dl$, and $K = \int dK = \frac{Mv^2}{2L^3}\int_0^L l^2 dl = \frac{Mv^2}{6}$.

(b) $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, so $v = \sqrt{(k/m)x} = \sqrt{(3200\ \text{N/m})/(0.053\ \text{kg})}(2.50\times10^{-2}\ \text{m}) = 6.1\ \text{m/s}$.

(c) With the mass of the spring included, the work that the spring does goes into the kinetic energies of both the ball and the spring, so $\frac{1}{2}kx^2 = \frac{1}{2}mv^2 + \frac{1}{6}Mv^2$. Solving for *v*,

$$v = \sqrt{\frac{k}{m + M/3}}x = \sqrt{\frac{(3200\ \text{N/m})}{(0.053\ \text{kg}) + (0.243\ \text{kg})/3}}(2.50\times10^{-2}\text{m}) = 3.9\ \text{m/s}.$$

(d) Algebraically, $\frac{1}{2}mv^2 = \frac{(1/2)kx^2}{(1 + M/3m)} = 0.40\ \text{J}$ and $\frac{1}{6}Mv^2 = \frac{(1/2)kx^2}{(1 + 3m/M)} = 0.60\ \text{J}$.

EVALUATE: For this ball and spring, $\frac{K_{\text{ball}}}{K_{\text{spring}}} = \frac{3m}{M} = 3\left(\frac{0.053\ \text{kg}}{0.243\ \text{kg}}\right) = 0.65$. The percentage of the final kinetic energy that ends up with each object depends on the ratio of the masses of the two objects. As expected, when the mass of the spring is a small fraction of the mass of the ball, the fraction of the kinetic energy that ends up in the spring is small.

6.102. **IDENTIFY:** In both cases, a given amount of fuel represents a given amount of work W_0 that the engine does in moving the plane forward against the resisting force. Write W_0 in terms of the range *R* and speed *v* and in terms of the time of flight *T* and *v*.

SET UP: In both cases assume *v* is constant, so $W_0 = RF$ and $R = vT$.

EXECUTE: In terms of the range *R* and the constant speed *v*, $W_0 = RF = R\left(\alpha v^2 + \frac{\beta}{v^2}\right)$.

In terms of the time of flight $T, R = vt$, so $W_0 = vTF = T\left(\alpha v^3 + \frac{\beta}{v}\right)$.

(a) Rather than solve for *R* as a function of *v*, differentiate the first of these relations with respect to *v*, setting $\frac{dW_0}{dv} = 0$ to obtain $\frac{dR}{dv}F + R\frac{dF}{dv} = 0$. For the maximum range, $\frac{dR}{dv} = 0$, so $\frac{dF}{dv} = 0$. Performing the differentiation, $\frac{dF}{dv} = 2\alpha v - 2\beta/v^3 = 0$, which is solved for

$$v = \left(\frac{\beta}{\alpha}\right)^{1/4} = \left(\frac{3.5\times10^5\ \text{N}\cdot\text{m}^2/\text{s}^2}{0.30\ \text{N}\cdot\text{s}^2/\text{m}^2}\right)^{1/4} = 32.9\ \text{m/s} = 118\ \text{km/h}.$$

(b) Similarly, the maximum time is found by setting $\frac{d}{dv}(Fv) = 0$; performing the differentiation, $3\alpha v^2 - \beta/v^2 = 0$.

$$v = \left(\frac{\beta}{3\alpha}\right)^{1/4} = \left(\frac{3.5\times10^5\ \text{N}\cdot\text{m}^2/\text{s}^2}{3(0.30\ \text{N}\cdot\text{s}^2/\text{m}^2)}\right)^{1/4} = 25\ \text{m/s} = 90\ \text{km/h}.$$

EVALUATE: When $v = (\beta/\alpha)^{1/4}$, F_{air} has its minimum value $F_{\text{air}} = 2\sqrt{\alpha\beta}$. For this *v*, $R_1 = (0.50)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_1 = (0.50)\alpha^{-1/4}\beta^{-3/4}$. When $v = (\beta/3\alpha)^{1/4}$, $F_{\text{air}} = 2.3\sqrt{\alpha\beta}$. For this *v*, $R_2 = (0.43)\frac{W_0}{\sqrt{\alpha\beta}}$ and $T_2 = (0.57)\alpha^{-1/4}\beta^{-3/4}$.

$R_1 > R_2$ and $T_2 > T_1$, as they should be.

6.103. **IDENTIFY:** For each speed, calculate the time. Then use the graph to find the oxygen consumption and from that the energy consumption.

SET UP: $t = d/v$

EXECUTE: **(a)** The walk will take one-fifth of an hour, 12 min. From the graph, the oxygen consumption rate appears to be about $12\ \text{cm}^3/\text{kg}\cdot\text{min}$, and so the total energy is

$$(12\ \text{cm}^3/\text{kg}\cdot\text{min})\ (70\ \text{kg})\ (12\ \text{min})\ (20\ \text{J/cm}^3) = 2.0\times10^5\ \text{J}.$$

(b) The run will take 6 min. Using an estimation of the rate from the graph of about $33\ \text{cm}^3/\text{kg}\cdot\text{min}$ gives an energy consumption of about $2.8\times10^5\ \text{J}$.

(c) The run takes 4 min, and with an estimated rate of about $50\ \text{cm}^3/\text{kg}\cdot\text{min}$, the energy used is about $2.8\times10^5\ \text{J}$.

(d) Walking is the most efficient way to go. In general, the point where the slope of the line from the origin to the point on the graph is the smallest is the most efficient speed; about 5 km/h.

EVALUATE: In an exercise program, for a fixed distance, running burns more energy than walking.

6.104. **IDENTIFY:** Write equations similar to (6.11) for each component. Eq.(6.12) will now involve the sum of three integrals, one for each component.

SET UP: $v^2 = v_x^2 + v_y^2 + v_z^2$

EXECUTE: From $\vec{F} = m\vec{a}$, $F_x = ma_x, F_y = ma_y$ and $F_z = ma_z$. The generalization of Eq. (6.11) is then

$a_x = v_x \dfrac{dv_x}{dx}$, $a_y = v_y \dfrac{dv_y}{dy}$, $a_z = v_z \dfrac{dv_z}{dz}$. The total work is then

$$W_{\text{tot}} = \int_{(x_1,y_1,z_1)}^{(x_2,y_2,z_2)} F_x dx + F_y dy + F_z dz = m\left(\int_{x_1}^{x_2} v_x \frac{dv_x}{dx} dx + \int_{y_1}^{y_2} v_y \frac{dv_y}{dy} dy + \int_{z_1}^{z_2} v_z \frac{dv_z}{dz} dz\right).$$

$$W_{\text{tot}} = m\left(\int_{v_{x1}}^{v_{x2}} v_x dv_x + \int_{v_{y1}}^{v_{y2}} v_y dv_y + \int_{v_{z1}}^{v_{z2}} v_z dv_z\right) = \frac{1}{2}m(v_{x2}^2 - v_{x1}^2 + v_{y2}^2 - v_{y1}^2 + v_{z2}^2 - v_{z1}^2 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2.$$

EVALUATE: $\vec{F}$ and $d\vec{l}$ are vectors and have components. *W* and *K* are scalars and we never speak of their components.

7

POTENTIAL ENERGY AND ENERGY CONSERVATION

7.1. IDENTIFY: $U_{grav} = mgy$ so $\Delta U_{grav} = mg(y_2 - y_1)$

SET UP: $+y$ is upward.

EXECUTE: **(a)** $\Delta U = (75\text{ kg})(9.80\text{ m/s}^2)(2400\text{ m} - 1500\text{ m}) = +6.6\times10^5\text{ J}$

(b) $\Delta U = (75\text{ kg})(9.80\text{ m/s}^2)(1350\text{ m} - 2400\text{ m}) = -7.7\times10^5\text{ J}$

EVALUATE: U_{grav} increases when the altitude of the object increases.

7.2. IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to the sack to find the force. $W = Fs\cos\phi$.

SET UP: The lifting force acts in the same direction as the sack's motion, so $\phi = 0°$

EXECUTE: **(a)** For constant speed, the net force is zero, so the required force is the sack's weight, $(5.00\text{ kg})(9.80\text{ m/s}^2) = 49.0\text{ N}$.

(b) $W = (49.0\text{ N})\,(15.0\text{ m}) = 735\text{ J}$. This work becomes potential energy.

EVALUATE: The results are independent of the speed.

7.3. IDENTIFY: Use the free-body diagram for the bag and Newton's first law to find the force the worker applies. Since the bag starts and ends at rest, $K_2 - K_1 = 0$ and $W_{tot} = 0$.

SET UP: A sketch showing the initial and final positions of the bag is given in Figure 7.3a. $\sin\phi = \dfrac{2.0\text{ m}}{3.5\text{ m}}$ and $\phi = 34.85°$. The free-body diagram is given in Figure 7.3b. $\vec{F}$ is the horizontal force applied by the worker. In the calculation of U_{grav} take $+y$ upward and $y = 0$ at the initial position of the bag.

EXECUTE: **(a)** $\sum F_y = 0$ gives $T\cos\phi = mg$ and $\sum F_x = 0$ gives $F = T\sin\phi$. Combining these equations to eliminate T gives $F = mg\tan\phi = (120\text{ kg})(9.80\text{ m/s}^2)\tan 34.85° = 820\text{ N}$.

(b) (i) The tension in the rope is radial and the displacement is tangential so there is no component of T in the direction of the displacement during the motion and the tension in the rope does no work. (ii) $W_{tot} = 0$ so $W_{worker} = -W_{grav} = U_{grav,2} - U_{grav,1} = mg(y_2 - y_1) = (120\text{ kg})(9.80\text{ m/s}^2)(0.6277\text{ m}) = 740\text{ J}$.

EVALUATE: The force applied by the worker varies during the motion of the bag and it would be difficult to calculate W_{worker} directly.

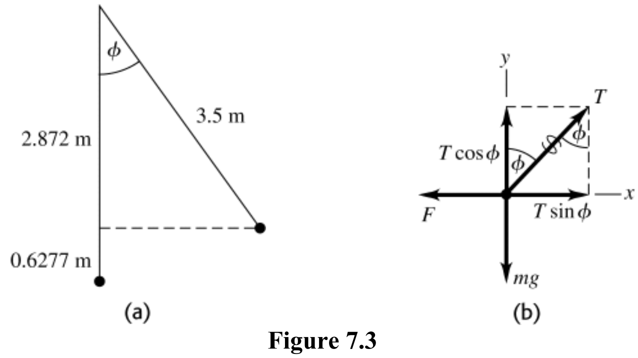


Figure 7.3

7.4. IDENTIFY: Only gravity does work on him from the point where he has just left the board until just before he enters the water, so Eq.(7.4) applies.

SET UP: Let point 1 be just after he leaves the board and point 2 be just before he enters the water. $+y$ is upward and $y = 0$ at the water.

EXECUTE: **(a)** $K_1 = 0$. $y_2 = 0$. $y_1 = 3.25$ m . $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $U_{\text{grav},1} = K_2$ and $mgy_1 = \frac{1}{2}mv_2^2$.

$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80 \text{ m/s}^2)(3.25 \text{ m})} = 7.98 \text{ m/s}$.

(b) $v_1 = 2.50$ m/s , $y_2 = 0$, $y_1 = 3.25$ m . $K_1 + U_{\text{grav},1} = K_2$ and $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$.

$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{(2.50 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(3.25 \text{ m})} = 8.36 \text{ m/s}$.

(c) $v_1 = 2.5$ m/s and $v_2 = 8.36$ m/s , the same as in part (b).

EVALUATE: Kinetic energy depends only on the speed, not on the direction of the velocity.

7.5. **IDENTIFY** and **SET UP:** Use energy methods.

(a) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Solve for K_2 and then use $K_2 = \frac{1}{2}mv_2^2$ to obtain v_2.

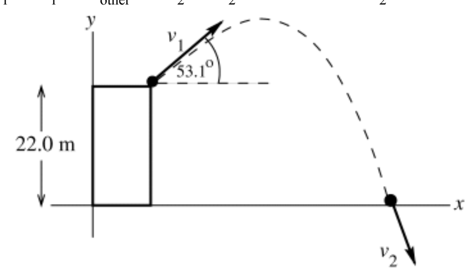


Figure 7.5

$W_{\text{other}} = 0$ (The only force on the ball while it is in the air is gravity.)
$K_1 = \frac{1}{2}mv_1^2$; $K_2 = \frac{1}{2}mv_2^2$
$U_1 = mgy_1$, $y_1 = 22.0$ m
$U_2 = mgy_2 = 0$, since $y_2 = 0$ for our choice of coordinates.

EXECUTE: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$

$$v_2 = \sqrt{v_1^2 + 2gy_1} = \sqrt{(12.0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(22.0 \text{ m})} = 24.0 \text{ m/s}$$

EVALUATE: The projection angle of $53.1°$ doesn't enter into the calculation. The kinetic energy depends only on the magnitude of the velocity; it is independent of the direction of the velocity.

(b) Nothing changes in the calculation. The expression derived in part (a) for v_2 is independent of the angle, so $v_2 = 24.0$ m/s, the same as in part (a).

(c) The ball travels a shorter distance in part (b), so in that case air resistance will have less effect.

7.6. **IDENTIFY:** The normal force does no work, so only gravity does work and Eq.(7.4) applies.

SET UP: $K_1 = 0$. The crate's initial point is at a vertical height of $d \sin\alpha$ above the bottom of the ramp.

EXECUTE: **(a)** $y_2 = 0$, $y_1 = d\sin\alpha$. $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $U_{\text{grav},1} = K_2$. $mgd\sin\alpha = \frac{1}{2}mv_2^2$ and $v_2 = \sqrt{2gd\sin\alpha}$.

(b) $y_1 = 0$, $y_2 = -d\sin\alpha$. $K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$ gives $0 = K_2 + U_{\text{grav},2}$. $0 = \frac{1}{2}mv_2^2 + (-mgd\sin\alpha)$ and $v_2 = \sqrt{2gd\sin\alpha}$, the same as in part (a).

(c) The normal force is perpendicular to the displacement and does no work.

EVALUATE: When we use $U_{\text{grav}} = mgy$ we can take any point as $y = 0$ but we must take $+y$ to be upward.

7.7. **IDENTIFY:** Apply Eq.(7.7) to points 2 and 3. Take results from Example 7.6. $W_{\text{other}} = -fs$, the work done by friction.

SET UP: As in Example 7.6, $K_2 = 0$, $U_2 = 94$ J, and $U_3 = 0$.

EXECUTE: The work done by friction is $-(35 \text{ N})(1.6 \text{ m}) = -56$ J . $K_3 = 38$ J, and $v_3 = \sqrt{\dfrac{2(38 \text{ J})}{12 \text{ kg}}} = 2.5$ m/s.

EVALUATE: The value of v_3 we obtained is the same as calculated in Example 7.6. For the motion from point 2 to point 3, gravity does positive work, friction does negative work and the net work is positive.

7.8. **IDENTIFY** and **SET UP:** Apply Eq.(7.7) and consider how each term depends on the mass.

EXECUTE: The speed is v and the kinetic energy is $4K$. The work done by friction is proportional to the normal force, and hence to the mass, and so each term in Eq. (7.7) is proportional to the total mass of the crate, and the speed at the bottom is the same for any mass. The kinetic energy is proportional to the mass, and for the same speed but four times the mass, the kinetic energy is quadrupled.

EVALUATE: The same result is obtained if we apply $\sum \vec{F} = m\vec{a}$ to the motion. Each force is proportional to m and m divides out, so a is independent of m.

7.9. IDENTIFY: $W_{tot} = K_B - K_A$. The forces on the rock are gravity, the normal force and friction.

SET UP: Let $y = 0$ at point *B* and let $+y$ be upward. $y_A = R = 0.50$ m. The work done by friction is negative; $W_f = -0.22$ J. $K_A = 0$. The free-body diagram for the rock at point *B* is given in Figure 7.9. The acceleration of the rock at this point is $a_{rad} = v^2/R$, upward.

EXECUTE: **(a)** (i) The normal force is perpendicular to the displacement and does zero work.

(ii) $W_{grav} = U_{grav,A} - U_{grav,B} = mgy_A = (0.20\text{ kg})(9.80\text{ m/s}^2)(0.50\text{ m}) = 0.98\text{ J}$.

(b) $W_{tot} = W_n + W_f + W_{grav} = 0 + (-0.22\text{ J}) + 0.98\text{ J} = 0.76\text{ J}$. $W_{tot} = K_B - K_A$ gives $\frac{1}{2}mv_B^2 = W_{tot}$.

$$v_B = \sqrt{\frac{2W_{tot}}{m}} = \sqrt{\frac{2(0.76\text{ J})}{0.20\text{ kg}}} = 2.8\text{ m/s}.$$

(c) Gravity is constant and equal to *mg*. *n* is not constant; it is zero at *A* and not zero at *B*. Therefore, $f_k = \mu_k n$ is also not constant.

(d) $\sum F_y = ma_y$ applied to Figure 7.9 gives $n - mg = ma_{rad}$.

$$n = m\left(g + \frac{v^2}{R}\right) = (0.20\text{ kg})\left(9.80\text{ m/s}^2 + \frac{[2.8\text{ m/s}]^2}{0.50\text{ m}}\right) = 5.1\text{ N}.$$

EVALUATE: In the absence of friction, the speed of the rock at point *B* would be $\sqrt{2gR} = 3.1$ m/s. As the rock slides through point *B*, the normal force is greater than the weight $mg = 2.0$ N of the rock.

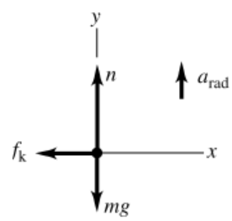


Figure 7.9

7.10. IDENTIFY: Only gravity does work, so Eq.(7.4) applies.

SET UP: Let point 1 be just after the rock leaves the thrower and point 2 be at the maximum height. Let $y_1 = 0$ and $+y$ be upward. $v_1 = v_0$. At the highest point, $v_2 = v_0\cos\theta$. $\sin^2\theta + \cos^2\theta = 1$.

EXECUTE: $K_1 + U_{grav,1} = K_2 + U_{grav,2}$ gives $\frac{1}{2}mv_0^2 = \frac{1}{2}m(v_0\cos\theta)^2 + mgy_2$. $y_2 = \frac{v_0^2}{2g}(1-\cos^2\theta) = \frac{v_0^2\sin^2\theta}{2g}$, was to be shown.

EVALUATE: The initial kinetic energy is independent of the angle θ but the kinetic energy at the maximum height depends on θ, so the maximum height depends on θ.

7.11. IDENTIFY: Apply Eq.(7.7) to the motion of the car.

SET UP: Take $y = 0$ at point *A*. Let point 1 be *A* and point 2 be *B*.

$$K_1 + U_1 + W_{other} = K_2 + U_2$$

EXECUTE: $U_1 = 0,\ U_2 = mg(2R) = 28{,}224\text{ J},\ W_{other} = W_f$

$K_1 = \frac{1}{2}mv_1^2 = 37{,}500\text{ J},\ K_2 = \frac{1}{2}mv_2^2 = 3840\text{ J}$

The work-energy relation then gives $W_f = K_2 + U_2 - K_1 = -5400$ J.

EVALUATE: Friction does negative work. The final mechanical energy $(K_2 + U_2 = 32{,}064\text{ J})$ is less than the initial mechanical energy $(K_1 + U_1 = 37{,}500\text{ J})$ because of the energy removed by friction work.

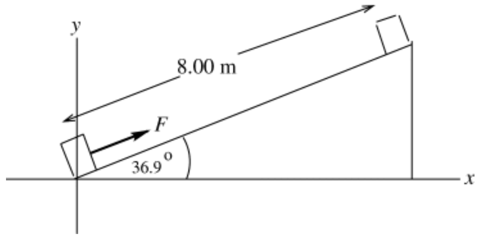
7.12. IDENTIFY: Only gravity does work, so apply Eq.(7.5).

SET UP: $v_1 = 0$, so $\frac{1}{2}mv_2^2 = mg(y_1 - y_2)$.

EXECUTE: Tarzan is lower than his original height by a distance $y_1 - y_2 = l(\cos 30° - \cos 45°)$ so his speed is $v = \sqrt{2gl(\cos 30° - \cos 45°)} = 7.9$ m/s, a bit quick for conversation.

EVALUATE: The result is independent of Tarzan's mass.

7.13.



$y_1 = 0$
$y_2 = (8.00\text{ m})\sin 36.9°$
$y_2 = 4.80\text{ m}$

Figure 7.13a

(a) IDENTIFY and SET UP: $\vec{F}$ is constant so Eq.(6.2) can be used. The situation is sketched in Figure 7.13a.

EXECUTE: $W_F = (F\cos\phi)s = (110\text{ N})(\cos 0°)(8.00\text{ m}) = 880\text{ J}$

EVALUATE: $\vec{F}$ is in the direction of the displacement and does positive work.

(b) IDENTIFY and SET UP: Calculate W using Eq.(6.2) but first must calculate the friction force. Use the free-body diagram for the oven sketched in Figure 7.13b to calculate the normal force n; then the friction force can be calculated from $f_k = \mu_k n$. For this calculation use coordinates parallel and perpendicular to the incline.

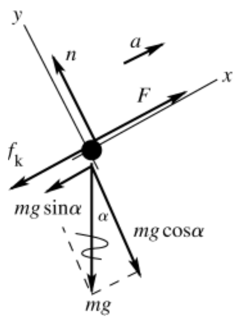


Figure 7.13b

EXECUTE: $\sum F_y = ma_y$

$n - mg\cos 36.9° = 0$

$n = mg\cos 36.9°$

$f_k = \mu_k n = \mu_k mg\cos 36.9°$

$f_k = (0.25)(10.0\text{ kg})(9.80\text{ m/s}^2)\cos 36.9° = 19.6\text{ N}$

$W_f = (f_k\cos\phi)s = (19.6\text{ N})(\cos 180°)(8.00\text{ m}) = -157\text{ J}$

EVALUATE: Friction does negative work.

(c) IDENTIFY and SET UP: $U = mgy$; take $y = 0$ at the bottom of the ramp.

EXECUTE: $\Delta U = U_2 - U_1 = mg(y_2 - y_1) = (10.0\text{ kg})(9.80\text{ m/s}^2)(4.80\text{ m} - 0) = 470\text{ J}$

EVALUATE: The object moves upward and U increases.

(d) IDENTIFY and SET UP: Use Eq.(7.7). Solve for ΔK.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$\Delta K = K_2 - K_1 = U_1 - U_2 + W_{\text{other}}$

$\Delta K = W_{\text{other}} - \Delta U$

$W_{\text{other}} = W_F + W_f = 880\text{ J} - 157\text{ J} = 723\text{ J}$

$\Delta U = 470\text{ J}$

Thus $\Delta K = 723\text{ J} - 470\text{ J} = 253\text{ J}$.

EVALUATE: W_{other} is positive. Some of W_{other} goes to increasing U and the rest goes to increasing K.

(e) IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the oven. Solve for $\vec{a}$ and then use a constant acceleration equation to calculate v_2.

SET UP: We can use the free-body diagram that is in part (b):

$\sum F_x = ma_x$

$F - f_k - mg\sin 36.9° = ma$

EXECUTE: $a = \dfrac{F - f_k - mg\sin 36.9°}{m} = \dfrac{110\text{ N} - 19.6\text{ N} - (10\text{ kg})(9.80\text{ m/s}^2)\sin 36.9°}{10.0\text{ kg}} = 3.16\text{ m/s}^2$

SET UP: $v_{1x} = 0$, $a_x = 3.16\text{ m/s}^2$, $x - x_0 = 8.00\text{ m}$, $v_{2x} = ?$

$v_{2x}^2 = v_{1x}^2 + 2a_x(x - x_0)$

EXECUTE: $v_{2x} = \sqrt{2a_x(x - x_0)} = \sqrt{2(3.16\text{ m/s}^2)(8.00\text{ m})} = 7.11\text{ m/s}^2$

Then $\Delta K = K_2 - K_1 = \frac{1}{2}mv_2^2 = \frac{1}{2}(10.0\text{ kg})(7.11\text{ m/s})^2 = 253\text{ J}$.

EVALUATE: This agrees with the result calculated in part (d) using energy methods.

7.14. **IDENTIFY:** Only gravity does work, so apply Eq.(7.4). Use $\sum \vec{F} = m\vec{a}$ to calculate the tension.

SET UP: Let $y = 0$ at the bottom of the arc. Let point 1 be when the string makes a $45°$ angle with the vertical and point 2 be where the string is vertical. The rock moves in an arc of a circle, so it has radial acceleration $a_{\text{rad}} = v^2/r$

EXECUTE: **(a)** At the top of the swing, when the kinetic energy is zero, the potential energy (with respect to the bottom of the circular arc) is $mgl(1-\cos\,\theta)$, where l is the length of the string and θ is the angle the string makes with the vertical. At the bottom of the swing, this potential energy has become kinetic energy, so $mgl(1-\cos\theta) = \frac{1}{2}mv^2$, or $v = \sqrt{2gl(1-\cos\theta)} = \sqrt{2(9.80\text{ m/s}^2)\,(0.80\text{ m})\,(1-\cos 45°)} = 2.1\text{ m/s}$.

(b) At $45°$ from the vertical, the speed is zero, and there is no radial acceleration; the tension is equal to the radial component of the weight, or $mg\cos\theta = (0.12\text{ kg})\,(9.80\text{ m/s}^2)\cos 45° = 0.83\text{ N}$.

(c) At the bottom of the circle, the tension is the sum of the weight and the mass times the radial acceleration,

$$mg + mv_2^2/l = mg(1+2(1-\cos 45°)) = 1.9\text{ N}$$

EVALUATE: When the string passes through the vertical, the tension is greater than the weight because the acceleration is upward.

7.15. **IDENTIFY:** Apply $U_{\text{el}} = \frac{1}{2}kx^2$.

SET UP: $kx = F$, so $U = \frac{1}{2}Fx$, where F is the magnitude of force required to stretch or compress the spring a distance x.

EXECUTE: **(a)** $(1/2)(800\text{ N})(0.200\text{ m}) = 80.0\text{ J}$.

(b) The potential energy is proportional to the square of the compression or extension; $(80.0\text{ J})\,(0.050\text{ m}/0.200\text{ m})^2 = 5.0\text{ J}$.

EVALUATE: We could have calculated $k = \dfrac{F}{x} = \dfrac{800\text{ N}}{0.200\text{ m}} = 4000\text{ N/m}$ and then used $U_{\text{el}} = \frac{1}{2}kx^2$ directly.

7.16. **IDENTIFY:** Use the information given in the problem with $F = kx$ to find k. Then $U_{\text{el}} = \frac{1}{2}kx^2$.

SET UP: x is the amount the spring is stretched. When the weight is hung from the spring, $F = mg$.

EXECUTE: $k = \dfrac{F}{x} = \dfrac{mg}{x} = \dfrac{(3.15\text{ kg})(9.80\text{ m/s}^2)}{0.1340\text{ m} - 0.1200\text{ m}} = 2205\text{ N/m}$.

$x = \pm\sqrt{\dfrac{2U_{\text{el}}}{k}} = \pm\sqrt{\dfrac{2(10.0\text{ J})}{2205\text{ N/m}}} = \pm 0.0952\text{ m} = \pm 9.52\text{ cm}$. The spring could be either stretched 9.52 cm or compressed 9.52 cm. If it were stretched, the total length of the spring would be $12.00\text{ cm} + 9.52\text{ cm} = 21.52\text{ cm}$. If it were compressed, the total length of the spring would be $12.00\text{ cm} - 9.52\text{ cm} = 2.48\text{ cm}$.

EVALUATE: To stretch or compress the spring 9.52 cm requires a force $F = kx = 210\text{ N}$.

7.17. **IDENTIFY:** Apply $U_{\text{el}} = \frac{1}{2}kx^2$.

SET UP: $U_0 = \frac{1}{2}kx_0^2$. x is the distance the spring is stretched or compressed.

EXECUTE: **(a)** (i) $x = 2x_0$ gives $U_{\text{el}} = \frac{1}{2}k(2x_0)^2 = 4(\frac{1}{2}kx_0^2) = 4U_0$. (ii) $x = x_0/2$ gives $U_{\text{el}} = \frac{1}{2}k(x_0/2)^2 = \frac{1}{4}(\frac{1}{2}kx_0^2) = U_0/4$.

(b) (i) $U = 2U_0$ gives $\frac{1}{2}kx^2 = 2(\frac{1}{2}kx_0^2)$ and $x = x_0\sqrt{2}$. (ii) $U = U_0/2$ gives $\frac{1}{2}kx^2 = \frac{1}{2}(\frac{1}{2}kx_0^2)$ and $x = x_0/\sqrt{2}$.

EVALUATE: U is proportional to x^2 and x is proportional to $\sqrt{U}$.

7.18. **IDENTIFY:** Apply Eq.(7.13).

SET UP: Initially and at the highest point, $v = 0$, so $K_1 = K_2 = 0$. $W_{\text{other}} = 0$.

EXECUTE: **(a)** In going from rest in the slingshot's pocket to rest at the maximum height, the potential energy stored in the rubber band is converted to gravitational potential energy; $U = mgy = (10\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)\,(22.0\text{ m}) = 2.16\text{ J}$.

(b) Because gravitational potential energy is proportional to mass, the larger pebble rises only 8.8 m.

(c) The lack of air resistance and no deformation of the rubber band are two possible assumptions.

EVALUATE: The potential energy stored in the rubber band depends on k for the rubber band and the maximum distance it is stretched.

7.19. IDENTIFY and SET UP: Use energy methods. There are changes in both elastic and gravitational potential energy; elastic; $U = \frac{1}{2}kx^2$, gravitational: $U = mgy$.

EXECUTE: **(a)** $U = \frac{1}{2}kx^2$ so $x = \sqrt{\frac{2U}{k}} = \sqrt{\frac{2(3.20\text{ J})}{1600\text{ N/m}}} = 0.0632\text{ m} = 6.32\text{ cm}$

(b) Points 1 and 2 in the motion are sketched in Figure 7.19.

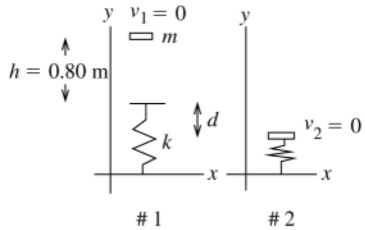


Figure 7.19

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$W_{\text{other}} = 0$ (Only work is that done by gravity and spring force)

$K_1 = 0,\ \ K_2 = 0$

$y = 0$ at final position of book

$U_1 = mg(h + d),\ \ U_2 = \frac{1}{2}kd^2$

$0 + mg(h + d) + 0 = \frac{1}{2}kd^2$

The original gravitational potential energy of the system is converted into potential energy of the compressed spring.

$\frac{1}{2}kd^2 - mgd - mgh = 0$

$$d = \frac{1}{k}\left(mg \pm \sqrt{(mg)^2 + 4\left(\frac{1}{2}k\right)(mgh)}\right)$$

d must be positive, so $d = \frac{1}{k}\left(mg + \sqrt{(mg)^2 + 2kmgh}\right)$

$$d = \frac{1}{1600\text{ N/m}}((1.20\text{ kg})(9.80\text{ m/s}^2) + \sqrt{((1.20\text{ kg})(9.80\text{ m/s}^2))^2 + 2(1600\text{ N/m})(1.20\text{ kg})(9.80\text{ m/s}^2)(0.80\text{ m})}$$

$d = 0.0074\text{ m} + 0.1087\text{ m} = 0.12\text{ m} = 12\text{ cm}$

EVALUATE: It was important to recognize that the total displacement was $h + d$; gravity continues to do work as the book moves against the spring. Also note that with the spring compressed 0.12 m it exerts an upward force (192 N) greater than the weight of the book (11.8 N). The book will be accelerated upward from this position.

7.20. IDENTIFY: Use energy methods. There are changes in both elastic and gravitational potential energy.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Points 1 and 2 in the motion are sketched in Figure 7.20.

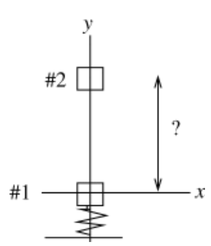


Figure 7.20

The spring force and gravity are the only forces doing work on the cheese, so $W_{\text{other}} = 0$ and $U = U_{\text{grav}} + U_{\text{el}}$.

EXECUTE: Cheese released from rest implies $K_1 = 0$.

At the maximum height $v_2 = 0$ so $K_2 = 0$.

$U_1 = U_{1,\text{el}} + U_{1,\text{grav}}$

$y_1 = 0$ implies $U_{1,\text{grav}} = 0$

$U_{1,\text{el}} = \frac{1}{2}kx_1^2 = \frac{1}{2}(1800\text{ N/m})(0.15\text{ m})^2 = 20.25\text{ J}$

(Here x_1 refers to the amount the spring is stretched or compressed when the cheese is at position 1; it is *not* the x-coordinate of the cheese in the coordinate system shown in the sketch.)

$U_2 = U_{2,\text{el}} + U_{2,\text{grav}}$

$U_{2,\text{grav}} = mgy_2$, where y_2 is the height we are solving for. $U_{2,\text{el}} = 0$ since now the spring is no longer compressed. Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $U_{1,\text{el}} = U_{2,\text{grav}}$

$$y_2 = \frac{20.25\text{ J}}{mg} = \frac{20.25\text{ J}}{(1.20\text{ kg})(9.80\text{ m/s}^2)} = 1.72\text{ m}$$

EVALUATE: The description in terms of energy is very simple; the elastic potential energy originally stored in the spring is converted into gravitational potential energy of the system.

7.21. **IDENTIFY:** Apply Eq.(7.13).

SET UP: $W_{\text{other}} = 0$. As in Example 7.7, $K_1 = 0$ and $U_1 = 0.0250$ J.

EXECUTE: For $v_2 = 0.20$ m/s, $K_2 = 0.0040$ J. $U_2 = 0.0210\text{ J} = \frac{1}{2}kx^2$, and $x = \pm\sqrt{\frac{2(0.0210\text{ J})}{5.00\text{ N/m}}} = \pm 0.092$ m. The glider has this speed when the spring is stretched 0.092 m or compressed 0.092 m.

EVALUATE: Example 7.7 showed that $v_x = 0.30$ m/s when $x = 0.0800$ m. As x increases, v_x decreases, so our result of $v_x = 0.20$ m/s at $x = 0.092$ m is consistent with the result in the example.

7.22. **IDENTIFY** and **SET UP:** Use energy methods. The elastic potential energy changes. In part (a) solve for K_2 and from this obtain v_2. In part (b) solve for U_1 and from this obtain x_1.

(a) $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

point 1: the glider is at its initial position, where $x_1 = 0.100$ m and $v_1 = 0$

point 2: the glider is at $x = 0$

EXECUTE: $K_1 = 0$ (released from rest), $K_2 = \frac{1}{2}mv_2^2$

$U_1 = \frac{1}{2}kx_1^2$, $U_2 = 0$, $W_{\text{other}} = 0$ (only the spring force does work)

Thus $\frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2$. (The initial potential energy of the stretched spring is converted entirely into kinetic energy of the glider.)

$$v_2 = x_1\sqrt{\frac{k}{m}} = (0.100\text{ m})\sqrt{\frac{5.00\text{ N/m}}{0.200\text{ kg}}} = 0.500\text{ m/s}$$

(b) The maximum speed occurs at $x = 0$, so the same equation applies.

$$\tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2$$

$$x_1 = v_2\sqrt{\frac{m}{k}} = 2.50\text{ m/s}\sqrt{\frac{0.200\text{ kg}}{5.00\text{ N/m}}} = 0.500\text{ m}$$

EVALUATE: Elastic potential energy is converted into kinetic energy. A larger x_1 gives a larger v_2.

7.23. **IDENTIFY:** Only the spring does work and Eq.(7.11) applies. $a = \frac{F}{m} = \frac{-kx}{m}$, where F is the force the spring exerts on the mass.

SET UP: Let point 1 be the initial position of the mass against the compressed spring, so $K_1 = 0$ and $U_1 = 11.5$ J. Let point 2 be where the mass leaves the spring, so $U_{\text{el},2} = 0$.

EXECUTE: **(a)** $K_1 + U_{\text{el},1} = K_2 + U_{\text{el},2}$ gives $U_{\text{el},1} = K_2$. $\frac{1}{2}mv_2^2 = U_{\text{el},1}$ and $v_2 = \sqrt{\frac{2U_{\text{el},1}}{m}} = \sqrt{\frac{2(11.5\text{ J})}{2.50\text{ kg}}} = 3.03$ m/s.

K is largest when U_{el} is least and this is when the mass leaves the spring. The mass achieves its maximum speed of 3.03 m/s as it leaves the spring and then slides along the surface with constant speed.

(b) The acceleration is greatest when the force on the mass is the greatest, and this is when the spring has its maximum compression. $U_{\text{el}} = \frac{1}{2}kx^2$ so $x = -\sqrt{\frac{2U_{\text{el}}}{k}} = -\sqrt{\frac{2(11.5\text{ J})}{2500\text{ N/m}}} = -0.0959$ m. The minus sign indicates compression. $F = -kx = ma_x$ and $a_x = -\frac{kx}{m} = -\frac{(2500\text{ N/m})(-0.0959\text{ m})}{2.50\text{ kg}} = 95.9\text{ m/s}^2$.

EVALUATE: If the end of the spring is displaced to the left when the spring is compressed, then a_x in part (b) is to the right, and vice versa.

7.24. **(a) IDENTIFY** and **SET UP:** Use energy methods. Both elastic and gravitational potential energy changes. Work is done by friction.

Choose point 1 as in Example 7.9 and let that be the origin, so $y_1 = 0$. Let point 2 be 1.00 m below point 1, so $y_2 = -1.00$ m.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(2000\text{ kg})(25\text{ m/s})^2 = 625{,}000\text{ J},\ U_1 = 0$

$W_{\text{other}} = -f|y_2| = -(17{,}000\text{ N})(1.00\text{ m}) = -17{,}000\text{ J}$

$K_2 = \frac{1}{2}mg_2^2$

$U_2 = U_{2,\text{grav}} + U_{2,\text{el}} = mgy_2 + \frac{1}{2}ky_2^2$

$U_2 = (2000\text{ kg})(9.80\text{ m/s}^2)(-1.00\text{ m}) + \frac{1}{2}(1.41\times10^5\text{ N/m})(1.00\text{ m})^2$

$U_2 = -19{,}600\text{ J} + 70{,}500\text{ J} = +50{,}900\text{ J}$

Thus $625{,}000\text{ J} - 17{,}000\text{ J} = \frac{1}{2}mv_2^2 + 50{,}900\text{ J}$

$\frac{1}{2}mv_2^2 = 557{,}100\text{ J}$

$$v_2 = \sqrt{\frac{2(557{,}100\text{ J})}{2000\text{ kg}}} = 23.6\text{ m/s}$$

EVALUATE: The elevator stops after descending 3.00 m. After descending 1.00 m it is still moving but has slowed down.

(b) IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the elevator. We know the forces and can solve for $\vec{a}$.

SET UP: The free-body diagram for the elevator is given in Figure 7.24.

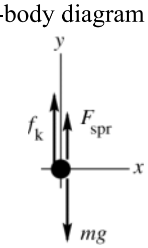


Figure 7.24

EXECUTE: $F_{\text{spr}} = kd$, where d is the distance the spring is compressed

$\sum F_y = ma_y$

$f_k + F_{\text{spr}} - mg = ma$

$f_k + kd - mg = ma$

$$a = \frac{f_k + kd - mg}{m} = \frac{17{,}000\text{ N} + (1.41\times10^5\text{ N/m})(1.00\text{ m}) - (2000\text{ kg})(9.80\text{ m/s}^2)}{2000\text{ kg}} = 69.2\text{ m/s}^2$$

We calculate that a is positive, so the acceleration is upward.

EVALUATE: The velocity is downward and the acceleration is upward, so the elevator is slowing down at this point. Note that $a = 7.1g$; this is unacceptably high for an elevator.

7.25. **IDENTIFY:** Apply Eq.(7.13) and $F = ma$.

SET UP: $W_{\text{other}} = 0$. There is no change in U_{grav}. $K_1 = 0$, $U_2 = 0$.

EXECUTE: $\frac{1}{2}kx^2 = \frac{1}{2}mv_x^2$. The relations for m, v_x, k and x are $kx^2 = mv_x^2$ and $kx = 5mg$.

Dividing the first equation by the second gives $x = \dfrac{v_x^2}{5g}$, and substituting this into the second gives $k = 25\dfrac{mg^2}{v_x^2}$.

(a) $k = 25\dfrac{(1160\text{ kg})(9.80\text{ m/s}^2)^2}{(2.50\text{ m/s})^2} = 4.46\times10^5\text{ N/m}$

(b) $x = \dfrac{(2.50\text{ m/s})^2}{5(9.80\text{ m/s}^2)} = 0.128\text{ m}$

EVALUATE: Our results for k and x do give the required values for a_x and v_x:

$$a_x = \frac{kx}{m} = \frac{(4.46\times10^5\text{ N/m})(0.128\text{ m})}{1160\text{ kg}} = 49.2\text{ m/s}^2 = 5.0g \text{ and } v_x = x\sqrt{\frac{k}{m}} = 2.5\text{ m/s}.$$

7.26. IDENTIFY: $W_{\text{grav}} = mg\cos\phi$.

SET UP: When he moves upward, $\phi = 180°$ and when he moves downward, $\phi = 0°$. When he moves parallel to the ground, $\phi = 90°$.

EXECUTE: (a) $W_{\text{grav}} = (75\text{ kg})(9.80\text{ m/s}^2)(7.0\text{ m})\cos180° = -5100\text{ J}$.

(b) $W_{\text{grav}} = (75\text{ kg})(9.80\text{ m/s}^2)(7.0\text{ m})\cos0° = +5100\text{ J}$.

(c) $\phi = 90°$ in each case and $W_{\text{grav}} = 0$ in each case.

(d) The total work done on him by gravity during the round trip is $-5100\text{ J} + 5100\text{ J} = 0$.

(e) Gravity is a conservative force since the total work done for a round trip is zero.

EVALUATE: The gravity force is independent of the position and motion of the object. When the object moves upward gravity does negative work and when the object moves downward gravity does positive work.

7.27. IDENTIFY: Apply $W_{f_k} = f_k s\cos\phi$. $f_k = \mu_k n$.

SET UP: For a circular trip the distance traveled is $d = 2\pi r$. At each point in the motion the friction force and the displacement are in opposite directions and $\phi = 180°$. Therefore, $W_{f_k} = -f_k d = -f_k(2\pi r)$. $n = mg$ so $f_k = \mu_k mg$.

EXECUTE: (a) $W_{f_k} = -\mu_k mg2\pi r = -(0.250)(10.0\text{ kg})(9.80\text{ m/s}^2)(2\pi)(2.00\text{ m}) = -308\text{ J}$.

(b) The distance along the path doubles so the work done doubles and becomes -616 J.

(c) The work done for a round trip displacement is not zero and friction is a nonconservative force.

EVALUATE: The direction of the friction force depends on the direction of motion of the object and that is why friction is a nonconservative force.

7.28. IDENTIFY and SET UP: The force is not constant so we must use Eq.(6.14) to calculate *W*. The properties of work done by a conservative force are described in Section 7.3.

$$W = \int_1^2 \vec{F}\cdot d\vec{l},\quad \vec{F} = -\alpha x^2\hat{i}$$

EXECUTE: (a) $d\vec{l} = dy\hat{j}$ (x is constant; the displacement is in the $+y$-direction)

$\vec{F}\cdot d\vec{l} = 0$ (since $\hat{i}\cdot\hat{j} = 0$) and thus $W = 0$.

(b) $d\vec{l} = dx\hat{i}$

$\vec{F}\cdot d\vec{l} = (-\alpha x^2\hat{i})\cdot(dx\hat{i}) = -\alpha x^2\ dx$

$$W = \int_{x_1}^{x_2}(-\alpha x^2)dx = -\tfrac{1}{3}\alpha x^3\Big|_{x_1}^{x_2} = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = -\frac{12\text{ N/m}^2}{3}((0.300\text{ m})^3 - (0.10\text{ m})^3) = -0.10\text{ J}$$

(c) $d\vec{l} = dx\hat{i}$ as in part (b), but now $x_1 = 0.30\text{ m}$ and $x_2 = 0.10\text{ m}$

$$W = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = +0.10\text{ J}$$

(d) EVALUATE: The total work for the displacement along the *x*-axis from 0.10 m to 0.30 m and then back to 0.10 m is the sum of the results of parts (b) and (c), which is zero. The total work is zero when the starting and ending points are the same, so the force is conservative.

EXECUTE: $W_{x_1\to x_2} = -\tfrac{1}{3}\alpha(x_2^3 - x_1^3) = \tfrac{1}{3}\alpha x_1^3 - \tfrac{1}{3}\alpha x_2^3$

The definition of the potential energy function is $W_{x_1\to x_2} = U_1 - U_2$. Comparison of the two expressions for *W* gives $U = \tfrac{1}{3}\alpha x^3$. This does correspond to $U = 0$ when $x = 0$.

EVALUATE: In part (a) the work done is zero because the force and displacement are perpendicular. In part (b) the force is directed opposite to the displacement and the work done is negative. In part (c) the force and displacement are in the same direction and the work done is positive.

7.29. IDENTIFY: Since the force is constant, use $W = Fs\cos\phi$.

SET UP: For both displacements, the direction of the friction force is opposite to the displacement and $\phi = 180°$.

EXECUTE: (a) When the book moves to the left, the friction force is to the right, and the work is $-(1.2\text{ N})(3.0\text{ m}) = -3.6\text{ J}$.

(b) The friction force is now to the left, and the work is again -3.6 J.

(c) -7.2 J.

(d) The net work done by friction for the round trip is not zero, and friction is not a conservative force.

EVALUATE: The direction of the friction force depends on the motion of the object. For the gravity force, which is conservative, the force does not depend on the motion of the object.

7.30. **IDENTIFY** and **SET UP:** The friction force is constant during each displacement and Eq.(6.2) can be used to calculate work, but the direction of the friction force can be different for different displacements.

$f = \mu_k mg = (0.25)(1.5\text{ kg})(9.80\text{ m/s}^2) = 3.675\text{ N}$; direction of $\vec{f}$ is opposite to the motion.

EXECUTE: **(a)** The path of the book is sketched in Figure 7.30a.

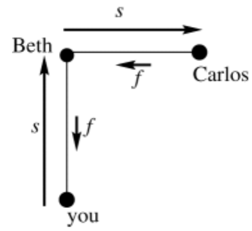


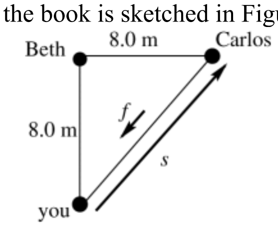
Figure 7.30a

For the motion from you to Beth the friction force is directed opposite to the displacement $\vec{s}$ and $W_1 = -fs = -(3.675\text{ N})(8.0\text{ m}) = -29.4\text{ J}$.

For the motion from Beth to Carlos the friction force is again directed opposite to the displacement and $W_2 = -29.4\text{ J}$.

$$W_{\text{tot}} = W_1 + W_2 = -29.4\text{ J} - 29.4\text{ J} = -59\text{ J}$$

(b) The path of the book is sketched in Figure 7.30b.



$s = \sqrt{2(8.0\text{ m})^2} = 11.3\text{ m}$

Figure 7.30b

$\vec{f}$ is opposite to $\vec{s}$, so $W = -fs = -(3.675\text{ N})(11.3\text{ m}) = -42\text{ J}$

(c)

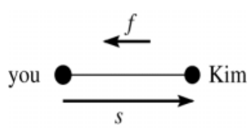


Figure 7.30c

For the motion from you to Kim
$W = -fs$
$W = -(3.675\text{ N})(8.0\text{ m}) = -29.4\text{ J}$

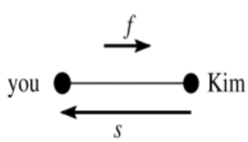


Figure 7.30d

For the motion from Kim to you
$W = -fs = -29.4\text{ J}$

The total work for the round trip is $-29.4\text{ J} - 29.4\text{ J} = -59\text{ J}$.

(d) EVALUATE: Parts (a) and (b) show that for two different paths between you and Carlos, the work done by friction is different. Part (c) shows that when the starting and ending points are the same, the total work is not zero. Both these results show that the friction force is nonconservative.

7.31. **IDENTIFY:** The work done by a spring on an object attached to its end when the object moves from x_i to x_f is $W = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$. This result holds for any x_i and x_f.

SET UP: Assume for simplicity that x_1, x_2 and x_3 are all positive, corresponding to the spring being stretched.

EXECUTE: **(a)** $\frac{1}{2}k(x_1^2 - x_2^2)$

(b) $-\frac{1}{2}k(x_1^2 - x_2^2)$. The total work is zero; the spring force is conservative.

(c) From x_1 to x_3, $W = -\frac{1}{2}k(x_3^2 - x_1^2)$. From x_3 to x_2, $W = -\frac{1}{2}k(x_2^2 - x_3^2)$. The net work is $-\frac{1}{2}k(x_2^2 - x_1^2)$. This is the same as the result of part (a).

EVALUATE: The results of part (c) illustrate that the work done by a conservative force is path independent.

7.32. **IDENTIFY and SET UP:** Use Eq.(7.17) to calculate the force from $U(x)$. Use coordinates where the origin is at one atom. The other atom then has coordinate x.

EXECUTE:

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left(-\frac{C_6}{x^6}\right) = +C_6\frac{d}{dx}\left(\frac{1}{x^6}\right) = -\frac{6C_6}{x^7}$$

The minus sign mean that F_x is directed in the $-x$-direction, toward the origin. The force has magnitude $6C_6/x^7$ and is attractive.

EVALUATE: U depends only on x so $\vec{F}$ is along the x-axis; it has no y or z components.

7.33. **IDENTIFY:** Apply Eq.(7.16).

SET UP: The sign of F_x indicates its direction.

EXECUTE: $F_x = -\frac{dU}{dx} = -4\alpha x^3 = -(4.8\ \text{J/m}^4)x^3$. $F_x(-0.800\ \text{m}) = -(4.8\ \text{J/m}^4)(-0.80\ \text{m})^3 = 2.46\ \text{N}$. The force is in the $+x$-direction.

EVALUATE: $F_x > 0$ when $x < 0$ and $F_x < 0$ when $x > 0$, so the force is always directed towards the origin.

7.34. **IDENTIFY:** Apply $F(x) = -\frac{dU(x)}{dx}$.

SET UP: $\frac{d(1/x)}{dx} = -\frac{1}{x^2}$

EXECUTE: $F_x(x) = -\frac{d(-Gm_1m_2/x)}{dx} = Gm_1m_2\left[\frac{d(1/x)}{dx}\right] = -\frac{Gm_1m_2}{x^2}$. The force on m_2 is in the $-x$-direction. This is toward m_1, so the force is attractive.

EVALUATE: By Newton's 3rd law the force on m_1 due to m_2 is Gm_1m_2/x^2, in the $+x$-direction (toward m_2). The gravitational potential energy belongs to the system of the two masses.

7.35. **IDENTIFY:** Apply $F_x = -\frac{\partial U}{\partial x}$ and $F_y = -\frac{\partial U}{\partial y}$.

SET UP: $r = (x^2 + y^2)^{1/2}$. $\frac{\partial(1/r)}{\partial x} = -\frac{x}{(x^2+y^2)^{3/2}}$ and $\frac{\partial(1/r)}{\partial y} = -\frac{y}{(x^2+y^2)^{3/2}}$.

EXECUTE: **(a)** $U(r) = -\frac{Gm_1m_2}{r}$. $F_x = -\frac{\partial U}{\partial x} = +Gm_1m_2\left[\frac{\partial(1/r)}{\partial x}\right] = -\frac{Gm_1m_2x}{(x^2+y^2)^{3/2}}$ and

$F_y = -\frac{\partial U}{\partial y} = +Gm_1m_2\left[\frac{\partial(1/r)}{\partial y}\right] = -\frac{Gm_1m_2y}{(x^2+y^2)^{3/2}}$.

(b) $(x^2+y^2)^{3/2} = r^3$ so $F_x = -\frac{Gm_1m_2x}{r^3}$ and $F_y = -\frac{Gm_1m_2y}{r^3}$. $F = \sqrt{F_x^2 + F_y^2} = \frac{Gm_1m_2}{r^3}\sqrt{x^2+y^2} = \frac{Gm_1m_2}{r^2}$.

(c) F_x and F_y are negative. $F_x = \alpha x$ and $F_y = \alpha y$, where α is a constant, so $\vec{F}$ and the vector $\vec{r}$ from m_1 to m_2 are in the same direction. Therefore, $\vec{F}$ is directed toward m_1 at the origin and $\vec{F}$ is attractive.

EVALUATE: If θ is the angle between the vector $\vec{r}$ that points from m_1 to m_2, then $\frac{x}{r} = \cos\theta$ and $\frac{y}{r} = \sin\theta$. This gives $F_x = -F\cos\theta$ and $F_y = -F\sin\theta$, our more usual way of writing the components of a vector.

7.36. **IDENTIFY:** Apply Eq.(7.18).

SET UP: $\frac{d}{dx}\left(\frac{1}{x^2}\right) = -\frac{2}{x^3}$ and $\frac{d}{dy}\left(\frac{1}{y^2}\right) = -\frac{2}{y^3}$.

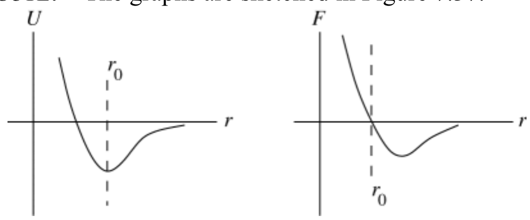
EXECUTE: $\vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j}$ since U has no z-dependence. $\frac{\partial U}{\partial x} = \frac{-2\alpha}{x^3}$ and $\frac{\partial U}{\partial y} = \frac{-2\alpha}{y^3}$, so

$$\vec{F} = -\alpha\left(\frac{-2}{x^3}\hat{i} + \frac{-2}{y^3}\hat{j}\right) = 2\alpha\left(\frac{\hat{i}}{x^3} + \frac{\hat{j}}{y^3}\right).$$

EVALUATE: F_x and x have the same sign and F_y and y have the same sign. When $x > 0$, F_x is in the $+x$-direction, and so forth.

7.37. **IDENTIFY** and **SET UP:** Use Eq.(7.17) to calculate the force from U. At equilibrium $F = 0$.

(a) EXECUTE: The graphs are sketched in Figure 7.37.



$$U = \frac{a}{r^{12}} - \frac{b}{r^6}$$

$$F = -\frac{dU}{dr} = +\frac{12a}{r^{13}} - \frac{6b}{r^7}$$

Figure 7.37

(b) At equilibrium $F = 0$, so $\frac{dU}{dr} = 0$

$$F = 0 \text{ implies } \frac{+12a}{r^{13}} - \frac{6b}{r^7} = 0$$

$6br^6 = 12a$; solution is the equilibrium distance $r_0 = (2a/b)^{1/6}$

U is a minimum at this r; the equilibrium is stable.

(c) At $r = (2a/b)^{1/6}$, $U = a/r^{12} - b/r^6 = a(b/2a)^2 - b(b/2a) = -b^2/4a$.

At $r \to \infty$, $U = 0$. The energy that must be added is $-\Delta U = b^2/4a$.

(d) $r_0 = (2a/b)^{1/6} = 1.13 \times 10^{-10}$ m gives that

$2a/b = 2.082 \times 10^{-60}$ m^6 and $b/4a = 2.402 \times 10^{59}$ m^{-6}

$b^2/4a = b(b/4a) = 1.54 \times 10^{-18}$ J

$b(2.402 \times 10^{59}\ \text{m}^{-6}) = 1.54 \times 10^{-18}$ J and $b = 6.41 \times 10^{-78}\ \text{J} \cdot \text{m}^6$.

Then $2a/b = 2.082 \times 10^{-60}$ m^6 gives $a = (b/2)(2.082 \times 10^{-60}\ \text{m}^6) =$

$\frac{1}{2}(6.41 \times 10^{-78}\ \text{J} \cdot \text{m}^6)(2.082 \times 10^{-60}\ \text{m}^6) = 6.67 \times 10^{-138}\ \text{J} \cdot \text{m}^{12}$

EVALUATE: As the graphs in part (a) show, $F(r)$ is the slope of $U(r)$ at each r. $U(r)$ has a minimum where $F = 0$.

7.38. **IDENTIFY:** Apply Eq.(7.16).

SET UP: $\frac{dU}{dx}$ is the slope of the U versus x graph.

EXECUTE: **(a)** Considering only forces in the x-direction, $F_x = -\frac{dU}{dx}$ and so the force is zero when the slope of the U *vs* x graph is zero, at points b and d.

(b) Point b is at a potential minimum; to move it away from b would require an input of energy, so this point is stable.

(c) Moving away from point d involves a decrease of potential energy, hence an increase in kinetic energy, and the marble tends to move further away, and so d is an unstable point.

EVALUATE: At point b, F_x is negative when the marble is displaced slightly to the right and F_x is positive when the marble is displaced slightly to the left, the force is a restoring force, and the equilibrium is stable. At point d, a small displacement in either direction produces a force directed away from d and the equilibrium is unstable.

7.39. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the bag and to the box. Apply Eq.(7.7) to the motion of the system of the box and bucket after the bag is removed.

SET UP: Let $y = 0$ at the final height of the bucket, so $y_1 = 2.00$ m and $y_2 = 0$. $K_1 = 0$. The box and the bucket move with the same speed v, so $K_2 = \frac{1}{2}(m_{\text{box}} + m_{\text{bucket}})v^2$. $W_{\text{other}} = -f_k d$, with $d = 2.00$ m and $f_k = \mu_k m_{\text{box}} g$. Before the bag is removed, the maximum possible friction force the roof can exert on the box is $(0.700)(80.0\ \text{kg} + 50.0\ \text{kg})(9.80\ \text{m/s}^2) = 892$ N. This is larger than the weight of the bucket (637 N), so before the bag is removed the system is at rest.

EXECUTE: **(a)** The friction force on the bag of gravel is zero, since there is no other horizontal force on the bag for friction to oppose. The static friction force on the box equals the weight of the bucket, 637 N.

(b) Eq.(7.7) gives $m_{\text{bucket}}gy_1 - f_k d = \frac{1}{2}m_{\text{tot}}v^2$, with $m_{\text{tot}} = 145.0 \text{ kg}$. $v = \sqrt{\frac{2}{m_{\text{tot}}}(m_{\text{bucket}}gy_1 - \mu_k m_{\text{box}}gd)}$.

$$v = \sqrt{\frac{2}{145.0 \text{ kg}}\left[(65.0 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m}) - (0.400)(80.0 \text{ kg})(9.80 \text{ m/s}^2)(2.00 \text{ m})\right]}.$$

$v = 2.99 \text{ m/s}$.

EVALUATE: If we apply $\sum \vec{F} = m\vec{a}$ to the box and to the bucket we can calculate their common acceleration a. Then a constant acceleration equation applied to either object gives $v = 2.99 \text{ m/s}$, in agreement with our result obtained using energy methods.

7.40. **IDENTIFY:** For the system of two blocks, only gravity does work. Apply Eq.(7.5).

SET UP: Call the blocks A and B, where A is the more massive one. $v_{A1} = v_{B1} = 0$. Let $y = 0$ for each block to be at the initial height of that block, so $y_{A1} = y_{B1} = 0$. $y_{A2} = -1.20 \text{ m}$ and $y_{B2} = +1.20 \text{ m}$. $v_{A2} = v_{B2} = v_2 = 3.00 \text{ m/s}$.

EXECUTE: Eq.(7.5) gives $0 = \frac{1}{2}(m_A + m_B)v_2^2 + g(1.20 \text{ m})(m_B - m_A)$. $m_A + m_B = 15.0 \text{ kg}$.

$\frac{1}{2}(15.0 \text{ kg})(3.00 \text{ m/s})^2 + (9.80 \text{ m/s}^2)(1.20 \text{ m})(15.0 \text{ kg} - 2m_A)$. Solving for m_A gives $m_A = 10.4 \text{ kg}$. And then $m_B = 4.6 \text{ kg}$.

EVALUATE: The final kinetic energy of the two blocks is 68 J. The potential energy of block A decreases by 122 J. The potential energy of block B increases by 54 J. The total decrease in potential energy is $122 \text{ J} - 54 \text{ J} = 68 \text{ J}$, and this equals the increase in kinetic energy of the system.

7.41. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

SET UP: $U_1 = U_2 = K_2 = 0$. $W_{\text{other}} = W_f = -\mu_k mgs$, with $s = 280 \text{ ft} = 85.3 \text{ m}$

EXECUTE: **(a)** The work-energy expression gives $\frac{1}{2}mv_1^2 - \mu_k mgs = 0$.

$v_1 = \sqrt{2\mu_k gs} = 22.4 \text{ m/s} = 50 \text{ mph}$; the driver was speeding.

(b) 15 mph over speed limit so $150 ticket.

EVALUATE: The negative work done by friction removes the kinetic energy of the object.

7.42. **IDENTIFY:** Apply Eq.(7.14).

SET UP: Only the spring force and gravity do work, so $W_{\text{other}} = 0$. Let $y = 0$ at the horizontal surface.

EXECUTE: **(a)** Equating the potential energy stored in the spring to the block's kinetic energy, $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$, or

$$v = \sqrt{\frac{k}{m}}x = \sqrt{\frac{400 \text{ N/m}}{2.00 \text{ kg}}}(0.220 \text{ m}) = 3.11 \text{ m/s}.$$

(b) Using energy methods directly, the initial potential energy of the spring equals the final gravitational potential energy, $\frac{1}{2}kx^2 = mgL\sin\theta$, or $L = \frac{\frac{1}{2}kx^2}{mg\sin\theta} = \frac{\frac{1}{2}(400 \text{ N/m})(0.220 \text{ m})^2}{(2.00 \text{ kg})(9.80 \text{ m/s}^2)\sin 37.0°} = 0.821 \text{ m}.$

EVALUATE: The total energy of the system is constant. Initially it is all elastic potential energy stored in the spring, then it is all kinetic energy and finally it is all gravitational potential energy.

7.43. **IDENTIFY:** Use the work-energy theorem, Eq(7.7). The target variable μ_k will be a factor in the work done by friction.

SET UP: Let point 1 be where the block is released and let point 2 be where the block stops, as shown in Figure 7.43.

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

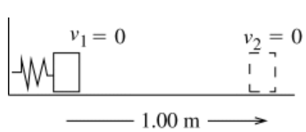


Figure 7.43

Work is done on the block by the spring and by friction, so $W_{\text{other}} = W_f$ and $U = U_{\text{el}}$.

EXECUTE: $K_1 = K_2 = 0$

$U_1 = U_{1,\text{el}} = \frac{1}{2}kx_1^2 = \frac{1}{2}(100 \text{ N/m})(0.200 \text{ m})^2 = 2.00 \text{ J}$

$U_2 = U_{2,\text{el}} = 0$, since after the block leaves the spring has given up all its stored energy

$W_{\text{other}} = W_f = (f_k\cos\phi)s = \mu_k mg(\cos\phi)s = -\mu_k mgs$, since $\phi = 180°$ (The friction force is directed opposite to the displacement and does negative work.)

Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$U_{1,\text{el}} + W_f = 0$

$\mu_k mgs = U_{1,\text{el}}$

$$\mu_k = \frac{U_{1,\text{el}}}{mgs} = \frac{200\text{ J}}{(0.50\text{ kg})(9.80\text{ m/s}^2)(1.00\text{ m})} = 0.41.$$

EVALUATE: $U_{1,\text{el}} + W_f = 0$ says that the potential energy originally stored in the spring is taken out of the system by the negative work done by friction.

7.44. **IDENTIFY:** Apply Eq.(7.14). Calculate f_k from the fact that the crate slides a distance $x = 5.60$ m before coming to rest. Then apply Eq.(7.14) again, with $x = 2.00$ m .

SET UP: $U_1 = U_{\text{el}} = 360$ J . $U_2 = 0$. $K_1 = 0$. $W_{\text{other}} = -f_k x$.

EXECUTE: Work done by friction against the crate brings it to a halt: $U_1 = -W_{\text{other}}$.

$f_k x =$ potential energy of compressed spring , and $f_k = \dfrac{360\text{ J}}{5.60\text{ m}} = 64.29$ N .

The friction force working over a 2.00-m distance does work equal to $-f_k x = -(64.29\text{ N})(2.00\text{ m}) = -128.6$ J. The kinetic energy of the crate at this point is thus $360\text{ J} - 128.6\text{ J} = 231.4$ J, and its speed is found from

$mv^2/2 = 231.4$ J , so $v = \sqrt{\dfrac{2(231.4\text{ J})}{50.0\text{ kg}}} = 3.04$ m/s .

EVALUATE: The energy of the compressed spring goes partly into kinetic energy of the crate and is partly removed by the negative work done by friction. After the crate leaves the spring the crate slows down as friction does negative work on it.

7.45. **IDENTIFY:** At its highest point between bounces all the mechanical energy of the ball is in the form of gravitational potential energy.

SET UP: $E = U = mgh$, where h is the height at the highest point of the motion.

EXECUTE: **(a)** $mgh = (0.650\text{ kg})(9.80\text{ m/s}^2)(2.50\text{ m}) = 15.9$ J

(b) The second height is $0.75(2.50\text{ m}) = 1.875$ m, so the second $mgh = 11.9$ J ; it loses $15.9\text{ J} - 11.9\text{ J} = 4.0$ J on first bounce. This energy is converted to thermal energy.

(c) The third height is $0.75(1.875\text{ m}) = 1.40$ m, , so third $mgh = 8.9$ J ; it loses $11.9\text{ J} - 8.9\text{ J} = 3.0$ J on second bounce.

EVALUATE: In each bounce the ball loses 25% of its mechanical energy.

7.46. **IDENTIFY:** Apply Eq.(7.14) to relate h and v_B . Apply $\sum \vec{F} = m\vec{a}$ at point B to find the minimum speed required at B for the car not to fall off the track.

SET UP: At B, $a = v_B^2/R$, downward. The minimum speed is when $n \to 0$ and $mg = mv_B^2/R$. The minimum speed required is $v_B = \sqrt{gR}$. $K_1 = 0$ and $W_{\text{other}} = 0$.

EXECUTE: **(a)** Eq.(7.14) applied to points A and B gives $U_A - U_B = \frac{1}{2}mv_B^2$. The speed at the top must be at least $\sqrt{gR}$. Thus, $mg(h - 2R) > \dfrac{1}{2}mgR,$ or $h > \dfrac{5}{2}R.$

(b) Apply Eq.(7.14) to points A and C. $U_A - U_C = (2.50)Rmg = K_C$, so

$$v_C = \sqrt{(5.00)gR} = \sqrt{(5.00)(9.80\text{ m/s}^2)(20.0\text{ m})} = 31.3\text{ m/s}.$$

The radial acceleration is $a_{\text{rad}} = \dfrac{v_C^2}{R} = 49.0\text{ m/s}^2.$ The tangential direction is down, the normal force at point C is horizontal, there is no friction, so the only downward force is gravity, and $a_{\tan} = g = 9.80\text{ m/s}^2.$

EVALUATE: If $h > \frac{5}{2}R$, then the downward acceleration at B due to the circular motion is greater than g and the track must exert a downward normal force n. n increases as h increases and hence v_B increases.

7.47. **(a) IDENTIFY:** Use work-energy relation to find the kinetic energy of the wood as it enters the rough bottom.

SET UP: Let point 1 be where the piece of wood is released and point 2 be just before it enters the rough bottom. Let $y = 0$ be at point 2.

EXECUTE: $U_1 = K_2$ gives $K_2 = mgy_1 = 78.4$ J.

IDENTIFY: Now apply work-energy relation to the motion along the rough bottom.

SET UP: Let point 1 be where it enters the rough bottom and point 2 be where it stops.

$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$$

EXECUTE: $W_{\text{other}} = W_f = -\mu_k mgs$, $K_2 = U_1 = U_2 = 0$; $K_1 = 78.4\text{ J}$

$78.4\text{ J} - \mu_k mgs = 0$; solving for s gives $s = 20.0\text{ m}$.

The wood stops after traveling 20.0 m along the rough bottom.

(b) Friction does -78.4 J of work.

EVALUATE: The piece of wood stops before it makes one trip across the rough bottom. The final mechanical energy is zero. The negative friction work takes away all the mechanical energy initially in the system.

7.48. **IDENTIFY:** Apply Eq.(7.14) to the rock. $W_{\text{other}} = W_{f_k}$.

SET UP: Let $y = 0$ at the foot of the hill, so $U_1 = 0$ and $U_2 = mgh$, where h is the vertical height of the rock above the foot of the hill when it stops.

EXECUTE: **(a)** At the maximum height, $K_2 = 0$. Eq.(7.14) gives $K_{\text{Bottom}} + W_{f_k} = U_{\text{Top}}$.

$\frac{1}{2}mv_0^2 - \mu_k mg\cos\theta\, d = mgh$. $d = h/\sin\theta$, so $\frac{1}{2}v_0^2 - \mu_k g\cos\theta\frac{h}{\sin\theta} = gh$.

$\frac{1}{2}(15\text{ m/s})^2 - (0.20)(9.8\text{ m/s}^2)\frac{\cos 40°}{\sin 40°}h = (9.8\text{ m/s}^2)h$ and $h = 9.3\text{ m}$.

(b) Compare maximum static friction force to the weight component down the plane.

$f_s = \mu_s mg\cos\theta = (0.75)(28\text{ kg})(9.8\text{ m/s}^2)\cos 40° = 158\text{ N}$. $mg\sin\theta = (28\text{ kg})(9.8\text{ m/s}^2)(\sin 40°) = 176\text{ N} > f_s$, so the rock will slide down.

(c) Use same procedure as in part (a), with $h = 9.3\text{ m}$ and v_B being the speed at the bottom of the hill.

$U_{\text{Top}} + W_{f_k} = K_B$. $mgh - \mu_k mg\cos\theta\frac{h}{\sin\theta} = \frac{1}{2}mv_B^2$ and

$v_B = \sqrt{2gh - 2\mu_k gh\cos\theta/\sin\theta} = 11.8\text{ m/s}$.

EVALUATE: For the round trip up the hill and back down, there is negative work done by friction and the speed of the rock when it returns to the bottom of the hill is less than the speed it had when it started up the hill.

7.49. **IDENTIFY:** Apply Eq.(7.7) to the motion of the stone.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

Let point 1 be point *A* and point 2 be point *B*. Take $y = 0$ at point *B*.

EXECUTE: $mgy_1 + \frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2$, with $h = 20.0\text{ m}$ and $v_1 = 10.0\text{ m/s}$

$v_2 = \sqrt{v_1^2 + 2gh} = 22.2\text{ m/s}$

EVALUATE: The loss of gravitational potential energy equals the gain of kinetic energy.

(b) IDENTIFY: Apply Eq.(7.8) to the motion of the stone from point *B* to where it comes to rest against the spring.

SET UP: Use $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, with point 1 at *B* and point 2 where the spring has its maximum compression *x*.

EXECUTE: $U_1 = U_2 = K_2 = 0$; $K_1 = \frac{1}{2}mv_1^2$ with $v_1 = 22.2\text{ m/s}$

$W_{\text{other}} = W_f + W_{\text{el}} = -\mu_k mgs - \frac{1}{2}kx^2$, with $s = 100\text{ m} + x$

The work-energy relation gives $K_1 + W_{\text{other}} = 0$.

$\frac{1}{2}mv_1^2 - \mu_k mgs - \frac{1}{2}kx^2 = 0$

Putting in the numerical values gives $x^2 + 29.4x - 750 = 0$. The positive root to this equation is $x = 16.4\text{ m}$.

EVALUATE: Part of the initial mechanical (kinetic) energy is removed by friction work and the rest goes into the potential energy stored in the spring.

(c) IDENTIFY and **SET UP:** Consider the forces.

EXECUTE: When the spring is compressed $x = 16.4\text{ m}$ the force it exerts on the stone is $F_{\text{el}} = kx = 32.8\text{ N}$. The maximum possible static friction force is

$\max f_s = \mu_s mg = (0.80)(15.0\text{ kg})(9.80\text{ m/s}^2) = 118\text{ N}$.

EVALUATE: The spring force is less than the maximum possible static friction force so the stone remains at rest.

7.50. **IDENTIFY:** Once the block leaves the top of the hill it moves in projectile motion. Use Eq.(7.14) to relate the speed v_B at the bottom of the hill to the speed v_{Top} at the top and the 70 m height of the hill.

SET UP: For the projectile motion, take $+y$ to be downward. $a_x = 0$, $a_y = g$. $v_{0x} = v_{Top}$, $v_{0y} = 0$. For the motion up the hill only gravity does work. Take $y = 0$ at the base of the hill.

EXECUTE: First get speed at the top of the hill for the block to clear the pit. $y = \frac{1}{2}gt^2$. $20\text{ m} = \frac{1}{2}(9.8\text{ m/s}^2)t^2$.

$t = 2.0\text{ s}$. Then $v_{Top}t = 40\text{ m}$ gives $v_{Top} = \frac{40\text{ m}}{2.0\text{ s}} = 20\text{ m/s}$.

Energy conservation applied to the motion up the hill: $K_{Bottom} = U_{Top} + K_{Top}$ gives

$\frac{1}{2}mv_B^2 = mgh + \frac{1}{2}mv_{Top}^2$. $v_B = \sqrt{v_{Top}^2 + 2gh} = \sqrt{(20\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(70\text{ m})} = 42\text{ m/s}$.

EVALUATE: The result does not depend on the mass of the block.

7.51. **IDENTIFY:** Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to the motion of the person.

SET UP: Point 1 is where he steps off the platform and point 2 is where he is stopped by the cord. Let $y = 0$ at point 2. $y_1 = 41.0\text{ m}$. $W_{other} = -\frac{1}{2}kx^2$, where $x = 11.0\text{ m}$ is the amount the cord is stretched at point 2. The cord does negative work.

EXECUTE: $K_1 = K_2 = U_2 = 0$, so $mgy_1 - \frac{1}{2}kx^2 = 0$ and $k = 631\text{ N/m}$.

Now apply $F = kx$ to the test pulls:

$F = kx$ so $x = F/k = 0.602\text{ m}$.

EVALUATE: All his initial gravitational potential energy is taken away by the negative work done by the force exerted by the cord, and this amount of energy is stored as elastic potential energy in the stretched cord.

7.52. **IDENTIFY:** Apply Eq.(7.14) to the motion of the skier from the gate to the bottom of the ramp.

SET UP: $W_{other} = -4000\text{ J}$. Let $y = 0$ at the bottom of the ramp.

EXECUTE: For the skier to be moving at no more than 30.0 m/s; his kinetic energy at the bottom of the ramp can be no bigger than $\frac{mv^2}{2} = \frac{(85.0\text{ kg})(30.0\text{ m/s})^2}{2} = 38{,}250\text{ J}$. Friction does -4000 J of work on him during his run, which means his combined *U* and *K* at the top of the ramp must be no more than $38{,}250\text{ J} + 4000\text{ J} = 42{,}250\text{ J}$. His *K* at the top is $\frac{mv^2}{2} = \frac{(85.0\text{ kg})(2.0\text{ m/s})^2}{2} = 170\text{ J}$. His *U* at the top should thus be no more than $42{,}250\text{ J} - 170\text{ J} = 42{,}080\text{ J}$, which gives a height above the bottom of the ramp of $h = \frac{42{,}080\text{ J}}{mg} = \frac{42{,}080\text{ J}}{(85.0\text{ kg})(9.80\text{ m/s}^2)} = 50.5\text{ m}$.

EVALUATE: In the absence of air resistance, for this *h* his speed at the bottom of the ramp would be 31.5 m/s. The work done by air resistance is small compared to the kinetic and potential energies that enter into the calculation.

7.53. **IDENTIFY:** Use the work-energy theorem, Eq.(7.7). Solve for K_2 and then for v_2.

SET UP: Let point 1 be at his initial position against the compressed spring and let point 2 be at the end of the barrel, as shown in Figure 7.53. Use $F = kx$ to find the amount the spring is initially compressed by the 4400 N force.

$K_1 + U_1 + W_{other} = K_2 + U_2$

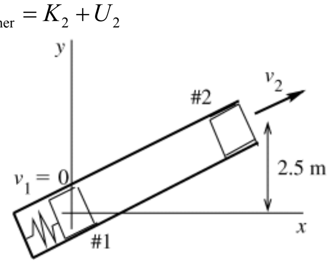


Figure 7.53

Take $y = 0$ at his initial position.

EXECUTE: $K_1 = 0$, $K_2 = \frac{1}{2}mv_2^2$

$W_{other} = W_{fric} = -fs$

$W_{other} = -(40\text{ N})(4.0\text{ m}) = -160\text{ J}$

$U_{1,grav} = 0$, $U_{1,el} = \frac{1}{2}kd^2$, where *d* is the distance the spring is initially compressed.

$F = kd$ so $d = \frac{F}{k} = \frac{4400\text{ N}}{1100\text{ N/m}} = 4.00\text{ m}$

and $U_{1,el} = \frac{1}{2}(1100\text{ N/m})(4.00\text{ m})^2 = 8800\text{ J}$

$U_{2,grav} = mgy_2 = (60\text{ kg})(9.80\text{ m/s}^2)(2.5\text{ m}) = 1470\text{ J}$, $U_{2,el} = 0$

Then $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$$8800\text{ J} - 160\text{ J} = \tfrac{1}{2}mv_2^2 + 1470\text{ J}$$

$$\tfrac{1}{2}mv_2^2 = 7170\text{ J} \text{ and } v_2 = \sqrt{\frac{2(7170\text{ J})}{60\text{ kg}}} = 15.5\text{ m/s}$$

EVALUATE: Some of the potential energy stored in the compressed spring is taken away by the work done by friction. The rest goes partly into gravitational potential energy and partly into kinetic energy.

7.54. **IDENTIFY:** To be at equilibrium at the bottom, with the spring compressed a distance x_0, the spring force must balance the component of the weight down the ramp plus the largest value of the static friction, or $kx_0 = w\sin\theta + f$. Apply Eq.(7.14) to the motion down the ramp.

SET UP: $K_2 = 0$, $K_1 = \tfrac{1}{2}mv^2$, where v is the speed at the top of the ramp. Let $U_2 = 0$, so $U_1 = wL\sin\theta$, where L is the total length traveled down the ramp.

EXECUTE: Eq.(7.14) gives $\frac{1}{2}kx_0^2 = (w\sin\theta - f)L + \frac{1}{2}mv^2$. With the given parameters, $\tfrac{1}{2}kx_0^2 = 248\text{ J}$ and $kx_0 = 1.10\times10^3\text{ N}$. Solving for k gives $k = 2440\text{ N/m}$.

EVALUATE: $x_0 = 0.451\text{ m}$. $w\sin\theta = 551\text{ N}$. The decrease in gravitational potential energy is only slightly larger than the amount of mechanical energy removed by the negative work done by friction. $\tfrac{1}{2}mv^2 = 243\text{ J}$. The energy stored in the spring is only slightly larger than the initial kinetic energy of the crate at the top of the ramp.

7.55. **IDENTIFY:** Apply Eq.(7.7) to the system consisting of the two buckets. If we ignore the inertia of the pulley we ignore the kinetic energy it has.

SET UP: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Points 1 and 2 in the motion are sketched in Figure 7.55.

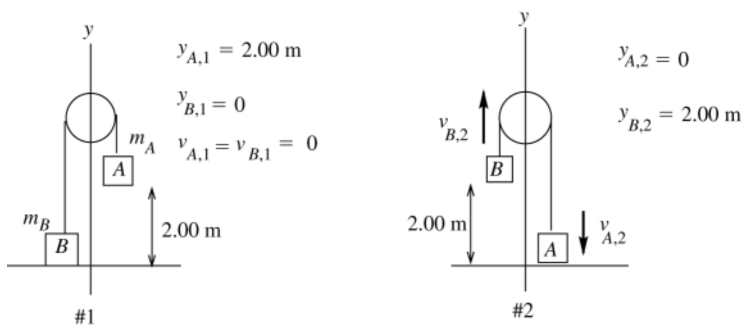


Figure 7.55

The tension force does positive work on the 4.0 kg bucket and an equal amount of negative work on the 12.0 kg bucket, so the net work done by the tension is zero.

Work is done on the system only by gravity, so $W_{\text{other}} = 0$ and $U = U_{\text{grav}}$

EXECUTE: $K_1 = 0$

$K_2 = \tfrac{1}{2}m_A v_{A,2}^2 + \tfrac{1}{2}m_B v_{B,2}^2$ But since the two buckets are connected by a rope they move together and have the same speed: $v_{A,2} = v_{B,2} = v_2$.

Thus $K_2 = \tfrac{1}{2}(m_A + m_B)v_2^2 = (8.00\text{ kg})v_2^2$.

$U_1 = m_A g y_{A,1} = (12.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 235.2\text{ J}.$

$U_2 = m_B g y_{B,2} = (4.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 78.4\text{ J}.$

Putting all this into $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$$U_1 = K_2 + U_2$$

$$235.2\text{ J} = (8.00\text{ kg})v_2^2 + 78.4\text{ J}$$

$$v_2 = \sqrt{\frac{235.2\text{ J} - 78.4\text{ J}}{8.00\text{ kg}}} = 4.4\text{ m/s}$$

EVALUATE: The gravitational potential energy decreases and the kinetic energy increases by the same amount. We could apply Eq.(7.7) to one bucket, but then we would have to include in W_{other} the work done on the bucket by the tension T.

7.56. **IDENTIFY:** Apply $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ to the motion of the rocket from the starting point to the base of the ramp. W_{other} is the work done by the thrust and by friction.

SET UP: Let point 1 be at the starting point and let point 2 be at the base of the ramp. $v_1 = 0$, $v_2 = 50.0\text{ m/s}$. Let $y = 0$ at the base and take $+y$ upward. Then $y_2 = 0$ and $y_1 = d\sin 53°$, where d is the distance along the ramp from the base to the starting point. Friction does negative work.

EXECUTE: $K_1 = 0$, $U_2 = 0$. $U_1 + W_{\text{other}} = K_2$. $W_{\text{other}} = (2000\text{ N})d - (500\text{ N})d = (1500\text{ N})d$.

$mgd\sin 53° + (1500\text{ N})d = \frac{1}{2}mv_2^2$.

$$d = \frac{mv_2^2}{2[mg\sin 53° + 1500\text{ N}]} = \frac{(1500\text{ kg})(50.0\text{ m/s})^2}{2[(1500\text{ kg})(9.80\text{ m/s}^2)\sin 53° + 1500\text{ N}]} = 142\text{ m}.$$

EVALUATE: The initial height is $y_1 = (142\text{ m})\sin 53° = 113\text{ m}$. An object free-falling from this distance attains a speed $v = \sqrt{2gy_1} = 47.1\text{ m/s}$. The rocket attains a greater speed than this because the forward thrust is greater than the friction force.

7.57. **IDENTIFY:** The force exerted by a spring is $F_x = -kx$. The acceleration of the object is given by $F_x = ma_x$. Apply Eq.(7.14) to relate position and speed.

SET UP: Let $+x$ be when the spring is stretched.

EXECUTE: **(a)** $U = \frac{1}{2}kx^2$. Let point 1 be when the spring is initially compressed a distance x_0, so $x_1 = -x_0$. $K_1 = 0$. $W_{\text{other}} = 0$. $\frac{1}{2}kx_0^2 = U_2 + K_2$. The speed is maximum when $x = 0$, so $U_2 = 0$. Then $\frac{1}{2}kx_0^2 = \frac{1}{2}mv_2^2$ and $v_2 = x_0\sqrt{k/m}$ is this maximum speed.

(b) $F_x = -kx$ and $F_x = ma_x$ give $a_x = -\dfrac{k}{m}x$. a is maximum when $|x|$ is maximum, so $a = \dfrac{k}{m}x_0$.

(c) The speed is maximum when $x = 0$, when the spring has returned to its natural length, and the acceleration is maximum when $x = -x_0$, at the initial compression of the spring.

(d) When the spring has maximum extension, $v_2 = 0$. $\frac{1}{2}kx_0^2 = \frac{1}{2}kx^2$ and $x = x_0$. The magnitude of the maximum extension equals the magnitude of the maximum compression.

(e) The machine part oscillates between $x = -x_0$ and $x = +x_0$ and never stops permanently.

EVALUATE: In any real system there are mechanical energy losses, for example due to negative work done by friction, and the object eventually comes to rest.

7.58. **IDENTIFY:** Conservation of energy says the decrease in potential energy equals the gain in kinetic energy.

SET UP: Since the two animals are equidistant from the axis, they each have the same speed v.

EXECUTE: One mass rises while the other falls, so the net loss of potential energy is $(0.500\text{ kg} - 0.200\text{ kg})(9.80\text{ m/s}^2)(0.400\text{ m}) = 1.176\text{ J}$. This is the sum of the kinetic energies of the animals and is equal to $\frac{1}{2}m_{\text{tot}}v^2$, and $v = \sqrt{\dfrac{2(1.176\text{ J})}{(0.700\text{ kg})}} = 1.83\text{ m/s}.$

EVALUATE: The mouse gains both gravitational potential energy and kinetic energy. The rat's gain in kinetic energy is less than its decrease of potential energy, and the energy difference is transferred to the mouse.

7.59. **(a) IDENTIFY** and **SET UP:** Apply Eq.(7.7) to the motion of the potato.

Let point 1 be where the potato is released and point 2 be at the lowest point in its motion, as shown in Figure 7.59a.

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

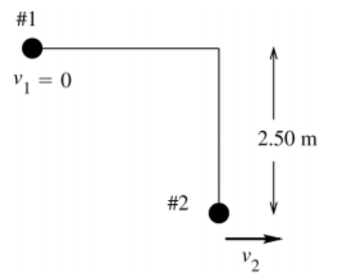


Figure 7.59a

$y_1 = 2.50\text{ m}$

$y_2 = 0$

The tension in the string is at all points in the motion perpendicular to the displacement, so $W_T = 0$

The only force that does work on the potato is gravity, so $W_{\text{other}} = 0$.

EXECUTE: $K_1 = 0,\; K_2 = \frac{1}{2}mv_2^2,\; U_1 = mgy_1,\; U_2 = 0$

Thus $U_1 = K_2$.

$mgy_1 = \frac{1}{2}mv_2^2$

$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\text{ m/s}^2)(2.50\text{ m})} = 7.00\text{ m/s}$

EVALUATE: v_2 is the same as if the potato fell through 2.50 m.

(b) IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to the potato. The potato moves in an arc of a circle so its acceleration is $\vec{a}_{\text{rad}}$, where $a_{\text{rad}} = v^2/R$ and is directed toward the center of the circle. Solve for one of the forces, the tension T in the string.

SET UP: The free-body diagram for the potato as it swings through its lowest point is given in Figure 7.59b.

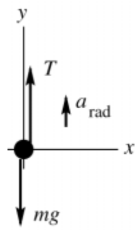


Figure 7.59b

The acceleration $\vec{a}_{\text{rad}}$ is directed in toward the center of the circular path, so at this point it is upward.

EXECUTE: $\sum F_y = ma_y$

$T - mg = ma_{\text{rad}}$

$T = m(g + a_{\text{rad}}) = m\left(g + \dfrac{v_2^2}{R}\right)$, where the radius R for the circular motion is the length L of the string.

It is instructive to use the algebraic expression for v_2 from part (a) rather than just putting in the numerical value:

$v_2 = \sqrt{2gy_1} = \sqrt{2gL}$, so $v_2^2 = 2gL$

Then $T = m\left(g + \dfrac{v_2^2}{L}\right) = m\left(g + \dfrac{2gL}{L}\right) = 3mg$; the tension at this point is three times the weight of the potato.

$$T = 3mg = 3(0.100\text{ kg})(9.80\text{ m/s}^2) = 2.94\text{ N}$$

EVALUATE: The tension is greater than the weight; the acceleration is upward so the net force must be upward.

7.60. IDENTIFY: Eq.(7.14) says $W_{\text{other}} = K_2 + U_2 - (K_1 + U_1)$. W_{other} is the work done on the baseball by the force exerted by the air.

SET UP: $U = mgy$. $K = \frac{1}{2}mv^2$, where $v^2 = v_x^2 + v_y^2$.

EXECUTE: **(a)** The change in total energy is the work done by the air,

$W_{\text{other}} = (K_2 + U_2) - (K_1 + U_1) = m\left(\dfrac{1}{2}(v_2^2 - v_1^2) + gy_2\right)$.

$W_{\text{other}} = (0.145\text{ kg})\Big((1/2\big[(18.6\text{ m/s})^2 - (30.0\text{ m/s})^2 - (40.0\text{ m/s})^2\big] + (9.80\text{ m/s}^2)(53.6\text{ m})\Big)$.

$W_{\text{other}} = -80.0\text{ J}$.

(b) Similarly, $W_{\text{other}} = (K_3 + U_3) - (K_2 + U_2)$.

$W_{\text{other}} = (0.145\text{ kg})\Big((1/2)\big[(11.9\text{ m/s})^2 + (-28.7\text{ m/s})^2 - (18.6\text{ m/s})^2\big] - (9.80\text{ m/s}^2)(53.6\text{ m})\Big)$.

$W_{\text{other}} = -31.3\text{ J}$.

(c) The ball is moving slower on the way down, and does not go as far (in the x-direction), and so the work done by the air is smaller in magnitude.

EVALUATE: The initial kinetic energy of the baseball is $\frac{1}{2}(0.145\text{ kg})(50.0\text{ m/s})^2 = 181\text{ J}$. For the total motion from the ground, up to the maximum height, and back down the total work done by the air is 111 J. The ball returns to the ground with $181\text{ J} - 111\text{ J} = 70\text{ J}$ of kinetic energy and a speed of 31 m/s, less than its initial speed of 50 m/s.

7.61. **IDENTIFY** and **SET UP:** There are two situations to compare: stepping off a platform and sliding down a pole. Apply the work-energy theorem to each.

(a) EXECUTE: Speed at ground if steps off platform at height h:

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$mgh = \frac{1}{2}mv_2^2$, so $v_2^2 = 2gh$

Motion from top to bottom of pole: (take $y = 0$ at bottom)

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$mgd - fd = \frac{1}{2}mv_2^2$

Use $v_2^2 = 2gh$ and get $mgd - fd = mgh$

$fd = mg(d - h)$

$f = mg(d - h)/d = mg(1 - h/d)$

EVALUATE: For $h = d$ this gives $f = 0$ as it should (friction has no effect).

For $h = 0$, $v_2 = 0$ (no motion). The equation for f gives $f = mg$ in this special case. When $f = mg$ the forces on him cancel and he doesn't accelerate down the pole, which agrees with $v_2 = 0$.

(b) EXECUTE: $f = mg(1 - h/d) = (75\text{ kg})(9.80\text{ m/s}^2)(1 - 1.0\text{ m}/2.5\text{ m}) = 441\text{ N}$.

(c) Take $y = 0$ at bottom of pole, so $y_1 = d$ and $y_2 = y$.

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$0 + mgd - f(d - y) = \frac{1}{2}mv^2 + mgy$

$\frac{1}{2}mv^2 = mg(d - y) - f(d - y)$

Using $f = mg(1 - h/d)$ gives $\frac{1}{2}mv^2 = mg(d - y) - mg(1 - h/d)(d - y)$

$\frac{1}{2}mv^2 = mg(h/d)(d - y)$ and $v = \sqrt{2gh(1 - y/d)}$

EVALUATE: This gives the correct results for $y = 0$ and for $y = d$.

7.62. **IDENTIFY:** Apply Eq.(7.14) to each stage of the motion.

SET UP: Let $y = 0$ at the bottom of the slope. In part (a), W_{other} is the work done by friction. In part (b), W_{other} is the work done by friction and the air resistance force. In part (c), W_{other} is the work done by the force exerted by the snowdrift.

EXECUTE: **(a)** The skier's kinetic energy at the bottom can be found from the potential energy at the top minus the work done by friction, $K_1 = mgh - W_f = (60.0\text{ kg})(9.8\text{ N/kg})(65.0\text{ m}) - 10{,}500\text{ J}$, or

$K_1 = 38{,}200\text{ J} - 10{,}500\text{ J} = 27{,}720\text{ J}$. Then $v_1 = \sqrt{\dfrac{2K_1}{m}} = \sqrt{\dfrac{2(27{,}720\text{ J})}{60\text{ kg}}} = 30.4\text{ m/s}$.

(b) $K_2 = K_1 - (W_f + W_{\text{air}}) = 27{,}720\text{ J} - (\mu_k mgd + f_{\text{air}}d)$. $K_2 = 27{,}720\text{ J} - [(0.2)(588\text{ N})(82\text{ m}) + (160\text{ N})(82\text{ m})]$ or

$K_2 = 27{,}720\text{ J} - 22{,}763\text{ J} = 4957\text{ J}$. Then, $v_2 = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(4957\text{ J})}{60\text{ kg}}} = 12.9\text{ m/s}$

(c) Use the Work-Energy Theorem to find the force. $W = \Delta K$, $F = K/d = (4957\text{ J})/(2.5\text{ m}) = 2000\text{ N}$.

EVALUATE: In each case, W_{other} is negative and removes mechanical energy from the system.

7.63. **IDENTIFY** and **SET UP:** First apply $\sum \vec{F} = m\vec{a}$ to the skier.

Find the angle α where the normal force becomes zero, in terms of the speed v_2 at this point. Then apply the work-energy theorem to the motion of the skier to obtain another equation that relates v_2 and α. Solve these two equations for α.

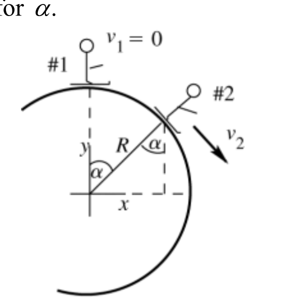


Figure 7.63a

Let point 2 be where the skier loses contact with the snowball, as sketched in Figure 7.63a Loses contact implies $n \to 0$.

$y_1 = R$, $y_2 = R\cos\alpha$

First, analyze the forces on the skier when she is at point 2. The free-body diagram is given in Figure 7.63b. For this use coordinates that are in the tangential and radial directions. The skier moves in an arc of a circle, so her acceleration is $a_{\text{rad}} = v^2/R$, directed in towards the center of the snowball.

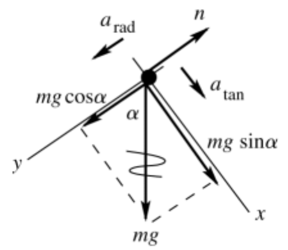


Figure 7.63b

EXECUTE: $\sum F_y = ma_y$

$mg\cos\alpha - n = mv_2^2/R$

But $n = 0$ so $mg\cos\alpha = mv_2^2/R$

$v_2^2 = Rg\cos\alpha$

Now use conservation of energy to get another equation relating v_2 to α:

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

The only force that does work on the skier is gravity, so $W_{\text{other}} = 0$.

$K_1 = 0,\ \ K_2 = \frac{1}{2}mv_2^2$

$U_1 = mgy_1 = mgR,\ \ U_2 = mgy_2 = mgR\cos\alpha$

Then $mgR = \frac{1}{2}mv_2^2 + mgR\cos\alpha$

$v_2^2 = 2gR(1-\cos\alpha)$

Combine this with the $\sum F_y = ma_y$ equation:

$Rg\cos\alpha = 2gR(1-\cos\alpha)$

$\cos\alpha = 2 - 2\cos\alpha$

$3\cos\alpha = 2$ so $\cos\alpha = 2/3$ and $\alpha = 48.2°$

EVALUATE: She speeds up and her a_{rad} increases as she loses gravitational potential energy. She loses contact when she is going so fast that the radially inward component of her weight isn't large enough to keep her in the circular path. Note that α where she loses contact does not depend on her mass or on the radius of the snowball.

7.64. **IDENTIFY:** Use conservation of energy to relate the speed at the lowest point to the speed at the highest point. Use $\sum \vec{F} = m\vec{a}$ to calculate the tension.

SET UP: The rock has acceleration $a_{\text{rad}} = v^2/R$, directed toward the center of the circle.

EXECUTE: If the speed of the rock at the top is v_{t}, then conservation of energy gives the speed v_{b} at the bottom from $\frac{1}{2}mv_{\text{b}}^2 = \frac{1}{2}mv_{\text{t}}^2 + mg(2R)$, R being the radius of the circle, and so $v_{\text{b}}^2 = v_{\text{t}}^2 + 4gR$. The tension at the top and bottom are found from $T_{\text{t}} + mg = \dfrac{mv_{\text{t}}^2}{R}$ and $T_{\text{b}} - mg = \dfrac{mv_{\text{b}}^2}{R}$, so $T_{\text{b}} - T_{\text{t}} = \dfrac{m}{R}(v_{\text{b}}^2 - v_{\text{t}}^2) + 2mg = 6mg = 6w$.

EVALUATE: The tensions T_{t} and T_{b} depend on the speed of the rock and on R, but the difference $T_{\text{b}} - T_{\text{t}}$ is independent of the speed of the rock and the radius of the circle.

7.65. **IDENTIFY** and **SET UP:**

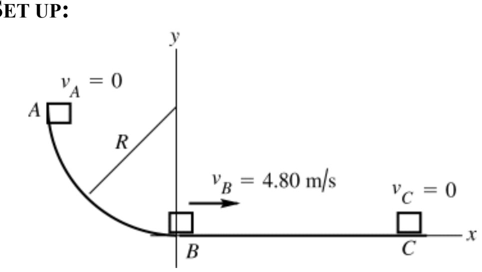


Figure 7.65

$y_A = R$

$y_B = y_C = 0$

(a) Apply conservation of energy to the motion from B to C:

$K_B + U_B + W_{\text{other}} = K_C + U_C$. The motion is described in Figure 7.65.

EXECUTE: The only force that does work on the package during this part of the motion is friction, so

$W_{\text{other}} = W_f = f_{\text{k}}(\cos\phi)s = \mu_{\text{k}}mg(\cos 180°)s = -\mu_{\text{k}}mgs$

$K_B = \frac{1}{2}mv_B^2,\ \ K_C = 0$

$U_B = 0,\ \ U_C = 0$

Thus $K_B + W_f = 0$

$\frac{1}{2}mv_B^2 - \mu_k mgs = 0$

$$\mu_k = \frac{\mu_B^2}{2gs} = \frac{(4.80\text{ m/s})^2}{2(9.80\text{ m/s}^2)(3.00\text{ m})} = 0.392$$

EVALUATE: The negative friction work takes away all the kinetic energy.

(b) IDENTIFY and **SET UP:** Apply conservation of energy to the motion from *A* to *B*:

$$K_A + U_A + W_{\text{other}} = K_B + U_B$$

EXECUTE: Work is done by gravity and by friction, so $W_{\text{other}} = W_f$.

$K_A = 0$, $K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(0.200\text{ kg})(4.80\text{ m/s})^2 = 2.304\text{ J}$

$U_A = mgy_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(1.60\text{ m}) = 3.136\text{ J}$, $U_B = 0$

Thus $U_A + W_f = K_B$

$W_f = K_B - U_A = 2.304\text{ J} - 3.136\text{ J} = -0.83\text{ J}$

EVALUATE: W_f is negative as expected; the friction force does negative work since it is directed opposite to the displacement.

7.66. **IDENTIFY:** Apply Eq.(7.14) to the initial and final positions of the truck.

SET UP: Let $y = 0$ at the lowest point of the path of the truck. W_{other} is the work done by friction. $f_r = \mu_r n = \mu_r mg\cos\beta$.

EXECUTE: Denote the distance the truck moves up the ramp by *x*. $K_1 = \frac{1}{2}mv_0^2$, $U_1 = mgL\sin\alpha$, $K_2 = 0$, $U_2 = mgx\sin\beta$ and $W_{\text{other}} = -\mu_r mgx\cos\beta$. From $W_{\text{other}} = (K_2 + U_2) - (K_1 + U_1)$, and solving for *x*,

$$x = \frac{K_1 + mgL\sin\alpha}{mg(\sin\beta + \mu_r\cos\beta)} = \frac{(v_0^2/2g) + L\sin\alpha}{\sin\beta + \mu_r\cos\beta}.$$

EVALUATE: *x* increases when v_0 increases and decreases when μ_r increases.

7.67. $F_x = -\alpha x - \beta x^2$, $\alpha = 60.0\text{ N/m}$ and $\beta = 18.0\text{ N/m}^2$

(a) IDENTIFY: Use Eq.(6.7) to calculate *W* and then use $W = -\Delta U$ to identify the potential energy function $U(x)$.

SET UP: $W_{F_x} = U_1 - U_2 = \int_{x_1}^{x_2} F_x(x)\,dx$

Let $x_1 = 0$ and $U_1 = 0$. Let x_2 be some arbitrary point *x*, so $U_2 = U(x)$.

EXECUTE: $U(x) = -\int_0^x F_x(x)\,dx = -\int_0^x (-\alpha x - \beta x^2)\,dx = \int_0^x (\alpha x + \beta x^2)\,dx = \frac{1}{2}\alpha x^2 + \frac{1}{3}\beta x^3$.

EVALUATE: If $\beta = 0$, the spring does obey Hooke's law, with $k = \alpha$, and our result reduces to $\frac{1}{2}kx^2$.

(b) IDENTIFY: Apply Eq.(7.15) to the motion of the object.

SET UP: The system at points 1 and 2 is sketched in Figure 7.67.

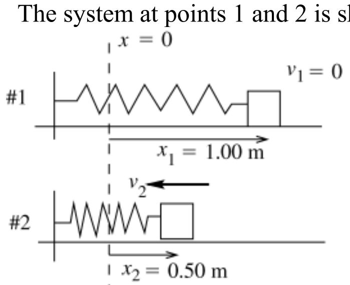


Figure 7.67

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

The only force that does work on the object is the spring force, so $W_{\text{other}} = 0$.

EXECUTE: $K_1 = 0$, $K_2 = \frac{1}{2}mv_2^2$

$U_1 = U(x_1) = \frac{1}{2}\alpha x_1^2 + \frac{1}{3}\beta x_1^3 = \frac{1}{2}(60.0\text{ N/m})(1.00\text{ m})^2 + \frac{1}{3}(18.0\text{ N/m}^2)(1.00\text{ m})^3 = 36.0\text{ J}$

$U_2 = U(x_2) = \frac{1}{2}\alpha x_2^2 + \frac{1}{3}\beta x_2^3 = \frac{1}{2}(60.0\text{ N/m})(0.500\text{ m})^2 + \frac{1}{3}(18.0\text{ N/m}^2)(0.500\text{ m})^3 = 8.25\text{ J}$

Thus $36.0\text{ J} = \frac{1}{2}mv_2^2 + 8.25\text{ J}$

$$v_2 = \sqrt{\frac{2(36.0\text{ J} - 8.25\text{ J})}{0.900\text{ kg}}} = 7.85\text{ m/s}$$

EVALUATE: The elastic potential energy stored in the spring decreases and the kinetic energy of the object increases.

7.68. **IDENTIFY:** Apply Eq.(7.14). W_{other} is the work done by F.

SET UP: $W_{other} = \Delta K + \Delta U$. The distance the spring stretches is $a\theta$. $y_2 - y_1 = a\sin\theta$.

EXECUTE: The force increases both the gravitational potential energy of the block and the potential energy of the spring. If the block is moved slowly, the kinetic energy can be taken as constant, so the work done by the force is the increase in potential energy, $\Delta U = mga\sin\theta + \frac{1}{2}k(a\theta)^2$.

EVALUATE: The force is kept tangent to the surface so the block will stay in contact with the surface.

7.69. **IDENTIFY:** Apply Eq.(7.14) to the motion of the block.

SET UP: Let $y = 0$ at the floor. Let point 1 be the initial position of the block against the compressed spring and let point 2 be just before the block strikes the floor.

EXECUTE: With $U_2 = 0, K_1 = 0$, $K_2 = U_1$. $\frac{1}{2}mv_2^2 = \frac{1}{2}kx^2 + mgh$. Solving for v_2 ,

$$v_2 = \sqrt{\frac{kx^2}{m} + 2gh} = \sqrt{\frac{(1900\text{ N/m})(0.045\text{ m})^2}{(0.150\text{ kg})} + 2(9.80\text{ m/s}^2)(1.20\text{ m})} = 7.01\text{ m/s}.$$

EVALUATE: The potential energy stored in the spring and the initial gravitational potential energy all go into the final kinetic energy of the block.

7.70. **IDENTIFY:** Apply Eq.(7.14). U is the total elastic potential energy of the two springs.

SET UP: Call the two points in the motion where Eq.(7.14) is applied A and B to avoid confusion with springs 1 and 2, that have force constants k_1 and k_2 . At any point in the motion the distance one spring is stretched equals the distance the other spring is compressed. Let $+x$ be to the right. Let point A be the initial position of the block, where it is released from rest, so $x_{1A} = +0.150$ m and $x_{2A} = -0.150$ m .

EXECUTE: **(a)** With no friction, $W_{other} = 0$. $K_A = 0$ and $U_A = K_B + U_B$. The maximum speed is when $U_B = 0$ and this is at $x_{1B} = x_{2B} = 0$, when both springs are at their natural length. $\frac{1}{2}k_1x_{1A}^2 + \frac{1}{2}k_2x_{2A}^2 = \frac{1}{2}mv_B^2$.

$$x_{1A}^2 = x_{2A}^2 = (0.150\text{ m})^2\text{, so } v_B = \sqrt{\frac{k_1 + k_2}{m}}(0.150\text{ m}) = \sqrt{\frac{2500\text{ N/m} + 2000\text{ N/m}}{3.00\text{ kg}}}(0.150\text{ m}) = 5.81\text{ m/s}.$$

(b) At maximum compression of spring 1, spring 2 has its maximum extension and $v_B = 0$. Therefore, at this point $U_A = U_B$. The distance spring 1 is compressed equals the distance spring 2 is stretched, and vice versa: $x_{1A} = -x_{2A}$ and $x_{1B} = -x_{2B}$. Then $U_A = U_B$ gives $\frac{1}{2}(k_1 + k_2)x_{1A}^2 = \frac{1}{2}(k_1 + k_2)x_{1B}^2$ and $x_{1B} = -x_{1A} = -0.150$ m . The maximum compression of spring 1 is 15.0 cm.

EVALUATE: When friction is not present mechanical energy is conserved and is continually transformed between kinetic energy of the block and potential energy in the springs. If friction is present, its work removes mechanical energy from the system.

7.71. **IDENTIFY:** Apply conservation of energy to relate x and h. Apply $\sum \vec{F} = m\vec{a}$ to relate a and x.

SET UP: The first condition, that the maximum height above the release point is h, is expressed as $\frac{1}{2}kx^2 = mgh$. The magnitude of the acceleration is largest when the spring is compressed to a distance x; at this point the net upward force is $kx - mg = ma$, so the second condition is expressed as $x = (m/k)(g + a)$.

EXECUTE: **(a)** Substituting the second expression into the first gives

$$\frac{1}{2}k\left(\frac{m}{k}\right)^2(g + a)^2 = mgh, \quad\text{or}\quad k = \frac{m(g + a)^2}{2gh}.$$

(b) Substituting this into the expression for x gives $x = \dfrac{2gh}{g + a}$.

EVALUATE: When $a \to 0$, our results become $k = \dfrac{mg}{2h}$ and $x = 2h$. The initial spring force is $kx = mg$ and the net upward force approaches zero. But $\frac{1}{2}kx^2 = mgh$ and sufficient potential energy is stored in the spring to move the mass to height h.

7.72. **IDENTIFY:** At equilibrium the upward spring force equals the weight mg of the object. Apply conservation of energy to the motion of the fish.

SET UP: The distance that the mass descends equals the distance the spring is stretched. $K_1 = K_2 = 0$, so $U_1(\text{gravitational}) = U_2(\text{spring})$

EXECUTE: Following the hint, the force constant k is found from $mg = kd$, or $k = mg/d$. When the fish falls from rest, its gravitational potential energy decreases by mgy; this becomes the potential energy of the spring, which is $\frac{1}{2}ky^2 = \frac{1}{2}(mg/d)y^2$. Equating these, $\dfrac{1}{2}\dfrac{mg}{d}y^2 = mgy$, or $y = 2d$.

EVALUATE: At its lowest point the fish is not in equilibrium. The upward spring force at this point is $ky = 2kd$, and this is equal to twice the weight. At this point the net force is *mg*, upward, and the fish has an upward acceleration equal to *g*.

7.73. **IDENTIFY:** Apply Eq.(7.15) to the motion of the block.

SET UP: The motion from *A* to *B* is described in Figure 7.73.

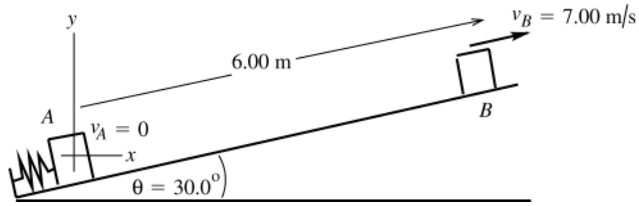


Figure 7.73

The normal force is $n = mg\cos\theta$, so $f_k = \mu_k n = \mu_k mg\cos\theta$.

$y_A = 0;\ \ y_B = (60.0\text{ m})\sin 30.0° = 3.00\text{ m}$

$K_A + U_A + W_{\text{other}} = K_B + U_B$

EXECUTE: Work is done by gravity, by the spring force, and by friction, so $W_{\text{other}} = W_f$ and $U = U_{\text{el}} + U_{\text{grav}}$

$K_A = 0,\ \ K_B = \frac{1}{2}mv_B^2 = \frac{1}{2}(1.50\text{ kg})(7.00\text{ m/s})^2 = 36.75\text{ J}$

$U_A = U_{\text{el},A} + U_{\text{grav},A} = U_{\text{el},A}$, since $U_{\text{grav},A} = 0$

$U_B = U_{\text{el},B} + U_{\text{grav},B} = 0 + mgy_B = (1.50\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m}) = 44.1\text{ J}$

$W_{\text{other}} = W_f = (f_k\cos\phi)s = \mu_k mg\cos\theta(\cos 180°)s = -\mu_k mg\cos\theta s$

$W_{\text{other}} = -(0.50)(1.50\text{ kg})(9.80\text{ m/s}^2)(\cos 30.0°)(6.00\text{ m}) = -38.19\text{ J}$

Thus $U_{\text{el},A} - 38.19\text{ J} = 36.75\text{ J} + 44.10\text{ J}$

$U_{\text{el},A} = 38.19\text{ J} + 36.75\text{ J} + 44.10\text{ J} = 119\text{ J}$

EVALUATE: U_{el} must always be positive. Part of the energy initially stored in the spring was taken away by friction work; the rest went partly into kinetic energy and partly into an increase in gravitational potential energy.

7.74. **IDENTIFY:** Apply Eq.(7.14) to the motion of the package. $W_{\text{other}} = W_{f_k}$, the work done by the kinetic friction force.

SET UP: $f_k = \mu_k n = \mu_k mg\cos\theta$, with $\theta = 53.1°$. Let $L = 4.00\text{ m}$, the distance the package moves before reaching the spring and let *d* be the maximum compression of the spring. Let point 1 be the initial position of the package, point 2 be just as it contacts the spring, point 3 be at the maximum compression of the spring, and point 4 be the final position of the package after it rebounds.

EXECUTE: **(a)** $K_1 = 0$, $U_2 = 0$, $W_{\text{other}} = -f_k L = -\mu_k L\cos\theta$. $U_1 = mgL\sin\theta$. $K_2 = \frac{1}{2}mv^2$, where *v* is the speed before the block hits the spring. Eq.(7.14) applied to points 1 and 2, with $y_2 = 0$, gives $U_1 + W_{\text{other}} = K_2$. Solving for *v*,

$v = \sqrt{2gL(\sin\theta - \mu_k\cos\theta)} = \sqrt{2(9.80\text{ m/s}^2)(4.00\text{ m})(\sin 53.1° - (0.20)\cos 53.1°)} = 7.30\text{ m/s}.$

(b) Apply Eq.(7.14) to points 1 and 3. Let $y_3 = 0$. $K_1 = K_3 = 0$. $U_1 = mg(L+d)\sin\theta$. $U_2 = \frac{1}{2}kd^2$.

$W_{\text{other}} = -f_k(L+d)$. Eq.(7.14) gives $mg(L+d)\sin\theta - \mu_k mg\cos\theta(L+d) = \frac{1}{2}kd^2$. This can be written as

$d^2\dfrac{k}{2mg(\sin\theta - \mu_k\cos\theta)} - d - L = 0.$ The factor multiplying d^2 is 4.504 m^{-1}, and use of the quadratic formula gives $d = 1.06\text{ m}$.

(c) The easy thing to do here is to recognize that the presence of the spring determines *d*, but at the end of the motion the spring has no potential energy, and the distance below the starting point is determined solely by how much energy has been lost to friction. If the block ends up a distance *y* below the starting point, then the block has moved a distance $L + d$ down the incline and $L + d - y$ up the incline. The magnitude of the friction force is the same in both directions, $\mu_k mg\cos\theta$, and so the work done by friction is $-\mu_k(2L + 2d - y)mg\cos\theta$. This must be equal to the change in gravitational potential energy, which is $-mgy\sin\theta$. Equating these and solving for *y* gives

$y = (L+d)\dfrac{2\mu_k\cos\theta}{\sin\theta + \mu_k\cos\theta} = (L+d)\dfrac{2\mu_k}{\tan\theta + \mu_k}.$ Using the value of *d* found in part (b) and the given values for μ_k and θ gives $y = 1.32\text{ m}$.

EVALUATE: Our expression for y gives the reasonable results that $y = 0$ when $\mu_k = 0$; in the absence of friction the package returns to its starting point.

7.75. **(a) IDENTIFY** and **SET UP:** Apply $K_A + U_A + W_{\text{other}} = K_B + U_B$ to the motion from A to B.

EXECUTE: $K_A = 0,\ K_B = \frac{1}{2}mv_B^2$

$U_A = 0,\ U_B = U_{\text{el},B} = \frac{1}{2}kx_B^2$, where $x_B = 0.25\text{ m}$

$W_{\text{other}} = W_F = Fx_B$

Thus $Fx_B = \frac{1}{2}mv_B^2 + \frac{1}{2}kx_B^2$. (The work done by F goes partly to the potential energy of the stretched spring and partly to the kinetic energy of the block.)

$Fx_B = (20.0\text{ N})(0.25\text{ m}) = 5.0\text{ J}$ and $\frac{1}{2}kx_B^2 = \frac{1}{2}(40.0\text{ N/m})(0.25\text{ m})^2 = 1.25\text{ J}$

Thus $5.0\text{ J} = \frac{1}{2}mv_B^2 + 1.25\text{ J}$ and $v_B = \sqrt{\dfrac{2(3.75\text{ J})}{0.500\text{ kg}}} = 3.87\text{ m/s}$

(b) IDENTIFY: Apply Eq.(7.15) to the motion of the block. Let point C be where the block is closest to the wall. When the block is at point C the spring is compressed an amount $|x_C|$, so the block is $0.60\text{ m} - |x_C|$ from the wall, and the distance between B and C is $x_B + |x_C|$.

SET UP: The motion from A to B to C is described in Figure 7.75.

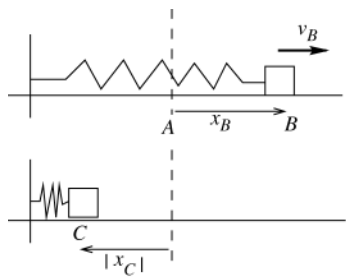


Figure 7.75

$K_B + U_B + W_{\text{other}} = K_C + U_C$

EXECUTE: $W_{\text{other}} = 0$

$K_B = \frac{1}{2}mv_B^2 = 5.0\text{ J} - 1.25\text{ J} = 3.75\text{ J}$ (from part (a))

$U_B = \frac{1}{2}kx_B^2 = 1.25\text{ J}$

$K_C = 0$ (instantaneously at rest at point closest to wall)

$U_C = \frac{1}{2}k|x_C|^2$

Thus $3.75\text{ J} + 1.25\text{ J} = \frac{1}{2}k|x_C|^2$

$|x_C| = \sqrt{\dfrac{2(5.0\text{ J})}{40.0\text{ N/m}}} = 0.50\text{ m}$

The distance of the block from the wall is $0.60\text{ m} - 0.50\text{ m} = 0.10\text{ m}$.

EVALUATE: The work $(20.0\text{ N})(0.25\text{ m}) = 5.0\text{ J}$ done by F puts 5.0 J of mechanical energy into the system. No mechanical energy is taken away by friction, so the total energy at points B and C is 5.0 J.

7.76. **IDENTIFY:** Apply Eq.(7.14) to the motion of the student.

SET UP: Let $x_0 = 0.18\text{ m}$, $x_1 = 0.71\text{ m}$. The spring constants (assumed identical) are then known in terms of the unknown weight w, $4kx_0 = w$. Let $y = 0$ at the initial position of the student.

EXECUTE: **(a)** The speed of the brother at a given height h above the point of maximum compression is then found from $\frac{1}{2}(4k)x_1^2 = \frac{1}{2}\left(\frac{w}{g}\right)v^2 + mgh$, or $v^2 = \frac{(4k)g}{w}x_1^2 - 2gh = g\left(\frac{x_1^2}{x_0} - 2h\right)$. Therefore,

$v = \sqrt{(9.80\text{ m/s}^2)((0.71\text{ m})^2/(0.18\text{ m}) - 2(0.90\text{ m}))} = 3.13\text{ m/s}$, or 3.1 m/s to two figures.

(b) Setting $v = 0$ and solving for h, $h = \frac{2kx_1^2}{mg} = \frac{x_1^2}{2x_0} = 1.40\text{ m}$, or 1.4 m to two figures.

(c) No; the distance x_0 will be different, and the ratio $\frac{x_1^2}{x_0} = \frac{(x_1 + 0.53\text{ m})^2}{x_1} = x_1\left(1 + \frac{0.53\text{ m}}{x_1}\right)^2$ will be different.

Note that on a planet with lower g, x_1 will be smaller and h will be larger.

EVALUATE: We are able to solve the problem without knowing either the mass of the student or the force constant of the spring.

7.77. IDENTIFY: $a_x = d^2x/dt^2$, $a_y = d^2y/dt^2$. $F_x = ma_x$, $F_y = ma_y$. $U = \int F_x dx + \int F_y dy$.

SET UP: $\dfrac{d}{dt}(\cos\omega_0 t) = -\omega_0 \sin\omega_0 t$. $\dfrac{d}{dt}(\sin\omega_0 t) = \omega_0 \cos\omega_0 t$. $\int \cos\omega_0 t\; dt = \dfrac{1}{\omega_0}\sin\omega_0 t$, $\int \sin\omega_0 t\; dt = -\dfrac{1}{\omega_0}\cos\omega_0 t$. $v_x = dx/dt$, $v_y = dy/dt$. $E = K + U$.

EXECUTE: (a) $a_x = d^2x/dt^2 = -\omega_0^2 x,\ F_x = ma_x = -m\omega_0^2 x.\ \ a_y = d^2y/dt^2 = -\omega_0^2 y,\ \ F_y = ma_y = -m\omega_0^2 y$

(b) $U = -\left[\int F_x dx + \int F_y dy\right] = m\omega_0^2\left[\int x dx + \int y dy\right] = \dfrac{1}{2}m\omega_0^2(x^2 + y^2)$

(c) $v_x = dx/dt = -x_0\omega_0 \sin\omega_0 t = -x_0\omega_0(y/y_0)$. $v_y = dy/dt = +y_0\omega_0\cos\omega_0 t = +y_0\omega_0(x/x_0)$.

(i) When $x = x_0$ and $y = 0$, $v_x = 0$ and $v_y = y_0\omega_0$,

$$K = \frac{1}{2}m(v_x^2 + v_y^2) = \frac{1}{2}my_0^2\omega_0^2,\ U = \frac{1}{2}\omega_0^2 m x_0^2 \text{ and } E = K + U = \frac{1}{2}m\omega_0^2(x_0^2 + y_0^2)$$

(ii) When $x = 0$ and $y = y_0$, $v_x = -x_0\omega_0$ and $v_y = 0$,

$$K = \frac{1}{2}\omega_0^2 m x_0^2,\ U = \frac{1}{2}m\omega_0^2 y_0^2 \text{ and } E = K + U = \frac{1}{2}m\omega_0^2(x_0^2 + y_0^2)$$

EVALUATE: The total energy is the same at the two points in part (c); the total energy of the system is constant.

7.78. IDENTIFY: Calculate the increase in kinetic energy for the car.

SET UP: The car gets $(0.15)(1.3\times10^8\ \text{J})$ of energy from one gallon of gasoline.

EXECUTE: (a) The mechanical energy increase of the car is $K_2 - K_1 = \frac{1}{2}(1500\ \text{kg})(37\ \text{m/s})^2 = 1.027\times10^6\ \text{J}$. Let α be the number of gallons of gasoline consumed. $\alpha(1.3\times10^8\ \text{J})(0.15) = 1.027\times10^6\ \text{J}$ and $\alpha = 0.053$ gallons.

(b) $(1.00\ \text{gallons})/\alpha = 19$ accelerations

EVALUATE: The time over which the increase in velocity occurs doesn't enter into the calculation.

7.79. IDENTIFY: $U = mgh$. Use $h = 150$ m for all the water that passes through the dam.

SET UP: $m = \rho V$ and $V = A\Delta h$ is the volume of water in a height Δh of water in the lake.

EXECUTE: (a) Stored energy $= mgh = (\rho V)gh = \rho A(1\ \text{m})gh$.

stored energy $= (1000\ \text{kg/m}^3)(3.0\times10^6\ \text{m}^2)(1\ \text{m})(9.8\ \text{m/s}^2)(150\ \text{m}) = 4.4\times10^{12}$ J.

(b) 90% of the stored energy is converted to electrical energy, so $(0.90)(mgh) = 1000$ kWh.

$(0.90)\rho Vgh = 1000$ kWh. $V = \dfrac{(1000\ \text{kWh})((3600\ \text{s})/(1\ \text{h}))}{(0.90)(1000\ \text{kg/m}^3)(150\ \text{m})(9.8\ \text{m/s}^2)} = 2.7\times10^3\ \text{m}^3$.

Change in level of the lake: $A\Delta h = V_{\text{water}}$. $\Delta h = \dfrac{V}{A} = \dfrac{2.7\times10^3\ \text{m}^3}{3.0\times10^6\ \text{m}^2} = 9.0\times10^{-4}$ m.

EVALUATE: Δh is much less than 150 m, so using $h = 150$ m for all the water that passed through the dam was a very good approximation.

7.80. IDENTIFY and SET UP: The potential energy of a horizontal layer of thickness *dy*, area *A*, and height *y* is $dU = (dm)gy$. Let ρ be the density of water.

EXECUTE: $dm = \rho\, dV = \rho A\, dy$, so $dU = \rho Agy\, dy$.

The total potential energy *U* is

$U = \int_0^h dU = \rho Ag\int_0^h y\, dy = \frac{1}{2}\rho Agh^2.$

$A = 3.0\times10^6\ \text{m}^2$ and $h = 150$ m, so $U = 3.3\times10^{14}\ \text{J} = 9.2\times10^7$ kWh

EVALUATE: The volume is Ah and the mass of water is $\rho V = \rho Ah$. The average depth is $h_{\text{av}} = h/2$, so $U = mgh_{\text{av}}$.

7.81. IDENTIFY: Apply $F_x = -\dfrac{\partial U}{\partial x}$, $F_y = -\dfrac{\partial U}{\partial y}$ and $F_z = -\dfrac{\partial U}{\partial z}$.

SET UP: $r = (x^2 + y^2 + z^2)^{1/2}$. $\dfrac{\partial(1/r)}{\partial x} = -\dfrac{x}{(x^2 + y^2)^{3/2}}$, $\dfrac{\partial(1/r)}{\partial y} = -\dfrac{y}{(x^2 + y^2)^{3/2}}$ and $\dfrac{\partial(1/r)}{\partial z} = -\dfrac{z}{(x^2 + y^2)^{3/2}}$.

EXECUTE: (a) $U(r) = -\frac{Gm_1m_2}{r}$. $F_x = -\frac{\partial U}{\partial x} = +Gm_1m_2\left[\frac{\partial(1/r)}{\partial x}\right] = -\frac{Gm_1m_2x}{(x^2+y^2+z^2)^{3/2}}$. Similarly,

$F_y = -\frac{Gm_1m_2y}{(x^2+y^2+z^2)^{3/2}}$ and $F_z = -\frac{Gm_1m_2z}{(x^2+y^2+z^2)^{3/2}}$.

(b) $(x^2+y^2+z^2)^{3/2} = r^3$ so $F_x = -\frac{Gm_1m_2x}{r^3}$, $F_y = -\frac{Gm_1m_2y}{r^3}$ and $F_z = -\frac{Gm_1m_2z}{r^3}$.

$F = \sqrt{F_x^2+F_y^2+F_z^2} = \frac{Gm_1m_2}{r^3}\sqrt{x^2+y^2+z^2} = \frac{Gm_1m_2}{r^2}$.

(c) F_x, F_y and F_z are negative. $F_x = \alpha x$, $F_y = \alpha y$ and $F_z = \alpha z$, where α is a constant, so $\vec{F}$ and the vector $\vec{r}$ from m_1 to m_2 are in the same direction. Therefore, $\vec{F}$ is directed toward m_1 at the origin and $\vec{F}$ is attractive.

EVALUATE: When m_2 moves to larger *r*, the work done on it by the attractive gravity force is negative. Since $W = -\Delta U$, negative work done by gravity means the gravitational potential energy increases.

$U(r) = -\frac{Gm_1m_2}{r}$ does increase (becomes less negative) as *r* increases. For an object near the surface of the earth,

$U(r) = -\frac{Gm_1m_2}{r}$ will be shown in Chapter 12 to be equivalent to $U_{\text{grav}} = mgy$.

7.82. IDENTIFY: Calculate the work *W* done by this force. If the force is conservative, the work is path independent.

SET UP: $W = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{l}$.

EXECUTE: (a) $W = \int_{P_1}^{P_2} F_y dy = C\int_{P_1}^{P_2} y^2 dy$. *W* doesn't depend on *x*, so it is the same for all paths between P_1 and P_2. The force is conservative.

(b) $W = \int_{P_1}^{P_2} F_x dx = C\int_{P_1}^{P_2} y^2 dx$. *W* will be different for paths between points P_1 and P_2 for which *y* has different values. For example, if *y* has the constant value y_0 along the path, then $W = Cy_0(x_2 - x_1)$. *W* depends on the value of y_0. The force is not conservative.

EVALUATE: $\vec{F} = Cy^2\hat{j}$ has the potential energy function $U(y) = -\frac{Cy^3}{3}$. We cannot find a potential energy function for $\vec{F} = Cy^2\hat{i}$.

7.83. $\vec{F} = -\alpha xy^2\hat{j}$, $\alpha = 2.50\ \text{N/m}^3$

IDENTIFY: $\vec{F}$ is not constant so use Eq.(6.14) to calculate *W*. $\vec{F}$ must be evaluated along the path.

(a) SET UP: The path is sketched in Figure 7.83a.

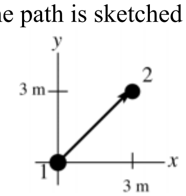


Figure 7.83a

$d\vec{l} = dx\hat{i} + dy\hat{j}$

$\vec{F}\cdot d\vec{l} = -\alpha xy^2\ dy$

On the path, $x = y$ so $\vec{F}\cdot d\vec{l} = -\alpha y^3\ dy$

EXECUTE: $W = \int_1^2 \vec{F}\cdot d\vec{l} = \int_{y_1}^{y_2}(-\alpha y^3)\ dy = -(\alpha/4)\left(y^4\Big|_{y_1}^{y_2}\right) = -(\alpha/4)(y_2^4 - y_1^4)$

$y_1 = 0$, $y_2 = 3.00$ m, so $W = -\frac{1}{4}(2.50\ \text{N/m}^3)(3.00\ \text{m})^4 = -50.6\ \text{J}$

(b) SET UP: The path is sketched in Figure 7.83b.

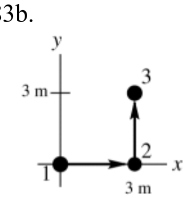


Figure 7.83b

For the displacement from point 1 to point 2, $d\vec{l} = dx\hat{i}$, so $\vec{F}\cdot d\vec{l} = 0$ and $W = 0$. (The force is perpendicular to the displacement at each point along the path, so $W = 0$.)

For the displacement from point 2 to point 3, $d\vec{l} = dy\hat{j}$, so $\vec{F}\cdot d\vec{l} = -\alpha xy^2\ dy$. On this path, $x = 3.00$ m, so

$$\vec{F}\cdot d\vec{l} = -(2.50\ \text{N/m}^3)(3.00\ \text{m})y^2\ dy = -(7.50\ \text{N/m}^2)y^2\ dy.$$

EXECUTE: $W = \int_2^3 \vec{F}\cdot d\vec{l} = -(7.50\ \text{N/m}^2)\int_{y_2}^{y_3} y^2\ dy = -(7.50\ \text{N/m}^2)\frac{1}{3}(y_3^3 - y_2^3)$

$W = -(7.50\ \text{N/m}^2)\left(\frac{1}{3}\right)(3.00\ \text{m})^3 = -67.5$ J

(c) EVALUATE: For these two paths between the same starting and ending points the work is different, so the force is nonconservative.

7.84. **IDENTIFY:** Use $W = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{l}$ to calculate W for each segment of the path.

SET UP: $\vec{F}\cdot d\vec{l} = F_x dx = \alpha xy\ dx$

EXECUTE: **(a)** The path is sketched in Figure 7.84.

(b) (1): $x = 0$ along this leg, so $\vec{F} = 0$ and $W = 0$. (2)**:** Along this leg, $y = 1.50$ m, so $\vec{F}\cdot d\vec{l} = (3.00\ \text{N/m})x dx$, and $W = (1.50\ \text{N/m})((1.50\ \text{m})^2 - 0) = 3.38$ J (3) $\vec{F}\cdot d\vec{l} = 0$, so $W = 0$ (4) $y = 0$, so $\vec{F} = 0$ and $W = 0$. The work done in moving around the closed path is 3.38 J.

(c) The work done in moving around a closed path is not zero, and the force is not conservative.

EVALUATE: There is no potential energy function for this force.

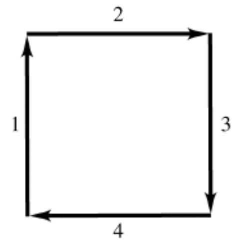


Figure 7.84

7.85. **IDENTIFY:** Use Eq.(7.16) to relate F_x and $U(x)$. The equilibrium is stable where $U(x)$ is a local minimum and the equilibrium is unstable where $U(x)$ is a local maximum.

SET UP: The maximum and minimum values of x are those for which $U(x) = E$. $K = E - U$, so the maximum speed is where U is a minimum.

EXECUTE: **(a)** For the given proposed potential $U(x), -\dfrac{dU}{dx} = -kx + F$, so this is a possible potential function. For this potential, $U(0) = -F^2/2k$, not zero. Setting the zero of potential is equivalent to adding a constant to the potential; any additive constant will not change the derivative, and will correspond to the same force.

(b) At equilibrium, the force is zero; solving $-kx + F = 0$ for x gives $x_0 = F/k$. $U(x_0) = -F^2/k$, and this is a minimum of U, and hence a stable point.

(c) The graph is given in Figure 7.85.

(d) No; $F_{\text{tot}} = 0$ at only one point, and this is a stable point.

(e) The extreme values of x correspond to zero velocity, hence zero kinetic energy, so $U(x_\pm) = E$, where $x_\pm$ are the extreme points of the motion. Rather than solve a quadratic, note that $\frac{1}{2}k(x - F/k)^2 - F^2/k$, so $U(x_\pm) = E$ becomes $\dfrac{1}{2}k\left(x_\pm - \dfrac{F}{k}\right)^2 - F^2/k = \dfrac{F^2}{k}$. $x_\pm - \dfrac{F}{k} = \pm 2\dfrac{F}{k}$, so $x_+ = 3\dfrac{F}{k}$ $x_- = -\dfrac{F}{k}$.

(f) The maximum kinetic energy occurs when $U(x)$ is a minimum, the point $x_0 = F/k$ found in part (b). At this point $K = E - U = (F^2/k) - (-F^2/k) = 2F^2/k$, so $v = 2F/\sqrt{mk}$.

EVALUATE: As E increases, the magnitudes of x_+ and x_- increase. The particle cannot reach values of x for which $E < U(x)$ because K cannot be negative.

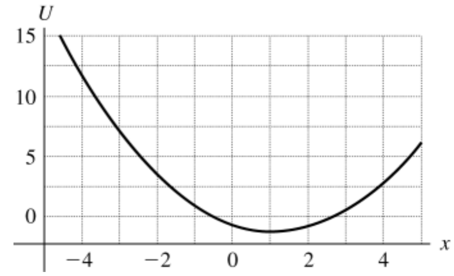


Figure 7.85

7.86. **IDENTIFY:** Use Eq.(7.16) to relate F_x and $U(x)$. The equilibrium is stable where $U(x)$ is a local minimum and the equilibrium is unstable where $U(x)$ is a local maximum.

SET UP: dU/dx is the slope of the graph of U versus x. $K = E - U$, so K is a maximum when U is a minimum. The maximum x is where $E = U$.

EXECUTE: **(a)** The slope of the U *vs.* x curve is negative at point A, so F_x is positive (Eq. (7.16)).

(b) The slope of the curve at point B is positive, so the force is negative.

(c) The kinetic energy is a maximum when the potential energy is a minimum, and that figures to be at around 0.75 m.

(d) The curve at point C looks pretty close to flat, so the force is zero.

(e) The object had zero kinetic energy at point A, and in order to reach a point with more potential energy than $U(A)$, the kinetic energy would need to be negative. Kinetic energy is never negative, so the object can never be at any point where the potential energy is larger than $U(A)$. On the graph, that looks to be at about 2.2 m.

(f) The point of minimum potential (found in part (c)) is a stable point, as is the relative minimum near 1.9 m.

(g) The only potential maximum, and hence the only point of unstable equilibrium, is at point C.

EVALUATE: If E is less than U at point C, the particle is trapped in one or the other of the potential "wells" and cannot move from one allowed region of x to the other.

7.87. **IDENTIFY:** $K = E - U$ determines $v(x)$.

SET UP: v is a maximum when U is a minimum and v is a minimum when U is a maximum. $F_x = -dU/dx$. The extreme values of x are where $E = U(x)$.

EXECUTE: **(a)** Eliminating β in favor of α and $x_0 (\beta = \alpha/x_0)$,

$$U(x) = \frac{\alpha}{x^2} - \frac{\beta}{x} = \frac{\alpha}{x_0^2}\frac{x_0^2}{x^2} - \frac{\alpha}{x_0 x} = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \left(\frac{x_0}{x}\right)\right].$$

$U(x_0) = \left(\frac{\alpha}{x_0^2}\right)(1-1) = 0$. $U(x)$ is positive for $x < x_0$ and negative for $x > x_0$ (α and β must be taken as positive). The graph of $U(x)$ is sketched in Figure 7.87a.

(b) $v(x) = \sqrt{-\frac{2}{m}U} = \sqrt{\left(\frac{2\alpha}{mx_0^2}\right)\left(\left(\frac{x_0}{x}\right) - \left(\frac{x_0}{x}\right)^2\right)}$. The proton moves in the positive x-direction, speeding up until it reaches a maximum speed (see part (c)), and then slows down, although it never stops. The minus sign in the square root in the expression for $v(x)$ indicates that the particle will be found only in the region where $U < 0$, that is, $x > x_0$. The graph of $v(x)$ is sketched in Figure 7.87b.

(c) The maximum speed corresponds to the maximum kinetic energy, and hence the minimum potential energy. This minimum occurs when $\frac{dU}{dx} = 0$, or $\frac{dU}{dx} = \frac{\alpha}{x_0} 3\left[-2\left(\frac{x_0}{x}\right)^3 + \left(\frac{x_0}{x}\right)^2\right] = 0$,

which has the solution $x = 2x_0$. $U(2x_0) = -\frac{\alpha}{4x_0^2}$, so $v = \sqrt{\frac{\alpha}{2mx_0^2}}$.

(d) The maximum speed occurs at a point where $\frac{dU}{dx} = 0$, and from Eq. (7.15), the force at this point is zero.

(e) $x_1 = 3x_0$, and $U(3x_0) = -\dfrac{2\alpha}{9x_0^2}$.

$$v(x) = \sqrt{\frac{2}{m}(U(x_1) - U(x))} = \sqrt{\frac{2}{m}\left[\left(\frac{-2}{9}\frac{\alpha}{x_0^2}\right) - \frac{\alpha}{x_0^2}\left(\left(\frac{x_0}{x}\right)^2 - \frac{x_0}{x}\right)\right]} = \sqrt{\frac{2\alpha}{mx_0^2}\left(\left(\frac{x_0}{x}\right) - \left(\frac{x_0}{x}\right)^2 - 2/9\right)}.$$

The particle is confined to the region where $U(x) < U(x_1)$. The maximum speed still occurs at $x = 2x_0$, but now the particle will oscillate between x_1 and some minimum value (see part (f)).

(f) Note that $U(x) - U(x_1)$ can be written as

$$\frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \left(\frac{x_0}{x}\right) + \left(\frac{2}{9}\right)\right] = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right) - \frac{1}{3}\right]\left[\left(\frac{x_0}{x}\right) - \frac{2}{3}\right],$$

which is zero (and hence the kinetic energy is zero) at $x = 3x_0 = x_1$ and $x = \frac{3}{2}x_0$. Thus, when the particle is released from x_0, it goes on to infinity, and doesn't reach any maximum distance. When released from x_1, it oscillates between $\frac{3}{2}x_0$ and $3x_0$.

EVALUATE: In each case the proton is released from rest and $E = U(x_i)$, where x_i is the point where it is released. When $x_i = x_0$ the total energy is zero. When $x_i = x_1$ the total energy is negative. $U(x) \to 0$ as $x \to \infty$, so for this case the proton can't reach $x \to \infty$ and the maximum x it can have is limited.

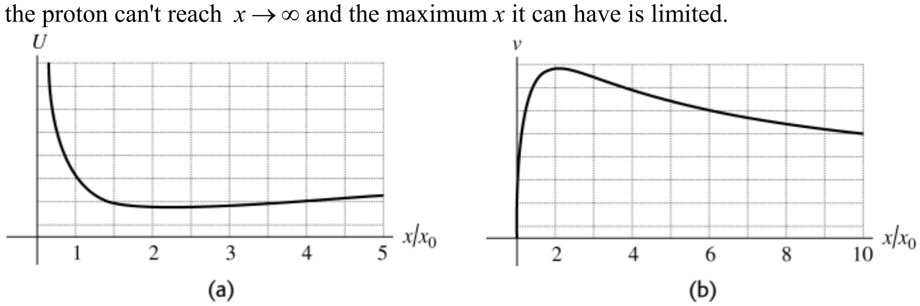


Figure 7.87

MOMENTUM, IMPULSE, AND COLLISIONS

8

8.1. **IDENTIFY** and **SET UP:** $p = mv$. $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** $p = (10{,}000\text{ kg})(12.0\text{ m/s}) = 1.20\times10^5\text{ kg}\cdot\text{m/s}$

(b) (i) $v = \dfrac{p}{m} = \dfrac{1.20\times10^5\text{ kg}\cdot\text{m/s}}{2000\text{ kg}} = 60.0\text{ m/s}$. (ii) $\frac{1}{2}m_\text{T}v_\text{T}^2 = \frac{1}{2}m_\text{SUV}v_\text{SUV}^2$, so

$$v_\text{SUV} = \sqrt{\frac{m_\text{T}}{m_\text{SUV}}}v_\text{T} = \sqrt{\frac{10{,}000\text{ kg}}{2000\text{ kg}}}(12.0\text{ m/s}) = 26.8\text{ m/s}$$

EVALUATE: The SUV must have less speed to have the same kinetic energy as the truck than to have the same momentum as the truck.

8.2. **IDENTIFY:** Example 8.1 shows that the two iceboats have the same kinetic energy at the finish line. $K = \frac{1}{2}mv^2$. $p = mv$.

SET UP: Let A be the iceboat with mass m and let B be the iceboat with mass $2m$, so $m_B = 2m_A$.

EXECUTE: $K_A = K_B$ gives $\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2$. $v_A = \sqrt{\dfrac{m_B}{m_A}}v_B = \sqrt{2}v_B$.

$$p_A = m_Av_A.\ \ p_B = m_Bv_B = (2m_A)\left(v_A/\sqrt{2}\right) = \sqrt{2}m_Av_A = \sqrt{2}p_A.$$

EVALUATE: The more massive boat must have less speed but greater momentum than the other boat in order to have the same kinetic energy.

8.3. **IDENTIFY** and **SET UP:** $p = mv$. $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** $v = \dfrac{p}{m}$ and $K = \frac{1}{2}m\left(\dfrac{p}{m}\right)^2 = \dfrac{p^2}{2m}$.

(b) $K_\text{c} = K_\text{b}$ and the result from part (a) gives $\dfrac{p_\text{c}^2}{2m_\text{c}} = \dfrac{p_\text{b}^2}{2m_\text{b}}$. $p_\text{b} = \sqrt{\dfrac{m_\text{b}}{m_\text{c}}}p_\text{c} = \sqrt{\dfrac{0.145\text{ kg}}{0.040\text{ kg}}}p_\text{c} = 1.90p_\text{c}$. The baseball has the greater magnitude of momentum. $p_\text{c}/p_\text{b} = 0.526$.

(c) $p^2 = 2mK$ so $p_\text{m} = p_\text{w}$ gives $2m_\text{m}K_\text{m} = 2m_\text{w}K_\text{w}$. $w = mg$, so $w_\text{m}K_\text{m} = w_\text{w}K_\text{w}$.

$$K_\text{w} = \left(\frac{w_\text{m}}{w_\text{w}}\right)K_\text{m} = \left(\frac{700\text{ N}}{450\text{ N}}\right)K_\text{m} = 1.56K_\text{m}.$$

The woman has greater kinetic energy. $K_\text{m}/K_\text{w} = 0.641$.

EVALUATE: For equal kinetic energy, the more massive object has the greater momentum. For equal momenta, the less massive object has the greater kinetic energy.

8.4. **IDENTIFY:** Each momentum component is the mass times the corresponding velocity component.

SET UP: Let $+x$ be along the horizontal motion of the shotput. Let $+y$ be vertically upward. $v_x = v\cos\theta$, $v_y = v\sin\theta$.

EXECUTE: The horizontal component of the initial momentum is

$$p_x = mv_x = mv\cos\theta = (7.30\text{ kg})(15.0\text{ m/s})\cos40.0^\circ = 83.9\text{ kg}\cdot\text{m/s}.$$

The vertical component of the initial momentum is $p_y = mv_y = mv\sin\theta = (7.30\text{ kg})(15.0\text{ m/s})\sin40.0^\circ = 70.4\text{ kg}\cdot\text{m/s}$

EVALUATE: The initial momentum is directed at 40.0° above the horizontal.

8.5. **IDENTIFY:** For each object, $\vec{p} = m\vec{v}$ and $K = \frac{1}{2}mv^2$. The total momentum is the vector sum of the momenta of each object. The total kinetic energy is the scalar sum of the kinetic energies of each object.

SET UP: Let object A be the 110 kg lineman and object B the 125 kg lineman. Let $+x$ be the object to the right, so $v_{Ax} = +2.75$ m/s and $v_{Bx} = -2.60$ m/s.

EXECUTE: **(a)** $P_x = m_A v_{Ax} + m_B v_{Bx} = (110\text{ kg})(2.75\text{ m/s}) + (125\text{ kg})(-2.60\text{ m/s}) = -22.5\text{ kg}\cdot\text{m/s}$. The net momentum has magnitude $22.5\text{ kg}\cdot\text{m/s}$ and is directed to the left.

(b) $K = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 = \frac{1}{2}(110\text{ kg})(2.75\text{ m/s})^2 + \frac{1}{2}(125\text{ kg})(2.60\text{ m/s})^2 = 838\text{ J}$

EVALUATE: The kinetic energy of an object is a scalar and is never negative. It depends only on the magnitude of the velocity of the object, not on its direction. The momentum of an object is a vector and has both magnitude and direction. When two objects are in motion, their total kinetic energy is greater than the kinetic energy of either one. But if they are moving in opposite directions, the net momentum of the system has a smaller magnitude than the magnitude of the momentum of either object.

8.6. **IDENTIFY:** For each object $\vec{p} = m\vec{v}$ and the net momentum of the system is $\vec{P} = \vec{p}_A + \vec{p}_B$. The momentum vectors are added by adding components. The magnitude and direction of the net momentum is calculated from its x and y components.

SET UP: Let object A be the pickup and object B be the sedan. $v_{Ax} = -14.0$ m/s, $v_{Ay} = 0$. $v_{Bx} = 0$, $v_{By} = +23.0$ m/s.

EXECUTE: **(a)** $P_x = p_{Ax} + p_{Bx} = m_A v_{Ax} + m_B v_{Bx} = (2500\text{ kg})(-14.0\text{ m/s}) + 0 = -3.50\times10^4\text{ kg}\cdot\text{m/s}$

$$P_y = p_{Ay} + p_{By} = m_A v_{Ay} + m_B v_{By} = (1500\text{ kg})(+23.0\text{ m/s}) = +3.45\times10^4\text{ kg}\cdot\text{m/s}$$

(b) $P = \sqrt{P_x^2 + P_y^2} = 4.91\times10^4\text{ kg}\cdot\text{m/s}$. From Figure 8.6, $\tan\theta = \dfrac{|P_x|}{|P_y|} = \dfrac{3.50\times10^4\text{ kg}\cdot\text{m/s}}{3.45\times10^4\text{ kg}\cdot\text{m/s}}$ and $\theta = 45.4°$. The net momentum has magnitude $4.91\times10^4\text{ kg}\cdot\text{m/s}$ and is directed at $45.4°$ west of north.

EVALUATE: The momenta of the two objects must be added as vectors. The momentum of one object is west and the other is north. The momenta of the two objects are nearly equal in magnitude, so the net momentum is directed approximately midway between west and north.

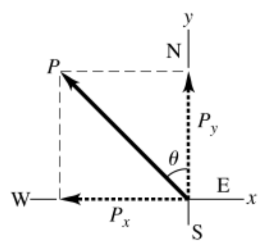


Figure 8.6

8.7. **IDENTIFY:** The average force on an object and the object's change in momentum are related by Eq. 8.9. The weight of the ball is $w = mg$.

SET UP: Let $+x$ be in the direction of the final velocity of the ball, so $v_{1x} = 0$ and $v_{2x} = 25.0$ m/s.

EXECUTE: $(F_{\text{av}})_x(t_2 - t_1) = mv_{2x} - mv_{1x}$ gives $(F_{\text{av}})_x = \dfrac{mv_{2x} - mv_{1x}}{t_2 - t_1} = \dfrac{(0.0450\text{ kg})(25.0\text{ m/s})}{2.00\times10^{-3}\text{ s}} = 562\text{ N}$.

$w = (0.0450\text{ kg})(9.80\text{ m/s}^2) = 0.441\text{ N}$. The force exerted by the club is much greater than the weight of the ball, so the effect of the weight of the ball during the time of contact is not significant.

EVALUATE: Forces exerted during collisions typically are very large but act for a short time.

8.8. **IDENTIFY:** The change in momentum, the impulse and the average force are related by Eq. 8.9.

SET UP: Let the direction in which the batted ball is traveling be the $+x$ direction, so $v_{1x} = -45.0$ m/s and $v_{2x} = 55.0$ m/s.

EXECUTE: **(a)** $\Delta p_x = p_{2x} - p_{1x} = m(v_{2x} - v_{1x}) = (0.145\text{ kg})(55.0\text{ m/s} - [-45.0\text{ m/s}]) = 14.5\text{ kg}\cdot\text{m/s}$. $J_x = \Delta p_x$, so $J_x = 14.5\text{ kg}\cdot\text{m/s}$. Both the change in momentum and the impulse have magnitude $14.5\text{ kg}\cdot\text{m/s}$.

(b) $(F_{\text{av}})_x = \dfrac{J_x}{\Delta t} = \dfrac{14.5\text{ kg}\cdot\text{m/s}}{2.00\times10^{-3}\text{ s}} = 7250\text{ N}$.

EVALUATE: The force is in the direction of the momentum change.

8.9. **IDENTIFY:** Use Eq. 8.9. We know the intial momentum and the impluse so can solve for the final momentum and then the final velocity.

SET UP: Take the x-axis to be toward the right, so $v_{1x} = +3.00\ \text{m/s}$. Use Eq. 8.5 to calculate the impulse, since the force is constant.

EXECUTE: **(a)** $J_x = p_{2x} - p_{1x}$

$$J_x = F_x(t_2 - t_1) = (+25.0\ \text{N})(0.050\ \text{s}) = +1.25\ \text{kg}\cdot\text{m/s}$$

Thus $p_{2x} = J_x + p_{1x} = +1.25\ \text{kg}\cdot\text{m/s} + (0.160\ \text{kg})(+3.00\ \text{m/s}) = +1.73\ \text{kg}\cdot\text{m/s}$

$$v_{2x} = \frac{p_{2x}}{m} = \frac{1.73\ \text{kg}\cdot\text{m/s}}{0.160\ \text{kg}} = +10.8\ \text{kg}\cdot\text{m/s (to the right)}$$

(b) $J_x = F_x(t_2 - t_1) = (-12.0\ \text{N})(0.050\ \text{s}) = -0.600\ \text{kg}\cdot\text{m/s}$ (negative since force is to left)

$$p_{2x} = J_x + p_{1x} = -0.600\ \text{kg}\cdot\text{m/s} + (0.160\ \text{kg})(+3.00\ \text{m/s}) = -0.120\ \text{kg}\cdot\text{m/s}$$

$$v_{2x} = \frac{p_{2x}}{m} = \frac{-0.120\ \text{kg}\cdot\text{m/s}}{0.160\ \text{kg}} = -0.75\ \text{m/s (to the left)}$$

EVALUATE: In part (a) the impulse and initial momentum are in the same direction and v_x increases. In part (b) the impulse and initial momentum are in opposite directions and the velocity decreases.

8.10. IDENTIFY: The impulse, change in momentum and change in velocity are related by Eq. 8.9.

SET UP: $F_y = 26{,}700\ \text{N}$ and $F_x = 0$. The force is constant, so $(F_{\text{av}})_y = F_y$.

EXECUTE: **(a)** $J_y = F_y\Delta t = (26{,}700\ \text{N})(3.90\ \text{s}) = 1.04\times10^5\ \text{N}\cdot\text{s}$.

(b) $\Delta p_y = J_y = 1.04\times10^5\ \text{kg}\cdot\text{m/s}$.

(c) $\Delta p_y = m\Delta v_y$. $\Delta v_y = \dfrac{\Delta p_y}{m} = \dfrac{1.04\times10^5\ \text{kg}\cdot\text{m/s}}{95{,}000\ \text{kg}} = 1.09\ \text{m/s}$.

(d) The initial velocity of the shuttle isn't known. The change in kinetic energy is $\Delta K = K_2 - K_1 = \frac{1}{2}m(v_2^2 - v_1^2)$. It depends on the initial and final speeds and isn't determined solely by the change in speed.

EVALUATE: The force in the $+y$ direction produces an increase of the velocity in the $+y$ direction.

8.11. IDENTIFY: The force is not constant so $\vec{J} = \int_{t_1}^{t_2} \vec{F}dt$. The impulse is related to the change in velocity by Eq. 8.9.

SET UP: Only the x component of the force is nonzero, so $J_x = \int_{t_1}^{t_2} F_x dt$ is the only nonzero component of $\vec{J}$.

$$J_x = m(v_{2x} - v_{1x}).\ t_1 = 2.00\ \text{s},\ t_2 = 3.50\ \text{s}.$$

EXECUTE: **(a)** $A = \dfrac{F_x}{t^2} = \dfrac{781.25\ \text{N}}{(1.25\ \text{s})^2} = 500\ \text{N/s}^2$.

(b) $J_x = \int_{t_1}^{t_2} At^2 dt = \frac{1}{3}A(t_2^3 - t_1^3) = \frac{1}{3}(500\ \text{N/s}^2)([3.50\ \text{s}]^3 - [2.00\ \text{s}]^3) = 5.81\times10^3\ \text{N}\cdot\text{s}$.

(c) $\Delta v_x = v_{2x} - v_{1x} = \dfrac{J_x}{m} = \dfrac{5.81\times10^3\ \text{N}\cdot\text{s}}{2150\ \text{kg}} = 2.70\ \text{m/s}$. The x component of the velocity of the rocket increases by 2.70 m/s.

EVALUATE: The change in velocity is in the same direction as the impulse, which in turn is in the direction of the net force. In this problem the net force equals the force applied by the engine, since that is the only force on the rocket.

8.12. IDENTIFY: Apply Eq. 8.9 to relate the change in momentum of the momentum to the components of the average force on it.

SET UP: Let $+x$ be to the right and $+y$ be upward.

EXECUTE: **(a)** $J_x = \Delta p_x = mv_{2x} - mv_{1x} = (0.145\ \text{kg})(-[65.0\ \text{m/s}]\cos30^\circ - 50.0\ \text{m/s}) = -15.4\ \text{kg}\cdot\text{m/s}$.

$$J_y = \Delta p_y = mv_{2y} - mv_{1y} = (0.145\ \text{kg})([65.0\ \text{m/s}]\sin30^\circ - 0) = 4.71\ \text{kg}\cdot\text{m/s}$$

The horizontal component is $15.4\ \text{kg}\cdot\text{m/s}$, to the left and the vertical component is $4.71\ \text{kg}\cdot\text{m/s}$, upward.

(b) $F_{\text{av-}x} = \dfrac{J_x}{\Delta t} = \dfrac{-15.4\ \text{kg}\cdot\text{m/s}}{1.75\times10^{-3}\ \text{s}} = -8800\ \text{N}$. $F_{\text{av-}y} = \dfrac{J_y}{\Delta t} = \dfrac{4.71\ \text{kg}\cdot\text{m/s}}{1.75\times10^{-3}\ \text{s}} = 2690\ \text{N}$.

The horizontal component is 8800 N, to the left, and the vertical component is 2690 N, upward.

EVALUATE: The ball gains momentum to the left and upward and the force components are in these directions.

8.13. IDENTIFY: The force is constant during the 1.0 ms interval that it acts, so $\vec{J} = \vec{F}\Delta t$. $\vec{J} = \vec{p}_2 - \vec{p}_1 = m(\vec{v}_2 - \vec{v}_1)$.

SET UP: Let $+x$ be to the right, so $v_{1x} = +5.00\ \text{m/s}$. Only the x component of $\vec{J}$ is nonzero, and $J_x = m(v_{2x} - v_{1x})$.

EXECUTE: **(a)** The magnitude of the impulse is $J = F\Delta t = (2.50\times 10^3\ \text{N})(1.00\times 10^{-3}\ \text{s}) = 2.50\ \text{N}\cdot\text{s}$. The direction of the impulse is the direction of the force.

(b) (i) $v_{2x} = \dfrac{J_x}{m} + v_{1x}$. $J_x = +2.50\ \text{N}\cdot\text{s}$. $v_{2x} = \dfrac{+2.50\ \text{N}\cdot\text{s}}{2.00\ \text{kg}} + 5.00\ \text{m/s} = 6.25\ \text{m/s}$. The stone's velocity has magnitude 6.25 m/s and is directed to the right. (ii) Now $J_x = -2.50\ \text{N}\cdot\text{s}$ and $v_{2x} = \dfrac{-2.50\ \text{N}\cdot\text{s}}{2.00\ \text{kg}} + 5.00\ \text{m/s} = 3.75\ \text{m/s}$. The stone's velocity has magnitude 3.75 m/s and is directed to the right.

EVALUATE: When the force and initial velocity are in the same direction the speed increases and when they are in opposite directions the speed decreases.

8.14. **IDENTIFY:** Apply conservation of momentum to the system of the astronaut and tool.

SET UP: Let *A* be the astronaut and *B* be the tool. Let +*x* be the direction in which she throws the tool, so $v_{B2x} = +3.20\ \text{m/s}$. Assume she is initially at rest, so $v_{A1x} = v_{B1x} = 0$. Solve for v_{A2x}.

EXECUTE: $P_{1x} = P_{2x}$. $P_{1x} = m_A v_{A1x} + m_B v_{B1x} = 0$. $P_{2x} = m_A v_{A2x} + m_B v_{B2x} = 0$ and

$v_{A2x} = -\dfrac{m_B v_{A2x}}{m_A} = -\dfrac{(2.25\ \text{kg})(3.20\ \text{m/s})}{68.5\ \text{kg}} = -0.105\ \text{m/s}$. Her speed is 0.105 m/s and she moves opposite to the direction in which she throws the tool.

EVALUATE: Her mass is much larger than that of the tool so to have the same magnitude of momentum as the tool her speed is much less.

8.15. **IDENTIFY:** Since drag effects are neglected there is no net external force on the system of squid plus expelled water and the total momentum of the system is conserved. Since the squid is initially at rest, with the water in its cavity, the initial momentum of the system is zero. For each object, $K = \frac{1}{2}mv^2$.

SET UP: Let *A* be the squid and *B* be the water it expels, so $m_A = 6.50\ \text{kg} - 1.75\ \text{kg} = 4.75\ \text{kg}$. Let +*x* be the direction in which the water is expelled. $v_{A2x} = -2.50\ \text{m/s}$. Solve for v_{B2x}.

EXECUTE: **(a)** $P_{1x} = 0$. $P_{2x} = P_{1x}$, so $0 = m_A v_{A2x} + m_B v_{B2x}$. $v_{B2x} = -\dfrac{m_A v_{A2x}}{m_B} = -\dfrac{(4.75\ \text{kg})(-2.50\ \text{m/s})}{1.75\ \text{kg}} = +6.79\ \text{m/s}$.

(b) $K_2 = K_{A2} + K_{B2} = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 = \frac{1}{2}(4.75\ \text{kg})(2.50\ \text{m/s})^2 + \frac{1}{2}(1.75\ \text{kg})(6.79\ \text{m/s})^2 = 55.2\ \text{J}$ The initial kinetic energy is zero, so the kinetic energy produced is $K_2 = 55.2\ \text{J}$.

EVALUATE: The two objects end up with momenta that are equal in magnitude and opposite in direction, so the total momentum of the system remains zero. The kinetic energy is created by the work done by the squid as it expels the water.

8.16. **IDENTIFY:** Apply conservation of momentum to the system of you and the ball. In part (a) both objects have the same final velocity.

SET UP: Let +*x* be in the direction the ball is traveling initially. $m_A = 0.400\ \text{kg}$ (ball). $m_B = 70.0\ \text{kg}$ (you).

EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $(0.400\ \text{kg})(10.0\ \text{m/s}) = (0.400\ \text{kg} + 70.0\ \text{kg})v_2$ and $v_2 = 0.0568\ \text{m/s}$.

(b) $P_{1x} = P_{2x}$ gives $(0.400\ \text{kg})(10.0\ \text{m/s}) = (0.400\ \text{kg})(-8.00\ \text{m/s}) + (70.0\ \text{kg})v_{B2}$ and $v_{B2} = 0.103\ \text{m/s}$.

EVALUATE: When the ball bounces off it has a greater change in momentum and you acquire a greater final speed.

8.17. **IDENTIFY:** Apply conservation of momentum to the system of the two pucks.

SET UP: Let +*x* be to the right.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ says $(0.250)v_{A1} = (0.250\ \text{kg})(-0.120\ \text{m/s}) + (0.350\ \text{kg})(0.650\ \text{m/s})$ and $v_{A1} = 0.790\ \text{m/s}$.

(b) $K_1 = \frac{1}{2}(0.250\ \text{kg})(0.790\ \text{m/s})^2 = 0.0780\ \text{J}$.

$K_2 = \frac{1}{2}(0.250\ \text{kg})(0.120\ \text{m/s})^2 + \frac{1}{2}(0.350\ \text{kg})(0.650\ \text{m/s})^2 = 0.0757\ \text{J}$ and $\Delta K = K_2 - K_1 = -0.0023\ \text{J}$.

EVALUATE: The total momentum of the system is conserved but the total kinetic energy decreases.

8.18. **IDENTIFY:** Since road friction is neglected, there is no net external force on the system of the two cars and the total momentum of the system is conserved. For each object, $K = \frac{1}{2}mv^2$.

SET UP: Let *A* be the 1750 kg car and *B* be the 1450 kg car. Let +*x* be to the right, so $v_{A1x} = +1.50\ \text{m/s}$, $v_{B1x} = -1.10\ \text{m/s}$, and $v_{A2x} = +0.250\ \text{m/s}$. Solve for v_{B2x}.

EXECUTE: **(a)** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$. $v_{B2x} = \dfrac{m_A v_{A1x} + m_B v_{B1x} - m_A v_{A2x}}{m_B}$.

$$v_{B2x} = \frac{(1750\ \text{kg})(1.50\ \text{m/s}) + (1450\ \text{kg})(-1.10\ \text{m/s}) - (1750\ \text{kg})(0.250\ \text{m/s})}{1450\ \text{kg}} = 0.409\ \text{m/s}.$$

After the collision the lighter car is moving to the right with a speed of 0.409 m/s.

(b) $K_1 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(1750\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}(1450\text{ kg})(1.10\text{ m/s})^2 = 2846\text{ J}$.

$$K_2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 = \frac{1}{2}(1750\text{ kg})(0.250\text{ m/s})^2 + \frac{1}{2}(1450\text{ kg})(0.409\text{ m/s})^2 = 176\text{ J}.$$

The change in kinetic energy is $\Delta K = K_2 - K_1 = 176\text{ J} - 2846\text{ J} = -2670\text{ J}$.

EVALUATE: The total momentum of the system is constant because there is no net external force during the collision. The kinetic energy of the system decreases because of negative work done by the forces the cars exert on each other during the collision.

8.19. **IDENTIFY:** Since the rifle is loosely held there is no net external force on the system consisting of the rifle, bullet and propellant gases and the momentum of this system is conserved. Before the rifle is fired everything in the system is at rest and the initial momentum of the system is zero.

SET UP: Let $+x$ be in the direction of the bullet's motion. The bullet has speed $601\text{ m/s} - 1.85\text{ m/s} = 599\text{ m/s}$ relative to the earth. $P_{2x} = p_{\text{r}x} + p_{\text{b}x} + p_{\text{g}x}$, the momenta of the rifle, bullet and gases. $v_{\text{r}x} = -1.85\text{ m/s}$ and $v_{\text{b}x} = +599\text{ m/s}$.

EXECUTE: $P_{2x} = P_{1x} = 0$. $p_{\text{r}x} + p_{\text{b}x} + p_{\text{g}x} = 0$. $p_{\text{g}x} = -p_{\text{r}x} - p_{\text{b}x} = -(2.80\text{ kg})(-1.85\text{ m/s}) - (0.00720\text{ kg})(599\text{ m/s})$ and $p_{\text{g}x} = +5.18\text{ kg}\cdot\text{m/s} - 4.31\text{ kg}\cdot\text{m/s} = 0.87\text{ kg}\cdot\text{m/s}$. The propellant gases have momentum $0.87\text{ kg}\cdot\text{m/s}$, in the same direction as the bullet is traveling.

EVALUATE: The magnitude of the momentum of the recoiling rifle equals the magnitude of the momentum of the bullet plus that of the gases as both exit the muzzle.

8.20. **IDENTIFY:** In part (a) no horizontal force implies P_x is constant. In part (b) use the energy expression, Eq. 7.14, to find the potential energy intially in the spring.

SET UP: Initially both blocks are at rest.

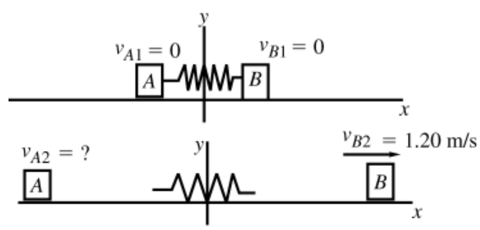


Figure 8.20

EXECUTE: **(a)** $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$

$$0 = m_A v_{A2x} + m_B v_{B2x}$$

$$v_{A2x} = -\left(\frac{m_B}{m_A}\right)v_{B2x} = -\left(\frac{3.00\text{ kg}}{1.00\text{ kg}}\right)(+1.20\text{ m/s}) = -3.60\text{ m/s}$$

Block A has a final speed of 3.60 m/s, and moves off in the opposite direction to B.

(b) Use energy conservation: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

Only the spring force does work so $W_{\text{other}} = 0$ and $U = U_{\text{el}}$.

$K_1 = 0$ (the blocks initially are at rest)

$U_2 = 0$ (no potential energy is left in the spring)

$$K_2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2 = \frac{1}{2}(1.00\text{ kg})(3.60\text{ m/s})^2 + \frac{1}{2}(3.00\text{ kg})(1.20\text{ m/s})^2 = 8.64\text{ J}$$

$U_1 = U_{1,\text{el}}$ the potential energy stored in the compressed spring.

Thus $U_{1,\text{el}} = K_2 = 8.64\text{ J}$

EVALUATE: The blocks have equal and opposite momenta as they move apart, since the total momentum is zero. The kinetic energy of each block is positive and doesn't depend on the direction of the block's velocity, just on its magnitude.

8.21. **IDENTIFY:** Since friction at the pond surface is neglected, there is no net external horizontal force and the horizontal component of the momentum of the system of hunter plus bullet is conserved. Both objects are initially at rest, so the initial momentum of the system is zero. Gravity and the normal force exerted by the ice together produce a net vertical force while the rifle is firing, so the vertical component of momentum is not conserved.

SET UP: Let object A be the hunter and object B be the bullet. Let $+x$ be the direction of the horizontal component of velocity of the bullet. Solve for v_{A2x}.

EXECUTE: **(a)** $v_{B2x} = +965\text{ m/s}$. $P_{1x} = P_{2x} = 0$. $0 = m_A v_{A2x} + m_B v_{B2x}$ and

$$v_{A2x} = -\frac{m_B}{m_A} v_{B2x} = -\left(\frac{4.20\times10^{-3}\text{ kg}}{72.5\text{ kg}}\right)(965\text{ m/s}) = -0.0559\text{ m/s}.$$

(b) $v_{B2x} = v_{B2}\cos\theta = (965\text{ m/s})\cos 56.0° = 540\text{ m/s}$. $v_{A2x} = -\left(\frac{4.20\times10^{-3}\text{ kg}}{72.5\text{ kg}}\right)(540\text{ m/s}) = -0.0313\text{ m/s}$.

EVALUATE: The mass of the bullet is much less than the mass of the hunter, so the final mass of the hunter plus gun is still 72.5 kg, to three significant figures. Since the hunter has much larger mass, her final speed is much less than the speed of the bullet.

8.22. **IDENTIFY:** Assume the nucleus is initially at rest. $K = \frac{1}{2}mv^2$.

SET UP: Let $+x$ be to the right. $v_{A2x} = -v_A$ and $v_{B2x} = +v_B$.

EXECUTE: **(a)** $P_{2x} = P_{1x} = 0$ gives $m_A v_{A2x} + m_B v_{B2x} = 0$. $v_B = \left(\frac{m_A}{m_B}\right) v_A$.

(b) $\frac{K_A}{K_B} = \frac{\frac{1}{2}m_A v_A^2}{\frac{1}{2}m_B v_B^2} = \frac{m_A v_A^2}{m_B\left(m_A v_A/m_B\right)^2} = \frac{m_B}{m_A}$.

EVALUATE: The lighter fragment has the greater kinetic energy.

8.23. **IDENTIFY:** Apply conservation of momentum to the nucleus and its fragments. The initial momentum is zero. The ^{214}Po nucleus has mass $214(1.67\times10^{-27}\text{ kg}) = 3.57\times10^{-25}\text{ kg}$, where $1.67\times10^{-27}\text{ kg}$ is the mass of a nucleon (proton or neutron). $K = \frac{1}{2}mv^2$.

SET UP: Let $+x$ be the direction in which the alpha particle is emitted. The nucleus that is left after the decay has mass $m_n = 3.75\times10^{-25}\text{ kg} - m_\alpha = 3.57\times10^{-25}\text{ kg} - 6.65\times10^{-27}\text{ kg} = 3.50\times10^{-25}\text{ kg}$.

EXECUTE: $P_{2x} = P_{1x} = 0$ gives $m_\alpha v_\alpha + m_n v_n = 0$. $v_n = \frac{m_\alpha}{m_n} v_\alpha$. $v_\alpha = \sqrt{\frac{2K_\alpha}{m_\alpha}} = \sqrt{\frac{2(1.23\times10^{-12}\text{ J})}{6.65\times10^{-27}\text{ kg}}} = 1.92\times10^7\text{ m/s}$.

$v_n = \left(\frac{6.65\times10^{-27}\text{ kg}}{3.50\times10^{-25}\text{ kg}}\right)(1.92\times10^7\text{ m/s}) = 3.65\times10^5\text{ m/s}$.

EVALUATE: The recoil velocity of the more massive nucleus is much less than the speed of the emitted alpha particle.

8.24. **IDENTIFY** and **SET UP:** Let the $+x$-direction be horizontal, along the direction the rock is thrown. There is no net horizontal force, so P_x is constant. Let object A be you and object B be the rock.

EXECUTE: $0 = -m_A v_A + m_B v_B \cos 35.0°$

$$v_A = \frac{m_B v_B \cos 35.0°}{m_A} = 2.11\text{ m/s}$$

EVALUATE: P_y is not conserved because there is a net external force in the vertical direction; as you throw the rock the normal force exerted on you by the ice is larger than the total weight of the system.

8.25. **IDENTIFY:** Each horizontal component of momentum is conserved. $K = \frac{1}{2}mv^2$.

SET UP: Let $+x$ be the direction of Rebecca's initial velocity and let the $+y$ axis make an angle of $36.9°$ with respect to the direction of her final velocity. $v_{D1x} = v_{D1y} = 0$. $v_{R1x} = 13.0\text{ m/s}$; $v_{R1y} = 0$.

$v_{R2x} = (8.00\text{ m/s})\cos 53.1° = 4.80\text{ m/s}$; $v_{R2y} = (8.00\text{ m/s})\sin 53.1° = 6.40\text{ m/s}$. Solve for v_{D2x} and v_{D2y}.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $m_R v_{R1x} = m_R v_{R2x} + m_D v_{D2x}$.

$$v_{D2x} = \frac{m_R(v_{R1x} - v_{R2x})}{m_D} = \frac{(45.0\text{ kg})(13.0\text{ m/s} - 4.80\text{ m/s})}{65.0\text{ kg}} = 5.68\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $0 = m_R v_{R2y} + m_D v_{D2y}$. $v_{D2y} = -\frac{m_R}{m_D} v_{R2y} = -\left(\frac{45.0\text{ kg}}{65.0\text{ kg}}\right)(6.40\text{ m/s}) = -4.43\text{ m/s}$.

The directions of $\vec{v}_{R1}$, $\vec{v}_{R2}$ and $\vec{v}_{D2}$ are sketched in Figure 8.25. $\tan\theta = \left|\frac{v_{D2y}}{v_{D2x}}\right| = \frac{4.43\text{ m/s}}{5.68\text{ m/s}}$ and $\theta = 38.0°$.

$v_D = \sqrt{v_{D2x}^2 + v_{D2y}^2} = 7.20\text{ m/s}$.

(b) $K_1 = \frac{1}{2}m_R v_{R1}^2 = \frac{1}{2}(45.0\text{ kg})(13.0\text{ m/s})^2 = 3.80\times10^3\text{ J}$.

$K_2 = \frac{1}{2}m_R v_{R2}^2 + \frac{1}{2}m_D v_{D2}^2 = \frac{1}{2}(45.0\text{ kg})(8.00\text{ m/s})^2 + \frac{1}{2}(65.0\text{ kg})(7.20\text{ m/s})^2 = 3.12\times10^3\text{ J}$.

$\Delta K = K_2 - K_1 = -680\text{ J}$.

EVALUATE: Each component of momentum is separately conserved. The kinetic energy of the system increases.

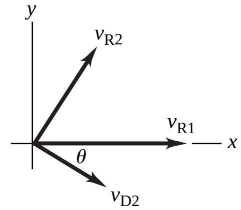


Figure 8.25

8.26. **IDENTIFY:** There is no net external force on the system of astronaut plus canister, so the momentum of the system is conserved.

SET UP: Let object A be the astronaut and object B be the canister. Assume the astronaut is initially at rest. After the collision she must be moving in the same direction as the canister. Let $+x$ be the direction in which the canister is traveling initially, so $v_{A1x} = 0$, $v_{A2x} = +2.40\text{ m/s}$, $v_{B1x} = +3.50\text{ m/s}$, and $v_{B2x} = +1.20\text{ m/s}$. Solve for m_B.

EXECUTE: $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$. $m_B = \dfrac{m_A(v_{A2x} - v_{A1x})}{v_{B1x} - v_{B2x}} = \dfrac{(78.4\text{ kg})(2.40\text{ m/s} - 0)}{3.50\text{ m/s} - 1.20\text{ m/s}} = 81.8\text{ kg}$.

EVALUATE: She must exert a force on the canister in the $-x$ direction to reduce its velocity component in the $+x$ direction. By Newton's third law, the canister exerts a force on her that is in the $+x$ direction and she gains velocity in that direction.

8.27. **IDENTIFY:** The horizontal component of the momentum of the system of the rain and freight car is conserved.

SET UP: Let $+x$ be the direction the car is moving initially. Before it lands in the car the rain has no momentum along the x axis.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ says $(24{,}000\text{ kg})(4.00\text{ m/s}) = (27{,}000\text{ kg})v_{2x}$ and $v_{2x} = 3.56\text{ m/s}$.

(b) After it lands in the car the water must gain horizontal momentum, so the car loses horizontal momentum.

EVALUATE: The vertical component of the momentum is not conserved, because of the vertical external force exerted by the track.

8.28. **IDENTIFY:** The x and y components of the momentum of the system of the two asteroids are separately conserved.

SET UP: The before and after diagrams are given in Figure 8.28 and the choice of coordinates is indicated. Each asteroid has mass m.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $mv_{A1} = mv_{A2}\cos30.0° + mv_{B2}\cos45.0°$. $40.0\text{ m/s} = 0.866v_{A2} + 0.707v_{B2}$ and $0.707v_{B2} = 40.0\text{ m/s} - 0.866v_{A2}$.

$$P_{2y} = P_{2y} \text{ gives } 0 = mv_{A2}\sin30.0° - mv_{B2}\sin45.0° \text{ and } 0.500v_{A2} = 0.707v_{B2}.$$

Combining these two equations gives $0.500v_{A2} = 40.0\text{ m/s} - 0.866v_{A2}$ and $v_{A2} = 29.3\text{ m/s}$. Then

$$v_{B2} = \left(\frac{0.500}{0.707}\right)(29.3\text{ m/s}) = 20.7\text{ m/s}.$$

(b) $K_1 = \frac{1}{2}mv_{A1}^2$. $K_2 = \frac{1}{2}mv_{A2}^2 + \frac{1}{2}mv_{B2}^2$. $\dfrac{K_2}{K_1} = \dfrac{v_{A2}^2 + v_{B2}^2}{v_{A1}^2} = \dfrac{(29.3\text{ m/s})^2 + (20.7\text{ m/s})^2}{(40.0\text{ m/s})^2} = 0.804$.

$$\frac{\Delta K}{K_1} = \frac{K_2 - K_1}{K_1} = \frac{K_2}{K_1} - 1 = -0.196.$$

19.6% of the original kinetic energy is dissipated during the collision.

EVALUATE: We could use any directions we wish for the x and y coordinate directions, but the particular choice we have made is especially convenient.

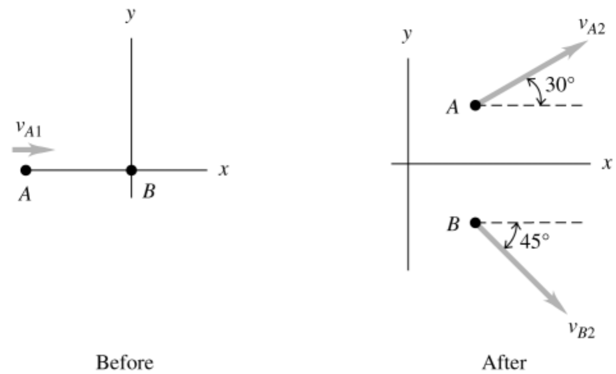


Figure 8.28

8.29. **IDENTIFY:** Since drag effects are neglected there is no net external force on the system of two fish and the momentum of the system is conserved. The mechanical energy equals the kinetic energy, which is $K = \frac{1}{2}mv^2$ for each object.

SET UP: Let object *A* be the 15.0 kg fish and *B* be the 4.50 kg fish. Let +*x* be the direction the large fish is moving initially, so $v_{A1x} = 1.10$ m/s and $v_{B1x} = 0$. After the collision the two objects are combined and move with velocity $\vec{v}_2$. Solve for v_{2x}.

EXECUTE: **(a)** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(15.0\text{ kg})(1.10\text{ m/s}) + 0}{15.0\text{ kg} + 4.50\text{ kg}} = 0.846\text{ m/s}.$$

(b) $K_1 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(15.0\text{ kg})(1.10\text{ m/s})^2 = 9.08\text{ J}$. $K_2 = \frac{1}{2}(m_A + m_B)v_2^2 = \frac{1}{2}(19.5\text{ kg})(0.846\text{ m/s})^2 = 6.98\text{ J}$. $\Delta K = K_2 - K_1 = -2.10\text{ J}$. 2.10 J of mechanical energy is dissipated.

EVALUATE: The total kinetic energy always decreases in a collision where the two objects become combined.

8.30. **IDENTIFY:** There is no net external force on the system of the two otters and the momentum of the system is conserved. The mechanical energy equals the kinetic energy, which is $K = \frac{1}{2}mv^2$ for each object.

SET UP: Let *A* be the 7.50 kg otter and *B* be the 5.75 kg otter. After the collision their combined velocity is $\vec{v}_2$. Let +*x* be to the right, so $v_{A1x} = -5.00$ m/s and $v_{B1x} = +6.00$ m/s. Solve for v_{2x}.

EXECUTE: **(a)** $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(7.50\text{ kg})(-5.00\text{ m/s}) + (5.75)(+6.00\text{ m/s})}{7.50\text{ kg} + 5.75\text{ kg}} = -0.226\text{ m/s}.$$

(b) $K_1 = \frac{1}{2}m_A v_{A1}^2 + \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(7.50\text{ kg})(5.00\text{ m/s})^2 + \frac{1}{2}(5.75\text{ kg})(6.00\text{ m/s})^2 = 197.2\text{ J}$.

$K_2 = \frac{1}{2}(m_A + m_B)v_2^2 = \frac{1}{2}(13.25\text{ kg})(0.226\text{ m/s})^2 = 0.338\text{ J}$.

$\Delta K = K_2 - K_1 = -197\text{ J}$. 197 J of mechanical energy is dissipated.

EVALUATE: The total kinetic energy always decreases in a collision where the two objects become combined.

8.31. **IDENTIFY:** Treat the comet and probe as an isolated system for which momentum is conserved.

SET UP: In part (a) let object *A* be the probe and object *B* be the comet. Let $-x$ be the direction the probe is traveling just before the collision. After the collision the combined object moves with speed v_2. The change in velocity is $\Delta v = v_{2x} - v_{B1x}$. In part (a) the impact speed of 37,000 km/h is the speed of the probe relative to the comet just before impact: $v_{A1x} - v_{B1x} = -37{,}000$ km/h. In part (b) let object *A* be the comet and object *B* be the earth. Let $-x$ be the direction the comet is traveling just before the collision. The impact speed is 40,000 km/h, so $v_{A1x} - v_{B1x} = -40{,}000$ km/h.

EXECUTE: **(a)** $P_{1x} = P_{2x}$. $v_{2x} = \dfrac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B}$.

$$\Delta v = v_{2x} - v_{B1x} = \left(\frac{m_A}{m_A + m_B}\right)v_{A1x} + \left(\frac{m_B - m_A - m_B}{m_A + m_B}\right)v_{B1x} = \left(\frac{m_A}{m_A + m_B}\right)(v_{A1x} - v_{B1x}).$$

$$\Delta v = \left(\frac{372\text{ kg}}{372\text{ kg} + 0.10\times 10^{14}\text{ kg}}\right)(-37{,}000\text{ km/h}) = -1.4\times 10^{-6}\text{ km/h}.$$

The speed of the comet decreased by 1.4×10^{-6} km/h. This change is not noticeable.

(b) $\Delta v = \left(\dfrac{0.10\times 10^{14}\text{ kg}}{0.10\times 10^{14}\text{ kg} + 5.97\times 10^{24}\text{ kg}}\right)(-40{,}000\text{ km/h}) = -6.7\times 10^{-8}\text{ km/h}$. The speed of the earth would change by 6.7×10^{-8} km/h. This change is not noticeable.

EVALUATE: $v_{A1x} - v_{B1x}$ is the velocity of the projectile (probe or comet) relative to the target (comet or earth). The expression for Δv can be derived directly by applying momentum conservation in coordinates in which the target is initially at rest.

8.32. **IDENTIFY:** The forces the two vehicles exert on each other during the collision are much larger than the horizontal forces exerted by the road, and it is a good approximation to assume momentum conservation.

SET UP: Let $+x$ be eastward. After the collision two vehicles move with a common velocity $\vec{v}_2$.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $m_{\text{SC}} v_{\text{SC}x} + m_{\text{T}} v_{\text{T}x} = (m_{\text{SC}} + m_{\text{T}}) v_{2x}$.

$$v_{2x} = \frac{m_{\text{SC}} v_{\text{SC}x} + m_{\text{T}} v_{\text{T}x}}{m_{\text{SC}} + m_{\text{T}}} = \frac{(1050\text{ kg})(-15.0\text{ m/s}) + (6320\text{ kg})(+10.0\text{ m/s})}{1050\text{ kg} + 6320\text{ kg}} = 6.44\text{ m/s}.$$

The final velocity is 6.44 m/s, eastward.

(b) $P_{1x} = P_{2x} = 0$ gives $m_{\text{SC}} v_{\text{SC}x} + m_{\text{T}} v_{\text{T}x} = 0$. $v_{\text{T}x} = -\left(\dfrac{m_{\text{SC}}}{m_{\text{T}}}\right) v_{\text{SC}x} = -\left(\dfrac{1050\text{ kg}}{6320\text{ kg}}\right)(-15.0\text{ m/s}) = 2.50\text{ m/s}$. The truck would need to have initial speed 2.50 m/s.

(c) part (a): $\Delta K = \tfrac{1}{2}(7370\text{ kg})(6.44\text{ m/s})^2 - \tfrac{1}{2}(1050\text{ kg})(15.0\text{ m/s})^2 - \tfrac{1}{2}(6320\text{ kg})(10.0\text{ m/s})^2 = -2.81\times 10^5\text{ J}$

part (b): $\Delta K = 0 - \tfrac{1}{2}(1050\text{ kg})(15.0\text{ m/s})^2 - \tfrac{1}{2}(6320\text{ kg})(2.50\text{ m/s})^2 = -1.38\times 10^5\text{ J}$. The change in kinetic energy has the greater magnitude in part (a).

EVALUATE: In part (a) the eastward momentum of the truck has a greater magnitude than the westward momentum of the car and the wreckage moves eastward after the collision. In part (b) the two vehicles have equal magnitudes of momentum, the total momentum of the system is zero, and the wreckage is at rest after the collision.

8.33. **IDENTIFY:** The forces the two players exert on each other during the collision are much larger than the horizontal forces exerted by the slippery ground and it is a good approximation to assume momentum conservation. Each component of momentum is separately conserved.

SET UP: Let $+x$ be east and $+y$ be north. After the collision the two players have velocity $\vec{v}_2$. Let the linebacker be object A and the halfback be object B, so $v_{A1x} = 0$, $v_{A1y} = 8.8\text{ m/s}$, $v_{B1x} = 7.2\text{ m/s}$ and $v_{B1y} = 0$. Solve for v_{2x} and v_{2y}.

EXECUTE: $P_{1x} = P_{2x}$ gives $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B) v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(85\text{ kg})(7.2\text{ m/s})}{110\text{ kg} + 85\text{ kg}} = 3.14\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_A v_{A1y} + m_B v_{B1y} = (m_A + m_B) v_{2y}$.

$$v_{2y} = \frac{m_A v_{A1y} + m_B v_{B1y}}{m_A + m_B} = \frac{(110\text{ kg})(8.8\text{ m/s})}{110\text{ kg} + 85\text{ kg}} = 4.96\text{ m/s}.$$

$v = \sqrt{v_{2x}^2 + v_{2y}^2} = 5.9\text{ m/s}$.

$$\tan\theta = \frac{v_{2y}}{v_{2x}} = \frac{4.96\text{ m/s}}{3.14\text{ m/s}} \text{ and } \theta = 58^\circ.$$

The players move with a speed of 5.9 m/s and in a direction 58° north of east.

EVALUATE: Each component of momentum is separately conserved.

8.34. **IDENTIFY:** There is no net external force on the system of the two skaters and the momentum of the system is conserved.

SET UP: Let object A be the skater with mass 70.0 kg and object B be the skater with mass 65.0 kg. Let $+x$ be to the right, so $v_{A1x} = +2.00\text{ m/s}$ and $v_{B1x} = -2.50\text{ m/s}$. After the collision the two objects are combined and move with velocity $\vec{v}_2$. Solve for v_{2x}.

EXECUTE: $P_{1x} = P_{2x}$. $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B) v_{2x}$.

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B} = \frac{(70.0\text{ kg})(2.00\text{ m/s}) + (65.0)(-2.50\text{ m/s})}{70.0\text{ kg} + 65.0\text{ kg}} = -0.167\text{ m/s}.$$

The two skaters move to the left at 0.167 m/s.

EVALUATE: There is a large decrease in kinetic energy.

8.35. IDENTIFY: Neglect external forces during the collision. Then the momentum of the system of the two cars is conserved.

SET UP: $m_S = 1200\text{ kg}$, $m_L = 3000\text{ kg}$. The small car has velocity v_S and the large car has velocity v_L.

EXECUTE: **(a)** The total momentum of the system is conserved, so the momentum lost by one car equals the momentum gained by the other car. They have the same magnitude of change in momentum. Since $\vec{p} = m\vec{v}$ and $\Delta\vec{p}$ is the same, the car with the smaller mass has a greater change in velocity.

$$m_S \Delta v_S = m_L \Delta v_L \text{ and } \Delta v_S = \left(\frac{m_L}{m_S}\right)\Delta v_L = \left(\frac{3000\text{ kg}}{1200\text{ kg}}\right)\Delta v = 2.50\Delta v.$$

(b) The acceleration of the small car is greater, since it has a greater change in velocity during the collision. The large acceleration means a large force on the occupants of the small car and they would sustain greater injuries.

EVALUATE: Each car exerts the same magnitude of force on the other car but the force on the compact has a greater effect on its velocity since its mass is less.

8.36. IDENTIFY: The collision forces are large so gravity can be neglected during the collision. Therefore, the horizontal and vertical components of the momentum of the system of the two birds are conserved.

SET UP: The system before and after the collision is sketched in Figure 8.36. Use the coordinates shown.

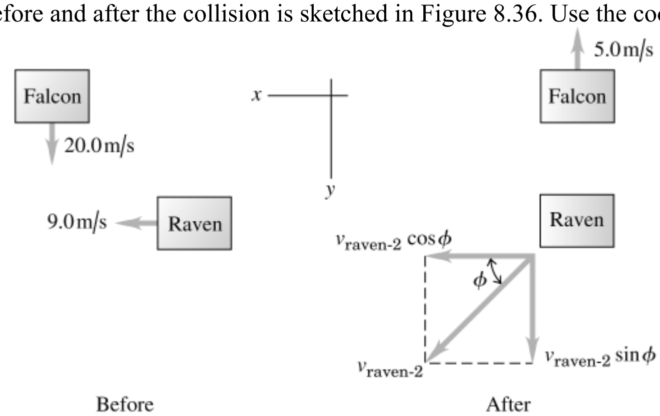


Figure 8.36

EXECUTE: There is no external force on the system so $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

$P_{1x} = P_{2x}$ gives $(1.5\text{ kg})(9.0\text{ m/s}) = (1.5\text{ kg})v_{\text{raven-2}}\cos\phi$ and $v_{\text{raven-2}}\cos\phi = 9.0\text{ m/s}$.

$P_{1y} = P_{2y}$ gives $(0.600\text{ kg})(20.0\text{ m/s}) = (0.600\text{ kg})(-5.0\text{ m/s}) + (1.5\text{ kg})v_{\text{raven-2}}\sin\phi$ and $v_{\text{raven-2}}\sin\phi = 10.0\text{ m/s}$.

Combining these two equations gives $\tan\phi = \dfrac{10.0\text{ m/s}}{9.0\text{ m/s}}$ and $\phi = 48°$.

EVALUATE: Due to its large initial speed the lighter falcon was able to produce a large change in the raven's direction of motion.

8.37. IDENTIFY: Since friction forces from the road are ignored, the *x* and *y* components of momentum are conserved.

SET UP: Let object *A* be the subcompact and object *B* be the truck. After the collision the two objects move together with velocity $\vec{v}_2$. Use the *x* and *y* coordinates given in the problem. $v_{A1y} = v_{B1y} = 0$.

$v_{2x} = (16.0\text{ m/s})\sin 24.0° = 6.5\text{ m/s}$; $v_{2y} = (16.0\text{ m/s})\cos 24.0° = 14.6\text{ m/s}$.

EXECUTE: $P_{1x} = P_{2x}$ gives $m_A v_{A1x} = (m_A + m_B)v_{2x}$.

$$v_{A1x} = \left(\frac{m_A + m_B}{m_A}\right)v_{2x} = \left(\frac{950\text{ kg} + 1900\text{ kg}}{950\text{ kg}}\right)(6.5\text{ m/s}) = 19.5\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_A v_{B1y} = (m_A + m_B)v_{2y}$.

$$v_{B1y} = \left(\frac{m_A + m_B}{m_A}\right)v_{2y} = \left(\frac{950\text{ kg} + 1900\text{ kg}}{1900\text{ kg}}\right)(14.6\text{ m/s}) = 21.9\text{ m/s}.$$

Before the collision the subcompact car has speed 19.5 m/s and the truck has speed 21.9 m/s.

EVALUATE: Each component of momentum is independently conserved.

8.38. IDENTIFY: Apply conservation of momentum to the collision. Apply conservation of energy to the motion of the block after the collision.

SET UP: Conservation of momentum applied to the collision between the bullet and the block: Let object A be the bullet and object B be the block. Let v_A be the speed of the bullet before the collision and let V be the speed of the block with the bullet inside just after the collision.

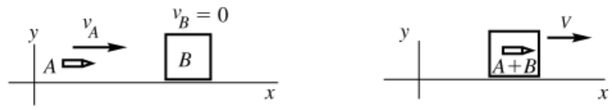


Figure 8.38a

P_x is constant gives $m_A v_A = (m_A + m_B)V$.
Conservation of energy applied to the motion of the block after the collision:

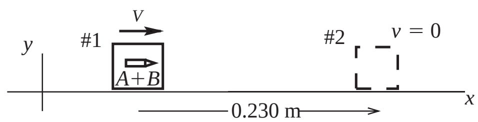


Figure 8.38b

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

EXECUTE: Work is done by friction so $W_{\text{other}} = W_f = (f_k \cos\phi)s = -f_k s = -\mu_k mgs$

$U_1 = U_2 = 0$ (no work done by gravity)

$K_1 = \frac{1}{2}mV^2$; $K_2 = 0$ (block has come to rest)

Thus $\frac{1}{2}mV^2 - \mu_k mgs = 0$

$V = \sqrt{2\mu_k gs} = \sqrt{2(0.20)(9.80 \text{ m/s}^2)(0.230 \text{ m})} = 0.9495$ m/s

Use this in the conservation of momentum equation

$$v_A = \left(\frac{m_A + m_B}{m_A}\right)V = \left(\frac{5.00\times10^{-3} \text{ kg} + 1.20 \text{ kg}}{5.00\times10^{-3} \text{ kg}}\right)(0.9495 \text{ m/s}) = 229 \text{ m/s}$$

EVALUATE: When we apply conservation of momentum to the collision we are ignoring the impulse of the friction force exerted by the surface during the collision. This is reasonable since this force is much smaller than the forces the bullet and block exert on each other during the collision. This force does work as the block moves after the collision, and takes away all the kinetic energy.

8.39. **IDENTIFY:** Apply conservation of momentum to the collision and conservation of energy to the motion after the collision. After the collision the kinetic energy of the combined object is converted to gravitational potential energy.

SET UP: Immediately after the collision the combined object has speed V. Let h be the vertical height through which the pendulum rises.

EXECUTE: **(a)** Conservation of momentum applied to the collision gives

$(12.0\times10^{-3} \text{ kg})(380 \text{ m/s}) = (6.00 \text{ kg} + 12.0\times10^{-3} \text{ kg})V$ and $V = 0.758$ m/s.

Conservation of energy applied to the motion after the collision gives $\frac{1}{2}m_{\text{tot}}V^2 = m_{\text{tot}}gh$ and

$$h = \frac{V^2}{2g} = \frac{(0.758 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = 0.0293 \text{ m} = 2.93 \text{ cm}.$$

(b) $K = \frac{1}{2}m_b v_b^2 = \frac{1}{2}(12.0\times10^{-3} \text{ kg})(380 \text{ m/s})^2 = 866$ J.

(c) $K = \frac{1}{2}m_{\text{tot}}V^2 = \frac{1}{2}(6.00 \text{ kg} + 12.0\times10^{-3} \text{ kg})(0.758 \text{ m/s})^2 = 1.73$ J.

EVALUATE: Most of the initial kinetic energy of the bullet is dissipated in the collision.

8.40. **IDENTIFY:** Each component of horizontal momentum is conserved.

SET UP: Let $+x$ be east and $+y$ be north. $v_{S1y} = v_{A1x} = 0$. $v_{S2x} = (6.00 \text{ m/s})\cos 37.0° = 4.79$ m/s, $v_{S2y} = (6.00 \text{ m/s})\sin 37.0° = 3.61$ m/s, $v_{A2x} = (9.00 \text{ m/s})\cos 23.0° = 8.28$ m/s and $v_{A2y} = -(9.00 \text{ m/s})\sin 23.0° = -3.52$ m/s.

EXECUTE: $P_{1x} = P_{2x}$ gives $m_S v_{S1x} = m_S v_{S2x} + m_A v_{A2x}$.

$$v_{S1x} = \frac{m_S v_{S2x} + m_A v_{A2x}}{m_S} = \frac{(80.0 \text{ kg})(4.79 \text{ m/s}) + (50.0 \text{ kg})(8.28 \text{ m/s})}{80.0 \text{ kg}} = 9.97 \text{ m/s}.$$

Sam's speed before the collision was 9.97 m/s.

$P_{1y} = P_{2y}$ gives $m_A v_{A1y} = m_S v_{S2y} + m_A v_{A2y}$.

$$v_{A1y} = \frac{m_S v_{S2y} + m_A v_{A2y}}{m_S} = \frac{(80.0\text{ kg})(3.61\text{ m/s}) + (50.0\text{ kg})(-3.52\text{ m/s})}{50.0\text{ kg}} = 2.26\text{ m/s}.$$

Abigail's speed before the collision was 2.26 m/s.

(b) $\Delta K = \frac{1}{2}(80.0\,\text{kg})(6.00\,\text{m/s})^2 + \frac{1}{2}(50.0\,\text{kg})(9.00\,\text{m/s})^2 - \frac{1}{2}(80.0\,\text{kg})(9.97\,\text{m/s})^2 - \frac{1}{2}(50.0\,\text{kg})(2.26\,\text{m/s})^2$. $\Delta K = -639\text{ J}$.

EVALUATE: The total momentum is conserved because there is no net external horizontal force. The kinetic energy decreases because the forces between the objects do negative work during the collision.

8.41. IDENTIFY: When the spring is compressed the maximum amount the two blocks aren't moving relative to each other and have the same velocity $\vec{V}$ relative to the surface. Apply conservation of momentum to find V and conservation of energy to find the energy stored in the spring. Since the collision is elastic, Eqs. 8.24 and 8.25 give the final velocity of each block after the collision.

SET UP: Let $+x$ be the direction of the initial motion of A.

EXECUTE: **(a)** Momentum conservation gives $(2.00\text{ kg})(2.00\text{ m/s}) = (12.0\text{ kg})V$ and $V = 0.333\text{ m/s}$. Both blocks are moving at 0.333 m/s, in the direction of the initial motion of block A. Conservation of energy says the initial kinetic energy of A equals the total kinetic energy at maximum compression plus the potential energy U_b stored in the bumpers: $\frac{1}{2}(2.00\text{ kg})(2.00\text{ m/s})^2 = U_b + \frac{1}{2}(12.0\text{ kg})(0.333\text{ m/s})^2$ and $U_b = 3.33\text{ J}$.

(b) $v_{A2x} = \left(\frac{m_A - m_B}{m_A + m_B}\right)v_{A1x} = \left(\frac{2.00\text{ kg} - 10.0\text{ kg}}{12.0\text{ kg}}\right)(2.00\text{ m/s}) = -1.33\text{ m/s}$. Block A is moving in the $-x$ direction at 1.33 m/s.

$v_{B2x} = \left(\frac{2m_A}{m_A + m_B}\right)v_{A1x} = \frac{2(2.00\text{ kg})}{12.0\text{ kg}}(2.00\text{ m/s}) = +0.667\text{ m/s}$. Block B is moving in the $+x$ direction at 0.667 m/s.

EVALUATE: When the spring is compressed the maximum amount the system must still be moving in order to conserve momentum.

8.42. IDENTIFY: No net external horizontal force so P_x is conserved. Elastic collision so $K_1 = K_2$ and can use Eq. 8.27.

SET UP:

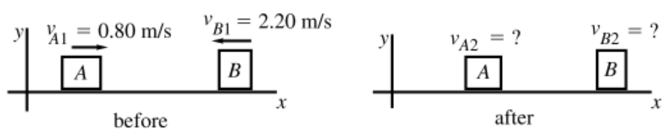


Figure 8.42

EXECUTE: From conservation of x-component of momentum:

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

$$m_A v_{A1} - m_B v_{B1} = m_A v_{A2x} + m_B v_{B2x}$$

$$(0.150\text{ kg})(0.80\text{ m/s}) - (0.300\text{ kg})(2.20\text{ m/s}) = (0.150\text{ kg})v_{A2x} + (0.300\text{ kg})v_{B2x}$$

$$-3.60\text{ m/s} = v_{A2x} + 2v_{B2x}$$

From the relative velocity equation for an elastic collision Eq. 8.27:

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x}) = -(-2.20\text{ m/s} - 0.80\text{ m/s}) = +3.00\text{ m/s}$$

$$3.00\text{ m/s} = -v_{A2x} + v_{B2x}$$

Adding the two equations gives $-0.60\text{ m/s} = 3v_{B2x}$ and $v_{B2x} = -0.20\text{ m/s}$. Then $v_{A2x} = v_{B2x} - 3.00\text{ m/s} = -3.20\text{ m/s}$. The 0.150 kg glider ($A$) is moving to the left at 3.20 m/s and the 0.300 kg glider (B) is moving to the left at 0.20 m/s.

EVALUATE: We can use our v_{A2x} and v_{B2x} to show that P_x is constant and $K_1 = K_2$

8.43. IDENTIFY: Since the collision is elastic, both momentum conservation and Eq. 8.27 apply.

SET UP: Let object A be the 30.0 kg marble and let object B be the 10.0 g marble. Let $+x$ be to the right.

EXECUTE: **(a)** Conservation of momentum gives

$(0.0300\text{ kg})(0.200\text{ m/s}) + (0.0100\text{ kg})(-0.400\text{ m/s}) = (0.0300\text{ kg})v_{A2x} + (0.0100\text{ kg})v_{B2x}$.

$3v_{A2x} + v_{B2x} = 0.200\text{ m/s}$. Eq. 8.27 says $v_{B2x} - v_{A2x} = -(-0.400\text{ m/s} - 0.200\text{ m/s}) = +0.600\text{ m/s}$. Solving this pair of equations gives $v_{A2x} = -0.100\text{ m/s}$ and $v_{B2x} = +0.500\text{ m/s}$. The 30.0 g marble is moving to the left at 0.100 m/s and the 10.0 g marble is moving to the right at 0.500 m/s.

(b) For marble A, $\Delta P_{Ax} = m_A v_{A2x} - m_A v_{A1x} = (0.0300 \text{ kg})(-0.100 \text{ m/s} - 0.200 \text{ m/s}) = -0.00900 \text{ kg}\cdot\text{m/s}$.
For marble B, $\Delta P_{Bx} = m_B v_{B2x} - m_B v_{B1x} = (0.0100 \text{ kg})(0.500 \text{ m/s} - [-0.400 \text{ m/s}]) = +0.00900 \text{ kg}\cdot\text{m/s}$.
The changes in momentum have the same magnitude and opposite sign.
(c) For marble A, $\Delta K_A = \frac{1}{2}m_A v_{A2}^2 - \frac{1}{2}m_A v_{A1}^2 = \frac{1}{2}(0.0300 \text{ kg})([0.100 \text{ m/s}]^2 - [0.200 \text{ m/s}]^2) = -4.5\times10^{-4} \text{ J}$.
For marble B, $\Delta K_B = \frac{1}{2}m_B v_{B2}^2 - \frac{1}{2}m_B v_{B1}^2 = \frac{1}{2}(0.0100 \text{ kg})([0.500 \text{ m/s}]^2 - [0.400 \text{ m/s}]^2) = +4.5\times10^{-4} \text{ J}$.
The changes in kinetic energy have the same magnitude and opposite sign.
EVALUATE: The results of parts **(b)** and **(c)** show that momentum and kinetic energy are conserved in the collision.

8.44. **IDENTIFY** and **SET UP:** Without rounding, the calculation in Example 8.12 gives $v_{B2} = \sqrt{20}$ m/s.
EXECUTE: The two equations in Example 8.12 for α and β are

$$(0.500 \text{ kg})(4.00 \text{ m/s}) = (0.500 \text{ kg})(2.00 \text{ m/s})(\cos\alpha) + (0.300 \text{ kg})(\sqrt{20} \text{ m/s})(\cos\beta) \text{ Eq. 1}$$

and

$$0 = (0.500 \text{ kg})(2.00 \text{ m/s})(\sin\alpha) - (0.300 \text{ kg})(\sqrt{20} \text{ m/s})\sin\beta \text{ Eq. 2.}$$

Dividing each equation by $(0.500 \text{ kg})(1.00 \text{ m/s})$ gives

$$4.00 = 2.00\cos\alpha + 0.6\sqrt{20}\cos\beta \text{ Eq. 3}$$

and

$$0 = 2.00\sin\alpha - 0.6\sqrt{20}\sin\beta \text{ Eq. 4.}$$

Eq. 3 gives $\cos\beta = \dfrac{4.00 - 2.00\cos\alpha}{0.6\sqrt{20}}$ and $\cos^2\beta = 2.222 - 2.222\cos\alpha + 0.5556\cos^2\alpha$.
Eq. 4 gives $\sin\beta = 0.7454\sin\alpha$ and $\sin^2\beta = 0.5556\sin^2\alpha = 0.5556 - 0.5556\cos^2\alpha$.
Adding the two equations and using $\sin^2\beta + \cos^2\beta = 1$ gives $1 = 2.778 - 2.222\cos\alpha$ and $\cos\alpha = 0.8002$.
$\alpha = 36.9°$. Then $\sin\beta = 0.7454\sin\alpha$ gives $\beta = 26.6°$.
EVALUATE: For these values of α and β, the *x* component of momentum, the *y* component of momentum and the kinetic energy are all conserved in the collision.

8.45. **IDENTIFY:** Eqs. 8.24 and 8.25 apply, with object *A* being the neutron.
SET UP: Let +*x* be the direction of the initial momentum of the neutron. The mass of a neutron is $m_n = 1.0$ u.
EXECUTE: **(a)** $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_{A1x} = \dfrac{1.0 \text{ u} - 2.0 \text{ u}}{1.0 \text{ u} + 2.0 \text{ u}}v_{A1x} = -v_{A1x}/3.0$. The speed of the neutron after the collision is one-third its initial speed.
(b) $K_2 = \frac{1}{2}m_n v_n^2 = \frac{1}{2}m_n(v_{A1}/3.0)^2 = \dfrac{1}{9.0}K_1$.
(c) After *n* collisions, $v_{A2} = \left(\dfrac{1}{3.0}\right)^n v_{A1}$. $\left(\dfrac{1}{3.0}\right)^n = \dfrac{1}{59{,}000}$, so $3.0^n = 59{,}000$. $n\log 3.0 = \log 59{,}000$ and $n = 10$.
EVALUATE: Since the collision is elastic, in each collision the kinetic energy lost by the neutron equals the kinetic energy gained by the deuteron.

8.46. **IDENTIFY:** Elastic collision. Solve for mass and speed of target nucleus.
SET UP: (a) Let *A* be the proton and *B* be the target nucleus. The collision is elastic, all velocities lie along a line, and *B* is at rest before the collision. Hence the results of Eqs. 8.24 and 8.25 apply.
EXECUTE: Eq. 8.24: $m_B(v_x + v_{Ax}) = m_A(v_x - v_{Ax})$, where v_x is the velocity component of *A* before the collision and v_{Ax} is the velocity component of *A* after the collision. Here, $v_x = 1.50\times10^7$ m/s (take direction of incident beam to be positive) and $v_{Ax} = -1.20\times10^7$ m/s (negative since traveling in direction opposite to incident beam).

$$m_B = m_A\left(\frac{v_x - v_{Ax}}{v_x + v_{Ax}}\right) = m\left(\frac{1.50\times10^7 \text{ m/s} + 1.20\times10^7 \text{ m/s}}{1.50\times10^7 \text{ m/s} - 1.20\times10^7 \text{ m/s}}\right) = m\left(\frac{2.70}{0.30}\right) = 9.00m.$$

(b) Eq. 8.25: $v_{Bx} = \left(\dfrac{2m_A}{m_A + m_B}\right)v = \left(\dfrac{2m}{m + 9.00m}\right)(1.50\times10^7 \text{ m/s}) = 3.00\times10^6 \text{ m/s}.$

EVALUATE: Can use our calculated v_{Bx} and m_B to show that P_x is constant and that $K_1 = K_2$.

8.47. **IDENTIFY:** Apply Eq. 8.28.

SET UP: $m_A = 0.300\text{ kg}$, $m_B = 0.400\text{ kg}$, $m_C = 0.200\text{ kg}$.

EXECUTE: $x_{\text{cm}} = \dfrac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C}$.

$$x_{\text{cm}} = \frac{(0.300\text{ kg})(0.200\text{ m}) + (0.400\text{ kg})(0.100\text{ m}) + (0.200\text{ kg})(-0.300\text{ m})}{0.300\text{ kg} + 0.400\text{ kg} + 0.200\text{ kg}} = 0.0444\text{ m}.$$

$$y_{\text{cm}} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C}.$$

$$y_{\text{cm}} = \frac{(0.300\text{ kg})(0.300\text{ m}) + (0.400\text{ kg})(-0.400\text{ m}) + (0.200\text{ kg})(0.600\text{ m})}{0.300\text{ kg} + 0.400\text{ kg} + 0.200\text{ kg}} = 0.0556\text{ m}.$$

EVALUATE: There is mass at both positive and negative x and at positive and negative y and therefore the center of mass is close to the origin.

8.48. **IDENTIFY:** Calculate x_{cm}.

SET UP: Apply Eq. 8.28 with the sun as mass 1 and Jupiter as mass 2. Take the origin at the sun and let Jupiter lie on the positive x-axis.

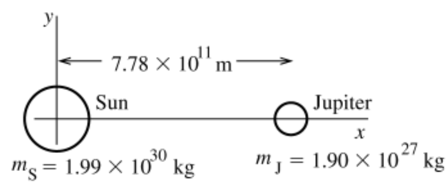


Figure 8.48

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

EXECUTE: $x_1 = 0$ and $x_2 = 7.78\times10^{11}\text{ m}$

$$x_{\text{cm}} = \frac{\left(1.90\times10^{27}\text{ kg}\right)\left(7.78\times10^{11}\text{ m}\right)}{1.99\times10^{30}\text{ kg} + 1.90\times10^{27}\text{ kg}} = 7.42\times10^{8}\text{ m}$$

The center of mass is $7.42\times10^{8}\text{ m}$ from the center of the sun and is on the line connecting the centers of the sun and Jupiter. The sun's radius is $6.96\times10^{8}\text{ m}$ so the center of mass lies just outside the sun.

EVALUATE: The mass of the sun is much greater than the mass of Jupiter so the center of mass is much closer to the sun. For each object we have considered all the mass as being at the center of mass (geometrical center) of the object.

8.49. **IDENTIFY:** The location of the center of mass is given by Eq. 8.48. The mass can be expressed in terms of the diameter. Each object can be replaced by a point mass at its center.

SET UP: Use coordinates with the origin at the center of Pluto and the +x direction toward Charon, so $x_{\text{P}} = 0$ $x_{\text{C}} = 19{,}700\text{ km}$. $m = \rho V = \rho\frac{4}{3}\pi r^3 = \frac{1}{6}\rho\pi d^3$.

EXECUTE: $x_{\text{cm}} = \dfrac{m_{\text{P}}x_{\text{P}} + m_{\text{C}}x_{\text{C}}}{m_{\text{P}} + m_{\text{C}}} = \left(\dfrac{m_{\text{C}}}{m_{\text{P}} + m_{\text{C}}}\right)x_{\text{C}} = \left(\dfrac{\frac{1}{6}\rho\pi d_{\text{C}}^3}{\frac{1}{6}\rho\pi d_{\text{P}}^3 + \frac{1}{6}\rho\pi d_{\text{C}}^3}\right)x_{\text{C}} = \left(\dfrac{d_{\text{C}}^3}{d_{\text{P}}^3 + d_{\text{C}}^3}\right)x_{\text{C}}$.

$$x_{\text{cm}} = \left(\frac{[1250\text{ km}]^3}{[2370\text{ km}]^3 + [1250\text{ km}]^3}\right)(19{,}700\text{ km}) = 2.52\times10^{3}\text{ km}.$$

The center of mass of the system is $2.52\times10^{3}\text{ km}$ from the center of Pluto.

EVALUATE: The center of mass is closer to Pluto because Pluto has more mass than Charon.

8.50. **IDENTIFY:** Apply Eqs. 8.28, 8.30 and 8.32. There is only one component of position and velocity.

SET UP: $m_A = 1200\text{ kg}$, $m_B = 1800\text{ kg}$. $M = m_A + m_B = 3000\text{ kg}$. Let +$x$ be to the right and let the origin be at the center of mass of the station wagon.

EXECUTE: **(a)** $x_{\text{cm}} = \dfrac{m_A x_A + m_B x_B}{m_A + m_B} = \dfrac{0 + (1800\text{ kg})(40.0\text{ m})}{1200\text{ kg} + 1800\text{ kg}} = 24.0\text{ m}.$

The center of mass is between the two cars, 24.0 m to the right of the station wagon and 16.0 m behind the lead car.

(b) $P_x = m_A v_{A1} + m_B v_{B1} = (1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s}) = 5.04\times10^4\text{ kg}\cdot\text{m/s}.$

(c) $v_{\text{cm},x} = \dfrac{m_A v_{A,x} + m_B v_{B,x}}{m_A + m_B} = \dfrac{(1200\text{ kg})(12.0\text{ m/s}) + (1800\text{ kg})(20.0\text{ m/s})}{1200\text{ kg} + 1800\text{ kg}} = 16.8\text{ m/s}.$

(d) $P_x = Mv_{\text{cm-}x} = (3000\text{ kg})(16.8\text{ m/s}) = 5.04\times10^4\text{ kg}\cdot\text{m/s}$, the same as in part (b).

EVALUATE: The total momentum can be calculated either as the vector sum of the momenta of the individual objects in the system, or as the total mass of the system times the velocity of the center of mass.

8.51. **IDENTIFY:** Use Eq. 8.28 to find the *x* and *y* coordinates of the center of mass of the machine part for each configuration of the part. In calculating the center of mass of the machine part, each uniform bar can be represented by a point mass at its geometrical center.

SET UP: Use coordinates with the axis at the hinge and the +*x* and +*y* axes along the horizontal and vertical bars in the figure in the problem. Let (x_i, y_i) and (x_f, y_f) be the coordinates of the bar before and after the vertical bar is pivoted. Let object 1 be the horizontal bar, object 2 be the vertical bar and 3 be the ball.

EXECUTE: $x_\text{i} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \dfrac{(4.00\text{ kg})(0.750\text{ m}) + 0 + 0}{4.00\text{ kg} + 3.00\text{ kg} + 2.00\text{ kg}} = 0.333\text{ m}.$

$$y_\text{i} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{0 + (3.00\text{ kg})(0.900\text{ m}) + (2.00\text{ kg})(1.80\text{ m})}{9.00\text{ kg}} = 0.700\text{ m}.$$

$$x_\text{f} = \frac{(4.00\text{ kg})(0.750\text{ m}) + (3.00\text{ kg})(-0.900\text{ m}) + (2.00\text{ kg})(-1.80\text{ m})}{9.00\text{ kg}} = -0.366\text{ m}.$$

$y_\text{f} = 0$. $x_\text{f} - x_\text{i} = -0.700\text{ m}$ and $y_\text{f} - y_\text{i} = -0.700\text{ m}$. The center of mass moves 0.700 m to the right and 0.700 m upward.

EVALUATE: The vertical bar moves upward and to the right so it is sensible for the center of mass of the machine part to move in these directions.

8.52. **(a) IDENTIFY:** Use Eq. 8.28.

SET UP: The target variable is m_1.

EXECUTE: $x_\text{cm} = 2.0\text{ m}$, $y_\text{cm} = 0$

$$x_\text{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{m_1(0) + (0.10\text{ kg})(8.0\text{ m})}{m_1 + (0.10\text{ kg})} = \frac{0.80\text{ kg}\cdot\text{m}}{m_1 + 0.10\text{ kg}}.$$

$$x_\text{cm} = 2.0\text{ m gives } 2.0\text{ m} = \frac{0.80\text{ kg}\cdot\text{m}}{m_1 + 0.10\text{ kg}}.$$

$$m_1 + 0.10\text{ kg} = \frac{0.80\text{ kg}\cdot\text{m}}{2.0\text{ m}} = 0.40\text{ kg}.$$

$$m_1 = 0.30\text{ kg}.$$

EVALUATE: The cm is closer to m_1 so its mass is larger then m_2.

(b) IDENTIFY: Use Eq. 8.32 to calculate $\vec{P}$.

SET UP: $\vec{v}_\text{cm} = (5.0\text{ m/s})\hat{j}$.

$$\vec{P} = M\vec{v}_\text{cm} = (0.10\text{ kg} + 0.30\text{ kg})(5.0\text{ m/s})\hat{i} = (2.0\text{ kg}\cdot\text{m/s})\hat{i}.$$

(c) IDENTIFY: Use Eq. 8.31.

SET UP: $\vec{v}_\text{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$. The target variable is $\vec{v}_1$. Particle 2 at rest says $v_2 = 0$.

EXECUTE: $\vec{v}_1 = \left(\dfrac{m_1 + m_2}{m_1}\right)\vec{v}_\text{cm} = \left(\dfrac{0.30\text{ kg} + 0.10\text{ kg}}{0.30\text{ kg}}\right)(5.00\text{ m/s})\hat{i} = (6.7\text{ m/s})\hat{i}.$

EVALUATE: Using the result of part (c) we can calculate $\vec{p}_1$ and $\vec{p}_2$ and show that $\vec{P}$ as calculated in part (b) does equal $\vec{p}_1 + \vec{p}_2$.

8.53. **IDENTIFY:** There is no net external force on the system of James, Ramon and the rope and the momentum of the system is conserved and the velocity of its center of mass is constant. Initially there is no motion, and the velocity of the center of mass remains zero after Ramon has started to move.

SET UP: Let $+x$ be in the direction of Ramon's motion. Ramon has mass $m_R = 60.0$ kg and James has mass $m_J = 90.0$ kg.

EXECUTE: $v_{cm\text{-}x} = \dfrac{m_R v_{Rx} + m_J v_{Jx}}{m_R + m_J} = 0$.

$$v_{Jx} = -\left(\frac{m_R}{m_J}\right)v_{Rx} = -\left(\frac{60.0\text{ kg}}{90.0\text{ kg}}\right)(0.700\text{ m/s}) = -0.47\text{ m/s}$$. James' speed is 0.47 m/s.

EVALUATE: As they move, the two men have momenta that are equal in magnitude and opposite in direction, and the total momentum of the system is zero. Also, Example 8.14 shows that Ramon moves farther than James in the same time interval. This is consistent with Ramon having a greater speed.

8.54. **(a) IDENTIFY** and **SET UP:** Apply Eq. 8.28 and solve for m_1 and m_2.

EXECUTE: $y_{cm} = \dfrac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$

$$m_1 + m_2 = \frac{m_1 y_1 + m_2 y_2}{y_{cm}} = \frac{m_1(0) + (0.50\text{ kg})(6.0\text{ m})}{2.4\text{ m}} = 1.25\text{ kg} \text{ and } m_1 = 0.75\text{ kg}.$$

EVALUATE: y_{cm} is closer to m_1 since $m_1 > m_2$.

(b) IDENTIFY and **SET UP:** Apply $\vec{a} = d\vec{v}/dt$ for the cm motion.

EXECUTE: $\vec{a}_{cm} = \dfrac{d\vec{v}_{cm}}{dt} = (1.5\text{ m/s}^3)t\hat{i}$.

(c) IDENTIFY and **SET UP:** Apply Eq. 8.34.

EXECUTE: $\sum \vec{F}_{ext} = M\vec{a}_{cm} = (1.25\text{ kg})(1.5\text{ m/s}^3)t\hat{i}$.

At $t = 3.0$ s, $\sum \vec{F}_{ext} = (1.25\text{ kg})(1.5\text{ m/s}^3)(3.0\text{ s})\hat{i} = (5.6\text{ N})\hat{i}$.

EVALUATE: $v_{cm\text{-}x}$ is positive and increasing so a_{cm-x} is positive and $\vec{F}_{ext}$ is in the $+x$-direction. There is no motion and no force component in the y-direction.

8.55. **IDENTIFY:** Apply $\sum \vec{F} = \dfrac{d\vec{P}}{dt}$ to the airplane.

SET UP: $\dfrac{d}{dt}(t^n) = nt^{n-1}$. $1\text{ N} = 1\text{ kg}\cdot\text{m/s}^2$.

EXECUTE: $\dfrac{d\vec{P}}{dt} = [-(1.50\text{ kg}\cdot\text{m/s}^3)t]\hat{i} + (0.25\text{ kg}\cdot\text{m/s}^2)\hat{j}$. $F_x = -(1.50\text{ N/s})t$, $F_y = 0.25$ N, $F_z = 0$.

EVALUATE: There is no momentum or change in momentum in the z direction and there is no force component in this direction.

8.56. **IDENTIFY:** Use Eq. 8.38, applied to a finite time interval.

SET UP: $v_{ex} = 1600$ m/s

EXECUTE: **(a)** $F = -v_{ex}\dfrac{\Delta m}{\Delta t} = -(1600\text{ m/s})\dfrac{-0.0500\text{ kg}}{1.00\text{ s}} = +80.0\text{ N}$.

(b) The absence of atmosphere would not prevent the rocket from operating. The rocket could be steered by ejecting the gas in a direction with a component perpendicular to the rocket's velocity and braked by ejecting it in a direction parallel (as opposed to antiparallel) to the rocket's velocity.

EVALUATE: The thrust depends on the speed of the ejected gas relative to the rocket and on the mass of gas ejected per second.

8.57. **IDENTIFY:** $a = -\dfrac{v_{ex}}{m}\dfrac{dm}{dt}$. Assume that dm/dt is constant over the 5.0 s interval, since m doesn't change much during that interval. The thrust is $F = -v_{ex}\dfrac{dm}{dt}$.

SET UP: Take m to have the constant value $110\text{ kg} + 70\text{ kg} = 180\text{ kg}$. dm/dt is negative since the mass of the MMU decreases as gas is ejected.

EXECUTE: **(a)** $\dfrac{dm}{dt} = -\dfrac{m}{v_{ex}}a = -\left(\dfrac{180\text{ kg}}{490\text{ m/s}}\right)(0.029\text{ m/s}^2) = -0.0106\text{ kg/s}$. In 5.0 s the mass that is ejected is $(0.0106\text{ kg/s})(5.0\text{ s}) = 0.053\text{ kg}$.

(b) $F = -v_{ex}\dfrac{dm}{dt} = -(490\text{ m/s})(-0.0106\text{ kg/s}) = 5.19\text{ N}$.

EVALUATE: The mass change in the 5.0 s is a very small fraction of the total mass *m*, so it is accurate to take *m* to be constant.

8.58. **IDENTIFY** and **SET UP:** Apply Eq. 8.39: $a = -\frac{v_{ex}}{m}\frac{dm}{dt}$. Solve for dm/dt.

EXECUTE:

$$\frac{dm}{dt} = -\frac{ma}{v_{ex}} = -\frac{(6000\text{ kg})(25.0\text{ m/s}^2)}{2000\text{ m/s}} = -75.0\text{ kg/s}\,.$$

So in 1 s the rocket must eject 75.0 kg of gas.

EVALUATE: We have approximated dm/dt by $\Delta m/\Delta t$. We have assumed that 25.0 m/s^2 is the average acceleration for the first second.

8.59. **IDENTIFY:** Use Eq. 8.39, applied to a finite time interval. Solve for v_{ex}.

SET UP: $\frac{\Delta m}{\Delta t} = -\frac{m}{160}$.

EXECUTE: $a = -\frac{v_{ex}}{m}\frac{\Delta m}{\Delta t}$. $v_{ex} = -\frac{a}{\left(\frac{\Delta m}{\Delta t}/m\right)} = \frac{-15.0\text{ m/s}^2}{\left(-\frac{m}{160}\right)/m} = 2.40\times10^3\text{ m/s} = 2.40\text{ km/s}$

EVALUATE: The acceleration is proportional to the speed of the exhaust gas and to the rate at which mass is ejected.

8.60. **IDENTIFY** and **SET UP:** $(F_{av})\Delta t = J$ relates the impulse *J* to the average thrust F_{av}. Eq. 8.38 applied to a finite time interval gives $F_{av} = -v_{ex}\frac{\Delta m}{\Delta t}$. $v - v_0 = v_{ex}\ln\left(\frac{m_0}{m}\right)$. The remaining mass *m* after 1.70 s is 0.0133 kg.

EXECUTE: **(a)** $F = \frac{J}{\Delta t} = \frac{10.0\text{ N}\cdot\text{s}}{1.70\text{ s}} = 5.88\text{ N}$. $F_{av}/F_{max} = 0.442$.

(b) $v_{ex} = -\frac{F_{av}\Delta t}{-0.0125\text{ kg}} = 800\text{ m/s}$.

(c) $v_0 = 0$ and $v = v_{ex}\ln\left(\frac{m_0}{m}\right) = (800\text{ m/s})\ln\left(\frac{0.0258\text{ kg}}{0.0133\text{ kg}}\right) = 530\text{ m/s}$.

EVALUATE: The acceleration of the rocket is not constant. It increases as the mass remaining decreases.

8.61. **IDENTIFY:** $v - v_0 = v_{ex}\ln\left(\frac{m_0}{m}\right)$.

SET UP: $v_0 = 0$.

EXECUTE: $\ln\left(\frac{m_0}{m}\right) = \frac{v}{v_{ex}} = \frac{8.00\times10^3\text{ m/s}}{2100\text{ m/s}} = 3.81$ and $\frac{m_0}{m} = e^{3.81} = 45.2$.

EVALUATE: Note that the final speed of the rocket is greater than the relative speed of the exhaust gas.

8.62. **IDENTIFY** and **SET UP:** Use Eq. 8.40: $v - v_0 = v_{ex}\ln(m_0/m)$.

$v_0 = 0$ ("fired from rest"), so $v/v_{ex} = \ln(m_0/m)$.

Thus $m_0/m = e^{v/v_{ex}}$, or $m/m_0 = e^{-v/v_{ex}}$.

If *v* is the final speed then *m* is the mass left when all the fuel has been expended; m/m_0 is the fraction of the initial mass that is not fuel.

(a) EXECUTE: $v = 1.00\times10^{-3}c = 3.00\times10^5\text{ m/s}$ gives

$$m/m_0 = e^{-(3.00\times10^5\text{ m/s})/(2000\text{ m/s})} = 7.2\times10^{-66}\,.$$

EVALUATE: This is clearly not feasible, for so little of the initial mass to not be fuel.

(b) EXECUTE: $v = 3000\text{ m/s}$ gives $m/m_0 = e^{-(3000\text{ m/s})/(2000\text{ m/s})} = 0.223$.

EVALUATE: 22.3% of the total initial mass not fuel, so 77.7% is fuel; this is possible.

8.63. **IDENTIFY:** Use the heights to find v_{1y} and v_{2y}, the velocity of the ball just before and just after it strikes the slab. Then apply $J_y = F_y\Delta t = \Delta p_y$.

SET UP: Let +*y* be downward.

EXECUTE: **(a)** $\frac{1}{2}mv^2 = mgh$ so $v = \pm\sqrt{2gh}$.

$v_{1y} = +\sqrt{2(9.80\text{ m/s}^2)(2.00\text{ m})} = 6.26\text{ m/s}$. $v_{2y} = -\sqrt{2(9.80\text{ m/s}^2)(1.60\text{ m})} = -5.60\text{ m/s}$.

$$J_y = \Delta p_y = m(v_{2y} - v_{1y}) = (40.0\times10^{-3}\text{ kg})(-5.60\text{ m/s} - 6.26\text{ m/s}) = -0.474\text{ kg}\cdot\text{m/s}.$$

The impulse is $0.474\text{ kg}\cdot\text{m/s}$, upward.

(b) $F_y = \dfrac{J_y}{\Delta t} = \dfrac{-0.474\text{ kg}\cdot\text{m/s}}{2.00\times10^{-3}\text{ s}} = -237\text{ N}$. The average force on the ball is 237 N, upward.

EVALUATE: The upward force on the ball changes the direction of its momentum.

8.64. **IDENTIFY:** Momentum is conserved in the explosion. At the highest point the velocity of the boulder is zero. Since one fragment moves horizontally the other fragment also moves horizontally. Use projectile motion to relate the initial horizontal velocity of each fragment to its horizontal displacement.

SET UP: Use coordinates where $+x$ is north. Since both fragments start at the same height with zero vertical component of velocity, the time in the air, t, is the same for both. Call the fragments A and B, with A being the one that lands to the north. Therefore, $m_B = 3m_A$.

EXECUTE: Apply $P_{1x} = P_{2x}$ to the collision: $0 = m_A v_{Ax} + m_B v_{Bx}$. $v_{Bx} = -\dfrac{m_A}{m_B}v_{Ax} = -v_{Ax}/3$. Apply projectile motion to the motion after the collision: $x - x_0 = v_{0x}t$. Since t is the same, $\dfrac{(x-x_0)_A}{v_{Ax}} = \dfrac{(x-x_0)_B}{v_{Bx}}$ and

$(x-x_0)_B = \left(\dfrac{v_{Bx}}{v_{Ax}}\right)(x-x_0)_A = \left(\dfrac{-v_{Ax}/3}{v_{Ax}}\right)(x-x_0)_A = -(274\text{ m})/3 = -91.3\text{ m}$. The other fragment lands 91.3 m directly south of the point of explosion.

EVALUATE: The fragment that has three times the mass travels one-third as far.

8.65. **IDENTIFY:** The impulse, force and change in velocity are related by Eq. 8.9

SET UP: $m = w/g = 0.0571\text{ kg}$. Since the force is constant, $\vec{F} = \vec{F}_{\text{av}}$.

EXECUTE: **(a)** $J_x = F_x\Delta t = (-380\text{ N})(3.00\times10^{-3}\text{ s}) = -1.14\text{ N}\cdot\text{s}$. $J_y = F_y\Delta t = (110\text{ N})(3.00\times10^{-3}\text{ s}) = 0.330\text{ N}\cdot\text{s}$.

(b) $v_{2x} = \dfrac{J_x}{m} + v_{1x} = \dfrac{-1.14\text{ N}\cdot\text{s}}{0.0571\text{ kg}} + 20.0\text{ m/s} = 0.04\text{ m/s}$. $v_{2y} = \dfrac{J_y}{m} - v_{1y} = \dfrac{0.330\text{ N}\cdot\text{s}}{0.0571\text{ kg}} + (-4.0\text{ m/s}) = +1.8\text{ m/s}$.

EVALUATE: The change in velocity $\Delta\vec{v}$ is in the same direction as the force, so $\Delta\vec{v}$ has a negative x component and a positive y component.

8.66. **IDENTIFY:** The horizontal component of the momentum of the system of cars is conserved.

SET UP: Let $+x$ be the direction the cars are traveling. Each car has mass m. Let v_1 be the initial speed of the three cars. $v_2 = \frac{1}{5}v_1$. Let N be the number of cars in the final collection.

EXECUTE: $P_{1x} = P_{2x}$. $(3m)v_1 = (Nm)v_2$. $N = \dfrac{3v_1}{v_2} = 3\dfrac{v_1}{v_1/5} = 15$.

EVALUATE: In the complete absence of friction or other external horizontal forces this process of adding cars and slowing down continues forever.

8.67. **IDENTIFY:** $P_x = p_{Ax} + p_{Bx}$ and $P_y = p_{Ay} + p_{By}$.

SET UP: Let object A be the convertible and object B be the SUV. Let $+x$ be west and $+y$ be south, $p_{Ax} = 0$ and $p_{By} = 0$.

EXECUTE: $P_x = (8000\text{ kg}\cdot\text{m/s})\sin 60.0° = 6928\text{ kg}\cdot\text{m/s}$, so $p_{Bx} = 6928\text{ kg}\cdot\text{m/s}$ and

$v_{Bx} = \dfrac{6928\text{ kg}\cdot\text{m/s}}{2000\text{ kg}} = 3.46\text{ m/s}$.

$P_y = (8000\text{ kg}\cdot\text{m/s})\cos 60.0° = 4000\text{ kg}\cdot\text{m/s}$, so $p_{Bx} = 4000\text{ kg}\cdot\text{m/s}$ and $v_{Ay} = \dfrac{4000\text{ kg}\cdot\text{m/s}}{1500\text{ kg}} = 2.67\text{ m/s}$.

The convertible has speed 2.67 m/s and the SUV has speed 3.46 m/s.

EVALUATE: Each component of the total momentum arises from a single vehicle.

8.68. **IDENTIFY:** The total momentum of the system is conserved and is equal to zero, since the pucks are released from rest.

SET UP: Each puck has the same mass m. Let $+x$ be east and $+y$ be north. Let object A be the puck that moves west. All three pucks have the same speed v.

EXECUTE: $P_{1x} = P_{2x}$ gives $0 = -mv + mv_{Bx} + mv_{Cx}$ and $v = v_{Bx} + v_{Cx}$. $P_{1y} = P_{2y}$ gives $0 = mv_{By} + mv_{Cy}$ and $v_{By} = -v_{Cy}$. Since $v_B = v_C$ and the *y* components are equal in magnitude, the *x* components must also be equal: $v_{Bx} = v_{Cx}$ and $v = v_{Bx} + v_{Cx}$ says $v_{Bx} = v_{Cx} = v/2$. If v_{By} is positive then v_{Cy} is negative. The angle θ that puck *B* makes with the *x* axis is given by $\cos\theta = \dfrac{v/2}{v}$ and $\theta = 60°$. One puck moves in a direction $60°$ north of east and the other puck moves in a direction $60°$ south of east.

EVALUATE: Each component of momentum is separately conserved.

8.69. IDENTIFY: The *x* and *y* components of the momentum of the system are conserved.

Set Up: After the collision the combined object with mass $m_{\text{tot}} = 0.100$ kg moves with velocity $\vec{v}_2$. Solve for v_{Cx} and v_{Cy}.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $m_A v_{Ax} + m_B v_{Bx} + m_C v_{Cx} = m_{\text{tot}} v_{2x}$.

$$v_{Cx} = -\frac{m_A v_{Ax} + m_B v_{Bx} - m_{\text{tot}} v_{2x}}{m_C}$$

$$v_{Cx} = -\frac{(0.020\text{ kg})(-1.50\text{ m/s}) + (0.030\text{ kg})(-0.50\text{ m/s})\cos 60° - (0.100\text{ kg})(0.50\text{ m/s})}{0.050\text{ kg}}.$$

$$v_{Cx} = 1.75\text{ m/s}.$$

$P_{1y} = P_{2y}$ gives $m_A v_{Ay} + m_B v_{By} + m_C v_{Cy} = m_{\text{tot}} v_{2y}$.

$$v_{Cy} = -\frac{m_A v_{Ay} + m_B v_{By} - m_{\text{tot}} v_{2y}}{m_C} = -\frac{(0.030\text{ kg})(-0.50\text{ m/s})\sin 60°}{0.050\text{ kg}} = +0.260\text{ m/s}.$$

(b) $v_C = \sqrt{v_{Cx}^2 + v_{Cy}^2} = 1.77$ m/s. $\Delta K = K_2 - K_1$.

$\Delta K = \frac{1}{2}(0.100\text{ kg})(0.50\text{ m/s})^2 - [\frac{1}{2}(0.020\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}(0.030)(0.50\text{ m/s})^2 + \frac{1}{2}(0.050\text{ kg})(1.77\text{ m/s})^2]$

$\Delta K = -0.092$ J.

EVALUATE: Since there is no horizontal external force the vector momentum of the system is conserved. The forces the spheres exert on each other do negative work during the collision and this reduces the kinetic energy of the system.

8.70. IDENTIFY: Use a coordinate system attached to the ground. Take the *x*-axis to be east (along the tracks) and the *y*-axis to be north (parallel to the ground and perpendicular to the tracks). Then P_x is conserved and P_y is *not* conserved, due to the sideways force exerted by the tracks, the force that keeps the handcar on the tracks.

(a) SET UP: Let *A* be the 25.0 kg mass and *B* be the car (mass 175 kg). After the mass is thrown sideways relative to the car it still has the same eastward component of velocity, 5.00 m/s, as it had before it was thrown.

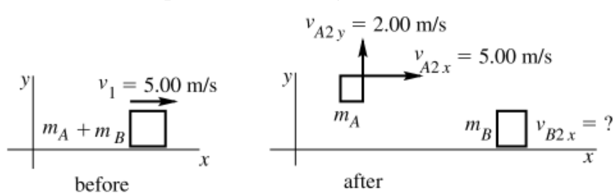


Figure 8.70a

P_x is conserved so $(m_A + m_B)v_1 = m_A v_{A2x} + m_B v_{B2x}$

EXECUTE: $(200\text{ kg})(5.00\text{ m/s}) = (25.0\text{ kg})(5.00\text{ m/s}) + (175\text{ kg})v_{B2x}$.

$$v_{B2x} = \frac{1000\text{ kg}\cdot\text{m/s} - 125\text{ kg}\cdot\text{m/s}}{175\text{ kg}} = 5.00\text{ m/s}.$$

The final velocity of the car is 5.00 m/s, east (unchanged).

EVALUATE: The thrower exerts a force on the mass in the *y*-direction and by Newton's 3rd law the mass exerts an equal and opposite force in the $-y$-direction on the thrower and car.

(b) SET UP: We are applying P_x = constant in coordinates attached to the ground, so we need the final velocity of *A* relative to the ground. Use the relative velocity addition equation. Then use P_x = constant to find the final velocity of the car.

EXECUTE: $\vec{v}_{A/E} = \vec{v}_{A/B} + \vec{v}_{B/E}$

$v_{B/E} = +5.00$ m/s

$v_{A/B} = -5.00$ m/s (minus since the mass is moving west relative to the car). This gives $v_{A/E} = 0;$ the mass is at rest relative to the earth after it is thrown backwards from the car.

As in part (a), $(m_A + m_B)v_1 = m_A v_{A2x} + m_B v_{B2x}$.

Now $v_{A2x} = 0,$ so $(m_A + m_B)v_1 = m_B v_{B2x}$.

$$v_{B2x} = \left(\frac{m_A + m_B}{m_B}\right)v_1 = \left(\frac{200 \text{ kg}}{175 \text{ kg}}\right)(5.00 \text{ m/s}) = 5.71 \text{ m/s}.$$

The final velocity of the car is 5.71 m/s, east.

EVALUATE: The thrower exerts a force in the $-x$-direction so the mass exerts a force on him in the $+x$-direction and he and the car speed up.

(c) SET UP: Let A be the 25.0 kg mass and B be the car (mass $m_B = 200$ kg).

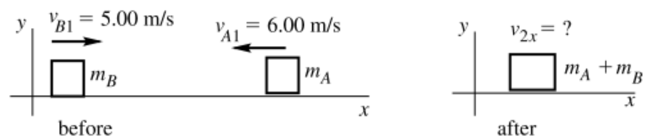


Figure 8.70b

P_x is conserved so $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

EXECUTE: $-m_A v_{A1} + m_B v_{B1} = (m_A + m_B)v_{2x}$.

$$v_{2x} = \frac{m_B v_{B1} - m_A v_{A1}}{m_A + m_B} = \frac{(200 \text{ kg})(5.00 \text{ m/s}) - (25.0 \text{ kg})(6.00 \text{ m/s})}{200 \text{ kg} + 25.0 \text{ kg}} = 3.78 \text{ m/s}.$$

The final velocity of the car is 3.78 m/s, east.

EVALUATE: The mass has negative p_x so reduces the total P_x of the system and the car slows down.

8.71. IDENTIFY: The horizontal component of the momentum of the sand plus railroad system is conserved.

SET UP: As the sand leaks out it retains its horizontal velocity of 15.0 m/s.

EXECUTE: The horizontal component of the momentum of the sand doesn't change when it leaks out so the speed of the railroad car doesn't change; it remains 15.0 m/s. In Exercise 8.27 the rain is falling vertically and initially has no horizontal component of momentum. Its momentum changes as it lands in the freight car. Therefore, in order to conserve the horizontal momentum of the system the freight car must slow down.

EVALUATE: The horizontal momentum of the sand does change when it strikes the ground, due to the force that is external to the system of sand plus railroad car.

8.72. IDENTIFY: Kinetic energy is $K = \frac{1}{2}mv^2$ and the magnitude of the momentum is $p = mv$. The force and the time t it acts are related to the change in momentum whereas the force and distance d it acts are related to the change in kinetic energy.

SET UP: Assume the net forces are constant and let the forces and the motion be along the x axis. The impulse-momentum theorem then says $Ft = \Delta p$ and the work-energy theorem says $Fd = \Delta K$.

EXECUTE: **(a)** $K_{\text{N}} = \frac{1}{2}(840 \text{ kg})(9.0 \text{ m/s})^2 = 3.40 \times 10^4 \text{ J}$. $K_{\text{P}} = \frac{1}{2}(1620 \text{ kg})(5.0 \text{ m/s})^2 = 2.02 \times 10^4 \text{ J}$. The Nash has the greater kinetic energy and $\frac{K_{\text{N}}}{K_{\text{P}}} = 1.68$.

(b) $p_{\text{N}} = (840 \text{ kg})(9.0 \text{ m/s}) = 7.56 \times 10^3 \text{ kg} \cdot \text{m/s}$. $p_{\text{P}} = (1620 \text{ kg})(5.0 \text{ m/s}) = 8.10 \times 10^3 \text{ kg} \cdot \text{m/s}$. The Packard has the greater magnitude of momentum and $\frac{p_{\text{N}}}{p_{\text{P}}} = 0.933$.

(c) Since the cars stop the magnitude of the change in momentum equals the initial momentum. Since $p_{\text{P}} > p_{\text{N}}$, $F_{\text{P}} > F_{\text{N}}$ and $\frac{F_{\text{N}}}{F_{\text{P}}} = \frac{p_{\text{N}}}{p_{\text{P}}} = 0.933$.

(d) Since the cars stop the magnitude of the change in kinetic energy equals the initial kinetic energy. Since $K_{\text{N}} > K_{\text{P}}$, $F_{\text{N}} > F_{\text{P}}$ and $\frac{F_{\text{N}}}{F_{\text{P}}} = \frac{K_{\text{N}}}{K_{\text{P}}} = 1.68$.

EVALUATE: If the stopping forces were the same, the Packard would have a larger stopping time but would travel a shorter distance while stopping. This consistent with it having a smaller initial speed.

8.73. **IDENTIFY:** Use the impulse-momentum theorem to relate the average force on the bullets to their rate of change in momentum. By Newton's third law, the average force the weapon exerts on the bullets is equal in magnitude and opposite in direction to the recoil force the bullets exert on the weapon.

SET UP: Consider a time interval of 1.00 minute. Let $+x$ be the direction of motion of the bullets and use coordinated fixed to the ground. The bullets start from rest.

EXECUTE: $F_{\text{av}}\Delta t = \Delta p$ gives $F_{\text{av}} = \dfrac{(1000)(7.45\times 10^{-3}\text{ kg})(293\text{ m/s})}{60.0\text{ s}} = 36.4\text{ N}$. The recoil force is 36.4 N.

EVALUATE: The change in momentum for each bullet is small since the mass is small, but over 16 bullets are fired per second.

8.74. **IDENTIFY:** Find k for the spring from the forces when the frame hangs at rest, use constant acceleration equations to find the speed of the putty just before it strikes the frame, apply conservation of momentum to the collision between the putty and the frame and then apply conservation of energy to the motion of the frame after the collision.

SET UP: Use the free-body diagram for the frame when it hangs at rest on the end of the spring to find the force constant k of the spring. Let s be the amount the spring is stretched.

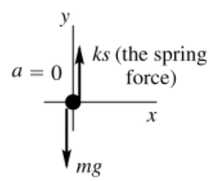


Figure 8.74a

EXECUTE: $\sum F_y = ma_y$.

$-mg + ks = 0$.

$$k = \frac{mg}{s} = \frac{(0.150\text{ kg})(9.80\text{ m/s}^2)}{0.050\text{ m}} = 29.4\text{ N/m}.$$

SET UP: Next find the speed of the putty when it reaches the frame. The putty falls with acceleration $a = g$, downward.

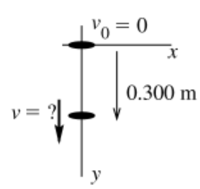


Figure 8.74b

$v_0 = 0$

$y - y_0 = 0.300\text{ m}$

$a = +9.80\text{ m/s}^2$

$v = ?$

$v^2 = v_0^2 + 2a(y - y_0)$

EXECUTE: $v = \sqrt{2a(y - y_0)} = \sqrt{2(9.80\text{ m/s}^2)(0.300\text{ m})} = 2.425\text{ m/s}$.

SET UP: Apply conservation of momentum to the collision between the putty (A) and the frame (B):

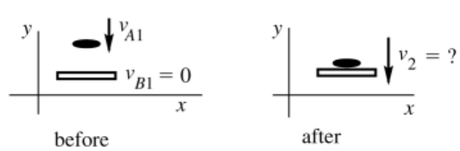


Figure 8.74c

P_y is conserved, so $-m_A v_{A1} = -(m_A + m_B)v_2$.

EXECUTE: $v_2 = \left(\dfrac{m_A}{m_A + m_B}\right)v_{A1} = \left(\dfrac{0.200\text{ kg}}{0.350\text{ kg}}\right)(2.425\text{ m/s}) = 1.386\text{ m/s}$.

SET UP: Apply conservation of energy to the motion of the frame on the end of the spring after the collision. Let point 1 be just after the putty strikes and point 2 be when the frame has its maximum downward displacement. Let d be the amount the frame moves downward.

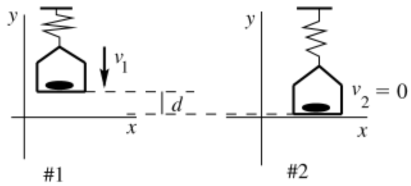


Figure 8.74d

When the frame is at position 1 the spring is stretched a distance $x_1 = 0.050$ m. When the frame is at position 2 the spring is stretched a distance $x_2 = 0.050\text{ m} + d$. Use coordinates with the y-direction upward and $y = 0$ at the lowest point reached by the frame, so that $y_1 = d$ and $y_2 = 0$. Work is done on the frame by gravity and by the spring force, so $W_{\text{other}} = 0$, and $U = U_{\text{el}} + U_{\text{gravity}}$.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

$W_{\text{other}} = 0$.

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.350\text{ kg})(1.386\text{ m/s})^2 = 0.3362\text{ J}$.

$U_1 = U_{1,\text{el}} + U_{1,\text{grav}} = \frac{1}{2}kx_1^2 + mgy_1 = \frac{1}{2}(29.4\text{ N/m})(0.050\text{ m})^2 + (0.350\text{ kg})(9.80\text{ m/s}^2)d$.

$U_1 = 0.03675\text{ J} + (3.43\text{ N})d$.

$U_2 = U_{2,\text{el}} + U_{2,\text{grav}} = \frac{1}{2}kx_2^2 + mgy_2 = \frac{1}{2}(29.4\text{ N/m})(0.050\text{ m} + d)^2$.

$U_2 = 0.03675\text{ J} + (1.47\text{ N})d + (14.7\text{ N/m})d^2$.

Thus $0.3362\text{ J} + 0.03675\text{ J} + (3.43\text{ N})d = 0.03675\text{ J} + (1.47\text{ N})d + (14.7\text{ N/m})d^2$.

$(14.7\text{ N/m})d^2 - (1.96\text{ N})d - 0.3362\text{ J} = 0$.

$d = (1/29.4)\left[1.96 \pm \sqrt{(1.96)^2 - 4(14.7)(-0.3362)}\right]\text{ m} = 0.0667\text{ m} \pm 0.1653\text{ m}$.

The solution we want is a positive (downward) distance, so $d = 0.0667\text{ m} + 0.1653\text{ m} = 0.232\text{ m}$.

EVALUATE: The collision is inelastic and mechanical energy is lost. Thus the decrease in gravitational potential energy is *not* equal to the increase in potential energy stored in the spring.

8.75. **IDENTIFY:** Apply conservation of momentum to the collision and conservation of energy to the motion after the collision.

SET UP: Let $+x$ be to the right. The total mass is $m = m_{\text{bullet}} + m_{\text{block}} = 1.00\text{ kg}$. The spring has force constant $k = \frac{|F|}{|x|} = \frac{0.750\text{ N}}{0.250 \times 10^{-2}\text{ m}} = 300\text{ N/m}$. Let V be the velocity of the block just after impact.

EXECUTE: **(a)** Conservation of energy for the motion after the collision gives $K_1 = U_{\text{el2}}$. $\frac{1}{2}mV^2 = \frac{1}{2}kx^2$ and

$$V = x\sqrt{\frac{k}{m}} = (0.150\text{ m})\sqrt{\frac{300\text{ N/m}}{1.00\text{ kg}}} = 2.60\text{ m/s}.$$

(b) Conservation of momentum applied to the collision gives $m_{\text{bullet}}v_1 = mV$.

$$v_1 = \frac{mV}{m_{\text{bullet}}} = \frac{(1.00\text{ kg})(2.60\text{ m/s})}{8.00 \times 10^{-3}\text{ kg}} = 325\text{ m/s}.$$

EVALUATE: The initial kinetic energy of the bullet is 422 J. The energy stored in the spring at maximum compression is 3.38 J. Most of the initial mechanical energy of the bullet is dissipated in the collision.

8.76. **IDENTIFY:** The horizontal components of momentum of the system of bullet plus stone are conserved. The collision is elastic if $K_1 = K_2$.

SET UP: Let A be the bullet and B be the stone.

(a)

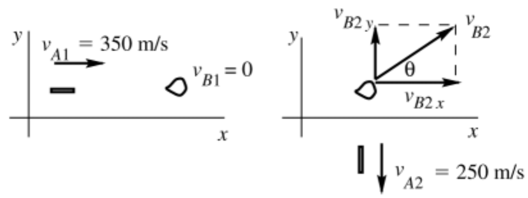


Figure 8.76

EXECUTE: P_x is conserved so $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$.

$m_A v_{A1} = m_B v_{B2x}$.

$$v_{B2x} = \left(\frac{m_A}{m_B}\right) v_{A1} = \left(\frac{6.00\times 10^{-3}\ \text{kg}}{0.100\ \text{kg}}\right)(350\ \text{m/s}) = 21.0\ \text{m/s}$$

P_y is conserved so $m_A v_{A1y} + m_B v_{B1y} = m_A v_{A2y} + m_B v_{B2y}$.

$0 = -m_A v_{A2} + m_B v_{B2y}$.

$$v_{B2y} = \left(\frac{m_A}{m_B}\right) v_{A2} = \left(\frac{6.00\times 10^{-3}\ \text{kg}}{0.100\ \text{kg}}\right)(250\ \text{m/s}) = 15.0\ \text{m/s}.$$

$$v_{B2} = \sqrt{v_{B2x}^2 + v_{B2y}^2} = \sqrt{(21.0\ \text{m/s})^2 + (15.0\ \text{m/s})^2} = 25.8\ \text{m/s}.$$

$\tan\theta = \dfrac{v_{B2y}}{v_{B2x}} = \dfrac{15.0\ \text{m/s}}{21.0\ \text{m/s}} = 0.7143;\ \ \theta = 35.5°$ (defined in the sketch).

(b) To answer this question compare K_1 and K_2 for the system:

$$K_1 = \tfrac{1}{2}m_A v_{A1}^2 + \tfrac{1}{2}m_B v_{B1}^2 = \tfrac{1}{2}(6.00\times 10^{-3}\ \text{kg})(350\ \text{m/s})^2 = 368\ \text{J}.$$

$$K_2 = \tfrac{1}{2}m_A v_{A2}^2 + \tfrac{1}{2}m_B v_{B2}^2 = \tfrac{1}{2}(6.00\times 10^{-3}\ \text{kg})(250\ \text{m/s})^2 + \tfrac{1}{2}(0.100\ \text{kg})(25.8\ \text{m/s})^2 = 221\ \text{J}.$$

$$\Delta K = K_2 - K_1 = 221\ \text{J} - 368\ \text{J} = -147\ \text{J}.$$

EVALUATE: The kinetic energy of the system decreases by 147 J as a result of the collision; the collision is *not* elastic. Momentum is conserved because $\sum F_{\text{ext},x} = 0$ and $\sum F_{\text{ext},y} = 0$. But there are internal forces between the bullet and the stone. These forces do negative work that reduces K.

8.77. **IDENTIFY:** Apply conservation of momentum to the collision between the two people. Apply conservation of energy to the motion of the stuntman before the collision and to the entwined people after the collision.

SET UP: For the motion of the stuntman, $y_1 - y_2 = 5.0$ m. Let v_S be the magnitude of his horizontal velocity just before the collision. Let V be the speed of the entwined people just after the collision. Let d be the distance they slide along the floor.

EXECUTE: **(a)** Motion before the collision: $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$ and $\tfrac{1}{2}mv_S^2 = mg(y_1 - y_2)$.

$v_S = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.80\ \text{m/s}^2)(5.0\ \text{m})} = 9.90\ \text{m/s}$.

Collision: $m_S v_S = m_{\text{tot}} V$. $V = \dfrac{m_S}{m_{\text{tot}}} v_S = \left(\dfrac{80.0\ \text{kg}}{150.0\ \text{kg}}\right)(9.90\ \text{m/s}) = 5.28\ \text{m/s}$.

(b) Motion after the collision: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives $\tfrac{1}{2}m_{\text{tot}}V^2 - \mu_k m_{\text{tot}} g d = 0$.

$d = \dfrac{V^2}{2\mu_k g} = \dfrac{(5.28\ \text{m/s})^2}{2(0.250)(9.80\ \text{m/s}^2)} = 5.7\ \text{m}$.

EVALUATE: Mechanical energy is dissipated in the inelastic collision, so the kinetic energy just after the collision is less than the initial potential energy of the stuntman.

8.78. **IDENTIFY:** Apply conservation of energy to the motion before and after the collision and apply conservation of momentum to the collision.

SET UP: Let v be the speed of the mass released at the rim just before it strikes the second mass. Let each object have mass m.

EXECUTE: Conservation of energy says $\tfrac{1}{2}mv^2 = mgR$; $v = \sqrt{2gR}$.

SET UP: This is speed v_1 for the collision. Let v_2 be the speed of the combined object just after the collision.

EXECUTE: Conservation of momentum applied to the collision gives $mv_1 = 2mv_2$ so $v_2 = v_1/2 = \sqrt{gR/2}$

SET UP: Apply conservation of energy to the motion of the combined object after the collision. Let y_3 be the final height above the bottom of the bowl.

EXECUTE: $\frac{1}{2}(2m)v_2^2 = (2m)gy_3$.

$$y_3 = \frac{v_2^2}{2g} = \frac{1}{2g}\left(\frac{gR}{2}\right) = R/4.$$

EVALUATE: Mechanical energy is lost in the collision, so the final gravitational potential energy is less than the initial gravitational potential energy.

8.79. **IDENTIFY:** Eqs. 8.24 and 8.25 give the outcome of the elastic collision. Apply conservation of energy to the motion of the block after the collision.

SET UP: Object *B* is the block, initially at rest. If *L* is the length of the wire and θ is the angle it makes with the vertical, the height of the block is $y = L(1-\cos\theta)$. Initially, $y_1 = 0$.

EXECUTE: Eq. 8.25 gives $v_B = \left(\frac{2m_A}{m_A + m_B}\right)v_A = \left(\frac{2M}{M+3M}\right)(5.00\text{ m/s}) = 2.50\text{ m/s}$. Conservation of energy gives $\frac{1}{2}m_B v_B^2 = m_B gL(1-\cos\theta)$. $\cos\theta = 1 - \frac{v_B^2}{2gL} = 1 - \frac{(2.50\text{ m/s})^2}{2(9.80\text{ m/s}^2)(0.500\text{ m})} = 0.362$ and $\theta = 68.8°$.

EVALUATE: Only a portion of the initial kinetic energy of the ball is transferred to the block in the collision.

8.80. **IDENTIFY:** Apply conservation of energy to the motion before and after the collision. Apply conservation of momentum to the collision.

SET UP: First consider the motion after the collision. The combined object has mass $m_{\text{tot}} = 25.0$ kg. Apply $\sum\vec{F} = \text{m}\vec{a}$ to the object at the top of the circular loop, where the object has speed v_3. The acceleration is $a_{\text{rad}} = v_3^2/R$, downward.

EXECUTE: $T + mg = m\frac{v_3^2}{R}$.

The minimum speed v_3 for the object not to fall out of the circle is given by setting $T = 0$. This gives $v_3 = \sqrt{Rg}$, where $R = 3.50$ m.

SET UP: Next, use conservation of energy with point 2 at the bottom of the loop and point 3 at the top of the loop. Take $y = 0$ at point 2. Only gravity does work, so $K_2 + U_2 = K_3 + U_3$

EXECUTE: $\frac{1}{2}m_{\text{tot}}v_2^2 = \frac{1}{2}m_{\text{tot}}v_3^2 + m_{\text{tot}}g(2R)$.

Use $v_3 = \sqrt{Rg}$ and solve for v_2: $v_2 = \sqrt{5gR} = 13.1$ m/s.

SET UP: Now apply conservation of momentum to the collision between the dart and the sphere. Let v_1 be the speed of the dart before the collision.

EXECUTE: $(5.00\text{ kg})v_1 = (25.0\text{ kg})(13.1\text{ m/s})$.

$v_1 = 65.5$ m/s.

EVALUATE: The collision is inelastic and mechanical energy is removed from the system by the negative work done by the forces between the dart and the sphere.

8.81. **IDENTIFY:** Use Eq. 8.25 to find the speed of the hanging ball just after the collision. Apply $\sum\vec{F} = m\vec{a}$ to find the tension in the wire. After the collision the hanging ball moves in an arc of a circle with radius $R = 1.35$ m and acceleration $a_{\text{rad}} = v^2/R$.

SET UP: Let *A* be the 2.00 kg ball and *B* be the 8.00 kg ball. For applying $\sum\vec{F} = m\vec{a}$ to the hanging ball, let +*y* be upward, since $\vec{a}_{\text{rad}}$ is upward. The free-body force diagram for the 8.00 kg ball is given in Figure 8.81.

EXECUTE: $v_{B2x} = \left(\frac{2m_A}{m_A + m_B}\right)v_{A1x} = \left(\frac{2[2.00\text{kg}]}{2.00\text{ kg} + 8.00\text{ kg}}\right)(5.00\text{ m/s}) = 2.00\text{ m/s}$. Just after the collision the 8.00 kg ball has speed $v = 2.00$ m/s. Using the free-body diagram, $\sum F_y = ma_y$ gives $T - mg = ma_{\text{rad}}$.

$$T = m\left(g + \frac{v^2}{R}\right) = (8.00\text{ kg})\left(9.80\text{ m/s}^2 + \frac{[2.00\text{ m/s}]^2}{1.35\text{ m}}\right) = 102\text{ N}.$$

EVALUATE: The tension before the collision is the weight of the ball, 78.4 N. Just after the collision, when the ball has started to move, the tension is greater than this.

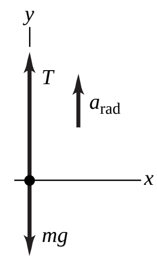


Figure 8.81

8.82. **IDENTIFY:** The impulse applied to the ball equals its change in momentum. The height of the ball and its speed are related by conservation of energy.

SET UP: Let +*y* be upward.

EXECUTE: Applying conservation of energy to the motion of the ball from its height *h* to the floor gives $\frac{1}{2}mv_1^2 = mgh$, where v_1 is its speed just before it hits the floor. Just before it hits, it is traveling downward, so the velocity of the ball just before it hits the floor is $v_{1y} = -\sqrt{2gh}$. Applying conservation of energy to the motion of the ball from just after it bounces off the floor with speed v_2 to its maximum height of $0.90h$ gives $\frac{1}{2}mv_2^2 = mg(0.90h)$. It is moving upward, so $v_{2y} = +\sqrt{2g(0.90h)}$. The impulse applied to the ball is $J_y = p_{2y} - p_{1y} = m(v_{2y} - v_{1y}) =$ $m\sqrt{2g(0.90h)} + m\sqrt{2gh} = 2.76m\sqrt{gh}$. The floor exerts an upward impulse of $2.76m\sqrt{gh}$ to the ball.

EVALUATE: The impulse increases when *m* increases and when *h* increases. The ball does not return to its initial height because some mechanical energy is dissipated during the collision with the floor.

8.83. **IDENTIFY:** Apply conservation of momentum to the collision between the bullet and the block and apply conservation of energy to the motion of the block after the collision.

(a) SET UP: Collision between the bullet and the block: Let object *A* be the bullet and object *B* be the block. Apply momentum conservation to find the speed v_{B2} of the block just after the collision.

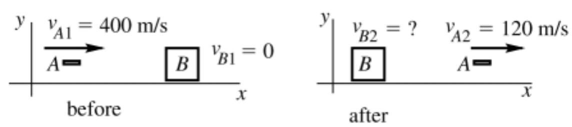


Figure 8.83a

EXECUTE: P_x is conserved so $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$.

$m_A v_{A1} = m_A v_{A2} + m_B v_{B2x}$.

$$v_{B2x} = \frac{m_A(v_{A1} - v_{A2})}{m_B} = \frac{4.00 \times 10^{-3}\ \text{kg}(400\ \text{m/s} - 120\ \text{m/s})}{0.800\ \text{kg}} = 1.40\ \text{m/s}.$$

SET UP: Motion of the block after the collision.

Let point 1 in the motion be just after the collision, where the block has the speed 1.40 m/s calculated above, and let point 2 be where the block has come to rest.

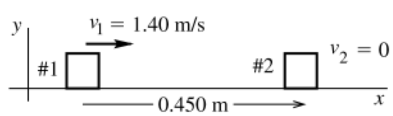


Figure 8.83b

$K_1 + U_1 + W_{other} = K_2 + U_2$.

EXECUTE: Work is done on the block by friction, so $W_{other} = W_f$.

$W_{other} = W_f = (f_k \cos\phi)s = -f_k s = -\mu_k mgs$, where $s = 0.450$ m

$U_1 = 0, \qquad U_2 = 0$

$K_1 = \frac{1}{2}mv_1^2, \qquad K_2 = 0$ (block has come to rest)

Thus $\frac{1}{2}mv_1^2 - \mu_k mgs = 0$.

$$\mu_k = \frac{v_1^2}{2gs} = \frac{(1.40\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)(0.450\ \text{m})} = 0.222.$$

(b) For the bullet,

$$K_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}\left(4.00\times10^{-3}\text{ kg}\right)\left(400\text{ m/s}\right)^2 = 320\text{ J}.$$

$$K_2 = \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}\left(4.00\times10^{-3}\text{ kg}\right)\left(120\text{ m/s}\right)^2 = 28.8\text{ J}.$$

$$\Delta K = K_2 - K_1 = 28.8\text{ J} - 320\text{ J} = -291\text{ J}.$$

The kinetic energy of the bullet decreases by 291 J.

(c) Immediately after the collision the speed of the block is 1.40 m/s so its kinetic energy is $K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.800\text{ kg})(1.40\text{ m/s})^2 = 0.784\text{ J}.$

EVALUATE: The collision is highly inelastic. The bullet loses 291 J of kinetic energy but only 0.784 J is gained by the block. But momentum is conserved in the collision. All the momentum lost by the bullet is gained by the block.

8.84. IDENTIFY: Apply conservation of momentum to the collision and conservation of energy to the motion of the block after the collision.

SET UP: Let $+x$ be to the right. Let the bullet be A and the block be B. Let V be the velocity of the block just after the collision.

EXECUTE: Motion of block after the collision: $K_1 = U_{\text{grav2}}$. $\tfrac{1}{2}m_BV^2 = m_Bgh$.

$$V = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(0.450\times10^{-2}\text{ m})} = 0.297\text{ m/s}.$$

Collision: $v_{B2} = 0.297$ m/s. $P_{1x} = P_{2x}$ gives $m_Av_{A1} = m_Av_{A2} + m_Bv_{B2}$.

$$v_{A2} = \frac{m_Av_{A1} - m_Bv_{B2}}{m_A} = \frac{(5.00\times10^{-3}\text{ kg})(450\text{ m/s}) - (1.00\text{ kg})(0.297\text{ m/s})}{5.00\times10^{-3}\text{ kg}} = 391\text{ m/s}.$$

EVALUATE: We assume the block moves very little during the time it takes the bullet to pass through it.

8.85. IDENTIFY: Eqs. 8.24 and 8.25 give the outcome of the elastic collision. The value of M where the kinetic energy loss K_{loss} of the neutron is a maximum satisfies $dK_{\text{loss}}/dM = 0$.

SET UP: Let object A be the neutron and object B be the nucleus. Let the initial speed of the neutron be v_{A1}. All motion is along the x-axis. $K_0 = \tfrac{1}{2}mv_{A1}^2$.

EXECUTE: **(a)** $v_{A2} = \dfrac{m-M}{m+M}v_{A1}$. $K_{\text{loss}} = \tfrac{1}{2}mv_{A1}^2 - \tfrac{1}{2}mv_{A2}^2 = \tfrac{1}{2}m\left(1 - \left[\dfrac{m-M}{m+M}\right]^2\right)v_{A1}^2 = \dfrac{2m^2M}{(M+m)^2}v_{A1}^2 = \dfrac{4K_0mM}{(M+m)^2}$, as was to be shown.

(b) $\dfrac{dK_{\text{loss}}}{dM} = 4K_0m\left[\dfrac{1}{(M+m)^2} - \dfrac{2M}{(M+m)^3}\right] = 0$. $\dfrac{2M}{M+m} = 1$ and $M = m$. The incident neutron loses the most kinetic energy when the target has the same mass as the neutron.

(c) When $m_A = m_B$, Eq. 8.24 says $v_{A2} = 0$. The final speed of the neutron is zero and the neutron loses all of its kinetic energy.

EVALUATE: When $M >> m$, $v_{A2x} \approx -v_{A1x}$ and the neutron rebounds with speed almost equal to its initial speed. In this case very little kinetic energy is lost; $K_{\text{loss}} = 4K_0m/M$, which is very small.

8.86. IDENTIFY: Eqs. 8.24 and 8.25 give the outcome of the elastic collision.

SET UP: Let all the motion be along the x axis. $v_{A1x} = v_0$.

EXECUTE: **(a)** $v_{A2x} = \left(\dfrac{m_A - m_B}{m_A + m_B}\right)v_0$ and $v_{B2x} = \left(\dfrac{2m_A}{m_A + m_B}\right)v_0$. $K_1 = \tfrac{1}{2}m_Av_0^2$.

$$K_{A2} = \tfrac{1}{2}m_Av_{A2x}^2 = \tfrac{1}{2}m_A\left(\frac{m_A - m_B}{m_A + m_B}\right)^2 v_0^2 = \left(\frac{m_A - m_B}{m_A + m_B}\right)^2 K_1 \text{ and } \frac{K_{A2}}{K_1} = \left(\frac{m_A - m_B}{m_A + m_B}\right)^2.$$

$$K_{B2} = \tfrac{1}{2}m_Bv_{B2x}^2 = \tfrac{1}{2}m_B\left(\frac{2m_A}{m_A + m_B}\right)^2 v_0^2 = \frac{4m_Am_B}{\left(m_A + m_B\right)^2}K_1 \text{ and } \frac{K_{B2}}{K_1} = \frac{4m_Am_B}{\left(m_A + m_B\right)^2}.$$

(b) (i) For $m_A = m_B$, $\dfrac{K_{A2}}{K_1} = 0$ and $\dfrac{K_{B2}}{K_1} = 1$. (ii) For $m_A = 5m_B$, $\dfrac{K_{A2}}{K_1} = \dfrac{4}{9}$ and $\dfrac{K_{B2}}{K_1} = \dfrac{5}{9}$.

(c) Equal sharing of the kinetic energy means $\frac{K_{A2}}{K_1} = \frac{K_{B2}}{K_1} = \frac{1}{2}$. $\left(\frac{m_A - m_B}{m_A + m_B}\right)^2 = \frac{1}{2}$.

$2m_A^2 + 2m_B^2 - 4m_A m_B = m_A^2 + 2m_A m_B + m_B^2$. $m_A^2 - 6m_A m_B + m_B^2 = 0$. The quadratic formula gives $\frac{m_A}{m_B} = 5.83$ or $\frac{m_A}{m_B} = 0.172$. We can also verify that these values give $\frac{K_{B2}}{K_1} = \frac{1}{2}$.

EVALUATE: When $m_A << m_B$ or when $m_A >> m_B$, object *A* retains almost all of the original kinetic energy.

8.87. IDENTIFY: Apply conservation of energy to the motion of the package before the collision and apply conservation of the horizontal component of momentum to the collision.

(a) SET UP: Apply conservation of energy to the motion of the package from point 1 as it leaves the chute to point 2 just before it lands in the cart. Take $y = 0$ at point 2, so $y_1 = 4.00$ m. Only gravity does work, so

$$K_1 + U_1 = K_2 + U_2.$$

EXECUTE: $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2$.

$v_2 = \sqrt{v_1^2 + 2gy_1} = 9.35$ m/s.

(b) SET UP: In the collision between the package and the cart momentum is conserved in the horizontal direction. (But not in the vertical direction, due to the vertical force the floor exerts on the cart.) Take $+x$ to be to the right. Let *A* be the package and *B* be the cart.

EXECUTE: P_x is constant gives $m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$.

$v_{B1x} = -5.00$ m/s.

$v_{A1x} = (3.00 \text{ m/s})\cos 37.0°$. (The horizontal velocity of the package is constant during its free-fall.)

Solving for v_{2x} gives $v_{2x} = -3.29$ m/s. The cart is moving to the left at 3.29 m/s after the package lands in it.

EVALUATE: The cart is slowed by its collision with the package, whose horizontal component of momentum is in the opposite direction to the motion of the cart.

8.88. IDENTIFY: Eqs. 8.24, 8.25, and 8.27 give the outcome of the elastic collision.

SET UP: The blue puck is object *A* and the red puck is object *B*. Let $+x$ be the direction of the initial motion of *A*. $v_{A1x} = 0.200$ m/s, $v_{A2x} = 0.050$ m/s and $v_{B1x} = 0$

EXECUTE: **(a)** Eq. 8.27 gives $v_{B2x} = v_{A2x} - v_{B1x} + v_{A1x} = 0.250$ m/s.

(b) Eq. 8.25 gives $m_B = m_A\left(2\frac{v_{A1x}}{v_{B2x}} - 1\right) = (0.0400 \text{ kg})\left(2\left[\frac{0.200 \text{ m/s}}{0.250 \text{ m/s}}\right] - 1\right) = 0.024$ kg.

EVALUATE: We can verify that our results give $K_1 = K_2$ and $P_{1x} = P_{2x}$, as required in an elastic collision.

8.89. (a) IDENTIFY and **SET UP:** $K = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$.

Use $\vec{v}_A = \vec{v}'_A + \boldsymbol{v}_{\text{cm}}$ and $\vec{v}_B = \vec{v}'_B + \boldsymbol{v}_{\text{cm}}$ to replace v_A and v_B in this equation. Note $\vec{v}'_A$ and $\vec{v}'_B$ as defined in the problem are the velocities of *A* and *B* in coordinates moving with the center of mass. Note also that $m_A\vec{v}'_A + m_B\vec{v}'_B = M\vec{v}'_{\text{cm}}$ where $\vec{v}'_{\text{cm}}$ is the velocity of the car in these coordinates. But that's zero, so $m_A\vec{v}'_A + m_B\vec{v}'_B = 0$; we can use this in the proof.

In part (b), use that $\vec{P}$ is conserved in a collision.

EXECUTE: $\vec{v}_A = \vec{v}'_A + \boldsymbol{v}_{\text{cm}}$, so $v_A^2 = v_A'^2 + v_{\text{cm}}^2 + 2\vec{v}'_A \cdot \vec{v}_{\text{cm}}$.

$$\vec{v}_B = \vec{v}'_B + \boldsymbol{v}_{\text{cm}}, \text{ so } v_B^2 = v_B'^2 + v_{\text{cm}}^2 + 2\vec{v}'_B \cdot \vec{v}_{\text{cm}}.$$

(We have used that for a vector $\vec{A}$, $A^2 = \vec{A} \cdot \vec{A}$.)

Thus $K = \frac{1}{2}m_A v_A'^2 + \frac{1}{2}m_A v_{\text{cm}}^2 + m_A\vec{v}'_A \cdot \vec{v}_{\text{cm}} + \frac{1}{2}m_B v_B'^2 + \frac{1}{2}m_B v_{\text{cm}}^2 + m_B\vec{v}'_B \cdot \vec{v}_{\text{cm}}$.

$$K = \tfrac{1}{2}(m_A + m_B)v_{\text{cm}}^2 + \tfrac{1}{2}\left(m_A v_A'^2 + m_B v_B'^2\right) + \left(m_A\vec{v}'_A + m_B\vec{v}'_B\right) \cdot \vec{v}_{\text{cm}}.$$

But $m_A + m_B = M$ and as noted earlier $m_A\vec{v}'_A + m_B\vec{v}'_B = 0$, so $K = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}\left(m_A v_A'^2 + m_B v_B'^2\right)$. This is the result the problem asked us to derive.

(b) EVALUATE: In the collision $\vec{P} = M\vec{v}_{\text{cm}}$ is constant, so $\frac{1}{2}Mv_{\text{cm}}^2$ stays constant. The asteroids can lose all their relative kinetic energy but the $\frac{1}{2}Mv_{\text{cm}}^2$ must remain.

8.90. **IDENTIFY:** Eq. 8.27 describes the elastic collision, with *x* replaced by *y*. Speed and height are related by conservation of energy.

SET UP: Let +*y* be upward. Let *A* be the large ball and *B* be the small ball, so $v_{B1y} = -v$ and $v_{A1y} = +v$. If the large ball has much greater mass than the small ball its speed is changed very little in the collision and $v_{A2y} = +v$.

EXECUTE: **(a)** $v_{B2y} - v_{A2y} = -(v_{B1y} - v_{A1y})$ gives $v_{B2y} = +v_{A2y} - v_{B1y} + v_{A1y} = v - (-v) + v = +3v$. The small ball moves upward with speed 3*v* after the collision.

(b) Let h_1 be the height the small ball fell before the collision. Conservation of energy applied to the motion from the release point to the floor gives $U_1 = K_2$ and $mgh_1 = \frac{1}{2}mv^2$. $h_1 = \dfrac{v^2}{2g}$. Conservation of energy applied to the motion of the small ball from immediately after the collision to its maximum height h_2 (rebound distance) gives $K_1 = U_2$ and $\frac{1}{2}m(3v)^2 = mgh_2$. $h_2 = \dfrac{9v^2}{2g} = 9h_1$. The ball's rebound distance is nine times the distance it fell.

EVALUATE: The mechanical energy gained by the small ball comes from the energy of the large ball. But since the large ball's mass is much larger it can give up this energy with very little decrease in speed.

8.91. **IDENTIFY:** Apply conservation of momentum to the system consisting of Jack, Jill and the crate. The speed of Jack or Jill relative to the ground will be different from 4.00 m/s.

SET UP: Use an inertial coordinate system attached to the ground. Let +*x* be the direction in which the people jump. Let Jack be object *A*, Jill be *B*, and the crate be *C*.

EXECUTE: **(a)** If the final speed of the crate is *v*, $v_{C2x} = -v$, and $v_{A2x} = v_{B2x} = 4.00 \text{ m/s} - v$. $P_{2x} = P_{1x}$ gives $m_A v_{A2x} + m_B v_{Bx2} + m_C v_{Cx2} = 0$. $(75.0 \text{ kg})(4.00 \text{ m/s} - v) + (45.0 \text{ kg})(4.00 \text{ m/s} - v) + (15.0 \text{ kg})(-v) = 0$ and

$$v = \frac{(75.0 \text{ kg} + 45.0 \text{ kg})(4.00 \text{ m/s})}{75.0 \text{ kg} + 45.0 \text{ kg} + 15.0 \text{ kg}} = 3.56 \text{ m/s}.$$

(b) Let v' be the speed of the crate after Jack jumps. Apply momentum conservation to Jack jumping: $(75.0 \text{ kg})(4.00 \text{ m/s} - v') + (60.0 \text{ kg})(-v') = 0$ and $v' = \dfrac{(75.0 \text{ kg})(4.00 \text{ m/s})}{135.0 \text{ kg}} = 2.22 \text{ m/s}$. Then apply momentum conservation to Jill jumping, with *v* being the final speed of the crate: $P_{1x} = P_{2x}$ gives $(60.0 \text{ kg})(-v') = (45.0 \text{ kg})(4.00 \text{ m/s} - v) + (15.0 \text{ kg})(-v)$.

$$v = \frac{(45.0 \text{ kg})(4.00 \text{ m/s}) + (60.0 \text{ kg})(2.22 \text{ m/s})}{60.0 \text{ kg}} = 5.22 \text{ m/s}.$$

(c) Repeat the calculation in **(b)**, but now with Jill jumping first.
Jill jumps: $(45.0 \text{ kg})(4.00 \text{ m/s} - v') + (90.0 \text{ kg})(-v') = 0$ and $v' = 1.33 \text{ m/s}$.
Jack jumps: $(90.0 \text{ kg})(-v') = (75.0 \text{ kg})(4.00 \text{ m/s} - v) + (15.0 \text{ kg})(-v)$.

$$v = \frac{(75.0 \text{ kg})(4.00 \text{ m/s}) + (90.0 \text{ kg})(1.33 \text{ m/s})}{90.0 \text{ kg}} = 4.66 \text{ m/s}.$$

EVALUATE: The final speed of the crate is greater when Jack jumps first, then Jill. In this case Jack leaves with a speed of 1.78 m/s relative to the ground, whereas when they both jump simultaneously Jack and Jill each leave with a speed of only 0.44 m/s relative to the ground.

8.92. **IDENTIFY:** Momentum is conserved in the explosion. The total kinetic energy of the two fragments is *Q*.

SET UP: Let the final speed of the two fragments be v_A and v_B. They must move in opposite directions after the explosion.

EXECUTE: **(a)** Since the initial momentum of the system is zero, conservation of momentum says $m_A v_A = m_B v_B$ and $v_B = \left(\dfrac{m_A}{m_B}\right) v_A$. $K_A + K_B = Q$ gives $\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B\left(\dfrac{m_A}{m_B}\right)^2 v_A^2 = Q$. $\frac{1}{2}m_A v_A^2\left(1 + \dfrac{m_A}{m_B}\right) = Q$.

$K_A = \dfrac{Q}{1 + m_A/m_B} = \left(\dfrac{m_B}{m_A + m_B}\right)Q$. $K_B = Q - K_A = Q\left(1 - \dfrac{m_B}{m_A + m_B}\right) = \left(\dfrac{m_A}{m_A + m_B}\right)Q$.

(b) If $m_B = 4m_A$, then $K_A = \dfrac{4}{5}Q$ and $K_B = \dfrac{1}{5}Q$. The lighter fragment gets 80% of the energy that is released.

EVALUATE: If $m_A = m_B$ the fragments share the energy equally. In the limit that $m_B >> m_A$, the lighter fragment gets almost all of the released energy.

8.93. **IDENTIFY:** Apply conservation of momentum to the system of the neutron and its decay products.

SET UP: Let the proton be moving in the +x direction with speed v_p after the decay. The initial momentum of the neutron is zero, so to conserve momentum the electron must be moving in the $-x$ direction after the decay. Let the speed of the electron be v_e.

EXECUTE: $P_{1x} = P_{2x}$ gives $0 = m_\text{p} v_\text{p} - m_\text{e} v_\text{e}$ and $v_\text{e} = \left(\dfrac{m_\text{p}}{m_\text{e}}\right) v_\text{p}$. The total kinetic energy after the decay is

$$K_\text{tot} = \tfrac{1}{2} m_\text{e} v_\text{e}^2 + \tfrac{1}{2} m_\text{p} v_\text{p}^2 = \tfrac{1}{2} m_\text{e} \left(\frac{m_\text{p}}{m_\text{e}}\right)^2 v_\text{p}^2 + \tfrac{1}{2} m_\text{p} v_\text{p}^2 = \tfrac{1}{2} m_\text{p} v_\text{p}^2 \left(1 + \frac{m_\text{p}}{m_\text{e}}\right).$$

Thus, $\dfrac{K_\text{p}}{K_\text{tot}} = \dfrac{1}{1 + m_\text{p}/m_\text{e}} = \dfrac{1}{1 + 1836} = 5.44 \times 10^{-4} = 0.0544\%$.

EVALUATE: Most of the released energy goes to the electron, since it is much lighter than the proton.

8.94. **IDENTIFY:** Momentum is conserved in the decay. The results of Problem 8.92 give the kinetic energy of each fragment.

SET UP: Let A be the alpha particle and let B be the radium nucleus, so $m_A / m_B = 0.0176$. $Q = 6.54 \times 10^{-13}$ J.

EXECUTE: $K_A = \dfrac{Q}{1 + m_A / m_B} = \dfrac{6.54 \times 10^{-13}\ \text{J}}{1 + 0.0176} = 6.43 \times 10^{-13}$ J and $K_B = 0.11 \times 10^{-13}$ J.

EVALUATE: The lighter particle receives most of the released energy.

8.95. **IDENTIFY:** The momentum of the system is conserved.

SET UP: Let +x be to the right. $P_{1x} = 0$. p_{ex}, $p_{\text{n}x}$ and $p_{\text{an}x}$ are the momenta of the electron, polonium nucleus and antineutrino, respectively.

EXECUTE: $P_{1x} = P_{2x}$ gives $p_{\text{ex}} + p_{\text{n}x} + p_{\text{an}x} = 0$. $p_{\text{an}x} = -(p_{\text{ex}} + p_{\text{n}x})$.

$p_{\text{an}x} = -(5.60 \times 10^{-22}\ \text{kg} \cdot \text{m/s} + [3.50 \times 10^{-25}\ \text{kg}][-1.14 \times 10^{3}\ \text{m/s}]) = -1.66 \times 10^{-22}\ \text{kg} \cdot \text{m/s}$.

The antineutrino has momentum to the left with magnitude $1.66 \times 10^{-22}\ \text{kg} \cdot \text{m/s}$.

EVALUATE: The antineutrino interacts very weakly with matter and most easily shows its presence by the momentum it carries away.

8.96. **IDENTIFY:** Momentum components in the x and y directions are separately conserved. For an elastic collision $K_1 = K_2$.

SET UP: $v_{A1x} = +v_{A1}$, $v_{B1x} = 0$. $v_{A2x} = v_{A2} \cos\alpha$, $v_{A2y} = v_{A2} \sin\alpha$. $v_{B2x} = v_{B2} \cos\alpha$, $v_{B2y} = -v_{B2} \sin\alpha$.

$\sin^2\theta + \cos^2\theta = 1$, for any angle θ. $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$.

EXECUTE: **(a)** $P_{1x} = P_{2x}$ gives $m_A v_{A1} = m_A v_{A2} \cos\alpha + m_B v_{B2} \cos\beta$.

$P_{1y} = P_{2y}$ gives $0 = m_A v_{A2} \sin\alpha - m_B v_{B2} \sin\beta$.

(b) $m_A^2 v_{A1}^2 = m_A^2 v_{A2}^2 \cos^2\alpha + m_B^2 v_{B2}^2 \cos^2\beta + 2 m_A m_B v_{A2} v_{B2} \cos\alpha \cos\beta$ and

$0 = m_A^2 v_{A2}^2 \sin^2\alpha + m_B^2 v_{B2}^2 \sin^2\beta - 2 m_A m_B v_{A2} v_{B2} \sin\alpha \sin\beta$. Adding these two equations and using the trig identities in the SET UP step gives $m_A^2 v_{A1}^2 = m_A^2 v_{A2}^2 + m_B^2 v_{B2}^2 + 2 m_A m_B v_{A2} v_{B2} \cos(\alpha + \beta)$.

(c) $K_1 = K_2$ says $\tfrac{1}{2} m_A v_{A1}^2 = \tfrac{1}{2} m_A v_{A2}^2 + \tfrac{1}{2} m_B v_{B2}^2$. The result in part (b) agrees with this expression only if

$\cos(\alpha + \beta) = 0$. This requires that $\alpha + \beta = 90° = \dfrac{\pi}{2}$ rad.

EVALUATE: The result of part (c) says that the two protons move in perpendicular directions after the collision.

8.97. **IDENTIFY** and **SET UP:**

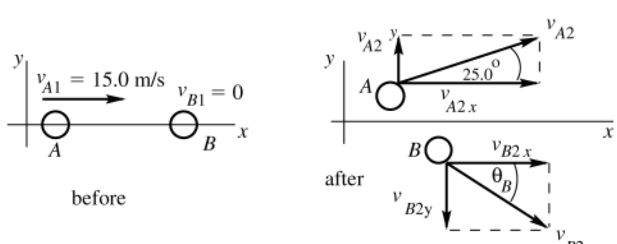


Figure 8.97

P_x and P_y are conserved in the collision since there is no external horizontal force.

The result of Problem 8.96 part (d) applies here since the collision is elastic This says that $25.0° + \theta_B = 90°$, so that $\theta_B = 65.0°$. (*A* and *B* move off in perpendicular directions.)

EXECUTE: P_x is conserved so $m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$.

But $m_A = m_B$ so $v_{A1} = v_{A2}\cos 25.0° + v_{B2}\cos 65.0°$.

$$P_y \text{ is conserved so } m_A v_{A1y} + m_B v_{B1y} = m_A v_{A2y} + m_B v_{B2y}.$$

$0 = v_{A2y} + v_{B2y}$.

$0 = v_{A2}\sin 25.0° - v_{B2}\sin 65.0°$.

$v_{B2} = (\sin 25.0°/\sin 65.0°)v_{A2}$.

This result in the first equation gives $v_{A1} = v_{A2}\cos 25.0° + \left(\dfrac{\sin 25.0°\cos 65.0°}{\sin 65.0°}\right)v_{A2}$.

$v_{A1} = 1.103 v_{A2}$.

$v_{A2} = v_{A1}/1.103 = (15.0\text{ m/s})/1.103 = 13.6\text{ m/s}$.

And then $v_{B2} = (\sin 25.0°/\sin 65.0°)(13.6\text{ m/s}) = 6.34\text{ m/s}$.

EVALUATE: We can use our numerical results to show that $K_1 = K_2$ and that $P_{1x} = P_{2x}$ and $P_{1y} = P_{2y}$.

8.98. **IDENTIFY:** Since there is no friction, the horizontal component of momentum of the system of Jonathan, Jane and the sleigh is conserved.

SET UP: Let +*x* be to the right. $w_A = 800\text{ N}$, $w_B = 600\text{ N}$ and $w_C = 1000\text{ N}$.

EXECUTE: $P_{1x} = P_{2x}$ gives $0 = m_A v_{A2x} + m_B v_{B2x} + m_C v_{C2x}$. $v_{C2x} = \dfrac{m_A v_{A2x} + m_B v_{B2x}}{m_C} = \dfrac{w_A v_{A2x} + w_B v_{B2x}}{w_C}$.

$$v_{C2x} = \frac{(800\text{ N})(-[5.00\text{ m/s}]\cos 30.0°) + (600\text{ N})(+[7.00\text{ m/s}]\cos 36.9°)}{1000\text{ N}} = -0.105\text{ m/s}.$$

The sleigh's velocity is 0.105 m/s, to the left.

EVALUATE: The vertical component of the momentum of the system consisting of the two people and the sleigh is not conserved, because of the net force exerted on the sleigh by the ice while they jump.

8.99. **IDENTIFY:** In Eq. 8.28 treat each straight piece as an object in the system.

SET UP: The center of mass of each piece of length *L* is at its center.

EXECUTE: **(a)** From symmetry, the center of mass is on the vertical axis, a distance $(L/2)\cos(\alpha/2)$ below the apex.

(b) The center of mass is on the vertical axis of symmetry, a distance $2(L/2)/3 = L/3$ above the center of the horizontal segment.

(c) Using the wire frame as a coordinate system, the coordinates of the center of mass are equal and each is equal to $(L/2)/2 = L/4$. The center of mass is along the bisector of the angle, a distance $L/\sqrt{8}$ from the corner.

(d) By symmetry, the center of mass is at the center of the equilateral triangle, a distance $(L/3)\sin 60° = L/\sqrt{12}$ above the center of the horizontal segment.

EVALUATE: The center of mass need not lie on any point of the object, it can be in empty space.

8.100. **IDENTIFY:** There is no net horizontal external force so v_{cm} is constant.

SET UP: Let +*x* be to the right, with the origin at the initial position of the left-hand end of the canoe. $m_A = 45.0\text{ kg}$, $m_B = 60.0\text{ kg}$. The center of mass of the canoe is at its center.

EXECUTE: Initially, $v_{\text{cm}} = 0$, so the center of mass doesn't move. Initially, $x_{\text{cm1}} = \dfrac{m_A x_{A1} + m_B x_{B1}}{m_A + m_B}$. After she walks, $x_{\text{cm2}} = \dfrac{m_A x_{A2} + m_B x_{B2}}{m_A + m_B}$. $x_{\text{cm1}} = x_{\text{cm2}}$ gives $m_A x_{A1} + m_B x_{B1} = m_A x_{A2} + m_B x_{B2}$. She walks to a point 1.00 m from the right-hand end of the canoe, so she is 1.50 m to the right of the center of mass of the canoe and $x_{A2} = x_{B2} + 1.50\text{ m}$.

$$(45.0\text{ kg})(1.00\text{ m}) + (60.0\text{ kg})(2.50\text{ m}) = (45.0\text{ kg})(x_{B2} + 1.50\text{ m}) + (60.0\text{ kg})x_{B2}.$$

$(105.0\text{ kg})x_{B2} = 127.5\text{ kg}\cdot\text{m}$ and $x_{B2} = 1.21\text{ m}$. $x_{B2} - x_{B1} = 1.21\text{ m} - 2.50\text{ m} = -1.29\text{ m}$. The canoe moves 1.29 m to the left.

EVALUATE: When the woman walks to the right, the canoe moves to the left. The woman walks 3.00 m to the right relative to the canoe and the canoe moves 1.29 m to the left, so she moves $3.00\text{ m} - 1.29\text{ m} = 1.71\text{ m}$ to the right relative to the water. Note that this distance is $(60.0\text{ kg}/45.0\text{ kg})(1.29\text{ m})$.

8.101. **IDENTIFY:** Take as the system you and the slab. There is no horizontal force, so horizontal momentum is conserved. By Eq. 8.32, $\vec{P}$ is constant $\vec{v}_{\text{cm}}$ is constant (for a system of constant mass). Use coordinates fixed to the ice, with the direction you walk as the x-direction. $\vec{v}_{\text{cm}}$ is constant and initially $\vec{v}_{\text{cm}} = 0$.

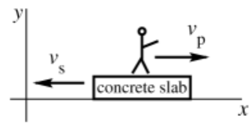


Figure 8.101

$$\vec{v}_{\text{cm}} = \frac{m_{\text{p}}\vec{v}_{\text{p}} + m_{\text{s}}\vec{v}_{\text{s}}}{m_{\text{p}} + m_{\text{s}}} = \mathbf{0}.$$

$$m_{\text{p}}\vec{v}_{\text{cm}} + m_{\text{s}}\vec{v}_{\text{s}} = \mathbf{0}.$$

$$m_{\text{p}}v_{\text{p}x} + m_{\text{s}}v_{\text{s}x} = 0.$$

$$v_{\text{s}x} = -\left(m_{\text{p}}/m_{\text{s}}\right)v_{\text{p}x} = -\left(m_{\text{p}}/5m_{\text{p}}\right)2.00\text{ m/s} = -0.400\text{ m/s}.$$

The slab moves at $0.400\text{ m/s},$ in the direction opposite to the direction you are walking.

EVALUATE: The initial momentum of the system is zero. You gain momentum in the $+x$-direction so the slab gains momentum in the $-x$-direction. The slab exerts a force on you in the $+x$-direction so you exert a force on the slab in the $-x$-direction.

8.102. **IDENTIFY:** Conservation of x and y components of momentum applies to the collision. At the highest point of the trajectory the vertical component of the velocity of the projectile is zero.

SET UP: Let $+y$ be upward and $+x$ be horizontal and to the right. Let the two fragments be A and B, each with mass m. For the projectile before the explosion and the fragments after the explosion. $a_x = 0$, $a_y = -9.80\text{ m/s}^2$.

EXECUTE: **(a)** $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ with $v_y = 0$ gives that the maximum height of the projectile is

$h = -\dfrac{v_{0y}^2}{2a_y} = -\dfrac{([80.0\text{ m/s}]\sin 60.0°)^2}{2(-9.80\text{ m/s}^2)} = 244.9\text{ m}$. Just before the explosion the projectile is moving to the right with horizontal velocity $v_x = v_{0x} = v_0\cos 60.0° = 40.0\text{ m/s}$. After the explosion $v_{Ax} = 0$ since fragment A falls vertically. Conservation of momentum applied to the explosion gives $(2m)(40.0\text{ m/s}) = mv_{Bx}$ and $v_{Bx} = 80.0\text{ m/s}$. Fragment B has zero initial vertical velocity so $y - y_0 = v_{0y}t + \frac{1}{2}a_yt^2$ gives a time of fall of

$t = \sqrt{-\dfrac{2h}{a_y}} = \sqrt{-\dfrac{2(244.9\text{ m})}{-9.80\text{ m/s}^2}} = 7.07\text{ s}$. During this time the fragment travels horizontally a distance $(80.0\text{ m/s})(7.07\text{ s}) = 566\text{ m}$. It also took the projectile 7.07 s to travel from launch to maximum height and during this time it travels a horizontal distance of $([80.0\text{ m/s}]\cos 60.0°)(7.07\text{ s}) = 283\text{ m}$. The second fragment lands $283\text{ m} + 566\text{ m} = 849\text{ m}$ from the firing point.

(b) For the explosion, $K_1 = \frac{1}{2}(20.0\text{ kg})(40.0\text{ m/s})^2 = 1.60 \times 10^4\text{ J}$. $K_2 = \frac{1}{2}(10.0\text{ kg})(80.0\text{ m/s})^2 = 3.20 \times 10^4\text{ J}$. The energy released in the explosion is $1.60 \times 10^4\text{ J}$.

EVALUATE: The kinetic energy of the projectile just after it is launched is $6.40 \times 10^4\text{ J}$. We can calculate the speed of each fragment just before it strikes the ground and verify that the total kinetic energy of the fragments just before they strike the ground is $6.40 \times 10^4\text{ J} + 1.60 \times 10^4\text{ J} = 8.00 \times 10^4\text{ J}$. Fragment A has speed 69.3 m/s just before it strikes the ground, and hence has kinetic energy $2.40 \times 10^4\text{ J}$. Fragment B has speed $\sqrt{(80.0\text{ m/s})^2 + (69.3\text{ m/s})^2} = 105.8\text{ m/s}$ just before it strikes the ground, and hence has kinetic energy $5.60 \times 10^4\text{ J}$. Also, the center of mass of the system has the same horizontal range $R = \dfrac{v_0^2}{g}\sin(2\alpha_0) = 565\text{ m}$ that the projectile would have had if no explosion had occurred. One fragment lands at $R/2$ so the other, equal mass fragment lands at a distance $3R/2$ from the launch point.

8.103. **IDENTIFY:** The rocket moves in projectile motion before the explosion and its fragments move in projectile motion after the explosion. Apply conservation of energy and conservation of momentum to the explosion.

SET UP: Apply conservation of energy to the explosion. Just before the explosion the shell is at its maximum height and has zero kinetic energy. Let *A* be the piece with mass 1.40 kg and *B* be the piece with mass 0.28 kg. Let v_A and v_B be the speeds of the two pieces immediately after the collision.

EXECUTE: $\frac{1}{2}m_Av_A^2+\frac{1}{2}m_Bv_B^2=860\text{ J}$

SET UP: Since the two fragments reach the ground at the same time, their velocities just after the explosion must be horizontal. The initial momentum of the shell before the explosion is zero, so after the explosion the pieces must be moving in opposite horizontal directions and have equal magnitude of momentum: $m_Av_A=m_Bv_B$.

EXECUTE: Use this to eliminate v_A in the first equation and solve for v_B:

$$\tfrac{1}{2}m_Bv_B^2\left(1+m_B/m_A\right)=860\text{ J and } v_B=71.6\text{ m/s.}$$

Then $v_A=\left(m_B/m_A\right)v_B=14.3\text{ m/s.}$

(b) SET UP: Use the vertical motion from the maximum height to the ground to find the time it takes the pieces to fall to the ground after the explosion. Take $+y$ downward.

$$v_{0y}=0,\quad a_y=+9.80\text{ m/s}^2,\quad y-y_0=80.0\text{ m},\quad t=?$$

EXECUTE: $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $t=4.04$ s.

During this time the horizontal distance each piece moves is $x_A=v_At=57.8\text{ m}$ and $x_B=v_Bt=289.1\text{ m}$. They move in opposite directions, so they are $x_A+x_B=347\text{ m}$ apart when they land.

EVALUATE: Fragment *A* has more mass so it is moving slower right after the collision, and it travels horizontally a smaller distance as it falls to the ground.

8.104. IDENTIFY: Apply conservation of momentum to the collision. At the highest point of its trajectory the shell is moving horizontally. If one fragment received some upward momentum in the collision, the other fragment would have had to receive a downward component. Since they each the ground at the same time, each must have zero vertical velocity immediately after the explosion.

SET UP: Let +*x* be horizontal, along the initial direction of motion of the projectile and let +*y* be upward. At its maximum height the projectile has $v_x=v_0\cos 55.0°=86.0\text{ m/s}$. Let the heavier fragment be *A* and the lighter fragment be *B*. $m_A=9.00\text{ kg}$ and $m_B=3.00\text{ kg}$.

EXECUTE: Since fragment *A* returns to the launch point, immediately after the explosion it has $v_{Ax}=-86.0\text{ m/s}$. Conservation of momentum applied to the explosion gives $(12.0\text{ kg})(86.0\text{ m/s})=(9.00\text{ kg})(-86.0\text{ m/s})+(3.00\text{ kg})v_{Bx}$ and $v_{Bx}=602\text{ m/s}$. The horizontal range of the projectile, if no explosion occurred, would be $R=\dfrac{v_0^2}{g}\sin(2\alpha_0)=2157\text{ m}$. The horizontal distance each fragment travels is proportional to its initial speed and the heavier fragment travels a horizontal distance $R/2=1078\text{ m}$ after the explosion, so the lighter fragment travels a horizontal distance $\left(\dfrac{602\text{ m}}{86\text{ m}}\right)(1078\text{ m})=7546\text{ m}$ from the point of explosion and $1078\text{ m}+7546\text{ m}=8624\text{ m}$ from the launch point. The energy released in the explosion is $K_2-K_1=\frac{1}{2}(9.00\text{ kg})(86.0\text{ m/s})^2+\frac{1}{2}(3.00\text{ kg})(602\text{ m/s})^2-\frac{1}{2}(12.0\text{ kg})(86.0\text{ m/s})^2=5.33\times10^5\text{ J}$.

EVALUATE: The center of mass of the system has the same horizontal range $R=2157\text{ m}$ as if the explosion didn't occur. This gives $(12.0\text{ kg})(2157\text{ m})=(9.00\text{ kg})(0)+(3.00\text{ kg})d$ and $d=8630\text{ m}$, where *d* is the distance from the launch point to where the lighter fragment lands. This agrees with our calculation.

8.105. IDENTIFY: No external force, so $\vec{P}$ is conserved in the collision.

SET UP: Apply momentum conservation in the *x* and *y* directions:

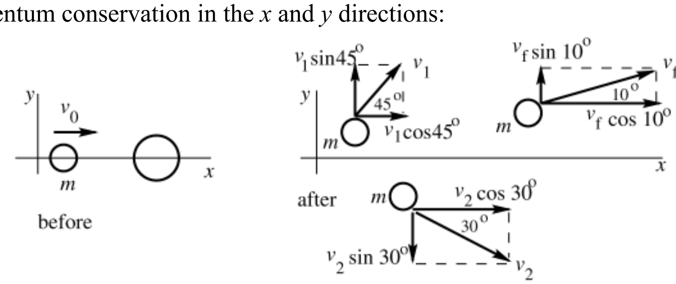


Figure 8.105

Solve for v_1 and v_2.

EXECUTE: P_x is conserved so $mv_0 = m\left(v_1\cos45° + v_f\cos10° + v_2\cos30°\right)$.

$v_0 - v_f\cos10° = v_1\cos45° + v_2\cos30°$.

$1030.4\ \text{m/s} = v_1\cos45° + v_2\cos30°$.

P_x is conserved so $0 = m\left(v_1\sin45° - v_2\sin30° + v_f\sin10°\right)$.

$v_1\sin45° = v_2\sin30° - 347.3\ \text{m/s}$.

$\sin45° = \cos45°$ so

$1030.4\ \text{m/s} = v_2\sin30° - 347.3\ \text{m/s} + v_2\cos30°$.

$v_2 = \dfrac{1030.4\ \text{m/s} + 347.3\ \text{m/s}}{\sin30° + \cos30.0°} = 1010\ \text{m/s}$.

And then $v_1 = \dfrac{v_2\sin30° - 347.3\ \text{m/s}}{\sin45°} = 223\ \text{m/s}$. Then two emitted neutrons have speeds of $223\ \text{m/s}$ and $1010\ \text{m/s}$.

The speeds of the Ba and Kr nuclei are related by P_z conservation.

P_z is constant implies that $0 = m_{\text{Ba}}v_{\text{Ba}} - m_{\text{Kr}}v_{\text{Kr}}$

$v_{\text{Kr}} = \left(\dfrac{m_{\text{Ba}}}{m_{\text{Kr}}}\right)v_{\text{Ba}} = \left(\dfrac{2.3\times10^{-25}\ \text{kg}}{1.5\times10^{-25}\ \text{kg}}\right)v_{\text{Ba}} = 1.5v_{\text{Ba}}$.

We can't say what these speeds are but they must satisfy this relation. The value of v_{Ba} depends on energy considerations.

EVALUATE: $K_1 = \tfrac{1}{2}m_n\left(3.0\times10^3\ \text{m/s}\right)^2 = \left(4.5\times10^6\ \text{J/kg}\right)m_n$.

$$K_2 = \tfrac{1}{2}m_n\left(2.0\times10^3\ \text{m/s}\right)^2 + \tfrac{1}{2}m_n\left(223\ \text{m/s}\right)^2 + \tfrac{1}{2}m_n\left(1010\ \text{m/s}\right)^2 + K_{\text{Ba}} + K_{\text{Kr}} = \left(2.5\times10^6\ \text{J/kg}\right)m_n + K_{\text{Ba}} + K_{\text{Kr}}.$$

We don't know what K_{Ba} and K_{Kr} are, but they are positive. We will study such nuclear reactions further in Chapter 43 and will find that energy is released in this process; $K_2 > K_1$. Some of the potential energy stored in the ^{235}U nucleus is released as kinetic energy and shared by the collision fragments.

8.106. **IDENTIFY:** The velocity of the center of mass of the system of the two blocks is given by Eq. 8.30. Conservation of momentum says the center of mass moves at constant speed.

SET UP: $v_{A1x} = v_{A1}$, $v_{B1x} = 0$. The velocity $\vec{u}$ in the center of mass frame is related to the velocity $\vec{v}$ in the stationary frame by $\vec{u} = \vec{v} - \vec{v}_{\text{cm}}$. We can express kinetic energy as $K = \dfrac{p^2}{2m}$.

EXECUTE: **(a)** $v_{\text{cm-}x} = \dfrac{m_A v_{A1}}{m_A + m_B}$.

(b) The center of mass moves with constant speed so this coordinate system is an inertial frame.

(c) $u_{A1x} = v_{A1x} - v_{\text{cm-}x} = \dfrac{m_B v_{A1}}{m_A + m_B}$. $u_{B1x} = v_{B1x} - v_{\text{cm-}x} = -\dfrac{m_A v_{A1}}{m_A + m_B}$. In this frame $P_{1x} = m_A u_{A1x} + m_B u_{B1x} = 0$.

(d) $P_{2x} = P_{1x} = 0$ gives $p_{A1x} + p_{B1x} = 0$ and $p_{A2x} + p_{B2x} = 0$, so $p_{B1x} = -p_{A1x}$ and $p_{B2x} = -p_{A2x}$. Conservation of kinetic energy gives $\dfrac{p_{A2x}^2}{2m_A} + \dfrac{p_{B2x}^2}{2m_B} = \dfrac{p_{A1x}^2}{2m_A} + \dfrac{p_{B1x}^2}{2m_B}$. Using $p_{B2x} = -p_{A2x}$ and $p_{B1x} = -p_{A1x}$ gives $p_{A2x}^2 = p_{A1x}^2$ and $p_{A2x} = \pm p_{A1x}$. If a collision occurs p_{Ax} changes and $p_{A2x} = -p_{A1x}$. But $p_{B2x} = -p_{A2x}$ and $p_{B1x} = -p_{A1x}$, so $p_{B2x} = -p_{B1x}$. In the center of mass frame the momentum and hence the velocity of each puck keeps the same magnitude and reverses direction.

(e) $v_{\text{cm-}x} = \left(\dfrac{0.400\ \text{kg}}{0.600\ \text{kg}}\right)(6.00\ \text{m/s}) = 4.00\ \text{m/s}$. $u_{A1x} = 6.00\ \text{m/s} - 4.00\ \text{m/s} = 2.00\ \text{m/s}$.

$u_{B1x} = 0 - 4.00\ \text{m/s} = -4.00\ \text{m/s}$. $u_{A2x} = -2.00\ \text{m/s}$ and $u_{B2x} = +4.00\ \text{m/s}$.

$v_{A2x} = u_{A2x} + v_{\text{cm-}x} = -2.00\ \text{m/s} + 4.00\ \text{m/s} = 2.00\ \text{m/s}$. $v_{B2x} = u_{B2x} + v_{\text{cm-}x} = 4.00\ \text{m/s} + 4.00\ \text{m/s} = 8.00\ \text{m/s}$.

Eq. 8.24 says $v_{A2x} = \left(\dfrac{0.400\ \text{kg} - 0.200\ \text{kg}}{0.400\ \text{kg} + 0.200\ \text{kg}}\right)(6.00\ \text{m/s}) = 2.00\ \text{m/s}$. Eq. 8.25 says

$v_{A2x} = \left(\dfrac{2[0.400\ \text{kg}]}{0.400\ \text{kg} + 0.200\ \text{kg}}\right)(6.00\ \text{m/s}) = 8.00\ \text{m/s}$. Our result agrees with Eqs. 8.24 and 8.25.

EVALUATE: Eqs. 8.24 and 8.25 apply only when $v_{B1} = 0$. The result that the velocity of each puck in the center of mass frame reverses direction and retains the same magnitude applies to all elastic collisions, even when both are moving initially.

8.107. **IDENTIFY** and **SET UP:** Apply conservation of energy to find the total energy before and after the collision with the floor from the initial and final maximum heights.

EXECUTE: **(a)** Objects stick together says that the relative speed after the collision is zero, so $\epsilon = 0$.

(b) In an elastic collision the relative velocity of the two bodies has the same magnitude before and after the collision, so $\epsilon = 1$.

(c) Speed of ball just before collision: $mgh = \frac{1}{2}mv_1^2$.

$v_1 = \sqrt{2gh}$

Speed of ball just after collision: $mgH_1 = \frac{1}{2}mv_2^2$.

$v_2 = \sqrt{2gH_1}$

The second object (the surface) is stationary, so $\epsilon = v_2 / v_1 = \sqrt{H_1 / h}$.

(d) $\epsilon = \sqrt{H_1 / h}$ implies $H_1 = h\epsilon^2 = (1.2\text{ m})(0.85)^2 = 0.87\text{ m}$.

(e) $H_1 = h\epsilon^2$.

$H_2 = H_1\epsilon^2 = h\epsilon^4$.

$H_3 = H_2\epsilon^2 = \left(h\epsilon^4\right)\epsilon^2 = h\epsilon^6$.

Generalize to $H_n = H_{n-1}\epsilon^2 = h\epsilon^{2(n-1)}\epsilon^2 = h\epsilon^{2n}$.

(f) 8th bounce implies $n = 8$.

$H_8 = h\epsilon^{16} = 1.2\text{ m}(0.85)^{16} = 0.089\text{ m}$.

EVALUATE: ϵ is a measure of the kinetic energy lost in the collision. The collision here is between a ball and the earth. Momentum lost by the ball is gained by the earth, but the velocity gained by the earth is very small and can be taken to be zero.

8.108. **IDENTIFY:** Momentum is conserved in the collision. Conservation of energy says $K_2 = K_1 + \Delta$.

SET UP: For part (b) let v_0 be the common speed of each atom before the collision and let $\vec{V}$ and $\vec{v}_3$ be the velocities after the collision of the molecule and the atom that remains. $m = 1.67 \times 10^{-27}\text{ kg}$ is the mass of one hydrogen atom.

EXECUTE: **(a)** In the center of mass frame $P_{1x} = 0$ so $P_{2x} = 0$ and $v_{\text{cm2}} = 0$. But in this frame the potential energy decreases and the kinetic energy increases. This is inconsistent with $K_{2\text{cm}} = \frac{1}{2}m_{\text{tot}}v_{\text{cm2}}^2 = 0$.

(b) Before the collision $v_{\text{cm}} = 0$. After the collision the molecule and remaining atom move in opposite directions and $(2m)V = mv_3$; $v_3 = 2V$. Conservation of energy gives $\frac{1}{2}(2m)V^2 + \frac{1}{2}mv_3^2 = 3(\frac{1}{2}mv_0^2)^2 + \Delta$. With $v_3 = 2V$ this becomes $V^2 = \frac{1}{2}v_0^2 + \dfrac{\Delta}{3m}$. $V = \sqrt{\frac{1}{2}(1.00 \times 10^3\text{ m/s})^2 + \dfrac{7.23 \times 10^{-19}\text{ J}}{3(1.67 \times 10^{-27})}} = 1.20 \times 10^4\text{ m/s}$ and $v_3 = 2V = 2.40 \times 10^4\text{ m/s}$.

EVALUATE: $K = 3\left(\frac{1}{2}mv_0^2\right) = 2.50 \times 10^{-21}\text{ J}$, which is much less than the binding energy of the molecule. Other initial conditions also lead to molecule formation; the one of zero initial momentum is just particularly simple to analyze.

8.109. **IDENTIFY:** Apply conservation of energy to the motion of the wagon before the collision. After the collision the combined object moves with constant speed on the level ground. In the collision the horizontal component of momentum is conserved.

SET UP: Let the wagon be object *A* and treat the two people together as object *B*. Let $+x$ be horizontal and to the right. Let *V* be the speed of the combined object after the collision.

EXECUTE: **(a)** The speed v_{A1} of the wagon just before the collision is given by conservation of energy applied to the motion of the wagon prior to the collision. $U_1 = K_2$ says $m_A g([50\text{ m}][\sin 6.0°]) = \frac{1}{2}m_A v_{A1}^2$. $v_{A1} = 10.12\text{ m/s}$.

$P_{1x} = P_{2x}$ for the collision says $m_A v_{A1} = (m_A + m_B)V$ and $V = \left(\dfrac{300\text{ kg}}{300\text{ kg} + 75.0\text{ kg} + 60.0\text{ kg}}\right)(10.12\text{ m/s}) = 6.98\text{ m/s}$.

In 5.0 s the wagon travels $(6.98\text{ m/s})(5.0\text{ s}) = 34.9\text{ m}$, and the people will have time to jump out of the wagon before it reaches the edge of the cliff.

(b) For the wagon, $K_1 = \frac{1}{2}(300\text{ kg})(10.12\text{ m/s})^2 = 1.54\times10^4\text{ J}$. Assume that the two heroes drop from a small height, so their kinetic energy just before the wagon can be neglected compared to K_1 of the wagon. $K_2 = \frac{1}{2}(435\text{ kg})(6.98\text{ m/s})^2 = 1.06\times10^4\text{ J}$. The kinetic energy of the system decreases by $K_1 - K_2 = 4.8\times10^3\text{ J}$.

EVALUATE: The wagon slows down when the two heroes drop into it. The mass that is moving horizontally increases, so the speed decreases to maintain the same horizontal momentum. In the collision the vertical momentum is not conserved, because of the net external force due to the ground.

8.110. **IDENTIFY:** Gravity gives a downward external force of magnitude *mg*. The impulse of this force equals the change in momentum of the rocket.

SET UP: Let +*y* be upward. Consider an infinitesimal time interval *dt*. In Example 8.15, $v_{ex} = 2400\text{ m/s}$ and $\frac{dm}{dt} = -\frac{m_0}{120\text{ s}}$. In Example 8.16, $m = m_0/4$ after $t = 90\text{ s}$.

EXECUTE: **(a)** The impulse-momentum theorem gives $-mgdt = (m+dm)(v+dv)+(dm)(v-v_{ex})-mv$. This simplifies to $-mgdt = mdv + v_{ex}dm$ and $m\frac{dv}{dt} = -v_{ex}\frac{dm}{dt} - mg$.

(b) $a = \frac{dv}{dt} = -\frac{v_{ex}}{m}\frac{dm}{dt} - g$.

(c) At $t = 0$, $a = -\frac{v_{ex}}{m_0}\frac{dm}{dt} - g = -(2400\text{ m/s})\left(-\frac{1}{120\text{ s}}\right) - 9.80\text{ m/s}^2 = 10.2\text{ m/s}^2$.

(d) $dv = -\frac{v_{ex}}{m}dm - gdt$. Integrating gives $v - v_0 = +v_{ex}\ln\frac{m_0}{m} - gt$. $v_0 = 0$ and $v = +(2400\text{ m/s})\ln 4 - (9.80\text{ m/s}^2)(90\text{ s}) = 2445\text{ m/s}$.

EVALUATE: Both the initial acceleration in Example 8.15 and the final speed of the rocket in Example 8.16 are reduced by the presence of gravity.

8.111. **IDENTIFY** and **SET UP:** Apply Eq. 8.40 to the single-stage rocket and to each stage of the two-stage rocket.

(a) EXECUTE: $v - v_0 = v_{ex}\ln(m_0/m)$; $v_0 = 0$ so $v = v_{ex}\ln(m_0/m)$

The total initial mass of the rocket is $m_0 = 12{,}000\text{ kg} + 1000\text{ kg} = 13{,}000\text{ kg}$. Of this, $9000\text{ kg} + 700\text{ kg} = 9700\text{ kg}$ is fuel, so the mass *m* left after all the fuel is burned is $13{,}000\text{ kg} - 9700\text{ kg} = 3300\text{ kg}$.

$$v = v_{ex}\ln(13{,}000\text{ kg}/3300\text{ kg}) = 1.37v_{ex}.$$

(b) First stage: $v = v_{ex}\ln(m_0/m)$

$m_0 = 13{,}000\text{ kg}$

The first stage has 9000 kg of fuel, so the mass left after the first stage fuel has burned is $13{,}000\text{ kg} - 9000\text{ kg} = 4000\text{ kg}$.

$$v = v_{ex}\ln(13{,}000\text{ kg}/4000\text{ kg}) = 1.18v_{ex}.$$

(c) Second stage: $m_0 = 1000\text{ kg}$, $m = 1000\text{ kg} - 700\text{ kg} = 300\text{ kg}$.

$$v = v_0 + v_{ex}\ln(m_0/m) = 1.18v_{ex} + v_{ex}\ln(1000\text{ kg}/300\text{ kg}) = 2.38v_{ex}.$$

(d) $v = 7.00\text{ km/s}$

$$v_{ex} = v/2.38 = (7.00\text{ km/s})/2.38 = 2.94\text{ km/s}. \quad \text{q}$$

EVALUATE: The two-stage rocket achieves a greater final speed because it jetisons the left-over mass of the first stage before the second-state fires and this reduces the final *m* and increases m_0/m.

8.112. **IDENTIFY:** During an interval where the mass is constant the speed of the rocket is constant. During an interval where the mass is changing at a constant rate, the equations of Section 8.6 apply.

SET UP: For $0 \le t \le 90\text{ s}$, $\frac{dm}{dt} = -\frac{m_0}{120\text{ s}}$. From Example 8.15, $v_{ex} = 2400\text{ m/s}$.

EXECUTE: **(a)** For $t \le 0$, $v = 0$. For $0 \le t \le 90\text{ s}$, Eq. 8.40 says $v = (2400\text{ m/s})\ln 4 = 3327\text{ m/s}$. For $t > 90\text{ s}$, *v* has the constant value 3327 m/s. The graph of $v(t)$ is given in Fig. 8.112a.

(b) For $0 \le t \le 90\text{ s}$, Eq. 8.39 gives $a = -\frac{v_{ex}}{m}\frac{dm}{dt} = -\frac{2400\text{ m/s}}{m_0(1-t/[120\text{ s}])}\left(-\frac{m_0}{120\text{ s}}\right) = \frac{20\text{ m/s}^2}{1-t/[120\text{ s}]}$. $a = 20\text{ m/s}^2$ at $t = 0$ (as in Example 8.15) and $a = 80\text{ m/s}^2$ at $t = 90\text{ s}$. For $t > 90\text{ s}$, $a = 0$. The graph of $a(t)$ is given in Fig. 8.112b.

(c) The astronaut has the same acceleration as the rocket. This is maximum at $t = 90\text{ s}$ and $F_{max} = m_{astronaut}a_{max} = (75\text{ kg})(80\text{ m/s}^2) = 6.0\times10^3\text{ N}$. This is 8.2 times her weight on earth, since a_{max} is 8.2 times *g*.

EVALUATE: The acceleration increases because the mass decreases while the thrust $F = -v_{\text{ex}}\dfrac{dm}{dt}$ remains constant.

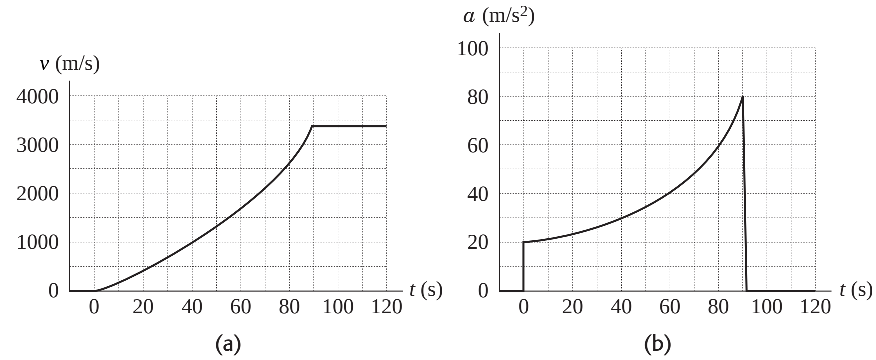


Figure 8.112

8.113. IDENTIFY and SET UP: $dm = \rho dV$. $dV = Adx$. Since the thin rod lies along the x axis, $y_{\text{cm}} = 0$. The mass of the rod is given by $M = \int dm$.

EXECUTE: (a) $x_{\text{cm}} = \dfrac{1}{M}\int_0^L x dm = \dfrac{\rho}{M}A\int_0^L x dx = \dfrac{\rho A}{M}\dfrac{L^2}{2}$. The volume of the rod is AL and $M = \rho AL$.

$x_{\text{cm}} = \dfrac{\rho AL^2}{2\rho AL} = \dfrac{L}{2}$. The center of mass of the uniform rod is at its geometrical center, midway between its ends.

(b) $x_{\text{cm}} = \dfrac{1}{M}\int_0^L x dm = \dfrac{1}{M}\int_0^L x\rho A dx = \dfrac{A\alpha}{M}\int_0^L x^2 dx = \dfrac{A\alpha L^3}{3M}$. $M = \int dm = \int_0^L \rho A dx = \alpha A\int_0^L x dx = \dfrac{\alpha AL^2}{2}$. Therefore,

$x_{\text{cm}} = \left(\dfrac{A\alpha L^3}{3}\right)\left(\dfrac{2}{\alpha AL^2}\right) = \dfrac{2L}{3}$.

EVALUATE: When the density increases with x, the center of mass is to the right of the center of the rod.

8.114. IDENTIFY: $x_{\text{cm}} = \dfrac{1}{M}\int x dm$ and $y_{\text{cm}} = \dfrac{1}{M}\int y dm$. At the upper surface of the plate, $y^2 + x^2 = a^2$.

SET UP: To find x_{cm}, divide the plate into thin strips parallel to the y-axis, as shown in Fig. 8.114a. To find y_{cm}, divide the plate into thin strips parallel to the x-axis as shown in Fig. 8.114b. The plate has volume one-half that of a circular disk, so $V = \tfrac{1}{2}\pi a^2 t$ and $M = \tfrac{1}{2}\rho\pi a^2 t$.

EXECUTE: In Fig.114a each strip has length $y = \sqrt{a^2 - x^2}$. $x_{\text{cm}} = \dfrac{1}{M}\int x dm$, where $dm = \rho t y dx = \rho t\sqrt{a^2 - x^2}dx$.

$x_{\text{cm}} = \dfrac{\rho t}{M}\int_{-a}^{a} x\sqrt{a^2 - x^2}dx = 0$, since the integrand is an odd function of x. $x_{\text{cm}} = 0$ because of symmetry. In Fig.114b each strip has length $2x = 2\sqrt{a^2 - y^2}$. $y_{\text{cm}} = \dfrac{1}{M}\int y dm$, where $dm = 2\rho t x dy = 2\rho t\sqrt{a^2 - y^2}dy$.

$y_{\text{cm}} = \dfrac{2\rho t}{M}\int_0^a y\sqrt{a^2 - y^2}dy$. The integral can be evaluated using $u = a^2 - y^2$, $du = -2ydy$. This substitution gives

$y_{\text{cm}} = \dfrac{2\rho t}{M}\left(-\dfrac{1}{2}\right)\int_{a^2}^{0} u^{1/2}du = \dfrac{2\rho t a^3}{3M} = \left(\dfrac{2\rho t a^3}{3}\right)\left(\dfrac{2}{\rho\pi a^2 t}\right) = \dfrac{4a}{3\pi}$.

EVALUATE: $\dfrac{4}{3\pi} = 0.424$. y_{cm} is less than $a/2$, as expected, since the plate becomes wider as y decreases.

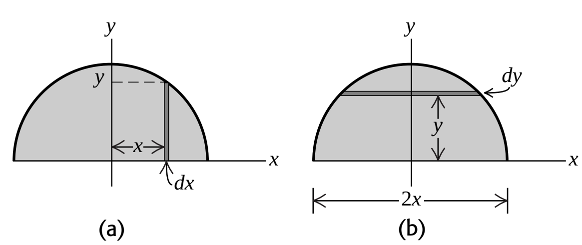


Figure 8.114

8.115. **IDENTIFY:** The work is related to the force by $W = \int_{x_1}^{x_2} F dx$. The force the person must apply equals the weight of the hanging portion. Since the rope is uniform, the center of mass of the hanging portion is at its geometrical center.

SET UP: Let *y* be the length of the rope hanging over the edge and use coordinates where the origin is at the edge of the table and +*y* is downward. When the rope is pulled onto the table, *y* goes from $l/4$ to zero. A length *y* of the rope has mass λy.

EXECUTE: **(a)** When a length *y* hangs over the edge, the person must apply an upward force

$F_y = -m(y)g = -\lambda y g$. $W = \int_{l/4}^{0} F_y(y)dy = -\lambda g \int_{l/4}^{0} y dy = \dfrac{\lambda g l^2}{32}$.

(b) Initially, $y_{cm} = l/8$. The work done to raise an object of mass *M* a distance y_{cm} is $W = Mgy_{cm}$.

$W = \left(\dfrac{\lambda l}{4}\right) g \left(\dfrac{l}{8}\right) = \dfrac{\lambda g l^2}{32}$.

EVALUATE: The answers from methods (a) and (b) agree. The change in gravitational potential energy of the rope can be calculated by considering all its mass acting at its center of mass, and the work done by the person equals the increase in gravitational potential energy of the rope.

8.116. **IDENTIFY:** From our analysis of motion with constant acceleration, if $v = at$ and *a* is constant, then $x - x_0 = v_0 t + \frac{1}{2}at^2$.

SET UP: Take $v_0 = 0$, $x_0 = 0$ and let +*x* downward.

EXECUTE: **(a)** $\dfrac{dv}{dt} = a$, $v = at$ and $x = \frac{1}{2}at^2$. Substituting into $xg = x\dfrac{dv}{dt} + v^2$ gives

$\frac{1}{2}at^2 g = \frac{1}{2}at^2 a + a^2t^2 = \frac{3}{2}a^2t^2$. The nonzero solution is $a = g/3$.

(b) $x = \frac{1}{2}at^2 = \frac{1}{6}gt^2 = \frac{1}{6}(9.80\text{ m/s}^2)(3.00\text{ s})^2 = 14.7\text{ m}$.

(c) $m = kx = (2.00\text{ g/m})(14.7\text{ m}) = 29.4\text{ g}$.

EVALUATE: The acceleration is less than *g* because the small water droplets are initially at rest, before they adhere to the falling drop. The small droplets are suspended by buoyant forces that we ignore for the raindrops.

ROTATION OF RIGID BODIES

9

9.1. **IDENTIFY:** $s = r\theta$, with θ in radians.

SET UP: π rad $= 180°$.

EXECUTE: **(a)** $\theta = \dfrac{s}{r} = \dfrac{1.50\text{ m}}{2.50\text{ m}} = 0.600\text{ rad} = 34.4°$

(b) $r = \dfrac{s}{\theta} = \dfrac{14.0\text{ cm}}{(128°)(\pi\text{ rad}/180°)} = 6.27\text{ cm}$

(c) $s = r\theta = (1.50\text{ m})(0.700\text{ rad}) = 1.05\text{ m}$

EVALUATE: An angle is the ratio of two lengths and is dimensionless. But, when $s = r\theta$ is used, θ must be in radians. Or, if $\theta = s/r$ is used to calculate θ, the calculation gives θ in radians.

9.2. **IDENTIFY:** $\theta - \theta_0 = \omega t$, since the angular velocity is constant.

SET UP: $1\text{ rpm} = (2\pi/60)\text{ rad/s}$.

EXECUTE: **(a)** $\omega = (1900)(2\pi\text{ rad}/60\text{ s}) = 199\text{ rad/s}$

(b) $35° = (35°)(\pi/180°) = 0.611\text{ rad}$. $t = \dfrac{\theta - \theta_0}{\omega} = \dfrac{0.611\text{ rad}}{199\text{ rad/s}} = 3.1\times10^{-3}\text{ s}$

EVALUATE: In $t = \dfrac{\theta - \theta_0}{\omega}$ we must use the same angular measure (radians, degrees or revolutions) for both $\theta - \theta_0$ and ω.

9.3. **IDENTIFY:** $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. Writing Eq.(2.16) in terms of angular quantities gives $\theta - \theta = \int_{t_1}^{t_2} \omega_z dt$.

SET UP: $\dfrac{d}{dt}t^n = nt^{n-1}$ and $\int t^n dt = \dfrac{1}{n+1}t^{n+1}$

EXECUTE: **(a)** A must have units of rad/s and B must have units of rad/s^3.

(b) $\alpha_z(t) = 2Bt = (3.00\text{ rad/s}^3)t$. (i) For $t = 0$, $\alpha_z = 0$. (ii) For $t = 5.00\text{ s}$, $\alpha_z = 15.0\text{ rad/s}^2$.

(c) $\theta_2 - \theta_1 = \int_{t_1}^{t_2}(A + Bt^2)dt = A(t_2 - t_1) + \frac{1}{3}B(t_2^3 - t_1^3)$. For $t_1 = 0$ and $t_2 = 2.00\text{ s}$,

$\theta_2 - \theta_1 = (2.75\text{ rad/s})(2.00\text{ s}) + \frac{1}{3}(1.50\text{ rad/s}^3)(2.00\text{ s})^3 = 9.50\text{ rad}$.

EVALUATE: Both α_z and ω_z are positive and the angular speed is increasing.

9.4. **IDENTIFY:** $\alpha_z = d\omega_z/dt$. $\alpha_{\text{av-}z} = \dfrac{\Delta\omega_z}{\Delta t}$.

SET UP: $\dfrac{d}{dt}(t^2) = 2t$

EXECUTE: **(a)** $\alpha_z(t) = \dfrac{d\omega_z}{dt} = -2\beta t = (-1.60\text{ rad/s}^3)t.$

(b) $\alpha_z(3.0\text{ s}) = (-1.60\text{ rad/s}^3)(3.0\text{ s}) = -4.80\text{ rad/s}^2.$

$$\alpha_{\text{av-}z} = \frac{\omega_z(3.0\text{ s}) - \omega_z(0)}{3.0\text{ s}} = \frac{-2.20\text{ rad/s} - 5.00\text{ rad/s}}{3.0\text{ s}} = -2.40\text{ rad/s}^2,$$

which is half as large (in magnitude) as the acceleration at $t = 3.0$ s.

EVALUATE: $\alpha_z(t)$ increases linearly with time, so $\alpha_{\text{av-}z} = \dfrac{\alpha_z(0) + \alpha_z(3.0\text{ s})}{2}$. $\alpha_z(0) = 0$.

9.5. **IDENTIFY** and **SET UP:** Use Eq.(9.3) to calculate the angular velocity and Eq.(9.2) to calculate the average angular velocity for the specified time interval.

EXECUTE: $\theta = \gamma t + \beta t^3$; $\gamma = 0.400$ rad/s, $\beta = 0.0120$ rad/s^3

(a) $\omega_z = \dfrac{d\theta}{dt} = \gamma + 3\beta t^2$

(b) At $t = 0$, $\omega_z = \gamma = 0.400$ rad/s

(c) At $t = 5.00$ s, $\omega_z = 0.400 \text{ rad/s} + 3(0.0120 \text{ rad/s}^3)(5.00 \text{ s})^2 = 1.30 \text{ rad/s}$

$$\omega_{\text{av-}z} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

For $t_1 = 0$, $\theta_1 = 0$.

For $\theta_2 = 5.00$ s, $\theta_2 = (0.400 \text{ rad/s})(5.00 \text{ s}) + (0.012 \text{ rad/s}^3)(5.00 \text{ s})^3 = 3.50 \text{ rad}$

So $\omega_{\text{av-}z} = \dfrac{3.50 \text{ rad} - 0}{5.00 \text{ s} - 0} = 0.700 \text{ rad/s}.$

EVALUATE: The average of the instantaneous angular velocities at the beginning and end of the time interval is $\frac{1}{2}(0.400 \text{ rad/s} + 1.30 \text{ rad/s}) = 0.850 \text{ rad/s}$. This is larger than $\omega_{\text{av-}z}$, because $\omega_z(t)$ is increasing faster than linearly.

9.6. **IDENTIFY:** $\omega_z(t) = \dfrac{d\theta}{dt}$. $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. $\omega_{\text{av-}z} = \dfrac{\Delta\theta}{\Delta t}$.

SET UP: $\omega_z = (250 \text{ rad/s}) - (40.0 \text{ rad/s}^2)t - (4.50 \text{ rad/s}^3)t^2$. $\alpha_z = -(40.0 \text{ rad/s}^2) - (9.00 \text{ rad/s}^3)t$.

EXECUTE: **(a)** Setting $\omega_z = 0$ results in a quadratic in *t*. The only positive root is $t = 4.23$ s.

(b) At $t = 4.23$ s, $\alpha_z = -78.1 \text{ rad/s}^2$.

(c) At $t = 4.23$ s, $\theta = 586 \text{ rad} = 93.3 \text{ rev}$.

(d) At $t = 0$, $\omega_z = 250$ rad/s.

(e) $\omega_{\text{av-}z} = \dfrac{586 \text{ rad}}{4.23 \text{ s}} = 138 \text{ rad/s}.$

EVALUATE: Between $t = 0$ and $t = 4.23$ s, ω_z decreases from 250 rad/s to zero. ω_z is not linear in *t*, so $\omega_{\text{av-}z}$ is not midway between the values of ω_z at the beginning and end of the interval.

9.7. **IDENTIFY:** $\omega_z(t) = \dfrac{d\theta}{dt}$. $\alpha_z(t) = \dfrac{d\omega_z}{dt}$. Use the values of θ and ω_z at $t = 0$ and α_z at 1.50 s to calculate *a*, *b*, and *c*.

SET UP: $\dfrac{d}{dt}t^n = nt^{n-1}$

EXECUTE: **(a)** $\omega_z(t) = b - 3ct^2$. $\alpha_z(t) = -6ct$. At $t = 0$, $\theta = a = \pi/4$ rad and $\omega_z = b = 2.00$ rad/s. At $t = 1.50$ s, $\alpha_z = -6c(1.50 \text{ s}) = 1.25 \text{ rad/s}^2$ and $c = -0.139 \text{ rad/s}^3$.

(b) $\theta = \pi/4$ rad and $\alpha_z = 0$ at $t = 0$.

(c) $\alpha_z = 3.50 \text{ rad/s}^2$ at $t = -\dfrac{\alpha_z}{6c} = -\dfrac{3.50 \text{ rad/s}^2}{6(-0.139 \text{ rad/s}^3)} = 4.20$ s. At $t = 4.20$ s,

$$\theta = \frac{\pi}{4} \text{ rad} + (2.00 \text{ rad/s})(4.20 \text{ s}) - (-0.139 \text{ rad/s}^3)(4.20 \text{ s})^3 = 19.5 \text{ rad}.$$

$$\omega_z = 2.00 \text{ rad/s} - 3(-0.139 \text{ rad/s}^3)(4.20 \text{ s})^2 = 9.36 \text{ rad/s}.$$

EVALUATE: θ, ω_z and α_z all increase as *t* increases.

9.8. **IDENTIFY:** $\alpha_z = \dfrac{d\omega_z}{dt}$. $\theta - \theta_0 = \omega_{\text{av-}z}t$. When ω_z is linear in *t*, $\omega_{\text{av-}z}$ for the time interval t_1 to t_2 is

$$\omega_{\text{av-}z} = \frac{\omega_{z1} + \omega_{z2}}{t_2 - t_1}.$$

SET UP: From the information given, $\omega_z(t) = -6.00 \text{ rad/s} + (2.00 \text{ rad/s}^2)t$

EXECUTE: **(a)** The angular acceleration is positive, since the angular velocity increases steadily from a negative value to a positive value.

(b) It takes 3.00 seconds for the wheel to stop $(\omega_z = 0)$. During this time its speed is decreasing. For the next 4.00 s its speed is increasing from 0 rad/s to +8.00 rad/s.

(c) The average angular velocity is $\dfrac{-6.00\ \text{rad/s} + 8.00\ \text{rad/s}}{2} = 1.00\ \text{rad/s}$. $\theta - \theta_0 = \omega_{\text{av-}z} t$ then leads to displacement of 7.00 rad after 7.00 s.

EVALUATE: When α_z and ω_z have the same sign, the angular speed is increasing; this is the case for $t = 3.00$ s to $t = 7.00$ s . When α_z and ω_z have opposite signs, the angular speed is decreasing; this is the case between $t = 0$ and $t = 3.00$ s .

9.9. **IDENTIFY:** Apply the constant angular acceleration equations.

SET UP: Let the direction the wheel is rotating be positive.

EXECUTE: **(a)** $\omega_z = \omega_{0z} + \alpha_z t = 1.50\ \text{rad/s} + (0.300\ \text{rad/s}^2)(2.50\ \text{s}) = 2.25\ \ \text{rad/s}.$

(b) $\theta - \theta_0 = \omega_{0z} t + \frac{1}{2}\alpha_z t^2 = (1.50\ \text{rad/s})(2.50\ \text{s}) + \frac{1}{2}(0.300\ \text{rad/s}^2)(2.50\ \text{s})^2 = 4.69\ \text{rad}$.

EVALUATE: $\theta - \theta_0 = \left(\dfrac{\omega_{0z} + \omega_z}{2}\right)t = \left(\dfrac{1.50\ \text{rad/s} + 2.25\ \text{rad/s}}{2}\right)(2.50\ \text{s}) = 4.69\ \text{rad}$, the same as calculated with another equation in part (b).

9.10. **IDENTIFY:** Apply the constant angular acceleration equations to the motion of the fan.

(a) SET UP: $\omega_{0z} = (500\ \text{rev/min})(1\ \text{min}/60\ \text{s}) = 8.333\ \text{rev/s},\ \ \omega_z = (200\ \text{rev/min})(1\ \text{min}/60\ \text{s}) = 3.333\ \text{rev/s},$
$t = 4.00\ \text{s},\ \ \alpha_z = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{3.333\ \text{rev/s} - 8.333\ \text{rev/s}}{4.00\ \text{s}} = -1.25\ \text{rev/s}^2$

$\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z} t + \frac{1}{2}\alpha_z t^2 = (8.333\ \text{rev/s})(4.00\ \text{s}) + \frac{1}{2}(-1.25\ \text{rev/s}^2)(4.00\ \text{s})^2 = 23.3\ \text{rev}$

(b) SET UP: $\omega_z = 0$ (comes to rest); $\omega_{0z} = 3.333\ \text{rev/s};\ \ \alpha_z = -1.25\ \text{rev/s}^2;$
$t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{0 - 3.333\ \text{rev/s}}{-1.25\ \text{rev/s}^2} = 2.67\ \text{s}$

EVALUATE: The angular acceleration is negative because the angular velocity is decreasing. The average angular velocity during the 4.00 s time interval is 350 rev/min and $\theta - \theta_0 = \omega_{\text{av-}z} t$ gives $\theta - \theta_0 = 23.3\ \text{rev},$ which checks.

9.11. **IDENTIFY:** Apply the constant angular acceleration equations to the motion. The target variables are *t* and $\theta - \theta_0$.

SET UP: **(a)** $\alpha_z = 1.50\ \text{rad/s}^2;\ \ \omega_{0z} = 0$ (starts from rest); $\omega_z = 36.0\ \text{rad/s};\ \ t = ?$

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{36.0\ \text{rad/s} - 0}{1.50\ \text{rad/s}^2} = 24.0\ \text{s}$

(b) $\theta - \theta_0 = ?$

$\theta - \theta_0 = \omega_{0z} t + \frac{1}{2}\alpha_z t^2 = 0 + \frac{1}{2}(1.50\ \text{rad/s}^2)(2.40\ \text{s})^2 = 432\ \text{rad}$

$\theta - \theta_0 = 432\ \text{rad}(1\ \text{rev}/2\pi\ \text{rad}) = 68.8\ \text{rev}$

EVALUATE: We could use $\theta - \theta_0 = \frac{1}{2}(\omega_z + \omega_{0z})t$ to calculate $\theta - \theta_0 = \frac{1}{2}(0 + 36.0\ \text{rad/s})(24.0\ \text{s}) = 432\ \text{rad},$ which checks.

9.12. **IDENTIFY:** In part (b) apply the equation derived in part (a).

SET UP: Let the direction the propeller is rotating be positive.

EXECUTE: **(a)** Solving Eq. (9.7) for *t* gives $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z}$. Rewriting Eq. (9.11) as $\theta - \theta_0 = t(\omega_{0z} + \frac{1}{2}\omega_z t)$ and substituting for *t* gives

$$\theta - \theta_0 = \left(\frac{\omega_z - \omega_{0z}}{\alpha_z}\right)\left(\omega_{0z} + \frac{1}{2}(\omega_z - \omega_{0z})\right) = \frac{1}{\alpha_z}(\omega_z - \omega_{0z})\left(\frac{\omega_z + \omega_{0z}}{2}\right) = \frac{1}{2\alpha_z}(\omega_z^2 - \omega_{0z}^2),$$

which when rearranged gives Eq. (9.12).

(b) $\alpha_z = \frac{1}{2}\left(\dfrac{1}{\theta - \theta_0}\right)\left(\omega_z^2 - \omega_{0z}^2\right) = \frac{1}{2}\left(\dfrac{1}{7.00\ \text{rad}}\right)\left((16.0\ \text{rad/s})^2 - (12.0\ \text{rad/s})^2\right) = 8.00\ \text{rad/s}^2$

EVALUATE: We could also use $\theta-\theta_0=\left(\frac{\omega_{0z}+\omega_z}{2}\right)t$ to calculate $t=0.500\text{ s}$. Then $\omega_z=\omega_{0z}+\alpha_z t$ gives $\alpha_z=8.00\text{ rad/s}^2$, which agrees with our results in part (b).

9.13. **IDENTIFY:** Use a constant angular acceleration equation and solve for ω_{0z}.

SET UP: Let the direction of rotation of the flywheel be positive.

EXECUTE: $\theta-\theta_0=\omega_{0z}t+\frac{1}{2}\alpha_z^2$ gives $\omega_{0z}=\frac{\theta-\theta_0}{t}-\frac{1}{2}\alpha_z=\frac{60.0\text{ rad}}{4.00\text{ s}}-\frac{1}{2}(2.25\text{ rad/s}^2)(4.00\text{ s})=10.5\text{ rad/s}$.

EVALUATE: At the end of the 4.00 s interval, $\omega_z=\omega_{0z}+\alpha_z t=19.5\text{ rad/s}$.

$\theta-\theta_0=\left(\frac{\omega_{0z}+\omega_z}{2}\right)t=\left(\frac{10.5\text{ rad/s}+19.5\text{ rad/s}}{2}\right)(4.00\text{ s})=60.0\text{ rad}$, which checks.

9.14. **IDENTIFY:** Apply the constant angular acceleration equations.

SET UP: Let the direction of the rotation of the blade be positive. $\omega_{0z}=0$.

EXECUTE: $\omega_z=\omega_{0z}+\alpha_z$ gives $\alpha_z=\frac{\omega_z-\omega_{0z}}{t}=\frac{140\text{ rad/s}-0}{6.00\text{ s}}=23.3\text{ rad/s}^2$.

$(\theta-\theta_0)=\left(\frac{\omega_{0z}+\omega_z}{2}\right)t=\left(\frac{0+140\text{ rad/s}}{2}\right)(6.00\text{ s})=420\text{ rad}$

EVALUATE: We could also use $\theta-\theta_0=\omega_{0z}t+\frac{1}{2}\alpha_z t^2$. This equation gives $\theta-\theta_0=\frac{1}{2}(23.3\text{ rad/s}^2)(6.00\text{ s})^2=419\text{ rad}$, in agreement with the result obtained above.

9.15. **IDENTIFY:** Apply constant angular acceleration equations.

SET UP: Let the direction the flywheel is rotating be positive.

$\theta-\theta_0=200\text{ rev},\ \omega_{0z}=500\text{ rev/min}=8.333\text{ rev/s},\ t=30.0\text{ s}$.

EXECUTE: **(a)** $\theta-\theta_0=\left(\frac{\omega_{0z}+\omega_z}{2}\right)t$ gives $\omega_z=5.00\text{ rev/s}=300\text{ rpm}$

(b) Use the information in part (a) to find α_z: $\omega_z=\omega_{0z}+\alpha_z t$ gives $\alpha_z=-0.1111\text{ rev/s}^2$. Then $\omega_z=0$, $\alpha_z=-0.1111\text{ rev/s}^2$, $\omega_{0z}=8.333\text{ rev/s}$ in $\omega_z=\omega_{0z}+\alpha_z t$ gives $t=75.0\text{ s}$ and $\theta-\theta_0=\left(\frac{\omega_{0z}+\omega_z}{2}\right)t$ gives $\theta-\theta_0=312\text{ rev}$.

EVALUATE: The mass and diameter of the flywheel are not used in the calculation.

9.16. **IDENTIFY:** Use the constant angular acceleration equations, applied to the first revolution and to the first two revolutions.

SET UP: Let the direction the disk is rotating be positive. $1\text{ rev}=2\pi\text{ rad}$. Let t be the time for the first revolution. The time for the first two revolutions is $t+0.750\text{ s}$.

EXECUTE: **(a)** $\theta-\theta_0=\omega_{0z}t+\frac{1}{2}\alpha_z t^2$ applied to the first and to the first two revolutions gives $2\pi\text{ rad}=\frac{1}{2}\alpha_z t^2$ and $4\pi\text{ rad}=\frac{1}{2}\alpha_z(t+0.750\text{ s})^2$. Eliminating α_z between these equations gives $4\pi\text{ rad}=\frac{2\pi\text{ rad}}{t^2}(t+0.750\text{ s})^2$.

$2t^2=(t+0.750\text{ s})^2$. $\sqrt{2}t=\pm(t+0.750\text{ s})$. The positive root is $t=\frac{0.750\text{ s}}{\sqrt{2}-1}=1.81\text{ s}$.

(b) $2\pi\text{ rad}=\frac{1}{2}\alpha_z t^2$ and $t=1.81\text{ s}$ gives $\alpha_z=3.84\text{ rad/s}^2$

EVALUATE: At the start of the second revolution, $\omega_{0z}=(3.84\text{ rad/s}^2)(1.81\text{ s})=6.95\text{ rad/s}$. The distance the disk rotates in the next 0.750 s is $\theta-\theta_0=\omega_{0z}t+\frac{1}{2}\alpha_z t^2=(6.95\text{ rad/s})(0.750\text{ s})+\frac{1}{2}(3.84\text{ rad/s}^2)(0.750\text{ s})^2=6.29\text{ rad}$, which is two revolutions.

9.17. **IDENTIFY:** Apply Eq.(9.12) to relate ω_z to $\theta-\theta_0$.

SET UP: Establish a proportionality.

EXECUTE: From Eq.(9.12), with $\omega_{0z}=0$, the number of revolutions is proportional to the square of the initial angular velocity, so tripling the initial angular velocity increases the number of revolutions by 9, to 9.00 rev.

EVALUATE: We don't have enough information to calculate α_z; all we need to know is that it is constant.

9.18. IDENTIFY: In each case we apply constant acceleration equations to determine $\theta(t)$ and $\omega_z(t)$.

SET UP: Let $\theta_0 = 0$. The following table gives the revolutions and the angle θ (in degrees) through which the wheel has rotated for each instant in time (in seconds) and each of the three situations:

t	(a)		(b)		(c)	
	rev	θ	rev	θ	rev	θ
0.05	0.50	180	0.03	11.3	0.44	158
0.10	1.00	360	0.13	45	0.75	270
0.15	1.50	540	0.28	101	0.94	338
0.20	2.00	720	0.50	180	1.00	360

EXECUTE: The θ and ω_z graphs for each case are given in Figures 9.18 a–c.

EVALUATE: The slope of the $\theta(t)$ graph is $\omega_z(t)$ and the slope of the $\omega_z(t)$ graph is $\alpha_z(t)$.

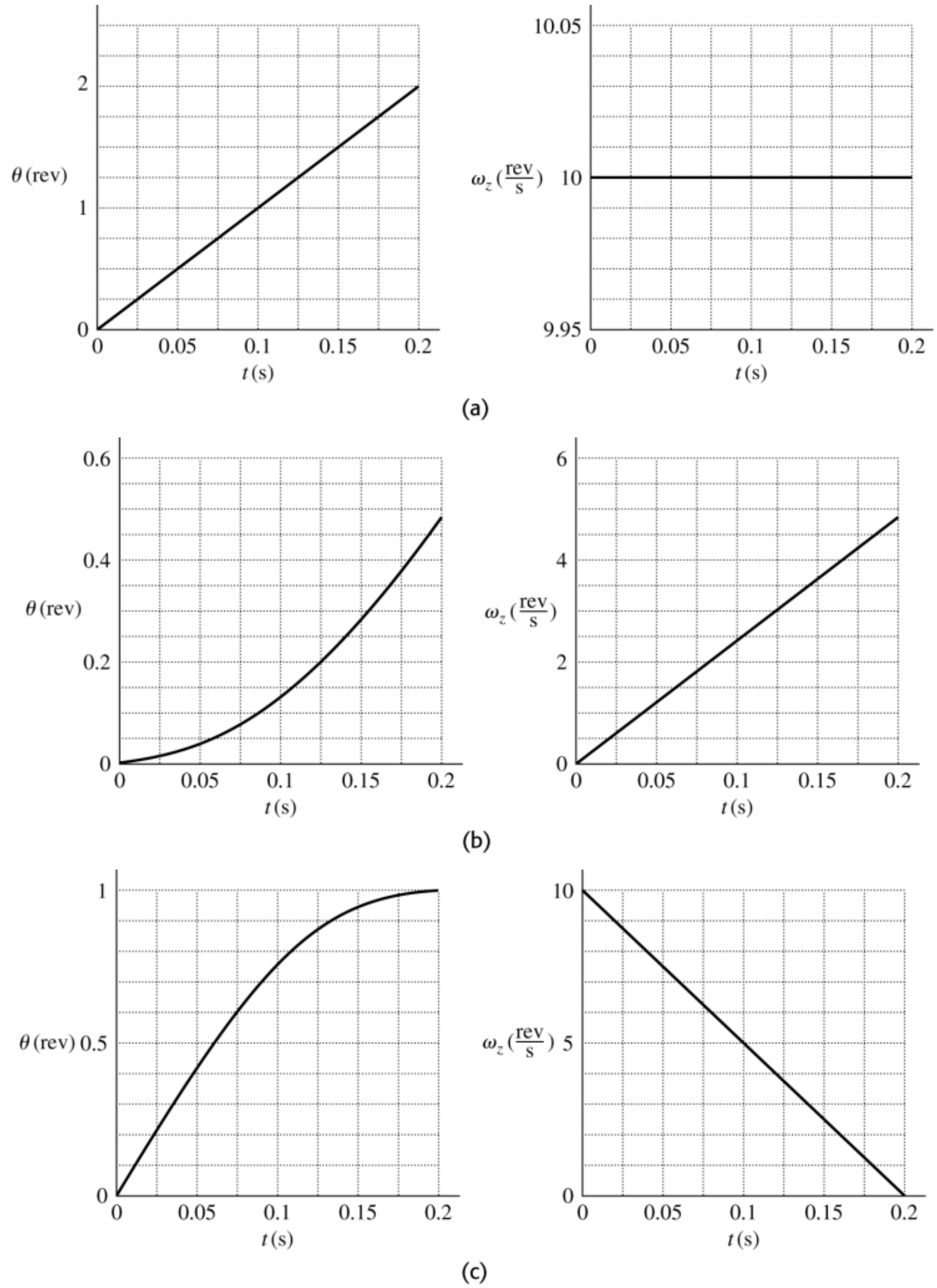


Figure 9.18

9.19. IDENTIFY: Apply the constant angular acceleration equations separately to the time intervals 0 to 2.00 s and 2.00 s until the wheel stops.

(a) SET UP: Consider the motion from $t = 0$ to $t = 2.00$ s:

$\theta - \theta_0 = ?$; $\omega_{0z} = 24.0$ rad/s; $\alpha_z = 30.0$ rad/s^2; $t = 2.00$ s

EXECUTE: $\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2 = (24.0 \text{ rad/s})(2.00 \text{ s}) + \frac{1}{2}(30.0 \text{ rad/s}^2)(2.00 \text{ s})^2$

$\theta - \theta_0 = 48.0 \text{ rad} + 60.0 \text{ rad} = 108 \text{ rad}$

Total angular displacement from $t = 0$ until stops: $108 \text{ rad} + 432 \text{ rad} = 540 \text{ rad}$

Note: At $t = 2.00$ s, $\omega_z = \omega_{0z} + \alpha_z t = 24.0 \text{ rad/s} + (30.0 \text{ rad/s}^2)(2.00 \text{ s}) = 84.0 \text{ rad/s}$; angular speed when breaker trips.

(b) SET UP: Consider the motion from when the circuit breaker trips until the wheel stops. For this calculation let $t = 0$ when the breaker trips.

$t = ?$; $\theta - \theta_0 = 432$ rad; $\omega_z = 0$; $\omega_{0z} = 84.0$ rad/s (from part (a))

$$\theta - \theta_0 = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t$$

EXECUTE: $t = \dfrac{2(\theta - \theta_0)}{\omega_{0z} + \omega_z} = \dfrac{2(432 \text{ rad})}{84.0 \text{ rad/s} + 0} = 10.3 \text{ s}$

The wheel stops 10.3 s after the breaker trips so $2.00 \text{ s} + 10.3 \text{ s} = 12.3 \text{ s}$ from the beginning.

(c) SET UP: $\alpha_z = ?$; consider the same motion as in part (b):

$\omega_z = \omega_{0z} + \alpha_z t$

EXECUTE: $\alpha_z = \dfrac{\omega_z - \omega_{0z}}{t} = \dfrac{0 - 84.0 \text{ rad/s}}{10.3 \text{ s}} = -8.16 \text{ rad/s}^2$

EVALUATE: The angular acceleration is positive while the wheel is speeding up and negative while it is slowing down. We could also use $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ to calculate $\alpha_z = \dfrac{\omega_z^2 - \omega_{0z}^2}{2(\theta - \theta_0)} = \dfrac{0 - (84.0 \text{ rad/s})^2}{2(432 \text{ rad})} = -8.16 \text{ rad/s}^2$ for the acceleration after the breaker trips.

9.20. IDENTIFY: The linear distance the elevator travels, its speed and the magnitude of its acceleration are equal to the tangential displacement, speed and acceleration of a point on the rim of the disk. $s = r\theta$, $v = r\omega$ and $a = r\alpha$. In these equations the angular quantities must be in radians.

SET UP: $1 \text{ rev} = 2\pi \text{ rad}$. $1 \text{ rpm} = 0.1047 \text{ rad/s}$. $\pi \text{ rad} = 180°$. For the disk, $r = 1.25$ m.

EXECUTE: **(a)** $v = 0.250$ m/s so $\omega = \dfrac{v}{r} = \dfrac{0.250 \text{ m/s}}{1.25 \text{ m}} = 0.200 \text{ rad/s} = 1.91 \text{ rpm}$.

(b) $a = \frac{1}{8}g = 1.225 \text{ m/s}^2$. $\alpha = \dfrac{a}{r} = \dfrac{1.225 \text{ m/s}^2}{1.25 \text{ m}} = 0.980 \text{ rad/s}^2$.

(c) $s = 3.25$ m. $\theta = \dfrac{s}{r} = \dfrac{3.25 \text{ m}}{1.25 \text{ m}} = 2.60 \text{ rad} = 149°$.

EVALUATE: When we use $s = r\theta$, $v = r\omega$ and $a_{\tan} = r\alpha$ to solve for θ, ω and α, the results are in rad, rad/s and rad/s^2.

9.21. IDENTIFY: When the angular speed is constant, $\omega = \theta / t$. $v_{\tan} = r\omega$, $a_{\tan} = r\alpha$ and $a_{\text{rad}} = r\omega^2$. In these equations radians must be used for the angular quantities.

SET UP: The radius of the earth is $R_E = 6.38 \times 10^6$ m and the earth rotates once in $1 \text{ day} = 86{,}400 \text{ s}$. The orbit radius of the earth is 1.50×10^{11} m and the earth completes one orbit in $1 \text{ y} = 3.156 \times 10^7$ s. When ω is constant, $\omega = \theta / t$.

EXECUTE: **(a)** $\theta = 1 \text{ rev} = 2\pi$ rad in $t = 3.156 \times 10^7$ s. $\omega = \dfrac{2\pi \text{ rad}}{3.156 \times 10^7 \text{ s}} = 1.99 \times 10^{-7} \text{ rad/s}$.

(b) $\theta = 1 \text{ rev} = 2\pi$ rad in $t = 86{,}400$ s. $\omega = \dfrac{2\pi \text{ rad}}{86{,}400 \text{ s}} = 7.27 \times 10^{-5} \text{ rad/s}$

(c) $v = r\omega = (1.50 \times 10^{11} \text{ m})(1.99 \times 10^{-7} \text{ rad/s}) = 2.98 \times 10^4 \text{ m/s}$.

(d) $v = r\omega = (6.38 \times 10^6 \text{ m})(7.27 \times 10^{-5} \text{ rad/s}) = 464 \text{ m/s}$.

(e) $a_{\text{rad}} = r\omega^2 = (6.38 \times 10^6 \text{ m})(7.27 \times 10^{-5} \text{ rad/s})^2 = 0.0337 \text{ m/s}^2$. $a_{\tan} = r\alpha = 0$. $\alpha = 0$ since the angular velocity is constant.

EVALUATE: The tangential speeds associated with these motions are large even though the angular speeds are very small, because the radius for the circular path in each case is quite large.

9.22. IDENTIFY: Linear and angular velocities are related by $v = r\omega$. Use $\omega_z = \omega_{0z} + \alpha_z t$ to calculate α_z.

SET UP: $\omega = v/r$ gives ω in rad/s.

EXECUTE: **(a)** $\dfrac{1.25\text{ m/s}}{25.0\times 10^{-3}\text{ m}} = 50.0\text{ rad/s},\ \dfrac{1.25\text{ m/s}}{58.0\times 10^{-3}\text{ m}} = 21.6\text{ rad/s}.$

(b) $(1.25\text{ m/s})(74.0\text{ min})(60\text{ s/min}) = 5.55\text{ km}.$

(c) $\alpha_z = \dfrac{21.55\text{ rad/s} - 50.0\text{ rad/s}}{(74.0\text{ min})(60\text{ s/min})} = -6.41\times 10^{-3}\text{ rad/s}^2.$

EVALUATE: The width of the tracks is very small, so the total track length on the disc is huge.

9.23. IDENTIFY: Use constant acceleration equations to calculate the angular velocity at the end of two revolutions. $v = r\omega$.

SET UP: $2\text{ rev} = 4\pi\text{ rad}.\ r = 0.200\text{ m}.$

EXECUTE: **(a)** $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta-\theta_0).\ \omega_z = \sqrt{2\alpha_z(\theta-\theta_0)} = \sqrt{2(3.00\text{ rad/s}^2)(4\pi\text{ rad})} = 8.68\text{ rad/s}.$

$a_{\text{rad}} = r\omega^2 = (0.200\text{ m})(8.68\text{ rad/s})^2 = 15.1\text{ m/s}^2.$

(b) $v = r\omega = (0.200\text{ m})(8.68\text{ rad/s}) = 1.74\text{ m/s}.\ a_{\text{rad}} = \dfrac{v^2}{r} = \dfrac{(1.74\text{ m/s})^2}{0.200\text{ m}} = 15.1\text{ m/s}^2.$

EVALUATE: $r\omega^2$ and v^2/r are completely equivalent expressions for a_{rad}.

9.24. IDENTIFY: $a_{\text{rad}} = r\omega^2$, with ω in rad/s. Solve for ω.

SET UP: $1\text{ rpm} = (2\pi/60)\text{ rad/s}$

EXECUTE: $\omega = \sqrt{\dfrac{a_{\text{rad}}}{r}} = \sqrt{\dfrac{(400{,}000)(9.80\text{ m/s}^2)}{0.0250\text{ m}}} = 1.25\times 10^4\text{ rad/s} = 1.20\times 10^5\text{ rpm}$

EVALUATE: In $a_{\text{rad}} = r\omega^2$, ω must be in rad/s.

9.25. IDENTIFY and SET UP: Use constant acceleration equations to find ω and α after each displacement. The use Eqs.(9.14) and (9.15) to find the components of the linear acceleration.

EXECUTE: **(a)** <u>at the start</u> $t = 0$

flywheel starts from rest so $\omega_z = \omega_{0z} = 0$

$a_{\tan} = r\alpha = (0.300\text{ m})(0.600\text{ rad/s}^2) = 0.180\text{ m/s}^2$

$a_{\text{rad}} = r\omega^2 = 0$

$a = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2} = 0.180\text{ m/s}^2$

(b) $\underline{\theta-\theta_0 = 60^\circ}$

$a_{\tan} = r\alpha = 0.180\text{ m/s}^2$

Calculate ω:

$\theta-\theta_0 = 60^\circ(\pi\text{ rad}/180^\circ) = 1.047\text{ rad};\ \omega_{0z} = 0;\ \alpha_z = 0.600\text{ rad/s}^2;\ \omega_z = ?$

$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta-\theta_0)$

$\omega_z = \sqrt{2\alpha_z(\theta-\theta_0)} = \sqrt{2(0.600\text{ rad/s}^2)(1.047\text{ rad})} = 1.121\text{ rad/s}$ and $\omega = \omega_z$.

Then $a_{\text{rad}} = r\omega^2 = (0.300\text{ m})(1.121\text{ rad/s})^2 = 0.377\text{ m/s}^2.$

$a = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2} = \sqrt{(0.377\text{ m/s}^2)^2 + (0.180\text{ m/s}^2)^2} = 0.418\text{ m/s}^2$

(c) $\underline{\theta-\theta_0 = 120^\circ}$

$a_{\tan} = r\alpha = 0.180\text{ m/s}^2$

Calculate ω:

$\theta-\theta_0 = 120^\circ(\pi\text{ rad}/180^\circ) = 2.094\text{ rad};\ \omega_{0z} = 0;\ \alpha_z = 0.600\text{ rad/s}^2;\ \omega_z = ?$

$\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta-\theta_0)$

$\omega_z = \sqrt{2\alpha_z(\theta-\theta_0)} = \sqrt{2(0.600\text{ rad/s}^2)(2.094\text{ rad})} = 1.585\text{ rad/s}$ and $\omega = \omega_z$.

Then $a_{\text{rad}} = r\omega^2 = (0.300\text{ m})(1.585\text{ rad/s})^2 = 0.754\text{ m/s}^2.$

$a = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2} = \sqrt{(0.754\text{ m/s}^2)^2 + (0.180\text{ m/s}^2)^2} = 0.775\text{ m/s}^2$

EVALUATE: α is constant so $\alpha_{\tan}$ is constant. ω increases so a_{rad} increases.

9.26. IDENTIFY: Apply constant angular acceleration equations. $v = r\omega$. A point on the rim has both tangential and radial components of acceleration.

SET UP: $a_{\tan} = r\alpha$ and $a_{\rm rad} = r\omega^2$.

EXECUTE: **(a)** $\omega_z = \omega_{0z} + \alpha_z t = 0.250\ \text{rev/s} + (0.900\ \text{rev/s}^2)(0.200\ \text{s}) = 0.430\ \text{rev/s}$

(Note that since ω_{0z} and α_z are given in terms of revolutions, it's not necessary to convert to radians).

(b) $\omega_{\text{av-}z}\Delta t = (0.340\ \text{rev/s})(0.2\ \text{s}) = 0.068\ \text{rev}$.

(c) Here, the conversion to radians must be made to use Eq. (9.13), and

$$v = r\omega = \left(\frac{0.750\ \text{m}}{2}\right)(0.430\ \text{rev/s})(2\pi\ \text{rad/rev}) = 1.01\ \text{m/s}.$$

(d) Combining equations (9.14) and (9.15),

$a = \sqrt{a_{\rm rad}^2 + a_{\tan}^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2}$.

$a = \sqrt{\left[((0.430\ \text{rev/s})(2\pi\ \text{rad/rev}))^2(0.375\ \text{m})\right]^2 + \left[(0.900\ \text{rev/s}^2)(2\pi\ \text{rad/rev})(0.375\ \text{m})\right]^2}$.

$a = 3.46\ \text{m/s}^2$.

EVALUATE: If the angular acceleration is constant, $a_{\tan}$ is constant but $a_{\rm rad}$ increases as ω increases.

9.27. IDENTIFY: Use Eq.(9.15) and solve for *r*.

SET UP: $a_{\rm rad} = r\omega^2$ so $r = a_{\rm rad}/\omega^2$, where ω must be in rad/s

EXECUTE: $a_{\rm rad} = 3000g = 3000(9.80\ \text{m/s}^2) = 29{,}400\ \text{m/s}^2$

$\omega = (5000\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 523.6\ \text{rad/s}$

Then $r = \frac{a_{\rm rad}}{\omega^2} = \frac{29{,}400\ \text{m/s}^2}{(523.6\ \text{rad/s})^2} = 0.107\ \text{m}.$

EVALUATE: The diameter is then 0.214 m, which is larger than 0.127 m, so the claim is *not* realistic.

9.28. IDENTIFY: In part (b) apply the result derived in part (a).

SET UP: $a_{\rm rad} = r\omega^2$ and $v = r\omega$; combine to eliminate *r*.

EXECUTE: **(a)** $a_{\rm rad} = \omega^2 r = \omega^2\left(\frac{v}{\omega}\right) = \omega v.$

(b) From the result of part (a), $\omega = \frac{a_{\rm rad}}{v} = \frac{0.500\ \text{m/s}^2}{2.00\ \text{m/s}} = 0.250\ \text{rad/s}.$

EVALUATE: $a_{\rm rad} = r\omega^2$ and $v = r\omega$ both require that ω be in rad/s, so in $a_{\rm rad} = \omega v$, ω is in rad/s.

9.29. IDENTIFY: $v = r\omega$ and $a_{\rm rad} = r\omega^2 = v^2/r$.

SET UP: $2\pi\ \text{rad} = 1\ \text{rev}$, so $\pi\ \text{rad/s} = 30\ \text{rev/min}$.

EXECUTE: **(a)** $\omega r = (1250\ \text{rev/min})\left(\frac{\pi\ \text{rad/s}}{30\ \text{rev/min}}\right)\left(\frac{12.7\times10^{-3}\text{m}}{2}\right) = 0.831\ \text{m/s}.$

(b) $\frac{v^2}{r} = \frac{(0.831\ \text{m/s})^2}{(12.7\times10^{-3}\ \text{m})/2} = 109\ \text{m/s}^2.$

EVALUATE: In $v = r\omega$, ω must be in rad/s.

9.30. IDENTIFY: $a_{\tan} = r\alpha$, $v = r\omega$ and $a_{\rm rad} = v^2/r$. $\theta - \theta_0 = \omega_{\text{av-}z}t$.

SET UP: When α_z is constant, $\omega_{\text{av-}z} = \frac{\omega_{0z} + \omega_z}{2}$. Let the direction the wheel is rotating be positive.

EXECUTE: **(a)** $\alpha = \frac{a_{\tan}}{r} = \frac{-10.0\ \text{m/s}^2}{0.200\ \text{m}} = -50.0\ \text{rad/s}^2$

(b) At $t = 3.00$ s, $v = 50.0$ m/s and $\omega = \frac{v}{r} = \frac{50.0\ \text{m/s}}{0.200\ \text{m}} = 250\ \text{rad/s}$ and at $t = 0$,

$v = 50.0\ \text{m/s} + (-10.0\ \text{m/s}^2)(0 - 3.00\ \text{s}) = 80.0\ \text{m/s}$, $\omega = 400\ \text{rad/s}.$

(c) $\omega_{\text{av-}z}t = (325\ \text{rad/s})(3.00\ \text{s}) = 975\ \text{rad} = 155\ \text{rev}$.

(d) $v=\sqrt{a_{\text{rad}}r}=\sqrt{(9.80\text{ m/s}^2)(0.200\text{ m})}=1.40\text{ m/s}$. This speed will be reached at time $\dfrac{50.0\text{ m/s}-1.40\text{ m/s}}{10.0\text{ m/s}}=4.86\text{ s}$ after $t=3.00$ s, or at $t=7.86$ s. (There are many equivalent ways to do this calculation.)

EVALUATE: At $t=0$, $a_{\text{rad}}=r\omega^2=3.20\times10^4\text{ m/s}^2$. At $t=3.00$ s, $a_{\text{rad}}=1.25\times10^4\text{ m/s}^2$. For $a_{\text{rad}}=g$ the wheel must be rotating more slowly than at 3.00 s so it occurs some time after 3.00 s.

9.31. IDENTIFY and SET UP: Use Eq.(9.15) to relate ω to a_{rad} and $\sum\vec{F}=m\vec{a}$ to relate a_{rad} to F_{rad}. Use Eq.(9.13) to relate ω and v, where v is the tangential speed.

EXECUTE: **(a)** $a_{\text{rad}}=r\omega^2$ and $F_{\text{rad}}=ma_{\text{rad}}=mr\omega^2$

$$\frac{F_{\text{rad},2}}{F_{\text{rad},1}}=\left(\frac{\omega_2}{\omega_1}\right)^2=\left(\frac{640\text{ rev/min}}{423\text{ rev/min}}\right)^2=2.29$$

(b) $v=r\omega$

$$\frac{v_2}{v_1}=\frac{\omega_2}{\omega_1}=\frac{640\text{ rev/min}}{423\text{ rev/min}}=1.51$$

(c) $v=r\omega$

$$\omega=(640\text{ rev/min})\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)=67.0\text{ rad/s}$$

Then $v=r\omega=(0.235\text{ m})(67.0\text{ rad/s})=15.7\text{ m/s}$.

$$a_{\text{rad}}=r\omega^2=(0.235\text{ m})(67.0\text{ rad/s})^2=1060\text{ m/s}^2$$

$$\frac{a_{\text{rad}}}{g}=\frac{1060\text{ m/s}^2}{9.80\text{ m/s}^2}=108;\quad a=108g$$

EVALUATE: In parts (a) and (b), since a ratio is used the units cancel and there is no need to convert ω to rad/s. In part (c), v and a_{rad} are calculated from ω, and ω must be in rad/s.

9.32. IDENTIFY: $v=r\omega$ and $a_{\tan}=r\alpha$.

SET UP: The linear acceleration of the bucket equals $a_{\tan}$ for a point on the rim of the axle.

EXECUTE: **(a)** $v=R\omega$. $2.00\text{ cm/s}=R\left(\dfrac{7.5\text{ rev}}{\text{min}}\right)\left(\dfrac{1\text{ min}}{60\text{ s}}\right)\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)$ gives $R=2.55$ cm.

$D=2R=5.09$ cm.

(b) $a_{\tan}=R\alpha$. $\alpha=\dfrac{a_{\tan}}{R}=\dfrac{0.400\text{ m/s}^2}{0.0255\text{ m}}=15.7\text{ rad/s}^2$.

EVALUATE: In $v=R\omega$ and $a_{\tan}=R\alpha$, ω and α must be in radians.

9.33. IDENTIFY: Apply $v=r\omega$.

SET UP: Points on the chain all move at the same speed, so $r_{\text{r}}\omega_{\text{r}}=r_{\text{f}}\omega_{\text{f}}$.

EXECUTE: The angular velocity of the rear wheel is $\omega_{\text{r}}=\dfrac{v_{\text{r}}}{r}=\dfrac{5.00\text{ m/s}}{0.330\text{ m}}=15.15\text{ rad/s}$.

The angular velocity of the front wheel is $\omega_{\text{f}}=0.600\text{ rev/s}=3.77\text{ rad/s}$. $r_{\text{r}}=r_{\text{f}}(\omega_{\text{f}}/\omega_{\text{r}})=2.99$ cm.

EVALUATE: The rear sprocket and wheel have the same angular velocity and the front sprocket and wheel have the same angular velocity. $r\omega$ is the same for both, so the rear sprocket has a smaller radius since it has a larger angular velocity. The speed of a point on the chain is $v=r_{\text{r}}\omega_{\text{r}}=(2.99\times10^{-2}\text{ m})(15.15\text{ rad/s})=0.453\text{ m/s}$. The linear speed of the bicycle is 5.50 m/s.

9.34. IDENTIFY and SET UP: Use Eq.(9.16). Treat the spheres as point masses and ignore I of the light rods.

EXECUTE: The object is shown in Figure 9.34a.

(a)

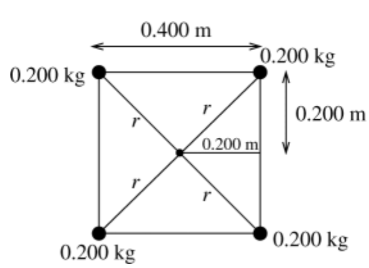


Figure 9.34a

$$r=\sqrt{(0.200\text{ m})^2+(0.200\text{ m})^2}=0.2828\text{ m}$$

$$I=\sum m_ir_i^2=4(0.200\text{ kg})(0.2828\text{ m})^2$$

$$I=0.0640\text{ kg}\cdot\text{m}^2$$

(b) The object is shown in Figure 9.34b.

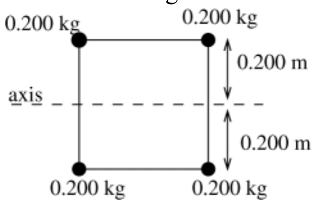


Figure 9.34b

$r = 0.200$ m

$I = \sum m_i r_i^2 = 4(0.200 \text{ kg})(0.200 \text{ m})^2$

$I = 0.0320 \text{ kg}\cdot\text{m}^2$

(c) The object is shown in Figure 9.34c.

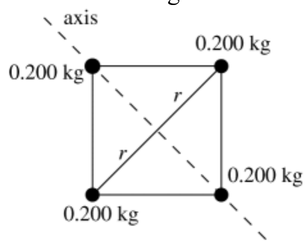


Figure 9.34c

$r = 0.2828$ m

$I = \sum m_i r_i^2 = 2(0.200 \text{ kg})(0.2828 \text{ m})^2$

$I = 0.0320 \text{ kg}\cdot\text{m}^2$

EVALUATE: In general I depends on the axis and our answer for part (a) is larger than for parts (b) and (c). It just happens that I is the same in parts (b) and (c).

9.35. IDENTIFY: Use Table 9.2. The correct expression to use in each case depends on the shape of the object and the location of the axis.

SET UP: In each case express the mass in kg and the length in m, so the moment of inertia will be in $\text{kg}\cdot\text{m}^2$.

EXECUTE: **(a)** (i) $I = \frac{1}{3}ML^2 = \frac{1}{3}(2.50 \text{ kg})(0.750 \text{ m})^2 = 0.469 \text{ kg}\cdot\text{m}^2$.

(ii) $I = \frac{1}{12}ML^2 = \frac{1}{4}(0.469 \text{ kg}\cdot\text{m}^2) = 0.117 \text{ kg}\cdot\text{m}^2$. (iii) For a very thin rod, all of the mass is at the axis and $I = 0$.

(b) (i) $I = \frac{2}{5}MR^2 = \frac{2}{5}(3.00 \text{ kg})(0.190 \text{ m})^2 = 0.0433 \text{ kg}\cdot\text{m}^2$.

(ii) $I = \frac{2}{3}MR^2 = \frac{5}{3}(0.0433 \text{ kg}\cdot\text{m}^2) = 0.0722 \text{ kg}\cdot\text{m}^2$.

(c) (i) $I = MR^2 = (8.00 \text{ kg})(0.0600 \text{ m})^2 = 0.0288 \text{ kg}\cdot\text{m}^2$.

(ii) $I = \frac{1}{2}MR^2 = \frac{1}{2}(8.00 \text{ kg})(0.0600 \text{ m})^2 = 0.0144 \text{ kg}\cdot\text{m}^2$.

EVALUATE: I depends on how the mass of the object is distributed relative to the axis.

9.36. IDENTIFY: Treat each block as a point mass, so for each block $I = mr^2$, where r is the distance of the block from the axis. The total I for the object is the sum of the I for each of its pieces.

SET UP: In part (a) two blocks are a distance $L/2$ from the axis and the third block is on the axis. In part (b) two blocks are a distance $L/4$ from the axis and one is a distance $3L/4$ from the axis.

EXECUTE: **(a)** $I = 2m(L/2)^2 = \frac{1}{2}mL^2$.

(b) $I = 2m(L/4)^2 + m(3L/4)^2 = \dfrac{1}{16}mL^2(2+9) = \dfrac{11}{16}mL^2$.

EVALUATE: For the same object I is in general different for different axes.

9.37. IDENTIFY: I for the object is the sum of the values of I for each part.

SET UP: For the bar, for an axis perpendicular to the bar, use the appropriate expression from Table 9.2. For a point mass, $I = mr^2$, where r is the distance of the mass from the axis.

EXECUTE: **(a)** $I = I_{\text{bar}} + I_{\text{balls}} = \dfrac{1}{12}M_{\text{bar}}L^2 + 2m_{\text{balls}}\left(\dfrac{L}{2}\right)^2$.

$$I = \frac{1}{12}(4.00 \text{ kg})(2.00 \text{ m})^2 + 2(0.500 \text{ kg})(1.00 \text{ m})^2 = 2.33 \text{ kg}\cdot\text{m}^2$$

(b) $I = \dfrac{1}{3}m_{\text{bar}}L^2 + m_{\text{ball}}L^2 = \dfrac{1}{3}(4.00 \text{ kg})(2.00 \text{ m})^2 + (0.500 \text{ kg})\ (2.00 \text{ m})^2 = 7.33 \text{ kg}\cdot\text{m}^2$

(c) $I = 0$ because all masses are on the axis.

(d) All the mass is a distance $d = 0.500$ m from the axis and

$I = m_{\text{bar}}d^2 + 2m_{\text{ball}}d^2 = M_{\text{Total}}d^2 = (5.00 \text{ kg})(0.500 \text{ m})^2 = 1.25 \text{ kg}\cdot\text{m}^2$.

EVALUATE: I for an object depends on the location and direction of the axis.

9.38. **IDENTIFY** and **SET UP:** According to Eq.(9.16), I for the entire object equals the sum of I for each piece, the rod plus the end caps. The object is shown in Figure 9.38.

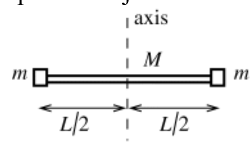


Figure 9.38

EXECUTE: $I = I_{\text{rod}} + 2I_{\text{cap}}$

$I = \frac{1}{12}ML^2 + 2(m)(L/2)^2 = \left(\frac{1}{12}M + \frac{1}{2}m\right)L^2$

EVALUATE: Table 9.2 was used for I_{rod} and $I = mr^2$ for the end caps, since they are treated as point particles.

9.39. **IDENTIFY** and **SET UP:** $I = \sum m_i r_i^2$ implies $I = I_{\text{rim}} + I_{\text{spokes}}$

EXECUTE: $I_{\text{rim}} = MR^2 = (1.40 \text{ kg})(0.300 \text{ m})^2 = 0.126 \text{ kg} \cdot \text{m}^2$

Each spoke can be treated as a slender rod with the axis through one end, so

$I_{\text{spokes}} = 8\left(\frac{1}{3}ML^2\right) = \frac{8}{3}(0.280 \text{ kg})(0.300 \text{ m})^2 = 0.0672 \text{ kg} \cdot \text{m}^2$

$I = I_{\text{rim}} + I_{\text{spokes}} = 0.126 \text{ kg} \cdot \text{m}^2 + 0.0672 \text{ kg} \cdot \text{m}^2 = 0.193 \text{ kg} \cdot \text{m}^2$

EVALUATE: Our result is smaller than $m_{\text{tot}}R^2 = (3.64 \text{ kg})(0.300 \text{ m})^2 = 0.328 \text{ kg} \cdot \text{m}^2$, since the mass of each spoke is distributed between $r = 0$ and $r = R$.

9.40. **IDENTIFY:** Compare this object to a uniform disk of radius R and mass $2M$.

SET UP: With an axis perpendicular to the round face of the object at its center, I for a uniform disk is the same as for a solid cylinder.

EXECUTE: **(a)** The total I for a disk of mass $2M$ and radius R, $I = \frac{1}{2}(2M)R^2 = MR^2$. Each half of the disk has the same I, so for the half-disk, $I = \frac{1}{2}MR^2$.

(b) The same mass M is distributed the same way as a function of distance from the axis.

(c) The same method as in part (a) says that I for a quarter-disk of radius R and mass M is half that of a half-disk of radius R and mass $2M$, so $I = \frac{1}{2}(\frac{1}{2}[2M]R^2) = \frac{1}{2}MR^2$.

EVALUATE: I depends on how the mass of the object is distributed relative to the axis, and this is the same for any segment of a disk.

9.41. **IDENTIFY:** I for the compound disk is the sum of I of the solid disk and of the ring.

SET UP: For the solid disk, $I = \frac{1}{2}m_{\text{d}}r_{\text{d}}^2$. For the ring, $I_{\text{r}} = \frac{1}{2}m_{\text{r}}(r_1^2 + r_2^2)$, where $r_1 = 50.0 \text{ cm}, r_2 = 70.0 \text{ cm}$. The mass of the disk and ring is their area times their area density.

EXECUTE: $I = I_{\text{d}} + I_{\text{r}}$.

Disk: $m_{\text{d}} = (3.00 \text{ g/cm}^2)\pi r_{\text{d}}^2 = 23.56 \text{ kg}$. $I_{\text{d}} = \frac{1}{2}m_{\text{d}}r_{\text{d}}^2 = 2.945 \text{ kg} \cdot \text{m}^2$.

Ring: $m_{\text{r}} = (2.00 \text{ g/cm}^2)\pi(r_2^2 - r_1^2) = 15.08 \text{ kg}$. $I_{\text{r}} = \frac{1}{2}m_{\text{r}}(r_1^2 + r_2^2) = 5.580 \text{ kg} \cdot \text{m}^2$.

$I = I_{\text{d}} + I_{\text{r}} = 8.52 \text{ kg} \cdot \text{m}^2$.

EVALUATE: Even though $m_{\text{r}} < m_{\text{d}}$, $I_{\text{r}} > I_{\text{d}}$ since the mass of the ring is farther from the axis.

9.42. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2b to calculate I.

SET UP: $I = \frac{1}{12}ML^2$. $1 \text{ rpm} = 0.1047 \text{ rad/s}$

EXECUTE: **(a)** $I = \frac{1}{12}(117 \text{ kg})(2.08 \text{ m})^2 = 42.2 \text{ kg} \cdot \text{m}^2$. $\omega = (2400 \text{ rev/min})\left(\frac{0.1047 \text{ rad/s}}{1 \text{ rev/min}}\right) = 251 \text{ rad/s}$.

$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(42.2 \text{ kg} \cdot \text{m}^2)(251 \text{ rad/s})^2 = 1.33 \times 10^6 \text{ J}$.

(b) $K_1 = \frac{1}{12}M_1L_1^2\omega_1^2$, $K_2 = \frac{1}{12}M_2L_2^2\omega_2^2$. $L_1 = L_2$ and $K_1 = K_2$, so $M_1\omega_1^2 = M_2\omega_2^2$.

$\omega_2 = \omega_1\sqrt{\frac{M_1}{M_2}} = (2400 \text{ rpm})\sqrt{\frac{M_1}{0.750M_1}} = 2770 \text{ rpm}$

EVALUATE: The rotational kinetic energy is proportional to the square of the angular speed and directly proportional to the mass of the object.

9.43. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2 to calculate I.

SET UP: $I = \frac{2}{5}MR^2$. For the moon, $M = 7.35 \times 10^{22} \text{ kg}$ and $R = 1.74 \times 10^6 \text{ m}$. The moon moves through $1 \text{ rev} = 2\pi \text{ rad}$ in 27.3 d. $1 \text{ d} = 8.64 \times 10^4 \text{ s}$.

EXECUTE: **(a)** $I = \frac{2}{5}(7.35\times10^{22}\ \text{kg})(1.74\times10^{6}\ \text{m})^2 = 8.90\times10^{34}\ \text{kg}\cdot\text{m}^2$.

$$\omega = \frac{2\pi\ \text{rad}}{(27.3\ \text{d})(8.64\times10^{4}\ \text{s/d})} = 2.66\times10^{-6}\ \text{rad/s}.$$

$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(8.90\times10^{34}\ \text{kg}\cdot\text{m}^2)(2.66\times10^{-6}\ \text{rad/s})^2 = 3.15\times10^{23}\ \text{J}$.

(b) $\dfrac{3.15\times10^{23}\ \text{J}}{5(4.0\times10^{20}\ \text{J})} = 158$ years. Considering the expense involved in tapping the moon's rotational energy, this does not seem like a worthwhile scheme for only 158 years worth of energy.

EVALUATE: The moon has a very large amount of kinetic energy due to its motion. The earth has even more, but changing the rotation rate of the earth would change the length of a day.

9.44. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$. Use Table 9.2 to relate *I* to the mass *M* of the disk.

SET UP: 45.0 rpm = 4.71 rad/s. For a uniform solid disk, $I = \frac{1}{2}MR^2$.

EXECUTE: **(a)** $I = \dfrac{2K}{\omega^2} = \dfrac{2(0.250\ \text{J})}{(4.71\ \text{rad/s})^2} = 0.0225\ \text{kg}\cdot\text{m}^2$.

(b) $I = \frac{1}{2}MR^2$ and $M = \dfrac{2I}{R^2} = \dfrac{2(0.0225\ \text{kg}\cdot\text{m}^2)}{(0.300\ \text{m})^2} = 0.500\ \text{kg}$.

EVALUATE: No matter what the shape is, the rotational kinetic energy is proportional to the mass of the object.

9.45. **IDENTIFY:** $K = \frac{1}{2}I\omega^2$, with ω in rad/s. Solve for *I*.

SET UP: 1 rev/min $= (2\pi/60)$ rad/s. $\Delta K = -500$ J

EXECUTE: $\omega_\text{i} = 650$ rev/min $= 68.1$ rad/s. $\omega_\text{f} = 520$ rev/min $= 54.5$ rad/s. $\Delta K = K_\text{f} - K_\text{i} = \frac{1}{2}I(\omega_\text{f}^2 - \omega_\text{i}^2)$ and

$$I = \frac{2(\Delta K)}{\omega_\text{f}^2 - \omega_\text{i}^2} = \frac{2(-500\ \text{J})}{(54.5\ \text{rad/s})^2 - (68.1\ \text{rad/s})^2} = 0.600\ \text{kg}\cdot\text{m}^2.$$

EVALUATE: In $K = \frac{1}{2}I\omega^2$, ω must be in rad/s.

9.46. **IDENTIFY:** The work done on the cylinder equals its gain in kinetic energy.

SET UP: The work done on the cylinder is *PL*, where *L* is the length of the rope. $K_1 = 0$. $K_2 = \frac{1}{2}I\omega^2$.

$I = \frac{1}{2}mr^2 = \frac{1}{2}\left(\dfrac{w}{g}\right)r^2$.

EXECUTE: $PL = \dfrac{1}{2}\dfrac{w}{g}v^2$, or $P = \dfrac{1}{2}\dfrac{w}{g}\dfrac{v^2}{L} = \dfrac{(40.0\ \text{N})(6.00\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)(5.00\ \text{m})} = 14.7\ \text{N}$.

EVALUATE: The linear speed *v* of the end of the rope equals the tangential speed of a point on the rim of the cylinder. When *K* is expressed in terms of *v*, the radius *r* of the cylinder doesn't appear.

9.47. **IDENTIFY** and **SET UP:** Combine Eqs.(9.17) and (9.15) to solve for *K*. Use Table 9.2 to get *I*.

EXECUTE: $K = \frac{1}{2}I\omega^2$

$a_\text{rad} = R\omega^2$, so $\omega = \sqrt{a_\text{rad}/R} = \sqrt{(3500\ \text{m/s}^2)/1.20\ \text{m}} = 54.0\ \text{rad/s}$

For a disk, $I = \frac{1}{2}MR^2 = \frac{1}{2}(70.0\ \text{kg})(1.20\ \text{m})^2 = 50.4\ \text{kg}\cdot\text{m}^2$

Thus $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(50.4\ \text{kg}\cdot\text{m}^2)(54.0\ \text{rad/s})^2 = 7.35\times10^4\ \text{J}$

EVALUATE: The limit on a_rad limits ω which in turn limits *K*.

9.48. **IDENTIFY:** Repeat the calculation in Example 9.9, but with a different expression for *I*.

SET UP: For the solid cylinder in Example 9.9, $I = \frac{1}{2}MR^2$. For the thin-walled, hollow cylinder, $I = MR^2$.

EXECUTE: **(a)** With $I = MR^2$, the expression for *v* is $v = \sqrt{\dfrac{2gh}{1 + M/m}}$.

(b) This expression is smaller than that for the solid cylinder; more of the cylinder's mass is concentrated at its edge, so for a given speed, the kinetic energy of the cylinder is larger. A larger fraction of the potential energy is converted to the kinetic energy of the cylinder, and so less is available for the falling mass.

EVALUATE: When *M* is much larger than *m*, *v* is very small. When *M* is much less than *m*, *v* becomes $v = \sqrt{2gh}$, the same as for a mass that falls freely from a height *h*.

9.49. **IDENTIFY:** Apply conservation of energy to the system of stone plus pulley. $v = r\omega$ relates the motion of the stone to the rotation of the pulley.

SET UP: For a uniform solid disk, $I = \frac{1}{2}MR^2$. Let point 1 be when the stone is at its initial position and point 2 be when it has descended the desired distance. Let $+y$ be upward and take $y = 0$ at the initial position of the stone, so $y_1 = 0$ and $y_2 = -h$, where h is the distance the stone descends.

EXECUTE: **(a)** $K_\text{p} = \frac{1}{2}I_\text{p}\omega^2$. $I_\text{p} = \frac{1}{2}M_\text{p}R^2 = \frac{1}{2}(2.50\text{ kg})(0.200\text{ m})^2 = 0.0500\text{ kg}\cdot\text{m}^2$.

$\omega = \sqrt{\dfrac{2K_\text{p}}{I_\text{p}}} = \sqrt{\dfrac{2(4.50\text{ J})}{0.0500\text{ kg}\cdot\text{m}^2}} = 13.4\text{ rad/s}$. The stone has speed $v = R\omega = (0.200\text{ m})(13.4\text{ rad/s}) = 2.68\text{ m/s}$. The stone has kinetic energy $K_\text{s} = \frac{1}{2}mv^2 = \frac{1}{2}(1.50\text{ kg})(2.68\text{ m/s})^2 = 5.39\text{ J}$. $K_1 + U_1 = K_2 + U_2$ gives $0 = K_2 + U_2$.

$0 = 4.50\text{ J} + 5.39\text{ J} + mg(-h)$. $h = \dfrac{9.89\text{ J}}{(1.50\text{ kg})(9.80\text{ m/s}^2)} = 0.673\text{ m}$.

(b) $K_\text{tot} = K_\text{p} + K_\text{s} = 9.89\text{ J}$. $\dfrac{K_\text{p}}{K_\text{tot}} = \dfrac{4.50\text{ J}}{9.89\text{ J}} = 45.5\%$.

EVALUATE: The gravitational potential energy of the pulley doesn't change as it rotates. The tension in the wire does positive work on the pulley and negative work of the same magnitude on the stone, so no net work on the system.

9.50. **IDENTIFY:** $K_\text{p} = \frac{1}{2}I\omega^2$ for the pulley and $K_\text{b} = \frac{1}{2}mv^2$ for the bucket. The speed of the bucket and the rotational speed of the pulley are related by $v = R\omega$.

SET UP: $K_\text{p} = \frac{1}{2}K_\text{b}$

EXECUTE: $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{1}{2}mv^2) = \frac{1}{4}mR^2\omega^2$. $I = \frac{1}{2}mR^2$.

EVALUATE: The result is independent of the rotational speed of the pulley and the linear speed of the mass.

9.51. **IDENTIFY:** The general expression for I is Eq.(9.16). $K = \frac{1}{2}I\omega^2$.

SET UP: R will be multiplied by f.

EXECUTE: **(a)** In the expression of Eq. (9.16), each term will have the mass multiplied by f^3 and the distance multiplied by f, and so the moment of inertia is multiplied by $f^3(f)^2 = f^5$.

(b) $(2.5\text{ J})(48)^5 = 6.37\times 10^8\text{ J}$.

EVALUATE: Mass and volume are proportional to each other so both scale by the same factor.

9.52. **IDENTIFY:** The work the person does is the negative of the work done by gravity. $W_\text{grav} = U_\text{grav,1} - U_\text{grav,2}$. $U_\text{grav} = Mgy_\text{cm}$.

SET UP: The center of mass of the ladder is at its center, 1.00 m from each end. $y_\text{cm,1} = (1.00\text{ m})\sin 53.0° = 0.799\text{ m}$. $y_\text{cm,2} = 1.00\text{ m}$.

EXECUTE: $W_\text{grav} = (9.00\text{ kg})(9.80\text{ m/s}^2)(0.799\text{ m} - 1.00\text{ m}) = -17.7\text{ J}$. The work done by the person is 17.7 J.

EVALUATE: The gravity force is downward and the center of mass of the ladder moves upward, so gravity does negative work. The person pushes upward and does positive work.

9.53. **IDENTIFY:** $U = Mgy_\text{cm}$. $\Delta U = U_2 - U_1$.

SET UP: Half the rope has mass 1.50 kg and length 12.0 m. Let $y = 0$ at the top of the cliff and take $+y$ to be upward. The center of mass of the hanging section of rope is at its center and $y_\text{cm,2} = -6.00\text{ m}$.

EXECUTE: $\Delta U = U_2 - U_1 = mg(y_\text{cm,2} - y_\text{cm,1}) = (1.50\text{ kg})(9.80\text{ m/s}^2)(-6.00\text{ m} - 0) = -88.2\text{ J}$.

EVALUATE: The potential energy of the rope decreases when part of the rope moves downward.

9.54. **IDENTIFY:** Apply Eq.(9.19), the parallel-axis theorem.

SET UP: The center of mass of the hoop is at its geometrical center.

EXECUTE: In Eq. (9.19), $I_\text{cm} = MR^2$ and $d = R^2$, so $I_P = 2MR^2$.

EVALUATE: I is larger for an axis at the edge than for an axis at the center. Some mass is closer than distance R from the axis but some is also farther away. Since I for each piece of the hoop is proportional to the square of the distance from the axis, the increase in distance has a larger effect.

9.55. **IDENTIFY:** Use Eq.(9.19) to relate I for the wood sphere about the desired axis to I for an axis along a diameter.

SET UP: For a thin-walled hollow sphere, axis along a diameter, $I = \frac{2}{3}MR^2$.

For a solid sphere with mass M and radius R, $I_\text{cm} = \frac{2}{5}MR^2$, for an axis along a diameter.

EXECUTE: Find d such that $I_P = I_{\text{cm}} + Md^2$ with $I_P = \frac{2}{3}MR^2$:

$\frac{2}{3}MR^2 = \frac{2}{5}MR^2 + Md^2$

The factors of M divide out and the equation becomes $\left(\frac{2}{3} - \frac{2}{5}\right)R^2 = d^2$

$d = \sqrt{(10-6)/15}R = 2R/\sqrt{15} = 0.516R.$

The axis is parallel to a diameter and is $0.516R$ from the center.

EVALUATE: $I_{\text{cm}}(\text{lead}) > I_{\text{cm}}(\text{wood})$ even though M and R are the same since for a hollow sphere all the mass is a distance R from the axis. Eq.(9.19) says $I_P > I_{\text{cm}}$, so there must be a d where $I_P(\text{wood}) = I_{\text{cm}}(\text{lead})$.

9.56. **IDENTIFY:** Using the parallel-axis theorem to find the moment of inertia of a thin rod about an axis through its end and perpendicular to the rod.

SET UP: The center of mass of the rod is at its center, and $I_{\text{cm}} = \frac{1}{12}ML^2$.

EXECUTE: $I_p = I_{\text{cm}} + Md^2 = \dfrac{M}{12}L^2 + M\left(\dfrac{L}{2}\right)^2 = \dfrac{M}{3}L^2.$

EVALUATE: I is larger when the axis is not at the center of mass.

9.57. **IDENTIFY** and **SET UP:** Use Eq.(9.19). The cm of the sheet is at its geometrical center. The object is sketched in Figure 9.57.

EXECUTE: $I_P = I_{\text{cm}} + Md^2.$

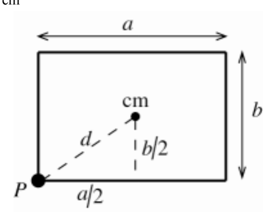


Figure 9.57

From part (c) of Table 9.2, $I_{\text{cm}} = \frac{1}{12}M(a^2 + b^2).$
The distance d of P from the cm is $d = \sqrt{(a/2)^2 + (b/2)^2}.$

Thus $I_P = I_{\text{cm}} + Md^2 = \frac{1}{12}M(a^2+b^2) + M\left(\frac{1}{4}a^2 + \frac{1}{4}b^2\right) = \left(\frac{1}{12} + \frac{1}{4}\right)M(a^2+b^2) =$

$\frac{1}{3}M(a^2+b^2)$

EVALUATE: $I_P = 4I_{\text{cm}}$. For an axis through P mass is farther from the axis.

9.58. **IDENTIFY:** Consider the plate as made of slender rods placed side-by-side.

SET UP: The expression in Table 9.2(a) gives I for a rod and an axis through the center of the rod.

EXECUTE: **(a)** I is the same as for a rod with length a: $I = \frac{1}{12}Ma^2$.

(b) I is the same as for a rod with length b: $I = \frac{1}{12}Mb^2$.

EVALUATE: I is smaller when the axis is through the center of the plate than when it is along one edge.

9.59. **IDENTIFY:** Use the equations in Table 9.2. I for the rod is the sum of I for each segment. The parallel-axis theorem says $I_{\text{p}} = I_{\text{cm}} + Md^2$.

SET UP: The bent rod and axes a and b are shown in Figure 9.59. Each segment has length $L/2$ and mass $M/2$.

EXECUTE: **(a)** For each segment the moment of inertia is for a rod with mass $M/2$, length $L/2$ and the axis through one end. For one segment, $I_{\text{s}} = \dfrac{1}{3}\left(\dfrac{M}{2}\right)\left(\dfrac{L}{2}\right)^2 = \dfrac{1}{24}ML^2$. For the rod, $I_{\text{a}} = 2I_{\text{s}} = \dfrac{1}{12}ML^2$.

(b) The center of mass of each segment is at the center of the segment, a distance of $L/4$ from each end. For each segment, $I_{\text{cm}} = \dfrac{1}{12}\left(\dfrac{M}{2}\right)\left(\dfrac{L}{2}\right)^2 = \dfrac{1}{96}ML^2$. Axis b is a distance $L/4$ from the cm of each segment, so for each segment the parallel axis theorem gives I for axis b to be $I_{\text{s}} = \dfrac{1}{96}ML^2 + \dfrac{M}{2}\left(\dfrac{L}{4}\right)^2 = \dfrac{1}{24}ML^2$ and $I_{\text{b}} = 2I_{\text{s}} = \dfrac{1}{12}ML^2$.

EVALUATE: I for these two axes are the same.

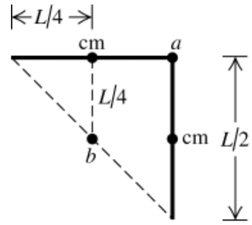


Figure 9.59

9.60. IDENTIFY: Apply the parallel-axis theorem.

SET UP: In Eq.(9.19), $I_{cm} = \frac{M}{12}L^2$ and $d = (L/2 - h)$.

EXECUTE: $I_P = M\left[\frac{1}{12}L^2 + \left(\frac{L}{2} - h\right)^2\right] = M\left[\frac{1}{12}L^2 + \frac{1}{4}L^2 - Lh + h^2\right] = M\left[\frac{1}{3}L^2 - Lh + h^2\right]$,

which is the same as found in Example 9.11.

EVALUATE: Example 9.11 shows that this result gives the expected result for $h = 0$, $h = L$ and $h = L/2$.

9.61. IDENTIFY: Apply Eq.(9.20).

SET UP: $dm = \rho dV = \rho(2\pi rL\ dr)$, where L is the thickness of the disk. $M = \pi L\rho R^2$.

EXECUTE: The analysis is identical to that of Example 9.12, with the lower limit in the integral being zero and the upper limit being R. The result is $I = \frac{1}{2}MR^2$.

EVALUATE: Our result agrees with Table 9.2(f).

9.62. IDENTIFY: Eq.(9.20), $I = \int r^2\ dm$

SET UP:

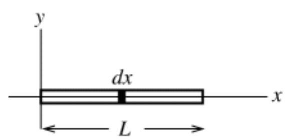


Figure 9.62

Take the x-axis to lie along the rod, with the origin at the left end. Consider a thin slice at coordinate x and width dx, as shown in Figure 9.62. The mass per unit length for this rod is M/L, so the mass of this slice is $dm = (M/L)\ dx$.

EXECUTE: $I = \int_0^L x^2(M/L)\ dx = (M/L)\int_0^L x^2\ dx = (M/L)(L^3/3) = \frac{1}{3}ML^2$

EVALUATE: This result agrees with Table 9.2.

9.63. IDENTIFY: Apply Eq.(9.20).

SET UP: For this case, $dm = \gamma\ dx$.

EXECUTE: **(a)** $M = \int dm = \int_0^L \gamma x\ dx = \gamma \frac{x^2}{2}\Big|_0^L = \frac{\gamma L^2}{2}$

(b) $I = \int_0^L x^2(\gamma x)dx = \gamma \frac{x^4}{4}\Big|_0^L = \frac{\gamma L^4}{4} = \frac{M}{2}L^2$. This is larger than the moment of inertia of a uniform rod of the same mass and length, since the mass density is greater further away from the axis than nearer the axis.

(c) $I = \int_0^L (L-x)^2\gamma x dx = \gamma\int_0^L (L^2x - 2Lx^2 + x^3)dx = \gamma\left(L^2\frac{x^2}{2} - 2L\frac{x^3}{3} + \frac{x^4}{4}\right)\Bigg|_0^L = \gamma\frac{L^4}{12} = \frac{M}{6}L^2$.

This is a third of the result of part (b), reflecting the fact that more of the mass is concentrated at the right end.

EVALUATE: For a uniform rod with an axis at one end, $I = \frac{1}{3}ML^2$. The result in (b) is larger than this and the result in (a) is smaller than this.

9.64. IDENTIFY: We know that $v = r\omega$ and $\vec{v}$ is tangential. We know that $a_{rad} = r\omega^2$ and $\vec{a}_{rad}$ is in toward the center of the wheel. See if the vector product expressions give these results.

SET UP: $|\boldsymbol{A}\times\boldsymbol{B}| = AB\sin\phi$, where ϕ is the angle between $\vec{A}$ and $\vec{B}$.

EXECUTE: **(a)** For a counterclockwise rotation, $\vec{\omega}$ will be out of the page.

(b) The upward direction crossed into the radial direction is, by the right-hand rule, counterclockwise. $\vec{\omega}$ and $\vec{r}$ are perpendicular, so the magnitude of $\vec{\omega}\times\vec{r}$ is $\omega r = v$.

(c) $\vec{\omega}$ is perpendicular to $\vec{v}$ and so $\vec{\omega}\times\vec{v}$ has magnitude $\omega v = a_{\text{rad}},$ and from the right-hand rule, the upward direction crossed into the counterclockwise direction is inward, the direction of $\vec{a}_{\text{rad}}$.

EVALUATE: If the wheel rotates clockwise, the directions of $\vec{\omega}$ and $\vec{v}$ are reversed, but $\vec{a}_{\text{rad}}$ is still inward.

9.65. IDENTIFY: Apply $\theta = \omega t$.

SET UP: For alignment, the earth must move through $60°$ more than Mars, in the same time *t*. $\omega_{\text{e}} = 360°/\text{yr}$. $\omega_{\text{M}} = 360°/(1.9\text{ yr})$.

EXECUTE: $\theta_{\text{e}} = \theta_{\text{M}} + 60°$. $\omega_{\text{e}} t = \omega_{\text{M}} t + 60°$.

$$t = \frac{60°}{\omega_{\text{e}} - \omega_{\text{M}}} = \frac{60°}{\dfrac{360°}{1\text{ yr}} - \dfrac{360°}{1.9\text{ yr}}} = \frac{60°}{360°}(1/[0.9\text{yr}/1.9\text{ yr}^2]) = 0.352\text{ yr} = 128\text{ days}.$$

EVALUATE: Earth has a larger angular velocity than Mars, and completes one orbit in less time.

9.66. IDENTIFY and SET UP: Use Eqs.(9.3) and (9.5). As long as $\alpha_z > 0$, ω_z increases. At the *t* when $\alpha_z = 0$, ω_z is at its maximum positive value and then starts to decrease when α_z becomes negative.

$\theta(t) = \gamma t^2 - \beta t^3$; $\gamma = 3.20\text{ rad/s}^2$, $\beta = 0.500\text{ rad/s}^3$

EXECUTE: **(a)** $\omega_z(t) = \dfrac{d\theta}{dt} = \dfrac{d(\gamma t^2 - \beta t^3)}{dt} = 2\gamma t - 3\beta t^2$

(b) $\alpha_z(t) = \dfrac{d\omega_z}{dt} = \dfrac{d(2\gamma t - 3\beta t^2)}{dt} = 2\gamma - 6\beta t$

(c) The maximum angular velocity occurs when $\alpha_z = 0$.

$2\gamma - 6\beta t = 0$ implies $t = \dfrac{2\gamma}{6\beta} = \dfrac{\gamma}{3\beta} = \dfrac{3.20\text{ rad/s}^2}{3(0.500\text{ rad/s}^3)} = 2.133\text{ s}$

At this *t*, $\omega_z = 2\gamma t - 3\beta t^2 = 2(3.20\text{ rad/s}^2)(2.133\text{ s}) - 3(0.500\text{ rad/s}^3)(2.133\text{ s})^2 = 6.83\text{ rad/s}$

The maximum positive angular velocity is 6.83 rad/s and it occurs at 2.13 s.

EVALUATE: For large *t* both ω_z and α_z are negative and ω_z increases in magnitude. In fact, $\omega_z \to -\infty$ at $t \to \infty$. So the answer in (c) is not the largest angular speed, just the largest positive angular velocity.

9.67. IDENTIFY: The angular acceleration α of the disk is related to the linear acceleration a of the ball by $a = R\alpha$. Since the acceleration is not constant, use $\omega_z - \omega_{0z} = \int_0^t \alpha_z dt$ and $\theta - \theta_0 = \int_0^t \omega_z dt$ to relate θ, ω_z, α_z and *t* for the disk. $\omega_{0z} = 0$.

SET UP: $\int t^n dt = \dfrac{1}{n+1} t^{n+1}$. In $a = R\alpha$, α is in rad/s^2.

EXECUTE: **(a)** $A = \dfrac{a}{t} = \dfrac{1.80\text{ m/s}^2}{3.00\text{ s}} = 0.600\text{ m/s}^3$

(b) $\alpha = \dfrac{a}{R} = \dfrac{(0.600\text{ m/s}^3)t}{0.250\text{ m}} = (2.40\text{ rad/s}^3)t$

(c) $\omega_z = \int_0^t (2.40\text{ rad/s}^3)t dt = (1.20\text{ rad/s}^3)t^2$. $\omega_z = 15.0\text{ rad/s}$ for $t = \sqrt{\dfrac{15.0\text{ rad/s}}{1.20\text{ rad/s}^3}} = 3.54\text{ s}$.

(d) $\theta - \theta_0 = \int_0^t \omega_z dt = \int_0^t (1.20\text{ rad/s}^3)t^2 dt = (0.400\text{ rad/s}^3)t^3$. For $t = 3.54\text{ s}$, $\theta - \theta_0 = 17.7\text{ rad}$.

EVALUATE: If the disk had turned at a constant angular velocity of 15.0 rad/s for 3.54 s it would have turned through an angle of 53.1 rad in 3.54 s. It actually turns through less than half this because the angular velocity is increasing in time and is less than 15.0 rad/s at all but the end of the interval.

9.68. IDENTIFY and SET UP: The translational kinetic energy is $K = \frac{1}{2}mv^2$ and the kinetic energy of the rotating flywheel is $K = \frac{1}{2}I\omega^2$. Use the scale speed to calculate the actual speed *v*. From that calculate *K* for the car and then solve for ω that gives this *K* for the flywheel.

EXECUTE: **(a)** $\dfrac{v_{\text{toy}}}{v_{\text{scale}}} = \dfrac{L_{\text{toy}}}{L_{\text{real}}}$

$$v_{\text{toy}} = v_{\text{scale}}\left(\frac{L_{\text{toy}}}{L_{\text{real}}}\right) = (700\ \text{km/h})\left(\frac{0.150\ \text{m}}{3.0\ \text{m}}\right) = 35.0\ \text{km/h}$$

$v_{\text{toy}} = (35.0\ \text{km/h})(1000\ \text{m/1 km})(1\ \text{h/3600 s}) = 9.72\ \text{m/s}$

(b) $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.180\ \text{kg})(9.72\ \text{m/s})^2 = 8.50\ \text{J}$

(c) $K = \frac{1}{2}I\omega^2$ gives that $\omega = \sqrt{\dfrac{2K}{I}} = \sqrt{\dfrac{2(8.50\ \text{J})}{4.00\times10^{-5}\ \text{kg}\cdot\text{m}^2}} = 652\ \text{rad/s}$

EVALUATE: $K = \frac{1}{2}I\omega^2$ gives ω in rad/s. $652\ \text{rad/s} = 6200\ \text{rev/min}$ so the rotation rate of the flywheel is very large.

9.69. **IDENTIFY:** $a_{\text{tan}} = r\alpha$, $a_{\text{rad}} = r\omega^2$. Apply the constant acceleration equations and $\sum\vec{F} = m\vec{a}$.

SET UP: a_{tan} and a_{rad} are perpendicular components of $\vec{a}$, so $a = \sqrt{a_{\text{rad}}^2 + a_{\text{tan}}^2}$.

EXECUTE: **(a)** $\alpha = \dfrac{a_{\text{tan}}}{r} = \dfrac{3.00\ \text{m/s}^2}{60.0\ \text{m}} = 0.050\ \text{rad/s}^2$

(b) $\alpha t = (0.05\ \text{rad/s}^2)(6.00\ \text{s}) = 0.300\ \text{rad/s}$.

(c) $a_{\text{rad}} = \omega^2 r = (0.300\ \text{rad/s})^2(60.0\ \text{m}) = 5.40\ \text{m/s}^2$.

(d) The sketch is given in Figure 9.69.

(e) $a = \sqrt{a^2_{\text{rad}} + a^2_{\text{tan}}} = \sqrt{(5.40\ \text{m/s}^2)^2 + (3.00\ \text{m/s}^2)^2} = 6.18\ \text{m/s}^2$, and the magnitude of the force is $F = ma = (1240\ \text{kg})(6.18\ \text{m/s}^2) = 7.66\ \text{kN}$.

(f) $\arctan\left(\dfrac{a_{\text{rad}}}{a_{\text{tan}}}\right) = \arctan\left(\dfrac{5.40}{3.00}\right) = 60.9°$.

EVALUATE: a_{tan} is constant and a_{rad} increases as ω increases. At $t = 0$, $\vec{a}$ is parallel to $\vec{v}$. As t increases, $\vec{a}$ moves toward the radial direction and the angle between $\vec{a}$ increases toward $90°$.

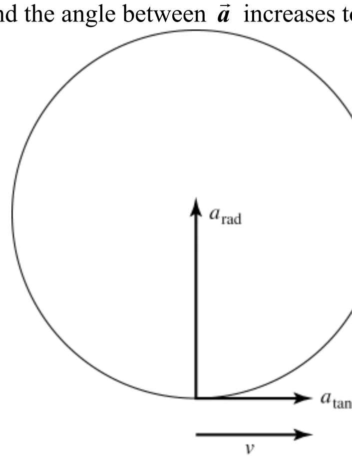


Figure 9.69

9.70. **IDENTIFY:** Apply conservation of energy to the system of drum plus falling mass, and compare the results for earth and for Mars.

SET UP: $K_{\text{drum}} = \frac{1}{2}I\omega^2$. $K_{\text{mass}} = \frac{1}{2}mv^2$. $v = R\omega$ so if K_{drum} is the same, ω is the same and v is the same on both planets. Therefore, K_{mass} is the same. Let $y = 0$ at the initial height of the mass and take $+y$ upward.
Configuration 1 is when the mass is at its initial position and 2 is when the mass has descended 5.00 m, so $y_1 = 0$ and $y_2 = -h$, where h is the height the mass descends.

EXECUTE: **(a)** $K_1 + U_1 = K_2 + U_2$ gives $0 = K_{\text{drum}} + K_{\text{mass}} - mgh$. $K_{\text{drum}} + K_{\text{mass}}$ are the same on both planets, so $mg_{\text{E}}h_{\text{E}} = mg_{\text{M}}h_{\text{M}}$. $h_{\text{M}} = h_{\text{E}}\left(\dfrac{g_{\text{E}}}{g_{\text{M}}}\right) = (5.00\ \text{m})\left(\dfrac{9.80\ \text{m/s}^2}{3.71\ \text{m/s}^2}\right) = 13.2\ \text{m}$.

(b) $mg_{\rm M}h_{\rm M} = K_{\rm drum} + K_{\rm mass}$. $\frac{1}{2}mv^2 = mg_{\rm M}h_{\rm M} - K_{\rm drum}$ and

$$v = \sqrt{2g_{\rm M}h_{\rm M} - \frac{2K_{\rm drum}}{m}} = \sqrt{2(3.71\ {\rm m/s^2})(13.2\ {\rm m}) - \frac{2(250.0\ {\rm J})}{15.0\ {\rm kg}}} = 8.04\ {\rm m/s}$$

EVALUATE: We did the calculations without knowing the moment of inertia I of the drum, or the mass and radius of the drum.

9.71. **IDENTIFY** and **SET UP:** All points on the belt move with the same speed. Since the belt doesn't slip, the speed of the belt is the same as the speed of a point on the rim of the shaft and on the rim of the wheel, and these speeds are related to the angular speed of each circular object by $v = r\omega$.

EXECUTE:

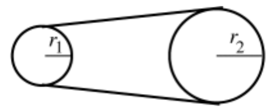


Figure 9.71

(a) $v_1 = r_1\omega_1$

$\omega_1 = (60.0\ {\rm rev/s})(2\pi\ {\rm rad/1\ rev}) = 377\ {\rm rad/s}$

$v_1 = r_1\omega_1 = (0.45\times10^{-2}\ {\rm m})(377\ {\rm rad/s}) = 1.70\ {\rm m/s}$

(b) $v_1 = v_2$

$r_1\omega_1 = r_2\omega_2$

$\omega_2 = (r_1/r_2)\omega_1 = (0.45\ {\rm cm}/2.00\ {\rm cm})(377\ {\rm rad/s}) = 84.8\ {\rm rad/s}$

EVALUATE: The wheel has a larger radius than the shaft so turns slower to have the same tangential speed for points on the rim.

9.72. **IDENTIFY:** The speed of all points on the belt is the same, so $r_1\omega_1 = r_2\omega_2$ applies to the two pulleys.

SET UP: The second pulley, with half the diameter of the first, must have twice the angular velocity, and this is the angular velocity of the saw blade. π rad/s $= 30$ rev/min .

EXECUTE: **(a)** $v_2 = (2(3450\ {\rm rev/min}))\left(\frac{\pi}{30}\frac{\rm rad/s}{\rm rev/min}\right)\left(\frac{0.208\ {\rm m}}{2}\right) = 75.1\ {\rm m/s}.$

(b) $a_{\rm rad} = \omega^2 r = \left(2(3450\ {\rm rev/min})\left(\frac{\pi}{30}\ \frac{\rm rad/s}{\rm rev/min}\right)\right)^2\left(\frac{0.208\ {\rm m}}{2}\right) = 5.43\times10^4\ {\rm m/s^2},$

so the force holding sawdust on the blade would have to be about 5500 times as strong as gravity.

EVALUATE: In $v = r\omega$ and $a_{\rm rad} = r\omega^2$, ω must be in rad/s.

9.73. **IDENTIFY** and **SET UP:** Use Eq.(9.15) to relate $a_{\rm rad}$ to ω and then use a constant acceleration equation to replace ω.

EXECUTE: **(a)** $a_{\rm rad} = r\omega^2$, $a_{\rm rad,1} = r\omega_1^2$, $a_{\rm rad,2} = r\omega_2^2$

$\Delta a_{\rm rad} = a_{\rm rad,2} - a_{\rm rad,1} = r(\omega_2^2 - \omega_1^2)$

One of the constant acceleration equations can be written

$\omega_{2z}^2 = \omega_{1z}^2 + 2\alpha(\theta_2 - \theta_1)$, or $\omega_{2z}^2 - \omega_{1z}^2 = 2\alpha_z(\theta_2 - \theta_1)$

Thus $\Delta a_{\rm rad} = r2\alpha_z(\theta_2 - \theta_1) = 2r\alpha_z(\theta_2 - \theta_1)$, as was to be shown.

(b) $\alpha_z = \dfrac{\Delta a_{\rm rad}}{2r(\theta_2 - \theta_1)} = \dfrac{85.0\ {\rm m/s^2} - 25.0\ {\rm m/s^2}}{2(0.250\ {\rm m})(15.0\ {\rm rad})} = 8.00\ {\rm rad/s^2}$

Then $a_{\rm tan} = r\alpha = (0.250\ {\rm m})(8.00\ {\rm rad/s^2}) = 2.00\ {\rm m/s^2}$

EVALUATE: ω^2 is proportional to α_z and $(\theta - \theta_0)$ so $a_{\rm rad}$ is also proportional to these quantities. $a_{\rm rad}$ increases while r stays fixed, ω_z increases, and α_z is positive.

IDENTIFY and **SET UP:** Use Eq.(9.17) to relate K and ω and then use a constant acceleration equation to replace ω.

EXECUTE: **(c)** $K = \frac{1}{2}I\omega^2$; $K_2 = \frac{1}{2}I\omega_2^2$, $K_1 = \frac{1}{2}I\omega_1^2$

$\Delta K = K_2 - K_1 = \frac{1}{2}I(\omega_2^2 - \omega_1^2) = \frac{1}{2}I(2\alpha_z(\theta_2 - \theta_1)) = I\alpha_z(\theta_2 - \theta_1)$, as was to be shown.

(d) $I = \dfrac{\Delta K}{\alpha_z(\theta_2 - \theta_1)} = \dfrac{45.0\ {\rm J} - 20.0\ {\rm J}}{(8.00\ {\rm rad/s^2})(15.0\ {\rm rad})} = 0.208\ {\rm kg\cdot m^2}$

EVALUATE: α_z is positive, ω increases, and K increases.

9.74. IDENTIFY: $I = I_{\text{wood}} + I_{\text{lead}}$. $m = \rho V$, where ρ is the volume density and $m = \sigma A$, where σ is the area density.

SET UP: For a solid sphere, $I = \frac{2}{5}mR^2$. For the hollow sphere (foil), $I = \frac{2}{3}mR^2$. For a sphere, $V = \frac{4}{3}\pi R^3$ and $A = 4\pi R^2$. $m_{\text{w}} = \rho_{\text{w}} V_{\text{w}} = \rho_{\text{w}} \frac{4}{3}\pi R^3$. $m_{\text{L}} = \sigma_{\text{L}} A_{\text{L}} = \sigma_{\text{L}} 4\pi R^2$.

EXECUTE: $I = \frac{2}{5}m_{\text{w}}R^2 + \frac{2}{3}m_{\text{L}}R^2 = \frac{2}{5}\left(\rho_{\text{w}}\frac{4}{3}\pi R^3\right)R^2 + \frac{2}{3}(\sigma_{\text{L}} 4\pi R^2)R^2 = \frac{8}{3}\pi R^4\left(\frac{\rho_{\text{w}} R}{5} + \sigma_{\text{L}}\right)$.

$I = \frac{8\pi}{3}(0.20\text{ m})^4\left[\frac{(800\text{ kg/m}^3)(0.20\text{ m})}{5} + 20\text{ kg/m}^2\right] = 0.70\text{ kg}\cdot\text{m}^2$.

EVALUATE: $m_{\text{W}} = 26.8$ kg and $I_{\text{W}} = 0.429\text{ kg}\cdot\text{m}^2$. $m_{\text{L}} = 10.1$ kg and $I_{\text{L}} = 0.268\text{ kg}\cdot\text{m}^2$. Even though the foil is only 27% of the total mass its contribution to I is about 38% of the total.

9.75. IDENTIFY: Estimate the shape and dimensions of your body and apply the approximate expression from Table 9.2.

SET UP: I approximate my body as a vertical cylinder with mass 80 kg, length 1.7 m, and diameter 0.30 m (radius 0.15 m)

EXECUTE: $I = \frac{1}{2}mR^2 = \frac{1}{2}(80\text{ kg})\,(0.15\text{ m})^2 = 0.9\text{ kg}\cdot\text{m}^2$

EVALUATE: I depends on your mass and width but not on your height.

9.76. IDENTIFY: Treat the V like two thin 0.160 kg bars, each 25 cm long.

SET UP: For a slender bar with the axis at one end, $I = \frac{1}{3}mL^2$.

EXECUTE: $I = 2\left(\frac{1}{3}mL^2\right) = 2\left(\frac{1}{3}\right)(0.160\text{ kg})(0.250\text{ m})^2 = 6.67\times10^{-3}\text{ kg}\cdot\text{m}^2$

EVALUATE: The value of I is independent of the angle between the two sides of the V; the angle 70.0° didn't enter into the calculation.

9.77. IDENTIFY: $K = \frac{1}{2}I\omega^2$. $a_{\text{rad}} = r\omega^2$. $m = \rho V$.

SET UP: For a disk with the axis at the center, $I = \frac{1}{2}mR^2$. $V = t\pi R^2$, where $t = 0.100$ m is the thickness of the flywheel. $\rho = 7800\text{ kg/m}^3$ is the density of the iron.

EXECUTE: **(a)** $\omega = 90.0\text{ rpm} = 9.425\text{ rad/s}$. $I = \frac{2K}{\omega^2} = \frac{2(10.0\times10^6\text{ J})}{(9.425\text{ rad/s})^2} = 2.252\times10^5\text{ kg}\cdot\text{m}^2$.

$m = \rho V = \rho\pi R^2 t$. $I = \frac{1}{2}mR^2 = \frac{1}{2}\rho\pi t R^4$. This gives $R = (2I/\rho\pi t)^{1/4} = 3.68$ m and the diameter is 7.36 m.

(b) $a_{\text{rad}} = R\omega^2 = 327\text{ m/s}^2$

EVALUATE: In $K = \frac{1}{2}I\omega^2$, ω must be in rad/s. a_{rad} is about 33*g*; the flywheel material must have large cohesive strength to prevent the flywheel from flying apart.

9.78. IDENTIFY: $K = \frac{1}{2}I\omega^2$**.** To have the same K for any ω the two parts must have the same I. Use Table 9.2 for I.

SET UP: For a solid sphere, $I_{\text{solid}} = \frac{2}{5}M_{\text{solid}}R^2$. For a hollow sphere, $I_{\text{hollow}} = \frac{2}{3}M_{\text{hollow}}R^2$.

EXECUTE: $I_{\text{solid}} = I_{\text{hollow}}$ gives $\frac{2}{5}M_{\text{solid}}R^2 = \frac{2}{3}M_{\text{hollow}}R^2$ and $M_{\text{hollow}} = \frac{3}{5}M_{\text{solid}} = \frac{3}{5}M$.

EVALUATE: The hollow sphere has less mass since all its mass is distributed farther from the rotation axis.

9.79. IDENTIFY: $K = \frac{1}{2}I\omega^2$. $\omega = \frac{2\pi\text{ rad}}{T}$, where T is the period of the motion. For the earth's orbital motion it can be treated as a point mass and $I = MR^2$.

SET UP: The earth's rotational period is $24\text{ h} = 86{,}164\text{ s}$. Its orbital period is $1\text{ yr} = 3.156\times10^7\text{ s}$. $M = 5.97\times10^{24}\text{ kg}$. $R = 6.38\times10^6\text{ m}$.

EXECUTE: **(a)** $K = \frac{2\pi^2 I}{T^2} = \frac{2\pi^2(0.3308)(5.97\times10^{24}\text{ kg})(6.38\times10^6\text{m})^2}{(86{,}164\text{ s})^2} = 2.14\times10^{29}\text{ J}$.

(b) $\frac{1}{2}M\left(\frac{2\pi R}{T}\right)^2 = \frac{2\pi^2(5.97\times10^{24}\text{ kg})(1.50\times10^{11}\text{ m})^2}{(3.156\times10^7\text{ s})^2} = 2.66\times10^{33}\text{ J}$.

(c) Since the Earth's moment of inertia is less than that of a uniform sphere, more of the Earth's mass must be concentrated near its center.

EVALUATE: These kinetic energies are very large, because the mass of the earth is very large.

9.80. **IDENTIFY:** Using energy considerations, the system gains as kinetic energy the lost potential energy, mgR.

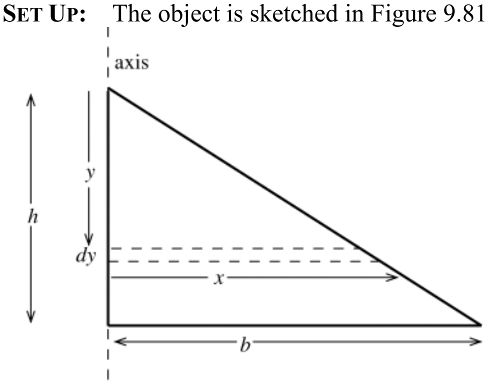
SET UP: The kinetic energy is $K=\frac{1}{2}I\omega^2+\frac{1}{2}mv^2$, with $I=\frac{1}{2}mR^2$ for the disk. $v=R\omega$.

EXECUTE: $K=\frac{1}{2}I\omega^2+\frac{1}{2}m(\omega R)^2=\frac{1}{2}(I+mR^2)$. Using $I=\frac{1}{2}mR^2$ and solving for ω, $\omega^2=\frac{4}{3}\frac{g}{R}$ and $\omega=\sqrt{\frac{4}{3}\frac{g}{R}}$.

EVALUATE: The small object has speed $v=\sqrt{\frac{2}{3}}\sqrt{2gR}$. If it was not attached to the disk and was dropped from a height h, it would attain a speed $\sqrt{2gR}$. Being attached to the disk reduces its final speed by a factor of $\sqrt{\frac{2}{3}}$.

9.81. **IDENTIFY:** Use Eq.(9.20) to calculate I. Then use $K=\frac{1}{2}I\omega^2$ to calculate K.

(a) SET UP: The object is sketched in Figure 9.81.



Consider a small strip of width dy and a distance y below the top of the triangle.
The length of the strip is $x=(y/h)b$.

Figure 9.81

EXECUTE: The strip has area $x\,dy$ and the area of the sign is $\frac{1}{2}bh$, so the mass of the strip is

$$dm=M\left(\frac{x\,dy}{\frac{1}{2}bh}\right)=M\left(\frac{yb}{h}\right)\left(\frac{2\,dy}{bh}\right)=\left(\frac{2M}{h^2}\right)y\,dy$$

$$dI=\tfrac{1}{3}(dm)x^2=\left(\frac{2Mb^2}{3h^4}\right)y^3\,dy$$

$$I=\int_0^h dI=\frac{2Mb^2}{3h^4}\int_0^h y^3\,dy=\frac{2Mb^2}{3h^4}\left(\frac{1}{h}y^4\Big|_0^h\right)=\frac{1}{6}Mb^2$$

(b) $I=\frac{1}{6}Mb^2=2.304\ \text{kg}\cdot\text{m}^2$

$\omega=2.00\ \text{rev/s}=4.00\pi\ \text{rad/s}$

$K=\frac{1}{2}I\omega^2=182\ \text{J}$

EVALUATE: From Table (9.2), if the sign were rectangular, with length b, then $I=\frac{1}{3}Mb^2$. Our result is one-half this, since mass is closer to the axis for the triangular than for the rectangular shape.

9.82. **IDENTIFY:** Apply conservation of energy to the system.

SET UP: For the falling mass $K=\frac{1}{2}mv^2$. For the wheel $K=\frac{1}{2}I\omega^2$.

EXECUTE: **(a)** The kinetic energy of the falling mass after 2.00 m is $K=\frac{1}{2}mv^2=\frac{1}{2}(8.00\ \text{kg})(5.00\ \text{m/s})^2=100\ \text{J}$. The change in its potential energy while falling is $mgh=(8.00\ \text{kg})(9.8\ \text{m/s}^2)(2.00\ \text{m})=156.8\ \text{J}$. The wheel must have the "missing" 56.8 J in the form of rotational kinetic energy. Since its outer rim is moving at the same speed as the falling mass, 5.00 m/s, $v=r\omega$ gives $\omega=\frac{v}{r}=\frac{5.00\ \text{m/s}}{0.370\ \text{m}}=13.51\ \text{rad/s}$. $K=\frac{1}{2}I\omega^2$; therefore

$$I=\frac{2K}{\omega^2}=\frac{2(56.8\ \text{J})}{(13.51\ \text{rad/s})^2}=0.622\ \text{kg}\cdot\text{m}^2.$$

(b) The wheel's mass is $(280\ \text{N})/(9.8\ \text{m/s}^2)=28.6\ \text{kg}$. The wheel with the largest possible moment of inertia would have all this mass concentrated in its rim. Its moment of inertia would be $I=MR^2=(28.6\ \text{kg})(0.370\ \text{m})^2=3.92\ \text{kg}\cdot\text{m}^2$. The boss's wheel is physically impossible.

EVALUATE: If the mass falls from rest in free-fall its speed after it has descended 2.00 m is $v=\sqrt{2g(2.00\text{ m})}=6.26\text{ m/s}$. Its actual speed is less because some of the energy of the system is in the form of rotational kinetic energy of the wheel.

9.83. **IDENTIFY:** Use conservation of energy. The stick rotates about a fixed axis so $K=\frac{1}{2}I\omega^2$. Once we have ω use $v=r\omega$ to calculate v for the end of the stick.

SET UP: The object is sketched in Figure 9.83.

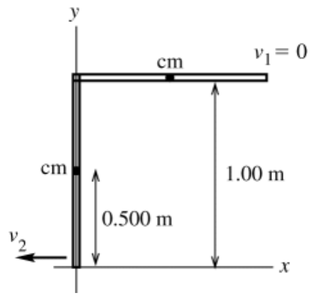


Figure 9.83

Take the origin of coordinates at the lowest point reached by the stick and take the positive y-direction to be upward.

EXECUTE: **(a)** Use Eq.(9.18): $U=Mgy_{\text{cm}}$

$\Delta U=U_2-U_1=Mg(y_{\text{cm2}}-y_{\text{cm1}})$

The center of mass of the meter stick is at its geometrical center, so

$y_{\text{cm1}}=1.00\text{ m}$ and $y_{\text{cm2}}=0.50\text{ m}$

Then $\Delta U=(0.160\text{ kg})(9.80\text{ m/s}^2)(0.50\text{ m}-1.00\text{ m})=-0.784\text{ J}$

(b) Use conservation of energy: $K_1+U_1+W_{\text{other}}=K_2+U_2$

Gravity is the only force that does work on the meter stick, so $W_{\text{other}}=0$.

$K_1=0$.

Thus $K_2=U_1-U_2=-\Delta U$, where ΔU was calculated in part (a).

$K_2=\frac{1}{2}I\omega_2^2$ so $\frac{1}{2}I\omega_2^2=-\Delta U$ and $\omega_2=\sqrt{2(-\Delta U)/I}$

For stick pivoted about one end, $I=\frac{1}{3}ML^2$ where $L=1.00\text{ m}$, so

$$\omega_2=\sqrt{\frac{6(-\Delta U)}{ML^2}}=\sqrt{\frac{6(0.784\text{ J})}{(0.160\text{ kg})(1.00\text{ m})^2}}=5.42\text{ rad/s}$$

(c) $v=r\omega=(1.00\text{ m})(5.42\text{ rad/s})=5.42\text{ m/s}$

(d) For a particle in free-fall, with $+y$ upward,

$v_{0y}=0;\ y-y_0=-1.00\text{ m};\ a_y=-9.80\text{ m/s}^2;\ v_y=?$

$v_y^2=v_{0y}^2+2a_y(y-y_0)$

$v_y=-\sqrt{2a_y(y-y_0)}=-\sqrt{2(-9.80\text{ m/s}^2)(-1.00\text{ m})}=-4.43\text{ m/s}$

EVALUATE: The magnitude of the answer in part (c) is larger. $U_{1,\text{grav}}$ is the same for the stick as for a particle falling from a height of 1.00 m. For the stick $K=\frac{1}{2}I\omega_2^2=\frac{1}{2}\left(\frac{1}{3}ML^2\right)(v/L)^2=\frac{1}{6}Mv^2$. For the stick and for the particle, K_2 is the same but the same K gives a larger v for the end of the stick than for the particle. The reason is that all the other points along the stick are moving slower than the end opposite the axis.

9.84. **IDENTIFY:** Apply conservation of energy to the system of cylinder and rope.

SET UP: Taking the zero of gravitational potential energy to be at the axle, the initial potential energy is zero (the rope is wrapped in a circle with center on the axle).When the rope has unwound, its center of mass is a distance πR below the axle, since the length of the rope is $2\pi R$ and half this distance is the position of the center of the mass. Initially, every part of the rope is moving with speed $\omega_0 R$, and when the rope has unwound, and the cylinder has angular speed ω, the speed of the rope is ωR (the upper end of the rope has the same tangential speed at the edge of the cylinder). $I=(1/2)MR^2$ for a uniform cylinder,

EXECUTE: $K_1 = K_2 + U_2$. $\left(\frac{M}{4}+\frac{m}{2}\right)R^2\omega_0^2 = \left(\frac{M}{4}+\frac{m}{2}\right)R^2\omega^2 - mg\pi R$. Solving for ω gives

$\omega = \sqrt{\omega_0^2 + \frac{(4\pi mg/R)}{(M+2m)}}$, and the speed of any part of the rope is $v = \omega R$.

EVALUATE: When $m \to 0$, $\omega \to \omega_0$. When $m >> M$, $\omega = \sqrt{\omega_0^2 + \frac{2\pi g}{R}}$ and $v = \sqrt{v_0^2 + 2\pi gR}$. This is the final speed when an object with initial speed v_0 descends a distance πR.

9.85. **IDENTIFY:** Apply conservation of energy to the system consisting of blocks *A* and *B* and the pulley.
SET UP: The system at points 1 and 2 of its motion is sketched in Figure 9.85.

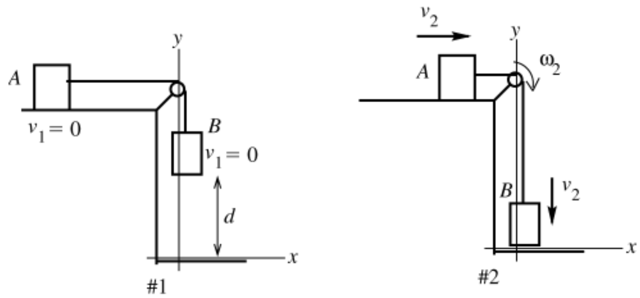


Figure 9.85

Use the work-energy relation $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$. Use coordinates where $+y$ is upward and where the origin is at the position of block *B* after it has descended. The tension in the rope does positive work on block *A* and negative work of the same magnitude on block *B*, so the net work done by the tension in the rope is zero. Both blocks have the same speed.

EXECUTE: Gravity does work on block *B* and kinetic friction does work on block *A*. Therefore
$W_{\text{other}} = W_f = -\mu_k m_A gd$.

$K_1 = 0$ (system is released from rest)

$U_1 = m_B g y_{B1} = m_B gd$; $U_2 = m_B g y_{B2} = 0$

$K_2 = \frac{1}{2}m_A v_2^2 + \frac{1}{2}m_B v_2^2 + \frac{1}{2}I\omega_2^2$.

But $v(\text{blocks}) = R\omega(\text{pulley})$, so $\omega_2 = v_2/R$ and

$K_2 = \frac{1}{2}(m_A + m_B)v_2^2 + \frac{1}{2}I(v_2/R)^2 = \frac{1}{2}(m_A + m_B + I/R^2)v_2^2$

Putting all this into the work-energy relation gives

$m_B gd - \mu_k m_A gd = \frac{1}{2}(m_A + m_B + I/R^2)v_2^2$

$(m_A + m_B + I/R^2)v_2^2 = 2gd(m_B - \mu_k m_A)$

$v_2 = \sqrt{\frac{2gd(m_B - \mu_k m_A)}{m_A + m_B + I/R^2}}$

EVALUATE: If $m_B >> m_A$ and I/R^2, then $v_2 = \sqrt{2gd}$; block *B* falls freely. If *I* is very large, v_2 is very small. Must have $m_B > \mu_k m_A$ for motion, so the weight of *B* will be larger than the friction force on *A*. I/R^2 has units of mass and is in a sense the "effective mass" of the pulley.

9.86. **IDENTIFY:** Apply conservation of energy to the system of two blocks and the pulley.
SET UP: Let the potential energy of each block be zero at its initial position. The kinetic energy of the system is the sum of the kinetic energies of each object. $v = R\omega$, where *v* is the common speed of the blocks and ω is the angular velocity of the pulley.

EXECUTE: The amount of gravitational potential energy which has become kinetic energy is
$K = (4.00\text{ kg} - 2.00\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m}) = 98.0\text{ J}$. In terms of the common speed *v* of the blocks, the kinetic energy of the system is $K = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2$.

$K = v^2\frac{1}{2}\left(4.00\text{ kg} + 2.00\text{ kg} + \frac{(0.480\text{ kg}\cdot\text{m}^2)}{(0.160\text{ m})^2}\right) = v^2(12.4\text{ kg})$. Solving for *v* gives $v = \sqrt{\frac{98.0\text{ J}}{12.4\text{ kg}}} = 2.81\text{ m/s}$.

EVALUATE: If the pulley is massless, $98.0\text{ J} = \frac{1}{2}(4.00\text{ kg} + 2.00\text{ kg})v^2$ and $v = 5.72\text{ m/s}$. The moment of inertia of the pulley reduces the final speed of the blocks.

9.87. IDENTIFY and SET UP: Apply conservation of energy to the motion of the hoop. Use Eq.(9.18) to calculate U_{grav}. Use $K = \frac{1}{2}I\omega^2$ for the kinetic energy of the hoop. Solve for ω. The center of mass of the hoop is at its geometrical center.

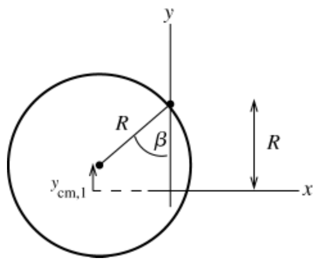


Figure 9.87

Take the origin to be at the original location of the center of the hoop, before it is rotated to one side, as shown in Figure 9.87.

$y_{\text{cm1}} = R - R\cos\beta = R(1-\cos\beta)$

$y_{\text{cm2}} = 0$ (at equilibrium position hoop is at original position)

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$W_{\text{other}} = 0$ (only gravity does work)

$K_1 = 0$ (released from rest), $K_2 = \frac{1}{2}I\omega_2^2$

For a hoop, $I_{\text{cm}} = MR^2$, so $I = Md^2 + MR^2$ with $d = R$ and $I = 2MR^2$, for an axis at the edge. Thus

$K_2 = \frac{1}{2}(2MR^2)\omega_2^2 = MR^2\omega_2^2.$

$U_1 = Mgy_{\text{cm1}} = MgR(1-\cos\beta)$, $U_2 = mgy_{\text{cm2}} = 0$

Thus $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$ gives

$MgR(1-\cos\beta) = MR^2\omega_2^2$ and $\omega = \sqrt{g(1-\cos\beta)/R}$

EVALUATE: If $\beta = 0$, then $\omega_2 = 0$. As β increases, ω_2 increases.

9.88. IDENTIFY: $K = \frac{1}{2}I\omega^2$, with ω in rad/s. $P = \dfrac{\text{energy}}{t}$

SET UP: For a solid cylinder, $I = \frac{1}{2}MR^2$. $1\text{ rev/min} = (2\pi/60)\text{ rad/s}$

EXECUTE: **(a)** $\omega = 3000\text{ rev/min} = 314\text{ rad/s}$. $I = \frac{1}{2}(1000\text{ kg})(0.900\text{ m})^2 = 405\text{ kg}\cdot\text{m}^2$

$K = \frac{1}{2}(405\text{ kg}\cdot\text{m}^2)(314\text{ rad/s})^2 = 2.00\times10^7\text{ J}$.

(b) $t = \dfrac{K}{P} = \dfrac{2.00\times10^7\text{ J}}{1.86\times10^4\text{ W}} = 1.08\times10^3\text{ s} = 17.9\text{ min}$.

EVALUATE: In $K = \frac{1}{2}I\omega^2$, we must use ω in rad/s.

9.89. IDENTIFY: $I = I_1 + I_2$. Apply conservation of energy to the system. The calculation is similar to Example 9.9.

SET UP: $\omega = \dfrac{v}{R_1}$ for part (b) and $\omega = \dfrac{v}{R_2}$ for part (c).

EXECUTE: **(a)** $I = \dfrac{1}{2}M_1R_1^2 + \dfrac{1}{2}M_2R_2^2 = \dfrac{1}{2}((0.80\text{ kg})(2.50\times10^{-2}\text{ m})^2 + (1.60\text{ kg})(5.00\times10^{-2}\text{ m})^2)$

$I = 2.25\times10^{-3}\text{ kg}\cdot\text{m}^2.$

(b) The method of Example 9.9 yields $v = \sqrt{\dfrac{2gh}{1+(I/mR_1^2)}}$.

$$v = \sqrt{\frac{2(9.80\text{ m/s}^2)(2.00\text{ m})}{(1+((2.25\times10^{-3}\text{ kg}\cdot\text{m}^2)/(1.50\text{ kg})(0.025\text{ m})^2))}} = 3.40\text{ m/s}.$$

The same calculation, with R_2 instead of R_1 gives $v = 4.95\text{ m/s}$.

EVALUATE: The final speed of the block is greater when the string is wrapped around the larger disk. $v = R\omega$, so when $R = R_2$ the factor that relates v to ω is larger. For $R = R_2$ a larger fraction of the total kinetic energy resides

with the block. The total kinetic energy is the same in both cases (equal to mgh), so when $R = R_2$ the kinetic energy and speed of the block are greater.

9.90. **IDENTIFY:** Apply conservation of energy to the motion of the mass after it hits the ground.

SET UP: From Example 9.9, the speed of the mass just before it hits the ground is $v = \sqrt{\dfrac{2gh}{1+M/2m}}$.

EXECUTE: **(a)** In the case that no energy is lost, the rebound height h' is related to the speed v by $h' = \dfrac{v^2}{2g}$, and with the form for v given in Example 9.9, $h' = \dfrac{h}{1+ M/2m}$.

(b) Considering the system as a whole, some of the initial potential energy of the mass went into the kinetic energy of the cylinder. Considering the mass alone, the tension in the string did work on the mass, so its total energy is not conserved.

EVALUATE: If $m >> M$, $h' = h$ and the mass does rebound to its initial height.

9.91. **IDENTIFY:** Apply conservation of energy to relate the height of the mass to the kinetic energy of the cylinder.

SET UP: First use $K(\text{cylinder}) = 250\text{ J}$ to find ω for the cylinder and v for the mass.

EXECUTE: $I = \frac{1}{2}MR^2 = \frac{1}{2}(10.0\text{ kg})(0.150\text{ m})^2 = 0.1125\text{ kg}\cdot\text{m}^2$

$K = \frac{1}{2}I\omega^2$ so $\omega = \sqrt{2K/I} = 66.67\text{ rad/s}$

$v = R\omega = 10.0\text{ m/s}$

SET UP: Use conservation of energy $K_1 + U_1 = K_2 + U_2$ to solve for the distance the mass descends. Take $y = 0$ at lowest point of the mass, so $y_2 = 0$ and $y_1 = h$, the distance the mass descends.

EXECUTE: $K_1 = U_2 = 0$ so $U_1 = K_2$.

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, where $m = 12.0\text{ kg}$

For the cylinder, $I = \frac{1}{2}MR^2$ and $\omega = v/R$, so $\frac{1}{2}I\omega^2 = \frac{1}{4}Mv^2$.

$mgh = \frac{1}{2}mv^2 + \frac{1}{4}Mv^2$

$$h = \frac{v^2}{2g}\left(1 + \frac{M}{2m}\right) = 7.23\text{ m}$$

EVALUATE: For the cylinder $K_{\text{cyl}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)(v/R)^2 = \frac{1}{4}Mv^2$.

$K_{\text{mass}} = \frac{1}{2}mv^2$, so $K_{\text{mass}} = (2m/M)K_{\text{cyl}} = [2(12.0\text{ kg})/10.0\text{ kg}](250\text{ J}) = 600\text{ J}$. The mass has 600 J of kinetic energy when the cylinder has 250 J of kinetic energy and at this point the system has total energy 850 J since $U_2 = 0$. Initially the total energy of the system is $U_1 = mgy_1 = mgh = 850\text{ J}$, so the total energy is shown to be conserved.

9.92. **IDENTIFY:** Energy conservation: Loss of U of box equals gain in K of system. Both the cylinder and pulley have kinetic energy of the form $K = \frac{1}{2}I\omega^2$. $m_{\text{box}}gh = \dfrac{1}{2}m_{\text{box}}v_{\text{box}}^2 + \dfrac{1}{2}I_{\text{pulley}}\omega_{\text{pulley}}^2 + \dfrac{1}{2}I_{\text{cylinder}}\omega_{\text{cylinder}}^2$.

SET UP: $\omega_{\text{pulley}} = \dfrac{v_{\text{Box}}}{r_{\text{p}}}$ and $\omega_{\text{cylinder}} = \dfrac{v_{\text{Box}}}{r_{\text{cylinder}}}$.

EXECUTE: $m_{\text{B}}gh = \dfrac{1}{2}m_{\text{B}}v_{\text{B}}^2 + \dfrac{1}{2}\left(\dfrac{1}{2}m_{\text{P}}r_{\text{p}}^2\right)\left(\dfrac{v_B}{r_{\text{p}}}\right)^2 + \dfrac{1}{2}\left(\dfrac{1}{2}m_{\text{C}}r_{\text{C}}^2\right)\left(\dfrac{v_{\text{B}}}{r_{\text{C}}}\right)^2$. $m_{\text{B}}gh = \dfrac{1}{2}m_{\text{B}}v_{\text{B}}^2 + \dfrac{1}{4}m_{\text{P}}v_{\text{B}}^2 + \dfrac{1}{4}m_{\text{C}}v_{\text{B}}^2$ and

$$v_{\text{B}} = \sqrt{\frac{m_{\text{B}}gh}{\frac{1}{2}m_{\text{B}} + \frac{1}{4}m_{\text{p}} + \frac{1}{4}m_{\text{C}}}} = \sqrt{\frac{(3.00\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m})}{1.50\text{ kg} + \frac{1}{4}(7.00\text{ kg})}} = 3.68\text{ m/s}.$$

EVALUATE: If the box was disconnected from the rope and dropped from rest, after falling 1.50 m its speed would be $v = \sqrt{2g(1.50\text{ m})} = 5.42\text{ m/s}$. Since in the problem some of the energy of the system goes into kinetic energy of the cylinder and of the pulley, the final speed of the box is less than this.

9.93. **IDENTIFY:** $I = I_{\text{disk}} - I_{\text{hole}}$, where I_{hole} is I for the piece punched from the disk. Apply the parallel-axis theorem to calculate the required moments of inertia.

SET UP: For a uniform disk, $I = \frac{1}{2}MR^2$.

EXECUTE: **(a)** The initial moment of inertia is $I_0 = \frac{1}{2}MR^2$. The piece punched has a mass of $\frac{M}{16}$ and a moment of inertia with respect to the axis of the original disk of

$$\frac{M}{16}\left[\frac{1}{2}\left(\frac{R}{4}\right)^2 + \left(\frac{R}{2}\right)^2\right] = \frac{9}{512}MR^2.$$

The moment of inertia of the remaining piece is then $I = \frac{1}{2}MR^2 - \frac{9}{512}MR^2 = \frac{247}{512}MR^2$.

(b) $I = \frac{1}{2}MR^2 + M(R/2)^2 - \frac{1}{2}(M/16)(R/4)^2 = \frac{383}{512}MR^2$.

EVALUATE: For a solid disk and an axis at a distance $R/2$ from the disk's center, the parallel-axis theorem gives $I = \frac{1}{2}MR^2 = \frac{3}{4}MR^2 = \frac{384}{512}MR^2$. For both choices of axes the presence of the hole reduces *I*, but the effect of the hole is greater in part (a), when it is farther from the axis.

9.94. **IDENTIFY:** In part (a) use the parallel-axis theorem to relate the moment of inertia I_{cm} for an axis through the center of the sphere to I_P, the moment of inertia for an axis at the pivot.

SET UP: *I* for a uniform solid sphere and the axis through its center is $\frac{2}{5}MR^2$. *I* for a slender rod and an axis at one end is $\frac{1}{3}mL^2$, where *m* is the mass of the rod and *L* is its length.

EXECUTE: **(a)** From the parallel-axis theorem, the moment of inertia is $I_P = (2/5)MR^2 + ML^2$, and $\frac{I_P}{ML^2} = \left(1 + \left(\frac{2}{5}\right)\left(\frac{R}{L}\right)^2\right)$. If $R = (0.05)L$, the difference is $(2/5)(0.05)^2 = 0.001 = 0.1\%$.

(b) $(I_{rod}/ML^2) = (m_{rod}/3M)$, which is 0.33% when $m_{rod} = (0.01)M$.

EVALUATE: In both these cases the correction to $I = ML^2$ is very small.

9.95. **IDENTIFY:** Follow the instructions in the problem to derive the perpendicular-axis theorem. Then apply that result in part (b).

SET UP: $I = \sum_i m_i r_i^2$. The moment of inertia for the washer and an axis perpendicular to the plane of the washer at its center is $\frac{1}{2}M(R_1^2 + R_2^2)$. In part (b), *I* for an axis perpendicular to the plane of the square at its center is $\frac{1}{12}M(L^2 + L^2) = \frac{1}{6}ML^2$.

EXECUTE: **(a)** With respect to *O*, $r_i^2 = x_i^2 + y_i^2$, and so

$$I_O = \sum_i m_i r_i^2 = \sum_i m_i(x_i^2 + y_i^2) = \sum_i m_i x_i^2 + \sum_i m_i y_i^2 = I_x + I_y.$$

(b) Two perpendicular axes, both perpendicular to the washer's axis, will have the same moment of inertia about those axes, and the perpendicular-axis theorem predicts that they will sum to the moment of inertia about the washer axis, which is $I = \frac{1}{2}M(R_1^2 + R_2^2)$, and so $I_x = I_y = \frac{1}{4}M(R_1^2 + R_2^2)$.

(c) $I_0 = \frac{1}{6}mL^2$. Since $I_0 = I_x + I_y$, and $I_x = I_y$, both I_x and I_y must be $\frac{1}{12}mL^2$.

EVALUATE: The result in part (c) says that *I* is the same for an axis that bisects opposite sides of the square as for an axis along the diagonal of the square, even though the distribution of mass relative to the two axes is quite different in these two cases.

9.96. **IDENTIFY:** Apply the parallel-axis theorem to each side of the square.

SET UP: Each side has length *a* and mass $M/4$, and the moment of inertia of each side about an axis perpendicular to the side and through its center is $\frac{1}{12}\frac{1}{4}Ma^2 = \frac{1}{48}Ma^2$.

EXECUTE: The moment of inertia of each side about the axis through the center of the square is, from the perpendicular axis theorem, $\frac{Ma^2}{48} + \frac{M}{4}\left(\frac{a}{2}\right)^2 = \frac{Ma^2}{12}$. The total moment of inertia is the sum of the contributions from the four sides, or $4 \times \frac{Ma^2}{12} = \frac{Ma^2}{3}$.

EVALUATE: If all the mass of a side were at its center, a distance $a/2$ from the axis, we would have $I = 4\left(\frac{M}{4}\right)\left(\frac{a}{2}\right)^2 = \frac{1}{4}Ma^2$. If all the mass was divided equally among the four corners of the square, a distance $a/\sqrt{2}$ from the axis, we would have $I = 4\left(\frac{M}{4}\right)\left(\frac{a}{\sqrt{2}}\right)^2 = \frac{1}{2}Ma^2$. The actual *I* is between these two values.

9.97. **IDENTIFY:** Use Eq.(9.20) to calculate *I*.

(a) SET UP: Let *L* be the length of the cylinder. Divide the cylinder into thin cylindrical shells of inner radius *r* and outer radius $r + dr$. An end view is shown in Figure 9.97.

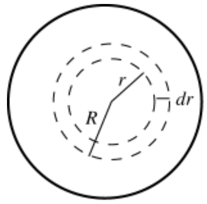


Figure 9.97

$\rho = \alpha r$

The mass of the thin cylindrical shell is

$dm = \rho\ dV = \rho(2\pi r\ dr)L = 2\pi\alpha L r^2\ dr$

EXECUTE: $I = \int r^2\ dm = 2\pi\alpha L \int_0^R r^4\ dr = 2\pi\alpha L\left(\frac{1}{5}R^5\right) = \frac{2}{5}\pi\alpha L R^5$

Relate *M* to α: $M = \int dm = 2\pi\alpha L \int_0^R r^2\ dr = 2\pi\alpha L\left(\frac{1}{3}R^3\right) = \frac{2}{3}\pi\alpha L R^3$, so $\pi\alpha L R^3 = 3M/2$.

Using this in the above result for *I* gives $I = \frac{2}{5}(3M/2)R^2 = \frac{3}{5}MR^2$.

(b) EVALUATE: For a cylinder of uniform density $I = \frac{1}{2}MR^2$. The answer in (a) is larger than this. Since the density increases with distance from the axis the cylinder in (a) has more mass farther from the axis than for a cylinder of uniform density.

9.98. **IDENTIFY:** Write *K* in terms of the period *T* and take derivatives of both sides of this equation to relate dK/dt to dT/dt.

SET UP: $\omega = \dfrac{2\pi}{T}$ and $K = \frac{1}{2}I\omega^2$. The speed of light is $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $K = \dfrac{2\pi^2 I}{T^2}$. $\dfrac{dK}{dt} = -\dfrac{4\pi^2 I}{T^3}\dfrac{dT}{dt}$. The rate of energy loss is $\dfrac{4\pi^2 I}{T^3}\dfrac{dT}{dt}$. Solving for the moment of inertia *I* in terms of the power *P*,

$$I = \frac{PT^3}{4\pi}\ \frac{1}{dT/dt} = \frac{(5\times10^{31}\ \text{W})(0.0331\ \text{s})^3}{4\pi^2}\ \frac{1\ \text{s}}{4.22\times10^{-13}\ \text{s}} = 1.09\times10^{38}\ \text{kg}\cdot\text{m}^2$$

(b) $R = \sqrt{\dfrac{5I}{2M}} = \sqrt{\dfrac{5(1.08\times10^{38}\ \text{kg}\cdot\text{m}^2)}{2(1.4)(1.99\times10^{30}\ \text{kg})}} = 9.9\times10^3$ m, about 10 km.

(c) $v = \dfrac{2\pi R}{T} = \dfrac{2\pi(9.9\times10^3\text{m})}{(0.0331\ \text{s})} = 1.9\times10^6\ \text{m/s} = 6.3\times10^{-3}c.$

(d) $\rho = \dfrac{M}{V} = \dfrac{M}{(4\pi/3)R^3} = 6.9\times10^{17}\ \text{kg/m}^3$, which is much higher than the density of ordinary rock by 14 orders of magnitude, and is comparable to nuclear mass densities.

EVALUATE: *I* is huge because *M* is huge. A small rate of change in the period corresponds to a large release of energy.

9.99. **IDENTIFY:** In part (a), do the calculations as specified in the hint. In part (b) calculate the mass of each shell of inner radius R_1 and outer radius R_2 and sum to get the total mass. In part (c) use the expression in part (a) to calculate *I* for each shell and sum to get the total *I*.

SET UP: $m = \rho V$. For a solid sphere, $V = \frac{4}{3}\pi R^3$.

EXECUTE: **(a)** Following the hint, the moment of inertia of a uniform sphere in terms of the mass density is $I = \frac{2}{5}MR^2 = \frac{8}{15}\pi\rho R^5$, and so the difference in the moments of inertia of two spheres with the same density ρ but different radii R_2 and R_1 is $I = \rho(8\pi/15)(R_2^5 - R_1^5)$.

(b) A rather tedious calculation, summing the product of the densities times the difference in the cubes of the radii that bound the regions and multiplying by $4\pi/3$, gives $M = 5.97\times10^{24}$ kg.

(c) A similar calculation, summing the product of the densities times the difference in the fifth powers of the radii that bound the regions and multiplying by $8\pi/15$, gives $I = 8.02\times10^{22}\ \text{kg}\cdot\text{m}^2 = 0.334MR^2$.

EVALUATE: The calculated value of $I = 0.334MR^2$ agrees closely with the measured value of $0.3308MR^2$. This simple model is fairly accurate.

9.100. IDENTIFY: Apply Eq.(9.20)

SET UP: Let z be the coordinate along the vertical axis. $r(z)=\dfrac{zR}{h}$. $dm=\pi\rho\dfrac{R^2z^2}{h^2}$ and $dI=\dfrac{\pi\rho}{2}\dfrac{R^4}{h^4}z^4\,dz$.

EXECUTE: $I=\int dI=\dfrac{\pi\rho}{2}\dfrac{R^4}{h}\int_0^h z^4\,dz=\dfrac{\pi\rho}{10}\dfrac{R^4}{h^4}\left[z^5\right]_0^h=\dfrac{1}{10}\pi\rho R^4h$. The volume of a right circular cone is $V=\frac{1}{3}\pi R^2h$, the mass is $\frac{1}{3}\pi\rho R^2h$ and so $I=\dfrac{3}{10}\left(\dfrac{\pi\rho R^2h}{3}\right)R^2=\dfrac{3}{10}MR^2$.

EVALUATE: For a uniform cylinder of radius R and for an axis through its center, $I=\frac{1}{2}MR^2$. I for the cone is less, as expected, since the cone is constructed from a series of parallel discs whose radii decrease from R to zero along the vertical axis of the cone.

9.101. IDENTIFY: Follow the steps outlined in the problem.

SET UP: $\omega_z=d\theta/dt$. $\alpha_z=d^2\omega_z/dt^2$.

EXECUTE: (a) $ds=r\,d\theta=r_0d\theta+\beta\theta\,d\theta$ so $s(\theta)=r_0\theta+\dfrac{\beta}{2}\theta^2$. θ must be in radians.

(b) Setting $s=vt=r_0\theta+\dfrac{\beta}{2}\theta^2$ gives a quadratic in θ. The positive solution is

$$\theta(t)=\frac{1}{\beta}\left[\sqrt{r_0^{\,2}+2\beta vt}-r_0\right].$$

(The negative solution would be going backwards, to values of r smaller than r_0.)

(c) Differentiating, $\omega_z(t)=\dfrac{d\theta}{dt}=\dfrac{v}{\sqrt{r_0^2+2\beta vt}}$, $\alpha_z=\dfrac{d\omega_z}{dt}=-\dfrac{\beta v^2}{\left(r_0^2+2\beta vt\right)^{3/2}}$. The angular acceleration α_z is not constant.

(d) $r_0=25.0$ mm. θ must be measured in radians, so $\beta=(1.55\ \mu\text{m/rev})(1\ \text{rev}/2\pi\ \text{rad})=0.247\ \mu\text{m/rad}$. Using $\theta(t)$ from part (b), the total angle turned in $74.0\ \text{min}=4440\ \text{s}$ is

$$\theta=\frac{1}{2.47\times10^{-7}\,\text{m/rad}}\left(\sqrt{2\left(2.47\times10^{-7}\,\text{m/rad}\right)\left(1.25\ \text{m/s}\right)\left(4440\ \text{s}\right)+\left(25.0\times10^{-3}\text{m}\right)^2}-25.0\times10^{-3}\ \text{m}\right)$$

$\theta=1.337\times10^5$ rad, which is 2.13×10^4 rev.

(e) The graphs are sketched in Figure 9.101.

EVALUATE: ω_z must decrease as r increases, to keep $v=r\omega$ constant. For ω_z to decrease in time, α_z must be negative.

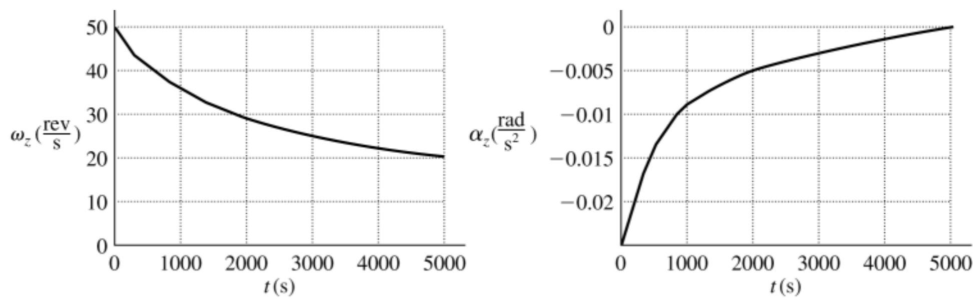


Figure 9.101

DYNAMICS OF ROTATIONAL MOTION

10

10.1. **IDENTIFY:** Use Eq.(10.2) to calculate the magnitude of the torque and use the right-hand rule illustrated in Fig.(10.4) to calculate the torque direction.

(a) SET UP: Consider Figure 10.1a.

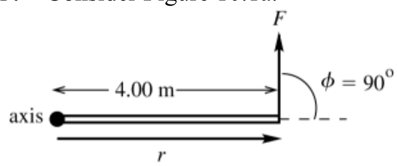


Figure 10.1a

EXECUTE: $\tau = Fl$

$l = r\sin\phi = (4.00\text{ m})\sin 90^\circ$

$l = 4.00\text{ m}$

$\tau = (10.0\text{ N})(4.00\text{ m}) = 40.0\text{ N}\cdot\text{m}$

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

(b) SET UP: Consider Figure 10.1b.

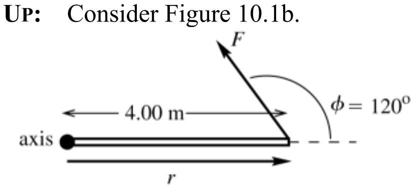


Figure 10.1b

EXECUTE: $\tau = Fl$

$l = r\sin\phi = (4.00\text{ m})\sin 120^\circ$

$l = 3.464\text{ m}$

$\tau = (10.0\text{ N})(3.464\text{ m}) = 34.6\text{ N}\cdot\text{m}$

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

(c) SET UP: Consider Figure 10.1c.

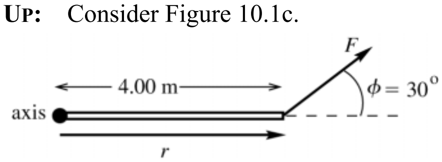


Figure 10.1c

EXECUTE: $\tau = Fl$

$l = r\sin\phi = (4.00\text{ m})\sin 30^\circ$

$l = 2.00\text{ m}$

$\tau = (10.0\text{ N})(2.00\text{ m}) = 20.0\text{ N}\cdot\text{m}$

This force tends to produce a counterclockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed out of the plane of the figure.

(d) SET UP: Consider Figure 10.1d.

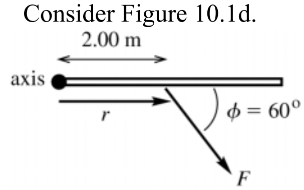


Figure 10.1d

EXECUTE: $\tau = Fl$

$l = r\sin\phi = (2.00\text{ m})\sin 60^\circ = 1.732\text{ m}$

$\tau = (10.0\text{ N})(1.732\text{ m}) = 17.3\text{ N}\cdot\text{m}$

This force tends to produce a clockwise rotation about the axis; by the right-hand rule the vector $\vec{\tau}$ is directed into the plane of the figure.

(e) SET UP: Consider Figure 10.1e.

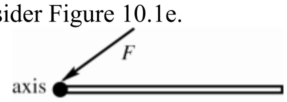


Figure 10.1e

EXECUTE: $\tau = Fl$

$r = 0$ so $l = 0$ and $\tau = 0$

(f) SET UP: Consider Figure 10.1f.

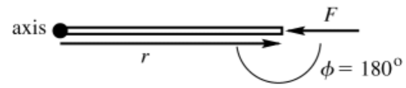


Figure 10.1f

EXECUTE: $\tau = Fl$

$l = r\sin\phi,\ \ \phi = 180°$,

so $l = 0$ and $\tau = 0$

EVALUATE: The torque is zero in parts (e) and (f) because the moment arm is zero; the line of action of the force passes through the axis.

10.2. **IDENTIFY:** $\tau = Fl$ with $l = r\sin\phi$. Add the two torques to calculate the net torque.

SET UP: Let counterclockwise torques be positive.

EXECUTE: $\tau_1 = -F_1 l_1 = -(8.00\text{ N})(5.00\text{ m}) = -40.0\text{ N}\cdot\text{m}$. $\tau_2 = +F_2 l_2 = (12.0\text{ N})(2.00\text{ m})\sin 30.0° = +12.0\text{ N}\cdot\text{m}$.

$\sum\tau = \tau_1 + \tau_2 = -28.0\text{ N}\cdot\text{m}$. The net torque is $28.0\text{ N}\cdot\text{m}$, clockwise.

EVALUATE: Even though $F_1 < F_2$, the magnitude of τ_1 is greater than the magnitude of τ_2, because F_1 has a larger moment arm.

10.3. **IDENTIFY** and **SET UP:** Use Eq.(10.2) to calculate the magnitude of each torque and use the right-hand rule (Fig.10.4) to determine the direction. Consider Figure 10.3

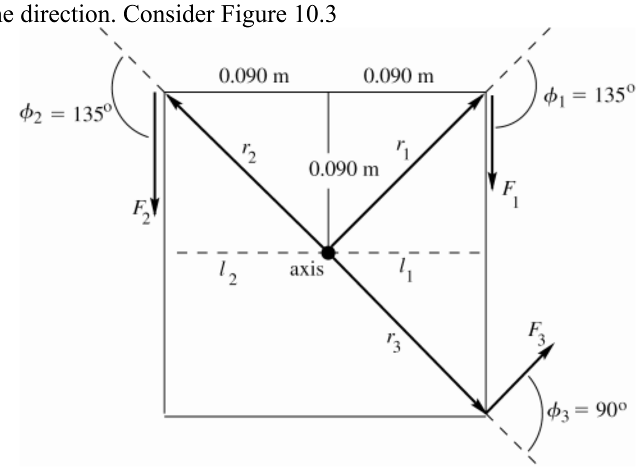


Figure 10.3

Let counterclockwise be the positive sense of rotation.

EXECUTE: $r_1 = r_2 = r_3 = \sqrt{(0.090\text{ m})^2 + (0.090\text{ m})^2} = 0.1273\text{ m}$

$\tau_1 = -F_1 l_1$

$l_1 = r_1\sin\phi_1 = (0.1273\text{ m})\sin 135° = 0.0900\text{ m}$

$\tau_1 = -(18.0\text{ N})(0.0900\text{ m}) = -1.62\text{ N}\cdot\text{m}$

$\vec{\tau}_1$ is directed into paper

$\tau_2 = +F_2 l_2$

$l_2 = r_2\sin\phi_2 = (0.1273\text{ m})\sin 135° = 0.0900\text{ m}$

$\tau_2 = +(26.0\text{ N})(0.0900\text{ m}) = +2.34\text{ N}\cdot\text{m}$

$\vec{\tau}_2$ is directed out of paper

$\tau_3 = +F_3 l_3$

$l_3 = r_3\sin\phi_3 = (0.1273\text{ m})\sin 90° = 0.1273\text{ m}$

$\tau_3 = +(14.0\text{ N})(0.1273\text{ m}) = +1.78\text{ N}\cdot\text{m}$

$\vec{\tau}_3$ is directed out of paper

$\sum\tau = \tau_1 + \tau_2 + \tau_3 = -1.62\text{ N}\cdot\text{m} + 2.34\text{ N}\cdot\text{m} + 1.78\text{ N}\cdot\text{m} = 2.50\text{ N}\cdot\text{m}$

EVALUATE: The net torque is positive, which means it tends to produce a counterclockwise rotation; the vector torque is directed out of the plane of the paper. In summing the torques it is important to include $+$ or $-$ signs to show direction.

10.4. **IDENTIFY:** Use $\tau = Fl = rF\sin\phi$ to calculate the magnitude of each torque and use the right-hand rule to determine the direction of each torque. Add the torques to find the net torque.

SET UP: Let counterclockwise torques be positive. For the 11.9 N force (F_1), $r = 0$. For the 14.6 N force (F_2), $r = 0.350$ m and $\phi = 40.0°$. For the 8.50 N force (F_3), $r = 0.350$ m and $\phi = 90.0°$

EXECUTE: $\tau_1 = 0$. $\tau_2 = -(14.6\text{ N})(0.350\text{ m})\sin 40.0° = -3.285\text{ N}\cdot\text{m}$.

$\tau_3 = +(8.50\text{ N})(0.350\text{ m})\sin 90.0° = +2.975\text{ N}\cdot\text{m}$. $\sum\tau = -3.285\text{ N}\cdot\text{m} + 2.975\text{ N}\cdot\text{m} = -0.31\text{ N}\cdot\text{m}$. The net torque is $0.31\text{ N}\cdot\text{m}$ and is clockwise.

EVALUATE: If we treat the torques as vectors, $\vec{\boldsymbol{\tau}}_2$ is into the page and $\vec{\boldsymbol{\tau}}_3$ is out of the page.

10.5. **IDENTIFY** and **SET UP:** Calculate the torque using Eq.(10.3) and also determine the direction of the torque using the right-hand rule.

(a) $\vec{\boldsymbol{r}} = (-0.450\text{ m})\hat{\boldsymbol{i}} + (0.150\text{ m})\hat{\boldsymbol{j}}$; $\vec{\boldsymbol{F}} = (-5.00\text{ N})\hat{\boldsymbol{i}} + (4.00\text{ N})\hat{\boldsymbol{j}}$. The sketch is given in Figure 10.5.

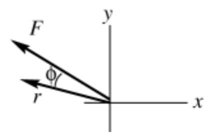


Figure 10.5

EXECUTE: **(b)** When the fingers of your right hand curl from the direction of $\vec{\boldsymbol{r}}$ into the direction of $\vec{\boldsymbol{F}}$ (through the smaller of the two angles, angle ϕ) your thumb points into the page (the direction of $\vec{\tau}$, the $-z$-direction).

(c) $\vec{\tau} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{F}} = \left[(-0.450\text{ m})\hat{\boldsymbol{i}} + (0.150\text{ m})\hat{\boldsymbol{j}}\right] \times \left[(-5.00\text{ N})\hat{\boldsymbol{i}} + (4.00\text{ N})\hat{\boldsymbol{j}}\right]$

$\vec{\tau} = +(2.25\text{ N}\cdot\text{m})\hat{\boldsymbol{i}} \times \hat{\boldsymbol{i}} - (1.80\text{ N}\cdot\text{m})\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} - (0.750\text{ N}\cdot\text{m})\hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}} + (0.600\text{ N}\cdot\text{m})\hat{\boldsymbol{j}} \times \hat{\boldsymbol{j}}$

$\hat{\boldsymbol{i}} \times \hat{\boldsymbol{i}} = \hat{\boldsymbol{j}} \times \hat{\boldsymbol{j}} = \boldsymbol{0}$

$\hat{\boldsymbol{i}} \times \hat{\boldsymbol{j}} = \hat{\boldsymbol{k}}$, $\hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}} = -\hat{\boldsymbol{k}}$

Thus $\vec{\tau} = -(1.80\text{ N}\cdot\text{m})\hat{\boldsymbol{k}} - (0.750\text{ N}\cdot\text{m})(-\hat{\boldsymbol{k}}) = (-1.05\text{ N}\cdot\text{m})\hat{\boldsymbol{k}}$.

EVALUATE: The calculation gives that $\vec{\tau}$ is in the $-z$-direction. This agrees with what we got from the right-hand rule.

10.6. **IDENTIFY:** Use $\tau = Fl = rF\sin\phi$ for the magnitude of the torque and the right-hand rule for the direction.

SET UP: In part (a), $r = 0.250$ m and $\phi = 37°$

EXECUTE: (a) $\tau = (17.0\text{ N})(0.250\text{ m})\sin 37° = 2.56\text{ N}\cdot\text{m}$. The torque is counterclockwise.

(b) The torque is maximum when $\phi = 90°$ and the force is perpendicular to the wrench. This maximum torque is $(17.0\text{ N})(0.250\text{ m}) = 4.25\text{ N}\cdot\text{m}$.

EVALUATE: If the force is directed along the handle then the torque is zero. The torque increases as the angle between the force and the handle increases.

10.7. **IDENTIFY:** Apply $\sum\tau_z = I\alpha_z$.

SET UP: $\omega_{0z} = 0$. $\omega_z = (400\text{ rev/min})\left(\dfrac{2\pi\text{ rad/rev}}{60\text{ s/min}}\right) = 41.9\text{ rad/s}$

EXECUTE: $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t} = \left(2.50\text{ kg}\cdot\text{m}^2\right)\dfrac{41.9\text{ rad/s}}{8.00\text{ s}} = 13.1\text{ N}\cdot\text{m}.$

EVALUATE: In $\tau_z = I\alpha_z$, α_z must be in rad/s^2.

10.8. **IDENTIFY:** Use a constant acceleration equation to calculate α_z and then apply $\sum\tau_z = I\alpha_z$.

SET UP: $I = \frac{2}{3}MR^2 + 2mR^2$, where $M = 8.40$ kg, $m = 2.00$ kg, so $I = 0.600\text{ kg}\cdot\text{m}^2$.

$\omega_{0z} = 75.0\text{ rpm} = 7.854\text{ rad/s}$; $\omega_z = 50.0\text{ rpm} = 5.236\text{ rad/s}$; $t = 30.0$ s.

EXECUTE: $\omega_z = \omega_{0_z} + \alpha_z t$ gives $\alpha_z = -0.08726\text{ rad/s}^2$. $\tau_z = I\alpha_z = -0.0524\text{ N}\cdot\text{m}$

EVALUATE: The torque is negative because its direction is opposite to the direction of rotation, which must be the case for the speed to decrease.

10.9. **IDENTIFY:** Use $\sum\tau_z = I\alpha_z$ to calculate α. Use a constant angular acceleration kinematic equation to relate α_z, ω_z and t.

SET UP: For a solid uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$. Let the direction the sphere is spinning be the positive sense of rotation. The moment arm for the friction force is $l = 0.0150$ m and the torque due to this force is negative.

EXECUTE: **(a)** $\alpha_z = \dfrac{\tau_z}{I} = \dfrac{-(0.0200\text{ N})(0.0150\text{ m})}{\frac{2}{5}(0.225\text{ kg})(0.0150\text{ m})^2} = -14.8\text{ rad/s}^2$

(b) $\omega_z - \omega_{0z} = -22.5\text{ rad/s}$. $\omega_z = \omega_{0z} + \alpha_z t$ gives $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{-22.5\text{ rad/s}}{-14.8\text{ rad/s}^2} = 1.52\text{ s}$.

EVALUATE: The fact that α_z is negative means its direction is opposite to the direction of spin. The negative α_z causes ω_z to decrease.

10.10. IDENTIFY: Apply $\sum \tau_z = I\alpha_z$ to the wheel. The acceleration a of a point on the cord and the angular acceleration α of the wheel are related by $a = R\alpha$.

SET UP: Let the direction of rotation of the wheel be positive. The wheel has the shape of a disk and $I = \frac{1}{2}MR^2$. The free-body diagram for the wheel is sketched in Figure 10.10a for a horizontal pull and in Figure 10.10b for a vertical pull. P is the pull on the cord and F is the force exerted on the wheel by the axle.

EXECUTE: **(a)** $\alpha_z = \dfrac{\tau_z}{I} = \dfrac{(40.0\text{ N})(0.250\text{ m})}{\frac{1}{2}(9.20\text{ kg})(0.250\text{ m})^2} = 34.8\text{ rad/s}^2$. $a = R\alpha = (0.250\text{ m})(34.8\text{ rad/s}^2) = 8.70\text{ m/s}^2$.

(b) $F_x = -P$, $F_y = -Mg$. $F = \sqrt{P^2 + (Mg)^2} = \sqrt{(40.0\text{ N})^2 + ([9.20\text{ kg}][9.80\text{ m/s}^2])^2} = 98.6\text{ N}$.

$\tan\phi = \dfrac{|F_y|}{|F_x|} = \dfrac{Mg}{P} = \dfrac{(9.20\text{ kg})(9.80\text{ m/s}^2)}{40.0\text{ N}}$ and $\phi = 66.1°$. The force exerted by the axle has magnitude 98.6 N and is directed at $66.1°$ above the horizontal, away from the direction of the pull on the cord.

(c) The pull exerts the same torque as in part (a), so the answers to part (a) don't change. In part (b), $F + P = Mg$ and $F = Mg - P = (9.20\text{ kg})(9.80\text{ m/s}^2) - 40.0\text{ N} = 50.2\text{ N}$. The force exerted by the axle has magnitude 50.2 N and is upward.

EVALUATE: The weight of the wheel and the force exerted by the axle produce no torque because they act at the axle.

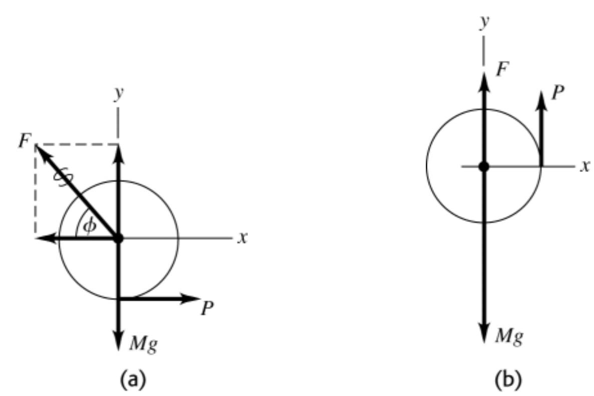


Figure 10.10

10.11. IDENTIFY: Use a constant angular acceleration equation to calculate α_z and then apply $\sum \tau_z = I\alpha_z$ to the motion of the cylinder. $f_k = \mu_k n$.

SET UP: $I = \frac{1}{2}mR^2 = \frac{1}{2}(8.25\text{ kg})(0.0750\text{ m})^2 = 0.02320\text{ kg}\cdot\text{m}^2$. Let the direction the cylinder is rotating be positive. $\omega_{0z} = 220\text{ rpm} = 23.04\text{ rad/s}$; $\omega_z = 0$; $\theta - \theta_0 = 5.25\text{ rev} = 33.0\text{ rad}$.

EXECUTE: $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives $\alpha_z = -8.046\text{ rad/s}^2$. $\sum\tau_z = \tau_f = -f_k R = -\mu_k nR$. Then $\sum \tau_z = I\alpha_z$ gives $-\mu_k nR = I\alpha_z$ and $n = \dfrac{I\alpha_z}{\mu_k R} = 7.47\text{ N}$.

EVALUATE: The friction torque is directed opposite to the direction of rotation and therefore produces an angular acceleration that slows the rotation.

10.12. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the stone and $\sum \tau_z = I\alpha_z$ to the pulley. Use a constant acceleration equation to find a for the stone.

SET UP: For the motion of the stone take $+y$ to be downward. The pulley has $I = \frac{1}{2}MR^2$. $a = R\alpha$.

EXECUTE: **(a)** $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $12.6\text{ m} = \frac{1}{2}a_y(3.00\text{ s})^2$ and $a_y = 2.80\text{ m/s}^2$. Then $\sum F_y = ma_y$ applied to the stone gives $mg - T = ma$. $\sum \tau_z = I\alpha_z$ applied to the pulley gives $TR = \frac{1}{2}MR^2\alpha = \frac{1}{2}MR^2(a/R)$. $T = \frac{1}{2}Ma$. Combining these two equations to eliminate T gives

$$M = \frac{M}{2}\left(\frac{a}{g-a}\right) = \left(\frac{10.0\text{ kg}}{2}\right)\left(\frac{2.80\text{ m/s}^2}{9.80\text{ m/s}^2 - 2.80\text{ m/s}^2}\right) = 2.00\text{ kg}.$$

(b) $T = \dfrac{1}{2}Ma = \dfrac{1}{2}(10.0\text{ kg})(2.80\text{ m/s}^2) = 14.0\text{ N}$

EVALUATE: The tension in the wire is less than the weight $mg = 19.6$ N of the stone, because the stone has a downward acceleration.

10.13. **IDENTIFY:** Use the kinematic information to solve for the angular acceleration of the grindstone. Assume that the grindstone is rotating counterclockwise and let that be the positive sense of rotation. Then apply Eq.(10.7) to calculate the friction force and use $f_k = \mu_k n$ to calculate μ_k.

SET UP: $\omega_{0z} = 850\text{ rev/min}(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 89.0\text{ rad/s}$

$t = 7.50$ s; $\omega_z = 0$ (comes to rest); $\alpha_z = ?$

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t$

$$\alpha_z = \frac{0 - 89.0\text{ rad/s}}{7.50\text{ s}} = -11.9\text{ rad/s}^2$$

SET UP: Apply $\sum \tau_z = I\alpha_z$ to the grindstone. The free-body diagram is given in Figure 10.13.

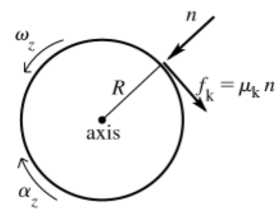


Figure 10.13

The normal force has zero moment arm for rotation about an axis at the center of the grindstone, and therefore zero torque. The only torque on the grindstone is that due to the friction force f_k exerted by the ax; for this force the moment arm is $l = R$ and the torque is negative.

EXECUTE: $\sum \tau_z = -f_k R = -\mu_k nR$

$I = \frac{1}{2}MR^2$ (solid disk, axis through center)

Thus $\sum \tau_z = I\alpha_z$ gives $-\mu_k nR = \left(\frac{1}{2}MR^2\right)\alpha_z$

$$\mu_k = -\frac{MR\alpha_z}{2n} = -\frac{(50.0\text{ kg})(0.260\text{ m})(-11.9\text{ rad/s}^2)}{2(160\text{ N})} = 0.483$$

EVALUATE: The friction torque is clockwise and slows down the counterclockwise rotation of the grindstone.

10.14. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the bucket, with $+y$ downward. Apply $\sum \tau_z = I\alpha_z$ to the cylinder, with the direction the cylinder rotates positive.

SET UP: The free-body diagram for the bucket is given in Fig.10.14a and the free-body diagram for the cylinder is given in Fig.10.14b. $I = \frac{1}{2}MR^2$. $a(\text{bucket}) = R\alpha(\text{cylinder})$

EXECUTE: **(a)** For the bucket, $mg - T = ma$. For the cylinder, $\sum \tau_z = I\alpha_z$ gives $TR = \frac{1}{2}MR^2\alpha$. $\alpha = a/R$ then gives $T = \frac{1}{2}Ma$. Combining these two equations gives $mg - \frac{1}{2}Ma = ma$ and

$$a = \frac{mg}{m + M/2} = \left(\frac{15.0\text{ kg}}{15.0\text{ kg} + 6.0\text{ kg}}\right)(9.80\text{ m/s}^2) = 7.00\text{ m/s}^2.$$

$T = m(g - a) = (15.0\text{ kg})(9.80\text{ m/s}^2 - 7.00\text{ m/s}^2) = 42.0\text{ N}$.

(b) $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$ gives $v_y = \sqrt{2(7.00\text{ m/s}^2)(10.0\text{ m})} = 11.8\text{ m/s}$.

(c) $a_y = 7.00\text{ m/s}^2$, $v_{0y} = 0$, $y - y_0 = 10.0$ m. $y - y_0 = v_{0y}t + \frac{1}{2}\alpha_y t^2$ gives $t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = \sqrt{\dfrac{2(10.0\text{ m})}{7.00\text{ m/s}^2}} = 1.69\text{ s}$

(d) $\sum F_y = ma_y$ applied to the cylinder gives $n - T - Mg = 0$ and

$n = T + mg = 42.0\ \text{N} + (12.0\ \text{kg})(9.80\ \text{m/s}^2) = 160\ \text{N}$.

EVALUATE: The tension in the rope is less than the weight of the bucket, because the bucket has a downward acceleration. If the rope were cut, so the bucket would be in free-fall, the bucket would strike the water in $t = \sqrt{\dfrac{2(10.0\ \text{m})}{9.80\ \text{m/s}^2}} = 1.43$ s and would have a final speed of 14.0 m/s. The presence of the cylinder slows the fall of the bucket.

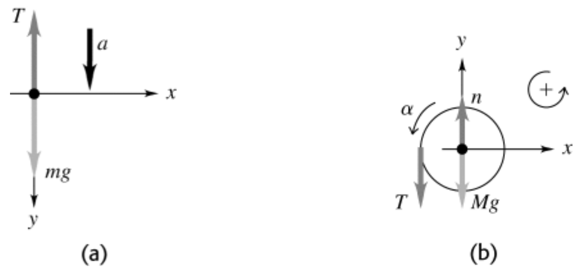


Figure 10.14

10.15. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each book and apply $\sum \tau_z = I\alpha_z$ to the pulley. Use a constant acceleration equation to find the common acceleration of the books.

SET UP: $m_1 = 2.00\ \text{kg}$, $m_2 = 3.00\ \text{kg}$. Let T_1 be the tension in the part of the cord attached to m_1 and T_2 be the tension in the part of the cord attached to m_2. Let the $+x$-direction be in the direction of the acceleration of each book. $a = R\alpha$.

EXECUTE: **(a)** $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $a_x = \dfrac{2(x - x_0)}{t^2} = \dfrac{2(1.20\ \text{m})}{(0.800\ \text{s})^2} = 3.75\ \text{m/s}^2$. $a_1 = 3.75\ \text{m/s}^2$ so

$T_1 = m_1 a_1 = 7.50$ N and $T_2 = m_2(g - a_1) = 18.2$ N.

(b) The torque on the pulley is $(T_2 - T_1)R = 0.803\ \text{N} \cdot \text{m}$, and the angular acceleration is

$\alpha = a_1/R = 50\ \text{rad/s}^2$, so $I = \tau/\alpha = 0.016\ \text{kg} \cdot \text{m}^2$.

EVALUATE: The tensions in the two parts of the cord must be different, so there will be a net torque on the pulley.

10.16. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each box and $\sum \tau_z = I\alpha_z$ to the pulley. The magnitude a of the acceleration of each box is related to the magnitude of the angular acceleration α of the pulley by $a = R\alpha$.

SET UP: The free-body diagrams for each object are shown in Figure 10.16a-c. For the pulley, $R = 0.250$ m and $I = \frac{1}{2}MR^2$. T_1 and T_2 are the tensions in the wire on either side of the pulley. $m_1 = 12.0\ \text{kg}$, $m_2 = 5.00\ \text{kg}$ and $M = 2.00\ \text{kg}$. $\vec{F}$ is the force that the axle exerts on the pulley. For the pulley, let clockwise rotation be positive.

EXECUTE: **(a)** $\sum F_x = ma_x$ for the 12.0 kg box gives $T_1 = m_1 a$. $\sum F_y = ma_y$ for the 5.00 kg weight gives $m_2 g - T_2 = m_2 a$. $\sum \tau_z = I\alpha_z$ for the pulley gives $(T_2 - T_1)R = (\frac{1}{2}MR^2)\alpha$. $a = R\alpha$ and $T_2 - T_1 = \frac{1}{2}Ma$. Adding these three equations gives $m_2 g = (m_1 + m_2 + \frac{1}{2}M)a$ and

$$a = \left(\frac{m_2}{m_1 + m_2 + \frac{1}{2}M}\right)g = \left(\frac{5.00\ \text{kg}}{12.0\ \text{kg} + 5.00\ \text{kg} + 1.00\ \text{kg}}\right)(9.80\ \text{m/s}^2) = 2.72\ \text{m/s}^2$$. Then

$T_1 = m_1 a = (12.0\ \text{kg})(2.72\ \text{m/s}^2) = 32.6\ \text{N}$. $m_2 g - T_2 = m_2 a$ gives

$T_2 = m_2(g - a) = (5.00\ \text{kg})(9.80\ \text{m/s}^2 - 2.72\ \text{m/s}^2) = 35.4\ \text{N}$. The tension to the left of the pulley is 32.6 N and below the pulley it is 35.4 N.

(b) $a = 2.72\ \text{m/s}^2$

(c) For the pulley, $\sum F_x = ma_x$ gives $F_x = T_1 = 32.6$ N and $\sum F_y = ma_y$ gives

$F_y = Mg + T_2 = (2.00\ \text{kg})(9.80\ \text{m/s}^2) + 35.4\ \text{N} = 55.0\ \text{N}$.

EVALUATE: The equation $m_2g = (m_1 + m_2 + \frac{1}{2}M)a$ says that the external force m_2g must accelerate all three objects.

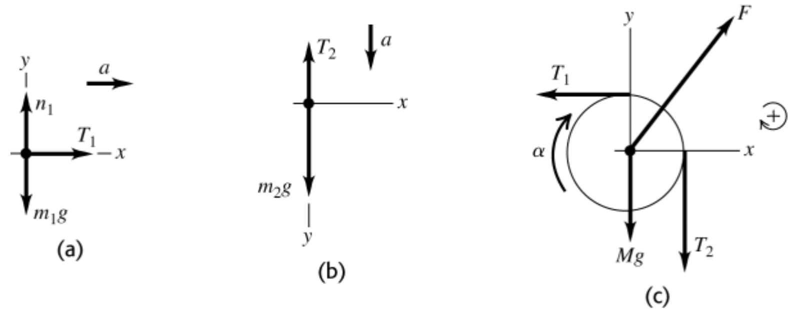


Figure 10.16

10.17. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the post and $\sum \vec{F} = m\vec{a}$ to the hanging mass. The acceleration $\vec{a}$ of the mass has the same magnitude as the tangential acceleration $a_{tan} = r\alpha$ of the point on the post where the string is attached; $r = 1.75\text{ m} - 0.500\text{ m} = 1.25\text{ m}$.

SET UP: The free-body diagrams for the post and mass are given in Figures 10.17a and b. The post has $I = \frac{1}{3}ML^2$, with $M = 15.0\text{ kg}$ and $L = 1.75\text{ m}$.

EXECUTE: **(a)** $\sum \tau_z = I\alpha_z$ for the post gives $Tr = \left(\frac{1}{3}ML^2\right)\alpha$. $a = r\alpha$ so $\alpha = \dfrac{a}{r}$ and $T = \left(\dfrac{ML^2}{3r^2}\right)a$. $\sum F_y = ma_y$ for the mass gives $mg - T = ma$. These two equations give $mg = (m + ML^2/[3r^2])a$ and

$$a = \left(\frac{m}{m + ML^2/[3r^2]}\right)g = \left(\frac{5.00\text{ kg}}{5.00\text{ kg} + [15.0\text{ kg}][1.75\text{ m}]^2/3[1.25\text{ m}]^2}\right)(9.80\text{ m/s}^2) = 3.31\text{ m/s}^2.$$

$$\alpha = \frac{a}{r} = \frac{3.31\text{ m/s}^2}{1.25\text{ m}} = 2.65\text{ rad/s}^2.$$

(b) No. As the post rotates and the point where the string is attached moves in an arc of a circle, the string is no longer perpendicular to the post. The torque due to this tension changes and the acceleration due to this torque is not constant.

(c) From part (a), $a = 3.31\text{ m/s}^2$. The acceleration of the mass is not constant. It changes as α for the post changes.

EVALUATE: At the instant the cable breaks the tension in the string is less than the weight of the mass because the mass accelerates downward and there is a net downward force on it.

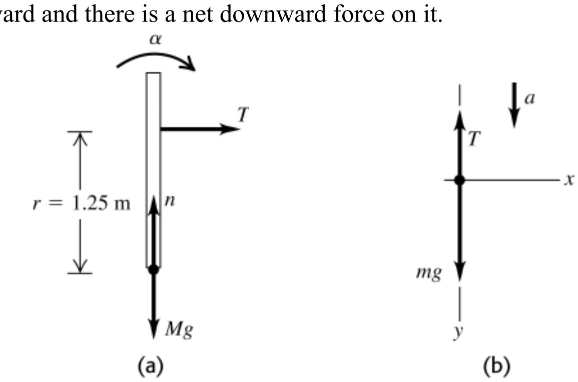


Figure 10.17

10.18. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the rod.

SET UP: For the rod and axis at one end, $I = \frac{1}{3}Ml^2$.

EXECUTE: $\alpha = \dfrac{\tau}{I} = \dfrac{Fl}{\frac{1}{3}Ml^2} = \dfrac{3F}{Ml}$.

EVALUATE: Note that α decreases with the length of the rod, even though the torque increases.

10.19. **IDENTIFY:** Since there is rolling without slipping, $v_{cm} = R\omega$. The kinetic energy is given by Eq.(10.8). The velocities of points on the rim of the hoop are as described in Figure 10.13 in chapter 10.

SET UP: $\omega = 3.00\text{ rad/s}$ and $R = 0.600\text{ m}$. For a hoop rotating about an axis at its center, $I = MR^2$.

EXECUTE: **(a)** $v_{cm} = R\omega = (0.600 \text{ m})(3.00 \text{ rad/s}) = 1.80 \text{ m/s}$.

(b) $K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}(MR^2)(v_{cm}/R^2) = Mv_{cm}^2 = (2.20 \text{ kg})(1.80 \text{ m/s})^2 = 7.13 \text{ J}$

(c) (i) $v = 2v_{cm} = 3.60 \text{ m/s}$. $\vec{v}$ is to the right. (ii) $v = 0$

(iii) $v = \sqrt{v_{cm}^2 + v_{tan}^2} = \sqrt{v_{cm}^2 + (R\omega)^2} = \sqrt{2}v_{cm} = 2.55 \text{ m/s}$. $\vec{v}$ at this point is at $45°$ below the horizontal.

(d) To someone moving to the right at $v = v_{cm}$, the hoop appears to rotate about a stationary axis at its center.

(i) $v = R\omega = 1.80 \text{ m/s}$, to the right. (ii) $v = 1.80 \text{ m/s}$, to the left. (iii) $v = 1.80 \text{ m/s}$, downward.

EVALUATE: For the special case of a hoop, the total kinetic energy is equally divided between the motion of the center of mass and the rotation about the axis through the center of mass. In the rest frame of the ground, different points on the hoop have different speed.

10.20. **IDENTIFY:** Only gravity does work, so $W_{other} = 0$ and conservation of energy gives $K_i + U_i = K_f + U_f$. $K_f = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$.

SET UP: Let $y_f = 0$, so $U_f = 0$ and $y_i = 0.750 \text{ m}$. The hoop is released from rest so $K_i = 0$. $v_{cm} = R\omega$. For a hoop with an axis at its center, $I_{cm} = MR^2$.

EXECUTE: **(a)** Conservation of energy gives $U_i = K_f$. $K_f = \frac{1}{2}MR^2\omega^2 + \frac{1}{2}(MR^2)\omega^2 = MR^2\omega^2$, so $MR^2\omega^2 = Mgy_i$.

$$\omega = \frac{\sqrt{gy_i}}{R} = \frac{\sqrt{(9.80 \text{ m/s}^2)(0.750 \text{ m})}}{0.0800 \text{ m}} = 33.9 \text{ rad/s}.$$

(b) $v = R\omega = (0.0800 \text{ m})(33.9 \text{ rad/s}) = 2.71 \text{ m/s}$

EVALUATE: An object released from rest and falling in free-fall for 0.750 m attains a speed of $\sqrt{2g(0.750 \text{ m})} = 3.83 \text{ m/s}$. The final speed of the hoop is less than this because some of its energy is in kinetic energy of rotation. Or, equivalently, the upward tension causes the magnitude of the net force of the hoop to be less than its weight.

10.21. **IDENTIFY:** Apply Eq.(10.8).

SET UP: For an object that is rolling without slipping, $v_{cm} = R\omega$.

EXECUTE: The fraction of the total kinetic energy that is rotational is

$$\frac{(1/2)I_{cm}\omega^2}{(1/2)Mv_{cm}^2 + (1/2)I_{cm}\omega^2} = \frac{1}{1(M/I_{cm})v_{cm}^2/\omega^2} = \frac{1}{1 + (MR^2/I_{cm})}$$

(a) $I_{cm} = (1/2)MR^2$, so the above ratio is $1/3$.

(b) $I_{cm} = (2/5)MR^2$ so the above ratio is $2/7$.

(c) $I_{cm} = (2/3)MR^2$ so the ratio is $2/5$.

(d) $I_{cm} = (5/8)MR^2$ so the ratio is $5/13$.

EVALUATE: The moment of inertia of each object takes the form $I = \beta MR^2$. The ratio of rotational kinetic energy to total kinetic energy can be written as $\dfrac{1}{1+1/\beta} = \dfrac{\beta}{1+\beta}$. The ratio increases as β increases.

10.22. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the translational motion of the center of mass and $\sum \tau_z = I\alpha_z$ to the rotation about the center of mass.

SET UP: Let $+x$ be down the incline and let the shell be turning in the positive direction. The free-body diagram for the shell is given in Fig.10.22. From Table 9.2, $I_{cm} = \frac{2}{3}mR^2$.

EXECUTE: $\sum F_x = ma_x$ gives $mg\sin\beta - f = ma_{cm}$. $\sum \tau_z = I\alpha_z$ gives $fR = (\frac{2}{3}mR^2)\alpha$. With $\alpha = a_{cm}/R$ this becomes $f = \frac{2}{3}ma_{cm}$. Combining the equations gives $mg\sin\beta - \frac{2}{3}ma_{cm} = ma_{cm}$ and

$a_{cm} = \dfrac{3g\sin\beta}{5} = \dfrac{3(9.80 \text{ m/s}^2)(\sin 38.0°)}{5} = 3.62 \text{ m/s}^2$. $f = \frac{2}{3}ma_{cm} = \frac{2}{3}(2.00 \text{ kg})(3.62 \text{ m/s}^2) = 4.83 \text{ N}$. The friction is static since there is no slipping at the point of contact. $n = mg\cos\beta = 15.45 \text{ N}$. $\mu_s = \dfrac{f}{n} = \dfrac{4.83 \text{ N}}{15.45 \text{ N}} = 0.313$.

(b) The acceleration is independent of *m* and doesn't change. The friction force is proportional to *m* so will double; $f = 9.66 \text{ N}$. The normal force will also double, so the minimum μ_s required for no slipping wouldn't change.

EVALUATE: If there is no friction and the object slides without rolling, the acceleration is $g\sin\beta$. Friction and rolling without slipping reduce a to 0.60 times this value.

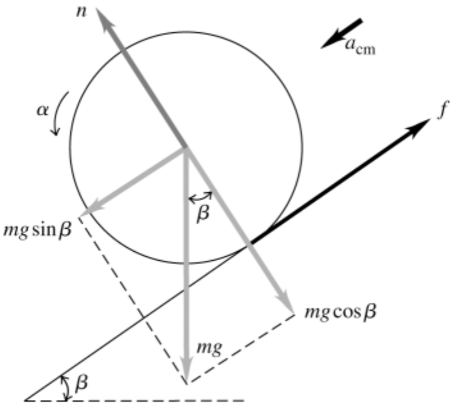


Figure 10.22

10.23. **IDENTIFY:** Apply $\sum \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$ and $\sum \tau_z = I_{\text{cm}}\alpha_z$ to the motion of the ball.

(a) SET UP: The free-body diagram is given in Figure 10.23a.

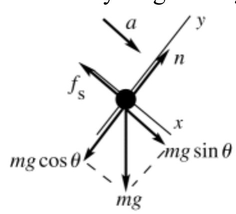


Figure 10.23a

EXECUTE: $\sum F_y = ma_y$

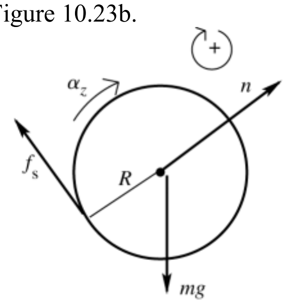
$n = mg\cos\theta$ and $f_s = \mu_s mg\cos\theta$

$\sum F_x = ma_x$

$mg\sin\theta - \mu_s mg\cos\theta = ma$

$g(\sin\theta - \mu_s\cos\theta) = a$ (eq. 1)

SET UP: Consider Figure 10.23b.



n and mg act at the center of the ball and provide no torque

Figure 10.23b

EXECUTE: $\sum \tau = \tau_f = \mu_s mg\cos\theta R$; $I = \frac{2}{5}mR^2$

$\sum \tau_z = I_{\text{cm}}\alpha_z$ gives $\mu_s mg\cos\theta R = \frac{2}{5}mR^2\alpha$

No slipping means $\alpha = a/R$, so $\mu_s g\cos\theta = \frac{2}{5}a$ (eq.2)

We have two equations in the two unknowns a and μ_s. Solving gives $a = \frac{5}{7}g\sin\theta$ and $\mu_s = \frac{2}{7}\tan\theta = \frac{2}{7}\tan 65.0° = 0.613$

(b) Repeat the calculation of part (a), but now $I = \frac{2}{3}mR^2$. $a = \frac{3}{5}g\sin\theta$ and $\mu_s = \frac{2}{5}\tan\theta = \frac{2}{5}\tan 65.0° = 0.858$

The value of μ_s calculated in part (a) is not large enough to prevent slipping for the hollow ball.

(c) EVALUATE: There is no slipping at the point of contact. More friction is required for a hollow ball since for a given m and R it has a larger I and more torque is needed to provide the same α. Note that the required μ_s is independent of the mass or radius of the ball and only depends on how that mass is distributed.

10.24. **IDENTIFY:** Apply conservation of energy to the motion of the marble.

SET UP: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, with $I = \frac{2}{5}MR^2$. $v_{\text{cm}} = R\omega$ for no slipping. Let $y = 0$ at the bottom of the bowl. The marble at its initial and final locations is sketched in Figure 10.24.

EXECUTE: **(a)** Motion from the release point to the bottom of the bowl: $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$.

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2$ and $v = \sqrt{\frac{10}{7}gh}$.

Motion along the smooth side: The rotational kinetic energy does not change, since there is no friction torque on the marble, $\frac{1}{2}mv^2 + K_{\text{rot}} = mgh' + K_{\text{rot}}$. $h' = \frac{v^2}{2g} = \frac{\frac{10}{7}gh}{2g} = \frac{5}{7}h$

(b) $mgh = mgh'$ so $h' = h$.

EVALUATE: **(c)** With friction on both halves, all the initial potential energy gets converted back to potential energy. Without friction on the right half some of the energy is still in rotational kinetic energy when the marble is at its maximum height.

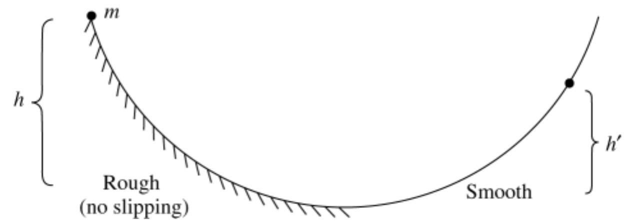


Figure 10.24

10.25. **IDENTIFY:** Apply conservation of energy to the motion of the wheel.

SET UP: The wheel at points 1 and 2 of its motion is shown in Figure 10.25.

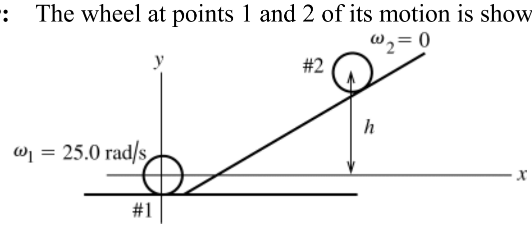


Figure 10.25

Take $y = 0$ at the center of the wheel when it is at the bottom of the hill.

The wheel has both translational and rotational motion so its kinetic energy is $K = \frac{1}{2}I_{\text{cm}}\omega^2 + \frac{1}{2}Mv_{\text{cm}}^2$.

EXECUTE: $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

$W_{\text{other}} = W_{\text{fric}} = -3500$ J (the friction work is negative)

$K_1 = \frac{1}{2}I\omega_1^2 + \frac{1}{2}Mv_1^2$; $v = R\omega$ and $I = 0.800MR^2$ so

$K_1 = \frac{1}{2}(0.800)MR^2\omega_1^2 + \frac{1}{2}MR^2\omega_1^2 = 0.900MR^2\omega_1^2$

$K_2 = 0$, $U_1 = 0$, $U_2 = Mgh$

Thus $0.900MR^2\omega_1^2 + W_{\text{fric}} = Mgh$

$M = w/g = 392\text{ N}/(9.80\text{ m/s}^2) = 40.0\text{ kg}$

$$h = \frac{0.900MR^2\omega_1^2 + W_{\text{fric}}}{Mg}$$

$$h = \frac{(0.900)(40.0\text{ kg})(0.600\text{ m})^2(25.0\text{ rad/s})^2 - 3500\text{ J}}{(40.0\text{ kg})(9.80\text{ m/s}^2)} = 11.7\text{ m}$$

EVALUATE: Friction does negative work and reduces h.

10.26. **IDENTIFY:** Apply $\sum\tau_z = I\alpha_z$ and $\sum\vec{F} = m\vec{a}$ to the motion of the bowling ball.

SET UP: $a_{\text{cm}} = R\alpha$. $f_s = \mu_s n$. Let $+x$ be directed down the incline.

EXECUTE: **(a)** The free-body diagram is sketched in Figure 10.26.

The angular speed of the ball must decrease, and so the torque is provided by a friction force that acts up the hill.

(b) The friction force results in an angular acceleration, given by $I\alpha = fR$. $\sum\vec{F} = m\vec{a}$ applied to the motion of the center of mass gives $mg\sin\beta - f = ma_{\text{cm}}$, and the acceleration and angular acceleration are related by $a_{\text{cm}} = R\alpha$.

Combining, $mg\sin\beta = ma\left(1 + \frac{I}{mR^2}\right) = ma(7/5)$. $a_{\text{cm}} = (5/7)g\sin\beta$.

(c) From either of the above relations between f and a_{cm}, $f = \frac{2}{5}ma_{\text{cm}} = \frac{2}{7}mg\sin\beta \le \mu_s n = \mu_s mg\cos\beta$.

$\mu_s \ge (2/7)\tan\beta.$

EVALUATE: If $\mu_s = 0$, $a_{\text{cm}} = mg\sin\beta$. a_{cm} is less when friction is present. The ball rolls farther uphill when friction is present, because the friction removes the rotational kinetic energy and converts it to gravitational potential energy. In the absence of friction the ball retains the rotational kinetic energy that is has initially.

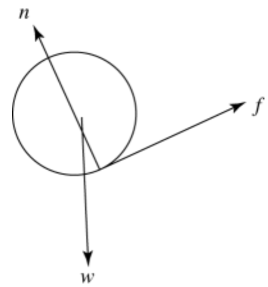


Figure 10.26

10.27. **(a) IDENTIFY:** Use Eq.(10.7) to find α_z and then use a constant angular acceleration equation to find ω_z.

SET UP: The free-body diagram is given in Figure 10.27.

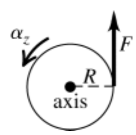


Figure 10.27

EXECUTE: Apply $\sum \tau_z = I\alpha_z$ to find the angular acceleration:

$FR = I\alpha_z$

$$\alpha_z = \frac{FR}{I} = \frac{(18.0\text{ N})(2.40\text{ m})}{2100\text{ kg}\cdot\text{m}^2} = 0.02057\text{ rad/s}^2$$

SET UP: Use the constant α_z kinematic equations to find ω_z.

$\omega_z = ?$; ω_{0z} (initially at rest); $\alpha_z = 0.02057\text{ rad/s}^2$; $t = 15.0\text{ s}$

EXECUTE: $\omega_z = \omega_{0z} + \alpha_z t = 0 + (0.02057\text{ rad/s}^2)(15.0\text{ s}) = 0.309\text{ rad/s}$

(b) IDENTIFY and SET UP: Calculate the work from Eq.(10.21), using a constant angular acceleration equation to calculate $\theta - \theta_0$, or use the work-energy theorem. We will do it both ways.

EXECUTE: (1) $W = \tau_z \Delta\theta$ (Eq.(10.21))

$\Delta\theta = \theta - \theta_0 = \omega_{0z}t + \tfrac{1}{2}\alpha_z t^2 = 0 + \tfrac{1}{2}(0.02057\text{ rad/s}^2)(15.0\text{ s})^2 = 2.314\text{ rad}$

$t_z = FR = (18.0\text{ N})(2.40\text{ m}) = 43.2\text{ N}\cdot\text{m}$

Then $W = \tau_z\Delta\theta = (43.2\text{ N}\cdot\text{m})(2.314\text{ rad}) = 100\text{ J}$.

or

(2) $W_{\text{tot}} = K_2 - K_1$ (the work-energy relation from chapter 6)

$W_{\text{tot}} = W$, the work done by the child

$K_1 = 0$; $K_2 = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(2100\text{ kg}\cdot\text{m}^2)(0.309\text{ rad/s})^2 = 100\text{ J}$

Thus $W = 100\text{ J}$, the same as before.

EVALUATE: Either method yields the same result for W.

(c) IDENTIFY and SET UP: Use Eq.(6.15) to calculate P_{av}

EXECUTE: $P_{\text{av}} = \dfrac{\Delta W}{\Delta t} = \dfrac{100\text{ J}}{15.0\text{ s}} = 6.67\text{ W}$

EVALUATE: Work is in joules, power is in watts.

10.28. **IDENTIFY:** Apply $P = \tau\omega$ and $W = \tau\Delta\theta$.

SET UP: P must be in watts, $\Delta\theta$ must be in radians, and ω must be in rad/s. $1\text{ rev} = 2\pi\text{ rad}$. $1\text{ hp} = 746\text{ W}$. $\pi\text{ rad/s} = 30\text{ rev/min}$.

EXECUTE: **(a)** $\tau = \dfrac{P}{\omega} = \dfrac{(175\text{ hp})(746\text{ W/hp})}{(2400\text{ rev/min})\left(\dfrac{\pi}{30}\dfrac{\text{rad/s}}{\text{rev/min}}\right)} = 519\text{ N}\cdot\text{m}.$

(b) $W = \tau\Delta\theta = (519\text{ N}\cdot\text{m})(2\pi\text{ rad}) = 3260\text{ J}$

EVALUATE: $\omega = 40$ rev/s, so the time for one revolution is 0.025 s. $P = 1.306\times10^5$ W, so in one revolution, $W = Pt = 3260$ J, which agrees with our previous result.

10.29. IDENTIFY: Apply $\sum\tau_z = I\alpha_z$ and constant angular acceleration equations to the motion of the wheel.

SET UP: 1 rev $= 2\pi$ rad. π rad/s $= 30$ rev/min.

EXECUTE: **(a)** $\tau_z = I\alpha_z = I\dfrac{\omega_z - \omega_{0z}}{t}$.

$$\tau_z = \frac{\left((1/2)(1.50\text{ kg})(0.100\text{ m})^2\right)(1200\text{ rev/min})\left(\dfrac{\pi\text{ rad/s}}{30\text{ rev/min}}\right)}{2.5\text{ s}} = 0.377\text{ N}\cdot\text{m}$$

(b) $\omega_{\text{av}}\Delta t = \dfrac{(600\text{ rev/min})(2.5\text{ s})}{60\text{ s/min}} = 25.0\text{ rev} = 157$ rad.

(c) $W = \tau\Delta\theta = (0.377\text{ N}\cdot\text{m})(157\text{ rad}) = 59.2$ J.

(d) $K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\left((1/2)(1.5\text{ kg})(0.100\text{ m})^2\right)\left((1200\text{ rev/min})\left(\dfrac{\pi\text{ rad/s}}{30\text{ rev/min}}\right)\right)^2 = 59.2$ J.

the same as in part (c).

EVALUATE: The agreement between the results of parts (c) and (d) illustrates the work-energy theorem

10.30. IDENTIFY: The power output of the motor is related to the torque it produces and to its angular velocity by $P = \tau_z\omega_z$, where ω_z must be in rad/s.

SET UP: The work output of the motor in 60.0 s is $\dfrac{2}{3}(9.00\text{ kJ}) = 6.00$ kJ, so $P = \dfrac{6.00\text{ kJ}}{60.0\text{ s}} = 100$ W.

$\omega_z = 2500\text{ rev/min} = 262$ rad/s.

EXECUTE: $\tau_z = \dfrac{P}{\omega_z} = \dfrac{100\text{ W}}{262\text{ rad/s}} = 0.382\text{ N}\cdot\text{m}$

EVALUATE: For a constant power output, the torque developed decreases and the rotation speed of the motor increases.

10.31. IDENTIFY: Apply $\tau = FR$ and $P = \tau\omega$.

SET UP: 1 hp $= 746$ W. π rad/s $= 30$ rev/min

EXECUTE: **(a)** With no load, the only torque to be overcome is friction in the bearings (neglecting air friction), and the bearing radius is small compared to the blade radius, so any frictional torque can be neglected.

(b) $F = \dfrac{\tau}{R} = \dfrac{P/\omega}{R} = \dfrac{(1.9\text{ hp})(746\text{ W/hp})}{(2400\text{ rev/min})\left(\dfrac{\pi\text{ rad/s}}{30\text{ rev/min}}\right)(0.086\text{ m})} = 65.6$ N.

EVALUATE: In $P = I\omega$, τ must be in watts and ω must be in rad/s.

10.32. IDENTIFY: Apply $\sum\tau_z = I\alpha_z$ to the motion of the propeller and then use constant acceleration equations to analyze the motion. $W = \tau\Delta\theta$.

SET UP: $I = \frac{1}{2}mL^2 = \frac{1}{2}(117\text{ kg})(2.08\text{ m})^2 = 42.2\text{ kg}\cdot\text{m}^2$.

EXECUTE: **(a)** $\alpha = \dfrac{\tau}{I} = \dfrac{1950\text{ N}\cdot\text{m}}{42.2\text{ kg}\cdot\text{m}^2} = 46.2\text{ rad/s}^2$.

(b) $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives $\omega = \sqrt{2\alpha\theta} = \sqrt{2(46.2\text{ rad/s}^2)(5.0\text{ rev})(2\pi\text{ rad/rev})} = 53.9$ rad/s.

(c) $W = \tau\theta = (1950\text{ N}\cdot\text{m})(5.00\text{ rev})(2\pi\text{ rad/rev}) = 6.13\times10^4$ J.

(d) $t = \dfrac{\omega_z - \omega_{0z}}{\alpha_z} = \dfrac{53.9\text{ rad/s}}{46.2\text{ rad/s}^2} = 1.17$ s. $P_{\text{av}} = \dfrac{W}{\Delta t} = \dfrac{6.13\times10^4\text{ J}}{1.17\text{ s}} = 52.5$ kW.

EVALUATE: $P = \tau\omega$. τ is constant and ω is linear in *t*, so P_{av} is half the instantaneous power at the end of the 5.00 revolutions. We could also calculate W from $W = \Delta K = \frac{1}{2}I\omega^2 = \frac{1}{2}(42.2\text{ kg}\cdot\text{m}^2)(53.9\text{ rad/s})^2 = 6.13\times10^4$ J.

10.33. (a) IDENTIFY and SET UP: Use Eq.(10.23) and solve for τ_z.

$P = \tau_z\omega_z$, where ω_z must be in rad/s

EXECUTE: $\omega_z = (4000\text{ rev/min})(2\pi\text{ rad/1 rev})(1\text{ min/60 s}) = 418.9$ rad/s

$\tau_z = \dfrac{P}{\omega_z} = \dfrac{1.50\times10^5\text{ W}}{418.9\text{ rad/s}} = 358\text{ N}\cdot\text{m}$

(b) IDENTIFY and SET UP: Apply $\sum \vec{F} = m\vec{a}$ to the drum. Find the tension T in the rope using τ_z from part (a). The system is sketched in Figure 10.33.

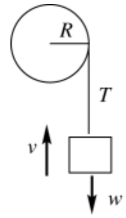


Figure 10.33

EXECUTE: v constant implies $a = 0$ and $T = w$

$\tau_z = TR$ implies

$T = \tau_z / R = 358\ \text{N}\cdot\text{m}/0.200\ \text{m} = 1790\ \text{N}$

Thus a weight $w = 1790\ \text{N}$ can be lifted.

(c) IDENTIFY and SET UP: Use $v = R\omega$.

EXECUTE: The drum has $\omega = 418.9\ \text{rad/s}$, so $v = (0.200\ \text{m})(418.9\ \text{rad/s}) = 83.8\ \text{m/s}$

EVALUATE: The rate at which T is doing work on the drum is $P = Tv = (1790\ \text{N})(83.8\ \text{m/s}) = 150\ \text{kW}$. This agrees with the work output of the motor.

10.34. **IDENTIFY:** $L = I\omega$ and $I = I_{\text{disk}} + I_{\text{woman}}$.

SET UP: $\omega = 0.50\ \text{rev/s} = 3.14\ \text{rad/s}$. $I_{\text{disk}} = \frac{1}{2} m_{\text{disk}} R^2$ and $I_{\text{woman}} = m_{\text{woman}} R^2$.

EXECUTE: $I = (55\ \text{kg} + 50.0\ \text{kg})(4.0\ \text{m})^2 = 1680\ \text{kg}\cdot\text{m}^2$. $L = (1680\ \text{kg}\cdot\text{m}^2)(3.14\ \text{rad/s}) = 5.28\times10^3\ \text{kg}\cdot\text{m}^2/\text{s}$

EVALUATE: The disk and the woman have similar values of I, even though the disk has twice the mass.

10.35. **(a) IDENTIFY:** Use $L = mvr\sin\phi$ (Eq.(10.25)):

SET UP: Consider Figure 10.35.

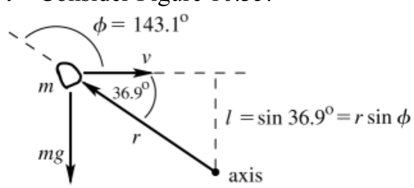


Figure 10.35

EXECUTE: $L = mvr\sin\phi =$

$(2.00\ \text{kg})(12.0\ \text{m/s})(8.00\ \text{m})\sin 143.1°$

$L = 115\ \text{kg}\cdot\text{m}^2/\text{s}$

To find the direction of $\vec{L}$ apply the right-hand rule by turning $\vec{r}$ into the direction of $\vec{v}$ by pushing on it with the fingers of your right hand. Your thumb points into the page, in the direction of $\vec{L}$.

(b) IDENTIFY and SET UP: By Eq.(10.26) the rate of change of the angular momentum of the rock equals the torque of the net force acting on it.

EXECUTE: $\tau = mg(8.00\ \text{m})\cos 36.9° = 125\ \text{kg}\cdot\text{m}^2/\text{s}^2$

To find the direction of $\vec{\tau}$ and hence of $d\vec{L}/dt$, apply the right-hand rule by turning $\vec{r}$ into the direction of the gravity force by pushing on it with the fingers of your right hand. Your thumb points out of the page, in the direction of $d\vec{L}/dt$.

EVALUATE: $\vec{L}$ and $d\vec{L}/dt$ are in opposite directions, so L is decreasing. The gravity force is accelerating the rock downward, toward the axis. Its horizontal velocity is constant but the distance l is decreasing and hence L is decreasing.

10.36. **IDENTIFY:** $L_z = I\omega_z$

SET UP: For a particle of mass m moving in a circular path at a distance r from the axis, $I = mr^2$ and $v = r\omega$. For a uniform sphere of mass M and radius R and an axis through its center, $I = \frac{2}{5}MR^2$. The earth has mass $m_{\text{E}} = 5.97\times10^{24}\ \text{kg}$, radius $R_{\text{E}} = 6.38\times10^{6}\ \text{m}$ and orbit radius $r = 1.50\times10^{11}\ \text{m}$. The earth completes one rotation on its axis in $24\ \text{h} = 86{,}400\ \text{s}$ and one orbit in $1\ \text{y} = 3.156\times10^{7}\ \text{s}$.

EXECUTE: **(a)** $L_z = I\omega_z = mr^2\omega_z = (5.97\times10^{24}\ \text{kg})(1.50\times10^{11}\ \text{m})^2\left(\dfrac{2\pi\ \text{rad}}{3.156\times10^{7}\ \text{s}}\right) = 2.67\times10^{40}\ \text{kg}\cdot\text{m}^2/\text{s}$.

The radius of the earth is much less than its orbit radius, so it is very reasonable to model it as a particle for this calculation.

(b) $L_z = I\omega_z = \left(\frac{2}{5}MR^2\right)\omega = \frac{2}{5}(5.97\times10^{24}\ \text{kg})(6.38\times10^{6}\ \text{m})^2\left(\dfrac{2\pi\ \text{rad}}{86{,}400\ \text{s}}\right) = 7.07\times10^{33}\ \text{kg}\cdot\text{m}^2/\text{s}$

EVALUATE: The angular momentum associated with each of these motions is very large.

10.37. **IDENTIFY** and **SET UP:** Use $L = I\omega$

EXECUTE: The second hand makes 1 revolution in 1 minute, so
$\omega = (1.00 \text{ rev/min})(2\pi \text{ rad/1 rev})(1 \text{ min/60 s}) = 0.1047 \text{ rad/s}$

For a slender rod, with the axis about one end,
$I = \frac{1}{3}ML^2 = \frac{1}{3}(6.00\times10^{-3} \text{ kg})(0.150 \text{ m})^2 = 4.50\times10^{-5} \text{ kg}\cdot\text{m}^2$

Then $L = I\omega = (4.50\times10^{-5} \text{ kg}\cdot\text{m}^2)(0.1047 \text{ rad/s}) = 4.71\times10^{-6} \text{ kg}\cdot\text{m}^2/\text{s}.$

EVALUATE: $\vec{L}$ is clockwise.

10.38. **IDENTIFY:** $\omega_z = d\theta/dt$. $L_z = I\omega_z$ and $\tau_z = dL_z dt$.

SET UP: For a hollow, thin-walled sphere rolling about an axis through its center, $I = \frac{2}{3}MR^2$. $R = 0.240 \text{ m}$.

EXECUTE: **(a)** $A = 1.50 \text{ rad/s}^2$ and $B = 1.10 \text{ rad/s}^4$, so that $\theta(t)$ will have units of radians.

(b) (i) $\omega_z = \dfrac{d\theta}{dt} = 2At + 4Bt^3$. At $t = 3.00 \text{ s}$, $\omega_z = 2(1.50 \text{ rad/s}^2)(3.00 \text{ s}) + 4(1.10 \text{ rad/s}^4)(3.00 \text{ s})^3 = 128 \text{ rad/s}$.

$L_z = (\frac{2}{3}MR^2)\omega_z = \frac{2}{3}(12.0 \text{ kg})(0.240 \text{ m})^2(128 \text{ rad/s}) = 59.0 \text{ kg}\cdot\text{m}^2/\text{s}$.

(ii) $\tau_z = \dfrac{dL_z}{dt} = I\dfrac{d\omega_z}{dt} = I(2A + 12Bt^2)$ and

$\tau_z = \frac{2}{3}(12.0 \text{ kg})(0.240 \text{ m})^2(2[1.50 \text{ rad/s}^2] + 12[1.10 \text{ rad/s}^4][3.00 \text{ s}]^2) = 56.1 \text{ N}\cdot\text{m}$.

EVALUATE: The angular speed of rotation is increasing. This increase is due to an acceleration α_z that is produced by the torque on the sphere. When I is constant, as it is here, $\tau_z = dL_z dt = Id\omega_z/dt = I\alpha_z$ and Equations (10.29) and (10.7) are identical.

10.39. **IDENTIFY:** Apply conservation of angular momentum.

SET UP: For a uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$.

EXECUTE: The moment of inertia is proportional to the square of the radius, and so the angular velocity will be proportional to the inverse of the square of the radius, and the final angular velocity is

$$\omega_2 = \omega_1\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{2\pi \text{ rad}}{(30 \text{ d})(86{,}400 \text{ s/d})}\right)\left(\frac{7.0\times10^5 \text{ km}}{16 \text{ km}}\right)^2 = 4.6\times10^3 \text{ rad/s}.$$

EVALUATE: $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega$. L is constant and ω increases by a large factor, so there is a large increase in the rotational kinetic energy of the star. This energy comes from potential energy associated with the gravity force within the star.

10.40. **IDENTIFY** and **SET UP:** $\vec{L}$ is conserved if there is no net external torque.
Use conservation of angular momentum to find ω at the new radius and use $K = \frac{1}{2}I\omega^2$ to find the change in kinetic energy, which is equal to the work done on the block.

EXECUTE: **(a)** Yes, angular momentum is conserved. The moment arm for the tension in the cord is zero so this force exerts no torque and there is no net torque on the block.

(b) $L_1 = L_2$ so $I_1\omega_1 = I_2\omega_2$. Block treated as a point mass, so $I = mr^2$, where r is the distance of the block from the hole.

$mr_1^2\omega_1 = mr_2^2\omega_2$

$\omega_2 = \left(\dfrac{r_1}{r_2}\right)^2 \omega_1 = \left(\dfrac{0.300 \text{ m}}{0.150 \text{ m}}\right)^2 (1.75 \text{ rad/s}) = 7.00 \text{ rad/s}$

(c) $K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}mr_1^2\omega_1^2 = \frac{1}{2}mv_1^2$

$v_1 = r_1\omega_1 = (0.300 \text{ m})(1.75 \text{ rad/s}) = 0.525 \text{ m/s}$

$K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(0.0250 \text{ kg})(0.525 \text{ m/s})^2 = 0.00345 \text{ J}$

$K_2 = \frac{1}{2}mv_2^2$

$v_2 = r_2\omega_2 = (0.150 \text{ m})(7.00 \text{ rad/s}) = 1.05 \text{ m/s}$

$K_2 = \frac{1}{2}mv_2^2 = \frac{1}{2}(0.0250 \text{ kg})(1.05 \text{ m/s})^2 = 0.01378 \text{ J}$

$\Delta K = K_2 - K_1 = 0.01378 \text{ J} - 0.00345 \text{ J} = 0.0103 \text{ J}$

(d) $W_{\text{tot}} = \Delta K$

But $W_{\text{tot}} = W$, the work done by the tension in the cord, so $W = 0.0103$ J

EVALUATE: Smaller *r* means smaller *I*. $L = I\omega$ is constant so ω increases and *K* increases. The work done by the tension is positive since it is directed inward and the block moves inward, toward the hole.

10.41. IDENTIFY: Apply conservation of angular momentum to the motion of the skater.

SET UP: For a thin-walled hollow cylinder $I = mR^2$. For a slender rod rotating about an axis through its center, $I = \frac{1}{12}ml^2$.

EXECUTE: $L_{\text{i}} = L_{\text{f}}$ so $I_{\text{i}}\omega_{\text{i}} = I_{\text{f}}\omega_{\text{f}}$.

$I_{\text{i}} = 0.40\ \text{kg}\cdot\text{m}^2 + \frac{1}{12}(8.0\ \text{kg})(1.8\ \text{m})^2 = 2.56\ \text{kg}\cdot\text{m}^2$. $I_{\text{f}} = 0.40\ \text{kg}\cdot\text{m}^2 + (8.0\ \text{kg})(0.25\ \text{m})^2 = 0.90\ \text{kg}\cdot\text{m}^2$.

$$\omega_{\text{f}} = \left(\frac{I_{\text{i}}}{I_{\text{f}}}\right)\omega_{\text{i}} = \left(\frac{2.56\ \text{kg}\cdot\text{m}^2}{0.90\ \text{kg}\cdot\text{m}^2}\right)(0.40\ \text{rev/s}) = 1.14\ \text{rev/s}.$$

EVALUATE: $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega$. ω increases and *L* is constant, so *K* increases. The increase in kinetic energy comes from the work done by the skater when he pulls in his hands.

10.42. IDENTIFY: Apply conservation of angular momentum to the diver.

SET UP: The number of revolutions she makes in a certain time is proportional to her angular velocity. The ratio of her untucked to tucked angular velocity is $(3.6\ \text{kg}\cdot\text{m}^2)/(18\ \text{kg}\cdot\text{m}^2)$.

EXECUTE: If she had tucked, she would have made $(2\ \text{rev})(3.6\ \text{kg}\cdot\text{m}^2)/(18\ \text{kg}\cdot\text{m}^2) = 0.40\ \text{rev}$ in the last 1.0 s, so she would have made $(0.40\ \text{rev})(1.5/1.0) = 0.60\ \text{rev}$ in the total 1.5 s.

EVALUATE: Untucked she rotates slower and completes fewer revolutions.

10.43. IDENTIFY and SET UP: There is no net external torque about the rotation axis so the angular momentum $L = I\omega$ is conserved.

EXECUTE: **(a)** $L_1 = L_2$ gives $I_1\omega_1 = I_2\omega_2$, so $\omega_2 = (I_1/I_2)\omega_1$

$I_1 = I_{\text{tt}} = \frac{1}{2}MR^2 = \frac{1}{2}(120\ \text{kg})(2.00\ \text{m})^2 = 240\ \text{kg}\cdot\text{m}^2$

$I_2 = I_{\text{tt}} + I_{\text{p}} = 240\ \text{kg}\cdot\text{m}^2 + mR^2 = 240\ \text{kg}\cdot\text{m}^2 + (70\ \text{kg})(2.00\ \text{m})^2 = 520\ \text{kg}\cdot\text{m}^2$

$\omega_2 = (I_1/I_2)\omega_1 = (240\ \text{kg}\cdot\text{m}^2/520\ \text{kg}\cdot\text{m}^2)(3.00\ \text{rad/s}) = 1.38\ \text{rad/s}$

(b) $K_1 = \frac{1}{2}I_1\omega_1^2 = \frac{1}{2}(240\ \text{kg}\cdot\text{m}^2)(3.00\ \text{rad/s})^2 = 1080\ \text{J}$

$K_2 = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}(520\ \text{kg}\cdot\text{m}^2)(1.38\ \text{rad/s})^2 = 495\ \text{J}$

EVALUATE: The kinetic energy decreases because of the negative work done on the turntable and the parachutist by the friction force between these two objects.

The angular speed decreases because *I* increases when the parachutist is added to the system.

10.44. IDENTIFY: Apply conservation of angular momentum to the collision.

SET UP: Let the width of the door be *l*. The initial angular momentum of the mud is $mv(l/2)$, since it strikes the door at its center. For the axis at the hinge, $I_{\text{door}} = \frac{1}{3}Ml^2$ and $I_{\text{mud}} = m(l/2)^2$.

EXECUTE: $$\omega = \frac{L}{I} = \frac{mv(l/2)}{(1/3)Ml^2 + m(l/2)^2}.$$

$$\omega = \frac{(0.500\ \text{kg})(12.0\ \text{m/s})(0.500\ \text{m})}{(1/3)(40.0\ \text{kg})(1.00\ \text{m})^2 + (0.500\ \text{kg})(0.500\ \text{m})^2} = 0.223\ \text{rad/s}.$$

Ignoring the mass of the mud in the denominator of the above expression gives $\omega = 0.225\ \text{rad/s}$, so the mass of the mud in the moment of inertia does affect the third significant figure.

EVALUATE: Angular momentum is conserved but there is a large decrease in the kinetic energy of the system.

10.45. (a) IDENTIFY and SET UP: Apply conservation of angular momentum $\vec{L}$, with the axis at the nail. Let object *A* be the bug and object *B* be the bar. Initially, all objects are at rest and $L_1 = 0$. Just after the bug jumps, it has angular momentum in one direction of rotation and the bar is rotating with angular velocity ω_B in the opposite direction.

EXECUTE: $L_2 = m_A v_A r - I_B\omega_B$ where $r = 1.00$ m and $I_B = \frac{1}{3}m_B r^2$

$L_1 = L_2$ gives $m_A v_A r = \frac{1}{3}m_B r^2\omega_B$

$$\omega_B = \frac{3m_A v_A}{m_B r} = 0.120\ \text{rad/s}$$

(b) $K_1 = 0$; $K_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}I_B\omega_B^2 =$

$\frac{1}{2}(0.0100\text{ kg})(0.200\text{ m/s})^2 + \frac{1}{2}\left(\frac{1}{3}[0.0500\text{ kg}][1.00\text{ m}]^2\right)(0.120\text{ rad/s})^2 = 3.2\times10^{-4}\text{ J}.$

The increase in kinetic energy comes from work done by the bug when it pushes against the bar in order to jump.

EVALUATE: There is no external torque applied to the system and the total angular momentum of the system is constant. There are internal forces, forces the bug and bar exert on each other. The forces exert torques and change the angular momentum of the bug and the bar, but these changes are equal in magnitude and opposite in direction. These internal forces do positive work on the two objects and the kinetic energy of each object and of the system increases.

10.46. **IDENTIFY:** Apply conservation of angular momentum to the system of earth plus asteroid.

SET UP: Take the axis to be the earth's rotation axis. The asteroid may be treated as a point mass and it has zero angular momentum before the collision, since it is headed toward the center of the earth. For the earth, $L_z = I\omega_z$ and $I = \frac{2}{5}MR^2$,where M is the mass of the earth and R is its radius. The length of a day is $T = \dfrac{2\pi\text{ rad}}{\omega}$, where ω is the earth's angular rotation rate.

EXECUTE: Conservation of angular momentum applied to the collision between the earth and asteroid gives $\frac{2}{5}MR^2\omega_1 = (mR^2 + \frac{2}{5}MR^2)\omega_2$ and $m = \frac{2}{5}M\left(\dfrac{\omega_1 - \omega_2}{\omega_2}\right)$. $T_2 = 1.250T_1$ gives $\dfrac{1}{\omega_2} = \dfrac{1.250}{\omega_1}$ and $\omega_1 = 1.250\omega_2$.

$\dfrac{\omega_1 - \omega_2}{\omega_2} = 0.250$. $m = \frac{2}{5}(0.250)M = 0.100M$.

EVALUATE: If the asteroid hit the surface of the earth tangentially it could have some angular momentum with respect to the earth's rotation axis, and could either speed up or slow down the earth's rotation rate.

10.47. **IDENTIFY:** Apply conservation of angular momentum to the collision.

SET UP: The system before and after the collision is sketched in Figure 10.47. Let counterclockwise rotation be positive. The bar has $I = \frac{1}{3}m_2L^2$.

EXECUTE: **(a)** Conservation of angular momentum: $m_1v_0d = -m_1vd + \frac{1}{3}m_2L^2\omega$.

$$(3.00\text{ kg})(10.0\text{ m/s})(1.50\text{ m}) = -(3.00\text{ kg})(6.00\text{ m/s})(1.50\text{ m}) + \frac{1}{3}\left(\frac{90.0\text{ N}}{9.80\text{ m/s}^2}\right)(2.00\text{ m})^2\omega$$

$\omega = 5.88\text{ rad/s}$.

(b) There are no unbalanced torques about the pivot, so angular momentum is conserved. But the pivot exerts an unbalanced horizontal external force on the system, so the linear momentum is not conserved.

EVALUATE: Kinetic energy is not conserved in the collision.

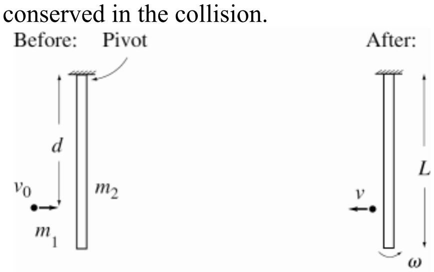


Figure 10.47

10.48. **IDENTIFY:** $d\vec{L} = \vec{\tau}dt$, so $d\vec{L}$ is in the direction of $\vec{\tau}$.

SET UP: The direction of $\vec{\omega}$ is given by the right-hand rule, as described in Figure 10.26 in the textbook.

EXECUTE: The sketches are given in Figures 10.48a–d.

EVALUATE: In figures (a) and (c) the precession is counterclockwise and in figures (b) and (d) it is clockwise. When the direction of either $\vec{\omega}$ or $\vec{\tau}$ reverses, the direction of precession reverses.

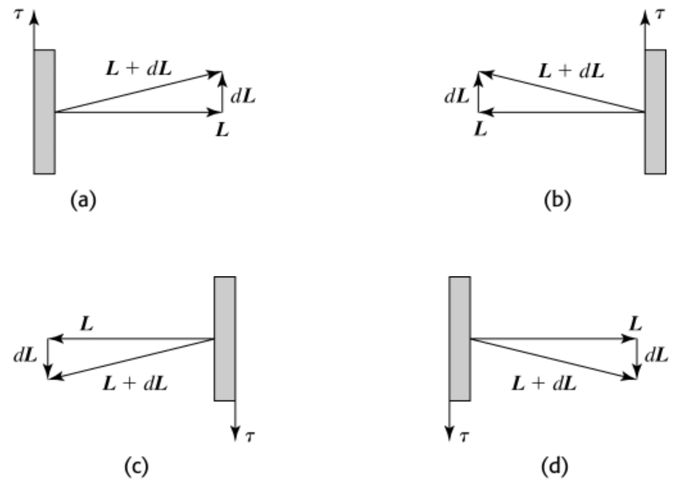


Figure 10.48

10.49. **IDENTIFY:** The precession angular velocity is $\Omega = \dfrac{wr}{I\omega}$, where ω is in rad/s. Also apply $\sum \vec{F} = m\vec{a}$ to the gyroscope.

SET UP: The total mass of the gyroscope is $m_r + m_f = 0.140\text{ kg} + 0.0250\text{ kg} = 0.165\text{ kg}$.

$\Omega = \dfrac{2\pi\text{ rad}}{T} = \dfrac{2\pi\text{ rad}}{2.20\text{ s}} = 2.856\text{ rad/s}$.

EXECUTE: **(a)** $F_p = w_{tot} = (0.165\text{ kg})(9.80\text{ m/s}^2) = 1.62\text{ N}$

(b) $\omega = \dfrac{wr}{I\Omega} = \dfrac{(0.165\text{ kg})(9.80\text{ m/s}^2)(0.0400\text{ m})}{(1.20\times10^{-4}\text{ kg}\cdot\text{m}^2)(2.856\text{ rad/s})} = 189\text{ rad/s} = 1.80\times10^3\text{ rev/min}$

(c) If the figure in the problem is viewed from above, $\vec{\tau}$ is in the direction of the precession and $\vec{L}$ is along the axis of the rotor, away from the pivot.

EVALUATE: There is no vertical component of acceleration associated with the motion, so the force from the pivot equals the weight of the gyroscope. The larger ω is, the slower the rate of precession.

10.50. **IDENTIFY:** The precession angular speed is related to the acceleration due to gravity by Eq.(10.33), with $w = mg$.

SET UP: $\Omega_E = 0.50\text{ rad/s}$, $g_E = g$ and $g_M = 0.165g$. For the gyroscope, *m*, *r*, *I*, and ω are the same on the moon as on the earth.

EXECUTE: $\Omega = \dfrac{mgr}{I\omega}$. $\dfrac{\Omega}{g} = \dfrac{mr}{I\omega} = \text{constant}$, so $\dfrac{\Omega_E}{g_E} = \dfrac{\Omega_M}{g_M}$.

$\Omega_M = \Omega_E\left(\dfrac{g_M}{g_E}\right) = 0.165\Omega_E = (0.165)(0.50\text{ rad/s}) = 0.0825\text{ rad/s}$.

EVALUATE: In the limit that $g \to 0$ the precession rate $\to 0$.

10.51. **IDENTIFY** and **SET UP:** Apply Eq.(10.33).

EXECUTE: **(a)** halved

(b) doubled (assuming that the added weight is distributed in such a way that *r* and *I* are not changed)

(c) halved (assuming that w and r are not changed)

(d) doubled

(e) unchanged.

EVALUATE: Ω is directly proportional to *w* and *r* and is inversely proportional to *I* and ω.

10.52. **IDENTIFY:** Apply Eq.(10.33), where $\tau = wr$.

SET UP: $1\text{ day} = 86{,}400\text{ s}$. $1\text{ yr} = 3.156\times10^7\text{ s}$. The earth has mass $M = 5.97\times10^{24}\text{ kg}$ and radius $R = 6.38\times10^6\text{ m}$. For a uniform sphere and an axis through its center, $I = \frac{2}{5}MR^2$.

EXECUTE: **(a)** $\tau = I\omega\Omega = (2/5)MR^2\omega\Omega$. Using $\omega = \dfrac{2\pi\text{ rad}}{86{,}400\text{ s}}$ and $\Omega = \dfrac{2\pi\text{ rad}}{(26{,}000\text{ y})(3.156\times10^7\text{ s/y})}$, and the mass and radius of the earth from Appendix F, $\tau = 5.4\text{ N}\cdot\text{m}$.

EVALUATE: If the torque is applied by the sun, $r = 1.5\times10^{11}\text{ m}$ and $F_\perp = 3.6\times10^{11}\text{ N}$.

10.53. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ and constant acceleration equations to the motion of the grindstone.

SET UP: Let the direction of rotation of the grindstone be positive. The friction force is $f = \mu_k n$ and produces torque fR. $\omega = \left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 4\pi$ rad. $I = \frac{1}{2}MR^2 = 1.69 \text{ kg}\cdot\text{m}^2$.

EXECUTE: **(a)** The net torque must be

$$\tau = I\alpha = I\frac{\omega_z - \omega_{0z}}{t} = (1.69 \text{ kg}\cdot\text{m}^2)\frac{4\pi \text{ rad/s}}{9.00 \text{ s}} = 2.36 \text{ N}\cdot\text{m}.$$

This torque must be the sum of the applied force FR and the opposing frictional torques τ_f at the axle and $fR = \mu_k nR$ due to the knife. $F = \frac{1}{R}(\tau + \tau_f + \mu_k nR)$.

$$F = \frac{1}{0.500 \text{ m}}\big((2.36 \text{ N}\cdot\text{m}) + (6.50 \text{ N}\cdot\text{m}) + (0.60)(160 \text{ N})(0.260 \text{ m})\big) = 67.6 \text{ N}.$$

(b) To maintain a constant angular velocity, the net torque τ is zero, and the force F' is

$$F' = \frac{1}{0.500 \text{ m}}(6.50 \text{ N}\cdot\text{m} + 24.96 \text{ N}\cdot\text{m}) = 62.9 \text{ N}.$$

(c) The time t needed to come to a stop is found by taking the magnitudes in Eq.(10.27), with $\tau = \tau_f$ constant;

$$t = \frac{L}{\tau_f} = \frac{\omega I}{\tau_f} = \frac{(4\pi \text{ rad/s})\left(1.69 \text{ kg}\cdot\text{m}^2\right)}{6.50 \text{ N}\cdot\text{m}} = 3.27 \text{ s}.$$

EVALUATE: The time for a given change in ω is proportional to α, which is in turn proportional to the net torque, so the time in part (c) can also be found as $t = (9.00 \text{ s})\frac{2.36 \text{ N}\cdot\text{m}}{6.50 \text{ N}\cdot\text{m}}$.

10.54. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ and use the constant acceleration equations to relate α to the motion.

SET UP: Let the direction the wheel is rotating be positive. 100 rev/min = 10.47 rad/s

EXECUTE: **(a)** $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = \frac{\omega_z - \omega_{0z}}{t} = \frac{10.47 \text{ rad/s} - 0}{2.00 \text{ s}} = 5.23 \text{ rad/s}^2$.

$$I = \frac{\sum \tau_z}{\alpha_z} = \frac{5.00 \text{ N}\cdot\text{m}}{5.23 \text{ rad/s}^2} = 0.956 \text{ kg}\cdot\text{m}^2$$

(b) $\omega_{0z} = 10.47$ rad/s, $\omega_z = 0$, $t = 125$ s. $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = \frac{\omega_z - \omega_{0z}}{t} = \frac{0 - 10.47 \text{ rad/s}}{125 \text{ s}} = -0.0838 \text{ rad/s}^2$

$$\sum \tau_z = I\alpha_z = (0.956 \text{ kg}\cdot\text{m}^2)(-0.0838 \text{ rad/s}^2) = -0.0801 \text{ N}\cdot\text{m}$$

(c) $\theta = \left(\frac{\omega_{0z} + \omega_z}{2}\right)t = \left(\frac{10.47 \text{ rad/s} + 0}{2}\right)(125 \text{ s}) = 654 \text{ rad} = 104 \text{ rev}$

EVALUATE: The applied net torque ($5.00 \text{ N}\cdot\text{m}$) is much larger than the magnitude of the friction torque ($0.0801 \text{ N}\cdot\text{m}$), so the time of 2.00 s that it takes the wheel to reach an angular speed of 100 rev/min is much less than the 125 s it takes the wheel to be brought to rest by friction.

10.55. **IDENTIFY** and **SET UP:** Apply $v = r\omega$. v is the tangential speed of a point on the rim of the wheel and equals the linear speed of the car.

EXECUTE: **(a)** $v = 60 \text{ mph} = 26.82 \text{ m/s}$

$r = 12 \text{ in.} = 0.3048 \text{ m}$

$$\omega = \frac{v}{r} = 88.0 \text{ rad/s} = 14.0 \text{ rev/s} = 840 \text{ rpm}$$

(b) Same ω as in part (a) since speedometer reads same.

$r = 15 \text{ in.} = 0.381 \text{ m}$

$v = r\omega = (0.381 \text{ m})(88.0 \text{ rad/s}) = 33.5 \text{ m/s} = 75 \text{ mph}$

(c) $v = 50 \text{ mph} = 22.35 \text{ m/s}$

$r = 10 \text{ in.} = 0.254 \text{ m}$

$\omega = \frac{v}{r} = 88.0$ rad/s. This is the same as for 60 mph with correct tires, so speedometer read 60 mph.

EVALUATE: For a given ω, v increases when r increases.

10.56. IDENTIFY: The kinetic energy of the disk is $K = \frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I\omega^2$. As it falls its gravitational potential energy decreases and its kinetic energy increases. The only work done on the disk is the work done by gravity, so $K_1 + U_1 = K_2 + U_2$.

SET UP: $I_{cm} = \frac{1}{2}M(R_2^2 + R_1^2)$, where $R_1 = 0.300$ m and $R_2 = 0.500$ m. $v_{cm} = R_2\omega$. Take $y_1 = 0$, so $y_2 = -1.20$ m.

EXECUTE: $K_1 + U_1 = K_2 + U_2$. $K_1 = 0$, $U_1 = 0$. $K_2 = -U_2$. $\frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2 = -Mgy_2$.

$\frac{1}{2}I_{cm}\omega^2 = \frac{1}{4}M\left(1 + [R_1/R_2]^2\right)v_{cm}^2 = 0.340Mv_{cm}^2$. Then $0.840Mv_{cm}^2 = -Mgy_2$ and

$$v_{cm} = \sqrt{\frac{-gy_2}{0.840}} = \sqrt{\frac{-(9.80\text{ m/s}^2)(-1.20\text{ m})}{0.840}} = 3.74\text{ m/s}$$

EVALUATE: A point mass in free-fall acquires a speed of 4.85 m/s after falling 1.20 m. The disk has a value of v_{cm} that is less than this, because some of the original gravitational potential energy has been converted to rotational kinetic energy.

10.57. IDENTIFY: Use $\sum \tau_z = I\alpha_z$ to find the angular acceleration just after the ball falls off and use conservation of energy to find the angular velocity of the bar as it swings through the vertical position.

SET UP: The axis of rotation is at the axle. For this axis the bar has $I = \frac{1}{12}m_{bar}L^2$, where $m_{bar} = 3.80$ kg and $L = 0.800$ m. Energy conservation gives $K_1 + U_1 = K_2 + U_2$. The gravitational potential energy of the bar doesn't change. Let $y_1 = 0$, so $y_2 = -L/2$.

EXECUTE: **(a)** $\tau_z = m_{ball}g(L/2)$ and $I = I_{ball} + I_{bar} = \frac{1}{12}m_{bar}L^2 + m_{ball}(L/2)^2$. $\sum \tau_z = I\alpha_z$ gives

$$\alpha_z = \frac{m_{ball}g(L/2)}{\frac{1}{12}m_{bar}L^2 + m_{ball}(L/2)^2} = \frac{2g}{L}\left(\frac{m_{ball}}{m_{ball} + m_{bar}/3}\right) \text{ and } \alpha_z = \frac{2(9.80\text{ m/s}^2)}{0.800\text{ m}}\left(\frac{2.50\text{ kg}}{2.50\text{ kg} + [3.80\text{ kg}]/3}\right) = 16.3\text{ rad/s}^2.$$

(b) As the bar rotates, the moment arm for the weight of the ball decreases and the angular acceleration of the bar decreases.

(c) $K_1 + U_1 = K_2 + U_2$. $0 = K_2 + U_2$. $\frac{1}{2}(I_{bar} + I_{ball})\omega^2 = -m_{ball}g(-L/2)$.

$$\omega = \sqrt{\frac{m_{ball}gL}{m_{ball}L^2/4 + m_{bar}L^2/12}} = \sqrt{\frac{g}{L}\left(\frac{4m_{ball}}{m_{ball} + m_{bar}/3}\right)} = \sqrt{\frac{9.80\text{ m/s}^2}{0.800\text{ m}}\left(\frac{4[2.50\text{ kg}]}{2.50\text{ kg} + [3.80\text{ kg}]/3}\right)}$$

$\omega = 5.70$ rad/s.

EVALUATE: As the bar swings through the vertical, the linear speed of the ball that is still attached to the bar is $v = (0.400\text{ m})(5.70\text{ rad/s}) = 2.28$ m/s. A point mass in free-fall acquires a speed of 2.80 m/s after falling 0.400 m; the ball on the bar acquires a speed less than this.

10.58. IDENTIFY: Use $\sum \tau_z = I\alpha_z$ to find α_z, and then use the constant α_z kinematic equations to solve for *t*.

SET UP: The door is sketched in Figure 10.58.

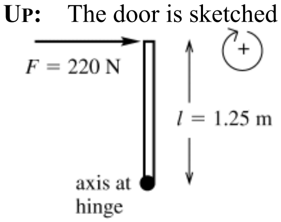


Figure 10.58

EXECUTE: $\sum \tau_z = Fl = (220\text{ N})(1.25\text{ m}) = 275\text{ N}\cdot\text{m}$

From Table 9.2(d), $I = \frac{1}{3}Ml^2$

$I = \frac{1}{3}(750\text{ N}/9.80\text{ m/s}^2)(1.25\text{ m})^2 = 39.9\text{ kg}\cdot\text{m}^2$

$$\sum \tau_z = I\alpha_z \text{ so } \alpha_z = \frac{\sum \tau_z}{I} = \frac{275\text{ N}\cdot\text{m}}{39.9\text{ kg}\cdot\text{m}^2} = 6.89\text{ rad/s}^2$$

SET UP: $\alpha_z = 6.89\text{ rad/s}^2$; $\theta - \theta_0 = 90°(\pi\text{ rad}/180°) = \pi/2$ rad; $\omega_{0z} = 0$ (door initially at rest); $t = ?$

$\theta - \theta_0 = \omega_{0z}t + \frac{1}{2}\alpha_z t^2$

EXECUTE: $t = \sqrt{\dfrac{2(\theta - \theta_0)}{\alpha_z}} - \sqrt{\dfrac{2(\pi/2\text{ rad})}{6.89\text{ rad/s}^2}} = 0.675\text{ s}$

EVALUATE: The forces and the motion are connected through the angular acceleration.

10.59. IDENTIFY: $\tau = rF\sin\phi$

SET UP: Let *x* be the distance from the left end of the rod where the string is attached. For the value of *x* where $f(x)$ is a maximum, $df/dx = 0$.

EXECUTE: **(a)** From geometric consideration, the lever arm and the sine of the angle between $\vec{F}$ and $\vec{r}$ are both maximum if the string is attached at the end of the rod.

(b) In terms of the distance x where the string is attached, the magnitude of the torque is $Fxh/\sqrt{x^2+h^2}$. This function attains its maximum at the boundary, where $x = h$, so the string should be attached at the right end of the rod.

(c) As a function of x, l and h, the torque has magnitude $\tau = F\dfrac{xh}{\sqrt{(x-l/2)^2+h^2}}$. Differentiating τ with respect to x and setting equal to zero gives $x_{\text{max}} = (l/2)(1+(2h/l)^2)$. This will be the point at which to attach the string unless $2h > l$, in which case the string should be attached at the furthest point to the right, $x = l$.

EVALUATE: In part (a) the maximum torque is independent of h. In part (b) the maximum torque is independent of l. In part (c) the maximum torque depends on both h and l.

10.60. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$, where τ_z is due to the gravity force on the object.

SET UP: $I = I_{\text{rod}} + I_{\text{clay}}$. $I_{\text{rod}} = \frac{1}{3}ML^2$. In part (b), $I_{\text{clay}} = ML^2$. In part (c), $I_{\text{clay}} = 0$.

EXECUTE: **(a)** A distance $L/4$ from the end with the clay.

(b) In this case $I = (4/3)ML^2$ and the gravitational torque is $(3L/4)(2Mg)\sin\theta = (3MgL/2)\sin\theta$, so $\alpha = (9g/8L)\sin\theta$.

(c) In this case $I = (1/3)ML^2$ and the gravitational torque is $(L/4)(2Mg)\sin\theta = (MgL/2)\sin\theta$, so $\alpha = (3g/2L)\sin\theta$. This is greater than in part (b).

(d) The greater the angular acceleration of the upper end of the cue, the faster you would have to react to overcome deviations from the vertical.

EVALUATE: In part (b), I is 4 times larger than in part (c) and τ is 3 times larger. $\alpha = \tau / I$, so the net effect is that α is smaller in (b) than in (c).

10.61. **IDENTIFY:** Calculate W using the procedure specified in the problem. In part (c) apply the work-energy theorem. In part (d), $a_{\text{tan}} = R\alpha$ and $\sum \tau_z = I\alpha_z$. $a_{\text{rad}} = R\omega^2$.

SET UP: Let θ be the angle the disk has turned through. The moment arm for F is $R\cos\theta$.

EXECUTE: **(a)** The torque is $\tau = FR\cos\theta$. $W = \int_0^{\pi/2} FR\cos\theta\, d\theta = FR$.

(b) In Eq.(6.14), dl is the horizontal distance the point moves, and so $W = F\int dl = FR$, the same as part (a).

(c) From $K_2 = W = (MR^2/4)\omega^2$, $\omega = \sqrt{4F/MR}$.

(d) The torque, and hence the angular acceleration, is greatest when $\theta = 0$, at which point $\alpha = (\tau/I) = 2F/MR$, and so the maximum tangential acceleration is $2F/M$.

(e) Using the value for ω found in part (c), $a_{\text{rad}} = \omega^2 R = 4F/M$.

EVALUATE: $a_{\text{tan}} = \omega^2 R$ is maximum initially, when the moment arm for F is a maximum, and it is zero after the disk has rotated one-quarter of a revolution. a_{rad} is zero initially and is a maximum at the end of the motion, after the disk has rotated one-quarter of a revolution.

10.62. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the crate and $\sum \tau_z = I\alpha_z$ to the cylinder. The motions are connected by $a(\text{crate}) = R\alpha(\text{cylinder})$.

SET UP: The force diagram for the crate is given in Figure 10.62a.

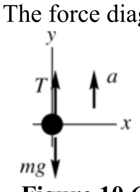


Figure 10.62a

EXECUTE: $\sum F_y = ma_y$

$T - mg = ma$

$T = m(g+a) = 50\text{ kg}(9.80\text{ m/s}^2 + 0.80\text{ m/s}^2) = 530\text{ N}$

SET UP: The force diagram for the cylinder is given in Figure 10.62b.

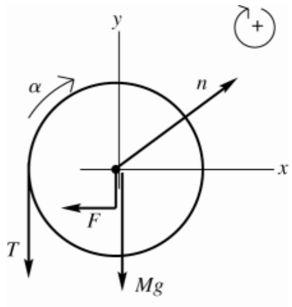


Figure 10.62b

EXECUTE: $\sum \tau_z = I\alpha_z$

$Fl - TR = I\alpha_z$, where $l = 0.12$ m and $R = 0.25$ m

$a = R\alpha$ so $\alpha_z = a/R$

$Fl = TR + Ia/R$

$$F = T\left(\frac{R}{l}\right) + \frac{Ia}{Rl} = 530\text{ N}\left(\frac{0.25\text{ m}}{0.12\text{ m}}\right) + \frac{(2.9\text{ kg}\cdot\text{m}^2)(0.80\text{ m/s}^2)}{(0.25\text{ m})(0.12\text{ m})} = 1200\text{ N}$$

EVALUATE: The tension in the rope is greater than the weight of the crate since the crate accelerates upward. If F were applied to the rim of the cylinder ($l = 0.25$ m), it would have the value $F = 567$ N. This is greater than T because it must accelerate the cylinder as well as the crate. And F is larger than this because it is applied closer to the axis than R so has a smaller moment arm and must be larger to give the same torque.

10.63. **IDENTIFY:** Apply $\sum \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$ and $\sum \tau_z = I_{\text{cm}}\alpha_z$ to the roll.

SET UP: At the point of contact, the wall exerts a friction force f directed downward and a normal force n directed to the right. This is a situation where the net force on the roll is zero, but the net torque is *not* zero.

EXECUTE: **(a)** Balancing vertical forces, $F_{\text{rod}}\cos\theta = f + w + F$, and balancing horizontal forces $F_{\text{rod}}\sin\theta = n$. With $f = \mu_k n$, these equations become $F_{\text{rod}}\cos\theta = \mu_k n + F + w$, $F_{\text{rod}}\sin\theta = n$. Eliminating n and solving for F_{rod} gives

$$F_{\text{rod}} = \frac{w + F}{\cos\theta - \mu_k\sin\theta} = \frac{(16.0\text{ kg})\,(9.80\text{ m/s}^2) + (40.0\text{ N})}{\cos 30° - (0.25)\sin 30°} = 266\text{ N}.$$

(b) With respect to the center of the roll, the rod and the normal force exert zero torque. The magnitude of the net torque is $(F - f)R$, and $f = \mu_k n$ may be found by insertion of the value found for F_{rod} into either of the above relations; *i.e.*, $f = \mu_k F_{\text{rod}}\sin\theta = 33.2$ N. Then,

$$\alpha = \frac{\tau}{I} = \frac{(40.0\text{ N} - 31.54\text{ N})(18.0\times10^{-2}\text{ m})}{(0.260\text{ kg}\cdot\text{m}^2)} = 4.71\text{ rad/s}^2.$$

EVALUATE: If the applied force F is increased, F_{rod} increases and this causes n and f to increase. The angle ϕ changes as the amount of paper unrolls and this affects α for a given F.

10.64. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the flywheel and $\sum \vec{F} = m\vec{a}$ to the block. The target variables are the tension in the string and the acceleration of the block.

(a) SET UP: Apply $\sum \tau_z = I\alpha_z$ to the rotation of the flywheel about the axis. The free-body diagram for the flywheel is given in Figure 10.64a.

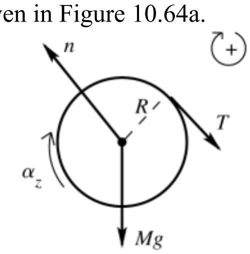


Figure 10.64a

EXECUTE: The forces n and Mg act at the axis so have zero torque.

$\sum \tau_z = TR$

$TR = I\alpha_z$

Set Up: Apply $\sum \vec{F} = m\vec{a}$ to the translational motion of the block. The free-body diagram for the block is given in Figure 10.64b.

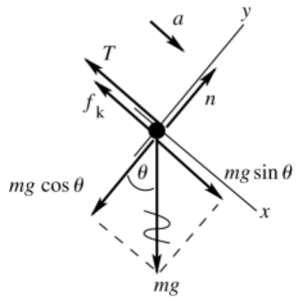


Figure 10.64b

Execute: $\sum F_y = ma_y$

$n - mg\cos 36.9° = 0$

$n = mg\cos 36.9°$

$f_k = \mu_k n = \mu_k mg\cos 36.9°$

$\sum F_x = ma_x$

$mg\sin 36.9° - T - \mu_k mg\cos 36.9° = ma$

$mg(\sin 36.9° - \mu_k\cos 36.9°) - T = ma$

But we also know that $a_{block} = R\alpha_{wheel}$, so $\alpha = a/R$. Using this in the $\sum \tau_z = I\alpha_z$ equation gives $TR = Ia/R$ and $T = (I/R^2)a$. Use this to replace T in the $\sum F_x = ma_x$ equation:

$mg(\sin 36.9° - \mu_k\cos 36.9°) - (I/R^2)a = ma$

$$a = \frac{mg(\sin 36.9° - \mu_k\cos 36.9°)}{m + I/R^2}$$

$$a = \frac{(5.00\text{ kg})(9.80\text{ m/s}^2)(\sin 36.9° - (0.25)\cos 36.9°)}{5.00\text{ kg} + 0.500\text{ kg}\cdot\text{m}^2/(0.200\text{ m})^2} = 1.12\text{ m/s}^2$$

(b) $T = \dfrac{0.500\text{ kg}\cdot\text{m}^2}{(0.200\text{ m})^2}(1.12\text{ m/s}^2) = 14.0\text{ N}$

Evaluate: If the string is cut the block will slide down the incline with $a = g\sin 36.9° - \mu_k g\cos 36.9° = 3.92\text{ m/s}^2$. The actual acceleration is less than this because $mg\sin 36.9°$ must also accelerate the flywheel. $mg\sin 36.9° - f_k = 19.6$ N. T is less than this; there must be more force on the block directed down the incline than up then incline since the block accelerates down the incline.

10.65. Identify: Apply $\sum \vec{F} = m\vec{a}$ to the block and $\sum \tau_z = I\alpha_z$ to the combined disks.

Set Up: For a disk, $I_{disk} = \frac{1}{2}MR^2$, so I for the disk combination is $I = 2.25\times 10^{-3}\text{ kg}\cdot\text{m}^2$.

Execute: For a tension T in the string, $mg - T = ma$ and $TR = I\alpha = I\dfrac{a}{R}$. Eliminating T and solving for a gives

$a = g\dfrac{m}{m + I/R^2} = \dfrac{g}{1 + I/mR^2}$, where m is the mass of the hanging block and R is the radius of the disk to which the string is attached.

(a) With $m = 1.50$ kg and $R = 2.50\times 10^{-2}$ m, $a = 2.88\text{ m/s}^2$.

(b) With $m = 1.50$ kg and $R = 5.00\times 10^{-2}$ m, $a = 6.13\text{ m/s}^2$.

The acceleration is larger in case (b); with the string attached to the larger disk, the tension in the string is capable of applying a larger torque.

Evaluate: $\omega = v/R$, where v is the speed of the block and ω is the angular speed of the disks. When R is larger, in part (b), a smaller fraction of the kinetic energy resides with the disks. The block gains more speed as it falls a certain distance and therefore has a larger acceleration.

10.66. Identify: Apply both $\sum \vec{F} = m\vec{a}$ and $\sum \tau_z = I\alpha_z$ to the motion of the roller. Rolling without slipping means $a_{cm} = R\alpha$. Target variables are a_{cm} and f.

SET UP: The free-body diagram for the roller is given in Figure 10.66.

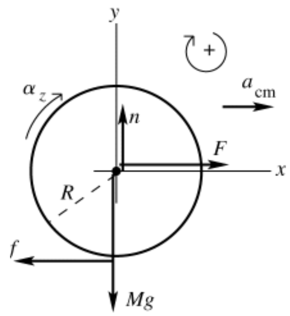


Figure 10.66

EXECUTE: Apply $\sum \vec{F} = m\vec{a}$ to the translational motion of the center of mass:

$\sum F_x = ma_x$

$F - f = Ma_{cm}$

Apply $\sum \tau_z = I\alpha_z$ to the rotation about the center of mass:

$\sum \tau_z = fR$

thin-walled hollow cylinder: $I = MR^2$

Then $\sum \tau_z = I\alpha_z$ implies $fR = MR^2\alpha.$

But $\alpha_{cm} = R\alpha,$ so $f = Ma_{cm}.$

Using this in the $\sum F_x = ma_x$ equation gives $F - Ma_{cm} = Ma_{cm}$

$a_{cm} = F/2M,$ and then $f = Ma_{cm} = M(F/2M) = F/2.$

EVALUATE: If the surface were frictionless the object would slide without rolling and the acceleration would be $a_{cm} = F/M.$ The acceleration is less when the object rolls.

10.67. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to each object and apply $\sum \tau_z = I\alpha_z$ to the pulley.

SET UP: Call the 75.0 N weight *A* and the 125 N weight *B*. Let T_A and T_B be the tensions in the cord to the left and to the right of the pulley. For the pulley, $I = \frac{1}{2}MR^2$, where $Mg = 50.0$ N and $R = 0.300$ m. The 125 N weight accelerates downward with acceleration *a*, the 75.0 N weight accelerates upward with acceleration *a* and the pulley rotates clockwise with angular acceleration α, where $a = R\alpha$.

EXECUTE: $\sum \vec{F} = m\vec{a}$ applied to the 75.0 N weight gives $T_A - w_A = m_A a$. $\sum \vec{F} = m\vec{a}$ applied to the 125.0 N weight gives $w_B - T_B = m_B a$. $\sum \tau_z = I\alpha_z$ applied to the pulley gives $(T_B - T_A)R = (\frac{1}{2}MR^2)\alpha_z$ and $T_B - T_A = \frac{1}{2}M$.

Combining these three equations gives $w_B - w_A = (m_A + m_B + M/2)a$ and

$a = \left(\dfrac{w_B - w_A}{w_A + w_B + w_{\text{pulley}}/2}\right) g = \left(\dfrac{125\text{ N} - 75.0\text{ N}}{75.0\text{ N} + 125\text{ N} + 25.0\text{ N}}\right) g = 0.222g$. $T_A = w_A(1 + a/g) = 1.222 w_A = 91.65$ N.

$T_B = w_B(1 - a/g) = 0.778 w_B = 97.25$ N. $\sum \vec{F} = m\vec{a}$ applied to the pulley gives that the force *F* applied by the hook to the pulley is $F = T_A + T_B + w_{\text{pulley}} = 239$ N. The force the ceiling applies to the hook is 239 N.

EVALUATE: The force the hook exerts on the pulley is less than the total weight of the system, since the net effect of the motion of the system is a downward acceleration of mass.

10.68. **IDENTIFY:** This problem can be done either with conservation of energy or with $\sum \vec{F}_{\text{ext}} = m\vec{a}$. We will do it both ways.

(a) SET UP: (1) *Conservation of energy:* $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$.

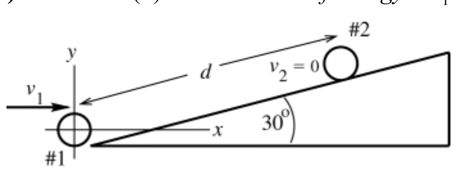


Figure 10.68a

Take position 1 to be the location of the disk at the base of the ramp and 2 to be where the disk momentarily stops before rolling back down, as shown in Figure 10.68a.

Take the origin of coordinates at the center of the disk at position 1 and take $+y$ to be upward. Then $y_1 = 0$ and $y_2 = d\sin 30°,$ where *d* is the distance that the disk rolls up the ramp. "Rolls without slipping" and neglect rolling friction says $W_f = 0;$ only gravity does work on the disk, so $W_{\text{other}} = 0$

EXECUTE: $U_1 = Mgy_1 = 0$

$K_1 = \frac{1}{2}Mv_1^2 + \frac{1}{2}I_{cm}\omega_1^2$ (Eq.10.11). But $\omega_1 = v_1/R$ and $I_{cm} = \frac{1}{2}MR^2$, so $\frac{1}{2}I_{cm}\omega_1^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)(v_1/R)^2 = \frac{1}{4}Mv_1^2$. Thus

$K_1 = \frac{1}{2}Mv_1^2 + \frac{1}{4}Mv_1^2 = \frac{3}{4}Mv_1^2$.

$U_2 = Mgy_2 = Mgd\sin 30°$

$K_2 = 0$ (disk is at rest at point 2).

Thus $\frac{3}{4}Mv_1^2 = Mgd\sin 30°$

$$d = \frac{3v_1^2}{4g\sin 30°} = \frac{3(2.50\text{ m/s})^2}{4(9.80\text{ m/s}^2)\sin 30°} = 0.957\text{ m}$$

SET UP: (2) *force and acceleration* The free-body diagram is given in Figure 10.68b.

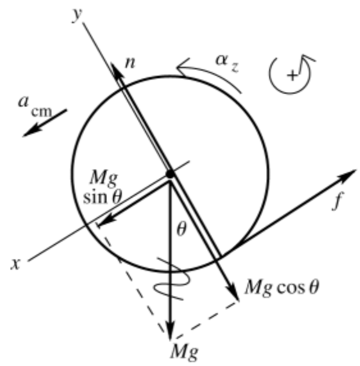


Figure 10.68b

EXECUTE: Apply $\sum F_x = ma_x$ to the translational motion of the center of mass:

$Mg\sin\theta - f = Ma_{cm}$

Apply $\sum \tau_z = I\alpha_z$ to the rotation about the center of mass:

$fR = \left(\frac{1}{2}MR^2\right)\alpha_z$

$f = \frac{1}{2}MR\alpha_z$

But $a_{cm} = R\alpha$ in this equation gives $f = \frac{1}{2}Ma_{cm}$. Use this in the $\sum F_x = ma_x$ equation to eliminate *f*.

$Mg\sin\theta - \frac{1}{2}Ma_{cm} = Ma_{cm}$

M divides out and $\frac{3}{2}a_{cm} = g\sin\theta$. $a_{cm} = \frac{2}{3}g\sin\theta = \frac{2}{3}(9.80\text{ m/s}^2)\sin 30° = 3.267\text{ m/s}^2$

SET UP: Apply the constant acceleration equations to the motion of the center of mass. Note that in our coordinates the positive *x*-direction is down the incline.

$v_{0x} = -2.50$ m/s (directed up the incline); $a_x = +3.267\text{ m/s}^2$;

$v_x = 0$ (momentarily comes to rest); $x - x_0 = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

EXECUTE: $x - x_0 = -\dfrac{v_{0x}^2}{2a_x} = -\dfrac{(-2.50\text{ m/s})^2}{2(3.267\text{ m/s}^2)} = -0.957\text{ m}$

(b) EVALUATE: The results from the two methods agree; the disk rolls 0.957 m up the ramp before it stops. The mass *M* enters both in the linear inertia and in the gravity force so divides out. The mass *M* and radius *R* enter in both the rotational inertia and the gravitational torque so divide out.

10.69. IDENTIFY: Apply $\sum \vec{F}_{ext} = m\vec{a}_{cm}$ to the motion of the center of mass and apply $\sum \tau_z = I_{cm}\alpha_z$ to the rotation about the center of mass.

SET UP: $I = 2\left(\frac{1}{2}MR^2\right) = MR^2$. The moment arm for *T is b*.

EXECUTE: The tension is related to the acceleration of the yo-yo by $(2m)g - T = (2m)a$, and to the angular acceleration by $Tb = I\alpha = I\dfrac{a}{b}$. Dividing the second equation by *b* and adding to the first to eliminate *T* yields

$a = g\dfrac{2m}{(2m + I/b^2)} = g\dfrac{2}{2 + (R/b)^2}$, $\alpha = g\dfrac{2}{2b + R^2/b}$. The tension is found by substitution into either of the two equations:

$$T = (2m)(g - a) = (2mg)\left(1 - \frac{2}{2 + (R/b)^2}\right) = 2mg\frac{(R/b)^2}{2 + (R/b)^2} = \frac{2mg}{(2(b/R)^2 + 1)}.$$

EVALUATE: $a \to 0$ when $b \to 0$. As $b \to R$, $a \to 2g/3$.

10.70. **IDENTIFY:** Apply conservation of energy to the motion of the shell, to find its linear speed v at points A and B. Apply $\sum \vec{F} = m\vec{a}$ to the circular motion of the shell in the circular part of the track to find the normal force exerted by the track at each point. Since $r << R$ the shell can be treated as a point mass moving in a circle of radius R when applying $\sum \vec{F} = m\vec{a}$. But as the shell rolls along the track, it has both translational and rotational kinetic energy.

SET UP: $K_1 + U_1 = K_2 + U_2$. Let 1 be at the starting point and take $y = 0$ to be at the bottom of the track, so $y_1 = h_0$. $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $I = \frac{2}{3}mr^2$ and $\omega = v/r$, so $K = \frac{5}{6}mv^2$. During the circular motion, $a_{\text{rad}} = v^2/R$.

EXECUTE: **(a)** $\sum \vec{F} = m\vec{a}$ at point A gives $n + mg = m\dfrac{v^2}{R}$. The minimum speed for the shell not to fall off the track is when $n \to 0$ and $v^2 = gR$. Let point 2 be A, so $y_2 = 2R$ and $v_2^2 = mR$. Then $K_1 + U_1 = K_2 + U_2$ gives $mgh_0 = 2mgR + \frac{5}{6}m(gR)$. $h_0 = (2 + \frac{5}{6})R = \frac{17}{6}R$.

(b) Let point 2 be B, so $y_2 = R$. Then $K_1 + U_1 = K_2 + U_2$ gives $mgh_0 = mgR + \frac{5}{6}mv_2^2$. With $h = \frac{17}{6}R$ this gives $v^2 = \frac{11}{5}gR$. Then $\sum \vec{F} = m\vec{a}$ at B gives $n = m\dfrac{v^2}{R} = \frac{11}{5}mg$.

(c) Now $K = \frac{1}{2}mv^2$ instead of $\frac{5}{6}mv^2$. The shell would be moving faster at A than with friction and would still make the complete loop.

(d) In part (c): $mgh_0 = mg(2R) + \frac{1}{2}mv^2$. $h_0 = \frac{17}{6}R$ gives $v^2 = \frac{5}{3}gR$. $\sum \vec{F} = m\vec{a}$ at point A gives $mg + n = m\dfrac{v^2}{R}$ and $n = m\left(\dfrac{v^2}{R} - g\right) = \frac{2}{3}mg$. In part (a), $n = 0$, since at this point gravity alone supplies the net downward force that is required for the circular motion.

EVALUATE: The normal force at A is greater when friction is absent because the speed of the shell at A is greater when friction is absent than when there is rolling without slipping.

10.71. **IDENTIFY:** Consider the direction of the net force and the sense of the net torque in each case.

SET UP: The free-body diagram in each case is shown in Figure 10.71.

EXECUTE: In the first case, $\vec{F}$ and the friction force act in opposite directions, and the friction force causes a larger torque to tend to rotate the yo-yo to the right. The net force to the right is the difference $F - f$, so the net force is to the right while the net torque causes a clockwise rotation. For the second case, both the torque and the friction force tend to turn the yo-yo clockwise, and the yo-yo moves to the right. In the third case, friction tends to move the yo-yo to the right, and since the applied force is vertical, the yo-yo moves to the right.

EVALUATE: In the first case the torque due to friction must be larger than the torque due to F, so the net torque is clockwise. In the third case the torque due to F must be larger than the torque due to f, so the net torque will be clockwise.

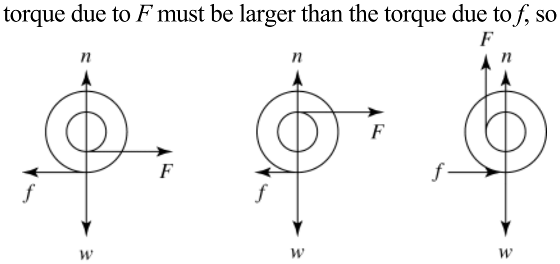


Figure 10.71

10.72. **IDENTIFY:** Apply $\sum \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$ to the motion of the center of mass and $\sum \tau_z = I_{\text{cm}}\alpha_z$ to the rotation about the center of mass.

SET UP: For a hoop, $I = MR^2$. For a solid disk, $I = \frac{1}{2}MR^2$.

EXECUTE: **(a)** Because there is no vertical motion, the tension is just the weight of the hoop: $T = Mg = (0.180\text{ kg})(9.8\text{ N/kg}) = 1.76\text{ N}$.

(b) Use $\tau = I\alpha$ to find α. The torque is RT, so $\alpha = RT/I = RT/MR^2 = T/MR = Mg/MR$, so $\alpha = g/R = (9.8\text{ m/s}^2)/(0.08\text{ m}) = 122.5\text{ rad/s}^2$.

(c) $a = R\alpha = 9.8\text{ m/s}^2$

(d) T would be unchanged because the mass M is the same, α and a would be twice as great because I is now $\frac{1}{2}MR^2$.

EVALUATE: $a_{\tan}$ for a point on the rim of the hoop or disk equals a for the free end of the string. Since I is smaller for the disk, the same value of T produces a greater angular acceleration.

10.73. **IDENTIFY:** Apply $\sum \tau_z = I\alpha_z$ to the cylinder or hoop. Find a for the free end of the cable and apply constant acceleration equations.

SET UP: $a_{\tan}$ for a point on the rim equals a for the free end of the cable, and $a_{\tan} = R\alpha$.

EXECUTE: **(a)** $\sum \tau_z = I\alpha_z$ and $a_{\tan} = R\alpha$ gives $FR = \dfrac{1}{2}MR^2\alpha = \dfrac{1}{2}MR^2\left(\dfrac{a_{\tan}}{R}\right)$. $a_{\tan} = \dfrac{2F}{M} = \dfrac{200\text{ N}}{4.00\text{ kg}} = 50\text{ m/s}^2$.

Distance the cable moves: $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $50\text{ m} = \dfrac{1}{2}\left(50\text{ m/s}^2\right)t^2$ and $t = 1.41\text{ s}$.

$v_x = v_{0x} + a_x t = 0 + \left(50\text{ m/s}^2\right)\left(1.41\text{ s}\right) = 70.5\text{ m/s}$.

(b) For a hoop, $I = MR^2$, which is twice as large as before, so α and $a_{\tan}$ would be half as large. Therefore the time would be longer by a factor of $\sqrt{2}$. For the speed, $v_x^2 = v_{0x}^2 + 2a_x x$, in which x is the same, so v_x would be half as large since a_x is smaller.

EVALUATE: The acceleration a that is produced depends on the mass of the object but is independent of its radius. But a depends on how the mass is distributed and is different for a hoop versus a cylinder.

10.74. **IDENTIFY:** Use projectile motion to find the speed v the marble needs at the edge of the pit to make it to the level ground on the other side. Apply conservation of energy to the motion down the hill in order to relate the initial height to the speed v at the edge of the pit. $W_{\text{other}} = 0$ so conservation of energy gives $K_{\text{i}} + U_{\text{i}} = K_{\text{f}} + U_{\text{f}}$.

SET UP: In the projectile motion the marble must travel 36 m horizontally while falling vertically 20 m. Let $+y$ be downward. For the motion down the hill, let $y_{\text{f}} = 0$ so $U_{\text{f}} = 0$ and $y_{\text{i}} = h$. $K_{\text{i}} = 0$. Rolling without slipping means $v = R\omega$. $K = \frac{1}{2}I_{\text{cm}}\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}(\frac{2}{5}mR^2)\omega^2 + \frac{1}{2}mv^2 = \frac{7}{10}mv^2$.

EXECUTE: **(a)** Projectile motion: $v_{0y} = 0$. $a_y = 9.80\text{ m/s}^2$. $y - y_0 = 20\text{ m}$. $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives

$t = \sqrt{\dfrac{2(y - y_0)}{a_y}} = 2.02\text{ s}$. Then $x - x_0 = v_{0x}t$ gives $v = v_{0x} = \dfrac{x - x_0}{t} = \dfrac{36\text{ m}}{2.02\text{ s}} = 17.8\text{ m/s}$.

Motion down the hill: $U_{\text{i}} = K_{\text{f}}$. $mgh = \frac{7}{10}mv^2$. $h = \dfrac{7v^2}{10g} = \dfrac{7(17.8\text{ m/s})^2}{10(9.80\text{ m/s}^2)} = 22.6\text{ m}$.

(b) $\frac{1}{2}I\omega^2 = \frac{1}{5}mv^2$, independent of R. I is proportional to R^2 but ω^2 is proportional to $1/R^2$ for a given translational speed v.

(c) The object still needs $v = 17.8\text{ m/s}$ at the bottom of the hill in order to clear the pit. But now $K_{\text{f}} = \frac{1}{2}mv^2$ and

$h = \dfrac{v^2}{2g} = 16.6\text{ m}$.

EVALUATE: The answer to part (a) also does not depend on the mass of the marble. But, it does depend on how the mass is distributed within the object. The answer would be different if the object were a hollow spherical shell. In part (c) less height is needed to give the object the same translational speed because in (c) none of the energy goes into rotational motion.

10.75. **IDENTIFY:** Apply conservation of energy to the motion of the boulder.

SET UP: $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ and $v = R\omega$ when there is rolling without slipping. $I = \frac{2}{5}mR^2$.

EXECUTE: Break into 2 parts, the rough and smooth sections.

Rough: $mgh_1 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$. $mgh_1 = \dfrac{1}{2}mv^2 + \dfrac{1}{2}\left(\dfrac{2}{5}mR^2\right)\left(\dfrac{v}{R}\right)^2$. $v^2 = \dfrac{10}{7}gh_1$.

Smooth: Rotational kinetic energy does not change. $mgh_2 + \dfrac{1}{2}mv^2 + K_{\text{rot}} = \dfrac{1}{2}mv_{\text{Bottom}}^2 + K_{\text{rot}}$. $gh_2 + \dfrac{1}{2}\left(\dfrac{10}{7}gh_1\right) = \dfrac{1}{2}v_{\text{B}}^2$.

$v_{\text{B}} = \sqrt{\dfrac{10}{7}gh_1 + 2gh_2} = \sqrt{\dfrac{10}{7}(9.80\text{ m/s}^2)(25\text{ m}) + 2(9.80\text{ m/s}^2)(25\text{ m})} = 29.0\text{ m/s}$.

EVALUATE: If all the hill was rough enough to cause rolling without slipping, $v_{\text{B}} = \sqrt{\dfrac{10}{7}g(50\text{ m})} = 26.5\text{ m/s}$. A smaller fraction of the initial gravitational potential energy goes into translational kinetic energy of the center of mass than if part of the hill is smooth. If the entire hill is smooth and the boulder slides without slipping, $v_{\text{B}} = \sqrt{2g(50\text{ m})} = 31.3\text{ m/s}$. In this case all the initial gravitational potential energy goes into the kinetic energy of the translational motion.

10.76. IDENTIFY: Apply conservation of energy to the motion of the ball as it rolls up the hill. After the ball leaves the edge of the cliff it moves in projectile motion and constant acceleration equations can be used.

(a) SET UP: Use conservation of energy to find the speed v_2 of the ball just before it leaves the top of the cliff. Let point 1 be at the bottom of the hill and point 2 be at the top of the hill. Take $y=0$ at the bottom of the hill, so $y_1=0$ and $y_2=28.0$ m.

EXECUTE: $K_1=U_1=K_2+U_2$

$\frac{1}{2}mv_1^2+\frac{1}{2}I\omega_1^2=mgy_2+\frac{1}{2}mv_2^2+\frac{1}{2}I\omega_2^2$

Rolling without slipping means $\omega=v/r$ and $\frac{1}{2}I\omega^2=\frac{1}{2}\left(\frac{2}{5}mr^2\right)(v/r)^2=\frac{1}{5}mv^2$

$\frac{7}{10}mv_1^2=mgy_2+\frac{7}{10}mv_2^2$

$v_2=\sqrt{v_1^2-\frac{10}{7}gy_2}=15.26$ m/s

SET UP: Consider the projectile motion of the ball, from just after it leaves the top of the cliff until just before it lands. Take $+y$ to be downward. Use the vertical motion to find the time in the air:

$v_{0y}=0,\ a_y=9.80\ \text{m/s}^2,\ y-y_0=28.0\ \text{m},\ t=?$

EXECUTE: $y-y_0=v_{0y}t+\frac{1}{2}a_yt^2$ gives $t=2.39$ s

During this time the ball travels horizontally

$x-x_0=v_{0x}t=(15.26\ \text{m/s})(2.39\ \text{s})=36.5\ \text{m}.$

Just before it lands, $v_y=v_{0y}+a_yt=23.4$ m/s and $v_x=v_{0x}=15.3$ m/s

$v=\sqrt{v_x^2+v_y^2}=28.0$ m/s

(b) EVALUATE: At the bottom of the hill, $\omega=v/r=(25.0\ \text{m/s})/r.$ The rotation rate doesn't change while the ball is in the air, after it leaves the top of the cliff, so just before it lands $\omega=(15.3\ \text{m/s})/r.$ The total kinetic energy is the same at the bottom of the hill and just before it lands, but just before it lands less of this energy is rotational kinetic energy, so the translational kinetic energy is greater.

10.77. IDENTIFY: Apply conservation of energy to the motion of the wheel. $K=\frac{1}{2}mv^2+\frac{1}{2}I\omega^2$.

SET UP: No slipping means that $\omega=v/R$. Uniform density means $m_{\text{r}}=\lambda 2\pi R$ and $m_{\text{s}}=\lambda R$, where m_{r} is the mass of the rim and m_{s} is the mass of each spoke. For the wheel, $I=I_{\text{rim}}+I_{\text{spokes}}$. For each spoke, $I=\frac{1}{3}m_{\text{s}}R^2$.

EXECUTE: (a) $mgh=\dfrac{1}{2}mv^2+\dfrac{1}{2}I\omega^2$. $I=I_{\text{rim}}+I_{\text{spokes}}=m_{\text{r}}R^2+6\left(\dfrac{1}{3}m_{\text{s}}R^2\right)$

Also, $m=m_{\text{r}}+m_{\text{s}}=2\pi R\lambda+6R\lambda=2R\lambda(\pi+3)$. Substituting into the conservation of energy equation gives

$$2R\lambda(\pi+3)gh=\frac{1}{2}(2R\lambda)(\pi+3)(R\omega)^2+\frac{1}{2}\left[2\pi R\lambda R^2+6\left(\frac{1}{3}\pi RR^2\right)\right]\omega^2.$$

$$\omega=\sqrt{\frac{(\pi+3)gh}{R^2(\pi+2)}}=\sqrt{\frac{(\pi+3)(9.80\ \text{m/s}^2)(58.0\ \text{m})}{(0.210\ \text{m})^2(\pi+2)}}=124\ \text{rad/s and } v=R\omega=26.0\ \text{m/s}$$

(b) Doubling the density would have no effect because it does not appear in the answer. ω is inversely proportional to R so doubling the diameter would double the radius which would reduce ω by half, but $v=R\omega$ would be unchanged.

EVALUATE: Changing the masses of the rim and spokes by different amounts would alter the speed v at the bottom of the hill.

10.78. IDENTIFY: Apply $v=R\omega$.

SET UP: For the antique bike, v is the same for points on the rim of each wheel and equals the linear speed of the bike. $1\ \text{rev}=2\pi\ \text{rad}$.

EXECUTE: (a) The front wheel is turning at $\omega=1.00\ \text{rev/s}=2\pi\ \text{rad/s}$. $v=r\omega=(0.330\ \text{m})(2\pi\ \text{rad/s})=2.07\ \text{s}$.

(b) $\omega=v/r=(2.07\ \text{m/s})/(0.655\ \text{m})=3.16\ \text{rad/s}=0.503\ \text{rev/s}$

(c) $\omega=v/r=(2.07\ \text{m/s})/(0.220\ \text{m})=9.41\ \text{rad/s}\ =1.50\ \text{rev/s}$

EVALUATE: Since the front wheel has a larger radius for the antique bike, that wheel doesn't have to rotate at as many rev/s to achieve the same linear speed of the bike.

10.79. **IDENTIFY:** Apply conservation of energy to the motion of the ball. Once the ball leaves the track the ball moves in projectile motion.

SET UP: The ball has $I = \frac{2}{5}mR^2$; the silver dollar has $I = \frac{1}{2}mR^2$. For the projectile motion take $+y$ downward, so $a_x = 0$ and $a_y = +g$.

EXECUTE: **(a)** The kinetic energy of the ball when it leaves the track (when it is still rolling without slipping) is $(7/10)mv^2$ and this must be the work done by gravity, $W = mgh$, so $v = \sqrt{10gh/7}$. The ball is in the air for a time $t = \sqrt{2y/g}$, so $x = vt = \sqrt{20hy/7}$.

(b) The answer does not depend on g, so the result should be the same on the moon.

(c) The presence of rolling friction would decrease the distance.

(e) For the dollar coin, modeled as a uniform disc, $K = (3/4)mv^2$, and so $x = \sqrt{8hy/3}$.

EVALUATE: The sphere travels a little farther horizontally, because its moment of inertia is a smaller fraction of MR^2 than for the disk. The result is independent of the mass and radius of the object but it does depend on how that mass is distributed within the object.

10.80. **IDENTIFY** and **SET UP:** Apply conservation of energy to the motion of the ball. The ball ends up with both translational and rotational kinetic energy. Use Fig.(10.13) in the textbook to relate the speed of different points on the ball to v_{cm}.

EXECUTE: **(a)** $U_{el} = \frac{1}{2}kx^2 = \frac{1}{2}(400\ \text{N}\cdot\text{m})(0.15\ \text{m})^2 = 4.50\ \text{J}$ and $K_1 = 0.800U_{el} = 3.60\ \text{J}$

$K_1 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$ rolling without slipping says $\omega = v_{cm}/R$

$I_{cm} = \frac{2}{5}mR^2$

Thus $K_1 = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)(v_{cm}/R)^2 = mv_{cm}^2\left(\frac{1}{2}+\frac{1}{5}\right) = \frac{7}{10}mv_{cm}^2$

and $v_{cm} = \sqrt{\dfrac{10K_1}{7m}} = \sqrt{\dfrac{10(3.60\ \text{J})}{7(0.0590\ \text{kg})}} = 9.34\ \text{m/s}.$

(b) Consider Figure 10.80a.

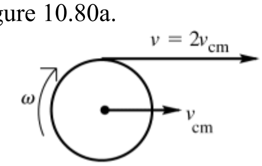


Figure 10.80a

From Fig.(10.13) in the textbook, at the top of the ball $v = 2v_{cm} = 18.7\ \text{m/s}$

(c)

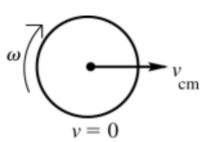


Figure 10.80b

From Fig.(10.13) in the textbook, $v = 0$ at the bottom of the ball.

(d) The problem says that $U_2 = 0.900K_1 = 3.24\ \text{J}$. Thus $U_2 = mgh = 3.24\ \text{J}$ and

$$h = \frac{3.24\ \text{J}}{mg} = \frac{3.24\ \text{J}}{(0.0590\ \text{kg})(9.80\ \text{m/s}^2)} = 5.60\ \text{m}$$

EVALUATE: Not all the potential energy stored in the spring goes into kinetic energy at the base of the ramp or into gravitational potential energy at the top of the ramp because of loss of mechanical energy due to negative work done by friction. If the ball slides without rolling, then $K_1 = \frac{1}{2}mv_{cm}^2$ and $v_{cm} = 11.0\ \text{m/s}$. v_{cm} is less than this when the ball rolls and some of its total kinetic energy is rotational.

10.81. **IDENTIFY:** $v_x = dx/dt$, $v_y = dy/dt$. $a_x = dv_x/dt$, $a_y = dv_y/dt$.

SET UP: $d\cos(\omega t)/dt = -\omega\sin(\omega t)$. $d\sin(\omega t)/dt = \omega\cos(\omega t)$.

EXECUTE: **(a)** The sketch is shown in Figure 10.81.

(b) R is the radius of the wheel (y varies from 0 to $2R$) and T is the period of the wheel's rotation.

(c) Differentiating, $v_x = \dfrac{2\pi R}{T}\left[1-\cos\left(\dfrac{2\pi t}{T}\right)\right]$, $a_x = \left(\dfrac{2\pi}{T}\right)^2 R\sin\left(\dfrac{2\pi t}{T}\right)$ and $v_y = \dfrac{2\pi R}{T}\sin\left(\dfrac{2\pi t}{T}\right)$,

$a_y = \left(\dfrac{2\pi}{T}\right)^2 R\cos\left(\dfrac{2\pi t}{T}\right).$

(d) $v_x = v_y = 0$ when $\left(\frac{2\pi t}{T}\right) = 2\pi$ or any multiple of 2π, so the times are integer multiples of the period *T*. The acceleration components at these times are $a_x = 0,\ a_y = \frac{4\pi^2 R}{T^2}.$

(e) $a = \sqrt{a_x^2 + a_y^2} = \left(\frac{2\pi}{T}\right)^2 R\sqrt{\cos^2\left(\frac{2\pi t}{T}\right) + \sin^2\left(\frac{2\pi t}{T}\right)} = \frac{4\pi^2 R}{T^2}$, independent of time. This is the magnitude of the radial acceleration for a point moving on a circle of radius *R* with constant angular velocity $2\pi/T$. For motion that consists of this circular motion superimposed on motion with constant velocity $(\vec{a} = 0)$, the acceleration due to the circular motion will be the total acceleration.

EVALUATE: *a* is independent of time, but *v* does depend on time.

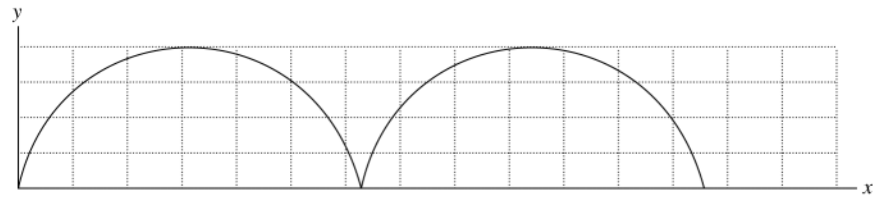


Figure 10.81

10.82. **IDENTIFY:** Apply the work-energy theorem to the motion of the basketball. $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ and $v = R\omega$.

SET UP: For a thin-walled, hollow sphere $I = \frac{2}{3}mR^2$.

EXECUTE: For rolling without slipping, the kinetic energy is $(1/2)(m + I/R^2)v^2 = (5/6)mv^2$; initially, this is 32.0 J and at the return to the bottom it is 8.0 J. Friction has done -24.0 J of work, -12.0 J each going up and down. The potential energy at the highest point was 20.0 J, so the height above the ground was $\frac{20.0\text{ J}}{(0.600\text{ kg})(9.80\text{ m/s}^2)} = 3.40\text{ m}.$

EVALUATE: All of the kinetic energy of the basketball, translational and rotational, has been removed at the point where the basketball is at its maximum height up the ramp.

10.83. **IDENTIFY:** Use conservation of energy to relate the speed of the block to the distance it has descended. Then use a constant acceleration equation to relate these quantities to the acceleration.

SET UP: For the cylinder, $I = \frac{1}{2}M(2R)^2$, and for the pulley, $I = \frac{1}{2}MR^2$.

EXECUTE: Doing this problem using kinematics involves four unknowns (six, counting the two angular accelerations), while using energy considerations simplifies the calculations greatly. If the block and the cylinder both have speed *v*, the pulley has angular velocity *v*/*R* and the cylinder has angular velocity *v*/2*R*, the total kinetic energy is

$$K = \frac{1}{2}\left[Mv^2 + \frac{M(2R)^2}{2}(v/2R)^2 + \frac{MR^2}{2}(v/R)^2 + Mv^2\right] = \frac{3}{2}Mv^2.$$

This kinetic energy must be the work done by gravity; if the hanging mass descends a distance *y*, $K = Mgy$, or $v^2 = (2/3)gy$. For constant acceleration, $v^2 = 2ay$, and comparison of the two expressions gives $a = g/3$.

EVALUATE: If the pulley were massless and the cylinder slid without rolling, $Mg = 2Ma$ and $a = g/2$. The rotation of the objects reduces the acceleration of the block.

10.84. **IDENTIFY:** Apply $\sum\tau_z = I\alpha_z$ to the drawbridge and calculate α_z. For part (c) use conservation of energy.

SET UP: The free-body diagram for the drawbridge is given in Fig.10.84. For an axis at the lower end, $I = \frac{1}{3}ml^2$.

EXECUTE: **(a)** $\sum\tau_z = I\alpha_z$ gives $mg(4.00\text{ m})(\cos 60.0°) = \frac{1}{3}ml^2\alpha_z$ and $\alpha_z = \frac{3g(4.00\text{ m})(\cos 60.0°)}{(8.00\text{ m})^2} = 0.919\text{ rad/s}^2$.

(b) α_z depends on the angle the bridge makes with the horizontal. α_z is not constant during the motion and $\omega_z = \omega_{0z} + \alpha_z t$ cannot be used.

(c) Use conservation of energy. Take $y = 0$ at the lower end of the drawbridge, so $y_\text{i} = (4.00\text{ m})(\sin 60.0°)$ and $y_\text{f} = 0$. $K_\text{f} + U_\text{f} = K_\text{i} + U_\text{i} + W_\text{other}$ gives $U_\text{i} = K_\text{f}$, $mgy_\text{i} = \frac{1}{2}I\omega^2$. $mgy_\text{i} = \frac{1}{2}(\frac{1}{3}ml^2)\omega^2$ and

$$\omega = \frac{\sqrt{6gy_\text{i}}}{l} = \frac{\sqrt{6(9.80\text{ m/s}^2)(4.00\text{ m})(\sin 60.0°)}}{8.00\text{ m}} = 1.78\text{ rad/s}.$$

Evaluate: If we incorrectly assume that α_z is constant and has the value calculated in part (a), then $\omega_z^2 = \omega_{0z}^2 + 2\alpha_z(\theta - \theta_0)$ gives $\omega = 139$ rad/s . The angular acceleration increases as the bridge rotates and the actual angular velocity is larger than this.

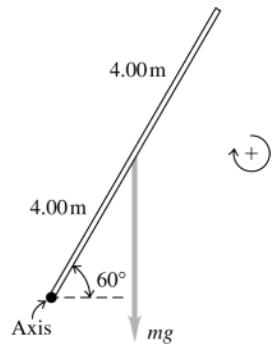


Figure 10.84

10.85. **Identify:** Apply conservation of energy to the motion of the first ball before the collision and to the motion of the second ball after the collision. Apply conservation of angular momentum to the collision between the first ball and the bar.

Set Up: The speed of the ball just before it hits the bar is $v = \sqrt{2gy} = 15.34$ m/s. Use conservation of angular momentum to find the angular velocity ω of the bar just after the collision. Take the axis at the center of the bar.

Execute: $L_1 = mvr = (5.00 \text{ kg})(15.34 \text{ m/s})(2.00 \text{ m}) = 153.4 \text{ kg}\cdot\text{m}^2$

Immediately after the collision the bar and both balls are rotating together.

$L_2 = I_{\text{tot}}\omega$

$I_{\text{tot}} = \frac{1}{12}Ml^2 + 2mr^2 = \frac{1}{12}(8.00 \text{ kg})(4.00 \text{ m})^2 + 2(5.00 \text{ kg})(2.00 \text{ m})^2 = 50.67 \text{ kg}\cdot\text{m}^2$

$L_2 = L_1 = 153.4 \text{ kg}\cdot\text{m}^2$

$\omega = L_2 / I_{\text{tot}} = 3.027$ rad/s

Just after the collision the second ball has linear speed $v = r\omega = (2.00 \text{ m})(3.027 \text{ rad/s}) = 6.055 \text{ m/s}$ and is moving upward. $\frac{1}{2}mv^2 = mgy$ gives $y = 1.87$ m for the height the second ball goes.

Evaluate: Mechanical energy is lost in the inelastic collision and some of the final energy is in the rotation of the bar with the first ball stuck to it. As a result, the second ball does not reach the height from which the first ball was dropped.

10.86. **Identify:** The rings and the rod exert forces on each other, but there is no net force or torque on the system, and so the angular momentum will be constant.

Set Up: For the rod, $I = \frac{1}{12}ML^2$. For each ring, $I = mr^2$, where *r* is their distance from the axis.

Execute: **(a)** As the rings slide toward the ends, the moment of inertia changes, and the final angular velocity is given by $\omega_2 = \omega_1 \dfrac{I_1}{I_2} = \omega_1 \left[\dfrac{\frac{1}{12}ML^2 + 2mr_1^2}{\frac{1}{12}ML^2 + 2mr_2^2}\right] = \omega_1 \dfrac{5.00\times10^{-4} \text{ kg}\cdot\text{m}^2}{2.00\times10^{-3} \text{ kg}\cdot\text{m}^2} = \dfrac{\omega_1}{4}$, so $\omega_2 = 7.5$ rev/min.

(b) The forces and torques that the rings and the rod exert on each other will vanish, but the common angular velocity will be the same, 7.5 rev/min.

Evaluate: Note that conversion from rev/min to rad/s was not necessary. The angular velocity of the rod decreases as the rings move away from the rotation axis.

10.87. **Identify:** Apply conservation of angular momentum to the collision. Linear momentum is not conserved because of the force applied to the rod at the axis. But since this external force acts at the axis, it produces no torque and angular momentum is conserved.

Set Up: The system before and after the collision is sketched in Figure 10.87.

Execute: **(a)** $m_b = \frac{1}{4}m_{\text{rod}}$

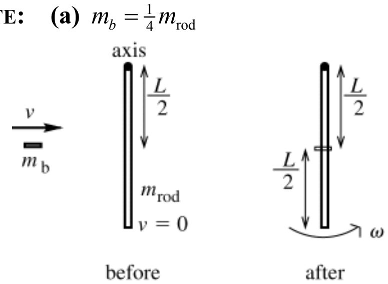


Figure 10.87

Execute: $L_1 = m_b vr = \frac{1}{4}m_{\text{rod}}v(L/2)$

$L_1 = \frac{1}{8}m_{\text{rod}}vL$

$L_2 = (I_{\text{rod}} + I_b)\omega$

$I_{\text{rod}} = \frac{1}{3}m_{\text{rod}}L^2$

$I_b = m_b r^2 = \frac{1}{4}m_{\text{rod}}(L/2)^2$

$I_b = \frac{1}{16}m_{\text{rod}}L^2$

Thus $L_1 = L_2$ gives $\frac{1}{8}m_{\text{rod}}vL = \left(\frac{1}{3}m_{\text{rod}}L^2 + \frac{1}{16}m_{\text{rod}}L^2\right)\omega$

$\frac{1}{8}v = \frac{19}{48}L\omega$

$\omega = \frac{6}{19}v/L$

(b) $K_1 = \frac{1}{2}mv^2 = \frac{1}{8}m_{\text{rod}}v^2$

$K_2 = \frac{1}{2}I\omega^2 = \frac{1}{2}(I_{\text{rod}} + I_{\text{b}})\omega^2 = \frac{1}{2}\left(\frac{1}{3}m_{\text{rod}}L^2 + \frac{1}{16}m_{\text{rod}}L^2\right)(6v/19L)^2$

$K_2 = \frac{1}{2}\left(\frac{19}{48}\right)\left(\frac{6}{19}\right)^2 m_{\text{rod}}v^2 = \frac{3}{152}m_{\text{rod}}v^2$

Then $\dfrac{K_2}{K_1} = \dfrac{\frac{3}{152}m_{\text{rod}}v^2}{\frac{1}{8}m_{\text{rod}}v^2} = 3/19.$

EVALUATE: The collision is inelastic and $K_2 < K_1$.

10.88. **IDENTIFY:** Apply Eq.(10.29).

SET UP: The door has $I = \frac{1}{3}ml^2$. The torque applied by the force is rF_{av}, where $r = l/2$.

EXECUTE: $\Sigma\tau_{\text{av}} = rF_{\text{av},}$ and $\Delta L = rF_{\text{av}}\Delta t = rJ.$ The angular velocity ω is then

$\omega = \dfrac{\Delta L}{I} = \dfrac{rF_{\text{av}}\Delta t}{I} = \dfrac{(l/2)F_{\text{av}}\Delta t}{\frac{1}{3}ml^2} = \dfrac{3}{2}\dfrac{F_{\text{av}}\Delta t}{ml}$, where l is the width of the door. Substitution of the given numeral values gives $\omega = 0.514\,\text{rad/s}$.

EVALUATE: The final angular velocity of the door is proportional to both the magnitude of the average force and also to the time it acts.

10.89. **(a) IDENTIFY:** Apply conservation of angular momentum to the collision between the bullet and the board:

SET UP: The system before and after the collision is sketched in Figure 10.89a.

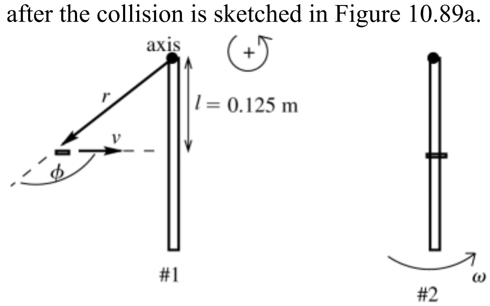


Figure 10.89a

EXECUTE: $L_1 = L_2$

$L_1 = mvr\sin\phi = mvl = (1.90\times10^{-3}\ \text{kg})(360\ \text{m/s})(0.125\ \text{m}) = 0.0855\ \text{kg}\cdot\text{m}^2/\text{s}$

$L_2 = I_2\omega_2$

$I_2 = I_{\text{board}} + I_{\text{bullet}} = \frac{1}{3}ML^2 + mr^2$

$I_2 = \frac{1}{3}(0.750\ \text{kg})(0.250\ \text{m})^2 + (1.90\times10^{-3}\ \text{kg})(0.125\ \text{m})^2 = 0.01565\ \text{kg}\cdot\text{m}^2$

Then $L_1 = L_2$ gives that $\omega_2 = \dfrac{L_1}{I_2} = \dfrac{0.0855\ \text{kg}\cdot\text{m}^2/\text{s}}{0.1565\ \text{kg}\cdot\text{m}^2} = 5.46\ \text{rad/s}$

(b) IDENTIFY: Apply conservation of energy to the motion of the board after the collision.

SET UP: The position of the board at points 1 and 2 in its motion is shown in Figure 10.89b. Take the origin of coordinates at the center of the board and $+y$ to be upward, so $y_{\text{cm},1} = 0$ and $y_{\text{cm},2} = h$, the height being asked for.

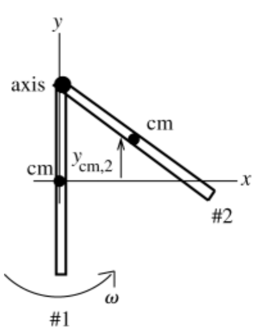


Figure 10.89b

$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$

EXECUTE: Only gravity does work, so $W_{\text{other}} = 0$.

$K_1 = \frac{1}{2}I\omega^2$

$U_1 = mgy_{\text{cm},1} = 0$

$K_2 = 0$

$U_2 = mgy_{\text{cm},2} = mgh$

Thus $\frac{1}{2}I\omega^2 = mgh.$

$$h = \frac{I\omega^2}{2mg} = \frac{(0.01565\ \text{kg}\cdot\text{m}^2)(5.46\ \text{rad/s})^2}{2(0.750\ \text{kg} + 1.90\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)} = 0.0317\ \text{m} = 3.17\ \text{cm}$$

(c) IDENTIFY and **SET UP:** The position of the board at points 1 and 2 in its motion is shown in Figure 10.89c.

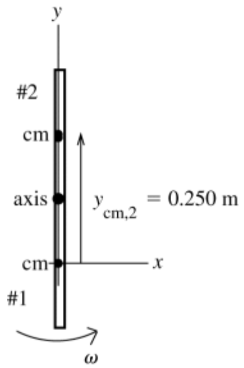


Figure 10.89c

Apply conservation of energy as in part (b), except now we want $y_{\text{cm},2} = h = 0.250$ m. Solve for the ω after the collision that is required for this to happen.

EXECUTE: $\frac{1}{2}I\omega^2 = mgh$

$$\omega = \sqrt{\frac{2mgh}{I}} = \sqrt{\frac{2(0.750\ \text{kg} + 1.90\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)(0.250\ \text{m})}{0.01565\ \text{kg}\cdot\text{m}^2}}$$

$\omega = 15.34$ rad/s

Now go back to the equation that results from applying conservation of angular momentum to the collision and solve for the initial speed of the bullet. $L_1 = L_2$ implies $m_{\text{bullet}}vl = I_2\omega_2$

$$v = \frac{I_2\omega_2}{m_{\text{bullet}}l} = \frac{(0.01565\ \text{kg}\cdot\text{m}^2)(15.34\ \text{rad/s})}{(1.90\times10^{-3}\ \text{kg})(0.125\ \text{m})} = 1010\ \text{m/s}$$

EVALUATE: We have divided the motion into two separate events: the collision and the motion after the collision. Angular momentum is conserved in the collision because the collision happens quickly. The board doesn't move much until after the collision is over, so there is no gravity torque about the axis. The collision is inelastic and mechanical energy is lost in the collision. Angular momentum of the system is not conserved during this motion, due to the external gravity torque. Our answer to parts (b) and (c) say that a bullet speed of 360 m/s causes the board to swing up only a little and a speed of 1010 m/s causes it to swing all the way over.

10.90. IDENTIFY: Angular momentum is conserved, so $I_0\omega_0 = I_2\omega_2$.

SET UP: For constant mass the moment of inertia is proportional to the square of the radius.

EXECUTE: $R_0^2\omega_0 = R_2^2\omega_2$, or $R_0^2\omega_0 = \left(R_0 + \Delta R\right)^2\left(\omega_0 + \Delta\omega\right) = R_0^2\omega_0 + 2R_0\Delta R\omega_0 + R_0^2\Delta\omega,$ where the terms in $\Delta R\Delta\omega$ and $(\Delta\omega)^2$ have been omitted. Canceling the $R_0^2\omega_0$ term gives

$$\Delta R = -\frac{R_0}{2}\frac{\Delta\omega}{\omega_0} = -1.1\ \text{cm}.$$

EVALUATE: $\Delta R/R_0$ and $\Delta\omega/\omega_0$ are each very small so the neglect of terms containing $\Delta R\Delta\omega$ or $(\Delta\omega)^2$ is an accurate simplifying approximation.

10.91. IDENTIFY: Apply conservation of angular momentum to the collision between the bird and the bar and apply conservation of energy to the motion of the bar after the collision.

SET UP: For conservation of angular momentum take the axis at the hinge. For this axis the initial angular momentum of the bird is $m_{\text{bird}}(0.500\ \text{m})v$, where $m_{\text{bird}} = 0.500$ kg and $v = 2.25$ m/s . For this axis the moment of inertia is $I = \frac{1}{3}m_{\text{bar}}L^2 = \frac{1}{3}(1.50\ \text{kg})(0.750\ \text{m})^2 = 0.281\ \text{kg}\cdot\text{m}^2$. For conservation of energy, the gravitational potential energy of the bar is $U = m_{\text{bar}}gy_{\text{cm}}$, where y_{cm} is the height of the center of the bar. Take $y_{\text{cm},1} = 0$, so $y_{\text{cm},2} = -0.375$ m .

EXECUTE: **(a)** $L_1 = L_2$ gives $m_{\text{bird}}(0.500\ \text{m})v = (\frac{1}{3}m_{\text{bar}}L^2)\omega$.

$$\omega = \frac{3m_{\text{bird}}(0.500\ \text{m})v}{m_{\text{bar}}L^2} = \frac{3(0.500\ \text{kg})(0.500\ \text{m})(2.25\ \text{m/s})}{(1.50\ \text{kg})(0.750\ \text{m})^2} = 2.00\ \text{rad/s}.$$

(b) $U_1 + K_1 = U_2 + K_2$ applied to the motion of the bar after the collision gives $\frac{1}{2}I\omega_1^2 = m_{\text{bar}}g(-0.375\text{ m}) + \frac{1}{2}I\omega_2^2$.

$\omega_2 = \sqrt{\omega_1^2 + \frac{2}{I}m_{\text{bar}}g(0.375\text{ m})}$. $\omega_2 = \sqrt{(2.00\text{ rad/s})^2 + \frac{2}{0.281\text{ kg}\cdot\text{m}^2}(1.50\text{ kg})(9.80\text{ m/s}^2)(0.375\text{ m})} = 6.58\text{ rad/s}$

EVALUATE: Mechanical energy is not conserved in the collision. The kinetic energy of the bar just after the collision is less than the kinetic energy of the bird just before the collision.

10.92. **IDENTIFY:** Angular momentum is conserved, since the tension in the string is in the radial direction and therefore produces no torque. Apply $\sum\vec{F} = m\vec{a}$ to the block, with $a = a_{\text{rad}} = v^2/r$.

SET UP: The block's angular momentum with respect to the hole is $L = mvr$.

EXECUTE: The tension is related to the block's mass and speed, and the radius of the circle, by $T = m\frac{v^2}{r}$.

$T = mv^2\frac{1}{r} = \frac{m^2v^2}{m}\frac{r^2}{r^3} = \frac{(mvr)^2}{mr^3} = \frac{L^2}{mr^3}$. The radius at which the string breaks is

$r^3 = \frac{L^2}{mT_{\max}} = \frac{(mv_1r_1)^2}{mT_{\max}} = \frac{\left((0.250\text{ kg})(4.00\text{ m/s})(0.800\text{ m})\right)^2}{(0.250\text{ kg})(30.0\text{ N})}$, from which $r = 0.440$ m.

EVALUATE: Just before the string breaks the speed of the rock is $(4.00\text{ m/s})\left(\frac{0.800\text{ m}}{0.440\text{ m}}\right) = 7.27\text{ m/s}$. We can verify that $v = 7.27$ m/s and $r = 0.440$ m do give $T = 30.0$ N.

10.93. **IDENTIFY** and **SET UP:** Apply conservation of angular momentum to the system consisting of the disk and train.

SET UP: $L_1 = L_2$, counterclockwise positive. The motion is sketched in Figure 10.93.

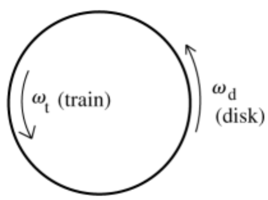


Figure 10.93

$L_1 = 0$ (before you switch on the train's engine; both the train and the platform are at rest)

$L_2 = L_{\text{train}} + L_{\text{disk}}$

EXECUTE: The train is $\frac{1}{2}(0.95\text{ m}) = 0.475\text{ m}$ from the axis of rotation, so for it

$I_t = m_tR_t^2 = (1.20\text{ kg})(0.475\text{ m})^2 = 0.2708\text{ kg}\cdot\text{m}^2$

$\omega_{\text{rel}} = v_{\text{rel}}/R_t = (0.600\text{ m/s})/0.475\text{ s} = 1.263\text{ rad/s}$

This is the angular velocity of the train relative to the disk. Relative to the earth $\omega_t = \omega_{\text{rel}} + \omega_d$.

Thus $L_{\text{train}} = I_t\omega_t = I_t(\omega_{\text{rel}} + \omega_d)$.

$L_2 = L_1$ says $L_{\text{disk}} = -L_{\text{train}}$

$L_{\text{disk}} = I_d\omega_d$, where $I_d = \frac{1}{2}m_dR_d^2$

$\frac{1}{2}m_dR_d^2\omega_d = -I_t(\omega_{\text{rel}} + \omega_d)$

$$\omega_d = -\frac{I_t\omega_{\text{rel}}}{\frac{1}{2}m_dR_d^2 + I_t} = -\frac{(0.2708\text{ kg}\cdot\text{m}^2)(1.263\text{ rad/s})}{\frac{1}{2}(7.00\text{ kg})(0.500\text{ m})^2 + 0.2708\text{ kg}\cdot\text{m}^2} = -0.30\text{ rad/s}.$$

EVALUATE: The minus sign tells us that the disk is rotating clockwise relative to the earth. The disk and train rotate in opposite directions, since the total angular momentum of the system must remain zero. Note that we applied $L_1 = L_2$ in an inertial frame attached to the earth.

10.94. **IDENTIFY:** I for the wheel is the sum of the values of I for each of its parts, the rim and each spoke. The total length of wire is constant. The motion is related to the friction torque by $\sum\tau_z = I\alpha_z$.

SET UP: $4R + 2\pi R = L_0$, where R is the radius of the wheel and therefore the length of each of the four spokes. The mass of a piece is proportional to the length of that piece.

EXECUTE: **(a)** $R = \frac{L_0}{4+2\pi}$. $I_{\text{rim}} = m_{\text{rim}}R^2$. $m_{\text{rim}} = \frac{2\pi R}{L_0}M_0 = \left(\frac{2\pi}{4+2\pi}\right)M_0$.

$I_{\text{rim}} = M_0L_0^2\frac{2\pi}{(2\pi+4)^3} = (5.778\times10^{-3})M_0L_0^2$. $I_{\text{spoke}} = \frac{1}{3}m_{\text{spoke}}R^2$. $m_{\text{spoke}} = \frac{R}{L_0}M_0 = \frac{M_0}{2\pi+4}$ and

$I_{\text{spoke}} = M_0L_0^2\frac{1}{3(2\pi+4)^3} = (3.065\times10^{-4})M_0L_0^2$. $I = I_{\text{rim}} + 4I_{\text{spoke}} = (7.00\times10^{-3})M_0L_0^2$.

(b) $\omega_z = \omega_{0z} + \alpha_z t$ gives $\alpha_z = -\dfrac{\omega_0}{T}$. Then $\sum \tau_z = I\alpha_z$ gives $\tau_\text{f} = (7.00\times10^{-3})M_0 L_0^2 \dfrac{\omega_0}{T}$

EVALUATE: If the wire were bent into a circle, without spokes, the moment of inertia would be $M_0 R^2 = \dfrac{M_0 L_0^2}{(4+2\pi)^2} = (9.46\times10^{-3})M_0 L_0^2$. The actual value of I for the wheel is less than this because the mass in the spokes is closer to the axis than the rim.

10.95. **IDENTIFY** and **SET UP:** Use the methods stipulated in the problem.

EXECUTE: **(a)** The initial angular momentum with respect to the pivot is mvr, and the final total moment of inertia is $I + mr^2$, so the final angular velocity is $\omega = mvr/\left(mr^2 + I\right)$.

(b) The kinetic energy after the collision is converted to gravitational potential energy, so

$$\frac{1}{2}\omega^2\left(mr^2 + I\right) = \left(M+m\right)gh \text{, or } \omega = \sqrt{\frac{2\left(M+m\right)gh}{\left(mr^2+I\right)}}.$$

(c) Substitution of $I = Mr^2$ into the result of part (a) gives $\omega = \left(\dfrac{m}{m+M}\right)\left(v/r\right)$, and into the result of part (b), $\omega = \sqrt{2gh}(1/r)$, which are consistent with the forms for v.

EVALUATE: $I = Mr^2$ applies approximately when the pendulum consists of a heavy catcher mounted on a light arm. In the actual apparatus some of the mass is distributed closer to the axis and $I < Mr^2$.

10.96. **IDENTIFY:** Apply conservation of momentum to the system of the runner and turntable

SET UP: Let the positive sense of rotation be the direction the turntable is rotating initially.

EXECUTE: The initial angular momentum is $I\omega_1 - mRv_1$, with the minus sign indicating that runner's motion is opposite the motion of the part of the turntable under his feet. The final angular momentum is $\omega_2(I + mR^2)$, so $\omega_2 = \dfrac{I\omega_1 - mRv_1}{I + mR^2}$.

$$\omega_2 = \frac{(80\ \text{kg}\cdot\text{m}^2)(0.200\ \text{rad/s}) - (55.0\ \text{kg})(3.00\ \text{m})(2.8\ \text{m/s})}{(80\ \text{kg}\cdot\text{m}^2) + (55.0\ \text{kg})(3.00\ \text{m})^2} = -0.776\ \text{rad/s}.$$

EVALUATE: The minus sign indicates that the turntable has reversed its direction of motion. This happened because the man had the larger magnitude of angular momentum initially.

10.97. **IDENTIFY:** Treat the moon as a point mass, so $L = I\omega = mr^2\omega$, where r is the distance of the moon from the center of the earth. Conservation of angular momentum says $dL/dt = 0$.

SET UP: $dr/dt = 3.0\ \text{cm/y} = 3.0\times10^{-2}\ \text{m/y}$. The period of the moon's orbital motion is $27.3\ \text{d} = 2.36\times10^6\ \text{s}$. $r = 3.84\times10^8\ \text{m}$.

EXECUTE: $dL/dt = \dfrac{d}{dt}(mr^2\omega) = m\omega(2r)\dfrac{dr}{dt} + mr^2\dfrac{d\omega}{dt} = 0$, so $\dfrac{d\omega}{dt} = -\dfrac{2\omega}{r}\dfrac{dr}{dt}$.

$\omega = \dfrac{2\pi\ \text{rad}}{T} = \dfrac{2\pi\ \text{rad}}{2.36\times10^6\ \text{s}} = 2.66\times10^{-6}\ \text{rad/s}$. $\dfrac{d\omega}{dt} = -\dfrac{2(2.66\times10^{-6}\ \text{rad/s})}{3.84\times10^8\ \text{m}}(3.0\times10^{-2}\ \text{m/y}) = -4.2\times10^{-16}\ \text{rad/s per year}$.

$\dfrac{d\omega}{dt}$ is negative, so the angular velocity is decreasing.

EVALUATE: $L = mr^2\omega$. If L is constant, then ω decreases when r increases. The fractional changes in r and ω are very, very small.

10.98. **IDENTIFY:** Follow the method outlined in the hint.

SET UP: $J = m\Delta v_\text{cm}$. $\Delta L = J(x - x_\text{cm})$.

EXECUTE: The velocity of the center of mass will change by $\Delta v_\text{cm} = J/m$ and the angular velocity will change by $\Delta\omega = \dfrac{J(x - x_\text{cm})}{I}$. The change is velocity of the end of the bat will then be $\Delta v_\text{end} = \Delta v_\text{cm} - \Delta\omega x_\text{cm} = \dfrac{J}{m} - \dfrac{J(x - x_\text{cm})x_\text{cm}}{I}$.
Setting $\Delta v_\text{end} = 0$ allows cancellation of J and gives $I = (x - x_\text{cm})x_\text{cm}m$, which when solved for x is

$$x = \frac{I}{x_\text{cm}m} + x_\text{cm} = \frac{(5.30\times10^{-2}\text{kg}\cdot\text{m}^2)}{(0.600\ \text{m})(0.800\ \text{kg})} + (0.600\ \text{m}) = 0.710\ \text{m}.$$

EVALUATE: The center of percussion is farther from the handle than the center of mass.

10.99. **IDENTIFY** and **SET UP:** Follow the analysis that led to Eq.(10.33).

EXECUTE: In Figure 10.33a in the textbook, if the vector $\vec{r}$ and hence the vector $\vec{L}$ are not horizontal but make an angle β with the horizontal, the torque will still be horizontal (the torque must be perpendicular to the vertical weight). The magnitude of the torque will be $\omega r\cos\beta$, and this torque will change the direction of the horizontal component of the angular momentum, which has magnitude $L\cos\beta$. Thus, the situation of Figure 10.35 in the textbook is reproduced, but with $\vec{L}_{\text{horiz}}$ instead of $\vec{L}$. Then, the expression found in Eq. (10.33) becomes

$$\Omega = \frac{d\phi}{dt} = \frac{|d\vec{L}| \big/ |\vec{L}_{\text{horiz}}|}{dt} = \frac{\tau}{|\vec{L}_{\text{horiz}}|} = \frac{mgr\cos\beta}{L\cos\beta} = \frac{wr}{I\omega}.$$

EVALUATE: The torque and the horizontal component of $\vec{L}$ both depend on β by the same factor, $\cos\beta$.

10.100. **IDENTIFY:** Apply conservation of energy to the motion of the ball.

SET UP: In relating $\frac{1}{2}mv_{\text{cm}}^2$ and $\frac{1}{2}I\omega^2$, instead of $v_{\text{cm}} = R\omega$ use the relation derived in part (a). $I = \frac{2}{5}mR^2$.

EXECUTE: **(a)** Consider the sketch in Figure 10.100.
The distance from the center of the ball to the midpoint of the line joining the points where the ball is in contact with the rails is $\sqrt{R^2 - (d/2)^2}$, so $v_{\text{cm}} = \omega\sqrt{R^2 - d^2/4}$. When $d = 0$, this reduces to $v_{\text{cm}} = \omega R$, the same as rolling on a flat surface. When $d = 2R$, the rolling radius approaches zero, and $v_{\text{cm}} \to 0$ for any ω.

(b) $K = \dfrac{1}{2}mv^2 + \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\left[mv_{\text{cm}}^2 + (2/5)mR^2\left(\dfrac{v_{\text{cm}}}{\sqrt{R^2 - (d^2/4)}}\right)^2\right] = \dfrac{mv_{\text{cm}}^2}{10}\left[5 + \dfrac{2}{(1 - d^2/4R^2)}\right]$

Setting this equal to mgh and solving for v_{cm} gives the desired result.

(c) The denominator in the square root in the expression for v_{cm} is larger than for the case $d = 0$, so v_{cm} is smaller. For a given speed, ω is larger than in the $d = 0$ case, so a larger fraction of the kinetic energy is rotational, and the translational kinetic energy, and hence v_{cm} , is smaller.

(d) Setting the expression in part (b) equal to 0.95 of that of the $d = 0$ case and solving for the ratio d/R gives $d/R = 1.05$. Setting the ratio equal to 0.995 gives $d/R = 0.37$.

EVALUATE: If we set $d = 0$ in the expression in part (b), $v_{\text{cm}} = \sqrt{\dfrac{10gh}{7}}$, the same as for a sphere rolling down a ramp. When $d \to 2R$, the expression gives $v_{\text{cm}} = 0$, as it should.

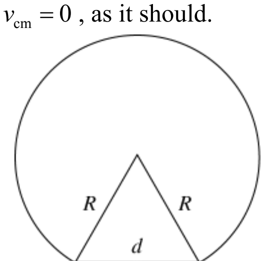


Figure 10.100

10.101. **IDENTIFY:** Apply $\sum \vec{F}_{\text{ext}} = m\vec{a}_{\text{cm}}$ and $\sum \tau_z = I_{\text{cm}}\alpha_z$ to the motion of the cylinder. Use constant acceleration equations to relate a_x to the distance the object travels. Use the work-energy theorem to find the work done by friction.

SET UP: The cylinder has $I_{\text{cm}} = \frac{1}{2}MR^2$.

EXECUTE: **(a)** The free-body diagram is sketched in Figure 10.101. The friction force is $f = \mu_k n = \mu_k Mg$, so $a = \mu_k g$. The magnitude of the angular acceleration is $\dfrac{fR}{I} = \dfrac{\mu_k MgR}{(1/2)MR^2} = \dfrac{2\mu_k g}{R}$.

(b) Setting $v = at = \omega R = (\omega_0 - \omega t)R$ and solving for t gives $t = \dfrac{R\omega_0}{a + R\alpha} = \dfrac{R\omega_0}{\mu_k g + 2\mu_k g} = \dfrac{R\omega_0}{3\mu_k g}$,

and $d = \dfrac{1}{2}at^2 = \dfrac{1}{2}(\mu_k g)\left(\dfrac{R\omega_0}{3\mu_k g}\right)^2 = \dfrac{R^2\omega_0^2}{18\mu_k g}$.

(c) The final kinetic energy is $(3/4)Mv^2 = (3/4)M(at)^2$, so the change in kinetic energy is

$$\Delta K = \frac{3}{4}M\left(\mu_k g\frac{R\omega_0}{3\mu_k g}\right)^2 - \frac{1}{4}MR^2\omega_0^2 = -\frac{1}{6}MR^2\omega_0^2.$$

EVALUATE: The fraction of the initial kinetic energy that is removed by friction work is $\frac{\frac{1}{6}MR\omega_0^2}{\frac{1}{4}MR\omega_0^2} = \frac{2}{3}$. This fraction is independent of the initial angular speed ω_0.

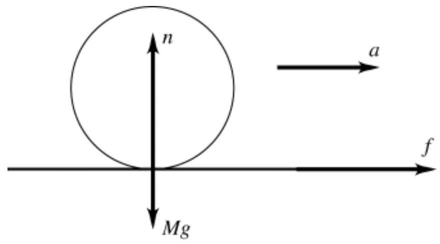


Figure 10.101

10.102. IDENTIFY: The vertical forces must sum to zero. Apply Eq.(10.33).

SET UP: Denote the upward forces that the hands exert as F_L and F_R. $\tau = (F_L - F_R)r$, where $r = 0.200$ m.

EXECUTE: The conditions that F_L and F_R must satisfy are $F_L + F_R = w$ and $F_L - F_R = \Omega\frac{I\omega}{r}$, where the second equation is $\tau = \Omega L$, divided by *r*. These two equations can be solved for the forces by first adding and then subtracting, yielding $F_L = \frac{1}{2}\left(w + \Omega\frac{I\omega}{r}\right)$ and $F_R = \frac{1}{2}\left(w - \Omega\frac{I\omega}{r}\right)$. Using the values $w = mg = (8.00\text{ kg})(9.80\text{ m/s}^2) = 78.4\text{ N}$ and

$$\frac{I\omega}{r} = \frac{(8.00\text{ kg})(0.325\text{ m})^2(5.00\text{ rev/s}\times 2\pi\text{ rad/rev})}{(0.200\text{ m})} = 132.7\text{ kg}\cdot\text{m/s} \text{ gives}$$

$F_L = 39.2\text{ N} + \Omega(66.4\text{ N}\cdot\text{s}),\ F_R = 39.2\text{ N} - \Omega(66.4\text{ N}\cdot\text{s}).$

(a) $\Omega = 0, F_L = F_R = 39.2$ N.

(b) $\Omega = 0.05\text{ rev/s} = 0.314\text{ rad/s}, F_L = 60.0\text{ N}, F_R = 18.4\text{ N}.$

(c) $\Omega = 0.3\text{ rev/s} = 1.89\text{ rad/s}, F_L = 165\text{ N}, F_R = -86.2\text{ N}$, with the minus sign indicating a downward force.

(e) $F_R = 0$ gives $\Omega = \frac{39.2\text{ N}}{66.4\text{ N}\cdot\text{s}} = 0.575\text{ rad/s}$, which is 0.0916 rev/s.

EVALUATE: The larger the precession rate Ω, the greater the torque on the wheel and the greater the difference between the forces exerted by the two hands.

10.103. IDENTIFY: The answer to part (a) can be taken from the solution to Problem 10.92. The work-energy theorem says $W = \Delta K$.

SET UP: Problem 10.92 uses conservation of angular momentum to show that $r_1 v_1 = r_2 v_2$.

EXECUTE: **(a)** $T = mv_1^2 r_1^2/r^3$.

(b) $\vec{T}$ and $d\vec{r}$ are always antiparallel. $\vec{T}\cdot d\vec{r} = -Tdr$.

$$W = -\int_{r_1}^{r_2} T\,dr = mv_1^2 r_1^2 \int_{r_2}^{r_1}\frac{dr}{r^3} = \frac{mv_1^2}{2}r_1^2\left[\frac{1}{r_2^2} - \frac{1}{r_1^2}\right].$$

(c) $v_2 = v_1(r_1/r_2)$, so $\Delta K = \frac{1}{2}m(v_2^2 - v_1^2) = \frac{mv_1^2}{2}\left[(r_1/r_2)^2 - 1\right]$, which is the same as the work found in part (b).

EVALUATE: The work done by *T* is positive, since $\vec{T}$ is toward the hole in the surface and the block moves toward the hole. Positive work means the kinetic energy of the object increases.

11

EQUILIBRIUM AND ELASTICITY

11.1. **IDENTIFY:** Use Eq.(11.3) to calculate x_{cm}. The center of gravity of the bar is at its center and it can be treated as a point mass at that point.

SET UP: Use coordinates with the origin at the left end of the bar and the $+x$ axis along the bar. $m_1 = 2.40$ kg, $m_2 = 1.10$ kg, $m_3 = 2.20$ kg.

EXECUTE: $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \dfrac{(2.40\text{ kg})(0.250\text{ m}) + 0 + (2.20\text{ kg})(0.500\text{ m})}{2.40\text{ kg} + 1.10\text{ kg} + 2.20\text{ kg}} = 0.298\text{ m}$. The fulcrum should be placed 29.8 cm to the right of the left-hand end.

EVALUATE: The mass at the right-hand end is greater than the mass at the left-hand end. So the center of gravity is to the right of the center of the bar.

11.2. **IDENTIFY:** Use Eq.(11.3) to calculate x_{cm} of the composite object.

SET UP: Use coordinates where the origin is at the original center of gravity of the object and $+x$ is to the right. With the 1.50 g mass added, $x_{cm} = -2.20\text{ cm}$, $m_1 = 5.00\text{ g}$ and $m_2 = 1.50\text{ g}$. $x_1 = 0$.

EXECUTE: $x_{cm} = \dfrac{m_2x_2}{m_1 + m_2}$. $x_2 = \left(\dfrac{m_1 + m_2}{m_2}\right)x_{cm} = \left(\dfrac{5.00\text{ g} + 1.50\text{ g}}{1.50\text{ g}}\right)(-2.20\text{ cm}) = -9.53\text{ cm}$.

The additional mass should be attached 9.53 cm to the left of the original center of gravity.

EVALUATE: The new center of gravity is somewhere between the added mass and the original center of gravity.

11.3. **IDENTIFY:** The center of gravity of the combined object must be at the fulcrum. Use Eq.(11.3) to calculate x_{cm}

SET UP: The center of gravity of the sand is at the middle of the box. Use coordinates with the origin at the fulcrum and $+x$ to the right. Let $m_1 = 25.0\text{ kg}$, so $x_1 = 0.500\text{ m}$. Let $m_2 = m_{sand}$, so $x_2 = -0.625\text{ m}$. $x_{cm} = 0$.

EXECUTE: $x_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2} = 0$ and $m_2 = -m_1\dfrac{x_1}{x_2} = -(25.0\text{ kg})\left(\dfrac{0.500\text{ m}}{-0.625\text{ m}}\right) = 20.0\text{ kg}$.

EVALUATE: The mass of sand required is less than the mass of the plank since the center of the box is farther from the fulcrum than the center of gravity of the plank is.

11.4. **IDENTIFY:** Apply the first and second conditions for equilibrium to the trap door.

SET UP: For $\sum\tau_z = 0$ take the axis at the hinge. Then the torque due to the applied force must balance the torque due to the weight of the door.

EXECUTE: **(a)** The force is applied at the center of gravity, so the applied force must have the same magnitude as the weight of the door, or 300 N. In this case the hinge exerts no force.

(b) With respect to the hinges, the moment arm of the applied force is twice the distance to the center of mass, so the force has half the magnitude of the weight, or 150 N. The hinges supply an upward force of $300\text{ N} - 150\text{ N} = 150\text{ N}$.

EVALUATE: Less force must be applied when it is applied farther from the hinges.

11.5. **IDENTIFY:** Apply $\sum\tau_z = 0$ to the ladder.

SET UP: Take the axis to be at point A. The free-body diagram for the ladder is given in Figure 11.5. The torque due to F must balance the torque due to the weight of the ladder.

EXECUTE: $F(8.0\text{ m})\sin 40° = (2800\text{ N})(10.0\text{ m})$, so $F = 5.45\text{ kN}$.

EVALUATE: The force required is greater than the weight of the ladder, because the moment arm for F is less than the moment arm for w.

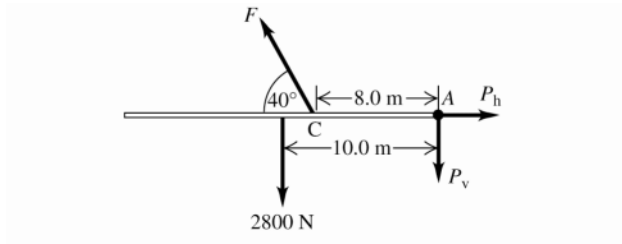


Figure 11.5

11.6. **IDENTIFY:** Apply the first and second conditions of equilibrium to the board.

SET UP: The free-body diagram for the board is given in Figure 11.6. Since the board is uniform its center of gravity is 1.50 m from each end. Apply $\sum F_y = 0$, with $+y$ upward. Apply $\sum \tau = 0$ with the axis at the end where the first person applies a force and with counterclockwise torques positive.

EXECUTE: $\sum F_y = 0$ gives $F_1 + F_2 - w = 0$ and $F_2 = w - F_1 = 160\text{ N} - 60\text{ N} = 100\text{ N}$. $\sum \tau = 0$ gives $F_2 x - w(1.50\text{ m}) = 0$ and $x = \left(\dfrac{w}{F_2}\right)(1.50\text{ m}) = \left(\dfrac{160\text{ N}}{100\text{ N}}\right)(1.50\text{ m}) = 2.40\text{ m}$. The other person lifts with a force of 100 N at a point 2.40 m from the end where the other person lifts.

EVALUATE: By considering the axis at the center of gravity we can see that a larger force is applied by the person who pushes closer to the center of gravity.

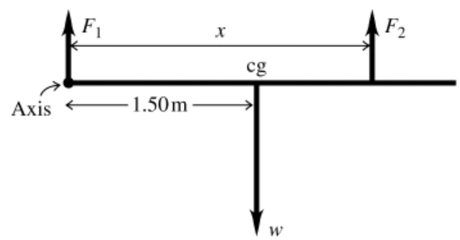


Figure 11.6

11.7. **IDENTIFY:** Apply $\sum F_y = 0$ and $\sum \tau_z = 0$ to the board.

SET UP: Let $+y$ be upward. Let x be the distance of the center of gravity of the motor from the end of the board where the 400 N force is applied.

EXECUTE: **(a)** If the board is taken to be massless, the weight of the motor is the sum of the applied forces, 1000 N. The motor is a distance $\dfrac{(2.00\text{ m})(600\text{ N})}{(1000\text{ N})} = 1.20\text{ m}$ from the end where the 400 N force is applied, and so is 0.800 m from the end where the 600 N force is applied.

(b) The weight of the motor is $400\text{ N} + 600\text{ N} - 200\text{ N} = 800\text{ N}$. Applying $\sum \tau_z = 0$ with the axis at the end of the board where the 400 N acts gives $(600\text{ N})(2.00\text{ m}) = (200\text{ N})(1.00\text{ m}) + (800\text{ N})x$ and $x = 1.25\text{ m}$. The center of gravity of the motor is 0.75 m from the end of the board where the 600 N force is applied.

EVALUATE: The motor is closest to the end of the board where the larger force is applied.

11.8. **IDENTIFY:** Apply the first and second conditions of equilibrium to the shelf.

SET UP: The free-body diagram for the shelf is given in Figure 11.8. Take the axis at the left-hand end of the shelf and let counterclockwise torque be positive. The center of gravity of the uniform shelf is at its center.

EXECUTE: **(a)** $\sum \tau_z = 0$ gives $-w_t(0.200\text{ m}) - w_s(0.300\text{ m}) + T(0.400\text{ m}) = 0$.

$$T = \frac{(25.0\text{ N})(0.200\text{ m}) + (50.0\text{ N})(0.300\text{ m})}{0.400\text{ m}} = 50.0\text{ N}$$

$\sum F_y = 0$ gives $T_1 + T_2 - w_t - w_s = 0$ and $T_1 = 25.0\text{ N}$. The tension in the left-hand wire is 25.0 N and the tension in the right-hand wire is 50.0 N.

EVALUATE: We can verify that $\sum \tau_z = 0$ is zero for any axis, for example for an axis at the right-hand end of the shelf.

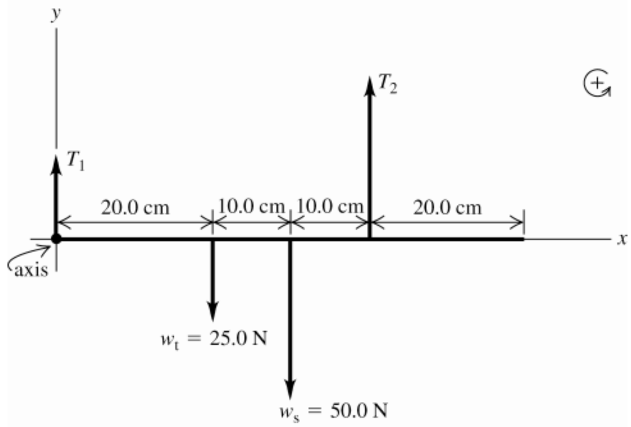


Figure 11.8

11.9. **IDENTIFY:** Apply the conditions for equilibrium to the bar. Set each tension equal to its maximum value.
SET UP: Let cable *A* be at the left-hand end. Take the axis to be at the left-hand end of the bar and *x* be the distance of the weight *w* from this end. The free-body diagram for the bar is given in Figure 11.9.
EXECUTE: **(a)** $\sum F_y = 0$ gives $T_A + T_B - w - w_{\text{bar}} = 0$ and
$w = T_A + T_B - w_{\text{bar}} = 500.0\text{ N} + 400.0\text{ N} - 350.0\text{ N} = 550\text{ N}$.
(b) $\sum \tau_z = 0$ gives $T_B(1.50\text{ m}) - wx - w_{\text{bar}}(0.750\text{ m}) = 0$.
$x = \dfrac{T_B(1.50\text{ m}) - w_{\text{bar}}(0.750\text{ m})}{w} = \dfrac{(400.0\text{ N})(1.50\text{ m}) - (350\text{ N})(0.750\text{ m})}{550\text{ N}} = 0.614\text{ m}$. The weight should be placed 0.614 m from the left-hand end of the bar.
EVALUATE: If the weight is moved to the left, T_A exceeds 500.0 N and if it is moved to the right T_B exceeds 400.0 N.

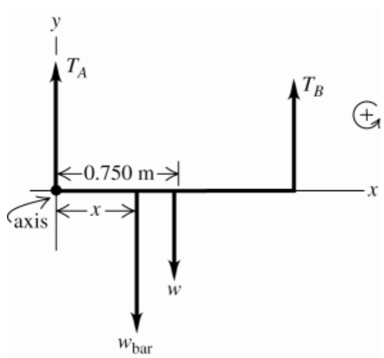


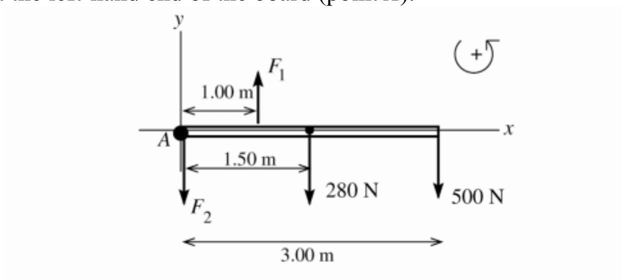
Figure 11.9

11.10. **IDENTIFY:** Apply the first and second conditions for equilibrium to the ladder.
SET UP: Let n_2 be the upward normal force exerted by the ground and let n_1 be the horizontal normal force exerted by the wall. The maximum possible static friction force that can be exerted by the ground is $\mu_s n_2$.
EXECUTE: **(a)** Since the wall is frictionless, the only vertical forces are the weights of the man and the ladder, and the normal force n_2 . For the vertical forces to balance, $n_2 = w_1 + w_m = 160\text{ N} + 740\text{ N} = 900\text{ N}$, and the maximum frictional force is $\mu_s n_2 = (0.40)(900\text{N}) = 360\text{N}$.
(b) Note that the ladder makes contact with the wall at a height of 4.0 m above the ground. Balancing torques about the point of contact with the ground, $(4.0\text{ m})n_1 = (1.5\text{ m})(160\text{ N}) + (1.0\text{ m})(3/5)(740\text{ N}) = 684\text{ N}\cdot\text{m}$, so $n_1 = 171.0\text{ N}$. This horizontal force about must be balanced by the friction force, which must then be 170 N to two figures.
(c) Setting the friction force, and hence n_1 , equal to the maximum of 360 N and solving for the distance *x* along the ladder, $(4.0\text{ m})(360\text{ N}) = (1.50\text{ m})(160\text{ N}) + x(3/5)(740\text{ N})$, so $x = 2.7\text{ m}$.

EVALUATE: The normal force exerted by the ground doesn't change as the man climbs up the ladder. But the normal force exerted by the wall and the friction force exerted by the ground both increase as he moves up the ladder.

11.11. **IDENTIFY:** The system of the person and diving board is at rest so the two conditions of equilibrium apply.

(a) SET UP: The free-body diagram for the diving board is given in Figure 11.11. Take the origin of coordinates at the left-hand end of the board (point A).



$\vec{F}_1$ is the force applied at the support point and $\vec{F}_2$ is the force at the end that is held down.

Figure 11.11

EXECUTE: $\sum \tau_A = 0$ gives $+F_1(1.0\text{ m}) - (500\text{ N})(3.00\text{ m}) - (280\text{ N})(1.50\text{ m}) = 0$

$$F_1 = \frac{(500\text{ N})(3.00\text{ m}) + (280\text{ N})(1.50\text{ m})}{1.00\text{ m}} = 1920\text{ N}$$

(b) $\sum F_y = ma_y$

$F_1 - F_2 - 280\text{ N} - 500\text{ N} = 0$

$F_2 = F_1 - 280\text{ N} - 500\text{ N} = 1920\text{ N} - 280\text{ N} - 500\text{ N} = 1140\text{ N}$

EVALUATE: We can check our answers by calculating the net torque about some point and checking that $\tau_z = 0$ for that point also. Net torque about the right-hand of the board:
$(1140\text{ N})(3.00\text{ m}) + (280\text{ N})(1.50\text{ m}) - (1920\text{ N})(2.00\text{ m}) = 3420\text{ N}\cdot\text{m} + 420\text{ N}\cdot\text{m} - 3840\text{ N}\cdot\text{m} = 0$, which checks.

11.12. **IDENTIFY:** Apply the first and second conditions of equilibrium to the beam.

SET UP: The boy exerts a downward force on the beam that is equal to his weight.

EXECUTE: **(a)** The graphs are given in Figure 11.12.

(b) $x = 6.25$ m when $F_A = 0$, which is 1.25 m beyond point B.

(c) Take torques about the right end. When the beam is just balanced, $F_A = 0$, so $F_B = 900$ N. The distance that point B must be from the right end is then $\dfrac{(300\text{ N})(4.50\text{ m})}{(900\text{ N})} = 1.50$ m.

EVALUATE: When the beam is on the verge of tipping it starts to lift off the support A and the normal force F_A exerted by the support goes to zero.

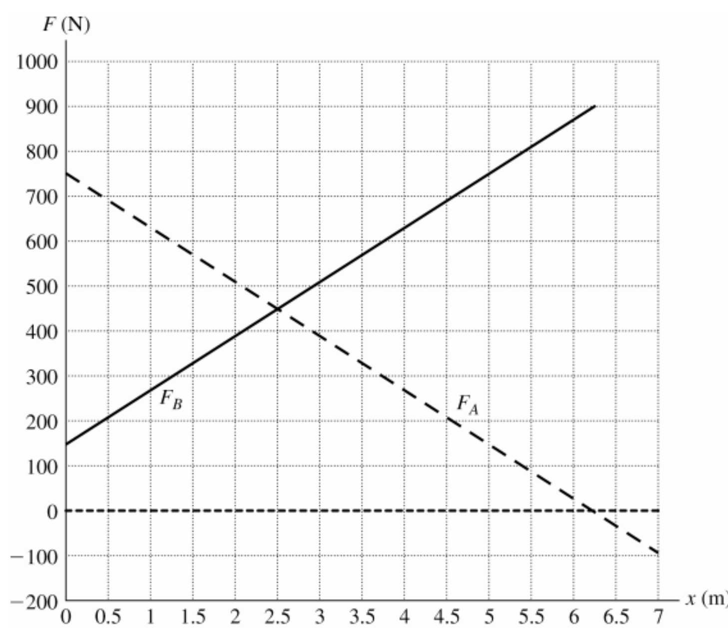


Figure 11.12

11.13. **IDENTIFY:** Apply the first and second conditions of equilibrium to the strut.

(a) SET UP: The free-body diagram for the strut is given in Figure 11.13a. Take the origin of coordinates at the hinge (point A) and $+y$ upward. Let F_{h} and F_{v} be the horizontal and vertical components of the force $\vec{F}$ exerted on the strut by the pivot. The tension in the vertical cable is the weight w of the suspended object. The weight w of the strut can be taken to act at the center of the strut. Let L be the length of the strut.

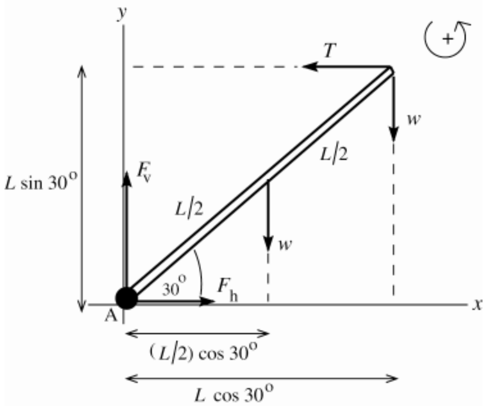


Figure 11.13a

EXECUTE:

$\sum F_y = ma_y$

$F_{\text{v}} - w - w = 0$

$F_{\text{v}} = 2w$

Sum torques about point A. The pivot force has zero moment arm for this axis and so doesn't enter into the torque equation.

$\tau_A = 0$

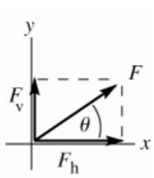
$TL\sin 30.0° - w\big((L/2)\cos 30.0°\big) - w(L\cos 30.0°) = 0$

$T\sin 30.0° - (3w/2)\cos 30.0° = 0$

$$T = \frac{3w\cos 30.0°}{2\sin 30.0°} = 2.60w$$

Then $\sum F_x = ma_x$ implies $T - F_{\text{h}} = 0$ and $F_{\text{h}} = 2.60w.$

We now have the components of $\vec{F}$ so can find its magnitude and direction (Figure 11.13b)



$F = \sqrt{F_{\text{h}}^2 + F_{\text{v}}^2}$

$F = \sqrt{(2.60w)^2 + (2.00w)^2}$

$F = 3.28w$

$$\tan\theta = \frac{F_{\text{v}}}{F_{\text{h}}} = \frac{2.00w}{2.60w}$$

$\theta = 37.6°$

Figure 11.13b

(b) SET UP: The free-body diagram for the strut is given in Figure 11.13c.

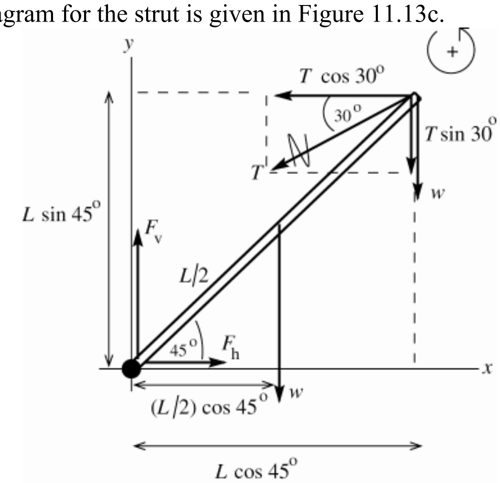


Figure 11.13c

The tension T has been replaced by its x and y components. The torque due to T equals the sum of the torques of its components, and the latter are easier to calculate.

EXECUTE: $\sum \tau_A = 0 + (T\cos 30.0°)(L\sin 45.0°) - (T\sin 30.0°)(L\cos 45.0°) -$
$w((L/2)\cos 45.0°) - w(L\cos 45.0°) = 0$

The length L divides out of the equation. The equation can also be simplified by noting that $\sin 45.0° = \cos 45.0°$. Then $T(\cos 30.0° - \sin 30.0°) = 3w/2$.

$$T = \frac{3w}{2(\cos 30.0° - \sin 30.0°)} = 4.10w$$

$\sum F_x = ma_x$

$F_h - T\cos 30.0° = 0$

$F_h = T\cos 30.0° = (4.10w)(\cos 30.0°) = 3.55w$

$\sum F_y = ma_y$

$F_v - w - w - T\sin 30.0° = 0$

$F_v = 2w + (4.10w)\sin 30.0° = 4.05w$

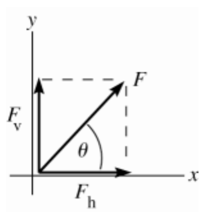


Figure 11.13d

From Figure 11.13d,

$F = \sqrt{F_h^2 + F_v^2}$

$F = \sqrt{(3.55w)^2 + (4.05w)^2} = 5.39w$

$\tan\theta = \frac{F_v}{F_h} = \frac{4.05w}{3.55w}$

$\theta = 48.8°$

EVALUATE: In each case the force exerted by the pivot does not act along the strut. Consider the net torque about the upper end of the strut. If the pivot force acted along the strut, it would have zero torque about this point. The two forces acting at this point also have zero torque and there would be one nonzero torque, due to the weight of the strut. The net torque about this point would then not be zero, violating the second condition of equilibrium.

11.14. IDENTIFY: Apply the first and second conditions of equilibrium to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.14. H_v and H_h are the vertical and horizontal components of the force exerted on the beam at the wall (by the hinge). Since the beam is uniform, its center of gravity is 2.00 m from each end. The angle θ has $\cos\theta = 0.800$ and $\sin\theta = 0.600$. The tension T has been replaced by its x and y components.

EXECUTE: **(a)** H_v, H_h and $T_x = T\cos\theta$ all produce zero torque. $\sum \tau = 0$ gives

$-w(2.00\text{ m}) - w_{load}(4.00\text{ m}) + T\sin\theta(4.00\text{ m}) = 0$ and $T = \frac{(150\text{ N})(2.00\text{ m}) + (300\text{ N})(4.00\text{ m})}{(4.00\text{ m})(0.600)} = 625\text{ N}$.

(b) $\sum F_x = 0$ gives $H_h = T\cos\theta = 0$ and $H_h = (625\text{ N})(0.800) = 500\text{ N}$. $\sum F_y = 0$ gives

$H_v - w - w_{load} + T\sin\theta = 0$ and $H_v = w + w_{load} - T\sin\theta = 150\text{ N} + 300\text{ N} - (625\text{ N})(0.600) = 75\text{ N}$.

EVALUATE: For an axis at the right-hand end of the beam, only w and H_v produce torque. The torque due to w is counterclockwise so the torque due to H_v must be clockwise. To produce a counterclockwise torque, H_v must be upward, in agreement with our result from $\sum F_y = 0$.

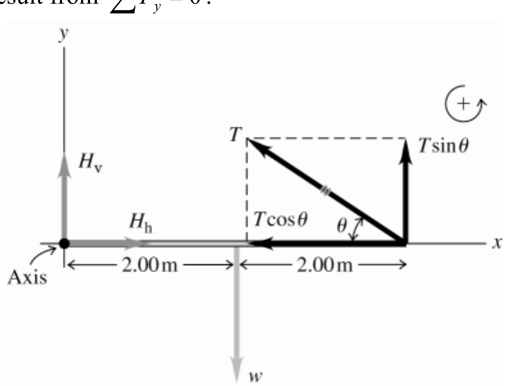


Figure 11.14

11.15. **IDENTIFY:** Apply the first and second conditions of equilibrium to the door.

SET UP: The free-body diagram for the door is given in Figure 11.15. Let $\vec{\boldsymbol{H}}_1$ and $\vec{\boldsymbol{H}}_2$ be the forces exerted by the upper and lower hinges. Take the origin of coordinates at the bottom hinge (point A) and $+y$ upward.

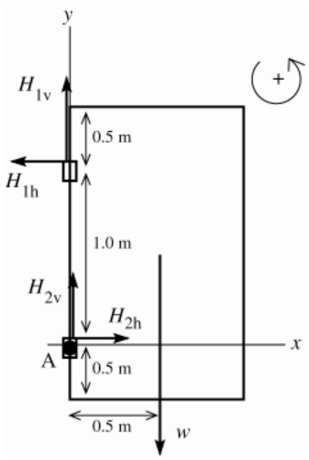


Figure 11.15

EXECUTE:
We are given that
$H_{1\text{v}} = H_{2\text{v}} = w/2 = 140\text{ N}.$
$\sum F_x = ma_x$
$H_{2\text{h}} - H_{1\text{h}} = 0$
$H_{1\text{h}} = H_{2\text{h}}$
The horizontal components of the hinge forces are equal in magnitude and opposite in direction.

Sum torques about point A. $H_{1\text{v}}$, $H_{2\text{v}}$, and $H_{2\text{h}}$ all have zero moment arm and hence zero torque about an axis at this point. Thus $\sum \tau_A = 0$ gives $H_{1\text{h}}(1.00\text{ m}) - w(0.50\text{ m}) = 0$

$$H_{1\text{h}} = w\left(\frac{0.50\text{ m}}{1.00\text{ m}}\right) - \tfrac{1}{2}(280\text{ N}) = 140\text{ N}.$$

The horizontal component of each hinge force is 140 N.

EVALUATE: The horizontal components of the force exerted by each hinge are the only horizontal forces so must be equal in magnitude and opposite in direction. With an axis at A, the torque due to the horizontal force exerted by the upper hinge must be counterclockwise to oppose the clockwise torque exerted by the weight of the door. So, the horizontal force exerted by the upper hinge must be to the left. You can also verify that the net torque is also zero if the axis is at the upper hinge.

11.16. **IDENTIFY:** Apply the conditions of equilibrium to the wheelbarrow plus its contents. The upward force applied by the person is 650 N.

SET UP: The free-body diagram for the wheelbarrow is given in Figure 11.16. $F = 650\text{ N}$, $w_{\text{wb}} = 80.0\text{ N}$ and w is the weight of the load placed in the wheelbarrow.

EXECUTE: **(a)** $\sum \tau_z = 0$ with the axis at the center of gravity gives $n(0.50\text{ m}) - F(0.90\text{ m}) = 0$ and

$n = F\left(\frac{0.90\text{ m}}{0.50\text{ m}}\right) = 1170\text{ N}$. $\sum F_y = 0$ gives $F + n - w_{\text{wb}} - w = 0$ and

$w = F + n - w_{\text{wb}} = 650\text{ N} + 1170\text{ N} - 80.0\text{ N} = 1740\text{ N}$.

(b) The extra force is applied by the ground pushing up on the wheel.

EVALUATE: You can verify that $\sum \tau_z = 0$ for any axis, for example for an axis where the wheel contacts the ground.

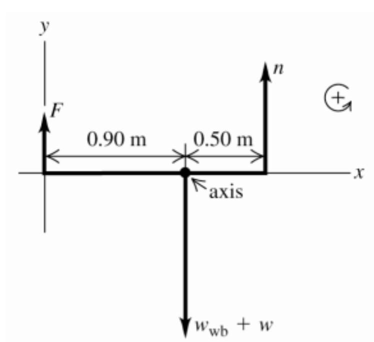


Figure 11.16

11.17. **IDENTIFY:** Apply the first and second conditions of equilibrium to Clea.

SET UP: Consider the forces on Clea. The free-body diagram is given in Figure 11.17

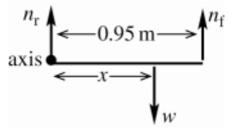


Figure 11.17

EXECUTE:

$n_r = 89$ N, $n_f = 157$ N

$n_r + n_f = w$ so $w = 246$ N

$\sum \tau_z = 0$, axis at rear feet

Let x be the distance from the rear feet to the center of gravity.

$n_f(0.95 \text{ m}) - xw = 0$

$x = 0.606$ m from rear feet so 0.34 m from front feet.

EVALUATE: The normal force at her front feet is greater than at her rear feet, so her center of gravity is closer to her front feet.

11.18. **IDENTIFY:** Apply the conditions for equilibrium to the crane.

SET UP: The free-body diagram for the crane is sketched in Figure 11.18. F_h and F_v are the components of the force exerted by the axle. $\vec{T}$ pulls to the left so F_h is to the right. $\vec{T}$ also pulls downward and the two weights are downward, so F_v is upward.

EXECUTE: **(a)** $\sum \tau_z = 0$ gives $T([13 \text{ m}]\sin 25° - w_c([7.0 \text{ m}]\cos 55°) - w_b([16.0 \text{ m}]\cos 55° = 0$.

$$T = \frac{(11{,}000 \text{ N})([16.0 \text{ m}]\cos 55°) + (15{,}000 \text{ N})([7.0 \text{ m}]\cos 55°)}{(13.0 \text{ m})\sin 25°} = 2.93 \times 10^4 \text{ N}.$$

(b) $\sum F_x = 0$ gives $F_h - T\cos 30° = 0$ and $F_h = 2.54 \times 10^4$ N.

$\sum F_y = 0$ gives $F_v - T\sin 30° - w_c - w_b = 0$ and $F_v = 4.06 \times 10^4$ N.

EVALUATE: $\tan\theta = \dfrac{F_v}{F_h} = \dfrac{4.06 \times 10^4 \text{ N}}{2.54 \times 10^4 \text{ N}}$ and $\theta = 58°$. The force exerted by the axle is not directed along the crane.

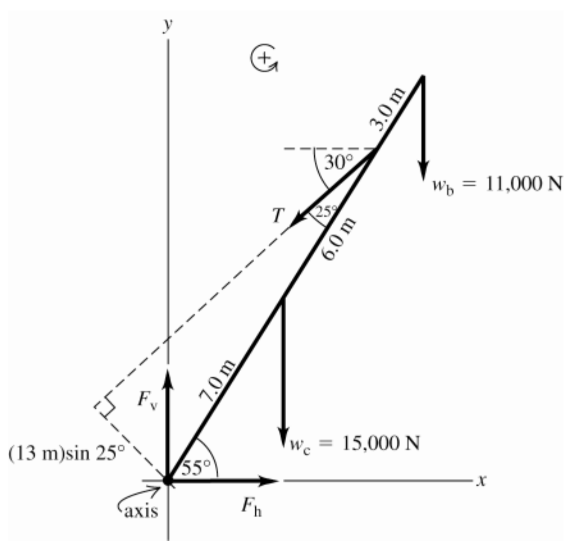


Figure 11.18

11.19. **IDENTIFY:** Apply the first and second conditions of equilibrium to the rod.

SET UP: The force diagram for the rod is given in Figure 11.19.

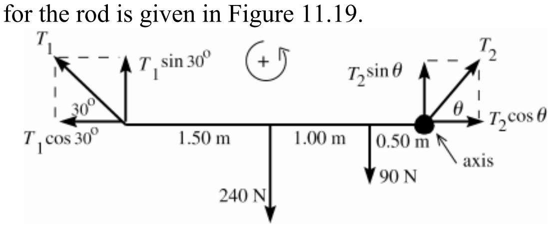


Figure 11.19

EXECUTE: $\sum \tau_z = 0$, axis at right end of rod, counterclockwise torque is positive

$(240\text{ N})(1.50\text{ m}) + (90\text{ N})(0.50\text{ m}) - (T_1 \sin 30.0°)(3.00\text{ m}) = 0$

$T_1 = \dfrac{360\text{ N}\cdot\text{m} + 45\text{ N}\cdot\text{m}}{1.50\text{ m}} = 270\text{ N}$

$\sum F_x = ma_x$

$T_2 \cos\theta - T_1 \cos 30° = 0$ and $T_2 \cos\theta = 234\text{ N}$

$\sum F_y = ma_y$

$T_1 \sin 30° + T_2 \sin\theta - 240\text{ N} - 90\text{ N} = 0$

$T_2 \sin\theta = 330\text{ N} - (270\text{ N})\sin 30° = 195\text{ N}$

Then $\dfrac{T_2 \sin\theta}{T_2 \cos\theta} = \dfrac{195\text{ N}}{234\text{ N}}$ gives $\tan\theta = 0.8333$ and $\theta = 40°$

And $T_2 = \dfrac{195\text{ N}}{\sin 40°} = 303\text{ N}.$

EVALUATE: The monkey is closer to the right rope than to the left one, so the tension is larger in the right rope. The horizontal components of the tensions must be equal in magnitude and opposite in direction. Since $T_2 > T_1$, the rope on the right must be at a greater angle above the horizontal to have the same horizontal component as the tension in the other rope.

11.20. **IDENTIFY:** Apply the first and second conditions for equilibrium to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.20.

EXECUTE: The cable is given as perpendicular to the beam, so the tension is found by taking torques about the pivot point; $T(3.00\text{ m}) = (1.00\text{ kN})(2.00\text{ m})\cos 25.0° + (5.00\text{ kN})(4.50\text{ m})\cos 25.0°$, and $T = 7.40\text{ kN}$. The vertical component of the force exerted on the beam by the pivot is the net weight minus the upward component of T, $6.00\text{ kN} - T\cos 25.0° = 0.17\text{ kN}$. The horizontal force is $T\sin 25.0° = 3.13\text{ kN}$.

EVALUATE: The vertical component of the tension is nearly the same magnitude as the total weight of the object and the vertical component of the force exerted by the pivot is much less than its horizontal component.

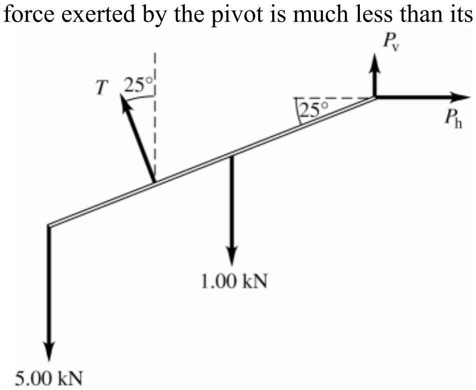


Figure 11.20

11.21. **(a) IDENTIFY** and **SET UP:** Use Eq.(10.3) to calculate the torque (magnitude and direction) for each force and add the torques as vectors. See Figure 11.21a.

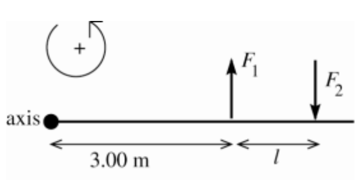


Figure 11.21a

EXECUTE:

$\tau_1 = F_1 l_1 = +(8.00\text{ N})(3.00\text{ m})$

$\tau_1 = +24.0\text{ N}\cdot\text{m}$

$\tau_2 = -F_2 l_2 = -(8.00\text{ N})(l + 3.00\text{ m})$

$\tau_2 = -24.0\text{ N}\cdot\text{m} - (8.00\text{ N})l$

$\sum \tau_z = \tau_1 + \tau_2 = +24.0\text{ N}\cdot\text{m} - 24.0\text{ N}\cdot\text{m} - (8.00\text{ N})l = -(8.00\text{ N})l$

Want l that makes $\sum \tau_z = -6.40\text{ N}\cdot\text{m}$ (net torque must be clockwise)

$-(8.00\text{ N})l = -6.40\text{ N}\cdot\text{m}$

$l = (6.40\text{ N}\cdot\text{m})/8.00\text{ N} = 0.800\text{ m}$

(b) $|\tau_2| > |\tau_1|$ since F_2 has a larger moment arm; the net torque is clockwise.

(c) See Figure 11.21b.

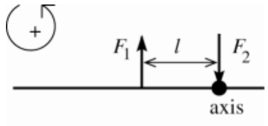


Figure 11.21b

$\tau_1 = -F_1 l_1 = -(8.00\text{ N})l$

$\tau_2 = 0$ since $\vec{F}_2$ is at the axis

$\sum \tau_z = -6.40\text{ N}\cdot\text{m}$ gives $-(8.00\text{ N})l = -6.40\text{ N}\cdot\text{m}$

$l = 0.800\text{ m},$ same as in part (a).

EVALUATE: The force couple gives the same magnitude of torque for the pivot at any point.

11.22. IDENTIFY: $Y = \dfrac{l_0 F_\perp}{A\Delta l}$

SET UP: $A = 50.0\text{ cm}^2 = 50.0\times10^{-4}\text{ m}^2$.

EXECUTE: relaxed: $Y = \dfrac{(0.200\text{ m})(25.0\text{ N})}{(50.0\times10^{-4}\text{ m}^2)(3.0\times10^{-2}\text{ m})} = 3.33\times10^4\text{ Pa}$

maximum tension: $Y = \dfrac{(0.200\text{ m})(500\text{ N})}{(50.0\times10^{-4}\text{ m}^2)(3.0\times10^{-2}\text{ m})} = 6.67\times10^5\text{ Pa}$

EVALUATE: The muscle tissue is much more difficult to stretch when it is under maximum tension.

11.23. IDENTIFY and SET UP: Apply Eq.(11.10) and solve for A and then use $A = \pi r^2$ to get the radius and $d = 2r$ to calculate the diameter.

EXECUTE: $Y = \dfrac{l_0 F_\perp}{A\,\Delta l}$ so $A = \dfrac{l_0 F_\perp}{Y\,\Delta l}$ (A is the cross-section area of the wire)

For steel, $Y = 2.0\times10^{11}\text{ Pa}$ (Table 11.1)

Thus $A = \dfrac{(2.00\text{ m})(400\text{ N})}{(2.0\times10^{11}\text{ Pa})(0.25\times10^{-2}\text{ m})} = 1.6\times10^{-6}\text{ m}^2.$

$A = \pi r^2,$ so $r = \sqrt{A/\pi} = \sqrt{1.6\times10^{-6}\text{ m}^2/\pi} = 7.1\times10^{-4}\text{ m}$

$d = 2r = 1.4\times10^{-3}\text{ m} = 1.4\text{ mm}$

EVALUATE: Steel wire of this diameter doesn't stretch much; $\Delta l/l_0 = 0.12\%.$

11.24. IDENTIFY: Apply Eq.(11.10).

SET UP: From Table 11.1, for steel, $Y = 2.0\times10^{11}$ Pa and for copper, $Y = 1.1\times10^{11}$ Pa.

$A = \pi(d^2/4) = 1.77\times10^{-4}\text{ m}^2$. $F_\perp = 4000$ N for each rod.

EXECUTE: **(a)** The strain is $\dfrac{\Delta l}{l_0} = \dfrac{F}{YA}$. For steel $\dfrac{\Delta l}{l_0} = \dfrac{(4000\text{N})}{(2.0\times10^{11}\text{ Pa})(1.77\times10^{-4}\text{ m}^2)} = 1.1\times10^{-4}.$ Similarly, the strain for copper is 2.1×10^{-4}.

(b) Steel: $(1.1\times10^{-4})(0.750\text{ m}) = 8.3\times10^{-5}\text{ m}$. Copper: $(2.1\times10^{-4})(0.750\text{ m}) = 1.6\times10^{-4}\text{ m}$.

EVALUATE: Copper has a smaller Y and therefore a greater elongation.

11.25. IDENTIFY: $Y = \dfrac{l_0 F_\perp}{A\Delta l}$

SET UP: $A = 0.50\text{ cm}^2 = 0.50\times10^{-4}\text{ m}^2$

EXECUTE: $Y = \dfrac{(4.00\text{ m})(5000\text{ N})}{(0.50\times10^{-4}\text{ m}^2)(0.20\times10^{-2}\text{m})} = 2.0\times10^{11}\text{ Pa}$

EVALUATE: Our result is the same as that given for steel in Table 11.1.

11.26. IDENTIFY: $Y = \dfrac{l_0 F_\perp}{A\Delta l}$

SET UP: $A = \pi r^2 = \pi(3.5\times10^{-3}\text{ m})^2 = 3.85\times10^{-5}\text{ m}^2$. The force applied to the end of the rope is the weight of the climber: $F_\perp = (65.0\text{ kg})(9.80\text{ m/s}^2) = 637\text{ N}$.

EXECUTE: $Y = \dfrac{(45.0\text{ m})(637\text{ N})}{(3.85\times10^{-5}\text{ m}^2)(1.10\text{ m})} = 6.77\times10^8\text{ Pa}$

EVALUATE: Our result is a lot smaller than the values given in Table 11.1. An object made of rope material is much easier to stretch than if the object were made of metal.

11.27. **IDENTIFY:** Use the first condition of equilibrium to calculate the tensions T_1 and T_2 in the wires (Figure 11.27a). Then use Eq.(11.10) to calculate the strain and elongation of each wire.

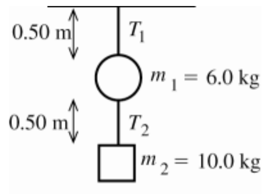


Figure 11.27a

SET UP: The free-body diagram for m_2 is given in Figure 11.27b.

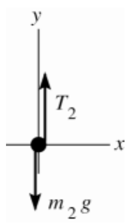


Figure 11.27b

EXECUTE:
$\sum F_y = ma_y$
$T_2 - m_2 g = 0$
$T_2 = 98.0\text{ N}$

SET UP: The free-body-diagram for m_1 is given in Figure 11.27c

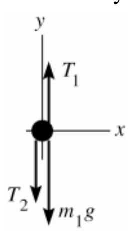


Figure 11.27c

EXECUTE:
$\sum F_y = ma_y$
$T_1 - T_2 - m_1 g = 0$
$T_1 = T_2 + m_1 g$
$T_1 = 98.0\text{ N} + 58.8\text{ N} = 157\text{ N}$

(a) $Y = \dfrac{\text{stress}}{\text{strain}}$ so $\text{strain} = \dfrac{\text{stress}}{Y} = \dfrac{F_\perp}{AY}$

upper wire: $\text{strain} = \dfrac{T_1}{AY} = \dfrac{157\text{ N}}{(2.5\times10^{-7}\text{ m}^2)(2.0\times10^{11}\text{ Pa})} = 3.1\times10^{-3}$

lower wire: $\text{strain} = \dfrac{T_2}{AY} = \dfrac{98\text{ N}}{(2.5\times10^{-7}\text{ m}^2)(2.0\times10^{11}\text{ Pa})} = 2.0\times10^{-3}$

(b) $\text{strain} = \Delta l / l_0$ so $\Delta l = l_0(\text{strain})$

upper wire: $\Delta l = (0.50\text{ m})(3.1\times10^{-3}) = 1.6\times10^{-3}\text{ m} = 1.6\text{ mm}$

lower wire: $\Delta l = (0.50\text{ m})(2.0\times10^{-3}) = 1.0\times10^{-3}\text{ m} = 1.0\text{ mm}$

EVALUATE: The tension is greater in the upper wire because it must support both objects. The wires have the same length and diameter, so the one with the greater tension has the greater strain and elongation.

11.28. **IDENTIFY:** Apply Eqs.(11.8), (11.9) and (11.10).

SET UP: The cross-sectional area of the post is $A = \pi r^2 = \pi(0.125\text{ m})^2 = 0.0491\text{ m}^2$. The force applied to the end of the post is $F_\perp = (8000\text{ kg})(9.80\text{ m/s}^2) = 7.84\times10^4\text{ N}$. The Young's modulus of steel is $Y = 2.0\times10^{11}\text{ Pa}$.

EXECUTE: **(a)** $\text{stress} = \dfrac{F_\perp}{A} = \dfrac{7.84\times10^4\text{ N}}{0.0491\text{ m}^2} = 1.60\times10^6\text{ Pa}$

(b) $\text{strain} = \dfrac{\text{stress}}{Y} = -\dfrac{1.60\times10^6\text{ Pa}}{2.0\times10^{11}\text{ Pa}} = -8.0\times10^{-6}$. The minus sign indicates that the length decreases.

(c) $\Delta l = l_0(\text{strain}) = (2.50\text{ m})(-8.0\times10^{-6}) = -2.0\times10^{-5}\text{ m}$

EVALUATE: The fractional change in length of the post is very small.

11.29. **IDENTIFY:** $F_\perp = pA$, so $F_{\text{net}} = (\Delta p)A$.

SET UP: $1\text{ atm} = 1.013\times10^5\text{ Pa}$.

EXECUTE: $(2.8\text{ atm} - 1.0\text{ atm})(1.013\times10^5\text{ Pa/atm})(50.0\text{ m}^2) = 9.1\times10^6\text{ N}$.

EVALUATE: This is a very large net force.

11.30. **IDENTIFY:** Apply Eq.(11.13).

SET UP: $\Delta V = -\dfrac{V_0 \Delta p}{B}$. Δp is positive when the pressure increases.

EXECUTE: **(a)** The volume would increase slightly.

(b) The volume change would be twice as great.

(c) The volume change is inversely proportional to the bulk modulus for a given pressure change, so the volume change of the lead ingot would be four times that of the gold.

EVALUATE: For lead, $B = 4.1 \times 10^{10}$ Pa, so $\Delta p / B$ is very small and the fractional change in volume is very small.

11.31. **IDENTIFY:** $p = F/A$

SET UP: $1\text{ cm}^2 = 1 \times 10^{-4}\text{ m}^2$

EXECUTE: **(a)** $\dfrac{250\text{ N}}{0.75 \times 10^{-4}\text{ m}^2} = 3.33 \times 10^6\text{ Pa}.$

(b) $(3.33 \times 10^6\text{ Pa})(2)(200 \times 10^{-4}\text{ m}^2) = 133\text{ kN}.$

EVALUATE: The pressure in part (a) is over 30 times larger than normal atmospheric pressure.

11.32. **IDENTIFY:** Apply Eq.(11.13). Density $= m/V$.

SET UP: At the surface the pressure is 1.0×10^5 Pa, so $\Delta p = 1.16 \times 10^8$ Pa. $V_0 = 1.00\text{ m}^3$. At the surface 1.00 m^3 of water has mass 1.03×10^3 kg.

EXECUTE: **(a)** $B = -\dfrac{(\Delta p)V_0}{\Delta V}$ gives $\Delta V = -\dfrac{(\Delta p)V_0}{B} = -\dfrac{(1.16 \times 10^8\text{ Pa})(1.00\text{ m}^3)}{2.2 \times 10^9\text{ Pa}} = -0.0527\text{ m}^3$

(b) At this depth 1.03×10^3 kg of seawater has volume $V_0 + \Delta V = 0.9473\text{ m}^3$. The density is

$\dfrac{1.03 \times 10^3\text{ kg}}{0.9473\text{ m}^3} = 1.09 \times 10^3\text{ kg/m}^3.$

EVALUATE: The density is increased because the volume is compressed due to the increased pressure.

11.33. **IDENTIFY** and **SET UP:** Use Eqs.(11.13) and (11.14) to calculate B and k.

EXECUTE: $B = -\dfrac{\Delta p}{\Delta V / V_0} = -\dfrac{(3.6 \times 10^6\text{ Pa})(600\text{ cm}^3)}{(-0.45\text{ cm}^3)} = +4.8 \times 10^9\text{ Pa}$

$k = 1/B = 1/4.8 \times 10^9\text{ Pa} = 2.1 \times 10^{-10}\text{ Pa}^{-1}$

EVALUATE: k is the same as for glycerine (Table 11.2).

11.34. **IDENTIFY:** Apply Eq.(11.17).

SET UP: $F_\parallel = 9.0 \times 10^5$ N. $A = (0.100\text{ m})(0.500 \times 10^{-2}\text{ m})$. $h = 0.100$ m. From Table 11.1, $S = 7.5 \times 10^{10}$ Pa for steel.

EXECUTE: **(a)** Shear strain $= \dfrac{F_\parallel}{AS} = \dfrac{(9 \times 10^5\text{ N})}{[(0.100\text{ m})(0.500 \times 10^{-2}\text{m})][7.5 \times 10^{10}\text{ Pa}]} = 2.4 \times 10^{-2}.$

(b) Using Eq.(11.16), $x = (\text{Shear strain}) \cdot h = (0.024)(0.100\text{ m}) = 2.4 \times 10^{-3}\text{m}$.

EVALUATE: This very large force produces a small displacement; $x/h = 2.4\%$.

11.35. **IDENTIFY:** The forces on the cube must balance. The deformation x is related to the force by $S = \dfrac{F_\parallel}{A}\dfrac{h}{x}$.

$F_\parallel = F$ since F is applied parallel to the upper face.

SET UP: $A = (0.0600\text{ m})^2$ and $h = 0.0600$ m. Table 11.1 gives $S = 4.4 \times 10^{10}$ Pa for copper and 0.6×10^{10} Pa for lead.

EXECUTE: **(a)** Since the horizontal forces balance, the glue exerts a force F in the opposite direction.

(b) $F = \dfrac{AxS}{h} = \dfrac{(0.0600\text{ m})^2(0.250 \times 10^{-3}\text{ m})(4.4 \times 10^{10}\text{ Pa})}{0.0600\text{ m}} = 6.6 \times 10^5\text{ N}$

(c) $x = \dfrac{Fh}{AS} = \dfrac{(6.6 \times 10^5\text{ N})(0.0600\text{ m})}{(0.0600\text{ m})^2(0.6 \times 10^{10}\text{ Pa})} = 1.8\text{ mm}$

EVALUATE: Lead has a smaller S than copper, so the lead cube has a greater deformation than the copper cube.

11.36. **IDENTIFY** and **SET UP:** Use Eq.(11.17). Same material implies same S

EXECUTE: $S = \dfrac{\text{stress}}{\text{strain}}$ so strain $= \dfrac{\text{stress}}{S} = \dfrac{F_\parallel / A}{S}$ and same forces implies same $F_\parallel$.

For the smaller object, $(\text{strain})_1 = F_\parallel / A_1 S$

For the larger object, $(\text{strain})_2 = F_\parallel / A_2 S$

$$\frac{(\text{strain})_2}{(\text{strain})_1} = \left(\frac{F_\parallel}{A_2 S}\right)\left(\frac{A_1 S}{F_\parallel}\right) = \frac{A_1}{A_2}$$

Larger solid has triple each edge length, so $A_2 = 9A_1$, and $\frac{(\text{strain})_2}{(\text{strain})_1} = \frac{1}{9}$

EVALUATE: The larger object has a smaller deformation.

11.37. **IDENTIFY** and **SET UP:** Use Eq.(11.8).

EXECUTE: Tensile stress $= \frac{F_\perp}{A} = \frac{F_\perp}{\pi r^2} = \frac{90.8\text{ N}}{\pi(0.92\times10^{-3}\text{ m})^2} = 3.41\times10^7\text{ Pa}$

EVALUATE: A modest force produces a very large stress because the cross-sectional area is small.

11.38. **IDENTIFY:** The proportional limit and breaking stress are values of the stress, $F_\perp/A$. Use Eq.(11.10) to calculate Δl.

SET UP: For steel, $Y = 20\times10^{10}\text{ Pa}$. $F_\perp = w$.

EXECUTE: **(a)** $w = (1.6\times10^{-3})(20\times10^{10}\text{ Pa})(5\times10^{-6}\text{ m}^2) = 1.60\times10^3\text{ N}$.

(b) $\Delta l = \left(\frac{F_\perp}{A}\right)\frac{l_0}{Y} = (1.6\times10^{-3})(4.0\text{ m}) = 6.4\text{ mm}$

(c) $(6.5\times10^{-3})(20\times10^{10}\text{ Pa})(5\times10^{-6}\text{ m}^2) = 6.5\times10^3\text{ N}$.

EVALUATE: At the proportional limit, the fractional change in the length of the wire is 0.16%.

11.39. **IDENTIFY:** The elastic limit is a value of the stress, $F_\perp / A$. Apply $\sum \vec{F} = m\vec{a}$ to the elevator in order to find the tension in the cable.

SET UP: $\frac{F_\perp}{A} = \frac{1}{3}(2.40\times10^8\text{ Pa}) = 0.80\times10^8\text{ Pa}$. The free-body diagram for the elevator is given in Figure 11.39. $F_\perp$ is the tension in the cable.

EXECUTE: $F_\perp = A(0.80\times10^8\text{ Pa}) = (3.00\times10^{-4}\text{ m}^2)(0.80\times10^8\text{ Pa}) = 2.40\times10^4\text{ N}$. $\sum F_y = ma_y$ applied to the elevator gives $F_\perp - mg = ma$ and $a = \frac{F_\perp}{m} - g = \frac{2.40\times10^4\text{ N}}{1200\text{ kg}} - 9.80\text{ m/s}^2 = 10.2\text{ m/s}^2$

EVALUATE: The tension in the cable is about twice the weight of the elevator.

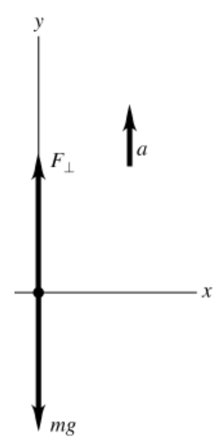


Figure 11.39

11.40. **IDENTIFY:** The breaking stress of the wire is the value of $F_\perp / A$ at which the wire breaks.

SET UP: From Table 11.3, the breaking stress of brass is $4.7\times10^8\text{ Pa}$. The area A of the wire is related to its diameter by $A = \pi d^2/4$.

EXECUTE: $A = \frac{350\text{ N}}{4.7\times10^8\text{ Pa}} = 7.45\times10^{-7}\text{ m}^2$, so $d = \sqrt{4A/\pi} = 0.97\text{ mm}$.

EVALUATE: The maximum force a wire can withstand without breaking is proportional to the square of its diameter.

11.41. **IDENTIFY:** Apply the conditions of equilibrium to the climber. For the minimum coefficient of friction the static friction force has the value $f_s = \mu_s n$.

SET UP: The free-body diagram for the climber is given in Figure 11.41. f_s and n are the vertical and horizontal components of the force exerted by the cliff face on the climber. The moment arm for the force T is $(1.4\text{ m})\cos 10°$.

EXECUTE: **(a)** $\sum \tau_z = 0$ gives $T(1.4\text{ m})\cos 10° - w(1.1\text{ m})\cos 35.0° = 0$.

$$T = \frac{(1.1\text{ m})\cos 35.0°}{(1.4\text{ m})\cos 10°}(82.0\text{ kg})(9.80\text{ m/s}^2) = 525\text{ N}$$

(b) $\sum F_x = 0$ gives $n = T\sin 25.0° = 222\text{ N}$. $\sum F_y = 0$ gives $f_s + T\cos 25° - w = 0$ and $f_s = (82.0\text{ kg})(9.80\text{ m/s}^2) - (525\text{ N})\cos 25° = 328\text{ N}$.

(c) $\mu_s = \frac{f_s}{n} = \frac{328\text{ N}}{222\text{ N}} = 1.48$

EVALUATE: To achieve this large value of μ_s the climber must wear special rough-soled shoes.

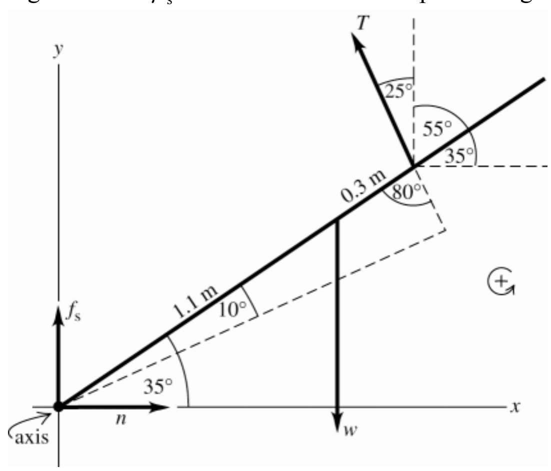


Figure 11.41

11.42. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the bridge.

SET UP: Let the axis of rotation be at the left end of the bridge and let counterclockwise torques be positive.
EXECUTE: If Lancelot were at the end of the bridge, the tension in the cable would be (from taking torques about the hinge of the bridge) obtained from $T(12.0\text{ N}) = (600\text{ kg})(9.80\text{ m/s}^2)(12.0\text{ m}) + (200\text{ kg})(9.80\text{ m/s}^2)(6.0\text{ m})$, so $T = 6860\text{ N}$. This exceeds the maximum tension that the cable can have, so Lancelot is going into the drink. To find the distance x Lancelot can ride, replace the 12.0 m multiplying Lancelot's weight by x and the tension T by $T_{max} = 5.80\times 10^3\text{ N}$ and solve for x;

$$x = \frac{(5.80\times 10^3\text{ N})(12.0\text{ m}) - (200\text{ kg})(9.80\text{ m/s}^2)(6.0\text{ m})}{(600\text{ kg})(9.80\text{ m/s}^2)} = 9.84\text{ m}.$$

EVALUATE: Before Lancelot goes onto the bridge, the tension in the supporting cable is $T = \frac{(6.0\text{ m})(200\text{ kg})(9.80\text{ m/s}^2)}{12.0\text{ m}} = 9800\text{ N}$, well below the breaking strength of the cable. As he moves along the bridge, the increase in tension is proportional to x, the distance he has moved along the bridge.

11.43. **IDENTIFY:** For the airplane to remain in level flight, both $\sum F_y = 0$ and $\sum \tau_z = 0$.

SET UP: The free-body diagram for the airplane is given in Figure 11.43. Let $+y$ be upward.

EXECUTE: $-F_{tail} - W + F_{wing} = 0$. Taking the counterclockwise direction as positive, and taking torques about the point where the tail force acts, $-(3.66\text{ m})(6700\text{ N}) + (3.36\text{ m})F_{wing} = 0.$ This gives $F_{wing} = 7300\text{ N(up)}$ and $F_{tail} = 7300\text{ N} - 6700\text{ N} = 600\text{ N(down)}$.

EVALUATE: We assumed that the wing force was upward and the tail force was downward. When we solved for these forces we obtained positive values for them, which confirms that they do have these directions. Note that the rear stabilizer provides a *downward* force. It does not hold up the tail of the aircraft, but serves to counter the torque produced by the wing. Thus balance, along with weight, is a crucial factor in airplane loading.

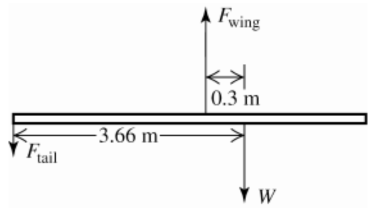


Figure 11.43

11.44. IDENTIFY: Apply the first and second conditions of equilibrium to the truck.

SET UP: The weight on the front wheels is n_f, the normal force exerted by the ground on the front wheels. The weight on the rear wheels is n_r, the normal force exerted by the ground on the rear wheels. When the front wheels come off the ground, $n_f \to 0$. The free-body diagram for the truck without the box is given in Figure 11.44a and with the box in Figure 11.44b. The center of gravity of the truck, without the box, is a distance x from the rear wheels.

EXECUTE: $\sum F_y = 0$ in Fig.11.44a gives $w = n_r + n_f = 8820 \text{ N} + 10{,}780 \text{ N} = 19{,}600 \text{ N}$

$\sum \tau = 0$ in Fig.11.44a, with the axis at the rear wheels and counterclockwise torques positive, gives

$n_f(3.00 \text{ m}) - wx = 0$ and $x = \dfrac{n_f(3.00 \text{ m})}{w} = \left(\dfrac{10{,}780 \text{ N}}{19{,}600 \text{ N}}\right)(3.00 \text{ m}) = 1.65 \text{ m}$.

(a) $\sum \tau = 0$ in Fig.11.44b, with the axis at the rear wheels and counterclockwise torques positive, gives $w_{box}(1.00 \text{ m}) + n_f(3.00 \text{ m}) - w(1.65 \text{ m}) = 0$.

$$n_f = \frac{-(3600 \text{ N})(1.00 \text{ m}) + (19{,}600 \text{ N})(1.65 \text{ m})}{3.00 \text{ m}} = 9{,}580 \text{ N}$$

$\sum F_y = 0$ gives $n_r + n_f = w_{box} + w$ and $n_r = 3600 \text{ N} + 19{,}600 \text{ N} - 9580 \text{ N} = 13{,}620 \text{ N}$. There is 9,580 N on the front wheels and 13,620 N on the rear wheels.

(b) $n_f \to 0$. $\sum \tau = 0$ gives $w_{box}(1.00 \text{ m}) - w(1.65 \text{ m}) = 0$ and $w_{box} = 1.65w = 3.23 \times 10^4 \text{ N}$.

EVALUATE: Placing the box on the tailgate in part (b) reduces the normal force exerted at the front wheels.

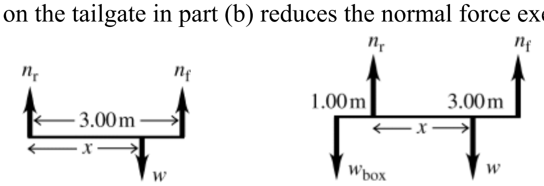


Figure 11.44a, b

11.45. IDENTIFY: In each case, to achieve balance the center of gravity of the system must be at the fulcrum. Use Eq.(11.3) to locate x_{cm}, with m_i replaced by w_i.

SET UP: Let the origin be at the left-hand end of the rod and take the $+x$ axis to lie along the rod. Let $w_1 = 255 \text{ N}$ (the rod) so $x_1 = 1.00 \text{ m}$, let $w_2 = 225 \text{ N}$ so $x_2 = 2.00 \text{ m}$ and let $w_3 = W$. In part (a) $x_3 = 0.500 \text{ m}$ and in part (b) $x_3 = 0.750 \text{ m}$.

EXECUTE: **(a)** $x_{cm} = 1.25 \text{ m}$. $x_{cm} = \dfrac{w_1x_1 + w_2x_2 + w_3x_3}{w_1 + w_2 + w_3}$ gives $w_3 = \dfrac{(w_1 + w_2)x_{cm} - w_1x_1 - w_2x_2}{x_3 - x_{cm}}$ and

$W = \dfrac{(480 \text{ N})(1.25 \text{ m}) - (255 \text{ N})(1.00 \text{ m}) - (225 \text{ N})(2.00 \text{ m})}{0.500 \text{ m} - 1.25 \text{ m}} = 140 \text{ N}$.

(b) Now $w_3 = W = 140 \text{ N}$ and $x_3 = 0.750 \text{ m}$.

$x_{cm} = \dfrac{(255 \text{ N})(1.00 \text{ m}) + (225 \text{ N})(2.00 \text{ m}) + (140 \text{ N})(0.750 \text{ m})}{255 \text{ N} + 225 \text{ N} + 140 \text{ N}} = 1.31 \text{ m}$. W must be moved $1.31 \text{ m} - 1.25 \text{ m} = 6 \text{ cm}$ to the right.

EVALUATE: Moving W to the right means x_{cm} for the system moves to the right.

11.46. **IDENTIFY:** The center of gravity of the object must have the same x coordinate as the hook. Use Eq.(11.3) for x_{cm}. The mass of a segment is proportional to its length. Define α to be the mass per unit length, so $m_i = \alpha l_i$, where l_i is the length of a piece that has mass m_i.

SET UP: Use coordinates with the origin at the right-hand edge of the object and $+x$ to the left. $x_{cm} = L$. The mass of each piece can be taken at its center of gravity, which is at its geometrical center. Let 1 be the horizontal piece of length L, 2 be the vertical piece of length L and 3 be the horizontal piece with length x.

EXECUTE: $x_{cm} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$ gives $L = \dfrac{\alpha L(L/2) + \alpha x(x/2)}{\alpha L + \alpha L + \alpha x}$. α divides out and the equation reduces to $x^2 - 2xL - 3L^2 = 0$. $x = \frac{1}{2}(2L \pm 4L)$. x must be positive, so $x = 3L$.

EVALUATE: $x_{cm} = L$ is equivalent to saying that the net torque is zero for an axis at the hook.

11.47. **IDENTIFY:** Apply the conditions of equilibrium to the horizontal beam. Since the two wires are symmetrically placed on either side of the middle of the sign, their tensions are equal and are each equal to $T_w = mg/2 = 137\ \text{N}$.

SET UP: The free-body diagram for the beam is given in Figure 11.47. F_v and F_h are the horizontal and vertical forces exerted by the hinge on the sign. Since the cable is 2.00 m long and the beam is 1.50 m long, $\cos\theta = \dfrac{1.50\ \text{m}}{2.00\ \text{m}}$ and $\theta = 41.4°$. The tension T_c in the cable has been replaced by its horizontal and vertical components.

EXECUTE: **(a)** $\sum \tau_z = 0$ gives $T_c(\sin 41.4°)(1.50\ \text{m}) - w_{beam}(0.750\ \text{m}) - T_w(1.50\ \text{m}) - T_w(0.60\ \text{m}) = 0$.

$$T_c = \frac{(18.0\ \text{kg})(9.80\ \text{m/s}^2)(0.750\ \text{m}) + (137\ \text{N})(1.50\ \text{m} + 0.60\ \text{m})}{(1.50\ \text{m})(\sin 41.4°)} = 423\ \text{N}.$$

(b) $\sum F_y = 0$ gives $F_v + T_c \sin 41.4° - w_{beam} - 2T_w = 0$ and

$F_v = 2T_w + w_{beam} - T_c \sin 41.4° = 2(137\ \text{N}) + (18.0\ \text{kg})(9.80\ \text{m/s}^2) - (423\ \text{N})(\sin 41.4°) = 171\ \text{N}$. The hinge must be able to supply a vertical force of 171 N.

EVALUATE: The force from the two wires could be replaced by the weight of the sign acting at a point 0.60 m to the left of the right-hand edge of the sign.

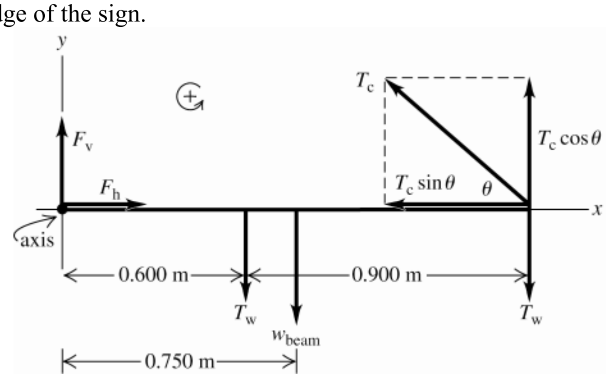


Figure 11.47

11.48. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the hammer.

SET UP: Take the axis of rotation to be at point A.

EXECUTE: The force $\vec{F}_1$ is directed along the length of the nail, and so has a moment arm of $(0.800\ \text{m})\sin 60°$. The moment arm of $\vec{F}_2$ is 0.300 m, so

$$F_2 = F_1 \frac{(0.0800\ \text{m})\sin 60°}{(0.300\ \text{m})} = (500\ \text{N})(0.231) = 116\ \text{N}.$$

EVALUATE: The force F_2 that must be applied to the hammer handle is much less than the force that the hammer applies to the nail, because of the large difference in the lengths of the moment arms.

11.49. **IDENTIFY:** Apply the first and second conditions of equilibrium to the bar.

SET UP: The free-body diagram for the bar is given in Figure 11.49. n is the normal force exerted on the bar by the surface. There is no friction force at this surface. H_h and H_v are the components of the force exerted on the

bar by the hinge. The components of the force of the bar on the hinge will be equal in magnitude and opposite in direction.

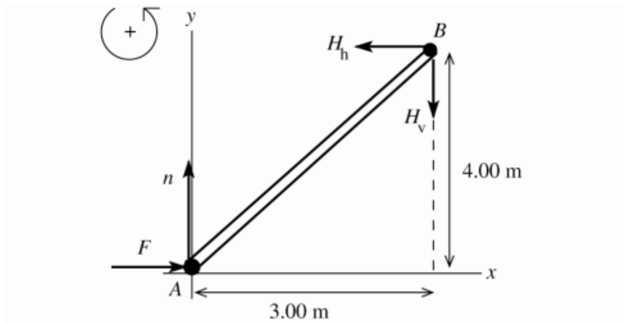


Figure 11.49

EXECUTE:
$\sum F_x = ma_x$
$F = H_h = 120$ N
$\sum F_y = ma_y$
$n - H_v = 0$
$H_v = n$, but we don't know either of these forces.

$\sum \tau_B = 0$ gives $F(4.00\text{ m}) - n(3.00\text{ m}) = 0$

$n = (4.00\text{ m}/3.00\text{ m})F = \frac{4}{3}(120\text{ N}) = 160\text{ N}$ and then $H_v = 160$ N

Force of bar on hinge:
horizontal component 120 N, to right
vertical component 160 N, upward

EVALUATE: $H_h / H_v = 120/160 = 3.00/4.00$, so the force the hinge exerts on the bar is directed along the bar. $\vec{n}$ and $\vec{F}$ have zero torque about point A, so the line of action of the hinge force $\vec{H}$ must pass through this point also if the net torque is to be zero.

11.50. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the piece of art.

SET UP: The free-body diagram for the piece of art is given in Figure 11.50.

EXECUTE: $\sum \tau_z = 0$ gives $T_B(1.25\text{ m}) - w(1.02\text{ m}) = 0$. $T_B = (358\text{ N})\left(\frac{1.02\text{ m}}{1.25\text{ m}}\right) = 292\text{ N}$. $\sum F_y = 0$ gives $T_A + T_B - w = 0$ and $T_A = w - T_B = 358\text{ N} - 292\text{ N} = 66\text{ N}$.

EVALUATE: If we consider the sum of torques about the center of gravity of the piece of art, T_A has a larger moment arm than T_B, and this is why $T_A < T_B$.

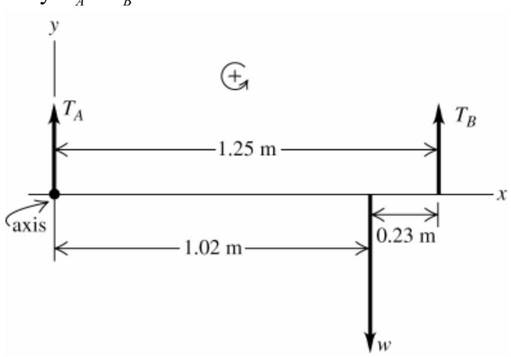


Figure 11.50

11.51. **IDENTIFY:** Apply the conditions of equilibrium to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.51. Let T_ϕ and T_θ be the tension in the two cables. Each tension has been replaced by its horizontal and vertical components.

EXECUTE: **(a)** The center of gravity of the beam is a distance $L/2$ from each end and $\sum \tau_z = 0$ with the axis at the center of gravity of the beam gives $-T_\phi \sin\phi(L/2) + T_\theta \sin\theta(L/2) = 0$. $T_\phi \sin\phi = T_\theta \sin\theta$. $\sum F_x = 0$ gives $T_\phi \cos\phi = T_\theta \cos\theta$. Dividing the first equation by the second gives $\tan\phi = \tan\theta$ and $\phi = \theta$. Then the equations also say $T_\phi = T_\theta$.

(b) The center of gravity of the beam is a distance $3L/4$ from the left-hand end so a distance $L/4$ from the right-hand end. $\sum \tau_z = 0$ with the axis at the center of gravity of the beam gives $-T_\phi \sin\phi(3L/2) + T_\theta \sin\theta(L/2) = 0$ and $3T_\phi \sin\phi = T_\theta \sin\theta$. $\sum F_x = 0$ gives $T_\phi \cos\phi = T_\theta \cos\theta$. Dividing the first equation by the second gives $3\tan\phi = \tan\theta$.

EVALUATE: $3\tan\phi = \tan\theta$ requires $\theta > \phi$. The cable closest to the center of gravity must be closer to the vertical direction. $T_\theta = T_\phi\left(\dfrac{\cos\phi}{\cos\theta}\right)$ and $\theta > \phi$ means the tension is greater in the wire that is closest to the center of gravity.

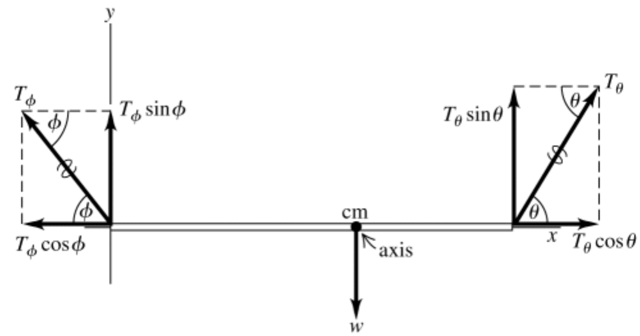


Figure 11.51

11.52. **IDENTIFY:** Apply the first and second conditions for equilibrium to the bridge.

SET UP: Find torques about the hinge. Use L as the length of the bridge and $w_{\rm T}$ and $w_{\rm B}$ for the weights of the truck and the raised section of the bridge. Take $+y$ to be upward and $+x$ to be to the right.

EXECUTE: **(a)** $TL\sin 70° = w_{\rm T}\left(\tfrac{3}{4}L\right)\cos 30° + w_{\rm B}\left(\tfrac{1}{2}L\right)\cos 30°$, so

$$T = \frac{\left(\tfrac{3}{4}m_{\rm T} + \tfrac{1}{2}m_{\rm B}\right)(9.80\ \text{m/s}^2)\cos 30°}{\sin 70°} = 2.57\times 10^5\ \text{N}.$$

(b) Horizontal: $T\cos(70° - 30°) = 1.97\times 10^5\ \text{N}$ (to the right). Vertical: $w_{\rm T} + w_{\rm B} - T\sin 40° = 2.46\times 10^5\ \text{N}$ (upward).

EVALUATE: If ϕ is the angle of the hinge force above the horizontal, $\tan\phi = \dfrac{2.46\times 10^5\ \text{N}}{1.97\times 10^5\ \text{N}}$ and $\phi = 51.3°$. The hinge force is not directed along the bridge.

11.53. **IDENTIFY:** Apply the conditions of equilibrium to the cylinder.

SET UP: The free-body diagram for the cylinder is given in Figure 11.53. The center of gravity of the cylinder is at its geometrical center. The cylinder has radius R.

EXECUTE: **(a)** T produces a clockwise torque about the center of gravity so there must be a friction force, that produces a counterclockwise torque about this axis.

(b) Applying $\sum\tau_z = 0$ to an axis at the center of gravity gives $-TR + fR = 0$ and $T = f$. $\sum\tau_z = 0$ applied to an axis at the point of contact between the cylinder and the ramp gives $-T(2R) + MgR\sin\theta = 0$. $T = (Mg/2)\sin\theta$.

EVALUATE: We can show that $\sum F_x = 0$ and $\sum F_y = 0$, for x and y axes parallel and perpendicular to the ramp, or for x and y axes that are horizontal and vertical.

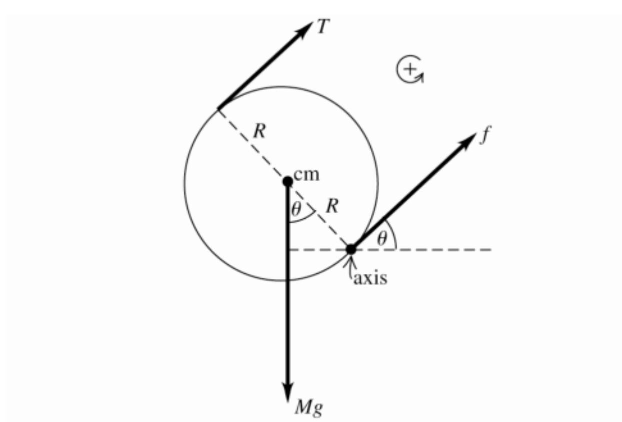


Figure 11.53

11.54. **IDENTIFY:** Apply the first and second conditions of equilibrium to the ladder.
SET UP: Take torques about the pivot. Let $+y$ be upward.
EXECUTE: **(a)** The force F_V that the ground exerts on the ladder is given to be vertical, so $\sum \tau_z = 0$ gives $F_V(6.0 \text{ m})\sin\theta = (250 \text{ N})(4.0 \text{ m})\sin\theta + (750 \text{ N})(1.50 \text{ m})\sin\theta$, so $F_V = 354 \text{ N}$.
(b) There are no other horizontal forces on the ladder, so the horizontal pivot force is zero. The vertical force that the pivot exerts on the ladder must be $(750 \text{ N}) + (250 \text{ N}) - (354 \text{ N}) = 646 \text{ N}$, up, so the ladder exerts a downward force of 646 N on the pivot.
(c) The results in parts (a) and (b) are independent of θ.
EVALUATE: All the forces on the ladder are vertical, so all the moment arms are vertical and are proportional to $\sin\theta$. Therefore, $\sin\theta$ divides out of the torque equations and the results are independent of θ.

11.55. **IDENTIFY:** Apply the first and second conditions for equilibrium to the strut.
SET UP: Denote the length of the strut by L.
EXECUTE: **(a)** $V = mg + w$ and $H = T$. To find the tension, take torques about the pivot point.

$$T\left(\frac{2}{3}L\right)\sin\theta = w\left(\frac{2}{3}L\right)\cos\theta + mg\left(\frac{L}{6}\right)\cos\theta \text{ and } T = \left(w + \frac{mg}{4}\right)\cot\theta .$$

(b) Solving the above for w, and using the maximum tension for T,

$$w = T\ \tan\theta - \frac{mg}{4} = (700 \text{ N})\tan 55.0° - (5.0 \text{ kg})(9.80 \text{ m/s}^2) = 951 \text{ N}.$$

(c) Solving the expression obtained in part (a) for $\tan\theta$ and letting $\omega \to 0$, $\tan\theta = \dfrac{mg}{4T} = 0.700$, so $\theta = 4.00°$.

EVALUATE: As the strut becomes closer to the horizontal, the moment arm for the horizontal tension force approaches zero and the tension approaches infinity.

11.56. **IDENTIFY:** Apply the first and second conditions of equilibrium to each rod.
SET UP: Apply $\sum F_y = 0$ with $+y$ upward and apply $\sum \tau = 0$ with the pivot at the point of suspension for each rod.
EXECUTE: **(a)** The free-body diagram for each rod is given in Figure 11.56.
(b) $\sum \tau = 0$ for the lower rod: $(6.0 \text{ N})(4.0 \text{ cm}) = w_A(8.0 \text{ cm})$ and $w_A = 3.0 \text{ N}$.

$\sum F_y = 0$ for the lower rod: $S_3 = 6.0 \text{ N} + w_A = 9.0 \text{ N}$

$\sum \tau = 0$ for the middle rod: $w_B(3.0 \text{ cm}) = (5.0 \text{ cm})S_3$ and $w_B = \left(\dfrac{5.0}{3.0}\right)(9.0 \text{ N}) = 15.0 \text{ N}$.

$\sum F_y = 0$ for the middle rod: $S_2 = 9.0 \text{ N} + S_3 = 24.0 \text{ N}$

$\sum \tau = 0$ for the upper rod: $S_2(2.0 \text{ cm}) = w_C(6.0 \text{ cm})$ and $w_C = \left(\dfrac{2.0}{6.0}\right)(24.0 \text{ N}) = 8.0 \text{ N}$.

$\sum F_y = 0$ for the upper rod: $S_1 = S_2 + w_C = 32.0 \text{ N}$.

In summary, $w_A = 3.0 \text{ N}$, $w_B = 15.0 \text{ N}$, $w_C = 8.0 \text{ N}$. $S_1 = 32.0 \text{ N}$, $S_2 = 24.0 \text{ N}$, $S_3 = 9.0 \text{ N}$.
(c) The center of gravity of the entire mobile must lie along a vertical line that passes through the point where S_1 is located.
EVALUATE: For the mobile as a whole the vertical forces must balance, so $S_1 = w_A + w_B + w_C + 6.0 \text{ N}$.

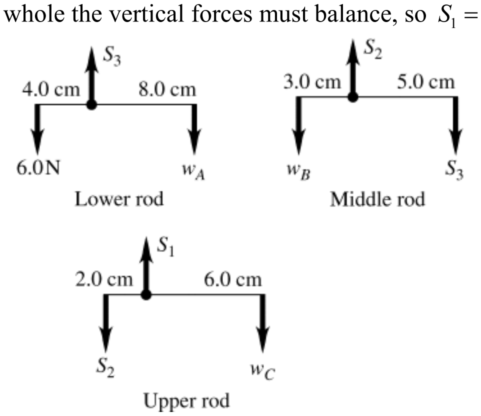


Figure 11.56

11.57. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.57.

EXECUTE: $\Sigma\tau_z = 0$, axis at hinge , gives $T(6.0\text{ m})(\sin 40°) - w(3.75\text{ m})(\cos 30°) = 0$ and $T = 7600\text{ N}$.

EVALUATE: The tension in the cable is less than the weight of the beam. $T\sin 40°$ is the component of T that is perpendicular to the beam.

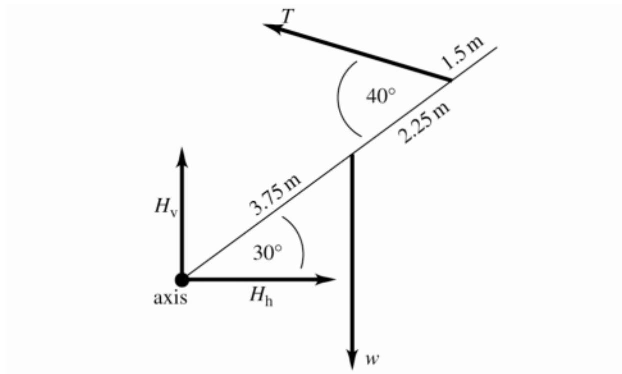


Figure 11.57

11.58. **IDENTIFY:** Apply the first and second conditions of equilibrium to the drawbridge.

SET UP: The free-body diagram for the drawbridge is given in Figure 11.58. H_v and H_h are the components of the force the hinge exerts on the bridge.

EXECUTE: **(a)** $\sum \tau_z = 0$ with the axis at the hinge gives $-w(7.0\text{ m})(\cos 37°) + T(3.5\text{ m})(\sin 37°) = 0$ and

$$T = 2w\frac{\cos 37°}{\sin 37°} = \frac{(45{,}000\text{ N})}{\tan 37°} = 1.19\times10^5\text{ N}$$

(b) $\sum F_x = 0$ gives $H_h = T = 1.19\times10^5\text{ N}$. $\sum F_y = 0$ gives $H_v = w = 4.50\times10^4\text{ N}$.

$H = \sqrt{H_h^2 + H_v^2} = 1.27\times10^5\text{ N}$. $\tan\theta = \dfrac{H_v}{H_h}$ and $\theta = 20.7°$. The hinge force has magnitude $1.27\times10^5\text{ N}$ and is directed at $20.7°$ above the horizontal.

EVALUATE: The hinge force is not directed along the bridge. If it were, it would have zero torque for an axis at the center of gravity of the bridge and for that axis the tension in the cable would produce a single, unbalanced torque.

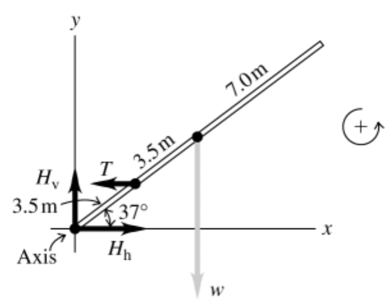


Figure 11.58

11.59. **IDENTIFY:** Apply the first and second conditions of equilibrium to the beam.

SET UP: The free-body diagram for the beam is given in Figure 11.59.

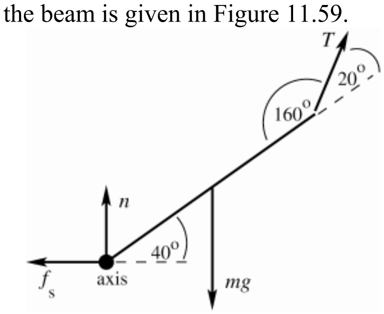


Figure 11.59

EXECUTE: **(a)** $\sum \tau_z = 0,$ axis at lower end of beam

Let the length of the beam be L.

$T(\sin 20°)L = -mg\left(\frac{L}{2}\right)\cos 40° = 0$

$T = \frac{\frac{1}{2}mg\cos 40°}{\sin 20°} = 2700\text{ N}$

(b) Take $+y$ upward.

$\sum F_y = 0$ gives $n - w + T\sin 60° = 0$ so $n = 73.6\text{ N}$

$\sum F_x = 0$ gives $f_s = T\cos 60° = 1372\text{ N}$

$f_s = \mu_s n,\;\; \mu_s = \frac{f_s}{n} = \frac{1372\text{ N}}{73.6\text{ N}} = 19$

EVALUATE: The floor must be very rough for the beam not to slip. The friction force exerted by the floor is to the left because T has a component that pulls the beam to the right.

11.60. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the beam.

SET UP: The center of mass of the beam is 1.0 m from the suspension point.

EXECUTE: **(a)** Taking torques about the suspension point,
$w(4.00\text{ m})\sin 30° + (140.0\text{ N})(1.00\text{ m})\sin 30° = (100\text{ N})(2.00\text{ m})\sin 30°$. The common factor of $\sin 30°$ divides out, from which $w = 15.0\text{ N}$.

(b) In this case, a common factor of $\sin 45°$ would be factored out, and the result would be the same.

EVALUATE: All the forces are vertical, so the moments are all horizontal and all contain the factor $\sin\theta$, where θ is the angle the beam makes with the horizontal.

11.61. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the flagpole.

SET UP: The free-body diagram for the flagpole is given in Figure 11.61. Let clockwise torques be positive. θ is the angle the cable makes with the horizontal pole.

EXECUTE: **(a)** Taking torques about the hinged end of the pole
$(200\text{ N})(2.50\text{ m}) + (600\text{ N})(5.00\text{ m}) - T_y(5.00\text{ m}) = 0$. $T_y = 700\text{ N}$. The x-component of the tension is then

$T_x = \sqrt{(1000\text{ N})^2 - (700\text{ N})^2} = 714\text{ N}$. $\tan\theta = \frac{h}{5.00\text{ m}} = \frac{T_y}{T_x}$. The height above the pole that the wire must be attached is $(5.00\text{ m})\frac{700}{714} = 4.90\text{ m}$.

(b) The y-component of the tension remains 700 N. Now $\tan\theta = \frac{4.40\text{ m}}{5.00\text{ m}}$ and $\theta = 41.35°$, so

$T = \frac{T_y}{\sin\theta} = \frac{700\text{ N}}{\sin 41.35°} = 1060\text{ N}$, an increase of 60 N.

EVALUATE: As the wire is fastened closer to the hinged end of the pole, the moment arm for T decreases and T must increase to produce the same torque about that end.

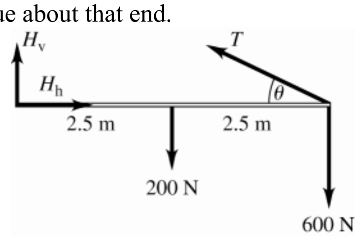


Figure 11.61

11.62. **IDENTIFY:** Apply $\sum \vec{F} = 0$ to each object, including the point where D, C and B are joined. Apply $\sum \tau_z = 0$ to the rod.

SET UP: To find T_C and T_D, use a coordinate system with axes parallel to the cords.

EXECUTE: A and B are straightforward, the tensions being the weights suspended:
$T_A = (0.0360\text{ kg})(9.80\text{ m/s}^2) = 0.353\text{ N}$ and $T_B = (0.0240\text{ kg} + 0.0360\text{ kg})(9.80\text{ m/s}^2) = 0.588\text{ N}$. Applying $\sum F_x = 0$ and $\sum F_y = 0$ to the point where the cords are joined, $T_C = T_B\cos 36.9° = 0.470\text{ N}$ and $T_D = T_B\cos 53.1° = 0.353\text{ N}$. To find T_E, take torques about the point where string F is attached.

$T_E(1.00\text{ m}) = T_D\sin 36.9°(0.800\text{ m}) + T_C\sin 53.1°(0.200\text{ m}) + (0.120\text{ kg})(9.80\text{ m/s}^2)(0.500\text{ m})$ and $T_E = 0.833$ N.

T_F may be found similarly, or from the fact that $T_E + T_F$ must be the total weight of the ornament.

$(0.180\text{kg})(9.80\text{m/s}^2) = 1.76$ N, from which $T_F = 0.931$ N.

EVALUATE: The vertical line through the spheres is closer to *F* than to *E*, so we expect $T_F > T_E$, and this is indeed the case.

11.63. **IDENTIFY:** Apply the equilibrium conditions to the plate. $\tau = Fr\sin\phi$.

SET UP: The free-body diagram for the plate is sketched in Figure 11.63. For the force *T* (tension in the cable), $\tau_T = Tr\sin\phi = T\sqrt{h^2+d^2}\sin\phi$.

EXECUTE: **(a)** $\sum\tau_z = 0$ gives $T\sqrt{h^2+d^2}\sin\phi - W\dfrac{d}{2} = 0$. *T* is least for $\phi = 90°$, and in that case $\tan\theta = \dfrac{h}{d}$ so $\theta = \tan^{-1}\left(\dfrac{h}{d}\right)$. Then $T = W\dfrac{d}{2\sqrt{h^2+d^2}}$.

(b) $\sum F_x = 0$ gives $F_\text{h} = T\sin\theta = \dfrac{Wd}{2\sqrt{h^2+d^2}}\left(\dfrac{h}{\sqrt{h^2+d^2}}\right) = \dfrac{Whd}{2(h^2+d^2)}$. $\sum F_y = 0$ gives $F_\text{v} + T\cos\theta - W = 0$ and

$$F_\text{v} = W - \left(\frac{Wd}{2\sqrt{h^2+d^2}}\right)\left(\frac{d}{\sqrt{h^2+d^2}}\right) = W\left(1 - \frac{d^2}{2(h^2+d^2)}\right) = W\frac{2h^2+d^2}{2(h^2+d^2)}$$

EVALUATE: The angle α that the net force exerted by the hinge makes with the horizontal is given by $\tan\alpha = \dfrac{F_\text{v}}{F_\text{h}} = \dfrac{W(2h^2+d^2)}{2(h^2+d^2)}\dfrac{2(h^2+d^2)}{Whd} = \dfrac{2h^2+d^2}{hd}$. This force does not lie along the diagonal of the plate.

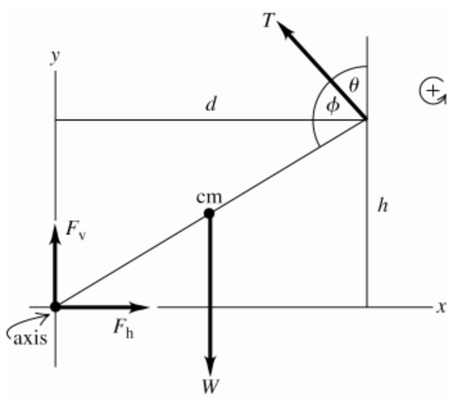


Figure 11.63

11.64. **IDENTIFY:** Apply Eq.(11.10) and the relation $\Delta w/w_0 = -\sigma\Delta l/l_0$ that is given in the problem.

SET UP: The steel rod in Example 11.5 has $\Delta l/l_0 = 9.0\times10^{-4}$. For nickel, $Y = 2.1\times10^{11}$ Pa. The width w_0 is $w_0 = \sqrt{4A/\pi}$.

EXECUTE: **(a)** $\Delta w = -\sigma\,(\Delta l/l)\,w_0 = -(0.23)(9.0\times10^{-4})\sqrt{4(0.30\times10^{-4}\text{m}^2)/\pi} = 1.3\ \mu$m.

(b) $F_\perp = AY\dfrac{\Delta l}{l} = AY\dfrac{1}{\sigma}\dfrac{\Delta w}{w}$ and $F_\perp = \dfrac{(2.1\times10^{11}\text{ Pa})\ (\pi\ (2.0\times10^{-2}\text{m})^2)}{0.42}\ \dfrac{0.10\times10^{-3}\text{m}}{2.0\times10^{-2}\text{ m}} = 3.1\times10^6$ N.

EVALUATE: For nickel and steel, $\sigma < 1$ and the fractional change in width is less than the fractional change in length.

11.65. **IDENTIFY:** Apply the equilibrium conditions to the crate. When the crate is on the verge of tipping it touches the floor only at its lower left-hand corner and the normal force acts at this point. The minimum coefficient of static friction is given by the equation $f_\text{s} = \mu_\text{s} n$.

SET UP: The free-body diagram for the crate when it is ready to tip is given in Figure 11.65.

EXECUTE: **(a)** $\sum\tau_z = 0$ gives $P(1.50\text{ m})\sin 53.0° - w(1.10\text{ m}) = 0$. $P = w\left(\dfrac{1.10\text{ m}}{[1.50\text{ m}][\sin 53.0°]}\right) = 1.15\times10^3$ N

(b) $\sum F_y = 0$ gives $n - w - P\cos 53.0° = 0$. $n = w + P\cos 53.0° = 1250\text{ N} + (1.15\times10^3\text{ N})\cos 53° = 1.94\times10^3$ N

(c) $\sum F_x = 0$ gives $f_\text{s} = P\sin 53.0° = (1.15\times10^3\text{ N})\sin 53.0° = 918$ N.

(d) $\mu_\text{s} = \dfrac{f_\text{s}}{n} = \dfrac{918\text{ N}}{1.94\times10^3\text{ N}} = 0.473$

EVALUATE: The normal force is greater than the weight because P has a downward component.

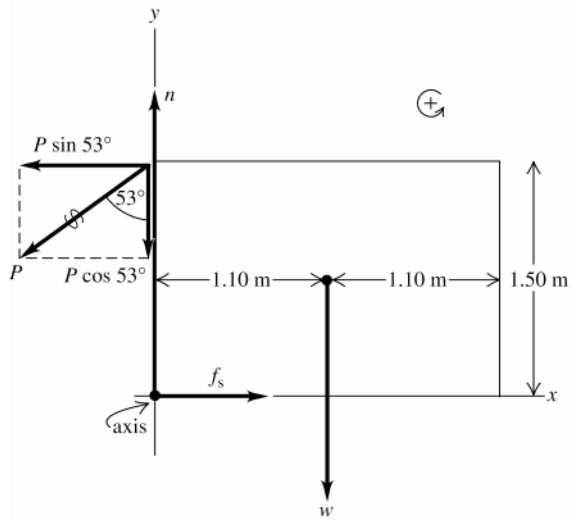


Figure 11.65

11.66. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the meter stick.

SET UP: The wall exerts an upward static friction force f and a horizontal normal force n on the stick. Denote the length of the stick by l. $f = \mu_s n$.

EXECUTE: **(a)** Taking torques about the right end of the stick, the friction force is half the weight of the stick, $f = w/2$. Taking torques about the point where the cord is attached to the wall (the tension in the cord and the friction force exert no torque about this point), and noting that the moment arm of the normal force is $l\tan\theta$, $n\tan\theta = w/2\cdot$ Then, $(f/n) = \tan\theta < 0.40$, so $\theta < \arctan(0.40) = 22°$.

(b) Taking torques as in part (a), $fl = w\frac{l}{2} + w(l-x)$ and $nl\tan\theta = w\frac{l}{2} + wx$. In terms of the coefficient of friction μ_s, $\mu_s > \frac{f}{n} = \frac{l/2 + (l-x)}{l/2 + x}\tan\theta = \frac{3l - 2x}{l + 2x}\tan\theta$. Solving for x, $x > \frac{l}{2}\frac{3\tan\theta - \mu_s}{\mu_s + \tan\theta} = 30.2$ cm.

(c) In the above expression, setting $x = 10$ cm and solving for μ_s gives $\mu_s > \frac{(3 - 20/l)\tan\theta}{1 + 20/l} = 0.625.$

EVALUATE: For $\theta = 15°$ and without the block suspended from the stick, a value of $\mu_s \geq 0.268$ is required to prevent slipping. Hanging the block from the stick increases the value of μ_s that is required.

11.67. **IDENTIFY:** Apply the first and second conditions of equilibrium to the crate.

SET UP: The free-body diagram for the crate is given in Figure 11.67.

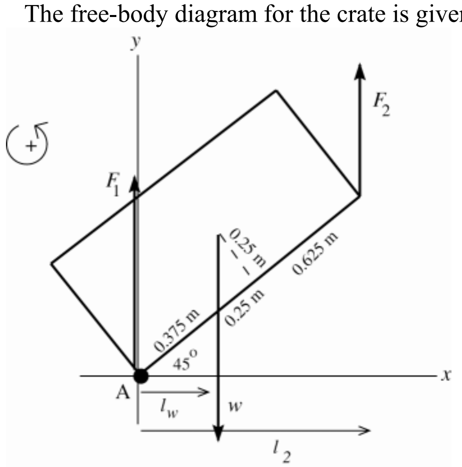


Figure 11.67

$l_w = (0.375\text{ m})\cos 45°$

$l_2 = (1.25\text{ m})\cos 45°$

Let $\vec{F}_1$ and $\vec{F}_2$ be the vertical forces exerted by you and your friend. Take the origin at the lower left-hand corner of the crate (point A).

EXECUTE: $\sum F_y = ma_y$ gives $F_1 + F_2 - w = 0$

$F_1 + F_2 = w = (200\text{ kg})(9.80\text{ m/s}^2) = 1960\text{ N}$

$\sum \tau_A = 0$ gives $F_2 l_2 - w l_w = 0$

$$F_2 = w\left(\frac{l_w}{l_2}\right) = 1960\text{ N}\left(\frac{0.375\text{ m}\cos 45°}{1.25\text{ m}\cos 45°}\right) = 590\text{ N}$$

Then $F_1 = w - F_2 = 1960\text{ N} - 590\text{ N} = 1370\text{ N}$.

EVALUATE: The person below (you) applies a force of 1370 N. The person above (your friend) applies a force of 590 N. It is better to be the person above. As the sketch shows, the moment arm for $\vec{F}_1$ is less than for $\vec{F}_2$, so must have $F_1 > F_2$ to compensate.

11.68. IDENTIFY: Apply the first and second conditions for equilibrium to the forearm.

SET UP: The free-body diagram is given in Figure 11.68a, and when holding the weight in Figure 11.68b. Let $+y$ be upward.

EXECUTE: **(a)** $\Sigma\tau_{\text{Elbow}} = 0$ gives $F_{\text{B}}(3.80\text{ cm}) = (15.0\text{ N})(15.0\text{ cm})$ and $F_{\text{B}} = 59.2\text{ N}$.

(b) $\Sigma\tau_E = 0$ gives $F_{\text{B}}(3.80\text{ cm}) = (15.0\text{ N})(15.0\text{ cm}) + (80.0\text{ N})(33.0\text{ cm})$ and $F_{\text{B}} = 754\text{ N}$. The biceps force has a short lever arm, so it must be large to balance the torques.

(c) $\Sigma F_y = 0$ gives $-F_{\text{E}} + F_{\text{B}} - 15.0\text{ N} - 80.0\text{ N} = 0$ and $F_{\text{E}} = 754\text{N} - 15.0\text{ N} - 80.0\text{ N} = 659\text{ N}$.

EVALUATE: **(d)** The biceps muscle acts perpendicular to the forearm, so its lever arm stays the same, but those of the other two forces *decrease* as the arm is raised. Therefore the tension in the biceps muscle *decreases.*

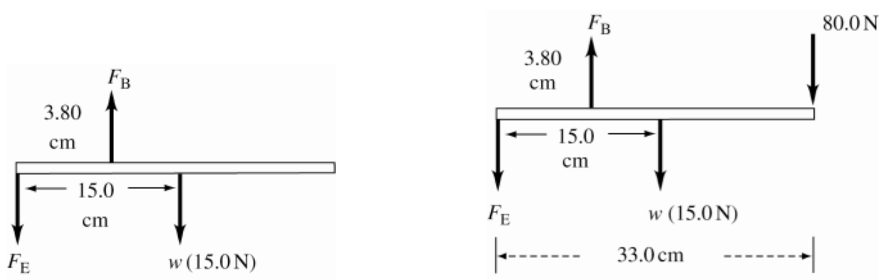


Figure 11.68a, b

11.69. IDENTIFY: Apply $\sum \tau_z = 0$ to the forearm.

SET UP: The free-body diagram for the forearm is given in Fig. 11.10 in the textbook.

EXECUTE: **(a)** $\Sigma\tau_z = 0$, axis at elbow gives $wL - (T\sin\theta)D = 0$. $\sin\theta = \dfrac{h}{\sqrt{h^2 + D^2}}$ so $w = T\dfrac{hD}{L\sqrt{h^2 + D^2}}$.

$$w_{\text{max}} = T_{\text{max}}\frac{hD}{L\sqrt{h^2 + D^2}}.$$

(b) $\dfrac{dw_{\text{max}}}{dD} = \dfrac{T_{\text{max}}h}{L\sqrt{h^2 + D^2}}\left(1 - \dfrac{D^2}{h^2 + D^2}\right)$; the derivative is positive

EVALUATE: **(c)** The result of part (b) shows that w_{max} increases when D increases, since the derivative is positive. w_{max} is larger for a chimp since D is larger.

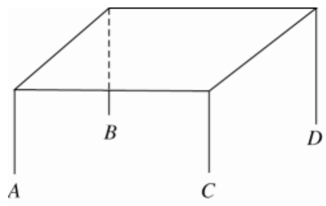
11.70. IDENTIFY: Apply the first and second conditions for equilibrium to the table.

SET UP: Label the legs as shown in Figure 11.70a. Legs A and C are 3.6 m apart. Let the weight be placed closest to legs C and D. By symmetry, $A = B$ and $C = D$. Redraw the table as viewed from the AC side. The free-body diagram in this view is given in Figure 11.70b.

EXECUTE: $\sum \tau_z(\text{about right end}) = 0$ gives $2A(3.6\text{ m}) = (90.0\text{ N})(1.8\text{ m}) + (1500\text{ N})(0.50\text{ m})$ and $A = 130\text{ N} = B$. $\sum F_y = 0$ gives $A + B + C + D = 1590\text{ N}$. Using $A = B = 130\text{ N}$ and $C = D$ gives $C = D = 670\text{ N}$.

By Newton's third law of motion, the forces A, B, C, and D on the table are the same magnitude as the forces the table exerts on the floor.

EVALUATE: As expected, the legs closest to the 1500 N weight exert a greater force on the floor.



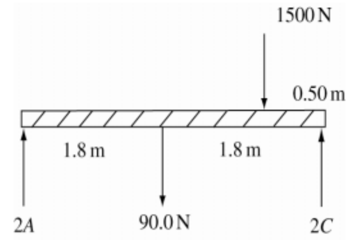


Figure 11.70a, b

11.71. **IDENTIFY:** Apply $\sum \tau_z = 0$ first to the roof and then to one wall.

(a) SET UP: Consider the forces on the roof; see Figure 11.71a.

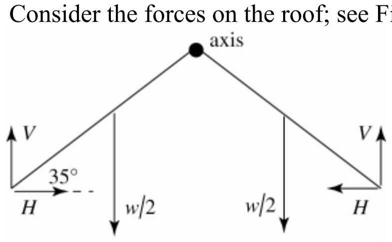


Figure 11.71a

V and H are the vertical and horizontal forces each wall exerts on the roof. $w = 20{,}000$ N is the total weight of the roof. $2V = w$ so $V = w/2$

Apply $\sum \tau_z = 0$ to one half of the roof, with the axis along the line where the two halves join. Let each half have length L.

EXECUTE: $(w/2)(L/2)(\cos 35.0°) + HL\sin 35.0° - VL\cos 35° = 0$

L divides out, and use $V = w/2$

$H\sin 35.0° = \frac{1}{4}w\cos 35.0°$

$$H = \frac{w}{4\tan 35.0°} = 7140 \text{ N}$$

EVALUATE: By Newton's 3rd law, the roof exerts a horizontal, outward force on the wall. For torque about an axis at the lower end of the wall, at the ground, this force has a larger moment arm and hence larger torque the taller the walls.

(b) SET UP: The force diagram for one wall is given in Figure 11.71b.

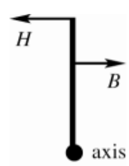


Figure 11.71b

Consider the torques on this wall.

H is the horizontal force exerted by the roof, as considered in part (a). B is the horizontal force exerted by the buttress. Now the angle is $40°$, so $H = \dfrac{w}{4\tan 40°} = 5959$ N

EXECUTE: $\sum \tau_z = 0$, axis at the ground

$H(40 \text{ m}) - B(30 \text{ m}) = 0$ and $B = 7900$ N.

EVALUATE: The horizontal force exerted by the roof is larger as the roof becomes more horizontal, since for torques applied to the roof the moment arm for H decreases. The force B required from the buttress is less the higher up on the wall this force is applied.

11.72. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the wheel.

SET UP: Take torques about the upper corner of the curb.

EXECUTE: The force $\vec{F}$ acts at a perpendicular distance $R - h$ and the weight acts at a perpendicular distance $\sqrt{R^2 - (R-h)^2} = \sqrt{2Rh - h^2}$. Setting the torques equal for the minimum necessary force, $F = mg\dfrac{\sqrt{2Rh - h^2}}{R - h}$.

(b) The torque due to gravity is the same, but the force $\vec{F}$ acts at a perpendicular distance $2R-h$, so the minimum force is $(mg)\sqrt{2Rh-hv}/(2R-h)$.

EVALUATE: **(c)** Less force is required when the force is applied at the top of the wheel, since in this case $\vec{F}$ has a larger moment arm.

11.73. **IDENTIFY:** Apply the first and second conditions of equilibrium to the gate.

SET UP: The free-body diagram for the gate is given in Figure 11.73.

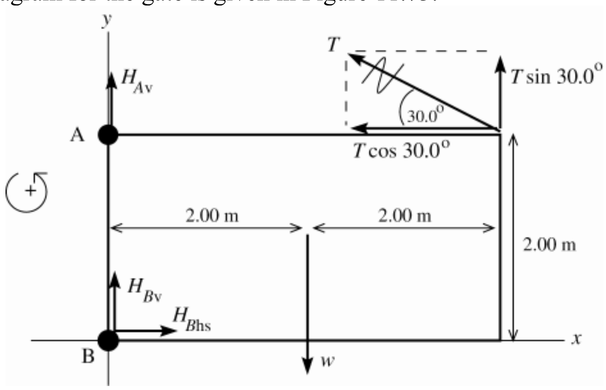


Figure 11.73

Use coordinates with the origin at B. Let $\vec{H}_A$ and $\vec{H}_B$ be the forces exerted by the hinges at A and B. The problem states that $\vec{H}_A$ has no horizontal component. Replace the tension $\vec{T}$ by its horizontal and vertical components.

EXECUTE: **(a)** $\sum\tau_B=0$ gives $+(T\sin 30.0°)(4.00\text{ m})+(T\cos 30.0°)(2.00\text{ m})-w(2.00\text{ m})=0$

$T(2\sin 30.0°+\cos 30.0°)=w$

$$T=\frac{w}{2\sin 30.0°+\cos 30.0°}=\frac{500\text{ N}}{2\sin 30.0°+\cos 30.0°}=268\text{ N}$$

(b) $\sum F_x=ma_x$ says $H_{Bh}-T\cos 30.0°=0$

$H_{Bh}=T\cos 30.0°=(268\text{ N})\cos 30.0°=232\text{ N}$

(c) $\sum F_y=ma_y$ says $H_{Av}+H_{Bv}+T\sin 30.0°-w=0$

$H_{Av}+H_{Bv}=w-T\sin 30.0°=500\text{ N}-(268\text{ N})\sin 30.0°=366\text{ N}$

EVALUATE: T has a horizontal component to the left so H_{Bh} must be to the right, as these are the only two horizontal forces. Note that we cannot determine H_{Av} and H_{Bv} separately, only their sum.

11.74. **IDENTIFY:** Use Eq.(11.3) to locate the *x*-coordinate of the center of gravity of the block combinations.

SET UP: The center of mass and the center of gravity are the same point. For two identical blocks, the center of gravity is midway between the center of the two blocks.

EXECUTE: **(a)** The center of gravity of top block can be as far out as the edge of the lower block. The center of gravity of this combination is then $3L/4$ to the left of the right edge of the upper block, so the overhang is $3L/4$.

(b) Take the two-block combination from part (a), and place it on top of the third block such that the overhang of $3L/4$ is from the right edge of the third block; that is, the center of gravity of the first two blocks is above the right edge of the third block. The center of mass of the three-block combination, measured from the right end of the bottom block, is $-L/6$ and so the largest possible overhang is $(3L/4)+(L/6)=11L/12$. Similarly, placing this three-block combination with its center of gravity over the right edge of the fourth block allows an extra overhang of $L/8$, for a total of $25L/24$.

(c) As the result of part (b) shows, with only four blocks, the overhang can be larger than the length of a single block.

EVALUATE: The sequence of maximum overhangs is $\frac{18L}{24}$, $\frac{22L}{24}$, $\frac{25L}{24}$,…. The increase of overhang when one more block is added is decreasing.

11.75. **IDENTIFY:** Apply the first and second conditions of equilibrium, first to both marbles considered as a composite object and then to the bottom marble.

(a) SET UP: The forces on each marble are shown in Figure 11.75.

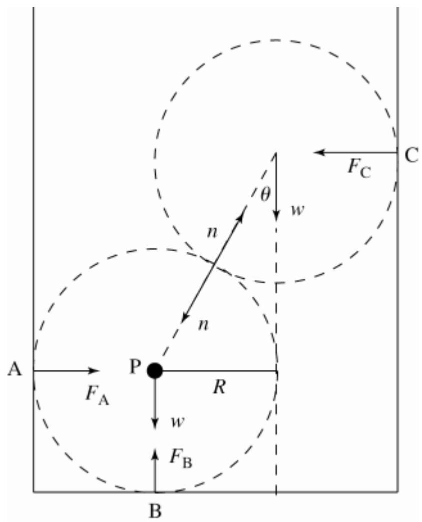


Figure 11.75

EXECUTE:

$F_B = 2w = 1.47\text{ N}$

$\sin\theta = R/2R$ so $\theta = 30°$

$\sum \tau_z = 0,$ axis at P

$F_C(2R\cos\theta) - wR = 0$

$$F_C = \frac{mg}{2\cos 30°} = 0.424\text{ N}$$

$F_A = F_C = 0.424\text{ N}$

(b) Consider the forces on the bottom marble. The horizontal forces must sum to zero, so

$F_A = n\sin\theta$

$$n = \frac{F_A}{\sin 30°} = 0.848\text{ N}$$

Could use instead that the vertical forces sum to zero

$F_B - mg - n\cos\theta = 0$

$$n = \frac{F_B - mg}{\cos 30°} = 0.848\text{ N, which checks.}$$

EVALUATE: If we consider each marble separately, the line of action of every force passes through the center of the marble so there is clearly no torque about that point for each marble. We can use the results we obtained to show that $\sum F_x = 0$ and $\sum F_y = 0$ for the top marble.

11.76. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the right-hand beam.

SET UP: Use the hinge as the axis of rotation and take counterclockwise rotation as positive. If F_{wire} is the tension in each wire and $w = 200\text{ N}$ is the weight of each beam, $2F_{\text{wire}} - 2w = 0$ and $F_{\text{wire}} = w$. Let L be the length of each beam.

EXECUTE: **(a)** $\sum \tau_z = 0$ gives $F_{\text{wire}} L\sin\frac{\theta}{2} - F_{\text{c}}\frac{L}{2}\cos\frac{\theta}{2} - w\frac{L}{2}\sin\frac{\theta}{2} = 0$, where θ is the angle between the beams and F_{c} is the force exerted by the cross bar. The length drops out, and all other quantities except F_{c} are known, so $F_{\text{c}} = \frac{F_{\text{wire}}\sin(\theta/2) - \frac{1}{2}w\sin(\theta/2)}{\frac{1}{2}\cos(\theta/2)} = (2F_{\text{wire}} - w)\tan\frac{\theta}{2}$. Therefore $F = (260\text{ N})\tan\frac{53°}{2} = 130\text{ N}$

(b) The crossbar is under compression, as can be seen by imagining the behavior of the two beams if the crossbar were removed. It is the crossbar that holds them apart.

(c) The upward pull of the wire on each beam is balanced by the downward pull of gravity, due to the symmetry of the arrangement. The hinge therefore exerts no vertical force. It must, however, balance the outward push of the crossbar. The hinge exerts a force 130 N horizontally to the left for the right-hand beam and 130 N to the right for the left-hand beam. Again, it's instructive to visualize what the beams would do if the hinge were removed.

EVALUATE: The force exerted on each beam increases as θ increases and exceeds the weight of the beam for $\theta \geq 90°$.

11.77. **IDENTIFY:** Apply the first and second conditions of equilibrium to the bale.

(a) SET UP: Find the angle where the bale starts to tip. When it starts to tip only the lower left-hand corner of the bale makes contact with the conveyor belt. Therefore the line of action of the normal force *n* passes through the left-hand edge of the bale. Consider $\sum \tau_A = 0$ with point A at the lower left-hand corner. Then $\tau_n = 0$ and $\tau_f = 0,$ so it must be that $\tau_{mg} = 0$ also. This means that the line of action of the gravity must pass through point A. Thus the free-body diagram must be as shown in Figure 11.77a

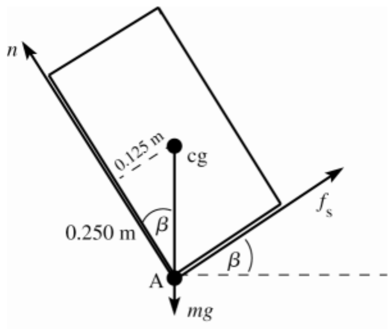


Figure 11.77a

EXECUTE:

$$\tan\beta = \frac{0.125\text{ m}}{0.250\text{ m}}$$

$\beta = 27°,$ angle where tips

SET UP: At the angle where the bale is ready to slip down the incline f_s has its maximum possible value, $f_s = \mu_s n.$ The free-body diagram for the bale, with the origin of coordinates at the cg is given in Figure 11.77b

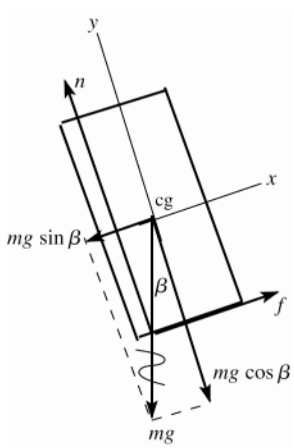


Figure 11.77b

EXECUTE:

$\sum F_y = ma_y$

$n - mg\cos\beta = 0$

$n = mg\cos\beta$

$f_s = \mu_s mg\cos\beta$

(f_s has maximum value when bale ready to slip)

$\sum F_x = ma_x$

$f_s - mg\sin\beta = 0$

$\mu_s mg\cos\beta - mg\sin\beta = 0$

$\tan\beta = \mu_s$

$\mu_s = 0.60$ gives that $\beta = 31°$

$\beta = 27°$ to tip; $\beta = 31°$ to slip, so tips first

(b) The magnitude of the friction force didn't enter into the calculation of the tipping angle; still tips at $\beta = 27°.$

For $\mu_s = 0.40$ tips at $\beta = \arctan(0.40) = 22°$

Now the bale will start to slide down the incline before it tips.

EVALUATE: With a smaller μ_s the slope angle β where the bale slips is smaller.

11.78. **IDENTIFY:** Apply $\sum \tau_z = 0$ and $\sum F_x = 0$ to the bale.

SET UP: Let $+x$ be horizontal to the right. Take the rotation axis to be at the forward edge of the bale, where it contacts the horizontal surface. When the bale just begins to tip, the only point of contact is this point and the normal force produces no torque.

EXECUTE: **(a)** $F = f = \mu_k n = \mu_k mg = (0.35)(30.0\text{ kg})(9.80\text{ m/s}^2) = 103\text{ N}$

(b) With respect to the forward edge of the bale, the lever arm of the weight is $\frac{0.250\text{ m}}{2} = 0.125\text{ m}$ and the lever arm *h* of the applied force is then $h = (0.125\text{ m})\frac{mg}{F} = (0.125\text{ m})\frac{1}{\mu_k} = \frac{0.125\text{ m}}{0.35} = 0.36\text{ m}$.

EVALUATE: As μ_k increases, *F* must increase and the bale tips at a smaller *h*.

11.79. **IDENTIFY:** Apply the first and second conditions of equilibrium to the door.

(a) SET UP: The free-body diagram for the door is given in Figure 11.79.

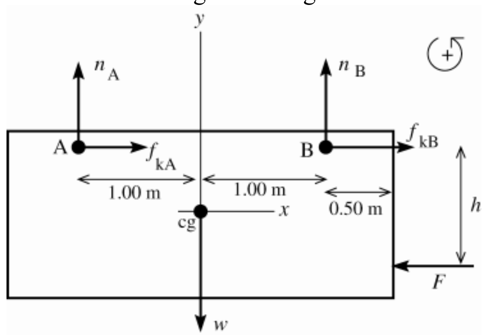


Figure 11.79

Take the origin of coordinates at the center of the door (at the cg). Let n_A f_{kA}, n_B, and f_{kB} be the normal and friction forces exerted on the door at each wheel.

EXECUTE: $\sum F_y = ma_y$

$n_A + n_B - w = 0$

$n_A + n_B = w = 950 \text{ N}$

$\sum F_x = ma_x$

$f_{kA} + f_{kB} - F = 0$

$F = f_{kA} + f_{kB}$

$f_{kA} = \mu_k n_A$, $f_{kB} = \mu_k n_B$, so $F = \mu_k(n_A + n_B) = \mu_k w = (0.52)(950 \text{ N}) = 494 \text{ N}$

$\sum \tau_B = 0$

n_B, f_{kA}, and f_{kB} all have zero moment arms and hence zero torque about this point.

Thus $+w(1.00 \text{ m}) - n_A(2.00 \text{ m}) - F(h) = 0$

$$n_A = \frac{w(1.00 \text{ m}) - F(h)}{2.00 \text{ m}} = \frac{(950 \text{ N})(1.00 \text{ m}) - (494 \text{ N})(1.60 \text{ m})}{2.00 \text{ m}} = 80 \text{ N}$$

And then $n_B = 950 \text{ N} - n_A = 950 \text{ N} - 80 \text{ N} = 870 \text{ N}$.

(b) SET UP: If h is too large the torque of F will cause wheel A to leave the track. When wheel A just starts to lift off the track n_A and f_{kA} both go to zero.

EXECUTE: The equations in part (a) still apply.

$n_A + n_B - w = 0$ gives $n_B = w = 950 \text{ N}$

Then $f_{kB} = \mu_k n_B = 0.52(950 \text{ N}) = 494 \text{ N}$

$F = f_{kA} + f_{kB} = 494 \text{ N}$

$+w(1.00 \text{ m}) - n_A(2.00 \text{ m}) - F(h) = 0$

$$h = \frac{w(1.00 \text{ m})}{F} = \frac{(950 \text{ N})(1.00 \text{ m})}{494 \text{ N}} = 1.92 \text{ m}$$

EVALUATE: The result in part (b) is larger than the value of h in part (a). Increasing h increases the clockwise torque about B due to F and therefore decreases the clockwise torque that n_A must apply.

11.80. **IDENTIFY:** Apply the first and second conditions for equilibrium to the boom.

SET UP: Take the rotation axis at the left end of the boom.

EXECUTE: **(a)** The magnitude of the torque exerted by the cable must equal the magnitude of the torque due to the weight of the boom. The torque exerted by the cable about the left end is $TL\sin\theta$. For any angle θ, $\sin(180° - \theta) = \sin\theta$, so the tension T will be the same for either angle. The horizontal component of the force that the pivot exerts on the boom will be $T\cos\theta$ or $T\cos(180° - \theta) = -T\cos\theta$.

(b) From the result of part (a), T is proportional to $\dfrac{1}{\sin\theta}$ and this becomes infinite as $\theta \to 0$ or $\theta \to 180°$.

(c) The tension is a minimum when $\sin\theta$ is a maximum, or $\theta = 90°$, a vertical cable.

(d) There are no other horizontal forces, so for the boom to be in equilibrium, the pivot exerts zero horizontal force on the boom.

EVALUATE: As the cable approaches the horizontal direction, its moment arm for the axis at the pivot approaches zero, so T must go to infinity in order for the torque due to the cable to continue to equal the gravity torque.

11.81. **IDENTIFY:** Apply the first and second conditions of equilibrium to the pole.

(a) SET UP: The free-body diagram for the pole is given in Figure 11.81.

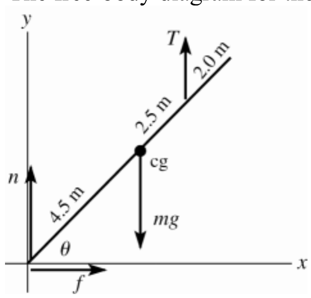


Figure 11.81

n and f are the vertical and horizontal components of the force the ground exerts on the pole.

$\sum F_x = ma_x$

$f = 0$

The force exerted by the ground has no horizontal component.

EXECUTE: $\sum \tau_A = 0$

$+T(7.0\text{ m})\cos\theta - mg(4.5\text{ m})\cos\theta = 0$

$T = mg(4.5\text{ m}/7.0\text{ m}) = (4.5/7.0)(5700\text{ N}) = 3700\text{ N}$

$\sum F_y = 0$

$n + T - mg = 0$

$n = mg - T = 5700\text{ N} - 3700\text{ N} = 2000\text{ N}$

The force exerted by the ground is vertical (upward) and has magnitude 2000 N.

EVALUATE: We can verify that $\sum \tau_z = 0$ for an axis at the cg of the pole. $T > n$ since T acts at a point closer to the cg and therefore has a smaller moment arm for this axis than n does.

(b) In the $\sum \tau_A = 0$ equation the angle θ divided out. All forces on the pole are vertical and their moment arms are all proportional to $\cos\theta$.

11.82. **IDENTIFY:** Apply the equilibrium conditions to the pole. The horizontal component of the tension in the wire is 22.0 N.

SET UP: The free-body diagram for the pole is given in Figure 11.82. The tension in the cord equals the weight W. F_v and F_h are the components of the force exerted by the hinge. If either of these forces is actually in the opposite direction to what we have assumed, we will get a negative value when we solve for it.

EXECUTE: **(a)** $T\sin 37.0° = 22.0\text{ N}$ so $T = 36.6\text{ N}$. $\sum \tau_z = 0$ gives $(T\sin 37.0°)(1.75\text{ m}) - W(1.35\text{ m}) = 0$.

$W = \dfrac{(22.0\text{ N})(1.75\text{ m})}{1.35\text{ m}} = 28.5\text{ N}$.

(b) $\sum F_y = 0$ gives $F_\text{v} - T\cos 37.0° - W = 0$ and $F_\text{v} = (36.6\text{ N})\cos 37.0° + 55.0\text{ N} = 84.2\text{ N}$. $\sum F_x = 0$ gives $W - T\sin 37.0° - F_\text{h} = 0$ and $F_\text{h} = 28.5\text{ N} - 22.0\text{ N} = 6.5\text{ N}$. The magnitude of the hinge force is $F = \sqrt{F_\text{h}^2 + F_\text{v}^2} = 84.5\text{ N}$.

EVALUATE: If we consider torques about an axis at the top of the plate, we see that F_h must be to the left in order for its torque to oppose the torque produced by the force W.

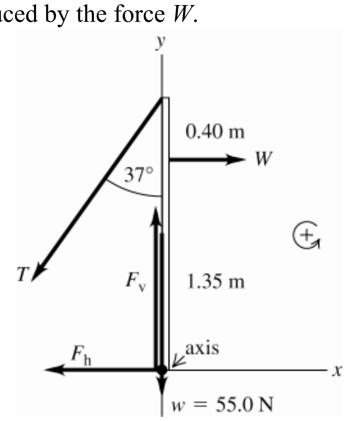


Figure 11.82

11.83. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the slab.

SET UP: The free-body diagram is given in Figure 11.83a. $\tan\beta = \dfrac{3.75\text{ m}}{1.75\text{ m}}$ so $\beta = 65.0°$. $20.0° + \beta + \alpha = 90°$ so $\alpha = 5.0°$. The distance from the axis to the center of the block is $\sqrt{\left(\dfrac{3.75\text{ m}}{2}\right)^2 + \left(\dfrac{1.75\text{ m}}{2}\right)^2} = 2.07\text{ m}$.

EXECUTE: **(a)** $w(2.07\text{ m})\sin 5.0° - T(3.75\text{ m})\sin 52.0° = 0$. $T = 0.061w$. Each worker must exert a force of $0.012w$, where *w* is the weight of the slab.

(b) As θ increases, the moment arm for *w* decreases and the moment arm for *T* increases, so the worker needs to exert less force.

(c) $T \to 0$ when *w* passes through the support point. This situation is sketched in Figure 11.83b.

$\tan\theta = \dfrac{(1.75\text{ m})/2}{(3.75\text{ m})/2}$ and $\theta = 25.0°$. If θ exceeds this value the gravity torque causes the slab to tip over.

EVALUATE: The moment arm for *T* is much greater than the moment arm for *w*, so the force the workers apply is much less than the weight of the slab.

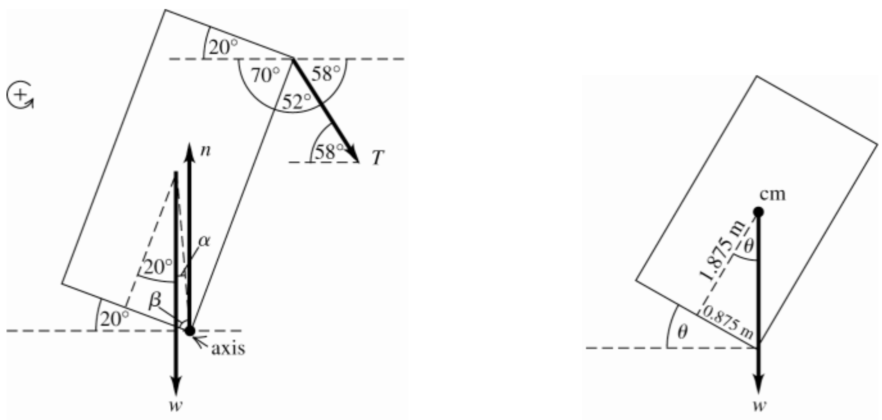


Figure 11.83a, b

11.84. **IDENTIFY:** For a spring, $F = kx$. $Y = \dfrac{F_\perp l_0}{A\Delta l}$.

SET UP: $F_\perp = F = W$ and $\Delta l = x$. For copper, $Y = 11\times 10^{10}\text{ Pa}$.

EXECUTE: **(a)** $F = \left(\dfrac{YA}{l_0}\right)\Delta l = \left(\dfrac{YA}{l_0}\right)x$. This in the form of $F = kx$, with $k = \dfrac{YA}{l_0}$.

(b) $k = \dfrac{YA}{l_0} = \dfrac{(11\times 10^{10}\text{ Pa})\pi(6.455\times 10^{-4}\text{ m})^2}{0.750\text{ m}} = 1.9\times 10^5\text{ N/m}$

(c) $W = kx = (1.9\times 10^5\text{ N/m})(1.25\times 10^{-3}\text{ m}) = 240\text{ N}$

EVALUATE: For the wire the force constant is very large, much larger than for a typical spring.

11.85. **IDENTIFY:** Apply Newton's 2nd law to the mass to find the tension in the wire. Then apply Eq.(11.10) to the wire to find the elongation this tensile force produces.

(a) SET UP: Calculate the tension in the wire as the mass passes through the lowest point. The free-body diagram for the mass is given in Figure 11.85a.

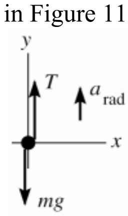


Figure 11.85a

The mass moves in an arc of a circle with radius $R = 0.50$ m. It has acceleration $\vec{a}_{rad}$ directed in toward the center of the circle, so at this point $\vec{a}_{rad}$ is upward.

EXECUTE: $\sum F_y = ma_y$

$T - mg = mR\omega^2$ so that $T = m(g + R\omega^2)$.

But ω must be in rad/s:

$\omega = (120 \text{ rev/min})(2\pi \text{ rad/1 rev})(1 \text{ min/60 s}) = 12.57 \text{ rad/s}$.

Then $T = (12.0 \text{ kg})(9.80 \text{ m/s}^2 + (0.50 \text{ m})(12.57 \text{ rad/s})^2) = 1066 \text{ N}$.

Now calculate the elongation Δl of the wire that this tensile force produces:

$$Y = \frac{F_\perp l_0}{A\,\Delta l} \text{ so } \Delta l = \frac{F_\perp l_0}{YA} = \frac{(1066 \text{ N})(0.50 \text{ m})}{(7.0\times10^{10} \text{ Pa})(0.014\times10^{-4} \text{ m}^2)} = 0.54 \text{ cm}.$$

(b) SET UP: The acceleration $\vec{a}_{\text{rad}}$ is directed in towards the center of the circular path, and at this point in the motion this direction is downward. The free-body diagram is given in Figure 11.85b.

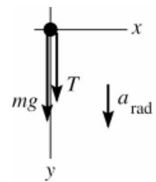


Figure 11.85b

EXECUTE:

$$\sum F_y = ma_y$$

$$mg + T = mR\omega^2$$

$$T = m(R\omega^2 - g)$$

$$T = (12.0 \text{ kg})((0.50 \text{ m})(12.57 \text{ rad/s})^2 - 9.80 \text{ m/s}^2) = 830 \text{ N}$$

$$\Delta l = \frac{F_\perp l_0}{YA} = \frac{(830 \text{ N})(0.50 \text{ m})}{(7.0\times10^{10} \text{ Pa})(0.014\times10^{-4} \text{ m}^2)} = 0.42 \text{ cm}.$$

EVALUATE: At the lowest point T and w are in opposite directions and at the highest point they are in the same direction, so T is greater at the lowest point and the elongation is greatest there. The elongation is at most 1% of the length.

11.86. IDENTIFY: $F_\perp = \left(\frac{YA}{l_0}\right)\Delta l$ so the slope of the graph in part (a) depends on Young's modulus.

SET UP: $F_\perp$ is the total load, 20 N plus the added load.

EXECUTE: **(a)** The graph is given in Figure 11.86.

(b) The slope is $\dfrac{60 \text{ N}}{(3.32 - 3.02)\times10^{-2} \text{ m}} = 2.0\times10^4 \text{ N/m}$.

$$Y = \left(\frac{l_0}{\pi r^2}\right)(2.0\times10^4 \text{ N/m}) = \left(\frac{3.50 \text{ m}}{\pi[0.35\times10^{-3} \text{ m}]^2}\right)(2.0\times10^4 \text{ N/m}) = 1.8\times10^{11} \text{ Pa}$$

(c) The stress is $F_\perp / A$. The total load at the proportional limit is $60 \text{ N} + 20 \text{ N} = 80 \text{ N}$.

$$\text{stress} = \frac{80 \text{ N}}{\pi(0.35\times10^{-3} \text{ m})^2} = 2.1\times10^8 \text{ Pa}$$

EVALUATE: The value of Y we calculated is close to the value for iron, nickel and steel in Table 11.1.

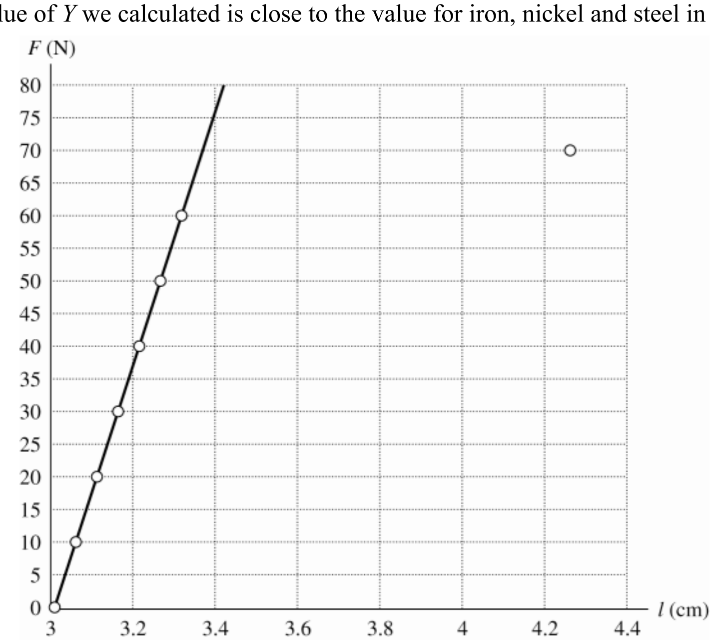


Figure 11.86

11.87. **IDENTIFY:** Use the second condition of equilibrium to relate the tension in the two wires to the distance *w* is from the left end. Use Eqs.(11.8) and (11.10) to relate the tension in each wire to its stress and strain.

(a) SET UP: stress $= F_\perp / A$, so equal stress implies T/A same for each wire.

$T_A / 2.00\ \text{mm}^2 = T_B / 4.00\ \text{mm}^2$ so $T_B = 2.00T_A$

The question is where along the rod to hang the weight in order to produce this relation between the tensions in the two wires. Let the weight be suspended at point *C*, a distance *x* to the right of wire *A*. The free-body diagram for the rod is given in Figure 11.87.

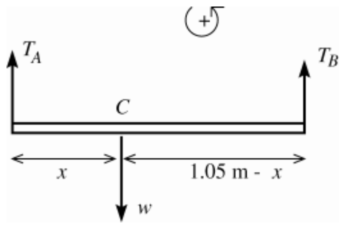


Figure 11.87

EXECUTE:

$\sum \tau_C = 0$

$+T_B(1.05\ \text{m} - x) - T_A x = 0$

But $T_B = 2.00T_A$ so $2.00T_A(1.05\ \text{m} - x) - T_A x = 0$

$2.10\ \text{m} - 2.00x = x$ and $x = 2.10\ \text{m}/3.00 = 0.70\ \text{m}$ (measured from *A*).

(b) SET UP: $Y = \text{stress/strain}$ gives that $\text{strain} = \text{stress}/Y = F_\perp / AY$.

EXECUTE: Equal strain thus implies

$$\frac{T_A}{(2.00\ \text{mm}^2)(1.80\times 10^{11}\ \text{Pa})} = \frac{T_B}{(4.00\ \text{mm}^2)(1.20\times 10^{11}\ \text{Pa})}$$

$$T_B = \left(\frac{4.00}{2.00}\right)\left(\frac{1.20}{1.80}\right)T_A = 1.333T_A.$$

The $\sum \tau_C = 0$ equation still gives $T_B(1.05\ \text{m} - x) - T_A x = 0$.

But now $T_B = 1.333T_A$ so $(1.333T_A)(1.05\ \text{m} - x) - T_A x = 0$

$1.40\ \text{m} = 2.33x$ and $x = 1.40\ \text{m}/2.33 = 0.60\ \text{m}$ (measured from *A*).

EVALUATE: Wire *B* has twice the diameter so it takes twice the tension to produce the same stress. For equal stress the moment arm for T_B (0.35 m) is half that for T_A (0.70 m), since the torques must be equal. The smaller *Y* for *B* partially compensates for the larger area in determining the strain and for equal strain the moment arms are closer to being equal.

11.88. **IDENTIFY:** Apply Eq.(11.10) and calculate Δl.

SET UP: When the ride is at rest the tension $F_\perp$ in the rod is the weight 1900 N of the car and occupants. When the ride is operating, the tension $F_\perp$ in the rod is obtained by applying $\sum \vec{F} = m\vec{a}$ to a car and its occupants. The free-body diagram is shown in Figure 11.88. The car travels in a circle of radius $r = l\sin\theta$, where *l* is the length of the rod and θ is the angle the rod makes with the vertical. For steel, $Y = 2.0\times 10^{11}\ \text{Pa}$.

$\omega = 8.00\ \text{rev/min} = 0.838\ \text{rad/s}$.

EXECUTE: **(a)** $\Delta l = \dfrac{l_0 F_\perp}{YA} = \dfrac{(15.0\ \text{m})(1900\ \text{N})}{(2.0\times 10^{11}\ \text{Pa})(8.00\times 10^{-4}\ \text{m}^2)} = 1.78\times 10^{-4}\ \text{m} = 0.18\ \text{mm}$

(b) $\sum F_x = ma_x$ gives $F_\perp \sin\theta = mr\omega^2 = ml\sin\theta\omega^2$ and

$F_\perp = ml\omega^2 = \left(\dfrac{1900\ \text{N}}{9.80\ \text{m/s}^2}\right)(15.0\ \text{m})(0.838\ \text{rad/s})^2 = 2.04\times 10^3\ \text{N}$. $\Delta l = \left(\dfrac{2.04\times 10^3\ \text{N}}{1900\ \text{N}}\right)(0.18\ \text{mm}) = 0.19\ \text{mm}$

EVALUATE: $\sum F_y = ma_y$ gives $F_\perp \cos\theta = mg$ and $\cos\theta = mg/F_\perp$. As ω increases $F_\perp$ increases and $\cos\theta$ becomes small. Smaller $\cos\theta$ means θ increases, so the rods move toward the horizontal as ω increases.

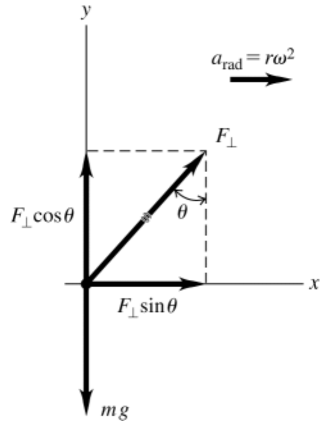


Figure 11.88

11.89. **IDENTIFY** and **SET UP:** The tension is the same at all points along the composite rod. Apply Eqs.(11.8) and (11.10) to relate the elongations, stresses, and strains for each rod in the compound.

EXECUTE: Each piece of the composite rod is subjected to a tensile force of 4.00×10^4 N.

(a) $Y = \dfrac{F_\perp l_0}{A\,\Delta l}$ so $\Delta l = \dfrac{F_\perp l_0}{YA}$

$\Delta l_{\rm b} = \Delta l_{\rm n}$ gives that $\dfrac{F_\perp l_{0,\rm b}}{Y_{\rm b}A_{\rm b}} = \dfrac{F_\perp l_{0,\rm n}}{Y_{\rm n}A_{\rm n}}$ (b for brass and n for nickel); $l_{0,\rm n} = L$

But the $F_\perp$ is the same for both, so

$$l_{0,\rm n} = \frac{Y_{\rm n}}{Y_{\rm b}}\frac{A_{\rm n}}{A_{\rm b}} l_{0,\rm b}$$

$$L = \left(\frac{21\times10^{10}\ \text{Pa}}{9.0\times10^{10}\ \text{Pa}}\right)\left(\frac{1.00\ \text{cm}^2}{2.00\ \text{cm}^2}\right)(1.40\ \text{m}) = 1.63\ \text{m}$$

(b) stress $= F_\perp / A = T/A$

brass: stress $= T/A = (4.00\times10^4\ \text{N})/(2.00\times10^{-4}\ \text{m}^2) = 2.00\times10^8$ Pa

nickel: stress $= T/A = (4.00\times10^4\ \text{N})/(1.00\times10^{-4}\ \text{m}^2) = 4.00\times10^8$ Pa

(c) $Y =$ stress/strain and strain $=$ stress/Y

brass: strain $= (2.00\times10^8\ \text{Pa})/(9.0\times10^{10}\ \text{Pa}) = 2.22\times10^{-3}$

nickel: strain $= (4.00\times10^8\ \text{Pa})/(21\times10^{10}\ \text{Pa}) = 1.90\times10^{-3}$

EVALUATE: Larger Y means less Δl and smaller A means greater Δl, so the two effects largely cancel and the lengths don't differ greatly. Equal Δl and nearly equal l means the strains are nearly the same. But equal tensions and A differing by a factor of 2 means the stresses differ by a factor of 2.

11.90. **IDENTIFY:** Apply $\dfrac{F_\perp}{A} = Y\left(\dfrac{\Delta l}{l_0}\right)$. The height from which he jumps determines his speed at the ground. The acceleration as he stops depends on the force exerted on his legs by the ground.

SET UP: In considering his motion take $+y$ downward. Assume constant acceleration as he is stopped by the floor.

EXECUTE: **(a)** $F_\perp = YA\left(\dfrac{\Delta l}{l_0}\right) = (3.0\times10^{-4}\ \text{m}^2)(14\times10^9\ \text{Pa})(0.010) = 4.2\times10^4$ N

(b) As he is stopped by the ground, the net force on him is $F_{\rm net} = F_\perp - mg$, where $F_\perp$ is the force exerted on him by the ground. From part (a), $F_\perp = 2(4.2\times10^4\ \text{N}) = 8.4\times10^4$ N and $F = 8.4\times10^4\ \text{N} - (70\ \text{kg})(9.80\ \text{m/s}^2) = 8.33\times10^4$ N. $F_{\rm net} = ma$ gives $a = 1.19\times10^3\ \text{m/s}^2$. $a_y = -1.19\times10^3\ \text{m/s}^2$ since the acceleration is upward. $v_y = v_{0y} + a_y t$ gives $v_{0y} = -a_y t = (-1.19\times10^3\ \text{m/s}^2)(0.030\ \text{s}) = 35.7$ m/s. His speed at the ground therefore is $v = 35.7$ m/s. This speed is related to his initial height h above the floor by $\frac{1}{2}mv^2 = mgh$ and $h = \dfrac{v^2}{2g} = \dfrac{(35.7\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 65$ m.

EVALUATE: Our estimate is based solely on compressive stress; other injuries are likely at a much lower height.

11.91. **IDENTIFY** and **SET UP:** $Y = F_{\perp} l_0 / A\,\Delta l$ (Eq.11.10 holds since the problem states that the stress is proportional to the strain.) Thus $\Delta l = F_{\perp} l_0 / AY$. Use proportionality to see how changing the wire properties affects Δl.

EXECUTE: **(a)** Change l_0 but $F_{\perp}$ (same floodlamp), A (same diameter wire), and Y (same material) all stay the same.

$$\frac{\Delta l}{l_0} = \frac{F_{\perp}}{AY} = \text{constant, so } \frac{\Delta l_1}{\Delta l_{01}} = \frac{\Delta l_2}{\Delta l_{02}}$$

$$\Delta l_2 = \Delta l_1 (l_{02}/l_{01}) = 2\Delta l_1 = 2(0.18 \text{ mm}) = 0.36 \text{ mm}$$

(b) $A = \pi(d/2)^2 = \frac{1}{4}\pi d^2$, so $\Delta l = \dfrac{F_{\perp} l_0}{\frac{1}{4}\pi d^2 Y}$

$F_{\perp}$, l_0, Y all stay the same, so $\Delta l(d^2) = F_{\perp} l_0 / \left(\frac{1}{4}\pi Y\right) = \text{constant}$

$$\Delta l_1(d_1^2) = \Delta l_2(d_2^2)$$

$$\Delta l_2 = \Delta l_1 (d_1/d_2)^2 = (0.18 \text{ mm})(1/2)^2 = 0.045 \text{ mm}$$

(c) $F_{\perp}$, l_0, A all stay the same so $\Delta l Y = F_{\perp} l_0 / A = \text{constant}$

$$\Delta l_1 Y_1 = \Delta l_2 Y_2$$

$$\Delta l_2 = \Delta l_1 (Y_1/Y_2) = (0.18 \text{ mm})(20\times10^{10} \text{ Pa}/11\times10^{10} \text{ Pa}) = 0.33 \text{ mm}$$

EVALUATE: Greater l means greater Δl, greater diameter means less Δl, and smaller Y means greater Δl.

11.92. **IDENTIFY:** Apply Eq.(11.13) and calculate ΔV.

SET UP: The pressure increase is w/A, where w is the weight of the bricks and A is the area πr^2 of the piston.

EXECUTE: $\Delta p = \dfrac{(1420 \text{ kg})(9.80 \text{ m/s}^2)}{\pi(0.150 \text{ m})^2} = 1.97\times10^5 \text{ Pa}$

$$\Delta p = -B\frac{\Delta V}{V_0} \text{ gives } \Delta V = -\frac{(\Delta p)V_0}{B} = -\frac{(1.97\times10^5 \text{ Pa})(250 \text{ L})}{9.09\times10^8 \text{ Pa}} = -0.0542 \text{ L}$$

EVALUATE: The fractional change in volume is only 0.022%, so this attempt is not worth the effort.

11.93. **IDENTIFY** and **SET UP:** Apply Eqs.(11.8) and (11.15). The tensile stress depends on the component of $\vec{F}$ perpendicular to the plane and the shear stress depends on the component of $\vec{F}$ parallel to the plane. The forces are shown in Figure 11.93a

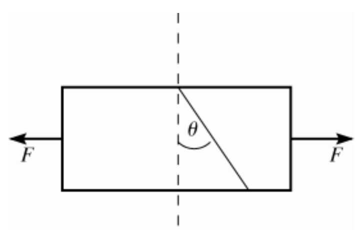


Figure 11.93a

(a) EXECUTE: The components of F are shown in Figure 11.93b.

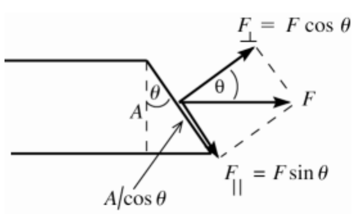


Figure 11.93b

The area of the diagonal face is $A/\cos\theta$.

$$\text{tensile stress} = \frac{F_{\perp}}{(A/\cos\theta)} = F\cos\theta/(A/\cos\theta) = \frac{F\cos^2\theta}{A}.$$

(b) $\text{shear stress} = \dfrac{F_{\parallel}}{(A/\cos\theta)} = F\sin\theta/(A/\cos\theta) = \dfrac{F\sin\theta\cos\theta}{A} = \dfrac{F\sin 2\theta}{2A}$ (using a trig identity).

EVALUATE: **(c)** From the result of part (a) the tensile stress is a maximum for $\cos\theta = 1$, so $\theta = 0°$.

(d) From the result of part (b) the shear stress is a maximum for $\sin 2\theta = 1$, so for $2\theta = 90°$ and thus $\theta = 45°$

11.94. **IDENTIFY:** Apply the first and second conditions of equilibrium to the rod. Then apply Eq.(11.10) to relate the compressive force on the rod to its change in length.

SET UP: For copper, $Y = 1.1\times10^{11}$ Pa .

EXECUTE: **(a)** Taking torques about the pivot, the tension T in the cable is related to the weight by $T(\sin\theta)l_0 = mgl_0/2$, so $T = \dfrac{mg}{2\sin\theta}$. The horizontal component of the force that the cable exerts on the rod, and hence the horizontal component of the force that the pivot exerts on the rod, is $\dfrac{mg}{2}\cot\theta$ and the stress is $\dfrac{mg}{2A}\cot\theta$.

(b) $\Delta l = \dfrac{l_0F}{AY} = \dfrac{mgl_0\cot\theta}{2AY}$. Δl corresponds to a decrease in length.

(c) In terms of the density and length, $(m/A) = \rho l_0$, so the stress is $(\rho l_0 g/2)\cot\theta$ and the change in length is $(\rho l_0^2 g/2Y)\cot\theta$.

(d) Using the numerical values, the stress is 1.4×10^5 Pa and the change in length is 2.2×10^{-6} m.

(e) The stress is proportional to the length and the change in length is proportional to the square of the length, and so the quantities change by factors of 2 and 4.

EVALUATE: The compressive force and therefore the decrease in length increase as θ decreases and the cable becomes more nearly horizontal.

11.95. **IDENTIFY:** Apply the first and second conditions for equilibrium to the bookcase.

SET UP: When the bookcase is on the verge of tipping, it contacts the floor only at its lower left-hand edge and the normal force acts at this point. When the bookcase is on the verge of slipping, the static friction force has its largest possible value, $\mu_s n$.

EXECUTE: **(a)** Taking torques about the left edge of the left leg, the bookcase would tip when $F = \dfrac{(1500\text{ N})(0.90\text{ m})}{(1.80\text{ m})} = 750$ N and would slip when $F = (\mu_s)(1500\text{ N}) = 600$ N, so the bookcase slides before tipping.

(b) If F is vertical, there will be no net horizontal force and the bookcase could not slide. Again taking torques about the left edge of the left leg, the force necessary to tip the case is $\dfrac{(1500\text{ N})(0.90\text{ m})}{(0.10\text{ m})} = 13.5$ kN .

(c) To slide, the friction force is $f = \mu_s(w + F\cos\theta)$, and setting this equal to $F\sin\theta$ and solving for F gives $F = \dfrac{\mu_s w}{\sin\theta - \mu_s\cos\theta}$ (to slide). To tip, the condition is that the normal force exerted by the right leg is zero, and taking torques about the left edge of the left leg, $F\sin\theta(1.80\text{ m}) + F\cos\theta(0.10\text{ m}) = w(0.90\text{ m})$, and solving for F gives $F = \dfrac{w}{(1/9)\cos\theta + 2\sin\theta}$ (to tip). Setting the two expressions equal to each other gives

$\mu_s((1/9)\cos\theta + 2\sin\theta) = \sin\theta - \mu_s\cos\theta$ and solving for θ gives $\theta = \arctan\left(\dfrac{(10/9)\mu_s}{(1-2\mu_s)}\right) = 66°$.

EVALUATE: The result in (c) depends not only on the numerical value of μ_s but also on the width and height of the bookcase.

11.96. **IDENTIFY:** Apply $\sum\tau_z = 0$ to the post, for various choices of the location of the rotation axis.

SET UP: When the post is on the verge of slipping, f_s has its largest possible value, $f_s = \mu_s n$.

EXECUTE: **(a)** Taking torques about the point where the rope is fastened to the ground, the lever arm of the applied force is $h/2$ and the lever arm of both the weight and the normal force is $h\tan\theta$, and so $F\dfrac{h}{2} = (n-w)h\tan\theta$.

Taking torques about the upper point (where the rope is attached to the post), $fh = F\dfrac{h}{2}$. Using $f \le \mu_s n$ and solving for F, $F \le 2w\left(\dfrac{1}{\mu_s} - \dfrac{1}{\tan\theta}\right)^{-1} = 2(400\text{ N})\left(\dfrac{1}{0.30} - \dfrac{1}{\tan 36.9°}\right)^{-1} = 400$ N .

(b) The above relations between F, n and f become $F\dfrac{3}{5}h = (n-w)h\tan\theta$, $f = \dfrac{2}{5}F$, and eliminating f and n and solving for F gives $F \le w\left(\dfrac{2/5}{\mu_s} - \dfrac{3/5}{\tan\theta}\right)^{-1}$, and substitution of numerical values gives 750 N to two figures.

(c) If the force is applied a distance y above the ground, the above relations become $Fy = (n-w)h\tan\theta,\ F(h-y) = fh,$ which become, on eliminating n and f, $w \ge F\left[\dfrac{(1-y/h)}{\mu_s} - \dfrac{(y/h)}{\tan\theta}\right].$ As the term in square brackets approaches zero, the necessary force becomes unboundedly large. The limiting value of y is found by setting the term in square brackets equal to zero. Solving for y gives

$$\frac{y}{h} = \frac{\tan\theta}{\mu_s + \tan\theta} = \frac{\tan 36.9°}{0.30 + \tan 36.9°} = 0.71.$$

EVALUATE: For the post to slip, for an axis at the top of the post the torque due to F must balance the torque due to the friction force. As the point of application of F approaches the top of the post, its moment arm for this axis approaches zero.

11.97. **IDENTIFY:** Apply $\sum \tau_z = 0$ to the girder.

SET UP: Assume that the center of gravity of the loaded girder is at $L/2$, and that the cable is attached a distance x to the right of the pivot. The sine of the angle between the lever arm and the cable is then $h\big/\sqrt{h^2 + ((L/2)-x)^2}$.

EXECUTE: The tension is obtained from balancing torques about the pivot;

$T\left[\dfrac{hx}{\sqrt{h^2 + ((L/2)-x)^2}}\right] = wL/2,$ where w is the total load . The minimum tension will occur when the term in square brackets is a maximum; differentiating and setting the derivative equal to zero gives a maximum, and hence a minimum tension, at $x_{\text{min}} = (h^2/L) + (L/2).$ However, if $x_{\text{min}} > L,$ which occurs if $h > L/\sqrt{2},$ the cable must be attached at L, the farthest point to the right.

EVALUATE: Note that x_{min} is greater than $L/2$ but approaches $L/2$ as $h \to 0$. The tension is a minimum when the cable is attached somewhere on the right-hand half of the girder.

11.98. **IDENTIFY:** Apply the equilibrium conditions to the ladder combination and also to each ladder.

SET UP: The geometry of the 3-4-5 right triangle simplifies some of the intermediate algebra. Denote the forces on the ends of the ladders by F_L and F_R (left and right). The contact forces at the ground will be vertical, since the floor is assumed to be frictionless.

EXECUTE: **(a)** Taking torques about the right end, $F_L(5.00\text{ m}) = (480\text{ N})(3.40\text{ m}) + (360\text{ N})(0.90\text{ m})$, so $F_L = 391\text{ N}$. F_R may be found in a similar manner, or from $F_R = 840\text{ N} - F_L = 449\text{ N}.$

(b) The tension in the rope may be found by finding the torque on each ladder, using the point A as the origin. The lever arm of the rope is 1.50 m. For the left ladder, $T(1.50\text{ m}) = F_L(3.20\text{ m}) - (480\text{ N})(1.60\text{ m}),$ so $T = 322.1\text{ N}$ (322 N to three figures). As a check, using the torques on the right ladder, $T(1.50\text{ m}) = F_R(1.80\text{ m}) - (360\text{ N})(0.90\text{ m})$ gives the same result.

(c) The horizontal component of the force at A must be equal to the tension found in part (b). The vertical force must be equal in magnitude to the difference between the weight of each ladder and the force on the bottom of each ladder, $480\text{ N} - 391\text{ N} = 449\text{ N} - 360\text{ N} = 89\text{ N}.$ The magnitude of the force at A is then $\sqrt{(322.1\text{ N})^2 + (89\text{ N})^2} = 334\text{ N}.$

(d) The easiest way to do this is to see that the added load will be distributed at the floor in such a way that $F'_L = F_L + (0.36)(800\text{ N}) = 679\text{ N},$ and $F'_R = F_R + (0.64)(800\text{ N}) = 961\text{ N}.$ Using these forces in the form for the tension found in part (b) gives

$$T = \frac{F'_L(3.20\text{ m}) - (480\text{ N})(1.60\text{ m})}{(1.50\text{ m})} = \frac{F'_R(1.80\text{ m}) - (360\text{ N})(0.90\text{ m})}{(1.50\text{ m})} = 937\text{ N}.$$

EVALUATE: The presence of the painter increases the tension in the rope, even though his weight is vertical and the tension force is horizontal.

11.99. **IDENTIFY:** Apply Eq.(11.14) to each material, the oil and the sodium. For each material, $\Delta p = F/A$.

SET UP: The total volume change, ΔV_{tot} , is related to the distance the piston moves by $\Delta V_{\text{tot}} = Ax$.

EXECUTE: The change in the volume of the oil is $k_O v_O \Delta p$ and the change in the volume of the sodium is $k_s v_s \Delta p$. Setting the total volume change equal to Ax (x is positive) and using $\Delta p = F/A,\quad Ax = (k_O V_O + k_s V_s)(F/A),$ and solving for k_s gives $k_s = \left(\dfrac{A^2 x}{F} - k_O V_O\right)\dfrac{1}{V_s}.$

EVALUATE: Neglecting the volume change of the oil corresponds to setting $k_O = 0$, and in that case $k_s = \dfrac{A^2 x}{FV_s}$. In either case, x is larger when k_s is larger.

11.100. IDENTIFY: Write $\Delta(pV)$ or $\Delta(pV^\gamma)$ in terms of Δp and ΔV and use the fact that pV or pV^γ is constant.

SET UP: B is given by Eq.(11.13).

EXECUTE: **(a)** For constant temperature $(\Delta T = 0)$, $\Delta(pV) = (\Delta p)V + p(\Delta V) = 0$ and $B = -\dfrac{(\Delta p)V}{(\Delta V)} = p$.

(b) In this situation, $(\Delta p)V^\gamma + \gamma p(\Delta V)\; V^{\gamma-1} = 0,\quad (\Delta p) + \gamma p\dfrac{\Delta V}{V} = 0,$ and $B = -\dfrac{(\Delta p)V}{\Delta V} = \gamma p.$

EVALUATE: We will see later that $\gamma > 1$, so B is larger in part (b).

11.101. IDENTIFY: Apply Eq.(11.10) to calculate Δl.

SET UP: For steel, $Y = 2.0 \times 10^{11}$ Pa.

EXECUTE: **(a)** From Eq.(11.10), $\Delta l = \dfrac{(4.50\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m})}{(20\times 10^{10}\text{ Pa})(5.00\times 10^{-7}\text{ m}_2)} = 6.62\times 10^{-4}$ m, or 0.66 mm to two figures.

(b) $(4.50\text{ kg})(9.80\text{ m/s}^2)(0.0500\times 10^{-2}\text{ m}) = 0.022$ J.

(c) The magnitude F will vary with distance; the average force is $YA(0.0250\text{ cm}/l_0) = 16.7$ N, and so the work done by the applied force is $(16.7\text{N})(0.0500\times 10^{-2}\text{ m}) = 8.35\times 10^{-3}$ J.

(d) The average force the wire exerts is $(450\text{ kg})g + 16.7\text{ N} = 60.8$ N. The work done is negative, and equal to $-(60.8\text{ N})(0.0500\times 10^{-2}\text{ m}) = -3.04\times 10^{-2}$ J.

(e) Eq.(11.10) is in the form of Hooke's law, with $k = \dfrac{YA}{l_0}$. $U_{el} = \frac{1}{2}kx^2$, so $\Delta U_{el} = \frac{1}{2}k(x_2^2 - x_1^2)$.

$x_1 = 6.62\times 10^{-4}$ m and $x_2 = 0.500\times 10^{-3}\text{ m} + x_1 = 11.62\times 10^{-4}$ m. The change in elastic potential energy is

$\dfrac{(20\times 10^{10}\text{ Pa})(5.00\times 10^{-7}\text{ m}^2)}{2(1.50\text{ m})}((11.62\times 10^{-4}\text{ m})^2 - (6.62\times 10^{-4}\text{ m})^2) = 3.04\times 10^{-2}$ J, the negative of the result of part (d).

EVALUATE: The tensile force in the wire is conservative and obeys the relation $W = -\Delta U$.

GRAVITATION

12

12.1. IDENTIFY and SET UP: Use the law of gravitation, Eq.(12.1), to determine F_g.

EXECUTE: $F_{\text{S on M}} G\dfrac{m_S m_M}{r_{SM}^2}$ (S = sun, M = moon); $F_{\text{E on M}} = G\dfrac{m_E m_M}{r_{EM}^2}$ (E = earth)

$$\frac{F_{\text{S on M}}}{F_{\text{E on M}}} = \left(G\frac{m_S m_M}{r_{SM}^2}\right)\left(\frac{r_{EM}^2}{G m_E m_M}\right) = \frac{m_S}{m_E}\left(\frac{r_{EM}}{r_{SM}}\right)^2$$

r_{EM}, the radius of the moon's orbit around the earth is given in Appendix F as 3.84×10^8 m. The moon is much closer to the earth than it is to the sun, so take the distance r_{SM} of the moon from the sun to be r_{SE}, the radius of the earth's orbit around the sun.

$$\frac{F_{\text{S on M}}}{F_{\text{E on M}}} = \left(\frac{1.99\times10^{30}\text{ kg}}{5.98\times10^{24}\text{ kg}}\right)\left(\frac{3.84\times10^8\text{ m}}{1.50\times10^{11}\text{ m}}\right)^2 = 2.18.$$

EVALUATE: The force exerted by the sun is larger than the force exerted by the earth. The moon's motion is a combination of orbiting the sun and orbiting the earth.

12.2. IDENTIFY: The gravity force between spherically symmetric spheres is $F_g = \dfrac{Gm_1m_2}{r^2}$, where r is the separation between their centers.

SET UP: $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$. The moment arm for the torque due to each force is 0.150 m.

EXECUTE: **(a)** For each pair of spheres, $F_g = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.10\text{ kg})(25.0\text{ kg})}{(0.120\text{ m})^2} = 1.27\times10^{-7}\text{ N}$. From Figure 12.4 in the textbook we see that the forces for each pair are in opposite directions, so $F_{net} = 0$.

(b) The net torque is $\tau_{net} = 2F_g l = 2(1.27\times10^{-7}\text{ N})(0.150\text{ m}) = 3.81\times10^{-8}\ \text{N}\cdot\text{m}$.

(c) The torque is very small and the apparatus must be very sensitive. The torque could be increased by increasing the mass of the spheres or by decreasing their separation.

EVALUATE: The quartz fiber must twist through a measurable angle when a small torque is applied to it.

12.3. IDENTIFY: The force exerted on the particle by the earth is $w = mg$, where m is the mass of the particle. The force exerted by the 100 kg ball is $F_g = \dfrac{Gm_1m_2}{r^2}$, where r is the distance of the particle from the center of the ball.

SET UP: $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$, $g = 9.80\text{ m/s}^2$.

EXECUTE: $F_g = w$ gives $\dfrac{Gmm_{ball}}{r^2} = mg$ and

$$r = \sqrt{\frac{Gm_{ball}}{g}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(100\text{ kg})}{9.80\text{ m/s}^2}} = 2.61\times10^{-5}\text{ m} = 0.0261\text{ mm}$$

It is not feasible to do this; a 100 kg ball would have a radius much larger than 0.0261 mm.

EVALUATE: The gravitational force between ordinary objects is very small. The gravitational force exerted by the earth on objects near its surface is large enough to be important because the mass of the earth is very large.

12.4. IDENTIFY: Apply Eq.(12.2), generalized to any pair of spherically symmetric objects.

SET UP: The separation of the centers of the spheres is $2R$.

EXECUTE: The magnitude of the gravitational attraction is $GM^2/(2R)^2 = GM^2/4R^2$.

EVALUATE: Eq.(12.2) applies to any pair of spherically symmetric objects; one of the objects doesn't have to be the earth.

12.5. **IDENTIFY:** Use Eq.(12.1) to calculate F_g exerted by the earth and by the sun and add these forces as vectors.

(a) SET UP: The forces and distances are shown in Figure 12.5.

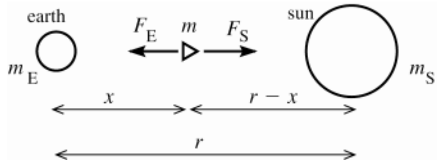


Figure 12.5

Let $\vec{F}_E$ and $\vec{F}_S$ be the gravitational forces exerted on the spaceship by the earth and by the sun.

EXECUTE: The distance from the earth to the sun is $r = 1.50 \times 10^{11}$ m. Let the ship be a distance x from the earth; it is then a distance $r - x$ from the sun.

$F_E = F_S$ says that $Gmm_E/x^2 = Gmm_S/(r-x)^2$

$m_E/x^2 = ms/\left(r-x\right)^2$ and $(r-x)^2 = x^2(m_S/m_E)$

$r - x = x\sqrt{m_S/m_E}$ and $r = x(1+\sqrt{m_S/m_E})$

$$x = \frac{r}{1+\sqrt{m_S/m_E}} = \frac{1.50\times10^{11}\ \text{m}}{1+\sqrt{1.99\times10^{30}\ \text{kg}/5.97\times10^{24}\ \text{kg}}} = 2.59\times10^{8}\ \text{m (from center of earth)}$$

(b) EVALUATE: At the instant when the spaceship passes through this point its acceleration is zero. Since $m_S \gg m_E$ this equal-force point is much closer to the earth than to the sun.

12.6. **IDENTIFY:** Apply Eq.(12.1) to calculate the magnitude of the gravitational force exerted by each sphere. Each force is attractive. The net force is the vector sum of the individual forces.

SET UP: Let $+x$ be to the right.

EXECUTE: **(a)** $F_{gx} = \left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(0.100\ \text{kg})\left[-\frac{(5.00\ \text{kg})}{(0.400\ \text{m})^2} + \frac{(10.0\ \text{kg})}{(0.600\ \text{m})^2}\right] = -2.32\times10^{-11}\text{N}$, with the minus sign indicating a net force to the left.

(b) No, the force found in part (a) is the *net* force due to the other two spheres.

EVALUATE: The force from the 5.00 kg sphere is greater than for the 10.0 kg sphere even though its mass is less, because r is smaller for this mass.

12.7. **IDENTIFY:** The force exerted by the moon is the gravitational force, $F_g = \frac{Gm_M m}{r^2}$. The force exerted on the person by the earth is $w = mg$.

SET UP: The mass of the moon is $m_M = 7.35\times10^{22}$ kg. $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** $F_{\text{moon}} = F_g = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(7.35\times10^{22}\ \text{kg})(70\ \text{kg})}{(3.78\times10^{8}\ \text{m})^2} = 2.4\times10^{-3}\ \text{N}$.

(b) $F_{\text{earth}} = w = (70\ \text{kg})(9.80\ \text{m/s}^2) = 690\ \text{N}$. $F_{\text{moon}}/F_{\text{earth}} = 3.5\times10^{-6}$.

EVALUATE: The force exerted by the earth is much greater than the force exerted by the moon. The mass of the moon is less than the mass of the earth and the center of the earth is much closer to the person than is the center of the moon.

12.8. **IDENTIFY:** Use Eq.(12.2) to find the force each point mass exerts on the particle, find the net force, and use Newton's second law to calculate the acceleration.

SET UP: Each force is attractive. The particle (mass m) is a distance $r_1 = 0.200$ m from $m_1 = 8.00$ kg and therefore a distance $r_2 = 0.300$ m from $m_2 = 15.0$ kg. Let $+x$ be toward the 15.0 kg mass.

EXECUTE: $F_1 = \frac{Gm_1 m}{r_1^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(8.00\ \text{kg})m}{(0.200\ \text{m})^2} = (1.334\times10^{-8}\ \text{N/kg})m$, in the $-x$-direction.

$F_2 = \frac{Gm_2 m}{r_2^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(15.0\ \text{kg})m}{(0.300\ \text{m})^2} = (1.112\times10^{-8}\ \text{N/kg})m$, in the $+x$-direction. The net force is

$F_x = F_{1x} + F_{2x} = (-1.334\times10^{-8}\ \text{N/kg} + 1.112\times10^{-8}\ \text{N/kg})m = (-2.2\times10^{-9}\ \text{N/kg})m$. $a_x = \frac{F_x}{m} = -2.2\times10^{-9}\ \text{m/s}^2$. The acceleration is $2.2\times10^{-9}\ \text{m/s}^2$, toward the 8.00 kg mass.

EVALUATE: The smaller mass exerts the greater force, because the particle is closer to the smaller mass.

12.9. **IDENTIFY:** Apply Eq.(12.1) to calculate the magnitude of each gravitational force. Each force is attractive.
SET UP: The masses are $m_M = 7.35\times10^{22}$ kg , $m_S = 1.99\times10^{30}$ kg and $m_E = 5.97\times10^{24}$ kg . Denote the earth-sun separation as r_1 and the earth-moon separation as r_2 .

EXECUTE: **(a)** $\left(Gm_M\right)\left[\dfrac{m_S}{(r_1+r_2)^2}+\dfrac{m_E}{r_2^2}\right]=6.30\times10^{20}$ N, toward the sun.

(b) The earth-moon distance is sufficiently small compared to the earth-sun distance ($r_2 << r_1$) that the vector from the earth to the moon can be taken to be perpendicular to the vector from the sun to the moon. The gravitational forces are then $\dfrac{Gm_Mm_S}{r_1^2}=4.34\times10^{20}$ N and $\dfrac{Gm_Mm_E}{r_2^2}=1.99\times10^{20}$ N , and so the force has magnitude 4.77×10^{20} N and is directed 24.6° from the direction toward the sun.

(c) $\left(Gm_M\right)\left[\dfrac{m_S}{\left(r_1-r_2\right)^2}-\dfrac{m_E}{r_2^2}\right]=2.37\times10^{20}$ N, toward the sun.

EVALUATE: The net force is very different in each of these three positions, even though the magnitudes of the forces from the sun and earth change very little.

12.10. **IDENTIFY:** Apply Eq.(12.1) to calculate the magnitude of each gravitational force. Each force is attractive.
SET UP: The forces on one of the masses are sketched in Figure 12.10. The figure shows that the vector sum of the three forces is toward the center of the square.

EXECUTE: $F_{onA} = 2F_B\cos 45° + F_D = 2\dfrac{Gm_Am_B\ \cos 45°}{r_{AB}^2}+\dfrac{Gm_Am_D}{r_{AD}^2}$.

$$F_{onA}=\frac{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(800\ \text{kg})^2\cos\ 45°}{(0.10\ \text{m})^2}+\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(800\ \text{kg})^2}{(0.10\ \text{m})^2}=8.2\times10^{-3}\ \text{N}$$ toward the center of the square.

EVALUATE: We have assumed each mass can be treated as a uniform sphere. Each mass must have an unusually large density in order to have mass 800 kg and still fit into a square of side length 10.0 cm.

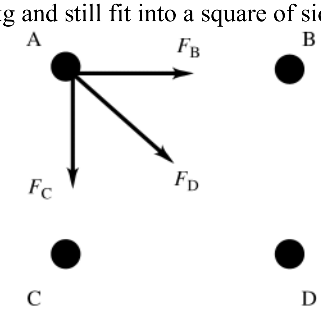


Figure 12.10

12.11. **IDENTIFY:** Use Eq.(12.2) to calculate the gravitational force each particle exerts on the third mass. The equilibrium is stable when for a displacement from equilibrium the net force is directed toward the equilibrium position and it is unstable when the net force is directed away from the equilibrium position.
SET UP: For the net force to be zero, the two forces on *M* must be in opposite directions. This is the case only when *M* is on the line connecting the two particles and between them. The free-body diagram for *M* is given in Figure 12.11. $m_1 = 3m$ and $m_2 = m$. If *M* is a distance *x* from m_1 , it is a distance $1.00\ \text{m} - x$ from m_2 .

EXECUTE: **(a)** $F_x = F_{1x}+F_{2x} = -G\dfrac{3mm}{x^2}+G\dfrac{mM}{(1.00\ \text{m}-x)^2}=0$. $3(1.00\ \text{m}-x)^2 = x^2$. $1.00\ \text{m}-x=\pm x/\sqrt{3}$. Since *M* is between the two particles, *x* must be less than 1.00 m and $x=\dfrac{1.00\ \text{m}}{1+1/\sqrt{3}}=0.634$ m . *M* must be placed at a point that is 0.634 m from the particle of mass 3*m* and 0.366 m from the particle of mass *m*.

(b) (i) If *M* is displaced slightly to the right in Figure 12.11, the attractive force from *m* is larger than the force from 3*m* and the net force is to the right. If *M* is displaced slightly to the left in Figure 12.11, the attractive force from 3*m* is larger than the force from *m* and the net force is to the left. In each case the net force is away from equilibrium and the equilibrium is unstable.
(ii) If *M* is displaced a very small distance along the *y* axis in Figure 12.11, the net force is directed opposite to the direction of the displacement and therefore the equilibrium is stable.

EVALUATE: The point where the net force on M is zero is closer to the smaller mass.

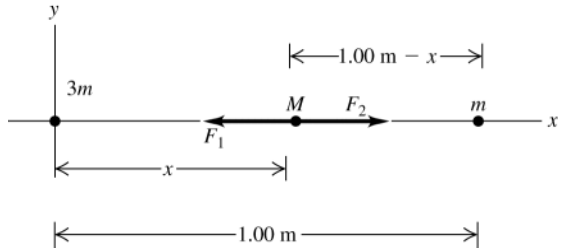


Figure 12.11

12.12. IDENTIFY: The force $\vec{F}_1$ exerted by m on M and the force $\vec{F}_2$ exerted by $2m$ on M are each given by Eq.(12.2) and the net force is the vector sum of these two forces.

SET UP: Each force is attractive. The forces on M in each region are sketched in Figure 12.12a. Let M be at coordinate x on the x-axis.

EXECUTE: **(a)** For the net force to be zero, $\vec{F}_1$ and $\vec{F}_2$ must be in opposite directions and this is the case only for $0 < x < L$. $\vec{F}_1 + \vec{F}_2 = 0$ then requires $F_1 = F_2$. $\dfrac{GmM}{x^2} = \dfrac{G(2m)M}{(L-x)^2}$. $2x^2 = (L-x)^2$ and $L - x = \pm\sqrt{2}x$. x must be less than L, so $x = \dfrac{L}{1+\sqrt{2}} = 0.414L$.

(b) For $x < 0$, $F_x > 0$. $F_x \to 0$ as $x \to -\infty$ and $F_x \to +\infty$ as $x \to 0$. For $x > L$, $F_x < 0$. $F_x \to 0$ as $x \to \infty$ and $F_x \to -\infty$ as $x \to L$. For $0 < x < 0.414L$, $F_x < 0$ and F_x increases from $-\infty$ to 0 as x goes from 0 to $0.414L$. For $0.414L < x < L$, $F_x > 0$ and F_x increases from 0 to $+\infty$ as x goes from $0.414L$ to L. The graph of F_x versus x is sketched in Figure 12.12b.

EVALUATE: Any real object is not exactly a point so it is not possible to have both m and M exactly at $x = 0$ or $2m$ and M both exactly at $x = L$. But the magnitude of the gravitational force between two objects approaches infinity as the objects get very close together.

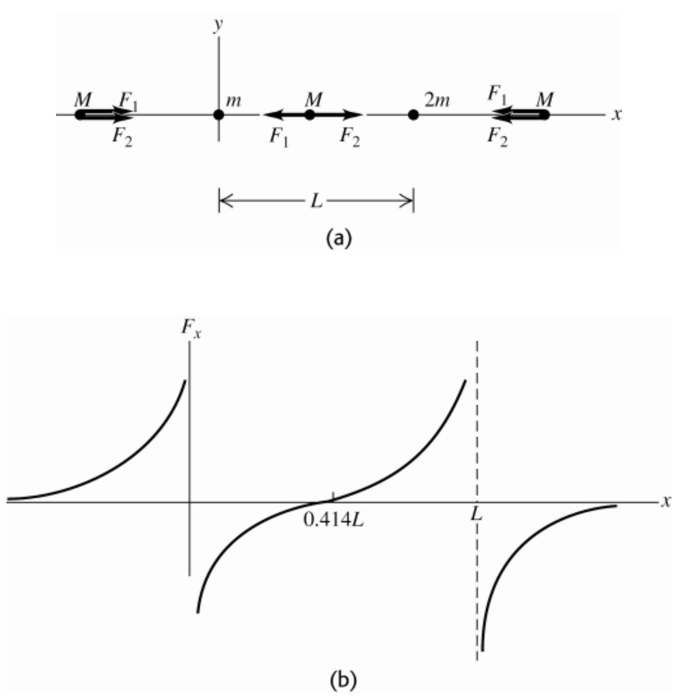


Figure 12.12

12.13. **IDENTIFY:** Use Eq.(12.1) to find the force exerted by each large sphere. Add these forces as vectors to get the net force and then use Newton's 2nd law to calculate the acceleration.

SET UP: The forces are shown in Figure 12.13.

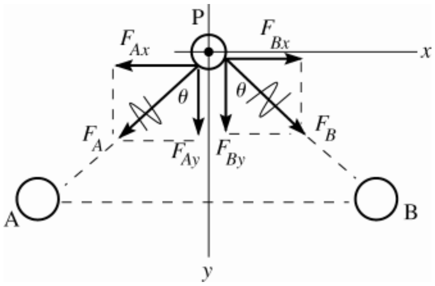


Figure 12.13

$\sin\theta = 0.80$
$\cos\theta = 0.60$
Take the origin of coordinate at point P.

EXECUTE: $F_A = G\dfrac{m_A m}{r^2} = G\dfrac{(0.26\text{ kg})(0.010\text{ kg})}{(0.100\text{ m})^2} = 1.735\times10^{-11}\text{ N}$

$F_B = G\dfrac{m_B m}{r^2} = 1.735\times10^{-11}\text{ N}$

$F_{Ax} = -F_A\sin\theta = -(1.735\times10^{-11}\text{ N})(0.80) = -1.39\times10^{-11}\text{ N}$

$F_{Ay} = -F_A\cos\theta = +(1.735\times10^{-11}\text{N})(0.60) = +1.04\times10^{-11}\text{ N}$

$F_{Bx} = +F_B\sin\theta = +1.39\times10^{-11}\text{ N}$

$F_{By} = +F_B\cos\theta = +1.04\times10^{-11}\text{ N}$

$\sum F_x = ma_x$ gives $F_{Ax} + F_{Bx} = ma_x$

$0 = ma_x$ so $a_x = 0$

$\sum F_y = ma_y$ gives $F_{Ay} + F_{By} = ma_y$

$2(1.04\times10^{-11}\text{ N}) = (0.010\text{ kg})a_y$

$a_y = 2.1\times10^{-9}\text{ m/s}^2$, directed downward midway between A and B

EVALUATE: For ordinary size objects the gravitational force is very small, so the initial acceleration is very small. By symmetry there is no *x*-component of net force and the *y*-component is in the direction of the two large spheres, since they attract the small sphere.

12.14. **IDENTIFY:** Apply Eq.(12.4) to Pluto.

SET UP: Pluto has mass $m = 1.5\times10^{22}$ kg and radius $R = 1.15\times10^6$ m .

EXECUTE: Equation (12.4) gives $g = \dfrac{\left(6.763\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.5\times10^{22}\text{kg}\right)}{\left(1.15\times10^6\text{m}\right)^2} = 0.757\text{ m}^2/\text{s}^2.$

EVALUATE: g at the surface of Pluto is much less than g at the surface of Earth. Eq.(12.4) applies to any spherically symmetric object.

12.15. **IDENTIFY:** $F_g = G\dfrac{mm_E}{r^2}$, so $a_g = G\dfrac{m_E}{r^2}$, where r is the distance of the object from the center of the earth.

SET UP: $r = h + R_E$, where h is the distance of the object above the surface of the earth and $R_E = 6.38\times10^6$ m is the radius of the earth.

EXECUTE: To decrease the acceleration due to gravity by one-tenth, the distance from the center of the earth must be increased by a factor of $\sqrt{10}$, and so the distance above the surface of the earth is

$\left(\sqrt{10}-1\right)R_E = 1.38\times10^7\text{m}.$

EVALUATE: This height is about twice the radius of the earth.

12.16. **IDENTIFY:** Apply Eq.(12.4) to the earth and to Venus. $w = mg$.

SET UP: $g = \dfrac{Gm_E}{R_E^2} = 9.80\text{ m/s}^2$. $m_V = 0.815m_E$ and $R_V = 0.949R_E$. $w_E = mg_E = 75.0$ N .

EXECUTE: **(a)** $g_V = \dfrac{Gm_V}{R_V^2} = \dfrac{G(0.815m_E)}{(0.949R_E)^2} = 0.905\dfrac{Gm_E}{R_E^2} = 0.905g_E$.

(b) $w_V = mg_V = 0.905mg_E = (0.905)(75.0\text{ N}) = 67.9\text{ N}$.

EVALUATE: The mass of the rock is independent of its location but its weight equals the gravitational force on it and that depends on its location.

12.17. **(a) IDENTIFY and SET UP:** Apply Eq.(12.4) to the earth and to Titania. The acceleration due to gravity at the surface of Titania is given by $g_T = Gm_T/R_T^2$, where m_T is its mass and R_T is its radius.

For the earth, $g_E = Gm_E/R_E^2$.

EXECUTE: For Titania, $m_T = m_E/1700$ and $R_T = R_E/8$, so $g_T = \dfrac{Gm_T}{R_T^2} = \dfrac{G(m_E/1700)}{(R_E/8)^2} = \left(\dfrac{64}{1700}\right)\dfrac{Gm_E}{R_E^2} = 0.0377 g_E$.

Since $g_E = 9.80\ \text{m/s}^2$, $g_T = (0.0377)(9.80\ \text{m/s}^2) = 0.37\ \text{m/s}^2$.

EVALUATE: g on Titania is much smaller than on earth. The smaller mass reduces g and is a greater effect than the smaller radius, which increases g.

(b) IDENTIFY and SET UP: Use density = mass/volume. Assume Titania is a sphere.

EXECUTE: From Section 12.2 we know that the average density of the earth is $5500\ \text{kg/m}^3$. For Titania

$$\rho_T = \frac{m_T}{\frac{4}{3}\pi R_T^3} = \frac{m_E/1700}{\frac{4}{3}\pi (R_E/8)^3} = \frac{512}{1700}\rho_E = \frac{512}{1700}(5500\ \text{kg/m}^3) = 1700\ \text{kg/m}^3$$

EVALUATE: The average density of Titania is about a factor of 3 smaller than for earth. We can write Eq.(12.4) for Titania as $g_T = \frac{4}{3}\pi G R_T \rho_T$. $g_T < g_E$ both because $\rho_T < \rho_E$ and $R_T < R_E$.

12.18. **IDENTIFY:** Apply Eq.(12.4) to Rhea.

SET UP: $\rho = m/V$. The volume of a sphere is $V = \frac{4}{3}\pi R^3$.

EXECUTE: $M = \dfrac{gR^2}{G} = 2.44\times 10^{21}\ \text{kg}$ and $\rho = \dfrac{M}{(4\pi/3)R^3} = 1.30\times 10^3\ \text{kg/m}^3$.

EVALUATE: The average density of Rhea is about one-fourth that of the earth.

12.19. **IDENTIFY:** Apply Eq.(12.2) to the astronaut.

SET UP: $m_E = 5.97\times 10^{24}\ \text{kg}$ and $R_E = 6.38\times 10^6\ \text{m}$.

EXECUTE: $F_g = G\dfrac{mm_E}{r^2}$. $r = 600\times 10^3\,\text{m} + R_E$ so $F_g = 610\ \text{N}$. At the surface of the earth, $w = mg = 735\ \text{N}$. The gravity force is not zero in orbit. The satellite and the astronaut have the same acceleration so the astronaut's apparent weight is zero.

EVALUATE: In Eq.(12.2), r is the distance of the object from the center of the earth.

12.20. **IDENTIFY:** $g_n = G\dfrac{m_n}{R_n^2}$, where the subscript n refers to the neutron star. $w = mg$.

SET UP: $R_n = 10.0\times 10^3\ \text{m}$. $m_n = 1.99\times 10^{30}\ \text{kg}$. Your mass is $m = \dfrac{w}{g} = \dfrac{675\ \text{N}}{9.80\ \text{m/s}^2} = 68.9\ \text{kg}$.

EXECUTE: $g_n = (6.673\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{1.99\times 10^{30}\ \text{kg}}{(10.0\times 10^3\ \text{m})^2} = 1.33\times 10^{12}\ \text{m/s}^2$

Your weight on the neutron star would be $w_n = mg_n = (68.9\ \text{kg})(1.33\times 10^{12}\ \text{m/s}^2) = 9.16\times 10^{13}\ \text{N}$.

EVALUATE: Since R_n is much less than the radius of the sun, the gravitational force exerted by the neutron star on an object at its surface is immense.

12.21. **IDENTIFY and SET UP:** Use the measured gravitational force to calculate the gravitational constant G, using Eq.(12.1). Then use Eq.(12.4) to calculate the mass of the earth:

EXECUTE: $F_g = G\dfrac{m_1 m_2}{r^2}$ so $G = \dfrac{F_g r^2}{m_1 m_2} = \dfrac{(8.00\times 10^{-10}\ \text{N})(0.0100\ \text{m})^2}{(0.400\ \text{kg})(3.00\times 10^{-3}\ \text{kg})} = 6.667\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

$g = \dfrac{Gm_E}{R_E^2}$ gives $m_E = \dfrac{R_E^2 g}{G} = \dfrac{(6.38\times 10^6\ \text{m})^2(9.80\ \text{m/s}^2)}{6.667\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 5.98\times 10^{24}\ \text{kg}$.

EVALUATE: Our result agrees with the value given in Appendix F.

12.22. **IDENTIFY:** Use Eq.(12.4) to calculate g for Europa. The acceleration of a particle moving in a circular path is $a_{rad} = r\omega^2$.

SET UP: In $a_{rad} = r\omega^2$, ω must be in rad/s. For Europa, $R = 1.569\times 10^6\ \text{m}$.

EXECUTE: $g = \dfrac{Gm}{R^2} = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(4.8\times10^{22}\ \text{kg})}{(1.569\times10^6\ \text{m})^2} = 1.30\ \text{m/s}^2$. $g = a_{\text{rad}}$ gives

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{1.30\ \text{m/s}^2}{4.25\ \text{m}}} = (0.553\ \text{rad/s})\left(\frac{60\ \text{s}}{1\ \text{min}}\right)\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = 5.28\ \text{rpm}.$$

EVALUATE: The radius of Europa is about one-fourth that of the earth and its mass is about one-hundredth that of earth, so g on Europa is much less than g on earth. The lander would have some spatial extent so different points on it would be different distances from the rotation axis and a_{rad} would have different values. For the ω we calculated, $a_{\text{rad}} = g$ at a point that is precisely 4.25 m from the rotation axis.

12.23. **IDENTIFY** and **SET UP:** Example 12.5 gives the escape speed as $v_1 = \sqrt{2GM/R}$, where M and R are the mass and radius of the astronomical object.

EXECUTE: $v_1 = \sqrt{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(3.6\times10^{12}\ \text{kg})/700\ \text{m}} = 0.83\ \text{m/s}.$

EVALUATE: At this speed a person can walk 100 m in 120 s; easily achieved for the average person. We can write the escape speed as $v_1 = \sqrt{\tfrac{4}{3}\pi\rho G R^2}$, where ρ is the average density of Dactyl. Its radius is much smaller than earth's and its density is about the same, so the escape speed is much less on Dactyl than on earth.

12.24. **IDENTIFY:** In part (a) use the expression for the escape speed that is derived in Example 12.5. In part (b) apply conservation of energy.

SET UP: $R = 4.5\times10^3$ m. In part (b) let point 1 be at the surface of the comet.

EXECUTE: **(a)** The escape speed is $v = \sqrt{\dfrac{2GM}{R}}$ so $M = \dfrac{Rv^2}{2G} = \dfrac{(4.5\times10^3\ \text{m})(1.0\ \text{m/s})^2}{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 3.37\times10^{13}\ \text{kg}$.

(b) (i) $K_1 = \tfrac{1}{2}mv_1^2$. $K_2 = 0.100K_1$. $U_1 = -\dfrac{GMm}{R}$; $U_2 = -\dfrac{GMm}{r}$. $K_1 + U_1 = K_2 + U_2$ gives

$\tfrac{1}{2}mv_1^2 - \dfrac{GMm}{R} = (0.100)(\tfrac{1}{2}mv_1^2) - \dfrac{GMm}{r}$. Solving for r gives

$\dfrac{1}{r} = \dfrac{1}{R} - \dfrac{0.450v_1^2}{GM} = \dfrac{1}{4.5\times10^3\ \text{m}} - \dfrac{0.450(1.0\ \text{m/s})^2}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(3.37\times10^{13}\ \text{kg})}$ and $r = 45$ km. (ii) The debris never loses all of its initial kinetic energy, but $K_2 \to 0$ as $r \to \infty$. The farther the debris are from the comet's center, the smaller is their kinetic energy.

EVALUATE: The debris will have lost 90.0% of their initial kinetic energy when they are at a distance from the comet's center of about ten times the radius of the comet.

12.25. **IDENTIFY:** The escape speed, from the results of Example 12.5, is $\sqrt{2GM/R}$.

SET UP: For Mars, $M = 6.42\times10^{23}$ kg and $R = 3.40\times10^6$ m. For Jupiter, $M = 1.90\times10^{27}$ kg and $R = 6.91\times10^7$ m.

EXECUTE: **(a)** $v = \sqrt{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.42\times10^{23}\ \text{kg})/(3.40\times10^6\ \text{m})} = 5.02\times10^3\ \text{m/s}.$

(b) $v = \sqrt{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2(1.90\times10^{27}\ \text{kg})/(6.91\times10^7\ \text{m})} = 6.06\times10^4\ \text{m/s}.$

(c) Both the kinetic energy and the gravitational potential energy are proportional to the mass of the spacecraft.

EVALUATE: Example 12.5 calculates the escape speed for earth to be 1.12×10^4 m/s. This is larger than our result for Mars and less than our result for Jupiter.

12.26. **IDENTIFY:** The kinetic energy is $K = \tfrac{1}{2}mv^2$ and the potential energy is $U = -\dfrac{GMm}{r}$

SET UP: The mass of the earth is $M_{\text{E}} = 5.97\times10^{24}$ kg.

EXECUTE: **(a)** $K = \tfrac{1}{2}(629\ \text{kg})(3.33\times10^3\ \text{m/s})^2 = 3.49\times10^9$ J

(b) $U = -\dfrac{GM_{\text{E}}m}{r} = -\dfrac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})(629\ \text{kg})}{2.87\times10^9\ \text{m}} = -8.73\times10^7$ J.

EVALUATE: The total energy $K + U$ is positive.

12.27. **IDENTIFY:** Apply Newton's 2nd law to the motion of the satellite and obtain and equation that relates the orbital speed v to the orbital radius *r*.

SET UP: The distances are shown in Figure 12.27a.

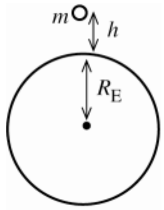


Figure 12.27a

The radius of the orbit is $r = h + R_\text{E}$.

$r = 7.80\times10^5\ \text{m} + 6.38\times10^6\ \text{m} = 7.16\times10^6\ \text{m}.$

The free-body diagram for the satellite is given in Figure 12.27b.

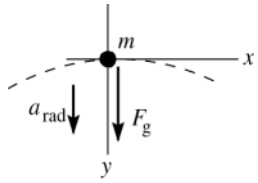


Figure 12.27b

(a) EXECUTE: $\sum F_y = ma_y$

$F_\text{g} = ma_\text{rad}$

$G\dfrac{mm_E}{r^2} = m\dfrac{v^2}{r}$

$$v = \sqrt{\frac{Gm_\text{E}}{r}} = \sqrt{\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}{7.16\times10^6\ \text{m}}} = 7.46\times10^3\ \text{m/s}$$

(b) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi(7.16\times10^6\ \text{m})}{7.46\times10^3\ \text{m/s}} = 6030\ \text{s} = 1.68\ \text{h}.$

EVALUATE: Note that $r = h + R_\text{E}$ is the radius of the orbit, measured from the center of the earth. For this satellite *r* is greater than for the satellite in Example 12.6, so its orbital speed is less.

12.28. **IDENTIFY:** The time to complete one orbit is the period *T*, given by Eq.(12.12). The speed *v* of the satellite is given by $v = \dfrac{2\pi r}{T}$.

SET UP: If *h* is the height of the orbit above the earth's surface, the radius of the orbit is $r = h + R_\text{E}$. $R_\text{E} = 6.38\times10^6\ \text{m}$ and $m_\text{E} = 5.97\times10^{24}\ \text{kg}$.

EXECUTE: **(a)** $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_\text{E}}} = \dfrac{2\pi(7.05\times10^5\ \text{m} + 6.38\times10^6\ \text{m})^{3/2}}{\sqrt{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}} = 5.94\times10^3\ \text{s} = 99.0\ \text{min}$

(b) $v = \dfrac{2\pi(7.05\times10^5\ \text{m} + 6.38\times10^6\ \text{m})}{5.94\times10^3\ \text{s}} = 7.49\times10^3\ \text{m/s} = 7.49\ \text{km/s}$

EVALUATE: The satellite in Example 12.6 is at a lower altitude and therefore has a smaller orbit radius than the satellite in this problem. Therefore, the satellite in this problem has a larger period and a smaller orbital speed. But a large percentage change in *h* corresponds to a small percentage change in *r* and the values of *T* and *v* for the two satellites do not differ very much.

12.29. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to the motion of the earth around the sun.

SET UP: For the earth, $T = 365.3\ \text{days} = 3.156\times10^7\ \text{s}$ and $r = 1.50\times10^{11}\ \text{m}$. $T = \dfrac{2\pi r}{v}$.

EXECUTE: $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(1.50\times10^{11}\ \text{m})}{3.156\times10^7\ \text{s}} = 2.99\times10^4\ \text{s}$. $F_\text{g} = ma_\text{rad}$ gives $G\dfrac{m_\text{E}m_\text{S}}{r^2} = m_\text{E}\dfrac{v^2}{r}$.

$$m_\text{S} = \frac{v^2 r}{G} = \frac{(2.99\times10^4\ \text{s})^2(1.50\times10^{11}\ \text{m})}{6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 2.01\times10^{30}\ \text{kg}$$

EVALUATE: Appendix F gives $m_\text{S} = 1.99\times10^{30}\ \text{kg}$, in good agreement with our calculation.

12.30. **IDENTIFY:** We can calculate the orbital period *T* from the number of revolutions per day. Then the period and the orbit radius are related by Eq.(12.12).

SET UP: $m_\text{E} = 5.97\times10^{24}\ \text{kg}$ and $R_\text{E} = 6.38\times10^6\ \text{m}$. The height *h* of the orbit above the surface of the earth is related to the orbit radius *r* by $r = h + R_\text{E}$. $1\ \text{day} = 8.64\times10^4\ \text{s}$.

EXECUTE: The satellite moves 15.65 revolutions in 8.64×10^4 s, so the time for 1.00 revolution is

$T = \dfrac{8.64\times10^4\text{s}}{15.65} = 5.52\times10^3$ s. $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ gives

$r = \left(\dfrac{Gm_E T^2}{4\pi^2}\right)^{1/3} = \left(\dfrac{[6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2][5.97\times10^{24}\ \text{kg}][5.52\times10^3\text{s}]^2}{4\pi^2}\right)^{1/3}$. $r = 6.75\times10^6$ m and

$h = r - R_E = 3.7\times10^5\ \text{m} = 370\ \text{km}$.

EVALUATE: The period of this satellite is slightly larger than the period for the satellite in Example 12.6 and the altitude of this satellite is therefore somewhat greater.

12.31. IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to the motion of the baseball. $v = \dfrac{2\pi r}{T}$.

SET UP: $r_D = 6\times10^3$ m.

EXECUTE: **(a)** $F_g = ma_{rad}$ gives $G\dfrac{m_D m}{r_D^2} = m\dfrac{v^2}{r_D}$. $v = \sqrt{\dfrac{Gm_D}{r_D}} = \sqrt{\dfrac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{15}\ \text{kg})}{6\times10^3\ \text{m}}} = 4.7$ m/s

$4.7\ \text{m/s} = 11$ mph, which is easy to achieve.

(b) $T = \dfrac{2\pi r}{v} = \dfrac{2\pi(6\times10^3\ \text{m})}{4.7\ \text{m/s}} = 8020\ \text{s} = 134$ min. The game would last a long time.

EVALUATE: The speed v is relative to the center of Deimos. The baseball would already have some speed before we throw it, because of the rotational motion of Deimos.

12.32. IDENTIFY: $T = \dfrac{2\pi r}{v}$ and $F_g = ma_{rad}$.

SET UP: The sun has mass $m_S = 1.99\times10^{30}$ kg. The radius of Mercury's orbit is 5.79×10^{10} m, so the radius of Vulcan's orbit is 3.86×10^{10} m.

EXECUTE: $F_g = ma_{rad}$ gives $G\dfrac{m_S m}{r^2} = m\dfrac{v^2}{r}$ and $v^2 = \dfrac{Gm_S}{r}$.

$$T = 2\pi r\sqrt{\frac{r}{Gm_S}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_S}} = \frac{2\pi(3.86\times10^{10}\ \text{m})^{3/2}}{\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}} = 4.13\times10^6\ \text{s} = 47.8\ \text{days}$$

EVALUATE: The orbital period of Mercury is 88.0 d, so we could calculate T for Vulcan as $T = (88.0\ \text{d})(2/3)^{3/2} = 47.9$ days.

12.33. IDENTIFY: The orbital speed is given by $v = \sqrt{Gm/r}$, where m is the mass of the star. The orbital period is given by $T = \dfrac{2\pi r}{v}$.

SET UP: The sun has mass $m_S = 1.99\times10^{30}$ kg. The orbit radius of the earth is 1.50×10^{11} m.

EXECUTE: **(a)** $v = \sqrt{Gm/r}$.

$v = \sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.85\times1.99\times10^{30}\ \text{kg})/((1.50\times10^{11}\ \text{m})(0.11))} = 8.27\times10^4$ m/s.

(b) $2\pi r/v = 1.25\times10^6\ \text{s} = 14.5$ days (about two weeks).

EVALUATE: The orbital period is less than the 88 day orbital period of Mercury; this planet is orbiting very close to its star, compared to the orbital radius of Mercury.

12.34. IDENTIFY: The period of each satellite is given by Eq.(12.12). Set up a ratio involving T and r.

SET UP: $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_p}}$ gives $\dfrac{T}{r^{3/2}} = \dfrac{2\pi}{\sqrt{Gm_p}} =$ constant, so $\dfrac{T_1}{r_1^{3/2}} = \dfrac{T_2}{r_2^{3/2}}$.

EXECUTE: $T_2 = T_1\left(\dfrac{r_2}{r_1}\right)^{3/2} = (6.39\ \text{days})\left(\dfrac{48{,}000\ \text{km}}{19{,}600\ \text{km}}\right)^{3/2} = 24.5$ days. For the other satellite,

$T_2 = (6.39\ \text{days})\left(\dfrac{64{,}000\ \text{km}}{19{,}600\ \text{km}}\right)^{3/2} = 37.7$ days.

EVALUATE: T increases when r increases.

12.35. IDENTIFY: In part (b) apply the results from part (a).

SET UP: For Pluto, $e = 0.248$ and $a = 5.92\times10^{12}$ m. For Neptune, $e = 0.010$ and $a = 4.50\times10^{12}$ m. The orbital period for Pluto is $T = 247.9$ y.

EXECUTE: **(a)** The result follows directly from Figure 12.19 in the textbook.

(b) The closest distance for Pluto is $(1-0.248)(5.92\times10^{12}\text{m}) = 4.45\times10^{12}\text{m}$. The greatest distance for Neptune is $(1+0.010)(4.50\times10^{12}\text{m}) = 4.55\times10^{12}\text{m}$.

(c) The time is the orbital period of Pluto, $T = 248\text{ y}$.

EVALUATE: Pluto's closest distance calculated in part (a) is $0.10\times10^{12}\text{ m} = 1.0\times10^{8}\text{ km}$, so Pluto is about 100 million km closer to the sun than Neptune, as is stated in the problem. The eccentricity of Neptune's orbit is small, so its distance from the sun is approximately constant.

12.36. **IDENTIFY:** $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}$, where m_{star} is the mass of the star. $v = \dfrac{2\pi r}{T}$.

SET UP: $3.09\text{ days} = 2.67\times10^{5}\text{ s}$. The orbit radius of Mercury is $5.79\times10^{10}\text{ m}$. The mass of our sun is $1.99\times10^{30}\text{ kg}$.

EXECUTE: **(a)** $T = 2.67\times10^{5}\text{ s}$. $r = (5.79\times10^{10}\text{ m})/9 = 6.43\times10^{9}\text{ m}$. $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_{\text{star}}}}$ gives

$$m_{\text{star}} = \frac{4\pi^2 r^3}{T^2 G} = \frac{4\pi^2(6.43\times10^{9}\text{ m})^3}{(2.67\times10^{5}\text{ s})^2(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)} = 2.21\times10^{30}\text{ kg}.\quad \frac{m_{\text{star}}}{m_{\text{sun}}} = 1.11,\text{ so } m_{\text{star}} = 1.11 m_{\text{sun}}.$$

(b) $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(6.43\times10^{9}\text{ m})}{2.67\times10^{5}\text{ s}} = 1.51\times10^{5}\text{ m/s}$

EVALUATE: The orbital period of Mercury is 88.0 d. The period for this planet is much less primarily because the orbit radius is much less and also because the mass of the star is greater than the mass of our sun.

12.37. **(a) IDENTIFY:** If the orbit is circular, Newton’s 2nd law requires a particular relation between its orbit radius and orbital speed.

SET UP: The gravitational force exedrted on the spacecraft by the sun is $F_{\text{g}} = Gm_{\text{S}}m_{\text{H}}/r^2,$ where m_{S} is the mass of the sun and m_{H} is the mass of the Helios B spacecraft.

For a circular orbit, $a_{\text{rad}} = v^2/r$ and $\sum F = m_{\text{H}}v^2/r.$ If we neglect all forces on the spacecraft except for the force exerted by the sun, $F_{\text{g}} = \sum F = m_{\text{H}}v^2/r,$ so $Gm_{\text{S}}m_{\text{H}}/r^2 = m_{\text{H}}v^2/r$

EXECUTE: $v = \sqrt{Gm_{\text{S}}/r} = \sqrt{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})/43\times10^{9}\text{ m}} = 5.6\times10^{4}\text{ m/s} = 56\text{ km/s}$

EVALUATE: The actual speed is 71 km/s, so the orbit cannot be circular.

(b) IDENTIFY and **SET UP:** The orbit is a circle or an ellipse if it is closed, a parabola or hyperbola if open. The orbit is closed if the total energy (kinetic + potential) is negative, so that the object cannot reach $r \to \infty.$

EXECUTE: For Helios B,

$K = \tfrac{1}{2}m_{\text{H}}v^2 = \tfrac{1}{2}m_{\text{H}}(71\times10^{3}\text{ m/s})^2 = (2.52\times10^{9}\text{m}^2/\text{s}^2)m_{\text{H}}$

$U = -Gm_{\text{S}}m_{\text{H}}/r = m_{\text{H}}(-(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})/(43\times10^{9}\text{ m})) = -(3.09\times10^{9}\text{ m}^2/\text{s}^2)m_{\text{H}}$

$E = K + U = (2.52\times10^{9}\text{ m}^2/\text{s}^2)m_{\text{H}} - (3.09\times10^{9}\text{ m}^2/\text{s}^2)m_{\text{H}} = -(5.7\times10^{8}\text{ m}^2/\text{s}^2)m_{\text{H}}$

EVALUATE: The total energy *E* is negative, so the orbit is closed. We know from part (a) that it is not circular, so it must be elliptical.

12.38. **IDENTIFY:** Section 12.6 states that for a point mass outside a spherical shell the gravitational force is the same as if all the mass of the shell were concentrated at its center. It also states that for a point inside a spherical shell the force is zero.

SET UP: For $r = 5.01\text{ m}$ the point mass is outside the shell and for $r = 4.99\text{ m}$ and $r = 2.12\text{ m}$ the point mass is inside the shell.

EXECUTE: **(a)** (i) $F_{\text{g}} = \dfrac{Gm_1m_2}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(1000.0\text{ kg})(2.00\text{ kg})}{(5.01\text{ m})^2} = 5.31\times10^{-9}\text{ N}$. (ii) $F_{\text{g}} = 0$. (iii) $F_{\text{g}} = 0$.

(b) For $r < 5.00\text{ m}$ the force is zero and for $r > 5.00\text{ m}$ the force is proportional to $1/r^2$. The graph of F_{g} versus *r* is sketched in Figure 12.38.

EVALUATE: Inside the shell the gravitational potential energy is constant and the force on a point mass inside the shell is zero.

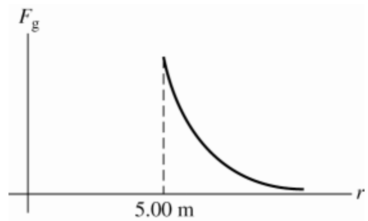


Figure 12.38

12.39. IDENTIFY: Section 12.6 states that for a point mass outside a uniform sphere the gravitational force is the same as if all the mass of the sphere were concentrated at its center. It also states that for a point mass a distance r from the center of a uniform sphere, where r is less than the radius of the sphere, the gravitational force on the point mass is the same as though we removed all the mass at points farther than r from the center and concentrated all the remaining mass at the center.

SET UP: The density of the sphere is $\rho = \dfrac{M}{\frac{4}{3}\pi R^3}$, where M is the mass of the sphere and R is its radius. The mass inside a volume of radius $r < R$ is $M_r = \rho V_r = \left(\dfrac{M}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = M\left(\dfrac{r}{R}\right)^3$. $r = 5.01$ m is outside the sphere and $r = 2.50$ m is inside the sphere.

EXECUTE: (a) (i) $F_g = \dfrac{GMm}{r^2} = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(1000.0\ \text{kg})(2.00\ \text{kg})}{(5.01\ \text{m})^2} = 5.31\times10^{-9}\ \text{N}$.

(ii) $F_g = \dfrac{GM'm}{r^2}$. $M' = M\left(\dfrac{r}{R}\right)^3 = (1000.0\ \text{kg})\left(\dfrac{2.50\ \text{m}}{5.00\ \text{m}}\right)^3 = 125\ \text{kg}$.

$F_g = (6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(125\ \text{kg})(2.00\ \text{kg})}{(2.50\ \text{m})^2} = 2.67\times10^{-9}\ \text{N}$.

(b) $F_g = \dfrac{GM(r/R)^3 m}{r^2} = \left(\dfrac{GMm}{R^3}\right)r$ for $r < R$ and $F_g = \dfrac{GMm}{r^2}$ for $r > R$. The graph of F_g versus r is sketched in Figure 12.39.

EVALUATE: At points outside the sphere the force on a point mass is the same as for a shell of the same mass and radius. For $r < R$ the force is different in the two cases of uniform sphere versus hollow shell.

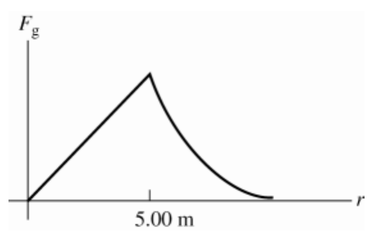


Figure 12.39

12.40. IDENTIFY: The gravitational potential energy of a point of point masses is $U = -G\dfrac{m_1 m_2}{r}$. Divide the rod into infinitesimal pieces and integrate to find U.

SET UP: Divide the rod into differential masses dm at position l, measured from the right end of the rod. $dm = dl(M/L)$.

EXECUTE: (a) $U = -\dfrac{Gm\,dm}{l+x} = -\dfrac{GmM}{L}\dfrac{dl}{l+x}$.

Integrating, $U - \dfrac{GmM}{L}\int_0^L \dfrac{dl}{l+x} = -\dfrac{GmM}{L}\ln\left(1+\dfrac{L}{x}\right)$. For $x >> L$, the natural logarithm is $\sim\left(L/x\right)$, and $U \to -GmM/x$.

(b) The x-component of the gravitational force on the sphere is $F_x = -\dfrac{\partial U}{\partial x} = \dfrac{GmM}{L}\dfrac{(-L/x^2)}{(1+(L/x))} = -\dfrac{GmM}{(x^2+Lx)}$, with the minus sign indicating an attractive force. As $x >> L$, the denominator in the above expression approaches x^2, and $F_x \to -GmM/x^2$, as expected.

EVALUATE: When x is much larger than L the rod can be treated as a point mass, and our results for U and F_x do reduce to the correct expression when $x >> L$.

12.41. **IDENTIFY:** Find the potential due to a small segment of the ring and integrate over the entire ring to find the total U.

(a) SET UP:

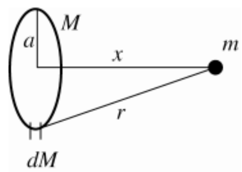


Figure 12.41

Divide the ring up into small segments dM, as indicated in Figure 12.41.

EXECUTE: The gravitational potential energy of dM and m is $dU = -GmdM/r$.

The total gravitational potential energy of the ring and particle is $U = \int dU = -Gm\int dM/r$.

But $r = \sqrt{x^2+a^2}$ is the same for all segments of the ring, so

$$U = -\frac{Gm}{r}\int dM = -\frac{GmM}{r} = -\frac{GmM}{\sqrt{x^2+a^2}}$$

(b) EVALUATE: When $x >> a$, $\sqrt{x^2+a^2} \to \sqrt{x^2} = x$ and $U = -GmM/x$. This is the gravitational potential energy of two point masses separated by a distance x. This is the expected result.

(c) IDENTIFY and SET UP: Use $F_x = -dU/dx$ with $U(x)$ from part (a) to calculate F_x.

EXECUTE: $F_x = -\dfrac{dU}{dx} = -\dfrac{d}{dx}\left(-\dfrac{GmM}{\sqrt{x^2+a^2}}\right)$

$$F_x = +GmM\frac{d}{dx}(x^2+a^2)^{-1/2} = GmM(-\frac{1}{2}(2x)(x^2+a^2)^{-3/2})$$

$F_x = -GmMx/(x^2+a^2)^{3/2}$; the minus sign means the force is attractive.

EVALUATE: **(d)** For $x >> a$, $(x^2+a^2)^{3/2} \to (x^2)^{3/2} = x^3$

Then $F_x = -GmMx/x^3 = -GmM/x^2$. This is the force between two point masses separated by a distance x and is the expected result.

(e) For $x = 0$, $U = -GMm/a$. Each small segment of the ring is the same distance from the center and the potential is the same as that due to a point charge of mass M located at a distance a.

For $x = 0$, $F_x = 0$. When the particle is at the center of the ring, symmetrically placed segments of the ring exert equal and opposite forces and the total force exerted by the ring is zero.

12.42. **IDENTIFY:** At the equator the object has inward acceleration $\dfrac{v^2}{R_E}$ and the reading w of the balance is related to the true weight w_0 (the gravitational force exerted by the earth) by $w_0 - w = \dfrac{mv^2}{R_E}$. At the North Pole, $a_{rad} = 0$ and $w = w_0$.

SET UP: As shown in Section 12.7, $v = 465$ m/s. $R_E = 6.38\times10^6$ m.

EXECUTE: $w_0 = 875$ N and $m = \dfrac{w_0}{g} = 89.29$ kg. $w = w_0 - \dfrac{mv^2}{R_E} = 875\text{ N} - (89.29\text{ kg})\dfrac{(465\text{ m/s})^2}{6.38\times10^6\text{ m}} = 872\text{ N}$

EVALUATE: The rotation of the earth causes the scale reading to be slightly less than the true weight, since there must be a net inward force on the object.

12.43. **IDENTIFY and SET UP:** Ate the north pole, $F_g = w_0 = mg_0$, where g_0 is given by Eq.(12.4) applied to Neptune. At the equator, the apparent weight is given by Eq.(12.28). The orbital speed v is obtained from the rotational period using Eq.(12.12).

EXECUTE: **(a)** $g_0 = Gm/R^2 = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.0\times10^{26}\ \text{kg})/(2.5\times10^7\ \text{m})^2 = 10.7\ \text{m/s}^2$. This agrees with the value of g given in the problem.

$F = w_0 = mg_0 = (5.0\ \text{kg})(10.7\ \text{m/s}^2) = 53\ \text{N}$; this is the true weight of the object.

(b) From Eq.(23.28), $w = w_0 - mv^2/R$

$$T = \frac{2\pi r}{v}\ \text{ gives }\ v = \frac{2\pi r}{T} = \frac{2\pi(2.5\times10^7\ \text{m})}{(16\ \text{h})(3600\ \text{s/1 h})} = 2.727\times10^3\ \text{m/s}$$

$v^2/R = (2.727\times10^3\ \text{s})^2/2.5\times10^7\ \text{m} = 0.297\ \text{m/s}^2$

Then $w = 53\ \text{N} - (5.0\ \text{kg})(0.297\ \text{m/s}^2) = 52\ \text{N}$.

EVALUATE: The apparent weight is less than the true weight. This effect is larger on Neptune than on earth.

12.44. **IDENTIFY:** The radius of a black hole and its mass are related by $R_S = \dfrac{2GM}{c^2}$.

SET UP: $R_S = 0.50\times10^{-15}\ \text{m}$, $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$ and $c = 3.00\times10^8\ \text{m/s}$.

EXECUTE: $M = \dfrac{c^2R_S}{2G} = \dfrac{(3.00\times10^8\ \text{m/s})^2(0.50\times10^{-15}\ \text{m})}{2(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 3.4\times10^{11}\ \text{kg}$

EVALUATE: The average density of the black hole would be

$\rho = \dfrac{M}{\frac{4}{3}\pi R_S^3} = \dfrac{3.4\times10^{11}\ \text{kg}}{\frac{4}{3}\pi(0.50\times10^{-15}\ \text{m})^3} = 6.49\times10^{56}\ \text{kg/m}^3$. We can combine $\rho = \dfrac{M}{\frac{4}{3}\pi R_S^3}$ and $R_S = \dfrac{2GM}{c^2}$ to give

$\rho = \dfrac{3c^6}{32\pi G^3M^2}$. The average density of a black hole increases when its mass decreases. The average density of this mini black hole is much greater than the average density of the much more massive black hole in Example 12.11.

12.45. **IDENTIFY** and **SET UP:** A black hole with the earth's mass M has the Schwarzschild radius R_S given by Eq.(12.30).

EXECUTE: $R_S = 2GM/c^2 = 2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})/(2.998\times10^8\ \text{m/s})^2 = 8.865\times10^{-3}\ \text{m}$

The ratio of R_S to the current radius R is $R_S/R = 8.865\times10^{-3}\ \text{m}/6.38\times10^6\ \text{m} = 1.39\times10^{-9}$.

EVALUATE: A black hole with the earth's radius is very small.

12.46. **IDENTIFY:** Apply Eq.(12.1) to calculate the gravitational force. For a black hole, the mass M and Schwarzschild radius R_S are related by Eq.(12.30).

SET UP: The speed of light is $c = 3.00\times10^8\ \text{m/s}$.

EXECUTE: **(a)** $\dfrac{GMm}{r^2} = \dfrac{(R_Sc^2/2)}{r^2} = \dfrac{mc^2R_S}{2r^2}$.

(b) $\dfrac{(5.00\ \text{kg})(3.00\times10^8\ \text{m/s})^2(1.4\times10^{-2}\text{m})}{2(3.00\times10^6\ \text{m})^2} = 350\ \text{N}$.

(c) Solving Eq.(12.30) for M, $M = \dfrac{R_Sc^2}{2G} = \dfrac{(14.00\times10^{-3}\ \text{m})(3.00\times10^8\ \text{m/s})^2}{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 9.44\times10^{24}\ \text{kg}$.

EVALUATE: The mass of the black hole is about twice the mass of the earth.

12.47. **IDENTIFY:** The orbital speed for an object a distance r from an object of mass M is $v = \dfrac{GM}{r}$. The mass M of a black hole and its Schwarzschild radius R_S are related by Eq.(12.30).

SET UP: $c = 3.00\times10^8\ \text{m/s}$. $1\ \text{ly} = 9.461\times10^{15}\ \text{m}$.

EXECUTE: **(a)**

$$M = \frac{Rv^2}{G} = \frac{(7.5\ \text{ly})(9.461\times10^{15}\ \text{m/ly})(200\times10^3\ \text{m/s})^2}{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 4.3\times10^{37}\ \text{kg} = 2.1\times10^7\ M_S.$$

(b) No, the object has a mass very much greater than 50 solar masses.

(c) $R_S = \dfrac{2GM}{c^2} = \dfrac{2v^2r}{c^2} = 6.32\times10^{10}\ \text{m}$, which does fit.

EVALUATE: The Schwarzschild radius of a black hole is approximately the same as the radius of Mercury's orbit around the sun.

12.48. **IDENTIFY:** The clumps orbit the black hole. Their speed, orbit radius and orbital period are related by $v = \dfrac{2\pi r}{T}$. Their orbit radius and period are related to the mass M of the black hole by $T = \dfrac{2\pi r^{3/2}}{\sqrt{GM}}$. The radius of the black hole's event horizon is related to the mass of the black hole by $R_S = \dfrac{2GM}{c^2}$.

SET UP: $v = 3.00\times10^7$ m/s. $T = 27$ h $= 9.72\times10^4$ s. $c = 3.00\times10^8$ m/s.

EXECUTE: **(a)** $r = \dfrac{vT}{2\pi} = \dfrac{(3.00\times10^7 \text{ m/s})(9.72\times10^4 \text{ s})}{2\pi} = 4.64\times10^{11} \text{ m}$.

(b) $T = \dfrac{2\pi r^{3/2}}{\sqrt{GM}}$ gives $M = \dfrac{4\pi^2 r^3}{GT^2} = \dfrac{4\pi^2(4.64\times10^{11} \text{ m})^3}{(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(9.72\times10^4 \text{ s})^2} = 6.26\times10^{36} \text{ kg}$.

(c) $R_S = \dfrac{2GM}{c^2} = \dfrac{2(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(6.26\times10^{36} \text{ kg})}{(3.00\times10^8 \text{ m/s})^2} = 9.28\times10^9 \text{ m}$

EVALUATE: The black hole has a mass that is about 3×10^6 solar masses.

12.49. **IDENTIFY:** Use Eq.(12.1) to find each gravitational force. Each force is attractive. In part (b) apply conservation of energy.

SET UP: For a pair of masses m_1 and m_2 with separation r, $U = -G\dfrac{m_1 m_2}{r}$.

EXECUTE: **(a)** From symmetry, the net gravitational force will be in the direction $45°$ from the x-axis (bisecting the x and y axes), with magnitude

$$F = (6.673\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(0.0150 \text{ kg})\left[\frac{(2.0 \text{ kg})}{(2(0.50 \text{ m})^2)} + 2\frac{(1.0 \text{ kg})}{(0.50 \text{ m})^2}\sin 45°\right] = 9.67\times10^{-12} \text{ N}$$

(b) The initial displacement is so large that the initial potential may be taken to be zero. From the work-energy theorem, $\dfrac{1}{2}mv^2 = Gm\left[\dfrac{(2.0 \text{ kg})}{\sqrt{2}\ (0.50 \text{ m})} + 2\dfrac{(1.0 \text{ kg})}{(0.50 \text{ m})}\right]$. Canceling the factor of m and solving for v, and using the numerical values gives $v = 3.02\times10^{-5}$ m/s.

EVALUATE: The result in part (b) is independent of the mass of the particle. It would take the particle a long time to reach point P.

12.50. **IDENTIFY:** Use Eq.(12.1) to calculate each gravitational force and add the forces as vectors.

(a) SET UP: The locations of the masses are sketched in Figure 12.50a.

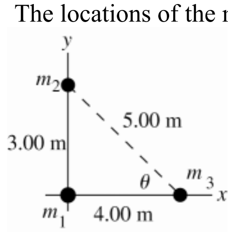


Figure 12.50a

Section 12.6 proves that any two spherically symmetric masses interact as though they were point masses with all the mass concentrated at their centers.

The force diagram for m_3 is given in Figure 12.50b

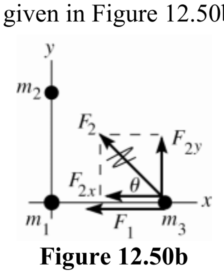


Figure 12.50b

$\cos\theta = 0.800$
$\sin\theta = 0.600$

EXECUTE: $F_1 = G\dfrac{m_1 m_3}{r_{13}^2} = \dfrac{(6.673\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(60.0 \text{ kg})(0.500 \text{ kg})}{(4.00 \text{ m})^2} = 1.251\times10^{-10} \text{ N}$

$F_2 = G\dfrac{m_2 m_3}{r_{23}^2} = \dfrac{(6.673\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(80.0 \text{ kg})(0.500 \text{ kg})}{(5.00 \text{ m})^2} = 1.068\times10^{-10} \text{ N}$

$F_{1x} = -1.251\times10^{-10}$ N, $F_{1y} = 0$

$F_{2x} = -F_2 \cos\theta = -(1.068\times10^{-10}\ \text{N})(0.800) = -8.544\times10^{-11}\ \text{N}$

$F_{2y} = +F_2 \sin\theta = +(1.068\times10^{-10}\ \text{N})(0.600) = +6.408\times10^{-11}\ \text{N}$

$F_x = F_{1x} + F_{2x} = -1.251\times10^{-10}\ \text{N} - 8.544\times10^{-11}\ \text{N} = -2.105\times10^{-10}\ \text{N}$

$F_y = F_{1y} + F_{2y} = 0 + 6.408\times10^{-11}\ \text{N} = +6.408\times10^{-11}\ \text{N}$

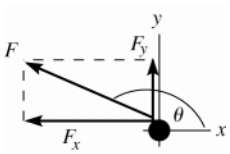


Figure 12.50c

F and its components are sketched in Figure 12.50c.

$F = \sqrt{F_x^2 + F_y^2}$

$F = \sqrt{(-2.105\times10^{-10}\ \text{N})^2 + (+6.408\times10^{-11}\ \text{N})^2}$

$F = 2.20\times10^{-10}\ \text{N}$

$\tan\theta = \dfrac{F_y}{F_x} = \dfrac{+6.408\times10^{-11}\ \text{N}}{-2.105\times10^{-10}\ \text{N}};\quad \theta = 163°$

EVALUATE: Both spheres attract the third sphere and the net force is in the second quadrant.

(b) SET UP: For the net force to be zero the forces from the two spheres must be equal in magnitude and opposite in direction. For the forces on it to be opposite in direction the third sphere must be on the *y*-axis and between the other two spheres. The forces on the third sphere are shown in Figure 12.50d.

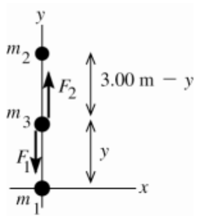


Figure 12.50d

EXECUTE: $F_{\text{net}} = 0$ if $F_1 = F_2$

$G\dfrac{m_1 m_3}{y^2} = G\dfrac{m_2 m_3}{(3.00\ \text{m} - y)^2}$

$\dfrac{60.0}{y^2} = \dfrac{80.0}{(3.00\ \text{m} - y)^2}$

$\sqrt{80.0}\,y = \sqrt{60.0}(3.00\ \text{m} - y)$

$(\sqrt{80.0} + \sqrt{60.0})y = (3.00\ \text{m})\sqrt{60.0}$ and $y = 1.39\ \text{m}$

Thus the sphere would have to be placed at the point $x = 0,\ \ y = 1.39\ \text{m}$

EVALUATE: For the forces to have the same magnitude the third sphere must be closer to the sphere that has smaller mass.

12.51. **IDENTIFY:** $\tau = Fr\sin\phi$. The net torque is the sum of the torques due to each force.

SET UP: From Example 12.3, using Newton's third law, the forces of the small star on each large star are $F_1 = 6.67\times10^{25}$ N and $F_2 = 1.33\times10^{26}$ N. Let counterclockwise torques be positive.

EXECUTE: **(a)** The direction from the origin to the point midway between the two large stars is $\arctan(\dfrac{0.100\ \text{m}}{0.200\ \text{m}}) = 26.6°$, which is not the angle (14.6°) found in the example.

(b) The common lever arm is 0.100 m, and the force on the upper mass is at an angle of 45° from the lever arm. The net torque is $\tau = +F_1(1.00\times10^{12}\ \text{m})\sin45° - F_2(1.00\times10^{12}\ \text{m}) = -8.58\times10^{37}\ \text{N}\cdot\text{m}$, with the minus sign indicating a clockwise torque.

EVALUATE: **(c)** There can be no net torque due to gravitational fields with respect to the center of gravity, and so the center of gravity in this case is not at the center of mass. For the center of gravity to be the same point as the center of mass, the gravity force on each mass must be proportional to the mass, with the same constant of proportionality, and that is not the case here.

12.52. **IDENTIFY:** The gravity force for each pair of objects is given by Eq.(12.1). The work done is $W = -\Delta U$.

SET UP: The simplest way to approach this problem is to find the force between the spacecraft and the center of mass of the earth-moon system, which is 4.67×10^6 m from the center of the earth. The distance from the spacecraft to the center of mass of the earth-moon system is 3.82×10^8 m (Figure 12.52). $m_{\text{E}} = 5.97\times10^{24}$ kg, $m_{\text{M}} = 7.35\times10^{22}$ kg.

EXECUTE: **(a)** Using the Law of Gravitation, the force on the spacecraft is 3.4 N, an angle of $0.61°$ from the earth-spacecraft line.

(b) $U = -G\dfrac{m_A m_B}{r}$. $U_2 = 0$ and $r_1 = 3.84\times10^8$ m for the spacecraft and the earth, and the spacecraft and the moon.

$$W = U_2 - U_1 = +\frac{GMm}{r_1} = +\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg} + 7.35\times10^{22}\ \text{kg})(1250\ \text{kg})}{3.84\times10^8\ \text{m}}.\ W = -1.31\times10^9\ \text{J}.$$

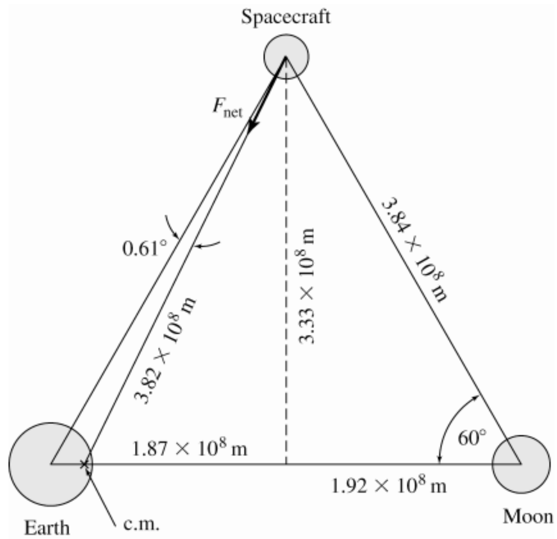


Figure 12.52

12.53. **IDENTIFY:** Apply conservation of energy and conservation of linear momentum to the motion of the two spheres.
SET UP: Denote the 25-kg sphere by a subscript 1 and the 100-kg sphere by a subscript 2.
EXECUTE: **(a)** Linear momentum is conserved because we are ignoring all other forces, that is, the net external force on the system is zero. Hence, $m_1v_1 = m_2v_2.$

(b) From the work-energy theorem in the form $K_\text{i} + U_\text{i} = K_\text{f} + U_\text{f}$, with the initial kinetic energy $K_\text{i} = 0$ and $U = -G\dfrac{m_1m_2}{r}$, $Gm_1m_2\left[\dfrac{1}{r_\text{f}} - \dfrac{1}{r_\text{i}}\right] = \dfrac{1}{2}\left(m_1v_1^2 + m_2v_2^2\right)$. Using the conservation of momentum relation $m_1v_1 = m_2v_2$ to eliminate v_2 in favor of v_1 and simplifying yields $v_1^2 = \dfrac{2Gm_2^2}{m_1+m_2}\left[\dfrac{1}{r_\text{f}} - \dfrac{1}{r_\text{i}}\right]$, with a similar expression for v_2.

Substitution of numerical values gives $v_1 = 1.63\times10^{-5}$ m/s, $v_2 = 4.08\times10^{-6}$ m/s. The magnitude of the relative velocity is the sum of the speeds, 2.04×10^{-5} m/s.

(c) The distance the centers of the spheres travel $(x_1 \text{ and } x_2)$ is proportional to their acceleration, and $\dfrac{x_1}{x_2} = \dfrac{a_1}{a_2} = \dfrac{m_2}{m_1}$, or $x_1 = 4x_2$. When the spheres finally make contact, their centers will be a distance of $2r$ apart, or $x_1 + x_2 + 2r = 40$ m, or $x_2 + 4x_2 + 2r = 40$ m. Thus, $x_2 = 8\ \text{m} - 0.4r$, and $x_1 = 32\ \text{m} - 1.6r$. The point of contact of the surfaces is $32\ \text{m} - 0.6r = 31.9$ m from the initial position of the center of the 25.0 kg sphere.

EVALUATE: The result $x_1/x_2 = 4$ can also be obtained from the conservation of momentum result that $\dfrac{v_1}{v_2} = \dfrac{m_2}{m_1}$, at every point in the motion.

12.54. **IDENTIFY:** Apply Eq.(12.12).
SET UP: $m_\text{E} = 5.97\times10^{24}$ kg

EXECUTE: Solving Eq. (12.14) for *R*, $R^3 = Gm_\text{E}\left(\dfrac{T}{2\pi}\right)^2$.

$$R = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})\left(\frac{(27.3\ \text{d})(86{,}400\ \text{s/d})}{2\pi}\right)^2 = 5.614\times10^{25}\ \text{m}^3,$$

from which $r = 3.83\times10^8$ m.
EVALUATE: The result we calculated is in very good agreement with the orbit radius given in Appendix F.

12.55. **IDENTIFY** and **SET UP:** **(a)** To stay above the same point on the surface of the earth the orbital period of the satellite must equal the orbital period of the earth:

$T = 1\text{ d}(24\text{ h/1 d})(3600\text{ s/1 h}) = 8.64\times 10^4\text{ s}$

Eq.(12.14) gives the relation between the orbit radius and the period:

EXECUTE: $T = \dfrac{2\pi r^{3/2}}{\sqrt{Gm_E}}$ and $T^2 = \dfrac{4\pi^2 r^3}{Gm_E}$

$$r = \left(\frac{T^2 G m_E}{4\pi^2}\right)^{1/3} = \left(\frac{(8.64\times 10^4\text{ s})^2(6.673\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97\times 10^{24}\text{ kg})}{4\pi^2}\right)^{1/3} = 4.23\times 10^7\text{ m}$$

This is the radius of the orbit; it is related to the height h above the earth's surface and the radius R_E of the earth by $r = h + R_E$. Thus $h = r - R_E = 4.23\times 10^7\text{ m} - 6.38\times 10^6\text{ m} = 3.59\times 10^7\text{ m}$.

EVALUATE: The orbital speed of the geosynchronous satellite is $2\pi r/T = 3080\text{ m/s}$. The altitude is much larger and the speed is much less than for the satellite in Example 12.6.

(b) Consider Figure 12.55.

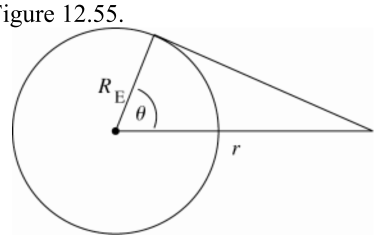


Figure 12.55

$$\cos\theta = \frac{R_E}{r} = \frac{6.38\times 10^6\text{ m}}{4.23\times 10^7\text{ m}}$$

$\theta = 81.3°$

A line from the satellite is tangent to a point on the earth that is at an angle of $81.3°$ above the equator. The sketch shows that points at higher latitudes are blocked by the earth from viewing the satellite.

12.56. **IDENTIFY:** Apply Eq.(12.12) to relate the orbital period T and M_P, the planet's mass, and then use Eq.(12.2) applied to the planet to calculate the astronaut's weight.

SET UP: The radius of the orbit of the lander is $5.75\times 10^5\text{ m} + 4.80\times 10^6\text{ m}$.

EXECUTE: From Eq.(12.14), $T^2 = \dfrac{4\pi^2 r^3}{GM_P}$ and

$$M_P = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(5.75\times 10^5\text{ m} + 4.80\times 10^6\text{ m})^3}{(6.673\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.8\times 10^3\text{ s})^2} = 2.731\times 10^{24}\text{ kg},$$

or about half the earth's mass. Now we can find the astronaut's weight on the surface from Eq.(12.2). (The landing on the north pole removes any need to account for centripetal acceleration.)

$$w = \frac{GM_P m_a}{r_P^2} = \frac{\left(6.673\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.731\times 10^{24}\text{ kg}\right)\left(85.6\text{ kg}\right)}{\left(4.80\times 10^6\text{ m}\right)^2} = 677\text{ N}.$$

EVALUATE: At the surface of the earth the weight of the astronaut would be 839 N.

12.57. **IDENTIFY:** From Example 12.5, the escape speed is $v = \sqrt{\dfrac{2GM}{R}}$. Use $\rho = M/V$ to write this expression in terms of ρ.

SET UP: For a sphere $V = \frac{4}{3}\pi R^3$.

EXECUTE: In terms of the density ρ, the ratio M/R is $(4\pi/3)\rho R^2$, and so the escape speed is

$$v = \sqrt{(8\pi/3)\left(6.673\times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(2500\text{ kg/m}^3\right)\left(150\times 10^3\text{ m}\right)^2} = 177\text{ m/s}.$$

EVALUATE: This is much less than the escape speed for the earth, 11,200 m/s.

12.58. **IDENTIFY:** From Example 12.5, the escape speed is $v = \sqrt{\dfrac{2GM}{R}}$. Use $\rho = M/V$ to write this expression in terms of ρ. On earth, the height h you can jump is related to your jump speed by $v = \sqrt{2gh}$ For part (b), apply Eq.(12.4) to Europa.

SET UP: For a sphere $V = \frac{4}{3}\pi R^3$

EXECUTE: $\rho = M/(\frac{4}{3}\pi R^3)$, so the escape speed can be written as $v = \sqrt{\dfrac{8\pi G\rho R^2}{3}}$. Equating the two expressions for *v* and squaring gives $2gh = \dfrac{8\pi}{3}\rho GR^2$, or $R^2 = \dfrac{3}{4\pi}\dfrac{gh}{\rho G}$, where $g = 9.80\ \text{m/s}^2$ is for the surface of the earth, not the asteroid. Estimate $h = 1$ m (variable for different people, of course), $R = 3.7$ km. For Europa,

$$g = \frac{GM}{R^2} = \frac{4\pi\rho RG}{3}.\ \rho = \frac{3g}{4\pi RG} = \frac{3(1.33\ \text{m/s}^2)}{4\pi(1.57\times10^6\ \text{m})(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)} = 3.03\times10^3\ \text{kg/m}^3.$$

EVALUATE: The earth has average density $5500\ \text{kg/m}^3$. The average density of Europa is about half that of the earth but a little larger than the average density of most asteroids.

12.59. **IDENTIFY** and **SET UP:** The observed period allows you to calculate the angular velocity of the satellite relative to you. You know your angular velocity as you rotate with the earth, so you can find the angular velocity of the satellite in a space-fixed reference frame. $v = r\omega$ gives the orbital speed of the satellite and Newton's second law relates this to the orbit radius of the satellite.

EXECUTE: **(a)** The satellite is revolving west to east, in the same direction the earth is rotating. If the angular speed of the satellite is ω_S and the angular speed of the earth is ω_E, the angular speed ω_{rel} of the satellite relative to you is $\omega_{rel} = \omega_s - \omega_E$.

$\omega_{rel} = (1\ \text{rev})/(12\ \text{h}) = (\frac{1}{12})\ \text{rev/h}$

$\omega_E = (\frac{1}{24})\ \text{rev/h}$

$\omega_s = \omega_{rel} + \omega_E = (\frac{1}{8})\ \text{rev/h} = 2.18\times10^{-4}\ \text{rad/s}$

$\sum \vec{F} = m\vec{a}$ says $G\dfrac{mm_E}{r^2} = m\dfrac{v^2}{r}$

$v^2 = \dfrac{Gm_E}{r}$ and with $v = r\omega$ this gives $r^3 = \dfrac{Gm_E}{\omega^2}$; $r = 2.03\times10^7$ m

This is the radius of the satellite's orbit. Its height *h* above the surface of the earth is $h = r - R_E = 1.39\times10^7$ m.

EVALUATE: In part (a) the satellite is revolving faster than the earth's rotation and in part (b) it is revolving slower. Slower *v* and ω means larger orbit radius *r*.

(b) Now the satellite is revolving opposite to the rotation of the earth. If west to east is positive, then

$\omega_{rel} = (-\frac{1}{12})\ \text{rev/h}$

$\omega_s = \omega_{rel} + \omega_E = (-\frac{1}{24})\ \text{rev/h} = -7.27\times10^{-5}\ \text{rad/s}$

$r^3 = \dfrac{Gm_E}{\omega^2}$ gives $r = 4.22\times10^7$ m and $h = 3.59\times10^7$ m

12.60. **IDENTIFY:** Apply the law of gravitation to the astronaut at the north pole to calculate the mass of planet . Then apply $\sum \vec{F} = m\vec{a}$ to the astronaut, with $a_{rad} = \dfrac{4\pi^2 R}{T^2}$, toward the center of the planet, to calculate the period *T*. Apply Eq.(12.12) to the satellite in order to calculate its orbital period.

SET UP: Get radius of X: $\frac{1}{4}(2\pi R) = 18{,}850$ km and $R = 1.20\times10^7$ m. Astronaut mass:

$$m = \frac{\omega}{g} = \frac{943\ \text{N}}{9.80\ \text{m/s}^2} = 96.2\ \text{kg}.$$

EXECUTE: $\dfrac{GmM_X}{R^2} = w$, where $w = 915.0$ N. $M_x = \dfrac{mg_x R^2}{Gm} = \dfrac{(915\ \text{N})(1.20\times10^7\ \text{m})^2}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(96.2\ \text{kg})} = 2.05\times10^{25}$ kg

Apply Newton's second law to astronaut on a scale at the equator of X. $F_{grav} - F_{scale} = ma_{rad}$, so

$$F_{grav} - F_{scale} = \frac{4\pi^2 mR}{T^2}.\ 915.0\ \text{N} - 850.0\ \text{N} = \frac{4\pi^2(96.2\ \text{kg})(1.20\times10^7\ \text{m})}{T^2}\ \text{and}\ T = 2.65\times10^4\,\text{s}\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 7.36\ \text{h}.$$

(b) For the satellite, $T = \sqrt{\dfrac{4\pi^2 r^3}{Gm_X}} = \sqrt{\dfrac{4\pi^2(1.20\times10^7\ \text{m} + 2.0\times10^6\ \text{m})^3}{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.05\times10^{25}\ \text{kg})}} = 8.90\times10^3\ \text{s} = 2.47$ hours.

EVALUATE: The acceleration of gravity at the surface of the planet is $g_X = \dfrac{915.0\ \text{N}}{96.2\ \text{kg}} = 9.51\ \text{m/s}^2$, similar to the value on earth. The radius of the planet is about twice that of earth. The planet rotates more rapidly than earth and the length of a day is about one-third what it is on earth.

12.61. **IDENTIFY:** Use $g = \frac{Gm_E}{R_E^2}$ and follow the procedure specified in the problem.

SET UP: $R_E = 6.38\times 10^6$ m

EXECUTE: The fractional error is $1 - \frac{mgh}{Gmm_E\left(1/R_E - 1/(R_E+h)\right)} = 1 - \frac{g}{Gm_E}(R_E+h)(R_E)$.

Using Eq.(12.4) for g the fractional difference is $1-(R_E+h)/R_E = -h/R_E$, so if the fractional difference is -1%. $h = (0.01)R_E = 6.4\times 10^4$ m.

EVALUATE: For $h = 1$ km, the fractional error is only 0.016%. Eq.(7.2) is very accurate for the motion of objects near the earth's surface.

12.62. **IDENTIFY:** Use the measurements of the motion of the rock to calculate g_M, the value of g on Mongo. Then use this to calculate the mass of Mongo. For the ship, $F_g = ma_{rad}$ and $T = \frac{2\pi r}{v}$.

SET UP: Take $+y$ upward. When the stone returns to the ground its velocity is 12.0 m/s, downward. $g_M = G\frac{m_M}{R_M^2}$.

The radius of Mongo is $R_M = \frac{c}{2\pi} = \frac{2.00\times 10^8\ \text{m}}{2\pi} = 3.18\times 10^7$ m. The ship moves in an orbit of radius $r = 3.18\times 10^7\ \text{m} + 3.00\times 10^7\ \text{m} = 6.18\times 10^7$ m.

EXECUTE: **(a)** $v_{0y} = +12.0$ m/s, $v_y = -12.0$ m/s, $a_y = -g_M$ and $t = 8.00$ s. $v_y = v_{0y} + a_y t$ gives

$$-g_M = \frac{v_y - v_{0y}}{t} = \frac{-12.0\ \text{m/s} - 12.0\ \text{m/s}}{8.00\ \text{s}} \text{ and } g_M = 3.00\ \text{m/s}^2.$$

$$m_M = \frac{g_M R_M^2}{G} = \frac{(3.00\ \text{m/s}^2)(3.18\times 10^7\ \text{m})^2}{6.673\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 4.55\times 10^{25}\ \text{kg}$$

(b) $F_g = ma_{rad}$ gives $G\frac{m_M m}{r^2} = m\frac{v^2}{r}$ and $v^2 = \frac{Gm_M}{r}$.

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_M}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_M}} = \frac{2\pi(6.18\times 10^7\ \text{m})^{3/2}}{\sqrt{(6.673\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(4.55\times 10^{25}\ \text{kg})}}$$

$T = 5.54\times 10^4\ \text{s} = 15.4\ \text{h}$

EVALUATE: $R_M = 5.0R_E$ and $m_M = 7.6m_E$, so $g_M = \frac{7.6}{(5.0)^2}g_E = 0.30g_E$, which agrees with the value calculated in part (a).

12.63. **IDENTIFY** and **SET UP:** Use Eq.(12.2) to calculate the gravity force at each location. For the top of Mount Everest write $r = h + R_E$ and use the fact that $h << R_E$ to obtain an expression for the difference in the two forces.

EXECUTE: At Sacramento, the gravity force on you is $F_1 = G\frac{mm_E}{R_E^2}$.

At the top of Mount Everest, a height of $h = 8800$ m above seal level, the gravity force on you is

$$F_2 = G\frac{mm_E}{(R_E+h)^2} = G\frac{mm_E}{R_E^2(1+h/R_E)^2}$$

$$(1+h/R_E)^{-2} \approx 1 - \frac{2h}{R_E},\quad F_2 = F_1\left(1 - \frac{2h}{R_E}\right)$$

$$\frac{F_1 - F_2}{F_1} = \frac{2h}{R_E} = 0.28\%$$

EVALUATE: The change in the gravitational force is very small, so for objects near the surface of the earth it is a good approximation to treat it as a constant.

12.64. **IDENTIFY:** Apply Eq.(12.9) to the particle-earth and particle-moon systems.

SET UP: When the particle is a distance r from the center of the earth, it is a distance $R_{EM} - r$ from the center of the moon.

EXECUTE: **(a)** The total gravitational potential energy in this model is $U = -Gm\left[\frac{m_E}{r} + \frac{m_M}{R_{EM} - r}\right]$.

(b) See Exercise 12.5. The point where the net gravitational force vanishes is $r = \frac{R_{EM}}{1+\sqrt{m_M/m_E}} = 3.46\times 10^8\ \text{m}$.

Using this value for r in the expression in part (a) and the work-energy theorem, including the initial potential energy of $-Gm(m_E/R_E + m_M/(R_{EM} - R_E))$ gives 11.1 km/s.

(c) The final distance from the earth is not R_M, but the Earth-moon distance minus the radius of the moon, or 3.823×10^8 m. From the work-energy theorem, the rocket impacts the moon with a speed of 2.9 km/s.

EVALUATE: The spacecraft has a greater gravitational potential energy at the surface of the moon than at the surface of the earth, so it reaches the surface of the moon with a speed that is less than its launch speed on earth.

12.65. **IDENTIFY** and **SET UP:** First use the radius of the orbit to find the initial orbital speed, from Eq.(12.10) applied to the moon.

EXECUTE: $v = \sqrt{Gm/r}$ and $r = R_M + h = 1.74\times 10^6\ \text{m} + 50.0\times 10^3\ \text{m} = 1.79\times 10^6\ \text{m}$

Thus $v = \sqrt{\frac{(6.673\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.35\times 10^{22}\ \text{kg})}{1.79\times 10^6\ \text{m}}} = 1.655\times 10^3\ \text{m/s}$

After the speed decreases by 20.0 m/s it becomes $1.655\times 10^3\ \text{m/s} - 20.0\ \text{m/s} = 1.635\times 10^3\ \text{m/s}$.

IDENTIFY and **SET UP:** Use conservation of energy to find the speed when the spacecraft reaches the lunar surface.

$K_1 + U_1 + W_{other} = K_2 + U_2$

Gravity is the only force that does work so $W_{other} = 0$ and $K_2 = K_1 + U_1 - U_2$

EXECUTE: $U_1 = -Gm_m m/r;\ U_2 = -Gm_m m/R_m$

$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 + Gmm_m(1/R_m - 1/r)$

And the mass m divides out to give $v_2 = \sqrt{v_1^2 + 2Gm_m(1/R_m - 1/r)}$

$v_2 = 1.682\times 10^3\ \text{m/s}(1\ \text{km}/1000\ \text{m})(3600\ \text{s}/1\ \text{h}) = 6060\ \text{km/h}$

EVALUATE: After the thruster fires the spacecraft is moving too slowly to be in a stable orbit; the gravitational force is larger than what is needed to maintain a circular orbit. The spacecraft gains energy as it is accelerated toward the surface.

12.66. **IDENTIFY:** $g = 0$ means the apparent weight is zero, so $a_{rad} = 9.80\ \text{m/s}^2$. $a_{rad} = \frac{4\pi^2 R}{T^2}$.

SET UP: The radius of the earth is $R_E = 6.38\times 10^6\ \text{m}$.

EXECUTE: $T = 2\pi\sqrt{\frac{R}{a_{rad}}} = 5.07\times 10^3\ \text{s}$, which is 84.5 min, or about an hour and a half.

EVALUATE: At the poles, g would still be $9.80\ \text{m/s}^2$.

12.67. **IDENTIFY** and **SET UP:** Apply conservation of energy. Must use Eq.(12.9) for the gravitational potential energy since h is not small compared to R_E.

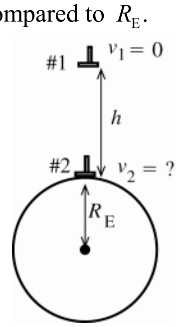


Figure 12.67

As indicated in Figure 12.67, take point 1 to be where the hammer is released and point 2 to be just above the surface of the earth, so $r_1 = R_E + h$ and $r_2 = R_E$.

EXECUTE: $K_1 + U_1 + W_{other} = K_2 + U_2$

Only gravity does work, so $W_{other} = 0$.

$K_1 = 0,\ \ K_2 = \frac{1}{2}mv_2^2$

$U_1 = -G\frac{mm_E}{r_1} = -\frac{Gmm_E}{h + R_E},\ \ U_2 = -G\frac{mm_E}{r_2} = -\frac{Gmm_E}{R_E}$

Thus, $-G\dfrac{mm_E}{h+R_E}=\dfrac{1}{2}mv_2^2-G\dfrac{mm_E}{R_E}$

$$v_2^2=2Gm_E\left(\frac{1}{R_E}-\frac{1}{R_E+h}\right)=\frac{2Gm_E}{R_E(R_E+h)}(R_E+h-R_E)=\frac{2Gm_Eh}{R_E(R_E+h)}$$

$$v_2=\sqrt{\frac{2Gm_Eh}{R_E(R_E+h)}}$$

EVALUATE: If $h\to\infty$, $v_2\to\sqrt{2Gm_E/R_E}$, which equals the escape speed. In this limit this event is the reverse of an object being projected upward from the surface with the escape speed. If $h\ll R_E$, then $v_2=\sqrt{2Gm_Eh/R_E^2}=\sqrt{2gh}$, the same result if used Eq.(7.2) for U.

12.68. IDENTIFY: In orbit the total mechanical energy of the satellite is $E=-\dfrac{Gm_Em}{2R_E}$. $U=-G\dfrac{m_Em}{r}$. $W=E_f-E_i$.

SET UP: $U\to 0$ as $r\to\infty$.

EXECUTE: **(a)** The energy the satellite has as it sits on the surface of the Earth is $E_i=\dfrac{-GmM_E}{R_E}$. The energy it has when it is in orbit at a radius $R\approx R_E$ is $E_f=\dfrac{-GmM_E}{2R_E}$. The work needed to put it in orbit is the difference between these: $W=E_f-E_i=\dfrac{GmM_E}{2R_E}$.

(b) The total energy of the satellite far away from the Earth is zero, so the additional work needed is $0-\left(\dfrac{-GmM_E}{2R_E}\right)=\dfrac{GmM_E}{2R_E}$.

EVALUATE: **(c)** The work needed to put the satellite into orbit was the same as the work needed to put the satellite from orbit to the edge of the universe.

12.69. IDENTIFY: At the escape speed, $E=K+U=0$.

SET UP: At the surface of the earth the satellite is a distance $R_E=6.38\times10^6$ m from the center of the earth and a distance $R_{ES}=1.50\times10^{11}$ m from the sun. The orbital speed of the earth is $\dfrac{2\pi R_{ES}}{T}$, where $T=3.156\times10^7$ s is the orbital period. The speed of a point on the surface of the earth at an angle ϕ from the equator is $v=\dfrac{2\pi R_E\cos\phi}{T}$, where $T=86{,}400$ s is the rotational period of the earth.

EXECUTE: **(a)** The escape speed will be $v=\sqrt{2G\left[\dfrac{m_E}{R_E}+\dfrac{m_s}{R_{ES}}\right]}=4.35\times10^4$ m/s. Making the simplifying assumption that the direction of launch is the direction of the earth's motion in its orbit, the speed relative to the center of the earth is $v-\dfrac{2\pi R_{ES}}{T}=4.35\times10^4\text{ m/s}-\dfrac{2\pi(1.50\times10^{11}\text{ m})}{(3.156\times10^7\text{s})}=1.37\times10^4$ m/s.

(b) The rotational speed at Cape Canaveral is $\dfrac{2\pi(6.38\times10^6\text{ m})\cos 28.5°}{86{,}400\text{ s}}=4.09\times10^2$ m/s, so the speed relative to the surface of the earth is 1.33×10^4 m/s.

(c) In French Guiana, the rotational speed is 4.63×10^2 m/s, so the speed relative to the surface of the earth is 1.32×10^4 m/s.

EVALUATE: The orbital speed of the earth is a large fraction of the escape speed, but the rotational speed of a point on the surface of the earth is much less.

12.70. IDENTIFY: From the discussion of Section 12.6, the force on a point mass at a distance r from the center of a spherically symmetric mass distribution is the same as though we removed all the mass at points farther than r from the center and concentrated all the remaining mass at the center.

SET UP: The mass M of a hollow sphere of density ρ, inner radius R_1 and outer radius R_2 is $M=\rho\frac{4}{3}\pi(R_2^3-R_1^3)$. From Figure 12.9 in the textbook, the inner core has outer radius 1.2×10^6 m, inner radius zero and density 1.3×10^4 kg/m^3. The outer core has inner radius 1.2×10^6 m, outer radius 3.6×10^6 m and density 1.1×10^4 kg/m^3. The total mass of the earth is $m_E=5.97\times10^{24}$ kg and its radius is $R_E=6.38\times10^6$ m.

EXECUTE: **(a)** $F_g = G\dfrac{m_E m}{R_E^2} = mg = (10.0\text{ kg})(9.80\text{ m/s}^2) = 98.0\text{ N}$.

(b) The mass of the inner core is $m_{\text{inner}} = \rho_{\text{inner}}\frac{4}{3}\pi(R_2^3 - R_1^3) = (1.3\times10^4\text{ kg/m}^3)\frac{4}{3}\pi(1.2\times10^6\text{ m})^3 = 9.4\times10^{22}\text{ kg}$. The mass of the outer core is $m_{\text{outer}} = (1.1\times10^4\text{ kg/m}^3)\frac{4}{3}\pi([3.6\times10^6\text{ m}]^3 - [1.2\times10^6\text{ m}]^3) = 2.1\times10^{24}\text{ kg}$. Only the inner and outer cores contribute to the force. $F_g = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(9.4\times10^{22}\text{ kg} + 2.1\times10^{24}\text{ kg})(10.0\text{ kg})}{(3.6\times10^6\text{ m})^2} = 110\text{ N}$.

(c) Only the inner core contributes to the force and $F_g = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\dfrac{(9.4\times10^{22}\text{ kg})(10.0\text{ kg})}{(1.2\times10^6\text{ m})^2} = 44\text{ N}$.

(d) At $r = 0$, $F_g = 0$.

EVALUATE: In this model the earth is spherically symmetric but not uniform, so the result of Example 12.10 doesn't apply. In particular, the force at the surface of the outer core is greater than the force at the surface of the earth.

12.71. **IDENTIFY:** Eq.(12.12) relates orbital period and orbital radius for a circular orbit.

SET UP: The mass of the sun is $M = 1.99\times10^{30}\text{ kg}$.

EXECUTE: **(a)** The period of the asteroid is $T = \dfrac{2\pi a^{3/2}}{GM}$. Inserting $3\times10^{11}\text{ m}$ for a gives 2.84 y and $5\times10^{11}\text{ m}$ gives a period of 6.11 y.

(b) If the period is 5.93 y, then $a = 4.90\times10^{11}$m.

(c) This happens because $0.4 = 2/5$, another ratio of integers. So once every 5 orbits of the asteroid and 2 orbits of Jupiter, the asteroid is at its perijove distance. Solving when $T = 4.74\ y$, $a = 4.22\times10^{11}$ m.

EVALUATE: The orbit radius for Jupiter is 7.78×10^{11} m and for Mars it is 2.21×10^{11} m. The asteroid belt lies between Mars and Jupiter. The mass of Jupiter is about 3000 times that of Mars, so the effect of Jupiter on the asteroids is much larger.

12.72. **IDENTIFY:** Apply the work-energy relation in the form $W = \Delta E$, where $E = K + U$. The speed v is related to the orbit radius by Eq.(12.10).

SET UP: $m_E = 5.97\times10^{24}\text{ kg}$

EXECUTE: **(a)** In moving to a lower orbit by whatever means, gravity does positive work, and so the speed does increase.

(b) $v = (Gm_E)^{1/2}r^{-1/2}$, so $\Delta v = (Gm_E)^{1/2}\left(-\dfrac{-\Delta r}{2}\right)r^{-3/2} = \left(\dfrac{\Delta r}{2}\right)\sqrt{\dfrac{Gm_E}{r^3}}$. Note that a positive Δr is given as a decrease in radius. Similarly, the kinetic energy is $K = (1/2)mv^2 = (1/2)Gm_E m/r$, and so

$\Delta K = (1/2)(Gm_E m/r^2)\Delta r$ and $\Delta U = -(Gm_E m/r^2)\Delta r$.

$W = \Delta U + \Delta K = -(Gm_E m/2r^2)\ \Delta r$

(c) $v = \sqrt{Gm_E/r} = 7.72\times10^3\text{ m/s}$, \ $\Delta v = (\Delta r/2)\sqrt{Gm_E/r^3} = 28.9\text{ m/s}$, $E = -Gm_E m/2r = -8.95\times10^{10}$ J (from Eq.(12.15)),

$\Delta K = (Gm_E m/2r^2)(\Delta r) = 6.70\times10^8\text{ J}$, $\Delta U = -2\Delta K = -1.34\times10^9\text{ J}$, and $W = -\Delta K = -6.70\times10^8\text{ J}$.

(d) As the term "burns up" suggests, the energy is converted to heat or is dissipated in the collisions of the debris with the ground.

EVALUATE: When r decreases, K increases and U decreases (becomes more negative).

12.73. **IDENTIFY:** Use Eq.(12.2) to calculate F_g. Apply Newton's 2nd law to circular motion of each star to find the orbital speed and period. Apply the conservation of energy expression, Eq.(7.13), to calculate the energy input (work) required to separate the two stars to infinity.

(a) SET UP: The cm is midway between the two stars since they have equal masses. Let R be the orbit radius for each star, as sketched in Figure 12.73.

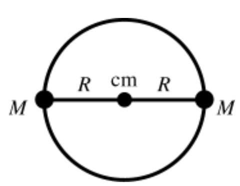


Figure 12.73

The two stars are separated by a distance $2R$, so $F_g = GM^2/(2R)^2 = GM^2/4R^2$

(b) EXECUTE: $F_g = ma_{rad}$

$GM^2/4R^2 = M(v^2/R)$ so $v = \sqrt{GM/4R}$

And $T = 2\pi R/v = 2\pi R\sqrt{4R/GM} = 4\pi\sqrt{R^3/GM}$

(c) SET UP: Apply $K_1 + U_1 + W_{other} = K_2 + U_2$ to the system of the two stars. Separate to infinity implies $K_2 = 0$ and $U_2 = 0$.

EXECUTE: $K_1 = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = 2(\frac{1}{2}M)(GM/4R) = GM^2/4R$

$U_1 = -GM^2/2R$

Thus the energy required is $W_{other} = -(K_1 + U_1) = -(GM^2/4R - GM^2/2R) = GM^2/4R.$

EVALUATE: The closer the stars are and the greater their mass, the larger their orbital speed, the shorter their orbital period and the greater the energy required to separate them.

12.74. IDENTIFY: In the center of mass coordinate system, $r_{cm} = 0$. Apply $\vec{F} = m\vec{a}$ to each star, where F is the gravitational force of one star on the other and $a = a_{rad} = \dfrac{4\pi^2 R}{T^2}$.

SET UP: $v = \dfrac{2\pi R}{T}$ allows R to be calculated from v and T.

IDENTIFY: **(a)** The radii R_1 and R_2 are measured with respect to the center of mass, and so $M_1R_1 = M_2R_2$, and $R_1/R_2 = M_2/M_1$.

(b) The forces on each star are equal in magnitude, so the product of the mass and the radial accelerations are equal: $\dfrac{4\pi^2 M_1R_1}{T_1^2} = \dfrac{4\pi^2 M_2R_2}{T_2^2}$. From the result of part (a), the numerators of these expressions are equal, and so the denominators are equal, and the periods are the same. To find the period in the symmetric form desired, there are many possible routes. An elegant method, using a bit of hindsight, is to use the above expressions to relate the periods to the force $F_g = \dfrac{GM_1M_2}{(R_1 + R_2)^2}$, so that equivalent expressions for the period are $M_2T^2 = \dfrac{4\pi^2 R_1(R_1 + R_2)^2}{G}$ and $M_1T^2 = \dfrac{4\pi^2 R_2(R_1 + R_2)^2}{G}$. Adding the expressions gives $(M_1 + M_2)T^2 = \dfrac{4\pi^2(R_1 + R_2)^3}{G}$ or $T = \dfrac{2\pi(R_1 + R_2)^{3/2}}{\sqrt{G(M_1 + M_2)}}$.

(c) First we must find the radii of each orbit given the speed and period data. In a circular orbit, $v = \dfrac{2\pi R}{T}$, or $R = \dfrac{vT}{2\pi}$. Thus $R_\alpha = \dfrac{(36\times 10^3 \text{ m/s})(137 \text{ d})(86{,}400 \text{ s/d})}{2\pi} = 6.78\times 10^{10}$ m and $R_\beta = \dfrac{(12\times 10^3 \text{ m/s})(137 \text{ d})(86{,}400 \text{ s/d})}{2\pi} = 2.26\times 10^{10}$ m. Now find the sum of the masses. Use $M_\alpha R_\alpha = M_\beta R_\beta$, and the fact that $R_\alpha = 3R_\beta (M_\alpha + M_\beta) = \dfrac{4\pi^2(R_\alpha + R_\beta)^3}{T^2 G}$, inserting the values of T, and the radii. This gives

$(M_\alpha + M_\beta) = \dfrac{4\pi^2(6.78\times 10^{10}\text{m} + 2.26\times 10^{10} \text{ m})^3}{[(137 \text{ d})(86{,}400 \text{ s/d})]^2(6.673\times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)}$. $M_\alpha + M_\beta = 3.12\times 10^{30}$ kg. Since

$M_\beta = M_\alpha R_\alpha/R_\beta = 3M_\alpha$, $4M_\alpha = 3.12\times 10^{30}$ kg, or $M_\alpha = 7.80\times 10^{29}$ kg and $M_\beta = 2.34\times 10^{30}$ kg.

(d) Let α refer to the star and β refer to the black hole. Use the relationships derived in parts (a) and (b):

$R_\beta = (M_\alpha/M_\beta)R_\alpha = (0.67/3.8)R_\alpha = (0.176)R_\alpha$, $R_\alpha + \text{R}_\beta = \sqrt[3]{\dfrac{(M_\alpha + M_\beta)T^2 G}{4\pi^2}}$. For Monocerotis, inserting the values for M and T and R_β gives $R_\alpha = 1.9\times 10^9$ m, $v_\alpha = 4.4\times 10^2$ km/s and for the black hole $R_\beta = 34\times 10^8$ m, $v_\beta = 77$ km/s.

EVALUATE: Since T is the same, v is smaller when R is smaller.

12.75. IDENTIFY and SET UP: Use conservation of energy, $K_1 + U_1 + W_{other} = K_2 + U_2$. The gravity force exerted by the sun is the only force that does work on the comet, so $W_{other} = 0$.

EXECUTE: $K_1 = \frac{1}{2}mv_1^2$, $v_1 = 2.0\times 10^4$ m/s

$U_1 = -Gm_Sm/r_1$, $r_1 = 2.5\times 10^{11}$ m

$K_2 = \frac{1}{2}mv_2^2$

$U_2 = -Gm_Sm/r_2$, $r_2 = 5.0\times 10^{10}$ m

$\frac{1}{2}mv_1^2 - Gm_S m/r_1 = \frac{1}{2}mv_2^2 - Gm_S m/r_2$

$v_2^2 = v_1^2 + 2Gm_S\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = v_1^2 + 2Gm_S\left(\frac{r_1 - r_2}{r_1 r_2}\right)$

$v_2 = 6.8\times10^4\ \text{m/s}$

EVALUATE: The comet has greater speed when it is closer to the sun.

12.76. IDENTIFY: Apply conservation of energy.

SET UP: Let m_M be the mass of Mars and M_S be the mass of the sun. The subscripts a and p denote aphelion and perihelion.

EXECUTE: $\frac{1}{2}m_M v_a^2 - \frac{GM_S m_M}{r_a} = \frac{1}{2}m_M v_p^2 - \frac{GM_S m_M}{r_p}$, or $v_p = \sqrt{v_a^2 - 2GM_S\left(\frac{1}{r_a} - \frac{1}{r_p}\right)} = 2.650\times10^4\ \text{m/s}.$

EVALUATE: We could instead use conservation of angular momentum. Note that at the extremes of distance (periheleion and aphelion), Mars' velocity vector must be perpendicular to its radius vector, and so the magnitude of the angular momentum is $L = mrv$. Since L is constant, the product rv must be a constant, and so

$v_p = v_a \frac{r_a}{r_p} = (2.198\times10^4\ \text{m/s})\frac{(2.492\times10^{11}\ \text{m})}{(2.067\times10^{11}\ \text{m})} = 2.650\times10^4\ \text{m/s}$. Mars has larger speed when it is closer to the sun.

12.77. (a) IDENTIFY and **SET UP:** Use Eq.(12.17), applied to the satellites orbiting the earth rather than the sun.

EXECUTE: Find the value of a for the elliptical orbit:

$2a = r_a + r_p = R_E + h_a + R_E + h_p$, where h_a and h_p are the heights at apogee and perigee, respectively.

$a = R_E + (h_a + h_p)/2$

$a = 6.38\times10^6\ \text{m} + (400\times10^3\ \text{m} + 4000\times10^3\ \text{m})/2 = 8.58\times10^6\ \text{m}$

$$T = \frac{2\pi a^{3/2}}{\sqrt{GM_E}} = \frac{2\pi(8.58\times10^6\ \text{m})^{3/2}}{\sqrt{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})}} = 7.91\times10^3\ \text{s}$$

(b) Conservation of angular momentum gives $r_a v_a = r_p v_p$

$$\frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{6.38\times10^6\ \text{m} + 4.00\times10^6\ \text{m}}{6.38\times10^6\ \text{m} + 4.00\times10^5\ \text{m}} = 1.53$$

(c) Conservation of energy applied to apogee and perigee gives $K_a + U_a = K_p + U_p$

$\frac{1}{2}mv_a^2 - Gm_E m/r_a = \frac{1}{2}mv_p^2 - Gm_E m/r_p$

$v_p^2 - v_a^2 = 2Gm_E(1/r_p - 1/r_a) = 2Gm_E(r_a - r_p)/r_a r_p$

But $v_p = 1.532 v_a$, so $1.347 v_a^2 = 2Gm_E(r_a - r_p)/r_a r_p$

$v_a = 5.51\times10^3\ \text{m/s}$, $v_p = 8.43\times10^3\ \text{m/s}$

(d) Need v so that $E = 0$, where $E = K + U$.

at perigee: $\frac{1}{2}mv_p^2 - Gm_E m/r_p = 0$

$v_p = \sqrt{2Gm_E/r_p} = \sqrt{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})/6.78\times10^6\ \text{m}} = 1.084\times10^4\ \text{m/s}$

This means an increase of $1.084\times10^4\ \text{m/s} - 8.43\times10^3\ \text{m/s} = 2.41\times10^3\ \text{m/s}.$

at apogee: $v_a = \sqrt{2Gm_E/r_a} = \sqrt{2(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\ \text{kg})/1.038\times10^7\ \text{m}} = 8.761\times10^3\ \text{m/s}$

This means an increase of $8.761\times10^3\ \text{m/s} - 5.51\times10^3\ \text{m/s} = 3.25\times10^3\ \text{m/s}.$

EVALUATE: Perigee is more efficient. At this point r is smaller so v is larger and the satellite has more kinetic energy and more total energy.

12.78. IDENTIFY: $g = \frac{GM}{R^2}$, where M and R are the mass and radius of the planet.

SET UP: Let m_U and R_U be the mass and radius of Uranus and let g_U be the acceleration due to gravity at its poles. The orbit radius of Miranda is $r = h + R_U$, where $h = 1.04\times10^8$ m is the altitude of Miranda above the surface of Uranus.

EXECUTE: **(a)** From the value of g at the poles,

$$m_U = \frac{g_U R_U^2}{G} = \frac{\left(11.1\ \text{m/s}^2\right)\left(2.556\times10^7\ \text{m}\right)^2}{\left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)} = 1.09\times10^{26}\ \text{kg}.$$

(b) $Gm_{\rm U}/r^2 = g_{\rm U}\left(R_{\rm U}/r\right)^2 = 0.432\ \text{m/s}^2$.

(c) $Gm_{\rm M}/R_{\rm M}^2 = 0.080\ \text{m/s}^2$.

EVALUATE: **(d)** No. Both the object and Miranda are in orbit together around Uranus, due to the gravitational force of Uranus. The object has additional force toward Miranda.

12.79. **IDENTIFY** and **SET UP:** Apply conservation of energy (Eq.7.13) and solve for $W_{\rm other}$. Only $r = h + R_{\rm E}$ is given, so use Eq.(12.10) to relate *r* and *v*.

EXECUTE: $K_1 + U_1 + W_{\rm other} = K_2 + U_2$

$U_1 = -Gm_{\rm M}m/r_1$, where $m_{\rm M}$ is the mass of Mars and $r_1 = R_{\rm M} + h$, where $R_{\rm M}$ is the radius of Mars and $h = 2000\times10^3$ m

$$U_1 = -(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.42\times10^{23}\ \text{kg})(3000\ \text{kg})}{3.40\times10^6\ \text{m} + 2000\times10^3\ \text{m}} = -2.380\times10^{10}\ \text{J}$$

$U_2 = -Gm_{\rm M}m/r_2$, where r_2 is the new orbit radius.

$$U_2 = -(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(6.42\times10^{23}\ \text{kg})(3000\ \text{kg})}{3.40\times10^6\ \text{m} + 4000\times10^3\ \text{m}} = -1.737\times10^{10}\ \text{J}$$

For a circular orbit $v = \sqrt{Gm_{\rm M}/r}$ (Eq.(12.10), with the mass of Mars rather than the mass of the earth).

Using this gives $K = \frac{1}{2}mv^2 = \frac{1}{2}m(Gm_{\rm M}/r) = \frac{1}{2}Gm_{\rm M}m/r$, so $K = -\frac{1}{2}U$

$K_1 = -\frac{1}{2}U_1 = +1.190\times10^{10}\ \text{J}$ and $K_2 = -\frac{1}{2}U_2 = +8.685\times10^9\ \text{J}$

Then $K_1 + U_1 + W_{\rm other} = K_2 + U_2$ gives

$W_{\rm other} = (K_2 - K_1) + (U_2 - U_1) = (8.685\times10^9\ \text{J} - 1.190\times10^{10}\ \text{J}) + (-2.380\times10^{10}\ \text{J} + 1.737\times10^{10}\ \text{J})$

$W_{\rm other} = -3.215\times10^9\ \text{J} + 6.430\times10^9\ \text{J} = 3.22\times10^9\ \text{J}$

EVALUATE: When the orbit radius increases the kinetic energy decreases and the gravitational potential energy increases. $K = -U/2$ so $E = K + U = -U/2$ and the total energy also increases (becomes less negative). Positive work must be done to increase the total energy of the satellite.

12.80. **IDENTIFY** and **SET UP:** Use Eq.(12.17) to calculate *a*. $T = 30{,}000\ \text{y}(3.156\times10^7\ \text{s/1 y}) = 9.468\times10^{11}\ \text{s}$

EXECUTE: Eq.(12.17): $T = \dfrac{2\pi a^{3/2}}{\sqrt{Gm_{\rm S}}}$, $T^2 = \dfrac{4\pi^2a^3}{Gm_{\rm S}}$

$$a = \left(\frac{Gm_{\rm S}T^2}{4\pi^2}\right)^{1/3} = 1.4\times10^{14}\ \text{m}.$$

EVALUATE: The average orbit radius of Pluto is 5.9×10^{12} m (Appendix F); the semi-major axis for this comet is larger by a factor of 24.

4.3 light years = 4.3 light years$(9.461\times10^{15}$ m/1 light year$) = 4.1\times10^{16}$ m

The distance of Alpha Centauri is larger by a factor of 300.

The orbit of the comet extends well past Pluto but is well within the distance to Alpha Centauri.

12.81. **IDENTIFY:** Integrate $dm = \rho dV$ to find the mass of the planet. Outside the planet, the planet behaves like a point mass, so at the surface $g = GM/R^2$.

SET UP: A thin spherical shell with thickness dr has volume $dV = 4\pi r^2 dr$. The earth has radius $R_{\rm E} = 6.38\times10^6$ m.

EXECUTE: Get $M : M = \int dm = \int \rho dV = \int \rho 4\pi r^2 dr$. The density is $\rho = \rho_0 - br$, where $\rho_0 = 15.0\times10^3\ \text{kg/m}^3$ at the center and at the surface, $\rho_{\rm S} = 2.0\times10^3\ \text{kg/m}^3$, so $b = \dfrac{\rho_0 - \rho_s}{R}$.

$$M = \int_0^R \left(\rho_0 - br\right) 4\pi r^2 dr = \frac{4\pi}{3}\rho_0 R^3 - \pi b R^4 = \frac{4}{3}\pi R^3\rho_0 - \pi R^4\left(\frac{\rho_0 - \rho_s}{R}\right) = \pi R^3\left(\frac{1}{3}\rho_0 + \rho_s\right) \text{and } M = 5.71\times10^{24}\ \text{kg}.$$

Then $g = \dfrac{GM}{R^2} = \dfrac{G\pi R^3\left(\frac{1}{3}\rho_0 + \rho_s\right)}{R^2} = \pi RG\left(\dfrac{1}{3}\rho_0 + \rho_s\right)$.

$$g = \pi\left(6.38\times10^6\text{m}\right)\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(\frac{15.0\times10^3\ \text{kg/m}^3}{3} + 2.0\times10^3\ \text{kg/m}^3\right).$$

$g = 9.36\ \text{m/s}^2$.

EVALUATE: The average density of the planet is $\rho_{av} = \dfrac{M}{V} = \dfrac{M}{\frac{4}{3}\pi R^3} = \dfrac{3(5.71\times10^{24}\ \text{kg})}{4\pi(6.38\times10^6\ \text{m})^3} = 5.25\times10^3\ \text{kg/m}^3$. Note that this is not $(\rho_0+\rho_s)/2$.

12.82. IDENTIFY and **SET UP:** Use Eq.(12.1) to calculate the force between the point mass and a small segment of the semicircle.

EXECUTE: The radius of the semicircle is $R = L/\pi$

Divide the semicircle up into small segments of length $R\,d\theta$, as shown in Figure 12.82.

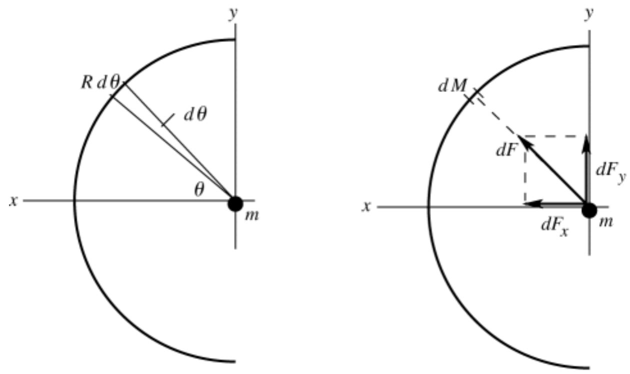


Figure 12.82

$dM = (M/L)R\,d\theta = (M/\pi)\,d\theta$

$d\vec{F}$ is the gravity force on m exerted by dM

$\int dF_y = 0$; the y-components from the upper half of the semicircle cancel the y-components from the lower half.

The x-components are all in the $+x$-direction and all add.

$$dF = G\frac{m\,dM}{R^2}$$

$$dF_x = G\frac{m\,dM}{R^2}\cos\theta = \frac{Gm\pi M}{L^2}\cos\theta\,d\theta$$

$$F_x = \int_{-\pi/2}^{\pi/2} dF_x = \frac{Gm\pi M}{L^2}\int_{-\pi/2}^{\pi/2}\cos\theta\,d\theta = \frac{Gm\pi M}{L^2}(2)$$

$$F = \frac{2\pi GmM}{L^2}$$

EVALUATE: If the semicircle were replaced by a point mass M at $x = R$, the gravity force would be $GmM/R^2 = \pi^2 GmM/L^2$. This is $\pi/2$ times larger than the force exerted by the semicirclar wire. For the semicircle it is the x-components that add, and the sum is less than if the force magnitudes were added.

12.83. IDENTIFY: The direct calculation of the force that the sphere exerts on the ring is slightly more involved than the calculation of the force that the ring exerts on the sphere. These forces are equal in magnitude but opposite in direction, so it will suffice to do the latter calculation. By symmetry, the force on the sphere will be along the axis of the ring in Figure 12.35 in the textbook, toward the ring.

SET UP: Divide the ring into infinitesimal elements with mass dM.

EXECUTE: Each mass element dM of the ring exerts a force of magnitude $\dfrac{(Gm)dM}{a^2+x^2}$ on the sphere, and the x-component of this force is $\dfrac{GmdM}{a^2+x^2}\dfrac{x}{\sqrt{a^2+x^2}} = \dfrac{GmdMx}{\left(a^2+x^2\right)^{3/2}}$.

Therefore, the force on the sphere is $GmMx/\left(a^2+x^2\right)^{3/2}$, in the $-x$-direction. The sphere attracts the ring with a force of the same magnitude.

EVALUATE: As $x >> a$ the denominator approaches x^3 and $F \to \dfrac{GMm}{x^2}$, as expected.

12.84. IDENTIFY: Use Eq.(12.1) for the force between a small segment of the rod and the particle. Integrate over the length of the rod to find the total force.

SET UP: Use a coordinate system with the origin at the left-hand end of the rod and the x'-axis along the rod, as shown in Figure 12.84. Divide the rod into small segments of length dx'. (Use x' for the coordinate so not to confuse with the distance x from the end of the rod to the particle.)

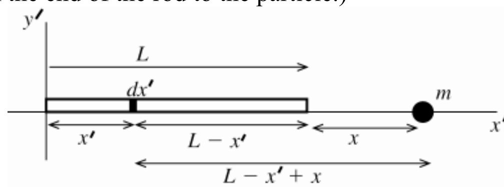


Figure 12.84

EXECUTE: The mass of each segment is $dM = dx'(M/L)$. Each segment is a distance $L - x' + x$ from mass m, so the force on the particle due to a segment is $dF = \dfrac{Gm\,dM}{(L-x'+x)^2} = \dfrac{GMm}{L}\dfrac{dx'}{(L-x'+x)^2}$.

$$F = \int_L^0 dF = \frac{GMm}{L}\int_L^0 \frac{dx'}{(L-x'+x)^2} = \frac{GMm}{L}\left(-\frac{1}{L-x'+x}\Big|_L^0\right)$$

$$F = \frac{GMm}{L}\left(\frac{1}{x} - \frac{1}{L+x}\right) = \frac{GMm}{L}\frac{(L+x-x)}{x(L+x)} = \frac{GMm}{x(L+x)}$$

EVALUATE: For $x >> L$ this result become $F = GMm/x^2$, the same as for a pair of point masses.

12.85. **IDENTIFY:** Compare F_E to Hooke's law.

SET UP: The earth has mass $m_E = 5.97\times10^{24}$ kg and radius $R_E = 6.38\times10^6$ m .

EXECUTE: For $F_x = -kx$, $U = \frac{1}{2}kx^2$. The force here is in the same form, so by analogy $U(r) = \dfrac{Gm_E m}{2R_E^3}r^2$. This is also given by the integral of F_g from 0 to r with respect to distance.

(b) From part (a), the initial gravitational potential energy is $\dfrac{Gm_E m}{2R_E}$. Equating initial potential energy and final kinetic energy (initial kinetic energy and final potential energy are both zero) gives

$v^2 = \dfrac{Gm_E}{R_E}$, so $v = 7.90\times10^3$ m/s.

EVALUATE: When $r = 0$, $U(r) = 0$, as specified in the problem.

12.86. **IDENTIFY:** In Eqs.(12.12) and (12.16) replace T by $T+\Delta T$ and r by $r+\Delta r$. Use the expression in the hint to simplifying the resulting equations.

SET UP: The earth has $m_E = 5.97\times10^{24}$ kg and $R = 6.38\times10^6$ m . $r = h + R_E$, where h is the altitude above the surface of the earth.

EXECUTE: **(a)** $T = \dfrac{2\pi\, r^{3/2}}{\sqrt{GM_E}}$ therefore

$$T+\Delta T = \frac{2\pi}{\sqrt{GM_E}}(r+\Delta r)^{3/2} = \frac{2\pi\, r^{3/2}}{\sqrt{GM_E}}\left(1+\frac{\Delta r}{r}\right)^{3/2} \approx \frac{2\pi\, r^{3/2}}{\sqrt{GM_E}}\left(1+\frac{3\Delta r}{2r}\right) = T + \frac{3\pi\, r^{1/2}\Delta r}{\sqrt{GM_E}}.$$

Since $v = \sqrt{\dfrac{GM_E}{r}}, \Delta T = \dfrac{3\pi\,\Delta r}{v}$. $v = \sqrt{GM_E}\; r^{-1/2}$, and therefore

$v - \Delta v = \sqrt{GM_E}\;(r+\Delta r)^{-1/2} = \sqrt{GM_E}\; r^{-1/2}\left(1+\dfrac{\Delta r}{r}\right)^{-1/2}$ and $v \approx \sqrt{GM_E}\; r^{-1/2}\left(1-\dfrac{\Delta r}{2r}\right) = v - \dfrac{\sqrt{GM_E}}{2r^{3/2}}\Delta r$. Since $T = \dfrac{2\pi\, r^{3/2}}{\sqrt{GM_E}}$, $\Delta v = \dfrac{\pi\Delta r}{T}$.

(b) Starting with $T = \dfrac{2\pi\, r^{3/2}}{\sqrt{GM}}$ (Eq.(12.12)), $T = 2\pi\, r/v$, and $v = \sqrt{\dfrac{GM}{r}}$ (Eq.(12.10)), find the velocity and period of the initial orbit: $v = \sqrt{\dfrac{(6.673\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times10^{24}\,\text{kg})}{6.776\times10^6\,\text{m}}} = 7.672\times10^3$ m/s, and

$T = 2\pi\, r/v = 5549$ s $= 92.5$ min. We then can use the two derived equations to approximate ΔT and Δv :

$\Delta T = \frac{3\pi\Delta r}{v} = \frac{3\pi\ (100\text{ m})}{7.672\times10^3\text{ m/s}} = 0.1228\text{ s}$ and $\Delta v = \frac{\pi\Delta r}{T} = \frac{\pi(100\text{ m})}{(5549\text{ s})} = 0.05662\text{ m/s}$. Before the cable breaks, the shuttle will have traveled a distance *d*, $d = \sqrt{(125\text{ m}^2) - (100\text{ m}^2)} = 75\text{ m}$.

$t = (75\text{ m})/(0.05662\text{ m/s}) = 1324.7\text{ s} = 22\text{ min}$. It will take 22 minutes for the cable to break.

(c) The ISS is moving faster than the space shuttle, so the total angle it covers in an orbit must be 2π radians more than the angle that the space shuttle covers before they are once again in line. Mathematically, $\frac{vt}{r} - \frac{(v-\Delta v)t}{(r+\Delta r)} = 2\pi$.

Using the binomial theorem and neglecting terms of order $\Delta v\Delta r$, $\frac{vt}{r} - \frac{(v-\Delta v)t}{r}\left(1+\frac{\Delta r}{r}\right)^{-1} \approx t\left(\frac{\Delta v}{r}+\frac{v\Delta r}{r^2}\right) = 2\pi$.

Therefore, $t = \frac{2\pi r}{\left(\Delta v + \frac{v\Delta r}{r}\right)} = \frac{vT}{\frac{\pi\ \Delta r}{T}+\frac{v\Delta r}{r}}$. Since $2\pi\ r = vT$ and $\Delta r = \frac{v\Delta T}{3\pi}$, $t = \frac{vT}{\frac{\pi}{t}\left(\frac{v\Delta T}{3\pi}\right)+\frac{2\pi}{T}\left(\frac{v\Delta T}{3\pi}\right)} = \frac{T^2}{\Delta T}$, as was to be shown. $t = \frac{T^2}{\Delta T} = \frac{(5549\text{ s})^2}{(0.1228\text{ s})} = 2.5\times10^8\text{ s} = 2900\text{ d} = 7.9\text{ y}$. It is highly doubtful the shuttle crew would survive the congressional hearings if they miss!

EVALUATE: When the orbit radius increases, the orbital period increases and the orbital speed decreases.

12.87. IDENTIFY: Apply Eq.(12.19) to the transfer orbit.

SET UP: The orbit radius for Earth is $r_E = 1.50\times10^{11}$ m and for Mars it is $r_M = 2.28\times10^{11}$ m. From Figure 12.19 in the textbook, $a = \frac{1}{2}(r_E + r_M)$

EXECUTE: **(a)** To get from the circular orbit of the earth to the transfer orbit, the spacecraft's energy must increase, and the rockets are fired in the direction opposite that of the motion, that is, in the direction that increases the speed. Once at the orbit of Mars, the energy needs to be increased again, and so the rockets need to be fired in the direction opposite that of the motion. From Figure 12.38 in the textbook, the semimajor axis of the transfer orbit is the arithmetic average of the orbit radii of the earth and Mars, and so from Eq.(12.13), the energy of the spacecraft while in the transfer orbit is intermediate between the energies of the circular orbits. Returning from Mars to the earth, the procedure is reversed, and the rockets are fired against the direction of motion.

(b) The time will be half the period as given in Eq. (12.17), with the semimajor axis equal to

$a = \frac{1}{2}(r_E + r_M) = 1.89\times10^{11}$ m so $t = \frac{T}{2} = \frac{\pi\ (1.89\times10^{11}\text{ m})^{3/2}}{\sqrt{(6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})}} = 2.24\times10^7\text{ s} = 263\text{ days}$,

which is more than $8\frac{1}{2}$ months.

(c) During this time, Mars will pass through an angle of $(360°)\frac{(2.24\times10^7\text{ s})}{(687\text{ d})(86{,}400\text{ s/d})} = 135.9°$, and the spacecraft passes through an angle of $180°$, so the angle between the earth-sun line and the Mars-sun line must be $44.1°$.

EVALUATE: The period *T* for the transfer orbit is 526 days, the average of the orbital periods for Earth and Mars.

12.88. IDENTIFY: Apply $\sum\vec{F} = m\vec{a}$ to each ear.

SET UP: Denote the orbit radius as r and the distance from this radius to either ear as δ. Each ear, of mass m, can be modeled as subject to two forces, the gravitational force from the black hole and the tension force (actually the force from the body tissues), denoted by F.

EXECUTE: The force equation for either ear is $\frac{GMm}{(r+\delta)^2} - F = m\omega^2(r+\delta)$, where δ can be of either sign.

Replace the product $m\omega^2$ with the value for $\delta = 0$, $m\omega^2 = GMm/r^3$, and solve for *F*:

$$F = (GMm)\left[\frac{r+\delta}{r^3} - \frac{1}{(r+\delta)^2}\right] = \frac{GMm}{r^3}\left[r+\delta - r\left(1+(\delta/r)\right)^{-2}\right].$$

Using the binomial theorem to expand the term in square brackets in powers of δ/r,

$$F \approx \frac{GMm}{r^3}\left[r+\delta - r\left(1-2(\delta/r)\right)\right] = \frac{GMm}{r^3}(3\delta) = 2.1\text{ kN}.$$

This tension is much larger than that which could be sustained by human tissue, and the astronaut is in trouble.

(b) The center of gravity is not the center of mass. The gravity force on the two ears is not the same.

EVALUATE: The tension between her ears is proportional to their separation.

12.89. IDENTIFY: As suggested in the problem, divide the disk into rings of radius *r* and thickness *dr*.

SET UP: Each ring has an area $dA = 2\pi r\ dr$ and mass $dM = \frac{M}{\pi\ a^2}dA = \frac{2M}{a^2}r\ dr$.

EXECUTE: The magnitude of the force that this small ring exerts on the mass m is then $(G\ m\ dM)(x/(r^2+x^2)^{3/2})$. The contribution dF to the force is $dF=\dfrac{2GMmx}{a^2}\dfrac{rdr}{(x^2+r^2)^{3/2}}$.

The total force F is then the integral over the range of r;

$$F=\int dF=\frac{2GMmx}{a^2}\int_0^a\frac{r}{(x^2+r^2)^{3/2}}dr.$$

The integral (either by looking in a table or making the substitution $u=r^2+a^2$) is

$$\int_0^a\frac{r}{(x^2+r^2)^{3/2}}dr=\left[\frac{1}{x}-\frac{1}{\sqrt{a^2+x^2}}\right]=\frac{1}{x}\left[1-\frac{x}{\sqrt{a^2+x^2}}\right].$$

Substitution yields the result $F=\dfrac{2GMm}{a^2}\left[1-\dfrac{x}{\sqrt{a^2+x^2}}\right]$. The force on m is directed toward the center of the ring.

The second term in brackets can be written as

$$\frac{1}{\sqrt{1+(a/x)^2}}=(1+(a/x)^2)^{-1/2}\approx 1-\frac{1}{2}\left(\frac{a}{x}\right)^2$$

if $x >> a$, where the binomial expansion has been used. Substitution of this into the above form gives $F\approx\dfrac{GMm}{x^2}$, as it should.

EVALUATE: As $x\to 0$, the force approaches a constant.

12.90. **IDENTIFY:** Divide the rod into infinitesimal segments. Calculate the force each segment exerts on m and integrate over the rod to find the total force.

SET UP: From symmetry, the component of the gravitational force parallel to the rod is zero. To find the perpendicular component, divide the rod into segments of length dx and mass $dm=dx\dfrac{M}{2L}$, positioned at a distance x from the center of the rod.

EXECUTE: The magnitude of the gravitational force from each segment is $dF=\dfrac{Gm\ dM}{x^2+a^2}=\dfrac{GmM}{2L}\dfrac{dx}{x^2+a^2}$. The component of dF perpendicular to the rod is $dF\dfrac{a}{\sqrt{x^2+a^2}}$ and so the net gravitational force is $F=\displaystyle\int_{-L}^{L}dF=\frac{GmMa}{2L}\int_{-L}^{L}\frac{dx}{(x^2+a^2)^{3/2}}$.

The integral can be found in a table, or found by making the substitution $x=a\ \tan\theta$. Then, $dx=a\ \sec^2\theta\ d\theta,\ (x^2+a^2)=a^2\ \sec^2\theta,$ and so

$$\int\frac{dx}{(x^2+a^2)^{3/2}}=\int\frac{a\ \sec^2\theta\ d\theta}{a^3\ \sec^3\theta}=\frac{1}{a^2}\int\cos\theta\ d\theta=\frac{1}{a^2}\sin\theta=\frac{x}{a^2\sqrt{x^2+a^2}},$$

and the definite integral is $F=\dfrac{GmM}{a\sqrt{a^2+L^2}}$.

EVALUATE: When a >> L, the term in the square root approaches a^2 and $F\to\dfrac{GmM}{a^2}$, as expected.

PERIODIC MOTION

13

13.1. **IDENTIFY** and **SET UP:** The target variables are the period T and angular frequency ω. We are given the frequency f, so we can find these using Eqs.(13.1) and (13.2)

EXECUTE: **(a)** $f = 220$ Hz

$T = 1/f = 1/220 \text{ Hz} = 4.54\times10^{-3} \text{ s}$

$\omega = 2\pi f = 2\pi(220 \text{ Hz}) = 1380 \text{ rad/s}$

(b) $f = 2(220 \text{ Hz}) = 440 \text{ Hz}$

$T = 1/f = 1/440 \text{ Hz} = 2.27\times10^{-3} \text{ s}$ (smaller by a factor of 2)

$\omega = 2\pi f = 2\pi(440 \text{ Hz}) = 2760 \text{ rad/s}$ (factor of 2 larger)

EVALUATE: The angular frequency is directly proportional to the frequency and the period is inversely proportional to the frequency.

13.2. **IDENTIFY** and **SET UP:** The amplitude is the maximum displacement from equilibrium. In one period the object goes from $x = +A$ to $x = -A$ and returns.

EXECUTE: **(a)** $A = 0.120$ m

(b) $0.800 \text{ s} = T/2$ so the period is 1.60 s

(c) $f = \dfrac{1}{T} = 0.625$ Hz

EVALUATE: Whenever the object is released from rest, its initial displacement equals the amplitude of its SHM.

13.3. **IDENTIFY:** The period is the time for one vibration and $\omega = \dfrac{2\pi}{T}$.

SET UP: The units of angular frequency are rad/s.

EXECUTE: The period is $\dfrac{0.50 \text{ s}}{440} = 1.14\times10^{-3}$ s and the angular frequency is $\omega = \dfrac{2\pi}{T} = 5.53\times10^{3}$ rad/s.

EVALUATE: There are 880 vibrations in 1.0 s, so $f = 880$ Hz. This is equal to $1/T$.

13.4. **IDENTIFY:** The period is the time for one cycle and the amplitude is the maximum displacement from equilibrium. Both these values can be read from the graph.

SET UP: The maximum x is 10.0 cm. The time for one cycle is 16.0 s.

EXECUTE: **(a)** $T = 16.0$ s so $f = \dfrac{1}{T} = 0.0625$ Hz.

(b) $A = 10.0$ cm.

(c) $T = 16.0$ s

(d) $\omega = 2\pi f = 0.393$ rad/s

EVALUATE: After one cycle the motion repeats.

13.5. **IDENTIFY:** This displacement is $\frac{1}{4}$ of a period.

SET UP: $T = 1/f = 0.200$ s.

EXECUTE: $t = 0.0500$ s

EVALUATE: The time is the same for $x = A$ to $x = 0$, for $x = 0$ to $x = -A$, for $x = -A$ to $x = 0$ and for $x = 0$ to $x = A$.

13.6. **IDENTIFY:** Apply Eq.(13.12).

SET UP: The period will be twice the interval between the times at which the glider is at the equilibrium position.

EXECUTE: $k = \omega^2 m = \left(\dfrac{2\pi}{T}\right)^2 m = \left(\dfrac{2\pi}{2(2.60 \text{ s})}\right)^2 (0.200 \text{ kg}) = 0.292$ N/m.

EVALUATE: $1 \text{ N} = 1 \text{ kg}\cdot\text{m/s}^2$, so $1 \text{ N/m} = 1 \text{ kg/s}^2$.

13.7. **IDENTIFY** and **SET UP:** Use Eq.(13.1) to calculate T, Eq.(13.2) to calculate ω, and Eq.(13.10) for m.

EXECUTE: **(a)** $T = 1/f = 1/6.00\text{ Hz} = 0.167\text{ s}$

(b) $\omega = 2\pi f = 2\pi(6.00\text{ Hz}) = 37.7\text{ rad/s}$

(c) $\omega = \sqrt{k/m}$ implies $m = k/\omega^2 = (120\text{ N/m})/(37.7\text{ rad/s})^2 = 0.0844\text{ kg}$

EVALUATE: We can verify that k/ω^2 has units of mass.

13.8. **IDENTIFY:** The mass and frequency are related by $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.

SET UP: $f\sqrt{m} = \dfrac{\sqrt{k}}{2\pi} =$ constant , so $f_1\sqrt{m_1} = f_2\sqrt{m_2}$.

EXECUTE: **(a)** $m_1 = 0.750\text{ kg}$, $f_1 = 1.33\text{ Hz}$ and $m_2 = 0.750\text{ kg} + 0.220\text{ kg} = 0.970\text{ kg}$.

$f_2 = f_1\sqrt{\dfrac{m_1}{m_2}} = (1.33\text{ Hz})\sqrt{\dfrac{0.750\text{ kg}}{0.970\text{ kg}}} = 1.17\text{ Hz}$.

(b) $m_2 = 0.750\text{ kg} - 0.220\text{ kg} = 0.530\text{ kg}$. $f_2 = (1.33\text{ Hz})\sqrt{\dfrac{0.750\text{ kg}}{0.530\text{ kg}}} = 1.58\text{ Hz}$

EVALUATE: When the mass increases the frequency decreases and when the mass decreases the frequency increases.

13.9. **IDENTIFY:** Apply Eqs.(13.11) and (13.12).

SET UP: $f = 1/T$

EXECUTE: **(a)** $T = 2\pi\sqrt{\dfrac{0.500\text{ kg}}{140\text{ N/m}}} = 0.375\text{ s}$.

(b) $f = \dfrac{1}{T} = 2.66\text{ Hz}$. **(c)** $\omega = 2\pi f = 16.7\text{ rad/s}$.

EVALUATE: We can verify that $1\text{ kg/(N/m)} = 1\text{ s}^2$.

13.10. **IDENTIFY** and **SET UP:** Use Eqs. (13.13), (13.15), and (13.16).

EXECUTE: $f = 440\text{ Hz}$, $A = 3.0\text{ mm}$, $\phi = 0$

(a) $x = A\cos(\omega t + \phi)$

$\omega = 2\pi f = 2\pi(440\text{ hz}) = 2.76\times10^3\text{ rad/s}$

$x = (3.0\times10^{-3}\text{ m})\cos((2.76\times10^3\text{ rad/s})t)$

(b) $v_x = -\omega A\sin(\omega t + \phi)$

$v_{max} = \omega A = (2.76\times10^3\text{ rad/s})(3.0\times10^{-3}\text{ m}) = 8.3\text{ m/s}$ (maximum magnitude of velocity)

$a_x = -\omega^2 A\cos(\omega t + \phi)$

$a_{max} = \omega^2 A = (2.76\times10^3\text{ rad/s})^2(3.0\times10^{-3}\text{ m}) = 2.3\times10^4\text{ m/s}^2$ (maximum magnitude of acceleration)

(c) $a_x = -\omega^2 A\cos\omega t$

$da_x/dt = +\omega^3 A\sin\omega t = [2\pi(440\text{ Hz})]^3\ (3.0\times10^{-3}\text{ m})\sin([2.76\times10^3\text{ rad/s}]t) = (6.3\times10^7\text{ m/s}^3)\sin([2.76\times10^3\text{rad/s}]t)$

Maximum magnitude of the jerk is $\omega^3 A = 6.3\times10^7\text{ m/s}^3$

EVALUATE: The period of the motion is small, so the maximum acceleration and jerk are large.

13.11. **IDENTIFY:** Use Eq.(13.19) to calculate A. The initial position and velocity of the block determine ϕ. $x(t)$ is given by Eq.(13.13).

SET UP: $\cos\theta$ is zero when $\theta = \pm\pi/2$ and $\sin(\pi/2) = 1$.

EXECUTE: **(a)** From Eq. (13.19), $A = \left|\dfrac{v_0}{\omega}\right| = \left|\dfrac{v_0}{\sqrt{k/m}}\right| = 0.98\text{ m}$.

(b) Since $x(0) = 0$, Eq.(13.14) requires $\phi = \pm\dfrac{\pi}{2}$. Since the block is initially moving to the left, $v_{0x} < 0$ and Eq.(13.7) requires that sin $\phi > 0$, so $\phi = +\dfrac{\pi}{2}$.

(c) $\cos(\omega t + (\pi/2)) = -\sin\omega t$, so $x = (-0.98\text{ m})\sin((12.2\text{ rad/s})t)$.

EVALUATE: The $x(t)$ result in part (c) does give $x = 0$ at $t = 0$ and $x < 0$ for t slightly greater than zero.

13.12. **IDENTIFY** and **SET UP:** We are given k, m, x_0, and v_0. Use Eqs.(13.19), (13.18), and (13.13).

EXECUTE: **(a)** Eq.(13.19): $A = \sqrt{x_0^2 + v_{0x}^2/\omega^2} = \sqrt{x_0^2 + mv_{0x}^2/k}$

$A = \sqrt{(0.200\text{ m})^2 + (2.00\text{ kg})(-4.00\text{ m/s})^2/(300\text{ N/m})} = 0.383\text{ m}$

(b) Eq.(13.18): $\phi = \arctan(-v_{0x}/\omega x_0)$

$\omega = \sqrt{k/m} = \sqrt{(300\ \text{N/m})/2.00\ \text{kg}} = 12.25\ \text{rad/s}$

$$\phi = \arctan\left(-\frac{(-4.00\ \text{m/s})}{(12.25\ \text{rad/s})(0.200\ \text{m})}\right) = \arctan(+1.633) = 58.5° \text{ (or 1.02 rad)}$$

(c) $x = A\cos(\omega t + \phi)$ gives $x = (0.383\ \text{m})\cos([12.2\text{rad/s}]t + 1.02\ \text{rad})$

EVALUATE: At $t = 0$ the block is displaced 0.200 m from equilibrium but is moving, so $A > 0.200$ m. According to Eq.(13.15), a phase angle ϕ in the range $0 < \phi < 90°$ gives $v_{0x} < 0$.

13.13. **IDENTIFY:** For SHM, $a_x = -\omega^2 x = -(2\pi f)^2 x$. Apply Eqs.(13.13), (13.15) and (13.16), with A and ϕ from Eqs.(13.18) and (13.19).

SET UP: $x = 1.1$ cm, $v_{0x} = -15$ cm/s. $\omega = 2\pi f$, with $f = 2.5$ Hz.

EXECUTE: **(a)** $a_x = -\left(2\pi(2.5\ \text{Hz})\right)^2 (1.1\times 10^{-2}\ \text{m}) = -2.71\ \text{m/s}^2$.

(b) From Eq. (13.19) the amplitude is 1.46 cm, and from Eq. (13.18) the phase angle is 0.715 rad. The angular frequency is $2\pi f = 15.7$ rad/s, so $x = (1.46\ \text{cm})\cos((15.7\ \text{rad/s})t + 0.715\ \text{rad})$,

$v_x = (-22.9\ \text{cm/s})\sin((15.7\ \text{rad/s})t + 0.715\ \text{rad})$ and $a_x = (-359\ \text{cm/s}^2)\cos((15.7\ \text{rad/s})t + 0.715\ \text{rad})$.

EVALUATE: We can verify that our equations for x, v_x and a_x give the specified values at $t = 0$.

13.14. **IDENTIFY** and **SET UP:** Calculate x using Eq.(13.13). Use T to calculate ω and x_0 to calculate ϕ.

EXECUTE: $x = 0$ at $t = 0$ implies that $\phi = \pm\pi/2$ rad

Thus $x = A\cos(\omega t \pm \pi/2)$.

$T = 2\pi/\omega$ so $\omega = 2\pi/T = 2\pi/1.20\ \text{s} = 5.236\ \text{rad/s}$

$x = (0.600\ \text{m})\cos([5.236\ \text{rad/s}][0.480\ \text{s}] \pm \pi/2) = \mp 0.353\ \text{m}$.

The distance of the object from the equilibrium position is 0.353 m.

EVALUATE: The problem doesn't specify whether the object is moving in the $+x$ or $-x$ direction at $t = 0$.

13.15. **IDENTIFY:** Apply $T = 2\pi\sqrt{\frac{m}{k}}$. Use the information about the empty chair to calculate k.

SET UP: When $m = 42.5$ kg, $T = 1.30$ s.

EXECUTE: Empty chair: $T = 2\pi\sqrt{\frac{m}{k}}$ gives $k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(42.5\ \text{kg})}{(1.30\ \text{s})^2} = 993\ \text{N/m}$

With person in chair: $T = 2\pi\sqrt{\frac{m}{k}}$ gives $m = \frac{T^2 k}{4\pi^2} = \frac{(2.54\ \text{s})^2(993\ \text{N/m})}{4\pi^2} = 162$ kg and

$m_{\text{person}} = 162\ \text{kg} - 42.5\ \text{kg} = 120\ \text{kg}$.

EVALUATE: For the same spring, when the mass increases, the period increases.

13.16. **IDENTIFY** and **SET UP:** Use Eq.(13.12) for T and Eq.(13.4) to relate a_x and k.

EXECUTE: $T = 2\pi\sqrt{m/k}$, $m = 0.400$ kg

Use $a_x = -2.70\ \text{m/s}^2$ to calculate k: $-kx = ma_x$ gives $k = -\frac{ma_x}{x} = -\frac{(0.400\ \text{kg})(-2.70\ \text{m/s}^2)}{0.300\ \text{m}} = +3.60\ \text{N/m}$

$T = 2\pi\sqrt{m/k} = 2.09$ s

EVALUATE: a_x is negative when x is positive. ma_x/x has units of N/m and $\sqrt{m/k}$ has units of s.

13.17. **IDENTIFY:** $T = 2\pi\sqrt{\frac{m}{k}}$. $a_x = -\frac{k}{m}x$ so $a_{\max} = \frac{k}{m}A$. $F = -kx$.

SET UP: a_x is proportional to x so a_x goes through one cycle when the displacement goes through one cycle. From the graph, one cycle of a_x extends from $t = 0.10$ s to $t = 0.30$ s, so the period is $T = 0.20$ s. $k = 2.50\ \text{N/cm} = 250\ \text{N/m}$. From the graph the maximum acceleration is $12.0\ \text{m/s}^2$.

EXECUTE: **(a)** $T = 2\pi\sqrt{\frac{m}{k}}$ gives $m = k\left(\frac{T}{2\pi}\right)^2 = (250\ \text{N/m})\left(\frac{0.20\ \text{s}}{2\pi}\right)^2 = 0.253\ \text{kg}$

(b) $A = \frac{ma_{\max}}{k} = \frac{(0.253\ \text{kg})(12.0\ \text{m/s}^2)}{250\ \text{N/m}} = 0.0121\ \text{m} = 1.21\ \text{cm}$

(c) $F_{\max} = kA = (250\ \text{N/m})(0.0121\ \text{m}) = 3.03\ \text{N}$

EVALUATE: We can also calculate the maximum force from the maximum acceleration: $F_{\max} = ma_{\max} = (0.253\text{ kg})(12.0\text{ m/s}^2) = 3.04\text{ N}$, which agrees with our previous results.

13.18. IDENTIFY: The general expression for $v_x(t)$ is $v_x(t) = -\omega A\sin(\omega t + \phi)$. We can determine ω and A by comparing the equation in the problem to the general form.

SET UP: $\omega = 4.71\text{ rad/s}$. $\omega A = 3.60\text{ cm/s} = 0.0360\text{ m/s}$.

EXECUTE: **(a)** $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi\text{ rad}}{4.71\text{ rad/s}} = 1.33\text{ s}$

(b) $A = \dfrac{0.0360\text{ m/s}}{\omega} = \dfrac{0.0360\text{ m/s}}{4.71\text{ rad/s}} = 7.64\times10^{-3}\text{ m} = 7.64\text{ mm}$

(c) $a_{\max} = \omega^2 A = (4.71\text{ rad/s})^2(7.64\times10^{-3}\text{ m}) = 0.169\text{ m/s}^2$

(d) $\omega = \sqrt{\dfrac{k}{m}}$ so $k = m\omega^2 = (0.500\text{ kg})(4.71\text{ rad/s})^2 = 11.1\text{ N/m}$.

EVALUATE: The overall positive sign in the expression for $v_x(t)$ and the factor of $-\pi/2$ both are related to the phase factor ϕ in the general expression.

13.19. IDENTIFY: Compare the specific $x(t)$ given in the problem to the general form of Eq.(13.13).

SET UP: $A = 7.40\text{ cm}$, $\omega = 4.16\text{ rad/s}$, and $\phi = -2.42\text{ rad}$.

EXECUTE: **(a)** $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4.16\text{ rad/s}} = 1.51\text{ s}$.

(b) $\omega = \sqrt{\dfrac{k}{m}}$ so $k = m\omega^2 = (1.50\text{ kg})(4.16\text{ rad/s})^2 = 26.0\text{ N/m}$

(c) $v_{\max} = \omega A = (4.16\text{ rad/s})(7.40\text{ cm}) = 30.8\text{ cm/s}$

(d) $F_x = -kx$ so $F = kA = (26.0\text{ N/m})(0.0740\text{ m}) = 1.92\text{ N}$.

(e) $x(t)$ evaluated at $t = 1.00\text{ s}$ gives $x = -0.0125\text{ m}$. $v_x = -\omega A\sin(\omega t + \phi) = 30.4\text{ cm/s}$.

$a_x = -kx/m = -\omega^2 x = +0.216\text{ m/s}^2$.

EVALUATE: The maximum speed occurs when $x = 0$ and the maximum force is when $x = \pm A$.

13.20. IDENTIFY: Apply $x(t) = A\cos(\omega t + \phi)$

SET UP: $x = A$ at $t = 0$, so $\phi = 0$. $A = 6.00\text{ cm}$. $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.300\text{ s}} = 20.9\text{ rad/s}$, so $x(t) = (6.00\text{ cm})\cos([20.9\text{ rad/s}]t)$.

EXECUTE: $t = 0$ at $x = 6.00\text{ cm}$. $x = -1.50\text{ cm}$ when $-1.50\text{ cm} = (6.00\text{ cm})\cos([20.9\text{ rad/s}]t)$.

$t = \left(\dfrac{1}{20.9\text{ rad/s}}\right)\arccos\left(-\dfrac{1.50\text{ cm}}{6.00\text{ cm}}\right) = 0.0872\text{ s}$. It takes 0.0872 s.

EVALUATE: It takes $t = T/4 = 0.075\text{ s}$ to go from $x = 6.00\text{ cm}$ to $x = 0$ and 0.150 s to go from $x = +6.00\text{ cm}$ to $x = -6.00\text{ cm}$. Our result is between these values, as it should be.

13.21. IDENTIFY: $v_{\max} = \omega A = 2\pi fA$. $K_{\max} = \tfrac{1}{2}mv_{\max}^2$

SET UP: The fly has the same speed as the tip of the tuning fork.

EXECUTE: **(a)** $v_{\max} = 2\pi fA = 2\pi(392\text{ Hz})(0.600\times10^{-3}\text{ m}) = 1.48\text{ m/s}$

(b) $K_{\max} = \tfrac{1}{2}mv_{\max}^2 = \tfrac{1}{2}(0.0270\times10^{-3}\text{ kg})(1.48\text{ m/s})^2 = 2.96\times10^{-5}\text{ J}$

EVALUATE: $v_{\max}$ is directly proportional to the frequency and to the amplitude of the motion.

13.22. IDENTIFY and SET UP: Use Eq.(13.21) to relate K and U. U depends on x and K depends on v_x.

EXECUTE: **(a)** $U + K = E$, so $U = K$ says that $2U = E$

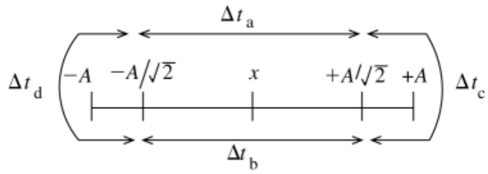
$2\left(\tfrac{1}{2}kx^2\right) = \tfrac{1}{2}kA^2$ and $x = \pm A/\sqrt{2}$; magnitude is $A/\sqrt{2}$

But $U = K$ also implies that $2K = E$

$2(\tfrac{1}{2}mv_x^2) = \tfrac{1}{2}kA^2$ and $v_x = \pm\sqrt{k/m}\,A/\sqrt{2} = \pm\omega A/\sqrt{2}$; magnitude is $\omega A/\sqrt{2}$.

(b) In one cycle x goes from A to 0 to $-A$ to 0 to $+A$. Thus $x = +A\sqrt{2}$ twice and $x = -A/\sqrt{2}$ twice in each cycle. Therefore, $U = K$ four times each cycle. The time between $U = K$ occurrences is the time Δt_a for $x_1 = +A/\sqrt{2}$ to

$x_2 = -A\sqrt{2}$, time Δt_b for $x_1 = -A/\sqrt{2}$ to $x_2 = +A/\sqrt{2}$, time Δt_c for $x_1 = +A/\sqrt{2}$ to $x_2 = +A\sqrt{2}$, or the time Δt_d for $x_1 = -A/\sqrt{2}$ to $x_2 = -A/\sqrt{2}$, as shown in Figure 13.22.



$\Delta t_a = \Delta t_b$
$\Delta t_c = \Delta t_d$

Figure 13.22

Calculation of Δt_a:

Specify *x* in $x = A\cos\omega t$ (choose $\phi = 0$ so $x = A$ at $t = 0$) and solve for *t*.

$x_1 = +A/\sqrt{2}$ implies $A/\sqrt{2} = A\cos(\omega t_1)$

$\cos\omega t_1 = 1/\sqrt{2}$ so $\omega t_1 = \arccos(1/\sqrt{2}) = \pi/4$ rad

$t_1 = \pi/4\omega$

$x_2 = -A/\sqrt{2}$ implies $-A/\sqrt{2} = A\cos(\omega t_2)$

$\cos\omega t_2 = -1/\sqrt{2}$ so $\omega t_1 = 3\pi/4$ rad

$t_2 = 3\pi/4\omega$

$\Delta t_a = t_2 - t_1 = 3\pi/4\omega - \pi/4\omega = \pi/2\omega$ (Note that this is $T/4$, one fourth period.)

Calculation of Δt_d:

$x_1 = -A/\sqrt{2}$ implies $t_1 = 3\pi/4\omega$

$x_2 = -A/\sqrt{2}$, t_2 is the *next* time after t_1 that gives $\cos\omega t_2 = -1/\sqrt{2}$

Thus $\omega t_2 = \omega t_1 + \pi/2 = 5\pi/4$ and $t_2 = 5\pi/4\omega$

$\Delta t_d = t_2 - t_1 = 5\pi/4\omega - 3\pi/4\omega = \pi/2\omega$, so is the same as Δt_a.

Therfore the occurrences of $K = U$ are equally spaced in time, with a time interval between them of $\pi/2\omega$.

EVALUATE: This is one-fourth *T*, as it must be if there are 4 equally spaced occurrences each period.

(c) EXECUTE: $x = A/2$ and $U + K = E$

$K = E - U = \frac{1}{2}kA^2 - \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}k(A/2)^2 = \frac{1}{2}kA^2 - \frac{1}{8}kA^2 = 3kA^2/8$

Then $\dfrac{K}{E} = \dfrac{3kA^2/8}{\frac{1}{2}kA^2} = \dfrac{3}{4}$ and $\dfrac{U}{E} = \dfrac{\frac{1}{8}kA^2}{\frac{1}{2}kA^2} = \dfrac{1}{4}$

EVALUATE: At $x = 0$ all the energy is kinetic and at $x = \pm A$ all the energy is potiential. But $K = U$ does not occur at $x = \pm A/2$, since *U* is not linear in *x*.

13.23. IDENTIFY: Velocity and position are related by $E = \frac{1}{2}kA^2 = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$. Acceleration and position are related by $-kx = ma_x$.

SET UP: The maximum speed is at $x = 0$ and the maximum magnitude of acceleration is at $x = \pm A$,

EXECUTE: **(a)** For $x = 0$, $\frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$ and $v_{max} = A\sqrt{\dfrac{k}{m}} = (0.040\text{ m})\sqrt{\dfrac{450\text{ N/m}}{0.500\text{ kg}}} = 1.20\text{ m/s}$

(b) $v_x = \pm\sqrt{\dfrac{k}{m}}\sqrt{A^2 - x^2} = \pm\sqrt{\dfrac{450\text{ N/m}}{0.500\text{ kg}}}\sqrt{(0.040\text{ m})^2 - (0.015\text{ m})^2} = \pm 1.11\text{ m/s}$.

The speed is $v = 1.11\text{ m/s}$.

(c) For $x = \pm A$, $a_{max} = \dfrac{k}{m}A = \left(\dfrac{450\text{ N/m}}{0.500\text{ kg}}\right)(0.040\text{ m}) = 36\text{ m/s}^2$

(d) $a_x = -\dfrac{kx}{m} = -\dfrac{(450\text{ N/m})(-0.015\text{ m})}{0.500\text{ kg}} = +13.5\text{ m/s}^2$

(e) $E = \frac{1}{2}kA^2 = \frac{1}{2}(450\text{ N/m})(0.040\text{ m})^2 = 0.360\text{ J}$

EVALUATE: The speed and acceleration at $x = -0.015$ m are less than their maximum values.

13.24. IDENTIFY and SET UP: a_x is related to *x* by Eq.(13.4) and v_x is related to *x* by Eq.(13.21). a_x is a maximum when $x = \pm A$ and v_x is a maximum when $x = 0$. *t* is related to *x* by Eq.(13.13).

EXECUTE: **(a)** $-kx = ma_x$ so $a_x = -(k/m)x$ (Eq.13.4). But the maximum $|x|$ is A, so $a_{max} = (k/m)A = \omega^2 A$.

$f = 0.850$ Hz implies $\omega = \sqrt{k/m} = 2\pi f = 2\pi(0.850 \text{ Hz}) = 5.34$ rad/s.

$a_{max} = \omega^2 A = (5.34 \text{ rad/s})^2(0.180 \text{ m}) = 5.13 \text{ m/s}^2$.

$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$v_x = v_{max}$ when $x = 0$ so $\frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$

$v_{max} = \sqrt{k/m}A = \omega A = (5.34 \text{ rad/s})(0.180 \text{ m}) = 0.961$ m/s

(b) $a_x = -(k/m)x = -\omega^2 x = -(5.34 \text{ rad/s})^2(0.090 \text{ m}) = -2.57 \text{ m/s}^2$

$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ says that $v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2} = \pm\omega\sqrt{A^2 - x^2}$

$v_x = \pm(5.34 \text{ rad/s})\sqrt{(0.180 \text{ m})^2 - (0.090 \text{ m})^2} = \pm 0.832$ m/s

The speed is 0.832 m/s.

(c) $x = A\cos(\omega t + \phi)$

Let $\phi = -\pi/2$ so that $x = 0$ at $t = 0$.

Then $x = A\cos(\omega t - \pi/2) = A\sin(\omega t)$ [Using the trig identity $\cos(a - \pi/2) = \sin a$]

Find the time t that gives $x = 0.120$ m.

$0.120 \text{ m} = (0.180 \text{ m})\sin(\omega t)$

$\sin\omega t = 0.6667$

$t = \arcsin(0.6667)/\omega = 0.7297 \text{ rad}/(5.34 \text{ rad/s}) = 0.137$ s

EVALUATE: It takes one-fourth of a period for the object to go from $x = 0$ to $x = A = 0.180$ m. So the time we have calculated should be less than $T/4$. $T = 1/f = 1/0.850 \text{ Hz} = 1.18$ s, $T/4 = 0.295$ s, and the time we calculated is less than this. Note that the a_x and v_x we calculated in part (b) are smaller in magnitude than the maximum values we calculated in part (b).

(d) The conservation of energy equation relates v and x and $F = ma$ relates a and x. So the speed and acceleration can be found by energy methods but the time cannot.

Specifying x uniquely determines a_x but determines only the magnitude of v_x; at a given x the object could be moving either in the $+x$ or $-x$ direction.

13.25. **IDENTIFY:** Use the results of Example 13.15 and also that $E = \frac{1}{2}kA^2$.

SET UP: In the example, $A_2 = A_1\sqrt{\frac{M}{M+m}}$ and now we want $A_2 = \frac{1}{2}A_1$. Therefore, $\frac{1}{2} = \sqrt{\frac{M}{M+m}}$, or $m = 3M$. For the energy, $E_2 = \frac{1}{2}kA_2^2$, but since $A_2 = \frac{1}{2}A_1$, $E_2 = \frac{1}{4}E_1$, and $\frac{3}{4}E_1$ is lost to heat.

EVALUATE: The putty and the moving block undergo a totally inelastic collision and the mechanical energy of the system decreases.

13.26. **IDENTIFY** and **SET UP:** Use Eq.(13.21). $x = \pm A\omega$ when $v_x = 0$ and $v_x = \pm v_{max}$ when $x = 0$.

EXECUTE: **(a)** $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$E = \frac{1}{2}(0.150 \text{ kg})(0.300 \text{ m/s})^2 + \frac{1}{2}(300 \text{ N/m})(0.012 \text{ m})^2 = 0.0284$ J

(b) $E = \frac{1}{2}kA^2$ so $A = \sqrt{2E/k} = \sqrt{2(0.0284 \text{ J})/300 \text{ N/m}} = 0.014$ m

(c) $E = \frac{1}{2}mv_{max}^2$ so $v_{max} = \sqrt{2E/m} = \sqrt{2(0.0284 \text{ J})/0.150 \text{ kg}} = 0.615$ m/s

EVALUATE: The total energy E is constant but is transferred between kinetic and potential energy during the motion.

13.27. **IDENTIFY:** Conservation of energy says $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ and Newton's second law says $-kx = ma_x$.

SET UP: Let $+x$ be to the right. Let the mass of the object be m.

EXECUTE: $k = -\frac{ma_x}{x} = -m\left(\frac{-8.40 \text{ m/s}^2}{0.600 \text{ m}}\right) = (14.0 \text{ s}^{-2})m$.

$A = \sqrt{x^2 + (m/k)v^2} = \sqrt{(0.600 \text{ m})^2 + \left(\frac{m}{[14.0 \text{ s}^{-2}]m}\right)(2.20 \text{ m/s})^2} = 0.840$ m. The object will therefore travel $0.840 \text{ m} - 0.600 \text{ m} = 0.240$ m to the right before stopping at its maximum amplitude.

EVALUATE: The acceleration is not constant and we cannot use the constant acceleration kinematic equations.

13.28. **IDENTIFY:** When the box has its maximum speed all of the energy of the system is in the form of kinetic energy. When the stone is removed the oscillating mass is decreased and the speed of the remaining mass is unchanged. The period is given by $T = 2\pi\sqrt{\frac{m}{k}}$.

SET UP: The maximum speed is $v_{\text{max}} = \omega A = \sqrt{\frac{k}{m}}A$. With the stone in the box $m = 8.64\text{ kg}$ and $A = 0.0750\text{ m}$.

EXECUTE: **(a)** $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{5.20\text{ kg}}{375\text{ N/m}}} = 0.740\text{ s}$

(b) Just before the stone is removed, the speed is $v_{\text{max}} = \sqrt{\frac{375\text{ N/m}}{8.64\text{ kg}}}(0.0750\text{ m}) = 0.494\text{ m/s}$. The speed of the box isn't altered by removing the stone but the mass on the spring decreases to 5.20 kg. The new amplitude is $A = \sqrt{\frac{m}{k}}v_{\text{max}} = \sqrt{\frac{5.20\text{ kg}}{375\text{ N/m}}}(0.494\text{ m/s}) = 0.0582\text{ m}$. The new amplitude can also be calculated as $\sqrt{\frac{5.20\text{ kg}}{8.64\text{ kg}}}(0.0750\text{ m}) = 0.0582\text{ m}$.

(c) $T = 2\pi\sqrt{\frac{m}{k}}$. The force constant remains the same. *m* decreases, so *T* decreases.

EVALUATE: After the stone is removed, the energy left in the system is $\frac{1}{2}m_{\text{box}}v_{\text{max}}^2 = \frac{1}{2}(5.20\text{ kg})(0.494\text{ m/s})^2 = 0.6345\text{ J}$. This then is the energy stored in the spring at its maximum extension or compression and $\frac{1}{2}kA^2 = 0.6345\text{ J}$. This gives the new amplitude to be 0.0582 m, in agreement with our previous calculation.

13.29. **IDENTIFY:** Work in an inertial frame moving with the vehicle after the engines have shut off. The acceleration before engine shut-off determines the amount the spring is initially stretched. The initial speed of the ball relative to the vehicle is zero.

SET UP: Before the engine shut-off the ball has acceleration $a = 5.00\text{ m/s}^2$.

EXECUTE: **(a)** $F_x = -kx = ma_x$ gives $A = \frac{ma}{k} = \frac{(3.50\text{ kg})(5.00\text{ m/s}^2)}{225\text{ N/m}} = 0.0778\text{ m}$. This is the amplitude of the subsequent motion.

(b) $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{225\text{ N/m}}{3.50\text{ kg}}} = 1.28\text{ Hz}$

(c) Energy conservation gives $\frac{1}{2}kA^2 = \frac{1}{2}mv_{\text{max}}^2$ and $v_{\text{max}} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{225\text{ N/m}}{3.50\text{ kg}}}(0.0778\text{ m}) = 0.624\text{ m/s}$.

EVALUATE: During the simple harmonic motion of the ball its maximum acceleration, when $x = \pm A$, continues to have magnitude 5.00 m/s^2.

13.30. **IDENTIFY:** Use the amount the spring is stretched by the weight of the fish to calculate the force constant *k* of the spring. $T = 2\pi\sqrt{m/k}$. $v_{\text{max}} = \omega A = 2\pi fA.$

SET UP: When the fish hangs at rest the upward spring force $|F_x| = kx$ equals the weight *mg* of the fish. $f = 1/T$. The amplitude of the SHM is 0.0500 m.

EXECUTE: **(a)** $mg = kx$ so $k = \frac{mg}{x} = \frac{(65.0\text{ kg})(9.80\text{ m/s}^2)}{0.120\text{ m}} = 5.31\times10^3\text{ N/m}$.

(b) $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{65.0\text{ kg}}{5.31\times10^3\text{ N/m}}} = 0.695\text{ s}$.

(c) $v_{\text{max}} = 2\pi fA = \frac{2\pi A}{T} = \frac{2\pi(0.0500\text{ m})}{0.695\text{ s}} = 0.452\text{ m/s}$

EVALUATE: Note that *T* depends only on *m* and *k* and is independent of the distance the fish is pulled down. But v_{max} does depend on this distance.

13.31. **IDENTIFY:** Initially part of the energy is kinetic energy and part is potential energy in the stretched spring. When $x = \pm A$ all the energy is potential energy and when the glider has its maximum speed all the energy is kinetic energy. The total energy of the system remains constant during the motion.

SET UP: Initially $v_x = \pm 0.815\text{ m/s}$ and $x = \pm 0.0300\text{ m}$.

EXECUTE: **(a)** Initially the energy of the system is

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}(0.175\text{ kg})(0.815\text{ m/s})^2 + \frac{1}{2}(155\text{ N/m})(0.0300\text{ m})^2 = 0.128\text{ J}$. $\frac{1}{2}kA^2 = E$ and

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.128\text{ J})}{155\text{ N/m}}} = 0.0406\text{ m} = 4.06\text{ cm}.$$

(b) $\frac{1}{2}mv_{max}^2 = E$ and $v_{max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.128\text{ J})}{0.175\text{ kg}}} = 1.21\text{ m/s}$.

(c) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{155\text{ N/m}}{0.175\text{ kg}}} = 29.8\text{ rad/s}$

EVALUATE: The amplitude and the maximum speed depend on the total energy of the system but the angular frequency is independent of the amount of energy in the system and just depends on the force constant of the spring and the mass of the object.

13.32. **IDENTIFY:** $K = \frac{1}{2}mv^2$, $U_{grav} = mgy$ and $U_{el} = \frac{1}{2}kx^2$.

SET UP: At the lowest point of the motion, the spring is stretched an amount $2A$.

EXECUTE: **(a)** At the top of the motion, the spring is unstretched and so has no potential energy, the cat is not moving and so has no kinetic energy, and the gravitational potential energy relative to the bottom is $2mgA = 2(4.00\text{ kg})(9.80\text{ m/s}^2)(0.050\text{ m}) = 3.92\text{ J}$. This is the total energy, and is the same total for each part.

(b) $U_{grav} = 0, K = 0,$ so $U_{spring} = 3.92\text{ J}$.

(c) At equilibrium the spring is stretched half as much as it was for part (a), and so $U_{spring} = \frac{1}{4}(3.92\text{ J}) = 0.98\text{ J}$,

$U_{grav} = \frac{1}{2}(3.92\text{ J}) = 1.96\text{ J},$ and so $K = 0.98\text{ J}.$

EVALUATE: During the motion, work done by the forces transfers energy among the forms kinetic energy, gravitational potential energy and elastic potential energy.

13.33. **IDENTIFY:** The location of the equilibrium position, the position where the downward gravity force is balanced by the upward spring force, changes when the mass of the suspended object changes.

SET UP: At the equilibrium position, the spring is stretched a distance *d*. The amplitude is the maximum distance of the object from the equilibrium position.

EXECUTE: **(a)** The force of the glue on the lower ball is the upward force that accelerates that ball upward. The upward acceleration of the two balls is greatest when they have the greatest downward displacement, so this is when the force of the glue must be greatest.

(b) With both balls, the distance d_1 that the spring is stretched at equilibrium is given by $kd_1 = (1.50\text{ kg} + 2.00\text{ kg})g$ and $d_1 = 20.8\text{ cm}$. At the lowest point the spring is stretched $20.8\text{ cm} + 15.0\text{ cm} = 35.8\text{ cm}$. After the 1.50 kg ball falls off the distance d_2 that the spring is stretched at equilibrium is given by $kd_2 = (2.00\text{ kg})g$ and $d_2 = 11.9\text{ cm}$.

The new amplitude is $35.8\text{ cm} - 11.9\text{ cm} = 23.9\text{ cm}$. The new frequency is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{165\text{ N/m}}{2.00\text{ kg}}} = 1.45\text{ Hz}$.

EVALUATE: The potential energy stored in the spring doesn't change when the lower ball comes loose.

13.34. **IDENTIFY:** The torsion constant κ is defined by $\tau_z = -\kappa\theta$. $f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$ and $T = 1/f$. $\theta(t) = \Theta\cos(\omega t + \phi)$.

SET UP: For the disk, $I = \frac{1}{2}MR^2$. $\tau_z = -FR$. At $t = 0$, $\theta = \Theta = 3.34° = 0.0583\text{ rad}$, so $\phi = 0$.

EXECUTE: **(a)** $\kappa = -\frac{\tau_z}{\theta} = -\frac{-FR}{0.0583\text{ rad}} = +\frac{(4.23\text{ N})(0.120\text{ m})}{0.0583\text{ rad}} = 8.71\text{ N}\cdot\text{m/rad}$

(b) $f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}} = \frac{1}{2\pi}\sqrt{\frac{2\kappa}{MR^2}} = \frac{1}{2\pi}\sqrt{\frac{2(8.71\text{ N}\cdot\text{m/rad})}{(6.50\text{ kg})(0.120\text{ m})^2}} = 2.71\text{ Hz}$. $T = 1/f = 0.461\text{ s}$.

(c) $\omega = 2\pi f = 13.6\text{ rad/s}$. $\theta(t) = (3.34°)\cos([13.6\text{ rad/s}]t)$.

EVALUATE: The frequency and period are independent of the initial angular displacement, so long as this displacement is small.

13.35. **IDENTIFY** and **SET UP:** The number of ticks per second tells us the period and therefore the frequency. We can use a formula form Table 9.2 to calculate *I*. Then Eq.(13.24) allows us to calculate the torsion constant κ.

EXECUTE: Ticks four times each second implies 0.25 s per tick. Each tick is half a period, so $T = 0.50\text{ s}$ and $f = 1/T = 1/0.50\text{ s} = 2.00\text{ Hz}$

(a) Thin rim implies $I = MR^2$ (from Table 9.2). $I = (0.900\times10^{-3}\text{ kg})(0.55\times10^{-2}\text{ m})^2 = 2.7\times10^{-8}\text{ kg}\cdot\text{m}^2$

(b) $T = 2\pi\sqrt{I/\kappa}$ so $\kappa = I(2\pi/T)^2 = (2.7\times10^{-8}\text{ kg}\cdot\text{m}^2)(2\pi/0.50\text{ s})^2 = 4.3\times10^{-6}\text{ N}\cdot\text{m/rad}$

EVALUATE: Both *I* and κ are small numbers.

13.36. IDENTIFY: Eq.(13.24) and $T = 1/f$ says $T = 2\pi\sqrt{\frac{I}{\kappa}}$.

SET UP: $I = \frac{1}{2}mR^2$.

EXECUTE: Solving Eq. (13.24) for κ in terms of the period,

$$\kappa = \left(\frac{2\pi}{T}\right)^2 I = \left(\frac{2\pi}{1.00\text{ s}}\right)^2 ((1/2)(2.00\times10^{-3}\text{ kg})(2.20\times10^{-2}\text{ m})^2) = 1.91\times10^{-5}\text{ N}\cdot\text{m/rad}.$$

EVALUATE: The longer the period, the smaller the torsion constant.

13.37. IDENTIFY: $f = \frac{1}{2\pi}\sqrt{\frac{\kappa}{I}}$.

SET UP: $f = 125/(265\text{ s})$, the number of oscillations per second.

EXECUTE: $I = \frac{\kappa}{(2\pi f)^2} = \frac{0.450\text{ N}\cdot\text{m/rad}}{(2\pi(125)/(265\text{ s}))^2} = 0.0152\text{ kg}\cdot\text{m}^2.$

EVALUATE: For a larger I, f is smaller.

13.38. IDENTIFY: $\theta(t)$ is given by $\theta(t) = \Theta\cos(\omega t + \phi)$. Evaluate the derivatives specified in the problem.

SET UP: $d(\cos\omega t)/dt = -\omega\sin\omega t$. $d(\sin\omega t)/dt = \omega\cos\omega t$. $\sin^2\theta + \cos^2\theta = 1$

In this problem, $\varphi = 0$.

EXECUTE: **(a)** $\omega = \frac{d\theta}{dt} = -\omega\,\Theta\sin(\omega\,t)$ and $\frac{d^2\theta}{dt^2} = -\omega^2\,\Theta\cos(\omega\,t)$.

(b) When the angular displacement is Θ, $\Theta = \Theta\cos(\omega t)$. This occurs at $t = 0$, so $\omega = 0$. $\alpha = -\omega^2\Theta$. When the angular displacement is $\Theta/2$, $\frac{\Theta}{2} = \Theta\cos(\omega t)$, or $\frac{1}{2} = \cos(\omega t)$. $\omega = \frac{-\omega\,\Theta\sqrt{3}}{2}$ since $\sin(\omega t) = \frac{\sqrt{3}}{2}$. $\alpha = \frac{-\omega^2\Theta}{2}$, since $\cos(\omega t) = 1/2$.

EVALUATE: $\cos(\omega t) = \frac{1}{2}$ when $\omega t = \pi/3\text{ rad} = 60°$. At this t, $\cos(\omega t)$ is decreasing and θ is decreasing, as required. There are other, larger values of ωt for which $\theta = \Theta/2$, but θ is increasing.

13.39. IDENTIFY and SET UP: Follow the procedure outlined in the problem.

EXECUTE: Eq.(13.25): $U = U_0[(R_0/r)^{12} - 2(R_0/r)^6]$. Let $r = R_0 + x$.

$$U = U_0\left[\left(\frac{R_0}{R_0+x}\right)^{12} - 2\left(\frac{R_0}{R_0+x}\right)^6\right] = U_0\left[\left(\frac{1}{1+x/R_0}\right)^{12} - 2\left(\frac{1}{1+x/R_0}\right)^6\right]$$

$$\left(\frac{1}{1+x/R_0}\right)^{12} = (1+x/R_0)^{-12};\ \ |x/R_0| << 1$$

Apply Eq.(13.28) with $n = -12$ and $u = +x/R_0$:

$$\left(\frac{1}{1+x/R_0}\right)^{12} = 1 - 12x/R_0 + 66x^2/R_0^2 - \ldots$$

For $\left(\frac{1}{1+x/R_0}\right)^6$ apply Eq.(13.28) with $n = -6$ and $u = +x/R_0$:

$$\left(\frac{1}{1+x/R_0}\right)^6 = 1 - 6x/R_0 + 15x^2/R_0^2 - \ldots$$

Thus $U = U_0(1 - 12x/R_0 + 66x^2/R_0^2 - 2 + 12x/R_0 - 30x^2/R_0^2) = -U_0 + 36U_0x^2/R_0^2$. This is in the form $U = \frac{1}{2}kx^2 - U_0$ with $k = 72U_0/R_0^2$, which is the same as the force constant in Eq.(13.29).

EVALUATE: $F_x = -dU/dx$ so $U(x)$ contains an additive constant that can be set to any value we wish. If $U_0 = 0$ then $U = 0$ when $x = 0$.

13.40. IDENTIFY: Example 13.7 tells us that $f = \frac{1}{2\pi}\sqrt{\frac{k}{(m/2)}}$.

SET UP: $1\text{ u} = 1.66\times10^{-27}\text{ kg}$

EXECUTE: $f = \frac{1}{2\pi}\sqrt{\frac{k}{(m/2)}} = \frac{1}{2\pi}\sqrt{\frac{2(580\text{ N/m})}{(1.008)(1.66\times10^{-27}\text{ kg})}} = 1.33\times10^{14}\text{ Hz}.$

EVALUATE: This frequency is much larger than f calculated in Example 13.7. Here m is smaller by a factor of 1/40 but k is smaller by a factor of 1/700.

13.41. IDENTIFY: $T = 2\pi\sqrt{L/g}$ is the time for one complete swing.

SET UP: The motion from the maximum displacement on either side of the vertical to the vertical position is one-fourth of a complete swing.

EXECUTE: **(a)** To the given precision, the small-angle approximation is valid. The highest speed is at the bottom of the arc, which occurs after a quarter period, $\frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{L}{g}} = 0.25\text{ s}$.

(b) The same as calculated in (a), 0.25 s. The period is independent of amplitude.

EVALUATE: For small amplitudes of swing, the period depends on L and g.

13.42. IDENTIFY: Since the rope is long compared to the height of a person, the system can be modeled as a simple pendulum. Since the amplitude is small, the period of the motion is $T = 2\pi\sqrt{\frac{L}{g}}$.

SET UP: From his initial position to his lowest point is one-fourth of a cycle. He returns to this lowest point in time $T/2$ from when he was previously there.

EXECUTE: **(a)** $T = 2\pi\sqrt{\frac{6.50\text{ m}}{9.80\text{ m/s}^2}} = 5.12\text{ s}$. $t = T/4 = 1.28\text{ s}$.

(b) $t = 3T/4 = 3.84\text{ s}$.

EVALUATE: The period is independent of his mass.

13.43. IDENTIFY: Since the cord is much longer than the height of the object, the system can be modeled as a simple pendulum. We will assume the amplitude of swing is small, so that $T = 2\pi\sqrt{\frac{L}{g}}$.

SET UP: The number of swings per second is the frequency $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$.

EXECUTE: $f = \frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{1.50\text{ m}}} = 0.407\text{ swings per second}$.

EVALUATE: The period and frequency are both independent of the mass of the object.

13.44. IDENTIFY: Use Eq.(13.34) to relate the period to g.

SET UP: Let the period on earth be $T_E = 2\pi\sqrt{L/g_E}$, where $g_E = 9.80\text{ m/s}^2$, the value on earth.

Let the period on Mars be $T_M = 2\pi\sqrt{L/g_M}$, where $g_M = 3.71\text{ m/s}^2$, the value on Mars.

We can eliminate L, which we don't know, by taking a ratio:

EXECUTE: $\frac{T_M}{T_E} = 2\pi\sqrt{\frac{L}{g_M}}\frac{1}{2\pi}\sqrt{\frac{g_E}{L}} = \sqrt{\frac{g_E}{g_M}}$.

$$T_M = T_E\sqrt{\frac{g_E}{g_M}} = (1.60\text{ s})\sqrt{\frac{9.80\text{ m/s}^2}{3.71\text{ m/s}^2}} = 2.60\text{ s}.$$

EVALUATE: Gravity is weaker on Mars so the period of the pendulum is longer there.

13.45. IDENTIFY and **SET UP:** The bounce frequency is given by Eq.(13.11) and the pendulum frequency by Eq.(13.33). Use the relation between these two frequencies that is specified in the problem to calculate the equilibrium length L of the spring, when the apple hangs at rest on the end of the spring.

EXECUTE: vertical SHM: $f_b = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

pendulum motion (small amplitude): $f_p = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

The problem specifies that $f_p = \frac{1}{2}f_b$.

$$\frac{1}{2\pi}\sqrt{\frac{g}{L}} = \frac{1}{2}\frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$g/L = k/4m$ so $L = 4gm/k = 4w/k = 4(1.00\text{ N})/1.50\text{ N/m} = 2.67\text{ m}$

EVALUATE: This is the *stretched* length of the spring, its length when the apple is hanging from it. (Note: Small angle of swing means v is small as the apple passes through the lowest point, so a_{rad} is small and the component of mg perpendicular to the spring is small. Thus the amount the spring is stretched changes very little as the apple swings back and forth.)

IDENTIFY: Use Newton's second law to calculate the distance the spring is stretched form its unstretched length when the apple hangs from it.

SET UP: The free-body diagram for the apple hanging at rest on the end of the spring is given in Figure 13.45.

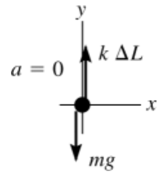


Figure 13.45

EXECUTE: $\sum F_y = ma_y$

$k\Delta L - mg = 0$

$\Delta L = mg/k = w/k = 1.00\ \text{N}/1.50\ \text{N/m} = 0.667\ \text{m}$

Thus the unstretched length of the spring is $2.67\ \text{m} - 0.67\ \text{m} = 2.00\ \text{m}$.

EVALUATE: The spring shortens to its unstretched length when the apple is removed.

13.46. IDENTIFY: $a_{\tan} = L\alpha$, $a_{\text{rad}} = L\omega^2$ and $a = \sqrt{a_{\tan}^2 + a_{\text{rad}}^2}$. Apply conservation of energy to calculate the speed in part (c).

SET UP: Just after the sphere is released, $\omega = 0$ and $a_{\text{rad}} = 0$. When the rod is vertical, $a_{\tan} = 0$.

EXECUTE: **(a)** The forces and acceleration are shown in Figure 13.46a. $a_{\text{rad}} = 0$ and $a = a_{\tan} = g\sin\theta$.

(b) The forces and acceleration are shown in Figure 13.46b.

(c) The forces and acceleration are shown in Figure 13.46c. $U_{\text{i}} = K_{\text{f}}$ gives $mgL(1-\cos\Theta) = \tfrac{1}{2}mv^2$ and $v = \sqrt{2gL(1-\cos\Theta)}$.

EVALUATE: As the rod moves toward the vertical, v increases, a_{rad} increases and $a_{\tan}$ decreases.

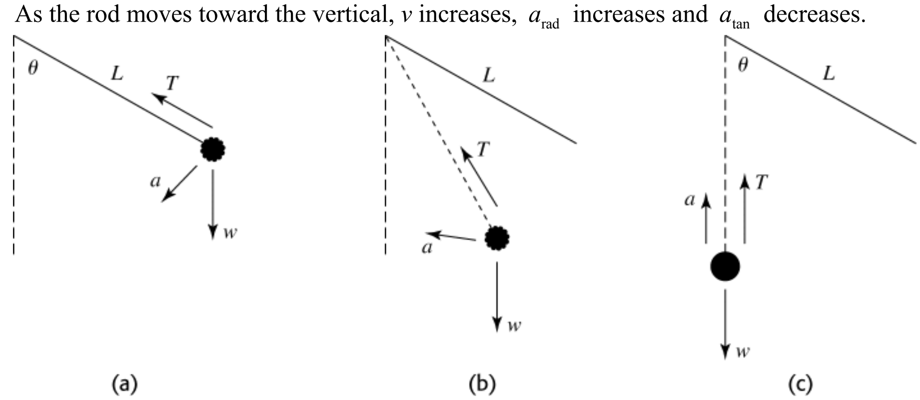


Figure 13.46

13.47. IDENTIFY: Apply $T = 2\pi\sqrt{L/g}$

SET UP: The period of the pendulum is $T = (136\ \text{s})/100 = 1.36\ \text{s}$.

EXECUTE: $g = \dfrac{4\pi^2 L}{T^2} = \dfrac{4\pi^2(0.500\ \text{m})}{(1.36\ \text{s})^2} = 10.7\ \text{m/s}^2$.

EVALUATE: The same pendulum on earth, where g is smaller, would have a larger period.

13.48. IDENTIFY: If a small amplitude is assumed, $T = 2\pi\sqrt{\dfrac{L}{g}}$.

SET UP: The fourth term in Eq.(13.35) would be $\dfrac{1^2\cdot 3^2\cdot 5^2}{2^2\cdot 4^2\cdot 6^2}\sin^6\dfrac{\Theta}{2}$.

EXECUTE: **(a)** $T = 2\pi\sqrt{\dfrac{2.00\ \text{m}}{9.80\ \text{m/s}^2}} = 2.84\ \text{s}$

(b) $T = (2.84\ \text{s})\left(1 + \dfrac{1}{4}\sin^2 15.0° + \dfrac{9}{64}\sin^4 15.0° + \dfrac{225}{2305}\sin^6 15.0°\right) = 2.89\ \text{s}$

(c) Eq.(13.35) is more accurate. Eq.(13.34) is in error by $\dfrac{2.84\ \text{s} - 2.89\ \text{s}}{2.89\ \text{s}} = -2\%$.

EVALUATE: As Figure 13.22 in Section 13.5 shows, the approximation $F_\theta = -mg\theta$ is larger in magnitude than the true value as θ increases. Eq.(13.34) therefore over-estimates the restoring force and this results in a value of T that is smaller than the actual value.

13.49. **IDENTIFY:** $T = 2\pi\sqrt{I/mgd}$.

SET UP: $d = 0.200$ m. $T = (120\text{ s})/100$.

EXECUTE: $I = mgd\left(\frac{T}{2\pi}\right)^2 = (1.80\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})\left(\frac{120\text{ s}/100}{2\pi}\right)^2 = 0.129\text{ kg.m}^2.$

EVALUATE: If the rod were uniform, its center of gravity would be at its geometrical center and it would have length $l = 0.400$ m. For a uniform rod with an axis at one end, $I = \frac{1}{3}ml^2 = 0.096\text{ kg}\cdot\text{m}^2$. The value of *I* for the actual rod is about 34% larger than this value.

13.50. **IDENTIFY:** $T = 2\pi\sqrt{I/mgd}$

SET UP: From the parallel axis theorem, the moment of inertia of the hoop about the nail is $I = MR^2 + MR^2 = 2MR^2$. $d = R$.

EXECUTE: Solving for *R*, $R = gT^2/8\pi^2 = 0.496$ m.

EVALUATE: A simple pendulum of length $L = R$ has period $T = 2\pi\sqrt{R/g}$. The hoop has a period that is larger by a factor of $\sqrt{2}$.

13.51. **IDENTIFY:** For a physical pendulum, $T = 2\pi\sqrt{I/mgd}$ and for a simple pendulum $T = 2\pi\sqrt{L/g}$.

SET UP: For the situation described, $I = mL^2$ and $d = L$.

EXECUTE: $T = 2\pi\sqrt{\frac{mL^2}{mgL}} = 2\pi\sqrt{L/g}$, so the two expressions are the same.

EVALUATE: Eq.(13.39) applies to any pendulum and reduces to Eq.(13.34) when the conditions for the object to be a simple pendulum are satisfied.

13.52. **IDENTIFY:** Apply Eq.(13.39) to calculate *I* and conservation of energy to calculate the maximum angular speed, Ω_{max}.

SET UP: $d = 0.250$ m. In part (b), $y_i = d(1-\cos\Theta)$, with $\Theta = 0.400$ rad and $y_f = 0$.

EXECUTE: **(a)** Solving Eq.(13.39) for *I*,

$$I = \left(\frac{T}{2\pi}\right)^2 mgd = \left(\frac{0.940\text{ s}}{2\pi}\right)^2 (1.80\text{ kg})(9.80\text{ m/s}^2)(0.250\text{ m}) = 0.0987\text{ kg}\cdot\text{m}^2.$$

(b) The small-angle approximation will not give three-figure accuracy for $\Theta = 0.400$ rad. From energy considerations, $mgd(1-\cos\Theta) = \frac{1}{2}I\Omega_{max}^2$. Expressing Ω_{max} in terms of the period of small-angle oscillations, this becomes

$$\Omega_{max} = \sqrt{2\left(\frac{2\pi}{T}\right)^2(1-\cos\Theta)} = \sqrt{2\left(\frac{2\pi}{0.940\text{ s}}\right)^2(1-\cos(0.400\text{ rad}))} = 2.66\text{ rad/s}.$$

EVALUATE: The time for the motion in part (b) is $t = T/4$, so $\Omega_{av} = \Delta\theta/\Delta t = (0.400\text{ rad})/(0.235\text{ s}) = 1.70\text{ rad/s}$. Ω increases during the motion and the final Ω is larger than the average Ω.

13.53. **IDENTIFY:** Pendulum *A* can be treated as a simple pendulum. Pendulum *B* is a physical pendulum.

SET UP: For pendulum *B* the distance *d* from the axis to the center of gravity is $3L/4$. $I = \frac{1}{3}(m/2)L^2$ for a bar of mass *m*/2 and the axis at one end. For a small ball of mass *m*/2 at a distance *L* from the axis, $I_{ball} = (m/2)L^2$.

EXECUTE: Pendulum *A*: $T_A = 2\pi\sqrt{\frac{L}{g}}$.

Pendulum *B*: $I = I_{bar} + I_{ball} = \frac{1}{3}(m/2)L^2 + (m/2)L^2 = \frac{2}{3}mL^2$.

$T_B = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{\frac{2}{3}mL^2}{mg(3L/4)}} = 2\pi\sqrt{\frac{L}{g}}\sqrt{\frac{2}{3}\cdot\frac{4}{3}} = \sqrt{\frac{8}{9}}\left(2\pi\sqrt{\frac{L}{g}}\right) = 0.943T_A$. The period is longer for pendulum *A*.

EVALUATE: Example 13.9 shows that for the bar alone, $T = \sqrt{\frac{2}{3}}T_A = 0.816T_A$. Adding the ball of equal mass to the end of the rod increases the period compared to that for the rod alone.

13.54. **IDENTIFY:** The ornament is a physical pendulum: $T = 2\pi\sqrt{I/mgd}$ (Eq.13.39). T is the target variable.

SET UP: $I = 5MR^2/3,$ the moment of inertia about an axis at the edge of the sphere. d is the distance from the axis to the center of gravity, which is at the center of the sphere, so $d = R.$

EXECUTE: $T = 2\pi\sqrt{5/3}\sqrt{R/g} = 2\pi\sqrt{5/3}\sqrt{0.050\text{ m}/(9.80\text{m/s}^2)} = 0.58\text{ s}.$

EVALUATE: A simple pendulum of length $R = 0.050\text{ m}$ has period 0.45 s; the period of the physical pendulum is longer.

13.55. **IDENTIFY:** Pendulum A can be treated as a simple pendulum. Pendulum B is a physical pendulum. Use the parallel-axis theorem to find the moment of inertia of the ball in B for an axis at the top of the string.

SET UP: For pendulum B the center of gravity is at the center of the ball, so $d = L$. For a solid sphere with an axis through its center, $I_{\text{cm}} = \frac{2}{5}MR^2$. $R = L/2$ and $I_{\text{cm}} = \frac{1}{10}ML^2$.

EXECUTE: Pendulum A: $T_A = 2\pi\sqrt{\dfrac{L}{g}}$.

Pendulum B: The parallel-axis theorem says $I = I_{\text{cm}} + ML^2 = \frac{11}{10}ML^2$.

$T = 2\pi\sqrt{\dfrac{I}{mgd}} = 2\pi\sqrt{\dfrac{11ML^2}{10MgL}} = \sqrt{\dfrac{11}{10}}\left(2\pi\sqrt{\dfrac{L}{g}}\right) = \sqrt{\dfrac{11}{10}}T_A = 1.05T_A$. It takes pendulum B longer to complete a swing.

EVALUATE: The center of the ball is the same distance from the top of the string for both pendulums, but the mass is distributed differently and I is larger for pendulum B, even though the masses are the same.

13.56. **IDENTIFY:** If the system is critically damped or overdamped it doesn't oscillate. With no damping, $\omega = \sqrt{m/k}$.

With underdamping, the angular frequency has the smaller value $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$.

SET UP: $m = 2.20\text{ kg}$, $k = 250.0\text{ N/m}$. $T' = \dfrac{2\pi}{\omega'}$ and $\omega' = \dfrac{2\pi}{T'} = \dfrac{2\pi}{0.615\text{ s}} = 10.22\text{ rad/s}$.

EXECUTE: **(a)** $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{250.0\text{ N/m}}{2.20\text{ kg}}} = 10.66\text{ rad/s}$. $\omega' < \omega$ so the system is damped. $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$ gives

$b = 2m\sqrt{\dfrac{k}{m} - \omega'^2} = 2(2.20\text{ kg})\sqrt{\dfrac{250.0\text{ N/m}}{2.20\text{ kg}} - (10.22\text{ rad/s})^2} = 13.3\text{ kg/s}$.

(b) Since the motion has a period the system oscillates and is underdamped.

EVALUATE: The critical value of the damping constant is $b = 2\sqrt{km} = 2\sqrt{(250.0\text{ N/m})(2.20\text{ kg})} = 46.9\text{ kg/s}$. In this problem b is much less than its critical value.

13.57. **IDENTIFY** and **SET UP:** Use Eq.(13.43) to calculate ω', and then $f' = \omega'/2\pi.$

(a) EXECUTE: $\omega' = \sqrt{(k/m) - (b^2/4m^2)} = \sqrt{\dfrac{2.50\text{ N/m}}{0.300\text{ kg}} - \dfrac{(0.900\text{ kg/s})^2}{4(0.300\text{ kg})^2}} = 2.47\text{ rad/s}$

$f' = \omega'/2\pi = (2.47\text{ rad/s})/2\pi = 0.393\text{ Hz}$

(b) IDENTIFY and **SET UP:** The condition for critical damping is $b = 2\sqrt{km}$ (Eq.13.44)

EXECUTE: $b = 2\sqrt{(2.50\text{ N/m})(0.300\text{ kg})} = 1.73\text{ kg/s}$

EVALUATE: The value of b in part (a) is less than the critical damping value found in part (b). With no damping, the frequency is $f = 0.459\text{ Hz};$ the damping reduces the oscillation frequency.

13.58. **IDENTIFY:** From Eq.(13.42) $A_2 = A_1\ \exp\left(-\dfrac{b}{2m}t\right)$.

SET UP: $\ln(e^{-x}) = -x$

EXECUTE: $b = \dfrac{2m}{t}\ln\left(\dfrac{A_1}{A_2}\right) = \dfrac{2(0.050\text{ kg})}{(5.00\text{ s})}\ln\left(\dfrac{0.300\text{ m}}{0.100\text{ m}}\right) = 0.0220\text{ kg/s}.$

EVALUATE: As a check, note that the oscillation frequency is the same as the undamped frequency to 4.8×10^{-3}%, so Eq. (13.42) is valid.

13.59. **IDENTIFY:** $x(t)$ is given by Eq.(13.42). $v_x = dx/dt$ and $a_x = dv_x/dt$.

SET UP: $d(\cos\omega' t)/dt = -\omega'\sin\omega' t$. $d(\sin\omega' t)/dt = \omega'\cos\omega' t$. $d(e^{-\alpha t})/dt = -\alpha e^{-\alpha t}$.

EXECUTE: **(a)** With $\phi = 0$, $x(0) = A$.

(b) $v_x = \dfrac{dx}{dt} = Ae^{-(b/2m)t}\left[-\dfrac{b}{2m}\cos\omega' t - \omega' \sin\omega' t\right]$, and at $t = 0$, $v_x = -Ab/2m$; the graph of x versus t near $t = 0$ slopes down.

(c) $a_x = \dfrac{dv_x}{dt} = Ae^{-(b/2m)t}\left[\left(\dfrac{b^2}{4m^2} - \omega'^2\right)\cos\omega' t + \dfrac{\omega' b}{2m}\sin\omega' t\right]$, and at $t = 0$, $a_x = A\left(\dfrac{b^2}{4m^2} - \omega'^2\right) = A\left(\dfrac{b^2}{2m^2} - \dfrac{k}{m}\right)$.

(Note that this is $(-bv_0 - kx_0)/m$.) This will be negative if $b < \sqrt{2km}$, zero if $b = \sqrt{2km}$ and positive if $b > \sqrt{2km}$. The graph in the three cases will be curved down, not curved, or curved up, respectively.

EVALUATE: $a_x(0) = 0$ corresponds to the situation of critical damping.

13.60. **IDENTIFY:** Apply Eq.(13.46).

SET UP: $\omega_d = \sqrt{k/m}$ corresponds to resonance, and in this case Eq.(13.46) reduces to $A = F_{max}/b\omega_d$.

EXECUTE: **(a)** $A_1/3$

(b) $2A_1$

EVALUATE: Note that the resonance frequency is independent of the value of b. (See Figure 13.28 in the textbook).

13.61 **IDENTIFY** and **SET UP:** Apply Eq.(13.46): $A = \dfrac{F_{max}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$

EXECUTE: **(a)** Consider the special case where $k - m\omega_d^2 = 0$, so $A = F_{max}/b\omega_d$ and $b = F_{max}/A\omega_d$. Units of $\dfrac{F_{max}}{A\omega_d}$ are $\dfrac{\text{kg}\cdot\text{m/s}^2}{(\text{m})(\text{s}^{-1})} = \text{kg/s}$. For units consistency the units of b must be kg/s.

(b) Units of $\sqrt{km}$: $[(\text{N/m})\text{kg}]^{1/2} = (\text{N kg/m})^{1/2} = [(\text{kg}\cdot\text{m/s}^2)(\text{kg})/\text{m}]^{1/2} = (\text{kg}^2/\text{s}^2)^{1/2} = \text{kg/s}$, the same as the units for b.

(c) For $\omega_d = \sqrt{k/m}$ (at resonance) $A = (F_{max}/b)\sqrt{m/k}$.

(i) $b = 0.2\sqrt{km}$

$$A = F_{max}\sqrt{\frac{m}{k}}\frac{1}{0.2\sqrt{km}} = \frac{F_{max}}{0.2k} = 5.0\frac{F_{max}}{k}.$$

(ii) $b = 0.4\sqrt{km}$

$$A = F_{max}\sqrt{\frac{m}{k}}\frac{1}{0.4\sqrt{km}} = \frac{F_{max}}{0.4k} = 2.5\frac{F_{max}}{k}.$$

EVALUATE: Both these results agree with what is shown in Figure 13.28 in the textbook. As b increases the maximum amplitude decreases.

13.62. **IDENTIFY:** Calculate the resonant frequency and compare to 35 Hz.

SET UP: ω in rad/s is related to f in Hz by $\omega = 2\pi f$.

EXECUTE: The resonant frequency is $\sqrt{k/m} = \sqrt{(2.1\times10^6\ \text{N/m})/108\ \text{kg}} = 139\ \text{rad/s} = 22.2\ \text{Hz}$, and this package does not meet the criterion.

EVALUATE: To make the package meet the requirement, increase the resonant frequency by increasing the force constant k.

13.63. **IDENTIFY:** $ma_x = -kx$ so $a_{max} = \dfrac{k}{m}A = \omega^2 A$ is the magnitude of the acceleration when $x = \pm A$. $v_{max} = \sqrt{\dfrac{k}{m}}A = \omega A$. $P = \dfrac{W}{t} = \dfrac{\Delta K}{t}$.

SET UP: $A = 0.0500$ m. $\omega = 3500\ \text{rpm} = 366.5\ \text{rad/s}$.

EXECUTE: **(a)** $a_{max} = \omega^2 A = (366.5\ \text{rad/s})^2(0.0500\ \text{m}) = 6.72\times10^3\ \text{m/s}^2$

(b) $F_{max} = ma_{max} = (0.450\ \text{kg})(6.72\times10^3\ \text{m/s}^2) = 3.02\times10^3\ \text{N}$

(c) $v_{max} = \omega A = (366.5\ \text{rad/s})(0.0500\ \text{m}) = 18.3\ \text{m/s}$. $K_{max} = \frac{1}{2}mv_{max}^2 = \frac{1}{2}(0.450\ \text{kg})(18.3\ \text{m/s})^2 = 75.4\ \text{J}$

(d) $P = \dfrac{\frac{1}{2}mv^2}{t}$. $t = \dfrac{T}{4} = \dfrac{2\pi}{4\omega} = 4.286\times10^{-3}\ \text{s}$. $P = \dfrac{(0.450\ \text{kg})(18.3\ \text{m/s})^2}{2(4.286\times10^{-3}\ \text{s})} = 1.76\times10^4\ \text{W}$.

(e) a_{max} is proportional to ω^2, so F_{max} increases by a factor of 4, to 1.21×10^4 N. v_{max} is proportional to ω, so v_{max} doubles, to 36.6 m/s, and K_{max} increases by a factor of 4, to 302 J. In part (d). t is halved and K is quadrupled, so P_{max} increases by a factor of 8 and becomes 1.41×10^5 W.

EVALUATE: For a given amplitude, the maximum acceleration and maximum velocity increase when the frequency of the motion increases and the period decreases.

13.64. **IDENTIFY:** $T = 2\pi\sqrt{\frac{m}{k}}$. The period changes when the mass changes.

SET UP: M is the mass of the empty car and the mass of the loaded car is $M + 250\text{ kg}$.

EXECUTE: The period of the empty car is $T_{\text{E}} = 2\pi\sqrt{\frac{M}{k}}$. The period of the loaded car is $T_{\text{L}} = 2\pi\sqrt{\frac{M + 250\text{ kg}}{k}}$.

$$k = \frac{(250\text{ kg})(9.80\text{ m/s}^2)}{4.00\times10^{-2}\text{ m}} = 6.125\times10^4\text{ N/m}$$

$$M = \left(\frac{T_{\text{L}}}{2\pi}\right)^2 k - 250\text{ kg} = \left(\frac{1.08\text{ s}}{2\pi}\right)^2(6.125\times10^4\text{ N/m}) - 250\text{ kg} = 1.56\times10^3\text{ kg} .\ T_{\text{E}} = 2\pi\sqrt{\frac{1.56\times10^3\text{ kg}}{6.125\times10^4\text{ N/m}}} = 1.00\text{ s} .$$

EVALUATE: When the mass decreases, the period decreases.

13.65. **IDENTIFY** and **SET UP:** Use Eqs. (13.12), (13.21) and (13.22) to relate the various quantities to the amplitude.

EXECUTE: **(a)** $T = 2\pi\sqrt{m/k}$; independent of A so period doesn't change

$f = 1/T$; doesn't change

$\omega = 2\pi f$; doesn't change

(b) $E = \frac{1}{2}kA^2$ when $x = \pm A$. When A is halved E decreases by a factor of 4; $E_2 = E_1/4$.

(c) $v_{\text{max}} = \omega A = 2\pi fA$

$v_{\text{max},1} = 2\pi fA_1$, $\quad v_{\text{max},2} = 2\pi fA_2 \quad$ (f doesn't change)

Since $A_2 = \frac{1}{2}A_1, v_{\text{max},2} = 2\pi f(\frac{1}{2}A_1) = \frac{1}{2}2\pi fA_1 = \frac{1}{2}v_{\text{max},1}; v_{\text{max}}$ is one-half as great

(d) $v_x = \pm\sqrt{k/m}\sqrt{A^2 - x^2}$

$x = \pm A_1/4$ gives $v_x = \pm\sqrt{k/m}\sqrt{A^2 - A_1^2/16}$

With the original amplitude $v_{1x} = \pm\sqrt{k/m}\sqrt{A_1^2 - A_1^2/16} = \pm\sqrt{15/16}(\sqrt{k/m})A_1$

With the reduced amplitude $v_{2x} = \pm\sqrt{k/m}\sqrt{A_2^2 - A_1^2/16} = \pm\sqrt{k/m}\sqrt{(A_1/2)^2 - A_1^2/16} = \pm\sqrt{3/16}(\sqrt{k/m})A_1$

$v_{1x}/v_{2x} = \sqrt{15/3} = \sqrt{5}$, so $v_2 = v_1/\sqrt{5}$; the speed at this x is $1/\sqrt{5}$ times as great.

(e) $U = \frac{1}{2}kx^2$; same x so same U.

$K = \frac{1}{2}mv_x^2$; $K_1 = \frac{1}{2}mv_{1x}^2$

$K_2 = \frac{1}{2}mv_{2x}^2 = \frac{1}{2}m(v_{1x}/\sqrt{5})^2 = \frac{1}{5}(\frac{1}{2}mv_{1x}^2) = K_1/5$; 1/5 times as great.

EVALUATE: Reducing A reduces the total energy but doesn't affect the period and the frequency.

13.66. **(a) IDENTIFY** and **SET UP:** Combine Eqs. (13.12) and (13.21) to relate v_x and x to T.

EXECUTE: $T = 2\pi\sqrt{m/k}$

We are given information about v_x at a particular x. The expression relating these two quantities comes from conservation of energy: $\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

We can solve this equation for $\sqrt{m/k}$, and then use that result to calculate T. $mv_x^2 = k(A^2 - x^2)$

$$\sqrt{\frac{m}{k}} = \frac{\sqrt{A^2 - x^2}}{v_x} = \frac{\sqrt{(0.100\text{ m})^2 - (0.060\text{ m})^2}}{0.300\text{ m/s}} = 0.267\text{ s}$$

Then $T = 2\pi\sqrt{m/k} = 2\pi(0.267\text{ s}) = 1.68\text{ s}.$

(b) IDENTIFY and **SET UP:** We are asked to relate x and v_x, so use conservation of energy equation:

$\frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$kx^2 = kA^2 - mv_x^2$

$$x = \sqrt{A^2 - (m/k)v_x^2} = \sqrt{(0.100\text{ m})^2 - (0.267\text{ s})^2(0.160\text{ m/s})^2} = 0.090\text{ m}$$

EVALUATE: Smaller $|v_x|$ means larger x.

(c) IDENTIFY: If the slice doesn't slip the maximum acceleration of the plate (Eq.13.4) equals the maximum acceleration of the slice, which is determined by applying Newton's 2nd law to the slice.

SET UP: For the plate, $-kx = ma_x$ and $a_x = -(k/m)x$. The maximum $|x|$ is A, so $a_{max} = (k/m)A$. If the carrot slice doesn't slip then the static friction force must be able to give it this much acceleration. The free-body diagram for the carrot slice (mass m') is given in Figure 13.66.

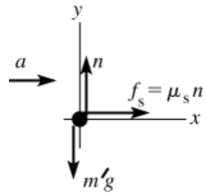


Figure 13.66

EXECUTE: $\sum F_y = ma_y$

$n - m'g = 0$

$n = m'g$

$\sum F_x = ma_x$

$\mu_s n = m'a$

$\mu_s m'g = m'a$ and $a = \mu_s g$

But we require that $a = a_{max} = (k/m)A = \mu_s g$ and $\mu_s = \frac{k}{m}\frac{A}{g} = \left(\frac{1}{0.267\ \text{s}}\right)^2\left(\frac{0.100\ \text{m}}{9.80\ \text{m/s}^2}\right) = 0.143$

EVALUATE: We can write this as $\mu_s = \omega^2 A/g$. More friction is required if the frequency or the amplitude is increased.

13.67. IDENTIFY: The largest downward acceleration the ball can have is g whereas the downward acceleration of the tray depends on the spring force. When the downward acceleration of the tray is greater than g, then the ball leaves the tray. $y(t) = A\cos(\omega t + \phi)$.

SET UP: The downward force exerted by the spring is $F = kd$, where d is the distance of the object above the equilibrium point. The downward acceleration of the tray has magnitude $\frac{F}{m} = \frac{kd}{m}$, where m is the total mass of the ball and tray. $x = A$ at $t = 0$, so the phase angle ϕ is zero and $+x$ is downward.

EXECUTE: **(a)** $\frac{kd}{m} = g$ gives $d = \frac{mg}{k} = \frac{(1.775\ \text{kg})(9.80\ \text{m/s}^2)}{185\ \text{N/m}} = 9.40\ \text{cm}$. This point is 9.40 cm above the equilibrium point so is $9.40\ \text{cm} + 15.0\ \text{cm} = 24.4\ \text{cm}$ above point A.

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{185\ \text{N/m}}{1.775\ \text{kg}}} = 10.2\ \text{rad/s}$. The point in (a) is above the equilibrium point so $x = -9.40\ \text{cm}$.

$x = A\cos(\omega t)$ gives $\omega t = \arccos\left(\frac{x}{A}\right) = \arccos\left(\frac{-9.40\ \text{cm}}{15.0\ \text{cm}}\right) = 2.25\ \text{rad}$. $t = \frac{2.25\ \text{rad}}{10.2\ \text{rad/s}} = 0.221\ \text{s}$.

(c) $\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2$ gives $v = \sqrt{\frac{k}{m}(A^2 - x^2)} = \sqrt{\frac{185\ \text{N/m}}{1.775\ \text{kg}}([0.150\ \text{m}]^2 - [-0.0940\ \text{m}]^2)} = 1.19\ \text{m/s}$.

EVALUATE: The period is $T = 2\pi\sqrt{\frac{m}{k}} = 0.615\ \text{s}$. To go from the lowest point to the highest point takes time $T/2 = 0.308\ \text{s}$. The time in (b) is less than this, as it should be.

13.68. IDENTIFY: In SHM, $a_{max} = \frac{k}{m_{tot}}A$. Apply $\sum \vec{F} = m\vec{a}$ to the top block.

SET UP: The maximum acceleration of the lower block can't exceed the maximum acceleration that can be given to the other block by the friction force.

EXECUTE: For block m, the maximum friction force is $f_s = \mu_s n = \mu_s mg$. $\sum F_x = ma_x$ gives $\mu_s mg = ma$ and $a = \mu_s g$. Then treat both blocks together and consider their simple harmonic motion. $a_{max} = \left(\frac{k}{M+m}\right)A$. Set $a_{max} = a$ and solve for A: $\mu_s g = \left(\frac{k}{M+m}\right)A$ and $A = \frac{\mu_s g(M+m)}{k}$.

EVALUATE: If A is larger than this the spring gives the block with mass M a larger acceleration than friction can give the other block, and the first block accelerates out from underneath the other block.

13.69. IDENTIFY: Apply conservation of linear momentum to the collision and conservation of energy to the motion after the collision. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ and $T = \frac{1}{f}$.

SET UP: The object returns to the equilibrium position in time $T/2$.

EXECUTE: **(a)** Momentum conservation during the collision: $mv_0 = (2m)V$. $V = \frac{1}{2}v_0 = \frac{1}{2}(2.00\ \text{m/s}) = 1.00\ \text{m/s}$.

Energy conservation after the collision: $\frac{1}{2}MV^2 = \frac{1}{2}kx^2$.

$$x = \sqrt{\frac{MV^2}{k}} = \sqrt{\frac{(20.0\ \text{kg})(1.00\ \text{m/s})^2}{80.0\ \text{N/m}}} = 0.500\ \text{m (amplitude)}$$

$$\omega = 2\pi f = \sqrt{k/M}.\ f = \frac{1}{2\pi}\sqrt{k/M} = \frac{1}{2\pi}\sqrt{\frac{80.0\ \text{N/m}}{20.0\ \text{kg}}} = 0.318\ \text{Hz}.\ T = \frac{1}{f} = \frac{1}{0.318\ \text{Hz}} = 3.14\ \text{s}.$$

(b) It takes $1/2$ period to first return: $\frac{1}{2}(3.14\ \text{s}) = 1.57\ \text{s}$

EVALUATE: The total mechanical energy of the system determines the amplitude. The frequency and period depend only on the force constant of the spring and the mass that is attached to the spring.

13.70. IDENTIFY: The upward acceleration of the rocket produces an effective downward acceleration for objects in its frame of reference that is equal to $g' = a + g$.

SET UP: The amplitude is the maximum displacement from equilibrium and is unaffected by the motion of the rocket. The period is affected and is given by $T = 2\pi\sqrt{\frac{L}{g'}}$.

EXECUTE: The amplitude is $8.50°$. $T = 2\pi\sqrt{\frac{1.10\ \text{m}}{4.00\ \text{m/s}^2 + 9.80\ \text{m/s}^2}} = 1.77\ \text{s}$.

EVALUATE: For a pendulum of the same length and with its point of support at rest relative to the earth, $T = 2\pi\sqrt{\frac{L}{g}} = 2.11\ \text{s}$. The upward acceleration decreases the period of the pendulum. If the rocket were instead accelerating downward, the period would be greater than 2.11 s.

13.71. IDENTIFY: The object oscillates as a physical pendulum, so $f = \frac{1}{2\pi}\sqrt{\frac{m_{\text{object}}gd}{I}}$. Use the parallel-axis theorem, $I = I_{\text{cm}} + Md^2$, to find the moment of inertia of each stick about an axis at the hook.

SET UP: The center of mass of the square object is at its geometrical center, so its distance from the hook is $L\cos 45° = L/\sqrt{2}$. The center of mass of each stick is at its geometrical center. For each stick, $I_{\text{cm}} = \frac{1}{12}mL^2$.

EXECUTE: The parallel-axis theorem gives I for each stick for an axis at the center of the square to be $\frac{1}{12}mL^2 + m(L/2)^2 = \frac{1}{3}mL^2$ and the total I for this axis is $\frac{4}{3}mL^2$. For the entire object and an axis at the hook, applying the parallel-axis theorem again to the object of mass $4m$ gives $I = \frac{4}{3}mL^2 + 4m(L/\sqrt{2})^2 = \frac{10}{3}mL^2$.

$$f = \frac{1}{2\pi}\sqrt{\frac{m_{\text{object}}gd}{I}} = \frac{1}{2\pi}\sqrt{\frac{4mgL/\sqrt{2}}{\frac{10}{3}mL^2}} = \sqrt{\frac{6}{5\sqrt{2}}}\left(\frac{1}{2\pi}\sqrt{\frac{g}{L}}\right) = 0.921\left(\frac{1}{2\pi}\sqrt{\frac{g}{L}}\right).$$

EVALUATE: Just as for a simple pendulum, the frequency is independent of the mass. A simple pendulum of length L has frequency $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$ and this object has a frequency that is slightly less than this.

13.72. IDENTIFY: Conservation of energy says $K + U = E$.

SET UP: $U = \frac{1}{2}kx^2$ and $E = U_{\text{max}} = \frac{1}{2}kA^2$.

EXECUTE: **(a)** The graph is given in Figure 13.72. The following answers are found algebraically, to be used as a check on the graphical method.

(b) $A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.200\ \text{J})}{(10.0\ \text{N/m})}} = 0.200\ \text{m}.$

(c) $\frac{E}{4} = 0.050\ \text{J}$.

(d) $U = \frac{1}{2}E$. $x = \frac{A}{\sqrt{2}} = 0.141\ \text{m}$.

(e) From Eq. (13.18), using $v_0 = -\sqrt{\frac{2K_0}{m}}$ and $x_0 = \sqrt{\frac{2U_0}{k}}$, $-\frac{v_0}{\omega x_0} = \frac{\sqrt{(2K_0/m)}}{\sqrt{(k/m)}\sqrt{(2U_0/k)}} = \sqrt{\frac{K_0}{U_0}} = \sqrt{0.429}$ and

$\phi = \arctan\left(\sqrt{0.429}\right) = 0.580\ \text{rad}$.

EVALUATE: The dependence of U on x is not linear and $U=\frac{1}{2}U_{max}$ does not occur at $x=\frac{1}{2}x_{max}$.

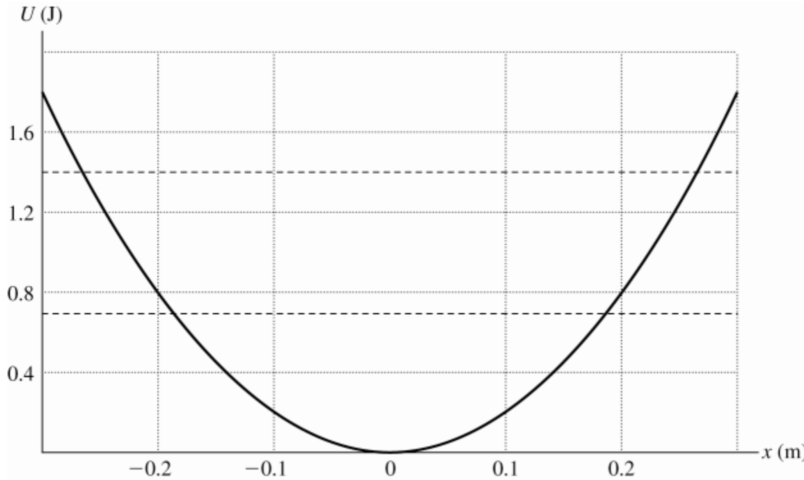


Figure 13.72

13.73. **IDENTIFY:** $T=2\pi\sqrt{\dfrac{m}{k}}$ so the period changes because the mass changes.

SET UP: $\dfrac{dm}{dt}=-2.00\times10^{-3}$ kg/s. The rate of change of the period is $\dfrac{dT}{dt}$.

EXECUTE: **(a)** When the bucket is half full, $m=7.00$ kg. $T=2\pi\sqrt{\dfrac{7.00\text{ kg}}{125\text{ N/m}}}=1.49$ s.

(b) $\dfrac{dT}{dt}=\dfrac{2\pi}{\sqrt{k}}\dfrac{d}{dt}(m^{1/2})=\dfrac{2\pi}{\sqrt{k}}\frac{1}{2}m^{-1/2}\dfrac{dm}{dt}=\dfrac{\pi}{\sqrt{mk}}\dfrac{dm}{dt}$.

$\dfrac{dT}{dt}=\dfrac{\pi}{\sqrt{(7.00\text{ kg})(125\text{ N/m})}}(-2.00\times10^{-3}\text{ kg/s})=-2.12\times10^{-4}$ s per s. $\dfrac{dT}{dt}$ is negative; the period is getting shorter.

(c) The shortest period is when all the water has leaked out and $m=2.00$ kg. Then $T=0.795$ s.

EVALUATE: The rate at which the period changes is not constant but instead increases in time, even though the rate at which the water flows out is constant.

13.74. **IDENTIFY:** Use $F_x=-kx$ to determine k for the wire. Then $f=\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.

SET UP: $F=mg$ moves the end of the wire a distance Δl.

EXECUTE: The force constant for this wire is $k=\dfrac{mg}{\Delta l}$, so $f=\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}=\dfrac{1}{2\pi}\sqrt{\dfrac{g}{\Delta l}}=\dfrac{1}{2\pi}\sqrt{\dfrac{9.80\text{ m/s}^2}{2.00\times10^{-3}\text{ m}}}=11.1$ Hz.

EVALUATE: The frequency is independent of the additional distance the ball is pulled downward, so long as that distance is small.

13.75. **IDENTIFY** and **SET UP:** Measure x from the equilibrium position of the object, where the gravity and spring forces balance. Let $+x$ be downward.

(a) Use conservation of energy (Eq.13.21) to relate v_x and x. Use Eq. (13.12) to relate T to k/m.

EXECUTE: $\frac{1}{2}mv_x^2+\frac{1}{2}kx^2=\frac{1}{2}kA^2$

For $x=0, \frac{1}{2}mv_x^2=\frac{1}{2}kA^2$ and $v=A\sqrt{k/m}$, just as for horizontal SHM. We can use the period to calculate $\sqrt{k/m}: T=2\pi\sqrt{m/k}$ implies $\sqrt{k/m}=2\pi/T$. Thus $v=2\pi A/T=2\pi(0.100\text{ m})/4.20\text{ s}=0.150$ m/s.

(b) IDENTIFY and **SET UP:** Use Eq.(13.4) to relate a_x and x.

EXECUTE: $ma_x=-kx$ so $a_x=-(k/m)x$

$+x$-direction is downward, so here $x=-0.050$ m

$a_x=-(2\pi/T)^2(-0.050\text{ m})=+(2\pi/4.20\text{ s})^2(0.050\text{ m})=0.112\text{ m/s}^2$ (positive, so direction is downward)

(c) IDENTIFY and **SET UP:** Use Eq.(13.13) to relate x and t. The time asked for is twice the time it takes to go from $x=0$ to $x=+0.050$ m.

EXECUTE: $x(t) = A\cos(\omega t + \phi)$

Let $\phi = -\pi/2$, so $x = 0$ at $t = 0$. Then $x = A\cos(\omega t - \pi/2) = A\sin\omega t = A\sin(2\pi t/T)$. Find the time t that gives $x = +0.050$ m: $0.050 \text{ m} = (0.100 \text{ m}) \sin(2\pi t/T)$

$2\pi t/T = \arcsin(0.50) = \pi/6$ and $t = T/12 = 4.20 \text{ s}/12 = 0.350$ s

The time asked for in the problem is twice this, 0.700 s.

(d) IDENTIFY: The problem is asking for the distance d that the spring stretches when the object hangs at rest from it. Apply Newton's 2nd law to the object.

SET UP: The free-body diagram for the object is given in Figure 13.75.

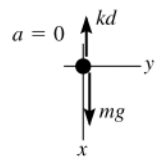


Figure 13.75

EXECUTE: $\sum F_x = ma_x$

$mg - kd = 0$

$d = (m/k)g$

But $\sqrt{k/m} = 2\pi/T$ (part (a)) and $m/k = (T/2\pi)^2$

$$d = \left(\frac{T}{2\pi}\right)^2 g = \left(\frac{4.20 \text{ s}}{2}\right)^2 (9.80 \text{ m/s}^2) = 4.38 \text{ m}.$$

EVALUATE: When the displacement is upward (part (b)), the acceleration is downward. The mass of the partridge is never entered into the calculation. We used just the ratio k/m, that is determined from T.

13.76. **IDENTIFY:** $x(t) = A\cos(\omega t + \phi)$, $v_x = -A\omega\sin(\omega t + \phi)$ and $a_x = -\omega^2 x$. $\omega = 2\pi/T$.

SET UP: $x = A$ when $t = 0$ gives $\phi = 0$.

EXECUTE: $x = (0.240 \text{ m})\cos\left(\frac{2\pi t}{1.50 \text{ s}}\right)$. $v_x = -\left(\frac{2\pi(0.240 \text{ m})}{(1.50 \text{ s})}\right)\sin\left(\frac{2\pi t}{1.50 \text{ s}}\right) = -(1.00530 \text{ m/s})\sin\left(\frac{2\pi t}{1.50 \text{ s}}\right)$.

$$a_x = -\left(\frac{2\pi}{1.50 \text{ s}}\right)^2 (0.240 \text{ m})\cos\left(\frac{2\pi t}{1.50 \text{ s}}\right) = -(4.2110 \text{ m/s}^2)\cos\left(\frac{2\pi t}{1.50 \text{ s}}\right).$$

(a) Substitution gives $x = -0.120$ m, or using $t = \frac{T}{3}$ gives $x = A\cos 120° = \frac{-A}{2}$.

(b) Substitution gives $ma_x = +(0.0200 \text{ kg})(2.106 \text{ m/s}^2) = 4.21\times10^{-2}$ N, in the $+x$-direction.

(c) $t = \frac{T}{2\pi} \arccos\left(\frac{-3A/4}{A}\right) = 0.577$ s.

(d) Using the time found in part (c), $v = 0.665$ m/s.

EVALUATE: We could also calculate the speed in part (d) from the conservation of energy expression, Eq.(13.22).

13.77. **IDENTIFY:** Apply conservation of linear momentum to the collision between the steak and the pan. Then apply conservation of energy to the motion after the collision to find the amplitude of the subsequent SHM. Use Eq.(13.12) to calculate the period.

(a) SET UP: First find the speed of the steak just before it strikes the pan. Use a coordinate system with $+y$ downward.

$v_{0y} = 0$ (released from the rest); $y - y_0 = 0.40$ m; $a_y = +9.80 \text{ m/s}^2$; $v_y = ?$

$v_y^2 = v_{0_y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_y = +\sqrt{2a_y(y - y_0)} = +\sqrt{2(9.80 \text{ m/s}^2)(0.40 \text{ m})} = +2.80$ m/s

SET UP: Apply conservation of momentum to the collision between the steak and the pan. After the collision the steak and the pan are moving together with common velocity v_2. Let A be the steak and B be the pan. The system before and after the collision is shown in Figure 13.77.

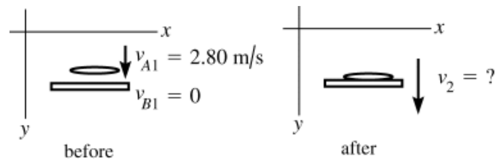


Figure 13.77

EXECUTE: P_y conserved: $m_A v_{A1y} + m_B v_{B1y} = (m_A + m_B)v_{2y}$

$m_A v_{A1} = (m_A + m_B)v_2$

$$v_2 = \left(\frac{m_A}{m_A + m_B}\right)v_{A1} = \left(\frac{2.2\text{ kg}}{2.2\text{ kg} + 0.20\text{ kg}}\right)(2.80\text{ m/s}) = 2.57\text{ m/s}$$

(b) SET UP: Conservation of energy applied to the SHM gives: $\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}kA^2$ where v_0 and x_0 are the initial speed and displacement of the object and where the displacement is measured from the equilibrium position of the object.

EXECUTE: The weight of the steak will stretch the spring an additional distance d given by $kd = mg$ so $d = \frac{mg}{k} = \frac{(2.2\text{ kg})(9.80\text{ m/s}^2)}{400\text{ N/m}} = 0.0539\text{ m}$. So just after the steak hits the pan, before the pan has had time to move, the steak plus pan is 0.0539 m above the equilibrium position of the combined object. Thus $x_0 = 0.0539$ m. From part (a) $v_0 = 2.57$ m/s, the speed of the combined object just after the collision. Then $\frac{1}{2}mv_0^2 + \frac{1}{2}kx_0^2 = \frac{1}{2}kA^2$ gives

$$A = \sqrt{\frac{mv_0^2 + kx_0^2}{k}} = \sqrt{\frac{2.4\text{ kg}(2.57\text{ m/s})^2 + (400\text{ N/m})(0.0539\text{ m})^2}{400\text{ N/m}}} = 0.21\text{ m}$$

(c) $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{\frac{2.4\text{ kg}}{400\text{ N/m}}} = 0.49\text{ s}$

EVALUATE: The amplitude is less than the initial height of the steak above the pan because mechanical energy is lost in the inelastic collision.

13.78. **IDENTIFY:** $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. Use energy considerations to find the new amplitude.

SET UP: $f = 0.600$ Hz, $m = 400$ kg; $f = \frac{1}{2}\sqrt{\frac{k}{m}}$ gives $k = 5685$ N/m. This is the effective force constant of the two springs.

(a) After the gravel sack falls off, the remaining mass attached to the springs is 225 kg. The force constant of the springs is unaffected, so $f = 0.800$ Hz. To find the new amplitude use energy considerations to find the distance downward that the beam travels after the gravel falls off. Before the sack falls off, the amount x_0 that the spring is stretched at equilibrium is given by $mg - kx_0$, so $x_0 = mg/k = (400\text{ kg})(9.80\text{ m/s}^2)/(5685\text{ N/m}) = 0.6895$ m. The maximum upward displacement of the beam is $A = 0.400$ m above this point, so at this point the spring is stretched 0.2895 m. With the new mass, the mass 225 kg of the beam alone, at equilibrium the spring is stretched $mg/k = (225\text{ kg})(9.80\text{ m/s}^2)/(5685\text{ N/m}) = 0.6895$ m. The new amplitude is therefore $0.3879\text{ m} - 0.2895\text{ m} = 0.098$ m. The beam moves 0.098 m above and below the new equilibrium position. Energy calculations show that $v = 0$ when the beam is 0.098 m above and below the equilibrium point.

(b) The remaining mass and the spring constant is the same in part (a), so the new frequency is again 0.800 Hz. The sack falls off when the spring is stretched 0.6895 m. And the speed of the beam at this point is $v = A\sqrt{k/m} = (0.400\text{ m})\sqrt{(5685\text{ N/m})/(400\text{ kg})} = 1.508$ m/s. Take $y = 0$ at this point. The total energy of the beam at this point, just after the sack falls off, is $E = K + U_{\text{el}} + U_{\text{grav}} = \frac{1}{2}(225\text{ kg})(1.508\text{ m/s}^2) + \frac{1}{2}(5695\text{ N/m})(0.6895\text{ m})^2 + 0 = 1608$ J. Let this be point 1. Let point 2 be where the beam has moved upward a distance d and where $v = 0$. $E_2 = \frac{1}{2}k(0.6985\text{ m} - d)^2 + mgd$. $E_1 = E_2$ gives $d = 0.7275$ m. At this end point of motion the spring is compressed 0.7275 m – 0.6895 m = 0.0380 m. At the new equilibrium position the spring is stretched 0.3879 m, so the new amplitude is 0.3789 m + 0.0380 m = 0.426 m. Energy calculations show that v is also zero when the beam is 0.426 m below the equilibrium position.

EVALUATE: The new frequency is independent of the point in the motion at which the bag falls off. The new amplitude is smaller than the original amplitude when the sack falls off at the maximum upward displacement of the beam. The new amplitude is larger than the original amplitude when the sack falls off when the beam has maximum speed.

13.79. **IDENTIFY** and **SET UP:** Use Eq.(13.12) to calculate g and use Eq.(12.4) applied to Newtonia to relate g to the mass of the planet.

EXECUTE: The pendulum swings through $\frac{1}{2}$ cycle in 1.42 s, so T = 2.84 s. L = 1.85 m. Use T to find g:

$T = 2\pi\sqrt{L/\text{g}}$ so $\text{g} = L(2\pi/T)^2 = 9.055\text{ m/s}^2$

Use g to find the mass M_{p} of Newtonia: $g = GM_{\text{p}}/R_{\text{p}}^2$

$2\pi R_{\text{p}} = 5.14\times10^7$ m, so $R_{\text{p}} = 8.18\times10^6$ m

$$m_{\text{p}} = \frac{gR_{\text{p}}^2}{G} = 9.08\times10^{24}\text{ kg}$$

EVALUATE: g is similar to that at the surface of the earth. The radius of Newtonia is a little less than earth's radius and its mass is a little more.

13.80. **IDENTIFY:** $F_x = -kx$ allows us to calculate k. $T = 2\pi\sqrt{m/k}$. $x(t) = A\cos(\omega t + \phi)$. $F_{net} = -kx$.

SET UP: Let $\phi = \pi/2$ so $x(t) = A\sin(\omega t)$. At $t = 0$, $x = 0$ and the object is moving downward. When the object is below the equilibrium position, F_{spring} is upward.

EXECUTE: **(a)** Solving Eq. (13.12) for m, and using $k = \dfrac{F}{\Delta l}$

$$m = \left(\frac{T}{2\pi}\right)^2 \frac{F}{\Delta l} = \left(\frac{1}{2\pi}\right)^2 \frac{40.0\text{ N}}{0.250\text{ m}} = 4.05\text{ kg}.$$

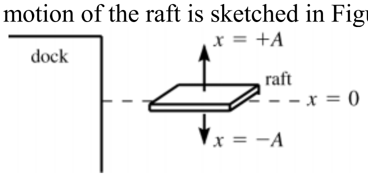
(b) $t = (0.35)T$, and so $x = -A\sin[2\pi(0.35)] = -0.0405$ m. Since $t > T/4$, the mass has already passed the lowest point of its motion, and is on the way up.

(c) Taking upward forces to be positive, $F_{spring} - mg = -kx$, where x is the displacement from equilibrium, so $F_{spring} = -(160\text{ N/m})(-0.030\text{ m}) + (4.05\text{ kg})(9.80\text{ m/s}^2) = 44.5\text{ N}$.

EVALUATE: When the object is below the equilibrium position the net force is upward and the upward spring force is larger in magnitude than the downward weight of the object.

13.81. **IDENTIFY:** Use Eq.(13.13) to relate x and t. $T = 3.5$ s.

SET UP: The motion of the raft is sketched in Figure 13.81.



Let the raft be at $x = +A$ when $t = 0$. Then $\phi = 0$ and $x(t) = A\cos\omega t$.

Figure 13.81

EXECUTE: Calculate the time it takes the raft to move from $x = +A = +0.200$ m to $x = A - 0.100\text{ m} = 0.100$ m. Write the equation for $x(t)$ in terms of T rather than ω: $\omega = 2\pi/T$ gives that $x(t) = A\cos(2\pi t/T)$

$x = A$ at $t = 0$

$x = 0.100$ m implies $0.100\text{ m} = (0.200\text{ m})\cos(2\pi t/T)$

$\cos(2\pi t/T) = 0.500$ so $2\pi t/T = \arccos(0.500) = 1.047$ rad

$t = (T/2\pi)(1.047\text{ rad}) = (3.5\text{ s}/2\pi)(1.047\text{ rad}) = 0.583$ s

This is the time for the raft to move down from $x = 0.200$ m to $x = 0.100$ m. But people can also get off while the raft is moving up from $x = 0.100$ m to $x = 0.200$ m, so during each period of the motion the time the people have to get off is $2t = 2(0.583\text{ s}) = 1.17$ s.

EVALUATE: The time to go from $x = 0$ to $x = A$ and return is $T/2 = 1.75$ s. The time to go from $x = A/2$ to A and return is less than this.

13.82. **IDENTIFY:** $T = 2\pi/\omega$. $F_r(r) = -kr$ to determine k.

SET UP: Example 12.10 derives $F_r(r) = -\dfrac{GM_E m}{R_E^3} r$.

EXECUTE: $a_r = F_r/m$ is in the form of Eq.(13.8), with x replaced by r, so the motion is simple harmonic.

$k = \dfrac{GM_E m}{R_E^3}$. $\omega^2 = \dfrac{k}{m} = \dfrac{GM_E}{R_E^3} = \dfrac{g}{R_E}$. The period is then $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{R_E}{g}} = 2\pi\sqrt{\dfrac{6.38\times 10^6\text{ m}}{9.80\text{ m/s}^2}} = 5070$ s, or 84.5 min.

EVALUATE: The period is independent of the mass of the object but does depend on R_E, which is also the amplitude of the motion.

13.83. **IDENTIFY:** If $F_{net} = kx$, then $\omega = \sqrt{\dfrac{k}{m}}$. Calculate F_{net}. If it is of this form, calculate k.

SET UP: The gravitational force between two point masses is $F_g = G\dfrac{m_1 m_2}{r^2}$ and is attractive. The forces on M are sketched in Figure 13.83.

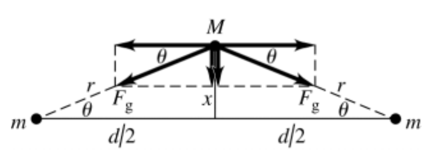


Figure 13.83

EXECUTE: **(a)** $r=\sqrt{(d/2)^2+x^2}\approx d/2$, if $x<<d$. $\tan\theta=\dfrac{x}{d/2}=\dfrac{2x}{d}$. The net force is toward the original position of M and has magnitude $F_{\text{net}}=2G\dfrac{mM}{(d/2)^2}\sin\theta$. Since θ is small, $\sin\theta\approx\tan\theta=\dfrac{2x}{d}$ and $F_{\text{net}}=\left(\dfrac{16GmM}{d^3}\right)x$. This is a restoring force.

(b) Comparing the result in part (a) to $F_{\text{net}}=kx$ gives $k=\dfrac{16GmM}{d^3}$. $\omega=\sqrt{\dfrac{k}{m}}=\dfrac{4}{d}\sqrt{\dfrac{GM}{d}}$. $T=\dfrac{2\pi}{\omega}=\dfrac{\pi d}{2}\sqrt{\dfrac{d}{GM}}$.

(c) $T=\dfrac{\pi(0.250\text{ m})}{2}\sqrt{\dfrac{0.250\text{ m}}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(100\text{ kg})}}=2.40\times10^3\text{ s}=40\text{ min}$. This period is short enough that a patient person could measure it. The experiment would have to be done such that the gravitational forces are much larger than any other forces on M. The gravitational forces are very weak, so other forces, such as friction, forces from air currents, etc., would have to be kept extremely small.

(d) If M is displaced toward one of the fixed masses there is a net force on M toward that mass and therefore away from the equilibrium position of M. The net force is not a restoring force and M would not oscillate, it would continue to move in the direction in which it was displaced.

EVALUATE: The period is very long because the restoring force is very small.

13.84. **IDENTIFY:** $U(x)-U(x_0)=\int_{x_0}^{x}F_x dx$. In part (b) follow the steps outlined in the hint.

SET UP: In part (a), let $x_0=0$ and $U(x_0)=U(0)=0$. The time for the object to go from $x=0$ to $x=A$ is $T/4$.

EXECUTE: **(a)** $U=-\int_0^x F_x dx=c\int_0^x x^3 dx=\dfrac{c}{4}x^4$.

(b) From conservation of energy, $\tfrac{1}{2}mv_x^2=\dfrac{c}{4}(A^4-x^4)$. $v_x=\dfrac{dx}{dt}$, so $\dfrac{dx}{\sqrt{A^4-x^4}}=\sqrt{\dfrac{c}{2m}}dt$. Integrating from 0 to A with respect to x and from 0 to $T/4$ with respect to t, $\int_0^A\dfrac{dx}{\sqrt{A^4-x^4}}=\sqrt{\dfrac{c}{2m}}\dfrac{T}{4}$ To use the hint, let $u=\dfrac{x}{A}$, so that $dx=a\ du$ and the upper limit of the u-integral is $u=1$. Factoring A^2 out of the square root,

$\dfrac{1}{A}\int_0^1\dfrac{du}{\sqrt{1-u^4}}=\dfrac{1.31}{A}=\sqrt{\dfrac{c}{32m}}T$, which may be expressed as $T=\dfrac{7.41}{A}\sqrt{\dfrac{m}{c}}$.

(c) The period does depend on amplitude, and the motion is not simple harmonic.

EVALUATE: Simple harmonic motion requires $F_x=-kx$, where k is a constant, and that is not the case here.

13.85. **IDENTIFY:** Find the x-component of the vector $\vec{v}_Q$ in Figure 13.6a in the textbook.

SET UP: $v_x=-v_{\tan}\sin\theta$ and $\theta=\omega t+\phi$.

EXECUTE: $v_x=-v_{\tan}\sin\theta$. Substituting for $v_{\tan}$ and θ gives Eq.(13.15).

EVALUATE: At $t=0$, Q is on the x-axis and has zero component of velocity. This corresponds to $v_x=0$ in Eq.(13.15).

13.86. **IDENTIFY:** $mV_{\text{cm}}=P_{\text{tot}}$. $K=p^2/2m$ for a single object and the total kinetic energy of the two masses is just the sum of their individual kinetic energies.

SET UP: Momentum is a vector and kinetic energy is a scalar.

EXECUTE: **(a)** For the center of mass to be at rest, the total momentum must be zero, so the momentum vectors must be of equal magnitude but opposite directions, and the momenta can be represented as $\vec{p}$ and $-\vec{p}$.

(b) $K_{\text{tot}}=2\dfrac{p^2}{2m}=\dfrac{p^2}{2(m/2)}$.

(c) The argument of part (a) is valid for any masses. The kinetic energy is

$K_{\text{tot}}=\dfrac{p^2}{2m_1}+\dfrac{p^2}{2m_2}=\dfrac{p^2}{2}\left(\dfrac{m_1+m_2}{m_1m_2}\right)=\dfrac{p^2}{2(m_1m_2/(m_1+m_2))}$.

EVALUATE: If $m_1=m_2=m$, the reduced mass is $m/2$. If $m_1>>m_2$, then the reduced mass is m_2.

13.87. **IDENTIFY:** $F_r=-dU/dr$. The equilibrium separation r_{eq} is given by $F(r_{\text{eq}})=0$. The force constant k is defined by $F_r=-kx$. $f=\dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$, where m is the reduced mass.

SET UP: $d(r^{-n})/dr = -nr^{-(n+1)}$, for $n \ge 1$.

EXECUTE: **(a)** $F_r = -\dfrac{dU}{dr} = A\left[\left(\dfrac{R_0^7}{r^9}\right) - \dfrac{1}{r^2}\right]$.

(b) Setting the above expression for F_r equal to zero, the term in square brackets vanishes, so that $\dfrac{R_0^7}{r_{\text{eq}}^9} = \dfrac{1}{r_{\text{eq}}^2}$, or

$R_0^7 = r_{\text{eq}}^7$, and $r_{\text{eq}} = R_0$.

(c) $U(R_0) = -\dfrac{7A}{8R_0} = -7.57 \times 10^{-19}$ J.

(d) The above expression for F_r can be expressed as

$$F_r = \frac{A}{R_0^2}\left[\left(\frac{r}{R_0}\right)^{-9} - \left(\frac{r}{R_0}\right)^{-2}\right] = \frac{A}{R_0^2}\left[(1+(x/R_0))^{-9} - (1+(x/R_0))^{-2}\right]$$

$$F_r \approx \frac{A}{R_0^2}[(1-9(x/R_0)) - (1-2(x/R_0))] = \frac{A}{R_0^2}(-7\ x/R_0) = -\left(\frac{7A}{R_0^3}\right)x.$$

(e) $f = \dfrac{1}{2\pi}\sqrt{k/m} = \dfrac{1}{2\pi}\sqrt{\dfrac{7A}{R_0^3 m}} = 8.39 \times 10^{12}\,\text{Hz}.$

EVALUATE: The force constant depends on the parameters A and R_0 in the expression for $U(r)$. The minus sign in the expression in part (d) shows that for small displacements from equilibrium, F_r is a restoring force.

13.88. **IDENTIFY:** Apply $\sum \tau_z = I_{\text{cm}}\alpha_z$ and $\sum F_x = Ma_{\text{cm}-x}$ to the cylinders. Solve for $a_{\text{cm}-x}$. Compatre to Eq.(13.8) to find the angular frequency and period, $T = 2\pi\omega$.

SET UP:

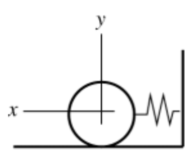


Figure 13.88a

Let the origin of coordinate be at the center of the cylinders when they are at their equilibrium position.

The free-body diagram for the cylinders when they are displaced a distance x to the left is given in Figure 13.88b.

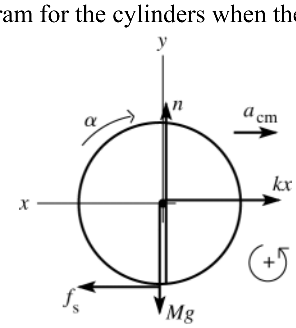


Figure 13.88b

EXECUTE:

$\sum \tau_z = I_{\text{cm}}\alpha_z$

$f_s R = (\tfrac{1}{2}MR^2)\alpha$

$f_s = \tfrac{1}{2}MR\alpha$

But $R\alpha = a_{\text{cm}}$ so

$f_s = \tfrac{1}{2}Ma_{\text{cm}}$

$\sum F_x = ma_x$

$f_s - kx = -Ma_{\text{cm}}$

$\tfrac{1}{2}Ma_{\text{cm}} - kx = -Ma_{\text{cm}}$

$kx = \tfrac{3}{2}Ma_{\text{cm}}$

$(2k/3M)x = a_{\text{cm}}$

Eq. (13.8): $a_x = -\omega^2 x$ (The minus sign says that x and a_x have opposite directions, as our diagram shows.) Our result for a_{cm} is of this form, with $\omega^2 = 2k/3M$ and $\omega = \sqrt{2k/3M}$. Thus $T = 2\pi/\omega = 2\pi\sqrt{3M/2k}$.

EVALUATE: If there were no friction and the cylinder didn't roll, the period would be $2\pi\sqrt{M/k}$. The period when there is rolling without slipping is larger than this.

13.89. **IDENTIFY:** Apply conservation of energy to the motion before and after the collision. Apply conservation of linear momentum to the collision. After the collision the system moves as a simple pendulum. If the maximum angular displacement is small, $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$.

SET UP: In the motion before and after the collision there is energy conversion between gravitational potential energy mgh, where h is the height above the lowest point in the motion, and kinetic energy.

EXECUTE: Energy conservation during downward swing: $m_2gh_0 = \frac{1}{2}m_2v^2$ and

$v = \sqrt{2gh_0} = \sqrt{2(9.8\text{ m/s}^2)(0.100\text{ m})} = 1.40\text{ m/s}$.

Momentum conservation during collision: $m_2v = (m_2 + m_3)V$ and $V = \frac{m_2v}{m_2+m_3} = \frac{(2.00\text{ kg})(1.40\text{ m/s})}{5.00\text{ kg}} = 0.560\text{ m/s}$.

Energy conservation during upward swing: $Mgh_f = \frac{1}{2}MV^2$ and $h_f = V^2/2g = \frac{(0.560\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 0.0160\text{ m} = 1.60\text{ cm}$.

Figure 13.89 shows how the maximum angular displacement is calculated from h_f. $\cos\theta = \frac{48.4\text{ cm}}{50.0\text{ cm}}$ and $\theta = 14.5°$.

$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = \frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{0.500\text{ m}}} = 0.705\text{ Hz}$.

EVALUATE: $14.5° = 0.253$ rad. $\sin(0.253\text{ rad}) = 0.250$. $\sin\theta \approx \theta$ and Eq.(13.34) is accurate.

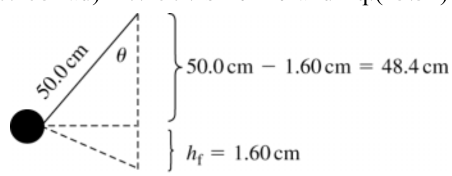


Figure 13.89

13.90. **IDENTIFY:** $T = 2\pi\sqrt{I/mgd}$

SET UP: The model for the leg is sketched in Figure 13.90. $T = 2\pi\sqrt{I/mgd}$, $m = 3M$. $d = y_{cg} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2}$. For a rod with the axis at one end, $I = \frac{1}{3}ML^2$. For a rod with the axis at its center, $I = \frac{1}{12}ML^2$.

EXECUTE: $d = \frac{2M([1.55\text{ m}]/2) + M(1.55\text{ m} + [1.55\text{ m}]/2)}{3M} = 1.292\text{ m}$. $I + I_1 + I_2$.

$I_1 = \frac{1}{3}(2M)(1.55\text{ m})^2 = (1.602\text{ m}^2)M$. $I_{2,cm} = \frac{1}{12}M(1.55\text{ m})^2$. The parallel-axis theorem (Eq. 9.19) gives

$I_2 = I_{2,cm} + M(1.55\text{ m} + [1.55\text{ m}]/2)^2 = (5.06\text{ m}^2)M$. $I = I_1 + I_2 = (7.208\text{ m}^2)M$. Then

$T = 2\pi\sqrt{I/mgd} = 2\pi\sqrt{\frac{(7.208\text{ m}^2)M}{(3M)(9.80\text{ m/s}^2)(1.292\text{ m})}} = 2.74\text{ s}$.

EVALUATE: This is a little smaller than $T = 2.9$ s found in Example 13.10.

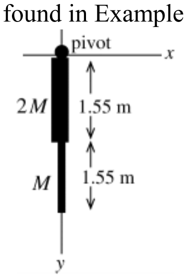


Figure 13.90

13.91: **IDENTIFY:** The motion is simple harmonic if the equation of motion for the angular oscillations is of the form $\frac{d^2\theta}{dt^2} = -\frac{\kappa}{I}\theta$, and in this case the period is $T = 2\pi\sqrt{I/\kappa}$

SET UP: For a slender rod pivoted about its center, $I = \frac{1}{12}ML^2$

EXECUTE: The torque on the rod about the pivot is $\tau = -\left(k\dfrac{L}{2}\theta\right)\dfrac{L}{2}$. $\tau = I\alpha = I\dfrac{d^2\theta}{dt^2}$ gives

$\dfrac{d^2\theta}{dt^2} = -k\dfrac{L^2/4}{I}\theta = -\dfrac{3k}{M}\theta$. $\dfrac{d^2\theta}{dt^2}$ is proportional to θ and the motion is angular SHM. $\dfrac{\kappa}{I} = \dfrac{3k}{M}$, $T = 2\pi\sqrt{\dfrac{M}{3k}}$.

EVALUATE: The expression we used for the torque, $\tau = -\left(k\dfrac{L}{2}\theta\right)\dfrac{L}{2}$, is valid only when θ is small enough for $\sin\theta \approx \theta$ and $\cos\theta \approx 1$.

13.92. **IDENTIFY** and **SET UP:** Eq. (13.39) gives the period for the bell and Eq. (13.34) gives the period for the clapper.
EXECUTE: The bell swings as a physical pendulum so its period of oscillation is given by

$T = 2\pi\sqrt{I/mgd} = 2\pi\sqrt{18.0\ \text{kg}\cdot\text{m}^2/(34.0\ \text{kg})(9.80\ \text{m/s}^2)(0.60\ \text{m})} = 1.885\ \text{s}$

The clapper is a simple pendulum so its period is given by $T = 2\pi\sqrt{L/g}$.

Thus $L = g(T/2\pi)^2 = (9.80\ \text{m/s}^2)(1.885\ \text{s}/2\pi)^2 = 0.88\ \text{m}$.

EVALUATE: If the cm of the bell were at the geometrical center of the bell, the bell would extend 1.20 m from the pivot, so the clapper is well inside tbe bell.

13.93. **IDENTIFY:** The object oscillates as a physical pendulum, with $f = \dfrac{1}{2\pi}\sqrt{\dfrac{Mgd}{I}}$, where M is the total mass of the object.

SET UP: The moment of inertia about the pivot is $2(1/3)ML^2 = (2/3)\ ML^2$, and the center of gravity when balanced is a distance $d = L/(2\sqrt{2})$ below the pivot .

EXECUTE: The frequency is $f = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{6g}{4\sqrt{2}L}} = \dfrac{1}{4\pi}\sqrt{\dfrac{6g}{\sqrt{2}L}}$.

EVALUATE: If $f_{sp} = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{L}}$ is the frequency for a simple pendulum of length L, $f = \dfrac{1}{2}\sqrt{\dfrac{6}{\sqrt{2}}}f_{sp} = 1.03 f_{sp}$.

13.94. **IDENTIFY** and **SET UP:** Use Eq. (13.34) for the simple pendulum. Use a physical pendulum (Eq.13.39) for the pendulum in the case.
EXECUTE: (a) $T = 2\pi\sqrt{L/g}$ and $L = g(T/2\pi)^2 = (9.80\ \text{m/s}^2)(4.00\ \text{s}/2\pi)^2 = 3.97\ \text{m}$

(b) Use a uniform slender rod of mass M and length $L = 0.50$ m. Pivot the rod about an axis that is a distance d above the center of the rod. The rod will oscillate as a physical pendulum with period $T = 2\pi\sqrt{I/Mgd}$.

Choose d so that $T = 4.00$ s.

$I = I_{cm} + Md^2 = \frac{1}{12}ML^2 + Md^2 = M(\frac{1}{12}L^2 + d^2)$

$T = 2\pi\sqrt{\dfrac{I}{Mgd}} = 2\pi\sqrt{\dfrac{M(\frac{1}{12}L^2 + d^2)}{Mgd}} = 2\pi\sqrt{\dfrac{\frac{1}{12}L^2 + d^2}{gd}}$.

Solve for d and set $L = 0.50$ m and $T = 4.00$ s:

$gd(T/2\pi)^2 = \frac{1}{12}L^2 + d^2$

$d^2 - (T/2\pi)^2 gd + L^2/12 = 0$

$d^2 - (4.00\ \text{s}/2\pi)^2(9.80\ \text{m/s}^2)d + (0.50\ \text{m})^2/12 = 0$

$d^2 - 3.9718d + 0.020833 = 0$

The quadratic formula gives

$d = \frac{1}{2}[3.9718 \pm \sqrt{(3.9718)^2 - 4(0.020833)}]\ \text{m}$

$d = (1.9859 \pm 1.9806)\ \text{m}$ so $d = 3.97$ m or $d = 0.0053$ m.

The maximum value d can have is $L/2 = 0.25$ m, so the answer we want is $d = 0.0053\ \text{m} = 0.53\ \text{cm}$.

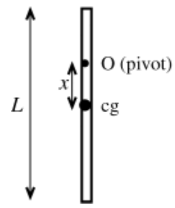
Therefore, take a slender rod of length 0.50 m and pivot it about an axis that is 0.53 cm above its center.

EVALUATE: Note that $T \to \infty$ as $d \to 0$ (pivot at center of rod) and that if the pivot is at the top of rod then

$d = L/2$ and $T = 2\pi\sqrt{\dfrac{\frac{1}{12}L^2 + \frac{1}{4}L^2}{Lg/2}} = 2\pi\sqrt{\dfrac{L}{g}\dfrac{4}{6}} = 2\pi\sqrt{\dfrac{2L}{3g}} = 2\pi\sqrt{\dfrac{2(0.50\ \text{m})}{3(9.80\ \text{m/s}^2)}} = 1.16$ s, which is less than the desired 4.00 s. Thus it is reasonable to expect that there is a value of d between 0 and $L/2$ for which $T = 4.00$ s.

13.95. IDENTIFY: The angular frequency is given by Eq.(13.38). Use the parallel-axis theorem to calculate I in terms of x.

(a) SET UP:



$\omega = \sqrt{mgd/I}$

Figure 13.95

$d = x,$ the distance from the cg of the object (which is at its geometrical center) from the pivot

EXECUTE: I is the moment of inertia about the axis of rotation through O. By the parallel axis theorem

$I_0 = md^2 + I_{\text{cm}}$. $I_{\text{cm}} = \tfrac{1}{12}mL^2$ (Table 9.2), so $I_0 = mx^2 + \tfrac{1}{12}mL^2$. $\omega = \sqrt{\dfrac{mgx}{mx^2 + \tfrac{1}{12}mL^2}} = \sqrt{\dfrac{gx}{x^2 + L^2/12}}$

(b) The maximum ω as x varies occurs when $d\omega/dx = 0.$ $\dfrac{d\omega}{dx} = 0$ gives $\sqrt{g}\,\dfrac{d}{dx}\left(\dfrac{x^{1/2}}{(x^2 + L^2/12)^{1/2}}\right) = 0.$

$$\frac{\tfrac{1}{2}x^{-1/2}}{(x^2 + L^2/12)^{1/2}} - \frac{1}{2}\frac{2x}{(x^2 + L^2/12)^{3/2}}(x^{1/2}) = 0$$

$$x^{-1/2} - \frac{2x^{3/2}}{x^2 + L^2/12} = 0$$

$x^2 + L^2/12 = 2x^2$ so $x = L/\sqrt{12}.$ Get maximum ω when the pivot is a distance $L/\sqrt{12}$ above the center of the rod.

(c) To answer this question we need an expression for $\omega_{\max}$:

In $\omega = \sqrt{\dfrac{gx}{x^2 + L^2/12}}$ substitute $x = L/\sqrt{12}.$

$$\omega_{\max} = \sqrt{\frac{g(L/\sqrt{12})}{L^2/12 + L^2/12}} = \frac{g^{1/2}(12)^{-1/4}}{(L/6)^{1/2}} = \sqrt{g/L}(12)^{-1/4}(6)^{1/2} = \sqrt{g/L}(3)^{1/4}$$

$\omega_{\max}^2 = (g/L)\sqrt{3}$ and $L = g\sqrt{3}/\omega_{\max}^2$

$\omega_{\max} = 2\pi$ rad/s gives $L = \dfrac{(9.80\text{ m/s}^2)\sqrt{3}}{(2\pi\text{ rad/s})^2} = 0.430\text{ m}.$

EVALUATE: $\omega \to 0$ as $x \to 0$ and $\omega \to \sqrt{3g/(2L)} = 1.225\sqrt{g/L}$ when $x \to L/2.$ $\omega_{\max}$ is greater than the $x = L/2$ value. A simple pendulum has $\omega = \sqrt{g/L};$ $\omega_{\max}$ is greater than this.

13.96. IDENTIFY: Calculate F_{net} and define k_{eff} by $F_{\text{net}} = -k_{\text{eff}}x$. $T = 2\pi\sqrt{m/k_{\text{eff}}}$.

SET UP: If the elongations of the springs are x_1 and x_2, they must satisfy $x_1 + x_2 = 0.200$ m

EXECUTE: **(a)** The net force on the block at equilibrium is zero, and so $k_1x_1 = k_2x_2$ and one spring (the one with $k_1 = 2.00$ N/m) must be stretched three times as much as the one with $k_2 = 6.00$ N/m. The sum of the elongations is 0.200 m, and so one spring stretches 0.150 m and the other stretches 0.050 m, and so the equilibrium lengths are 0.350 m and 0.250 m.

(b) When the block is displaced a distance x to the right, the net force on the block is $-k_1(x_1 + x) + k_2(x_2 - x) = [k_1x_1 - k_2x_2] - (k_1 + k_2)x.$ From the result of part (a), the term in square brackets is zero, and so the net force is $-(k_1 + k_2)x,$ the effective spring constant is $k_{\text{eff}} = k_1 + k_2$ and the period of vibration is

$$T = 2\pi\sqrt{\frac{0.100\text{ kg}}{8.00\text{ N/m}}} = 0.702\text{ s}.$$

EVALUATE: The motion is the same as if the block were attached to a single spring that has force constant k_{eff}.

13.97. IDENTIFY: In each situation, imagine the mass moves a distance $\Delta x,$ the springs move distances Δx_1 and $\Delta x_2,$ with forces $F_1 = -k_1\Delta x_1,$ $F_2 = -k_2\Delta x_2.$

SET UP: Let Δx_1 and Δx_2 be positive if the springs are stretched, negative if compressed.

EXECUTE: **(a)** $\Delta x = \Delta x_1 = \Delta x_2,$ $F = F_1 + F_2 = -(k_1 + k_2)\Delta x,$ so $k_{\text{eff}} = k_1 + k_2.$

(b) Despite the orientation of the springs, and the fact that one will be compressed when the other is extended, $\Delta x = \Delta x_1 - \Delta x_2$ and both spring forces are in the same direction. The above result is still valid; $k_{\text{eff}} = k_1 + k_2$.

(c) For massless springs, the force on the block must be equal to the tension in any point of the spring combination, and $F = F_1 = F_2$. $\Delta x_1 = -\dfrac{F}{k_1}$, $\Delta x_2 = -\dfrac{F}{k_2}$, $\Delta x = -\left(\dfrac{1}{k_1} + \dfrac{1}{k_2}\right) F = -\dfrac{k_1 + k_2}{k_1 k_2} F$ and $k_{\text{eff}} = \dfrac{k_1 k_2}{k_1 + k_2}$.

(d) The result of part (c) shows that when a spring is cut in half, the effective spring constant doubles, and so the frequency increases by a factor of $\sqrt{2}$.

EVALUATE: In cases (a) and (b) the effective force constant is greater than either k_1 or k_2 and in case (c) it is less.

13.98. **IDENTIFY:** Follow the procedure specified in the hint.

SET UP: $T = 2\pi\sqrt{L/g}$

EXECUTE: **(a)** $T + \Delta T \approx 2\pi\sqrt{L}\left(g^{-1/2} - \dfrac{1}{2} g^{-3/2}\Delta g\right) = T - T\dfrac{\Delta g}{2g}$, so $\Delta T = -(1/2)(T/g)\Delta g$.

(b) The clock runs slow; $\Delta T > 0$, $\Delta g < 0$ and $g + \Delta g = g\left(1 - \dfrac{2\Delta T}{T}\right) = (9.80\ \text{m/s}^2)\left(1 - \dfrac{2(4.00\ \text{s})}{(86{,}400\ \text{s})}\right) = 9.7991\ \text{m/s}^2$.

EVALUATE: The result in part (a) says that T increases when g decreases, and the magnitude of the fractional change in T is one-half of the magnitude of the fractional change in g.

13.99. **IDENTIFY:** Follow the procedure specified in the hint.

SET UP: Denote the position of a piece of the spring by l; $l = 0$ is the fixed point and $l = L$ is the moving end of the spring. Then the velocity of the point corresponding to l, denoted u, is $u(l) = v\dfrac{l}{L}$ (when the spring is moving, l will be a function of time, and so u is an implicit function of time).

(a) $dm = \dfrac{M}{L} dl$, and so $dK = \dfrac{1}{2} dm\, u^2 = \dfrac{1}{2}\dfrac{Mv^2}{L^3} l^2\, dl$ and $K = \int dK = \dfrac{Mv^2}{2L^3}\int_0^L l^2\, dl = \dfrac{Mv^2}{6}$.

(b) $mv\dfrac{dv}{dt} + kx\dfrac{dx}{dt} = 0$, or $ma + kx = 0$, which is Eq. (13.4).

(c) m is replaced by $\dfrac{M}{3}$, so $\omega = \sqrt{\dfrac{3k}{M}}$ and $M' = \dfrac{M}{3}$.

EVALUATE: The effective mass of the spring is only one-third of its actual mass.

13.100. **IDENTIFY:** $T = 2\pi\sqrt{I/mgd}$

SET UP: With $I = (1/3)ML^2$ and $d = L/2$ in Eq. (13.39), $T_0 = 2\pi\sqrt{2L/3g}$. With the added mass, $I = M\left((L^2/3) + y^2\right)$, $m = 2M$ and $d = (L/4) + y/2$. $T = 2\pi\sqrt{(L^2/3 + y^2)/(g(L/2 + y))}$ and

$r = \dfrac{T}{T_0} = \sqrt{\dfrac{L^2 + 3y^2}{L^2 + 2yL}}$. The graph of the ratio r versus y is given in Figure 13.100.

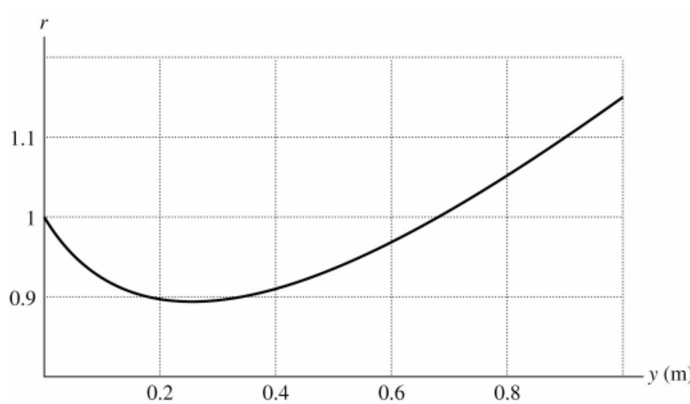


Figure 13.100

(b) From the expression found in part (a), $T = T_0$ when $y = \frac{2}{3}L$. At this point, a simple pendulum with length y would have the same period as the meter stick without the added mass; the two bodies oscillate with the same period and do not affect the other's motion.

EVALUATE: Adding the mass can either increase or decrease the period, depending on where the added mass is placed.

13.101. IDENTIFY: Eq.(13.39) says $T = 2\pi\sqrt{I/mgd}$.

SET UP: Let the two distances from the center of mass be d_1 and d_2. There are then two relations of the form of Eq. (13.39); with $I_1 = I_{\text{cm}} + md_1^2$ and $I_2 = I_{\text{cm}} + md_2^2$.

EXECUTE: These relations may be rewritten as $mgd_1T^2 = 4\pi^2\left(I_{\text{cm}} + md_1^2\right)$ and $mgd_2T^2 = 4\pi^2\left(I_{\text{cm}} + md_2^2\right)$. Subtracting the expressions gives $mg\left(d_1 - d_2\right)T^2 = 4\pi^2 m\left(d_1^2 - d_2^2\right) = 4\pi^2 m\left(d_1 - d_2\right)\left(d_1 + d_2\right)$. Dividing by the common factor of $m\left(d_1 - d_2\right)$ and letting $d_1 + d_2 = L$ gives the desired result.

EVALUATE: The procedure works in practice only if both pivot locations give rise to SHM for small oscillations.

13.102. IDENTIFY: Apply $\sum \vec{F} = m\vec{a}$ to the mass, with $a = a_{\text{rad}} = r\omega^2$.

SET UP: The spring, when stretched, provides an inward force.

EXECUTE: Using $\omega'^2 l$ for the magnitude of the inward radial acceleration, $m\omega'^2 l = k\left(l - l_0\right)$, or $l = \dfrac{kl_0}{k - m\omega'^2}$.

(b) The spring will tend to become unboundedly long.

EXECUTE: As resonance is approached and l becomes very large, both the spring force and the radial acceleration become large.

13.103. IDENTIFY: For a small displacement x, the force constant k is defined by $F_x = -kx$.

SET UP: Let $r = R_0 + x,$ so that $r - R_0 = x$ and $F = A[e^{-2bx} - e^{-bx}]$.

EXECUTE: When x is small compared to b^{-1}, expanding the exponential function gives $F \approx A\ [(1 - 2bx) - (1 - bx)] = -Abx,$ corresponding to a force constant of $Ab = 579\ \text{N/m}$.

EVALUATE: Our result is very close to the value given in Exercise 13.40.

FLUID MECHANICS

14

14.1. **IDENTIFY:** Use Eq.(14.1) to calculate the mass and then use $w = mg$ to calculate the weight.

SET UP: $\rho = m/V$ so $m = \rho V$ From Table 14.1, $\rho = 7.8\times10^3 \text{ kg/m}^3$.

EXECUTE: For a cylinder of length L and radius R, $V = (\pi R^2)L = \pi(0.01425 \text{ m})^2(0.858 \text{ m}) = 5.474\times10^{-4} \text{ m}^3$.

Then $m = \rho V = (7.8\times10^3 \text{ kg/m}^3)(5.474\times10^{-4} \text{ m}^3) = 4.27 \text{ kg}$, and $w = mg = (4.27 \text{ kg})(9.80 \text{ m/s}^2) = 41.8 \text{ N}$ (about 9.4 lbs). A cart is not needed.

EVALUATE: The rod is less than 1m long and less than 3 cm in diameter, so a weight of around 10 lbs seems reasonable.

14.2. **IDENTIFY:** Convert gallons to kg. The mass m of a volume V of gasoline is $m = \rho V$.

SET UP: $1 \text{ gal} = 3.788 \text{ L} = 3.788\times10^{-3} \text{ m}^3$. 1 m^3 of gasoline has a mass of 737 kg.

EXECUTE: $45.0 \text{ mi/gal} = (45.0 \text{ mi/gal})\left(\dfrac{1 \text{ gal}}{3.788\times10^{-3} \text{ m}^3}\right)\left(\dfrac{1 \text{ m}^3}{737 \text{ kg}}\right) = 16.1 \text{ mi/kg}$

EVALUATE: 1 gallon of gasoline has a mass of 2.79 kg. The car goes fewer miles on 1 kg than on 1 gal, since 1 kg of gasoline is less gasoline than 1 gal of gasoline.

14.3. **IDENTIFY:** $\rho = m/V$

SET UP: The density of gold is $19.3\times10^3 \text{ kg/m}^3$.

EXECUTE: $V = (5.0\times10^{-3} \text{ m})(15.0\times10^{-3} \text{ m})(30.0\times10^{-3} \text{ m}) = 2.25\times10^{-6} \text{ m}^3$.

$\rho = \dfrac{m}{V} = \dfrac{0.0158 \text{ kg}}{2.25\times10^{-6} \text{ m}^3} = 7.02\times10^3 \text{ kg/m}^3$. The metal is not pure gold.

EVALUATE: The average density is only 36% that of gold, so at most 36% of the mass is gold.

14.4. **IDENTIFY:** Find the mass of gold that has a value of $\$1.00\times10^6$. Then use the density of gold to find the volume of this mass of gold.

SET UP: For gold, $\rho = 19.3\times10^3 \text{ kg/m}^3$. The volume V of a cube is related to the length L of one side by $V = L^3$.

EXECUTE: $m = (\$1.00\times10^6)\left(\dfrac{1 \text{ troy ounce}}{\$426.60}\right)\left(\dfrac{31.1035\times10^{-3} \text{ kg}}{1 \text{ troy ounce}}\right) = 72.9 \text{ kg}$. $\rho = \dfrac{m}{V}$ so

$V = \dfrac{m}{\rho} = \dfrac{72.9 \text{ kg}}{19.3\times10^3 \text{ kg/m}^3} = 3.78\times10^{-3} \text{ m}^3$. $L = V^{1/3} = 0.156 \text{ m} = 15.6 \text{ cm}$.

EVALUATE: The cube of gold would weigh about 160 lbs.

14.5. **IDENTIFY:** Apply $\rho = m/V$ to relate the densities and volumes for the two spheres.

SET UP: For a sphere, $V = \frac{4}{3}\pi r^3$. For lead, $\rho_1 = 11.3\times10^3 \text{ kg/m}^3$ and for aluminum, $\rho_a = 2.7\times10^3 \text{ kg/m}^3$.

EXECUTE: $m = \rho V = \frac{4}{3}\pi r^3 \rho$. Same mass means $r_a^3\rho_a = r_1^3\rho_1$. $\dfrac{r_a}{r_1} = \left(\dfrac{\rho_1}{\rho_a}\right)^{1/3} = \left(\dfrac{11.3\times10^3}{2.7\times10^3}\right)^{1/3} = 1.6$.

EVALUATE: The aluminum sphere is larger, since its density is less.

14.6. **IDENTIFY:** Average density is $\rho = m/V$.

SET UP: For a sphere, $V = \frac{4}{3}\pi R^3$. The sun has mass $M_{sun} = 1.99\times10^{30} \text{ kg}$ and radius $6.96\times10^8 \text{ m}$.

EXECUTE: **(a)** $\rho = \dfrac{M_{sun}}{V_{sun}} = \dfrac{1.99\times10^{30} \text{ kg}}{\frac{4}{3}\pi(6.96\times10^8 \text{m})^3} = \dfrac{1.99\times10^{30} \text{ kg}}{1.412\times10^{27} \text{ m}^3} = 1.409\times10^3 \text{ kg/m}^3$

(b) $\rho = \dfrac{1.99\times10^{30} \text{ kg}}{\frac{4}{3}\pi(2.00\times10^4 \text{ m})^3} = \dfrac{1.99\times10^{30} \text{ kg}}{3.351\times10^{13} \text{ m}^3} = 5.94\times10^{16} \text{ kg/m}^3$

EVALUATE: For comparison, the average density of the earth is $5.5\times10^3\ \text{kg/m}^3$. A neutron star is extremely dense.

14.7. **IDENTIFY:** $w = mg$ and $m = \rho V$. Find the volume V of the pipe.

SET UP: For a hollow cylinder with inner radius R_1 , outer radius R_2 , and length L the volume is $V = \pi(R_2^2 - R_1^2)L$. $R_1 = 1.25\times10^{-2}$ m and $R_2 = 1.75\times10^{-2}$ m

EXECUTE: $V = \pi([0.0175\ \text{m}]^2 - [0.0125\ \text{m}]^2)(1.50\ \text{m}) = 7.07\times10^{-4}\ \text{m}^3$.

$m = \rho V = (8.9\times10^3\ \text{kg/m}^3)(7.07\times10^{-4}\ \text{m}^3) = 6.29\ \text{kg}$. $w = mg = 61.6\ \text{N}$.

EVALUATE: The pipe weights about 14 pounds.

14.8. **IDENTIFY:** The gauge pressure $p - p_0$ at depth h is $p - p_0 = \rho gh$.

SET UP: Ocean water is seawater and has a density of $1.03\times10^3\ \text{kg/m}^3$.

EXECUTE: $p - p_0 = (1.03\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(3200\ \text{m}) = 3.23\times10^7\ \text{Pa}$.

$$p - p_0 = (3.23\times10^7\ \text{Pa})\left(\frac{1\ \text{atm}}{1.013\times10^5\ \text{Pa}}\right) = 319\ \text{atm} .$$

EVALUATE: The gauge pressure is about 320 times the atmospheric pressure at the surface.

14.9. **IDENTIFY:** The gauge pressure $p - p_0$ at depth h is $p - p_0 = \rho gh$.

SET UP: Freshwater has density $1.00\times10^3\ \text{kg/m}^3$ and seawater has density $1.03\times10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** $p - p_0 = (1.00\times10^3\ \text{kg/m}^3)(3.71\ \text{m/s}^2)(500\ \text{m}) = 1.86\times10^6\ \text{Pa}$.

(b) $h = \dfrac{p - p_0}{\rho g} = \dfrac{1.86\times10^6\ \text{Pa}}{(1.03\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 184\ \text{m}$

EVALUATE: The pressure at a given depth is greater on earth because a cylinder of water of that height weighs more on earth than on Mars.

14.10. **IDENTIFY:** The difference in pressure at points with heights y_1 and y_2 is $p - p_0 = \rho g(y_1 - y_2)$. The outward force $F_\perp$ is related to the surface area A by $F_\perp = pA$.

SET UP: For blood, $\rho = 1.06\times10^3\ \text{kg/m}^3$. $y_1 - y_2 = 1.65\ \text{m}$. The surface area of the segment is πDL , where $D = 1.50\times10^{-3}$ m and $L = 2.00\times10^{-2}$ m .

EXECUTE: **(a)** $p_1 - p_2 = (1.06\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.65\ \text{m}) = 1.71\times10^4\ \text{Pa}$.

(b) The additional force due to this pressure difference is $\Delta F_\perp = (p_1 - p_2)A$.

$A = \pi DL = \pi(1.50\times10^{-3}\ \text{m})(2.00\times10^{-2}\ \text{m}) = 9.42\times10^{-5}\ \text{m}^2$. $\Delta F_\perp = (1.71\times10^4\ \text{Pa})(9.42\times10^{-5}\ \text{m}^2) = 1.61\ \text{N}$.

EVALUATE: The pressure difference is about $\frac{1}{6}$ atm .

14.11. **IDENTIFY:** Apply $p = p_0 + \rho gh$.

SET UP: Gauge pressure is $p - p_{\text{air}}$.

EXECUTE: The pressure difference between the top and bottom of the tube must be at least 5980 Pa in order to force fluid into the vein: $\rho gh = 5980$ Pa and

$$h = \frac{5980\ \text{Pa}}{gh} = \frac{5980\ \text{N/m}^2}{(1050\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 0.581\ \text{m} .$$

EVALUATE: The bag of fluid is typically hung from a vertical pole to achieve this height above the patient's arm.

14.12. **IDENTIFY:** $p_0 = p_{\text{surface}} + \rho gh$ where p_{surface} is the pressure at the surface of a liquid and p_0 is the pressure at a depth h below the surface.

SET UP: The density of water is $1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** For the oil layer, $p_{\text{surface}} = p_{\text{atm}}$ and p_0 is the pressure at the oil-water interface.

$p_0 - p_{\text{atm}} = p_{\text{gauge}} = \rho gh = (600\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.120\ \text{m}) = 706\ \text{Pa}$

(b) For the water layer, $p_{\text{surface}} = 706\ \text{Pa} + p_{\text{atm}}$.

$p_0 - p_{\text{atm}} = p_{\text{gauge}} = 706\ \text{Pa} + \rho gh = 706\ \text{Pa} + (1.00\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.250\ \text{m}) = 3.16\times10^3\ \text{Pa}$

EVALUATE: The gauge pressure at the bottom of the barrel is due to the combined effects of the oil layer and water layer. The pressure at the bottom of the oil layer is the pressure at the top of the water layer

14.13. **IDENTIFY:** An inflation to 32.0 pounds means a gauge pressure of 32.0 lb/in.^2. The contact area A with the pavement is related to the gauge pressure $p - p_0$ in the tire and the force $F_\perp$ the tire exerts on the pavement by $F_\perp = (p - p_0)A$. By Newton's third law the magnitude of the force the tire exerts on the pavement equals the magnitude of the force the pavement exerts on the car, and this must equal the weight of the car.

SET UP: $14.7 \text{ lb/in.}^2 = 1.013\times10^5 \text{ Pa} = 1 \text{ atm}$. Assume $p_0 = 1 \text{ atm}$.

EXECUTE: **(a)** The gauge pressure is $32.0 \text{ lb/in.}^2 = 2.21\times10^5 \text{ Pa} = 2.18 \text{ atm}$. The absolute pressure is $46.7 \text{ lb/in.}^2 = 3.22\times10^5 \text{ Pa} = 3.18 \text{ atm}$.

(b) No, the tire would touch the pavement at a single point and the contact area would be zero.

(c) $F_\perp = mg = 9.56\times10^3 \text{ N}$. $A = \dfrac{F_\perp}{p - p_0} = \dfrac{9.56\times10^3 \text{ N}}{2.21\times10^5 \text{ Pa}} = 0.0433 \text{ m}^2 = 433 \text{ cm}^2$.

EVALUATE: If the contact area is square, the length of each side for each tire is $\sqrt{\dfrac{433 \text{ cm}^2}{4}} = 10.4 \text{ cm}$. This is a realistic value, based on our observation of the tires of cars.

14.14. **IDENTIFY** and **SET UP:** Use Eq.(14.8) to calculate the gauge pressure at this depth. Use Eq.(14.3) to calculate the force the inside and outside pressures exert on the window, and combine the forces as vectors to find the net force.

EXECUTE: **(a)** gauge pressure $= p - p_0 = \rho gh$ From Table 14.1 the density of seawater is $1.03\times10^3 \text{ kg/m}^3$, so

$$p - p_0 = \rho gh = (1.03\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(250 \text{ m}) = 2.52\times10^6 \text{ Pa}$$

(b) The force on each side of the window is $F = pA$. Inside the pressure is p_0 and outside in the water the pressure is $p = p_0 + \rho gh$. The forces are shown in Figure 14.14.

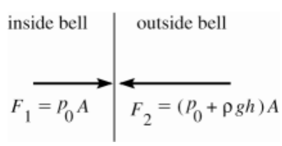


Figure 14.14

The net force is

$F_2 - F_1 = (p_0 + \rho gh)A - p_0 A = (\rho gh)A$

$F_2 - F_1 = (2.52\times10^6 \text{ Pa})\pi(0.150 \text{ m})^2$

$F_2 - F_1 = 1.78\times10^5 \text{ N}$

EVALUATE: The pressure at this depth is very large, over 20 times normal air pressure, and the net force on the window is huge. Diving bells used at such depths must be constructed to withstand these large forces.

14.15. **IDENTIFY:** $p_{\text{gauge}} = p_0 - p_{\text{atm}} = \rho gh$.

SET UP: $1 \text{ atm} = 1.013\times10^5 \text{ Pa}$. The density of water is $1.00\times10^3 \text{ kg/m}^3$. The gauge pressure must equal the pressure difference due to a column of water $1370 \text{ m} - 730 \text{ m} = 640 \text{ m}$ tall.

EXECUTE: $(1.00\times10^3 \text{ m}^3)(9.80 \text{ m/s}^2)(640 \text{ m}) = 6.27\times10^6 \text{ Pa} = 61.9 \text{ atm}$

EVALUATE: The gauge pressure required is directly proportional to the height to which the water is pumped.

14.16. **IDENTIFY** and **SET UP:** Use Eq.(14.6) to calculate the pressure at the specified depths in the open tube. The pressure is the same at all points the same distance from the bottom of the tubes, so the pressure calculated in part (b) is the pressure in the tank. Gauge pressure is the difference between the absolute pressure and air pressure.

EXECUTE: $p_\text{a} = 980 \text{ millibar} = 9.80\times10^4 \text{ Pa}$

(a) Apply $p = p_0 + \rho gh$ to the right-hand tube. The top of this tube is open to the air so $p_0 = p_\text{a}$. The density of the liquid (mercury) is $13.6\times10^3 \text{ kg/m}^3$.

Thus $p = 9.80\times10^4 \text{ Pa} + (13.6\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.0700 \text{ m}) = 1.07\times10^5 \text{ Pa}$.

(b) $p = p_0 + \rho gh = 9.80\times10^4 \text{ Pa} + (13.6\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.0400 \text{ m}) = 1.03\times10^5 \text{ Pa}$.

(c) Since $y_2 - y_1 = 4.00 \text{ cm}$ the pressure at the mercury surface in the left-hand end tube equals that calculated in part (b). Thus the absolute pressure of gas in the tank is $1.03\times10^5 \text{ Pa}$.

(d) $p - p_0 = \rho gh = (13.6\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.0400 \text{ m}) = 5.33\times10^3 \text{ Pa}$.

EVALUATE: If Eq.(14.8) is evaluated with the density of mercury and $p - p_\text{a} = 1 \text{ atm} = 1.01\times10^5 \text{ Pa}$, then $h = 76 \text{ cm}$. The mercury columns here are much shorter than 76 cm, so the gauge pressures are much less than $1.0\times10^5 \text{ Pa}$.

14.17. **IDENTIFY:** Apply $p = p_0 + \rho gh$.

SET UP: For water, $\rho = 1.00\times10^3 \text{ kg/m}^3$.

EXECUTE: $p - p_{\text{air}} = \rho gh = (1.00\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(6.1 \text{ m}) = 6.0\times10^4 \text{ Pa}$.

EVALUATE: The pressure difference increases linearly with depth.

14.18. **IDENTIFY** and **SET UP:** Apply Eq.(14.6) to the water and mercury columns. The pressure at the bottom of the water column is the pressure at the top of the mercury column.

EXECUTE: With just the mercury, the gauge pressure at the bottom of the cylinder is $p = p_0 + \rho_m g h_m$. With the water to a depth h_w, the gauge pressure at the bottom of the cylinder is $p = p_0 + \rho_m g h_m + \rho_w g h_w$. If this is to be double the first value, then $\rho_w g h_w = \rho_m g h_m$.

$$h_w = h_m(\rho_m/\rho_w) = (0.0500 \text{ m})(13.6\times10^3/1.00\times10^3) = 0.680 \text{ m}$$

The volume of water is $V = hA = (0.680 \text{ m})(12.0\times10^{-4} \text{ m}^2) = 8.16\times10^{-4} \text{ m}^3 = 816 \text{ cm}^3$

EVALUATE: The density of mercury is 13.6 times the density of water and $(13.6)(5 \text{ cm}) = 68 \text{ cm}$, so the pressure increase from the top to the bottom of a 68-cm tall column of water is the same as the pressure increase from top to bottom for a 5-cm tall column of mercury.

14.19. **IDENTIFY:** Assume the pressure at the upper surface of the ice is $p_0 = 1.013\times10^5$ Pa. The pressure at the surface of the water is increased from p_0 by $\rho_{ice} g h_{ice}$ and then increases further with depth in the water.

SET UP: $\rho_{ice} = 0.92\times10^3 \text{ kg/m}^3$ and $\rho_{water} = 1.00\times10^3 \text{ kg/m}^3$.

EXECUTE: $p - p_0 = \rho_{ice} g h_{ice} + \rho_{water} g h_{water}$.

$p - p_0 = (0.92\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1.75 \text{ m}) + (1.00\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(2.50 \text{ m})$.

$p - p_0 = 4.03\times10^4$ Pa.

$p = p_0 + 4.03\times10^4 \text{ Pa} = 1.42\times10^5$ Pa.

EVALUATE: The gauge pressure at the surface of the water must be sufficient to apply an upward force on a section of ice equal to the weight of that section.

14.20. **IDENTIFY:** Apply $p = p_0 + \rho g h$, where p_0 is the pressure at the surface of the fluid. Gauge pressure is $p - p_{air}$.

SET UP: For water, $\rho = 1.00\times10^3 \text{ kg/m}^3$.

EXECUTE: **(a)** The pressure difference between the surface of the water and the bottom is due to the weight of the water and is still 2500 Pa after the pressure increase above the surface. But the surface pressure increase is also transmitted to the fluid, making the total difference from atmospheric pressure $2500 \text{ Pa} + 1500 \text{ Pa} = 4000 \text{ Pa}$.

(b) Initially, the pressure due to the water alone is 2500 Pa $= \rho g h$. Thus $h = \dfrac{2500 \text{ N/m}^2}{(1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 0.255 \text{ m}.$

To keep the bottom gauge pressure at 2500 Pa after the 1500 Pa increase at the surface, the pressure due to the water's weight must be reduced to 1000 Pa: $h = \dfrac{1000 \text{ N/m}^2}{(1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 0.102 \text{ m}.$ Thus the water must be lowered by $0.255 \text{ m} - 0.102 \text{ m} = 0.153 \text{ m}.$.

EVALUATE: Note that $\rho g h$, with $h = 0.153$ m, is 1500 Pa.

14.21. **IDENTIFY:** $p = p_0 + \rho g h$. $F = pA$.

SET UP: For seawater, $\rho = 1.03\times10^3 \text{ kg/m}^3$

EXECUTE: The force F that must be applied is the difference between the upward force of the water and the downward forces of the air and the weight of the hatch. The difference between the pressure inside and out is the gauge pressure, so

$$F = (\rho g h)A - w = (1.03\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(30 \text{ m})(0.75 \text{ m}^2) - 300 \text{ N} = 2.27\times10^5 \text{ N}.$$

EVALUATE: The force due to the gauge pressure of the water is much larger than the weight of the hatch and would be impossible for the crew to apply it just by pushing.

14.22. **IDENTIFY:** The force on an area A due to pressure p is $F_\perp = pA$. Use $p - p_0 = \rho g h$ to find the pressure inside the tank, at the bottom.

SET UP: $1 \text{ atm} = 1.013\times10^5$ Pa. For benzene, $\rho = 0.90\times10^3 \text{ kg/m}^3$. The area of the bottom of the tank is $\pi D^2/4$, where $D = 1.72$ m. The area of the vertical walls of the tank is πDL, where $L = 11.50$ m.

EXECUTE: **(a)** At the bottom of the tank,

$p = p_0 + \rho g h = 92(1.013\times10^5 \text{ Pa}) + (0.90\times10^3 \text{ kg/m}^3)(0.894)(9.80 \text{ m/s}^2)(11.50 \text{ m}).$

$p = 9.32\times10^6 \text{ Pa} + 9.07\times10^4 \text{ Pa} = 9.41\times10^6$ Pa. $F_\perp = pA = (9.41\times10^6 \text{ Pa})\pi(1.72 \text{ m})^2/4 = 2.19\times10^7$ N.

(b) At the outside surface of the bottom of the tank, the air pressure is $p = (92)(1.013\times10^5 \text{ Pa}) = 9.32\times10^6$ Pa.

$F_\perp = pA = (9.32\times10^6 \text{ Pa})\pi(1.72 \text{ m})^2/4 = 2.17\times10^7$ N.

(c) $F_{\perp} = pA = 92(1.013\times10^5\ \text{Pa})\pi(1.72\ \text{m})(11.5\ \text{m}) = 5.79\times10^8\ \text{N}$

EVALUATE: Most of the force in part (a) is due to the 92 atm of air pressure above the surface of the benzene and the net force on the bottom of the tank is much less than the inward and outward forces.

14.23. **IDENTIFY:** The gauge pressure at the top of the oil column must produce a force on the disk that is equal to its weight.

SET UP: The area of the bottom of the disk is $A = \pi r^2 = \pi(0.150\ \text{m})^2 = 0.0707\ \text{m}^2$.

EXECUTE: **(a)** $p - p_0 = \dfrac{w}{A} = \dfrac{45.0\ \text{N}}{0.0707\ \text{m}^2} = 636\ \text{Pa}$.

(b) The increase in pressure produces a force on the disk equal to the increase in weight. By Pascal's law the increase in pressure is transmitted to all points in the oil.

(i) $\Delta p = \dfrac{83.0\ \text{N}}{0.0707\ \text{m}^2} = 1170\ \text{Pa}$. (ii) 1170 Pa

EVALUATE: The absolute pressure at the top of the oil produces an upward force on the disk but this force is partially balanced by the force due to the air pressure at the top of the disk.

14.24. **IDENTIFY:** $F_2 = \dfrac{A_2}{A_1}F_1$. F_2 must equal the weight $w = mg$ of the car.

SET UP: $A = \pi D^2/4$. D_1 is the diameter of the vessel at the piston where F_1 is applied and D_2 of the diameter at the car.

EXECUTE: $mg = \dfrac{\pi D_2^2/4}{\pi D_1^2/4}F_1$. $\dfrac{D_2}{D_1} = \sqrt{\dfrac{mg}{F_1}} = \sqrt{\dfrac{(1520\ \text{kg})(9.80\ \text{m/s}^2)}{125\ \text{N}}} = 10.9$

EVALUATE: The diameter is smaller where the force is smaller, so the pressure will be the same at both pistons.

14.25. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the piston, with $+y$ upward. $F = pA$.

SET UP: $1\ \text{atm} = 1.013\times10^5\ \text{Pa}$. The force diagram for the piston is given in Figure 14.25. p is the absolute pressure of the hydraulic fluid.

EXECUTE: $pA - w - p_{\text{atm}}A = 0$ and $p - p_{\text{atm}} = p_{\text{gauge}} = \dfrac{w}{A} = \dfrac{mg}{\pi r^2} = \dfrac{(1200\ \text{kg})(9.80\ \text{m/s}^2)}{\pi(0.15\ \text{m})^2} = 1.7\times10^5\ \text{Pa} = 1.7\ \text{atm}$

EVALUATE: The larger the diameter of the piston, the smaller the gauge pressure required to lift the car.

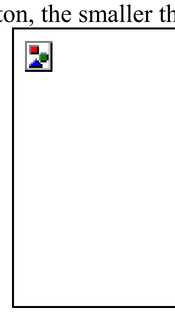


Figure 14.25

14.26. **IDENTIFY:** Apply Newton's 2nd law to the woman plus slab. The buoyancy force exerted by the water is upward and given by $B = \rho_{\text{water}}V_{\text{displ}}g$, where V_{displ} is the volume of water displaced.

SET UP: The floating object is the slab of ice plus the woman; the buoyant force must support both. The volume of water displaced equals the volume V_{ice} of the ice. The free-body diagram is given in Figure 14.26.

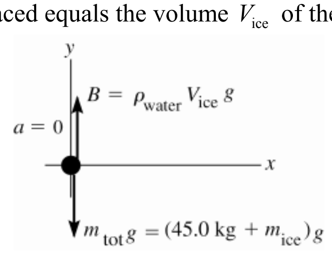


Figure 14.26

EXECUTE: $\sum F_y = ma_y$

$B - m_{\text{tot}}g = 0$

$\rho_{\text{water}}V_{\text{ice}}g = (45.0\ \text{kg} + m_{\text{ice}})g$

But $\rho = m/V$ so $m_{\text{ice}} = p_{\text{ice}}V_{\text{ice}}$

$V_{\text{ice}} = \dfrac{45.0\ \text{kg}}{\rho_{\text{water}} - \rho_{\text{ice}}} = \dfrac{45.0\ \text{kg}}{1000\ \text{kg/m}^3 - 920\ \text{kg/m}^3} = 0.562\ \text{m}^3$.

EVALUATE: The mass of ice is $m_{\text{ice}} = \rho_{\text{ice}}V_{\text{ice}} = 517\ \text{kg}$.

14.27. IDENTIFY: Apply $\sum F_y = ma_y$ to the sample, with $+y$ upward. $B = \rho_{\text{water}} V_{\text{obj}} g$.

SET UP: $w = mg = 17.50\text{ N}$ and $m = 1.79\text{ kg}$.

EXECUTE: $T + B - mg = 0$. $B = mg - T = 17.50\text{ N} - 11.20\text{ N} = 6.30\text{ N}$.

$$V_{\text{obj}} = \frac{B}{\rho_{\text{water}} g} = \frac{6.30\text{ N}}{(1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 6.43\times10^{-4}\text{ m}^3.$$

$$\rho = \frac{m}{V} = \frac{1.79\text{ kg}}{6.43\times10^{-4}\text{ m}^3} = 2.78\times10^3\text{ kg/m}^3.$$

EVALUATE: The density of the sample is greater than that of water and it doesn't float.

14.28. IDENTIFY: The upward buoyant force *B* exerted by the liquid equals the weight of the fluid displaced by the object. Since the object floats the buoyant force equals its weight.

SET UP: Glycerin has density $\rho_{\text{gly}} = 1.26\times10^3\text{ kg/m}^3$ and seawater has density $\rho_{\text{sw}} = 1.03\times10^3\text{ kg/m}^3$. Let V_{obj} be the volume of the apparatus. $g_{\text{E}} = 9.80\text{ m/s}^2$; $g_{\text{C}} = 4.15\text{ m/s}^2$. Let V_{sub} be the volume submerged on Caasi.

EXECUTE: On earth $B = \rho_{\text{sw}}(0.250V_{\text{obj}})g_{\text{E}} = mg_{\text{E}}$. $m = (0.250)\rho_{\text{obj}}V_{\text{sw}}$. On Caasi, $B = \rho_{\text{gly}}V_{\text{sub}}g_{\text{C}} = mg_{\text{C}}$. $m = \rho_{\text{gyl}}V_{\text{sub}}$. The two expressions for *m* must be equal, so $(0.250)V_{\text{obj}}\rho_{\text{sw}} = \rho_{\text{gly}}V_{\text{sub}}$ and

$$V_{\text{sub}} = \left(\frac{0.250\rho_{\text{sw}}}{\rho_{\text{gly}}}\right)V_{\text{obj}} = \left(\frac{[0.250][1.03\times10^3\text{ kg/m}^3]}{1.26\times10^3\text{ kg/m}^3}\right)V_{\text{obj}} = 0.204V_{\text{obj}}.$$ 20.4% of the volume will be submerged on Caasi.

EVALUATE: Less volume is submerged in glycerin since the density of glycerin is greater than the density of seawater. The value of *g* on each planet cancels out and has no effect on the answer. The value of *g* changes the weight of the apparatus and the buoyant force by the same factor.

14.29. IDENTIFY: For a floating object, the weight of the object equals the upward buoyancy force, *B*, exerted by the fluid.

SET UP: $B = \rho_{\text{fluid}}V_{\text{submerged}}g$. The weight of the object can be written as $w = \rho_{\text{object}}V_{\text{object}}g$. For seawater, $\rho = 1.03\times10^3\text{ kg/m}^3$.

EXECUTE: **(a)** The displaced fluid must weigh more than the object, so $\rho < \rho_{\text{fluid}}$.

(b) If the ship does not leak, much of the water will be displaced by air or cargo, and the average density of the floating ship is less than that of water.

(c) Let the portion submerged have volume *V*, and the total volume be V_0. Then $\rho V_0 = \rho_{\text{fluid}}V$, so $\frac{V}{V_0} = \frac{\rho}{\rho_{\text{fluid}}}$ The fraction above the fluid surface is then $1 - \frac{\rho}{\rho_{\text{fluid}}}$. If $\rho \to 0$, the entire object floats, and if $\rho \to \rho_{\text{fluid}}$, none of the object is above the surface.

(d) Using the result of part (c), $1 - \frac{\rho}{\rho_{\text{fluid}}} = 1 - \frac{(0.042\text{ kg})/([5.0][4.0][3.0]\times10^{-6}\text{m}^3)}{1030\text{kg/m}^3} = 0.32 = 32\%.$

EVALUATE: For a given object, the fraction of the object above the surface increases when the density of the fluid in which it floats increases.

14.30. IDENTIFY: $B = \rho_{\text{water}}V_{\text{obj}}g$. The net force on the sphere is zero.

SET UP: The density of water is $1.00\times10^3\text{ kg/m}^3$.

EXECUTE: **(a)** $B = (1000\text{ kg/m}^3)(0.650\text{ m}^3)(9.80\text{ m/s}^2) = 6.37\times10^3\text{ N}$

(b) $B = T + mg$ and $m = \frac{B-T}{g} = \frac{6.37\times10^3\text{ N} - 900\text{ N}}{9.80\text{ m/s}^2} = 558\text{ kg}$.

(c) Now $B = \rho_{\text{water}}V_{\text{sub}}g$, where V_{sub} is the volume of the sphere that is submerged. $B = mg$. $\rho_{\text{water}}V_{\text{sub}} = mg$ and

$$V_{\text{sub}} = \frac{m}{\rho_{\text{water}}} = \frac{558\text{ kg}}{1000\text{ kg/m}^3} = 0.558\text{ m}^3.\ \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{0.558\text{ m}^3}{0.650\text{ m}^3} = 0.858 = 85.8\%.$$

EVALUATE: The average density of the sphere is $\rho_{\text{sph}} = \frac{m}{V} = \frac{558\text{ kg}}{0.650\text{ m}^3} = 858\text{ kg/m}^3$. $\rho_{\text{sph}} < \rho_{\text{water}}$, and that is why it floats with 85.8% of its volume submerged.

14.31. IDENTIFY and SET UP: Use Eq.(14.8) to calculate the gauge pressure at the two depths.

(a) The distances are shown in Figure 14.31a.

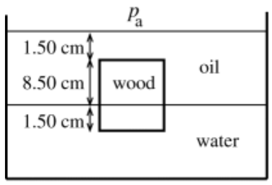


Figure 14.31a

EXECUTE: $p - p_0 = \rho g h$

The upper face is 1.50 cm below the top of the oil, so

$p - p_0 = (7.90\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0150\ \text{m})$

$p - p_0 = 116\ \text{Pa}$

(b) The pressure at the interface is $p_{\text{interface}} = p_a + \rho_{\text{oil}} g (0.100\ \text{m})$. The lower face of the block is 1.50 cm below the interface, so the pressure there is $p = p_{\text{interface}} + \rho_{\text{water}} g (0.0150\ \text{m})$. Combining these two equations gives

$p - p_a = \rho_{\text{oil}} g (0.100\ \text{m}) + \rho_{\text{water}} g (0.0150\ \text{m})$

$p - p_a = [(790\ \text{kg/m}^3)(0.100\ \text{m}) + (1000\ \text{kg/m}^3)(0.0150\ \text{m})](9.80\ \text{m/s}^2)$

$p - p_a = 921\ \text{Pa}$

(c) IDENTIFY and SET UP: Consider the forces on the block. The area of each face of the block is $A = (0.100\ \text{m})^2 = 0.0100\ \text{m}^2$. Let the absolute pressure at the top face be p_t and the pressure at the bottom face be p_b. In Eq.(14.3) use these pressures to calculate the force exerted by the fluids at the top and bottom of the block. The free-body diagram for the block is given in Figure 14.31b.

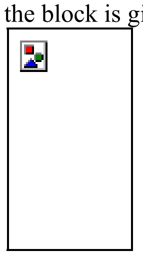


Figure 14.31b

EXECUTE: $\sum F_y = m a_y$

$p_b A - p_t A - mg = 0$

$(p_b - p_t) A = mg$

Note that $(p_b - p_t) = (p_b - p_a) - (p_t - p_a) = 921\ \text{Pa} - 116\ \text{Pa} = 805\ \text{Pa}$; the difference in absolute pressures equals the difference in gauge pressures.

$$m = \frac{(p_b - p_t)A}{g} = \frac{(805\ \text{Pa})(0.0100\ \text{m}^2)}{9.80\ \text{m/s}^2} = 0.821\ \text{kg}.$$

And then $\rho = m/V = 0.821\ \text{kg}/(0.100\ \text{m})^3 = 821\ \text{kg/m}^3$.

EVALUATE: We can calculate the buoyant force as $B = (\rho_{\text{oil}} V_{\text{oil}} + \rho_{\text{water}} V_{\text{water}}) g$ where $V_{\text{oil}} = (0.0100\ \text{m}^2)(0.850\ \text{m}) = 8.50 \times 10^{-4}\ \text{m}^3$ is the volume of oil displaced by the block and $V_{\text{water}} = (0.0100\ \text{m}^2)(0.0150\ \text{m}) = 1.50 \times 10^{-4}\ \text{m}^3$ is the volume of water displaced by the block. This gives $B = (0.821\ \text{kg})g$. The mass of water displaced equals the mass of the block.

14.32. IDENTIFY: The sum of the vertical forces on the ingot is zero. $\rho = m/V$. The buoyant force is $B = \rho_{\text{water}} V_{\text{obj}} g$.

SET UP: The density of aluminum is $2.7 \times 10^3\ \text{kg/m}^3$. The density of water is $1.00 \times 10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** $T = mg = 89\ \text{N}$ so $m = 9.08\ \text{kg}$. $V = \dfrac{m}{\rho} = \dfrac{9.08\ \text{kg}}{2.7 \times 10^3\ \text{kg/m}^3} = 3.36 \times 10^{-3}\ \text{m}^3 = 3.4\ \text{L}$.

(b) When the ingot is totally immersed in the water while suspended, $T + B - mg = 0$.

$B = \rho_{\text{water}} V_{\text{obj}} g = (1.00 \times 10^3\ \text{kg/m}^3)(3.36 \times 10^{-3}\ \text{m}^3)(9.80\ \text{m/s}^2) = 32.9\ \text{N}$. $T = mg - B = 89\ \text{N} - 32.9\ \text{N} = 56\ \text{N}$.

EVALUATE: The buoyant force is equal to the difference between the apparent weight when the object is submerged in the fluid and the actual gravity force on the object.

14.33. IDENTIFY: The vertical forces on the rock sum to zero. The buoyant force equals the weight of liquid displaced by the rock. $V = \frac{4}{3}\pi R^3$.

SET UP: The density of water is $1.00 \times 10^3\ \text{kg/m}^3$.

EXECUTE: The rock displaces a volume of water whose weight is $39.2\ \text{N} - 28.4\ \text{N} = 10.8\ \text{N}$. The mass of this much water is thus $10.8\ \text{N}/(9.80\ \text{m/s}^2) = 1.102\ \text{kg}$ and its volume, equal to the rock's volume, is $\dfrac{1.102\ \text{kg}}{1.00 \times 10^3\ \text{kg/m}^3} = 1.102 \times 10^{-3}\ \text{m}^3$. The weight of unknown liquid displaced is $39.2\ \text{N} - 18.6\ \text{N} = 20.6\ \text{N}$, and its mass is $20.6\ \text{N}/(9.80\ \text{m/s}^2) = 2.102\ \text{kg}$. The liquid's density is thus $2.102\ \text{kg}/(1.102 \times 10^{-3}\ \text{m}^3) = 1.91 \times 10^3\ \text{kg/m}^3$.

EVALUATE: The density of the unknown liquid is roughly twice the density of water.

14.34. IDENTIFY: The volume flow rate is Av.

SET UP: $Av = 0.750\ \text{m/s}^3$. $A = \pi D^2/4$.

EXECUTE: **(a)** $v\pi D^2/4 = 0.750\ \text{m/s}^3$. $v = \dfrac{4(0.750\ \text{m/s}^3)}{\pi(4.50\times10^{-2}\ \text{m})^2} = 472\ \text{m/s}$.

(b) vD^2 must be constant, so $v_1D_1^2 = v_2D_2^2$. $v_2 = v_1\left(\dfrac{D_1}{D_2}\right)^2 = (472\ \text{m/s})\left(\dfrac{D_1}{3D_1}\right)^2 = 52.4\ \text{m/s}$.

EVALUATE: The larger the hole, the smaller the speed of the fluid as it exits.

14.35. IDENTIFY: Apply the equation of continuity, $v_1A_1 = v_2A_2$.

SET UP: $A = \pi r^2$

EXECUTE: $v_2 = v_1(A_1/A_2)$. $A_1 = \pi(0.80\ \text{cm})^2$, $A_2 = 20\pi(0.10\ \text{cm})^2$. $v_2 = (3.0\ \text{m/s})\dfrac{\pi(0.80)^2}{20\pi(0.10)^2} = 9.6\ \text{m/s}$.

EVALUATE: The total area of the shower head openings is less than the cross section area of the pipe, and the speed of the water in the shower head opening is greater than its speed in the pipe.

14.36. IDENTIFY: $v_1A_1 = v_2A_2$. The volume flow rate is vA.

SET UP: $1.00\ \text{h} = 3600\ \text{s}$.

EXECUTE: **(a)** $v_2 = v_1\left(\dfrac{A_1}{A_2}\right) = (3.50\ \text{m/s})\left(\dfrac{0.070\ \text{m}^2}{0.105\ \text{m}^2}\right) = 2.33\ \text{m/s}$

(b) $v_2 = v_1\left(\dfrac{A_1}{A_2}\right) = (3.50\ \text{m/s})\left(\dfrac{0.070\ \text{m}^2}{0.047\ \text{m}^2}\right) = 5.21\ \text{m/s}$

(c) $V = v_1A_1t = (3.50\ \text{m/s})(0.070\ \text{m}^2)(3600\ \text{s}) = 882\ \text{m}^3$.

EVALUATE: The equation of continuity says the volume flow rate is the same at all points in the pipe.

14.37. IDENTIFY and SET UP: Apply Eq.(14.10). In part (a) the target variable is V. In part (b) solve for A and then from that get the radius of the pipe.

EXECUTE: **(a)** $vA = 1.20\ \text{m}^3\text{/s}$

$$v = \frac{1.20\ \text{m}^3\text{/s}}{A} = \frac{1.20\ \text{m}^3\text{/s}}{\pi r^2} = \frac{1.20\ \text{m}^3\text{/s}}{\pi(0.150\ \text{m})^2} = 17.0\ \text{m/s}$$

(b) $vA = 1.20\ \text{m}^3\text{/s}$

$v\pi r^2 = 1.20\ \text{m}^3\text{/s}$

$$r = \sqrt{\frac{1.20\ \text{m}^3\text{/s}}{v\pi}} = \sqrt{\frac{1.20\ \text{m}^3\text{/s}}{(3.80\ \text{m/s})\pi}} = 0.317\ \text{m}$$

EVALUATE: The speed is greater where the area and radius are smaller.

14.38. IDENTIFY: The volume flow rate is equal to Av .

SET UP: In the equation preceding Eq.(14.10), label the densities of the two points ρ_1 and ρ_2 .

EXECUTE: **(a)** From the equation preceding Eq.(14.10), dividing by the time interval dt gives Eq.(14.12).

(b) The volume flow rate decreases by 1.50%.

EVALUATE: When the density increases, the volume flow rate decreases; it is the mass flow rate that remains constant.

14.39. IDENTIFY and SET UP:

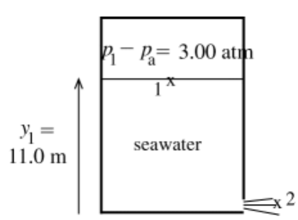


Figure 14.39

Apply Bernoulli's equation with points 1 and 2 chosen as shown in Figure 14.39. Let $y = 0$ at the bottom of the tank so $y_1 = 11.0\ \text{m}$ and $y_2 = 0$. The target variable is v_2.

$p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$A_1v_1 = A_2v_2$, so $v_1 = (A_2/A_1)v_2$. But the cross-section area of the tank (A_1) is much larger than the cross-section area of the hole (A_2), so $v_1 << v_2$ and the $\frac{1}{2}\rho v_1^2$ term can be neglected.

EXECUTE: This gives $\frac{1}{2}\rho v_2^2 = (p_1 - p_2) + \rho g y_1$.

Use $p_2 = p_a$ and solve for v_2:

$$v_2 = \sqrt{2(p_1 - p_a)/\rho + 2gy_1} = \sqrt{\frac{2(3.039\times 10^5\ \text{Pa})}{1030\ \text{kg/m}^3} + 2(9.80\ \text{m/s}^2)(11.0\ \text{m})}$$

$v_2 = 28.4$ m/s

EVALUATE: If the pressure at the top surface of the water were air pressure, then Toricelli's theorem (Example 14.8) gives $v_2 = \sqrt{2g(y_1 - y_2)} = 14.7$ m/s. The actual afflux speed is much larger than this due to the excess pressure at the top of the tank.

14.40. **IDENTIFY:** Toricelli's theorem says the speed of efflux is $v = \sqrt{2gh}$, where h is the distance of the small hole below the surface of the water in the tank. The volume flow rate is vA.

SET UP: $A = \pi D^2/4$, with $D = 6.00\times 10^{-3}$ m.

EXECUTE: **(a)** $v = \sqrt{2(9.80\ \text{m/s}^2)(14.0\ \text{m})} = 16.6$ m/s

(b) $vA = (16.6\ \text{m/s})\pi(6.00\times 10^{-3}\ \text{m})^2/4 = 4.69\times 10^{-4}\ \text{m}^3/\text{s}$. A volume of $4.69\times 10^{-4}\ \text{m}^3 = 0.469$ L is discharged each second.

EVALUATE: We have assumed that the diameter of the hole is much less than the diameter of the tank.

14.41. **IDENTIFY** and **SET UP:**

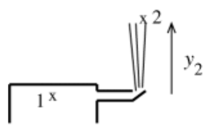


Figure 14.41

Apply Bernoulli's equation to points 1 and 2 as shown in Figure 14.41. Point 1 is in the mains and point 2 is at the maximum height reached by the stream, so $v_2 = 0$.

Solve for p_1 and then convert this absolute pressure to gauge pressure.

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

Let $y_1 = 0$, $y_2 = 15.0$ m. The mains have large diameter, so $v_1 \approx 0$.

Thus $p_1 = p_2 + \rho g y_2$.

But $p_2 = p_a$, so $p_1 - p_a = \rho g y_2 = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(15.0\ \text{m}) = 1.47\times 10^5$ Pa.

EVALUATE: This is the gauge pressure at the bottom of a column of water 15.0 m high.

14.42. **IDENTIFY:** Apply Bernoulli's equation to the two points.

SET UP: The continuity equation says $v_1A_1 = v_2A_2$. In Eq.(14.17) either absolute or gauge pressures can be used at both points.

EXECUTE: Using $v_2 = \frac{1}{4}v_1$,

$$p_2 = p_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(y_1 - y_2) = p_1 + \rho\left[\left(\frac{15}{32}\right)v_1^2 + g(y_1 - y_2)\right]$$

$$p_2 = 5.00\times 10^4\ \text{Pa} + (1.00\times 10^3\ \text{kg/m}^3)\left(\frac{15}{32}(3.00\ \text{m/s})^2 + (9.80\ \text{m/s}^2)(11.0\ \text{m})\right) = 1.62\times 10^5\ \text{Pa}.$$

EVALUATE: The decrease in speed and the decrease in height at point 2 both cause the pressure at point 2 to be greater than the pressure at point 1.

14.43. **IDENTIFY:** Apply Bernoulli's equation to the air flowing past the wing. $F = pA$.

SET UP: Let point 1 be at the top surface and point 2 be at the bottom surface. Neglect the $\rho g(y_1 - y_2)$ term in Bernoulli's equation. In calculating the net force take $+y$ to be upward.

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$.

$p_2 - p_1 = \frac{1}{2}\rho(v_1^2 - v_2^2) = \frac{1}{2}(1.20\ \text{kg/m}^3)([70.0\ \text{m/s}]^2 - [60.0\ \text{m/s}]^2) = 780$ Pa.

The net force exerted by the air is $p_2A - p_1A = (780\ \text{Pa})(16.2\ \text{m}^2) = 12{,}600$ N. The net force is upward.

EVALUATE: The pressure is lower where the fluid speed is higher.

14.44. **IDENTIFY:** $\rho = m/V$. Apply the equation of continuity and Bernoulli's equation to points 1 and 2.

SET UP: The density of water is 1 kg/L.

EXECUTE: **(a)** $\dfrac{(220)(0.355\ \text{kg})}{60.0\ \text{s}} = 1.30$ kg/s.

(b) The density of the liquid is $\dfrac{0.355\text{ kg}}{0.355\times10^{-3}\text{ m}^3}=1000\text{ kg/m}^3$, and so the volume flow rate is

$\dfrac{1.30\text{ kg/s}}{1000\text{ kg/m}^3}=1.30\times10^{-3}\text{ m}^3\text{/s}=1.30\text{ L/s}$. This result may also be obtained from $\dfrac{(220)(0.355\text{ L})}{60.0\text{ s}}=1.30\text{ L/s}$.

(c) $v_1=\dfrac{1.30\times10^{-3}\text{ m}^3\text{/s}}{2.00\times10^{-4}\text{m}^2}=6.50\text{ m/s}$. $v_2=v_1/4=1.63\text{ m/s}$.

(d) $p_1=p_2+\dfrac{1}{2}\rho\left(v_2^2-v_1^2\right)+\rho g\left(y_2-y_1\right)$.

$p_1=152\text{ kPa}+(1000\text{ kg/m}^3)\left(\tfrac{1}{2}[(1.63\text{ m/s})^2-(6.50\text{ m/s})^2]+(9.80\text{ m/s}^2)(-1.35\text{ m})\right)$. $p_1=119\text{ kPa}$.

EVALUATE: The increase in height and the increase in fluid speed at point 1 both cause the pressure at point 1 to be less than the pressure at point 2.

14.45. **IDENTIFY:** Apply Bernoulli's equation to the two points.

SET UP: $y_1=y_2$. $v_1A_1=v_2A_2$. $A_2=2A_1$.

EXECUTE: $p_1+\rho gy_1+\tfrac{1}{2}\rho v_1^2=p_2+\rho gy_2+\tfrac{1}{2}\rho v_2^2$. $v_2=v_1\left(\dfrac{A_1}{A_2}\right)=(2.50\text{ m/s})\left(\dfrac{A_1}{2A_1}\right)=1.25\text{ m/s}$.

$p_2=p_1+\tfrac{1}{2}\rho(v_1^2-v_2^2)=1.80\times10^4\text{ Pa}+\tfrac{1}{2}(1000\text{ kg/m}^3)([2.50\text{ m/s}]^2-[1.25\text{ m/s}]^2)=2.03\times10^4\text{ Pa}$

EVALUATE: The gauge pressure is higher at the second point because the water speed is less there.

14.46. **IDENTIFY** and **SET UP:** Let point 1 be where $r_1=4.00$ cm and point 2 be where $r_2=2.00$ cm. The volume flow rate vA has the value $7200\text{ cm}^3\text{/s}$ at all points in the pipe. Apply Eq.(14.10) to find the fluid speed at points 1 and 2 and then use Bernoulli's equation for these two points to find p_2.

EXECUTE: $v_1A_1=v_1\pi r_1^2=7200\text{ cm}^3$, so $v_1=1.43\text{ m/s}$

$v_2A_2=v_2\pi r_2^2=7200\text{ cm}^3$, so $v_2=5.73\text{ m/s}$

$p_1+\rho gy_1+\tfrac{1}{2}\rho v_1^2=p_2+\rho gy_2+\tfrac{1}{2}\rho v_2^2$

$y_1=y_2$ and $p_1=2.40\times10^5$ Pa, so $p_2=p_1+\tfrac{1}{2}\rho(v_1^2-v_2^2)=2.25\times10^5\text{ Pa}$

EVALUATE: Where the area decreases the speed increases and the pressure decreases.

14.47. **IDENTIFY:** $F=pA$, where A is the cross-sectional area presented by a hemisphere. The force F_{bb} that the body builder must apply must equal in magnitude the net force on each hemisphere due to the air inside and outside the sphere.

SET UP: $A=\pi\dfrac{D^2}{4}$.

EXECUTE: **(a)** $F_{bb}=\left(p_0-p\right)\pi\dfrac{D^2}{4}$.

(b) The force on each hemisphere due to the atmosphere is

$\pi(5.00\times10^{-2}\text{ m})^2(1.013\times10^5\text{ Pa/atm})(0.975\text{ atm})=776\text{ N}$. The bodybuilder must exert this force on each hemisphere to pull them apart.

EVALUATE: The force is about 170 lbs, feasible only for a very strong person. The force required is proportional to the square of the diameter of the hemispheres.

14.48. **IDENTIFY:** Apply $p=p_0+\rho gh$ and $\Delta V=-\dfrac{(\Delta p)V_0}{B}$,where B is the bulk modulus.

SET UP: Seawater has density $\rho=1.03\times10^3\text{ kg/m}^3$. The bulk modulus of water is $B=2.2\times10^9\text{ Pa}$.

$p_{air}=1.01\times10^5\text{ Pa}$.

EXECUTE: **(a)** $p_0=p_{air}+\rho gh=1.01\times10^5\text{ Pa}+(1.03\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(10.92\times10^3\text{ m})=1.10\times10^8\text{ Pa}$

(b) At the surface 1.00 m^3 of seawater has mass 1.03×10^3 kg. At a depth of 10.92 km the change in volume is

$\Delta V=-\dfrac{(\Delta p)V_0}{B}=-\dfrac{(1.10\times10^8\text{ Pa})(1.00\text{ m}^3)}{2.2\times10^9\text{ Pa}}=-0.050\text{ m}^3$. The volume of this mass of water at this depth therefore is $V=V_0+\Delta V=0.950\text{ m}^3$. $\rho=\dfrac{m}{V}=\dfrac{1.03\times10^3\text{ kg}}{0.950\text{ m}^3}=1.08\times10^3\text{ kg/m}^3$. The density is 5% larger than at the surface.

EVALUATE: For water B is small and a very large increase in pressure corresponds to a small fractional change in volume.

14.49. **IDENTIFY:** In part (a), the force is the weight of the water. In part (b), the pressure due to the water at a depth *h* is ρgh. $F = pA$ and $m = \rho V$.

SET UP: The density of water is $1.00 \times 10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** The weight of the water is

$\rho gV = (1.00 \times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)\big((5.00\ \text{m})(4.0\ \text{m})(3.0\ \text{m})\big) = 5.9 \times 10^5\ \text{N}$,

(b) Integration gives the expected result that the force is what it would be if the pressure were uniform and equal to the pressure at the midpoint. If *d* is the depth of the pool and *A* is the area of one end of the pool, then

$$F = \rho gA\frac{d}{2} = (1.00 \times 10^3\text{kg/m}^3)(9.80\ \text{m/s}^2)\big((4.0\ \text{m})(3.0\ \text{m})\big)(1.50\ \text{m}) = 1.76 \times 10^5\ \text{N}.$$

EVALUATE: The answer to part (a) can be obtained as $F = pA$, where $p = \rho gd$ is the gauge pressure at the bottom of the pool and $A = (5.0\ \text{m})(4.0\ \text{m})$ is the area of the bottom of the pool.

14.50. **IDENTIFY:** Use Eq.(14.8) to find the gauge pressure versus depth, use Eq.(14.3) to relate the pressure to the force on a strip of the gate, calculate the torque as force times moment arm, and follow the procedure outlined in the hint to calculate the total torque.

SET UP: The gate is sketched in Figure 14.50a

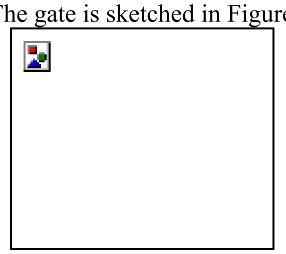


Figure 14.50a

Let τ_u be the torque due to the net force of the water on the upper half of the gate, and τ_1 be the torque due to the force on the lower half.

With the indicated sign convention, τ_1 is positive and τ_u is negative, so the net torque about the hinge is $\tau = \tau_1 - \tau_u$. Let *H* be the height of the gate.

Upper-half of gate:

Calculate the torque due to the force on a narrow strip of height *dy* located a distance *y* below the top of the gate, as shown in Figure 14.50b. Then integrate to get the total torque.

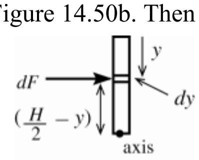


Figure 14.50b

The net force on the strip is $dF = p(y)\, dA$, where $p(y) = \rho gy$ is the pressure at this depth and $dA = W\, dy$ with $W = 4.00\ \text{m}$

$dF = \rho gyW\, dy$

The moment arm is $(H/2 - y)$, so $d\tau = pgW(H/2 - y)y\ dy$.

$$\tau_u = \int_0^{H/2} d\tau = \rho gW\int_0^{H/2}(H/2 - y)y\ dy = \rho gW((H/4)y^2 - y^3/3)\Big|_0^{H/2}$$

$$\tau_u = \rho gW(H^3/16 - H^3/24) = \rho gW(H^3/48)$$

$$\tau_u = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(4.00\ \text{m})(2.00\ \text{m})^3/48 = 6.533 \times 10^3\ \text{N}\cdot\text{m}$$

Lower-half of gate:

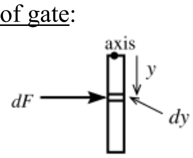


Figure 14.50c

Consider the narrow strip shown in Figure 14.50c
The depth of the strip is $(H/2 + y)$
so the force *dF* is

$dF = p(y)\ dA = \rho g(H/2 + y)W\ dy$

The moment arm is *y*, so $d\tau = \rho gW(H/2 + y)y\ dy$.

$$\tau_1 = \int_0^{H/2} d\tau = \rho gW\int_0^{H/2}(H/2 + y)y\ dy = \rho gW((H/4)y^2 + y^3/3)\Big|_0^{H/2}$$

$$\tau_1 = \rho gW(H^3/16 + H^3/24) = \rho gW(5H^3/48)$$

$$\tau_1 = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(4.00\ \text{m})5(2.00\ \text{m})^3/48 = 3.267 \times 10^4\ \text{N}\cdot\text{m}$$

Then $\tau = \tau_1 - \tau_u = 3.267 \times 10^4\ \text{N}\cdot\text{m} - 6.533 \times 10^3\ \text{N}\cdot\text{m} = 2.61 \times 10^4\ \text{N}\cdot\text{m}$.

EVALUATE: The forces and torques on the upper and lower halves of the gate are in opposite directions so find the net value by subtracting the magnitudes. The torque on the lower half is larger than the torque on the upper half since pressure increases with depth.

14.51. **IDENTIFY:** Compute the force and the torque on a thin, horizontal strip at a depth h and integrate to find the total force and torque.

SET UP: The strip has an area $dA = (dh)L$,where dh is the height of the strip and L is its length. $A = HL$. The height of the strip about the bottom of the dam is $H - h$.

EXECUTE: **(a)** $dF = pdA = \rho ghLdh$. $F = \int_0^H dF = \rho gL\int_0^H hdh = \rho gLH^2/2 = \rho gAH/2$.

(b) The torque about the bottom on a strip of vertical thickness dh is $d\tau = dF\left(H-h\right) = \rho gLh\left(H-h\right)dh,$ and integrating from $h = 0$ to $h = H$ gives $\tau = \rho gLH^3/6 = \rho gAH^2/6.$

(c) The force depends on the width and on the square of the depth, and the torque about the bottom depends on the width and the cube of the depth; the surface area of the lake does not affect either result (for a given width).

EVALUATE: The force is equal to the average pressure, at depth $H/2$, times the area A of the vertical side of the dam that faces the lake. But the torque is not equal to $F(H/2)$,where $H/2$ is the moment arm for a force acting at the center of the dam.

14.52. **IDENTIFY:** The information about Europa allows us to evaluate g at the surface of Europa. Since there is no atmosphere, $p_0 = 0$ at the surface. The pressure at depth h is $p = \rho gh$. The inward force on the window is $F_\perp = pA$.

SET UP: $g = \dfrac{Gm}{R^2}$, where $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$. $R = 1.565\times10^6\ \text{m}$. Assume the ocean water has density $\rho = 1.00\times10^3\ \text{kg/m}^3$.

EXECUTE: $g = \dfrac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(4.78\times10^{22}\ \text{kg})}{(1.565\times10^6\ \text{m})^2} = 1.30\ \text{m/s}^2$. The maximum pressure at the window is

$p = \dfrac{9750\ \text{N}}{(0.250\ \text{m})^2} = 1.56\times10^5\ \text{Pa}$. $p = \rho gh$ so $h = \dfrac{1.56\times10^5\ \text{Pa}}{(1.00\times10^3\ \text{kg/m}^3)(1.30\ \text{m/s}^2)} = 120\ \text{m}$.

EVALUATE: 9750 N is the inward force exerted by the surrounding water. This will also be the net force on the window if the pressure inside the submarine is essentially zero.

14.53. **IDENTIFY** and **SET UP:** Apply Eq.(14.6) and solve for g.
Then use Eq.(12.4) to relate g to the mass of the planet.

EXECUTE: $p - p_0 = \rho gd.$

This expression gives that $g = (p - p_0)/\rho d = (p - p_0)V/md.$

But also $g = Gm_\text{p}/R^2$ (Eq.(12.4) applied to the planet rather than to earth.)

Setting these two expressions for g equal gives $Gm_\text{p}/R^2 = (p - p_0)V/md$ and $m_\text{p} = (p - p_0)VR^2/Gmd.$

EVALUATE: The greater p is at a given depth, the greater g is for the planet and greater g means greater m_p.

14.54. **IDENTIFY:** The buoyant force B equals the weight of the air displaced by the balloon.

SET UP: $B = \rho_\text{air}Vg$. Let g_M be the value of g for Mars. For a sphere $V = \frac{4}{3}\pi R^3$. The surface area of a sphere is given by $A = 4\pi R^2$. The mass of the balloon is $(5.00\times10^{-3}\ \text{kg/m}^2)(4\pi R^2)$.

EXECUTE: **(a)** $B = mg_\text{M}$. $\rho_\text{air}Vg_\text{M} = mg_\text{M}$. $\rho_\text{air}\frac{4}{3}\pi R^3 = (5.00\times10^{-3}\ \text{kg/m}^2)(4\pi R^2)$.

$R = \dfrac{3(5.00\times10^{-3}\ \text{kg/m}^2)}{\rho_\text{air}} = 0.974\ \text{m}$. $m = (5.00\times10^{-3}\ \text{kg/m}^2)(4\pi R^2) = 0.0596\ \text{kg}$.

(b) $F_\text{net} = B - mg = ma$. $B = \rho_\text{air}Vg = \rho_\text{air}\frac{4}{3}\pi R^3 g = (1.20\ \text{kg/m}^3)\left(\dfrac{4\pi}{3}\right)(0.974\ \text{m})^3(9.80\ \text{m/s}^2) = 45.5\ \text{N}$.

$a = \dfrac{B - mg}{m} = \dfrac{45.5\ \text{N} - (0.0596\ \text{kg})(9.80\ \text{m/s}^2)}{0.0596\ \text{m}} = 754\ \text{m/s}^2$, upward.

(c) $B = m_\text{tot}g$. $\rho_\text{air}Vg = (m_\text{balloon} + m_\text{load})g$. $m_\text{load} = \rho_\text{air}\frac{4}{3}\pi R^3 - (5.00\times10^{-3}\ \text{kg/m}^2)4\pi R^2$.

$m_\text{load} = (0.0154\ \text{kg/m}^3)\left(\dfrac{4\pi}{3}\right)(5[0.974\ \text{m}])^3 - (5.00\times10^{-3}\ \text{kg/m}^2)(4\pi)(5[0.974\ \text{m}])^2$

$m_\text{load} = 7.45\ \text{kg} - 1.49\ \text{kg} = 5.96\ \text{kg}$

EVALUATE: The buoyant force is proportional to R^3 and the mass of the balloon is proportional to R^2 , so the load that can be carried increases when the radius of the balloon increases. We calculated the mass of the load. To find the weight of the load we would need to know the value of g for Mars.

14.55. **IDENTIFY:** Follow the procedure outlined in part (b). For a spherically symmetric object, with total mass m and radius r, at points on the surface of the object, $g(r) = Gm/r^2$.

SET UP: The earth has mass $m_E = 5.97\times10^{24}$ kg. If $g(r)$ is a maximum at $r = r_{max}$, then $\frac{dg}{dr} = 0$ for $r = r_{max}$.

EXECUTE: **(a)** At $r = 0$, the model predicts $\rho = A = 12{,}700\ \text{kg/m}^3$ and at $r = R$, the model predicts $\rho = A - BR = 12{,}700\ \text{kg/m}^3 - (1.50\times10^{-3}\ \text{kg/m}^4)(6.37\times10^6\ \text{m}) = 3.15\times10^3\ \text{kg/m}^3$.

(b) and **(c)** $M = \int dm = 4\pi\int_0^R [A - Br]r^2 dr = 4\pi\left[\frac{AR^3}{3} - \frac{BR^4}{4}\right] = \left(\frac{4\pi R^3}{3}\right)\left[A - \frac{3BR}{4}\right]$.

$$M = \left(\frac{4\pi(6.37\times10^6\ \text{m})^3}{3}\right)\left[12{,}700\ \text{kg/m}^3 - \frac{3(1.50\times10^{-3}\ \text{kg/m}^4)(6.37\times10^6\ \text{m})}{4}\right] = 5.99\times10^{24}\ \text{kg}$$

which is within 0.36% of the earth's mass.

(d) If $m(r)$ is used to denote the mass contained in a sphere of radius r, then $g = Gm(r)/r^2$. Using the same integration as that in part (b), with an upper limit of r instead of R gives the result.

(e) $g = 0$ at $r = 0$, and g at $r = R$,

$g = Gm(R)/R^2 = (6.673\times10^{-11}\ \text{N}\cdot\text{m}^2\text{kg}^2)(5.99\times10^{24}\ \text{kg})/(6.37\times10^6\ \text{m})^2 = 9.85\ \text{m/s}^2$.

(f) $\frac{dg}{dr} = \left(\frac{4\pi G}{3}\right)\frac{d}{dr}\left[Ar - \frac{3Br^2}{4}\right] = \left(\frac{4\pi G}{3}\right)\left[A - \frac{3Br}{2}\right]$. Setting this equal to zero gives $r = 2A/3B = 5.64\times10^6$ m, and at this radius $g = \left(\frac{4\pi G}{3}\right)\left(\frac{2A}{3B}\right)\left[A - \left(\frac{3}{4}\right)B\left(\frac{2A}{3B}\right)\right] = \frac{4\pi GA^2}{9B}$.

$$g = \frac{4\pi(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(12{,}700\ \text{kg/m}^3)^2}{9(1.50\times10^{-3}\ \text{kg/m}^4)} = 10.02\ \text{m/s}^2.$$

EVALUATE: If the earth were a uniform sphere of density ρ, then $g(r) = \frac{\rho V(r)}{r^2} = \left(\frac{4\pi\rho G}{3}\right)r$, the same as setting $B = 0$ and $A = \rho$ in $g(r)$ in part (d). If r_{max} is the value of r in part (f) where $g(r)$ is a maximum, then $r_{max}/R = 0.885$. For a uniform sphere, $g(r)$ is maximum at the surface.

14.56. **IDENTIFY:** Follow the procedure outlined in part (a).

SET UP: The earth has mass $M = 5.97\times10^{24}$ kg and radius $R = 6.38\times10^6$ m. Let $g_S = 9.80\ \text{m/s}^2$

EXECUTE: **(a)** Equation (14.4), with the radius r instead of height y, becomes $dp = -\rho g(r)\,dr = -\rho g_s(r/R)dr$. This form shows that the pressure decreases with increasing radius. Integrating, with $p = 0$ at $r = R$,

$$p = -\frac{\rho g_s}{R}\int_R^r r\,dr = \frac{\rho g_s}{R}\int_r^R r\,dr = \frac{\rho g_s}{2R}(R^2 - r^2).$$

(b) Using the above expression with $r = 0$ and $\rho = \frac{M}{V} = \frac{3M}{4\pi R^3}$,

$$p(0) = \frac{3(5.97\times10^{24}\ \text{kg})(9.80\ \text{m/s}^2)}{8\pi(6.38\times10^6\ \text{m})^2} = 1.71\times10^{11}\ \text{Pa}.$$

(c) While the same order of magnitude, this is not in very good agreement with the estimated value. In more realistic density models (see Problem 14.55), the concentration of mass at lower radii leads to a higher pressure.

EVALUATE: In this model, the pressure at the center of the earth is about 10^6 times what it is at the surface.

14.57. **(a) IDENTIFY** and **SET UP:**

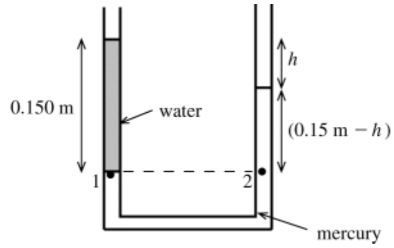


Figure 14.57

Apply $p = p_0 + \rho gh$ to the water in the left-hand arm of the tube. See Figure 14.57.

EXECUTE: $p_0 = p_a$, so the gauge pressure at the interface (point 1) is

$p - p_a = \rho g h = (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.150\ \text{m}) = 1470\ \text{Pa}$

(b) IDENTIFY and **SET UP:** The pressure at point 1 equals the pressure at point 2. Apply Eq.(14.6) to the right-hand arm of the tube and solve for h.

EXECUTE: $p_1 = p_a + p_w g(0.150\ \text{m})$ and $p_2 = p_a + \rho_{Hg} g(0.150\ \text{m} - h)$

$p_1 = p_2$ implies $\rho_w g(0.150\ \text{m}) = \rho_{Hg} g(0.150\ \text{m} - h)$

$$0.150\ \text{m} - h = \frac{\rho_w(0.150\ \text{m})}{\rho_{Hg}} = \frac{(1000\ \text{kg/m}^3)(0.150\ \text{m})}{13.6\times 10^3\ \text{kg/m}^3} = 0.011\ \text{m}$$

$h = 0.150\ \text{m} - 0.011\ \text{m} = 0.139\ \text{m} = 13.9\ \text{cm}$

EVALUATE: The height of mercury above the bottom level of the water is 1.1 cm. This height of mercury produces the same gauge pressure as a height of 15.0 cm of water.

14.58. **IDENTIFY:** Follow the procedure outlined in the hint. $F = pA$.

SET UP: The circular ring has area $dA = (2\pi R)dy$. The pressure due to the molasses at depth y is $\rho g y$.

EXECUTE: $F = \int_0^h (\rho g y)(2\pi R)dy = \rho g \pi R h^2$ where R and h are the radius and height of the tank. Using the given numerical values gives $F = 5.07\times 10^8\ \text{N}$.

EVALUATE: The net outward force is the area of the wall of the tank, $A = 2\pi R h$, times the average pressure, the pressure $\rho g h/2$ at depth $h/2$.

14.59. **IDENTIFY:** Apply Newton's 2nd law to the barge plus its contents. Apply Archimedes' principle to express the buoyancy force B in terms of the volume of the barge.

SET UP: The free-body diagram for the barge plus coal is given in Figure 14.59.

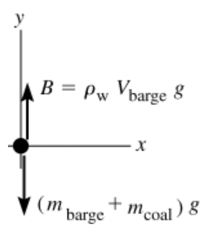


Figure 14.59

EXECUTE: $\sum F_y = ma_y$

$B - (m_{\text{barge}} + m_{\text{coal}})g = 0$

$\rho_w V_{\text{barge}} g = (m_{\text{marge}} + m_{\text{coal}})g$

$m_{\text{coal}} = \rho_w V_{\text{barge}} - m_{\text{barge}}$

$V_{\text{barge}} = (22\ \text{m})(12\ \text{m})(40\ \text{m}) = 1.056\times 10^4\ \text{m}^3$

The mass of the barge is $m_{\text{barge}} = \rho_s V_s$, where s refers to steel.

From Table 14.1, $\rho_s = 7800\ \text{kg/m}^3$. The volume V_s is 0.040 m times the total area of the five pieces of steel that make up the barge:

$$V_s = (0.040\ \text{m})\left[2(22\ \text{m})(12\ \text{m}) + 2(40\ \text{m})(12\ \text{m}) + (22\ \text{m})(40\ \text{m})\right] = 94.7\ \text{m}^3.$$

Therefore, $m_{\text{barge}} = \rho_s V_s = (7800\ \text{kg/m}^3)(94.7\ \text{m}^3) = 7.39\times 10^5\ \text{kg}$.

Then $m_{\text{coal}} = \rho_w V_{\text{barge}} - m_{\text{barge}} = (1000\ \text{kg/m}^3)(1.056\times 10^4\ \text{m}^3) - 7.39\times 10^5\ \text{kg} = 9.8\times 10^6\ \text{kg}$.

The volume of this mass of coal is $V_{\text{coal}} = m_{\text{coal}}/\rho_{\text{coal}} = 9.8\times 10^6\ \text{kg}/1500\ \text{kg/m}^3 = 6500\ \text{m}^3$; this is less that V_{barge} so it will fit into the barge.

EVALUATE: The buoyancy force B must support both the weight of the coal and also the weight of the barge. The weight of the coal is about 13 times the weight of the barge. The buoyancy force increases when more of the barge is submerged, so when it holds the maximum mass of coal the barge is fully submerged.

14.60. **IDENTIFY:** The buoyant force on the balloon must equal the total weight of the balloon fabric, the basket and its contents and the gas inside the balloon. $m_{\text{gas}} = \rho_{\text{gas}} V$. $B = \rho_{\text{air}} V g$.

SET UP: The total weight, exclusive of the gas inside the balloon, is $900\ \text{N} + 1700\ \text{N} + 3200\ \text{N} = 5800\ \text{N}$

EXECUTE: $5800\ \text{N} + \rho_{\text{gas}} V_g = \rho_{\text{air}} V_g$ and $\rho_{\text{gas}} = 1.23\ \text{kg/m}^3 - \dfrac{(5800\ \text{N})}{(9.80\ \text{m/s}^2)(2200\ \text{m}^3)} = 0.96\ \text{kg/m}^3$.

EVALUATE: The volume of a given mass of gas increases when the gas is heated, and the density of the gas therefore decreases.

14.61. **IDENTIFY:** Apply Newton's 2nd law to the car. The buoyancy force is given by Archimedes' principle.

(a) SET UP: The free-body diagram for the floating car is given in Figure 14.61. (V_{sub} is the volume that is submerged.)

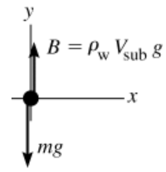


Figure 14.61

EXECUTE: $\sum F_y = ma_y$

$B - mg = 0$

$\rho_w V_{\text{sub}} g - mg = 0$

$V_{\text{sub}} = m/\rho_w = (900\text{ kg})/(1000\text{ kg/m}^3) = 0.900\text{ m}^3$

$V_{\text{sub}}/V_{\text{obj}} = (0.900\text{ m}^3)/(3.0\text{ m}^3) = 0.30 = 30\%$

EVALUATE: The average density of the car is $(900\text{ kg})/(3.0\text{ m}^3) = 300\text{ kg/m}^3$. $\rho_{\text{car}}/\rho_{\text{water}} = 0.30$; this equals $V_{\text{sub}}/V_{\text{obj}}$.

(b) SET UP: When the car starts to sink it is fully submerged and the buoyant force is equal to the weight of the car plus the water that is inside it.

EXECUTE: When the car is full submerged $V_{\text{sub}} = V$, the volume of the car and

$B = \rho_{\text{water}} V g = (1000\text{ kg/m}^3)(3.0\text{ m}^3)(9.80\text{ m/s}^2) = 2.94\times10^4\text{ N}$

The weight of the car is $mg = (900\text{ kg})(9.80\text{ m/s}^2) = 8820\text{ N}$.

Thus the weight of the water in the car when it sinks is the buoyant force minus the weight of the car itself:

$m_{\text{water}} = (2.94\times10^4\text{ N} - 8820\text{ N})/(9.80\text{ m/s}^2) = 2.10\times10^3\text{ kg}$

And $V_{\text{water}} = m_{\text{water}}/\rho_{\text{water}} = (2.10\times10^3\text{ kg})/(1000\text{ kg/m}^3) = 2.10\text{ m}^3$

The fraction this is of the total interior volume is $(2.10\text{ m}^3)/(3.00\text{ m}^3) = 0.70 = 70\%$

EVALUATE: The average density of the car plus the water inside it is $(900\text{ kg} + 2100\text{ kg})/(3.0\text{ m}^3) = 1000\text{ kg/m}^3$, so $\rho_{\text{car}} = \rho_{\text{water}}$ when the car starts to sink.

14.62. **IDENTIFY:** For a floating object, the buoyant force equals the weight of the object. $B = \rho_{\text{fluid}} V_{\text{submerged}} g$.

SET UP: Water has density $\rho = 1.00\text{ g/cm}^3$.

EXECUTE: **(a)** The volume displaced must be that which has the same weight and mass as the ice,
$\dfrac{9.70\text{ gm}}{1.00\text{ gm/cm}^3} = 9.70\text{ cm}^3$.

(b) No; when melted, the cube produces the same volume of water as was displaced by the floating cube, and the water level does not change.

(c) $\dfrac{9.70\text{ gm}}{1.05\text{ gm/cm}^3} = 9.24\text{ cm}^3$

(d) The melted water takes up more volume than the salt water displaced, and so 0.46 cm^3 flows over.

EVALUATE: The volume of water from the melted cube is less than the volume of the ice cube, but the cube floats with only part of its volume submerged.

14.63. **IDENTIFY:** For a floating object the buoyant force equals the weight of the object. The buoyant force when the wood sinks is $B = \rho_{\text{water}} V_{\text{tot}} g$, where V_{tot} is the volume of the wood plus the volume of the lead. $\rho = m/V$.

SET UP: The density of lead is $11.3\times10^3\text{ kg/m}^3$.

EXECUTE: $V_{\text{wood}} = (0.600\text{ m})(0.250\text{ m})(0.080\text{ m}) = 0.0120\text{ m}^3$.

$m_{\text{wood}} = \rho_{\text{wood}} V_{\text{wood}} = (600\text{ kg/m}^3)(0.0120\text{ m}^3) = 7.20\text{ kg}$.

$B = (m_{\text{wood}} + m_{\text{lead}})g$. Using $B = \rho_{\text{water}} V_{\text{tot}} g$ and $V_{\text{tot}} = V_{\text{wood}} + V_{\text{lead}}$ gives $\rho_{\text{water}}(V_{\text{wood}} + V_{\text{lead}})g = (m_{\text{wood}} + m_{\text{lead}})g$.

$m_{\text{lead}} = \rho_{\text{lead}} V_{\text{lead}}$ then gives $\rho_{\text{water}} V_{\text{wood}} + \rho_{\text{water}} V_{\text{lead}} = m_{\text{wood}} + \rho_{\text{lead}} V_{\text{lead}}$.

$V_{\text{lead}} = \dfrac{\rho_{\text{water}} V_{\text{wood}} - m_{\text{wood}}}{\rho_{\text{lead}} - \rho_{\text{water}}} = \dfrac{(1000\text{ kg/m}^3)(0.0120\text{ m}^3) - 7.20\text{ kg}}{11.3\times10^3\text{ kg/m}^3 - 1000\text{ kg/m}^3} = 4.66\times10^{-4}\text{ m}^3$. $m_{\text{lead}} = \rho_{\text{lead}} V_{\text{lead}} = 5.27\text{ kg}$.

EVALUATE: The volume of the lead is only 3.9% of the volume of the wood. If the contribution of the volume of the lead to F_B is neglected, the calculation is simplified: $\rho_{\text{water}} V_{\text{wood}} g = (m_{\text{wood}} + m_{\text{lead}})g$ and $m_{\text{lead}} = 4.8\text{ kg}$. The result of this calculation is in error by about 9%.

14.64. IDENTIFY: The fraction f of the volume that floats above the fluid is $f = 1 - \frac{\rho}{\rho_{\text{fluid}}}$, where ρ is the average density of the hydrometer (see Problem 14.29). This gives $\rho_{\text{fluid}} = \rho \frac{1}{1-f}$.

SET UP: The volume above the surface is hA, where h is the height of the stem above the surface and $A = 0.400\ \text{cm}^2$.

EXECUTE: If two fluids are observed to have floating fraction f_1 and f_2, $\rho_2 = \rho_1 \frac{1-f_1}{1-f_2}$. Using

$f_1 = \frac{(8.00\ \text{cm})(0.400\ \text{cm}^2)}{(13.2\ \text{cm}^3)} = 0.242$, $f_2 = \frac{(3.20\ \text{cm})(0.400\ \text{cm}^2)}{(13.2\ \text{cm}^3)} = 0.097$ gives $\rho_{\text{alcohol}} = (0.839)\rho_{\text{water}} = 839\ \text{kg/m}^3$.

EVALUATE: $\rho_{\text{alcohol}} < \rho_{\text{water}}$. When ρ_{fluid} increases, the fraction f of the object's volume that is above the surface increases.

14.65. (a) IDENTIFY: Apply Newton's 2nd law to the airship. The buoyancy force is given by Archimedes' principle; the fluid that exerts this force is the air.

SET UP: The free-body diagram for the dirigible is given in Figure 14.65. The lift corresponds to a mass $m_{\text{lift}} = (120\times10^3\ \text{N})/(9.80\ \text{m/s}^2) = 1.224\times10^4\ \text{kg}$. The mass m_{tot} is $1.224\times10^4\ \text{kg}$ plus the mass m_{gas} of the gas that fills the dirigible. B is the buoyant force exerted by the air.

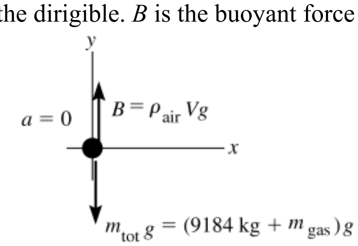


Figure 14.65

EXECUTE: $\sum F_y = ma_y$

$B - m_{\text{tot}}g = 0$

$\rho_{\text{air}}Vg = (1.224\times10^4\ \text{kg} + m_{\text{gas}})g$

Write m_{gas} in terms of V: $m_{\text{gas}} = \rho_{\text{gas}}V$

And let g divide out; the equation becomes $\rho_{\text{air}}V = 1.224\times10^4\ \text{kg} + \rho_{\text{gas}}V$

$$V = \frac{1.224\times10^4\ \text{kg}}{1.20\ \text{kg/m}^3 - 0.0899\ \text{kg/m}^3} = 1.10\times10^4\ \text{m}^3$$

EVALUATE: The density of the airship is less than the density of air and the airship is totally submerged in the air, so the buoyancy force exceeds the weight of the airship.

(b) SET UP: Let m_{lift} be the mass that could be lifted.

EXECUTE: From part (a), $m_{\text{lift}} = (\rho_{\text{air}} - \rho_{\text{gas}})V = (1.20\ \text{kg/m}^3 - 0.166\ \text{kg/m}^3)(1.10\times10^4\ \text{m}^3) = 1.14\times10^4\ \text{kg}$.

The lift force is $m_{\text{lift}} = (1.14\times10^4\ \text{kg})(9.80\ \text{m/s}^2) = 112\ \text{kN}$.

EVALUATE: The density of helium is less than that of air but greater than that of hydrogen. Helium provides lift, but less lift than hydrogen. Hydrogen is not used because it is highly explosive in air.

14.66. IDENTIFY: The vertical forces on the floating object must sum to zero. The buoyant force B applied to the object by the liquid is given by Archimedes's principle. The motion is SHM if the net force on the object is of the form $F_y = -ky$ and then $T = 2\pi\sqrt{m/k}$.

SET UP: Take $+y$ to be downward.

EXECUTE: (a) $V_{\text{submerged}} = LA$, where L is the vertical distance from the surface of the liquid to the bottom of the object. Archimedes' principle states $\rho gLA = Mg$, so $L = \frac{M}{\rho A}$.

(b) The buoyant force is $\rho gA(L+y) = Mg + F$, where y is the additional distance the object moves downward. Using the result of part (a) and solving for y gives $y = \frac{F}{\rho gA}$.

(c) The net force is $F_{\text{net}} = Mg - \rho gA(L+y) = -\rho gAy$. $k = \rho gA$, and the period of oscillation is

$$T = 2\pi\sqrt{\frac{M}{k}} = 2\pi\sqrt{\frac{M}{\rho gA}}.$$

EVALUATE: The force F determines the amplitude of the motion but the period does not depend on how much force was applied.

14.67. **IDENTIFY:** Apply the results of problem 14.66.

SET UP: The additional force F applied to the buoy is the weight $w = mg$ of the man.

EXECUTE: **(a)** $x = \dfrac{w}{\rho g A} = \dfrac{mg}{\rho g A} = \dfrac{m}{\rho A} = \dfrac{(70.0\text{ kg})}{(1.03\times10^3\text{ kg/m}^3)\pi(0.450\text{ m})^2} = 0.107\text{ m}.$

(b) Note that in part (c) of Problem 14.66, M is the mass of the buoy, not the mass of the man, and A is the cross-section area of the buoy, not the amplitude. The period is then

$$T = 2\pi\sqrt{\frac{(950\text{ kg})}{(1.03\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)\pi(0.450\text{ m})^2}} = 2.42\text{ s}$$

EVALUATE: The period is independent of the mass of the man.

14.68. **IDENTIFY:** After the water leaves the hose the only force on it is gravity. Use conservation of energy to relate the initial speed to the height the water reaches. The volume flow rate is Av.

SET UP: $A = \pi D^2/4$

EXECUTE: **(a)** $\frac{1}{2}mv^2 = mgh$. $v = \sqrt{2gh} = \sqrt{2(9.80\text{ m/s}^2)(35.0\text{ m})} = 26.2\text{ m/s}$. $(\pi D^2/4)v = 0.500\text{ m/s}^3$.

$D = \sqrt{\dfrac{4(0.500\text{ m/s}^3)}{\pi v}} = \sqrt{\dfrac{4(0.500\text{ m/s}^3)}{\pi(26.3\text{ m/s})}} = 0.156\text{ m} = 15.6\text{ cm}$.

(b) $D^2 v$ is constant so if D is twice as great then v is decreased by a factor of 4. h is proportional to v^2, so h is decreased by a factor of 16. $h = \dfrac{35.0\text{ m}}{16} = 2.19\text{ m}$.

EVALUATE: The larger the diameter of the nozzle the smaller the speed with which the water leaves the hose and the smaller the maximum height.

14.69. **IDENTIFY:** Find the horizontal range x as a function of the height y of the hole above the base of the cylinder. Then find the value of y for which x is a maximum. Once the water leaves the hole it moves in projectile motion.

SET UP: Apply Bernoulli's equation to points 1 and 2, where point 1 is at the surface of the water and point 2 is in the stream as the water leaves the hole. Since the hole is small the volume flow rate out the hole is small and $v_1 \approx 0$. $y_1 - y_2 = H - y$ and $p_1 = p_2 = p_{\text{air}}$. For the projectile motion, take $+y$ to be upward; $a_x = 0$ and $a_y = -9.80\text{ m/s}^2$.

EXECUTE: **(a)** $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$ gives $v_2 = \sqrt{2g(H-y)}$. In the projectile motion, $v_{0y} = 0$ and $y - y_0 = -y$, so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $t = \sqrt{\dfrac{2y}{g}}$. The horizontal range is $x = v_{0x}t = v_2 t = 2\sqrt{y(H-y)}$. The y that gives maximum x satisfies $\dfrac{dx}{dy} = 0$. $(Hy - y^2)^{-1/2}(H - 2y) = 0$ and $y = H/2$.

(b) $x = 2\sqrt{y(H-y)} = 2\sqrt{(H/2)(H - H/2)} = H$.

EVALUATE: A smaller y gives a larger v_2, but a smaller time in the air after the water leaves the hole.

14.70. **IDENTIFY:** Bernoulli's equation gives the speed at which water exits the hole, and from this we can calculate the volume flow rate. This will depend on the height h of the water remaining in the tank. Integrate to find h versus t. The time for the tank to empty is t for which $h = 0$.

SET UP: Apply Bernoulli's equation to point 1 at the top of the tank and point 2 at the hole. Assume the cross sectional area A_1 of the tank is much larger than the area A_2 of the hole. $v_1 = -\dfrac{dh}{dt}$, where the minus sign is because h is decreasing and dh/dt is negative, whereas v_1 is positive.

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$ gives $v_2^2 = 2gh + v_1^2$. $A_1 v_1 = A_2 v_2$ gives $v_1^2 = \left(\dfrac{A_2}{A_1}\right)^2 v_2^2$ and $v_2^2\left(1 - \left[\dfrac{A_2}{A_1}\right]^2\right) = 2gh$. $A_2 << A_1$ so $v_2 = \sqrt{2gh}$. $v_1 = -\dfrac{dh}{dt} = \dfrac{A_2}{A_1}v_2$ and $v_2 = -\dfrac{A_1}{A_2}\dfrac{dh}{dt}$. Combining these two equations for v_2 gives $\dfrac{dh}{dt} = -\dfrac{A_2}{A_1}\sqrt{2g}h^{1/2}$. $\dfrac{dh}{h^{1/2}} = -\left(\dfrac{A_2}{A_1}\sqrt{2g}\right)dt$. $\int_{h_0}^{h}\dfrac{dh}{h^{1/2}} = -\left(\dfrac{A_2}{A_1}\sqrt{2g}\right)\int_0^t dt$ gives $2\left(\sqrt{h} - \sqrt{h_0}\right) = -\dfrac{A_2}{A_1}\sqrt{2g}t$.

$h(t) = \left(\sqrt{h_0} - \dfrac{A_2}{A_1}\left(\sqrt{\dfrac{g}{2}}\right)t\right)^2$.

(b) $h = 0$ when $t = \dfrac{A_1}{A_2}\sqrt{\dfrac{2h_0}{g}}$.

EVALUATE: The time t for the tank to empty decreases when the area of the hole is larger. t increases when A_1 increases because for fixed h_0 an increase in A_1 corresponds to a greater volume of water initially in the tank.

14.71. **IDENTIFY:** Apply the 2nd condition of equilibrium to the balance arm and apply the first condition of equilibrium to the block and to the brass mass. The buoyancy force on the wood is given by Archimedes' principle and the buoyancy force on the brass mass is ignored.

SET UP: The objects and forces are sketched in Figure 14.71a.

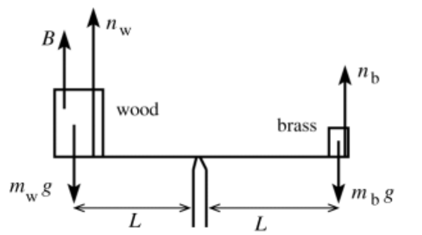


Figure 14.71a

The buoyant force on the brass is neglected, but we include the buoyant force B on the block of wood. n_w and n_b are the normal forces exerted by the balance arm on which the objects sit.

The free-body diagram for the balance arm is given in Figure 14.71b.



Figure 14.71b

EXECUTE: $\tau_P = 0$

$n_w L - n_b L = 0$

$n_w = n_b$

SET UP: The free-body diagram for the brass mass is given in Figure 14.71c.

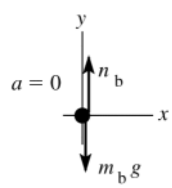


Figure 14.71c

EXECUTE: $\sum F_y = ma_y$

$n_b - m_b g = 0$

$n_b = m_b g$

The free-body diagram for the block of wood is given in Figure 14.71d.

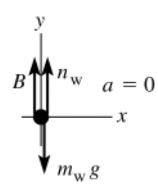


Figure 14.71d

$\sum F_y = ma_y$

$n_w + B - m_w g = 0$

$n_w = m_w g - B$

But $n_b = n_w$ implies $m_b g = m_w g - B$.

And $B = \rho_{air} V_w g = \rho_{air}(m_w/\rho_w)g$, so $m_b g = m_w g - \rho_{air}(m_w/\rho_w)g$.

$$m_w = \frac{m_b}{1-\rho_{air}/\rho_w} = \frac{0.0950\text{ kg}}{1-((1.20\text{ kg/m}^3)/(150\text{ kg/m}^3))} = 0.0958\text{ kg}$$

EVALUATE: The mass of the wood is greater than the mass of the brass; the wood is partially supported by the buoyancy force exerted by the air. The buoyancy in air of the brass can be neglected because the density of brass is much more than the density of air; the buoyancy force exerted on the brass by the air is much less than the weight of the brass. The density of the balsa wood is much less than the density of the brass, so the buoyancy force on the balsa wood is not such a small fraction of its weight.

14.72. **IDENTIFY:** $B = \rho V_A g$. Apply Newton's second law to the beaker, liquid and block as a combined object and also to the block as a single object.

SET UP: Take $+y$ upward. Let F_D and F_E be the forces corresponding to the scale reading.

EXECUTE: Forces on the combined object: $F_D + F_E - (w_A + w_B + w_C) = 0$. $w_A = F_D + F_E - w_B - w_C$.

D and E read mass rather than weight, so write the equation as $m_A = m_D + m_E - m_B - m_C$. $m_D = F_D/g$ is the reading in kg of scale D; a similar statement applies to m_E.

$m_A = 3.50\text{ kg} + 7.50\text{ kg} - 1.00\text{ kg} - 1.80\text{ kg} = 8.20\text{ kg}$.

Forces on A: $B + F_D - w_A = 0$. $\rho V_A g + F_D - m_A g = 0$. $\rho V_A + m_D = m_A$.

$$\rho = \frac{m_A - m_D}{V_A} = \frac{8.20\text{ kg} - 3.50\text{ kg}}{3.80\times 10^{-3}\text{ m}^3} = 1.24\times 10^3\text{ kg/m}^3$$

(b) D reads the mass of A: 8.20 kg. E reads the total mass of B and C: 2.80 kg.

EVALUATE: The sum of the readings of the two scales remains the same.

14.73. IDENTIFY: Apply Newton's 2nd law to the ingot. Use the expression for the buoyancy force given by Archimedes' principle to solve for the volume of the ingot. Then use the facts that the total mass is the mass of the gold plus the mass of the aluminum and that the volume of the ingot is the volume of the gold plus the volume of the aluminum.

SET UP: The free-body diagram for the piece of alloy is given in Figure 14.73.

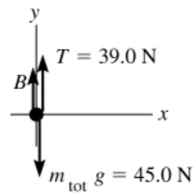


Figure 14.73

EXECUTE: $\sum F_y = ma_y$

$B + T - m_{\text{tot}}g = 0$

$B = m_{\text{tot}}g - T$

$B = 45.0\text{ N} - 39.0\text{ N} = 6.0\text{ N}$

Also, $m_{\text{tot}}g = 45.0\text{ N}$ so $m_{\text{tot}} = 45.0\text{ N}/(9.80\text{ m/s}^2) = 4.59\text{ kg}$.

We can use the known value of the buoyant force to calculate the volume of the object: $B = \rho_{\text{w}}V_{\text{obj}}g = 6.0\text{ N}$

$$V_{\text{obj}} = \frac{6.0\text{ N}}{\rho_{\text{w}}g} = \frac{6.0\text{ N}}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 6.122\times 10^{-4}\text{ m}^3$$

We know two things:

(1) The mass m_{g} of the gold plus the mass m_{a} of the aluminum must add to m_{tot}: $m_{\text{g}} + m_{\text{a}} = m_{\text{tot}}$

We write this in terms of the volumes V_{g} and V_{a} of the gold and aluminum: $\rho_{\text{g}}V_{\text{g}} + \rho_{\text{a}}V_{\text{a}} = m_{\text{tot}}$

(2) The volumes V_{a} and V_{g} must add to give V_{obj}: $V_{\text{a}} + V_{\text{g}} = V_{\text{obj}}$ so that $V_{\text{a}} = V_{\text{obj}} - V_{\text{g}}$

Use this in the equation in (1) to eliminate V_{a}: $\rho_{\text{g}}V_{\text{g}} + \rho_{\text{a}}(V_{\text{obj}} - V_{\text{g}}) = m_{\text{tot}}$

$$V_{\text{g}} = \frac{m_{\text{tot}} - \rho_{\text{a}}V_{\text{obj}}}{\rho_{\text{g}} - \rho_{\text{a}}} = \frac{4.59\text{ kg} - (2.7\times 10^3\text{ kg/m}^3)(6.122\times 10^{-4}\text{ m}^3)}{19.3\times 10^3\text{ kg/m}^3 - 2.7\times 10^3\text{ kg/m}^3} = 1.769\times 10^{-4}\text{ m}^3.$$

Then $m_{\text{g}} = \rho_{\text{g}}V_{\text{g}} = (19.3\times 10^3\text{ kg/m}^3)(1.769\times 10^{-4}\text{ m}^3) = 3.41\text{ kg}$ and the weight of gold is $w_{\text{g}} = m_{\text{g}}g = 33.4\text{ N}$.

EVALUATE: The gold is 29% of the volume but 74% of the mass, since the density of gold is much greater than the density of aluminum.

14.74. IDENTIFY: Apply $\sum F_y = ma_y$ to the ball, with $+y$ upward. The buoyant force is given by Archimedes's principle.

SET UP: The ball's volume is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12.0\text{ cm})^3 = 7238\text{ cm}^3$. As it floats, it displaces a weight of water equal to its weight.

EXECUTE: **(a)** By pushing the ball under water, you displace an additional amount of water equal to 84% of the ball's volume or $(0.84)(7238\text{ cm}^3) = 6080\text{ cm}^3$. This much water has a mass of $6080\text{ g} = 6.080\text{ kg}$ and weighs $(6.080\text{ kg})(9.80\text{ m/s}^2) = 59.6\text{ N}$, which is how hard you'll have to push to submerge the ball.

(b) The upward force on the ball in excess of its own weight was found in part (a): 59.6 N. The ball's mass is equal to the mass of water displaced when the ball is floating:

$$(0.16)(7238\text{ cm}^3)(1.00\text{ g/cm}^3) = 1158\text{ g} = 1.158\text{ kg},$$

and its acceleration upon release is thus $a = \frac{F_{\text{net}}}{m} = \frac{59.6\text{ N}}{1.158\text{ kg}} = 51.5\text{ m/s}^2$.

EVALUATE: When the ball is totally immersed the upward buoyant force on it is much larger than its weight.

14.75. **(a) IDENTIFY:** Apply Newton's 2nd law to the crown. The buoyancy force is given by Archimedes' principle. The target variable is the ratio ρ_c/ρ_w (c = crown, w = water).

SET UP: The free-body diagram for the crown is given in Figure 14.75.

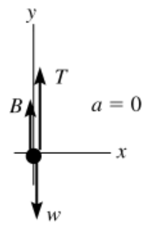


Figure 14.75

EXECUTE: $\sum F_y = ma_y$

$T + B - w = 0$

$T = fw$

$B = \rho_w V_c g$, where

$\rho_w =$ density of water,

$V_c =$ volume of crown

Then $fw + \rho_w V_c g - w = 0.$

$(1 - f)w = \rho_w V_c g$

Use $w = \rho_c V_c g$, where $\rho_c =$ density of crown.

$(1 - f)\rho_c V_c g = \rho_w V_c g$

$\dfrac{\rho_c}{\rho_w} = \dfrac{1}{1-f}$, as was to be shown.

$f \to 0$ gives $\rho_c / \rho_w = 1$ and $T = 0$. These values are consistent. If the density of the crown equals the density of the water, the crown just floats, fully submerged, and the tension should be zero.

When $f \to 1$, $\rho_c >> \rho_w$ and $T = w$. If $\rho_c >> \rho_w$ then B is negligible relative to the weight w of the crown and T should equal w.

(b) "apparent weight" equals T in rope when the crown is immersed in water. $T = fw$, so need to compute f.

$\rho_c = 19.3 \times 10^3 \text{ kg/m}^3$; $\rho_w = 1.00 \times 10^3 \text{ kg/m}^3$

$\dfrac{\rho_c}{\rho_w} = \dfrac{1}{1-f}$ gives $\dfrac{19.3 \times 10^3 \text{ kg/m}^3}{1.00 \times 10^3 \text{ kg/m}^3} = \dfrac{1}{1-f}$

$19.3 = 1/(1 - f)$ and $f = 0.9482$

Then $T = fw = (0.9482)(12.9 \text{ N}) = 12.2 \text{ N}.$

(c) Now the density of the crown is very nearly the density of lead;

$\rho_c = 11.3 \times 10^3 \text{ kg/m}^3.$

$\dfrac{\rho_c}{\rho_w} = \dfrac{1}{1-f}$ gives $\dfrac{11.3 \times 10^3 \text{ kg/m}^3}{1.00 \times 10^3 \text{ kg/m}^3} = \dfrac{1}{1-f}$

$11.3 = 1/(1 - f)$ and $f = 0.9115$

Then $T = fw = (0.9115)(12.9 \text{ N}) = 11.8 \text{ N}.$

EVALUATE: In part (c) the average density of the crown is less than in part (b), so the volume is greater. B is greater and T is less. These measurements can be used to determine if the crown is solid gold, without damaging the crown.

14.76. **IDENTIFY:** Problem 14.75 says $\dfrac{\rho_{\text{object}}}{\rho_{\text{fluid}}} = \dfrac{1}{1-f}$, where the apparent weight of the object when it is totally immersed in the fluid is fw.

SET UP: For the object in water, $f_{\text{water}} = w_{\text{water}}/w$ and for the object in the unknown fluid, $f_{\text{fluid}} = w_{\text{fluid}}/w$.

EXECUTE: **(a)** $\dfrac{\rho_{\text{steel}}}{\rho_{\text{fluid}}} = \dfrac{w}{w - w_{\text{fluid}}}$, $\dfrac{\rho_{\text{steel}}}{\rho_{\text{fluid}}} = \dfrac{w}{w - w_{\text{water}}}$. Dividing the second of these by the first gives

$\dfrac{\rho_{\text{fluid}}}{\rho_{\text{water}}} = \dfrac{w - w_{\text{fluid}}}{w - w_{\text{water}}}.$

(b) When w_{fluid} is greater than $w_{\text{water,}}$ the term on the right in the above expression is less than one, indicating that the fluids is less dense than water, and this is consistent with the buoyant force when suspended in liquid being less than that when suspended in water. If the density of the fluid is the same as that of water $w_{\text{fluid}} = w_{\text{water}}$, as expected. Similarly, if w_{fluid} is less than w_{water}, the term on the right in the above expression is greater than one, indicating that the fluid is denser than water.

(c) Writing the result of part (a) as $\dfrac{\rho_{\text{fluid}}}{\rho_{\text{water}}} = \dfrac{1-f_{\text{fluid}}}{1-f_{\text{water}}}$, and solving for f_{fluid},

$$f_{\text{fluid}} = 1 - \frac{\rho_{\text{fluid}}}{\rho_{\text{water}}}(1-f_{\text{water}}) = 1-(1.220)\ (0.128) = 0.844 = 84.4\%.$$

EVALUATE: Formic acid has density greater than the density of water. When the object is immersed in formic acid the buoyant force is greater and the apparent weight is less than when the object is immersed in water.

14.77. **IDENTIFY** and **SET UP:** Use Archimedes' principle for *B*.

(a) $B = \rho_{\text{water}} V_{\text{tot}} g$, where V_{tot} is the total volume of the object.

$V_{\text{tot}} = V_{\text{m}} + V_0$, where V_{m} is the volume of the metal.

EXECUTE: $V_{\text{m}} = w/g\rho_{\text{m}}$ so $V_{\text{tot}} = w/g\rho_{\text{m}} + V_0$

This gives $B = \rho_{\text{water}} g(w/g\rho_{\text{m}} + V_0)$

Solving for V_0 gives $V_0 = B/(\rho_{\text{water}} g) - w/(\rho_{\text{m}} g)$, as was to be shown.

(b) The expression derived in part (a) gives

$$V_0 = \frac{20\text{ N}}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)} - \frac{156\text{ N}}{(8.9\times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 2.52\times 10^{-4}\text{ m}^3$$

$$V_{\text{tot}} = \frac{B}{\rho_{\text{water}} g} = \frac{20\text{ N}}{(1000\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 2.04\times 10^{-3}\text{ m}^3 \text{ and } V_0/V_{\text{tot}} = (2.52\times 10^{-4}\text{ m}^3)/(2.04\times 10^{-3}\text{ m}^3) = 0.124.$$

EVALUATE: When $V_0 \to 0$, the object is solid and $V_{\text{obj}} = V_{\text{m}} = w/(\rho_{\text{m}} g)$. For $V_0 = 0$, the result in part (a) gives $B = (w/\rho_{\text{m}})\rho_{\text{w}} = V_{\text{m}}\rho_{\text{w}} g = V_{\text{obj}}\rho_{\text{w}} g$, which agrees with Archimedes' principle. As V_0 increases with the weight kept fixed, the total volume of the object increases and there is an increase in *B*.

14.78. **IDENTIFY:** For a floating object the buoyant force equals the weight of the object. Archimedes's principle says the buoyant force equals the weight of fluid displaced by the object. $m = \rho V$.

SET UP: Let *d* be the depth of the oil layer, *h* the depth that the cube is submerged in the water, and *L* be the length of a side of the cube.

EXECUTE: **(a)** Setting the buoyant force equal to the weight and canceling the common factors of *g* and the cross-section area, $(1000)h + (750)d = (550)L$. *d*, *h* and *L* are related by $d + h + 0.35L = L$, so $h = 0.65L - d$.

Substitution into the first relation gives $d = L\dfrac{(0.65)(1000)-(550)}{(1000)-(750)} = \dfrac{2L}{5.00} = 0.040\text{ m}$.

(b) The gauge pressure at the lower face must be sufficient to support the block (the oil exerts only sideways forces directly on the block), and $p = \rho_{\text{wood}} gL = (550\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.100\text{ m}) = 539\text{ Pa}$.

EVALUATE: As a check, the gauge pressure, found from the depths and densities of the fluids, is $[(0.040\text{ m})(750\text{ kg/m}^3) + (0.025\text{ m})(1000\text{ kg/m}^3)](9.80\text{ m/s}^2) = 539\text{ Pa}$.

14.79. **IDENTIFY** and **SET UP:** Apply the first condition of equilibrium to the barge plus the anchor. Use Archimedes' principle to relate the weight of the boat and anchor to the amount of water displaced. In both cases the total buoyant force must equal the weight of the barge plus the weight of the anchor. Thus the total amount of water displaced must be the same when the anchor is in the boat as when it is over the side. When the anchor is in the water the barge displaces less water, less by the amount the anchor displaces. Thus the barge rises in the water.

EXECUTE: The volume of the anchor is $V_{\text{anchor}} = m/\rho = (35.0\text{ kg})/(7860\text{ kg/m}^3) = 4.456\times 10^{-3}\text{ m}^3$. The barge rises in the water a vertical distance *h* given by $hA = 4.453\times 10^{-3}\text{ m}^3$, where *A* is the area of the bottom of the barge. $h = (4.453\times 10^{-3}\text{ m}^3)/(8.00\text{ m}^2) = 5.57\times 10^{-4}\text{ m}$.

EVALUATE: The barge rises a very small amount. The buoyancy force on the barge plus the buoyancy force on the anchor must equal the weight of the barge plus the weight of the anchor. When the anchor is in the water, the buoyancy force on it is less than its weight (the anchor doesn't float on its own), so part of the buoyancy force on the barge is used to help support the anchor. If the rope is cut, the buoyancy force on the barge must equal only the weight of the barge and the barge rises still farther.

14.80. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the barrel, with $+y$ upward. The buoyant force on the barrel is given by Archimedes's principle.

SET UP: $\rho_{\text{av}} = m_{\text{tot}}/V$. An object floats in a fluid if its average density is less than the density of the fluid. The density of seawater is 1030 kg/m^3 .

EXECUTE: **(a)** The average density of a filled barrel is $\dfrac{m_{\text{oil}} + m_{\text{steel}}}{V}\quad \rho_{\text{oil}} + \dfrac{m_{\text{steel}}}{V} = 750\text{ kg/m}^3 + \dfrac{15.0\text{ kg}}{0.120\text{ m}^3} = 875\text{ kg/m}^3$, which is less than the density of seawater, so the barrel floats.

(b) The fraction above the surface (see Problem 14.29) is

$$1-\frac{\rho_{\text{av}}}{\rho_{\text{water}}}=1-\frac{875\text{ kg/m}^3}{1030\text{ kg/m}^3}=0.150=15.0\%.$$

(c) The average density is $910\text{ kg/m}^3+\frac{32.0\text{ kg}}{0.120\text{ m}^3}=1172\text{ kg/m}^3$, which means the barrel sinks. In order to lift it, a tension $T=w_{\text{tot}}-B=(1177\text{ kg/m}^3)(0.120\text{ m}^3)(9.80\text{ m/s}^2)-(1030\text{ kg/m}^3)(0.120\text{ m}^3)(9.80\text{ m/s}^2)=173\text{ N}$ is required.

EVALUATE: When the barrel floats, the buoyant force B equals its weight, w. In part (c) the buoyant force is less than the weight and $T=w-B$.

14.81. IDENTIFY: Apply Newton's 2nd law to the block. In part (a), use Archimedes' principle for the buoyancy force. In part (b), use Eq.(14.6) to find the pressure at the lower face of the block and then use Eq.(14.3) to calculate the force the fluid exerts.

(a) SET UP: The free-body diagram for the block is given in Figure 14.81a.

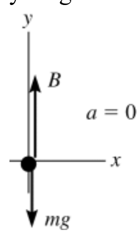


Figure 14.81a

EXECUTE: $\sum F_y=ma_y$

$B-mg=0$

$\rho_{\text{L}}V_{\text{sub}}g=\rho_{\text{B}}V_{\text{obj}}g$

The fraction of the volume that is submerged is $V_{\text{sub}}/V_{\text{obj}}=\rho_{\text{B}}/\rho_{\text{L}}$.

Thus the fraction that is *above* the surface is $V_{\text{above}}/V_{\text{obj}}=1-\rho_{\text{B}}/\rho_{\text{L}}$.

EVALUATE: If $\rho_{\text{B}}=\rho_{\text{L}}$ the block is totally submerged as it floats.

(b) SET UP: Let the water layer have depth d, as shown in Figure 14.81b.

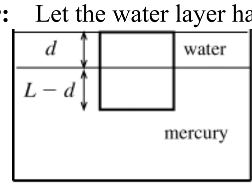


Figure 14.81b

EXECUTE: $p=p_0+\rho_{\text{w}}gd+\rho_{\text{L}}g(L-d)$

Applying $\sum F_y=ma_y$ to the block gives

$(p-p_0)A-mg=0.$

$[\rho_{\text{w}}gd+\rho_{\text{L}}g(L-d)]A=\rho_{\text{B}}LAg$

A and g divide out and $\rho_{\text{w}}d+\rho_{\text{L}}(L-d)=\rho_{\text{B}}L$

$d(\rho_{\text{w}}-\rho_{\text{L}})=(\rho_{\text{B}}-\rho_{\text{L}})L$

$$d=\left(\frac{\rho_{\text{L}}-\rho_{\text{B}}}{\rho_{\text{L}}-\rho_{\text{w}}}\right)L$$

(c) $d=\left(\frac{13.6\times10^3\text{ kg/m}^3-7.8\times10^3\text{ kg/m}^3}{13.6\times10^3\text{ kg/m}^3-1000\text{ kg/m}^3}\right)(0.100\text{ m})=0.0460\text{ m}=4.60\text{ cm}$

EVALUATE: In the expression derived in part (b), if $\rho_{\text{B}}=\rho_{\text{L}}$ the block floats in the liquid totally submerged and no water needs to be added. If $\rho_{\text{L}}\to\rho_{\text{w}}$ the block continues to float with a fraction $1-\rho_{\text{B}}/\rho_{\text{w}}$ above the water as water is added, and the water never reaches the top of the block $(d\to\infty)$.

14.82. IDENTIFY: For the floating tanker, the buoyant force equals its total weight. The buoyant force is given by Archimedes's principle.

SET UP: When the metal is in the tanker, it displaces its weight of water and after it has been pushed overboard it displaces its volume of water.

EXECUTE: **(a)** The change in height Δy is related to the displaced volume ΔV by $\Delta y=\frac{\Delta V}{A}$, where A is the surface area of the water in the lock. ΔV is the volume of water that has the same weight as the metal, so

$$\Delta y=\frac{\Delta V}{A}=\frac{w/(\rho_{\text{water}}g)}{A}=\frac{w}{\rho_{\text{water}}gA}=\frac{(2.50\times10^6\text{ N})}{(1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)[(60.0\text{ m})(20.0\text{ m})]}=0.213\text{ m}.$$

(b) In this case, ΔV is the volume of the metal; in the above expression, ρ_{water} is replaced by $\rho_{\text{metal}} = 9.00\rho_{\text{water}}$, which gives $\Delta y' = \frac{\Delta y}{9}$, and $\Delta y - \Delta y' = \frac{8}{9}\Delta y = 0.189$ m; the water level falls this amount.

EVALUATE: The density of the metal is greater than the density of water, so the volume of water that has the same weight as the steel is greater than the volume of water that has the same volume as the steel.

14.83. **IDENTIFY:** Consider the fluid in the horizontal part of the tube. This fluid, with mass ρAl, is subject to a net force due to the pressure difference between the ends of the tube

SET UP: The difference between the gauge pressures at the bottoms of the ends of the tubes is $\rho g(y_{\text{L}} - y_{\text{R}})$.

EXECUTE: The net force on the horizontal part of the fluid is $\rho g(y_{\text{L}} - y_{\text{R}})A = \rho Ala$, or, $(y_{\text{L}} - y_{\text{R}}) = \frac{a}{g}l.$

(b) Again consider the fluid in the horizontal part of the tube. As in part (a), the fluid is accelerating; the center of mass has a radial acceleration of magnitude $a_{\text{rad}} = \omega^2 l/2,$ and so the difference in heights between the columns is $(\omega^2 l/2)(l/g) = \omega^2 l^2/2g.$ An equivalent way to do part (b) is to break the fluid in the horizontal part of the tube into elements of thickness dr; the pressure difference between the sides of this piece is $dp = \rho(\omega^2 r)dr$ and integrating from $r = 0$ to $r = l$ gives $\Delta p = \rho\omega^2 l^2/2,$ the same result.

EVALUATE: **(c)** The pressure at the bottom of each arm is proportional to ρ and the mass of fluid in the horizontal portion of the tube is proportional to ρ , so ρ divides out and the results are independent of the density of the fluid. The pressure at the bottom of a vertical arm is independent of the cross-sectional area of the arm. Newton's second law could be applied to a cross-section of fluid smaller than that of the tubes. Therefore, the results are independent and of the size and shape of all parts of the tube.

14.84. **IDENTIFY:** Apply $\sum \vec{F} = m\vec{a}$ to a small fluid element located a distance r from the axis.

SET UP: For rotational motion, $a = \omega^2 r$.

EXECUTE: **(a)** The change in pressure with respect to the vertical distance supplies the force necessary to keep a fluid element in vertical equilibrium (opposing the weight). For the rotating fluid, the change in pressure with respect to radius supplies the force necessary to keep a fluid element accelerating toward the axis; specifically, $dp = \frac{\partial p}{\partial r}dr = \rho a\, dr$, and using $a = \omega^2 r$ gives $\frac{\partial p}{\partial r} = \rho\omega^2 r.$

(b) Let the pressure at $y = 0,\ r = 0$ be p_a (atmospheric pressure); integrating the expression for $\frac{\partial p}{\partial r}$ from part (a) gives $p(r,\ y = 0) = p_{\text{a}} + \frac{\rho\omega^2}{2}r^2$

(c) In Eq. (14.5), $p_2 = p_{\text{a}}$, $p = p_1 = p(r,\ y = 0)$ as found in part (b), $y_1 = 0$ and $y_2 = h(r)$, the height of the liquid above the $y = 0$ plane. Using the result of part (b) gives $h(r) = \omega^2 r^2/2g.$

EVALUATE: The curvature of the surface increases as the speed of rotation increases.

14.85. **IDENTIFY:** Follow the procedure specified in part (a) and integrate this result for part (b).

SET UP: A rotating particle a distance r' from the rotation axis has inward acceleration $\omega^2 r'$.

EXECUTE: **(a)** The net inward force is $(p + dp)A - pA = Adp,$ and the mass of the fluid element is $\rho Adr'$. Using Newton's second law, with the inward radial acceleration of $\omega^2 r'$, gives $dp = \rho\omega^2 r'dr'$.

(b) Integrating the above expression, $\int_{p_0}^{p} dp = \int_{r_0}^{r} \rho\omega^2 r'dr'$ and $p - p_0 = \left(\frac{\rho\omega^2}{2}\right)(r^2 - r^2{}_0),$ which is the desired result.

(c) The net force on the object must be the same as that on a fluid element of the same shape. Such a fluid element is accelerating inward with an acceleration of magnitude $\omega^2 R_{\text{cm,}}$ and so the force on the object is $\rho V\omega^2 R_{\text{cm}}$.

(d) If $\rho \text{R}_{\text{cm}} > \rho_{\text{ob}} R_{\text{cm ob}}$, the inward force is greater than that needed to keep the object moving in a circle with radius $R_{\text{cm ob}}$ at angular frequency ω , and the object moves inward. If $\rho R_{\text{cm}} < \rho_{\text{ob}} R_{\text{cm ob}}$, the net force is insufficient to keep the object in the circular motion at that radius, and the object moves outward.

(e) Objects with lower densities will tend to move toward the center, and objects with higher densities will tend to move away from the center.

EVALUATE: The pressure in the fluid increases as the distance r from the rotation axis increases.

14.86. **IDENTIFY:** Follow the procedure specified in the problem.

SET UP: Let increasing x correspond to moving toward the back of the car.

EXECUTE: **(a)** The mass of air in the volume element is $\rho dV = \rho A dx$, and the net force on the element in the forward direction is $\left(p + dp\right)A - pA = Adp$. From Newton's second law, $Adp = (\rho A dx)a$, from which $dp = \rho a dx$.

(b) With ρ given to be constant, and with $p = p_0$ at $x = 0$, $\quad p = p_0 + \rho a x$.

(c) Using $\rho = 1.2\ \text{kg/m}^3$ in the result of part (b) gives $(1.2\ \text{kg/m}^3)(5.0\ \text{m/s}^2)(2.5\ \text{m}) = 15.0\ \text{Pa} = 15\times 10^{-5} p_{\text{atm}}$, so the fractional pressure difference is negligible.

(d) Following the argument in Section 14.4, the force on the balloon must be the same as the force on the same volume of air; this force is the product of the mass ρV and the acceleration, or $\rho V a$.

(e) The acceleration of the balloon is the force found in part (d) divided by the mass $\rho_{\text{bal}}V$, or $(\rho/\rho_{\text{bal}})a$. The acceleration relative to the car is the difference between this acceleration and the car's acceleration, $a_{\text{rel}} = [(\rho/\rho_{\text{bal}}) - 1]a$.

(f) For a balloon filled with air, $\left(\rho/\rho_{\text{bal}}\right) < 1$ (air balloons tend to sink in still air), and so the quantity in square brackets in the result of part (e) is negative; the balloon moves to the back of the car. For a helium balloon, the quantity in square brackets is positive, and the balloon moves to the front of the car.

EVALUATE: The pressure in the air inside the car increases with distance from the windshield toward the rear of the car. This pressure increase is proportional to the acceleration of the car.

14.87. **IDENTIFY:** After leaving the tank, the water is in free fall, with $a_x = 0$ and $a_y = +g$.

SET UP: From Example 14.8, the speed of efflux is $\sqrt{2gh}$.

EXECUTE: **(a)** The time it takes any portion of the water to reach the ground is $t = \sqrt{\dfrac{2(H-h)}{g}}$, in which time the water travels a horizontal distance $R = vt = 2\sqrt{h(H-h)}$.

(b) Note that if $h' = H - h, h'(H - h') = (H - h)h$, and so $h' = H - h$ gives the same range. A hole $H - h$ below the water surface is a distance h above the bottom of the tank.

EVALUATE: For the special case of $h = H/2$, $h = h'$ and the two points coincide. For the upper hole the speed of efflux is less but the time in the air during the free-fall is greater.

14.88. **IDENTIFY:** Use Bernoulli's equation to find the velocity with which the water flows out the hole.

SET UP: The water level in the vessel will rise until the volume flow rate into the vessel, $2.40\times 10^{-4}\ \text{m}^3/\text{s}$, equals the volume flow rate out the hole in the bottom.

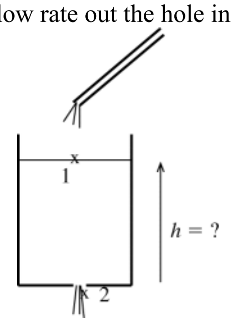


Figure 14.88

Let points 1 and 2 be chosen as in Figure 14.88.

EXECUTE: Bernoulli's equation: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

Volume flow rate out of hole equals volume flow rate from tube gives that $v_2 A_2 = 2.40\times 10^{-4}\ \text{m}^3/\text{s}$ and

$$v_2 = \frac{2.40\times 10^{-4}\ \text{m}^3/\text{s}}{1.50\times 10^{-4}\ \text{m}^2} = 1.60\ \text{m/s}$$

$A_1 \gg A_2$ and $v_1 A_1 = v_2 A_2$ says that $\frac{1}{2}\rho v_1^2 \ll \frac{1}{2}\rho v_2^2$; neglect the $\frac{1}{2}\rho v_1^2$ term.

Measure y from the bottom of the bucket, so $y_2 = 0$ and $y_1 = h$.

$p_1 = p_2 = p_\text{a}$ (air pressure)

Then $p_\text{a} + \rho g h = p_\text{a} + \frac{1}{2}\rho v_2^2$ and $h = v_2^2/2g = (1.60\ \text{m/s})^2/2(9.80\ \text{m/s}^2) = 0.131\ \text{m} = 13.1\ \text{cm}$

EVALUATE: The greater the flow rate into the bucket, the larger v_2 will be at equilibrium and the higher the water will rise in the bucket.

14.89. **IDENTIFY:** Apply Bernoulli's equation and the equation of continuity.

SET UP: Example 14.8 says the speed of efflux is $\sqrt{2gh}$, where h is the distance of the hole below the surface of the fluid.

EXECUTE: **(a)** $v_3A_3 = \sqrt{2g(y_1 - y_3)}A_3 = \sqrt{2(9.80\text{ m/s}^2)(8.00\text{ m})}(0.0160\text{ m}^2) = 0.200\text{ m}^3/\text{s}.$

(b) Since p_3 is atmospheric, the gauge pressure at point 2 is $p_2 = \frac{1}{2}\rho(v_3^2 - v_2^2) = \frac{1}{2}\rho v_3^2\left(1 - \left(\frac{A_3}{A_2}\right)^2\right) = \frac{8}{9}\rho g(y_1 - y_3),$

using the expression for v_3 found above. Substitution of numerical values gives $p_2 = 6.97\times10^4$ Pa.

EVALUATE: We could also calculate p_2 by applying Bernoulli's equation to points 1 and 2.

14.90. **IDENTIFY:** Apply Bernoulli's equation to the air in the hurricane.

SET UP: For a particle a distance r from the axis, the angular momentum is $L = mvr$.

EXECUTE: **(a)** Using the constancy of angular momentum, the product of the radius and speed is constant, so the speed at the rim is about $(200\text{ km/h})\left(\frac{30}{350}\right) = 17\text{ km/h}.$

(b) The pressure is lower at the eye, by an amount

$$\Delta p = \frac{1}{2}(1.2\text{ kg/m}^3)((200\text{ km/h})^2 - (17\text{ km/h})^2)\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right)^2 = 1.8\times10^3\text{ Pa}.$$

(c) $\frac{v^2}{2g} = 160\text{ m}$.

(d) The pressure difference at higher altitudes is even greater.

EVALUATE: According to Bernoulli's equation, the pressure decreases when the fluid velocity increases.

14.91. **IDENTIFY:** Apply Bernoulli's equation and the equation of continuity.

SET UP: Example 14.8 shows that the speed of efflux at point D is $\sqrt{2gh_1}$.

EXECUTE: Applying the equation of continuity to points at C and D gives that the fluid speed is $\sqrt{8gh_1}$ at C. Applying Bernoulli's equation to points A and C gives that the gauge pressure at C is $\rho gh_1 - 4\rho gh_1 = -3\rho gh_1$, and this is the gauge pressure at the surface of the fluid at E. The height of the fluid in the column is $h_2 = 3h_1$.

EVALUATE: The gauge pressure at C is less than the gauge pressure ρgh_1 at the bottom of tank A because of the speed of the fluid at C.

14.92. **IDENTIFY:** Apply Bernoulli's equation to points 1 and 2. Apply $p = p_0 + \rho gh$ to both arms of the U-shaped tube in order to calculate h.

SET UP: The discharge rate is $v_1A_1 = v_2A_2$. The density of mercury is $\rho_m = 13.6\times10^3\text{ kg/m}^3$ and the density of water is $\rho_w = 1.00\times10^3\text{ kg/m}^3$. Let point 1 be where $A_1 = 40.0\times10^{-4}\text{ m}^2$ and point 2 is where $A_2 = 10.0\times10^{-4}\text{ m}^2$. $y_1 = y_2$.

EXECUTE: **(a)** $v_1 = \frac{6.00\times10^{-3}\text{ kg/m}^3}{40.0\times10^{-4}\text{ m}^2} = 1.50\text{ m/s}$. $v_2 = \frac{6.00\times10^{-3}\text{ kg/m}^3}{10.0\times10^{-4}\text{ m}^2} = 6.00\text{ m/s}$

(b) $p_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$.

$p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}(1000\text{ kg/m}^3)([6.00\text{ m/s}]^2 - [1.50\text{ m/s}]^2) = 1.69\times10^4\text{ Pa}$

(c) $p_1 + \rho_w gh = p_2 + \rho_m gh$ and $h = \frac{p_1 - p_2}{(\rho_m - \rho_w)g} = \frac{1.69\times10^4\text{ Pa}}{(13.6\times10^3\text{ kg/m}^3 - 1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 0.137\text{ m} = 13.7\text{ cm}.$

EVALUATE: The pressure in the fluid decreases when the speed of the fluid increases.

14.93. **(a) IDENTIFY:** Apply constant acceleration equations to the falling liquid to find its speed as a function of the distance below the outlet. Then apply Eq.(14.10) to relate the speed to the radius of the stream.

SET UP:

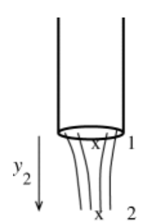


Figure 14.93

Let point 1 be at the end of the pipe and let point 2 be in the stream of liquid at a distance y_2 below the end of the tube, as shown in Figure 14.93.

Consider the free-fall of the liquid. Take $+y$ to be downward.

Free-fall implies $a_y = g$. v_y is positive, so replace it by the speed v.

EXECUTE: $v_2^2 = v_1^2 + 2a(y - y_0)$ gives $v_2^2 = v_1^2 + 2gy_2$ and $v_2 = \sqrt{v_1^2 + 2gy_2}$.

Equation of continuity says $v_1A_1 = v_2A_2$

And since $A = \pi r^2$ this becomes $v_1\pi r_1^2 = v_2\pi r_2^2$ and $v_2 = v_1(r_1/r_2)^2$.

Use this in the above to eliminate v_2: $v_1(r_1^2/r_2^2) = \sqrt{v_1^2 + 2gy_2}$

$r_2 = r_1\sqrt{v_1}/(v_1^2 + 2gy_2)^{1/4}$

To correspond to the notation in the problem, let $v_1 = v_0$ and $r_1 = r_0$, since point 1 is where the liquid first leaves the pipe, and let r_2 be *r* and y_2 be *y*. The equation we have derived then becomes $r = r_0\sqrt{v_0}/(v_0^2 + 2gy)^{1/4}$

(b) $v_0 = 1.20$ m/s

We want the value of *y* that gives $r = \frac{1}{2}r_0$, or $r_0 = 2r$

The result obtained in part (a) says $r^4(v_0^2 + 2gy) = r_0^4v_0^2$

Solving for *y* gives $y = \dfrac{[(r_0/r)^4 - 1]v_0^2}{2g} = \dfrac{(16-1)(1.20\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 1.10$ m

EVALUATE: The equation derived in part (a) says that *r* decreases with distance below the end of the pipe.

14.94. **IDENTIFY:** Apply $\sum F_y = ma_y$ to the rock.

SET UP: In the accelerated frame, all of the quantities that depend on *g* (weights, buoyant forces, gauge pressures and hence tensions) may be replaced by $g' = g + a$, with the positive direction taken upward.

EXECUTE: **(a)** The volume *V* of the rock is

$$V = \frac{B}{\rho_{\text{water}}g} = \frac{w - T}{\rho_{\text{water}}g} = \frac{((3.00\text{ kg})(9.80\text{ m/s}^2) - 21.0\text{ N})}{(1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = 8.57\times10^{-4}\text{ m}^3.$$

(b) The tension is $T = mg' - B' = (m - \rho V)g' = T_0\dfrac{g'}{g}$, where $T_0 = 21.0$N. $g' = g + a$. For $a = 2.50\text{ m/s}^2$,

$T = (21.0\text{ N})\dfrac{9.80 + 2.50}{9.80} = 26.4$ N.

(c) For $a = -2.50\text{ m/s}^2$, $T = (21.0\text{ N})\dfrac{9.80 - 2.50}{9.80} = 15.6$ N.

(d) If $a = -g$, $g' = 0$ and $T = 0$.

EVALUATE: The acceleration of the water alters the buoyant force it exerts.

14.95. **IDENTIFY:** The sum of the vertical forces on the object must be zero.

SET UP: The depth of the bottom of the styrofoam is not given; let this depth be h_0. Denote the length of the piece of foam by *L* and the length of the two sides by *l*. The volume of the object is $\frac{1}{2}l^2L$.

EXECUTE: **(a)** The tension in the cord plus the weight must be equal to the buoyant force, so

$T = Vg(\rho_{\text{water}} - \rho_{\text{foam}}) = \frac{1}{2}(0.20\text{ m})^2(0.50\text{ m})(9.80\text{ m/s}^2)(1000\text{ kg/m}^3 - 180\text{ kg/m}^3) = 80.4$ N.

(b) The pressure force on the bottom of the foam is $(p_0 + \rho gh_0)L\left(\sqrt{2}l\right)$ and is directed up. The pressure on each side is not constant; the force can be found by integrating, or using the results of Problem 14.49 or Problem 14.51. Although these problems found forces on vertical surfaces, the result that the force is the product of the average pressure and the area is valid. The average pressure is $p_0 + \rho g(h_0 - (l/(2\sqrt{2})))$, and the force on one side has magnitude $(p_0 + \rho g(h_0 - l/(2\sqrt{2})))Ll$

and is directed perpendicular to the side, at an angle of $45.0°$ from the vertical. The force on the other side has the same magnitude, but has a horizontal component that is opposite that of the other side. The horizontal component of the net buoyant force is zero, and the vertical component is

$$B = (p_0 + \rho gh_0)Ll\sqrt{2} - 2(\cos 45.0°)(p_0 + \rho g(h_0 - l/(2\sqrt{2})))Ll = \rho g\frac{Ll^2}{2},$$

the weight of the water displaced.

EVALUATE: The density of the object is less than the density of water, so if the cord were cut the object would float. When the object is fully submerged, the upward buoyant force is greater than its weight and the cord must pull downward on the object to hold it beneath the surface.

14.96. **IDENTIFY:** Use the efflux speed to calculate the volume flow rate and integrate to find the time for the entire volume of water to flow out of the tank.

SET UP: When the level of the water is a height y above the opening, the efflux speed is $\sqrt{2gy}$, and $\frac{dV}{dt} = \pi(d/2)^2\sqrt{2gy}.$

EXECUTE: As the tank drains, the height decreases, and $\frac{dy}{dt} = -\frac{dV/dt}{A} = -\frac{\pi(d/2)^2\sqrt{2gy}}{\pi(D/2)^2} = -\left(\frac{d}{D}\right)^2\sqrt{2gy}.$ This is a separable differential equation, and the time T to drain the tank is found from $\frac{dy}{\sqrt{y}} = -\left(\frac{d}{D}\right)^2\sqrt{2g}\,dt,$ which integrates to $\left[2\sqrt{y}\,\right]_H^0 = -\left(\frac{d}{D}\right)^2\sqrt{2g}T,$ or $T = \left(\frac{D}{d}\right)^2\frac{2\sqrt{H}}{\sqrt{2g}} = \left(\frac{D}{d}\right)^2\sqrt{\frac{2H}{g}}.$

EVALUATE: Even though the volume flow rate approaches zero as the tank drains, it empties in a finite amount of time. Doubling the height of the tank doubles the volume of water in the tank but increases the time to drain by only a factor of $\sqrt{2}$.

14.97. **IDENTIFY:** Apply Bernoulli's equation to the fluid in the siphon.

SET UP: Example 14.8 shows that the efflux speed from a small hole a distance h below the surface of fluid in a large open tank is $\sqrt{2gh}$.

EXECUTE: **(a)** The fact that the water first moves upwards before leaving the siphon does not change the efflux speed, $\sqrt{2gh}.$

(b) Water will not flow if the absolute (not gauge) pressure would be negative. The hose is open to the atmosphere at the bottom, so the pressure at the top of the siphon is $p_a - \rho g(H+h),$ where the assumption that the cross-section area is constant has been used to equate the speed of the liquid at the top and bottom. Setting $p = 0$ and solving for H gives $H = (p_a/\rho g) - h.$

EVALUATE: The analysis shows that $H + h < \frac{p_a}{\rho g}$, so there is also a limitation on $H + h$. For water and normal atmospheric pressure, $\frac{p_a}{\rho g} = 10.3$ m .

14.98. **IDENTIFY** and **SET UP:** Apply $p = p_0 + \rho gh$.

EXECUTE: Any bubbles will cause inaccuracies. At the bubble, the pressure at the surfaces of the water will be the same, but the levels need not be the same. The use of a hose as a level assumes that pressure is the same at all points that are at the same level, an assumption that is invalidated by the bubble.

EVALUATE: Larger bubbles can cause larger inaccuracies, because there can be greater changes in height across the length of the bubble.

15

MECHANICAL WAVES

15.1. **IDENTIFY:** $v = f\lambda$. $T = 1/f$ is the time for one complete vibration.

SET UP: The frequency of the note one octave higher is 1568 Hz.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{784\text{ Hz}} = 0.439\text{ m}$. $T = \dfrac{1}{f} = 1.28\text{ ms}$.

(b) $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{1568\text{ Hz}} = 0.219\text{ m}$.

EVALUATE: When *f* is doubled, λ is halved.

15.2. **IDENTIFY:** The distance between adjacent dots is λ . $v = f\lambda$. The long-wavelength sound has the lowest frequency, 20.0 Hz, and the short-wavelength sound has the highest frequency, 20.0 kHz.

SET UP: For sound in air, $v = 344\text{ m/s}$.

EXECUTE: **(a)** Red dots: $\lambda = \dfrac{v}{f} = \dfrac{344\text{ m/s}}{20.0\text{ Hz}} = 17.2\text{ m}$.

Blue dots: $\lambda = \dfrac{344\text{ m/s}}{20.0\times10^3\text{ Hz}} = 0.0172\text{ m} = 1.72\text{ cm}$.

(b) In each case the separation easily can be measured with a meterstick.

(c) Red dots: $\lambda = \dfrac{v}{f} = \dfrac{1480\text{ m/s}}{20.0\text{ Hz}} = 74.0\text{ m}$.

Blue dots: $\lambda = \dfrac{1480\text{ m/s}}{20.0\times10^3\text{ Hz}} = 0.0740\text{ m} = 7.40\text{ cm}$. In each case the separation easily can be measured with a meterstick, although for the red dots a long tape measure would be more convenient.

EVALUATE: Larger wavelengths correspond to smaller frequencies. When the wave speed increases, for a given frequency, the wavelength increases.

15.3. **IDENTIFY:** $v = f\lambda = \lambda/T$.

SET UP: $1.0\text{ h} = 3600\text{ s}$. The crest to crest distance is λ .

EXECUTE: $v = \dfrac{800\times10^3\text{ m}}{3600\text{ s}} = 220\text{ m/s}$. $v = \dfrac{800\text{ km}}{1.0\text{ h}} = 800\text{ km/h}$.

EVALUATE: Since the wave speed is very high, the wave strikes with very little warning.

15.4. **IDENTIFY:** $f\lambda = v$

SET UP: $1.0\text{ mm} = 0.0010\text{ m}$

EXECUTE: $f = \dfrac{v}{\lambda} = \dfrac{1500\text{ m/s}}{0.0010\text{ m}} = 1.5\times10^6\text{ Hz}$

EVALUATE: The frequency is much higher than the upper range of human hearing.

15.5. **IDENTIFY:** $v = f\lambda$. $T = 1/f$.

SET UP: $1\text{ nm} = 10^{-9}\text{ m}$

EXECUTE: **(a)** $\lambda = 400\text{ nm}$: $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\text{ m/s}}{400\times10^{-9}\text{ m}} = 7.50\times10^{14}\text{ Hz}$. $T = 1/f = 1.33\times10^{-15}\text{ s}$.

$\lambda = 700\text{ nm}$: $f = \dfrac{3.00\times10^8\text{ m/s}}{700\times10^{-9}\text{ m}} = 4.29\times10^{14}\text{ Hz}$. $T = 2.33\times10^{-15}\text{ s}$. The frequencies of visible light lie between $4.29\times10^{14}\text{ Hz}$ and $7.50\times10^{14}\text{ Hz}$. The periods lie between $1.33\times10^{-15}\text{ s}$ and $2.33\times10^{-15}\text{ s}$.

(b) *T* is very short and cannot be measured with a stopwatch.

EVALUATE: Longer wavelength corresponds to smaller frequency and larger period.

15.6. **IDENTIFY:** Compare $y(x,t)$ given in the problem to the general form of Eq.(15.4). $f = 1/T$ and $v = f\lambda$

SET UP: The comparison gives $A = 6.50\text{ mm}$, $\lambda = 28.0\text{ cm}$ and $T = 0.0360\text{ s}$.

EXECUTE: **(a)** 6.50 mm

(b) 28.0 cm

(c) $f = \dfrac{1}{0.0360\text{ s}} = 27.8\text{ Hz}$

(d) $v = (0.280\text{ m})(27.8\text{ Hz}) = 7.78\text{ m/s}$

(e) Since there is a minus sign in front of the t/T term, the wave is traveling in the $+x$-direction.

EVALUATE: The speed of propagation does not depend on the amplitude of the wave.

15.7. **IDENTIFY:** Use Eq.(15.1) to calculate v. $T = 1/f$ and k is defined by Eq.(15.5). The general form of the wave function is given by Eq.(15.8), which is the equation for the transverse displacement.

SET UP: $v = 8.00\text{ m/s}$, $A = 0.0700\text{ m}$, $\lambda = 0.320\text{ m}$

EXECUTE: **(a)** $v = f\lambda$ so $f = v/\lambda = (8.00\text{ m/s})/(0.320\text{ m}) = 25.0\text{ Hz}$

$T = 1/f = 1/25.0\text{ Hz} = 0.0400\text{ s}$

$k = 2\pi/\lambda = 2\pi\text{ rad/0.320 m} = 19.6\text{ rad/m}$

(b) For a wave traveling in the $-x$-direction,

$y(x,\ t) = A\cos 2\pi(x/\lambda + t/T)$ (Eq.(15.8).)

At $x = 0$, $y(0,\ t) = A\cos 2\pi(t/T)$, so $y = A$ at $t = 0$. This equation describes the wave specified in the problem.

Substitute in numerical values:

$y(x,\ t) = (0.0700\text{ m})\cos(2\pi(x/0.320\text{ m} + t/0.0400\text{ s}))$.

Or, $y(x,\ t) = (0.0700\text{ m})\cos((19.6\text{ m}^{-1})x + (157\text{ rad/s})t)$.

(c) From part (b), $y = (0.0700\text{ m})\cos(2\pi(x/0.320\text{ m} + t/0.0400\text{ s}))$.

Plug in $x = 0.360\text{ m}$ and $t = 0.150\text{ s}$:

$y = (0.0700\text{ m})\cos(2\pi(0.360\text{ m}/0.320\text{ m} + 0.150\text{ s}/0.0400\text{ s}))$

$y = (0.0700\text{ m})\cos[2\pi(4.875\text{ rad})] = +0.0495\text{ m} = +4.95\text{ cm}$

(d) In part (c) $t = 0.150\text{ s}$.

$y = A$ means $\cos(2\pi(x/\lambda + t/T)) = 1$

$\cos\theta = 1$ for $\theta = 0,\ 2\pi,\ 4\pi, \ldots = n(2\pi)$ or $n = 0,\ 1,\ 2, \ldots$

So $y = A$ when $2\pi(x/\lambda + t/T) = n(2\pi)$ or $x/\lambda + x/T = n$

$t = T(n - x/\lambda) = (0.0400\text{ s})(n - 0.360\text{ m}/0.320\text{ m}) = (0.0400\text{ s})(n - 1.125)$

For $n = 4$, $t = 0.1150\text{ s}$ (before the instant in part (c))

For $n = 5$, $t = 0.1550\text{ s}$ (the first occurrence of $y = A$ after the instant in part (c)) Thus the elapsed time is $0.1550\text{ s} - 0.1500\text{ s} = 0.0050\text{ s}$.

EVALUATE: Part (d) says $y = A$ at 0.115 s and next at 0.155 s; the difference between these two times is 0.040 s, which is the period. At $t = 0.150\text{ s}$ the particle at $x = 0.360\text{ m}$ is at $y = 4.95\text{ cm}$ and traveling upward. It takes $T/4 = 0.0100\text{ s}$ for it to travel from $y = 0$ to $y = A$, so our answer of 0.0050 s is reasonable.

15.8. **IDENTIFY:** The general form of the wave function for a wave traveling in the $-x$-direction is given by Eq.(15.8). The time for one complete cycle to pass a point is the period T and the number that pass per second is the frequency f. The speed of a crest is the wave speed v and the maximum speed of a particle in the medium is $v_{max} = \omega A$.

SET UP: Comparison to Eq.(15.8) gives $A = 3.75\text{ cm}$, $k = 0.450\text{ rad/cm}$ and $\omega = 5.40\text{ rad/s}$.

EXECUTE: **(a)** $T = \dfrac{2\pi\text{ rad}}{\omega} = \dfrac{2\pi\text{ rad}}{5.40\text{ rad/s}} = 1.16\text{ s}$. In one cycle a wave crest travels a distance

$\lambda = \dfrac{2\pi\text{ rad}}{k} = \dfrac{2\pi\text{ rad}}{0.450\text{ rad/cm}} = 0.140\text{ m}$.

(b) $k = 0.450\text{ rad/cm}$. $f = 1/T = 0.862\text{ Hz} = 0.862\text{ waves/second}$.

(c) $v = f\lambda = (0.862\text{ Hz})(0.140\text{ m}) = 0.121\text{ m/s}$. $v_{max} = \omega A = (5.40\text{ rad/s})(3.75\text{ cm}) = 0.202\text{ m/s}$.

EVALUATE: The transverse velocity of the particles in the medium (water) is not the same as the velocity of the wave.

15.9. **IDENTIFY:** Evaluate the partial derivatives and see if Eq.(15.12) is satisfied.

SET UP: $\frac{\partial}{\partial x}\cos(kx+\omega t) = -k\sin(kx+\omega t)$. $\frac{\partial}{\partial t}\cos(kx+\omega t) = -\omega\sin(kx+\omega t)$. $\frac{\partial}{\partial x}\sin(kx+\omega t) = k\cos(kx+\omega t)$. $\frac{\partial}{\partial t}\sin(kx+\omega t) = \omega\sin(kx+\omega t)$.

EXECUTE: **(a)** $\frac{\partial^2 y}{\partial x^2} = -Ak^2\cos(kx+\omega t)$. $\frac{\partial^2 y}{\partial t^2} = -A\omega^2\cos(kx+\omega t)$. Eq.(15.12) is satisfied, if $v = \omega/k$.

(b) $\frac{\partial^2 y}{\partial x^2} = -Ak^2\sin(kx+\omega t)$. $\frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(kx+\omega t)$. Eq.(15.12) is satisfied, if $v = \omega/k$.

(c) $\frac{\partial y}{\partial x} = -kA\sin(kx)$. $\frac{\partial^2 y}{\partial x^2} = -k^2A\cos(kx)$. $\frac{\partial y}{\partial t} = -\omega A\sin(\omega t)$. $\frac{\partial^2 y}{\partial t^2} = -\omega^2 A\cos(\omega t)$. Eq.(15.12) is not satisfied.

(d) $v_y = \frac{\partial y}{\partial t} = \omega A\cos(kx+\omega t)$. $a_y = \frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(kx+\omega t)$

EVALUATE: The functions $\cos(kx+\omega t)$ and $\sin(kx+\omega t)$ differ only in phase.

15.10. **IDENTIFY:** v_y and a_y are given by Eqs.(15.9) and (15.10).

SET UP: The sign of v_y determines the direction of motion of a particle on the string. If $v_y = 0$ and $a_y \neq 0$ the speed of the particle is increasing. If $v_y \neq 0$, the particle is speeding up if v_y and a_y have the same sign and slowing down if they have opposite signs.

EXECUTE: **(a)** The graphs are given in Figure 15.10.

(b) (i) $v_y = \omega A\sin(0) = 0$ and the particle is instantaneously at rest. $a_y = -\omega^2 A\cos(0) = -\omega^2 A$ and the particle is speeding up.

(ii) $v_y = \omega A\sin(\pi/4) = \omega A/\sqrt{2}$, and the particle is moving up. $a_y = -\omega^2 A\cos(\pi/4) = -\omega^2 A/\sqrt{2}$, and the particle is slowing down (v_y and a_y have opposite sign).

(iii) $v_y = \omega A\sin(\pi/2) = \omega A$ and the particle is moving up. $a_y = -\omega^2 A\cos(\pi/2) = 0$ and the particle is instantaneously not accelerating.

(iv) $v_y = \omega A\sin(3\pi/4) = \omega A/\sqrt{2}$, and the particle is moving up. $a_y = -\omega^2 A\cos(3\pi/4) = \omega^2 A/\sqrt{2}$, and the particle is speeding up.

(v) $v_y = \omega A\sin(\pi) = 0$ and the particle is instantaneously at rest. $a_y = -\omega^2 A\cos(\pi) = \omega^2 A$ and the particle is speeding up.

(vi) $v_y = \omega A\sin(5\pi/4) = -\omega A/\sqrt{2}$ and the particle is moving down. $a_y = -\omega^2 A\cos(5\pi/4) = \omega^2 A/\sqrt{2}$ and the particle is slowing down (v_y and a_y have opposite sign).

(vii) $v_y = \omega A\sin(3\pi/2) = -\omega A$ and the particle is moving down. $a_y = -\omega^2 A\cos(3\pi/2) = 0$ and the particle is instantaneously not accelerating.

(viii) $v_y = \omega A\sin(7\pi/4) = -\omega A/\sqrt{2}$, and the particle is moving down. $a_y = -\omega^2 A\cos(7\pi/4) = -\omega^2 A/\sqrt{2}$ and the particle is speeding up (v_y and a_y have the same sign).

EVALUATE: At $t = 0$ the wave is represented by Figure 15.10a in the textbook: point (i) in the problem corresponds to the origin, and points (ii)-(viii) correspond to the points in the figure labeled 1-7. Our results agree with what is shown in the figure.

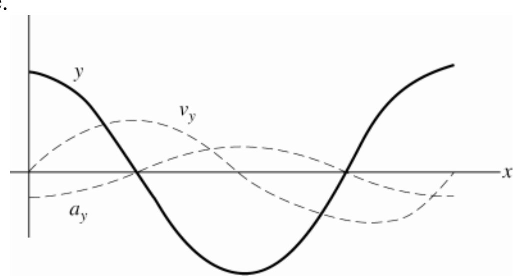


Figure 15.10

15.11. **IDENTIFY** and **SET UP:** Read A and T from the graph. Apply Eq.(15.4) to determine λ and then use Eq.(15.1) to calculate v.

EXECUTE: **(a)** The maximum y is 4 mm (read from graph).

(b) For either x the time for one full cycle is 0.040 s; this is the period.

(c) Since $y=0$ for $x=0$ and $t=0$ and since the wave is traveling in the $+x$-direction then $y(x,\ t)=A\sin[2\pi(t/T-x/\lambda)]$. (The phase is different from the wave described by Eq.(15.4); for that wave $y=A$ for $x=0,\ t=0$.) From the graph, if the wave is traveling in the $+x$-direction and if $x=0$ and $x=0.090$ m are within one wavelength the peak at $t=0.01$ s for $x=0$ moves so that it occurs at $t=0.035$ s (read from graph so is approximate) for $x=0.090$ m. The peak for $x=0$ is the first peak past $t=0$ so corresponds to the first maximum in $\sin[2\pi(t/T-x/\lambda)]$ and hence occurs at $2\pi(t/T-x/\lambda)=\pi/2$. If this same peak moves to $t_1=0.035$ s at $x_1=0.090$ m, then

$2\pi(t_1/T-x_1/\lambda)=\pi/2$

Solve for λ: $t_1/T-x_1/\lambda=1/4$

$x_1/\lambda=t_1/T-1/4=0.035\text{ s}/0.040\text{ s}-0.25=0.625$

$\lambda=x_1/0.625=0.090\text{ m}/0.625=0.14\text{ m}.$

Then $v=f\lambda=\lambda/T=0.14\text{ m}/0.040\text{ s}=3.5\text{ m/s}.$

(d) If the wave is traveling in the $-x$-direction, then $y(x,\ t)=A\sin(2\pi(t/T+x/\lambda))$ and the peak at $t=0.050$ s for $x=0$ corresponds to the peak at $t_1=0.035$ s for $x_1=0.090$ m. This peak at $x=0$ is the second peak past the origin so corresponds to $2\pi(t/T+x/\lambda)=5\pi/2$. If this same peak moves to $t_1=0.035$ s for $x_1=0.090$ m, then

$2\pi(t_1/T+x_1/\lambda)=5\pi/2.$

$t_1/T+x_1/\lambda=5/4$

$x_1/\lambda=5/4-t_1/T=5/4-0.035\text{ s}/0.040\text{ s}=0.375$

$\lambda=x_1/0.375=0.090\text{ m}/0.375=0.24\text{ m}.$

Then $v=f\lambda=\lambda/T=0.24\text{ m}/0.040\text{ s}=6.0\text{ m/s}.$

EVALUATE: No. Wouldn't know which point in the wave at $x=0$ moved to which point at $x=0.090$ m.

15.12. **IDENTIFY:** $v_y=\dfrac{\partial y}{\partial t}$. $v=f\lambda=\lambda/T$.

SET UP: $$\frac{\partial}{\partial t}A\cos\left(\frac{2\pi}{\lambda}(x-vt)\right)=+A\left(\frac{2\pi v}{\lambda}\right)\sin\left(\frac{2\pi}{\lambda}(x-vt)\right)$$

EXECUTE: **(a)** $A\cos 2\pi\left(\dfrac{x}{\lambda}-\dfrac{t}{T}\right)=+A\cos\dfrac{2\pi}{\lambda}\left(x-\dfrac{\lambda}{T}t\right)=+A\cos\dfrac{2\pi}{\lambda}(x-vt)$ where $\dfrac{\lambda}{T}=\lambda f=v$ has been used.

(b) $v_y=\dfrac{\partial y}{\partial t}=\dfrac{2\pi v}{\lambda}A\sin\dfrac{2\pi}{\lambda}(x-vt).$

(c) The speed is the greatest when the sine is 1, and that speed is $2\pi vA/\lambda$. This will be equal to v if $A=\lambda/2\pi$, less than v if $A<\lambda/2\pi$ and greater than v if $A>\lambda/2\pi$.

EVALUATE: The propagation speed applies to all points on the string. The transverse speed of a particle of the string depends on both x and t.

15.13. **IDENTIFY:** Follow the procedure specified in the problem.

SET UP: For λ and x in cm, v in cm/s and t in s, the argument of the cosine is in radians.

EXECUTE: **(a)** $t=0$:

***x* (cm)**	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
***y* (cm)**	0.300	0.212	0	–0.212	–0.300	–0.212	0	0.212	0.300

The graph is shown in Figure 15.13a.

(b) (i) $t=0.400$ s:

***x* (cm)**	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
***y* (cm)**	–0.221	–0.0131	0.203	0.300	0.221	0.0131	–0.203	–0.300	–0.221

The graph is shown in Figure 15.13b.

(ii) $t=0.800$ s:

***x* (cm)**	0.00	1.50	3.00	4.50	6.00	7.50	9.00	10.50	12.00
***y* (cm)**	0.0262	–0.193	–0.300	–0.230	–0.0262	0.193	0.300	0.230	0.0262

The graph is shown in Figure 15.13c.

(iii) The graphs show that the wave is traveling in $+x$-direction.

EVALUATE: We know that Eq.(15.3) is for a wave traveling in the $+x$-direction, and $y(x,t)$ is derived from this. This is consistent with the direction of propagation we deduced from our graph.

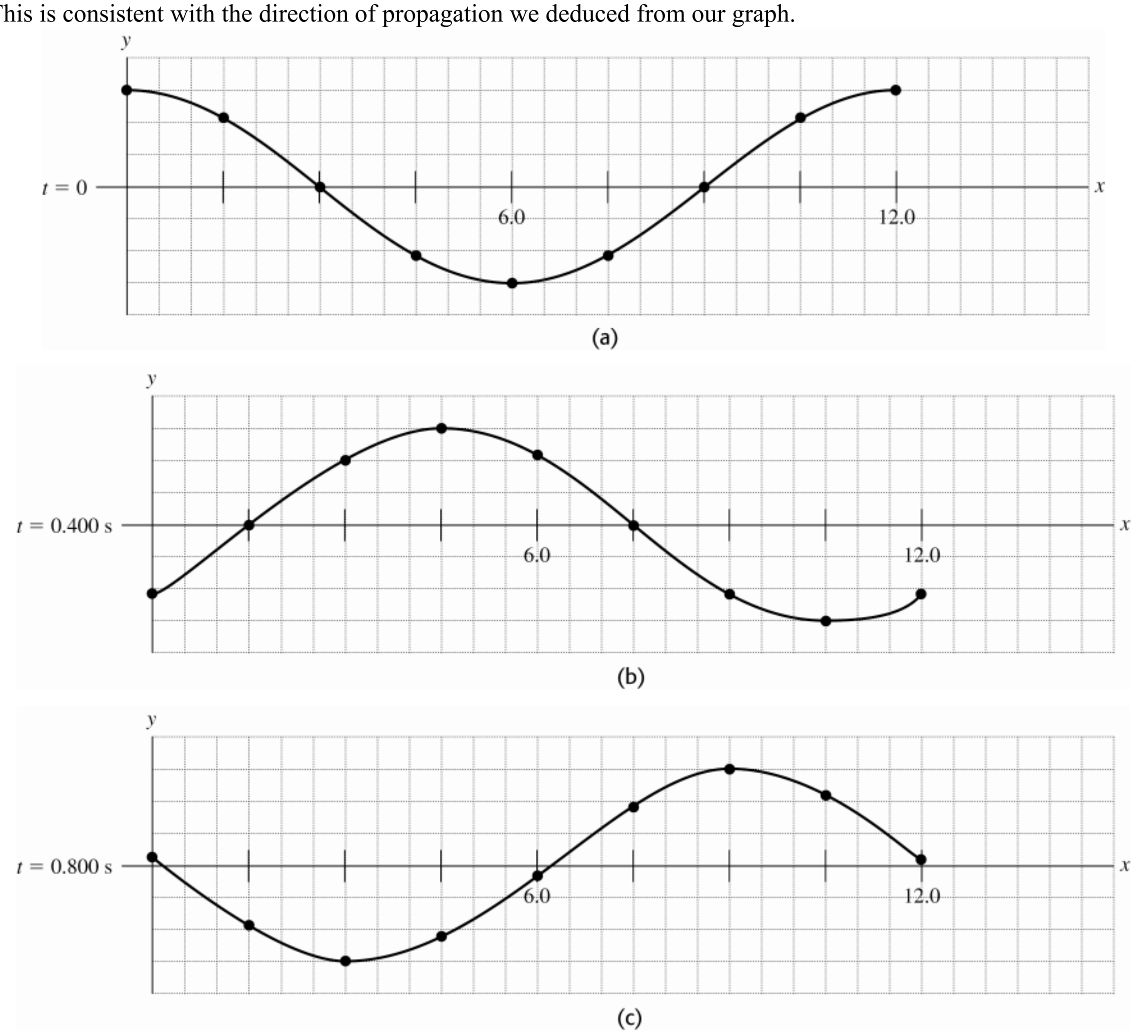


Figure 15.13

15.14. **IDENTIFY:** The frequency and wavelength determine the wave speed and the wave speed depends on the tension.

SET UP: $v = \sqrt{\dfrac{F}{\mu}}$. $\mu = m/L$. $v = f\lambda$.

EXECUTE: $F = \mu v^2 = \mu(f\lambda)^2 = \dfrac{0.120\text{ kg}}{2.50\text{ m}}([40.0\text{ Hz}][0.750\text{ m}])^2 = 43.2\text{ N}$

EVALUATE: If the frequency is held fixed, increasing the tension will increase the wavelength.

15.15. **IDENTIFY** and **SET UP:** Use Eq.(15.13) to calculate the wave speed. Then use Eq.(15.1) to calculate the wavelength.

EXECUTE: **(a)** The tension F in the rope is the weight of the hanging mass:

$F = mg = (1.50\text{ kg})(9.80\text{ m/s}^2) = 14.7\text{ N}$

$v = \sqrt{F/\mu} = \sqrt{14.7\text{ N}/(0.0550\text{ kg/m})} = 16.3\text{ m/s}$

(b) $v = f\lambda$ so $\lambda = v/f = (16.3\text{ m/s})/120\text{ Hz} = 0.136\text{ m}$.

(c) EVALUATE: $v = \sqrt{F/\mu}$, where $F = mg$. Doubling m increases v by a factor of $\sqrt{2}$. $\lambda = v/f$. f remains 120 Hz and v increases by a factor of $\sqrt{2}$, so λ increases by a factor of $\sqrt{2}$.

15.16. **IDENTIFY:** For transverse waves on a string, $v = \sqrt{F/\mu}$. The general form of the equation for waves traveling in the $+x$-direction is $y(x,t) = A\cos(kx - \omega t)$. For waves traveling in the $-x$-direction it is $y(x,t) = A\cos(kx + \omega t)$. $v = \omega/k$.

SET UP: Comparison to the general equation gives $A = 8.50$ mm, $k = 172$ rad/m and $\omega = 2730$ rad/s. The string has mass 0.128 kg and $\mu = m/L = 0.0850$ kg/m.

EXECUTE: **(a)** $v = \dfrac{\omega}{k} = \dfrac{2730\text{ rad/s}}{172\text{ rad/m}} = 15.9\text{ m/s}$. $t = \dfrac{d}{v} = \dfrac{1.50\text{ m}}{15.9\text{ m/s}} = 0.0943\text{ s}$.

(b) $W = F = \mu v^2 = (0.0850\text{ kg/m})(15.9\text{ m/s})^2 = 21.5\text{ N}$.

(c) $\lambda = \dfrac{2\pi\text{ rad}}{k} = \dfrac{2\pi\text{ rad}}{172\text{ rad/m}} = 0.0365\text{ m}$. The number of wavelengths along the length of the string is $\dfrac{1.50\text{ m}}{0.0365\text{ m}} = 41.1$.

(d) For a wave traveling in the opposite direction, $y(x,t) = (8.50\text{ mm})\cos([172\text{ rad/m}]x + [2730\text{ rad/s}]t)$

EVALUATE: We have assumed that the tension in the string is constant and equal to *W*. In reality the tension will vary along the length of the string because of the weight of the string and the wave speed will therefore vary along the string. The tension at the lower end of the string will be $W = 21.5$ N and at the upper end it is $W + 1.25\text{ N} = 22.8\text{ N}$, an increase of 6%.

15.17. **IDENTIFY:** For transverse waves on a string, $v = \sqrt{F/\mu}$. $v = f\lambda$.

SET UP: The wire has $\mu = m/L = (0.0165\text{ kg})/(0.750\text{ m}) = 0.0220\text{ kg/m}$.

EXECUTE: **(a)** $v = f\lambda = (875\text{ Hz})(3.33\times10^{-2}\text{ m}) = 29.1\text{ m/s}$. The tension is $F = \mu v^2 = (0.0220\text{ kg/m})(29.1\text{ m/s})^2 = 18.6\text{ N}$.

(b) $v = 29.1$ m/s

EVALUATE: If λ is kept fixed, the wave speed and the frequency increase when the tension is increased.

15.18. **IDENTIFY:** Apply $\sum F_y = 0$ to determine the tension at different points of the rope. $v = \sqrt{F/\mu}$.

SET UP: From Example 15.3, $m_{\text{samples}} = 20.0$ kg, $m_{\text{rope}} = 2.00$ kg and $\mu = 0.0250$ kg/m

EXECUTE: **(a)** The tension at the bottom of the rope is due to the weight of the load, and the speed is the same 88.5 m/s as found in Example 15.3.

(b) The tension at the middle of the rope is $(21.0\text{ kg})(9.80\text{ m/s}^2) = 205.8$ N and the wave speed is 90.7 m/s.

(c) The tension at the top of the rope is $(22.0\text{ kg})(9.80\text{ m/s}^2) = 215.6\text{ m/s}$ and the speed is 92.9 m/s. (See Challenge Problem (15.82) for the effects of varying tension on the time it takes to send signals.)

EVALUATE: The tension increases toward the top of the rope, so the wave speed increases from the bottom of the rope to the top of the rope.

15.19. **IDENTIFY:** $v = \sqrt{F/\mu}$. $v = f\lambda$. The general form for $y(x,t)$ is given in Eq.(15.4), where $T = 1/f$. Eq.(15.10) says that the maximum transverse acceleration is $a_{\max} = \omega^2 A = (2\pi f)^2 A$.

SET UP: $\mu = 0.0500$ kg/m

EXECUTE: **(a)** $v = \sqrt{F/\mu} = \sqrt{(5.00\text{ N})/(0.0500\text{ kg/m})} = 10.0\text{ m/s}$

(b) $\lambda = v/f = (10.0\text{ m/s})/(40.0\text{ Hz}) = 0.250\text{ m}$

(c) $y(x,t) = A\cos(kx - \omega t)$. $k = 2\pi/\lambda = 8.00\pi\text{ rad/m}$; $\omega = 2\pi f = 80.0\pi\text{ rad/s}$.

$y(x,t) = (3.00\text{ cm})\cos[\pi(8.00\text{ rad/m})x - (80.0\pi\text{ rad/s})t]$

(d) $v_y = +A\omega\sin(kx - \omega t)$ and $a_y = -A\omega^2\cos(kx - \omega t)$. $a_{y,\max} = A\omega^2 = A(2\pi f)^2 = 1890\text{ m/s}^2$.

(e) $a_{y,\max}$ is much larger than *g*, so it is a reasonable approximation to ignore gravity.

EVALUATE: $y(x,t)$ in part (c) gives $y(0,0) = A$, which does correspond to the oscillator having maximum upward displacement at $t = 0$.

15.20. **IDENTIFY:** Apply Eq.(15.25).

SET UP: $\omega = 2\pi f$. $\mu = m/L$.

EXECUTE: **(a)** $P_{\text{av}} = \dfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2$. $P_{\text{av}} = \dfrac{1}{2}\sqrt{\left(\dfrac{3.00\times10^{-3}\text{ kg}}{0.80\text{ m}}\right)(25.0\text{ N})}(2\pi(120.0\text{ Hz}))^2(1.6\times10^{-3}\text{ m})^2 = 0.223\text{ W}$ or 0.22 W to two figures.

(b) P_{av} is proportional to A^2, so halving the amplitude quarters the average power, to 0.056 W.

EVALUATE: The average power is also proportional to the square of the frequency.

15.21. **IDENTIFY:** For a point source, $I = \dfrac{P}{4\pi r^2}$ and $\dfrac{I_1}{I_2} = \dfrac{r_2^2}{r_1^2}$.

SET UP: $1\ \mu\text{W} = 10^{-6}\ \text{W}$

EXECUTE: **(a)** $r_2 = r_1\sqrt{\dfrac{I_1}{I_2}} = (30.0\ \text{m})\sqrt{\dfrac{10.0\ \text{W/m}^2}{1\times 10^{-6}\ \text{W/m}^2}} = 95\ \text{km}$

(b) $\dfrac{I_2}{I_3} = \dfrac{r_3^2}{r_2^2}$, with $I_2 = 1.0\ \mu\text{W/m}^2$ and $r_3 = 2r_2$. $I_3 = I_2\left(\dfrac{r_2}{r_3}\right)^2 = I_2/4 = 0.25\ \mu\text{W/m}^2$.

(c) $P = I(4\pi r^2) = (10.0\ \text{W/m}^2)(4\pi)(30.0\ \text{m})^2 = 1.1\times 10^5\ \text{W}$

EVALUATE: These are approximate calculations, that assume the sound is emitted uniformly in all directions and that ignore the effects of reflection, for example reflections from the ground.

15.22. **IDENTIFY:** Apply Eq.(15.26).

SET UP: $I_1 = 0.11\ \text{W/m}^2$. $r_1 = 7.5\ \text{m}$. Set $I_2 = 1.0\ \text{W/m}^2$ and solve for r_2.

EXECUTE: $r_2 = r_1\sqrt{\dfrac{I_1}{I_2}} = (7.5\ \text{m})\sqrt{\dfrac{0.11\ \text{W/m}^2}{1.0\ \text{W/m}^2}} = 2.5\ \text{m}$, so it is possible to move

$r_1 - r_2 = 7.5\ \text{m} - 2.5\ \text{m} = 5.0\ \text{m}$ closer to the source.

EVALUATE: I increases as the distance r of the observer from the source decreases.

15.23. **IDENTIFY** and **SET UP:** Apply Eq.(15.26) to relate I and r.

Power is related to intensity at a distance r by $P = I(4\pi r^2)$. Energy is power times time.

EXECUTE: **(a)** $I_1 r_1^2 = I_2 r_2^2$

$I_2 = I_1(r_1/r_2)^2 = (0.026\ \text{W/m}^2)(4.3\ \text{m}/3.1\ \text{m})^2 = 0.050\ \text{W/m}^2$

(b) $P = 4\pi r^2 I = 4\pi(4.3\ \text{m})^2(0.026\ \text{W/m}^2) = 6.04\ \text{W}$

Energy $= Pt = (6.04\ \text{W})(3600\ \text{s}) = 2.2\times 10^4\ \text{J}$

EVALUATE: We could have used $r = 3.1\ \text{m}$ and $I = 0.050\ \text{W/m}^2$ in $P = 4\pi r^2 I$ and would have obtained the same P. Intensity becomes less as r increases because the radiated power spreads over a sphere of larger area.

15.24. **IDENTIFY:** The tension and mass per unit length of the rope determine the wave speed. Compare $y(x,t)$ given in the problem to the general form given in Eq.(15.8). $v = \omega/k$. The average power is given by Eq. (15.25).

SET UP: Comparison with Eq.(15.8) gives $A = 2.33\ \text{mm}$, $k = 6.98\ \text{rad/m}$ and $\omega = 742\ \text{rad/s}$.

EXECUTE: **(a)** $A = 2.30\ \text{mm}$

(b) $f = \dfrac{\omega}{2\pi} = \dfrac{742\ \text{rad/s}}{2\pi} = 118\ \text{Hz}$.

(c) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{6.98\ \text{rad/m}} = 0.90\ \text{m}$

(d) $v = \dfrac{\omega}{k} = \dfrac{742\ \text{rad/s}}{6.98\ \text{rad/m}} = 106\ \text{m/s}$

(e) The wave is traveling in the $-x$ direction because the phase of $y(x,\ t)$ has the form $kx + \omega t$.

(f) The linear mass density is $\mu = (3.38\times 10^{-3}\ \text{kg})/(1.35\ \text{m}) = 2.504\times 10^{-3}\ \text{kg/m}$, so the tension is $F = \mu v^2 = (2.504\times 10^{-3}\ \text{kg/m})(106.3\ \text{m/s})^2 = 28.3\ \text{N}$.

(g) $P_{\text{av}} = \tfrac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = \tfrac{1}{2}\sqrt{(2.50\times 10^{-3}\ \text{kg/m})(28.3\ \text{N})}(742\ \text{rad/s})^2(2.30\times 10^{-3}\ \text{m})^2 = 0.39\ \text{W}$

EVALUATE: In part (d) we could also calculate the wave speed as $v = f\lambda$ and we would obtain the same result.

15.25. **IDENTIFY:** $P = 4\pi r^2 I$

SET UP: From Example 15.5, $I = 0.250\ \text{W/m}^2$ at $r = 15.0\ \text{m}$

EXECUTE: $P = 4\pi r^2 I = 4\pi(15.0\ \text{m})^2(0.250\ \text{W/m}^2) = 707\ \text{W}$

EVALUATE: $I = 0.010\ \text{W/m}^2$ at $75.0\ \text{m}$ and $4\pi(75.0\ \text{m})^2(0.010\ \text{W/m}^2) = 707\ \text{W}$. P is the average power of the sinusoidal waves emitted by the source.

15.26. **IDENTIFY:** The distance the wave shape travels in time t is vt. The wave pulse reflects at the end of the string, at point O.
SET UP: The reflected pulse is inverted when O is a fixed end and is not inverted when O is a free end.
EXECUTE: **(a)** The wave form for the given times, respectively, is shown in Figure 15.26a.
(b) The wave form for the given times, respectively, is shown in Figure 15.26b.
EVALUATE: For the fixed end the result of the reflection is an inverted pulse traveling to the left and for the free end the result is an upright pulse traveling to the left.

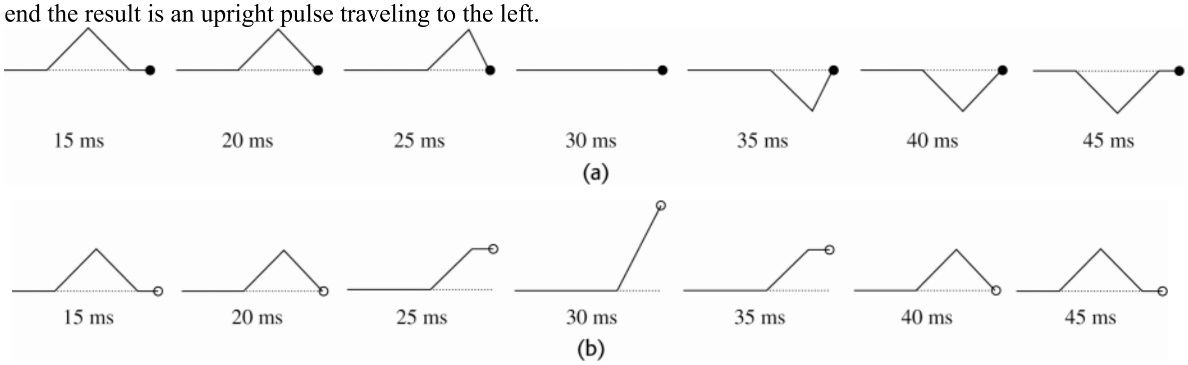


Figure 15.26

15.27. **IDENTIFY:** The distance the wave shape travels in time t is vt. The wave pulse reflects at the end of the string, at point O.
SET UP: The reflected pulse is inverted when O is a fixed end and is not inverted when O is a free end.
EXECUTE: **(a)** The wave form for the given times, respectively, is shown in Figure 15.27a.
(b) The wave form for the given times, respectively, is shown in Figure 15.27b.
EVALUATE: For the fixed end the result of the reflection is an inverted pulse traveling to the left and for the free end the result is an upright pulse traveling to the left.

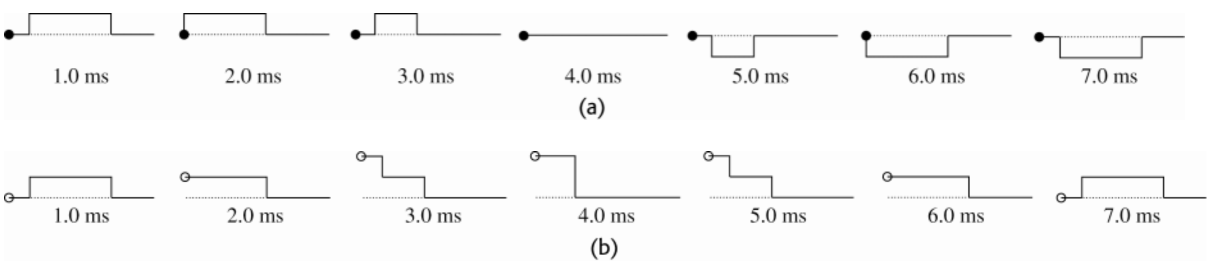


Figure 15.27

15.28. **IDENTIFY:** Apply the principle of superposition.
SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.
EXECUTE: The shape of the string at each specified time is shown in Figure 15.28.
EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.



Figure 15.28

15.29. **IDENTIFY:** Apply the principle of superposition.
SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.
EXECUTE: The shape of the string at each specified time is shown in Figure 15.29.

EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

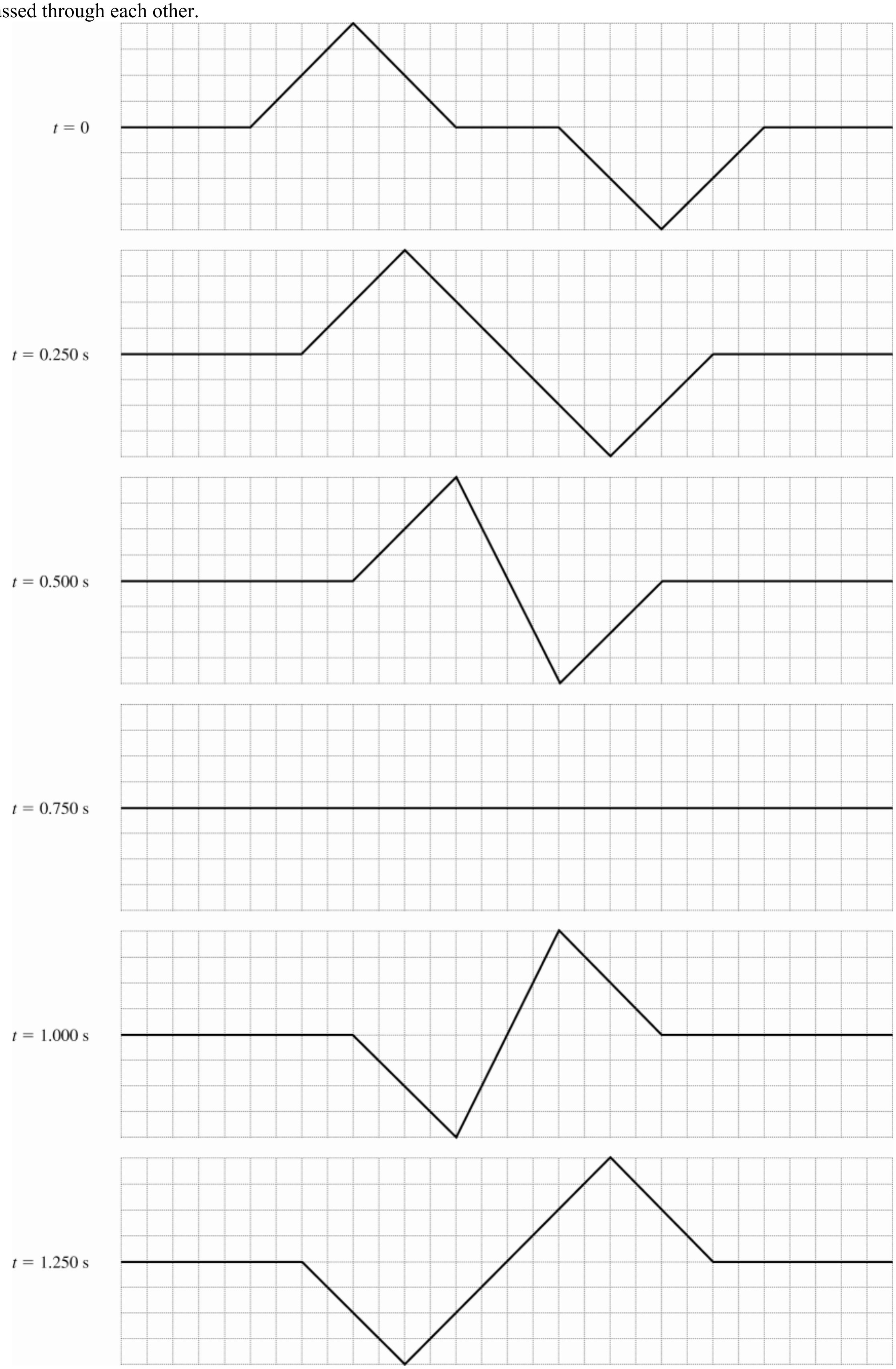


Figure 15.29

15.30. **IDENTIFY:** Apply the principle of superposition.
SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.
EXECUTE: The shape of the string at each specified time is shown in Figure 15.30.
EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

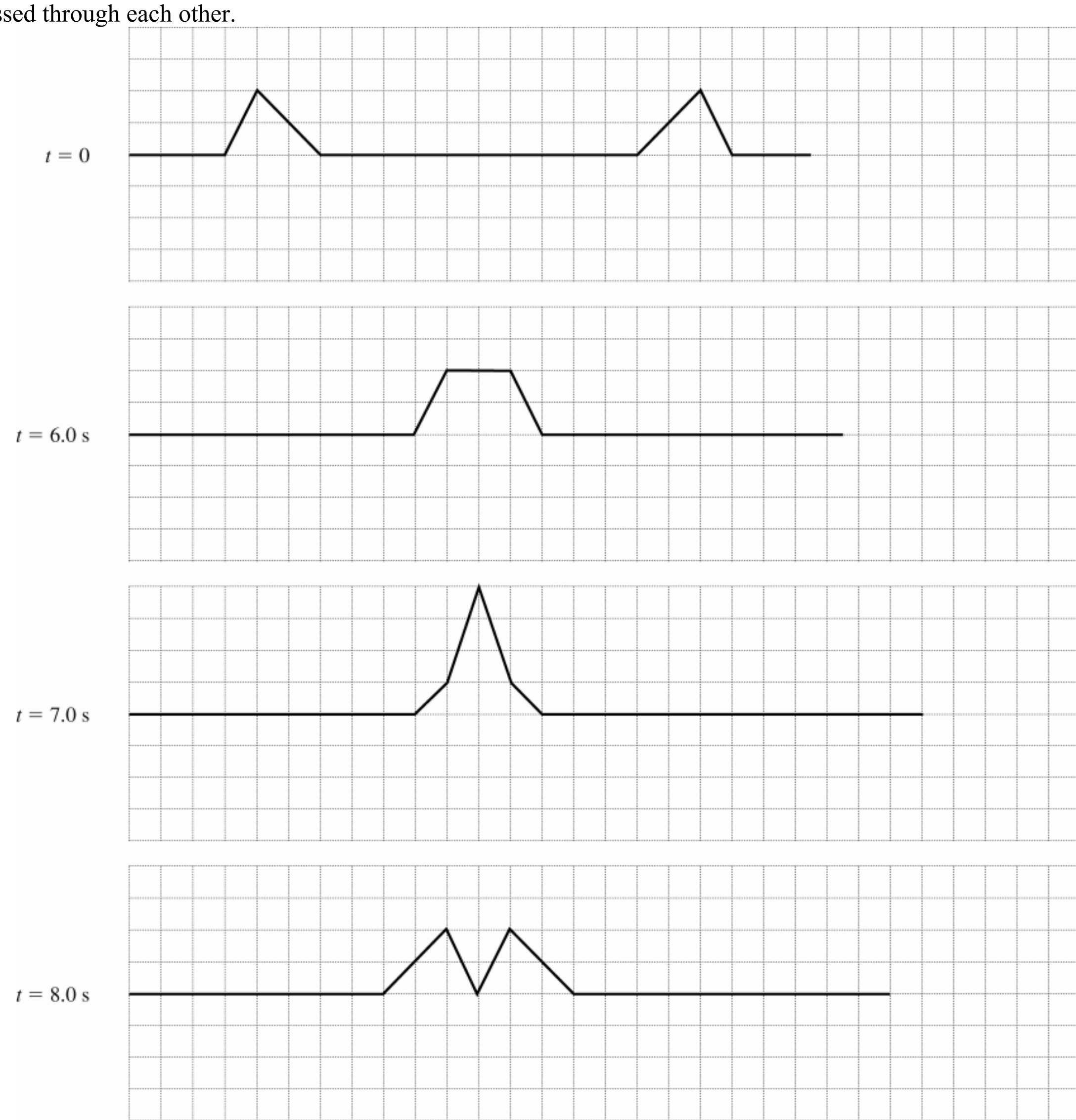


Figure 15.30

15.31. **IDENTIFY:** Apply the principle of superposition.
SET UP: The net displacement is the algebraic sum of the displacements due to each pulse.
EXECUTE: The shape of the string at each specified time is shown in Figure 15.31.
EVALUATE: The pulses interfere when they overlap but resume their original shape after they have completely passed through each other.

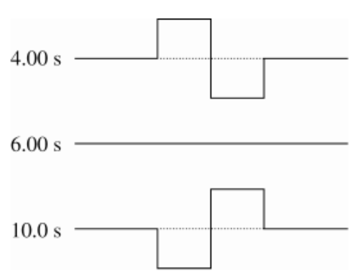


Figure 15.31

15.32. IDENTIFY: $y_{net} = y_1 + y_2$. The string never moves at values of x for which $\sin kx = 0$.

SET UP: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

EXECUTE: **(a)** $y_{net} = A\sin(kx + \omega t) + A\sin(kx - \omega t)$.

$y_{net} = A[\sin(kx)\cos(\omega t) + \cos(kx)\sin(\omega t) + \sin(kx)\cos(\omega t) - \cos(kx)\sin(\omega t)] = 2A\sin(kx)\cos(\omega t)$

(b) $\sin kx = 0$ for $kx = n\pi,\ n = 0, 1, 2, \ldots$. $x = \dfrac{n\pi}{k} = \dfrac{n\pi}{2\pi/\lambda} = \dfrac{n\lambda}{2}$.

EVALUATE: Using $y = A\sin(kx \pm \omega t)$ instead of $y = A\cos(kx \pm \omega t)$ corresponds to a particular choice of phase and corresponds to $y = 0$ at $x = 0$, for all t.

15.33. IDENTIFY and SET UP: Nodes occur where $\sin kx = 0$ and antinodes are where $\sin kx = \pm 1$.

EXECUTE: Eq.(15.28): $y = (A_{SW} \sin kx)\sin \omega t$

(a) At a node $y = 0$ for all t. This requires that $\sin kx = 0$ and this occurs for $kx = n\pi,\ n = 0, 1, 2, \ldots$

$x = n\pi / k = \dfrac{n\pi}{0.750\pi \text{ rad/m}} = (1.33 \text{ m})n,\ n = 0, 1, 2, \ldots$

(b) At an antinode $\sin kx = \pm 1$ so y will have maximum amplitude. This occurs when $kx = \left(n + \tfrac{1}{2}\right)\pi$, $n = 0, 1, 2, \ldots$

$x = \left(n + \tfrac{1}{2}\right)\pi / k = \left(n + \tfrac{1}{2}\right)\dfrac{\pi}{0.750\pi \text{ rad/m}} = (1.33 \text{ m})\left(n + \tfrac{1}{2}\right),\ n = 0, 1, 2, \ldots$

EVALUATE: $\lambda = 2\pi / k = 2.66$ m. Adjacent nodes are separated by $\lambda/2$, adjacent antinodes are separated by $\lambda/2$, and the node to antinode distance is $\lambda/4$.

15.34. IDENTIFY: Apply Eqs.(15.28) and (15.1). At an antinode, $y(t) = A_{SW} \sin \omega t$. k and ω for the standing wave have the same values as for the two traveling waves.

SET UP: $A_{SW} = 0.850$ cm. The antinode to antinode distance is $\lambda/2$, so $\lambda = 30.0$ cm. $v_y = \partial y / \partial t$.

EXECUTE: **(a)** The node to node distance is $\lambda/2 = 15.0$ cm.

(b) λ is the same as for the standing wave, so $\lambda = 30.0$ cm. $A = \tfrac{1}{2}A_{SW} = 0.425$ cm.

$v = f\lambda = \dfrac{\lambda}{T} = \dfrac{0.300 \text{ m}}{0.0750 \text{ s}} = 13.3 \text{ m/s}$.

(c) $v_y = \dfrac{\partial y}{\partial t} = A_{SW}\omega \sin kx \cos \omega t$. At an antinode $\sin kx = 1$, so $v_y = A_{SW}\omega \cos \omega t$. $v_{max} = A_{SW}\omega$.

$\omega = \dfrac{2\pi \text{ rad}}{T} = \dfrac{2\pi \text{ rad}}{0.0750 \text{ s}} = 83.8 \text{ rad/s}$. $v_{max} = (0.850 \times 10^{-2} \text{ m})(83.8 \text{ rad/s}) = 0.0712 \text{ m/s}$. $v_{min} = 0$.

(d) The distance from a node to an adjacent antinode is $\lambda/4 = 7.50$ cm.

EVALUATE: The maximum transverse speed for a point at an antinode of the standing wave is twice the maximum transverse speed for each traveling wave, since $A_{SW} = 2A$.

15.35. IDENTIFY: Evaluate $\partial^2 y/\partial x^2$ and $\partial^2 y/\partial t^2$ and see if Eq.(15.12) is satisfied for $v = \omega / k$.

SET UP: $\dfrac{\partial}{\partial x}\sin kx = k\cos kx$. $\dfrac{\partial}{\partial x}\cos kx = -k\sin kx$. $\dfrac{\partial}{\partial t}\sin \omega t = \omega \cos \omega t$. $\dfrac{\partial}{\partial t}\cos \omega t = -\omega \sin \omega t$

EXECUTE: **(a)** $\dfrac{\partial^2 y}{\partial x^2} = -k^2\left[A_{sw} \sin \omega t\right]\sin kx,\quad \dfrac{\partial^2 y}{\partial t^2} = -\omega^2\left[A_{sw} \sin \omega t\right]\sin kx,$ so for $y(x,t)$ to be a solution of Eq.(15.12), $-k^2 = \dfrac{-\omega^2}{v^2}$, and $v = \dfrac{\omega}{k}.$

(b) A standing wave is built up by the superposition of traveling waves, to which the relationship $v = \lambda/k$ applies.

EVALUATE: $y(x,t) = (A_{SW} \sin kx)\sin \omega t$ is a solution of the wave equation because it is a sum of solutions to the wave equation.

15.36. IDENTIFY and SET UP: $\cos(kx \pm \omega t) = \cos kx \cos \omega t \mp \sin kx \sin \omega t$

EXECUTE: $y_1 + y_2 = A\,[-\cos(kx + \omega t) + \cos(kx - \omega t)]$.

$y_1 + y_2 = A[-\cos kx \cos \omega t + \sin kx \sin \omega t + \cos kx \cos \omega t + \sin kx \sin \omega t] = 2A\sin kx \sin \omega t$.

EVALUATE: The derivation shows that the standing wave of Eq.(15.28) results from the combination of two waves with the same A, k, and ω that are traveling in opposite directions.

15.37. IDENTIFY: Evaluate $\partial^2 y/\partial x^2$ and $\partial^2 y/\partial t^2$ and show that Eq.(15.12) is satisfied.

SET UP: $\dfrac{\partial}{\partial x}(y_1+y_2)=\dfrac{\partial y_1}{\partial x}+\dfrac{\partial y_2}{\partial x}$ and $\dfrac{\partial}{\partial t}(y_1+y_2)=\dfrac{\partial y_1}{\partial t}+\dfrac{\partial y_2}{\partial t}$

EXECUTE: $\dfrac{\partial^2 y}{\partial x^2}=\dfrac{\partial^2 y_1}{\partial x^2}+\dfrac{\partial^2 y_2}{\partial x^2}$ and $\dfrac{\partial^2 y}{\partial t^2}=\dfrac{\partial^2 y_1}{\partial t^2}+\dfrac{\partial^2 y_2}{\partial t^2}$. The functions y_1 and y_2 are given as being solutions to the wave equation, so

$$\frac{\partial^2 y}{\partial x^2}=\frac{\partial^2 y_1}{\partial x^2}+\frac{\partial^2 y_2}{\partial x^2}=\left(\frac{1}{v^2}\right)\frac{\partial^2 y_1}{\partial t^2}+\left(\frac{1}{v^2}\right)\frac{\partial^2 y_2}{\partial t^2}=\left(\frac{1}{v^2}\right)\left[\frac{\partial^2 y_1}{\partial t^2}+\frac{\partial^2 y_2}{\partial t^2}\right]=\left(\frac{1}{v^2}\right)\frac{\partial^2 y}{\partial t^2}$$ and so $y=y_1+y_2$ is a solution of Eq. (15.12).

EVALUATE: The wave equation is a linear equation, as it is linear in the derivatives, and differentiation is a linear operation.

15.38. IDENTIFY: For a string fixed at both ends, $\lambda_n=\dfrac{2L}{n}$ and $f_n=n\left(\dfrac{v}{2L}\right)$.

SET UP: For the fundamental, $n=1$. For the second overtone, $n=3$. For the fourth harmonic, $n=4$.

EXECUTE: **(a)** $\lambda_1=2L=3.00\text{ m}$. $f_1=\dfrac{v}{2L}=\dfrac{(48.0\text{ m/s})}{2(1.50\text{ m})}=16.0\text{ Hz}$.

(b) $\lambda_3=\lambda_1/3=1.00\text{ m}$. $f_2=3f_1=48.0\text{ Hz}$.

(c) $\lambda_4=\lambda_1/4=0.75\text{ m}$. $f_3=4f_1=64.0\text{ Hz}$.

EVALUATE: As n increases, λ decreases and f increases.

15.39. IDENTIFY: Use Eq.(15.1) for v and Eq.(15.13) for the tension F. $v_y=\partial y/\partial t$ and $a_y=\partial v_y/\partial t$.

(a) SET UP: The fundamental standing wave is sketched in Figure 15.39.

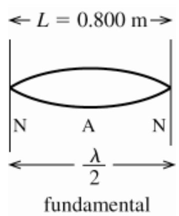


Figure 15.39

$f=60.0\text{ Hz}$
From the sketch,
$\lambda/2=L$ so
$\lambda=2L=1.60\text{ m}$

EXECUTE: $v=f\lambda=(60.0\text{ Hz})(1.60\text{ m})=96.0\text{ m/s}$

(b) The tension is related to the wave speed by Eq.(15.13):

$v=\sqrt{F/\mu}$ so $F=\mu v^2$.

$\mu=m/L=0.0400\text{ kg}/0.800\text{ m}=0.0500\text{ kg/m}$

$F=\mu v^2=(0.0500\text{ kg/m})(96.0\text{ m/s})^2=461\text{ N}.$

(c) $\omega=2\pi f=377\text{ rad/s}$ and $y(x,\ t)=A_{\text{SW}}\sin kx\sin\omega t$

$v_y=\omega A_{\text{SW}}\sin kx\cos\omega t$; $a_y=-\omega^2 A_{\text{SW}}\sin kx\sin\omega t$

$(v_y)_{\text{max}}=\omega A_{\text{SW}}=(377\text{ rad/s})(0.300\text{ cm})=1.13\text{ m/s}.$

$(a_y)_{\text{max}}=\omega^2 A_{\text{SW}}=(377\text{ rad/s})^2(0.300\text{ cm})=426\text{ m/s}^2.$

EVALUATE: The transverse velocity is different from the wave velocity. The wave velocity and tension are similar in magnitude to the values in the Examples in the text. Note that the transverse acceleration is quite large.

15.40. IDENTIFY: The fundamental frequency depends on the wave speed, and that in turn depends on the tension.

SET UP: $v=\sqrt{\dfrac{F}{\mu}}$ where $\mu=m/L$. $f_1=\dfrac{v}{2L}$. The nth harmonic has frequency $f_n=nf_1$.

EXECUTE: **(a)** $v=\sqrt{\dfrac{F}{m/L}}=\sqrt{\dfrac{FL}{m}}=\sqrt{\dfrac{(800\text{ N})(0.400\text{ m})}{3.00\times10^{-3}\text{ kg}}}=327\text{ m/s}$. $f_1=\dfrac{v}{2L}=\dfrac{327\text{ m/s}}{2(0.400\text{ m})}=409\text{ Hz}$.

(b) $n=\dfrac{10{,}000\text{ Hz}}{f_1}=24.4$. The 24th harmonic is the highest that could be heard.

EVALUATE: In part (b) we use the fact that a standing wave on the wire produces a sound wave in air of the same frequency.

15.41. **IDENTIFY:** Compare $y(x,\ t)$ given in the problem to Eq.(15.28). From the frequency and wavelength for the third harmonic find these values for the eighth harmonic.

(a) SET UP: The third harmonic standing wave pattern is sketched in Figure 15.41.

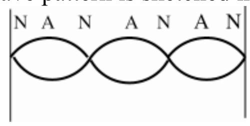


Figure 15.41

EXECUTE: **(b)** Eq. (15.28) gives the general equation for a standing wave on a string:

$y(x,\ t) = (A_{\text{SW}} \sin kx) \sin \omega t$

$A_{\text{SW}} = 2A$, so $A = A_{\text{SW}}/2 = (5.60 \text{ cm})/2 = 2.80 \text{ cm}$

(c) The sketch in part (a) shows that $L = 3(\lambda/2)$. $k = 2\pi/\lambda$, $\lambda = 2\pi/k$

Comparison of $y(x,\ t)$ given in the problem to Eq. (15.28) gives $k = 0.0340$ rad/cm. So,

$\lambda = 2\pi/(0.0340 \text{ rad/cm}) = 184.8 \text{ cm}$

$L = 3(\lambda/2) = 277 \text{ cm}$

(d) $\lambda = 185$ cm, from part (c)

$\omega = 50.0$ rad/s so $f = \omega/2\pi = 7.96$ Hz

period $T = 1/f = 0.126$ s

$v = f\lambda = 1470$ cm/s

(e) $v_y = dy/dt = \omega A_{\text{SW}} \sin kx \cos \omega t$

$v_{y,\ \max} = \omega A_{\text{SW}} = (50.0 \text{ rad/s})(5.60 \text{ cm}) = 280 \text{ cm/s}$

(f) $f_3 = 7.96 \text{ Hz} = 3f_1$, so $f_1 = 2.65$ Hz is the fundamental

$f_8 = 8f_1 = 21.2$ Hz; $\omega_8 = 2\pi f_8 = 133$ rad/s

$\lambda = v/f = (1470 \text{ cm/s})/(21.2 \text{ Hz}) = 69.3 \text{ cm}$ and $k = 2\pi/\lambda = 0.0906$ rad/cm

$y(x,\ t) = (5.60 \text{ cm}) \sin([0.0906 \text{ rad/cm}]x) \sin([133 \text{ rad/s}]t)$

EVALUATE: The wavelength and frequency of the standing wave equals the wavelength and frequency of the two traveling waves that combine to form the standing wave. In the 8th harmonic the frequency and wave number are larger than in the 3rd harmonic.

15.42. **IDENTIFY:** Compare the $y(x,t)$ specified in the problem to the general form of Eq.(15.28).

SET UP: The comparison gives $A_{\text{SW}} = 4.44$ mm, $k = 32.5$ rad/m and $\omega = 754$ rad/s.

EXECUTE: **(a)** $A = \frac{1}{2}A_{\text{SW}} = \frac{1}{2}(4.44 \text{ mm}) = 2.22 \text{ mm}$.

(b) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{32.5 \text{ rad/m}} = 0.193 \text{ m}.$

(c) $f = \dfrac{\omega}{2\pi} = \dfrac{754 \text{ rad/s}}{2\pi} = 120 \text{ Hz}.$

(d) $v = \dfrac{\omega}{k} = \dfrac{754 \text{ rad/s}}{32.5 \text{ rad/m}} = 23.2 \text{ m/s}.$

(e) If the wave traveling in the $+x$ direction is written as $y_1(x,t) = A\cos(kx - \omega t)$, then the wave traveling in the $-x$-direction is $y_2(x,t) = -A\cos(kx + \omega t)$, where $A = 2.22$ mm from part (a), $k = 32.5$ rad/m and $\omega = 754$ rad/s.

(f) The harmonic cannot be determined because the length of the string is not specified.

EVALUATE: The two traveling waves that produce the standing wave are identical except for their direction of propagation.

15.43. **(a) IDENTIFY** and **SET UP:** Use the angular frequency and wave number for the traveling waves in Eq.(15.28) for the standing wave.

EXECUTE: The traveling wave is $y(x,\ t) = (2.30 \text{ mm}) \cos([6.98 \text{ rad/m}]x) + [742 \text{ rad/s}]t)$

$A = 2.30$ mm so $A_{\text{SW}} = 4.60$ mm; $k = 6.98$ rad/m and $\omega = 742$ rad/s

The general equation for a standing wave is $y(x,\ t) = (A_{\text{SW}} \sin kx) \sin \omega t$, so

$y(x,\ t) = (4.60 \text{ mm}) \sin([6.98 \text{ rad/m}]x) \sin([742 \text{ rad/s}]t)$

(b) IDENTIFY and **SET UP:** Compare the wavelength to the length of the rope in order to identify the harmonic.

EXECUTE: $L = 1.35$ m (from Exercise 15.24)

$\lambda = 2\pi/k = 0.900$ m

$L = 3(\lambda/2)$, so this is the 3rd harmonic

(c) For this 3rd harmonic, $f=\omega/2\pi=118$ Hz

$f_3=3f_1$ so $f_1=(118\text{ Hz})/3=39.3$ Hz

EVALUATE: The wavelength and frequency of the standing wave equals the wavelength and frequency of the two traveling waves that combine to form the standing wave. The *n*th harmonic has *n* node-to-node segments and the node-to-node distance is $\lambda/2$, so the relation between L and λ for the *n*th harmonic is $L=n(\lambda/2)$.

15.44. **IDENTIFY:** $v=\sqrt{F/\mu}$. $v=f\lambda$. The standing waves have wavelengths $\lambda_n=\dfrac{2L}{n}$ and frequencies $f_n=nf_1$. The standing wave on the string and the sound wave it produces have the same frequency.

SET UP: For the fundamental $n=1$ and for the second overtone $n=3$. The string has $\mu=m/L=(8.75\times10^{-3}\text{ kg})/(0.750\text{ m})=1.17\times10^{-2}\text{ kg/m}$.

EXECUTE: **(a)** $\lambda=2L/3=2(0.750\text{ m})/3=0.500\text{ m}$. The sound wave has frequency

$f=\dfrac{v}{\lambda}=\dfrac{344\text{ m/s}}{3.35\times10^{-2}\text{ m}}=1.03\times10^4\text{ Hz}$. For waves on the string,

$v=f\lambda=(1.03\times10^4\text{ Hz})(0.500\text{ m})=5.15\times10^3\text{ m/s}$. The tension in the string is

$F=\mu v^2=(1.17\times10^{-2}\text{ kg/m})(5.15\times10^3\text{ m/s})^2=3.10\times10^5\text{ N}$.

(b) $f_1=f_3/3=(1.03\times10^4\text{ Hz})/3=3.43\times10^3\text{ Hz}$.

EVALUATE: The waves on the string have a much longer wavelength than the sound waves in the air because the speed of the waves on the string is much greater than the speed of sound in air.

15.45. **IDENTIFY** and **SET UP:** Use the information given about the A_4 note to find the wave speed, that depends on the linear mass density of the string and the tension. The wave speed isn't affected by the placement of the fingers on the bridge. Then find the wavelength for the D_5 note and relate this to the length of the vibrating portion of the string.

EXECUTE: **(a)** $f=440$ Hz when a length $L=0.600$ m vibrates; use this information to calculate the speed v of waves on the string. For the fundamental $\lambda/2=L$ so $\lambda=2L=2(0.600\text{ m})=1.20$ m. Then $v=f\lambda=(440\text{ Hz})(1.20\text{ m})=528$ m/s. Now find the length $L=x$ of the string that makes $f=587$ Hz.

$\lambda=\dfrac{v}{f}=\dfrac{528\text{ m/s}}{587\text{ Hz}}=0.900\text{ m}$

$L=\lambda/2=0.450$ m, so $x=0.450\text{ m}=45.0$ cm.

(b) No retuning means same wave speed as in part (a). Find the length of vibrating string needed to produce $f=392$ Hz.

$\lambda=\dfrac{v}{f}=\dfrac{528\text{ m/s}}{392\text{ Hz}}=1.35\text{ m}$

$L=\lambda/2=0.675$ m; string is shorter than this. No, not possible.

EVALUATE: Shortening the length of this vibrating string increases the frequency of the fundamental.

15.46. **IDENTIFY:** $y(x,t)=(A_{SW}\sin kx)\sin\omega t$. $v_y=\partial y/\partial t$. $a_y=\partial^2 y/\partial t^2$.

SET UP: $v_{max}=(A_{SW}\sin kx)\omega$. $a_{max}=(A_{SW}\sin kx)\omega^2$.

EXECUTE: **(a)** (i) $x=\dfrac{\lambda}{2}$ is a node, and there is no motion. (ii) $x=\dfrac{\lambda}{4}$ is an antinode, and $v_{max}=A(2\pi f)=2\pi fA$, $a_{max}=(2\pi f)v_{max}=4\pi^2f^2A$. (iii) $\cos\dfrac{\pi}{4}=\dfrac{1}{\sqrt{2}}$ and this factor multiplies the results of (ii), so $v_{max}=\sqrt{2}\pi fA$, $a_{max}=2\sqrt{2}\pi^2f^2A$.

(b) The amplitude is $2A\sin kx$, or (i) 0, (ii) $2A$, (iii) $2A/\sqrt{2}$.

(c) The time between the extremes of the motion is the same for any point on the string (although the period of the zero motion at a node might be considered indeterminate) and is $1/2f$.

EVALUATE: Any point in a standing wave moves in SHM. All points move with the same frequency but have different amplitude.

15.47. **IDENTIFY:** For the fundamental, $f_1=\dfrac{v}{2L}$. $v=\sqrt{F/\mu}$. A standing wave on a string with frequency *f* produces a sound wave that also has frequency *f*.

SET UP: $f_1=245$ Hz. $L=0.635$ m.

EXECUTE: **(a)** $v=2f_1L=2(245\text{ Hz})(0.635\text{ m})=311\text{ m/s}$.

(b) The frequency of the fundamental mode is proportional to the speed and hence to the square root of the tension; $(245\text{ Hz})\sqrt{1.01} = 246\text{ Hz}$.

(c) The frequency will be the same, 245 Hz. The wavelength will be $\lambda_{\text{air}} = v_{\text{air}}/f = (344\text{ m/s})/(245\text{ Hz}) = 1.40\text{ m}$, which is larger than the wavelength of standing wave on the string by a factor of the ratio of the speeds.

EVALUATE: Increasing the tension increases the wave speed and this in turn increases the frequencies of the standing waves. The wavelength of each normal mode depends only on the length of the string and doesn't change when the tension changes.

15.48. **IDENTIFY:** The ends of the stick are free, so they must be displacement antinodes. The first harmonic has one node, at the center of the stick, and each successive harmonic adds one node.

SET UP: The node to node and antinode to antinode distance is $\lambda/2$.

EXECUTE: The standing wave patterns for the first three harmonics are shown in Figure 15.48.

1st harmonic: $L = \frac{1}{2}\lambda_1 \rightarrow \lambda_1 = 2L = 4.0\text{ m}$. 2nd harmonic: $L = 1\lambda_2 \rightarrow \lambda_2 = L = 2.0\text{ m}$.

3rd harmonic: $L = \frac{3}{2}\lambda_3 \rightarrow \lambda_3 = \frac{2L}{3} = 1.33\text{ m}$.

EVALUATE: The higher the harmonic the shorter the wavelength.



Figure 15.48

15.49. **IDENTIFY** and **SET UP:** Calculate v, ω, and k from Eqs.(15.1), (15.5), and (15.6). Then apply Eq.(15.7) to obtain $y(x, t)$.

$A = 2.50\times10^{-3}\text{ m}$, $\lambda = 1.80\text{ m}$, $v = 36.0\text{ m/s}$

EXECUTE: **(a)** $v = f\lambda$ so $f = v/\lambda = (36.0\text{ m/s})/1.80\text{ m} = 20.0\text{ Hz}$

$\omega = 2\pi f = 2\pi(20.0\text{ Hz}) = 126\text{ rad/s}$

$k = 2\pi/\lambda = 2\pi\text{ rad}/1.80\text{ m} = 3.49\text{ rad/m}$

(b) For a wave traveling to the right, $y(x, t) = A\cos(kx - \omega t)$. This equation gives that the $x = 0$ end of the string has maximum upward displacement at $t = 0$.

Put in the numbers: $y(x, t) = (2.50\times10^{-3}\text{ m})\cos((3.49\text{ rad/m})x - (126\text{ rad/s})t)$.

(c) The left hand end is located at $x = 0$. Put this value into the equation of part (b):

$y(0, t) = +(2.50\times10^{-3}\text{ m})\cos((126\text{ rad/s})t)$.

(d) Put $x = 1.35\text{ m}$ into the equation of part (b):

$y(1.35\text{ m}, t) = (2.50\times10^{-3}\text{ m})\cos((3.49\text{ rad/m})(1.35\text{ m}) - (126\text{ rad/s})t)$.

$y(1.35\text{ m}, t) = (2.50\times10^{-3}\text{ m})\cos(4.71\text{ rad}) - (126\text{ rad/s})t$

$4.71\text{ rad} = 3\pi/2$ and $\cos(\theta) = \cos(-\theta)$, so $y(1.35\text{ m}, t) = (2.50\times10^{-3}\text{ m})\cos((126\text{ rad/s})t - 3\pi/2\text{ rad})$

(e) $y = A\cos(kx - \omega t)$ ((part b))

The transverse velocity is given by $v_y = \frac{\partial y}{\partial t} = A\frac{\partial}{\partial t}\cos(kx - \omega t) = +A\omega\sin(kx - \omega t)$.

The maximum v_y is $A\omega = (2.50\times10^{-3}\text{ m})(126\text{ rad/s}) = 0.315\text{ m/s}$.

(f) $y(x, t) = (2.50\times10^{-3}\text{ m})\cos((3.49\text{ rad/m})x - (126\text{ rad/s})t)$

$t = 0.0625\text{ s}$ and $x = 1.35\text{ m}$ gives

$y = (2.50\times10^{-3}\text{ m})\cos((3.49\text{ rad/m})(1.35\text{ m}) - (126\text{ rad/s})(0.0625\text{ s})) = -2.50\times10^{-3}\text{ m}$.

$v_y = +A\omega\sin(kx - \omega t) = +(0.315\text{ m/s})\sin((3.49\text{ rad/m})x - (126\text{ rad/s})t)$

$t = 0.0625\text{ s}$ and $x = 1.35\text{ m}$ gives

$v_y = (0.315\text{ m/s})\sin((3.49\text{ rad/m})(1.35\text{ m}) - (126\text{ rad/s})(0.0625\text{ s})) = 0.0$

EVALUATE: The results of part (f) illustrate that $v_y = 0$ when $y = \pm A$, as we saw from SHM in Chapter 13.

15.50. **IDENTIFY:** Compare $y(x,t)$ given in the problem to the general form given in Eq.(15.8).

SET UP: The comparison gives $A = 0.750\text{ cm}$, $k = 0.400\pi\text{ rad/cm}$ and $\omega = 250\pi\text{ rad/s}$.

EXECUTE: **(a)** $A = 0.750\text{ cm}$, $\lambda = \dfrac{2}{0.400\text{ rad/cm}} = 5.00\text{ cm}$, $f = 125\text{ Hz}$, $T = \dfrac{1}{f} = 0.00800\text{ s}$ and $v = \lambda f = 6.25\text{ m/s}$.

(b) The sketches of the shape of the rope at each time are given in Figure 15.50.

(c) To stay with a wavefront as t increases, x decreases and so the wave is moving in the $-x$-direction.

(d) From Eq. (15.13), the tension is $F = \mu v^2 = (0.50\text{ kg/m})\ (6.25\text{ m/s})^2 = 19.5\text{ N}$.

(e) $P_{\text{av}} = \frac{1}{2}\sqrt{\mu F}\,\omega^2 A^2 = 54.2\text{ W}$.

EVALUATE: The argument of the cosine is $(kx + \omega t)$ for a wave traveling in the $-x$-direction, and that is the case here.

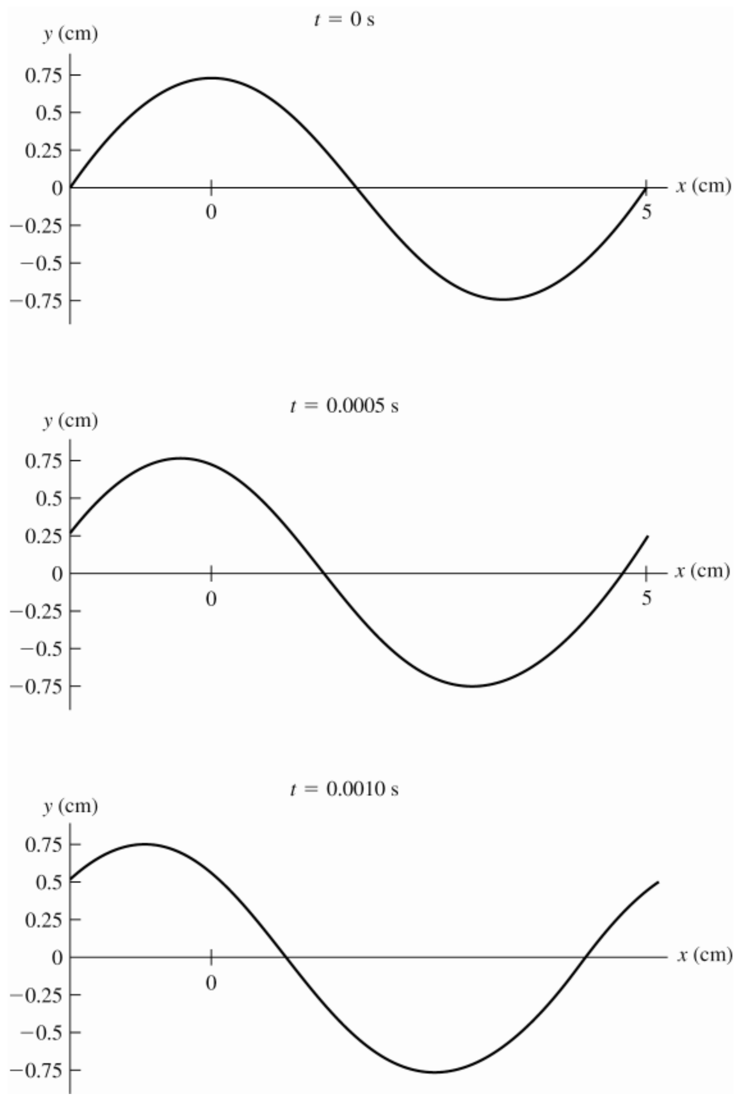


Figure 15.50

15.51. **IDENTIFY:** The speed in each segment is $v = \sqrt{F/\mu}$. The time to travel through a segment is $t = L/v$.

SET UP: The travel times for each segment are $t_1 = L\sqrt{\dfrac{\mu_1}{F}}$, $t_2 = L\sqrt{\dfrac{4\mu_1}{F}}$, and $t_3 = L\sqrt{\dfrac{\mu_1}{4F}}$.

EXECUTE: Adding the travel times gives $t_{\text{total}} = L\sqrt{\dfrac{\mu_1}{F}} + 2L\sqrt{\dfrac{\mu_1}{F}} + \frac{1}{2}L\sqrt{\dfrac{\mu_1}{F}} = \frac{7}{2}L\sqrt{\dfrac{\mu_1}{F}}$.

(b) No. The speed in a segment depends only on F and μ for that segment.

EVALUATE: The wave speed is greater and its travel time smaller when the mass per unit length of the segment decreases.

15.52. **IDENTIFY:** Apply $\sum \tau_z = 0$ to find the tension in each wire. Use $v = \sqrt{F/\mu}$ to calculate the wave speed for each wire and then $t = L/v$ is the time for each pulse to reach the ceiling, where $L = 1.25$ m .

SET UP: The wires have $\mu = \dfrac{m}{L} = \dfrac{2.50 \text{ N}}{(9.80 \text{ m/s}^2)(1.25 \text{ m})} = 0.204$ kg/m . The free-body diagram for the beam is given in Figure 15.52. Take the axis to be at the end of the beam where wire *A* is attached.

EXECUTE: $\sum \tau_z = 0$ gives $T_B L = w(L/3)$ and $T_B = w/3 = 583$ N . $T_A + T_B = 1750$ N , so $T_A = 1167$ N .

$v_A = \sqrt{\dfrac{T_A}{\mu}} = \sqrt{\dfrac{1167 \text{ N}}{0.204 \text{ kg/m}}} = 75.6$ m/s . $t_A = \dfrac{1.25 \text{ m}}{75.6 \text{ m/s}} = 0.0165$ s . $v_B = \sqrt{\dfrac{583 \text{ N}}{0.204 \text{ kg/m}}} = 53.5$ m/s .

$t_B = \dfrac{1.25 \text{ m}}{53.5 \text{ m/s}} = 0.0234$ s . $\Delta t = t_B - t_A = 6.9$ ms .

EVALUATE: The wave pulse travels faster in wire *A*, since that wire has the greater tension.

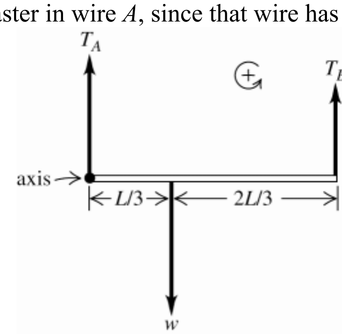


Figure 15.52

15.53. **IDENTIFY** and **SET UP:** The transverse speed of a point of the rope is $v_y = \partial y/\partial t$ where $y(x,\ t)$ is given by Eq.(15.7).

EXECUTE: **(a)** $y(x,\ t) = A\cos(kx - \omega t)$

$v_y = dy/dt = +A\omega\sin(kx - \omega t)$

$v_{y,\ \text{max}} = A\omega = 2\pi fA$

$f = \dfrac{v}{\lambda}$ and $v = \sqrt{\dfrac{F}{(m/L)}}$, so $f = \left(\dfrac{1}{\lambda}\right)\sqrt{\dfrac{FL}{M}}$

$v_{y,\ \text{max}} = \left(\dfrac{2\pi A}{\lambda}\right)\sqrt{\dfrac{FL}{M}}$

(b) To double $v_{y,\ \text{max}}$ increase *F* by a factor of 4.

EVALUATE: Increasing the tension increases the wave speed *v* which in turn increases the oscillation frequency. With the amplitude held fixed, increasing the number of oscillations per second increases the transverse velocity.

15.54. **IDENTIFY:** The maximum vertical acceleration must be at least g.

SET UP: $a_{\text{max}} = \omega^2 A$

EXECUTE: $g = \omega^2 A_{\text{min}}$ and thus $A_{\text{min}} = g/\omega^2$. Using $\omega = 2\pi f = 2\pi v/\lambda$ and $v = \sqrt{F/\mu}$, this becomes $A_{\text{min}} = \dfrac{g\lambda^2\mu}{4\pi^2 F}$.

EVALUATE: When the amplitude of the motion increases, the maximum acceleration of a point on the rope increases.

15.55. **IDENTIFY** and **SET UP:** Use Eq.(15.1) and $\omega = 2\pi f$ to replace *v* by ω in Eq.(15.13). Compare this equation to $\omega = \sqrt{k'/m}$ from Chapter 13 to deduce k'.

EXECUTE: **(a)** $\omega = 2\pi f$, $f = v/\lambda$, and $v = \sqrt{F/\mu}$ These equations combine to give

$\omega = 2\pi f = 2\pi(v/\lambda) = (2\pi/\lambda)\sqrt{F/\mu}.$

But also $\omega = \sqrt{k'/m}$. Equating these expressions for ω gives $k' = m(2\pi/\lambda)^2(F/\mu)$

But $m = \mu\ \Delta x$ so $k' = \Delta x(2\pi/\lambda)^2 F$

(b) EVALUATE: The "force constant" k' is independent of the amplitude *A* and mass per unit length μ, just as is the case for a simple harmonic oscillator. The force constant is proportional to the tension in the string *F* and inversely proportional to the wavelength λ. The tension supplies the restoring force and the $1/\lambda^2$ factor represents the dependence of the restoring force on the curvature of the string.

15.56. **IDENTIFY:** Apply $\sum \tau_z = 0$ to one post and calculate the tension in the wire. $v = \sqrt{F/\mu}$ for waves on the wire. $v = f\lambda$. The standing wave on the wire and the sound it produces have the same frequency. For standing waves on the wire, $\lambda_n = \frac{2L}{n}$.

SET UP: For the 7th overtone, $n = 8$. The wire has $\mu = m/L = (0.732\text{ kg})/(5.00\text{ m}) = 0.146\text{ kg/m}$. The free-body diagram for one of the posts is given in Figure 15.56. Forces at the pivot aren't shown. We take the rotation axis to be at the pivot, so forces at the pivot produce no torque.

EXECUTE: $\sum \tau_z = 0$ gives $w\left(\frac{L}{2}\cos 57.0°\right) - T(L\sin 57.0°) = 0$. $T = \frac{w}{2\tan 57.0°} = \frac{235\text{ N}}{2\tan 57.0°} = 76.3\text{ N}$. For waves on the wire, $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{76.3\text{ N}}{0.146\text{ kg/m}}} = 22.9\text{ m/s}$. For the 7th overtone standing wave on the wire, $\lambda = \frac{2L}{8} = \frac{2(5.00\text{ m})}{8} = 1.25\text{ m}$. $f = \frac{v}{\lambda} = \frac{22.9\text{ m/s}}{1.25\text{ m}} = 18.3\text{ Hz}$. The sound waves have frequency 18.3 Hz and wavelength $\lambda = \frac{344\text{ m/s}}{18.3\text{ Hz}} = 18.8\text{ m}$

EVALUATE: The frequency of the sound wave is at the lower limit of audible frequencies. The wavelength of the standing wave on the wire is much less than the wavelength of the sound waves, because the speed of the waves on the wire is much less than the speed of sound in air.

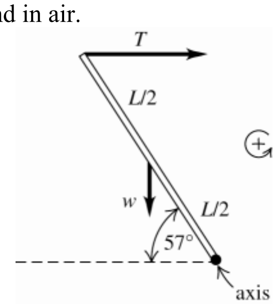


Figure 15.56

15.57. **IDENTIFY:** The magnitude of the transverse velocity is related to the slope of the *t* versus *x* curve. The transverse acceleration is related to the curvature of the graph, to the rate at which the slope is changing.

SET UP: If *y* increases as *t* increases, v_y is positive. a_y has the same sign as v_y if the transverse speed is increasing.

EXECUTE: **(a)** and **(b)** (1): The curve appears to be horizontal, and $v_y = 0$. As the wave moves, the point will begin to move downward, and $a_y < 0$. (2): As the wave moves in the $+x$-direction, the particle will move upward so $v_y > 0$. The portion of the curve to the left of the point is steeper, so $a_y > 0$. (3) The point is moving down, and will increase its speed as the wave moves; $v_y < 0$, $a_y < 0$. (4) The curve appears to be horizontal, and $v_y = 0$. As the wave moves, the point will move away from the *x*-axis, and $a_y > 0$. (5) The point is moving downward, and will increase its speed as the wave moves; $v_y < 0$, $a_y < 0$. (6) The particle is moving upward, but the curve that represents the wave appears to have no curvature, so $v_y > 0$ and $a_y = 0$.

(c) The accelerations, which are related to the curvatures, will not change. The transverse velocities will all change sign.

EVALUATE: At points 1, 3, and 5 the graph has negative curvature and $a_y < 0$. At points 2 and 4 the graph has positive curvature and $a_y > 0$.

15.58. **IDENTIFY:** The time it takes the wave to travel a given distance is determined by the wave speed *v*. A point on the string travels a distance 4*A* in time *T*.

SET UP: $v = f\lambda$. $T = 1/f$.

EXECUTE: **(a)** The wave travels a horizontal distance *d* in a time $t = \frac{d}{v} = \frac{d}{\lambda f} = \frac{8.00\text{ m}}{(0.600\text{ m})(40.0\text{ Hz})} = 0.333\text{ s}$.

(b) A point on the string will travel a vertical distance of $4A$ each cycle. Although the transverse velocity $v_y(x,t)$ is not constant, a distance of $h = 8.00\text{ m}$ corresponds to a whole number of cycles, $n = h/(4A) = (8.00\text{ m})/[4(5.00\times10^{-3}\text{ m})] = 400$, so the amount of time is $t = nT = n/f = (400)/(40.0\text{ Hz}) = 10.0\text{ s}$.

EVALUATE: **(c)** The time in part (a) is independent of amplitude but the time in part (b) depends on the amplitude of the wave. For (b), the time is halved if the amplitude is doubled.

15.59. **IDENTIFY:** $y^2(x,y)+z^2(x,y)=A^2$. The trajectory is a circle of radius A.

SET UP: $v_y = \partial y/\partial t$, $v_z = \partial z/\partial t$. $a_y = \partial v_y/\partial t$, $a_z = \partial v_z/\partial t$

EXECUTE: At $t=0$, $y(0,0)=A$, $z(0,0)=0$. At $t=\pi/2\omega$, $y(0,\pi/2\omega)=0$, $z(0,\pi/2\omega)=-A$. At $t=\pi/\omega$, $y(0,\pi/\omega)=-A$, $z(0,\pi/\omega)=0$. At $t=3\pi/2\omega$, $y(0,3\pi/2\omega)=0$, $z(0,3\pi/2\omega)=A$. The trajectory and these points are sketched in Figure 15.59.

(b) $v_y = \partial y/\partial t = +A\omega\sin(kx-\omega t)$, $v_z = \partial z/\partial t = -A\omega\cos(kx-\omega t)$.

$\vec{v} = v_y\hat{j} + v_z\hat{k} = A\omega[\sin(kx-\omega)\hat{j} - \cos(kx-\omega t)\hat{k}]$. $v=\sqrt{v_y^2+v_z^2}=A\omega$ so the speed is constant.

$\vec{r} = y\hat{j}+z\hat{k}$. $\vec{r}\cdot\vec{v} = yv_y + zv_z = A^2\omega\sin(kx-\omega t)\cos(kx-\omega t) - A^2\omega\cos(kx-\omega t)\sin(kx-\omega t) = 0$.

$\vec{r}\cdot\vec{v}=0$, so $\vec{v}$ is tangent to the circular path.

(c) $a_y = \partial v_y/\partial t = -A\omega^2\cos(kx-\omega t)$, $a_z = \partial v_z/\partial t = -A\omega^2\sin(kx-\omega t)$

$\vec{r}\cdot\vec{a} = ya_y + za_z = -A^2\omega^2[\cos^2(kx-\omega t)+\sin^2(kx-\omega t)] = -A^2\omega^2$. $r=A$, $\vec{r}\cdot\vec{a} = -ra$.

$\vec{r}\cdot\vec{a} = ra\cos\phi$ so $\phi = 180°$ and $\vec{a}$ is opposite in direction to $\vec{r}$; $\vec{a}$ is radially inward. For these $y(x,t)$ and $z(x,t)$, $y^2+z^2=A^2$, so the path is again circular, but the particle rotates in the opposite sense compared to part (a).

EVALUATE: The wave propagates in the $+x$-direction. The displacement is transverse, so $\vec{v}$ and $\vec{a}$ lie in the yz-plane.

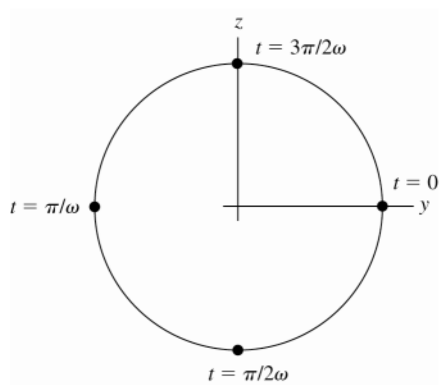


Figure 15.59

15.60. **IDENTIFY:** The wavelengths of the standing waves on the wire are given by $\lambda_n = \dfrac{2L}{n}$. When the ball is changed the wavelength changes because the length of the wire changes; $\Delta l = \dfrac{Fl_0}{AY}$.

SET UP: For the third harmonic, $n=3$. For copper, $Y = 11\times10^{10}\text{ Pa}$. The wire has cross-sectional area $A = \pi r^2 = \pi(0.512\times10^{-3}\text{ m})^2 = 8.24\times10^{-7}\text{ m}^2$

EXECUTE: **(a)** $\lambda_3 = \dfrac{2(1.20\text{ m})}{3} = 0.800\text{ m}$

(b) The increase in length when the 100.0 N ball is replaced by the 500.0 N ball is given by $\Delta l = \dfrac{(\Delta F)l_0}{AY}$, where $\Delta F = 400.0\text{ N}$ is the increase in the force applied to the end of the wire.

$\Delta l = \dfrac{(400.0\text{ N})(1.20\text{ m})}{(8.24\times10^{-7}\text{ m}^2)(11\times10^{10}\text{ Pa})} = 5.30\times10^{-3}\text{ m}$. The change in wavelength is $\Delta\lambda = \frac{2}{3}\Delta l = 3.5\text{ mm}$.

EVALUATE: The change in tension changes the wave speed and that in turn changes the frequency of the standing wave, but the problem asks only about the wavelength.

15.61. **IDENTIFY:** Follow the procedure specified in part (b).

SET UP: If $u = x - vt$, then $\frac{\partial u}{\partial t} = -v$ and $\frac{\partial u}{\partial x} = 1$.

EXECUTE: **(a)** As time goes on, someone moving with the wave would need to move in such a way that the wave appears to have the same shape. If this motion can be described by $x = vt + b$, with b a constant, then $y(x,t) = f(b)$, and the waveform is the same to such an observer.

(b) $\frac{\partial^2 y}{\partial x^2} = \frac{d^2 f}{du^2}$ and $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{d^2 f}{du^2}$, so $y(x,t) = f(x - vt)$ is a solution to the wave equation with wave speed v.

(c) This is of the form $y(x,t) = f(u)$, with $u = x - vt$ and $f(u) = De^{-B^2(x - Ct/B)^2}$. The result of part (b) may be used to determine the speed $v = C/B$.

EVALUATE: The wave in part (c) moves in the $+x$-direction. The speed of the wave is independent of the constant D.

15.62. **IDENTIFY:** The phase angle determines the value of y for $x = 0$, $t = 0$ but does not affect the shape of the $y(x,t)$ versus x or t graph.

SET UP: $\frac{\partial \cos(kx - \omega t + \phi)}{\partial t} = -\omega \sin(kx - \omega t + \phi)$.

EXECUTE: **(a)** The graphs for each ϕ are sketched in Figure 15.62.

(b) $\frac{\partial y}{\partial t} = -\omega A \sin(kx - \omega t + \phi)$

(c) No. $\phi = \pi/4$ or $\phi = 3\pi/4$ would both give $A/\sqrt{2}$. If the particle is known to be moving downward, the result of part (b) shows that $\cos\phi < 0$, and so $\phi = 3\pi/4$.

(d) To identify ϕ uniquely, the quadrant in which ϕ lies must be known. In physical terms, the signs of both the position and velocity, and the magnitude of either, are necessary to determine ϕ (within additive multiples of 2π).

EVALUATE: The phase $\phi = 0$ corresponds to $y = A$ at $x = 0$, $t = 0$.

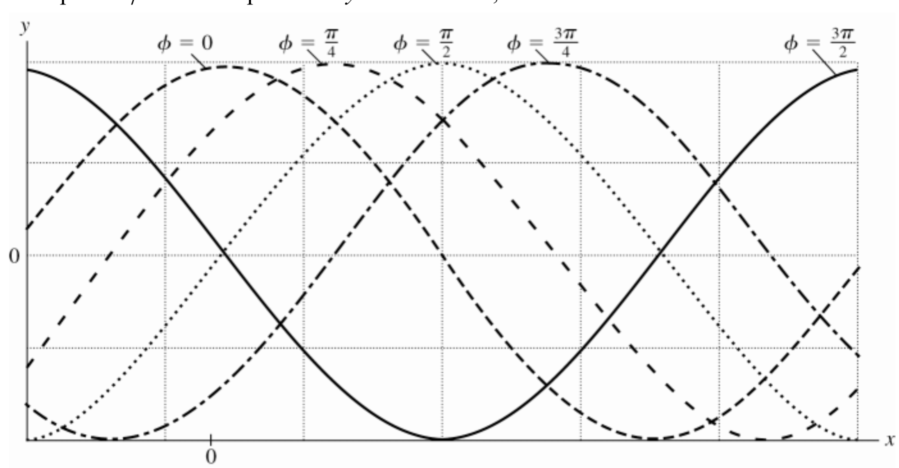


Figure 15.62

15.63. **IDENTIFY** and **SET UP:** Use Eq.(15.13) to replace μ, and then Eq.(15.6) to replace v.

EXECUTE: **(a)** Eq.(15.25): $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$

$v = \sqrt{F/\mu}$ says $\sqrt{\mu} = \sqrt{F}/v$ so $P_{av} = \frac{1}{2}\left(\sqrt{F}/v\right)\sqrt{F}\omega^2 A^2 = \frac{1}{2}F\omega^2 A^2/v$

$\omega = 2\pi f$ so $\omega/v = 2\pi f/v = 2\pi/\lambda = k$ and $P_{av} = \frac{1}{2}Fk\omega A^2$, as was to be shown.

(b) IDENTIFY: For the ω dependence, use Eq.(15.25) since it involves just ω, not k: $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$.

SET UP: P_{av}, μ, A all constant so $\sqrt{F}\omega^2$ is constant, and $\sqrt{F_1}\omega_1^2 = \sqrt{F_2}\omega_2^2$.

EXECUTE: $\omega_2 = \omega_1(F_1/F_2)^{1/4} = \omega_1(F_1/4F_1)^{1/4} = \omega_1(4)^{-1/4} = \omega_1/\sqrt{2}$

ω must be changed by a factor of $1/\sqrt{2}$ (decreased)

IDENTIFY: For the k dependence, use the equation derived in part (a), $P_{av} = \frac{1}{2}Fk\omega A^2$.

SET UP: If P_{av} and A are constant then $Fk\omega$ must be constant, and $F_1 k_1 \omega_1 = F_2 k_2 \omega_2$.

EXECUTE: $k_2 = k_1\left(\frac{F_1}{F_2}\right)\left(\frac{\omega_1}{\omega_2}\right) = k_1\left(\frac{F_1}{4F_1}\right)\left(\frac{\omega_1}{\omega_1/\sqrt{2}}\right) = k_1\frac{\sqrt{2}}{4} = k_1\sqrt{\frac{2}{16}} = k_1/\sqrt{8}$

k must be changed by a factor of $1/\sqrt{8}$ (decreased).

EVALUATE: Power is the transverse force times the transverse velocity. To keep P_{av} constant the transverse velocity must be decreased when F is increased, and this is done by decreasing ω.

15.64. **IDENTIFY:** The wave moves in the $+x$ direction with speed v, so to obtain $y(x,t)$ replace x with $x - vt$ in the expression for $y(x,0)$.

SET UP: $P(x,t)$ is given by Eq.(15.21).

EXECUTE: **(a)** The wave pulse is sketched in Figure 15.64.

(b)

$$y(x,t) = \begin{cases} 0 & \text{for } (x-vt) < -L \\ h(L+x-vt)/L & \text{for } -L < (x-vt) < 0 \\ h(L-x+vt)/L & \text{for } 0 < (x-vt) < L \\ 0 & \text{for } (x-vt) > L \end{cases}$$

(c) From Eq.(15.21):

$$P(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t} = \begin{cases} -F(0)(0) = 0 & \text{for } (x-vt) < -L \\ -F(h/L)(-hv/L) = Fv(h/L)^2 & \text{for } -L < (x-vt) < 0 \\ -F(-h/L)(hv/L) = Fv(h/L)^2 & \text{for } 0 < (x-vt) < L \\ -F(0)(0) = 0 & \text{for } (x-vt) > L \end{cases}$$

Thus the instantaneous power is zero except for $-L < (x-vt) < L$, where it has the constant value $Fv(h/L)^2$.

EVALUATE: For this pulse the transverse velocity v_y is constant in magnitude and has opposite sign on either side of the peak of the pulse.

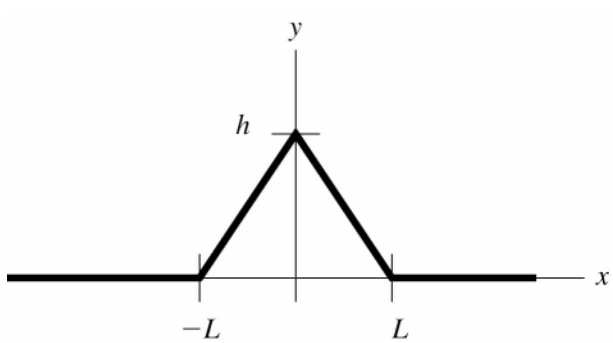


Figure 15.64

15.65. **IDENTIFY** and **SET UP:** The average power is given by Eq.(15.25). Rewrite this expression in terms of v and λ in place of F and ω.

EXECUTE: **(a)** $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$

$v = \sqrt{F/\mu}$ so $\sqrt{F} = v/\sqrt{\mu}$

$\omega = 2\pi f = 2\pi(v/\lambda)$

Using these two expressions to replace $\sqrt{F}$ and ω gives $P_{av} = 2\mu\pi^2 v^3 A^2/\lambda^2$; $\mu = (6.00\times10^{-3}\text{ kg})/(8.00\text{ m})$

$$A = \left(\frac{2\lambda^2 P_{av}}{4\pi^2 v^3 \mu}\right) = 7.07\text{ cm}$$

(b) EVALUATE: $P_{av} \sim v^3$ so doubling v increases P_{av} by a factor of 8.

$P_{av} = 8(50.0\text{ W}) = 400.0\text{ W}$

15.66. **IDENTIFY:** Draw the graphs specified in part (a).

SET UP: When $y(x,t)$ is a maximum, the slope $\partial y/\partial x$ is zero. The slope has maximum magnitude when $y(x,t) = 0$.

EXECUTE: **(a)** The graph is sketched in Figure 15.66a.

(b) The power is a maximum where the displacement is zero, and the power is a minimum of zero when the magnitude of the displacement is a maximum.

(c) The energy flow is always in the same direction.

(d) In this case, $\frac{\partial y}{\partial x} = -kA\sin(kx + \omega t)$ and Eq.(15.22) becomes $P(x,t) = -Fk\omega A^2\sin^2(kx + \omega t)$. The power is now negative (energy flows in the $-x$-direction), but the qualitative relations of part (b) are unchanged. The graph is sketched in Figure 15.66b.

EVALUATE: $\cos\theta$ and $\sin\theta$ are $180°$ out of phase, so for fixed *t*, maximum *y* corresponds to zero *P* and $y = 0$ corresponds to maximum *P*.

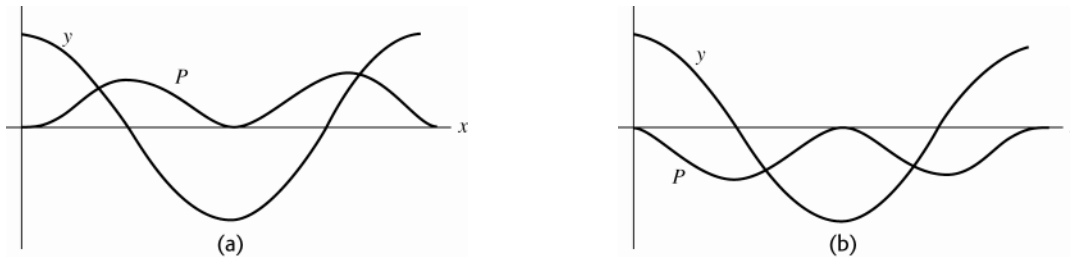


Figure 15.66

15.67. **IDENTIFY** and **SET UP:** $v = \sqrt{F/\mu}$. The coefficient of linear expansion α is defined by $\Delta L = L_0\alpha\ \Delta T$. This can be combined with $Y = \frac{F/A}{\Delta L/L_0}$ to give $\Delta F = -Y\alpha A\Delta T$ for the change in tension when the temperature changes by ΔT. Combine the two equations and solve for α.

EXECUTE: $v_1 = \sqrt{F/\mu}$, $v_1^2 = F/\mu$ and $F = \mu v_1^2$

The length and hence μ stay the same but the tension decreases by $\Delta F = -Y\alpha A\ \Delta T$.

$v_2 = \sqrt{(F + \Delta F)/\mu} = \sqrt{(F - Y\alpha A\ \Delta T)/\mu}$

$v_2^2 = F/\mu - Y\alpha A\ \Delta T/\mu = v_1^2 - Y\alpha A\ \Delta T/\mu$

And $\mu = m/L$ so $A/\mu = AL/m = V/m = 1/\rho$. (*A* is the cross-sectional area of the wire, *V* is the volume of a length *L*.) Thus $v_1^2 - v_2^2 = \alpha(Y\ \Delta T/\rho)$ and $\alpha = \frac{v_1^2 - v_2^2}{(Y/\rho)\ \Delta T}$

EVALUATE: When *T* increases the tension decreases and *v* decreases.

15.68. **IDENTIFY:** The time between positions 1 and 5 is equal to $T/2$. $v = f\lambda$. The velocity of points on the string is given by Eq.(15.9).

SET UP: Four flashes occur from position 1 to position 5, so the elapsed time is $4\left(\frac{60\text{ s}}{5000}\right) = 0.048\text{ s}$. The figure in the problem shows that $\lambda = L = 0.500\text{ m}$. At point *P* the amplitude of the standing wave is 1.5 cm.

EXECUTE: **(a)** $T/2 = 0.048\text{ s}$ and $T = 0.096\text{ s}$. $f = 1/T = 10.4\text{ Hz}$. $\lambda = 0.500\text{ m}$.

(b) The fundamental standing wave has nodes at each end and no nodes in between. This standing wave has one additional node. This is the 1st overtone and 2nd harmonic.

(c) $v = f\lambda = (10.4\text{ Hz})(0.500\text{ m}) = 5.20\text{ m/s}$.

(d) In position 1, point *P* is at its maximum displacement and its speed is zero. In position 3, point *P* is passing through its equilibrium position and its speed is $v_{max} = \omega A = 2\pi fA = 2\pi(10.4\text{ Hz})(0.015\text{ m}) = 0.980\text{ m/s}$.

(e) $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{FL}{m}}$ and $m = \frac{FL}{v^2} = \frac{(1.00\text{ N})(0.500\text{ m})}{(5.20\text{ m/s})^2} = 18.5\text{ g}$.

EVALUATE: The standing wave is produced by traveling waves moving in opposite directions. Each point on the string moves in SHM, and the amplitude of this motion varies with position along the string.

15.69. **IDENTIFY** and **SET UP:** There is a node at the post and there must be a node at the clothespin. There could be additional nodes in between. The distance between adjacent nodes is $\lambda/2$, so the distance between *any* two nodes is $n(\lambda/2)$ for $n = 1, 2, 3, \ldots$ This must equal 45.0 cm, since there are nodes at the post and clothespin. Use this in Eq.(15.1) to get an expression for the possible frequencies *f*.

EXECUTE: $45.0\text{ cm} = n(\lambda/2)$, $\lambda = v/f$, so $f = n[v/(90.0\text{ cm})] = (0.800\text{ Hz})n$, $n = 1, 2, 3, \ldots$

EVALUATE: Higher frequencies have smaller wavelengths, so more node-to-node segments fit between the post and clothespin.

15.70. **IDENTIFY:** The displacement of the string at any point is $y(x,t) = (A_{\text{SW}} \sin kx)\sin \omega t$. For the fundamental mode $\lambda = 2L$, so at the midpoint of the string $\sin kx = \sin(2\pi/\lambda)(L/2) = 1$, and $y = A_{\text{SW}} \sin \omega t$. The transverse velocity is $v_y = \partial y/\partial t$ and the transverse acceleration is $a_y = \partial v_y/\partial t$.

SET UP: Taking derivatives gives $v_y = \dfrac{\partial y}{\partial t} = \omega A_{\text{SW}} \cos \omega t$, with maximum value $v_{y,\max} = \omega A_{\text{SW}}$, and

$a_y = \dfrac{\partial v_y}{\partial t} = -\omega^2 A_{\text{SW}} \sin \omega t$, with maximum value $a_{y,\max} = \omega^2 A_{\text{SW}}$.

EXECUTE: $\omega = a_{y,\max}/v_{y,\max} = (8.40\times 10^3\ \text{m/s}^2)/(3.80\ \text{m/s}) = 2.21\times 10^3\ \text{rad/s}$, and then

$A_{\text{SW}} = v_{y,\max}/\omega = (3.80\ \text{m/s})/(2.21\times 10^3\ \text{rad/s}) = 1.72\times 10^{-3}\ \text{m}$.

(b) $v = \lambda f = (2L)(\omega/2\pi) = L\omega/\pi = (0.386\ \text{m})(2.21\times 10^3\ \text{rad/s})/\pi = 272\ \text{m/s}$.

EVALUATE: The maximum transverse velocity and acceleration will have different (smaller) values at other points on the string.

15.71. **IDENTIFY:** To show this relationship is valid, take the second time derivative.

SET UP: $\dfrac{\partial}{\partial t}\sin \omega t = \cos \omega t$. $\dfrac{\partial}{\partial t}\cos \omega t = -\omega \sin \omega t$.

EXECUTE: **(a)** $\dfrac{\partial^2 y(x,t)}{\partial t^2} = \dfrac{\partial^2}{\partial t^2}[(A_{\text{SW}} \sin kx)\sin \omega t] = \omega \dfrac{\partial}{\partial t}[(A_{\text{SW}} \sin kx)\cos \omega t]$

$\dfrac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 \left[(A_{\text{sw}} \sin kx)\sin \omega t\right] = -\omega^2 y(x,t)$. This equation shows that $a_y = -\omega^2 y$. This is characteristic of simple harmonic motion; each particle of the string moves in simple harmonic motion.

(b) Yes, the traveling wave is also a solution of this equation. When a string carries a traveling wave each point on the string moves in simple harmonic motion.

EVALUATE: A standing wave is the superposition of two traveling waves, so it is not surprising that for both types of waves the particles on the string move in SHM.

15.72. **IDENTIFY** and **SET UP:** Carry out the analysis specified in the problem.

EXECUTE: **(a)** The wave moving to the left is inverted and reflected; the reflection means that the wave moving to the left is the same function of $-x$, and the inversion means that the function is $-f(-x)$.

(b) The wave that is the sum is $f(x) - f(-x)$ (an inherently odd function), and for any f, $f(0) - f(-0) = 0$.

(c) The wave is reflected but not inverted (see the discussion in part (a) above), so the wave moving to the left in Figure 15.21 in the textbook is $+f(-x)$.

(d) $\dfrac{dy}{dx} = \dfrac{d}{dx}(f(x) + f(-x)) = \dfrac{df(x)}{dx} + \dfrac{df(-x)}{dx} = \dfrac{df(x)}{dx} + \dfrac{df(-x)}{d(-x)}\dfrac{d(-x)}{dx} = \dfrac{df}{dx} - \left.\dfrac{df}{dx}\right|_{x=-x}$.

At $x = 0$, the terms are the same and the derivative is zero.

EVALUATE: Our results verify the behavior shown in Figures 15.20 and 15.21 in the textbook.

15.73. **IDENTIFY:** Carry out the derivation as done in the text for Eq.(15.28). The transverse velocity is $v_y = \partial y/\partial t$ and the transverse acceleration is $a_y = \partial v_y/\partial t$.

(a) SET UP: For reflection from a free end of a string the reflected wave is *not* inverted, so

$y(x,\ t) = y_1(x,\ t) + y_2(x,\ t)$, where

$y_1(x,\ t) = A\cos(kx + \omega t)$ (traveling to the left)

$y_2(x,\ t) = A\cos(kx - \omega t)$ (traveling to the right)

Thus $y(x,\ t) = A[\cos(kx + \omega t) + \cos(kx - \omega t)]$.

EXECUTE: Apply the trig identity $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$ with $a = kx$ and $b = \omega t$:

$\cos(kx + \omega t) = \cos kx \cos \omega t - \sin kx \sin \omega t$ and

$\cos(kx - \omega t) = \cos kx \cos \omega t + \sin kx \sin \omega t$.

Then $y(x,\ t) = (2A\cos kx)\cos \omega t$ (the other two terms cancel)

(b) For $x = 0$, $\cos kx = 1$ and $y(x,\ t) = 2A\cos \omega t$. The amplitude of the simple harmonic motion at $x = 0$ is $2A$, which is the maximum for this standing wave, so $x = 0$ is an antinode.

(c) $y_{\max} = 2A$ from part (b).

$v_y = \dfrac{\partial y}{\partial t} = \dfrac{\partial}{\partial t}[(2A\cos kx)\cos \omega t] = 2A\cos kx \dfrac{\partial \cos \omega t}{\partial t} = -2A\omega \cos kx \sin \omega t$.

At $x = 0$, $v_y = -2A\omega \sin \omega t$ and $(v_y)_{\max} = 2A\omega$

$$a_y = \frac{\partial^2 y}{\partial t^2} = \frac{\partial v_y}{\partial t} = -2A\omega \cos kx \frac{\partial \sin \omega t}{\partial t} = -2A\omega^2 \cos kx \cos \omega t$$

At $x = 0$, $a_y = -2A\omega^2 \cos \omega t$ and $(a_y)_{\max} = 2A\omega^2$.

EVALUATE: The expressions for $(v_y)_{\max}$ and $(a_y)_{\max}$ are the same as at the antinodes for the standing wave of a string fixed at both ends.

15.74. IDENTIFY: The standing wave is given by Eq.(15.28).

SET UP: At an antinode, $\sin kx = 1$. $v_{y,\max} = \omega A$. $a_{y,\max} = \omega^2 A$.

EXECUTE: **(a)** $\lambda = v/f = (192.0\text{ m/s})/(240.0\text{ Hz}) = 0.800\text{ m}$, and the wave amplitude is $A_{\text{SW}} = 0.400\text{ cm}$. The amplitude of the motion at the given points is

(i) $(0.400\text{ cm})\sin(\pi) = 0$ (a node) (ii) $(0.400\text{ cm})\sin(\pi/2) = 0.400\text{ cm}$ (an antinode)

(iii) $(0.400\text{ cm})\sin(\pi/4) = 0.283\text{ cm}$

(b) The time is half of the period, or $1/(2f) = 2.08\times 10^{-3}\text{s}$.

(c) In each case, the maximum velocity is the amplitude multiplied by $\omega = 2\pi f$ and the maximum acceleration is the amplitude multiplied by $\omega^2 = 4\pi^2 f^2$:

(i) 0, 0; (ii) 6.03 m/s, $9.10\times 10^3\text{ m/s}^2$; (iii) 4.27 m/s, $6.43\times 10^3\text{ m/s}^2$.

EVALUATE: The amplitude, maximum transverse velocity, and maximum transverse acceleration vary along the length of the string. But the period of the simple harmonic motion of particles of the string is the same at all points on the string.

15.75. IDENTIFY: The standing wave frequencies are given by $f_n = n\left(\frac{v}{2L}\right)$. $v = \sqrt{F/\mu}$. Use the density of steel to calculate μ for the wire.

SET UP: For steel, $\rho = 7.8\times 10^3\text{ kg/m}^3$. For the first overtone standing wave, $n = 2$.

EXECUTE: $v = \frac{2Lf_2}{2} = (0.550\text{ m})(311\text{ Hz}) = 171\text{ m/s}$. The volume of the wire is $V = (\pi r^2)L$. $m = \rho V$ so

$\mu = \frac{m}{L} = \frac{\rho V}{L} = \rho\pi r^2 = (7.8\times 10^3\text{ kg/m}^3)\pi(0.57\times 10^{-3}\text{ m})^2 = 7.96\times 10^{-3}\text{ kg/m}$. The tension is

$F = \mu v^2 = (7.96\times 10^{-3}\text{ kg/m})(171\text{ m/s})^2 = 233\text{ N}$.

EVALUATE: The tension is not large enough to cause much change in length of the wire.

15.76. IDENTIFY: The mass and breaking stress determine the length and radius of the string. $f_1 = \frac{v}{2L}$, with $v = \sqrt{\frac{F}{\mu}}$.

SET UP: The tensile stress is $F/\pi r^2$.

EXECUTE: **(a)** The breaking stress is $\frac{F}{\pi r^2} = 7.0\times 10^8\text{ N/m}^2$ and the maximum tension is $F = 900\text{ N}$, so solving for r gives the minimum radius $r = \sqrt{\frac{900\text{ N}}{\pi(7.0\times 10^8\text{ N/m}^2)}} = 6.4\times 10^{-4}\text{ m}$. The mass and density are fixed,

$\rho = \frac{M}{\pi r^2 L}$ so the minimum radius gives the maximum length

$$L = \frac{M}{\pi r^2 \rho} = \frac{4.0\times 10^{-3}\text{ kg}}{\pi(6.4\times 10^{-4}\text{ m})^2(7800\text{ kg/m}^3)} = 0.40\text{ m}.$$

(b) The fundamental frequency is $f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}} = \frac{1}{2L}\sqrt{\frac{F}{M/L}} = \frac{1}{2}\sqrt{\frac{F}{ML}}$. Assuming the maximum length of the string is free to vibrate, the highest fundamental frequency occurs when $F = 900\text{ N}$ and

$$f_1 = \tfrac{1}{2}\sqrt{\frac{900\text{ N}}{(4.0\times 10^{-3}\text{ kg})(0.40\text{ m})}} = 376\text{ Hz}.$$

EVALUATE: If the radius was any smaller the breaking stress would be exceeded. If the radius were greater, so the stress was less than the maximum value, then the length would be less to achieve the same total mass.

15.77. IDENTIFY: At a node, $y(x,t) = 0$ for all t. $y_1 + y_2$ is a standing wave if the locations of the nodes don't depend on t.

SET UP: The string is fixed at each end so for all harmonics the ends are nodes. The second harmonic is the first overtone and has one additional node.

EXECUTE: **(a)** The fundamental has nodes only at the ends, $x = 0$ and $x = L$.
(b) For the second harmonic, the wavelength is the length of the string, and the nodes are at $x = 0, x = L/2$ and $x = L$.
(c) The graphs are sketched in Figure 15.77.
(d) The graphs in part (c) show that the locations of the nodes and antinodes between the ends vary in time.
EVALUATE: The sum of two standing waves of different frequencies is not a standing wave.

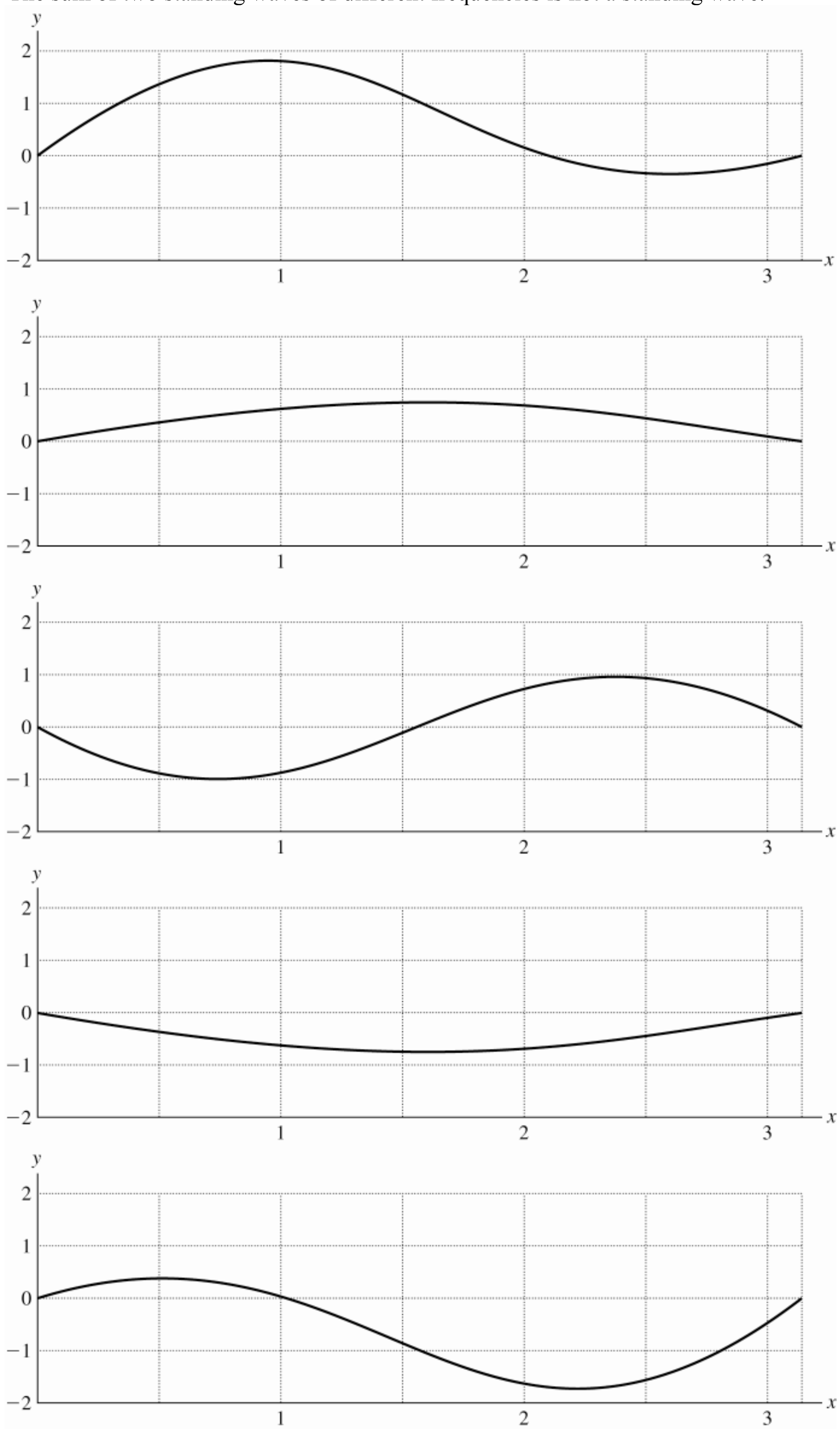


Figure 15.77

15.78. **IDENTIFY:** $f_1 = \dfrac{v}{2L}$. The buoyancy force B that the water exerts on the object reduces the tension in the wire. $B = \rho_{\text{fluid}} V_{\text{submerged}} g$.

SET UP: For aluminum, $\rho_{\text{a}} = 2700\ \text{kg/m}^3$. For water, $\rho_{\text{w}} = 1000\ \text{kg/m}^3$. Since the sculpture is completely submerged, $V_{\text{submerged}} = V_{\text{object}} = V$.

EXECUTE: L is constant, so $\dfrac{f_{\text{air}}}{v_{\text{air}}} = \dfrac{f_{\text{w}}}{v_{\text{w}}}$ and the fundamental frequency when the sculpture is submerged is $f_{\text{w}} = f_{\text{air}}\left(\dfrac{v_{\text{w}}}{v_{\text{air}}}\right)$, with $f_{\text{air}} = 250.0\ \text{Hz}$. $v = \sqrt{\dfrac{F}{\mu}}$ so $\dfrac{v_{\text{w}}}{v_{\text{air}}} = \sqrt{\dfrac{F_{\text{w}}}{F_{\text{air}}}}$. When the sculpture is in air, $F_{\text{air}} = w = mg = \rho_a V g$.

When the sculpture is submerged in water, $F_{\text{w}} = w - B = (\rho_{\text{a}} - \rho_{\text{w}})Vg$. $\dfrac{v_{\text{w}}}{v_{\text{air}}} = \sqrt{\dfrac{\rho_{\text{a}} - \rho_{\text{w}}}{\rho_{\text{a}}}}$ and

$$f_{\text{w}} = (250.0\ \text{Hz})\sqrt{1 - \frac{1000\ \text{kg/m}^3}{2700\ \text{kg/m}^3}} = 198\ \text{Hz}.$$

EVALUATE: We have neglected the buoyant force on the wire itself.

15.79. **IDENTIFY:** Compute the wavelength from the length of the string. Use Eq.(15.1) to calculate the wave speed and then apply Eq.(15.13) to relate this to the tension.

(a) SET UP: The tension F is related to the wave speed by $v = \sqrt{F/\mu}$ (Eq.(15.13)), so use the information given to calculate v.

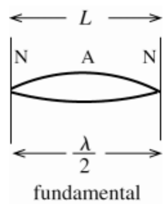


Figure 15.79

EXECUTE: $\lambda/2 = L$

$\lambda = 2L = 2(0.600\ \text{m}) = 1.20\ \text{m}$

$v = f\lambda = (65.4\ \text{Hz})(1.20\ \text{m}) = 78.5\ \text{m/s}$

$\mu = m/L = 14.4 \times 10^{-3}\ \text{kg}/0.600\ \text{m} = 0.024\ \text{kg/m}$

Then $F = \mu v^2 = (0.024\ \text{kg/m})(78.5\ \text{m/s})^2 = 148\ \text{N}$.

(b) SET UP: $F = \mu v^2$ and $v = f\lambda$ give $F = \mu f^2 \lambda^2$.

μ is a property of the string so is constant.

λ is determined by the length of the string so stays constant.

μ, λ constant implies $F/f^2 = \mu\lambda^2 = \text{constant}$, so $F_1/f_1^2 = F_2/f_2^2$

EXECUTE: $F_2 = F_1\left(\dfrac{f_2}{f_1}\right)^2 = (148\ \text{N})\left(\dfrac{73.4\ \text{Hz}}{65.4\ \text{Hz}}\right)^2 = 186\ \text{N}.$

The percent change in F is $\dfrac{F_2 - F_1}{F_1} = \dfrac{186\ \text{N} - 148\ \text{N}}{148\ \text{N}} = 0.26 = 26\%.$

EVALUATE: The wave speed and tension we calculated are similar in magnitude to values in the Examples. Since the frequency is proportional to $\sqrt{F}$, a 26% increase in tension is required to produce a 13% increase in the frequency.

15.80. **IDENTIFY** and **SET UP:** Consider the derivation of the speed of a longitudinal wave in Section 15.4.

EXECUTE: **(a)** The quantity of interest is the change in force per fractional length change. The force constant k' is the change in force per length change, so the force change per fractional length change is $k'L$, the applied force at one end is $F = (k'L)(v_y/v)$ and the longitudinal impulse when this force is applied for a time t is $k'Lt\,v_y/v$. The change in longitudinal momentum is $((vt)m/L)v_y$ and equating the expressions, canceling a factor of t and solving for v gives $v^2 = L^2 k'/m$.

(b) $v = (2.00\ \text{m})\sqrt{(1.50\ \text{N/m})/(0.250\ \text{kg})} = 4.90\ \text{m/s}$

EVALUATE: A larger k' corresponds to a stiffer spring and for a stiffer spring the wave speed is greater.

15.81. IDENTIFY: Carry out the analysis specified in the problem.

SET UP: The kinetic energy of a very short segment Δx is $\Delta K = \frac{1}{2}(\Delta m)v_y^2$. $v_y = \partial y/\partial t$. The work done by the tension is F times the increase in length of the segment. Let the potential energy be zero when the segment is unstretched.

EXECUTE: **(a)** $u_k = \dfrac{\Delta K}{\Delta x} = \dfrac{(1/2)\Delta m v_y^2}{\Delta m/\mu} = \dfrac{1}{2}\mu\left(\dfrac{\partial y}{\partial t}\right)^2$.

(b) $\dfrac{\partial y}{\partial t} = \omega A \sin(kx - \omega t)$ and so $u_k = \dfrac{1}{2}\mu\omega^2 A^2 \sin^2(kx - \omega t)$.

(c) The piece has width Δx and height $\Delta x \dfrac{\partial y}{\partial x}$, and so the length of the piece is

$\left((\Delta x)^2 + \left(\Delta x \dfrac{\partial y}{\partial x}\right)^2\right)^{1/2} = \Delta x\left(1 + \left(\dfrac{\partial y}{\partial x}\right)^2\right)^{1/2} \approx \Delta x\left[1 + \dfrac{1}{2}\left(\dfrac{\partial y}{\partial x}\right)^2\right]$, where the relation given in the hint has been used.

(d) $u_p = F\dfrac{\Delta x\left[1 + \frac{1}{2}(\partial y/\partial x)^2\right] - \Delta x}{\Delta x} = \dfrac{1}{2}F\left(\dfrac{\partial y}{\partial x}\right)^2$.

(e) $\dfrac{\partial y}{\partial x} = -kA \sin(kx - \omega t)$, and so $u_p = \dfrac{1}{2}Fk^2 A^2 \sin^2(kx - \omega t)$.

(f) Comparison with the result of part (c) with $k^2 = \omega^2/v^2 = \omega^2 \mu/F$ shows that for a sinusoidal wave $u_k = u_p$.

(g) The graph is given in Figure 15.81. In this graph, u_k and u_p coincide, as shown in part (f). At $y = 0$, the string is stretched the most, and is moving the fastest, so u_k and u_p are maximized. At the extremes of y, the string is unstretched and is not moving, so u_k and u_p are both at their minimum of zero.

(h) $u_k + u_p = Fk^2 A^2 \sin^2(kx - \omega t) = Fk(\omega/v) A^2 \sin^2(kx - \omega t) = \dfrac{P}{v}$.

EVALUATE: The energy density travels with the wave, and the rate at which the energy is transported is the product of the density per unit length and the speed.

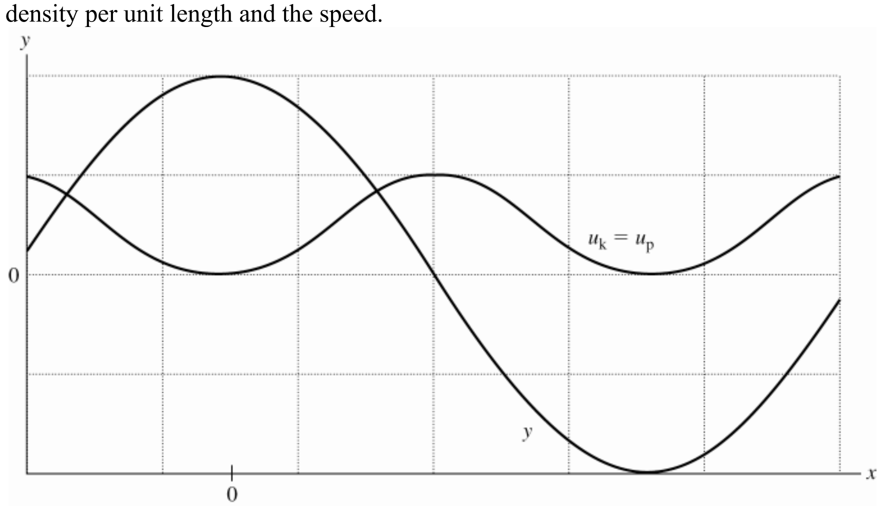


Figure 15.81

15.82. IDENTIFY: Apply $\sum F_y = 0$ to segments of the cable. The forces are the weight of the diver, the weight of the segment of the cable, the tension in the cable and the buoyant force on the segment of the cable and on the diver.

SET UP: The buoyant force on an object of volume V that is completely submerged in water is $B = \rho_{water} V g$.

EXECUTE: **(a)** The tension is the difference between the diver's weight and the buoyant force,
$F = (m - \rho_{water} V)g = (120 \text{ kg} - (1000 \text{ kg/m}^3)(0.0800 \text{ m}^3)(9.80 \text{ m/s}^2)) = 392 \text{ N}$.

(b) The increase in tension will be the weight of the cable between the diver and the point at x, minus the buoyant force. This increase in tension is then

$(\mu x - \rho(Ax))g = (1.10 \text{ kg/m} - (1000 \text{ kg/m}^3)\pi(1.00\times10^{-2} \text{ m})^2)(9.80 \text{ m/s}^2)x = (7.70 \text{ N/m})x$. The tension as a function of x is then $F(x) = (392 \text{ N}) + (7.70 \text{ N/m})x$.

(c) Denote the tension as $F(x) = F_0 + ax$, where $F_0 = 392\text{ N}$ and $a = 7.70\text{ N/m}$. Then the speed of transverse waves as a function of *x* is $v = \frac{dx}{dt} = \sqrt{(F_0 + ax)/\mu}$ and the time *t* needed for a wave to reach the surface is found from $t = \int dt = \int \frac{dx}{dx/dt} = \int \frac{\sqrt{\mu}}{\sqrt{F_0 + ax}} dx.$

Let the length of the cable be *L*, so $t = \sqrt{\mu}\int_0^L \frac{dx}{\sqrt{F_0 + ax}} = \sqrt{\mu}\frac{2}{a}\sqrt{F_0 + ax}\Big|_0^L = \frac{2\sqrt{\mu}}{a}\left(\sqrt{F_0 + aL} - \sqrt{F_0}\right)$.

$$t = \frac{2\sqrt{1.10\text{ kg/m}}}{7.70\text{ N/m}}(\sqrt{392\text{ N} + (7.70\text{ N/m})(100\text{ m})} - \sqrt{392\text{ N}}) = 3.98\text{ s}.$$

EVALUATE: If the weight of the cable and the buoyant force on the cable are neglected, then the tension would have the constant value calculated in part (a). Then $v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{392\text{ N}}{1.10\text{ kg/m}}} = 18.9\text{ m/s}$ and $t = \frac{L}{v} = 5.92\text{ s}$. The weight of the cable increases the tension along the cable and the time is reduced from this value.

15.83. **IDENTIFY:** The tension in the rope will vary with radius *r*.

SET UP: The tension at a distance *r* from the center must supply the force to keep the mass of the rope that is further out than *r* accelerating inward. The mass of this piece in $m\frac{L-r}{L}$, and its center of mass moves in a circle of radius $\frac{L+r}{2}$.

EXECUTE: $T(r) = \left[m\frac{L-r}{L}\right]\omega^2\left[\frac{L+r}{2}\right] = \frac{m\omega^2}{2L}(L^2 - r^2)$. The speed of propagation as a function of distance is $v(r) = \frac{dr}{dt} = \sqrt{\frac{T(r)}{\mu}} = \sqrt{\frac{TL}{m}} = \frac{\omega}{\sqrt{2}}\sqrt{L^2 - r^2}$, where $\frac{dr}{dt} > 0$ has been chosen for a wave traveling from the center to the edge. Separating variables and integrating, the time *t* is

$$t = \int dt = \frac{\sqrt{2}}{\omega}\int_0^L \frac{dr}{\sqrt{L^2 - r^2}}.$$

The integral may be found in a table, or in Appendix B. The integral is done explicitly by letting $r = L\sin\theta$, $dr = L\cos\theta\, d\theta$, $\sqrt{L^2 - r^2} = L\cos\theta$, so that $\int \frac{dr}{\sqrt{L^2 - r^2}} = \theta = \arcsin\frac{r}{L}$, and $t = \frac{\sqrt{2}}{\omega}\arcsin(1) = \frac{\pi}{\omega\sqrt{2}}$.

EVALUATE: An equivalent method for obtaining $T(r)$ is to consider the net force on a piece of the rope with length *dr* and mass $dm = dr\, m/L$. The tension must vary in such a way that $T(r) - T(r + dr) = -\omega^2 r\, dm$, or $\frac{dT}{dr} = -(m\omega^2/L)r\, dr$. This is integrated to obtained $T(r) = -(m\omega^2/2L)r^2 + C$, where *C* is a constant of integration. The tension must vanish at $r = L$, from which $C = (m\omega^2 L/2)$ and the previous result is obtained.

15.84. **IDENTIFY:** Carry out the calculation specified in part (a).

SET UP: $\frac{\partial y}{\partial x} = kA_{SW}\cos kx \sin\omega t$, $\frac{\partial y}{\partial t} = -\omega A_{SW}\sin kx\cos\omega t$. $\sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$.

EXECUTE: The instantaneous power is

$$P = FA_{SW}^2\omega k(\sin kx\cos kx)(\sin\omega t\cos\omega t) = \frac{1}{4}FA_{SW}^2\omega k\sin(2kx)\sin(2\omega t).$$

(b) The average value of *P* is proportional to the average value of $\sin(2\omega t)$, and the average of the sine function is zero; $P_{av} = 0$.

(c) The graphs are given in Figure 15.84. The waveform is the solid line, and the power is the dashed line. At time $t = 0$, $y = 0$ and $P = 0$ and the graphs coincide.

(d) When the standing wave is at its maximum displacement at all points, all of the energy is potential, and is concentrated at the places where the slope is steepest (the nodes). When the standing wave has zero displacement, all of the energy is kinetic, concentrated where the particles are moving the fastest (the antinodes). Thus, the energy must be transferred from the nodes to the antinodes, and back again, twice in each cycle. Note that $|P|$ is greatest midway between adjacent nodes and antinodes, and that *P* vanishes at the nodes and antinodes.

EVALUATE: There is energy flow back and forth between the nodes, but there is no net flow of energy along the string.

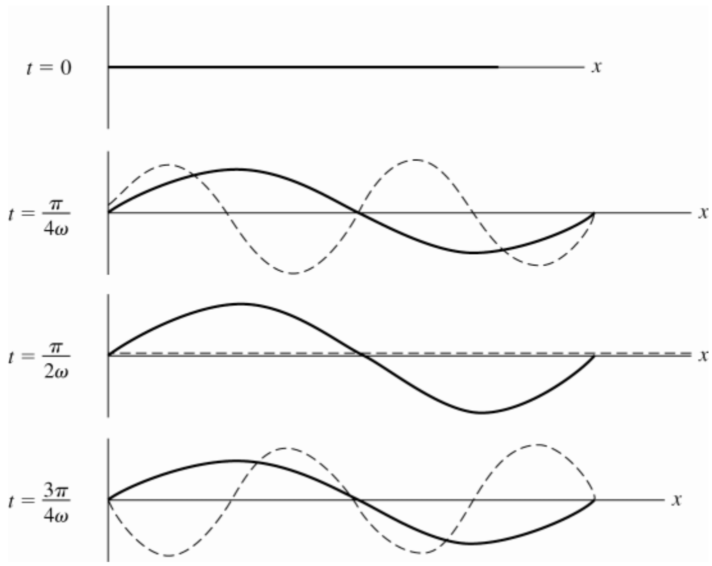


Figure 15.84

15.85. IDENTIFY: For a string, $f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$.

SET UP: For the fundamental, $n = 1$. Solving for F gives $F = \mu 4L^2 f^2$. Note that $\mu = \pi r^2 \rho$, so $\mu = \pi(0.203\times10^{-3}\text{ m})^2(7800\text{ kg/m}^3) = 1.01\times10^{-3}\text{ kg/m}$.

EXECUTE: **(a)** $F = (1.01\times10^{-3}\text{ kg/m})4(0.635\text{ m})^2(247.0\text{ Hz})^2 = 99.4\text{ N}$

(b) To find the fractional change in the frequency we must take the ratio of Δf to f: $f = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$ and

$$\Delta f = \Delta\left(\frac{1}{2L}\sqrt{\frac{F}{\mu}}\right) = \Delta\left(\frac{1}{2L\sqrt{\mu}}F^{\frac{1}{2}}\right) = \frac{1}{2L\sqrt{\mu}}\Delta\left(F^{\frac{1}{2}}\right) = \frac{1}{2L\sqrt{\mu}}\frac{1}{2}\frac{\Delta F}{\sqrt{F}}$$

Now divide both sides by the original equation for f and cancel terms: $\frac{\Delta f}{f} = \frac{\frac{1}{2L\sqrt{\mu}}\frac{1}{2}\frac{\Delta F}{\sqrt{F}}}{\frac{1}{2L}\sqrt{\frac{F}{\mu}}} = \frac{1}{2}\frac{\Delta F}{F}$.

(c) The coefficient of thermal expansion α is defined by $\Delta l = l_0\alpha\Delta T$. Combining this with $Y = \frac{F/A}{\Delta l/l_0}$ gives

$\Delta F = -Y\alpha A\Delta T$. $\Delta F = -\left(2.00\times10^{11}\text{ Pa}\right)\left(1.20\times10^{-5}/\text{C°}\right)\pi(0.203\times10^{-3}\text{ m})^2(11°\text{C}) = 3.4\text{ N}$. Then $\Delta F/F - 0.034$, $\Delta f/f = -0.017$ and $\Delta f = -4.2\text{ Hz}$. The pitch falls. This also explains the constant tuning in the string sections of symphonic orchestras.

EVALUATE: An increase in temperature causes a decrease in tension of the string, and this lowers the frequency of each standing wave.

16

SOUND AND HEARING

16.1. **IDENTIFY** and **SET UP:** Eq.(15.1) gives the wavelength in terms of the frequency. Use Eq.(16.5) to relate the pressure and displacement amplitudes.

EXECUTE: **(a)** $\lambda = v/f = (344\text{ m/s})/1000\text{ Hz} = 0.344\text{ m}$

(b) $p_{\text{max}} = BkA$ and Bk is constant gives $p_{\text{max1}}/A_1 = p_{\text{max2}}/A_2$

$$A_2 = A_1\left(\frac{p_{\text{max2}}}{p_{\text{max1}}}\right) = 1.2\times10^{-8}\text{ m}\left(\frac{30\text{ Pa}}{3.0\times10^{-2}\text{ Pa}}\right) = 1.2\times10^{-5}\text{ m}$$

(c) $p_{\text{max}} = BkA = 2\pi BA/\lambda$

$$p_{\text{max}}\lambda = 2\pi BA = \text{constant so } p_{\text{max1}}\lambda_1 = p_{\text{max2}}\lambda_2 \text{ and } \lambda_2 = \lambda_1\left(\frac{p_{\text{max1}}}{p_{\text{max2}}}\right) = (0.344\text{ m})\left(\frac{3.0\times10^{-2}\text{ Pa}}{1.5\times10^{-3}\text{ Pa}}\right) = 6.9\text{ m}$$

$f = v/\lambda = (344\text{ m/s})/6.9\text{ m} = 50\text{ Hz}$

EVALUATE: The pressure amplitude and displacement amplitude are directly proportional. For the same displacement amplitude, the pressure amplitude decreases when the frequency decreases and the wavelength increases.

16.2. **IDENTIFY:** Apply $p_{\text{max}} = BkA$ and solve for A.

SET UP: $k = \dfrac{2\pi}{\lambda}$ and $v = f\lambda$, so $k = \dfrac{2\pi f}{v}$ and $p = \dfrac{2\pi fBA}{v}$.

EXECUTE: $$A = \frac{p_{\text{max}}v}{2\pi Bf} = \frac{(3.0\times10^{-2}\text{ Pa})(1480\text{ m/s})}{2\pi(2.2\times10^{9}\text{ Pa})(1000\text{ Hz})} = 3.21\times10^{-12}\text{ m.}$$

EVALUATE: Both v and B are larger, but B is larger by a much greater factor, so v/B is a lot smaller and therefore A is a lot smaller.

16.3. **IDENTIFY:** Use Eq.(16.5) to relate the pressure and displacement amplitudes.

SET UP: As stated in Example 16.1 the adiabatic bulk modulus for air is $B = 1.42\times10^{5}\text{ Pa}$. Use Eq.(15.1) to calculate λ from f, and then $k = 2\pi/\lambda$.

EXECUTE: **(a)** $f = 150\text{ Hz}$

Need to calculate k: $\lambda = v/f$ and $k = 2\pi/\lambda$ so $k = 2\pi f/v = (2\pi\text{ rad})(150\text{ Hz})/344\text{ m/s} = 2.74\text{ rad/m}$. Then

$p_{\text{max}} = BkA = (1.42\times10^{5}\text{ Pa})(2.74\text{ rad/m})(0.0200\times10^{-3}\text{ m}) = 7.78\text{ Pa}$. This is below the pain threshold of 30 Pa.

(b) f is larger by a factor of 10 so $k = 2\pi f/v$ is larger by a factor of 10, and $p_{\text{max}} = BkA$ is larger by a factor of 10. $p_{\text{max}} = 77.8\text{ Pa}$, above the pain threshold.

(c) There is again an increase in f, k, and p_{max} of a factor of 10, so $p_{\text{max}} = 778\text{ Pa}$, far above the pain threshold.

EVALUATE: When f increases, λ decreases so k increases and the pressure amplitude increases.

16.4. **IDENTIFY:** Apply $p_{\text{max}} = BkA$. $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{v}$, so $p_{\text{max}} = \dfrac{2\pi fBA}{v}$.

SET UP: $v = 344\text{ m/s}$

EXECUTE: $$f = \frac{vp_{\text{max}}}{2\pi BA} = \frac{(344\text{ m/s})(10.0\text{ Pa})}{2\pi(1.42\times10^{5}\text{ Pa})(1.00\times10^{-6}\text{ m})} = 3.86\times10^{3}\text{ Hz}$$

EVALUATE: Audible frequencies range from about 20 Hz to about 20,000 Hz, so this frequency is audible.

16.5. **IDENTIFY:** $v = f\lambda$. Apply Eq.(16.7) for the waves in the liquid and Eq.(16.8) for the waves in the metal bar.

SET UP: In part (b) the wave speed is $v = \dfrac{d}{t} = \dfrac{1.50\text{ m}}{3.90\times10^{-4}\text{ s}}$

EXECUTE: **(a)** Using Eq.(16.7), $B = v^2\rho = (\lambda f)^2\rho$, so $B = [(8\text{ m})(400\text{ Hz})]^2(1300\text{ kg/m}^3) = 1.33\times10^{10}$ Pa.

(b) Using Eq.(16.8), $Y = v^2\rho = (L/t)^2\rho = [(1.50\text{ m})/(3.90\times10^{-4}\text{ s})]^2(6400\text{ kg/m}^3) = 9.47\times10^{10}$ Pa.

EVALUATE: In the liquid, $v = 3200$ m/s and in the metal, $v = 3850$ m/s. Both these speeds are much greater than the speed of sound in air.

16.6. **IDENTIFY:** $v = d/t$. Apply Eq.(16.7) to calculate B.

SET UP: $\rho = 3.3\times10^3\text{ kg/m}^3$

EXECUTE: **(a)** The time for the wave to travel to Caracas was $9\text{ min }39\text{ s} = 579\text{ s}$ and the speed was 1.08×10^4 m/s. Similarly, the time for the wave to travel to Kevo was 680 s for a speed of 1.28×10^4 m/s, and the time to travel to Vienna was 767 s for a speed of 1.26×10^4 m/s. The average for these three measurements is 1.21×10^4 m/s. Due to variations in density, or reflections (a subject addressed in later chapters), not all waves travel in straight lines with constant speeds.

(b) From Eq.(16.7), $B = v^2\rho$, and using the given value of $\rho = 3.3\times10^3\text{ kg/m}^3$ and the speeds found in part (a), the values for the bulk modulus are, respectively, 3.9×10^{11} Pa, 5.4×10^{11} Pa and 5.2×10^{11} Pa.

EVALUATE: These are larger, by a factor of 2 or 3, than the largest values in Table 11.1.

16.7. **IDENTIFY:** $d = vt$ for the sound waves in air an in water.

SET UP: Use $v_{\text{water}} = 1482$ m/s at 20°C, as given in Table 16.1. In air, $v = 344$ m/s.

EXECUTE: Since along the path to the diver the sound travels 1.2 m in air, the sound wave travels in water for the same time as the wave travels a distance $22.0\text{ m} - 1.20\text{ m} = 20.8\text{ m}$ in air. The depth of the diver is

$(20.8\text{ m})\dfrac{v_{\text{water}}}{v_{\text{air}}} = (20.8\text{ m})\dfrac{1482\text{ m/s}}{344\text{ m/s}} = 89.6\text{ m}$. This is the depth of the diver; the distance from the horn is 90.8 m.

EVALUATE: The time it takes the sound to travel from the horn to the person on shore is $t_1 = \dfrac{22.0\text{ m}}{344\text{ m/s}} = 0.0640$ s.

The time it takes the sound to travel from the horn to the diver is

$t_2 = \dfrac{1.2\text{ m}}{344\text{ m/s}} + \dfrac{89.6\text{ m}}{1482\text{ m/s}} = 0.0035\text{ s} + 0.0605\text{ s} = 0.0640\text{ s}$. These times are indeed the same. For three figure accuracy the distance of the horn above the water can't be neglected.

16.8. **IDENTIFY:** Apply Eq.(16.10) to each gas.

SET UP: In each case, express M in units of kg/mol. For H_2, $\gamma = 1.41$. For He and Ar, $\gamma = 1.67$.

EXECUTE: **(a)** $v_{H2} = \sqrt{\dfrac{(1.41)(8.3145\text{ J/mol}\cdot\text{K})(300.15\text{ K})}{(2.02\times10^{-3}\text{ kg/mol})}} = 1.32\times10^3\text{ m/s}$

(b) $v_{He} = \sqrt{\dfrac{(1.67)(8.3145\text{ J/mol}\cdot\text{K})(300.15\text{ K})}{(4.00\times10^{-3}\text{ kg/mol})}} = 1.02\times10^3\text{ m/s}$

(c) $v_{Ar} = \sqrt{\dfrac{(1.67)(8.3145\text{ J/mol}\cdot\text{K})(300.15\text{ K})}{(39.9\times10^{-3}\text{ kg/mol})}} = 323\text{ m/s}$.

(d) Repeating the calculation of Example 16.5 at $T = 300.15$ K gives $v_{\text{air}} = 348\text{ m/s}$, and so $v_{H_2} = 3.80v_{\text{air}}$, $v_{He} = 2.94v_{\text{air}}$ and $v_{Ar} = 0.928v_{\text{air}}$.

EVALUATE: v is larger for gases with smaller M.

16.9. **IDENTIFY:** $v = f\lambda$. The relation of v to gas temperature is given by $v = \sqrt{\dfrac{\gamma RT}{M}}$.

SET UP: Let $T = 22.0°\text{C} = 295.15$ K.

EXECUTE: At 22.0°C, $\lambda = \dfrac{v}{f} = \dfrac{325\text{ m/s}}{1250\text{ Hz}} = 0.260\text{ m} = 26.0\text{ cm}$. $\lambda = \dfrac{v}{f} = \dfrac{1}{f}\sqrt{\dfrac{\gamma RT}{M}}$. $\dfrac{\lambda}{\sqrt{T}} = \dfrac{1}{f}\sqrt{\dfrac{\gamma R}{M}}$, which is constant, so $\dfrac{\lambda_1}{\sqrt{T_1}} = \dfrac{\lambda_2}{\sqrt{T_2}}$. $T_2 = T_1\left(\dfrac{\lambda_2}{\lambda_1}\right)^2 = (295.15\text{ K})\left(\dfrac{28.5\text{ cm}}{26.0\text{ cm}}\right)^2 = 354.6\text{ K} = 81.4°\text{C}$.

EVALUATE: When T increases v increases and for fixed f, λ increases. Note that we did not need to know either γ or M for the gas.

16.10. **IDENTIFY:** $v=\sqrt{\dfrac{\gamma RT}{M}}$. Take the derivative of *v* with respect to *T*. In part (b) replace dv by Δv and dT by ΔT in the expression derived in part (a).

SET UP: $\dfrac{d(x^{1/2})}{dx}=\tfrac{1}{2}x^{-1/2}$. In Eq.(16.10), *T* must be in kelvins. $20°\text{C}=293\text{ K}$. $\Delta T=1\text{ C}°=1\text{ K}$.

EXECUTE: **(a)** $\dfrac{dv}{dT}=\sqrt{\dfrac{\gamma R}{M}}\dfrac{dT^{1/2}}{dT}=\sqrt{\dfrac{\gamma R}{M}}\tfrac{1}{2}T^{-1/2}=\dfrac{1}{2T}\sqrt{\dfrac{\gamma RT}{M}}=\dfrac{v}{2T}$. Rearranging gives $\dfrac{dv}{v}=\dfrac{1}{2}\dfrac{dT}{T}$, the desired result.

(b) $\dfrac{\Delta v}{v}=\dfrac{1}{2}\dfrac{\Delta T}{T}$. $\Delta v=\dfrac{v}{2}\dfrac{\Delta T}{T}=\left(\dfrac{344\text{ m/s}}{2}\right)\left(\dfrac{1\text{ K}}{293\text{ K}}\right)=0.59\text{ m/s}$.

EVALUATE: Since $\dfrac{\Delta T}{T}=3.4\times10^{-3}$ and $\dfrac{\Delta v}{v}$ is one-half this, replacing dT by ΔT and dv by Δv is accurate. Using the result from part (a) is much simpler than calculating *v* for $20°\text{C}$ and for $21°\text{C}$ and subtracting, and is not subject to round-off errors.

16.11. **IDENTIFY** and **SET UP:** Use $t=\text{distance/speed}$. Calculate the time it takes each sound wave to travel the $L=80.0\text{ m}$ length of the pipe. Use Eq.(16.8) to calculate the speed of sound in the brass rod.

EXECUTE: wave in air: $t=80.0\text{ m}/(344\text{ m/s})=0.2326\text{ s}$

wave in the metal: $v=\sqrt{\dfrac{Y}{\rho}}=\sqrt{\dfrac{9.0\times10^{10}\text{ Pa}}{8600\text{ kg/m}^3}}=3235\text{ m/s}$

$t=\dfrac{80.0\text{ m}}{3235\text{ m/s}}=0.0247\text{ s}$

The time interval between the two sounds is $\Delta t=0.2326\text{ s}-0.0247\text{ s}=0.208\text{ s}$

EVALUATE: The restoring forces that propagate the sound waves are much greater in solid brass than in air, so *v* is much larger in brass.

16.12. **IDENTIFY:** Repeat the calculation of Example 16.5 at each temperature.

SET UP: $27.0°\text{C}=300.15\text{ K}$ and $-13.0°\text{C}=260.15\text{ K}$

EXECUTE: $\sqrt{\dfrac{(1.40)(8.3145\text{ J/mol}\cdot\text{K})(300.15\text{ K})}{(28.8\times10^{-3}\text{ kg/mol})}}-\sqrt{\dfrac{(1.40)(8.3145\text{ J/mol}\cdot\text{K})(260.15\text{ K})}{(28.8\times10^{-3}\text{ kg/mol})}}=24\text{ m/s}$

EVALUATE: The speed is greater at the higher temperature. The difference in speeds corresponds to a 7% increase.

16.13. **IDENTIFY:** For transverse waves, $v_{\text{trans}}=\sqrt{\dfrac{F}{\mu}}$. For longitudinal waves, $v_{\text{long}}=\sqrt{\dfrac{Y}{\rho}}$.

SET UP: The mass per unit length μ is related to the density (assumed uniform) and the cross-section area *A* by $\mu=A\rho$.

EXECUTE: $v_{\text{long}}=30v_{\text{trans}}$ gives $\sqrt{\dfrac{Y}{\rho}}=30\sqrt{\dfrac{F}{\mu}}$ and $\dfrac{Y}{\rho}=900\dfrac{F}{A\rho}$. Therefore, $F/A=\dfrac{Y}{900}$.

EVALUATE: Typical values of *Y* are on the order of 10^{11} Pa, so the stress must be about 10^8 Pa. If *A* is on the order of $1\text{ mm}^2=10^{-6}\text{ m}^2$, this requires a force of about 100 N.

16.14. **IDENTIFY:** The intensity *I* is given in terms of the displacement amplitude by Eq.(16.12) and in terms of the pressure amplitude by Eq.(16.14). $\omega=2\pi f$. Intensity is energy per second per unit area.

SET UP: For part (a), $I=10^{-12}\text{ W/m}^2$. For part (b), $I=3.2\times10^{-3}\text{ W/m}^2$.

EXECUTE: **(a)** $I=\tfrac{1}{2}\sqrt{\rho B}\,\omega^2A^2$.

$A=\dfrac{1}{\omega}\sqrt{\dfrac{2I}{\sqrt{\rho B}}}=\dfrac{1}{2\pi(1000\text{ Hz})}\sqrt{\dfrac{2(1\times10^{-12}\text{ W/m}^2)}{\sqrt{(1.20\text{ kg/m}^3)(1.42\times10^5\text{ Pa})}}}=1.1\times10^{-11}\text{ m}$. $I=\dfrac{p_{\text{max}}^2}{2\sqrt{\rho B}}$.

$p_{\text{max}}=\sqrt{2I\sqrt{\rho B}}=\sqrt{2(1\times10^{-12}\text{ W/m}^2)\sqrt{(1.20\text{ kg/m}^3)(1.42\times10^5\text{ Pa})}}=2.9\times10^{-5}\text{ Pa}=2.8\times10^{-10}\text{ atm}$

(b) *A* is proportional to $\sqrt{I}$, so $A=(1.1\times10^{-11}\text{ m})\sqrt{\dfrac{3.2\times10^{-3}\text{ W/m}^2}{1\times10^{-12}\text{ W/m}^2}}=6.2\times10^{-7}\text{ m}$. p_{max} is also proportional to $\sqrt{I}$, so $p_{\text{max}}=(2.9\times10^{-5}\text{ Pa})\sqrt{\dfrac{3.2\times10^{-3}\text{ W/m}^2}{1\times10^{-12}\text{ W/m}^2}}=1.6\text{ Pa}=1.6\times10^{-5}\text{ atm}$.

(c) area $= (5.00 \text{ mm})^2 = 2.5\times10^{-5}\ \text{m}^2$. Part (a): $(1\times10^{-12}\ \text{W/m}^2)(2.5\times10^{-5}\ \text{m}^2) = 2.5\times10^{-17}\ \text{J/s}$.

Part (b): $(3.2\times10^{-3}\ \text{W/m}^2)(2.5\times10^{-5}\ \text{m}^2) = 8.0\times10^{-8}\ \text{J/s}$.

EVALUATE: For faint sounds the displacement and pressure variation amplitudes are very small. Intensities for audible sounds vary over a very wide range.

16.15. IDENTIFY: Apply Eq.(16.12) and solve for A. $\lambda = v/f$, with $v = \sqrt{B/\rho}$.

SET UP: $\omega = 2\pi f$. For air, $B = 1.42\times10^5$ Pa.

EXECUTE: **(a)** The amplitude is

$$A = \sqrt{\frac{2I}{\sqrt{\rho B}\omega^2}} = \sqrt{\frac{2(3.00\times10^{-6}\ \text{W/m}^2)}{\sqrt{(1000\ \text{kg/m}^3)(2.18\times10^9\ \text{Pa})}(2\pi(3400\ \text{Hz}))^2}} = 9.44\times10^{-11}\ \text{m}.$$

The wavelength is $\lambda = \dfrac{v}{f} = \dfrac{\sqrt{B/\rho}}{f} = \dfrac{\sqrt{(2.18\times10^9\ \text{Pa})/(1000\ \text{kg/m}^3)}}{3400\ \text{Hz}} = 0.434\ \text{m}$.

(b) Repeating the above with $B = 1.42\times10^5\,\text{Pa}$ and the density of air gives $A = 5.66\times10^{-9}$ m and $\lambda = 0.100$ m.

EVALUATE: **(c)** The amplitude is larger in air, by a factor of about 60. For a given frequency, the much less dense air molecules must have a larger amplitude to transfer the same amount of energy.

16.16. IDENTIFY and **SET UP:** Use Eq.(16.7) to eliminate either v or B in $I = \dfrac{vp_{\text{max}}^2}{2B}$.

EXECUTE: From Eq. (19.21), $v^2 = B/\rho$. Using Eq.(16.7) to eliminate $v, I = \left(\sqrt{B/\rho}\right)p_{\text{max}}^2/2B = p_{\text{max}}^2/2\sqrt{\rho B}$.

Using Eq. (16.7) to eliminate B, $I = vp_{\text{max}}^2/2(v^2\rho) = p_{\text{max}}^2/2\rho v$.

EVALUATE: The equation in this form shows the dependence of I on the density of the material in which the wave propagates.

16.17. IDENTIFY and **SET UP:** Apply Eqs.(16.5), (16.11) and (16.15).

EXECUTE: **(a)** $\omega = 2\pi f = (2\pi\ \text{rad})(150\ \text{Hz}) = 942.5\ \text{rad/s}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{\omega}{v} = \frac{942.5\ \text{rad/s}}{344\ \text{m/s}} = 2.74\ \text{rad/m}$$

$B = 1.42\times10^5$ Pa (Example 16.1)

Then $p_{\text{max}} = BkA = (1.42\times10^5\ \text{Pa})(2.74\ \text{rad/m})(5.00\times10^{-6}\ \text{m}) = 1.95\ \text{Pa}$.

(b) Eq.(16.11): $I = \frac{1}{2}\omega BkA^2$

$I = \frac{1}{2}(942.5\ \text{rad/s})(1.42\times10^5\ \text{Pa})(27.4\ \text{rad/m})(5.00\times10^{-6}\ \text{m})^2 = 4.58\times10^{-3}\ \text{W/m}^2$.

(c) Eq.(16.15): $\beta = (10\ \text{dB})\log(I/I_0)$, with $I_0 = 1\times10^{-12}\ \text{W/m}^2$.

$\beta = (10\ \text{dB})\log\left((4.58\times10^{-3}\ \text{W/m}^2)/(1\times10^{-12}\ \text{W/m}^2)\right) = 96.6\ \text{dB}$.

EVALUATE: Even though the displacement amplitude is very small, this is a very intense sound. Compare the sound intensity level to the values in Table 16.2.

16.18. IDENTIFY: Apply $\beta = (10\ \text{dB})\log(I/I_0)$. In part (b), use Eq.(16.14) to calculate I from the information that is given.

SET UP: $I_0 = 10^{-12}\ \text{W/m}^2$. From Table 16.1 the speed of sound in air at $20.0°\text{C}$ is 344 m/s. The density of air at that temperature is $1.20\,\text{kg/m}^3$.

EXECUTE: **(a)** $\beta = (10\ \text{dB})\log\left(\dfrac{0.500\ \mu\text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right) = 57\ \text{dB}$.

(b) $I = \dfrac{p_{\text{max}}^2}{2\rho v} = \dfrac{(0.150\ \text{N/m}^2)^2}{2(1.20\ \text{kg/m}^3)(344\ \text{m/s})} = 2.73\times10^{-5}\ \text{W/m}^2$. Using this in Equation (16.15),

$$\beta = (10\ \text{dB})\log\frac{2.73\times10^{-5}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2} = 74.4\ \text{dB}.$$

EVALUATE: As expected, the sound intensity is larger for the jack hammer.

16.19. IDENTIFY: Use Eq.(16.13) to relate I and p_{max}. $\beta = (10\ \text{dB})\log(I/I_0)$. Eq.(16.4) says the pressure amplitude and displacement amplitude are related by $p_{\text{max}} = BkA = B\left(\dfrac{2\pi f}{v}\right)A$.

SET UP: At $20°\text{C}$ the bulk modulus for air is 1.42×10^5 Pa and $v = 344$ m/s. $I_0 = 1\times10^{-12}\ \text{W/m}^2$.

EXECUTE: **(a)** $I = \dfrac{vp_{\max}^2}{2B} = \dfrac{(344\text{ m/s})(6.0\times10^{-5}\text{ Pa})^2}{2(1.42\times10^5\text{ Pa})} = 4.4\times10^{-12}\text{ W/m}^2$

(b) $\beta = (10\text{ dB})\log\left(\dfrac{4.4\times10^{-12}\text{ W/m}^2}{1\times10^{-12}\text{ W/m}^2}\right) = 6.4\text{ dB}$

(c) $A = \dfrac{vp_{\max}}{2\pi fB} = \dfrac{(344\text{ m/s})(6.0\times10^{-5}\text{ Pa})}{2\pi(400\text{ Hz})(1.42\times10^5\text{ Pa})} = 5.8\times10^{-11}\text{ m}$

EVALUATE: This is a very faint sound and the displacement and pressure amplitudes are very small. Note that the displacement amplitude depends on the frequency but the pressure amplitude does not.

16.20. **IDENTIFY** and **SET UP:** Apply the relation $\beta_2 - \beta_1 = (10\text{ dB})\log(I_2/I_1)$ that is derived in Example 16.10.

EXECUTE: **(a)** $\Delta\beta = (10\text{ dB})\log\left(\dfrac{4I}{I}\right) = 6.0\text{ dB}$

(b) The total number of crying babies must be multiplied by four, for an increase of 12 kids.

EVALUATE: For $I_2 = \alpha I_1$, where α is some factor, the increase in sound intensity level is $\Delta\beta = (10\text{ dB})\log\alpha$. For $\alpha = 4$, $\Delta\beta = 6.0\text{ dB}$.

16.21. **IDENTIFY** and **SET UP:** Let 1 refer to the mother and 2 to the father. Use the result derived in Example 16.11 for the difference in sound intensity level for the two sounds. Relate intensity to distance from the source using Eq.(15.26).

EXECUTE: From Example 16.11, $\beta_2 - \beta_1 = (10\text{ dB})\log(I_2/I_1)$

Eq.(15.26): $I_1/I_2 = r_2^2/r_1^2$ or $I_2/I_1 = r_1^2/r_2^2$

$\Delta\beta = \beta_2 - \beta_1 = (10\text{ dB})\log(I_2/I_1) = (10\text{ dB})\log(r_1/r_2)^2 = (20\text{ dB})\log(r_1/r_2)$

$\Delta\beta = (20\text{ dB})\log(1.50\text{ m}/0.30\text{ m}) = 14.0\text{ dB}.$

EVALUATE: The father is 5 times closer so the intensity at his location is 25 times greater.

16.22. **IDENTIFY:** $\beta = (10\text{ dB})\log\dfrac{I}{I_0}$. $\beta_2 - \beta_1 = (10\text{ dB})\log\dfrac{I_2}{I_1}$. Solve for $\dfrac{I_2}{I_1}$.

SET UP: If $\log y = x$ then $y = 10^x$. Let $\beta_2 = 70\text{ dB}$ and $\beta_1 = 95\text{ dB}$.

EXECUTE: $70.0\text{ dB} - 95.0\text{ dB} = -25.0\text{ dB} = (10\text{ dB})\log\dfrac{I_2}{I_1}$. $\log\dfrac{I_2}{I_1} = -2.5$ and $\dfrac{I_2}{I_1} = 10^{-2.5} = 3.2\times10^{-3}$.

EVALUATE: $I_2 < I_1$ when $\beta_2 < \beta_1$.

16.23. **(a) IDENTIFY** and **SET UP:** From Example 16.11 $\Delta\beta = (10\text{ dB})\log(I_2/I_1)$

Set $\Delta\beta = 13.0\text{ dB}$ and solve for I_2/I_1.

EXECUTE: $13.0\text{ dB} = 10\text{ dB}\log(I_2/I_1)$ so $1.3 = \log(I_2/I_1)$ and $I_2/I_1 = 20.0$.

(b) EVALUATE: According to the equation in part (a) the difference in two sound intensity levels is determined by the ratio of the sound intensities. So you don't need to know I_1, just the ratio I_2/I_1.

16.24. **IDENTIFY:** For an open pipe, $f_1 = \dfrac{v}{2L}$. For a stopped pipe, $f_1 = \dfrac{v}{4L}$. $v = f\lambda$.

SET UP: $v = 344\text{ m/s}$. For a pipe, there must be a displacement node at a closed end and an antinode at the open end.

EXECUTE: **(a)** $L = \dfrac{v}{2f_1} = \dfrac{344\text{ m/s}}{2(594\text{ Hz})} = 0.290\text{ m}.$

(b) There is a node at one end, an antinode at the other end and no other nodes or antinodes in between, so $\dfrac{\lambda_1}{4} = L$ and $\lambda_1 = 4L = 4(0.290\text{ m}) = 1.16\text{ m}.$

(c) $f_1 = \dfrac{v}{4L} = \dfrac{1}{2}\left(\dfrac{v}{2L}\right) = \dfrac{1}{2}(594\text{ Hz}) = 297\text{ Hz}.$

EVALUATE: We could also calculate f_1 for the stopped pipe as $f_1 = \dfrac{v}{\lambda_1} = \dfrac{344\text{ m/s}}{1.16\text{ m}} = 297\text{ Hz}$, which agrees with our result in part (a).

16.25. **IDENTIFY** and **SET UP:** An open end is a displacement antinode and a closed end is a displacement node. Sketch the standing wave pattern and use the sketch to relate the node-to-antinode distance to the length of the pipe. A displacement node is a pressure antinode and a displacement antinode is a pressure node.

EXECUTE: **(a)** The placement of the displacement nodes and antinodes along the pipe is as sketched in Figure 16.25a. The open ends are displacement antinodes.

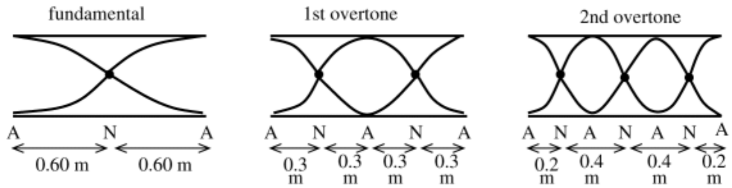


Figure 16.25a

Location of the displacement nodes (N) measured from the left end:
fundamental 0.60 m
1st overtone 0.30 m, 0.90 m
2nd overtone 0.20 m, 0.60 m, 1.00 m

Location of the pressure nodes (displacement antinodes (A)) measured from the left end:
fundamental 0, 1.20 m
1st overtone 0, 0.60 m, 1.20 m
2nd overtone 0, 0.40 m, 0.80 m, 1.20 m

(b) The open end is a displacement antinode and the closed end is a displacement node. The placement of the displacement nodes and antinodes along the pipe is sketched in Figure 16.25b.

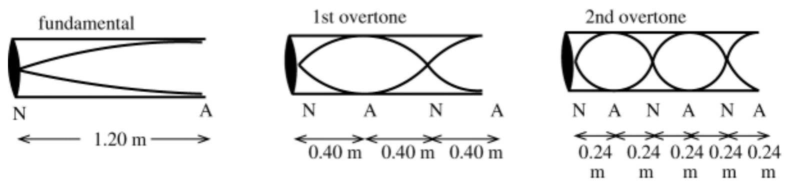


Figure 16.25b

Location of the displacement nodes (N) measured from the closed end:
fundamental 0
1st overtone 0, 0.80 m
2nd overtone 0, 0.48 m, 0.96 m

Location of the pressure nodes (displacement antinodes (A)) measured from the closed end:
fundamental 1.20 m
1st overtone 0.40 m, 1.20 m
2nd overtone 0.24 m, 0.72 m, 1.20 m

EVALUATE: The node-to-node or antinode-to-antinode distance is $\lambda/2$. For the higher overtones the frequency is higher and the wavelength is smaller.

16.26. **IDENTIFY:** A pipe closed at one end is a stopped pipe. Apply Eqs.(16.18) and (16.22) to find the frequencies and Eqs.(16.19) and (16.23) to find the highest audible harmonic in each case.

SET UP: For the open pipe $n = 1,\ 2,$ and 3 for the first three harmonics and for the stopped pipe $n = 1,\ 3,$ and 5.

EXECUTE: **(a)** $f_1 = \dfrac{v}{2L}$ and $f_n = nf_1$.

$$f_1 = \frac{344 \text{ m/s}}{2(0.450 \text{ m})} = 382 \text{ Hz}. \quad f_2 = 764 \text{ Hz}, \quad f_3 = 1146 \text{ Hz}, \quad f_4 = 1528 \text{ Hz}$$

(b) $f_1 = \dfrac{v}{4L}$ and $f_n = nf_1$, $n = 1,\ 3,\ 5,\ \ldots$.

$$f_1 = \frac{344 \text{ m/s}}{4(0.450 \text{ m})} = 191 \text{ Hz}. \quad f_3 = 573 \text{ Hz}, \quad f_5 = 955 \text{ Hz}, \quad f_7 = 1337 \text{ Hz}$$

(c) open pipe: $n = \dfrac{f}{f_1} = \dfrac{20{,}000 \text{ Hz}}{382 \text{ Hz}} = 52.$ closed pipe: $\dfrac{f}{f_1} = \dfrac{20{,}000 \text{ Hz}}{191 \text{ Hz}} = 104.$ But only odd *n* are present, so $n = 103$.

EVALUATE: For an open pipe all harmonics are present. For a stopped pipe only odd harmonics are present. For pipes of a given length, f_1 for a stopped pipe is half what it is for an open pipe.

16.27. **IDENTIFY:** For a stopped pipe, the standing wave frequencies are given by Eq.(16.22).

SET UP: The first three standing wave frequencies correspond to $n = 1,\ 3$ and 5.

EXECUTE: $f_1 = \dfrac{(344\text{ m/s})}{4(0.17\text{ m})} = 506\text{ Hz},\ f_3 = 3f_1 = 1517\text{ Hz},\ f_5 = 5f_1 = 2529\text{ Hz}.$

EVALUATE: All three of these frequencies are in the audible range, which is about 20 Hz to 20,000 Hz.

16.28. **IDENTIFY:** Model the auditory canal as a stopped pipe of length $L = 2.40$ cm. For a stopped pipe, $\lambda_1 = 4L$, $f_1 = \dfrac{v}{4L}$ and $f_n = nf_1,\ n = 1,\ 3,\ 5,\ \ldots.$

SET UP: Take the highest audible frequency to be 20,000 Hz. $v = 344$ m/s.

EXECUTE: **(a)** $f_1 = \dfrac{v}{4L} = \dfrac{344\text{ m/s}}{4(0.0240\text{ m})} = 3.58\times10^3\text{ Hz}.$ $\lambda_1 = 4L = 4(0.0240\text{ m}) = 0.0960\text{ m}.$ This frequency is audible.

(b) For $f = 20{,}000$ Hz, $\dfrac{f}{f_1} = \dfrac{20{,}000\text{ Hz}}{3580\text{ Hz}} = 5.6;$ the highest harmonic which is audible is for $n = 5$ (fifth harmonic). $f_5 = 5f_1 = 1.79\times10^4$ Hz.

EVALUATE: For a stopped pipe there are no even harmonics.

16.29. **IDENTIFY:** For either type of pipe, stopped or open, the fundamental frequency is proportional to the wave speed v. The wave speed is given in turn by Eq.(16.10).

SET UP: For He, $\gamma = 5/3$ and for air, $\gamma = 7/5$.

EXECUTE: **(a)** The fundamental frequency is proportional to the square root of the ratio $\dfrac{\gamma}{M}$, so

$$f_{\text{He}} = f_{\text{air}}\sqrt{\frac{\gamma_{\text{He}}}{\gamma_{\text{air}}}\cdot\frac{M_{\text{air}}}{M_{\text{He}}}} = (262\text{ Hz})\sqrt{\frac{(5/3)}{(7/5)}\cdot\frac{28.8}{4.00}} = 767\text{ Hz}.$$

(b) No. In either case the frequency is proportional to the speed of sound in the gas.

EVALUATE: The frequency is much higher for helium, since the rms speed is greater for helium.

16.30. **IDENTIFY:** There must be a node at each end of the pipe. For the fundamental there are no additional nodes and each successive overtone has one additional node. $v = f\lambda$.

SET UP: $v = 344$ m/s. The node to node distance is $\lambda/2$.

EXECUTE: **(a)** $\dfrac{\lambda_1}{2} = L$ so $\lambda_1 = 2L$. Each successive overtone adds an additional $\lambda/2$ along the pipe, so $n\left(\dfrac{\lambda_n}{2}\right) = L$ and $\lambda_n = \dfrac{2L}{n}$, where $n = 1,\ 2,\ 3,\ \ldots.$ $f_n = \dfrac{v}{\lambda_n} = \dfrac{nv}{2L}$.

(b) $f_1 = \dfrac{v}{2L} = \dfrac{344\text{ m/s}}{2(2.50\text{ m})} = 68.8\text{ Hz}.$ $f_2 = 2f_1 = 138\text{ Hz}.$ $f_3 = 3f_1 = 206\text{ Hz}.$ All three of these frequencies are audible.

EVALUATE: A pipe of length L closed at both ends has the same standing wave wavelengths, frequencies and nodal patterns as for a string of length L that is fixed at both ends.

16.31. **IDENTIFY** and **SET UP:** Use the standing wave pattern to relate the wavelength of the standing wave to the length of the air column and then use Eq.(15.1) to calculate f. There is a displacement antinode at the top (open) end of the air column and a node at the bottom (closed) end, as shown in Figure 16.31

EXECUTE: **(a)**

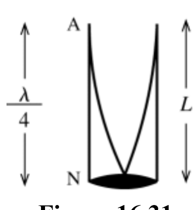


Figure 16.31

$\lambda/4 = L$

$\lambda = 4L = 4(0.140\text{ m}) = 0.560\text{ m}$

$f = \dfrac{v}{\lambda} = \dfrac{344\text{ m/s}}{0.560\text{ m}} = 614\text{ Hz}$

(b) Now the length L of the air column becomes $\frac{1}{2}(0.140\text{ m}) = 0.070\text{ m}$ and $\lambda = 4L = 0.280$ m.

$f = \dfrac{v}{\lambda} = \dfrac{344\text{ m/s}}{0.280\text{ m}} = 1230\text{ Hz}$

EVALUATE: Smaller L means smaller λ which in turn corresponds to larger f.

16.32. **IDENTIFY:** The wire will vibrate in its second overtone with frequency f_3^{wire} when $f_3^{\text{wire}} = f_1^{\text{pipe}}$. For a stopped pipe, $f_1^{\text{pipe}} = \dfrac{v}{4L_{\text{pipe}}}$. The first overtone standing wave frequency for a wire fixed at both ends is $f_3^{\text{wire}} = 3\left(\dfrac{v_{\text{wire}}}{2L_{\text{wire}}}\right)$. $v_{\text{wire}} = \sqrt{F/\mu}$.

SET UP: The wire has $\mu = \dfrac{m}{L_{\text{wire}}} = \dfrac{7.25\times10^{-3}\text{ kg}}{0.850\text{ m}} = 8.53\times10^{-3}$ kg/m. The speed of sound in air is $v = 344$ m/s.

EXECUTE: $v_{\text{wire}} = \sqrt{\dfrac{4110\text{ N}}{8.53\times10^{-3}\text{ kg/m}}} = 694$ m/s. $f_3^{\text{wire}} = f_1^{\text{pipe}}$ gives $3\dfrac{v_{\text{wire}}}{2L_{\text{wire}}} = \dfrac{v}{4L_{\text{pipe}}}$.

$$L_{\text{pipe}} = \frac{2L_{\text{wire}}v}{12v_{\text{wire}}} = \frac{2(0.850\text{ m})(344\text{ m/s})}{12(694\text{ m/s})} = 0.0702\text{ m} = 7.02\text{ cm}.$$

EVALUATE: The fundamental for the pipe has the same frequency as the third harmonic of the wire. But the wave speeds for the two objects are different and the two standing waves have different wavelengths.

16.33.

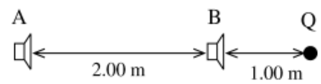


Figure 16.33

(a) IDENTIFY and **SET UP:** Path difference from points A and B to point Q is $3.00\text{ m} - 1.00\text{ m} = 2.00\text{ m}$, as shown in Figure 16.33. Constructive interference implies path difference $= n\lambda$, $n = 1, 2, 3, \ldots$

EXECUTE: $2.00\text{ m} = n\lambda$ so $\lambda = 2.00\text{ m}/n$

$$f = \frac{v}{\lambda} = \frac{nv}{2.00\text{ m}} = \frac{n(344\text{ m/s})}{2.00\text{ m}} = n(172\text{ Hz,})\quad n = 1, 2, 3, \ldots$$

The lowest frequency for which constructive interference occurs is 172 Hz.

(b) IDENTIFY and **SET UP:** Destructive interference implies path difference $= (n/2)\lambda$, $n = 1, 3, 5, \ldots$

EXECUTE: $2.00\text{ m} = (n/2)\lambda$ so $\lambda = 4.00\text{ m}/n$

$$f = \frac{v}{\lambda} = \frac{nv}{4.00\text{ m}} = \frac{n(344\text{ m/s})}{(4.00\text{ m})} = n(86\text{ Hz}),\quad n = 1, 3, 5, \ldots.$$

The lowest frequency for which destructive interference occurs is 86 Hz.

EVALUATE: As the frequency is slowly increased, the intensity at Q will fluctuate, as the interference changes between destructive and constructive.

16.34. **IDENTIFY:** Constructive interference occurs when the difference of the distances of each source from point P is an integer number of wavelengths. The interference is destructive when this difference of path lengths is a half integer number of wavelengths.

SET UP: The wavelength is $\lambda = v/f = (344\text{ m/s})/(206\text{ Hz}) = 1.67\text{ m}$. Since P is between the speakers, x must be in the range 0 to L, where $L = 2.00$ m is the distance between the speakers.

EXECUTE: The difference in path length is $\Delta l = (L - x) - x = L - 2x$, or $x = (L - \Delta l)/2$. For destructive interference, $\Delta l = (n + (1/2))\lambda$, and for constructive interference, $\Delta l = n\lambda$.

(a) Destructive interference: $n = 0$ gives $\Delta l = 0.835$ m and $x = 0.58$ m. $n = 1$ gives $\Delta l = -0.835$ m and $x = 1.42$ m. No other values of n place P between the speakers.

(b) Constructive interference: $n = 0$ gives $\Delta l = 0$ and $x = 1.00$ m. $n = 1$ gives $\Delta l = 1.67$ m and $x = 0.17$ m. $n = -1$ gives $\Delta l = -1.67$ m and $x = 1.83$ m. No other values of n place P between the speakers.

(c) Treating the speakers as point sources is a poor approximation for these dimensions, and sound reaches these points after reflecting from the walls, ceiling, and floor.

EVALUATE: Points of constructive interference are a distance $\lambda/2$ apart, and the same is true for the points of destructive interference.

16.35. **IDENTIFY:** For constructive interference the path difference is an integer number of wavelengths and for destructive interference the path difference is a half-integer number of wavelengths.

SET UP: $\lambda = v/f = (344\text{ m/s})/(688\text{ Hz}) = 0.500$ m

EXECUTE: To move from constructive interference to destructive interference, the path difference must change by $\lambda/2$. If you move a distance x toward speaker B, the distance to B gets shorter by x and the distance to A gets longer by x so the path difference changes by $2x$. $2x = \lambda/2$ and $x = \lambda/4 = 0.125$ m.

EVALUATE: If you walk an additional distance of 0.125 m farther, the interference again becomes constructive.

16.36. **IDENTIFY:** Destructive interference occurs when the path difference is a half integer number of wavelengths.

SET UP: $v = 344\ \text{m/s}$, so $\lambda = v/f = (344\ \text{m/s})/(172\ \text{Hz}) = 2.00\ \text{m}$. If $r_A = 8.00\ \text{m}$ and r_B are the distances of the person from each speaker, the condition for destructive interference is $r_B - r_A = \left(n + \frac{1}{2}\right)\lambda$, where *n* is any integer.

EXECUTE: Requiring $r_B = r_A + \left(n + \frac{1}{2}\right)\lambda > 0$ gives $n + \frac{1}{2} > -r_A/\lambda = -(8.00\ \text{m})/(2.00\ \text{m}) = -4$, so the smallest value of r_B occurs when $n = -4$, and the closest distance to *B* is $r_B = 8.00\ \text{m} + \left(-4 + \frac{1}{2}\right)(2.00\ \text{m}) = 1.00\ \text{m}$.

EVALUATE: For $r_B = 1.00\ \text{m}$, the path difference is $r_A - r_B = 7.00\ \text{m}$. This is 3.5λ.

16.37. **IDENTIFY:** Compare the path difference to the wavelength.

SET UP: $\lambda = v/f = (344\ \text{m/s})/(860\ \text{Hz}) = 0.400\ \text{m}$

EXECUTE: The path difference is $13.4\ \text{m} - 12.0\ \text{m} = 1.4\ \text{m}$. $\dfrac{\text{path difference}}{\lambda} = 3.5$. The path difference is a half-integer number of wavelengths, so the interference is destructive.

EVALUATE: The interference is destructive at any point where the path difference is a half-integer number of wavelengths.

16.38. **IDENTIFY:** $f_{\text{beat}} = |f_1 - f_2|$. $v = f\lambda$.

SET UP: $v = 344\ \text{m/s}$, Let $\lambda_1 = 6.50\ \text{cm}$ and $\lambda_2 = 6.52\ \text{cm}$. $\lambda_2 > \lambda_1$ so $f_1 > f_2$.

EXECUTE: $f_1 - f_2 = v\left(\dfrac{1}{\lambda_1} - \dfrac{1}{\lambda_2}\right) = \dfrac{v(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2} = \dfrac{(344\ \text{m/s})(0.02\times 10^{-2}\ \text{m})}{(6.50\times 10^{-2}\ \text{m})(6.52\times 10^{-2}\ \text{m})} = 16\ \text{Hz}$. There are 16 beats per second.

EVALUATE: We could have calculated f_1 and f_2 and subtracted, but doing it this way we would have to be careful to retain enough figures in intermediate calculations to avoid round-off errors.

16.39. **IDENTIFY:** $f_{\text{beat}} = |f_a - f_b|$. For a stopped pipe, $f_1 = \dfrac{v}{4L}$.

SET UP: $v = 344\ \text{m/s}$. Let $L_a = 1.14\ \text{m}$ and $L_b = 1.16\ \text{m}$. $L_b > L_a$ so $f_{1a} > f_{1b}$.

EXECUTE: $f_{1a} - f_{1b} = \dfrac{v}{4}\left(\dfrac{1}{L_a} - \dfrac{1}{L_b}\right) = \dfrac{v(L_b - L_a)}{4L_aL_b} = \dfrac{(344\ \text{m/s})(2.00\times 10^{-2}\ \text{m})}{4(1.14\ \text{m})(1.16\ \text{m})} = 1.3\ \text{Hz}$. There are 1.3 beats per second.

EVALUATE: Increasing the length of the pipe increases the wavelength of the fundamental and decreases the frequency.

16.40. **IDENTIFY:** $f_{\text{beat}} = |f - f_0|$. $f = \dfrac{v}{2L}$. Changing the tension changes the wave speed and this alters the frequency.

SET UP: $v = \sqrt{\dfrac{FL}{m}}$ so $f = \dfrac{1}{2}\sqrt{\dfrac{F}{mL}}$, where $F = F_0 + \Delta F$. Let $f_0 = \dfrac{1}{2}\sqrt{\dfrac{F_0}{mL}}$. We can assume that $\Delta F/F_0$ is very small. Increasing the tension increases the frequency, so $f_{\text{beat}} = f - f_0$.

EXECUTE: **(a)** $f_{\text{beat}} = f - f_0 = \dfrac{1}{2\sqrt{mL}}\left(\sqrt{F_0 + \Delta F} - \sqrt{F_0}\right) = \dfrac{1}{2}\sqrt{\dfrac{F_0}{mL}}\left(\left[1 + \dfrac{\Delta F}{F_0}\right]^{1/2} - 1\right)$. $\left[1 + \dfrac{\Delta F}{F_0}\right]^{1/2} = 1 + \dfrac{\Delta F}{2F_0}$ when $\Delta F/F_0$ is small. This gives that $f_{\text{beat}} = f_0\left(\dfrac{\Delta F}{2F_0}\right)$.

(b) $\dfrac{\Delta F}{F_0} = \dfrac{2f_{\text{beat}}}{f_0} = \dfrac{2(1.5\ \text{Hz})}{440\ \text{Hz}} = 0.68\%$.

EVALUATE: The fractional change in frequency is one-half the fractional change in tension.

16.41. **IDENTIFY:** Apply the Doppler shift equation $f_{\text{L}} = \left(\dfrac{v + v_{\text{L}}}{v + v_{\text{S}}}\right)f_{\text{S}}$.

SET UP: The positive direction is from listener to source. $f_{\text{S}} = 1200\ \text{Hz}$. $f_{\text{L}} = 1240\ \text{Hz}$.

EXECUTE: $v_{\text{L}} = 0$. $v_{\text{S}} = -25.0\ \text{m/s}$. $f_{\text{L}} = \left(\dfrac{v}{v + v_{\text{S}}}\right)f_{\text{S}}$ gives $v = \dfrac{v_{\text{S}}f_{\text{L}}}{f_{\text{S}} - f_{\text{L}}} = \dfrac{(-25\ \text{m/s})(1240\ \text{Hz})}{1200\ \text{Hz} - 1240\ \text{Hz}} = 780\ \text{m/s}$.

EVALUATE: $f_{\text{L}} > f_{\text{S}}$ since the source is approaching the listener.

16.42. **IDENTIFY:** Follow the steps of Example 16.19.

SET UP: In the first step, $v_S = +20.0 \text{ m/s}$ instead of -30.0 m/s. In the second step, $v_L = -20.0 \text{ m/s}$ instead of $+30.0 \text{ m/s}$.

EXECUTE: $f_W = \left(\dfrac{v}{v+v_S}\right) f_S = \left(\dfrac{340 \text{ m/s}}{340 \text{ m/s} + 20.0 \text{ m/s}}\right)(300 \text{ Hz}) = 283 \text{ Hz}$. Then

$f_L = \left(\dfrac{v+v_L}{v}\right) f_W = \left(\dfrac{340 \text{ m/s} - 20.0 \text{ m/s}}{340 \text{ m/s}}\right)(283 \text{ Hz}) = 266 \text{ Hz}$.

EVALUATE: When the car is moving toward the reflecting surface, the received frequency back at the source is higher than the emitted frequency. When the car is moving away from the reflecting surface, as is the case here, the received frequency back at the source is lower than the emitted frequency.

16.43. **IDENTIFY:** Apply the Doppler shift equation $f_L = \left(\dfrac{v+v_L}{v+v_S}\right) f_S$.

SET UP: The positive direction is from listener to source. $f_S = 392 \text{ Hz}$.

(a) $v_S = 0$. $v_L = -15.0 \text{ m/s}$. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right) f_S = \left(\dfrac{344 \text{ m/s} - 15.0 \text{ m/s}}{344 \text{ m/s}}\right)(392 \text{ Hz}) = 375 \text{ Hz}$

(b) $v_S = +35.0 \text{ m/s}$. $v_L = +15.0 \text{ m/s}$. $f_L = \left(\dfrac{v+v_L}{v+v_S}\right) f_S = \left(\dfrac{344 \text{ m/s} + 15.0 \text{ m/s}}{344 \text{ m/s} + 35.0 \text{ m/s}}\right)(392 \text{ Hz}) = 371 \text{ Hz}$

(c) $f_{\text{beat}} = f_1 - f_2 = 4 \text{ Hz}$

EVALUATE: The distance between whistle *A* and the listener is increasing, and for whistle *A* $f_L < f_S$. The distance between whistle *B* and the listener is also increasing, and for whistle *B* $f_L < f_S$.

16.44. **IDENTIFY** and **SET UP:** Apply Eqs.(16.27) and (16.28) for the wavelengths in front of and behind the source. Then $f = v/\lambda$. When the source is at rest $\lambda = \dfrac{v}{f_S} = \dfrac{344 \text{ m/s}}{400 \text{ Hz}} = 0.860 \text{ m}$.

EXECUTE: **(a)** Eq.(16.27): $\lambda = \dfrac{v - v_S}{f_S} = \dfrac{344 \text{ m/s} - 25.0 \text{ m/s}}{400 \text{ Hz}} = 0.798 \text{ m}$

(b) Eq.(16.28): $\lambda = \dfrac{v + v_S}{f_S} = \dfrac{344 \text{ m/s} + 25.0 \text{ m/s}}{400 \text{ Hz}} = 0.922 \text{ m}$

(c) $f_L = v/\lambda$ (since $v_L = 0$), so $f_L = (344 \text{ m/s})/0.798 \text{ m} = 431 \text{ Hz}$

(d) $f_L = v/\lambda = (344 \text{ m/s})/0.922 \text{ m} = 373 \text{ Hz}$

EVALUATE: In front of the source (source moving toward listener) the wavelength is decreased and the frequency is increased. Behind the source (source moving away from listener) the wavelength is increased and the frequency is decreased.

16.45. **IDENTIFY:** The distance between crests is λ. In front of the source $\lambda = \dfrac{v - v_S}{f_S}$ and behind the source $\lambda = \dfrac{v + v_S}{f_S}$. $f_S = 1/T$.

SET UP: $T = 1.6 \text{ s}$. $v = 0.32 \text{ m/s}$. The crest to crest distance is the wavelength, so $\lambda = 0.12 \text{ m}$.

EXECUTE: **(a)** $f_S = 1/T = 0.625 \text{ Hz}$. $\lambda = \dfrac{v - v_S}{f_S}$ gives $v_S = v - \lambda f_S = 0.32 \text{ m/s} - (0.12 \text{ m})(0.625 \text{ Hz}) = 0.25 \text{ m/s}$.

(b) $\lambda = \dfrac{v + v_S}{f_S} = \dfrac{0.32 \text{ m/s} + 0.25 \text{ m/s}}{0.625 \text{ Hz}} = 0.91 \text{ m}$

EVALUATE: If the duck was held at rest but still paddled its feet, it would produce waves of wavelength $\lambda = \dfrac{0.32 \text{ m/s}}{0.625 \text{ Hz}} = 0.51 \text{ m}$. In front of the duck the wavelength is decreased and behind the duck the wavelength is increased. The speed of the duck is 78% of the wave speed, so the Doppler effects are large.

16.46. **IDENTIFY:** Apply $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S.$

SET UP: $f_S = 1000$ Hz. The positive direction is from the listener to the source. $v = 344$ m/s.

(a) $v_S = -(344 \text{ m/s})/2 = -172 \text{ m/s}, \;\; v_L = 0.\;\; f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S = \left(\frac{344 \text{ m/s}}{344 \text{ m/s} - 172 \text{ m/s}}\right)(1000 \text{ Hz}) = 2000 \text{ Hz}$

(b) $v_S = 0, \;\; v_L = +172 \text{ m/s}.\;\; f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S = \left(\frac{344 \text{ m/s} + 172 \text{ m/s}}{344 \text{ m/s}}\right)(1000 \text{ Hz}) = 1500 \text{ Hz}$

EVALUATE: **(c)** The answer in (b) is much less than the answer in (a). It is the velocity of the source and listener relative to the air that determines the effect, not the relative velocity of the source and listener relative to each other.

16.47. **IDENTIFY:** Apply $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S.$

SET UP: The positive direction is from the motorcycle toward the car. The car is stationary, so $v_S = 0$.

EXECUTE: $f_L = \frac{v + v_L}{v + v_S} f_S = (1 + v_L/v) f_S$, which gives $v_L = v\left(\frac{f_L}{f_S} - 1\right) = (344 \text{ m/s})\left(\frac{490 \text{ Hz}}{520 \text{ Hz}} - 1\right) = -19.8 \text{ m/s}.$

You must be traveling at 19.8 m/s.

EVALUATE: $v_L < 0$ means that the listener is moving away from the source.

16.48. **IDENTIFY:** Apply the Doppler effect formula, Eq.(16.29).

(a) SET UP: The positive direction is from the listener toward the source, as shown in Figure 16.48a.

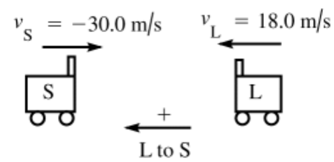


Figure 16.48a

EXECUTE: $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S = \left(\frac{344 \text{ m/s} + 18.0 \text{ m/s}}{344 \text{ m/s} - 30.0 \text{ m/s}}\right)(262 \text{ Hz}) = 302 \text{ Hz}$

EVALUATE: Listener and source are approaching and $f_L > f_S$.

(b) SET UP: See Figure 16.48b.

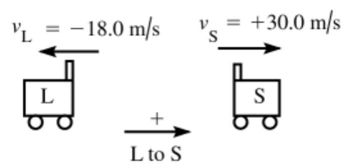


Figure 16.48b

EXECUTE: $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S = \left(\frac{344 \text{ m/s} - 18.0 \text{ m/s}}{344 \text{ m/s} + 30.0 \text{ m/s}}\right)(262 \text{ Hz}) = 228 \text{ Hz}$

EVALUATE: Listener and source are moving away from each other and $f_L < f_S$.

16.49. **IDENTIFY:** The radar beam consists of electromagnetic waves and Eq.(16.30) applies. Apply the Doppler formula twice, once with the storm as a receiver and then again with the storm as a source.

SET UP: $c = 3.00 \times 10^8$ m/s. When the source and receiver are moving toward each other, as is the case here, then v is negative.

EXECUTE: Let f' be the frequency received by the storm; $f' = \sqrt{\frac{c + |v|}{c - |v|}} f_S$. Then f' serves as the source frequency when the waves are reflected and $f_R = \sqrt{\frac{c + |v|}{c - |v|}}\left(\sqrt{\frac{c + |v|}{c - |v|}} f_S\right) = \left(\frac{c + |v|}{c - |v|}\right) f_S.$

$$\Delta f = f_R - f_S = \left(\frac{c + |v|}{c - |v|} - 1\right) f_S = \left(\frac{2|v|}{c - |v|}\right) f_S = \left[\frac{2(20.1 \text{ m/s})}{3.00 \times 10^8 \text{ m/s} - 20.1 \text{ m/s}}\right](200.0 \times 10^6 \text{ Hz}) = 26.8 \text{ Hz}$$

EVALUATE: Since $|v| \ll c$, in the expression $\Delta f = \left(\dfrac{2|v|}{c-|v|}\right) f_S$ it is a very good approximation to replace $c-|v|$ by c and then $\dfrac{\Delta f}{f_S} = 2\dfrac{v}{c}$. $\dfrac{v}{c}$ is very small, so $\dfrac{\Delta f}{f_S}$ is very small. Since the storm is approaching the station the final received frequency is larger than the original transmitted frequency.

16.50. IDENTIFY: Apply Eq.(16.30). The source is moving away, so v is positive.

SET UP: $c = 3.00\times10^8$ m/s. $v = +50.0\times10^3$ m/s.

EXECUTE: $f_R = \sqrt{\dfrac{c-v}{c+v}} f_S = \sqrt{\dfrac{3.00\times10^8 \text{ m/s} - 50.0\times10^3 \text{ m/s}}{3.00\times10^8 \text{ m/s} + 50.0\times10^3 \text{ m/s}}} (3.330\times10^{14} \text{ Hz}) = 3.329\times10^{14}$ Hz

EVALUATE: $f_R < f_S$ since the source is moving away. The difference between f_R and f_S is very small since $v \ll c$.

16.51. IDENTIFY: Apply Eq.(16.30).

SET UP: Require $f_R = 1.100 f_S$. Since $f_R > f_S$ the star would be moving toward us and $v < 0$, so $v = -|v|$. $c = 3.00\times10^8$ m/s.

EXECUTE: $f_R = \sqrt{\dfrac{c+|v|}{c-|v|}} f_S$. $f_R = 1.100 f_S$ gives $\dfrac{c+|v|}{c-|v|} = (1.100)^2$. Solving for $|v|$ gives

$|v| = \dfrac{\left[(1.100)^2 - 1\right]c}{1+(1.100)^2} = 0.0950c = 2.85\times10^7$ m/s.

EVALUATE: $\dfrac{v}{c} = 9.5\%$. $\dfrac{\Delta f}{f_S} = \dfrac{f_R - f_S}{f_S} = 10.0\%$. $\dfrac{v}{c}$ and $\dfrac{\Delta f}{f_S}$ are approximately equal.

16.52. IDENTIFY: Apply Eq.(16.31).

SET UP: The Mach number is the value of v_S/v, where v_S is the speed of the shuttle and v is the speed of sound at the altitude of the shuttle.

EXECUTE: **(a)** $\dfrac{v}{v_S} = \sin\alpha = \sin 58.0° = 0.848$. The Mach number is $\dfrac{v_S}{v} = \dfrac{1}{0.848} = 1.18$.

(b) $v_S = \dfrac{v}{\sin\alpha} = \dfrac{331 \text{ m/s}}{\sin 58.0°} = 390$ m/s

(c) $\dfrac{v_S}{v} = \dfrac{390 \text{ m/s}}{344 \text{ m/s}} = 1.13$. The Mach number would be 1.13. $\sin\alpha = \dfrac{v}{v_S} = \dfrac{344 \text{ m/s}}{390 \text{ m/s}}$ and $\alpha = 61.9°$

EVALUATE: The smaller the Mach number, the larger the angle of the shock-wave cone.

16.53. IDENTIFY: Apply Eq.(16.31) to calculate α. Use the method of Example 16.20 to calculate t.

SET UP: Mach 1.70 means $v_S/v = 1.70$.

EXECUTE: **(a)** In Eq.(16.31), $v/v_S = 1/1.70 = 0.588$ and $\alpha = \arcsin(0.588) = 36.0°$.

(b) As in Example 16.20, $t = \dfrac{(950 \text{ m})}{(1.70)(344 \text{ m/s})(\tan(36.0°))} = 2.23$ s.

EVALUATE: The angle α decreases when the speed v_S of the plane increases.

16.54. IDENTIFY: The displacement $y(x,t)$ is given in Eq.(16.1) and the pressure variation is given in Eq.(16.4). The pressure variation is related to the displacement by Eq.(16.3).

SET UP: $k = 2\pi/\lambda$

EXECUTE: **(a)** Mathematically, the waves given by Eq.(16.1) and Eq.(16.4) are out of phase. Physically, at a displacement node, the air is most compressed or rarefied on either side of the node, and the pressure gradient is zero. Thus, displacement nodes are pressure antinodes.

(b) The graphs have the same form as in Figure 16.3 in the textbook.

(c) $p(x,t) = -B\dfrac{\partial y(x,t)}{\partial x}$. When $y(x,t)$ versus x is a straight line with positive slope, $p(x,t)$ is constant and negative. When $y(x,t)$ versus x is a straight line with negative slope, $p(x,t)$ is constant and positive. The graph of $p(x,0)$ is given in Figure 16.54. The slope of the straightline segments for $y(x,0)$ is 1.6×10^{-4}, so for the wave in Figure 16.42 in the textbook, $p_{\text{max-non}} = (1.6\times10^{-4})B$. The sinusoidal wave has amplitude $p_{\text{max}} = BkA = (2.5\times10^{-4})B$. The difference in the pressure amplitudes is because the two $y(x,0)$ functions have different slopes.

EVALUATE: **(d)** $p(x,t)$ has its largest magnitude where $y(x,t)$ has the greatest slope. This is where $y(x,t)=0$ for a sinusoidal wave but it is not true in general.

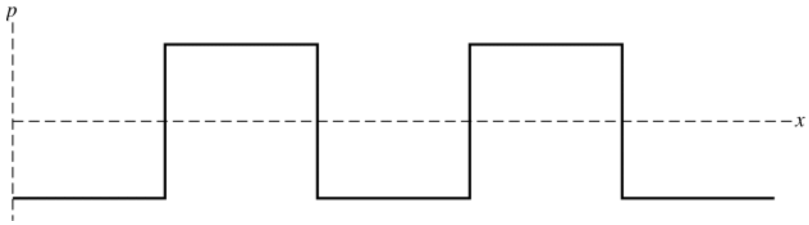


Figure 16.54

16.55. **IDENTIFY:** The sound intensity level is $\beta=(10\text{ dB})\log(I/I_0)$, so the same sound intensity level β means the same intensity *I*. The intensity is related to pressure amplitude by Eq.(16.13) and to the displacement amplitude by Eq.(16.12).

SET UP: $v=344\text{ m/s}$. $\omega=2\pi f$. Each octave higher corresponds to a doubling of frequency, so the note sung by the bass has frequency $(932\text{ Hz})/8=116.5\text{ Hz}$. Let 1 refer to the note sung by the soprano and 2 refer to the note sung by the bass. $I_0=1\times10^{-12}\text{ W/m}^2$.

EXECUTE: **(a)** $I=\dfrac{vp_{\max}^2}{2B}$ and $I_1=I_2$ gives $p_{\max,1}=p_{\max,2}$; the ratio is 1.00.

(b) $I=\frac{1}{2}\sqrt{\rho B}\omega^2A^2=\frac{1}{2}\sqrt{\rho B}4\pi^2f^2A^2$. $I_1=I_2$ gives $f_1A_1=f_2A_2$. $\dfrac{A_2}{A_1}=\dfrac{f_1}{f_2}=8.00$.

(c) $\beta=72.0\text{ dB}$ gives $\log(I/I_0)=7.2$. $\dfrac{I}{I_0}=10^{7.2}$ and $I=1.585\times10^{-5}\text{ W/m}^2$. $I=\frac{1}{2}\sqrt{\rho B}4\pi^2f^2A^2$.

$$A=\frac{1}{2\pi f}\sqrt{\frac{2I}{\sqrt{\rho B}}}=\frac{1}{2\pi(932\text{ Hz})}\sqrt{\frac{2(1.585\times10^{-5}\text{ W/m}^2)}{\sqrt{(1.20\text{ kg/m}^3)(1.42\times10^5\text{ Pa})}}}=4.73\times10^{-8}\text{ m}=47.3\text{ nm}.$$

EVALUATE: Even for this loud note the displacement amplitude is very small. For a given intensity, the displacement amplitude depends on the frequency of the sound wave but the pressure amplitude does not.

16.56. **IDENTIFY:** Use the equations that relate intensity level and intensity, intensity and pressure amplitude, pressure amplitude and displacement amplitude, and intensity and distance.

(a) SET UP: Use the intensity level β to calculate *I* at this distance. $\beta=(10\text{ dB})\log(I/I_0)$

EXECUTE: $52.0\text{ dB}=(10\text{ dB})\log(I/(10^{-12}\text{ W/m}^2))$

$\log(I/(10^{-12}\text{ W/m}^2))=5.20$ implies $I=1.585\times10^{-7}\text{ W/m}^2$

SET UP: Then use Eq.(16.14) to calculate $p_{\max}$:

$I=\dfrac{p_{\max}^2}{2\rho v}$ so $p_{\max}=\sqrt{2\rho vI}$

From Example 16.6, $\rho=1.20\text{ kg/m}^3$ for air at $20°\text{C}$.

EXECUTE: $p_{\max}=\sqrt{2\rho vI}=\sqrt{2(1.20\text{ kg/m}^3)(344\text{ m/s})(1.585\times10^{-7}\text{ W/m}^2)}=0.0114\text{ Pa}$

(b) SET UP: Eq.(16.5): $p_{\max}=BkA$ so $A=\dfrac{p_{\max}}{Bk}$

For air $B=1.42\times10^5\text{ Pa}$ (Example 16.1).

EXECUTE: $k=\dfrac{2\pi}{\lambda}=\dfrac{2\pi f}{v}=\dfrac{(2\pi\text{ rad})(587\text{ Hz})}{344\text{ m/s}}=10.72\text{ rad/m}$

$A=\dfrac{p_{\max}}{Bk}=\dfrac{0.0114\text{ Pa}}{(1.42\times10^5\text{ Pa})(10.72\text{ rad/m})}=7.49\times10^{-9}\text{ m}$

(c) SET UP: $\beta_2-\beta_1=(10\text{ dB})\log(I_2/I_1)$ (Example 16.11).

Eq.(15.26): $I_1/I_2=r_2^2/r_1^2$ so $I_2/I_1=r_1^2/r_2^2$

EXECUTE: $\beta_2-\beta_1=(10\text{ dB})\log(r_1/r_2)^2=(20\text{ dB})\log(r_1/r_2)$.

$\beta_2=52.0\text{ dB}$ and $r_2=5.00\text{ m}$. Then $\beta_1=30.0\text{ dB}$ and we need to calculate r_1.

$52.0\text{ dB}-30.0\text{ dB}=(20\text{ dB})\log(r_1/r_2)$

$22.0\text{ dB}=(20\text{ dB})\log(r_1/r_2)$

$\log(r_1/r_2)=1.10$ so $r_1=12.6r_2=63.0\text{ m}$.

EVALUATE: The decrease in intensity level corresponds to a decrease in intensity, and this means an increase in distance. The intensity level uses a logarithmic scale, so simple proportionality between r and β doesn't apply.

16.57. IDENTIFY: The sound is first loud when the frequency f_0 of the speaker equals the frequency f_1 of the fundamental standing wave for the gas in the tube. The tube is a stopped pipe, and $f_1 = \dfrac{v}{4L}$. $v = \sqrt{\dfrac{\gamma RT}{M}}$. The sound is next loud when the speaker frequency equals the first overtone frequency for the tube.

SET UP: A stopped pipe has only odd harmonics, so the frequency of the first overtone is $f_3 = 3f_1$.

EXECUTE: **(a)** $f_0 = f_1 = \dfrac{v}{4L} = \dfrac{1}{4L}\sqrt{\dfrac{\gamma RT}{M}}$. This gives $T = \dfrac{16L^2 M f_0^2}{\gamma R}$.

(b) $3f_0$.

EVALUATE: **(c)** Measure f_0 and L. Then $f_0 = \dfrac{v}{4L}$ gives $v = 4Lf_0$.

16.58. IDENTIFY: $f_{\text{beat}} = |f_A - f_B|$. $f_1 = \dfrac{v}{2L}$ and $v = \sqrt{\dfrac{FL}{m}}$ gives $f_1 = \dfrac{1}{2}\sqrt{\dfrac{F}{mL}}$. Apply $\sum \tau_z = 0$ to the bar to find the tension in each wire.

SET UP: For $\sum \tau_z = 0$ take the pivot at wire A and let counterclockwise torques be positive. The free-body diagram for the bar is given in Figure 16.58. Let L be the length of the bar.

EXECUTE: $\sum \tau_z = 0$ gives $F_B L - w_{\text{lead}}(3L/4) - w_{\text{bar}}(L/2) = 0$.

$F_B = 3w_{\text{lead}}/4 + w_{\text{bar}}/2 = 3(185\text{ N})/4 + (165\text{ N})/2 = 221\text{ N}$. $F_A + F_B = w_{\text{bar}} + w_{\text{lead}}$ so

$F_A = w_{\text{bar}} + w_{\text{lead}} - F_B = 165\text{ N} + 185\text{ N} - 221\text{ N} = 129\text{ N}$. $f_{1A} = \dfrac{1}{2}\sqrt{\dfrac{129\text{ N}}{(5.50\times 10^{-3}\text{ kg})(0.750\text{ m})}} = 88.4\text{ Hz}$.

$f_{1B} = f_{1A}\sqrt{\dfrac{221\text{ N}}{129\text{ N}}} = 115.7\text{ Hz}$. $f_{\text{beat}} = f_{1B} - f_{1A} = 27.3\text{ Hz}$.

EVALUATE: The frequency increases when the tension in the wire increases.

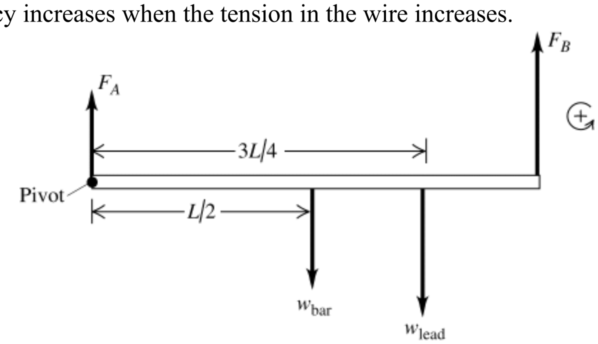


Figure 16.58

16.59. IDENTIFY: The flute acts as a stopped pipe and its harmonic frequencies are given by Eq.(16.23). The resonant frequencies of the string are $f_n = nf_1$, $n = 1, 2, 3, \ldots$ The string resonates when the string frequency equals the flute frequency.

SET UP: For the string $f_{1\text{s}} = 600.0\text{ Hz}$. For the flute, the fundamental frequency is $f_{1\text{f}} = \dfrac{v}{4L} = \dfrac{344.0\text{ m/s}}{4(0.1075\text{ m})} = 800.0\text{ Hz}$. Let n_f label the harmonics of the flute and let n_s label the harmonics of the string.

EXECUTE: For the flute and string to be in resonance, $n_\text{f} f_{1\text{f}} = n_\text{s} f_{1\text{s}}$, where $f_{1\text{s}} = 600.0\text{ Hz}$ is the fundamental frequency for the string. $n_\text{s} = n_\text{f}(f_{1\text{f}}/f_{1\text{s}}) = \frac{4}{3}n_\text{f}$. n_s is an integer when $n_\text{f} = 3N$, $N = 1, 3, 5, \ldots$ (the flute has only odd harmonics). $n_\text{f} = 3N$ gives $n_\text{s} = 4N$

Flute harmonic $3N$ resonates with string harmonic $4N$, $N = 1, 3, 5, \ldots$.

EVALUATE: We can check our results for some specific values of N. For $N = 1$, $n_\text{f} = 3$ and $f_{3\text{f}} = 2400\text{ Hz}$. For this N, $n_\text{s} = 4$ and $f_{4\text{s}} = 2400\text{ Hz}$. For $N = 3$, $n_\text{f} = 9$ and $f_{9\text{f}} = 7200\text{ Hz}$, and $n_\text{s} = 12$, $f_{12\text{s}} = 7200\text{ Hz}$. Our general results do give equal frequencies for the two objects.

16.60. **IDENTIFY:** The harmonics of the string are $f_n = nf_1 = n\left(\dfrac{v}{2l}\right)$, where l is the length of the string. The tube is a stopped pipe and its standing wave frequencies are given by Eq.(16.22). For the string, $v = \sqrt{F/\mu}$, where F is the tension in the string.

SET UP: The length of the string is $d = L/10$, so its third harmonic has frequency $f_3^{\text{string}} = 3\dfrac{1}{2d}\sqrt{F/\mu}$. The stopped pipe has length L, so its first harmonic has frequency $f_1^{\text{pipe}} = \dfrac{v_s}{4L}$.

EXECUTE: **(a)** Equating f_1^{string} and f_1^{pipe} and using $d = L/10$ gives $F = \dfrac{1}{3600}\mu v_s^2$.

(b) If the tension is doubled, all the frequencies of the string will increase by a factor of $\sqrt{2}$. In particular, the third harmonic of the string will no longer be in resonance with the first harmonic of the pipe because the frequencies will no longer match, so the sound produced by the instrument will be diminished.

(c) The string will be in resonance with a standing wave in the pipe when their frequencies are equal. Using $f_1^{\text{pipe}} = 3f_1^{\text{string}}$, the frequencies of the pipe are $nf_1^{\text{pipe}} = 3nf_1^{\text{string}}$, (where $n = 1, 3, 5, \ldots$). Setting this equal to the frequencies of the string $n'f_1^{\text{string}}$, the harmonics of the string are $n' = 3n = 3, 9, 15, \ldots$ The nth harmonic of the pipe is in resonance with the 3nth harmonic of the string.

EVALUATE: Each standing wave for the air column is in resonance with a standing wave on the string. But the reverse is not true; not all standing waves of the string are in resonance with a harmonic of the pipe.

16.61. **IDENTIFY** and **SET UP:** The frequency of any harmonic is an integer multiple of the fundamental. For a stopped pipe only odd harmonics are present. For an open pipe, all harmonics are present. See which pattern of harmonics fits to the observed values in order to determine which type of pipe it is. Then solve for the fundamental frequency and relate that to the length of the pipe.

EXECUTE: **(a)** For an open pipe the successive harmonics are $f_n = nf_1$, $n = 1, 2, 3, \ldots$. For a stopped pipe the successive harmonics are $f_n = nf_1$, $n = 1, 3, 5, \ldots$. If the pipe is open and these harmonics are successive, then $f_n = nf_1 = 1372\text{ Hz}$ and $f_{n+1} = (n+1)f_1 = 1764\text{ Hz}$. Subtract the first equation from the second: $(n+1)f_1 - nf_1 = 1764\text{ Hz} - 1372\text{ Hz}$. This gives $f_1 = 392\text{ Hz}$. Then $n = \dfrac{1372\text{ Hz}}{392\text{ Hz}} = 3.5$. But n must be an integer, so the pipe can't be open. If the pipe is stopped and these harmonics are successive, then $f_n = nf_1 = 1372\text{ Hz}$ and $f_{n+2} = (n+2)f_1 = 1764\text{ Hz}$ (in this case successive harmonics differ in n by 2). Subtracting one equation from the other gives $2f_1 = 392\text{ Hz}$ and $f_1 = 196\text{ Hz}$. Then $n = 1372\text{ Hz}/f_1 = 7$ so $1372\text{ Hz} = 7f_1$ and $1764\text{ Hz} = 9f_1$. The solution gives integer n as it should; the pipe is stopped.

(b) From part (a) these are the 7th and 9th harmonics.

(c) From part (a) $f_1 = 196\text{ Hz}$.

For a stopped pipe $f_1 = \dfrac{v}{4L}$ and $L = \dfrac{v}{4f_1} = \dfrac{344\text{ m/s}}{4(196\text{ Hz})} = 0.439\text{ m}$.

EVALUATE: It is essential to know that these are successive harmonics and to realize that 1372 Hz is not the fundamental. There are other lower frequency standing waves; these are just two successive ones.

16.62. **IDENTIFY:** The steel rod has standing waves much like a pipe open at both ends, since the ends are both displacement antinodes. An integral number of half wavelengths must fit on the rod, that is, $f_n = \dfrac{nv}{2L}$, with $n = 1, 2, 3, \ldots$.

SET UP: Table 16.1 gives $v = 5941\text{ m/s}$ for longitudinal waves in steel.

EXECUTE: **(a)** The ends of the rod are antinodes because the ends of the rod are free to oscillate.

(b) The fundamental can be produced when the rod is held at the middle because a node is located there.

(c) $f_1 = \dfrac{(1)(5941\text{ m/s})}{2(1.50\text{ m})} = 1980\text{ Hz}$

(d) The next harmonic is $n = 2$, or $f_2 = 3961\text{ Hz}$. We would need to hold the rod at an $n = 2$ node, which is located at $L/4 = 0.375\text{ m}$ from either end.

EVALUATE: For the 1.50 m long rod the wavelength of the fundamental is $x = 2L = 3.00\text{ m}$. The node to antinode distance is $\lambda/4 = 0.75\text{ m}$. For the second harmonic $\lambda = L = 1.50\text{ m}$ and the node to antinode distance is 0.375 m. There is a node at the middle of the rod, but forcing a node at 0.375 m from one end, by holding the rod there, prevents rod from vibrating in the fundamental.

16.63. **IDENTIFY:** The shower stall can be modeled as a pipe closed at both ends, and hence there are nodes at the two end walls. Figure 15.23 in the textbook shows standing waves on a *string* fixed at both ends but the sequence of harmonics is the same, namely that an integral number of half wavelengths must fit in the stall.

SET UP: The first three normal modes correspond to one half, two halves or three halves of a wavelength along the length of the air column.

EXECUTE: **(a)** The condition for standing waves is $f_n = \frac{nv}{2L}$, so the first three harmonics are for $n = 1,\ 2,\ 3.$

(b) A particular physics professor's shower has a length of $L = 1.48$ m. Using $f_n = \frac{nv}{2L}$ and $v = 344\ \text{m/s}$ gives resonant frequencies 116 Hz, 232 Hz and 349 Hz.

Note that the fundamental and second harmonic, which would have the greatest amplitude, are frequencies typically in the normal range of male singers. Hence, men do sing better in the shower! (For a further discussion of resonance and the human voice, see Thomas D. Rossing, *The Science of Sound,* Second Edition, Addison-Wesley, 1990, especially Chapters 4 and 17.)

EVALUATE: The standing wave frequencies for a pipe closed at both ends are the same as for an open pipe of the same length, even though the nodal patterns are different.

16.64. **IDENTIFY:** Stress is F/A, where F is the tension in the string and A is its cross sectional area.

SET UP: $A = \pi r^2$. For a string fixed at each end, $f_1 = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{F}{\mu}} = \frac{1}{2}\sqrt{\frac{F}{mL}}$

EXECUTE: **(a)** The cross-section area of the string would be $A = (900\ \text{N})/(7.0\times10^{8}\ \text{Pa}) = 1.29\times10^{-6}\ \text{m}^2$, corresponding to a radius of 0.640 mm. The length is the volume divided by the area, and the volume is $V = m/\rho$, so

$$L = \frac{V}{A} = \frac{m/\rho}{A} = \frac{(4.00\times10^{-3}\ \text{kg})}{(7.8\times10^{3}\ \text{kg/m}^3)(1.29\times10^{-6}\ \text{m}^2)} = 0.40\ \text{m}.$$

(b) For the maximum tension of 900 N, $f_1 = \frac{1}{2}\sqrt{\frac{900\ \text{N}}{(4.00\times10^{-3}\ \text{kg})(0.40\ \text{m})}} = 375\ \text{Hz}$, or 380 Hz to two figures.

EVALUATE: The string could be shorter and thicker. A shorter string of the same mass would have a higher fundamental frequency.

16.65. **IDENTIFY** and **SET UP:** There is a node at the piston, so the distance the piston moves is the node to node distance, $\lambda/2$. Use Eq.(15.1) to calculate v and Eq.(16.10) to calculate γ from v.

EXECUTE: **(a)** $\lambda/2 = 37.5$ cm, so $\lambda = 2(37.5\ \text{cm}) = 75.0\ \text{cm} = 0.750$ m.

$v = f\lambda = (500\ \text{Hz})(0.750\ \text{m}) = 375\ \text{m/s}$

(b) $v = \sqrt{\gamma RT/M}$ (Eq.16.10)

$\gamma = \frac{Mv^2}{RT} = \frac{(28.8\times10^{-3}\ \text{kg/mol})(375\ \text{m/s})^2}{(8.3145\ \text{J/mol}\cdot\text{K})(350\ \text{K})} = 1.39.$

(c) **EVALUATE:** There is a node at the piston so when the piston is 18.0 cm from the open end the node is inside the pipe, 18.0 cm from the open end. The node to antinode distance is $\lambda/4 = 18.8$ cm, so the antinode is 0.8 cm beyond the open end of the pipe.

The value of γ we calculated agrees with the value given for air in Example 16.5.

16.66. **IDENTIFY:** Model the auditory canal as a stopped pipe with length 2.5 cm.

SET UP: The frequencies of a stopped pipe are given by Eq.(16.22).

EXECUTE: **(a)** The frequency of the fundamental is $f_1 = v/4L = (344\ \text{m/s})/[4(0.025\ \text{m})] = 3440$ Hz. 3500 Hz is near the resonant frequency, and the ear will be sensitive to this frequency.

(b) The next resonant frequency would be $3f_1 = 10{,}500$ Hz and the ear would be sensitive to sounds with frequencies close to this value. But 7000 Hz is not a resonant frequency for a stopped pipe and the ear is not sensitive at this frequency.

EVALUATE: For a stopped pipe only odd harmonics are present.

16.67. **IDENTIFY:** The tuning fork frequencies that will cause this to happen are the standing wave frequencies of the wire. For a wire of mass m, length L and with tension F the fundamental frequency is $f_1 = \frac{v}{2L} = \sqrt{\frac{F}{4mL}}$. The standing wave frequencies are $f_n = nf_1,\ n = 1,\ 2,\ 3,\ \ldots$

SET UP: $F = Mg$, where $M = 0.420$ kg. The mass of the wire is $m = \rho V = \rho L\pi d^2/4$, where d is the diameter.

EXECUTE: **(a)** $f_1 = \sqrt{\frac{F}{4mL}} = \sqrt{\frac{Mg}{\pi d^2 L^2 \rho}} = \sqrt{\frac{(420.0\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{\pi(225\times10^{-6}\ \text{m})^2(0.45\ \text{m})^2(21.4\times10^{3}\ \text{kg/m}^3)}} = 77.3\ \text{Hz}.$

The tuning fork frequencies for which the fork would vibrate are integer multiples of 77.3 Hz.

EVALUATE: **(b)** The ratio $m/M \approx 9\times10^{-4}$, so the tension does not vary appreciably along the string due to the mass of the wire. Also, the suspended mass has a large inertia compared to the mass of the wire and assuming that it is stationary is an excellent approximation.

16.68. **IDENTIFY:** For a stopped pipe the frequency of the fundamental is $f_1 = \dfrac{v}{4L}$. The speed of sound in air depends on temperature, as shown by Eq.(16.10).

SET UP: Example 16.5 shows that the speed of sound in air at $20°\text{C}$ is 344 m/s.

EXECUTE: **(a)** $L = \dfrac{v}{4f} = \dfrac{344\ \text{m/s}}{4(349\ \text{Hz})} = 0.246\ \text{m}$

(b) The frequency will be proportional to the speed, and hence to the square root of the Kelvin temperature. The temperature necessary to have the frequency be higher is $(293.15\ \text{K})([370\ \text{Hz}]/[349\ \text{Hz}])^2 = 329.5\ \text{K}$, which is $56.3°\text{C}$.

EVALUATE: $56.3°\text{C} = 133°\text{F}$, so this extreme rise in pitch won't occur in practical situations. But changes in temperature can have noticeable effects on the pitch of the organ notes.

16.69. **IDENTIFY:** $v = f\lambda$. $v = \sqrt{\dfrac{\gamma RT}{M}}$. Solve for γ.

SET UP: The wavelength is twice the separation of the nodes, so $\lambda = 2L$, where $L = 0.200\ \text{m}$.

EXECUTE: $v = \lambda f = 2Lf = \sqrt{\dfrac{\gamma RT}{M}}$. Solving for γ,

$$\gamma = \frac{M}{RT}(2Lf)^2 = \frac{(16.0\times10^{-3}\ \text{kg/mol})}{(8.3145\ \text{J/mol}\cdot\text{K})\ (293.15\ \text{K})}\big(2(0.200\ \text{m})(1100\ \text{Hz})\big)^2 = 1.27.$$

EVALUATE: This value of γ is smaller than that of air. We will see in Chapter 19 that this value of γ is a typical value for polyatomic gases.

16.70. **IDENTIFY:** Destructive interference occurs when the path difference is a half-integer number of wavelengths. Constructive interference occurs when the path difference is an integer number of wavelengths.

SET UP: $\lambda = \dfrac{v}{f} = \dfrac{344\ \text{m/s}}{784\ \text{Hz}} = 0.439\ \text{m}$

EXECUTE: **(a)** If the separation of the speakers is denoted h, the condition for destructive interference is $\sqrt{x^2+h^2} - x = \beta\lambda$, where β is an odd multiple of one-half. Adding x to both sides, squaring, canceling the x^2 term from both sides and solving for x gives $x = \dfrac{h^2}{2\beta\lambda} - \dfrac{\beta}{2}\lambda$. Using $\lambda = 0.439\ \text{m}$ and $h = 2.00\ \text{m}$ yields 9.01 m for $\beta = \frac{1}{2}$, 2.71 m for $\beta = \frac{3}{2}$, 1.27 m for $\beta = \frac{5}{2}$, 0.53 m for $\beta = \frac{7}{2}$, and 0.026 m for $\beta = \frac{9}{2}$. These are the only allowable values of β that give positive solutions for x.

(b) Repeating the above for integral values of β, constructive interference occurs at 4.34 m, 1.84 m, 0.86 m, 0.26 m. Note that these are between, but not midway between, the answers to part (a).

(c) If $h = \lambda/2$, there will be destructive interference at speaker B. If $\lambda/2 > h$, the path difference can never be as large as $\lambda/2$. (This is also obtained from the above expression for x, with $x = 0$ and $\beta = \frac{1}{2}$.) The minimum frequency is then $v/2h = (344\ \text{m/s})/(4.0\ \text{m}) = 86\ \text{Hz}$.

EVALUATE: When f increases, λ is smaller and there are more occurrences of points of constructive and destructive interference.

16.71. **IDENTIFY:** Apply $f_\text{L} = \left(\dfrac{v+v_\text{L}}{v+v_\text{S}}\right)f_\text{S}$.

SET UP: The positive direction is from the listener to the source. (a) The wall is the listener. $v_\text{S} = -30\ \text{m/s}$. $v_\text{L} = 0$. $f_\text{L} = 600\ \text{Hz}$. (b) The wall is the source and the car is the listener. $v_\text{S} = 0$. $v_\text{L} = +30\ \text{m/s}$. $f_\text{S} = 600\ \text{Hz}$.

EXECUTE: **(a)** $f_\text{L} = \left(\dfrac{v+v_\text{L}}{v+v_\text{S}}\right)f_\text{S}$. $f_\text{S} = \left(\dfrac{v+v_\text{S}}{v+v_\text{L}}\right)f_\text{L} = \left(\dfrac{344\ \text{m/s} - 30\ \text{m/s}}{344\ \text{m/s}}\right)(600\ \text{Hz}) = 548\ \text{Hz}$

(b) $f_\text{L} = \left(\dfrac{v+v_\text{L}}{v+v_\text{S}}\right)f_\text{S} = \left(\dfrac{344\ \text{m/s} + 30\ \text{m/s}}{344\ \text{m/s}}\right)(600\ \text{Hz}) = 652\ \text{Hz}$

EVALUATE: Since the singer and wall are moving toward each other the frequency received by the wall is greater than the frequency sung by the soprano, and the frequency she hears from the reflected sound is larger still.

16.72. IDENTIFY: Apply $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S$. The wall first acts as a listener and then as a source.

SET UP: The positive direction is from listener to source. The bat is moving toward the wall so the Doppler effect increases the frequency and the final frequency received, f_{L2}, is greater than the original source frequency, f_{S1}. $f_{S1} = 2000$ Hz. $f_{L2} - f_{S1} = 10.0$ Hz.

EXECUTE: The wall receives the sound: $f_S = f_{S1}$. $f_L = f_{L1}$. $v_S = -v_{bat}$ and $v_L = 0$. $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S$ gives

$f_{L1} = \left(\frac{v}{v - v_{bat}}\right) f_{S1}$. The wall receives the sound: $f_{S2} = f_{L1}$. $v_S = 0$ and $v_L = +v_{bat}$.

$$f_{L2} = \left(\frac{v + v_{bat}}{v}\right) f_{S2} = \left(\frac{v + v_{bat}}{v}\right)\left(\frac{v}{v - v_{bat}}\right) f_{S1} = \left(\frac{v + v_{bat}}{v - v_{bat}}\right) f_{S1}.$$

$$f_{L2} - f_{S1} = \Delta f = \left(\frac{v + v_{bat}}{v - v_{bat}} - 1\right) f_{S1} = \left(\frac{2v_{bat}}{v - v_{bat}}\right) f_{S1}. \quad v_{bat} = \frac{v\Delta f}{2f_{S1} + \Delta f} = \frac{(344\text{ m/s})(10.0\text{ Hz})}{2(2000\text{ Hz}) + 10.0\text{ Hz}} = 0.858\text{ m/s}.$$

EVALUATE: $f_{S1} < \Delta f$, so we can write our result as the approximate but accurate expression $\Delta f = \left(\frac{2v_{bat}}{v}\right) f$.

16.73. IDENTIFY and **SET UP:** Use Eq.(16.12) for the intensity and Eq.(16.14) to relate the intensity and pressure amplitude.

EXECUTE: **(a)** The amplitude of the oscillations is ΔR.

$$I = \tfrac{1}{2}\sqrt{\rho B}(2\pi f)^2 A^2 = 2\sqrt{\rho B}\pi^2 f^2 (\Delta R)^2$$

(b) $P = I(4\pi R^2) = 8\pi^3\sqrt{\rho B} f^2 R^2 (\Delta R)^2$

(c) $I_R / I_d = d^2 / R^2$

$$I_d = (R/d)^2 I_R = 2\pi^2\sqrt{\rho B}(Rf/d)^2(\Delta R)^2$$

$I = p_{max}^2 / 2\sqrt{\rho B}$ so

$$p_{max} = \sqrt{\left(2\sqrt{\rho B} I\right)} = 2\pi\sqrt{\rho B}(Rf/d)\ \Delta R$$

$$A = \frac{p_{max}}{Bk} = \frac{p_{max}\lambda}{B2\pi} = \frac{p_{max}v}{B2\pi f} = v\sqrt{\rho/B}(R/d)\ \Delta R$$

But $v = \sqrt{B/\rho}$ so $v\sqrt{\rho/B} = 1$ so $A = (R/d)\ \Delta R$.

EVALUATE: The pressure amplitude and displacement amplitude fall off like $1/d$ and the intensity like $1/d^2$.

16.74. IDENTIFY: Apply $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S$. The heart wall first acts as the listener and then as the source.

SET UP: The positive direction is from listener to source. The heart wall is moving toward the receiver so the Doppler effect increases the frequency and the final frequency received, f_{L2}, is greater than the source frequency, f_{S1}. $f_{L2} - f_{S1} = 85$ Hz.

EXECUTE: Heart wall receives the sound: $f_S = f_{S1}$. $f_L = f_{L1}$. $v_S = 0$. $v_L = -v_{wall}$. $f_L = \left(\frac{v + v_L}{v + v_S}\right) f_S$ gives

$f_{L1} = \left(\frac{v - v_{wall}}{v}\right) f_{S1}$. Heart wall emits the sound: $f_{S2} = f_{L1}$. $v_S = +v_{wall}$. $v_L = 0$.

$$f_{L2} = \left(\frac{v}{v + v_{wall}}\right) f_{S2} = \left(\frac{v}{v + v_{wall}}\right)\left(\frac{v - v_{wall}}{v}\right) f_{S1} = \left(\frac{v - v_{wall}}{v + v_{wall}}\right) f_{S1}. \quad f_{L2} - f_{S1} = \left(1 - \frac{v - v_{wall}}{v + v_{wall}}\right) f_{S1} = \left(\frac{2v_{wall}}{v + v_{wall}}\right) f_{S1}.$$

$$v_{wall} = \frac{(f_{L2} - f_{S1})v}{2f_{S1} - (f_{L2} - f_{S1})}. \quad f_{S1} \gg f_{L2} - f_{S1} \text{ and } v_{wall} = \frac{(f_{L2} - f_{S1})v}{2f_{S1}} = \frac{(85\text{ Hz})(1500\text{ m/s})}{2(2.00\times 10^6\text{ Hz})} = 0.0319\text{ m/s} = 3.19\text{ cm/s}.$$

EVALUATE: $f_{S1} = 2.00\times 10^6$ Hz and $f_{L2} - f_{S1} = 85$ Hz, so the approximation we made is very accurate. Within this approximation, the frequency difference between the reflected and transmitted waves is directly proportional to the speed of the heart wall.

16.75. **(a) IDENTIFY** and **SET UP:** Use Eq.(15.1) to calculate λ.

EXECUTE: $\lambda = \dfrac{v}{f} = \dfrac{1482\text{ m/s}}{22.0\times10^3\text{ Hz}} = 0.0674\text{ m}$

(b) IDENTIFY: Apply the Doppler effect equation, Eq.(16.29). The Problem-Solving Strategy in the text (Section 16.8) describes how to do this problem. The frequency of the directly radiated waves is $f_S = 22{,}000$ Hz. The moving whale first plays the role of a moving listener, receiving waves with frequency f'_L. The whale then acts as a moving source, emitting waves with the same frequency, $f'_S = f'_L$ with which they are received. Let the speed of the whale be v_W.

SET UP: whale receives waves (Figure 16.75a)

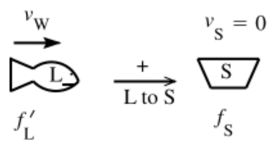


Figure 16.75a

EXECUTE: $v_L = +v_W$

$$f'_L = f_S\left(\frac{v+v_L}{v+v_S}\right) = f_S\left(\frac{v+v_W}{v}\right)$$

SET UP: whale re-emits the waves (Figure 16.75b)

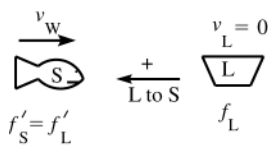


Figure 16.75b

EXECUTE: $v_S = -v_W$

$$f_L = f_S\left(\frac{v+v_L}{v+v_S}\right) = f'_S\left(\frac{v}{v-v_W}\right)$$

But $f'_S = f'_L$ so $f_L = f_S\left(\dfrac{v+v_W}{v}\right)\left(\dfrac{v}{v-v_W}\right) = f_S\left(\dfrac{v+v_W}{v-v_W}\right)$.

Then $\Delta f = f_S - f_L = f_S\left(1-\dfrac{v+v_W}{v-v_W}\right) = f_S\left(\dfrac{v-v_W-v-v_W}{v-v_W}\right) = \dfrac{-2f_S v_W}{v-v_W}$.

$$\Delta f = \frac{-2(2.20\times10^4\text{ Hz})(4.95\text{ m/s})}{1482\text{ m/s} - 4.95\text{ m/s}} = 147\text{ Hz}.$$

EVALUATE: Listener and source are moving toward each other so frequency is raised.

16.76. **IDENTIFY:** Apply the Doppler effect formula $f_L = \left(\dfrac{v+v_L}{v+v_S}\right)f_S$. In the SHM the source moves toward and away from the listener, with maximum speed $\omega_p A_p$.

SET UP: The direction from listener to source is positive.

EXECUTE: **(a)** The maximum velocity of the siren is $\omega_P A_P = 2\omega f_P A_P$. You hear a sound with frequency $f_L = f_{siren}\, v/(v+v_S)$, where v_S varies between $+2\pi f_P A_P$ and $-2\pi f_P A_P$. $f_{L-max} = f_{siren}\, v/(v-2\pi f_P A_P)$ and $f_{L-min} = f_{siren}\, v/(v+2\pi f_P A_P)$.

(b) The maximum (minimum) frequency is heard when the platform is passing through equilibrium and moving up (down).

EVALUATE: When the platform is moving upward the frequency you hear is greater than f_{siren} and when it is moving downward the frequency you hear is less than f_{siren}. When the platform is at its maximum displacement from equilibrium its speed is zero and the frequency you hear is f_{siren}.

16.77. **IDENTIFY:** Follow the method of Example 16.19 and apply the Doppler shift formula twice, once with the insect as the listener and again with the insect as the source.

SET UP: Let v_{bat} be the speed of the bat, v_{insect} be the speed of the insect, and f_i be the frequency with which the sound waves both strike and are reflected from the insect. The positive direction in each application of the Doppler shift formula is from the listener to the source.

EXECUTE: The frequencies at which the bat sends and receives the signals are related by $f_L = f_i\left(\frac{v+v_{bat}}{v-v_{insect}}\right) = f_S\left(\frac{v+v_{insect}}{v-v_{bat}}\right)\left(\frac{v+v_{bat}}{v-v_{insect}}\right)$. Solving for v_{insect},

$$v_{insect} = v\left[\frac{1-\frac{f_S}{f_L}\left(\frac{v+v_{bat}}{v-v_{bat}}\right)}{1+\frac{f_S}{f_L}\left(\frac{v+v_{bat}}{v-v_{bat}}\right)}\right] = v\left[\frac{f_L(v-v_{bat})-f_S(v+v_{bat})}{f_L(v-v_{bat})+f_S(v+v_{bat})}\right].$$

Letting $f_L = f_{refl}$ and $f_S = f_{bat}$ gives the result.

(b) If $f_{bat} = 80.7$ kHz, $f_{refl} = 83.5$ kHz, and $v_{bat} = 3.9$ m/s, then $v_{insect} = 2.0$ m/s.

EVALUATE: $f_{refl} > f_{bat}$ because the bat and insect are approaching each other.

16.78. **IDENTIFY:** Follow the steps specified in the problem. v is positive when the source is moving away from the receiver and v is negative when the source is moving toward the receiver. $|f_L - f_R|$ is the beat frequency.

SET UP: The source and receiver are approaching, so $f_R > f_S$ and $f_R - f_S = 46.0$ Hz.

EXECUTE: **(a)** $f_R = f_L\sqrt{\frac{c-v}{c+v}} = f_S\frac{\sqrt{1-v/c}}{\sqrt{1+v/c}} = f_S\left(1-\frac{v}{c}\right)^{1/2}\left(1+\frac{v}{c}\right)^{-1/2}$.

(b) For small x, the binomial theorem (see Appendix B) gives $(1-x)^{1/2} \approx 1-x/2$, $(1+x)^{-1/2} \approx 1-x/2$. Therefore $f_L \approx f_S\left(1-\frac{v}{2c}\right)^2 \approx f_S\left(1-\frac{v}{c}\right)$, where the binomial theorem has been used to approximate $(1-x/2)^2 \approx 1-x$.

(c) For an airplane, the approximation $v \ll c$ is certainly valid. Solving the expression found in part (b) for v, $v = c\frac{f_S - f_R}{f_S} = c\frac{f_{beat}}{f_S} = (3.00\times10^8\ \text{m/s})\frac{-46.0\ \text{Hz}}{2.43\times10^8\ \text{Hz}} = -56.8$ m/s. The speed of the aircraft is 56.8 m/s.

EVALUATE: The approximation $v \ll c$ is seen to be valid. v is negative because the source and receiver are approaching. Since $v \ll c$, the fractional shift in frequency, $\frac{\Delta f}{f}$, is very small.

16.79. **IDENTIFY:** Apply the result derived in part (b) of Problem 16.78. The radius of the nebula is $R = vt$, where t is the time since the supernova explosion.

SET UP: When the source and receiver are moving toward each other, v is negative and $f_R > f_S$. The light from the explosion reached earth 952 years ago, so that is the amount of time the nebula has expanded. $1\ \text{ly} = 9.46\times10^{15}$ m.

EXECUTE: **(a)** $v = c\frac{f_S - f_R}{f_S} = (3.00\times10^8\ \text{m/s})\frac{-0.018\times10^{14}\ \text{Hz}}{4.568\times10^8\ \text{Hz}} = -1.2\times10^6\ \text{m/s}$, with the minus sign indicating that the gas is approaching the earth, as is expected since $f_R > f_S$.

(b) The radius is $(952\ \text{yr})(3.156\times10^7\ \text{s/yr})(1.2\times10^6\ \text{m/s}) = 3.6\times10^{16}\ \text{m} = 3.8$ ly.

(c) The ratio of the width of the nebula to 2π times the distance from the earth is the ratio of the angular width (taken as 5 arc minutes) to an entire circle, which is 60×360 arc minutes. The distance to the nebula is then $2(3.75\ \text{ly})\frac{(60)(360)}{5} = 5.2\times10^3$ ly. The time it takes light to travel this distance is 5200 yr, so the explosion actually took place 5200 yr before 1054 C.E., or about 4100 B.C.E.

EVALUATE: $\left|\frac{v}{c}\right| = 4.0\times10^{-3}$, so even though $|v|$ is very large the approximation required for $v = c\frac{\Delta f}{f}$ is accurate.

16.80. **IDENTIFY** and **SET UP:** Use Eq.(16.30) that describes the Doppler effect for electromagnetic waves. $v \ll c$, so the simplified form derived in Problem 16.78b can be used.

(a) EXECUTE: From Problem 16.78b, $f_R = f_S(1 - v/c)$.

v is negative since the source is approaching:

$v = -(42.0\ \text{km/h})(1000\ \text{m/1 km})(1\ \text{h/3600 s}) = -11.67$ m/s

Approaching means that the frequency is increased.

$\Delta f = f_S\left(-\frac{v}{c}\right) = 2800\times10^6\ \text{Hz}\left(-\frac{-11.67\ \text{m/s}}{3.00\times10^8\ \text{m/s}}\right) = 109$ Hz

(b) EVALUATE: Approaching, so the frequency is increased. $v \ll c$ and therefore $\Delta f / f_S \ll 1$. The frequency of the waves received and reflected by the water is very close to 2880 MHz, so get an additional shift of 109 Hz and the total shift in frequency is $2(109\text{ Hz}) = 218\text{ Hz}$.

16.81. **IDENTIFY:** Follow the method of Example 16.19 and apply the Doppler shift formula twice, once for the wall as a listener and then again with the wall as a source.

SET UP: In each application of the Doppler formula, the positive direction is from the listener to the source

EXECUTE: **(a)** The wall will receive and reflect pulses at a frequency $\dfrac{v}{v - v_w} f_0$, and the woman will hear this reflected wave at a frequency $\dfrac{v + v_w}{v} \dfrac{v}{v - v_w} f_0 = \dfrac{v + v_w}{v - v_w} f_0$. The beat frequency is $f_{beat} = f_0\left(\dfrac{v + v_w}{v - v_w} - 1\right) = f_0\left(\dfrac{2v_w}{v - v_w}\right)$.

(b) In this case, the sound reflected from the wall will have a lower frequency, and using $f_0(v - v_w)/(v + v_w)$ as the detected frequency. v_w is replaced by $-v_w$ in the calculation of part (a) and $f_{beat} = f_0\left(1 - \dfrac{v - v_w}{v + v_w}\right) = f_0\left(\dfrac{2v_w}{v + v_w}\right)$.

EVALUATE: The beat frequency is larger when she runs toward the wall, even though her speed is the same in both cases.

16.82. **IDENTIFY** and **SET UP:** Use Fig.(16.37) to relate α and T.
Use this in Eq.(16.31) to eliminate $\sin\alpha$.

EXECUTE: Eq.(16.31): $\sin\alpha = v / v_S$ From Fig.16.37 $\tan\alpha = h / v_S T$. And $\tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{\sin\alpha}{\sqrt{1 - \sin^2\alpha}}$.

Combining these equations we get $\dfrac{h}{v_S T} = \dfrac{v / v_S}{\sqrt{1 - (v / v_S)^2}}$ and $\dfrac{h}{T} = \dfrac{v}{\sqrt{1 - (v / v_S)^2}}$.

$1 - (v / v_S)^2 = \dfrac{v^2 T^2}{h^2}$ and $v_S^2 = \dfrac{v^2}{1 - v^2 T^2 / h^2}$

$v_S = \dfrac{hv}{\sqrt{h^2 - v^2 T^2}}$ as was to be shown.

EVALUATE: For a given h, the faster the speed v_S of the plane, the greater is the delay time T. The maximum delay time is h / v, and T approaches this value as $v_S \to \infty$. $T \to 0$ as $v \to v_S$.

16.83. **IDENTIFY:** The phase of the wave is determined by the value of $x - vt$, so t increasing is equivalent to x decreasing with t constant. The pressure fluctuation and displacement are related by Eq.(16.3).

SET UP: $y(x,t) = -\dfrac{1}{B}\int p(x,t)dx$. If $p(x,t)$ versus x is a straight line, then $y(x,t)$ versus x is a parabola. For air, $B = 1.42 \times 10^5$ Pa.

EXECUTE: **(a)** The graph is sketched in Figure 16.83a.

(b) From Eq.(16.4), the function that has the given $p(x,\ 0)$ at $t = 0$ is given graphically in Figure 16.83b. Each section is a parabola, not a portion of a sine curve. The period is $\lambda/v = (0.200\text{ m})/(344\text{ m/s}) = 5.81 \times 10^{-4}$ s and the amplitude is equal to the area under the p versus x curve between $x = 0$ and $x = 0.0500$ m divided by B, or 7.04×10^{-6} m.

(c) Assuming a wave moving in the $+x$-direction, $y(0,\ t)$ is as shown in Figure 16.83c.

(d) The maximum velocity of a particle occurs when a particle is moving through the origin, and the particle speed is $v_y = -\dfrac{\partial y}{\partial x} v = \dfrac{pv}{B}$. The maximum velocity is found from the maximum pressure, and $v_{y\max} = (40\text{ Pa})(344\text{ m/s})/(1.42 \times 10^5\text{ Pa}) = 9.69\text{ cm/s}$. The maximum acceleration is the maximum pressure gradient divided by the density,

$$a_{\max} = \frac{(80.0\text{ Pa})/(0.100\text{ m})}{(1.20\text{ kg/m}^3)} = 6.67 \times 10^2\text{ m/s}^2.$$

(e) The speaker cone moves with the displacement as found in part (c); the speaker cone alternates between moving forward and backward with constant magnitude of acceleration (but changing sign). The acceleration as a function of time is a square wave with amplitude 667 m/s^2 and frequency $f = v/\lambda = (344\text{ m/s})/(0.200\text{ m}) = 1.72\text{ kHz}$.

EVALUATE: We can verify that $p(x,\ t)$ versus *x* has a shape proportional to the slope of the graph of $y(x,\ t)$ versus *x*. The same is also true of the graphs versus *t*.

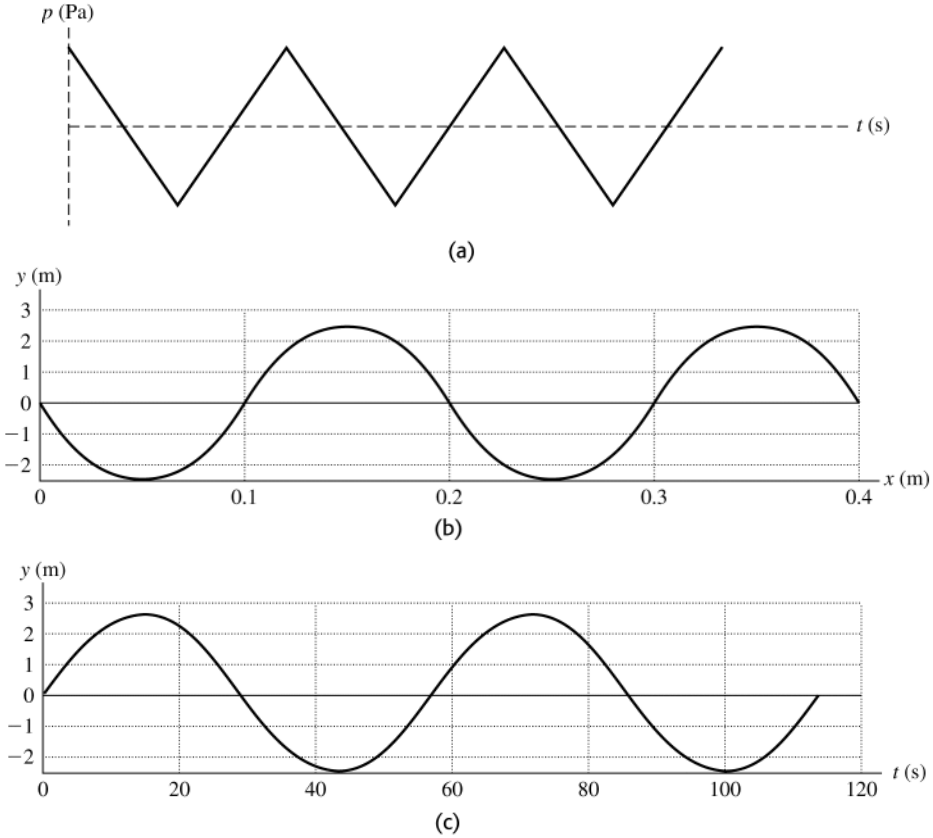


Figure 16.83

16.84. IDENTIFY: At a distance *r* from a point source with power output *P*, $I = \dfrac{P}{4\pi r^2}$. $\beta = (10\text{ dB})\log(I/I_0)$. For two sources the amplitudes are combined according to the phase difference.

SET UP: The amplitude is proportional to the square root of the intensity. Taking the speed of sound to be $344\ \text{m/s}$, the wavelength of the waves emitted by each speaker is $2.00\ \text{m}$.

EXECUTE: **(a)** Point C is two wavelengths from speaker A and one and one-half from speaker B, and so the phase difference is $180° = \pi$ rad.

(b) $I = \dfrac{P}{4\pi r^2} = \dfrac{8.00\times 10^{-4}\ \text{W}}{4\pi(4.00\ \text{m})^2} = 3.98\times 10^{-6}\ \text{W/m}^2$ and the sound intensity level is

$(10\text{ dB})\log(3.98\times 10^{6}) = 66.0\text{ dB}$. Repeating with $P = 6.00\times 10^{-5}\ \text{W}$ and $r = 3.00$ m gives

$I = 5.31\times 10^{-7}\ \text{W/m}^2$ and $\beta = 57.2$ dB.

(c) With the result of part (a), the amplitudes, either displacement or pressure, must be subtracted. That is, the intensity is found by taking the square roots of the intensities found in part (b), subtracting, and squaring the difference. The result is that $I = 1.60\times 10^{-6}\ \text{W/m}^2$ and $\beta = 62.1$ dB.

EVALUATE: Subtracting the intensities of *A* and *B* gives

$3.98\times 10^{-6}\ \text{W/m}^2 - 5.31\times 10^{-7}\ \text{W/m}^2 = 3.45\times 10^{-6}\ \text{W/m}^2$. This is very different from the correct intensity at *C*.

TEMPERATURE AND HEAT

17

17.1. IDENTIFY and SET UP: $T_F = \frac{9}{5}T_C + 32°$.

EXECUTE: (a) $T_F = (9/5)(-62.8) + 32 = -81.0°F$

(b) $T_F = (9/5)(56.7) + 32 = 134.1°F$

(c) $T_F = (9/5)(31.1) + 32 = 88.0°F$

EVALUATE: Fahrenheit degrees are smaller than Celsius degrees, so it takes more F° than C° to express the difference of a temperature from the ice point.

17.2. IDENTIFY and SET UP: $T_C = \frac{5}{9}(T_F - 32°)$

EXECUTE: (a) $T_C = (5/9)(41.0 - 32) = 5.0°C$

(b) $T_C = (5/9)(107 - 32) = 41.7°C$

(c) $T_C = (5/9)(-18 - 32) = -27.8°C$

EVALUATE: Fahrenheit degrees are smaller than Celsius degrees, so it takes more F° than C° to express the difference of a temperature from the ice point.

17.3. IDENTIFY: Convert each temperature from °C to °F.

SET UP: $T_F = \frac{9}{5}T_C + 32°C$

EXECUTE: $18°C$ equals $\frac{9}{5}(18°) + 32° = 64°F$ and $39°C$ equals $\frac{9}{5}(39°) + 32° = 102°F$. The temperature increase is $102°F - 64°F = 38\ F°$.

EVALUATE: The temperature increase is 21 C°, and this corresponds to $(21\ C°)\left(\frac{\frac{9}{5}\ F°}{1\ C°}\right) = 38\ F°$.

17.4. IDENTIFY: Convert $\Delta T = 10\ K$ to F°.

SET UP: $1\ K = 1\ C° = \frac{9}{5}F°$.

EXECUTE: A temperature increase of 10 K corresponds to an increase of 18 F°. Beaker *B* has the higher temperature.

EVALUATE: Kelvin and Celsius degrees are the same size. Fahrenheit degrees are smaller, so it takes more of them to express a given ΔT value.

17.5. IDENTIFY: Convert ΔT in kelvins to C° and to F°.

SET UP: $1\ K = 1\ C° = \frac{9}{5}F°$

EXECUTE: (a) $\Delta T_F = \frac{9}{5}\Delta T_C = \frac{9}{5}(-10.0\ C°) = -18.0\ F°$

(b) $\Delta T_C = \Delta T_K = -10.0\ C°$

EVALUATE: Kelvin and Celsius degrees are the same size. Fahrenheit degrees are smaller, so it takes more of them to express a given ΔT value.

17.6. IDENTIFY: Convert ΔT between different scales.

SET UP: ΔT is the same on the Celsius and Kelvin scales. $180\ F° = 100\ C°$, so $1\ C° = \frac{9}{5}F°$.

EXECUTE: (a) $\Delta T = 49.0\ F°$. $\Delta T = (49.0\ F°)\left(\frac{1\ C°}{\frac{9}{5}\ F°}\right) = 27.2\ C°$.

(b) $\Delta T = -100\ F°$. $\Delta T = (-100.0\ F°)\left(\frac{1\ C°}{\frac{9}{5}\ F°}\right) = -55.6\ C°$

EVALUATE: The magnitude of the temperature change is larger in F° than in C°.

17.7. **IDENTIFY:** Convert T in °C to °F.

SET UP: $T_F = \frac{9}{5}(T_C + 32°)$

EXECUTE: **(a)** $T_F = \frac{9}{5}(40.2°) + 32° = 104.4°\text{F}$. Yes, you should be concerned.

(b) $T_F = \frac{9}{5}(T_C + 32°) = \frac{9}{5}(12°\text{C}) + 32° = 54°\text{F}$.

EVALUATE: In doing the temperature conversion we account for both the size of the degrees and the different zero points on the two temperature scales.

17.8. **IDENTIFY:** Set $T_C = T_F$ and $T_F = T_K$.

SET UP: $T_F = \frac{9}{5}T_C + 32°\text{C}$ and $T_K = T_C + 273.15 = \frac{5}{9}(T_F - 32°) + 273.15$

EXECUTE: **(a)** $T_F = T_C = T$ gives $T = \frac{9}{5}T + 32°$ and $T = -40°$; $-40°\text{C} = -40°\text{F}$.

(b) $T_F = T_K = T$ gives $T = \frac{5}{9}(T - 32°) + 273.15$ and $T = \frac{9}{4}\left(-\left(\frac{5}{9}\right)(32°) + 273.15\right) = 575°$; $575°\text{F} = 575\text{ K}$.

EVALUATE: Since $T_K = T_C + 273.15$ there is no temperature at which Celsius and Kelvin thermometers agree.

17.9. **IDENTIFY:** Convert to the Celsius scale and then to the Kelvin scale.

SET UP: Combining Eq.(17.2) and Eq.(17.3), $T_K = \frac{5}{9}\left(T_F - 32°\right) + 273.15$,

EXECUTE: Substitution of the given Fahrenheit temperatures gives
(a) 216.5 K
(b) 325.9 K
(c) 205.4 K

EVALUATE: All temperatures on the Kelvin scale are positive.

17.10. **IDENTIFY:** Convert T_K to T_C and then convert T_C to T_F.

SET UP: $T_K = T_C + 273.15$ and $T_F = \frac{9}{5}T_C + 32°$.

EXECUTE: **(a)** $T_C = 400 - 273.15 = 127°\text{C}$, $T_F = (9/5)(126.85) + 32 = 260°\text{F}$

(b) $T_C = 95 - 273.15 = -178°\text{C}$, $T_F = (9/5)(-178.15) + 32 = -289°\text{F}$

(c) $T_C = 1.55\times10^7 - 273.15 = 1.55\times10^7°\text{C}$, $T_F = (9/5)(1.55\times10^7) + 32 = 2.79\times10^7°\text{F}$

EVALUATE: All temperatures on the Kelvin scale are positive. T_C is negative if the temperature is below the freezing point of water.

17.11. **IDENTIFY:** Convert T_F to T_C and then convert T_C to T_K.

SET UP: $T_C = \frac{5}{9}(T_F - 32°)$. $T_K = T_C + 273.15$.

EXECUTE: **(a)** $T_C = \frac{5}{9}(-346° - 32°) = -210°\text{C}$

(b) $T_K = -210° + 273.15 = 63\text{ K}$

EVALUATE: The temperature is negative on the Celsius and Fahrenheit scales but all temperatures are positive on the Kelvin scale.

17.12. **IDENTIFY:** Apply Eq.(17.5) and solve for p.

SET UP: $p_{\text{triple}} = 325\text{ mm of mercury}$

EXECUTE: $p = (325.0\text{ mm of mercury})\left(\frac{373.15\text{ K}}{273.16\text{ K}}\right) = 444\text{ mm of mercury}$

EVALUATE: mm of mercury is a unit of pressure. Since Eq.(17.5) involves a ratio of pressures, it is not necessary to convert the pressure to units of Pa.

17.13. **IDENTIFY:** When the volume is constant, $\frac{T_2}{T_1} = \frac{p_2}{p_1}$, for T in kelvins.

SET UP: $T_{\text{triple}} = 273.16\text{ K}$. Figure 17.7 in the textbook gives that the temperature at which CO_2 solidifies is $T_{CO_2} = 195\text{ K}$.

EXECUTE: $p_2 = p_1\left(\frac{T_2}{T_1}\right) = (1.35\text{ atm})\left(\frac{195\text{ K}}{273.16\text{ K}}\right) = 0.964\text{ atm}$

EVALUATE: The pressure decreases when T decreases.

17.14. **IDENTIFY:** $1\text{ K} = 1\text{ C}°$ and $1\text{ C}° = \frac{9}{5}\text{F}°$, so $1\text{ K} = \frac{9}{5}\text{R}°$.

SET UP: On the Kelvin scale, the triple point is 273.16 K.

EXECUTE: $T_{\text{triple}} = (9/5)273.16\text{ K} = 491.69°\text{R}$.

EVALUATE: One could also look at Figure 17.7 in the textbook and note that the Fahrenheit scale extends from $-460°\text{F}$ to $+32°\text{F}$ and conclude that the triple point is about $492°\text{R}$.

17.15. IDENTIFY and **SET UP:** Fit the data to a straight line for $p(T)$ and use this equation to find T when $p = 0$.

EXECUTE: **(a)** If the pressure varies linearly with temperature, then $p_2 = p_1 + \gamma(T_2 - T_1)$.

$$\gamma = \frac{p_2 - p_1}{T_2 - T_1} = \frac{6.50\times10^4\ \text{Pa} - 4.80\times10^4\ \text{Pa}}{100°\text{C} - 0.01°\text{C}} = 170.0\ \text{Pa/C°}$$

Apply $p = p_1 + \gamma(T - T_1)$ with $T_1 = 0.01°\text{C}$ and $p = 0$ to solve for T.

$$0 = p_1 + \gamma(T - T_1)$$

$$T = T_1 - \frac{p_1}{\gamma} = 0.01°\text{C} - \frac{4.80\times10^4\ \text{Pa}}{170\ \text{Pa/C°}} = -282°\text{C}.$$

(b) Let $T_1 = 100°\text{C}$ and $T_2 = 0.01°\text{C}$; use Eq.(17.4) to calculate p_2. Eq.(17.4) says $T_2/T_1 = p_2/p_1$, where T is in kelvins.

$p_2 = p_1\left(\frac{T_2}{T_1}\right) = 6.50\times10^4\ \text{Pa}\left(\frac{0.01 + 273.15}{100 + 273.15}\right) = 4.76\times10^4\ \text{Pa}$; this differs from the $4.80\times10^4\ \text{Pa}$ that was measured so Eq.(17.4) is not precisely obeyed.

EVALUATE: The answer to part (a) is in reasonable agreement with the accepted value of $-273°\text{C}$

17.16. IDENTIFY: Apply $\Delta L = \alpha L_0 \Delta T$ and calculate ΔT. Then $T_2 = T_1 + \Delta T$, with $T_1 = 15.5°\text{C}$.

SET UP: Table 17.1 gives $\alpha = 1.2\times10^{-5}\ (\text{C°})^{-1}$ for steel.

EXECUTE: $\Delta T = \frac{\Delta L}{\alpha L_0} = \frac{0.471\ \text{ft}}{[1.2\times10^{-5}\ (\text{C°})^{-1}][1671\ \text{ft}]} = 23.5\ \text{C°}$. $T_2 = 15.5°\text{C} + 23.5\ \text{C°} = 39.0°\text{C}$.

EVALUATE: Since then the lengths enter in the ratio $\Delta L / L_0$, we can leave the lengths in ft.

17.17. IDENTIFY: $\Delta L = L_0 \alpha \Delta T$

SET UP: For steel, $\alpha = 1.2\times10^{-5}\ (\text{C°})^{-1}$

EXECUTE: $\Delta L = (1.2\times10^{-5}(\text{C°})^{-1})(1410\ \text{m})(18.0°\text{C} - (-5.0°\text{C})) = +0.39\ \text{m}$

EVALUATE: The length increases when the temperature increases. The fractional increase is very small, since $\alpha\Delta T$ is small.

17.18. IDENTIFY: Apply $L = L_0(1 + \alpha\Delta T)$ to the diameter d of the rivet.

SET UP: For aluminum, $\alpha = 2.4\times10^{-5}\ (\text{C°})^{-1}$. Let d_0 be the diameter at –78.0°C and d be the diameter at 23.0°C.

EXECUTE: $d = d_0 + \Delta d = d_0(1 + \alpha\Delta T) = (0.4500\ \text{cm})(1 + (2.4\times10^{-5}(\text{C°})^{-1})(23.0°\text{C} - [-78.0°\text{C}]))$.

$d = 0.4511\ \text{cm} = 4.511\ \text{mm}$.

EVALUATE: We could have let d_0 be the diameter at $23.0°\text{C}$ and d be the diameter at $-78.0°\text{C}$. Then $\Delta T = -78.0°\text{C} - 23.0°\text{C}$.

17.19. IDENTIFY: Apply $L = L_0(1 + \alpha\Delta T)$ to the diameter D of the penny.

SET UP: $1\ \text{K} = 1\ \text{C°}$, so we can use temperatures in °C.

EXECUTE: Death Valley: $\alpha D_0 \Delta T = (2.6\times10^{-5}\ (\text{C°})^{-1})(1.90\ \text{cm})(28.0\ \text{C°}) = 1.4\times10^{-3}\ \text{cm}$, so the diameter is 1.9014 cm. Greenland: $\alpha D_0 \Delta T = -3.6\times10^{-3}\ \text{cm}$, so the diameter is 1.8964 cm.

EVALUATE: When T increases the diameter increases and when T decreases the diameter decreases.

17.20. IDENTIFY: $\Delta V = \beta V_0 \Delta T$. Use the diameter at $-15°\text{C}$ to calculate the value of V_0 at that temperature.

SET UP: For a hemisphere of radius R, the volume is $V = \frac{2}{3}\pi R^3$. Table 17.2 gives $\beta = 7.2\times10^{-5}\ (\text{C°})^{-1}$ for aluminum.

EXECUTE: $V_0 = \frac{2}{3}\pi R^3 = \frac{2}{3}\pi(27.5\ \text{m})^3 = 4.356\times10^4\ \text{m}^3$.

$\Delta V = (7.2\times10^{-5}\ (\text{C°})^{-1})(4.356\times10^4\ \text{m}^3)(35°\text{C} - [-15°\text{C}]) = 160\ \text{m}^3$

EVALUATE: We could also calculate $R = R_0(1 + \alpha\Delta T)$ and calculate the new V from R. The increase in volume is $V - V_0$, but we would have to be careful to avoid round-off errors when two large volumes of nearly the same size are subtracted.

17.21. IDENTIFY: Linear expansion; apply Eq.(17.6) and solve for α.

SET UP: Let $L_0 = 40.125\ \text{cm}$; $T_0 = 20.0°\text{C}$. $\Delta T = 45.0°\text{C} - 20.0°\text{C} = 25.0\ \text{C°}$ gives $\Delta L = 0.023\ \text{cm}$

EXECUTE: $\Delta L = \alpha L_0 \Delta T$ implies $\alpha = \frac{\Delta L}{L_0 \Delta T} = \frac{0.023\ \text{cm}}{(40.125\ \text{cm})(25.0\ \text{C°})} = 2.3\times10^{-5}\ (\text{C°})^{-1}$.

EVALUATE: The value we calculated is the same order of magnitude as the values for metals in Table 17.1.

17.22. **IDENTIFY:** Apply $\Delta V = V_0\beta\Delta T$.

SET UP: For copper, $\beta = 5.1\times10^{-5}\ (\text{C}°)^{-1}$. $\Delta V/V_0 = 0.150\times10^{-2}$.

EXECUTE: $\Delta T = \dfrac{\Delta V/V_0}{\beta} = \dfrac{0.150\times10^{-2}}{5.1\times10^{-5}\ (\text{C}°)^{-1}} = 29.4\ \text{C}°$. $T_\text{f} = T_\text{i} + \Delta T = 49.4°\text{C}$.

EVALUATE: The volume increases when the temperature increases.

17.23. **IDENTIFY:** Volume expansion; apply Eq.(17.8) to calculate ΔV for the ethanol.

SET UP: From Table 17.2, β for ethanol is $75\times10^{-5}\ \text{K}^{-1}$

EXECUTE: $\Delta T = 10.0°\text{C} - 19.0°\text{C} = -9.0\ \text{K}$. Then $\Delta V = \beta V_0\Delta T = (75\times10^{-5}\ \text{K}^{-1})(1700\ \text{L})(-9.0\ \text{K}) = -11\ \text{L}$. The volume of the air space will be $11\ \text{L} = 0.011\ \text{m}^3$.

EVALUATE: The temperature decreases, so the volume of the liquid decreases. The volume change is small, less than 1% of the original volume.

17.24. **IDENTIFY:** Apply $\Delta V = V_0\beta\Delta T$ to the tank and to the ethanol.

SET UP: For ethanol, $\beta_\text{e} = 75\times10^{-5}\ (\text{C}°)^{-1}$. For steel, $\beta_\text{s} = 3.6\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: The volume change for the tank is

$\Delta V_\text{s} = V_0\beta_\text{s}\Delta T = (2.80\ \text{m}^3)(3.6\times10^{-5}\ (\text{C}°)^{-1})(-14.0\ \text{C}°) = -1.41\times10^{-3}\ \text{m}^3 = -1.41\ \text{L}$.

The volume change for the ethanol is

$\Delta V_\text{e} = V_0\beta_\text{e}\Delta T = (2.80\ \text{m}^3)(75\times10^{-5}\ (\text{C}°)^{-1})(-14.0\ \text{C}°) = -2.94\times10^{-2}\ \text{m}^3 = -29.4\ \text{L}$.

The empty volume in the tank is $\Delta V_\text{e} - \Delta V_\text{s} = -29.4\ \text{L} - (-1.4\ \text{L}) = -28.0\ \text{L}$. 28.0 L of ethanol can be added to the tank.

EVALUATE: Both volumes decrease. But $\beta_\text{e} > \beta_\text{s}$, so the magnitude of the volume decrease for the ethanol is less than it is for the tank.

17.25. **IDENTIFY:** Apply $\Delta V = V_0\beta\Delta T$ to the volume of the flask and to the mercury. When heated, both the volume of the flask and the volume of the mercury increase.

SET UP: For mercury, $\beta_\text{Hg} = 18\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: $8.95\ \text{cm}^3$ of mercury overflows, so $\Delta V_\text{Hg} - \Delta V_\text{glass} = 8.95\ \text{cm}^3$.

EXECUTE: $\Delta V_\text{Hg} = V_0\beta_\text{Hg}\Delta T = (1000.00\ \text{cm}^3)(18\times10^{-5}\ (\text{C}°)^{-1})(55.0\ \text{C}°) = 9.9\ \text{cm}^3$.

$\Delta V_\text{glass} = \Delta V_\text{Hg} - 8.95\ \text{cm}^3 = 0.95\ \text{cm}^3$. $\beta_\text{glass} = \dfrac{\Delta V_\text{glass}}{V_0\Delta T} = \dfrac{0.95\ \text{cm}^3}{(1000.00\ \text{cm}^3)(55.0\ \text{C}°)} = 1.7\times10^{-5}\ (\text{C}°)^{-1}$.

EVALUATE: The coefficient of volume expansion for the mercury is larger than for glass. When they are heated, both the volume of the mercury and the inside volume of the flask increase. But the increase for the mercury is greater and it no longer all fits inside the flask.

17.26. **IDENTIFY:** Apply $\Delta L = L_0\alpha\Delta T$ to each linear dimension of the surface.

SET UP: The area can be written as $A = aL_1L_2$, where a is a constant that depends on the shape of the surface. For example, if the object is a sphere, $a = 4\pi$ and $L_1 = L_2 = r$. If the object is a cube, $a = 6$ and $L_1 = L_2 = L$, the length of one side of the cube. For aluminum, $\alpha = 2.4\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: **(a)** $A_0 = aL_{01}L_{02}$. $L_1 = L_{01}(1+\alpha\Delta T)$. $L_2 = L_{02}(1+\alpha\Delta T)$.

$A = aL_1L_2 = aL_{01}L_{02}(1+\alpha\Delta T)^2 = A_0(1 + 2\alpha\Delta T + [\alpha\Delta T]^2)$. $\alpha\Delta T$ is very small, so $[\alpha\Delta T]^2$ can be neglected and $A = A_0(1+2\alpha\Delta T)$. $\Delta A = A - A_0 = (2\alpha)A_0\Delta T$

(b) $\Delta A = (2\alpha)A_0\Delta T = (2)(2.4\times10^{-5}\ (\text{C}°)^{-1})(\pi(0.275\ \text{m})^2)(12.5\ \text{C}°) = 1.4\times10^{-4}\ \text{m}^2$

EVALUATE: The derivation assumes the object expands uniformly in all directions.

17.27. **IDENTIFY** and **SET UP:** Apply the result of Exercise 17.26a to calculate ΔA for the plate, and then $A = A_0 + \Delta A$.

EXECUTE: **(a)** $A_0 = \pi r_0^2 = \pi(1.350\ \text{cm}/2)^2 = 1.431\ \text{cm}^2$

(b) Exercise 17.26 says $\Delta A = 2\alpha A_0\Delta T$, so $\Delta A = 2(1.2\times10^{-5}\ \text{C}^{\circ-1})(1.431\ \text{cm}^2)(175°\text{C} - 25°\text{C}) = 5.15\times10^{-3}\ \text{cm}^2$

$A = A_0 + \Delta A = 1.436\ \text{cm}^2$

EVALUATE: A hole in a flat metal plate expands when the metal is heated just as a piece of metal the same size as the hole would expand.

17.28. **IDENTIFY:** Apply $\Delta L = L_0\alpha\Delta T$ to the diameter D_ST of the steel cylinder and the diameter D_BR of the brass piston.

SET UP: For brass, $\alpha_\text{BR} = 2.0\times10^{-5}\ (\text{C}°)^{-1}$. For steel, $\alpha_\text{ST} = 1.2\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: **(a)** No, the brass expands more than the steel.

(b) Call D_0 the inside diameter of the steel cylinder at 20°C. At 150°C, $D_{ST} = D_{BR}$.

$D_0 + \Delta D_{ST} = 25.000 \text{ cm} + \Delta D_{BR}$. This gives $D_0 + \alpha_{ST} D_0 \Delta T = 25.000 \text{ cm} + \alpha_{BR}(25.000 \text{ cm})\Delta T$.

$$D_0 = \frac{25.000 \text{ cm}(1 + \alpha_{BR}\Delta T)}{1 + \alpha_{ST}\Delta T} = \frac{(25.000 \text{ cm})\left[1 + (2.0\times10^{-5} \ (\text{C}°)^{-1})(130 \text{ C}°)\right]}{1 + (1.2\times10^{-5} \ (\text{C}°)^{-1})(130 \text{ C}°)} = 25.026 \text{ cm.}$$

EVALUATE: The space inside the steel cylinder expands just like a solid piece of steel of the same size.

17.29. **IDENTIFY:** Find the change ΔL in the diameter of the lid. The diameter of the lid expands according to Eq.(17.6).

SET UP: Assume iron has the same α as steel, so $\alpha = 1.2\times10^{-5} \ (\text{C}°)^{-1}$.

EXECUTE: $\Delta L = \alpha L_0 \Delta T = (1.2\times10^{-5} \ (\text{C}°)^{-1})(725 \text{ mm})(30.0 \text{ C}°) = 0.26 \text{ mm}$

EVALUATE: In Eq.(17.6), ΔL has the same units as L.

17.30. **IDENTIFY:** Apply Eq.(17.12) and solve for F.

SET UP: For brass, $Y = 0.9\times10^{11}$ Pa and $\alpha = 2.0\times10^{-5} \ (\text{C}°)^{-1}$.

EXECUTE: $F = -Y\alpha\Delta T A = -(0.9\times10^{11} \text{ Pa})(2.0\times10^{-5}(\text{C}°)^{-1})(-110 \text{ C}°)(2.01\times10^{-4} \text{ m}^2) = 4.0\times10^4 \text{ N}$

EVALUATE: A large force is required. ΔT is negative and a positive tensile force is required.

17.31. **IDENTIFY** and **SET UP:** For part (a), apply Eq.(17.6) to the linear expansion of the wire. For part (b), apply Eq.(17.12) and calculate F/A.

EXECUTE: **(a)** $\Delta L = \alpha L_0 \Delta T$

$$\alpha = \frac{\Delta L}{L_0 \Delta T} = \frac{1.9\times10^{-2} \text{ m}}{(1.50 \text{ m})(420°\text{C} - 20°\text{C})} = 3.2\times10^{-5} \ (\text{C}°)^{-1}$$

(b) Eq.(17.12): stress $F/A = -Y\alpha\Delta T$

$\Delta T = 20°\text{C} - 420°\text{C} = -400 \text{ C}°$ (ΔT always means final temperature minus initial temperature)

$F/A = -(2.0\times10^{11} \text{ Pa})(3.2\times10^{-5}(\text{C}°)^{-1})(-400 \text{ C}°) = +2.6\times10^9 \text{ Pa}$

EVALUATE: F/A is positive means that the stress is a tensile (stretching) stress. The answer to part (a) is consistent with the values of α for metals in Table 17.1. The tensile stress for this modest temperature decrease is huge.

17.32. **IDENTIFY:** Apply $\Delta L = L_0 \alpha \Delta T$ and $\text{stress} = F/A = -Y\alpha\Delta T$.

SET UP: For steel, $\alpha = 1.2\times10^{-5} \ (\text{C}°)^{-1}$ and $Y = 2.0\times10^{11}$ Pa.

EXECUTE: **(a)** $\Delta L = L_0 \alpha \Delta T = (12.0 \text{ m})(1.2\times10^{-5} \ (\text{C}°)^{-1})(35.0 \text{ C}°) = 5.0 \text{ mm}$

(b) $\text{stress} = -Y\alpha\Delta T = -(2.0\times10^{11} \text{ Pa})(1.2\times10^{-5} \ (\text{C}°)^{-1})(35.0 \text{ C}°) = -8.4\times10^7 \text{ Pa}$. The minus sign means the stress is compressive.

EVALUATE: Commonly occurring temperature changes result in very small fractional changes in length but very large stresses if the length change is prevented from occurring.

17.33. **IDENTIFY** and **SET UP:** Apply Eq.(17.13) to the kettle and water.

EXECUTE: <u>kettle</u>

$Q = mc\Delta T$, $c = 910 \text{ J/kg}\cdot\text{K}$ (from Table 17.3)

$Q = (1.50 \text{ kg})(910 \text{ J/kg}\cdot\text{K})(85.0°\text{C} - 20.0°\text{C}) = 8.873\times10^4 \text{ J}$

<u>water</u>

$Q = mc\Delta T$, $c = 4190 \text{ J/kg}\cdot\text{K}$ (from Table 17.3)

$Q = (1.80 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(85.0°\text{C} - 20.0°\text{C}) = 4.902\times10^5 \text{ J}$

Total $Q = 8.873\times10^4 \text{ J} + 4.902\times10^5 \text{ J} = 5.79\times10^5 \text{ J}$

EVALUATE: Water has a much larger specific heat capacity than aluminum, so most of the heat goes into raising the temperature of the water.

17.34. **IDENTIFY:** The heat required is $Q = mc\Delta T$. $P = 200 \text{ W} = 200 \text{ J/s}$, which is energy divided by time.

SET UP: For water, $c = 4.19\times10^3 \text{ J/kg}\cdot\text{K}$.

EXECUTE: **(a)** $Q = mc\Delta T = (0.320 \text{ kg})(4.19\times10^3 \text{ J/kg}\cdot\text{K})(60.0 \text{ C}°) = 8.04\times10^4 \text{ J}$

(b) $t = \dfrac{8.04\times10^4 \text{ J}}{200.0 \text{ J/s}} = 402 \text{ s} = 6.7 \text{ min}$

EVALUATE: 0.320 kg of water has volume 0.320 L. The time we calculated in part (b) is consistent with our everyday experience.

17.35. **IDENTIFY:** Apply $Q = mc\Delta T$. $m = w/g$.

SET UP: The temperature change is $\Delta T = 18.0$ K.

EXECUTE: $c = \dfrac{Q}{m\Delta T} = \dfrac{gQ}{w\Delta T} = \dfrac{(9.80 \text{ m/s}^2)(1.25\times10^4 \text{ J})}{(28.4 \text{ N})(18.0 \text{ K})} = 240 \text{ J/kg}\cdot\text{K}.$

EVALUATE: The value for c is similar to that for silver in Table 17.3, so it is a reasonable result.

17.36. **IDENTIFY** and **SET UP:** Use Eq.(17.13)

EXECUTE: **(a)** $Q = mc\Delta T$

$m = \frac{1}{2}(1.3\times10^{-3}\text{ kg}) = 0.65\times10^{-3}\text{ kg}$

$Q = (0.65\times10^{-3}\text{ kg})(1020\text{ J/kg}\cdot\text{K})(37°\text{C} - (-20°\text{C})) = 38\text{ J}$

(b) 20 breaths/min (60 min/1 h) = 1200 breaths/h

So $Q = (1200)(38\text{ J}) = 4.6\times10^4\text{ J}$.

EVALUATE: The heat loss rate is $Q/t = 13\text{ W}$.

17.37. **IDENTIFY:** Apply $Q = mc\Delta T$ to find the heat that would raise the temperature of the student's body $7\text{ C}°$.

SET UP: $1\text{ W} = 1\text{ J/s}$

EXECUTE: Find Q to raise the body temperature from $37°\text{C}$ to $44°\text{C}$.

$Q = mc\Delta T = (70\text{ kg})(3480\text{ J/kg}\cdot\text{K})(7\text{ C}°) = 1.7\times10^6\text{ J}$.

$$t = \frac{1.7\times10^6\text{ J}}{1200\text{ J/s}} = 1400\text{ s} = 23\text{ min}.$$

EVALUATE: Heat removal mechanisms are essential to the well-being of a person.

17.38. **IDENTIFY** and **SET UP:** Set the change in gravitational potential energy equal to the quantity of heat added to the water.

EXECUTE: The change in mechanical energy equals the decrease in gravitational potential energy, $\Delta U = -mgh$; $|\Delta U| = mgh$. $Q = |\Delta U| = mgh$ implies $mc\Delta T = mgh$

$\Delta T = gh/c = (9.80\text{ m/s}^2)(225\text{ m})/(4190\text{ J/kg}\cdot\text{K}) = 0.526\text{ K} = 0.526\text{ C}°$

EVALUATE: Note that the answer is independent of the mass of the object. Note also the small change in temperature that corresponds to this large change in height!

17.39. **IDENTIFY:** The work done by friction is the loss of mechanical energy. The heat input for a temperature change is $Q = mc\Delta T$

SET UP: The crate loses potential energy mgh, with $h = (8.00\text{ m})\sin 36.9°$, and gains kinetic energy $\frac{1}{2}mv_2^2$.

EXECUTE: **(a)** $W_f = mgh - \frac{1}{2}mv_2^2 = (35.0\text{ kg})\left((9.80\text{ m/s}^2)(8.00\text{ m})\sin 36.9° - \frac{1}{2}(2.50\text{ m/s})^2\right) = 1.54\times10^3\text{ J}$.

(b) Using the results of part (a) for Q gives $\Delta T = (1.54\times10^3\text{ J})/\left((35.0\text{ kg})(3650\text{ J/kg}\cdot\text{K})\right) = 1.21\times10^{-2}\text{ C}°$.

EVALUATE: The temperature rise is very small.

17.40. **IDENTIFY:** The work done by the brakes equals the initial kinetic energy of the train. Use the volume of the air to calculate its mass. Use $Q = mc\Delta T$ applied to the air to calculate ΔT for the air.

SET UP: $K = \frac{1}{2}mv^2$. $m = \rho V$.

EXECUTE: The initial kinetic energy of the train is $K = \frac{1}{2}(25{,}000\text{ kg})(15.5\text{ m/s})^2 = 3.00\times10^6\text{ J}$. Therefore, Q for the air is $3.00\times10^6\text{ J}$. $m = \rho V = (1.20\text{ kg/m}^3)(65.0\text{ m})(20.0\text{ m})(12.0\text{ m}) = 1.87\times10^4\text{ kg}$. $Q = mc\Delta T$ gives

$$\Delta T = \frac{Q}{mc} = \frac{3.00\times10^6\text{ J}}{(1.87\times10^4\text{ kg})(1020\text{ J/kg}\cdot\text{K})} = 0.157\text{ C}°.$$

EVALUATE: The mass of air in the station is comparable to the mass of the train and the temperature rise is small.

17.41. **IDENTIFY:** Set $K = \frac{1}{2}mv^2$ equal to $Q = mc\Delta T$ for the nail and solve for ΔT.

SET UP: For aluminum, $c = 0.91\times10^3\text{ J/kg}\cdot\text{K}$.

EXECUTE: The kinetic energy of the hammer before it strikes the nail is $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.80\text{ kg})(7.80\text{ m/s})^2 = 54.8\text{ J}$. Each strike of the hammer transfers $0.60(54.8\text{ J}) = 32.9\text{ J}$, and with 10 strikes $Q = 329\text{ J}$. $Q = mc\Delta T$ and $\Delta T = \dfrac{Q}{mc} = \dfrac{329\text{ J}}{(8.00\times10^{-3}\text{ kg})(0.91\times10^3\text{ J/kg}\cdot\text{K})} = 45.2\text{ C}°$

EVALUATE: This agrees with our experience that hammered nails get noticeably warmer.

17.42. **IDENTIFY** and **SET UP:** Use the power and time to calculate the heat input Q and then use Eq.(17.13) to calculate c.

(a) EXECUTE: $P = Q/t$, so the total heat transferred to the liquid is $Q = Pt = (65.0\text{ W})(120\text{ s}) = 7800\text{ J}$

Then $Q = mc\Delta T$ gives $c = \dfrac{Q}{m\Delta T} = \dfrac{7800\text{ K}}{0.780\text{ kg}(22.54°\text{C} - 18.55°\text{C})} = 2.51\times10^3\text{ J/kg}\cdot\text{K}$

(b) EVALUATE: Then the actual Q transferred to the liquid is less than 7800 J so the actual c is less than our calculated value; our result in part (a) is an overestimate.

17.43. **IDENTIFY:** $Q = mc\Delta T$. The mass of n moles is $m = nM$.

SET UP: For iron, $M = 55.845\times10^{-3}$ kg/mol and $c = 470$ J/kg·K.

EXECUTE: **(a)** The mass of 3.00 mol is $m = nM = (3.00 \text{ mol})(55.845\times10^{-3} \text{ kg/mol}) = 0.1675$ kg.

$\Delta T = Q/mc = (8950 \text{ J})/\left[(0.1675 \text{ kg})(470 \text{ J/kg}\cdot\text{K})\right] = 114 \text{ K} = 114 \text{ C}°$.

(b) For $m = 3.00$ kg, $\Delta T = Q/mc = 6.35$ C°.

EVALUATE: **(c)** The result of part (a) is much larger; 3.00 kg is more material than 3.00 mol.

17.44. **IDENTIFY:** The latent heat of fusion L_f is defined by $Q = mL_f$ for the solid → liquid phase transition. For a temperature change, $Q = mc\Delta T$.

SET UP: At $t = 1$ min the sample is at its melting point and at $t = 2.5$ min all the sample has melted.

EXECUTE: **(a)** It takes 1.5 min for all the sample to melt once its melting point is reached and the heat input during this time interval is $(1.5 \text{ min})(10.0\times10^3 \text{ J/min}) = 1.50\times10^4$ J. $Q = mL_f$.

$$L_f = \frac{Q}{m} = \frac{1.50\times10^4 \text{ J}}{0.500 \text{ kg}} = 3.00\times10^4 \text{ J/kg}.$$

(b) The liquid's temperature rises 30 C° in 1.5 min. $Q = mc\Delta T$.

$$c_{\text{liquid}} = \frac{Q}{m\Delta T} = \frac{1.50\times10^4 \text{ J}}{(0.500 \text{ kg})(30 \text{ C}°)} = 1.00\times10^3 \text{ J/kg}\cdot\text{K}.$$

The solid's temperature rises 15 C° in 1.0 min. $c_{\text{solid}} = \dfrac{Q}{m\Delta T} = \dfrac{1.00\times10^4 \text{ J}}{(0.500 \text{ kg})(15 \text{ C}°)} = 1.33\times10^3 \text{ J/kg}\cdot\text{K}.$

EVALUATE: The specific heat capacities for the liquid and solid states are different. The values of c and L_f that we calculated are within the range of values in Tables 17.3 and 17.4.

17.45. **IDENTIFY** and **SET UP:** Heat comes out of the metal and into the water. The final temperature is in the range $0 < T < 100°\text{C}$, so there are no phase changes. $Q_{\text{system}} = 0$.

(a) EXECUTE: $Q_{\text{water}} + Q_{\text{metal}} = 0$

$m_{\text{water}}c_{\text{water}}\Delta T_{\text{water}} + m_{\text{metal}}c_{\text{metal}}\Delta T_{\text{metal}} = 0$

$(1.00 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(2.0 \text{ C}°) + (0.500 \text{ kg})(c_{\text{metal}})(-78.0 \text{ C}°) = 0$

$c_{\text{metal}} = 215 \text{ J/kg}\cdot\text{K}$

(b) EVALUATE: Water has a larger specific heat capacity so stores more heat per degree of temperature change.
(c) If some heat went into the styrofoam then Q_{metal} should actually be larger than in part (a), so the true c_{metal} is larger than we calculated; the value we calculated would be smaller than the true value.

17.46. **IDENTIFY:** Apply $Q = mc\Delta T$ to each object. The net heat flow Q_{system} for the system (man, soft drink) is zero.

SET UP: The mass of 1.00 L of water is 1.00 kg. Let the man be designated by the subscript m and the "'water" by w. T is the final equilibrium temperature. $c_w = 4190$ J/kg·K. $\Delta T_K = \Delta T_C$.

EXECUTE: **(a)** $Q_{\text{system}} = 0$ gives $m_m C_m \Delta T_m + m_w C_w \Delta T_w = 0$. $m_m C_m (T - T_m) + m_w C_w (T - T_w) = 0$.

$m_m C_m (T_m - T) = m_w C_w (T - T_w)$. Solving for T, $T = \dfrac{m_m C_m T_m + m_w C_w T_w}{m_m C_m + m_w C_w}$.

$$T = \frac{(70.0 \text{ kg})\,(3480 \text{ J/kg}\cdot\text{K})\,(37.0°\text{C}) + (0.355 \text{ kg})\,(4190 \text{ J/kg}\cdot\text{C}°)\,(12.0°\text{C})}{(70.0 \text{ kg})(3480 \text{ J/kg}\cdot\text{C}°) + (0.355 \text{ kg})\,(4190 \text{ J/kg}\cdot\text{C}°)} = 36.85°\text{C}$$

(b) It is possible a sensitive digital thermometer could measure this change since they can read to 0.1°C. It is best to refrain from drinking cold fluids prior to orally measuring a body temperature due to cooling of the mouth.
EVALUATE: Heat comes out of the body and its temperature falls. Heat goes into the soft drink and its temperature rises.

17.47. **IDENTIFY:** For the man's body, $Q = mc\Delta T$.

SET UP: From Exercise 17.46, $\Delta T = 0.15$ C° when the body returns to 37.0°C.

EXECUTE: The rate of heat loss is Q/t. $\dfrac{Q}{t} = \dfrac{mC\Delta T}{t}$ and $t = \dfrac{mC\Delta T}{(Q/t)}$.

$$t = \frac{(70.355 \text{ kg})(3480 \text{ J/kg}\cdot\text{C}°)(0.15 \text{ C}°)}{7.00\times10^6 \text{ J/day}} = 0.00525 \text{ d} = 7.6 \text{ minutes}.$$

EVALUATE: Even if all the BMR energy stays in the body, it takes the body several minutes to return to its normal temperature.

17.48. **IDENTIFY:** For a temperature change $Q = mc\Delta T$ and for the liquid to solid phase change $Q = -mL_f$.

SET UP: For water, $c = 4.19\times10^3$ J/kg·K and $L_f = 3.34\times10^5$ J/kg.

EXECUTE: $Q = mc\Delta T - mL_f = (0.350 \text{ kg})([4.19\times10^3 \text{ J/kg}\cdot\text{K}][-18.0 \text{ C}°] - 3.34\times10^5 \text{ J/kg}) = -1.43\times10^5$ J. The minus sign says 1.43×10^5 J must be removed from the water. $(1.43\times10^5 \text{ J})\left(\frac{1 \text{ cal}}{4.186 \text{ J}}\right) = 3.42\times10^4 \text{ cal} = 34.2 \text{ kcal}$.

EVALUATE: $Q < 0$ when heat comes out of an object the equation $Q = mc\Delta T$ puts in the correct sign automatically, from the sign of $\Delta T = T_f - T_i$. But in $Q = \pm L$ we must select the correct sign.

17.49. **IDENTIFY** and **SET UP:** Use Eq.(17.13) for the temperature changes and Eq.(17.20) for the phase changes.
EXECUTE: Heat must be added to do the following
ice at $-10.0°\text{C} \to$ ice at 0°C

$Q_{ice} = mc_{ice}\Delta T = (12.0\times10^{-3} \text{ kg})(2100 \text{ J/kg}\cdot\text{K})(0°\text{C} - (-10.0°\text{C})) = 252 \text{ J}$

phase transition ice (0°C) $\to$ liquid water (0°C) (melting)

$Q_{melt} = +mL_f = (12.0\times10^{-3} \text{ kg})(334\times10^3 \text{ J/kg}) = 4.008\times10^3 \text{ J}$

water at 0°C (from melted ice $\to$ water at 100°C

$Q_{water} = mc_{water}\Delta T = (12.0\times10^{-3} \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(100°\text{C} - 0°\text{C}) = 5.028\times10^3 \text{ J}$

phase transition water (100°C) $\to$ steam (100°C) (boiling)

$Q_{boil} = +mL_v = (12.0\times10^{-3} \text{ kg})(2256\times10^3 \text{ J/kg}) = 2.707\times10^4 \text{ J}$

The total Q is $Q = 252 \text{ J} + 4.008\times10^3 \text{ J} + 5.028\times10^3 \text{ J} + 2.707\times10^4 \text{ J} = 3.64\times10^4 \text{ J}$

$(3.64\times10^4 \text{ J})(1 \text{ cal}/4.186 \text{ J}) = 8.70\times10^3 \text{ cal}$

$(3.64\times10^4 \text{ J})(1 \text{ Btu}/1055 \text{ J}) = 34.5 \text{ Btu}$

EVALUATE: Q is positive and heat must be added to the material. Note that more heat is needed for the liquid to gas phase change than for the temperature changes.

17.50. **IDENTIFY:** $Q = mc\Delta T$ for a temperature change and $Q = +mL_f$ for the solid to liquid phase transition. The ice starts to melt when its temperature reaches 0.0°C. The system stays at 0.00°C until all the ice has melted.

SET UP: For ice, $c = 2.01\times10^3$ J/kg·K. For water, $L_f = 3.34\times10^5$ J/kg.

EXECUTE: **(a)** Q to raise the temperature of ice to 0.00°C:

$Q = mc\Delta T = (0.550 \text{ kg})(2.01\times10^3 \text{ J/kg}\cdot\text{K})(15.0 \text{ C}°) = 1.66\times10^4$ J. $t = \frac{1.66\times10^4 \text{ J}}{800.0 \text{ J/min}} = 20.8$ min.

(b) To melt all the ice requires $Q = mL_f = (0.550 \text{ kg})(3.34\times10^5 \text{ J/kg}) = 1.84\times10^5$ J. $t = \frac{1.84\times10^5 \text{ J}}{800.0 \text{ J/min}} = 230$ min.

The total time after the start of the heating is 251 min.
(c) A graph of T versus t is sketched in Figure 17.50.
EVALUATE: It takes much longer for the ice to melt than it takes the ice to reach the melting point.

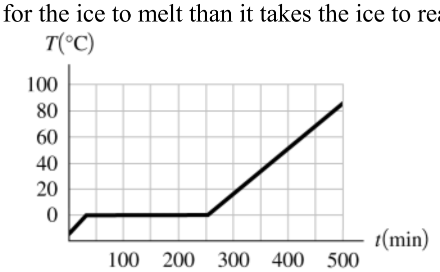


Figure 17.50

17.51. **IDENTIFY** and **SET UP:** Use Eq.(17.20) to calculate Q and then $P = Q/t$. Must convert the quantity of ice from lb to kg.

EXECUTE: "two-ton air conditioner" means 2 tons (4000 lbs) of ice can be frozen from water at 0°C in 24 h. Find the mass m that corresponds to 4000 lb (weight of water): $m = (4000 \text{ lb})(1 \text{ kg}/2.205 \text{ lb}) = 1814$ kg (The kg to lb equivalence from Appendix E has been used.) The heat that must be removed from the water to freeze it is $Q = -mL_f = -(1814 \text{ kg})(334\times10^3 \text{ J/kg}) = -6.06\times10^8$ J. The power required if this is to be done in 24 hours is

$P = \frac{|Q|}{t} = \frac{6.06\times10^8 \text{ J}}{(24 \text{ h})(3600 \text{ s/1 h})} = 7010$ W or $P = (7010 \text{ W})((1 \text{ Btu/h})/(0.293 \text{ W})) = 2.39\times10^4$ Btu/h.

EVALUATE: The calculated power, the rate at which heat energy is removed by the unit, is equivalent to seventy 100-W light bulbs.

17.52. **IDENTIFY:** For a temperature change, $Q = mc\Delta T$. For the vapor $\rightarrow$ liquid phase transition, $Q = -mL_v$.

SET UP: For water, $L_v = 2.256 \times 10^6$ J/kg and $c = 4.19 \times 10^3$ J/kg·K.

EXECUTE: **(a)** $Q = +m(-L_v + c\Delta T)$

$Q = +(25.0 \times 10^{-3}\text{ kg})(-2.256 \times 10^6\text{ J/kg} + [4.19 \times 10^3\text{ J/kg}\cdot\text{K}][-66.0\text{ C}°]) = -6.33 \times 10^4\text{ J}$

(b) $Q = mc\Delta T = (25.0 \times 10^{-3}\text{ kg})(4.19 \times 10^3\text{ J/kg}\cdot\text{K})(-66.0\text{ C}°) = -6.91 \times 10^3\text{ J}.$

(c) The total heat released by the water that starts as steam is nearly a factor of ten larger than the heat released by water that starts at 100°C. Steam burns are much more severe than hot-water burns.

EVALUATE: For a given amount of material, the heat for a phase change is typically much more than the heat for a temperature change.

17.53. **IDENTIFY** and **SET UP:** The heat that must be added to a lead bullet of mass *m* to melt it is $Q = mc\Delta t + mL_f$ ($mc\Delta T$ is the heat required to raise the temperature from 25°C to the melting point of 327.3°C; mL_f is the heat required to make the solid $\rightarrow$ liquid phase change.) The kinetic energy of the bullet if its speed is *v* is $K = \frac{1}{2}mv^2$.

EXECUTE: $K = Q$ says $\frac{1}{2}mv^2 = mc\Delta T + mL_f$

$v = \sqrt{2(c\Delta T + L_f)}$

$v = \sqrt{2[(130\text{ J/kg}\cdot\text{K})(327.3°\text{C} - 25°\text{C}) + 24.5 \times 10^3\text{ J/kg}]} = 357\text{ m/s}$

EVALUATE: This is a typical speed for a rifle bullet. A bullet fired into a block of wood does partially melt, but in practice not all of the initial kinetic energy is converted to heat that remains in the bullet.

17.54. **IDENTIFY:** For a temperature change, $Q = mc\Delta T$. For the liquid $\rightarrow$ vapor phase change, $Q = +mL_v$.

SET UP: The density of water is 1000 kg/m^3.

EXECUTE: **(a)** The heat that goes into mass *m* of water to evaporate it is $Q = +mL_v$. The heat flow for the man is $Q = m_{man}c\Delta T$, where $\Delta T = -1.00\text{ C}°$. $\sum Q = 0$ so $mL_v + m_{man}c\Delta T$ and

$$m = -\frac{m_{man}c\Delta T}{L_v} = -\frac{(70.0\text{ kg})(3480\text{ J/kg}\cdot\text{K})(-1.00\text{ C}°)}{2.42 \times 10^6\text{ J/kg}} = 0.101\text{ kg} = 101\text{ g}.$$

(b) $V = \frac{m}{\rho} = \frac{0.101\text{ kg}}{1000\text{ kg/m}^3} = 1.01 \times 10^{-4}\text{ m}^3 = 101\text{ cm}^3$. This is about 28% of the volume of a soft-drink can.

EVALUATE: Fluid loss by evaporation from the skin can be significant.

17.55. **IDENTIFY:** Use $Q = Mc\Delta T$ to find *Q* for a temperature rise from 34.0°C to 40.0°C. Set this equal to $Q = mL_v$ and solve for *m*, where *m* is the mass of water the camel would have to drink.

SET UP: $c = 3480$ J/kg·K and $L_v = 2.42 \times 10^6$ J/kg. For water, 1.00 kg has a volume 1.00 L. $M = 400$ kg is the mass of the camel.

EXECUTE: The mass of water that the camel saves is $m = \frac{Mc\Delta T}{L_v} = \frac{(400\text{ kg})(3480\text{ J/kg}\cdot\text{K})(6.0\text{ K})}{(2.42 \times 10^6\text{ J/kg})} = 3.45\text{ kg}$ which is a volume of 3.45 L.

EVALUATE: This is nearly a gallon of water, so it is an appreciable savings.

17.56. **IDENTIFY:** The asteroid's kinetic energy is $K = \frac{1}{2}mv^2$. To boil the water, its temperature must be raised to 100.0°C and the heat needed for the phase change must be added to the water.

SET UP: For water, $c = 4190$ J/kg·K and $L_v = 2256 \times 10^3$ J/kg.

EXECUTE: $K = \frac{1}{2}(2.60 \times 10^{15}\text{ kg})(32.0 \times 10^3\text{ m/s})^2 = 1.33 \times 10^{24}\text{ J}$. $Q = mc\Delta T + mL_v$.

$$m = \frac{Q}{c\Delta T + L_v} = \frac{1.33 \times 10^{22}\text{ J}}{(4190\text{ J/kg}\cdot\text{K})(90.0\text{ K}) + 2256 \times 10^3\text{ J/kg}} = 5.05 \times 10^{15}\text{ kg}.$$

EVALUATE: The mass of water boiled is 2.5 times the mass of water in Lake Superior.

17.57. **IDENTIFY:** Apply $Q = mc\Delta T$ to the air in the refrigerator and to the turkey.

SET UP: For the air $m_{air} = \rho V$

EXECUTE: $m_{air} = (1.20\text{ kg/m}^3)(1.50\text{ m}^3) = 1.80\text{ kg}$. $Q = m_{air}c_{air}\Delta T + m_t c_t \Delta T$.

$Q = ([1.80\text{ kg}][1020\text{ J/kg}\cdot\text{K}] + [10.0\text{ kg}][3480\text{ J/kg}\cdot\text{K}])(-15.0\text{ C}°) = -5.50 \times 10^5\text{ J}$

EVALUATE: *Q* is negative because heat is removed. 5% of the heat removed comes from the air.

17.58. **IDENTIFY:** $Q = mc\Delta T$ for a temperature change. The net *Q* for the system (sample, can and water) is zero.

SET UP: For water, $c_w = 4.19 \times 10^3$ J/kg·K. For copper, $c_c = 390$ J/kg·K.

EXECUTE: For the water, $Q_w = m_w c_w \Delta T_w = (0.200\text{ kg})(4.19\times10^3\text{ J/kg}\cdot\text{K})(7.1\text{ C}°) = 5.95\times10^3\text{ J}$.

For the copper can, $Q_c = m_c c_c \Delta T_c = (0.150\text{ kg})(390\text{ J/kg}\cdot\text{K})(7.1\text{ C}°) = 415\text{ J}$.

For the sample, $Q_s = m_s c_s \Delta T_s = (0.085\text{ kg})c_s(-73.9\text{ C}°)$.

$\sum Q = 0$ gives $(0.085\text{ kg})(-73.9\text{ C}°)c_s + 415\text{ J} + 5.95\times10^3\text{ J} = 0$. $c_s = 1.01\times10^3\text{ J/kg}\cdot\text{K}$.

EVALUATE: Heat comes out of the sample and goes into the water and the can. The value of c_s we calculated is consistent with the values in Table 17.3.

17.59. **IDENTIFY** and **SET UP:** Heat flows out of the water and into the ice. The net heat flow for the system is zero. The ice warms 0°C, melts, and then the water from the melted ice warms from 0°C to the final temperature.

EXECUTE: $Q_{\text{system}} = 0$; calculate Q for each component of the system: (Beaker has small mass says that $Q = mc\Delta T$ for beaker can be neglected.)

<u>0.250 kg of water</u> (cools from 75.0°C to 30.0°C)

$Q_{\text{water}} = mc\Delta T = (0.250\text{ kg})(4190\text{ J/kg}\cdot\text{K})(30.0°\text{C} - 75.0°\text{C}) = -4.714\times10^4\text{ J}$

<u>ice</u> (warms to 0°C; melts; water from melted ice warms to 30.0°C)

$Q_{\text{ice}} = mc_{\text{ice}}\Delta T + mL_f + mc_{\text{water}}\Delta T$

$Q_{\text{ice}} = m[(2100\text{ J/kg}\cdot\text{K})(0°\text{C} - (-20.0°\text{C})) + 334\times10^3\text{ J/kg} + (4190\text{ J/kg}\cdot\text{K})(30.0°\text{C} - 0°\text{C})]$

$Q_{\text{ice}} = (5.017\times10^5\text{ J/kg})m$

$Q_{\text{system}} = 0$ says $Q_{\text{water}} + Q_{\text{ice}} = 0$

$-4.714\times10^4\text{ J} + (5.017\times10^5\text{ J/kg})m = 0$

$$m = \frac{4.714\times10^4\text{ J}}{5.017\times10^5\text{ J/kg}} = 0.0940\text{ kg}$$

EVALUATE: Since the final temperature is 30.0°C we know that all the ice melts and the final system is all liquid water. The mass of ice added is much less than the mass of the 75°C water; the ice requires a large heat input for the phase change.

17.60. **IDENTIFY:** For a temperature change $Q = mc\Delta T$. For a melting phase transition $Q = mL_f$. The net Q for the system (sample, vial and ice) is zero.

SET UP: Ice remains, so the final temperature is 0.0°C. For water, $L_f = 3.34\times10^5\text{ J/kg}$.

EXECUTE: For the sample, $Q_s = m_s c_s \Delta T_s = (16.0\times10^{-3}\text{ kg})(2250\text{ J/kg}\cdot\text{K})(-19.5\text{ C}°) = -702\text{ J}$. For the vial, $Q_v = m_v c_v \Delta T_v = (6.0\times10^{-3}\text{ kg})(2800\text{ J/kg}\cdot\text{K})(-19.5\text{ C}°) = -328\text{ J}$. For the ice that melts, $Q_i = mL_f$. $\sum Q = 0$ gives $mL_f - 702\text{ J} - 328\text{ J} = 0$ and $m = 3.08\times10^{-3}\text{ kg} = 3.08\text{ g}$.

EVALUATE: Only a small fraction of the ice melts. The water for the melted ice remains at 0°C and has no heat flow.

17.61. **IDENTIFY** and **SET UP:** Large block of ice implies that ice is left, so $T_2 = 0°\text{C}$ (final temperature). Heat comes out of the ingot and into the ice. The net heat flow is zero. The ingot has a temperature change and the ice has a phase change.

EXECUTE: $Q_{\text{system}} = 0$; calculate Q for each component of the system:

<u>ingot</u>

$Q_{\text{ingot}} = mc\Delta T = (4.00\text{ kg})(234\text{ J/kg}\cdot\text{K})(0°\text{C} - 750°\text{C}) = -7.02\times10^5\text{ J}$

<u>ice</u>

$Q_{\text{ice}} = +mL_f$, where m is the mass of the ice that changes phase (melts)

$Q_{\text{system}} = 0$ says $Q_{\text{ingot}} + Q_{\text{ice}} = 0$

$-7.02\times10^5\text{ J} + m(334\times10^3\text{ J/kg}) = 0$

$$m = \frac{7.02\times10^5\text{ J}}{334\times10^3\text{ J/kg}} = 2.10\text{ kg}$$

EVALUATE: The liquid produced by the phase change remains at 0°C since it is in contact with ice.

17.62. **IDENTIFY:** The initial temperature of the ice and water mixture is 0.0°C. Assume all the ice melts. We will know that assumption is incorrect if the final temperature we calculate is less than 0.0°C. The net Q for the system (can, water, ice and lead) is zero.

SET UP: For copper, $c_c = 390\text{ J/kg}\cdot\text{K}$. For lead, $c_l = 130\text{ J/kg}\cdot\text{K}$. For water, $c_w = 4.19\times10^3\text{ J/kg}\cdot\text{K}$ and $L_f = 3.34\times10^5\text{ J/kg}$.

EXECUTE: For the copper can, $Q_c = m_c c_c \Delta T_c = (0.100\text{ kg})(390\text{ J/kg}\cdot\text{K})(T - 0.0°\text{C}) = (39.0\text{ J/K})T.$

For the water, $Q_w = m_w c_w \Delta T_w = (0.160\text{ kg})(4.19\times10^3\text{ J/kg}\cdot\text{K})(T - 0.0°\text{C}) = (670.4\text{ J/K})T.$

For the ice, $Q_i = m_i L_f + m_i c_w \Delta T_w$

$Q_i = (0.018\text{ kg})(3.34\times10^5\text{ J/kg}\cdot\text{K}) + (0.018\text{ kg})(4.19\times10^3\text{ J/kg}\cdot\text{K})(T - 0.0°\text{C}) = 6012\text{ J} + (75.4\text{ J/K})T$

For the lead, $Q_l = m_l c_l \Delta T_l = (0.750\text{ kg})(130\text{ J/kg}\cdot\text{K})(T - 255°\text{C}) = (97.5\text{ J/K})T - 2.486\times10^4\text{ J}$

$\sum Q = 0$ gives $(39.0\text{ J/K})T + (670.4\text{ J/K})T + 6012\text{ J} + (75.4\text{ J/K}) + (97.5\text{ J/K})T - 2.486\times10^4\text{ J} = 0.$

$T = \dfrac{1.885\times10^4\text{ J}}{882.3\text{ J/K}} = 21.4°\text{C}.$

EVALUATE: $T > 0.0°\text{C},$ which confirms that all the ice melts.

17.63. **IDENTIFY:** Set $Q_{\text{system}} = 0,$ for the system of water, ice and steam. $Q = mc\Delta T$ for a temperature change and $Q = \pm mL$ for a phase transition.

SET UP: For water, $c = 4190\text{ J/kg}\cdot\text{K},$ $L_f = 334\times10^3\text{ J/kg}$ and $L_v = 2256\times10^3\text{ J/kg}.$

EXECUTE: The steam both condenses and cools, and the ice melts and heats up along with the original water. $m_i L_f + m_i c(28.0\text{ C°}) + m_w c(28.0\text{ C°}) - m_{\text{steam}} L_v + m_{\text{steam}} c(-72.0\text{ C°}) = 0.$ The mass of steam needed is

$$m_{\text{steam}} = \frac{(0.450\text{ kg})(334\times10^3\text{ J/kg}) + (2.85\text{ kg})(4190\text{ J/kg}\cdot\text{K})(28.0\text{ C°})}{2256\times10^3\text{ J/kg} + (4190\text{ J/kg}\cdot\text{K})(72.0\text{ C°})} = 0.190\text{ kg}.$$

EVALUATE: Since the final temperature is greater than $0.0°\text{C},$ we know that all the ice melts.

17.64. **IDENTIFY:** $H = kA\Delta T/L$ and $k = \dfrac{HL}{A\Delta T}.$

SET UP: The SI units of H are watts, the units of area are $\text{m}^2,$ the temperature difference is in K, the length is in meters, so the SI units for thermal conductivity are $\dfrac{[\text{W}][\text{m}]}{[\text{m}^2][\text{K}]} = \dfrac{\text{W}}{\text{m}\cdot\text{K}}.$

EVALUATE: An equivalent way to express the units of k is $\text{J/(s}\cdot\text{m}\cdot\text{K)}.$

17.65. **IDENTIFY** and **SET UP:** The temperature gradient is $(T_H - T_C)/L$ and can be calculated directly. Use Eq.(17.21) to calculate the heat current H. In part (c) use H from part (b) and apply Eq.(17.21) to the 12.0-cm section of the left end of the rod. $T_2 = T_H$ and $T_1 = T,$ the target variable.

EXECUTE: **(a)** temperature gradient $= (T_H - T_C)/L = (100.0°\text{C} - 0.0°\text{C})/0.450\text{ m} = 222\text{ C°/m} = 222\text{ K/m}$

(b) $H = kA(T_H - T_C)/L.$ From Table 17.5, $k = 385\text{ W/m}\cdot\text{K},$ so

$H = (385\text{ W/m}\cdot\text{K})(1.25\times10^{-4}\text{ m}^2)(222\text{ K/m}) = 10.7\text{ W}$

(c) $H = 10.7\text{ W}$ for all sections of the rod.

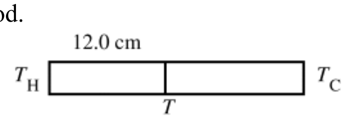


Figure 17.65

Apply $H = ka\Delta T/L$ to the 12.0 cm section (Figure 17.65): $T_H - T = LH/kA$ and

$T = T_H - LH/Ak = 100.0°\text{C} - \dfrac{(0.120\text{ m})(10.7\text{ W})}{(1.25\times10^{-4}\text{ m}^2)(385\text{ W/m}\cdot\text{K})} = 73.3°\text{C}$

EVALUATE: H is the same at all points along the rod, so $\Delta T/\Delta x$ is the same for any section of the rod with length $\Delta x.$ Thus $(T_H - T)/(12.0\text{ cm}) = (T_H - T_C)/(45.0\text{ cm})$ gives that $T_H = T = 26.7\text{ C°}$ and $T = 73.3°\text{C},$ as we already calculated.

17.66. **IDENTIFY:** For a melting phase transition, $Q = mL_f.$ The rate of heat conduction is $\dfrac{Q}{t} = \dfrac{kA(T_H - T_C)}{L}.$

SET UP: For water, $L_f = 3.34\times10^5\text{ J/kg}.$

EXECUTE: The heat conducted by the rod in 10.0 min is

$Q = mL_f = (8.50\times10^{-3}\text{ kg})(3.34\times10^5\text{ J/kg}) = 2.84\times10^3\text{ J}.$ $\dfrac{Q}{t} = \dfrac{2.84\times10^3\text{ J}}{600\text{ s}} = 4.73\text{ W}.$

$k = \dfrac{(Q/t)L}{A(T_H - T_C)} = \dfrac{(4.73\text{ W})(0.600\text{ m})}{(1.25\times10^{-4}\text{ m}^2)(100\text{ C°})} = 227\text{ W/m}\cdot\text{K}.$

EVALUATE: The heat conducted by the rod is the heat that enters the ice and produces the phase change.

17.67. **IDENTIFY** and **SET UP:** Call the temperature at the interface between the wood and the styrofoam *T*. The heat current in each material is given by $H = kA(T_H - T_C)/L.$

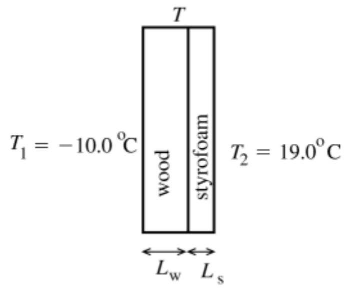


Figure 17.67

See Figure 17.67
Heat current through the wood: $H_w = k_w A(T - T_1)L_w$
Heat current through the styrofoam: $H_s = k_s A(T_2 - T)/L_s$

In steady-state heat does not accumulate in either material. The same heat has to pass through both materials in succession, so $H_w = H_s$.

EXECUTE: **(a)** This implies $k_w A(T - T_1)/L_w = k_s A(T_2 - T)/L_s$

$k_w L_s (T - T_1) = k_s L_w (T_2 - T)$

$$T = \frac{k_w L_s T_1 + k_s L_w T_2}{k_w L_s + k_s L_w} = \frac{-0.0176\text{ W}\cdot{}^{\circ}\text{C/K} + 00057\text{ W}\cdot{}^{\circ}\text{C/K}}{0.00206\text{ W/K}} = -5.8^{\circ}\text{C}$$

EVALUATE: The temperature at the junction is much closer in value to T_1 than to T_2. The styrofoam has a very large *k*, so a larger temperature gradient is required for than for wood to establish the same heat current.

(b) **IDENTIFY** and **SET UP:** Heat flow per square meter is $\frac{H}{A} = k\left(\frac{T_H - T_C}{L}\right)$. We can calculate this either for the wood or for the styrofoam; the results must be the same.

EXECUTE: wood

$$\frac{H_w}{A} = k_w \frac{T - T_1}{L_w} = (0.080\text{ W/m}\cdot\text{K})\frac{(-5.8^{\circ}\text{C} - (-10.0^{\circ}\text{C}))}{0.030\text{ m}} = 11\text{ W/m}^2.$$

styrofoam

$$\frac{H_s}{\text{A}} = k_s \frac{T_2 - T}{L_s} = (0.010\text{ W/m}\cdot\text{K})\frac{(19.0^{\circ}\text{C} - (-5.8^{\circ}\text{C}))}{0.022\text{ m}} = 11\text{ W/m}^2.$$

EVALUATE: *H* must be the same for both materials and our numerical results show this. Both materials are good insulators and the heat flow is very small.

17.68. **IDENTIFY:** $\frac{Q}{t} = \frac{kA(T_H - T_C)}{L}$

SET UP: $T_H - T_C = 175^{\circ}\text{C} - 35^{\circ}\text{C}$. $1\text{ K} = 1\text{ C}^{\circ}$, so there is no need to convert the temperatures to kelvins.

EXECUTE: **(a)** $\frac{Q}{t} = \frac{(0.040\text{ W/m}\cdot\text{K})(1.40\text{ m}^2)(175^{\circ}\text{C} - 35^{\circ}\text{C})}{4.0\times 10^{-2}\text{ m}} = 196\text{ W}.$

(b) The power input must be 196 W, to replace the heat conducted through the walls.

EVALUATE: The heat current is small because *k* is small for fiberglass.

17.69. **IDENTIFY:** Apply Eq.(17.23). $Q = Ht.$

SET UP: $1\text{ Btu} = 1055\text{ J}$

EXECUTE: The energy that flows in time *t* is $Q = Ht = \frac{A\Delta T}{R}t = \frac{(125\text{ ft}^2)(34\text{ F}^{\circ})}{(30\text{ ft}^2\cdot\text{F}^{\circ}\cdot\text{h/Btu})}(5.0\text{ h}) = 708\text{ Btu} = 7.5\times 10^5\text{ J}.$

EVALUATE: With the given units of *R*, we can use *A* in ft^2, ΔT in F° and *t* in h, and the calculation then gives *Q* in Btu.

17.70. **IDENTIFY:** $\frac{Q}{t} = \frac{kA\Delta T}{L}$. Q/t is the same for both sections of the rod.

SET UP: For copper, $k_c = 385\text{ W/m}\cdot\text{K}$. For steel, $k_s = 50.2\text{ W/m}\cdot\text{K}$.

EXECUTE: **(a)** For the copper section, $\frac{Q}{t} = \frac{(385\text{ W/m}\cdot\text{K})(4.00\times 10^{-4}\text{ m}^2)(100^{\circ}\text{C} - 65.0^{\circ}\text{C})}{1.00\text{ m}} = 5.39\text{ J/s}.$

(b) For the steel section, $L = \frac{kA\Delta T}{(Q/t)} = \frac{(50.2\text{ W/m}\cdot\text{K})(4.00\times 10^{-4}\text{ m}^2)(65.0^{\circ}\text{C} - 0^{\circ}\text{C})}{5.39\text{ J/s}} = 0.242\text{ m}.$

EVALUATE: The thermal conductivity for steel is much less than that for copper, so for the same ΔT and *A* a smaller *L* for steel would be needed for the same heat current as in copper.

17.71. **IDENTIFY** and **SET UP:** The heat conducted through the bottom of the pot goes into the water at 100°C to convert it to steam at 100°C. We can calculate the amount of heat flow from the mass of material that changes phase. Then use Eq.(17.21) to calculate T_H, the temperature of the lower surface of the pan.

EXECUTE: $Q = mL_v = (0.390 \text{ kg})(2256\times10^3 \text{ J/kg}) = 8.798\times10^5 \text{ J}$

$H = Q/t = 8.798\times10^5 \text{ J}/180 \text{ s} = 4.888\times10^3 \text{ J/s}$

Then $H = kA(T_H - T_C)/L$ says that $T_H - T_C = \dfrac{HL}{kA} = \dfrac{(4.888\times10^3 \text{ J/s})(8.50\times10^{-3} \text{ m})}{(50.2 \text{ W/m}\cdot\text{K})(0.150 \text{ m}^2)} = 5.52 \text{ C}°$

$T_H = T_C + 5.52 \text{ C}° = 100°\text{C} + 5.52 \text{ C}° = 105.5°\text{C}$

EVALUATE: The larger $T_H - T_C$ is the larger H is and the faster the water boils.

17.72. **IDENTIFY:** Apply Eq.(17.21) and solve for A.

SET UP: The area of each circular end of a cylinder is related to the diameter D by $A = \pi R^2 = \pi(D/2)^2$. For steel, $k = 50.2 \text{ W/m}\cdot\text{K}$. The boiling water has $T = 100°\text{C}$, so $\Delta T = 300 \text{ K}$.

EXECUTE: $\dfrac{Q}{t} = kA\dfrac{\Delta T}{L}$ and $150 \text{ J/s} = (50.2 \text{ W/m}\cdot\text{K})A\left(\dfrac{300 \text{ K}}{0.500 \text{ m}}\right)$. This gives $A = 4.98\times10^{-3} \text{ m}^2$, and

$D = \sqrt{4A/\pi} = \sqrt{4(4.98\times10^{-3} \text{ m}^2)/\pi} = 8.0\times10^{-2} \text{ m} = 8.0 \text{ cm}.$

EVALUATE: H increases when A increases.

17.73. **IDENTIFY:** Assume the temperatures of the surfaces of the window are the outside and inside temperatures. Use the concept of thermal resistance. For part (b) use the fact that when insulating materials are in layers, the R values are additive.

SET UP: From Table 17.5, $k = 0.8 \text{ W/m}\cdot\text{K}$ for glass. $R = L/k$.

EXECUTE: **(a)** For the glass, $R_{\text{glass}} = \dfrac{5.20\times10^{-3} \text{ m}}{0.8 \text{ W/m}\cdot\text{K}} = 6.50\times10^{-3} \text{ m}^2\cdot\text{K/W}$.

$H = \dfrac{A(T_H - T_C)}{R} = \dfrac{(1.40 \text{ m})(2.50 \text{ m})(39.5 \text{ K})}{6.50\times10^{-3} \text{ m}^2\cdot\text{K/W}} = 2.1\times10^4 \text{ W}$

(b) For the paper, $R_{\text{paper}} = \dfrac{0.750\times10^{-3} \text{ m}}{0.05 \text{ W/m}\cdot\text{K}} = 0.015 \text{ m}^2\cdot\text{K/W}$. The total R is $R = R_{\text{glass}} + R_{\text{paper}} = 0.0215 \text{ m}^2\cdot\text{K/W}$.

$H = \dfrac{A(T_H - T_C)}{R} = \dfrac{(1.40 \text{ m})(2.50 \text{ m})(39.5 \text{ K})}{0.0215 \text{ m}^2\cdot\text{K/W}} = 6.4\times10^3 \text{ W}.$

EVALUATE: The layer of paper decreases the rate of heat loss by a factor of about 3.

17.74. **IDENTIFY:** The rate of energy radiated per unit area is $\dfrac{H}{A} = e\sigma T^4$.

SET UP: A blackbody has $e = 1$.

EXECUTE: **(a)** $\dfrac{H}{A} = (1)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(273 \text{ K})^4 = 315 \text{ W/m}^2$

(b) $\dfrac{H}{A} = (1)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(2730 \text{ K})^4 = 3.15\times10^6 \text{ W/m}^2$

EVALUATE: When the Kelvin temperature increases by a factor of 10 the rate of energy radiation increases by a factor of 10^4.

17.75. **IDENTIFY:** Use Eq.(17.26) to calculate H_{net}.

SET UP: $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$ (Eq.(17.26); T must be in kelvins)

Example 17.16 gives $A = 1.2 \text{ m}^2$, $e = 1.0$, and $T = 30°\text{C} = 303 \text{ K}$ (body surface temperature)

$T_s = 5.0°\text{C} = 278 \text{ K}$

EXECUTE: $H_{\text{net}} = 573.5 \text{ W} - 406.4 \text{ W} = 167 \text{ W}$

EVALUATE: Note that this is larger than H_{net} calculated in Example 17.16. The lower temperature of the surroundings increases the rate of heat loss by radiation.

17.76. **IDENTIFY:** The net heat current is $H = Ae\sigma(T^4 - T_s^4)$. A power input equal to H is required to maintain constant temperature of the sphere.

SET UP: The surface area of a sphere is $4\pi r^2$.

EXECUTE: $H = 4\pi(0.0150 \text{ m})^2(0.35)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)([3000 \text{ K}]^4 - [290 \text{ K}]^4) = 4.54\times10^4 \text{ W}$

EVALUATE: Since $3000 \text{ K} > 290 \text{ K}$ and H is proportional to T^4, the rate of emission of heat energy is much greater than the rate of absorption of heat energy from the surroundings.

17.77. **IDENTIFY:** Use Eq.(17.26) to calculate A.

SET UP: $H = Ae\sigma T^4$ so $A = H/e\sigma T^4$

150-W and all electrical energy consumed is radiated says $H = 150$ W

EXECUTE: $A = \dfrac{150 \text{ W}}{(0.35)(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)(2450 \text{ K})^4} = 2.1\times10^{-4} \text{ m}^2(1\times10^4 \text{ cm}^2/1 \text{ m}^2) = 2.1 \text{ cm}^2$

EVALUATE: Light bulb filaments are often in the shape of a tightly wound coil to increase the surface area; larger A means a larger radiated power H.

17.78. **IDENTIFY:** Apply $H = Ae\sigma T^4$ and calculate A.

SET UP: For a sphere of radius R, $A = 4\pi R^2$. $\sigma = 5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4$. The radius of the earth is $R_\text{E} = 6.38\times10^6$ m, the radius of the sun is $R_\text{sun} = 6.96\times10^8$ m, and the distance between the earth and the sun is $r = 1.50\times10^{11}$ m.

EXECUTE: The radius is found from $R = \sqrt{\dfrac{A}{4\pi}} = \sqrt{\dfrac{H/(\sigma T^4)}{4\pi}} = \sqrt{\dfrac{H}{4\pi\sigma}\dfrac{1}{T^2}}$.

(a) $R_\text{a} = \sqrt{\dfrac{(2.7\times10^{32} \text{ W})}{4\pi(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)}}\dfrac{1}{(11{,}000 \text{ K})^2} = 1.61\times10^{11}$ m

(b) $R_\text{b} = \sqrt{\dfrac{(2.10\times10^{23} \text{ W})}{4\pi(5.67\times10^{-8} \text{ W/m}^2\cdot\text{K}^4)}}\dfrac{1}{(10{,}000 \text{ K})^2} = 5.43\times10^6$ m

EVALUATE: (c) The radius of Procyon B is comparable to that of the earth, and the radius of Rigel is comparable to the earth-sun distance.

17.79. **IDENTIFY** and **SET UP:** Use the temperature difference in M° and in C° between the melting and boiling points of mercury to relate M° to C°. Also adjust for the different zero points on the two scales to get an equation for T_M in terms of T_C.

(a) EXECUTE: normal melting point of mercury: $-39°\text{C} = 0.0°\text{M}$

normal boiling point of mercury: $357°\text{C} = 100.0°\text{M}$

$100.0°\text{M} = 396 \text{ C}°$ so $1 \text{ M}° = 3.96 \text{ C}°$

Zero on the M scale is -39 on the C scale, so to obtain T_C multiply T_M by 3.96 and then subtract 39°:

$T_\text{C} = 3.96T_M - 39°$

Solving for T_M gives $T_\text{M} = \frac{1}{3.96}(T_\text{C} + 39°)$

The normal boiling point of water is 100°C; $T_\text{M} = \frac{1}{3.96}(100° + 39°) = 35.1°\text{M}$

(b) $10.0 \text{ M}° = 39.6 \text{ C}°$

EVALUATE: A M° is larger than a C° since it takes fewer of them to express the difference between the boiling and melting points for mercury.

17.80. **IDENTIFY:** Apply $\Delta L = L_0\alpha\Delta T$ to the radius of the hoop. The thickness of the space equals the increase in radius of the hoop.

SET UP: The earth has radius $R_\text{E} = 6.38\times10^6$ m and this is the initial radius R_0 of the hoop. For steel, $\alpha = 1.2\times10^{-5} \text{ K}^{-1}$. $1 \text{ K} = 1 \text{ C}°$.

EXECUTE: The increase in the radius of the hoop would be

$$\Delta R = R\alpha\Delta T = (6.38\ \times10^6 \text{ m})(1.2\times10^{-5} \text{ K}^{-1})(0.5 \text{ K}) = 38 \text{ m}.$$

EVALUATE: Even though ΔR is large, the fractional change in radius, $\Delta R/R_0$, is very small.

17.81. **IDENTIFY:** The volume increases by $\Delta V = V_0\beta\Delta T$ and the mass is constant. $\rho = m/V$.

SET UP: Copper has density $\rho_0 = 8.9\times10^3 \text{ kg/m}^3$ and coefficient of volume expansion $\beta = 5.1\times10^{-5} (\text{C}°)^{-1}$. The tube is initially at temperature T_0, has sides of length L_0, volume V_0, density ρ_0, and coefficient of volume expansion β.

EXECUTE: **(a)** When the temperature increase to $T_0 + \Delta T$, the volume changes by an amount ΔV, where $\Delta V = \beta V_0\Delta T$. Then, $\rho = \dfrac{m}{V_0 + \Delta V}$, or eliminating ΔV, $\rho = \dfrac{m}{V_0 + \beta V_0\Delta T}$. Divide the top and bottom by V_0 and substitute $\rho_0 = m/V_0$. Then $\rho = \dfrac{\rho_0}{1 + \beta\Delta T}$. This can be rewritten as $\rho = \rho_0\left(1 + \beta\Delta T\right)^{-1}$. Then using the expression $\left(1 + x\right)^n \approx 1 + nx$, where $n = -1$, $\rho = \rho_0\left(1 - \beta\Delta T\right)$. This is accurate when $\beta\Delta T$ is small, which is the case if $\Delta T \ll 1/\beta$. $1/\beta$ is on the order of 10^4 C° and ΔT is typically about 10^2 C° or less, so this approximation is accurate.

(b) The copper cube has sides of length 1.25 cm = 0.0125 m and $\Delta T = 70.0°\text{C} - 20.0°\text{C} = 50.0\ \text{C}°$.
$\Delta V = \beta V_0 \Delta T = (5.1\times10^{-5}\ (\text{C}°)^{-1})(0.0125\ \text{m})^3(50.0\ \text{C}°) = 5\times10^{-9}\ \text{m}^3$. Similarly,
$\rho = (8.9\times10^3\ \text{kg/m}^3)(1-(5.1\times10^{-5}\ (\text{C}°)^{-1})(50.0\ \text{C}°)) = 8.877\times10^3\ \text{kg/m}^3$. Therefore, $\Delta\rho = -23\ \text{kg/m}^3$.

EVALUATE: When the temperature increases, the volume decreases and the density increases.

17.82. IDENTIFY: $v = \sqrt{F/\mu} = \sqrt{FL/m}$. For the fundamental, $\lambda = 2L$ and $f = \dfrac{v}{\lambda} = \dfrac{1}{2}\sqrt{\dfrac{F}{mL}}$. F, v and λ change when T changes because L changes. $\Delta L = L\alpha\Delta T$, where L is the original length.

SET UP: For copper, $\alpha = 1.7\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: (a) We can use differentials to find the frequency change because all length changes are small percents.
$\Delta f \approx \dfrac{\partial f}{\partial L}\Delta L$ (only L changes due to heating). $\Delta f = \frac{1}{2}\frac{1}{2}(F/mL)^{-1/2}(F/m)(-1/L^2)\Delta L = \frac{1}{2}\left(\frac{1}{2}\sqrt{\dfrac{F}{mL}}\right)\dfrac{\Delta L}{L} = \frac{1}{2} f \dfrac{\Delta L}{L}$.

$\Delta f = -\frac{1}{2}(\alpha\Delta T)f = -\frac{1}{2}(1.7\times10^{-5}\ (\text{C}°)^{-1})(40\ \text{C}°)(440\ \text{Hz}) = -0.15\ \text{Hz}$. The frequency decreases since the length increases.

(b) $\Delta v = \dfrac{\partial v}{\partial L}\Delta L$. $\dfrac{\Delta v}{v} = \dfrac{\frac{1}{2}(FL/m)^{-1/2}(F/m)\Delta L}{\sqrt{FL/m}} = \dfrac{\Delta L}{2L} = \dfrac{\alpha\Delta T}{2} = \dfrac{1}{2}(1.7\times10^{-5}\ (\text{C}°)^{-1})(40\ \text{C}°) = 3.4\times10^{-4} = 0.034\%$.

(d) $\lambda = 2L$ so $\Delta\lambda = 2\Delta L \rightarrow \dfrac{\Delta\lambda}{\lambda} = \dfrac{2\Delta L}{2L} = \dfrac{\Delta L}{L} = \alpha\Delta T$. $\dfrac{\Delta\lambda}{\lambda} = (1.7\times10^{-5}\ (\text{C}°)^{-1})(40\ \text{C}°) = 6.8\times10^{-4} = 0.068\%$.
λ increases.

EVALUATE: The wave speed and wavelength increase when the length increases and the frequency decreases. The percentage change in the frequency is -0.034%. The fractional change in all these quantities is very small.

17.83. IDENTIFY and SET UP: Use Eq.(17.8) for the volume expansion of the oil and of the cup. Both the volume of the cup and the volume of the olive oil increase when the temperature increases, but β is larger for the oil so it expands more. When the oil starts to overflow, $\Delta V_{\text{oil}} = \Delta V_{\text{glass}} + (1.00\times10^{-3}\ \text{m})A$, where A is the cross-sectional area of the cup.

EXECUTE: $\Delta V_{\text{oil}} = V_{0,\text{oil}}\beta_{\text{oil}}\Delta T = (9.9\ \text{cm})A\beta_{\text{oil}}\Delta T$

$\Delta V_{\text{glass}} = V_{0,\text{glass}}\beta_{\text{glass}}\Delta T = (10.0\ \text{cm})\text{A}\beta_{\text{glass}}\Delta T$

$(9.9\ \text{cm})\text{A}\beta_{\text{oil}}\Delta T - (10.0\ \text{cm})\text{A}\beta_{\text{glass}}\Delta T + (1.00\times10^{-3}\ \text{m})A$

The A divides out. Solving for ΔT gives $\Delta T = 15.5\ \text{C}°$

$T_2 = T_1 + \Delta T = 37.5°\text{C}$

EVALUATE: If the expansion of the cup is neglected, the olive oil will have expanded to fill the cup when $(0.100\ \text{cm})A = (9.9\ \text{cm})A\beta_{\text{oil}}\Delta T$, so $\Delta T = 15.0\ \text{C}°$ and $T_2 = 37.0°\text{C}$. Our result is slightly higher than this. The cup also expands but not very much since $\beta_{\text{glass}} \ll \beta_{\text{oil}}$.

17.84. IDENTIFY: Volume expansion: $dV = \beta V\ dT$. $\beta = \dfrac{dV/dT}{V}$.

SET UP: dV/dT is the slope of the graph of V versus T, the graph given in Figure 17.12 in the textbook.

EXECUTE: $\beta = \dfrac{\text{Slope of graph}}{V}$. Construct the tangent to the graph at 2°C and 8°C and measure the slope of this line.
At 22°C: Slope $\approx -\dfrac{0.10\ \text{cm}^3}{3\ \text{C}°}$ and $V \approx 1000\ \text{cm}^3$. $\beta \approx -\dfrac{0.10\ \text{cm}^3/3\ \text{C}°}{1000\ \text{cm}^3} \approx -3\times10^{-5}\ (\text{C}°)^{-1}$. The slope in negative, as the water contracts or it is heated. At 8°C: slope $\approx \dfrac{0.24\ \text{cm}^3}{4\ \text{C}°}$ and $V \approx 1000\ \text{cm}^3$. $\beta \approx \dfrac{0.24\ \text{cm}^3/4\ \text{C}°}{1000\ \text{cm}^3} \approx 6\times10^{-5}\ (\text{C}°)^{-1}$.
The water now expands when heated.

EVALUATE: $\beta > 0$ when the material expands when heated and $\beta < 0$ when the material contracts when it is heated. The minimum volume is at about 4°C and β has opposite signs above and below this temperature.

17.85. IDENTIFY: Use Eq.(17.6) to find the change in diameter of the sphere and the change in length of the cable. Set the sum of these two increases in length equal to 2.00 mm.

SET UP: $\alpha_{\text{brass}} = 2.0\times10^{-5}\ \text{K}^{-1}$ and $\alpha_{\text{steel}} = 1.2\times10^{-5}\ \text{K}^{-1}$.

EXECUTE: $\Delta L = (\alpha_{\text{brass}}L_{0,\text{brass}} + \alpha_{\text{steel}}L_{0,\text{steel}})\Delta T$.

$$\Delta T = \frac{2.00\times10^{-3}\ \text{m}}{(2.0\times10^{-5}\ \text{K}^{-1})(0.350\ \text{m}) + (1.2\times10^{-5}\ \text{K}^{-1})(10.5\ \text{m})} = 15.0\ \text{C}°.\quad T_2 = T_1 + \Delta T = 35.0°\text{C}.$$

EVALUATE: The change in diameter of the brass sphere is 0.10 mm. This is small, but should not be neglected.

17.86. **IDENTIFY:** Conservation of energy says $Q_e + Q_c = 0,$ where Q_e and Q_c are the heat changes for the ethanol and cylinder. To find the volume of ethanol that overflows calculate ΔV for the ethanol and for the cylinder.

SET UP: For ethanol, $c_e = 2428 \text{ J/kg}\cdot\text{K}$ and $\beta_e = 75\times 10^{-5}\ \text{K}^{-1}$.

EXECUTE: **(a)** $Q_e + Q_c = 0$ gives $m_e c_e (T_f - [-10.0°\text{C}]) + m_c c_c (T_f - 20.0°\text{C}]) = 0.$

$$T_f = \frac{(20.0°\text{C})m_c c_c - (10.0°\text{C})m_e c_e}{m_e c_e + m_c c_c}.\ \ T_f = \frac{(20.0°\text{C})(0.110 \text{ kg})(840 \text{ J/kg}\cdot\text{K}) - (10.0°\text{C})(0.0873 \text{ kg})(2428 \text{ J/kg}\cdot\text{K})}{(0.0873 \text{ kg})(2428 \text{ J/kg}\cdot\text{K}) + (0.110 \text{ kg})(840 \text{ J/kg}\cdot\text{K})}.$$

$$T_f = \frac{-271.6°\text{C}}{304.4} = -0.892°\text{C}.$$

(b) $\Delta V_e = \beta_e V_e \Delta T = (75\times 10^{-5}\ \text{K}^{-1})(108 \text{ cm}^3)(-0.892°\text{C} - [-10.0°\text{C}]) = +0.738 \text{ cm}^3.$

$\Delta V_c = \beta_c V_c \Delta T = (1.2\times 10^{-5}\ \text{K}^{-1})(108 \text{ cm}^3)(-0.892°\text{C} - 20.0°\text{C}) = -0.0271 \text{ cm}^3.$ The volume that overflows is $0.738 \text{ cm}^3 - (-0.0271 \text{ cm}^3) = 0.765 \text{ cm}^3.$

EVALUATE: The cylinder cools so its volume decreases. The ethanol warms, so its volume increases. The sum of the magnitudes of the two volume changes gives the volume that overflows.

17.87. **IDENTIFY** and **SET UP:** Call the metals A and B. Use the data given to calculate α for each metal.

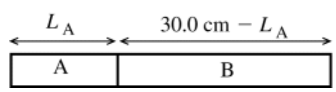
EXECUTE: $\Delta L = L_0 \alpha \Delta T$ so $\alpha = \Delta L/(L_0 \Delta T)$

metal A: $\alpha_A = \dfrac{\Delta L}{L_0 \Delta T} = \dfrac{0.0650 \text{ cm}}{(30.0 \text{ cm})(100 \text{ C}°)} = 2.167\times 10^{-5}\ (\text{C}°)^{-1}$

metal B: $\alpha_B = \dfrac{\Delta L}{L_0 \Delta T} = \dfrac{0.0350 \text{ cm}}{(30.0 \text{ cm})(100 \text{ C}°)} = 1.167\times 10^{-5}\ (\text{C}°)^{-1}$

EVALUATE: L_0 and ΔT are the same, so the rod that expands the most has the larger α.

IDENTIFY and **SET UP:** Now consider the composite rod (Figure 17.87). Apply Eq.(17.6). The target variables are L_A and L_B, the lengths of the metals A and B in the composite rod.



$\Delta T = 100 \text{ C}°$
$\Delta L = 0.058 \text{ cm}$

Figure 17.87

EXECUTE: $\Delta L = \Delta L_A + \Delta L_B = (\alpha_A L_A + \alpha_B L_B)\Delta T$

$\Delta L/\Delta T = \alpha_A L_A + \alpha_B (0.300 \text{ m} - L_A)$

$$L_A = \frac{\Delta L/\Delta T - (0.300 \text{ m})\alpha_B}{\alpha_A - \alpha_B} = \frac{(0.058\times 10^{-2} \text{ m}/100 \text{ C}°) - (0.300 \text{ m})(1.167\times 10^{-5}(\text{C}°)^{-1})}{1.00\times 10^{-5}\ (\text{C}°)^{-1}}$$

$L_B = 30.0 \text{ cm} - L_A = 30.0 \text{ cm} - 23.0 \text{ m} = 7.0 \text{ cm}$

EVALUATE: The expansion of the composite rod is similar to that of rod A, so the composite rod is mostly metal A.

17.88. **IDENTIFY:** Apply $\Delta V = V_0 \beta \Delta T$ to the gasoline and to the volume of the tank.

SET UP: For aluminum, $\beta = 7.2\times 10^{-5}\ \text{K}^{-1}$. $1 \text{ L} = 10^{-3} \text{ m}^3$.

EXECUTE: **(a)** The lost volume, 2.6 L, is the difference between the expanded volume of the fuel and the tanks, and the maximum temperature difference is

$$\Delta T = \frac{\Delta V}{(\beta_{\text{fuel}} - \beta_{\text{Al}})V_0} = \frac{(2.6\times 10^{-3} \text{ m}^3)}{(9.5\times 10^{-4}\ (\text{C}°)^{-1} - 7.2\times 10^{-5}\ (\text{C}°)^{-1})(106.0\times 10^{-3} \text{ m}^3)} = 28 \text{ C}°.$$

The maximum temperature was 32°C.

(b) No fuel can spill if the tanks are filled just before takeoff.

EVALUATE: Both the volume of the gasoline and the capacity of the tanks increased when T increased. But β is larger for gasoline than for aluminum so the volume of the gasoline increased more. When the tanks have returned to 4.0°C on Sunday morning there is 2.6 L of air space in the tanks.

17.89. **IDENTIFY:** The change in length due to heating is $\Delta L_T = L_0 \alpha \Delta T$ and this need not equal ΔL. The change in length due to the tension is $\Delta L_F = \dfrac{F L_0}{AY}$. Set $\Delta L = \Delta L_F + \Delta L_T$.

SET UP: $\alpha_{\text{brass}} = 2.0\times 10^{-5}\ (\text{C}°)^{-1}$. $\alpha_{\text{steel}} = 1.5\times 10^{-5}\ (\text{C}°)^{-1}$. $Y_{\text{steel}} = 20\times 10^{10}$ Pa.

EXECUTE: **(a)** The change in length is due to the tension and heating . $\dfrac{\Delta L}{L_0} = \dfrac{F}{AY} + \alpha \Delta T$. Solving for F/A,

$$\frac{F}{A} = Y\left(\frac{\Delta L}{L_0} - \alpha \Delta T\right).$$

(b) The brass bar is given as "heavy" and the wires are given as "fine," so it may be assumed that the stress in the bar due to the fine wires does not affect the amount by which the bar expands due to the temperature increase. This means that ΔL is not zero, but is the amount $\alpha_{brass}L_0\Delta T$ that the brass expands, and so

$$\frac{F}{A} = Y_{steel}(\alpha_{brass} - \alpha_{steel})\Delta T = (20\times10^{10}\ \text{Pa})(2.0\times10^{-5}\ (\text{C}°)^{-1} - 1.2\times10^{-5}\ (\text{C}°)^{-1})(120\ \text{C}°) = 1.92\times10^{8}\ \text{Pa}.$$

EVALUATE: The length of the brass bar increases more than the length of the steel wires. The wires remain taut and are under tension when the temperature of the system is raised above 20°C.

17.90. **IDENTIFY:** Apply the equation derived in part (a) of Problem 17.89 to the steel and aluminum sections. The sum of the ΔL values of the two sections must be zero.

SET UP: For steel, $Y = 20\times10^{10}$ Pa and $\alpha = 1.2\times10^{-5}\ (\text{C}°)^{-1}$. For aluminum, $Y = 7.0\times10^{10}$ Pa and $\alpha = 2.4\times10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: In deriving Eq.(17.12), it was assumed that $\Delta L = 0$; if this is not the case when there are both thermal and tensile stresses, Eq. (17.12) becomes $\Delta L = L_0\left(\alpha\Delta T + \frac{F}{AY}\right)$. (See Problem 17.89.) For the situation in this problem, there are two length changes which must sum to zero, and so Eq.(17.12) may be extended to two materials a and b in the form $L_{0a}\left(\alpha_a\,\Delta T + \frac{F}{AY_a}\right) + L_{0b}\left(\alpha_b\,\Delta T + \frac{F}{AY_b}\right) = 0$. Note that in the above, ΔT, F and A are the same for the two rods. Solving for the stress F/A, $\frac{F}{A} = -\frac{\alpha_a L_{0a} + \alpha_b L_{0b}}{((L_{oa}/Y_a) + (L_{0b}/Y_b))}\Delta T$.

$$\frac{F}{A} = \frac{(1.2\times10^{-5}\ (\text{C}°)^{-1})(0.350\ \text{m}) + (2.4\times10^{-5}\ (\text{C}°)^{-1})(0.250\ \text{m})}{((0.350\ \text{m}/20\times10^{10}\ \text{Pa}) + (0.250\ \text{m}/7\times10^{10}\ \text{Pa}))}(60.0\ \text{C}°) = -1.2\times10^{8}\ \text{Pa}.$$

EVALUATE: F/A is negative and the stress is compressive. If the steel rod was considered alone and its length was held fixed, the stress would be $-Y_{steel}\alpha_{steel}\Delta T = -1.4\times10^{8}$ Pa. For the aluminum rod alone the stress would be $-Y_{aluminum}\alpha_{aluminum}\Delta T = -1.0\times10^{8}$ Pa. The stress for the combined rod is the average of these two values.

17.91. **(a) IDENTIFY** and **SET UP:** The diameter of the ring undergoes linear expansion (increases with T) just like a solid steel disk of the same diameter as the hole in the ring. Heat the ring to make its diameter equal to 2.5020 in.

EXECUTE: $\Delta L = \alpha L_0\Delta T$ so $\Delta T = \frac{\Delta L}{L_0\alpha} = \frac{0.0020\ \text{in.}}{(2.5000\ \text{in.})(1.2\times10^{-5}(\text{C}°)^{-1})} = 66.7\ \text{C}°$

$T = T_0 + \Delta T = 20.0°\text{C} + 66.7\ \text{C}° = 87°\text{C}$

(b) IDENTIFY and **SET UP:** Apply the linear expansion equation to the diameter of the brass shaft and to the diameter of the hole in the steel ring.

EXECUTE: $L = L_0(1 + \alpha\Delta T)$

Want L_s (steel) $= L_b$ (brass) for the same ΔT for both materials: $L_{0s}(1 + \alpha_s\Delta T) = L_{0b}(1 + \alpha_b\Delta T)$ so

$L_{0s} + L_{0s}\alpha_s\Delta T = L_{0b} + L_{0b}\alpha_b\Delta T$

$$\Delta T = \frac{L_{0b} - L_{0s}}{L_{0s}\alpha_s - L_{0b}\alpha_b} = \frac{2.5020\ \text{in.} - 2.5000\ \text{in.}}{(2.5000\ \text{in.})(1.2\times10^{-5}(\text{C}°)^{-1}) - (2.5050\ \text{in.})(2.0\times10^{-5}(\text{C}°)^{-1})}$$

$$\Delta T = \frac{0.0020}{3.00\times10^{-5} - 5.00\times10^{-5}}\ \text{C}° = -100\ \text{C}°$$

$T = T_0 + \Delta T = 20.0°\text{C} - 100\ \text{C}° = -80°\text{C}$

EVALUATE: Both diameters decrease when the temperature is lowered but the diameter of the brass shaft decreases more since $\alpha_b > \alpha_s$; $|\Delta L_b| - |\Delta L_s| = 0.0020$ in.

17.92. **IDENTIFY:** Follow the derivation of Eq.(17.12).

SET UP: For steel, the bulk modulus is $B = 1.6\times10^{11}$ Pa and the volume expansion coefficient is $\beta = 3.0\times10^{-5}\ \text{K}^{-1}$.

EXECUTE: **(a)** The change in volume due to the temperature increase is $\beta V\Delta T$, and the change in volume due to the pressure increase is $-\frac{V}{B}\Delta p$. Setting the net change equal to zero, $\beta V\Delta T = V\frac{\Delta p}{B}$, or $\Delta p = B\beta\Delta V$.

(b) From the above, $\Delta p = (1.6\times10^{11}\ \text{Pa})(3.0\times10^{-5}\ \text{K}^{-1})(15.0\ \text{K}) = 8.6\times10^{7}\ \text{Pa}$.

EVALUATE: Δp in part (b) is about 850 atm. A small temperature increase corresponds to a very large pressure increase.

17.93. **IDENTIFY:** Apply Eq.(11.14) to the volume increase of the liquid due to the pressure decrease. Eq.(17.8) gives the volume decrease of the cylinder and liquid when they are cooled. Can think of the liquid expanding when the pressure is reduced and then contracting to the new volume of the cylinder when the temperature is reduced.

SET UP: Let β_l and β_m be the coefficients of volume expansion for the liquid and for the metal. Let ΔT be the (negative) change in temperature when the system is cooled to the new temperature.

EXECUTE: Change in volume of cylinder when cool: $\Delta V_m = \beta_m V_0 \Delta T$ (negative)

Change in volume of liquid when cool: $\Delta V_l = \beta_l V_0 \Delta T$ (negative)

The difference $\Delta V_l = \Delta V_m$ must be equal to the negative volume change due to the increase in pressure, which is $-\Delta p V_0 / B = -k \Delta p V_0$. Thus $\Delta V_l - \Delta V_m = -k \Delta p V_0$.

$$\Delta T = -\frac{k \Delta p}{\beta_l - \beta_m}$$

$$\Delta T = -\frac{(8.50\times10^{-10}\ \text{Pa}^{-1})(50.0\ \text{atm})(1.013\times10^{5}\ \text{Pa/1 atm})}{4.80\times10^{-4}\ \text{K}^{-1} - 3.90\times10^{-5}\ \text{K}^{-1}} = -9.8\ \text{C}°$$

$T = T_0 + \Delta T = 30.0°\text{C} - 9.8\ \text{C}° = 20.2°\text{C}.$

EVALUATE: A modest temperature change produces the same volume change as a large change in pressure; $B \gg \beta$ for the liquid.

17.94. **IDENTIFY:** $Q_{\text{system}} = 0.$ Assume that the normal melting point of iron is above $745°\text{C},$ so the iron initially is solid.

SET UP: For water, $c = 4190\ \text{J/kg}\cdot\text{K}$ and $L_v = 2256\times10^3\ \text{J/kg}.$ For solid iron, $c = 470\ \text{J/kg}\cdot\text{K}.$

EXECUTE: The heat released when the iron slug cools to $100°\text{C}$ is $Q = mc\Delta T = (0.1000\ \text{kg})(470\ \text{J/kg}\cdot\text{K})(645\ \text{K}) = 3.03\times10^4\ \text{J}.$ The heat absorbed when the temperature of the water is raised to $100°\text{C}$ is $Q = mc\Delta T = (0.0750\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(80.0\ \text{K}) = 2.51\times10^4\ \text{J}.$ This is less than the heat released from the iron and $3.03\times10^4\ \text{J} - 2.51\times10^4\ \text{J} = 5.20\times10^3\ \text{J}$ of heat is available for converting some of the liquid water at $100°\text{C}$ to vapor. The mass *m* of water that boils is $m = \dfrac{5.20\times10^3\ \text{J}}{2256\times10^3\ \text{J/kg}} = 2.30\times10^{-3}\ \text{kg} = 2.3\ \text{g}$

(a) The final temperature is $100°\text{C}.$

(b) There is $75.0\ \text{g} - 2.3\ \text{g} = 72.7\ \text{g}$ of liquid water remaining, so the final mass of the iron and remaining water is 172.7 g.

EVALUATE: If we ignore the phase change of the water and write $m_{\text{iron}} c_{\text{iron}} (T - 745°\text{C}) + m_{\text{water}} c_{\text{water}} (T - 200°\text{C}) = 0,$ when we solve for *T* we will get a value larger than $100°\text{C}.$ That result is unphysical and tells us that some of the water changes phase.

17.95. **(a) IDENTIFY:** Calculate K/Q. We don't know the mass *m* of the spacecraft, but it divides out of the ratio.

SET UP: The kinetic energy is $K = \frac{1}{2}mv^2.$ The heat required to raise its temperature by 600 C° (but not to melt it) is $Q = mc\Delta T.$

EXECUTE: The ratio is $\dfrac{K}{Q} = \dfrac{\frac{1}{2}mv^2}{mc\Delta T} = \dfrac{v^2}{2c\Delta T} = \dfrac{(7700\ \text{m/s})^2}{2(910\ \text{J/kg}\cdot\text{K})(600\ \text{C}°)} = 54.3.$

(b) EVALUATE: The heat generated when friction work (due to friction force exerted by the air) removes the kinetic energy of the spacecraft during reentry is very large, and could melt the spacecraft. Manned space vehicles must have heat shields made of very high melting temperature materials, and reentry must be made slowly.

17.96. **IDENTIFY:** The rate at which thermal energy is being generated equals the rate at which the net torque due to the rope is doing work. The energy input associated with a temperature change is $Q = mc\Delta T.$

SET UP: The rate at which work is being done is $P = \tau\omega.$ For iron, $c = 470\ \text{J/kg}\cdot\text{K}.$ $1\ \text{C}° = 1\ \text{K}$

EXECUTE: **(a)** The net torque that the rope exerts on the capstan, and hence the net torque that the capstan exerts on the rope, is the difference between the forces of the ends of the rope times the radius of the capstan. The capstan is doing work on the rope at a rate $P = \tau\omega = F_{\text{net}} r \dfrac{2\pi\ \text{rad}}{T} = (520\ \text{N})(5.0\times10^{-2}\ \text{m})\dfrac{2\pi\ \text{rad}}{(0.90\ \text{s})} = 182\ \text{W},$ or 180 W to two figures. A larger number of turns might increase the force, but for given forces, the torque is independent of the number of turns.

(b) $\dfrac{\Delta T}{t} = \dfrac{Q/t}{mc} = \dfrac{P}{mc} = \dfrac{(182\ \text{W})}{(6.00\ \text{kg})(470\ \text{J/kg}\cdot\text{K})} = 0.064\ \text{C}°/\text{s}.$

EVALUATE: The rate of temperature rise is proportional to the difference in tension between the ends of the rope and to the rate at which the capstan is rotating.

17.97. **IDENTIFY and SET UP:** To calculate Q, use Eq.(17.18) in the form $dQ = nC\,dT$ and integrate, using $C(T)$ given in the problem. C_{av} is obtained from Eq.(17.19) using the finite temperature range instead of an infinitesimal dT.

EXECUTE: **(a)** $dQ = mcdT$

$$Q = n\int_{T_1}^{T_2} C\ dT = n\int_{T_1}^{T_2} k(T^3/\Theta^3)dT = (nk/\Theta^3)\int_{T_1}^{T_2} T^3\ dt = (nk/\Theta^3)\left(\tfrac{1}{4}T^4\Big|_{T_1}^{T_2}\right)$$

$$Q = \frac{nk}{4\Theta^3}(T_2^4 - T_1^4) = \frac{(1.50\text{ mol})(1940\text{ J/mol}\cdot\text{K})}{4(281\text{ K})^3}((40.0\text{ K})^4 - (10.0\text{ K})^4) = 83.6\text{ J}$$

(b) $C_{av} = \dfrac{1}{n}\dfrac{\Delta Q}{\Delta T} = \dfrac{1}{1.50\text{ mol}}\left(\dfrac{83.6\text{ J}}{40.0\text{ K} - 10.0\text{ K}}\right) = 1.86\text{ J/mol}\cdot\text{K}$

(c) $C = k(t/\Theta)^3 = (1940\text{ J/mol}\cdot\text{K})(40.0\text{ K}/281\text{ K})^3 = 5.60\text{ J/mol}\cdot\text{K}$

EVALUATE: C is increasing with T, so C at the upper end of the temperature integral is larger than its average value over the interval.

17.98. **IDENTIFY:** For a temperature change, $Q = mc\Delta T$, and for the liquid $\rightarrow$ solid phase change, $Q = -mL_f$.

SET UP: The volume V_w of the water determines its mass. $m_w = \rho_w V_w$. For water, $\rho_w = 1000\text{ kg/m}^3$, $c = 4190\text{ J/kg}\cdot\text{K}$ and $L_f = 334\times10^3\text{ J/kg}$.

EXECUTE: Set the heat energy that flows into the water equal to the final gravitational potential energy. $L_f\rho_w V_w + c_w\rho_w V_w\Delta T = mgh$. Solving for h, and inserting numbers:

$$h = \frac{(1000\text{ kg/m}^3)(1.9\times0.8\times0.1\text{ m}^3)\left[334\times10^3\text{ J/kg} + (4190\text{ J/kg}\cdot\text{K})(37\text{ C}°)\right]}{(70\text{ kg})(9.8\text{ m/s}^2)}.$$

$h = 1.08\times10^5\text{ m} = 108\text{ km}.$

EVALUATE: The heat associated with temperature and phase changes corresponds to a large amount of mechanical energy.

17.99. **IDENTIFY:** Apply $Q = mc\Delta T$ to the air in the room.

SET UP: The mass if air in the room is $m = \rho V = (1.20\text{ kg/m}^3)(3200\text{ m}^3) = 3840\text{ kg}$. $1\text{ W} = 1\text{ J/s}$.

EXECUTE: **(a)** $Q = (3000\text{ s})(90\text{ students})(100\text{ J/s}\cdot\text{student}) = 2.70\times10^7\text{ J}.$

(b) $Q = mc\Delta T$. $\Delta T = \dfrac{Q}{mc} = \dfrac{2.70\times10^7\text{ J}}{(3840\text{ kg})(1020\text{ J/kg}\cdot\text{K})} = 6.89\text{ C}°$

(c) $\Delta T = (6.89\text{ C}°)\left(\dfrac{280\text{ W}}{100\text{ W}}\right) = 19.3\text{ C}°.$

EVALUATE: In the absence of a cooling mechanism for the air, the air temperature would rise significantly.

17.100. **IDENTIFY:** $dQ = nCdT$ so for the temperature change $T_1 \rightarrow T_2$, $Q = n\int_{T_1}^{T_2} C(T)dT$.

SET UP: $\int dT = T$ and $\int TdT = \tfrac{1}{2}T^2$. Express T_1 and T_2 in kelvins: $T_1 = 300\text{ K}$, $T_2 = 500\text{ K}$.

EXECUTE: Denoting C by $C = a + bT$, a and b independent of temperature, integration gives

$$Q = n\left(a(T_2 - T_1) + \frac{b}{2}(T_2^2 - T_1^2)\right).$$

$$Q = (3.00\text{ mol})(29.5\text{ J/mol}\cdot\text{K})(500\text{ K} - 300\text{ K}) + (4.10\times10^{-3}\text{ J/mol}\cdot\text{K}^2)((500\text{ K})^2 - (300\text{ K})^2)).$$

$Q = 1.97\times10^4\text{ J}.$

EVALUATE: If C is assumed to have the constant value $29.5\text{ J/mol}\cdot\text{K}$, then $Q = 1.77\times10^4\text{ J}$ for this temperature change. At $T_1 = 300\text{ K}$, $C = 32.0\text{ J/mol}\cdot\text{K}$ and at $T_2 = 500\text{ K}$, $C = 33.6\text{ J/mol}\cdot\text{K}$. The average value of C is $32.8\text{ J/mol}\cdot\text{K}$, If C is assumed to be constant and to have this average value, then $Q = 2.02\times10^4\text{J}$, which is within 3% of the correct value.

17.101. **IDENTIFY:** Use $Q = mL_f$ to find the heat that goes into the ice to melt it. This amount of heat must be conducted through the walls of the box; $Q = Ht$. Assume the surfaces of the styrofoam have temperatures of 5.00°C and 21.0°C.

SET UP: For water $L_f = 334\times10^3\text{J/kg}$. For Styrofoam $k = 0.01\text{ W/m}\cdot\text{K}$. One week is $6.048\times10^5\text{ s}$. The surface area of the box is $4(0.500\text{ m})(0.800\text{ m}) + 2(0.500\text{ m})^2 = 2.10\text{ m}^2$.

EXECUTE: $Q = mL_f = (25.0\text{ kg})(334\times10^3\text{ J/kg}) = 8.016\times10^6\text{ J}$. $H = kA\dfrac{T_H - T_C}{L}$. $Q = Ht$ gives

$$L = \frac{tkA(T_H - T_C)}{Q} = \frac{(6.048\times10^5\text{ s})(0.01\text{ W/m}\cdot\text{K})(2.10\text{ m}^2)(21.0°\text{C} - 5.00°\text{C})}{8.016\times10^6\text{ J}} = 2.5\text{ cm}$$

EVALUATE: We have assumed that the liquid water that is produced by melting the ice remains in thermal equilibrium with the ice so has a temperature of 0°C. The interior of the box and the ice are not in thermal equilibrium, since they have different temperatures.

17.102. IDENTIFY: For a temperature change $Q = mc\Delta T$. For the vapor $\rightarrow$ liquid phase transition, $Q = -mL_v$.

SET UP: For water, $c = 4190 \text{ J/kg}\cdot\text{K}$ and $L_v = 2256\times10^3 \text{ J/kg}$.

EXECUTE: The requirement that the heat supplied in each case is the same gives $m_w c_w \Delta T_w = m_s(c_w \Delta T_s + L_v)$, where $\Delta T_w = 42.0 \text{ K}$ and $\Delta T_s = 65.0 \text{ K}$. The ratio of the masses is

$$\frac{m_s}{m_w} = \frac{c_w \Delta T_w}{c_w \Delta T_s + L_v} = \frac{(4190 \text{ J/kg}\cdot\text{K})(42.0 \text{ K})}{(4190 \text{ J/kg}\cdot\text{K})(65.0 \text{ K}) + 2256\times10^3 \text{ J/kg}} = 0.0696,$$

so 0.0696 kg of steam supplies the same heat as 1.00 kg of water.

EVALUATE: Note the heat capacity of water is used to find the heat lost by the condensed steam, since the phase transition produces liquid water at an initial temperature of 100°C.

17.103. (a) IDENTIFY and SET UP: Assume that all the ice melts and that all the steam condenses. If we calculate a final temperature T that is outside the range 0°C to 100°C then we know that this assumption is incorrect. Calculate Q for each piece of the system and then set the total $Q_{\text{system}} = 0$.

EXECUTE: copper can (changes temperature form 0.0° to T; no phase change)

$Q_{\text{can}} = mc\Delta T = (0.446 \text{ kg})(390 \text{ J/kg}\cdot\text{K})(T - 0.0°\text{C}) = (173.9 \text{ J/K})T$

ice (melting phase change and then the water produced warms to T)

$Q_{\text{ice}} = +mL_f + mc\Delta T = (0.0950 \text{ kg})(334\times10^3 \text{J/kg}) + (0.0950 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(T - 0.0°\text{C})$

$Q_{\text{ice}} = 3.173\times10^4 \text{ J} + (398.0 \text{ J/K})T.$

steam (condenses to liquid and then water produced cools to T)

$Q_{\text{steam}} = -mL_v + mc\Delta T = -(0.0350 \text{ kg})(2256\times10^3 \text{ J/kg}) + (0.0350 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(T - 100.0°\text{C})$

$Q_{\text{steam}} = -7.896\times10^4 \text{ J} + (146.6 \text{ J/K})T - 1.466\times10^4 \text{ J} = -9.362\times10^4 \text{ J} + (146.6 \text{J/K})T$

$Q_{\text{system}} = 0$ implies $Q_{\text{can}} + Q_{\text{ice}} + Q_{\text{steam}} = 0.$

$(173.9 \text{ J/K})T + 3.173\times10^4 \text{ J} + (398.0 \text{ J/K})T - 9.362\times10^4 \text{ J} + (146.6 \text{ J/K})T = 0$

$(718.5 \text{ J/K})T = 6.189\times10^4 \text{ J}$

$$T = \frac{6.189\times10^4 \text{ J}}{718.5 \text{ J/K}} = 86.1°\text{C}$$

EVALUATE: This is between 0°C and 100°C so our assumptions about the phase changes being complete were correct.

(b) No ice, no steam $0.0950 \text{ kg} + 0.0350 \text{ kg} = 0.130 \text{ kg}$ of liquid water.

17.104. IDENTIFY: The final amount of ice is less than the initial mass of water, so water remains and the final temperature is 0°C. The ice added warms to 0°C and heat comes out of water to convert it to ice. Conservation of energy says $Q_i + Q_w = 0$, where Q_i and Q_w are the heat flows for the ice that is added and for the water that freezes.

SET UP: Let m_i be the mass of ice that is added and m_w is the mass of water that freezes. The mass of ice increases by 0.328 kg, so $m_i + m_w = 0.328 \text{ kg}$. For water, $L_f = 334\times10^3 \text{ J/kg}$ and for ice $c_i = 2100 \text{ J/kg}\cdot\text{K}$. Heat comes out of the water when it freezes, so $Q_w = -mL_f$

EXECUTE: $Q_i + Q_w = 0$ gives $m_i c_i (15.0 \text{ C°}) + (-m_w L_f) = 0$, $m_w = 0.328 \text{ kg} - m_i$, so

$$m_i c_i (15.0 \text{ C°}) + (-0.328 + m_i)L_f = 0. \quad m_i = \frac{(0.328 \text{ kg})L_f}{c_i(15.0 \text{ C°}) + L_f} = \frac{(0.328 \text{ kg})(334\times10^3 \text{ J/kg})}{(2100 \text{ J/kg}\cdot\text{K})(15.0 \text{ K}) + 334\times10^3 \text{ J/kg}} = 0.300 \text{ kg}.$$

0.300 kg of ice was added.

EVALUATE: The mass of water that froze when the ice at $-15.0°\text{C}$ was added was $0.778 \text{ kg} - 0.450 \text{ kg} - 0.300 \text{ kg} = 0.028 \text{ kg}.$

17.105. IDENTIFY and SET UP: Heat comes out of the steam when it changes phase and heat goes into the water and causes its temperature to rise. $Q_{\text{system}} = 0.$ First determine what phases are present after the system has come to a uniform final temperature.

(a) EXECUTE: Heat that must be removed from steam if all of it condenses is

$Q = -mL_v = -(0.0400 \text{ kg})(2256\times10^3 \text{ J/kg}) = -9.02\times10^4 \text{ J}$

Heat absorbed by the water if it heats all the way to the boiling point of 100°C:

$Q = mc\Delta T = (0.200 \text{ kg})(4190 \text{ J/kg}\cdot\text{K})(50.0 \text{ C°}) = 4.19\times10^4 \text{ J}$

EVALUATE: The water can't absorb enough heat for all the steam to condense. Steam is left and the final temperature then must be 100°C.

(b) EXECUTE: Mass of steam that condenses is $m = Q/L_v = 4.19\times10^4\ \text{J}/2256\times10^3\ \text{J/kg} = 0.0186\ \text{kg}$
Thus there is $0.0400\ \text{kg} - 0.0186\ \text{kg} = 0.0214\ \text{kg}$ of steam left. The amount of liquid water is $0.0186\ \text{kg} + 0.200\ \text{kg} = 0.219\ \text{kg}$.

17.106. IDENTIFY: $Q_{\text{system}} = 0$.

SET UP: The mass of the system increases by $0.525\ \text{kg} - 0.490\ \text{kg} = 0.035\ \text{kg}$, so the mass of the steam that condensed is 0.035 kg.

EVALUATE: The heat lost by the steam as it condenses and cools is $(0.035\ \text{kg})L_v + (0.035\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(29.0\ \text{K})$, and the heat gained by the original water and calorimeter is $((0.150\ \text{kg})(420\ \text{J/kg}\cdot\text{K}) + (0.340\ \text{kg})(4190\ \text{J/kg}\cdot\text{K}))(56.0\ \text{K}) = 8.33\times10^4\ \text{J}$. Setting the heat lost equal to the heat gained and solving for L_v gives $2.26\times10^6\ \text{J/kg}$, or $2.3\times10^6\ \text{J/kg}$ to two figures (the mass of steam condensed is known to only two figures).

EVALUATE: $Q_{\text{system}} = 0$ means the magnitude of the heat that flows out of the 0.035 kg of steam as it condenses and cools equals the heat that flows into the calorimeter and 0.340 kg of water as their temperature increases. To the accuracy of the calculation, our result agrees with the value of L_v given in Table 17.4.

17.107. IDENTIFY: Heat Q_l comes out of the lead when it solidifies and the solid lead cools to T_f. If mass m_s of steam is produced, the final temperature is $T_f = 100°\text{C}$ and the heat that goes into the water is $Q_w = m_w C_w (25.0\ \text{C°}) + m_s L_{v,w}$, where $m_w = 0.5000\ \text{kg}$. Conservation of energy says $Q_l + Q_w = 0$. Solve for m_s. The mass that remains is $1.250\ \text{kg} + 0.5000\ \text{kg} - m_s$.

SET UP: For lead, $L_{f,l} = 24.5\times10^3\ \text{J/kg}$, $c_l = 130\ \text{J/kg}\cdot\text{K}$ and the normal melting point of lead is $327.3°\text{C}$. For water, $c_w = 4190\ \text{J/kg}\cdot\text{K}$ and $L_{v,w} = 2256\times10^3\ \text{J/kg}$.

EXECUTE: $Q_l + Q_w = 0$. $-m_l L_{f,l} + m_l c_l(-227.3\ \text{C°}) + m_w c_w (25.0\ \text{C°}) + m_s L_{v,w} = 0$.

$$m_s = \frac{m_l L_{f,l} + m_l c_l(+227.3\ \text{C°}) - m_w c_w (25.0\ \text{C°})}{L_{v,w}}.$$

$$m_s = \frac{+(1.250\ \text{kg})(24.5\times10^3\ \text{J/kg}) + (1.250\ \text{kg})(130\ \text{J/kg}\cdot\text{K})(227.3\ \text{K}) - (0.5000\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(25.0\ \text{K})}{2256\times10^3\ \text{J/kg}}$$

$m_s = \dfrac{1.519\times10^4\ \text{J}}{2256\times10^3\ \text{J/kg}} = 0.0067\ \text{kg}$. The mass of water and lead that remains is 1.743 kg.

EVALUATE: The magnitude of heat that comes out of the lead when it goes from liquid at $327.3°\text{C}$ to solid at $100.0°\text{C}$ is $6.76\times10^4\ \text{J}$. The heat that goes into the water to warm it to $100°\text{C}$ is $5.24\times10^4\ \text{J}$. The additional heat that goes into the water, $6.76\times10^4\ \text{J} - 5.24\times10^4\ \text{J} = 1.52\times10^4\ \text{J}$ converts 0.0067 kg of water at $100°\text{C}$ to steam.

17.108. IDENTIFY: Apply $H = kA\dfrac{\Delta T}{L}$ and solve for k.

SET UP: H equals the power input required to maintain a constant interior temperature

EXECUTE: $k = H\dfrac{L}{A\Delta T} = (180\ \text{W})\dfrac{(3.9\times10^{-2}\ \text{m})}{(2.18\ \text{m}^2)(65.0\ \text{K})} = 5.0\times10^{-2}\ \text{W/m}\cdot\text{K}$.

EVALUATE: Our result is consistent with the values for insulating solids in Table 17.5.

17.109. IDENTIFY: Apply $H = kA\dfrac{\Delta T}{L}$.

SET UP: For the glass use $L = 12.45\ \text{cm}$, to account for the thermal resistance of the air films on either side of the glass.

EXECUTE: **(a)** $H = (0.120\ \text{J/mol.K})\ (2.00\times0.95\ \text{m}^2)\left(\dfrac{28.0\ \text{C°}}{5.0\times10^{-2}\ \text{m} + 1.8\times10^{-2}\ \text{m}}\right) = 93.9\ \text{W}$.

(b) The heat flow through the wood part of the door is reduced by a factor of $1 - \dfrac{(0.50)^2}{(2.00\times0.95)} = 0.868$, so it becomes $81.5\ \text{W}$. The heat flow through the glass is $H_{\text{glass}} = (0.80\ \text{J/mol}\cdot\text{K})(0.50\ \text{m})^2\left(\dfrac{28.0\ \text{C°}}{12.45\times10^{-2}\ \text{m}}\right) = 45.0\ \text{W}$, and so the ratio is $\dfrac{81.5 + 45.0}{93.9} = 1.35$.

EVALUATE: The single-pane window produces a significant increase in heat loss through the door. (See Problem 17.111).

17.110. IDENTIFY: Apply Eq.(17.23).

SET UP: Let $\Delta T_1 = \dfrac{HR_1}{A}$ be the temperature difference across the wood and let $\Delta T_2 = \dfrac{HR_2}{A}$ be the temperature difference across the insulation. The temperature difference across the combination is $\Delta T = \Delta T_1 + \Delta T_2$. The effective thermal resistance R of the combination is defined by $\Delta T = \dfrac{HR}{A}$.

EXECUTE: $\Delta T = \Delta T_1 + \Delta T_2$ gives $\dfrac{H}{A}(R_1 + R_2) = \dfrac{H}{A}R$, and $R = R_1 + R_2$.

EVALUATE: A good insulator has a large value of R. R for the combination is larger than the R for any one of the layers.

17.111. IDENTIFY and SET UP: Use H written in terms of the thermal resistance R: $H = A\Delta T/R$, where $R = L/k$ and $R = R_1 + R_2 + \ldots$ (additive).

EXECUTE: single pane $R_s = R_{\text{glass}} + R_{\text{film}}$, where $R_{\text{film}} = 0.15\ \text{m}^2 \cdot \text{K/W}$ is the combined thermal resistance of the air films on the room and outdoor surfaces of the window.

$R_{\text{glass}} = L/k = (4.2\times 10^{-3}\ \text{m})/(0.80\ \text{W/m}\cdot\text{K}) = 0.00525\ \text{m}^2\cdot\text{K/W}$

Thus $R_s = 0.00525\ \text{m}^2\cdot\text{K/W} + .15\ \text{m}^2\cdot\text{K/W} = 0.1553\ \text{m}^2\cdot\text{K/W}$.

double pane $R_d = 2R_{\text{glass}} + R_{\text{air}} + R_{\text{film}}$, where R_{air} is the thermal resistance of the air space between the panes.

$R_{\text{air}} = L/k = (7.0\times 10^{-3}\ \text{m})/(0.024\ \text{W/m}\cdot\text{K}) = 0.2917\ \text{m}^2\cdot\text{K/W}$

Thus $R_d = 2(0.00525\ \text{m}^2\cdot\text{K/W}) + 0.2917\ \text{m}^2\cdot\text{K/W} + 0.15\ \text{m}^2\cdot\text{K/W} = 0.4522\ \text{m}^2\cdot\text{K/W}$

$H_s = A\Delta T/R_s$, $H_d = A\Delta T/R_d$, so $H_s/H_d = R_d/R_s$ (since A and ΔT are same for both)

$H_s/H_d = (0.4522\ \text{m}^2\cdot\text{K/W})/(0.1553\ \text{m}^2\cdot\text{K/W}) = 2.9$

EVALUATE: The heat loss is about a factor of 3 less for the double-pane window. The increase in R for a double-pane is due mostly to the thermal resistance of the air space between the panes.

17.112. IDENTIFY: $H = \dfrac{kA\Delta T}{L}$ to each rod. Conservation of energy requires that the heat current through the copper equals the sum of the heat currents through the brass and the steel.

SET UP: Denote the quantities for copper, brass and steel by 1, 2 and 3, respectively, and denote the temperature at the junction by T_0.

EXECUTE: **(a)** $H_1 = H_2 + H_3$. Using Eq.(17.21) and dividing by the common area gives,

$\dfrac{k_1}{L_1}(100°\text{C} - T_0) = \dfrac{k_2}{L_2}T_0 + \dfrac{k_3}{L_3}T_0$. Solving for T_0 gives $T_0 = \dfrac{(k_1/L_1)}{(k_1/L_1) + (k_2/L_2) + (k_3/L_3)}(100°\text{C})$. Substitution of numerical values gives $T_0 = 78.4°\text{C}$.

(b) Using $H = \dfrac{kA}{L}\Delta T$ for each rod, with $\Delta T_1 = 21.6\ \text{C}°$, $\Delta T_2 = \Delta T_3 = 78.4\ \text{C}°$ gives $H_1 = 12.8\ \text{W}$, $H_2 = 9.50\ \text{W}$ and $H_3 = 3.30\ \text{W}$.

EVALUATE: In part (b), H_1 is seen to be the sum of H_2 and H_3.

17.113. (a) EXECUTE: Heat must be conducted from the water to cool it to 0°C and to cause the phase transition. The entire volume of water is not at the phase transition temperature, just the upper surface that is in contact with the ice sheet.

(b) IDENTIFY: The heat that must leave the water in order for it to freeze must be conducted through the layer of ice that has already been formed.

SET UP: Consider a section of ice that has area A. At time t let the thickness be h. Consider a short time interval t to $t + dt$. Let the thickness that freezes in this time be dh. The mass of the section that freezes in the time interval dt is $dm = \rho\, dV = \rho A\, dh$. The heat that must be conducted away from this mass of water to freeze it is $dQ = dmL_f = (\rho A L_f)dh$. $H = dQ/dt = kA(\Delta T/h)$, so the heat dQ conducted in time dt throughout the thickness h that is already there is $dQ = kA\left(\dfrac{T_H - T_C}{h}\right)dt$. Solve for dh in terms of dt and integrate to get an expression relating h and t.

EXECUTE: Equate these expressions for dQ.

$$\rho A L_{\mathrm{f}} dh = kA\left(\frac{T_{\mathrm{H}} - T_{\mathrm{C}}}{h}\right)dt$$

$$h\ dh = \left(\frac{k(T_{\mathrm{H}} - T_{\mathrm{C}})}{\rho L_{\mathrm{f}}}\right)dt$$

Integrate from $t = 0$ to time t. At $t = 0$ the thickness h is zero.

$$\int_0^h h\ dh = [k(T_{\mathrm{H}} - T_{\mathrm{C}})\rho L_{\mathrm{f}}]\int_0^t dt$$

$$\tfrac{1}{2}h^2 = \frac{k(T_{\mathrm{H}} - T_{\mathrm{C}})}{\rho L_{\mathrm{f}}}t \ \text{ and } \ h = \sqrt{\frac{2k(T_{\mathrm{H}} - T_{\mathrm{C}})}{\rho L_{\mathrm{f}}}}\sqrt{t}$$

The thickness after time t is proportional to $\sqrt{t}$.

(c) The expression in part (b) gives $t = \dfrac{h^2 \rho L_{\mathrm{f}}}{2k(T_{\mathrm{H}} - T_{\mathrm{C}})} = \dfrac{(0.25\ \text{m})^2(920\ \text{kg/m}^3)(334\times 10^3\ \text{J/kg})}{2(1.6\ \text{W/m}\cdot\text{K})(0°\text{C} - (-10°\text{C}))} = 6.0\times 10^5\ \text{s}$

$t = 170$ h.

(d) Find t for $h = 40$ m. t is proportional to h^2, so $t = (40\ \text{m}/0.25\ \text{m})^2(6.00\times 10^5\ \text{s}) = 1.5\times 10^{10}$ s. This is about 500 years. With our current climate this will not happen.

EVALUATE: As the ice sheet gets thicker, the rate of heat conduction through it decreases. Part (d) shows that it takes a very long time for a moderately deep lake to totally freeze.

17.114. IDENTIFY: Apply Eq.(17.22) at each end of the short element. In part (b) use the fact that the net heat current into the element provides the Q for the temperature increase, according to $Q = mc\Delta T$.

SET UP: dT/dx is the temperature gradient.

EXECUTE: **(a)** $H = (380\ \text{W/m}\cdot\text{K})(2.50\times 10^{-4}\ \text{m}^2)(140\ \text{C°/m}) =$ 13.3 W.

(b) Denoting the two ends of the element as 1 and 2, $H_2 - H_1 = \dfrac{Q}{t} = mc\dfrac{\Delta T}{t}$, where $\dfrac{\Delta T}{t} = 0.250$ C°/s.

$$kA\left.\frac{dT}{dx}\right|_2 - kA\left.\frac{dT}{dx}\right|_1 = mc\left(\frac{\Delta T}{t}\right).$$ The mass m is $\rho A\Delta x$, so $$kA\left.\frac{dT}{dx}\right|_2 = kA\left.\frac{dT}{dx}\right|_1 + \frac{\rho c\Delta x}{k}\left(\frac{\Delta T}{t}\right).$$

$$kA\left.\frac{dT}{dx}\right|_2 = 140\ \text{C°/m} + \frac{(1.00\times 10^4\ \text{kg/m}^3)(520\ \text{J/kg}\cdot\text{K})(1.00\times 10^{-2}\ \text{m})(0.250\ \text{C°/s})}{380\ \text{W/m}\cdot\text{K}} = 174\ \text{C°/m}.$$

EVALUATE: At steady-state temperature of the short element is no longer changing and $H_1 = H_2$.

17.115. IDENTIFY: The rate of heat conduction through the walls is 1.25 kW. Use the concept of thermal resistance and the fact that when insulating materials are in layers, the R values are additive.

SET UP: The total area of the four walls is $2(3.50\ \text{m})(2.50\ \text{m}) + 2(3.00\ \text{m})(2.50\ \text{m}) = 32.5\ \text{m}^2$

EXECUTE: $H = A\dfrac{T_{\mathrm{H}} - T_{\mathrm{C}}}{R}$ gives $R = \dfrac{A(T_{\mathrm{H}} - T_{\mathrm{C}})}{H} = \dfrac{(32.5\ \text{m}^2)(17.0\ \text{K})}{1.25\times 10^{-3}\ \text{W}} = 0.442\ \text{m}^2\cdot\text{K/W}$. For the wood,

$R_{\mathrm{w}} = \dfrac{L}{k} = \dfrac{1.80\times 10^{-2}\ \text{m}}{0.060\ \text{W/m}\cdot\text{K}} = 0.300\ \text{m}^2\cdot\text{K/W}$. For the insulating material, $R_{\mathrm{in}} = R - R_{\mathrm{w}} = 0.142\ \text{m}^2\cdot\text{K/W}$.

$R_{\mathrm{in}} = \dfrac{L_{\mathrm{in}}}{k_{\mathrm{in}}}$ and $k_{\mathrm{in}} = \dfrac{L_{\mathrm{in}}}{R_{\mathrm{in}}} = \dfrac{1.50\times 10^{-2}\ \text{m}}{0.142\ \text{m}^2\cdot\text{K/W}} = 0.106\ \text{W/m}\cdot\text{K}$.

EVALUATE: The thermal conductivity of the insulating material is larger than that of the wood, the thickness of the insulating material is less than that of the wood, and the thermal resistance of the wood is about three times that of the insulating material.

17.116. IDENTIFY: $I_1 r_1^2 = I_2 r_2^2$. Apply $H = Ae\sigma T^4$ (Eq.17.25) to the sun.

SET UP: $I_1 = 1.50\times 10^3\ \text{W/m}^2$ when $r = 1.50\times 10^{11}$ m.

EXECUTE: **(a)** The energy flux at the surface of the sun is

$$I_2 = (1.50\times 10^3\ \text{W/m}^2)\left(\frac{1.50\times 10^{11}\ \text{m}}{6.96\times 10^8\ \text{m}}\right)^2 = 6.97\times 10^7\ \text{W/m}^2.$$

(b) Solving Eq.(17.25) with $e = 1$, $T = \left[\dfrac{H}{A}\dfrac{1}{\sigma}\right]^{\frac{1}{4}} = \left[\dfrac{6.97\times 10^7\ \text{W/m}^2}{5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4}\right]^{\frac{1}{4}} = 5920$ K.

EVALUATE: The total power output of the sun is $P = 4\pi r_2^2 I_2 = 2.0\times 10^{31}$ W.

17.117. IDENTIFY and SET UP: Use Eq.(17.26) to find the net heat current into the can due to radiation. Use $Q = Ht$ to find the heat that goes into the liquid helium, set this equal to *mL* and solve for the mass *m* of helium that changes phase.

EXECUTE: Calculate the net rate of radiation of heat from the can. $H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$.

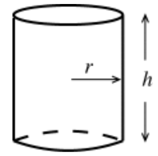


Figure 17.117

The surface area of the cylindrical can is $A = 2\pi rh + 2\pi r^2$. (See Figure 17.117.)

$A = 2\pi r(h + r) = 2\pi(0.045\ \text{m})(0.250\ \text{m} + 0.045\ \text{m}) = 0.08341\ \text{m}^2$.

$H_{\text{net}} = (0.08341\ \text{m}^2)(0.200)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)((4.22\ \text{K})^4 - (77.3\ \text{K})^4)$

$H_{\text{net}} = -0.0338\ \text{W}$ (the minus sign says that the net heat current is into the can). The heat that is put into the can by radiation in one hour is $Q = -(H_{\text{net}})t = (0.0338\ \text{W})(3600\ \text{s}) = 121.7\ \text{J}$. This heat boils a mass *m* of helium according to the equation $Q = mL_f$, so $m = \dfrac{Q}{L_f} = \dfrac{121.7\ \text{J}}{2.09\times10^4\ \text{J/kg}} = 5.82\times10^{-3}\ \text{kg} = 5.82\ \text{g}$.

EVALUATE: In the expression for the net heat current into the can the temperature of the surroundings is raised to the fourth power. The rate at which the helium boils away increases by about a factor of $(293/77)^4 = 210$ if the walls surrounding the can are at room temperature rather than at the temperature of the liquid nitrogen.

17.118. IDENTIFY: The coefficient of volume expansion β is defined by $\Delta V = V_0\beta\Delta T$.

SET UP: For copper, $\beta = 5.1\times10^{-5}\ \text{K}^{-1}$.

EXECUTE: **(a)** With $\Delta p = 0$, $p\Delta V = nR\Delta T = \dfrac{pV}{T}\Delta T$, or $\dfrac{\Delta V}{V} = \dfrac{\Delta T}{T}$, and $\beta = \dfrac{1}{T}$.

(b) $\dfrac{\beta_{\text{air}}}{\beta_{\text{copper}}} = \dfrac{1}{(293\ \text{K})(5.1\times10^{-5}\ \text{K}^{-1})} = 67$.

EVALUATE: The coefficient of volume expansion for air is much greater than that for copper. For a given ΔT, gases expand much more than solids do.

17.119. IDENTIFY: For the water, $Q = mc\Delta T$.

SET UP: For water, $c = 4190\ \text{J/kg}\cdot\text{K}$.

EXECUTE: **(a)** At steady state, the input power all goes into heating the water, so $P = \dfrac{Q}{t} = \dfrac{mc\Delta T}{t}$ and

$\Delta T = \dfrac{Pt}{cm} = \dfrac{(1800\ \text{W})(60\ \text{s/min})}{(4190\ \text{J/kg}\cdot\text{K})(0.500\ \text{kg/min})} = 51.6\ \text{K}$, and the output temperature is $18.0°\text{C} + 51.6°\text{C} = 69.6°\text{C}$.

EVALUATE: **(b)** At steady state, the temperature of the apparatus is constant and the apparatus will neither remove heat from nor add heat to the water.

17.120. IDENTIFY: For the air the heat input is related to the temperature change by $Q = mc\Delta T$.

SET UP: The rate *P* at which heat energy is generated is related to the rate P_0 at which food energy is consumed by the hamster by $P = 0.10P_0$.

EXECUTE: **(a)** The heat generated by the hamster is the heat added to the box;

$P = \dfrac{Q}{t} = mc\dfrac{\Delta T}{t} = (1.20\ \text{kg/m}^3)(0.0500\ \text{m}^3)(1020\ \text{J/kg}\cdot\text{K})(1.60\ \text{C°/h}) = 97.9\ \text{J/h}$.

(b) Taking the efficiency into account, the mass *M* of seed that must be eaten in time *t* is

$\dfrac{M}{t} = \dfrac{P_0}{L_c} = \dfrac{P/(10\%)}{L_c} = \dfrac{979\ \text{J/h}}{24\ \text{J/g}} = 40.8\ \text{g/h}$.

EVALUATE: This is about 1.5 ounces of seed consumed in one hour.

17.121. IDENTIFY: Heat Q_i goes into the ice when it warms to 0°C, melts, and the resulting water warms to the final temperature T_f. Heat Q_{ow} comes out of the ocean water when it cools to T_f. Conservation of energy gives $Q_i + Q_{ow} = 0$.

SET UP: For ice, $c_i = 2100\ \text{J/kg}\cdot\text{K}$. For water, $L_f = 334\times10^3\ \text{J/kg}$ and $c_w = 4190\ \text{J/kg}\cdot\text{K}$. Let *m* be the total mass of the water on the earth's surface. So $m_i = 0.0175m$ and $m_{ow} = 0.975m$.

EXECUTE: $Q_i + Q_{ow} = 0$ gives $m_i c_i (30\ \text{C}°) + m_i L_f + m_i c_w T_f + m_{ow} c_w (T_f - 5.00°\text{C}) = 0.$

$$T_f = \frac{-m_i c_i (30\ \text{C}°) - m_i L_f + m_{ow} c_w (5.00\ \text{C}°)}{(m_i + m_{ow}) c_w}.$$

$$T_f = \frac{-(0.0175m)(2100\ \text{J/kg}\cdot\text{K})(30\ \text{K}) - (0.0175m)(334\times 10^3\ \text{J/kg}) + (0.975m)(4190\ \text{J/kg}\cdot\text{K})(5.00\ \text{K})}{(0.0175m + 0.975m)(4190\ \text{J/kg}\cdot\text{K})}$$

$T_f = \dfrac{1.348\times 10^4\ \text{J/kg}}{4.159\times 10^3\ \text{J/kg}\cdot\text{K}} = 3.24°\text{C}.$ The temperature decrease is $1.76\ \text{C}°.$

EVALUATE: The mass of ice in the icecaps is much less than the mass of the water in the oceans, but much more heat is required to change the phase of 1 kg of ice than to change the temperature of 1 kg of water $1\ \text{C}°$, so the lowering of the temperature of the oceans would be appreciable.

17.122. IDENTIFY: Apply Eq.(17.21). For a spherical or cylindrical surface, the area A in Eq.(17.21) is not constant, and the material must be considered to consist of shells with thickness dr and a temperature difference between the inside and outside of the shell dT. The heat current will be a constant, and must be found by integrating a differential equation.

SET UP: The surface area of a sphere is $4\pi r^2$. The surface area of the curved side of a cylinder is $2\pi rl$. $\ln(1+\varepsilon) \approx \varepsilon$ when $\varepsilon \ll 1$.

(a) Equation (17.21) becomes $H = k(4\pi r^2)\dfrac{dT}{dr}$ or $\dfrac{H\ dr}{4\pi r^2} = k\ dT$. Integrating both sides between the appropriate limits, $\dfrac{H}{4\pi}\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = k(T_2 - T_1)$. In this case the "appropriate limits" have been chosen so that if the inner temperature T_2 is at the higher temperature T_1, the heat flows outward; that is, $\dfrac{dT}{dr} < 0$. Solving for the heat current, $H = \dfrac{k4\pi ab(T_2 - T_1)}{b - a}$.

(b) The rate of change of temperature with radius is of the form $\dfrac{dT}{dr} = \dfrac{B}{r^2}$, with B a constant. Integrating from $r = a$ to r and from $r = a$ to $r = b$ gives $T(r) - T_2 = B\left(\dfrac{1}{a} - \dfrac{1}{r}\right)$ and $T_1 - R_2 = B\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$. Using the second of these to eliminate B and solving for $T(r)$ gives $T(r) = T_2 - (T_2 - T_1)\left(\dfrac{r-a}{b-a}\right)\left(\dfrac{b}{r}\right)$. There are, of course, many equivalent forms. As a check, note that at $r = a$, $T = T_2$ and at $r = b$, $T = T_1$.

(c) As in part (a), the expression for the heat current is $H = k(2\pi rL)\dfrac{dT}{dr}$ or $\dfrac{H}{2\pi r} = kLdT$, which integrates, with the same condition on the limits, to $\dfrac{H}{2\pi}\ln(b/a) = kL(T_2 - T_1)$, or $H = \dfrac{2\pi kL(T_2 - T_1)}{\ln(b/a)}$.

(d) A method similar to that used in part (b) gives $T(r) = T_2 + (T_1 - T_2)\dfrac{\ln(r/a)}{\ln(b/a)}$.

EVALUATE: **(e)** For the sphere: Let $b - a = l$, and approximate $b \sim a$, with a the common radius. Then the surface area of the sphere is $A = 4\pi a^2$, and the expression for H is that of Eq. (17.21) (with l instead of L, which has another use in this problem). For the cylinder: with the same notation, consider $\ln\left(\dfrac{b}{a}\right) = \ln\left(1 + \dfrac{l}{a}\right) \sim \dfrac{l}{a}$, where the approximation for $\ln(1+\varepsilon)$ for small ε has been used. The expression for H then reduces to $k(2\pi La)(\Delta T/l)$, which is Eq. (17.21) with $A = 2\pi La$.

17.123. IDENTIFY: From the result of Problem 17.122, the heat current through each of the jackets is related to the temperature difference by $H = \dfrac{2\pi lk}{\ln(b/a)}\Delta T$, where l is the length of the cylinder and b and a are the inner and outer radii of the cylinder.

SET UP: Let the temperature across the cork be ΔT_1 and the temperature across the styrofoam be ΔT_2, with similar notation for the thermal conductivities and heat currents.

EXECUTE: **(a)** $\Delta T_1 + \Delta T_2 = \Delta T =$ 125 C°. Setting $H_1 = H_2 = H$ and canceling the common factors, $\frac{\Delta T_1 k_1}{\ln 2} = \frac{\Delta T_2 k_2}{\ln 1.5}$. Eliminating ΔT_2 and solving for ΔT_1 gives $\Delta T_1 = \Delta T\left(1 + \frac{k_1}{k_2}\frac{\ln 1.5}{\ln 2}\right)^{-1}$. Substitution of numerical values gives $\Delta T_1 = 37$ C°, and the temperature at the radius where the layers meet is $140°\text{C} - 37°\text{C} = 103°\text{C}$.

(b) Substitution of this value for ΔT_1 into the above expression for $H_1 = H$ gives

$$H = \frac{2\pi(2.00\text{ m})(0.0400\text{ W/m}\cdot\text{K})}{\ln 2}(37\text{ C°}) = 27\text{ W}.$$

EVALUATE: $\Delta T = 103°\text{C} - 15°\text{C} = 88\text{ C°}$. $H_2 = \frac{2\pi(2.00\text{ m})(0.0100\text{ W/m}\cdot\text{K})}{\ln(6.00/4.00)}(88\text{ C°}) = 27\text{ W}$. This is the same as H_1, as it should be.

17.124. IDENTIFY: Apply Eq.(17.22) to different points along the rod, where $\frac{dT}{dx}$ is the temperature gradient at each point.

SET UP: For copper, $k = 385\text{ W/m}\cdot\text{K}$.

EXECUTE: **(a)** The initial temperature distribution, $T = (100°\text{C})\sin \pi x/L$, is shown in Figure 17.124a.

(b) After a very long time, no heat will flow, and the entire rod will be at a uniform temperature which must be that of the ends, 0°C.

(c) The temperature distribution at successively greater times $T_1 < T_2 < T_3$ is sketched in Figure 17.124b.

(d) $\frac{dT}{dx} = (100°\text{C})(\pi/L)\cos \pi x/L$. At the ends, $x = 0$ and $x = L$, the cosine is ± 1 and the temperature gradient is $\pm(100°\text{C})(\pi/0.100\text{ m}) = \pm 3.14\times 10^3\text{ C°/m}$.

(e) Taking the phrase "into the rod" to mean an absolute value, the heat current will be $kA\frac{dT}{dx} = (385.0\text{ W/m}\cdot\text{K})(1.00\times 10^{-4}\text{ m}^2)(3.14\times 10^3\text{ C°/m}) = 121\text{ W}$.

(f) Either by evaluating $\frac{dT}{dx}$ at the center of the rod, where $\pi x/L = \pi/2$ and $\cos(\pi/2) = 0$, or by checking the figure in part (a), the temperature gradient is zero, and no heat flows through the center; this is consistent with the symmetry of the situation. There will not be any heat current at the center of the rod at any later time.

(g) $\frac{k}{\rho c} = \frac{(385\text{ W/m}\cdot\text{K})}{(8.9\times 10^3\text{ kg/m}^3)(390\text{ J/kg}\cdot\text{K})} = 1.1\times 10^{-4}\text{ m}^2/\text{s}$.

(h) Although there is no net heat current, the temperature of the center of the rod is decreasing; by considering the heat current at points just to either side of the center, where there is a non-zero temperature gradient, there must be a net flow of heat out of the region around the center. Specifically,

$H((L/2)+\Delta x) - H((L/2)-\Delta x) = \rho A(\Delta x)c\frac{\partial T}{\partial t} = kA\left(\left.\frac{\partial T}{\partial x}\right|_{(L/2)+\Delta x} - \left.\frac{\partial T}{\partial x}\right|_{(L/2)-\Delta x}\right) = kA\frac{\partial^2 T}{\partial x^2}\Delta x$, from which the Heat Equation, $\frac{\partial T}{\partial t} = \frac{k}{\rho c}\frac{\partial^2 T}{\partial x^2}$ is obtained. At the center of the rod, $\frac{\partial^2 T}{\partial x^2} = -(100\text{ C°})(\pi/L)^2$, and so

$$\frac{\partial T}{\partial t} = -(1.11\times 10^{-4}\text{ m}^2/\text{s})(100\text{ C°})\left(\frac{\pi}{0.100\text{ m}}\right)^2 = -10.9\text{ C°/s},\text{ or } -11\text{C°/s to two figures.}$$

(i) $\frac{100\text{ C°}}{10.9\text{ C°/s}} = 9.17\text{ s}$

(j) Decrease (that is, become less negative), since as T decreases, $\frac{\partial^2 T}{\partial x^2}$ decreases. This is consistent with the graphs, which correspond to equal time intervals.

(k) At the point halfway between the end and the center, at any given time $\frac{\partial^2 T}{\partial x^2}$ is a factor of $\sin(\pi/4) = 1/\sqrt{2}$ less than at the center, and so the initial rate of change of temperature is -7.71C°/s.

EVALUATE: A plot of temperature as a function of both position and time for $0 \le t \le 50$ s is shown in Figure 17.124c.

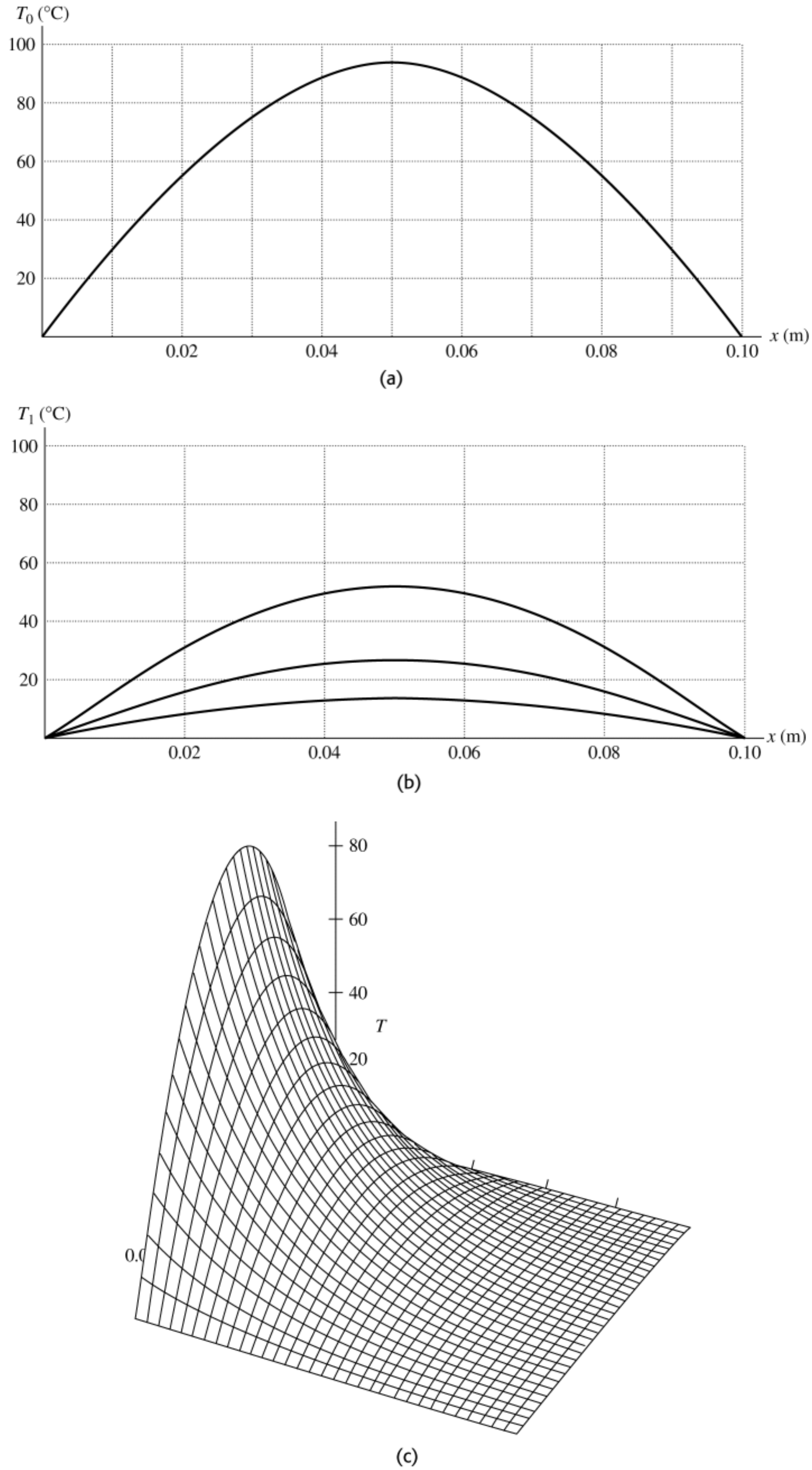


Figure 17.124

17.125. IDENTIFY: Apply the concept of thermal expansion. In part (b) the object can be treated as a simple pendulum.

SET UP: For steel $\alpha = 1.2\times10^{-5}\ (\text{C}°)^{-1}$. $1\ \text{yr} = 86{,}400\ \text{s}$.

EXECUTE: **(a)** In hot weather, the moment of inertia *I* and the length *d* in Eq.(13.39) will both increase by the same factor, and so the period will be longer and the clock will run slow (lose time). Similarly, the clock will run fast (gain time) in cold weather.

(b) $\dfrac{\Delta L}{L_0} = \alpha\Delta T = (1.2\times10^{-5}\ (\text{C}°)^{-1})(10.0\ \text{C}°) = 1.2\times10^{-4}.$

(c) See Problem 13.98. To avoid possible confusion, denote the pendulum period by τ. For this problem, $\dfrac{\Delta\tau}{\tau} = \dfrac{1}{2}\dfrac{\Delta L}{L} = 6.0\times10^{-5}$ so in one day the clock will gain $(86{,}400\ \text{s})(6.0\times10^{-5}) = 5.2\ \text{s}$.

(d) $\left|\dfrac{\Delta\tau}{\tau}\right| = \tfrac{1}{2}\alpha\Delta T$. $\left|\dfrac{\Delta\tau}{\tau}\right| = \dfrac{1.0\ \text{s}}{86{,}400\ \text{s}}$ gives $\Delta T = 2[(1.2\times10^{-5}\ (\text{C}°)^{-1})(86{,}400)]^{-1} = 1.9\ \text{C}°$. *T* must be controlled to within $1.9\ \text{C}°$.

EVALUATE: In part (d) the answer does not depend on the period of the pendulum. It depends only on the fractional change in the period.

17.126. IDENTIFY: The rate at which heat is absorbed at the blackened end is the heat current in the rod, $Ae\sigma(T_S^4 - T_2^4) = \dfrac{kA}{L}(T_2 - T_1)$ where $T_1 = 20.00\ \text{K}$ and T_2 is the temperature of the blackened end of the rod.

SET UP: Since the end is blackened, $e = 1$. $T_s = 500.0\ \text{K}$.

EXECUTE: If the equation were to be solved exactly for T_2, the equation would be a quartic, very likely not worth the trouble. Following the hint, approximate T_2 on the left side of the above expression as T_1 to obtain

$$T_2 = T_1 + \frac{\sigma L}{k}(T_s^2 - T_1^4) = T_1 + (6.79\times10^{-12}\ \text{K}^{-3})(T_s^4 - T_1^4) = T_1 + 0.424\ \text{K} = 20.42\ \text{K}.$$

EVALUATE: This approximation for T_2 is indeed only slightly than T_1, and is a good estimate of the temperature. Using this for T_2 in the original expression to find a better value of ΔT gives the same ΔT to eight figures, and further iterations are not worthwhile.

17.127. IDENTIFY: The rate in (iv) is given by Eq.(17.26), with $T = 309\ \text{K}$ and $T_s = 320\ \text{K}$. The heat absorbed in the evaporation of water is $Q = mL$.

SET UP: $m = \rho V$, so $\dfrac{m}{V} = \rho$.

EXECUTE: **(a)** The rates are: (i) 280 W,

(ii) $(54\ \text{J/h}\cdot\text{C}°\cdot\text{m}^2)(1.5\ \text{m}^2)(11\ \text{C}°)/(3600\ \text{s/h}) = 0.248\ \text{W}$,

(iii) $(1400\ \text{W/m}^2)(1.5\ \text{m}^2) = 2.10\times10^3\ \text{W}$,

(iv) $(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.5\ \text{m}^2)((320\ \text{K})^4 - (309\ \text{K})^4) = 116\ \text{W}$.

The total is 2.50 kW, with the largest portion due to radiation from the sun.

(b) $\dfrac{P}{\rho L_v} = \dfrac{2.50\times10^3\ \text{W}}{(1000\ \text{kg/m}^3)(2.42\times10^6\ \text{J/kg}\cdot\text{K})} = 1.03\times10^{-6}\ \text{m}^3/\text{s}$. This is equal to $= 3.72\ \text{L/h}$.

(c) Redoing the above calculations with $e = 0$ and the decreased area gives a power of 945 W and a corresponding evaporation rate of 1.4 L/h. Wearing reflective clothing helps a good deal. Large areas of loose weave clothing also facilitate evaporation.

EVALUATE: The radiant energy from the sun absorbed by the area covered by clothing is assumed to be zero, since $e \approx 0$ for the clothing and the clothing reflects almost all the radiant energy incident on it. For the same reason, the exposed skin area is the area used in Eq.(17.26).

THERMAL PROPERTIES OF MATTER

18

18.1. **(a) IDENTIFY:** We are asked about a single state of the system.

SET UP: Use Eq.(18.2) to calculate the number of moles and then apply the ideal-gas equation.

EXECUTE: $n = \dfrac{m_{\text{tot}}}{M} = \dfrac{0.225\text{ kg}}{4.00\times10^{-3}\text{ kg/mol}} = 56.2\text{ mol}$

(b) $pV = nRT$ implies $p = nRT/V$

T must be in kelvins; $T = (18+273)\text{ K} = 291\text{ K}$

$$p = \frac{(56.2\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})(291\text{ K})}{20.0\times10^{-3}\text{ m}^3} = 6.80\times10^6\text{ Pa}$$

$p = (6.80\times10^6\text{ Pa})(1.00\text{ atm}/1.013\times10^5\text{ Pa}) = 67.1\text{ atm}$

EVALUATE: Example 18.1 shows that 1.0 mol of an ideal gas is about this volume at STP. Since there are 56.2 moles the pressure is about 60 times greater than 1 atm.

18.2. **IDENTIFY:** $pV = nRT$.

SET UP: $T_1 = 41.0°\text{C} = 314\text{ K}$. $R = 0.08206\text{ L}\cdot\text{atm/mol}\cdot\text{K}$.

EXECUTE: n, R constant so $\dfrac{pV}{T} = nR$ is constant. $\dfrac{p_1V_1}{T_1} = \dfrac{p_2V_2}{T_2}$.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)\left(\frac{V_2}{V_1}\right) = (314\text{ K})(2)(2) = 1.256\times10^3\text{ K} = 983°\text{C}.$$

(b) $n = \dfrac{pV}{RT} = \dfrac{(1.30\text{ atm})(2.60\text{ L})}{(0.08206\text{ L}\cdot\text{atm/mol}\cdot\text{K})(314\text{ K})} = 0.131\text{ mol}$. $m_{\text{tot}} = nM = (0.131\text{ mol})(4.00\text{ g/mol}) = 0.524\text{ g}$.

EVALUATE: T is directly proportional to p and to V, so when p and V are each doubled the Kelvin temperature increases by a factor of 4.

18.3. **IDENTIFY:** $pV = nRT$.

SET UP: T is constant.

EXECUTE: nRT is constant so $p_1V_1 = p_2V_2$. $p_2 = p_1\left(\dfrac{V_1}{V_2}\right) = (3.40\text{ atm})\left(\dfrac{0.110\text{ m}^3}{0.390\text{ m}^3}\right) = 0.959\text{ atm}$.

EVALUATE: For T constant, p decreases.

18.4. **IDENTIFY:** $pV = nRT$.

SET UP: $T_1 = 20.0°\text{C} = 293\text{ K}$.

EXECUTE: **(a)** n, R, and V are constant. $\dfrac{p}{T} = \dfrac{nR}{V} = \text{constant}$. $\dfrac{p_1}{T_1} = \dfrac{p_2}{T_2}$.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right) = (293\text{ K})\left(\frac{1.00\text{ atm}}{3.00\text{ atm}}\right) = 97.7\text{ K} = -175°\text{C}.$$

(b) $p_2 = 1.00\text{ atm}$, $V_2 = 3.00\text{ L}$. $p_3 = 3.00\text{ atm}$. n, R, and T are constant so $pV = nRT = $ constant. $p_2V_2 = p_3V_3$.

$$V_3 = V_2\left(\frac{p_2}{p_3}\right) = (3.00\text{ L})\left(\frac{1.00\text{ atm}}{3.00\text{ atm}}\right) = 1.00\text{ L}.$$

EVALUATE: The final volume is one-third the initial volume. The initial and final pressures are the same, but the final temperature is one-third the initial temperature.

18.5. IDENTIFY: $pV = nRT$

SET UP: Assume a room size of $20\text{ ft}\times 20\text{ ft}\times 10\text{ ft}$. $V = 4000\text{ ft}^3 = 113\text{ m}^3$. Assume a temperature of $T = 20°\text{C} = 293\text{ K}$ and a pressure of $p = 1.01\times 10^5\text{ Pa}$. $1\text{ m}^3 = 10^6\text{ cm}^3$.

EXECUTE: **(a)** $n = \dfrac{pV}{RT} = \dfrac{(1.01\times 10^5\text{ Pa})(113\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(293\text{ K})} = 4.68\times 10^3\text{ mol}$.

$N = nN_\text{A} = (4.68\times 10^3\text{ mol})(6.022\times 10^{23}\text{ molecules/mol}) = 3\times 10^{27}\text{ molecules}$.

(b) $\dfrac{N}{V} = \dfrac{3\times 10^{27}\text{ molecules}}{113\text{ m}^3} = 3\times 10^{25}\text{ molecules/m}^3 = 3\times 10^{19}\text{ molecules/cm}^3$

EVALUATE: The solution doesn't rely on the assumption that air is all N_2.

18.6. IDENTIFY: $pV = nRT$ and the mass of the gas is $m_\text{tot} = nM$.

SET UP: The temperature is $T = 22.0°\text{C} = 295.15\text{K}$. The average molar mass of air is $M = 28.8\times 10^{-3}\text{ kg/mol}$. For helium $M = 4.00\times 10^{-3}\text{ kg/mol}$.

EXECUTE: **(a)** $m_\text{tot} = nM = \dfrac{pV}{RT}M = \dfrac{(1.00\text{ atm})(0.900\text{ L})(28.8\times 10^{-3}\text{ kg/mol})}{(0.08206\text{ L}\cdot\text{atm/mol}\cdot\text{K})(295.15\text{ K})} = 1.07\times 10^{-3}\text{ kg}$.

(b) $m_\text{tot} = nM = \dfrac{pV}{RT}M = \dfrac{(1.00\text{ atm})(0.900\text{ L})(4.00\times 10^{-3}\text{ kg/mol})}{(0.08206\text{ L}\cdot\text{atm/mol}\cdot\text{K})(295.15\text{ K})} = 1.49\times 10^{-4}\text{ kg}$.

EVALUATE: $n = \dfrac{N}{N_\text{A}} = \dfrac{pV}{RT}$ says that in each case the balloon contains the same number of molecules. The mass is greater for air since the mass of one molecule is greater than for helium.

18.7. IDENTIFY: We are asked to compare two states. Use the ideal gas law to obtain T_2 in terms of T_1 and ratios of pressures and volumes of the gas in the two states.

SET UP: $pV = nRT$ and *n*, *R* constant implies $pV/T = nR = \text{constant}$ and $p_1V_1/T_1 = p_2V_2/T_2$

EXECUTE: $T_1 = (27 + 273)\text{ K} = 300\text{ K}$

$p_1 = 1.01\times 10^5\text{ Pa}$

$p_2 = 2.72\times 10^6\text{ Pa} + 1.01\times 10^5\text{ Pa} = 2.82\times 10^6\text{ Pa}$ (in the ideal gas equation the pressures must be absolute, not gauge, pressures)

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)\left(\frac{V_2}{V_1}\right) = 300\text{ K}\left(\frac{2.82\times 10^6\text{ Pa}}{1.01\times 10^5\text{ Pa}}\right)\left(\frac{46.2\text{ cm}^3}{499\text{ cm}^3}\right) = 776\text{ K}$$

$T_2 = (776 - 273)°\text{C} = 503°\text{C}$

EVALUATE: The units cancel in the V_2/V_1 volume ratio, so it was not necessary to convert the volumes in cm^3 to m^3. It was essential, however, to use *T* in kelvins.

18.8. IDENTIFY: $pV = nRT$ and $m = nM$.

SET UP: We must use absolute pressure in $pV = nRT$. $p_1 = 4.01\times 10^5\text{ Pa}$, $p_2 = 2.81\times 10^5\text{ Pa}$. $T_1 = 310\text{ K}$, $T_2 = 295\text{ K}$.

EXECUTE: **(a)** $n_1 = \dfrac{p_1V_1}{RT_1} = \dfrac{(4.01\times 10^5\text{ Pa})(0.075\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(310\text{ K})} = 11.7\text{ mol}$. $m = nM = (11.7\text{ mol})(32.0\text{ g/mol}) = 374\text{ g}$.

(b) $n_2 = \dfrac{p_2V_2}{RT_2} = \dfrac{(2.81\times 10^5\text{ Pa})(0.075\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(295\text{ K})} = 8.59\text{ mol}$. $m = 275\text{ g}$.

The mass that has leaked out is $374\text{ g} - 275\text{ g} = 99\text{ g}$.

EVALUATE: In the ideal gas law we must use absolute pressure, expressed in Pa, and *T* must be in kelvins.

18.9. IDENTIFY: $pV = nRT$.

SET UP: $T_1 = 300\text{ K}$, $T_2 = 430\text{ K}$.

EXECUTE: **(a)** *n*, *R* are constant so $\dfrac{pV}{T} = nR = \text{constant}$. $\dfrac{p_1V_1}{T_1} = \dfrac{p_2V_2}{T_2}$.

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right) = (1.50\times 10^5\text{ Pa})\left(\frac{0.750\text{ m}^3}{0.480\text{ m}^3}\right)\left(\frac{430\text{ K}}{300\text{ K}}\right) = 3.36\times 10^5\text{ Pa}.$$

EVALUATE: In $pV = nRT$, *T* must be in kelvins, even if we use a ratio of temperatures.

18.10. **IDENTIFY:** Use the ideal-gas equation to calculate the number of moles, n. The mass m_{total} of the gas is $m_{\text{total}} = nM$.

SET UP: The volume of the cylinder is $V = \pi r^2 l$, where $r = 0.450$ m and $l = 1.50$ m. $T = 22.0°\text{C} = 293.15$ K. 1 atm $= 1.013 \times 10^5$ Pa. $M = 32.0 \times 10^{-3}$ kg/mol. $R = 8.314$ J/mol·K.

EXECUTE: **(a)** $pV = nRT$ gives $n = \dfrac{pV}{RT} = \dfrac{(21.0\text{ atm})(1.013\times10^5\text{ Pa/atm})\pi(0.450\text{ m})^2(1.50\text{ m})}{(8.314\text{ J/mol}\cdot\text{K})(295.15\text{ K})} = 827$ mol.

(b) $m_{\text{total}} = (827\text{ mol})(32.0\times10^{-3}\text{ kg/mol}) = 26.5$ kg

EVALUATE: In the ideal-gas law, T must be in kelvins. Since we used R in units of J/mol·K we had to express p in units of Pa and V in units of m^3.

18.11. **IDENTIFY:** We are asked to compare two states. Use the ideal-gas law to obtain V_1 in terms of V_2 and the ratio of the temperatures in the two states.

SET UP: $pV = nRT$ and n, R, p are constant so $V/T = nR/p = \text{constant}$ and $V_1/T_1 = V_2/T_2$

EXECUTE: $T_1 = (19 + 273)\text{ K} = 292$ K (T must be in kelvins)

$V_2 = V_1(T_2/T_1) = (0.600\text{ L})(77.3\text{ K}/292\text{ K}) = 0.159$ L

EVALUATE: p is constant so the ideal-gas equation says that a decrease in T means a decrease in V.

18.12. **IDENTIFY:** Apply $pV = nRT$ and the van der Waals equation (Eq.18.7) to calculate p.

SET UP: $400\text{ cm}^3 = 400\times10^{-6}\text{ m}^3$. $R = 8.314$ J/mol·K.

EXECUTE: **(a)** The ideal gas law gives $p = nRT/V = 7.28\times10^6$ Pa while Eq.(18.7) gives 5.87×10^6 Pa.

(b) The van der Waals equation, which accounts for the attraction between molecules, gives a pressure that is 20% lower.

(c) The ideal gas law gives $p = 7.28\times10^5$ Pa. Eq.(18.7) gives $p = 7.13\times10^5$ Pa, for a 2.1% difference.

EVALUATE: **(d)** As n/V decreases, the formulas and the numerical values for the two equations approach each other.

18.13. **IDENTIFY:** $pV = nRT$.

SET UP: T is constant.

EXECUTE: n, R, T are constant, so $pV = nRT = \text{constant}$. $p_1V_1 = p_2V_2$.

$$p_2 = p_1\left(\frac{V_1}{V_2}\right) = (1.00\text{ atm})\left(\frac{6.00\text{ L}}{5.70\text{ L}}\right) = 1.05\text{ atm}.$$

EVALUATE: For constant T, when V decreases, p increases. Since the volumes enter as a ratio we don't have to convert from L to m^3.

18.14. **IDENTIFY:** $pV = nRT$.

SET UP: $T_1 = 277$ K. $T_2 = 296$ K. Assume the number of moles of gas in the bubble remains constant.

EXECUTE: **(a)** n, R are constant so $\dfrac{pV}{T} = nR =$ constant. $\dfrac{p_1V_1}{T_1} = \dfrac{p_2V_2}{T_2}$ and

$$\frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)\left(\frac{T_2}{T_1}\right) = \left(\frac{3.50\text{ atm}}{1.00\text{ atm}}\right)\left(\frac{296\text{ K}}{277\text{ K}}\right) = 3.74.$$

(b) This increase in volume of air in the lungs would be dangerous.

EVALUATE: The large decrease in pressure results in a large increase in volume.

18.15. **IDENTIFY:** We are asked to compare two states. First use $pV = nRT$ to calculate p_1. Then use it to obtain T_2 in terms of T_1 and the ratio of pressures in the two states.

(a) SET UP: $pV = nRT$. Find the initial pressure p_1:

EXECUTE: $p_1 = \dfrac{nRT_1}{V} = \dfrac{(11.0\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})((23.0 + 273.13)\text{K})}{3.10\times10^{-3}\text{ m}^3} = 8.737\times10^6$ Pa

SET UP: $p_2 = 100\text{ atm}(1.013\times10^5\text{ Pa/1 atm}) = 1.013\times10^7$ Pa

$p/T = nR/V = \text{constant}$, so $p_1/T_1 = p_2/T_2$

EXECUTE: $T_2 = T_1\left(\dfrac{p_2}{p_1}\right) = (296.15\text{ K})\left(\dfrac{1.013\times10^7\text{ Pa}}{8.737\times10^6\text{ Pa}}\right) = 343.4\text{ K} = 70.2°\text{C}$

(b) EVALUATE: The coefficient of volume expansion for a gas is much larger than for a solid, so the expansion of the tank is negligible.

18.16. **IDENTIFY:** $F = pA$ and $pV = nRT$

SET UP: For a cube, $V/A = L$.

EXECUTE: **(a)** The force of any side of the cube is $F = pA = (nRT/V)A = (nRT)/L$, since the ratio of area to volume is $A/V = 1/L$. For $T = 20.0°\text{C} = 293.15\text{ K}$,

$$F = \frac{nRT}{L} = \frac{(3\text{ mol})\,(8.3145\text{ J/mol}\cdot\text{K})\,(293.15\text{ K})}{0.200\text{ m}} = 3.66\times 10^4\text{ N}.$$

(b) For $T = 100.00°\text{C} = 373.15\text{ K}$,

$$F = \frac{nRT}{L} = \frac{(3\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})(373.15\text{ K})}{0.200\text{ m}} = 4.65\times 10^4\text{ N}.$$

EVALUATE: When the temperature increases while the volume is kept constant, the pressure increases and therefore the force increases. The force increases by the factor T_2/T_1.

18.17. **IDENTIFY:** Example 18.4 assumes a temperature of $0°\text{C}$ at all altitudes and neglects the variation of *g* with elevation. With these approximations, $p = p_0 e^{-Mgy/RT}$.

SET UP: $\ln(e^{-x}) = -x$. For air, $M = 28.8\times 10^{-3}$ kg/mol.

EXECUTE: We want *y* for $p = 0.90 p_0$ so $0.90 = e^{-Mgy/RT}$ and $y = -\frac{RT}{Mg}\ln(0.90) = 850\text{ m}$.

EVALUATE: This is a commonly occurring elevation, so our calculation shows that 10% variations in atmospheric pressure occur at many locations.

18.18. **IDENTIFY:** From Example 18.4, the pressure at elevation *y* above sea level is $p = p_0 e^{-Mgy/RT}$.

SET UP: The average molar mass of air is $M = 28.8\times 10^{-3}$ kg/mol.

EXECUTE: At an altitude of 100 m, $\frac{Mgy_1}{RT} = \frac{(28.8\times 10^{-3}\text{ kg/mol})(9.80\text{ m/s}^2)(100\text{ m})}{(8.3145\text{ J/mol}\cdot\text{K})(273.15\text{ K})} = 0.01243$, and the percent decrease in pressure is $1 - p/p_0 = 1 - e^{-0.01243} = 0.0124 = 1.24\%$. At an altitude of 1000 m, $Mgy_2/RT = 0.1243$ and the percent decrease in pressure is $1 - e^{-0.1243} = 0.117 = 11.7\%$.

EVALUATE: These answers differ by a factor of $(11.7\%)/(1.24\%) = 9.44$, which is less than 10 because the variation of pressure with altitude is exponential rather than linear.

18.19. **IDENTIFY:** $p = p_0 e^{-Myg/RT}$ from Example 18.4. Eq.(18.5) says $p = (\rho/M)RT$. Example 18.4 assumes a constant $T = 273\text{ K}$, so *p* and *ρ* are directly proportional and we can write $\rho = \rho_0 e^{-Mgy/RT}$.

SET UP: From Example 18.4, $\frac{Mgy}{RT} = 1.10$ when $y = 8863\text{ m}$.

EXECUTE: For $y = 100\text{ m}$, $\frac{Mgy}{RT} = 0.0124$, so $\rho = \rho_0 e^{-0.0124} = 0.988\rho_0$. The density at sea level is 1.2% larger than the density at 100 m.

EVALUATE: The pressure decreases with altitude. $pV = \frac{m_{\text{tot}}}{M}RT$, so when the pressure decreases and *T* is constant the volume of a given mass of gas increases and the density decreases.

18.20. **IDENTIFY:** $p = p_0 e^{-Mgy/RT}$ from Example 18.4 gives the variation of air pressure with altitude. The density ρ of the air is $\rho = \frac{pM}{RT}$, so ρ is proportional to the pressure *p*. Let ρ_0 be the density at the surface, where the pressure is p_0.

SET UP: From Example 18.4, $\frac{Mg}{RT} = \frac{(28.8\times 10^{-3}\text{ kg/mol})(9.80\text{ m/s}^2)}{(8.314\text{ J/mol}\cdot\text{K})(273\text{ K})} = 1.244\times 10^{-4}\text{ m}^{-1}$.

EXECUTE: $p = p_0 e^{-(1.244\times 10^{-4}\text{ m}^{-1})(1.00\times 10^3\text{ m})} = 0.883 p_0$. $\frac{\rho}{p} = \frac{M}{RT} =$ constant, so $\frac{\rho}{p} = \frac{\rho_0}{p_0}$ and $\rho = \rho_0\left(\frac{p}{p_0}\right) = 0.883\rho_0$.

The density at an altitude of 1.00 km is 88.3% of its value at the surface.

EVALUATE: If the temperature is assumed to be constant, then the decrease in pressure with increase in altitude corresponds to a decrease in density.

18.21. IDENTIFY: Use Eq.(18.5) and solve for p.

SET UP: $\rho = pM/RT$ and $p = RT\rho/M$

$T = (-56.5 + 273.15)\text{ K} = 216.6\text{ K}$

For air $M = 28.8\times10^{-3}$ kg/mol (Example 18.3)

EXECUTE: $p = \dfrac{(8.3145\text{ J/mol}\cdot\text{K})(216.6\text{ K})(0.364\text{ kg/m}^3)}{28.8\times10^{-3}\text{ kg/mol}} = 2.28\times10^4\text{ Pa}$

EVALUATE: The pressure is about one-fifth the pressure at sea-level.

18.22. IDENTIFY: The molar mass is $M = N_A m$, where m is the mass of one molecule.

SET UP: $N_A = 6.02\times10^{23}$ molecules/mol.

EXECUTE: $M = N_A m = (6.02\times10^{23}\text{ molecules/mol})(1.41\times10^{-21}\text{ kg/molecule}) = 849\text{ kg/mol}$.

EVALUATE: For a carbon atom, $M = 12\times10^{-3}$ kg/mol. If this molecule is mostly carbon, so the average mass of its atoms is the mass of carbon, the molecule would contain $\dfrac{849\text{ kg/mol}}{12\times10^{-3}\text{ kg/mol}} = 71{,}000$ atoms.

18.23. IDENTIFY: The mass m_{tot} is related to the number of moles n by $m_{tot} = nM$. Mass is related to volume by $\rho = m/V$.

SET UP: For gold, $M = 196.97$ g/mol and $\rho = 19.3\times10^3\text{ kg/m}^3$. The volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.

EXECUTE: **(a)** $m_{tot} = nM = (3.00\text{ mol})(196.97\text{ g/mol}) = 590.9\text{ g}$. The value of this mass of gold is $(590.9\text{ g})(\$14.75/\text{g}) = \$8720$.

(b) $V = \dfrac{m}{\rho} = \dfrac{0.5909\text{ kg}}{19.3\times10^3\text{ kg/m}^3} = 3.06\times10^{-5}\text{ m}^3$. $V = \frac{4}{3}\pi r^3$ gives

$r = \left(\dfrac{3V}{4\pi}\right)^{1/3} = \left(\dfrac{3[3.06\times10^{-5}\text{ m}^3]}{4\pi}\right)^{1/3} = 0.0194\text{ m} = 1.94\text{ cm}$. The diameter is $2r = 3.88$ cm.

EVALUATE: The mass and volume are directly proportional to the number of moles.

18.24. IDENTIFY: Use $pV = nRT$ to calculate the number of moles and then the number of molecules would be $N = nN_A$.

SET UP: $1\text{ atm} = 1.013\times10^5\text{ Pa}$. $1.00\text{ cm}^3 = 1.00\times10^{-6}\text{ m}^3$. $N_A = 6.022\times10^{23}$ molecules/mol.

EXECUTE: **(a)** $n = \dfrac{pV}{RT} = \dfrac{(9.00\times10^{-14}\text{ atm})(1.013\times10^5\text{ Pa/atm})(1.00\times10^{-6}\text{ m}^3)}{(8.314\text{ J/mol}\cdot\text{K})(300.0\text{ K})} = 3.655\times10^{-18}\text{ mol}$.

$N = nN_A = (3.655\times10^{-18}\text{ mol})(6.022\times10^{23}\text{ molecules/mol}) = 2.20\times10^6$ molecules.

(b) $N = \dfrac{pVN_A}{RT}$ so $\dfrac{N}{p} = \dfrac{VN_A}{RT} =$ constant and $\dfrac{N_1}{p_1} = \dfrac{N_2}{p_2}$.

$N_2 = N_1\left(\dfrac{p_2}{p_1}\right) = (2.20\times10^6\text{ molecules})\left(\dfrac{1.00\text{ atm}}{9.00\times10^{-14}\text{ atm}}\right) = 2.44\times10^{19}$ molecules.

EVALUATE: The number of molecules in a given volume is directly proportional to the pressure. Even at the very low pressure in part (a) the number of molecules in 1.00 cm^3 is very large.

18.25. IDENTIFY: We are asked about a single state of the system.

SET UP: Use the ideal-gas law. Write n in terms of the number of molecules N.

(a) EXECUTE: $pV = nRT$, $n = N/N_A$ so $pV = (N/N_A)RT$

$$p = \left(\frac{N}{V}\right)\left(\frac{R}{N_A}\right)T$$

$$p = \left(\frac{80\text{ molecules}}{1\times10^{-6}\text{ m}^3}\right)\left(\frac{8.3145\text{ J/mol}\cdot\text{K}}{6.022\times10^{23}\text{ molecules/mol}}\right)(7500\text{ K}) = 8.28\times10^{-12}\text{ Pa}$$

$p = 8.2\times10^{-17}$ atm. This is much lower than the laboratory pressure of 1×10^{-13} atm in Exercise 18.24.

(b) EVALUATE: The Lagoon Nebula is a very rarefied low pressure gas. The gas would exert *very* little force on an object passing through it.

18.26. **IDENTIFY:** $pV = nRT = NkT$

SET UP: At STP, $T = 273\text{ K}$, $p = 1.01\times10^5\text{ Pa}$. $N = 6\times10^9$ molecules.

EXECUTE:. $V = \dfrac{NkT}{p} = \dfrac{(6\times10^9\text{ molecules})(1.381\times10^{-23}\text{ J/molecule}\cdot\text{K})(273\text{ K})}{1.01\times10^5\text{ Pa}} = 2.24\times10^{-16}\text{ m}^3$.

$L^3 = V$ so $L = V^{1/3} = 6.1\times10^{-6}\text{ m}$.

EVALUATE: This is a small cube.

18.27. **IDENTIFY:** $n = \dfrac{m}{M} = \dfrac{N}{N_A}$

SET UP: $N_A = 6.022\times10^{23}$ molecules/mol. For water, $M = 18\times10^{-3}\text{ kg/mol}$.

EXECUTE:. $n = \dfrac{m}{M} = \dfrac{1.00\text{ kg}}{18\times10^{-3}\text{ kg/mol}} = 55.6\text{ mol}$.

$N = nN_A = (55.6\text{ mol})(6.022\times10^{23}\text{ molecules/mol}) = 3.35\times10^{25}$ molecules.

EVALUATE: Note that we converted M to kg/mol.

18.28. **IDENTIFY:** Use $pV = nRT$ and $n = \dfrac{N}{N_A}$ with $N = 1$ to calculate the volume V occupied by 1 molecule. The length l of the side of the cube with volume V is given by $V = l^3$.

SET UP: $T = 27°\text{C} = 300\text{ K}$. $p = 1.00\text{ atm} = 1.013\times10^5\text{ Pa}$. $R = 8.314\text{ J/mol}\cdot\text{K}$. $N_A = 6.022\times10^{23}$ molecules/mol. The diameter of a typical molecule is about 10^{-10} m. $0.3\text{ nm} = 0.3\times10^{-9}\text{ m}$.

EXECUTE: **(a)** $pV = nRT$ and $n = \dfrac{N}{N_A}$ gives

$V = \dfrac{NRT}{N_A p} = \dfrac{(1.00)(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})}{(6.022\times10^{23}\text{ molecules/mol})(1.013\times10^5\text{ Pa})} = 4.09\times10^{-26}\text{ m}^3$. $l = V^{1/3} = 3.45\times10^{-9}\text{ m}$.

(b) The distance in part (a) is about 10 times the diameter of a typical molecule.

(c) The spacing is about 10 times the spacing of atoms in solids.

EVALUATE: There is space between molecules in a gas whereas in a solid the atoms are closely packed together.

18.29. **(a) IDENTIFY** and **SET UP:** Use the density and the mass of 5.00 mol to calculate the volume. $\rho = m/V$ implies $V = m/\rho$, where $m = m_{\text{tot}}$, the mass of 5.00 mol of water.

EXECUTE: $m_{\text{tot}} = nM = (5.00\text{ mol})(18.0\times10^{-3}\text{ kg/mol}) = 0.0900\text{ kg}$

Then $V = \dfrac{m}{\rho} = \dfrac{0.0900\text{ kg}}{1000\text{ kg/m}^3} = 9.00\times10^{-5}\text{ m}^3$

(b) One mole contains $N_A = 6.022\times10^{23}$ molecules, so the volume occupied by one molecule is

$\dfrac{9.00\times10^{-5}\text{ m}^3/\text{mol}}{(5.00\text{ mol})(6.022\times10^{23}\text{ molecules/mol})} = 2.989\times10^{-29}\text{ m}^3/\text{molecule}$

$V = a^3$, where a is the length of each side of the cube occupied by a molecule. $a^3 = 2.989\times10^{-29}\text{ m}^3$, so $a = 3.1\times10^{-10}$ m.

(c) EVALUATE: Atoms and molecules are on the order of 10^{-10} m in diameter, in agreement with the above estimates.

18.30. **IDENTIFY:** $K_{\text{av}} = \tfrac{3}{2}kT$. $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$.

SET UP: $M_{\text{Ne}} = 20.180\text{ g/mol}$, $M_{\text{Kr}} = 83.80\text{ g/mol}$ and $M_{\text{Rn}} = 222\text{ g/mol}$.

EXECUTE: **(a)** $K_{\text{av}} = \tfrac{3}{2}kT$ depends only on the temperature so it is the same for each species of atom in the mixture.

(b) $\dfrac{v_{\text{rms,Ne}}}{v_{\text{rms,Kr}}} = \sqrt{\dfrac{M_{\text{Kr}}}{M_{\text{Ne}}}} = \sqrt{\dfrac{83.80\text{ g/mol}}{20.18\text{ g/mol}}} = 2.04$. $\dfrac{v_{\text{rms,Ne}}}{v_{\text{rms,Rn}}} = \sqrt{\dfrac{M_{\text{Rn}}}{M_{\text{Ne}}}} = \sqrt{\dfrac{222\text{ g/mol}}{20.18\text{ g/mol}}} = 3.32$.

$\dfrac{v_{\text{rms,Kr}}}{v_{\text{rms,Rn}}} = \sqrt{\dfrac{M_{\text{Rn}}}{M_{\text{Kr}}}} = \sqrt{\dfrac{222\text{ g/mol}}{83.80\text{ g/mol}}} = 1.63$.

EVALUATE: The average kinetic energies are the same. The gas atoms with smaller mass have larger v_{rms}.

18.31. **IDENTIFY** and **SET UP:** $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$.

EXECUTE: **(a)** v_{rms} is different for the two different isotopes, so the 235 isotope diffuses more rapidly.

(b) $\dfrac{v_{\text{rms,235}}}{v_{\text{rms,238}}} = \sqrt{\dfrac{M_{238}}{M_{235}}} = \sqrt{\dfrac{0.352\text{ kg/mol}}{0.349\text{ kg/mol}}} = 1.004$.

EVALUATE: The v_{rms} values each depend on T but their ratio is independent of T.

18.32. **IDENTIFY** and **SET UP:** With the multiplicity of each score denoted by n_i, the average score is $\left(\dfrac{1}{150}\right)\sum n_i x_i$ and the rms score is $\left[\left(\dfrac{1}{150}\right)\sum n_i x_i^2\right]^{1/2}$.

EXECUTE: **(a)** 54.6
(b) 61.1
EVALUATE: The rms score is higher than the average score since the rms calculation gives more weight to the higher scores.

18.33. **IDENTIFY:** $pV = nRT = \dfrac{N}{N_{\text{A}}}RT = \dfrac{m_{\text{tot}}}{M}RT$.

SET UP: We known that $V_A = V_B$ and that $T_A > T_B$.

EXECUTE: **(a)** $p = nRT/V$; we don't know n for each box, so either pressure could be higher.

(b) $pV = \left(\dfrac{N}{N_A}\right)RT$ so $N = \dfrac{pVN_{\text{A}}}{RT}$, where N_{A} is Avogadro's number. We don't know how the pressures compare, so either N could be larger.

(c) $pV = (m_{\text{tot}}/M)RT$. We don't know the mass of the gas in each box, so they could contain the same gas or different gases.

(d) $\frac{1}{2}m\left(v^2\right)_{\text{av}} = \frac{3}{2}kT$. $T_{\text{A}} > T_{\text{B}}$ and the average kinetic energy per molecule depends only on T, so the statement **must** be true.

(e) $v_{\text{rms}} = \sqrt{3kT/m}$. We don't know anything about the masses of the atoms of the gas in each box, so either set of molecules could have a larger v_{rms}.

EVALUATE: Only statement (d) must be true. We need more information in order to determine whether the other statements are true or false.

18.34. **IDENTIFY:** Use $pV = nRT$ to solve for V.

SET UP: Use $R = 0.08206\text{ L}\cdot\text{atm/mol}\cdot\text{K}$. $T = 273.15\text{ K}$.

EXECUTE: **(a)** $V = \dfrac{nRT}{p} = \dfrac{(1.00\text{ mol})(0.08206\text{ L}\cdot\text{atm/mol}\cdot\text{K})(273.15\text{ K})}{1.00\text{ atm}} = 22.4\text{ L}$

(b) $pV = nRT =$ constant, so $p_1V_1 = p_2V_2$. $V_2 = \left(\dfrac{p_1}{p_2}\right)V_1 = \left(\dfrac{1.00\text{ atm}}{92\text{ atm}}\right)(22.4\text{ L}) = 0.243\text{ L}$.

EVALUATE: For constant T, the volume of 1.00 mol is inversely proportional to the pressure.

18.35. **IDENTIFY:** $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$

SET UP: The mass of a deuteron is $m = m_{\text{p}} + m_{\text{n}} = 1.673\times10^{-27}\text{ kg} + 1.675\times10^{-27}\text{ kg} = 3.35\times10^{-27}\text{ kg}$. $c = 3.00\times10^8\text{ m/s}$. $k = 1.381\times10^{-23}\text{ J/molecule}\cdot\text{K}$.

EXECUTE: **(a)** $v_{\text{rms}} = \sqrt{\dfrac{3(1.381\times10^{-23}\text{ J/molecule}\cdot\text{K})(300\times10^6\text{ K})}{3.35\times10^{-27}\text{ kg}}} = 1.93\times10^6\text{ m/s}$. $\dfrac{v_{\text{rms}}}{c} = 6.43\times10^{-3}$.

(b) $T = \left(\dfrac{m}{3k}\right)(v_{\text{rms}})^2 = \left(\dfrac{3.35\times10^{-27}\text{ kg}}{3(1.381\times10^{-23}\text{ J/molecule}\cdot\text{K})}\right)(3.0\times10^7\text{ m/s})^2 = 7.3\times10^{10}\text{ K}$.

EVALUATE: Even at very high temperatures and for this light nucleus, v_{rms} is a small fraction of the speed of light.

18.36. **IDENTIFY:** $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$, where T is in kelvins. $pV = nRT$ gives $\dfrac{n}{V} = \dfrac{p}{RT}$.

SET UP: $R = 8.314\ \text{J/mol}\cdot\text{K}$. $M = 44.0\times10^{-3}\ \text{kg/mol}$.

EXECUTE: **(a)** For $T = 0.0°\text{C} = 273.15\ \text{K}$, $v_{\text{rms}} = \sqrt{\dfrac{3(8.314\ \text{J/mol}\cdot\text{K})(273.15\ \text{K})}{44.0\times10^{-3}\ \text{kg/mol}}} = 393\ \text{m/s}$. For $T = -100.0°\text{C} = 173\ \text{K}$, $v_{\text{rms}} = 313\ \text{m/s}$. The range of speeds is 393 m/s to 313 m/s.

(b) For $T = 273.15\ \text{K}$, $\dfrac{n}{V} = \dfrac{650\ \text{Pa}}{(8.314\ \text{J/mol}\cdot\text{K})(273.15\ \text{K})} = 0.286\ \text{mol/m}^3$. For $T = 173.15\ \text{K}$, $\dfrac{n}{V} = 0.452\ \text{mol/m}^3$.

The range of densities is $0.286\ \text{mol/m}^3$ to $0.452\ \text{mol/m}^3$.

EVALUATE: When the temperature decreases the rms speed decreases and the density increases.

18.37. **IDENTIFY** and **SET UP:** Apply the analysis of Section 18.3.

EXECUTE: **(a)** $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT = \frac{3}{2}(1.38\times10^{-23}\ \text{J/molecule}\cdot\text{K})(300\ \text{K}) = 6.21\times10^{-21}\ \text{J}$

(b) We need the mass m of one atom: $m = \dfrac{M}{N_{\text{A}}} = \dfrac{32.0\times10^{-3}\ \text{kg/mol}}{6.022\times10^{23}\ \text{molecules/mol}} = 5.314\times10^{-26}\ \text{kg/molecule}$

Then $\frac{1}{2}m(v^2)_{\text{av}} = 6.21\times10^{-21}\ \text{J}$ (from part (a)) gives $(v^2)_{\text{av}} = \dfrac{2(6.21\times10^{-21}\ \text{J})}{m} = \dfrac{2(6.21\times10^{-21}\ \text{J})}{5.314\times10^{-26}\ \text{kg}} = 2.34\times10^{5}\ \text{m}^2/\text{s}^2$

(c) $v_{\text{rms}} = \sqrt{(v^2)_{\text{rms}}} = \sqrt{2.34\times10^{4}\ \text{m}^2/\text{s}^2} = 484\ \text{m/s}$

(d) $p = mv_{\text{rms}} = (5.314\times10^{-26}\ \text{kg})(484\ \text{m/s}) = 2.57\times10^{-23}\ \text{kg}\cdot\text{m/s}$

(e) Time between collisions with one wall is $t = \dfrac{0.20\ \text{m}}{v_{\text{rms}}} = \dfrac{0.20\ \text{m}}{484\ \text{m/s}} = 4.13\times10^{-4}\ \text{s}$

In a collision $\vec{v}$ changes direction, so $\Delta p = 2mv_{\text{rms}} = 2(2.57\times10^{-23}\ \text{kg}\cdot\text{m/s}) = 5.14\times10^{-23}\ \text{kg}\cdot\text{m/s}$

$F = \dfrac{dp}{dt}$ so $F_{\text{av}} = \dfrac{\Delta p}{\Delta t} = \dfrac{5.14\times10^{-23}\ \text{kg}\cdot\text{m/s}}{4.13\times10^{-4}\ \text{s}} = 1.24\times10^{-19}\ \text{N}$

(f) pressure $= F/A = 1.24\times10^{-19}\ \text{N}/(0.10\ \text{m})^2 = 1.24\times10^{-17}\ \text{Pa}$ (due to one atom)

(g) pressure $= 1\ \text{atm} = 1.013\times10^{5}\ \text{Pa}$

Number of atoms needed is $1.013\times10^{5}\ \text{Pa}/(1.24\times10^{-17}\ \text{Pa/atom}) = 8.17\times10^{21}$ atoms

(h) $pV = NkT$ (Eq.18.18), so $N = \dfrac{pV}{kT} = \dfrac{(1.013\times10^{5}\ \text{Pa})(0.10\ \text{m})^3}{(1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K})(300\ \text{K})} = 2.45\times10^{22}$ atoms

(i) From the factor of $\frac{1}{3}$ in $(v_x^2)_{\text{av}} = \frac{1}{3}(v^2)_{\text{av}}$.

EVALUATE: This Exercise shows that the pressure exerted by a gas arises from collisions of the molecules of the gas with the walls.

18.38. **IDENTIFY:** Apply Eq.(18.22) and calculate λ

SET UP: $1\ \text{atm} = 1.013\times10^{5}\ \text{Pa}$, so $p = 3.55\times10^{-8}\ \text{Pa}$. $r = 2.0\times10^{-10}\ \text{m}$ and $k = 1.38\times10^{-23}\ \text{J/K}$.

EXECUTE: $\lambda = \dfrac{kT}{4\pi\sqrt{2}r^2\rho} = \dfrac{(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})}{4\pi\sqrt{2}(2.0\times10^{-10}\ \text{m})^2(3.55\times10^{-8}\ \text{Pa})} = 1.5\times10^{5}\ \text{m}$

EVALUATE: At this very low pressure the mean free path is very large. If $v = 484\ \text{m/s}$, as in Example 18.8, then $t_{\text{mean}} = \dfrac{\lambda}{v} = 330\ \text{s}$. Collisions are infrequent.

18.39. **IDENTIFY** and **SET UP:** Use equal v_{rms} to relate T and M for the two gases. $v_{\text{rms}} = \sqrt{3RT/M}$ (Eq.18.19), so $v_{\text{rms}}^2/3R = T/M$, where T must be in kelvins. Same v_{rms} so same T/M for the two gases and $T_{N_2}/M_{N_2} = T_{H_2}/M_{H_2}$.

EXECUTE: $T_{N_2} = T_{H_2}\left(\dfrac{M_{N_2}}{M_{H_2}}\right) = ((20+273)\ \text{K})\left(\dfrac{28.014\ \text{g/mol}}{2.016\ \text{g/mol}}\right) = 4.071\times10^{3}\ \text{K}$

$T_{N_2} = (4071 - 273)°\text{C} = 3800°\text{C}$

EVALUATE: A N_2 molecule has more mass so N_2 gas must be at a higher temperature to have the same v_{rms}.

18.40. **IDENTIFY:** $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$.

SET UP: $k = 1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K}$.

EXECUTE: **(a)** $v_{\text{rms}} = \sqrt{\dfrac{3(1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K})(300\ \text{K})}{3.00\times10^{-16}\ \text{kg}}} = 6.44\times10^{-3}\ \text{m/s} = 6.44\ \text{mm/s}$

EVALUATE: **(b)** No. The rms speed depends on the average kinetic energy of the particles. At this T, H_2 molecules would have larger v_{rms} than the typical air molecules but would have the same average kinetic energy and the average kinetic energy of the smoke particles would be the same.

18.41. **IDENTIFY:** Use Eq.(18.24), applied to a finite temperature change.

SET UP: $C_V = 5R/2$ for a diatomic ideal gas and $C_V = 3R/2$ for a monatomic ideal gas.

EXECUTE: **(a)** $Q = nC_V\Delta T = n\left(\tfrac{5}{2}R\right)\ \Delta T$

$Q = (2.5\ \text{mol})\left(\tfrac{5}{2}\right)(8.3145\ \text{J/mol}\cdot\text{K})(30.0\ \text{K}) = 1560\ \text{J}$

(b) $Q = nC_V\Delta T = n\left(\tfrac{3}{2}R\right)\ \Delta T$

$Q = (2.5\ \text{mol})\left(\tfrac{3}{2}\right)(8.3145\ \text{J/mol}\cdot\text{K})(30.0\ \text{K}) = 935\ \text{J}$

EVALUATE: More heat is required for the diatomic gas; not all the heat that goes into the gas appears as translational kinetic energy, some goes into energy of the internal motion of the molecules (rotations).

18.42. **IDENTIFY:** The heat Q added is related to the temperature increase ΔT by $Q = nC_V\Delta T$.

SET UP: For H_2, $C_{V,\text{H}_2} = 20.42\ \text{J/mol}\cdot\text{K}$ and for Ne (a monatomic gas), $C_{V,\text{Ne}} = 12.47\ \text{J/mol}\cdot\text{K}$.

EXECUTE: $C_V\Delta T = \dfrac{Q}{n} = \text{constant}$, so $C_{V,\text{H}_2}\Delta T_{\text{H}_2} = C_{V,\text{Ne}}\Delta T_{\text{Ne}}$.

$$\Delta T_{\text{Ne}} = \left(\frac{C_{V,\text{H}_2}}{C_{V,\text{Ne}}}\right)\Delta T_{\text{H}_2} = \left(\frac{20.42\ \text{J/mol}\cdot\text{K}}{12.47\ \text{J/mol}\cdot\text{K}}\right)(2.50\ \text{C}^\circ) = 4.09\ \text{C}^\circ.$$

EVALUATE: The same amount of heat causes a smaller temperature increase for H_2 since some of the energy input goes into the internal degrees of freedom.

18.43. **IDENTIFY:** $C = Mc$, where C is the molar heat capacity and c is the specific heat capacity. $pV = nRT = \dfrac{m}{M}RT$.

SET UP: $M_{\text{N}_2} = 2(14.007\ \text{g/mol}) = 28.014\times10^{-3}\ \text{kg/mol}$. For water, $c_{\text{w}} = 4190\ \text{J/kg}\cdot\text{K}$. For N_2, $C_V = 20.76\ \text{J/mol}\cdot\text{K}$.

EXECUTE: **(a)** $c_{\text{N}_2} = \dfrac{C}{M} = \dfrac{20.76\ \text{J/mol}\cdot\text{K}}{28.014\times10^{-3}\ \text{kg/mol}} = 741\ \text{J/kg}\cdot\text{K}$. $\dfrac{c_{\text{w}}}{c_{\text{N}_2}} = 5.65$; c_{w} is over five time larger.

(b) To warm the water, $Q = mc_{\text{w}}\Delta T = (1.00\ \text{kg})(4190\ \text{J/mol}\cdot\text{K})(10.0\ \text{K}) = 4.19\times10^{4}\ \text{J}$. For air,

$$m = \frac{Q}{c_{\text{N}_2}\Delta T} = \frac{4.19\times10^4\ \text{J}}{(741\ \text{J/kg}\cdot\text{K})(10.0\ \text{K})} = 5.65\ \text{kg}.\ V = \frac{mRT}{Mp} = \frac{(5.65\ \text{kg})(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})}{(28.014\times10^{-3}\ \text{kg/mol})(1.013\times10^5\ \text{Pa})} = 4.85\ \text{m}^3.$$

EVALUATE: c is smaller for N_2, so less heat is needed for 1.0 kg of N_2 than for 1.0 kg of water.

18.44. **(a) IDENTIFY and SET UP:** $\tfrac{1}{2}R$ contribution to C_V for each degree of freedom. The molar heat capacity C is related to the specific heat capacity c by $C = Mc$.

EXECUTE: $C_V = 6\left(\tfrac{1}{2}R\right) = 3R = 3(8.3145\ \text{J/mol}\cdot\text{K}) = 24.9\ \text{J/mol}\cdot\text{K}$. The specific heat capacity is

$c_V = C_V/M = (24.9\ \text{J/mol}\cdot\text{K})/(18.0\times10^{-3}\ \text{kg/mol}) = 1380\ \text{J/kg}\cdot\text{K}$.

(b) For water vapor the specific heat capacity is $c = 2000\ \text{J/kg}\cdot\text{K}$. The molar heat capacity is

$C = Mc = (18.0\times10^{-3}\ \text{kg/mol})(2000\ \text{J/kg}\cdot\text{K}) = 36.0\ \text{J/mol}\cdot\text{K}$.

EVALUATE: The difference is $36.0\ \text{J/mol}\cdot\text{K} - 24.9\ \text{J/mol}\cdot\text{K} = 11.1\ \text{J/mol}\cdot\text{K}$, which is about $2.7\left(\tfrac{1}{2}R\right)$; the vibrational degrees of freedom make a significant contribution.

18.45. **IDENTIFY:** $C_V = 3R$ gives C_V in units of $\text{J/mol}\cdot\text{K}$. The atomic mass M gives the mass of one mole.

SET UP: For aluminum, $M = 26.982\times10^{-3}\ \text{kg/mol}$.

EXECUTE: **(a)** $C_V = 3R = 24.9\ \text{J/mol}\cdot\text{K}$. $c_V = \dfrac{24.9\ \text{J/mol}\cdot\text{K}}{26.982\times10^{-3}\ \text{kg/mol}} = 923\ \text{J/kg}\cdot\text{K}$.

(b) Table 17.3 gives $910\ \text{J/kg}\cdot\text{K}$. The value from Eq.(18.28) is too large by about 1.4%.

EVALUATE: As shown in Figure 18.21 in the textbook, C_V approaches the value $3R$ as the temperature increases. The values in Table 17.3 are at room temperature and therefore are somewhat smaller than $3R$.

18.46. **IDENTIFY:** Table 18.2 gives the value of v/v_{rms} for which 94.7% of the molecules have a smaller value of v/v_{rms}. $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$.

SET UP: For N_2, $M = 28.0\times10^{-3}$ kg/mol. $v/v_{\text{rms}} = 1.60$.

EXECUTE: $v_{\text{rms}} = \dfrac{v}{1.60} = \sqrt{\dfrac{3RT}{M}}$, so the temperature is

$$T = \frac{Mv^2}{3(1.60)^2 R} = \frac{(28.0\times10^{-3}\ \text{kg/mol})}{3(1.60)^2(8.3145\ \text{J/mol}\cdot\text{K})}v^2 = (4.385\times10^{-4}\ \text{K}\cdot\text{s}^2/\text{m}^2)v^2.$$

(a) $T = (4.385\times10^{-4}\ \text{K}\cdot\text{s}^2/\text{m}^2)(1500\ \text{m/s})^2 = 987\ \text{K}$

(b) $T = (4.385\times10^{-4}\text{K}\cdot\text{s}^2/\text{m}^2)(1000\ \text{m/s})^2 = 438\ \text{K}$

(c) $T = (4.385\times10^{-4}\ \text{K}\cdot\text{s}^2/\text{m}^2)(500\ \text{m/s})^2 = 110\ \text{K}.$

EVALUATE: As T decreases the distribution of molecular speeds shifts to lower values.

18.47. **IDENTIFY** and **SET UP:** Make the substitution $\epsilon = \frac{1}{2}mv^2$ in Eq.(18.32).

EXECUTE: $f(v) = 4\pi\left(\dfrac{m}{2\pi kT}\right)^{3/2}\dfrac{2\epsilon}{m}e^{-\epsilon/kT} = \dfrac{8\pi}{m}\left(\dfrac{m}{2\pi kT}\right)^{3/2}\epsilon e^{-\epsilon/kT}.$

EVALUATE: The shape of the distribution of molecular speeds versus the temperature is a function only of the kinetic energy of the molecules.

18.48. **IDENTIFY** and **SET UP:** Eq.(18.33): $f(v) = \dfrac{8\pi}{m}\left(\dfrac{m}{2\pi kT}\right)^{3/2}\epsilon e^{-\epsilon/kT}$

At the maximum of $f(\epsilon)$, $\dfrac{df}{d\epsilon} = 0.$

EXECUTE: $\dfrac{df}{d\epsilon} = \dfrac{8\pi}{m}\left(\dfrac{m}{2\pi kT}\right)^{3/2}\dfrac{d}{d\epsilon}(\epsilon e^{-\epsilon/kT}) = 0$

This requires that $\dfrac{d}{d\epsilon}(\epsilon e^{-\epsilon/kT}) = 0.$

$e^{-\epsilon/kT} - (\epsilon/kT)e^{-\epsilon/kT} = 0$

$(1-\epsilon/kT)e^{-\epsilon/kT} = 0$

This requires that $1-\epsilon/kT = 0$ so $\epsilon = kT$, as was to be shown. And then since $\epsilon = \frac{1}{2}mv^2$, this gives $\frac{1}{2}mv_{\text{mp}}^2 = kT$ and $v_{\text{mp}} = \sqrt{2kT/m}$, which is Eq.(18.34).

EVALUATE: $v_{\text{rms}} = \sqrt{\frac{3}{2}}v_{\text{mp}}$. The average of v^2 weights larger v.

18.49. **IDENTIFY:** Apply Eqs.(18.34) (18.35) and (18.36).

SET UP: Note that $\dfrac{k}{m} = \dfrac{R/N_{\text{A}}}{M/N_A} = \dfrac{R}{M}$. $M = 44.0\times10^{-3}$ kg/mol.

EXECUTE: **(a)** $v_{\text{mp}} = \sqrt{2(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})/(44.0\times10^{-3}\ \text{kg/mol})} = 3.37\times10^2\ \text{m/s}.$

(b) $v_{\text{av}} = \sqrt{8(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})/(\pi(44.0\times10^{-3}\ \text{kg/mol}))} = 3.80\times10^2\ \text{m/s}.$

(c) $v_{\text{rms}} = \sqrt{3(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})/(44.0\times10^{-3}\ \text{kg/mol})} = 4.12\times10^2\ \text{m/s}.$

EVALUATE: The average speed is greater than the most probable speed and the rms speed is greater than the average speed.

18.50. **IDENTIFY** and **SET UP:** If the temperature at altitude y is below the freezing point only cirrus clouds can form. Use $T = T_0 - \alpha y$ to find the y that gives $T = 0.0°\text{C}$.

EXECUTE: $y = \dfrac{T_0 - T}{\alpha} = \dfrac{15.0°\text{C} - 0.0°\text{C}}{6.0\ \text{C}°/\text{km}} = 2.5\ \text{km}$

EVALUATE: The solid-liquid phase transition occurs at $0°\text{C}$ only for $p = 1.01\times10^5$ Pa. Use the results of Example 18.4 to estimate the pressure at an altitude of 2.5 km.

$p_2 = p_1 e^{Mg(y_2-y_1)/RT}$

$Mg(y_2 - y_1)/RT = 1.10(2500\ \text{m}/8863\ \text{m}) = 0.310$ (using the calculation in Example 18.4)

Then $p_2 = (1.01\times10^5\ \text{Pa})e^{-0.31} = 0.74\times10^5\ \text{Pa}.$

This pressure is well above the triple point pressure for water. Figure 18.21 shows that the fusion curve has large slope and it takes a large change in pressure to change the phase transition temperature very much. Using 0.0°C introduces little error.

18.51. IDENTIFY: Refer to the phase diagram in Figure 18.24 in the textbook.

SET UP: For water the triple-point pressure is 610 Pa and the critical-point pressure is 2.212×10^7 Pa .

EXECUTE: **(a)** To observe a solid to liquid (melting) phase transition the pressure must be greater than the triple-point pressure, so $p_1 = 610$ Pa . For $p < p_1$ the solid to vapor (sublimation) phase transition is observed.

(b) No liquid to vapor (boiling) phase transition is observed if the pressure is greater than the critical-point pressure. $p_2 = 2.212\times10^7$ Pa . For $p_1 < p < p_2$ the sequence of phase transitions are solid to liquid and then liquid to vapor.

EVALUATE: Normal atmospheric pressure is approximately 1.0×10^5 Pa , so the solid to liquid to vapor sequence of phase transitions is normally observed when the material is water.

18.52. IDENTIFY: Refer to Figure 18.24 in the textbook.

SET UP: The triple-point temperature for water is $273.16\text{ K} = 0.01°\text{C}$.

EXECUTE: The temperature is less than the triple-point temperature so the solid and vapor phases are in equilibrium. The box contains ice and water vapor but no liquid water.

EVALUATE: The fusion curve terminates at the triple point.

18.53. IDENTIFY: Figure 18.24 in the textbook shows that there is no liquid phase below the triple point pressure.

SET UP: Table 18.3 gives the triple point pressure to be 610 Pa for water and 5.17×10^5 Pa for CO_2.

EXECUTE: The atmospheric pressure is below the triple point pressure of water, and there can be no liquid water on Mars. The same holds true for CO_2.

EVALUATE: On earth $p_{\text{atm}} = 1\times10^5$ Pa , so on the surface of the earth there can be liquid water but not liquid CO_2.

18.54. IDENTIFY: $\Delta V = \beta V_0 \Delta T - V_0 k \Delta p$

SET UP: For steel, $\beta = 3.6\times10^{-5}\text{ K}^{-1}$ and $k = 6.25\times10^{-12}\text{ Pa}^{-1}$.

EXECUTE: $\beta V_0 \Delta T = (3.6\times10^{-5}\text{ K}^{-1})(11.0\text{ L})(21\text{ C}°) = 0.0083\text{ L}$.

$-kV_0\Delta p = (6.25\times10^{-12}/\text{Pa})(11\text{ L})\ (2.1\times10^7\text{Pa}) = -0.0014\text{ L}$. The total change in volume is $\Delta V = 0.0083\text{ L} - 0.0014\text{ L} = 0.0069\text{ L}.$

(b) Yes; ΔV is much less than the original volume of 11.0 L.

EVALUATE: Even for a large pressure increase and a modest temperature increase, the magnitude of the volume change due to the temperature increase is much larger than that due to the pressure increase.

18.55. IDENTIFY: We are asked to compare two states. Use the ideal-gas law to obtain m_2 in terms of m_1 and the ratio of pressures in the two states. Apply Eq.(18.4) to the initial state to calculate m_1.

SET UP: $pV = nRT$ can be written $pV = (m/M)RT$

T, V, M, R are all constant, so $p/m = RT/MV = \text{constant}$.

So $p_1/m_1 = p_2/m_2$, where m is the mass of the gas in the tank.

EXECUTE: $p_1 = 1.30\times10^6\text{ Pa} + 1.01\times10^5\text{ Pa} = 1.40\times10^6\text{ Pa}$

$p_2 = 2.50\times10^5\text{ Pa} + 1.01\times10^5\text{ Pa} = 3.51\times10^5\text{ Pa}$

$m_1 = p_1VM/RT;\ \ V = hA = h\pi r^2 = (1.00\text{ m})\pi(0.060\text{ m})^2 = 0.01131\text{ m}^3$

$$m_1 = \frac{(1.40\times10^6\text{ Pa})(0.01131\text{ m}^3)(44.1\times10^{-3}\text{ kg/mol})}{(8.3145\text{ J/mol}\cdot\text{K})((22.0+273.15)\text{ K})} = 0.2845\text{ kg}$$

Then $m_2 = m_1\left(\dfrac{p_2}{p_1}\right) = (0.2845\text{ kg})\left(\dfrac{3.51\times10^5\text{ Pa}}{1.40\times10^6\text{ Pa}}\right) = 0.0713\text{ kg}.$

m_2 is the mass that remains in the tank. The mass that has been used is

$m_1 - m_2 = 0.2848\text{ kg} - 0.0713\text{ kg} = 0.213\text{ kg}.$

EVALUATE: Note that we have to use absolute pressures. The absolute pressure decreases by a factor of four and the mass of gas in the tank decreases by a factor of four.

18.56. IDENTIFY: Apply $pV = nRT$ to the air inside the diving bell. The pressure p at depth y below the surface of the water is $p = p_{\text{atm}} + \rho g y$.

SET UP: $p = 1.013\times10^5$ Pa . $T = 300.15$ K at the surface and $T' = 280.15$ K at the depth of 13.0 m.

EXECUTE: **(a)** The height h' of the air column in the diving bell at this depth will be proportional to the volume, and hence inversely proportional to the pressure and proportional to the Kelvin temperature:

$$h' = h\frac{p}{p'}\frac{T'}{T} = h\frac{p_{\text{atm}}}{p_{\text{atm}} + \rho g y}\frac{T'}{T}.$$

$$h' = (2.30\text{ m})\frac{(1.013\times10^5\text{ Pa})}{(1.013\times10^5\text{Pa}) + (1030\text{ kg/m}^3)(9.80\text{ m/s}^2)(73.0\text{ m})}\left(\frac{280.15\text{ K}}{300.15\text{ K}}\right) = 0.26\text{ m}.$$

The height of the water inside the diving bell is $h - h' = 2.04$ m .

(b) The necessary gauge pressure is the term $\rho g y$ from the above calculation, $p_{\text{gauge}} = 7.37\times10^{5}\ \text{Pa}.$.

EVALUATE: The gauge pressure required in part (b) is about 7 atm.

18.57. **IDENTIFY:** $pV = NkT$ gives $\dfrac{N}{V} = \dfrac{p}{kT}$.

SET UP: $1\ \text{atm} = 1.013\times10^{5}\ \text{Pa}$. $T_{\text{K}} = T_{\text{C}} + 273.15$. $k = 1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K}$.

EXECUTE: **(a)** $T_{\text{C}} = T_{\text{K}} - 273.15 = 94\ \text{K} - 273.15 = -179^\circ\text{C}$

(b) $\dfrac{N}{V} = \dfrac{p}{kT} = \dfrac{(1.5\ \text{atm})(1.013\times10^{5}\ \text{Pa/atm})}{(1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K})(94\ \text{K})} = 1.2\times10^{26}\ \text{molecules/m}^3$

(c) For the earth, $p = 1.0\ \text{atm} = 1.013\times10^{5}\ \text{Pa}$ and $T = 22^\circ\text{C} = 295\ \text{K}$.

$\dfrac{N}{V} = \dfrac{(1.0\ \text{atm})(1.013\times10^{5}\ \text{Pa/atm})}{(1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K})(295\ \text{K})} = 2.5\times10^{25}\ \text{molecules/m}^3$. The atmosphere of Titan is about five times denser than earth's atmosphere.

EVALUATE: Though it is smaller than Earth and has weaker gravity at its surface, Titan can maintain a dense atmosphere because of the very low temperature of that atmosphere.

18.58. **IDENTIFY:** For constant temperature, the variation of pressure with altitude is calculated in Example 18.4 to be $p = p_0 e^{-Mgy/RT}$. $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$.

SET UP: $g_{\text{Earth}} = 9.80\ \text{m/s}^2$. $T = 460^\circ\text{C} = 733\ \text{K}$. $M = 44.0\ \text{g/mol} = 44.0\times10^{-3}\ \text{kg/mol}$.

EXECUTE: **(a)** $\dfrac{Mgy}{RT} = \dfrac{(44.0\times10^{-3}\ \text{kg/mol})(0.894)(9.80\ \text{m/s}^2)(1.00\times10^{3}\ \text{m})}{(8.314\ \text{J/mol}\cdot\text{K})(733\ \text{K})} = 0.06326$.

$p = p_0 e^{-Mgy/RT} = (92\ \text{atm})e^{-0.06326} = 86\ \text{atm}$. The pressure is 86 Earth-atmospheres, or 0.94 Venus-atmospheres.

(b) $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3(8.314\ \text{J/mol}\cdot\text{K})(733\ \text{K})}{44.0\times10^{-3}\ \text{kg/mol}}} = 645\ \text{m/s}$. v_{rms} has this value both at the surface and at an altitude of 1.00 km.

EVALUATE: v_{rms} depends only on T and the molar mass of the gas. For Venus compared to earth, the surface temperature, in kelvins, is nearly a factor of three larger and the molecular mass of the gas in the atmosphere is only about 50% larger, so v_{rms} for the Venus atmosphere is larger than it is for the Earth's atmosphere.

18.59. **IDENTIFY:** $pV = nRT$

SET UP: In $pV = nRT$ we must use the absolute pressure. $T_1 = 278\ \text{K}$. $p_1 = 2.72\ \text{atm}$. $T_2 = 318\ \text{K}$.

EXECUTE: n, R constant, so $\dfrac{pV}{T} = nR = \text{constant}$. $\dfrac{p_1V_1}{T_1} = \dfrac{p_2V_2}{T_2}$ and

$p_2 = p_1\left(\dfrac{V_1}{V_2}\right)\left(\dfrac{T_2}{T_1}\right) = (2.72\ \text{atm})\left(\dfrac{0.0150\ \text{m}^3}{0.0159\ \text{m}^3}\right)\left(\dfrac{318\ \text{K}}{278\ \text{K}}\right) = 2.94\ \text{atm}$. The final gauge pressure is $2.94\ \text{atm} - 1.02\ \text{atm} = 1.92\ \text{atm}$.

EVALUATE: Since a ratio is used, pressure can be expressed in atm. But absolute pressures must be used. The ratio of gauge pressures is not equal to the ratio of absolute pressures.

18.60. **IDENTIFY:** In part (a), apply $pV = nRT$ to the ethane in the flask. The volume is constant once the stopcock is in place. In part (b) apply $pV = \dfrac{m_{\text{tot}}}{M}RT$ to the ethane at its final temperature and pressure.

SET UP: $1.50\ \text{L} = 1.50\times10^{-3}\ \text{m}^3$. $M = 30.1\times10^{-3}\ \text{kg/mol}$. Neglect the thermal expansion of the flask.

EXECUTE: **(a)** $p_2 = p_1(T_2/T_1) = (1.013\times10^{5}\ \text{Pa})(300\ \text{K}/380\ \text{K}) = 8.00\times10^{4}\ \text{Pa}$.

(b) $m_{\text{tot}} = \left(\dfrac{p_2V}{RT_2}\right)M = \left(\dfrac{(8.00\times10^{4}\ \text{Pa})(1.50\times10^{-3}\ \text{m}^3)}{(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})}\right)(30.1\times10^{-3}\ \text{kg/mol}) = 1.45\ \text{g}$.

EVALUATE: We could also calculate m_{tot} with $p = 1.013\times10^{5}\ \text{Pa}$ and $T = 380\ \text{K}$, and we would obtain the same result. Originally, before the system was warmed, the mass of ethane in the flask was

$m = (1.45\ \text{g})\left(\dfrac{1.013\times10^{5}\ \text{Pa}}{8.00\times10^{4}\ \text{Pa}}\right) = 1.84\ \text{g}$.

18.61. **(a) IDENTIFY:** Consider the gas in one cylinder. Calculate the volume to which this volume of gas expands when the pressure is decreased from $(1.20\times10^6\ \text{Pa}+1.01\times10^5\ \text{Pa})=1.30\times10^6\ \text{Pa}$ to $1.01\times10^5\ \text{Pa}$. Apply the ideal-gas law to the two states of the system to obtain an expression for V_2 in terms of V_1 and the ratio of the pressures in the two states.

SET UP: $pV = nRT$

n, R, T constant implies $pV = nRT = \text{constant}$, so $p_1V_1 = p_2V_2$.

EXECUTE: $V_2 = V_1(p_1/p_2) = (1.90\ \text{m}^3)\left(\dfrac{1.30\times10^6\ \text{Pa}}{1.01\times10^5\ \text{Pa}}\right) = 24.46\ \text{m}^3$

The number of cylinders required to fill a $750\ \text{m}^3$ balloon is $750\ \text{m}^3/24.46\ \text{m}^3 = 30.7$ cylinders.

EVALUATE: The ratio of the volume of the balloon to the volume of a cylinder is about 400. Fewer cylinders than this are required because of the large factor by which the gas is compressed in the cylinders.

(b) IDENTIFY: The upward force on the balloon is given by Archimedes' principle (Chapter 14): $B =$ weight of air displaced by balloon $= \rho_{\text{air}}Vg$. Apply Newton's 2nd law to the balloon and solve for the weight of the load that can be supported. Use the ideal-gas equation to find the mass of the gas in the balloon.

SET UP: The free-body diagram for the balloon is given in Figure 18.61.

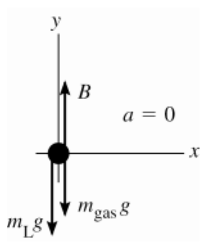


Figure 18.61

m_{gas} is the mass of the gas that is inside the balloon; m_{L} is the mass of the load that is supported by the balloon

EXECUTE: $\sum F_y = ma_y$

$B - m_{\text{L}}g - m_{\text{gas}}g = 0$

$\rho_{\text{air}}Vg - m_{\text{L}}g - m_{\text{gas}}g = 0$

$m_{\text{L}} = \rho_{\text{air}}V - m_{\text{gas}}$

Calculate m_{gas}, the mass of hydrogen that occupies $750\ \text{m}^3$ at 15°C and $p = 1.01\times10^5\ \text{Pa}$.

$pV = nRT = (m_{\text{gas}}/M)RT$ gives

$$m_{\text{gas}} = pVM/RT = \frac{(1.01\times10^5\ \text{Pa})(750\ \text{m}^3)(2.02\times10^{-3}\ \text{kg/mol})}{(8.3145\ \text{J/mol}\cdot\text{K})(288\ \text{K})} = 63.9\ \text{kg}$$

Then $m_{\text{L}} = (1.23\ \text{kg/m}^3)(750\ \text{m}^3) - 63.9\ \text{kg} = 859\ \text{kg}$, and the weight that can be supported is

$w_{\text{L}} = m_{\text{L}}g = (859\ \text{kg})(9.80\ \text{m/s}^2) = 8420\ \text{N}$.

(c) $m_{\text{L}} = \rho_{\text{air}}V - m_{\text{gas}}$

$m_{\text{gas}} = pVM/RT = (63.9\ \text{kg})((4.00\ \text{g/mol})/(2.02\ \text{g/mol})) = 126.5\ \text{kg}$ (using the results of part (b)).

Then $m_{\text{L}} = (1.23\ \text{kg/m}^3)(750\ \text{m}^3) - 126.5\ \text{kg} = 796\ \text{kg}$.

$w_{\text{L}} = m_{\text{L}}g = (796\ \text{kg})(9.80\ \text{m/s}^2) = 7800\ \text{N}$.

EVALUATE: A greater weight can be supported when hydrogen is used because its density is less.

18.62. **IDENTIFY:** The upward force exerted by the gas on the piston must equal the piston's weight. Use $pV = nRT$ to calculate the volume of the gas, and from this the height of the column of gas in the cylinder.

SET UP: $F = pA = p\pi r^2$, with $r = 0.100$ m and $p = 1.00\ \text{atm} = 1.013\times10^5\ \text{Pa}$. For the cylinder, $V = \pi r^2 h$.

EXECUTE: **(a)** $p\pi r^2 = mg$ and $m = \dfrac{p\pi r^2}{g} = \dfrac{(1.013\times10^5\ \text{Pa})\pi(0.100\ \text{m})^2}{9.80\ \text{m/s}^2} = 325\ \text{kg}$.

(b) $V = \dfrac{nRT}{p} - \dfrac{(1.80\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})(293.15\ \text{K})}{1.013\times10^5\ \text{Pa}} = 4.33\times10^{-2}\ \text{m}^3$. $h = \dfrac{V}{\pi r^2} = \dfrac{4.33\times10^{-2}\ \text{m}^3}{\pi(0.100\ \text{m})^2} = 1.38\ \text{m}$.

EVALUATE: The calculation assumes a vacuum $(p = 0)$ in the tank above the piston.

18.63. **IDENTIFY:** Apply Bernoulli's equation to relate the efflux speed of water out the hose to the height of water in the tank and the pressure of the air above the water in the tank. Use the ideal-gas equation to relate the volume of the air in the tank to the pressure of the air.

(a) SET UP: Points 1 and 2 are shown in Figure 18.63.

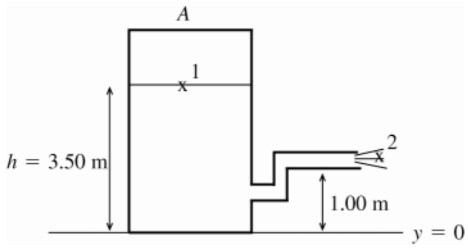


Figure 18.63

$p_1 = 4.20\times10^5$ Pa

$p_2 = p_{\text{air}} = 1.00\times10^5$ Pa

large tank implies $v_1 \approx 0$

EXECUTE: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

$\frac{1}{2}\rho v_2^2 = p_1 - p_2 + \rho g(y_1 - y_2)$

$v_2 = \sqrt{(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2)}$

$v_2 = 26.2$ m/s

(b) $\underline{h = 3.00 \text{ m}}$

The volume of the air in the tank increases so its pressure decreases. $pV = nRT =$ constant, so $pV = p_0V_0$ (p_0 is the pressure for $h_0 = 3.50$ m and p is the pressure for $h = 3.00$ m)

$p(4.00\text{ m} - h)A = p_0(4.00\text{ m} - h_0)A$

$$p = p_0\left(\frac{4.00\text{ m} - h_0}{4.00\text{ m} - h}\right) = (4.20\times10^5\text{ Pa})\left(\frac{4.00\text{ m} - 3.50\text{ m}}{4.00\text{ m} - 3.00\text{ m}}\right) = 2.10\times10^5\text{ Pa}$$

Repeat the calculation of part (a), but now $p_1 = 2.10\times10^5$ Pa and $y_1 = 3.00$ m.

$v_2 = \sqrt{(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2)}$

$v_2 = 16.1$ m/s

$\underline{h = 2.00 \text{ m}}$

$$p = p_0\left(\frac{4.00\text{ m} - h_0}{4.00\text{ m} - h}\right) = (4.20\times10^5\text{ Pa})\left(\frac{4.00\text{ m} - 3.50\text{ m}}{4.00\text{ m} - 2.00\text{ m}}\right) = 1.05\times10^5\text{ Pa}$$

$v_2 = \sqrt{(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2)}$

$v_2 = 5.44$ m/s

(c) $v_2 = 0$ means $(2/\rho)(p_1 - p_2) + 2g(y_1 - y_2) = 0$

$p_1 - p_2 = -\rho g(y_1 - y_2)$

$y_1 - y_2 = h - 1.00$ m

$p = p_0\left(\frac{0.50\text{ m}}{4.00\text{ m} - h}\right) = (4.20\times10^5\text{ Pa})\left(\frac{0.50\text{ m}}{4.00\text{ m} - h}\right)$. This is p_1, so

$$(4.20\times10^5\text{ Pa})\left(\frac{0.50\text{ m}}{4.00\text{ m} - h}\right) - 1.00\times10^5\text{ Pa} = (9.80\text{ m/s}^2)(1000\text{ kg/m}^3)(1.00\text{ m} - h)$$

$(210/(4.00 - h)) - 100 = 9.80 - 9.80h$, with h in meters.

$210 = (4.00 - h)(109.8 - 9.80h)$

$9.80h^2 - 149h + 229.2 = 0$ and $h^2 - 15.20h + 23.39 = 0$

quadratic formula: $h = \frac{1}{2}\left(15.20 \pm \sqrt{(15.20)^2 - 4(23.39)}\right) = (7.60 \pm 5.86)$ m

h must be less than 4.00 m, so the only acceptable value is $h = 7.60\text{ m} - 5.86\text{ m} = 1.74$ m

EVALUATE: The flow stops when $p + \rho g(y_1 - y_2)$ equals air pressure. For $h = 1.74$ m, $p = 9.3\times10^4$ Pa and $\rho g(y_1 - y_2) = 0.7\times10^4$ Pa, so $p + \rho g(y_1 - y_2) = 1.0\times10^5$ Pa, which is air pressure.

18.64. **IDENTIFY:** Use the ideal gas law to find the number of moles of air taken in with each breath and from this calculate the number of oxygen molecules taken in. Then find the pressure at an elevation of 2000 m and repeat the calculation.

SET UP: The number of molecules in a mole is $N_A = 6.022\times10^{23}$ molecules/mol. $R = 0.08206$ L·atm/mol·K. Example 18.4 shows that the pressure variation with altitude *y*, when constant temperature is assumed, is $p = p_0 e^{-Mgy/RT}$. For air, $M = 28.8\times10^{-3}$ kg/mol.

EXECUTE: **(a)** $pV = nRT$ gives $n = \dfrac{pV}{RT} = \dfrac{(1.00\text{ atm})(0.50\text{ L})}{(0.08206\text{ L}\cdot\text{atm/mol}\cdot\text{K})(293.15\text{ K})} = 0.0208\text{ mol}$.

$N = (0.210)nN_A = (0.210)(0.0208\text{ mol})(6.022\times10^{23}\text{ molecules/mol}) = 2.63\times10^{21}\text{ molecules}$.

(b) $\dfrac{Mgy}{RT} = \dfrac{(28.8\times10^{-3}\text{ kg/mol})(9.80\text{ m/s}^2)(2000\text{ m})}{(8.314\text{ J/mol}\cdot\text{K})(293.15\text{ K})} = 0.2316$. $p = p_0 e^{-Mgy/RT} = (1.00\text{ atm})e^{-0.2316} = 0.793\text{ atm}$.

N is proportional to *n*, which is in turn proportional to *p*, so

$N = \left(\dfrac{0.793\text{ atm}}{1.00\text{ atm}}\right)(2.63\times10^{21}\text{ molecules}) = 2.09\times10^{21}\text{ molecules}$.

(c) Less O_2 is taken in with each breath at the higher altitude, so the person must take more breaths per minute.

EVALUATE: A given volume of gas contains fewer molecules when the pressure is lowered and the temperature is kept constant.

18.65. **IDENTIFY** and **SET UP:** Apply Eq.(18.2) to find *n* and then use Avogadro's number to find the number of molecules.

EXECUTE: Calculate the number of water molecules *N*.

Number of moles: $n = \dfrac{m_{\text{tot}}}{M} = \dfrac{50\text{ kg}}{18.0\times10^{-3}\text{ kg/mol}} = 2.778\times10^3\text{ mol}$

$N = nN_A = (2.778\times10^3\text{ mol})(6.022\times10^{23}\text{ molecules/mol}) = 1.7\times10^{27}\text{ molecules}$

Each water molecule has three atoms, so the number of atoms is $3(1.7\times10^{27}) = 5.1\times10^{27}$ atoms

EVALUATE: We could also use the masses in Example 18.5 to find the mass *m* of one H_2O molecule: $m = 2.99\times10^{-26}$ kg. Then $N = m_{\text{tot}}/m = 1.7\times10^{27}$ molecules, which checks.

18.66. **IDENTIFY:** $pV = nRT = \dfrac{N}{N_A}RT$. Deviations will be noticeable when the volume *V* of a molecule is on the order of 1% of the volume of gas that contains one molecule.

SET UP: The volume of a sphere of radius *r* is $V = \dfrac{4}{3}\pi r^3$.

EXECUTE: The volume of gas per molecule is $\dfrac{RT}{N_A p}$, and the volume of a molecule is about $V_0 = \dfrac{4}{3}\pi(2.0\times10^{-10}\text{m})^3 = 3.4\times10^{-29}\text{m}^3$. Denoting the ratio of these volumes as *f*,

$$p = f\frac{RT}{N_A V_0} = f\frac{(8.3145\text{ J/mol}\cdot\text{K})(300\text{ K})}{(6.023\times10^{23}\text{ molecules/mol})(3.4\times10^{-29}\text{ m}^3)} = (1.2\times10^8\text{ Pa})f.$$

"Noticeable deviations" is a subjective term, but *f* on the order of 1.0% gives a pressure of 10^6 Pa.

EVALUATE: The forces between molecules also cause deviations from ideal-gas behavior.

18.67. **IDENTIFY:** Eq.(18.16) says that the average translational kinetic energy of each molecule is equal to $\frac{3}{2}kT$. $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$.

SET UP: $k = 1.381\times10^{-23}$ J/molecule·K.

EXECUTE: **(a)** $\frac{1}{2}m(v^2)_{\text{av}}$ depends only on *T* and both gases have the same *T*, so both molecules have the same average translational kinetic energy. v_{rms} is proportional to $m^{-1/2}$, so the lighter molecules, *A*, have the greater v_{rms}.

(b) The temperature of gas *B* would need to be raised.

(c) $\sqrt{\dfrac{T}{m}} = \dfrac{v_{\text{rms}}}{\sqrt{3k}} = \text{constant}$, so $\dfrac{T_A}{m_A} = \dfrac{T_B}{m_B}$. $T_B = \left(\dfrac{m_B}{m_A}\right)T_A = \left(\dfrac{5.34\times10^{-26}\text{ kg}}{3.34\times10^{-27}\text{ kg}}\right)(283.15\text{ K}) = 4.53\times10^3\text{ K} = 4250°\text{C}$.

(d) $T_B > T_A$ so the *B* molecules have greater translational kinetic energy per molecule.

EVALUATE: In $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT$ and $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$ the temperature *T* must be in kelvins.

18.68. **IDENTIFY:** The equations derived in the subsection Collisions between Molecules in Section 18.3 can be applied to the bees. The average distance a bee travels between collisions is the mean free path, λ. The average time between collisions is the mean free time, t_{mean}. The number of collisions per second is $\dfrac{dN}{dt} = \dfrac{1}{t_{\text{mean}}}$.

SET UP: $V = (1.25\text{ m})^3 = 1.95\text{ m}^3$. $r = 0.750\times10^{-2}\text{ m}$. $v = 1.10\text{ m/s}$. $N = 2500$.

EXECUTE: **(a)** $\lambda = \dfrac{V}{4\pi\sqrt{2}r^2N} = \dfrac{1.95\text{ m}^3}{4\pi\sqrt{2}(0.750\times10^{-2}\text{ m})^2(2500)} = 0.780\text{ m} = 78.0\text{ cm}$

(b) $\lambda = vt_{\text{mean}}$, so $t_{\text{mean}} = \dfrac{\lambda}{v} = \dfrac{0.780\text{ m}}{1.10\text{ m/s}} = 0.709\text{ s}$.

(c) $\dfrac{dN}{dt} = \dfrac{1}{t_{\text{mean}}} = \dfrac{1}{0.709\text{ s}} = 1.41\text{ collisions/s}$

EVALUATE: The calculation is valid only if the motion of each bee is random.

18.69. **IDENTIFY:** Apply the iteration procedure that is described in the problem.

SET UP: Let $x = n/V$. $T = 400.15\text{ K}$.

EXECUTE: **(a)** Dividing both sides of Eq.(18.7) by the product RTV gives the result.

(b) The algorithm described is best implemented on a programmable calculator or computer; for a calculator, the numerical procedure is an iteration of

$$x = \left[\frac{(9.80\times10^5)}{(8.3145)(400.15)} + \frac{(0.448)}{(8.3145)(400.15)}x^2\right]\left[1-(4.29\times10^{-5})x\right].$$

Starting at $x = 0$ gives a fixed point at $x = 3.03\times10^2$ after four iterations. The number density is $3.03\times10^2\text{ mol/m}^3$.

(c) The ideal-gas equation is the result after the first iteration, 295 mol/m^3.

EVALUATE: The van der Waals density is larger. The term corresponding to a represents the attraction of the molecules, and hence more molecules will be in a given volume for a given pressure.

18.70. **IDENTIFY:** Calculate v_{rms} and use conservation of energy to relate the initial speed of the molecules (v_{rms}) to the maximum height they reach.

SET UP: $T = 298.15\text{ K}$. $M = 28.0\times10^{-3}\text{ kg/mol}$.

EXECUTE: $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}} = \sqrt{\dfrac{3(8.314\text{ J/mol}\cdot\text{K})(298.15\text{ K})}{28.0\times10^{-3}\text{ kg/mol}}} = 515\text{ m/s}$. Conservation of energy gives

$\frac{1}{2}mv_{\text{rms}}^2 = mgy$ and $y = \dfrac{v_{\text{rms}}^2}{2g} = \dfrac{(515\text{ m/s})^2}{2(1.30\text{ m/s}^2)} = 1.02\times10^5\text{ m} = 102\text{ km}$

EVALUATE: The result does not depend on the amount of gas in the canister.

18.71. **IDENTIFY:** The mass of one molecule is the molar mass, M, divided by the number of molecules in a mole, N_A. The average translational kinetic energy of a single molecule is $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT$. Use $pV = NkT$ to calculate N, the number of molecules.

SET UP: $k = 1.381\times10^{-23}\text{ J/molecule}\cdot\text{K}$. $M = 28.0\times10^{-3}\text{ kg/mol}$. $T = 295.15\text{ K}$. The volume of the balloon is $V = \frac{4}{3}\pi(0.250\text{ m})^3 = 0.0654\text{ m}^3$. $p = 1.25\text{ atm} = 1.27\times10^5\text{ Pa}$.

EXECUTE: **(a)** $m = \dfrac{M}{N_A} = \dfrac{28.0\times10^{-3}\text{ kg/mol}}{6.022\times10^{23}\text{ molecules/mol}} = 4.65\times10^{-26}\text{ kg}$

(b) $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT = \frac{3}{2}(1.381\times10^{-23}\text{ J/molecule}\cdot\text{K})(295.15\text{ K}) = 6.11\times10^{-21}\text{ J}$

(c) $N = \dfrac{pV}{kT} = \dfrac{(1.27\times10^5\text{ Pa})(0.0654\text{ m}^3)}{(1.381\times10^{-23}\text{ J/molecule}\cdot\text{K})(295.15\text{ K})} = 2.04\times10^{24}\text{ molecules}$

(d) The total average translational kinetic energy is $N(\frac{1}{2}m(v^2)_{\text{av}}) = (2.04\times10^{24}\text{ molecules})(6.11\times10^{-21}\text{ J/molecule}) = 1.25\times10^4\text{ J}$.

EVALUATE: The number of moles is $n = \dfrac{N}{N_A} = \dfrac{2.04\times10^{24}\text{ molecules}}{6.022\times10^{23}\text{ molecules/mol}} = 3.39\text{ mol}$.

$K_{\text{tr}} = \frac{3}{2}nRT = \frac{3}{2}(3.39\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(295.15\text{ K}) = 1.25\times10^4\text{ J}$, which agrees with our results in part (d).

18.72. **IDENTIFY:** $U = mgy$. The mass of one molecule is $m = M/N_A$. $K_{av} = \frac{3}{2}kT$.

SET UP: Let $y = 0$ at the surface of the earth and $h = 400\text{ m}$. $N_A = 6.023\times10^{23}$ molecules/mol and $k = 1.38\times10^{-23}\text{ J/K}$. $15.0°\text{C} = 288\text{ K}$.

EXECUTE: **(a)** $U = mgh = \dfrac{M}{N_A}gh = \left(\dfrac{28.0\times10^{-3}\text{ kg/mol}}{6.023\times10^{23}\text{ molecules/mol}}\right)(9.80\text{ m/s}^2)(400\text{ m}) = 1.82\times10^{-22}\text{ J}.$

(b) Setting $U = \dfrac{3}{2}kT,\ T = \dfrac{2}{3}\left(\dfrac{1.82\times10^{-22}\text{ J}}{1.38\times10^{-23}\text{ J/K}}\right) = 8.80\text{ K}.$

EVALUATE: **(c)** The average kinetic energy at $15.0°\text{C}$ is much larger than the increase in gravitational potential energy, so it is energetically possible for a molecule to rise to this height. But Example 18.8 shows that the mean free path will be very much less than this and a molecule will undergo many collisions as it rises. These numerous collisions transfer kinetic energy between molecules and make it highly unlikely that a given molecule can have very much of its translational kinetic energy converted to gravitational potential energy.

18.73. **IDENTIFY** and **SET UP:** At equilibrium $F(r) = 0$. The work done to increase the separation from r_2 to ∞ is $U(\infty) - U(r_2)$.

(a) EXECUTE: $U(r) = U_0\left[(R_0/r)^{12} - 2(R_0/r)^6\right]$

Eq.(13.26): $F(r) = 12(U_0/R_0)\left[(R_0/r)^{13} - (R_0/r)^7\right]$. The graphs are given in Figure 18.73.

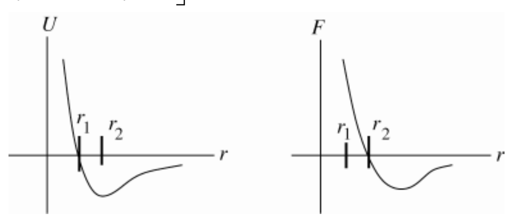


Figure 18.73

(b) equilibrium requires $F = 0$; occurs at point r_2. r_2 is where U is a minimum (stable equilibrium).

(c) $U = 0$ implies $\left[(R_0/r)^{12} - 2(R_0/r)^6\right] = 0$

$(r_1/R_0)^6 = 1/2$ and $r_1 = R_0/(2)^{1/6}$

$F = 0$ implies $\left[(R_0/r)^{13} - (R_0/r)^7\right] = 0$

$(r_2/R_0)^6 = 1$ and $r_2 = R_0$

Then $r_1/r_2 = (R_0/2^{1/6})/R_0 = 2^{-1/6}$

(d) $W_{\text{other}} = \Delta U$

At $r \to \infty$, $U = 0$, so $W = -U(R_0) = -U_0\left[(R_0/R_0)^{12} - 2(R_0/R_0)^6\right] = +U_0$

EVALUATE: The answer to part (d), U_0, is the depth of the potential well shown in the graph of $U(r)$.

18.74. **IDENTIFY:** Use $pV = nRT$ to calculate the number of moles, n. Then $K_{tr} = \frac{3}{2}nRT$. The mass of the gas, m_{tot}, is given by $m_{tot} = nM$.

SET UP: $5.00\text{ L} = 5.00\times10^{-3}\text{ m}^3$

EXECUTE: **(a)** $n = \dfrac{pV}{RT} = \dfrac{(1.01\times10^5\text{ Pa})(5.00\times10^{-3}\text{ m}^3)}{(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})} = 0.2025\text{ moles}.$

$K_{tr} = \frac{3}{2}(0.2025\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(300\text{ K}) = 758\text{ J}.$

(b) $m_{tot} = nM = (0.2025\text{ mol})(2.016\times10^{-3}\text{ kg/mol}) = 4.08\times10^{-4}\text{ kg}$. The kinetic energy due to the speed of the jet is $K = \frac{1}{2}mv^2 = \frac{1}{2}(4.08\times10^{-4}\text{ kg})(300.0\text{ m/s})^2 = 18.4\text{ J}$. The total kinetic energy is

$K_{tot} = K + K_{tr} = 18.4\text{ J} + 758\text{ J} = 776\text{ J}$. The percentage increase is $\dfrac{K}{K_{tot}}\times100\% = \dfrac{18.4\text{ J}}{776\text{ J}}\times100\% = 2.37\%$.

(c) No. The temperature is associated with the random translational motion, and that hasn't changed.

EVALUATE: Eq.(18.13) gives $K_{tr} = \frac{3}{2}pV = \frac{3}{2}(1.01\times10^5\text{ Pa})(5.00\times10^{-3}\text{ m}^3) = 758\text{ J}$, which agrees with our result in part (a). $v_{rms} = \sqrt{\dfrac{3RT}{M}} = 1.93\times10^3\text{ m/s}$. v_{rms} is a lot larger than the speed of the jet, so the percentage increase in the total kinetic energy, calculated in part (b), is small.

18.75. **IDENTIFY** and **SET UP:** Apply Eq.(18.19) for v_{rms}. The equation preceeding Eq.(18.12) relates v_{rms} and $(v_x)_{\text{rms}}$.

EXECUTE: **(a)** $v_{\text{rms}} = \sqrt{3RT/M}$

$$v_{\text{rms}} = \sqrt{\frac{3(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})}{28.0\times10^{-3}\ \text{kg/mol}}} = 517\ \text{m/s}$$

(b) $(v_x^2)_{\text{av}} = \frac{1}{3}(v^2)_{\text{av}}$ so $\sqrt{(v_x^2)_{\text{av}}} = \left(1/\sqrt{3}\right)\sqrt{(v^2)_{\text{av}}} = \left(1/\sqrt{3}\right)v_{\text{rms}} = \left(1/\sqrt{3}\right)(517\ \text{m/s}) = 298\ \text{m/s}$

EVALUATE: The speed of sound is approximately equal to $(v_x)_{\text{rms}}$ since it is the motion along the direction of propagation of the wave that transmits the wave.

18.76. **IDENTIFY:** $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}}$

SET UP: $M = 1.99\times10^{30}$ kg , $R = 6.96\times10^{8}$ m and $G = 6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** $v_{\text{rms}} = \sqrt{\dfrac{3kT}{m}} = \sqrt{\dfrac{3(1.38\times10^{-23}\,\text{J/K})\ (5800\ \text{K})}{(1.67\times10^{-27}\,\text{kg})}} = 1.20\times10^{4}\ \text{m/s}.$

(b) $v_{\text{escape}} = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2(6.673\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)\ (1.99\times10^{30}\text{kg})}{(6.96\times10^{8}\ \text{m})}} = 6.18\times10^{5}\ \text{m/s}.$

EVALUATE: **(c)** The escape speed is about 50 times the rms speed, and any of Figure 18.23 in the textbook, Eq.(18.32) or Table (18.2) will indicate that there is a negligibly small fraction of molecules with the escape speed.

18.77. **(a) IDENTIFY** and **SET UP:** Apply conservation of energy $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$, where $U = -Gmm_{\text{p}}/r$. Let point 1 be at the surface of the planet, where the projectile is launched, and let point 2 be far from the earth. Just barely escapes says $v_2 = 0$.

EXECUTE: Only gravity does work says $W_{\text{other}} = 0$.

$U_1 = -Gmm_{\text{p}}/R_{\text{p}}$; $r_2 \to \infty$ so $U_2 = 0$; $v_2 = 0$ so $K_2 = 0$.

The conservation of energy equation becomes $K_1 - Gmm_{\text{p}}/R_{\text{p}} = 0$ and $K_1 = Gmm_{\text{p}}/R_{\text{p}}$.

But $g = Gm_{\text{p}}/R_{\text{p}}^2$ so $Gm_{\text{p}}/R_{\text{p}} = R_{\text{p}}g$ and $K_1 = mgR_{\text{p}}$, as was to be shown.

EVALUATE: The greater gR_{p} is the more initial kinetic energy is required for escape.

(b) IDENTIFY and **SET UP:** Set K_1 from part (a) equal to the average kinetic energy of a molecule as given by Eq.(18.16). $\frac{1}{2}m(v^2)_{\text{av}} = mgR_{\text{p}}$ (from part (a)). But also, $\frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT$, so $mgR_{\text{p}} = \frac{3}{2}kT$

EXECUTE: $T = \dfrac{2mgR_{\text{p}}}{3k}$

<u>nitrogen</u>

$m_{\text{N}_2} = (28.0\times10^{-3}\ \text{kg/mol})/(6.022\times10^{23}\ \text{molecules/mol}) = 4.65\times10^{-26}\ \text{kg/molecule}$

$$T = \frac{2mgR_{\text{p}}}{3k} = \frac{2(4.65\times10^{-26}\ \text{kg/molecule})(9.80\ \text{m/s}^2)(6.38\times10^{6}\ \text{m})}{3(1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K})} = 1.40\times10^{5}\ \text{K}$$

<u>hydrogen</u>

$m_{\text{H}_2} = (2.02\times10^{-3}\ \text{kg/mol})/(6.022\times10^{23}\ \text{molecules/mol}) = 3.354\times10^{-27}\ \text{kg/molecule}$

$$T = \frac{2mgR_{\text{p}}}{3k} = \frac{2(3.354\times10^{-27}\ \text{kg/molecule})(9.80\ \text{m/s}^2)(6.38\times10^{6}\ \text{m})}{3(1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K})} = 1.01\times10^{4}\ \text{K}$$

(c) $T = \dfrac{2mgR_{\text{p}}}{3k}$

<u>nitrogen</u>

$$T = \frac{2(4.65\times10^{-26}\ \text{kg/molecule})(1.63\ \text{m/s}^2)(1.74\times10^{6}\ \text{m})}{3(1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K})} = 6730\ \text{K}$$

<u>hydrogen</u>

$$T = \frac{2(3.354\times10^{-27}\ \text{kg/molecule})(1.63\ \text{m/s}^2)(1.74\times10^{6}\ \text{m})}{3(1.381\times10^{-23}\ \text{J/molecule}\cdot\text{K})} = 459\ \text{K}$$

(d) EVALUATE: The "escape temperatures" are much less for the moon than for the earth. For the moon a larger fraction of the molecules at a given temperature will have speeds in the Maxwell-Boltzmann distribution larger than the escape speed. After the long time most of the molecules will have escaped from the moon.

18.78. **IDENTIFY:** $v_{rms} = \sqrt{\frac{3RT}{M}}$.

SET UP: $M_{H_2} = 2.02\times10^{-3}$ kg/mol. $M_{O_2} = 32.0\times10^{-3}$ kg/mol. For Earth, $M = 5.97\times10^{24}$ kg and $R = 6.38\times10^{6}$ m. For Jupiter, $M = 1.90\times10^{27}$ kg and $R = 6.91\times10^{7}$ m. For a sphere, $M = \rho V = \rho\frac{4}{3}\pi r^3$. The escape speed is $v_{escape} = \sqrt{\frac{2GM}{R}}$.

EXECUTE: **(a)** Jupiter: $v_{rms} = \sqrt{3(8.3145\,\text{J/mol}\cdot\text{K})(140\text{K})/(2.02\times10^{-3}\,\text{kg/mol})} = 1.31\times10^{3}\,\text{m/s}$.
$v_{escape} = 6.06\times10^{4}$ m/s. $v_{rms} = 0.022 v_{escape}$.
Earth: $v_{rms} = \sqrt{3(8.3145\,\text{J/mol}\cdot\text{K})(220\text{K})/(2.02\times10^{-3}\,\text{kg/mol})} = 1.65\times10^{3}\,\text{m/s}$. $v_{escape} = 1.12\times10^{4}$ m/s.
$v_{rms} = 0.15 v_{escape}$.
(b) Escape from Jupiter is not likely for any molecule, while escape from earth is much more probable.
(c) $v_{rms} = \sqrt{3(8.3145\,\text{J/mol}\cdot\text{K})(200\text{K})/(32.0\times10^{-3}\,\text{kg/mol})} = 395\,\text{m/s}$. The radius of the asteroid is $R = (3M/4\pi\rho)^{1/3} = 4.68\times10^{5}\,\text{m}$, and the escape speed is $v_{escape} = \sqrt{2GM/R} = 542\,\text{m/s}$. Over time the O_2 molecules would essentially all escape and there can be no such atmosphere.
EVALUATE: As Figure 18.23 in the textbook shows, there are some molecules in the velocity distribution that have speeds greater than v_{rms}. But as the speed increases above v_{rms} the number with speeds in that range decreases.

18.79. **IDENTIFY:** $v_{rms} = \sqrt{\frac{3kT}{m}}$. The number of molecules in an object of mass *m* is $N = nN_A = \frac{m}{M}N_A$.

SET UP: The volume of a sphere of radius *r* is $V = \frac{4}{3}\pi r^3$.

EXECUTE: **(a)** $m = \frac{3kT}{v_{rms}^2} = \frac{3(1.381\times10^{-23}\,\text{J/K})(300\text{K})}{(0.0010\,\text{m/s})^2} = 1.24\times10^{-14}\,\text{kg}$.
(b) $N = mN_A/M = (1.24\times10^{-14}\text{kg})(6.023\times10^{23}\,\text{molecules/mol})/(18.0\times10^{-3}\,\text{kg/mol})$
$N = 4.16\times10^{11}\,\text{molecules}$.
(c) The diameter is $D = 2r = 2\left(\frac{3V}{4\pi}\right)^{1/3} = 2\left(\frac{3m/\rho}{4\pi}\right)^{1/3} = 2\left(\frac{3(1.24\times10^{-14}\text{ kg})}{4\pi(920\text{ kg/m}^3)}\right)^{1/3} = 2.95\times10^{-6}$ m which is too small to see.
EVALUATE: v_{rms} decreases as *m* increases.

18.80. **IDENTIFY:** For a simple harmonic oscillator, $x = A\cos\omega t$ and $v_x = -\omega A\sin\omega t$, with $\omega = \sqrt{k/m}$.

SET UP: The average value of $\cos(2\omega t)$ over one period is zero, so $(\sin^2\omega t)_{av} = (\cos^2\omega t)_{av} = \frac{1}{2}$.

EXECUTE: $x = A\cos\omega t$, $v_x = -\omega A\sin\omega t$, $U_{av} = \frac{1}{2}kA^2(\cos^2\omega t)_{av}$, $K_{av} = \frac{1}{2}m\omega^2A^2(\sin^2\omega t)_{av}$. Using $(\sin^2\omega t)_{av} = (\cos^2\omega t)_{av} = \frac{1}{2}$ and $m\omega^2 = k$ shows that $K_{av} = U_{av}$.
EVALUATE: In general, at any given instant of time $U \neq K$. It is only the values averaged over one period that are equal.

18.81. **IDENTIFY:** The equipartition principle says that each atom has an average kinetic energy of $\frac{1}{2}kT$ for each degree of freedom. There is an equal average potential energy.
SET UP: The atoms in a three-dimensional solid have three degrees of freedom and the atoms in a two-dimensional solid have two degrees of freedom.
EXECUTE: **(a)** In the same manner that Eq.(18.28) was obtained, the heat capacity of the two-dimensional solid would be $2R = 16.6$ J/mol·K.
(b) The heat capacity would behave qualitatively like those in Figure 18.21 in the textbook, and the heat capacity would decrease with decreasing temperature.
EVALUATE: At very low temperatures the equipartition theorem doesn't apply. Most of the atoms remain in their lowest energy states because the next higher energy level is not accessible.

18.82. **IDENTIFY:** The equipartition principle says that each molecule has average kinetic energy of $\frac{1}{2}kT$ for each degree of freedom. $I = 2m(L/2)^2$, where L is the distance between the two atoms in the molecule. $K_{\text{rot}} = \frac{1}{2}I\omega^2$. $\omega_{\text{rms}} = \sqrt{(\omega^2)_{\text{av}}}$.

SET UP: The mass of one atom is $m = M/N_A = (16.0\times10^{-3}\text{ kg/mol})/(6.02\times10^{23}\text{ molecules/mol}) = 2.66\times10^{-26}\text{ kg}$.

EXECUTE: **(a)** The two degrees of freedom associated with the rotation for a diatomic molecule account for two-fifths of the total kinetic energy, so $K_{\text{rot}} = nRT = (1.00\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})(300\text{ K}) = 2.49\times10^3\text{ J}$.

(b) $I = 2m(L/2)^2 = 2\left(\dfrac{16.0\times10^{-3}\text{ kg/mol}}{6.023\times10^{23}\text{ molecules/mol}}\right)(6.05\times10^{-11}\text{ m})^2 = 1.94\times10^{-46}\text{ kg}\cdot\text{m}^2$

(c) Since the result in part (b) is for one mole, the rotational kinetic energy for one atom is K_{rot}/N_A and $\omega_{\text{rms}} = \sqrt{\dfrac{2K_{\text{rot}}/N_A}{I}} = \sqrt{\dfrac{2(2.49\times10^3\text{ J})}{(1.94\times10^{-46}\text{ kg}\cdot\text{m}^2)(6.023\times10^{23}\text{ molecules/mol})}} = 6.52\times10^{12}\text{ rad/s}$. This is much larger than the typical value for a piece of rotating machinery.

EVALUATE: The average rotational period, $T = \dfrac{2\pi\text{ rad}}{\omega_{\text{rms}}}$, for molecules is very short.

18.83. **IDENTIFY:** $C_V = N\left(\frac{1}{2}R\right)$, where N is the number of degrees of freedom.

SET UP: There are three translational degrees of freedom.

EXECUTE: For CO_2, $N = 5$ and the contribution to C_V other than from vibration is $\frac{5}{2}R = 20.79\text{ J/mol}\cdot\text{K}$ and $C_V - \frac{5}{2}R = 0.270\,C_V$. So 27% of C_V is due to vibration. For both SO_2 and H_2S, $N = 6$ and the contribution to C_V other than from vibration is $\frac{6}{2}R = 24.94\text{ J/mol}\cdot\text{K}$. The respective fractions of C_V from vibration are 21% and 3.9%.

EVALUATE: The vibrational contribution is much less for H_2S. In H_2S the vibrational energy steps are larger because the two hydrogen atoms have small mass and $\omega = \sqrt{k/m}$.

18.84. **IDENTIFY:** Evaluate the integral, as specified in the problem.

SET UP: Use the integral formula given in Problem 18.85, with $\alpha = m/2kT$.

EXECUTE: **(a)** $\displaystyle\int_0^\infty f(v)dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^2 e^{-mv^2/2kT}dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{1}{4(m/2kT)}\right)\sqrt{\frac{\pi}{m/2kT}} = 1$

EVALUATE: **(b)** $f(v)dv$ is the probability that a particle has speed between v and $v + dv$; the probability that the particle has some speed is unity, so the sum (integral) of $f(v)dv$ must be 1.

18.85. **IDENTIFY** and **SET UP:** Evaluate the integral in Eq.(18.31) as specified in the problem.

EXECUTE: $\int_0^\infty v^2 f(v)\,dv = 4\pi(m/2\pi kT)^{3/2}\int_0^\infty v^4 e^{-mv^2/2kT}\,dv$

The integral formula with $n = 2$ gives $\int_0^\infty v^4 e^{-av^2}\,dv = (3/8a^2)\sqrt{\pi/a}$

Apply with $a = m/2kT$, $\int_0^\infty v^2 f(v)\,dv = 4\pi(m/2\pi kT)^{3/2}(3/8)(2kT/m)^2\sqrt{2\pi kT/m} = (3/2)(2kT/m) = 3kT/m$

EVALUATE: Equation (18.16) says $\frac{1}{2}m(v^2)_{\text{av}} = 3kT/2$, so $(v^2)_{\text{av}} = 3kT/m$, in agreement with our calculation.

18.86. **IDENTIFY:** Follow the procedure specified in the problem.

SET UP: If $v^2 = x$, then $dx = 2vdv$.

EXECUTE: $\displaystyle\int_0^\infty vf(v)dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^3 e^{-mv^2/2kT}dv.$ Making the suggested change of variable, $v^2 = x$. $2vdv = dx$, $v^3 dv = (1/2)x\,dx$, and the integral becomes

$$\int_0^\infty vf(v)dv = 2\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty xe^{-mx/2kT}dx = 2\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{2kT}{m}\right)^2 = \frac{2}{\sqrt{\pi}}\sqrt{\frac{2KT}{m}} = \sqrt{\frac{8KT}{\pi m}}$$

which is Eq. (18.35).

EVALUATE: The integral $\int_0^\infty vf(v)dv$ is the definition of v_{av}.

18.87. **IDENTIFY:** $f(v)dv$ is the probability that a particle has a speed between v and $v+dv$. Eq.(18.32) gives $f(v)$. v_{mp} is given by Eq.(18.34).

SET UP: For O_2, the mass of one molecule is $m = M/N_A = 5.32\times10^{-26}$ kg.

EXECUTE: **(a)** $f(v)dv$ is the fraction of the particles that have speed in the range from v to $v+dv$. The number of particles with speeds between v and $v+dv$ is therefore $dN = Nf(v)dv$ and $\Delta N = N\int_v^{v+\Delta v} f(v)dv.$

(b) Setting $v = v_{\text{mp}} = \sqrt{\dfrac{2kT}{m}}$ in $f(v)$ gives $f(v_{\text{mp}}) = 4\pi\left(\dfrac{m}{2\pi kT}\right)^{3/2}\left(\dfrac{2kT}{m}\right)e^{-1} = \dfrac{4}{e\sqrt{\pi}v_{\text{mp}}}$. For oxygen gas at 300 K, $v_{\text{mp}} = 3.95\times10^2$ m/s and $f(v)\Delta v = 0.0421.$

(c) Increasing v by a factor of 7 changes f by a factor of 7^2e^{-48}, and $f(v)\Delta v = 2.94\times10^{-21}.$

(d) Multiplying the temperature by a factor of 2 increases the most probable speed by a factor of $\sqrt{2}$, and the answers are decreased by $\sqrt{2}$: 0.0297 and 2.08×10^{-21}.

(e) Similarly, when the temperature is one-half what it was parts (b) and (c), the fractions increase by $\sqrt{2}$ to 0.0595 and 4.15×10^{-21}.

EVALUATE: **(f)** At lower temperatures, the distribution is more sharply peaked about the maximum (the most probable speed), as is shown in Figure 18.23a in the textbook.

18.88. **IDENTIFY:** Apply the definition of relative humidity given in the problem. $pV = nRT = \dfrac{m_{\text{tot}}}{M}RT$.

SET UP: $M = 18.0\times10^{-3}$ kg/mol.

EXECUTE: **(a)** The pressure due to water vapor is $(0.60)(2.34\times10^3\ \text{Pa}) = 1.40\times10^3\ \text{Pa}$.

(b) $m_{\text{tot}} = \dfrac{MpV}{RT} = \dfrac{(18.0\times10^{-3}\ \text{kg/mol})(1.40\times10^3\ \text{Pa})(1.00\ \text{m}^3)}{(8.3145\ \text{J/mol}\cdot\text{K})(293.15\ \text{K})} = 10\ \text{g}$

EVALUATE: The vapor pressure of water vapor at this temperature is much less than the total atmospheric pressure of 1.0×10^5 Pa.

18.89. **IDENTIFY:** The measurement gives the dew point. Relative humidity is defined in Problem 18.88.

SET UP: $\text{relative humidity} = \dfrac{\text{partial pressure of water vapor at temperature } T}{\text{vapor pressure of water at temperature } T}$

EXECUTE: The experiment shows that the dew point is 16.0°C, so the partial pressure of water vapor at 30.0°C is equal to the vapor pressure at 16.0°C, which is 1.81×10^3 Pa.

Thus the relative humidity $= \dfrac{1.81\times10^3\ \text{Pa}}{4.25\times10^3\ \text{Pa}} = 0.426 = 42.6\%.$

EVALUATE: The lower the dew point is compared to the air temperature, the smaller the relative humidity.

18.90. **IDENTIFY:** Use the definition of relative humidity in Problem 18.88 and the vapor pressure table in Problem 18.89.

SET UP: At 28.0°C the vapor pressure of water is 3.78×10^3 Pa.

EXECUTE: For a relative humidity of 35%, the partial pressure of water vapor is $(0.35)(3.78\times10^3\ \text{Pa}) = 1.323\times10^3\ \text{Pa}$. This is close to the vapor pressure at 12°C, which would be at an altitude $(30°\text{C} - 12°\text{C})/(0.6°\text{C}/100\ \text{m}) = 3\ \text{km}$ above the ground. For a relative humidity of 80%, the vapor pressure will be the same as the water pressure at around 24°C, corresponding to an altitude of about 1 km.

EVALUATE: Clouds form at a lower height when the relative humidity at the surface is larger.

18.91. **IDENTIFY:** Eq.(18.21) gives the mean free path λ. In Eq.(18.20) use $v_{\text{rms}} = \sqrt{\dfrac{3RT}{M}}$ in place of v. $pV = nRT = NkT$. The escape speed is $v_{\text{escape}} = \sqrt{\dfrac{2GM}{R}}$.

SET UP: For atomic hydrogen, $M = 1.008\times10^{-3}$ kg/mol.

EXECUTE: **(a)** From Eq.(18.21), $\lambda = (4\pi\sqrt{2}r^2(N/V))^{-1} = (4\pi\sqrt{2}(5.0\times10^{-11}\ \text{m})^2(50\times10^6\ \text{m}^{-3}))^{-1} = 4.5\times10^{11}$ m.

(b) $v_{\text{rms}} = \sqrt{3RT/M} = \sqrt{3(8.3145\ \text{J/mol}\cdot\text{K})(20\ \text{K})/(1.008\times10^{-3}\ \text{kg/mol})} = 703\ \text{m/s}$, and the time between collisions is then $(4.5\times10^{11}\ \text{m})/(703\ \text{m/s}) = 6.4\times10^8$ s, about 20 yr. Collisions are not very important.

(c) $p = (N/V)kT = (50/1.0\times10^{-6}\ \text{m}^3)(1.381\times10^{-23}\ \text{J/K})(20\ \text{K}) = 1.4\times10^{-14}$ Pa.

(d) $v_{\text{escape}} = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2G(Nm/V)(4\pi R^3/3)}{R}} = \sqrt{(8\pi/3)G(N/V)mR^2}$

$v_{\text{escape}} = \sqrt{(8\pi/3)(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(50\times10^6\ \text{m}^{-3})(1.67\times10^{-27}\ \text{kg})(10\times9.46\times10^{15}\ \text{m})^2}$

$v_{\text{escape}} = 650\ \text{m/s}$. This is lower than v_{rms} and the cloud would tend to evaporate.

(e) In equilibrium (clearly not *thermal* equilibrium), the pressures will be the same; from $pV = NkT$, $kT_{\text{ISM}}(N/V)_{\text{ISM}} = kT_{\text{nebula}}(N/V)_{\text{nebula}}$ and the result follows.

(f) With the result of part (e),

$$T_{\text{ISM}} = T_{\text{nebula}}\left(\frac{(V/N)_{\text{nebula}}}{(V/N)_{\text{ISM}}}\right) = (20\ \text{K})\left(\frac{50\times10^6\ \text{m}^3}{(200\times10^{-6}\ \text{m}^3)^{-1}}\right) = 2\times10^5\ \text{K},$$

more than three times the temperature of the sun. This indicates a high average kinetic energy, but the thinness of the ISM means that a ship would not burn up.

EVALUATE: The temperature of a gas is determined by the average kinetic energy per atom of the gas. The energy density for the gas also depends on the number of atoms per unit volume, and this is very small for the ISM.

18.92. IDENTIFY: Follow the procedure of Example 18.4, but use $T = T_0 - \alpha y$.

SET UP: $\ln(1+x) \approx x$ when x is very small.

EXECUTE: **(a)** $\dfrac{dp}{dy} = -\dfrac{pM}{RT}$, which in this case becomes $\dfrac{dp}{p} = -\dfrac{Mg}{R}\dfrac{dy}{T_0 - \alpha y}$. This integrates to

$$\ln\left(\frac{p}{p_0}\right) = \frac{Mg}{R\alpha}\ln\left(1 - \frac{\alpha y}{T_0}\right), \text{ or } p = p_0\left(1 - \frac{\alpha y}{T_0}\right)^{Mg/R\alpha}.$$

(b) For sufficiently small α, $\ln(1 - \frac{\alpha y}{T_0}) \approx -\frac{\alpha y}{T_0}$, and this gives the expression derived in Example 18.4.

(c) $\left(1 - \dfrac{(0.6\times10^{-2}\ \text{C°/m})(8863\ \text{m})}{(288\ \text{K})}\right) = 0.8154$, $\dfrac{Mg}{R\alpha} = \dfrac{(28.8\times10^{-3})(9.80\ \text{m/s}^2)}{(8.3145\ \text{J/mol}\cdot\text{K})(0.6\times10^{-2}\ \text{C°/m})} = 5.6576$ and

$p_0(0.8154)^{5.6576} = 0.315$ atm, which is 0.95 of the result found in Example 18.4.

EVALUATE: The pressure is calculated to decrease more rapidly with altitude when we assume that T also decreases with altitude.

18.93. IDENTIFY and **SET UP:** The behavior of isotherms for a real gas above and below the critical point are shown in Figure 18.7 in the textbook.

EXECUTE: **(a)** A positive slope $\dfrac{\partial P}{\partial V}$ would mean that an increase in pressure causes an increase in volume, or that decreasing volume results in a decrease in pressure, which cannot be the case for any real gas.

(b) See Figure 18.7 in the textbook. From part (a), p cannot have a positive slope along an isotherm, and so can have no extremes (maxima or minima) along an isotherm. When $\dfrac{\partial p}{\partial V}$ vanishes along an isotherm, the point on the curve in a p-V diagram must be an inflection point, and $\dfrac{\partial^2 p}{\partial V^2} = 0$.

(c) $p = \dfrac{nRT}{V - nb} - \dfrac{an^2}{V^2}$. $\dfrac{\partial p}{\partial V} = -\dfrac{nRT}{(V - nb)^2} + \dfrac{2an^2}{V^3}$. $\dfrac{\partial^2 p}{\partial V^2} = \dfrac{2nRT}{(V - nb)^3} - \dfrac{6an^2}{V^4}$. Setting the last two of these equal to zero gives $V^3 nRT = 2an^2(V - nb)^2$ and $V^4 nRT = 3an^2(V - nb)^3$.

(d) Following the hint, $V = (3/2)(V - nb)$, which is solved for $(V/n)_c = 3b$. Substituting this into either of the last two expressions in part (c) gives $T_c = 8a/27Rb$.

(e) $p_c = \dfrac{RT}{(V/n)_c - b} - \dfrac{a}{(V/n)_c} = \dfrac{R(8a/27Rb)}{2b} - \dfrac{a}{9b^2} = \dfrac{a}{27b^2}$.

(f) $\dfrac{RT_c}{p_c(V/n)_c} = \dfrac{(8a/27b)}{(a/27b^2)3b} = \dfrac{8}{3}$.

(g) $H_2: 3.28$. $N_2: 3.44$. $H_2O: 4.35$.

EVALUATE: **(h)** While all are close to 8/3, the agreement is not good enough to be useful in predicting critical point data. The van der Waals equation models certain gases, and is not accurate for substances near critical points.

18.94. **IDENTIFY and SET UP:** For N particles, $v_{\text{av}} = \frac{\sum v_i}{N}$ and $v_{\text{rms}} = \sqrt{\frac{\sum v_i^2}{N}}$.

EXECUTE: **(a)** $v_{\text{av}} = \frac{1}{2}(v_1 + v_2)$, $v_{\text{rms}} = \frac{1}{\sqrt{2}}\sqrt{v_1^2 + v_2^2}$ and

$$v_{\text{rms}}^2 - v_{\text{av}}^2 = \frac{1}{2}(v_1^2 + v_2^2) - \frac{1}{4}(v_1^2 + v_2^2 + 2v_1v_2) = \frac{1}{4}(v_1^2 + v_2^2 - 2v_1v_2) = \frac{1}{4}(v_1 - v_2)^2$$

This shows that $v_{\text{rms}} \geq v_{\text{av}}$, with equality holding if and only if the particles have the same speeds.

(b) $v_{\text{rms}}'^2 = \frac{1}{N+1}(Nv_{\text{rms}}^2 + u^2), v_{\text{av}}' = \frac{1}{N+1}(Nv_{\text{av}} + u),$ and the given forms follow immediately.

(c) The algebra is similar to that in part (a); it helps somewhat to express

$$v_{\text{av}}'^2 = \frac{1}{(N+1)^2}(N((N+1)-1)v_{\text{av}}^2 + 2Nv_{\text{av}}u + ((N+1)-N)u^2) .$$

$$v_{\text{av}}'^2 = \frac{N}{N+1}v_{\text{av}}^2 + \frac{N}{(N+1)^2}(-v_{\text{av}}^2 + 2v_{\text{av}}u - u^2) + \frac{1}{N+1}u^2$$

Then,

$v_{\text{rms}}'^2 - v_{\text{av}}'^2 = \frac{N}{(N+1)}(v_{\text{rms}}^2 - v_{\text{av}}^2) + \frac{N}{(N+1)^2}(v_{\text{av}}^2 - 2v_{\text{av}}u + u^2) = \frac{N}{N+1}(v_{\text{rms}}^2 - v_{\text{av}}^2) + \frac{N}{(N+1)^2}(v_{\text{av}} - u)^2$. If $v_{\text{rms}} > v_{\text{av}}$, then this difference is necessarily positive, and $v_{\text{rms}}' > v_{\text{av}}'$.

(d) The result has been shown for $N = 1$, and it has been shown that validity for N implies validity for $N+1$; by induction, the result is true for all N.

EVALUATE: $v_{\text{rms}} > v_{\text{av}}$ because v_{rms} gives more weight to particles that have greater speed.

THE FIRST LAW OF THERMODYNAMICS

19

19.1. **(a) IDENTIFY and SET UP:** The pressure is constant and the volume increases.

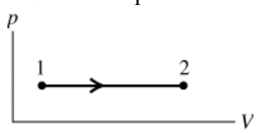


Figure 19.1

The pV-diagram is sketched in Figure 19.1

(b) $W = \int_{V_1}^{V_2} p\, dV$

Since p is constant, $W = p\int_{V_1}^{V_2} dV = p(V_2 - V_1)$

The problem gives T rather than p and V, so use the ideal gas law to rewrite the expression for W.

EXECUTE: $pV = nRT$ so $p_1V_1 = nRT_1,\ \ p_2V_2 = nRT_2;$ subtracting the two equations gives

$p(V_2 - V_1) = nR(T_2 - T_1)$

Thus $W = nR(T_2 - T_1)$ is an alternative expression for the work in a constant pressure process for an ideal gas.

Then $W = nR(T_2 - T_1) = (2.00\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})(107°\text{C} - 27°\text{C}) = +1330\text{ J}$

EVALUATE: The gas expands when heated and does positive work.

19.2. **IDENTIFY:** At constant pressure, $W = p\Delta V = nR\Delta T$.

SET UP: $R = 8.3145\text{ J/mol}\cdot\text{K}$. ΔT has the same numerical value in kelvins and in C°.

EXECUTE: $\Delta T = \dfrac{W}{nR} = \dfrac{1.75\times10^3\text{ J}}{(6\text{ mol})\ (8.3145\text{ J/mol}\cdot\text{K})} = 35.1\text{ K}.$ $\Delta T_\text{K} = \Delta T_\text{C}$ and $T_2 = 27.0°\text{C} + 35.1°\text{C} = 62.1°\text{C}.$

EVALUATE: When $W > 0$ the gas expands. When p is constant and V increases, T increases.

19.3. **IDENTIFY:** Example 19.1 shows that for an isothermal process $W = nRT\ln(p_1/p_2)$. $pV = nRT$ says V decreases when p increases and T is constant.

SET UP: $T = 358.15\text{ K}$. $p_2 = 3p_1$.

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.3.

(b) $W = (2.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(358.15\text{ K})\ln\left(\dfrac{p_1}{3p_1}\right) = -6540\text{ J}.$

EVALUATE: Since V decreases, W is negative.

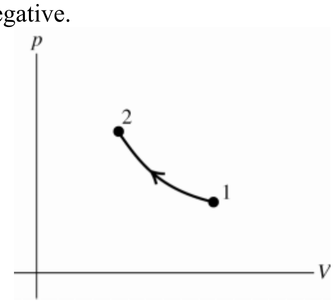


Figure 19.3

19.4. **IDENTIFY:** Use the expression for W that is appropriate to this type of process.

SET UP: The volume is constant.

EXECUTE: **(a)** The pV diagram is given in Figure 19.4.

(b) Since $\Delta V = 0,\ W = 0.$

EVALUATE: For any constant volume process the work done is zero.

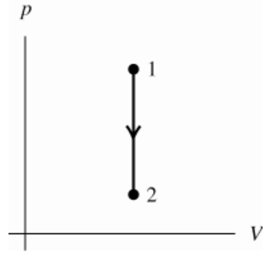


Figure 19.4

19.5. **IDENTIFY:** Example 19.1 shows that for an isothermal process $W = nRT\ln(p_1/p_2)$. Solve for p_1.

SET UP: For a compression (V decreases) W is negative, so $W = -518$ J. $T = 295.15$ K.

EXECUTE: **(a)** $\frac{W}{nRT} = \ln\left(\frac{p_1}{p_2}\right)$. $\frac{p_1}{p_2} = e^{W/nRT}$. $\frac{W}{nRT} = \frac{-518 \text{ J}}{(0.305 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(295.15 \text{ K})} = -0.692.$

$p_1 = p_2 e^{W/nRT} = (1.76 \text{ atm})e^{-0.692} = 0.881$ atm.

(b) In the process the pressure increases and the volume decreases. The pV-diagram is sketched in Figure 19.5.

EVALUATE: W is the work done by the gas, so when the surroundings do work on the gas, W is negative.

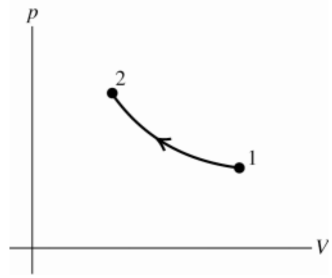


Figure 19.5

19.6. **(a) IDENTIFY** and **SET UP:** The pV-diagram is sketched in Figure 19.6.

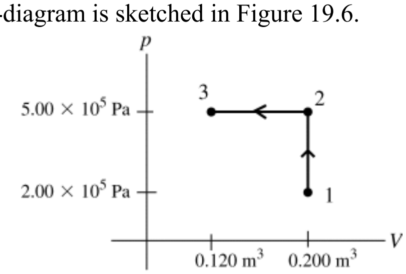


Figure 19.6

(b) Calculate W for each process, using the expression for W that applies to the specific type of process.

EXECUTE: $1 \to 2$, $\Delta V = 0$, so $W = 0$

$2 \to 3$

p is constant; so $W = p\,\Delta V = (5.00\times10^5 \text{ Pa})(0.120 \text{ m}^3 - 0.200 \text{ m}^3) = -4.00\times10^4$ J (W is negative since the volume decreases in the process.)

$W_{\text{tot}} = W_{1\to2} + W_{2\to3} = -4.00\times10^4$ J

EVALUATE: The volume decreases so the total work done is negative.

19.7. **IDENTIFY:** Calculate W for each step using the appropriate expression for each type of process.

SET UP: When p is constant, $W = p\Delta V$. When $\Delta V = 0$, $W = 0$.

EXECUTE: **(a)** $W_{13} = p_1(V_2 - V_1)$, $W_{32} = 0$, $W_{24} = p_2(V_1 - V_2)$ and $W_{41} = 0$. The total work done by the system is $W_{13} + W_{32} + W_{24} + W_{41} = (p_1 - p_2)(V_2 - V_1)$, which is the area in the pV plane enclosed by the loop.

(b) For the process in reverse, the pressures are the same, but the volume changes are all the negatives of those found in part (a), so the total work is negative of the work found in part (a).

EVALUATE: When $\Delta V > 0$, $W > 0$ and when $\Delta V < 0$, $W < 0$.

19.8. **IDENTIFY:** Apply $\Delta U = Q - W$.

SET UP: For an ideal gas, U depends only on T.

EXECUTE: **(a)** V decreases and W is negative.

(b) Since T is constant, $\Delta U = 0$ and $Q = W$. Since W is negative, Q is negative.

(c) $Q = W$, the magnitudes are the same.

EVALUATE: $Q < 0$ means heat flows out of the gas. The plunger does positive work on the gas. The energy added by the positive work done on the gas leaves as heat flow out of the gas and the internal energy of the gas is constant.

19.9. **IDENTIFY:** $\Delta U = Q - W$. For a constant pressure process, $W = p\Delta V$.

SET UP: $Q = +1.15\times10^5$ J, since heat enters the gas.

EXECUTE: **(a)** $W = p\Delta V = (1.80\times10^5 \text{ Pa})(0.320 \text{ m}^3 - 0.110 \text{ m}^3) = 3.78\times10^4$ J.

(b) $\Delta U = Q - W = 1.15\times10^5 \text{ J} - 3.78\times10^4 \text{ J} = 7.72\times10^4$ J.

EVALUATE: **(c)** $W = p\Delta V$ for a constant pressure process and $\Delta U = Q - W$ both apply to any material. The ideal gas law wasn't used and it doesn't matter if the gas is ideal or not.

19.10. **IDENTIFY:** The type of process is not specified. We can use $\Delta U = Q - W$ because this applies to all processes. Calculate ΔU and then from it calculate ΔT.

SET UP: Q is positive since heat goes into the gas; $Q = +1200$ J

W positive since gas expands; $W = +2100$ J

EXECUTE: $\Delta U = 1200 \text{ J} - 2100 \text{ J} = -900$ J

We can also use $\Delta U = n\left(\tfrac{3}{2}R\right)\Delta T$ since this is true for any process for an ideal gas.

$$\Delta T = \frac{2\,\Delta U}{3nR} = \frac{2(-900 \text{ J})}{3(5.00 \text{ mol})(8.3145 \text{ J/mol}\cdot\text{K})} = -14.4\text{C}^\circ$$

$T_2 = T_1 + \Delta T = 127^\circ\text{C} - 14.4\text{C}^\circ = 113^\circ\text{C}$

EVALUATE: More energy leaves the gas in the expansion work than enters as heat. The internal energy therefore decreases, and for an ideal gas this means the temperature decreases. We didn't have to convert ΔT to kelvins since ΔT is the same on the Kelvin and Celsius scales.

19.11. **IDENTIFY:** Apply $\Delta U = Q - W$ to the air inside the ball.

SET UP: Since the volume decreases, W is negative. Since the compression is sudden, $Q = 0$.

EXECUTE: $\Delta U = Q - W$ with $Q = 0$ gives $\Delta U = -W$. $W < 0$ so $\Delta U > 0$. $\Delta U = +410$ J.

(b) Since $\Delta U > 0$, the temperature increases.

EVALUATE: When the air is compressed, work is done on the air by the force on the air. The work done on the air increases its energy. No energy leaves the gas as a flow of heat, so the internal energy increases.

19.12. **IDENTIFY** and **SET UP:** Calculate W using the equation for a constant pressure process. Then use $\Delta U = Q - W$ to calculate Q.

(a) EXECUTE: $W = \int_{V_1}^{V_2} p\, dV = p(V_2 - V_1)$ for this constant pressure process.

$W = (2.3\times10^5 \text{ Pa})(1.20 \text{ m}^3 - 1.70 \text{ m}^3) = -1.15\times10^5$ J (The volume decreases in the process, so W is negative.)

(b) $\Delta U = Q - W$

$Q = \Delta U + W = -1.40\times10^5 \text{ J} + (-1.15\times10^5 \text{ J}) = -2.55\times10^5$ J

Q negative means heat flows out of the gas.

(c) EVALUATE: $W = \int_{V_1}^{V_2} p\, dV = p(V_2 - V_1)$ (constant pressure) and $\Delta U = Q - W$ apply to *any* system, not just to an ideal gas. We did not use the ideal gas equation, either directly or indirectly, in any of the calculations, so the results are the same whether the gas is ideal or not.

19.13. **IDENTIFY:** Calculate the total food energy value for one doughnut. $K = \tfrac{1}{2}mv^2$.

SET UP: 1 cal = 4.186 J

EXECUTE: **(a)** The energy is $(2.0 \text{ g})(4.0 \text{ kcal/g}) + (17.0 \text{ g})(4.0 \text{ kcal/g}) + (7.0 \text{ g})(9.0 \text{ kcal/g}) = 139$ kcal.

The time required is $(139 \text{ kcal})/(510 \text{ kcal/h}) = 0.273 \text{ h} = 16.4$ min.

(b) $v = \sqrt{2K/m} = \sqrt{2(139\times10^3 \text{ cal})(4.186 \text{ J/cal})/(60 \text{ kg})} = 139 \text{ m/s} = 501 \text{ km/h}$.

EVALUATE: When we set $K = Q$, we must express Q in J, so we can solve for v in m/s.

19.14. **IDENTIFY:** Apply $\Delta U = Q - W$.

SET UP: $W > 0$ when the system does work.

EXECUTE: **(a)** The container is said to be well-insulated, so there is no heat transfer.
(b) Stirring requires work. The stirring needs to be irregular so that the stirring mechanism moves against the water, not with the water.
(c) The work mentioned in part (b) is work done *on* the system, so $W < 0,$ and since no heat has been transferred, $\Delta U = -W > 0.$

EVALUATE: The stirring adds energy to the liquid and this energy stays in the liquid as an increase in internal energy.

19.15. **IDENTIFY:** Apply $\Delta U = Q - W$ to the gas.

SET UP: For the process, $\Delta V = 0.$ $Q = +400$ J since heat goes into the gas.

EXECUTE: **(a)** Since $\Delta V = 0,$ $W = 0.$

(b) $pV = nRT$ says $\dfrac{p}{T} = \dfrac{nR}{V} =$ constant. Since p doubles, T doubles. $T_b = 2T_a.$

(c) Since $W = 0,$ $\Delta U = Q = +400$ J. $U_b = U_a + 400$ J.

EVALUATE: For an ideal gas, when T increases, U increases.

19.16. **IDENTIFY:** Apply $\Delta U = Q - W.$ $|W|$ is the area under the path in the pV-plane.

SET UP: $W > 0$ when V increases.

EXECUTE: **(a)** The greatest work is done along the path that bounds the largest area above the V-axis in the p-V plane, which is path 1. The least work is done along path 3.
(b) $W > 0$ in all three cases; $Q = \Delta U + W,$ so $Q > 0$ for all three, with the greatest Q for the greatest work, that along path 1. When $Q > 0,$ heat is absorbed.

EVALUATE: ΔU is path independent and depends only on the initial and final states. W and Q are path independent and can have different values for different paths between the same initial and final states.

19.17. **IDENTIFY:** $\Delta U = Q - W.$ W is the area under the path in the pV-diagram. When the volume increases, $W > 0.$

SET UP: For a complete cycle, $\Delta U = 0.$

EXECUTE: **(a)** and **(b)** The clockwise loop (I) encloses a larger area in the p-V plane than the counterclockwise loop (II). Clockwise loops represent positive work and counterclockwise loops negative work, so $W_{\text{I}} > 0$ and $W_{\text{II}} < 0.$ Over one complete cycle, the net work $W_{\text{I}} + W_{\text{II}} > 0,$ and the net work done by the system is positive.
(c) For the complete cycle, $\Delta U = 0$ and so $W = Q.$ From part (a), $W > 0,$ so $Q > 0,$ and heat flows into the system.
(d) Consider each loop as beginning and ending at the intersection point of the loops. Around each loop, $\Delta U = 0,$ so $Q = W;$ then, $Q_{\text{I}} = W_{\text{I}} > 0$ and $Q_{\text{II}} = W_{\text{II}} < 0.$ Heat flows into the system for loop I and out of the system for loop II.

EVALUATE: W and Q are path dependent and are in general not zero for a cycle.

19.18. **IDENTIFY** and **SET UP:** Deduce information about Q and W from the problem statement and then apply the first law, $\Delta U = Q - W,$ to infer whether Q is positive or negative.

EXECUTE: **(a)** For the water $\Delta T > 0,$ so by $Q = mc\,\Delta T$ heat has been added to the water. Thus heat energy comes from the burning fuel-oxygen mixture, and Q for the system (fuel and oxygen) is negative.
(b) Constant volume implies $W = 0.$
(c) The 1st law (Eq.19.4) says $\Delta U = Q - W.$

$Q < 0,$ $W = 0$ so by the 1st law $\Delta U < 0.$ The internal energy of the fuel-oxygen mixture decreased.

EVALUATE: In this process internal energy from the fuel-oxygen mixture was transferred to the water, raising its temperature.

19.19. **IDENTIFY:** $\Delta U = Q - W.$ For a constant pressure process, $W = p\Delta V.$

SET UP: $Q = +2.20\times10^{6}$ J; $Q > 0$ since this amount of heat goes into the water. $p = 2.00\text{ atm} = 2.03\times10^{5}$ Pa.

EXECUTE: **(a)** $W = p\Delta V = (2.03\times10^{5}\text{ Pa})(0.824\text{ m}^3 - 1.00\times10^{-3}\text{ m}^3) = 1.67\times10^{5}\text{ J}$

(b) $\Delta U = Q - W = 2.20\times10^{6}\text{ J} - 1.67\times10^{5}\text{ J} = 2.03\times10^{6}\text{ J}.$

EVALUATE: 2.20×10^{6} J of energy enters the water. 1.67×10^{5} J of energy leaves the materials through expansion work and the remainder stays in the material as an increase in internal energy.

19.20. **IDENTIFY:** $\Delta U = Q - W$

SET UP: $Q < 0$ when heat leaves the gas.

EXECUTE: For an isothermal process, $\Delta U = 0,$ so $W = Q = -335$ J.

EVALUATE: In a compression the volume decreases and $W < 0.$

19.21. **IDENTIFY:** For a constant pressure process, $W = p\Delta V$, $Q = nC_p\Delta T$ and $\Delta U = nC_V\Delta T$. $\Delta U = Q - W$ and $C_p = C_V + R$. For an ideal gas, $p\Delta V = nR\Delta T$.

SET UP: From Table 19.1, $C_V = 28.46\text{ J/mol}\cdot\text{K}$.

EXECUTE: **(a)** The pV diagram is given in Figure 19.21.

(b) $W = pV_2 - pV_1 = nR(T_2 - T_1) = (0.250\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})(100.0\text{ K}) = 208\text{ J}$.

(c) The work is done on the piston.

(d) Since Eq. (19.13) holds for any process, $\Delta U = nC_V\Delta T = (0.250\text{ mol})(28.46\text{ J/mol}\cdot\text{K})(100.0\text{ K}) = 712\text{ J}$.

(e) Either $Q = nC_p\Delta T$ or $Q = \Delta U + W$ gives $Q = 920\text{ J}$ to three significant figures.

(f) The lower pressure would mean a correspondingly larger volume, and the net result would be that the work done would be the same as that found in part (b).

EVALUATE: $W = nR\Delta T$, so W, Q and ΔU all depend only on ΔT. When T increases at constant pressure, V increases and $W > 0$. ΔU and Q are also positive when T increases.

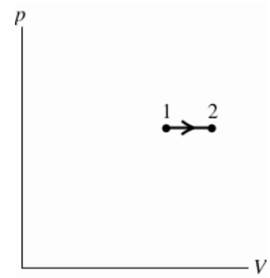


Figure 19.21

19.22. **IDENTIFY:** For constant volume $Q = nC_V\Delta T$. For constant pressure, $Q = nC_p\Delta T$. For any process of an ideal gas, $\Delta U = nC_V\Delta T$.

SET UP: $R = 8.315\text{ J/mol}\cdot\text{K}$. For helium, $C_V = 12.47\text{ J/mol}\cdot\text{K}$ and $C_p = 20.78\text{ J/mol}\cdot\text{K}$.

EXECUTE: **(a)** $Q = nC_V\Delta T = (0.0100\text{ mol})(12.47\text{ J/mol}\cdot\text{K})(40.0\text{ C}°) = 4.99\text{ J}$. The pV-diagram is sketched in Figure 19.22a.

(b) $Q = nC_p\Delta T = (0.0100\text{ mol})(20.78\text{ J/mol}\cdot\text{K})(40.0\text{ C}°) = 8.31\text{ J}$. The pV-diagram is sketched in Figure 19.22b.

(c) More heat is required for the constant pressure process. ΔU is the same in both cases. For constant volume $W = 0$ and for constant pressure $W > 0$. The additional heat energy required for constant pressure goes into expansion work.

(d) $\Delta U = nC_V\Delta T = 4.99\text{ J}$ for both processes. ΔU is path independent and for an ideal gas depends only on ΔT.

EVALUATE: $C_p = C_V + R$, so $C_p > C_V$.

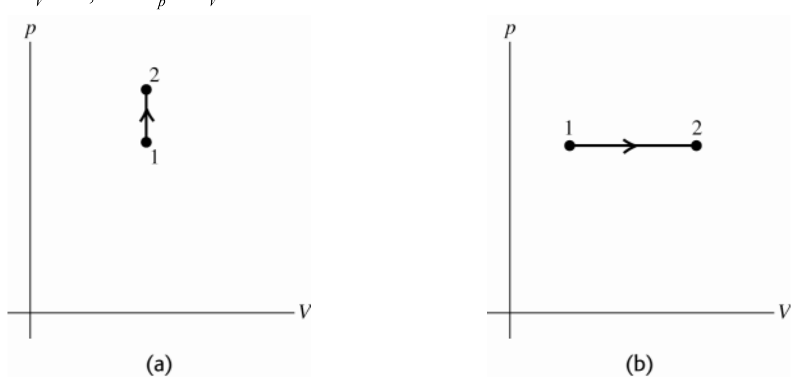


Figure 19.22

19.23. **IDENTIFY:** For constant volume, $Q = nC_V\Delta T$. For constant pressure, $Q = nC_p\Delta T$.

SET UP: From Table 19.1, $C_V = 20.76\text{ J/mol}\cdot\text{K}$ and $C_p = 29.07\text{ J/mol}\cdot\text{K}$

EXECUTE: **(a)** Using Equation (19.12), $\Delta T = \dfrac{Q}{nC_V} = \dfrac{645\text{ J}}{(0.185\text{ mol})(20.76\text{ J/mol}\cdot\text{K})} = 167.9\text{ K}$ and $T = 948\text{ K}$.

The pV-diagram is sketched in Figure 19.23a.

(b) Using Equation (19.14), $\Delta T = \dfrac{Q}{nC_p} = \dfrac{645 \text{ J}}{(0.185 \text{ mol})(29.07 \text{ J/mol}\cdot\text{K})} = 119.9 \text{ K}$ and $T = 900$ K.

The pV-diagram is sketched in Figure 19.23b.

EVALUATE: At constant pressure some of the heat energy added to the gas leaves the gas as expansion work and the internal energy change is less than if the same amount of heat energy is added at constant volume. ΔT is proportional to ΔU.

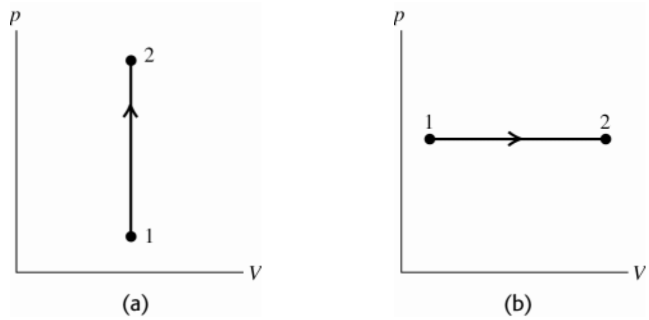


Figure 19.23

19.24. IDENTIFY and SET UP: Use information about the pressure and volume in the ideal gas law to determine the sign of ΔT, and from that the sign of Q.

EXECUTE: For constant p, $Q = nC_p \Delta T$

Since the gas is ideal, $pV = nRT$ and for constant p, $p\Delta V = nR\Delta T$.

$$Q = nC_p\left(\frac{p\Delta V}{nR}\right) = \left(\frac{C_p}{R}\right)p\Delta V$$

Since the gas expands, $\Delta V > 0$ and therefore $Q > 0$. $Q > 0$ means heat goes into gas.

EVALUATE: Heat flows into the gas, W is positive and the internal energy increases. It must be that $Q > W$.

19.25. IDENTIFY: $\Delta U = Q - W$. For an ideal gas, $\Delta U = C_V \Delta T$, and at constant pressure, $W = p\Delta V = nR\Delta T$.

SET UP: $C_V = \frac{3}{2}R$ for a monatomic gas.

EXECUTE: $\Delta U = n(\frac{3}{2}R)\Delta T = \frac{3}{2}p\Delta V = \frac{3}{2}W$. Then $Q = \Delta U + W = \frac{5}{2}W$, so $W/Q = \frac{2}{5}$.

EVALUATE: For diatomic or polyatomic gases, C_V is a different multiple of R and the fraction of Q that is used for expansion work is different.

19.26. IDENTIFY: For an ideal gas, $\Delta U = C_V \Delta T$, and at constant pressure, $p\Delta V = nR\Delta T$.

SET UP: $C_V = \frac{3}{2}R$ for a monatomic gas.

EXECUTE: $\Delta U = n\left(\frac{3}{2}R\right)\Delta T = \frac{3}{2}p\Delta V = \frac{3}{2}(4.00\times10^4 \text{ Pa})(8.00\times10^{-3} \text{ m}^3 - 2.00\times10^{-3} \text{ m}^3) = 360$ J.

EVALUATE: $W = nR\Delta T = \frac{2}{3}\Delta U = 240$ J. $Q = nC_p\Delta T = n(\frac{5}{2}R)\Delta T = \frac{5}{3}\Delta U = 600$ J. 600 J of heat energy flows into the gas. 240 J leaves as expansion work and 360 J remains in the gas as an increase in internal energy.

19.27. IDENTIFY: For a constant volume process, $Q = nC_V\Delta T$. For a constant pressure process, $Q = nC_p\Delta T$. For any process of an ideal gas, $\Delta U = nC_V\Delta T$.

SET UP: From Table 19.1, for N_2, $C_V = 20.76$ J/mol·K and $C_p = 29.07$ J/mol·K. Heat is added, so Q is positive and $Q = +1557$ J.

EXECUTE: **(a)** $\Delta T = \dfrac{Q}{nC_V} = \dfrac{1557 \text{ J}}{(3.00 \text{ mol})(20.76 \text{ J/mol}\cdot\text{K})} = +25.0 \text{ K}$

(b) $\Delta T = \dfrac{Q}{nC_p} = \dfrac{1557 \text{ J}}{(3.00 \text{ mol})(29.07 \text{ J/mol}\cdot\text{K})} = +17.9 \text{ K}$

(c) $\Delta U = nC_V\Delta T$ for either process, so ΔU is larger when ΔT is larger. The final internal energy is larger for the constant volume process in (a).

EVALUATE: For constant volume $W = 0$ and all the energy added as heat stays in the gas as internal energy. For the constant pressure process the gas expands and $W > 0$. Part of the energy added as heat leaves the gas as expansion work done by the gas.

19.28. **IDENTIFY:** Apply $pV = nRT$ to calculate T. For this constant pressure process, $W = p\Delta V$. $Q = nC_p\Delta T$. Use $\Delta U = Q - W$ to relate Q, W and ΔU.

SET UP: $2.50\text{ atm} = 2.53\times10^5\text{ Pa}$. For a monatomic ideal gas, $C_V = 12.47\text{ J/mol}\cdot\text{K}$ and $C_p = 20.78\text{ J/mol}\cdot\text{K}$.

EXECUTE: **(a)** $T_1 = \dfrac{pV_1}{nR} = \dfrac{(2.53\times10^5\text{ Pa})(3.20\times10^{-2}\text{ m}^2)}{(3.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})} = 325\text{ K}.$

$$T_2 = \frac{pV_2}{nR} = \frac{(2.53\times10^5\text{ Pa})(4.50\times10^{-2}\text{ m}^2)}{(3.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})} = 456\text{ K}.$$

(b) $W = p\Delta V = (2.53\times10^5\text{ Pa})(4.50\times10^{-2}\text{ m}^3 - 3.20\times10^{-2}\text{ m}^3) = 3.29\times10^3\text{ J}$

(c) $Q = nC_p\Delta T = (3.00\text{ mol})(20.78\text{ J/mol}\cdot\text{K})(456\text{ K} - 325\text{ K}) = 8.17\times10^3\text{ J}$

(d) $\Delta U = Q - W = 4.88\times10^3\text{ J}$

EVALUATE: We could also calculate ΔU as $\Delta U = nC_V\Delta T = (3.00\text{ mol})(12.47\text{ J/mol}\cdot\text{K})(456\text{ K} - 325\text{ K}) = 4.90\times10^3\text{ J}$, which agrees with the value we calculated in part (d).

19.29. **IDENTIFY:** Calculate W and ΔU and then use the first law to calculate Q.

(a) SET UP: $W = \int_{V_1}^{V_2} p\,dV$

$pV = nRT$ so $p = nRT/V$

$W = \int_{V_1}^{V_2} (nRT/V)\,dV = nRT\int_{V_1}^{V_2} dV/V = nRT\ln(V_2/V_1)$ (work done during an isothermal process).

EXECUTE: $W = (0.150\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})(350\text{ K})\ln(0.25V_1/V_1) = (436.5\text{ J})\ln(0.25) = -605\text{ J}.$

EVALUATE: W for the gas is negative, since the volume decreases.

(b) EXECUTE: $\Delta U = nC_V\Delta T$ for any ideal gas process.

$\Delta T = 0$ (isothermal) so $\Delta U = 0$.

EVALUATE: $\Delta U = 0$ for any ideal gas process in which T doesn't change.

(c) EXECUTE: $\Delta U = Q - W$

$\Delta U = 0$ so $Q = W = -605\text{ J}$. (Q is negative; the gas liberates 605 J of heat to the surroundings.)

EVALUATE: $Q = nC_V\Delta T$ is only for a constant volume process so doesn't apply here.

$Q = nC_p\Delta T$ is only for a constant pressure process so doesn't apply here.

19.30. **IDENTIFY:** $C_p = C_V + R$ and $\gamma = \dfrac{C_p}{C_V}$.

SET UP: $R = 8.315\text{ J/mol}\cdot\text{K}$

EXECUTE: $C_p = C_V + R$. $\gamma = \dfrac{C_p}{C_V} = 1 + \dfrac{R}{C_V}$. $C_V = \dfrac{R}{\gamma - 1} = \dfrac{8.315\text{ J/mol}\cdot\text{K}}{0.127} = 65.5\text{ J/mol}\cdot\text{K}$. Then $C_p = C_V + R = 73.8\text{ J/mol}\cdot\text{K}$.

EVALUATE: The value of C_V is about twice the values for the polyatomic gases in Table 19.1. A propane molecule has more atoms and hence more internal degrees of freedom than the polyatomic gases in the table.

19.31. **IDENTIFY:** $\Delta U = Q - W$. Apply $Q = nC_p\Delta T$ to calculate C_p. Apply $\Delta U = nC_V\Delta T$ to calculate C_V. $\gamma = C_p/C_V$.

SET UP: $\Delta T = 15.0\text{ C}^\circ = 15.0\text{ K}$. Since heat is added, $Q = +970\text{ J}$.

EXECUTE: **(a)** $\Delta U = Q - W = +970\text{ J} - 223\text{ J} = 747\text{ J}$

(b) $C_p = \dfrac{Q}{n\Delta T} = \dfrac{970\text{ J}}{(1.75\text{ mol})(15.0\text{ K})} = 37.0\text{ J/mol}\cdot\text{K}$. $C_V = \dfrac{\Delta U}{n\Delta T} = \dfrac{747\text{ J}}{(1.75\text{ mol})(15.0\text{ K})} = 28.5\text{ J/mol}\cdot\text{K}$.

$$\gamma = \frac{C_p}{C_V} = \frac{37.0\text{ J/mol}\cdot\text{K}}{28.5\text{ J/mol}\cdot\text{K}} = 1.30$$

EVALUATE: The value of γ we calculated is similar to the values given in Tables 19.1 for polyatomic gases.

19.32. **IDENTIFY** and **SET UP:** For an ideal gas $\Delta U = nC_V\Delta T$. The sign of ΔU is the same as the sign of ΔT. Combine Eq.(19.22) and the ideal gas law to obtain an equation relating T and p, and use it to determine the sign of ΔT.

EXECUTE: $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ and $V = nRT/p$ so, $T_1^\gamma p_1^{1-\gamma} = T_2^\gamma p_2^{1-\gamma}$ and $T_2^\gamma = T_1^\gamma (p_2/p_1)^{\gamma-1}$

$p_2 < p_1$ and $\gamma - 1$ is positive so $T_2 < T_1$. ΔT is negative so ΔU is negative; the energy of the gas decreases.

EVALUATE: Eq.(19.24) shows that the volume increases for this process, so it is an adiabatic expansion. In an adiabatic expansion the temperature decreases.

19.33. IDENTIFY: For an adiabatic process of an ideal gas, $p_1V_1^\gamma = p_2V_2^\gamma$, $W = \dfrac{1}{\gamma - 1}\left(p_1V_1 - p_2V_2\right)$ and $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$.

SET UP: For a monatomic ideal gas $\gamma = 5/3$.

EXECUTE: **(a)** $p_2 = p_1\left(\dfrac{V_1}{V_2}\right)^\gamma = \left(1.50\times10^5\ \text{Pa}\right)\left(\dfrac{0.0800\ \text{m}^3}{0.0400\ \text{m}^3}\right)^{5/3} = 4.76\times10^5\ \text{Pa}.$

(b) This result may be substituted into Eq.(19.26), or, substituting the above form for p_2,

$$W = \frac{1}{\gamma-1}p_1V_1\left(1-\left(V_1/V_2\right)^{\gamma-1}\right) = \frac{3}{2}\left(1.50\times10^5\ \text{Pa}\right)\left(0.0800\ \text{m}^3\right)\left(1-\left(\frac{0.0800}{0.0400}\right)^{2/3}\right) = -1.06\times10^4\ \text{J}.$$

(c) From Eq.(19.22), $\left(T_2/T_1\right) = \left(V_2/V_1\right)^{\gamma-1} = \left(0.0800/0.0400\right)^{2/3} = 1.59$, and since the final temperature is higher than the initial temperature, the gas is heated.

EVALUATE: In an adiabatic compression $W < 0$ since $\Delta V < 0$. $Q = 0$ so $\Delta U = -W$. $\Delta U > 0$ and the temperature increases.

19.34. IDENTIFY and SET UP: **(a)** In the process the pressure increases and the volume decreases. The pV-diagram is sketched in Figure 19.34.

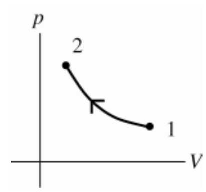


Figure 19.34

(b) For an adiabatic process for an ideal gas

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$, $p_1V_1^\gamma = p_2V_2^\gamma$, and $pV = nRT$

EXECUTE: From the first equation, $T_2 = T_1(V_1/V_2)^{\gamma-1} = (293\ \text{K})(V_1/0.0900V_1)^{1.4-1}$

$T_2 = (293\ \text{K})(11.11)^{0.4} = 768\ \text{K} = 495°\text{C}$

(Note: In the equation $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ the temperature *must* be in kelvins.)

$p_1V_1^\gamma = p_2V_2^\gamma$ implies $p_2 = p_1(V_1/V_2)^\gamma = (1.00\ \text{atm})(V_1/0.0900V_1)^{1.4}$

$p_2 = (1.00\ \text{atm})(11.11)^{1.4} = 29.1\ \text{atm}$

EVALUATE: Alternatively, we can use $pV = nRT$ to calculate p_2: n, R constant implies $pV/T = nR = \text{constant}$ so $p_1V_1/T_1 = p_2V_2/T_2$

$p_2 = p_1(V_1/V_2)(T_2/T_1) = (1.00\ \text{atm})(V_1/0.0900V_1)(768\ \text{K}/293\ \text{K}) = 29.1\ \text{atm}$, which checks.

19.35. IDENTIFY: For an adiabatic process of an ideal gas, $W = \dfrac{1}{\gamma-1}\left(p_1V_1 - p_2V_2\right)$ and $p_1V_1^\gamma = p_2V_2^\gamma$.

SET UP: $\gamma = 1.40$ for an ideal diatomic gas. $1\ \text{atm} = 1.013\times10^5\ \text{Pa}$ and $1\ \text{L} = 10^{-3}\ \text{m}^3$.

EXECUTE: $Q = \Delta U + W = 0$ for an adiabatic process, so $\Delta U = -W = \dfrac{1}{\gamma-1}\left(p_2V_2 - p_1V_1\right)$. $p_1 = 1.22\times10^5\ \text{Pa}$.

$p_2 = p_1\left(V_1/V_2\right)^\gamma = (1.22\times10^5\ \text{Pa})(3)^{1.4} = 5.68\times10^5\ \text{Pa}.$

$W = \dfrac{1}{0.40}\left([5.68\times10^5\ \text{Pa}][10\times10^{-3}\ \text{m}^{-3}] - [1.22\times10^5\ \text{Pa}][30\times10^{-3}\ \text{m}^{-3}]\right) = 5.05\times10^3\ \text{J}$. The internal energy increases because work is done *on* the gas $(\Delta U > 0)$ and $Q = 0$. The temperature increases because the internal energy has increased.

EVALUATE: In an adiabatic compression $W < 0$ since $\Delta V < 0$. $Q = 0$ so $\Delta U = -W$. $\Delta U > 0$ and the temperature increases.

19.36. IDENTIFY: Assume the expansion is adiabatic. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ relates V and T. Assume the air behaves as an ideal gas, so $\Delta U = nC_V\Delta T$. Use $pV = nRT$ to calculate n.

SET UP: For air, $C_V = 29.76\ \text{J/mol}\cdot\text{K}$ and $\gamma = 1.40$. $V_2 = 0.800V_1$. $T_1 = 293.15\ \text{K}$. $p_1 = 2.026\times10^5\ \text{Pa}$. For a sphere, $V = \frac{4}{3}\pi r^3$.

EXECUTE: **(a)** $T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = (293.15\text{ K})\left(\frac{V_1}{0.800V_1}\right)^{0.40} = 320.5\text{ K} = 47.4°\text{C}.$

(b) $V_1 = \frac{4}{3}\pi r^3 = \frac{4\pi}{3}(0.1195\text{ m})^3 = 7.15\times10^{-3}\text{ m}^3.$ $n = \frac{p_1V_1}{RT_1} = \frac{(2.026\times10^5\text{ Pa})(7.15\times10^{-3}\text{ m}^3)}{(8.314\text{ J/mol}\cdot\text{K})(293.15\text{ K})} = 0.594\text{ mol.}$

$\Delta U = nC_V\Delta T = (0.594\text{ mol})(20.76\text{ J/mol}\cdot\text{K})(321\text{ K} - 293\text{ K}) = 345\text{ J}.$

EVALUATE: We could also use $\Delta U = W = \frac{1}{\gamma-1}(p_1V_1 - p_2V_2)$ to calculate ΔU, if we first found p_2 from $pV = nRT$.

19.37. **(a) IDENTIFY** and **SET UP:** In the expansion the pressure decreases and the volume increases. The pV-diagram is sketched in Figure 19.37.

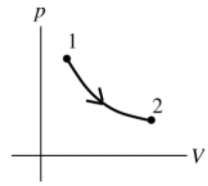


Figure 19.37

(b) Adiabatic means $Q = 0$.

Then $\Delta U = Q - W$ gives $W = -\Delta U = -nC_V\ \Delta T = nC_V(T_1 - T_2)$ (Eq.19.25).

$C_V = 12.47\text{ J/mol}\cdot\text{K}$ (Table 19.1)

EXECUTE: $W = (0.450\text{ mol})(12.47\text{ J/mol}\cdot\text{K})(50.0°\text{C} - 10.0°\text{C}) = +224\text{ J}$

W positive for $\Delta V > 0$ (expansion)

(c) $\Delta U = -W = -224\text{ J}.$

EVALUATE: There is no heat energy input. The energy for doing the expansion work comes from the internal energy of the gas, which therefore decreases. For an ideal gas, when T decreases, U decreases.

19.38. **IDENTIFY:** $pV = nRT$. For an adiabatic process, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$.

SET UP: For an ideal monatomic gas, $\gamma = 5/3$.

EXECUTE: **(a)** $T = \frac{pV}{nR} = \frac{(1.00\times10^5\text{ Pa})\,(2.50\times10^{-3}\text{ m}^3)}{(0.1\text{ mol})\,(8.3145\text{ J/mol}\cdot\text{K})} = 301\text{ K}.$

(b) (i) Isothermal: If the expansion is *isothermal,* the process occurs at constant temperature and the final temperature is the same as the initial temperature, namely 301 K. $p_2 = p_1(V_1/V_2) = \frac{1}{2}p_1 = 5.00\times10^4\text{ Pa}.$

(ii) Isobaric: $\Delta p = 0$ so $p_2 = 1.00\times10^5\text{ Pa}.$ $T_2 = T_1(V_2/V_1) = 2T_1 = 602\text{ K}.$

(iii) Adiabatic: Using Equation (19.22), $T_2 = \frac{T_1V_1^{\gamma-1}}{V_2^{\gamma-1}} = \frac{(301\text{ K})(V_1)^{0.67}}{(2V_1)^{0.67}} = (301\text{ K})\left(\frac{1}{2}\right)^{0.67} = 189\text{ K}.$

EVALUATE: In an isobaric expansion, T increases. In an adiabatic expansion, T decreases.

19.39. **IDENTIFY:** Combine $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ with $pV = nRT$ to obtain an expression relating T and p for an adiabatic process of an ideal gas.

SET UP: $T_1 = 299.15\text{ K}$

EXECUTE: $V = \frac{nRT}{p}$ so $T_1\left(\frac{nRT_1}{p_1}\right)^{\gamma-1} = T_2\left(\frac{nRT_2}{p_2}\right)^{\gamma-1}$ and $\frac{T_1^\gamma}{p_1^{\gamma-1}} = \frac{T_2^\gamma}{p_2^{\gamma-1}}.$

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma} = (299.15\text{ K})\left(\frac{0.850\times10^5\text{ Pa}}{1.01\times10^5\text{ Pa}}\right)^{0.4/1.4} = 284.8\text{ K} = 11.6\ °\text{C}$$

EVALUATE: For an adiabatic process of an ideal gas, when the pressure decreases the temperature decreases.

19.40. **IDENTIFY:** Apply $\Delta U = Q - W$. For any process of an ideal gas, $\Delta U = nC_V\Delta T$. For an isothermal expansion, $W = nRT\ln\left(\frac{V_2}{V_1}\right) = nRT\ln\left(\frac{p_1}{p_2}\right).$

SET UP: $T = 288.15\text{ K}.$ $\frac{p_1}{p_2} = \frac{V_2}{V_1} = 2.00.$

EXECUTE: **(a)** $\Delta U = 0$ since $\Delta T = 0$.

(b) $W = (1.50\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(288.15\text{ K})\ln(2.00) = 2.49\times10^3\text{ J}.$ $W > 0$ and work is done by the gas. Since $\Delta U = 0,$ $Q = W = +2.49\times10^3\text{ J}.$ $Q > 0$ so heat flows into the gas.

EVALUATE: When the volume increases, W is positive.

19.41. **IDENTIFY** and **SET UP:** For an ideal gas, $pV = nRT$. The work done is the area under the path in the pV-diagram.

EXECUTE: **(a)** The product pV increases and this indicates a temperature increase.

(b) The work is the area in the pV plane bounded by the blue line representing the process and the verticals at V_a and V_b. The area of this trapezoid is $\frac{1}{2}(p_b + p_a)(V_b - V_a) = \frac{1}{2}(2.40\times10^5\ \text{Pa})(0.0400\ \text{m}^3) = 4800\ \text{J}$.

EVALUATE: The work done is the average pressure, $\frac{1}{2}(p_1 + p_2)$, times the volume increase.

19.42. **IDENTIFY:** Use $pV = nRT$ to calculate T. W is the area under the process in the pV-diagram. Use $\Delta U = nC_V\Delta T$ and $\Delta U = Q - W$ to calculate Q.

SET UP: In state c, $p_c = 2.0\times10^5$ Pa and $V_c = 0.0040\ \text{m}^3$. In state a, $p_a = 4.0\times10^5$ Pa and $V_a = 0.0020\ \text{m}^3$.

EXECUTE: **(a)** $T_c = \dfrac{p_cV_c}{nR} = \dfrac{(2.0\times10^5\ \text{Pa})(0.0040\ \text{m}^3)}{(0.500\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})} = 192\ \text{K}$

(b) $W = \frac{1}{2}(4.0\times10^5\ \text{Pa} + 2.0\times10^5\ \text{Pa})(0.0030\ \text{m}^3 - 0.0020\ \text{m}^3) + (2.0\times10^5\ \text{Pa})(0.0040\ \text{m}^3 - 0.0030\ \text{m}^3)$

$W = +500$ J. 500 J of work is done by the gas.

(c) $T_a = \dfrac{p_aV_a}{nR} = \dfrac{(4.0\times10^5\ \text{Pa})(0.0020\ \text{m}^3)}{(0.500\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})} = 192\ \text{K}$. For the process, $\Delta T = 0$, so $\Delta U = 0$ and $Q = W = +500$ J.

500 J of heat enters the system.

EVALUATE: The work done by the gas is positive since the volume increases.

19.43. **IDENTIFY:** Use $\Delta U = Q - W$ and the fact that ΔU is path independent.

$W > 0$ when the volume increases, $W < 0$ when the volume decreases, and $W = 0$ when the volume is constant. $Q > 0$ if heat flows into the system.

SET UP: The paths are sketched in Figure 19.43.

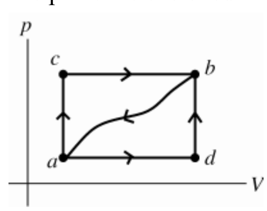


Figure 19.43

$Q_{acb} = +90.0$ J (positive since heat flows in)

$W_{acb} = +60.0$ J (positive since $\Delta V > 0$)

EXECUTE: **(a)** $\Delta U = Q - W$

ΔU is path independent; Q and W depend on the path.

$\Delta U = U_b - U_a$

This can be calculated for any path from a to b, in particular for path acb: $\Delta U_{a\to b} = Q_{acb} - W_{acb} = 90.0\ \text{J} - 60.0\ \text{J} = 30.0\ \text{J}$.

Now apply $\Delta U = Q - W$ to path adb; $\Delta U = 30.0$ J for this path also.

$W_{adb} = +15.0$ J (positive since $\Delta V > 0$)

$\Delta U_{a\to b} = Q_{adb} - W_{adb}$ so $Q_{acb} = \Delta U_{a\to b} + W_{adb} = 30.0\ \text{J} + 15.0\ \text{J} = +45.0\ \text{J}$

(b) Apply $\Delta U = Q - W$ to path ba: $\Delta U_{b\to a} = Q_{ba} - W_{ba}$

$W_{ba} = -35.0$ J (negative since $\Delta V < 0$)

$\Delta U_{b\to a} = U_a - U_b = -(U_b - U_a) = -\Delta U_{a\to b} = -30.0$ J

Then $Q_{ba} = \Delta U_{b\to a} + W_{ba} = -30.0\ \text{J} - 35.0\ \text{J} = -65.0\ \text{J}$.

($Q_{ba} < 0$; the system liberates heat.)

(c) $U_a = 0$, $U_d = 8.0$ J

$\Delta U_{a\to b} = U_b - U_a = +30.0$ J, so $U_b = +30.0$ J.

process $a \to d$

$\Delta U_{a\to d} = Q_{ad} - W_{ad}$

$\Delta U_{a\to d} = U_d - U_a = +8.0$ J

$W_{adb} = +15.0$ J and $W_{adb} = W_{ad} + W_{db}$. But the work W_{db} for the process $d \to b$ is zero since $\Delta V = 0$ for that process.

Therefore $W_{ad} = W_{adb} = +15.0$ J.

Then $Q_{ad} = \Delta U_{a\to d} + W_{ad} = +8.0\ \text{J} + 15.0\ \text{J} = +23.0\ \text{J}$ (positive implies heat absorbed).

process $d \to b$

$\Delta U_{d\to b} = Q_{db} - W_{db}$

$W_{db} = 0,$ as already noted.

$\Delta U_{d\to b} = U_b - U_d = 30.0\text{ J} - 8.0\text{ J} = +22.0\text{ J}.$

Then $Q_{db} = \Delta U_{d\to b} + W_{db} = +22.0\text{ J}$ (positive; heat absorbed).

EVALUATE: The signs of our calculated Q_{ad} and Q_{db} agree with the problem statement that heat is absorbed in these processes.

19.44. **IDENTIFY:** $\Delta U = Q - W.$

SET UP: $W = 0$ when $\Delta V = 0.$

EXECUTE: For each process, $Q = \Delta U + W.$ No work is done in the processes *ab* and *dc*, and so $W_{bc} = W_{abc} = 450\text{ J}$ and $W_{ad} = W_{adc} = 120\text{ J}.$ The heat flow for each process is: for ab, $Q = 90\text{ J}.$ For bc, $Q = 440\text{ J} + 450\text{ J} = 890\text{ J}.$ For ad, $Q = 180\text{ J} + 120\text{ J} = 300\text{ J}.$ For dc, $Q = 350\text{ J}.$ Heat is absorbed in each process. Note that the arrows representing the processes all point in the direction of increasing temperature (increasing U).

EVALUATE: ΔU is path independent so is the same for paths *adc* and *abc*. $Q_{adc} = 300\text{ J} + 350\text{ J} = 650\text{ J}.$ $Q_{abc} = 90\text{ J} + 890\text{ J} = 980\text{ J}.$ Q and W are path dependent and are different for these two paths.

19.45. **IDENTIFY:** Use $pV = nRT$ to calculate T_c/T_a. Calculate ΔU and W and use $\Delta U = Q - W$ to obtain Q.

SET UP: For path *ac*, the work done is the area under the line representing the process in the pV-diagram.

EXECUTE: **(a)** $\dfrac{T_c}{T_a} = \dfrac{p_c V_c}{p_a V_a} = \dfrac{(1.0\times 10^5\text{ J})(0.060\text{ m}^3)}{(3.0\times 10^5\text{ J})(0.020\text{ m}^3)} = 1.00.$ $T_c = T_a.$

(b) Since $T_c = T_a$, $\Delta U = 0$ for process *abc*. For *ab*, $\Delta V = 0$ and $W_{ab} = 0.$ For *bc*, p is constant and $W_{bc} = p\Delta V = (1.0\times 10^5\text{ Pa})(0.040\text{ m}^3) = 4.0\times 10^3\text{ J}.$ Therefore, $W_{abc} = +4.0\times 10^3\text{ J}.$ Since $\Delta U = 0,$ $Q = W = +4.0\times 10^3\text{ J}.$ $4.0\times 10^3\text{ J}$ of heat flows into the gas during process *abc*.

(c) $W = \tfrac{1}{2}(3.0\times 10^5\text{ Pa} + 1.0\times 10^5\text{ Pa})(0.040\text{ m}^3) = +8.0\times 10^3\text{ J}.$ $Q_{ac} = W_{ac} = +8.0\times 10^3\text{ J}.$

EVALUATE: The work done is path dependent and is greater for process *ac* than for process *abc*, even though the initial and final states are the same.

19.46. **IDENTIFY:** For a cycle, $\Delta U = 0$ and $Q = W$. Calculate W.

SET UP: The magnitude of the work done by the gas during the cycle equals the area enclosed by the cycle in the pV-diagram.

EXECUTE: **(a)** The cycle is sketched in Figure 19.46.

(b) $|W| = (3.50\times 10^4\text{ Pa} - 1.50\times 10^4\text{ Pa})(0.0435\text{ m}^3 - 0.0280\text{ m}^3) = +310\text{ J}.$ More negative work is done for *cd* than positive work for *ab* and the net work is negative. $W = -310\text{ J}.$

(c) $Q = W = -310\text{ J}.$ Since $Q < 0,$ the net heat flow is out of the gas.

EVALUATE: During each constant pressure process $W = p\Delta V$ and during the constant volume process $W = 0.$

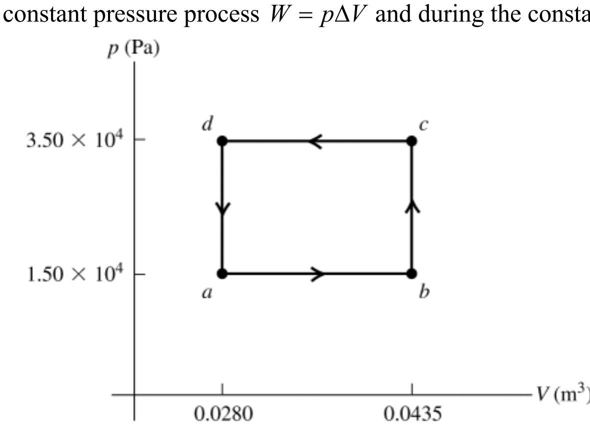


Figure 19.46

19.47. **IDENTIFY:** Use the 1st law to relate Q_{tot} to W_{tot} for the cycle.

Calculate W_{ab} and W_{bc} and use what we know about W_{tot} to deduce W_{ca}

(a) SET UP: We aren't told whether the pressure increases or decreases in process *bc*. The two possibilities for the cycle are sketched in Figure 19.47.

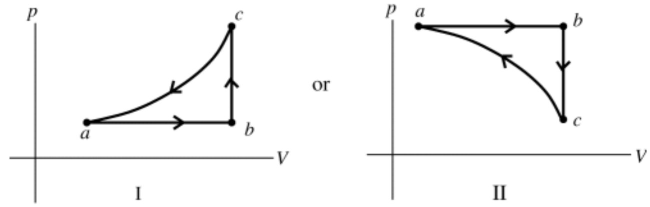


Figure 19.47

In cycle I, the total work is negative and in cycle II the total work is positive. For a cycle, $\Delta U = 0$, so $Q_{\text{tot}} = W_{\text{tot}}$ The net heat flow for the cycle is out of the gas, so heat $Q_{\text{tot}} < 0$ and $W_{\text{tot}} < 0$. Sketch I is correct.

(b) EXECUTE: $W_{\text{tot}} = Q_{\text{tot}} = -800\text{ J}$

$W_{\text{tot}} = W_{ab} + W_{bc} + W_{ca}$

$W_{bc} = 0$ since $\Delta V = 0$.

$W_{ab} = p\Delta V$ since *p* is constant. But since it is an ideal gas, $p\Delta V = nR\Delta T$

$W_{ab} = nR(T_b - T_a) = 1660\text{ J}$

$W_{ca} = W_{\text{tot}} - W_{ab} = -800\text{ J} - 1660\text{ J} = -2460\text{ J}$

EVALUATE: In process *ca* the volume decreases and the work *W* is negative.

19.48. **IDENTIFY:** Apply the appropriate expression for *W* for each type of process. $pV = nRT$ and $C_p = C_V + R$.

SET UP: $R = 8.315\text{ J/mol}\cdot\text{K}$

EXECUTE: Path ac has constant pressure, so $W_{ac} = p\Delta V = nR\Delta T$, and

$W_{ac} = nR(T_c - T_a) = (3\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})(492\text{ K} - 300\text{ K}) = 4.789\times10^3\text{ J}.$

Path cb is adiabatic $(Q = 0)$, so $W_{cb} = Q - \Delta U = -\Delta U = -nC_V\Delta T$, and using $C_V = C_p - R$,

$W_{cb} = -n(C_p - R)(T_b - T_c) = -(3\text{ mol})(29.1\text{ J/mol}\cdot\text{K} - 8.3145\text{ J/mol}\cdot\text{K})(600\text{ K} - 492\text{ K}) = -6.735\times10^3\text{ J}.$

Path ba has constant volume, so $W_{ba} = 0$. So the total work done is

$$W = W_{ac} + W_{cb} + W_{ba} = 4.789\times10^3\text{ J} - 6.735\times10^3\text{ J} + 0 = -1.95\times10^3\text{ J}.$$

EVALUATE: $W > 0$ when $\Delta V > 0$, $W < 0$ when $\Delta V < 0$ and $W = 0$ when $\Delta V = 0$.

19.49. **IDENTIFY:** Use $Q = nC_V\Delta T$ to calculate the temperature change in the constant volume process and use $pV = nRT$ to calculate the temperature change in the constant pressure process. The work done in the constant volume process is zero and the work done in the constant pressure process is $W = p\Delta V$. Use $Q = nC_p\Delta T$ to calculate the heat flow in the constant pressure process. $\Delta U = nC_V\Delta T$, or $\Delta U = Q - W$.

SET UP: For N_2, $C_V = 20.76\text{ J/mol}\cdot\text{K}$ and $C_p = 29.07\text{ J/mol}\cdot\text{K}$.

EXECUTE: **(a)** For process *ab*, $\Delta T = \dfrac{Q}{nC_V} = \dfrac{1.52\times10^4\text{ J}}{(2.50\text{ mol})(20.76\text{ J/mol}\cdot\text{K})} = 293\text{ K}.$ $T_a = 293\text{ K}$, so $T_b = 586\text{ K}$.

$pV = nRT$ says *T* doubles when *V* doubles and *p* is constant, so $T_c = 2(586\text{ K}) = 1172\text{ K} = 899°\text{C}.$

(b) For process *ab*, $W_{ab} = 0$. For process *bc*,

$W_{bc} = p\Delta V = nR\Delta T = (2.50\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(1172\text{ K} - 586\text{ K}) = 1.22\times10^4\text{ J}.$ $W = W_{ab} + W_{bc} = 1.22\times10^4\text{ J}.$

(c) For process *bc*, $Q = nC_p\Delta T = (2.50\text{ mol})(29.07\text{ J/mol}\cdot\text{K})(1172\text{ K} - 586\text{ K}) = 4.26\times10^4\text{ J}.$

(d) $\Delta U = nC_V\Delta T = (2.50\text{ mol})(20.76\text{ J/mol}\cdot\text{K})(1172\text{ K} - 293\text{ K}) = 4.56\times10^4\text{ J}.$

EVALUATE: The total *Q* is $1.52\times10^4\text{ J} + 4.26\times10^4\text{ J} = 5.78\times10^4\text{ J}.$

$\Delta U = Q - W = 5.78\times10^4\text{ J} - 1.22\times10^4\text{ J} = 4.56\times10^4\text{ J}$, which agrees with our results in part (d).

19.50. **IDENTIFY:** For a constant pressure process, $Q = nC_p\Delta T$. $\Delta U = Q - W$. $\Delta U = nC_V\Delta T$ for any ideal gas process.

SET UP: For N_2, $C_V = 20.76\text{ J/mol}\cdot\text{K}$ and $C_p = 29.07\text{ J/mol}\cdot\text{K}$. $Q < 0$ if heat comes out of the gas.

EXECUTE: **(a)** $n = \dfrac{Q}{C_p\Delta T} = \dfrac{+2.5\times10^4\text{ J}}{(29.07\text{ J/mol}\cdot\text{K})(40.0\text{ K})} = 21.5\text{ mol}.$

(b) $\Delta U = nC_V\Delta T = Q(C_V/C_p) = (-2.5\times10^4\text{ J})(20.76/29.07) = -1.79\times10^4\text{ J}.$

(c) $W = Q - \Delta U = -7.15\times10^3$ J.

(d) ΔU is the same for both processes, and if $\Delta V = 0$, $W = 0$ and $Q = \Delta U = -1.79\times10^4$ J.

EVALUATE: For a given ΔT, Q is larger when the pressure is constant than when the volume is constant.

19.51. **IDENTIFY** and **SET UP:** Use the first law to calculate W and then use $W = p\Delta V$ for the constant pressure process to calculate ΔV.

EXECUTE: $\Delta U = Q - W$

$Q = -2.15\times10^5$ J (negative since heat energy goes out of the system)

$\Delta U = 0$ so $W = Q = -2.15\times10^5$ J

Constant pressure, so $W = \int_{V_1}^{V_2} p\,dV = p(V_2 - V_1) = p\Delta V$.

Then $\Delta V = \dfrac{W}{p} = \dfrac{-2.15\times10^5 \text{ J}}{9.50\times10^5 \text{ Pa}} = -0.226 \text{ m}^3$.

EVALUATE: Positive work is done on the system by its surroundings; this inputs to the system the energy that then leaves the system as heat. Both Eq.(19.4) and (19.2) apply to all processes for any system, not just to an ideal gas.

19.52. **IDENTIFY:** $pV = nRT$. For an isothermal process $W = nRT\ln(V_2/V_1)$. For a constant pressure process, $W = p\Delta V$.

SET UP: $1 \text{ L} = 10^{-3} \text{ m}^3$.

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.52.

(b) At constant temperature, the product pV is constant, so $V_2 = V_1(p_1/p_2) = (1.5 \text{ L})\left(\dfrac{1.00\times10^5 \text{ Pa}}{2.50\times10^4 \text{ Pa}}\right) = 6.00$ L. The final pressure is given as being the same as $p_3 = p_2 = 2.5\times10^4$ Pa. The final volume is the same as the initial volume, so $T_3 = T_1(p_3/p_1) = 75.0$ K.

(c) Treating the gas as ideal, the work done in the first process is $W = nRT\ln(V_2/V_1) = p_1V_1\ln(p_1/p_2)$.

$W = (1.00\times10^5 \text{ Pa})(1.5\times10^{-3} \text{ m}^3)\ln\left(\dfrac{1.00\times10^5 \text{ Pa}}{2.50\times10^4 \text{ Pa}}\right) = 208$ J.

For the second process, $W = p_2(V_3 - V_2) = p_2(V_1 - V_2) = p_2V_1(1 - (p_1/p_2))$.

$W = (2.50\times10^4 \text{ Pa})(1.5\times10^{-3} \text{ m}^3)\left(1 - \dfrac{1.00\times10^5 \text{ Pa}}{2.50\times10^4 \text{ Pa}}\right) = -113$ J.

The total work done is $208 \text{ J} - 113 \text{ J} = 95$ J.

(d) Heat at constant volume. No work would be done by the gas or on the gas during this process.

EVALUATE: When the volume increases, $W > 0$. When the volume decreases, $W < 0$.

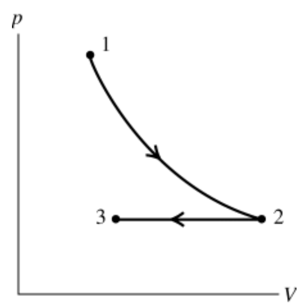


Figure 19.52

19.53. **IDENTIFY:** $\Delta V = V_0\beta\Delta T$. $W = p\Delta V$ since the force applied to the piston is constant. $Q = mc_p\Delta T$. $\Delta U = Q - W$.

SET UP: $m = \rho V$

EXECUTE: **(a)** The fractional change in volume is

$\Delta V = V_0\beta\Delta T = (1.20\times10^{-2} \text{ m}^3)(1.20\times10^{-3} \text{ K}^{-1})(30.0 \text{ K}) = 4.32\times10^{-4} \text{ m}^3$.

(b) $W = p\Delta V = (F/A)\Delta V = ((3.00\times10^4 \text{ N})/(0.0200 \text{ m}^2))(4.32\times10^{-4} \text{ m}^3) = 648$ J.

(c) $Q = mc_p\Delta T = V_0\rho c_p\Delta T = (1.20\times10^{-2} \text{ m}^3)(791 \text{ kg/m}^3)(2.51\times10^3 \text{ J/kg}\cdot\text{K})(30.0 \text{ K})$.

$Q = 7.15\times10^5$ J.

(d) $\Delta U = Q - W = 7.15\times10^5$ J to three figures.

(e) Under these conditions W is much less than Q and there is no substantial difference between c_V and c_p.

EVALUATE: $\Delta U = Q - W$ is valid for any material. For liquids the expansion work is much less than Q.

19.54. **IDENTIFY:** $\Delta V = \beta V_0 \Delta T$. $W = p\Delta V$ since the applied pressure (air pressure) is constant. $Q = mc_p\Delta T$. $\Delta U = Q - W$.

SET UP: For copper, $\beta = 5.1\times10^{-3}\ (\text{C}°)^{-1}$, $c_p = 390\ \text{J/kg}\cdot\text{K}$ and $\rho = 8.90\times10^3\ \text{kg/m}^3$.

EXECUTE: **(a)** $\Delta V = \beta\Delta T V_0 = (5.1\times10^{-5}(\text{C}°)^{-1})(70.0\ \text{C}°)(2.00\times10^{-2}\ \text{m})^3 = 2.86\times10^{-8}\ \text{m}^3$.

(b) $W = p\Delta V = 2.88\times10^{-3}$ J.

(c) $Q = mc_p\Delta T = \rho V_0 c_p \Delta T = (8.9\times10^3\ \text{kg/m}^3)(8.00\times10^{-6}\ \text{m}^3)(390\ \text{J/kg}\cdot\text{K})(70.0\ \text{C}°) = 1944$ J.

(d) To three figures, $\Delta U = Q = 1940$ J.

(e) Under these conditions, the difference is not substantial, since W is much less than Q.

EVALUATE: $\Delta U = Q - W$ applies to any material. For solids the expansion work is much less than Q.

19.55. **IDENTIFY** and **SET UP:** The heat produced from the reaction is $Q_{\text{reaction}} = mL_{\text{reaction}}$, where L_{reaction} is the heat of reaction of the chemicals.

$Q_{\text{reaction}} = W + \Delta U_{\text{spray}}$

EXECUTE: For a mass m of spray, $W = \frac{1}{2}mv^2 = \frac{1}{2}m(19\ \text{m/s})^2 = (180.5\ \text{J/kg})m$ and

$\Delta U_{\text{spray}} = Q_{\text{spray}} = mc\Delta T = m(4190\ \text{J/kg}\cdot\text{K})(100°\text{C} - 20°\text{C}) = (335{,}200\ \text{J/kg})m.$

Then $Q_{\text{reaction}} = (180\ \text{J/kg} + 335{,}200\ \text{J/kg})m = (335{,}380\ \text{J/kg})m$ and $Q_{\text{reaction}} = mL_{\text{reaction}}$ implies

$mL_{\text{reaction}} = (335{,}380\ \text{J/kg})m.$

The mass m divides out and $L_{\text{reaction}} = 3.4\times10^5$ J/kg

EVALUATE: The amount of energy converted to work is negligible for the two significant figures to which the answer should be expressed. Almost all of the energy produced in the reaction goes into heating the compound.

19.56. **IDENTIFY:** The process is adiabatic. Apply $p_1V_1^\gamma = p_2V_2^\gamma$ and $pV = nRT$. $Q = 0$ so

$$\Delta U = -W = -\frac{1}{\gamma - 1}(p_1V_1 - p_2V_2).$$

SET UP: For helium, $\gamma = 1.67$. $p_1 = 1.00\ \text{atm} = 1.013\times10^5$ Pa. $V_1 = 2.00\times10^3\ \text{m}^3$.

$p_2 = 0.900\ \text{atm} = 9.117\times10^4$ Pa. $T_1 = 288.15$ K.

EXECUTE: **(a)** $V_2^\gamma = V_1^\gamma\left(\dfrac{p_1}{p_2}\right)$. $V_2 = V_1\left(\dfrac{p_1}{p_2}\right)^{1/\gamma} = (2.00\times10^3\ \text{m}^3)\left(\dfrac{1.00\ \text{atm}}{0.900\ \text{atm}}\right)^{1/1.67} = 2.13\times10^3\ \text{m}^3$.

(b) $pV = nRT$ gives $\dfrac{T_1}{p_1V_1} = \dfrac{T_2}{p_2V_2}$.

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)\left(\frac{V_2}{V_1}\right) = (288.15\ \text{K})\left(\frac{0.900\ \text{atm}}{1.00\ \text{atm}}\right)\left(\frac{2.13\times10^3\ \text{m}^3}{2.00\times10^3\ \text{m}^3}\right) = 276.2\ \text{K} = 3.0°\text{C}.$$

(c) $\Delta U = -\dfrac{1}{0.67}([1.013\times10^5\ \text{Pa})(2.00\times10^3\ \text{m}^3)] - [9.117\times10^4\ \text{Pa})(2.13\times10^3\ \text{m}^3)] = -1.25\times10^7$ J.

EVALUATE: The internal energy decreases when the temperature decreases.

19.57. **IDENTIFY:** For an adiabatic process of an ideal gas, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$. $pV = nRT$.

SET UP: For air, $\gamma = 1.40 = \frac{7}{5}$.

EXECUTE: **(a)** As the air moves to lower altitude its density increases; under an adiabatic compression, the temperature rises. If the wind is fast-moving, Q is not as likely to be significant, and modeling the process as adiabatic (no heat loss to the surroundings) is more accurate.

(b) $V = \dfrac{nRT}{p}$, so $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ gives $T_1^\gamma p_1^{1-\gamma} = T_2^\gamma p_2^{1-\gamma}$. The temperature at the higher pressure is

$T_2 = T_1(p_1/p_2)^{(\gamma-1)/\gamma} = (258.15\ \text{K})([8.12\times10^4\ \text{Pa}]/[5.60\times10^4\ \text{Pa}])^{2/7} = 287.1\ \text{K} = 13.0°\text{C}$ so the temperature would rise by 11.9 C°.

EVALUATE: In an adiabatic compression, $Q = 0$ but the temperature rises because of the work done on the gas.

19.58. **IDENTIFY:** For constant pressure , $W = p\Delta V$. For an adiabatic process of an ideal gas, $W = \frac{C_V}{R}(p_1V_1 - p_2V_2)$ and $p_1V_1^\gamma = p_2V_2^\gamma$.

SET UP: $\gamma = \frac{C_p}{C_V} = \frac{C_p + C_V}{C_V} = 1 + \frac{R}{C_V}$

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.58.

(b) The work done is $W = p_0(2V_0 - V_0) + \frac{C_V}{R}(p_0(2V_0) - p_3(4V_0))$. $p_3 = p_0(2V_0/4V_0)^\gamma$ and so $W = p_0V_0\left[1 + \frac{C_V}{R}(2 - 2^{2-\gamma})\right]$. Note that p_0 is the absolute pressure.

(c) The most direct way to find the temperature is to find the ratio of the final pressure and volume to the original and treat the air as an ideal gas. $p_3 = p_2\left(\frac{V_2}{V_3}\right)^\gamma = p_1\left(\frac{V_2}{V_3}\right)^\gamma$, since $p_1 = p_2$. Then

$$T_3 = T_0\frac{p_3V_3}{p_1V_1} = T_0\left(\frac{V_2}{V_3}\right)^\gamma\left(\frac{V_3}{V_1}\right) = T_0\left(\frac{1}{2}\right)^\gamma 4 = T_0(2)^{2-\gamma}.$$

(d) Since $n = \frac{p_0V_0}{RT_0}$, $Q = \frac{p_0V_0}{RT_0}(C_V + R)(2T_0 - T_0) = p_0V_0\left(\frac{C_V}{R} + 1\right)$. This amount of heat flows into the gas, since $Q > 0$.

EVALUATE: In the isobaric expansion the temperature doubles and in the adiabatic expansion the temperature decreases. If the gas is diatomic, with $\gamma = \frac{7}{5}$, $2 - \gamma = \frac{3}{5}$ and $T_3 = 3.03T_0$, $W = 2.21p_0V_0$ and $Q = 3.50p_0V_0$. $\Delta U = 1.29p_0V_0$. $\Delta U > 0$ and this is consistent with an increase in temperature.

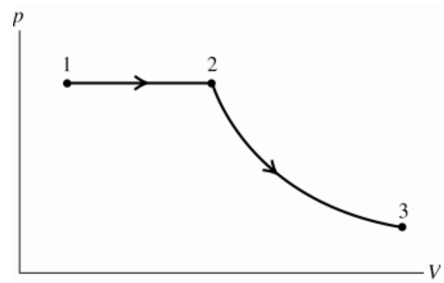


Figure 19.58

19.59. **IDENTIFY:** Assume that the gas is ideal and that the process is adiabatic. Apply Eqs.(19.22) and (19.24) to relate pressure and volume and temperature and volume. The distance the piston moves is related to the volume of the gas. Use Eq.(19.25) to calculate W.

(a) SET UP: $\gamma = C_p/C_V = (C_V + R)/C_V = 1 + R/C_V = 1.40$. The two positions of the piston are shown in Figure 19.59.

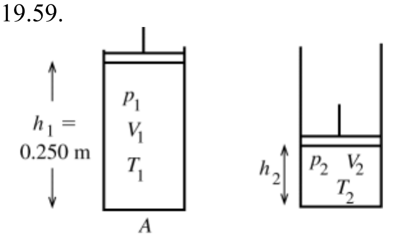


Figure 19.59

$p_1 = 1.01\times10^5$ Pa

$p_2 = 4.20\times10^5$ Pa $+ p_{\text{air}} = 5.21\times10^6$ Pa

$V_1 = h_1A$

$V_2 = h_2A$

EXECUTE: adiabatic process: $p_1V_1^\gamma = p_2V_2^\gamma$

$p_1h_1^\gamma A^\gamma = p_2h_2^\gamma A^\gamma$

$$h_2 = h_1\left(\frac{p_1}{p_2}\right)^{1/\gamma} = (0.250\text{ m})\left(\frac{1.01\times10^5\text{ Pa}}{5.21\times10^5\text{ Pa}}\right)^{1/1.40} = 0.0774\text{ m}$$

The piston has moved a distance $h_1 - h_2 = 0.250\text{ m} - 0.0774\text{ m} = 0.173\text{ m}$.

(b) $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_1h_1^{\gamma-1}A^{\gamma-1} = T_2h_2^{\gamma-1}A^{\gamma-1}$

$$T_2 = T_1\left(\frac{h_1}{h_2}\right)^{\gamma-1} = 300.1\text{ K}\left(\frac{0.250\text{ m}}{0.0774\text{ m}}\right)^{0.40} = 479.7\text{ K} = 207°\text{C}$$

(c) $W = nC_V(T_1 - T_2)$ (Eq.19.25)

$W = (20.0\text{ mol})(20.8\text{ J/mol}\cdot\text{K})(300.1\text{ K} - 479.7\text{ K}) = -7.47\times 10^4\text{ J}$

EVALUATE: In an adiabatic compression of an ideal gas the temperature increases. In any compression the work W is negative.

19.60. IDENTIFY: $m = \rho V$. The density of air is given by $\rho = \dfrac{pM}{RT}$. For an adiabatic process, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$.

$pV = nRT$

SET UP: Using $V = \dfrac{nRT}{p}$ in $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ gives $T_1p_1^{1-\gamma} = T_2p_2^{1-\gamma}$.

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.60.

(b) The final temperature is the same as the initial temperature, and the density is proportional to the absolute pressure. The mass needed to fill the cylinder is then

$$m = \rho_0 V\frac{p}{p_{\text{air}}} = (1.23\text{ kg/m}^3)(575\times 10^{-6}\text{ m}^3)\frac{1.45\times 10^5\text{ Pa}}{1.01\times 10^5\text{ Pa}} = 1.02\times 10^{-3}\text{ kg}.$$

Without the turbocharger or intercooler the mass of air at $T = 15.0°\text{C}$ and $p = 1.01\times 10^5$ Pa in a cylinder is $m = \rho_0 V = 7.07\times 10^{-4}$ kg. The increase in power is proportional to the increase in mass of air in the cylinder; the percentage increase is $\dfrac{1.02\times 10^{-3}\text{ kg}}{7.07\times 10^{-4}\text{ kg}} - 1 = 0.44 = 44\%$.

(c) The temperature after the adiabatic process is $T_2 = T_1\left(\dfrac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$. The density becomes

$\rho = \rho_0\left(\dfrac{T_1}{T_2}\right)\left(\dfrac{p_2}{p_1}\right) = \rho_0\left(\dfrac{p_2}{p_1}\right)^{(1-\gamma)/\gamma}\left(\dfrac{p_2}{p_1}\right) = \rho_0\left(\dfrac{p_2}{p_1}\right)^{1/\gamma}$. The mass of air in the cylinder is

$$m = (1.23\text{ kg/m}^3)(575\times 10^{-6}\text{ m}^3)\left(\frac{1.45\times 10^5\text{ Pa}}{1.01\times 10^5\text{ Pa}}\right)^{1/1.40} = 9.16\times 10^{-4}\text{ kg},$$

The percentage increase in power is $\dfrac{9.16\times 10^{-4}\text{ kg}}{7.07\times 10^{-4}\text{ kg}} - 1 = 0.30 = 30\%$.

EVALUATE: The turbocharger and intercooler each have an appreciable effect on the engine power.

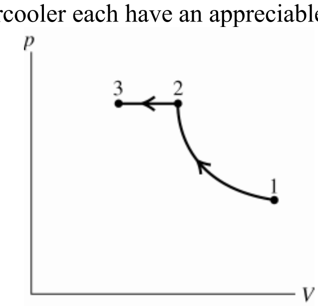


Figure 19.60

19.61. IDENTIFY: In each case calculate either ΔU or Q for the specific type of process and then apply the first law.

(a) SET UP: isothermal $(\Delta T = 0)$ $\Delta U = Q - W$; $W = +300$ J

For any process of an ideal gas, $\Delta U = nC_V\Delta T$.

EXECUTE: Therefore, for an ideal gas, if $\Delta T = 0$ then $\Delta U = 0$ and $Q = W = +300$ J.

(b) SET UP: adiabatic $(Q = 0)$

$\Delta U = Q - W$; $W = +300$ J

EXECUTE: $Q = 0$ says $\Delta U = -W = -300$ J

(c) SET UP: <u>isobaric</u> $\Delta p = 0$

Use W to calculate ΔT and then calculate Q.

EXECUTE: $W = p\Delta T = nR\Delta T;\quad \Delta T = W/nR$

$Q = nC_p\Delta T$ and for a monatomic ideal gas $C_p = \frac{5}{2}R$

Thus $Q = n\frac{5}{2}R\Delta T = (5Rn/2)(W/nR) = 5W/2 = +750$ J.

$\Delta U = nC_V\Delta T$ for any ideal gas process and $C_V = C_p - R = \frac{3}{2}R$.

Thus $\Delta U = 3W/2 = +450$ J

EVALUATE: 300 J of energy leaves the gas when it performs expansion work. In the isothermal process this energy is replaced by heat flow into the gas and the internal energy remains the same. In the adiabatic process the energy used in doing the work decreases the internal energy. In the isobaric process 750 J of heat energy enters the gas, 300 J leaves as the work done and 450 J remains in the gas as increased internal energy.

19.62. **IDENTIFY:** $pV = nRT$. For the isobaric process, $W = p\Delta V = nR\Delta T$. For the isothermal process, $W = nRT\ln\left(\frac{V_f}{V_i}\right)$.

SET UP: $R = 8.315$ J/mol·K

EXECUTE: **(a)** The pV diagram for these processes is sketched in Figure 19.62.

(b) Find T_2. For process $1 \to 2$, n, R, and p are constant so $\frac{T}{V} = \frac{p}{nR} = \text{constant}$. $\frac{T_1}{V_1} = \frac{T_2}{V_2}$ and

$$T_2 = T_1\left(\frac{V_2}{V_1}\right) = (355 \text{ K})(2) = 710 \text{ K}.$$

(c) The maximum pressure is for state 3. For process $2 \to 3$, n, R, and T are constant. $p_2V_2 = p_3V_3$ and

$$p_3 = p_2\left(\frac{V_2}{V_3}\right) = (2.40\times10^5 \text{ Pa})(2) = 4.80\times10^5 \text{ Pa}.$$

(d) process $1 \to 2$: $W = p\Delta V = nR\Delta T = (0.250 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})(710 \text{ K} - 355 \text{ K}) = 738$ K.

process $2 \to 3$: $W = nRT\ln\left(\frac{V_3}{V_2}\right) = (0.250 \text{ mol})(8.315 \text{ J/mol}\cdot\text{K})(710 \text{ K})\ln\left(\frac{1}{2}\right) = -1023$ J.

process $3 \to 1$: $\Delta V = 0$ and $W = 0$.

The total work done is $738 \text{ J} + (-1023 \text{ J}) = -285$ J. This is the work done by the gas. The work done on the gas is 285 J.

EVALUATE: The final pressure and volume are the same as the initial pressure and volume, so the final state is the same as the initial state. For the cycle, $\Delta U = 0$ and $Q = W = -285$ J. During the cycle, 285 J of heat energy must leave the gas.

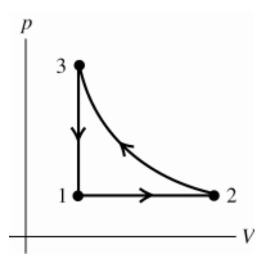


Figure 19.62

19.63. **IDENTIFY** and **SET UP:** Use the ideal gas law, the first law and expressions for Q and W for specific types of processes.

EXECUTE: **(a)** initial expansion (state $1 \to$ state 2)

$p_1 = 2.40\times10^5$ Pa, $T_1 = 355$ K, $p_2 = 2.40\times10^5$ Pa, $V_2 = 2V_1$

$pV = nRT$; $T/V = p/nR = \text{constant}$, so $T_1/V_1 = T_2/V_2$ and $T_2 = T_1(V_2/V_1) = 355 \text{ K}(2V_1/V_1) = 710$ K

$\Delta p = 0$ so $W = p\Delta V = nR\Delta T = (0.250 \text{ mol})(8.3145 \text{ J/mol}\cdot\text{K})(710 \text{ K} - 355 \text{ K}) = +738$ J

$Q = nC_p\Delta T = (0.250 \text{ mol})(29.17 \text{ J/mol}\cdot\text{K})(710 \text{ K} - 355 \text{ K}) = +2590$ J

$\Delta U = Q - W = 2590 \text{ J} - 738 \text{ J} = 1850$ J

(b) At the beginning of the final cooling process (cooling at constant volume), $T = 710$ K. The gas returns to its original volume and pressure, so also to its original temperature of 355 K.

$\Delta V = 0$ so $W = 0$

$Q = nC_V\Delta T = (0.250 \text{ mol})(20.85 \text{ J/mol}\cdot\text{K})(355 \text{ K} - 710 \text{ K}) = -1850 \text{ J}$

$\Delta U = Q - W = -1850$ J.

(c) For any ideal gas process $\Delta U = nC_V\Delta T$. For an isothermal process $\Delta T = 0$, so $\Delta U = 0$.

EVALUATE: The three processes return the gas to its initial state, so $\Delta U_{\text{total}} = 0$; our results agree with this.

19.64. IDENTIFY: $pV = nRT$. For an adiabatic process of an ideal gas, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$.

SET UP: For N_2, $\gamma = 1.40$.

EXECUTE: **(a)** The pV-diagram is sketched in Figure 19.64.

(b) At constant pressure, halving the volume halves the Kelvin temperature, and the temperature at the beginning of the adiabatic expansion is 150 K. The volume doubles during the adiabatic expansion, and from Eq. (19.22), the temperature at the end of the expansion is $(150 \text{ K})(1/2)^{0.40} = 114$ K.

(c) The minimum pressure occurs at the end of the adiabatic expansion (state 3). During the final heating the volume is held constant, so the minimum pressure is proportional to the Kelvin temperature,

$p_{\min} = (1.80\times10^5 \text{ Pa})(114 \text{ K}/300 \text{ K}) = 6.82\times10^4$ Pa.

EVALUATE: In the adiabatic expansion the temperature decreases.

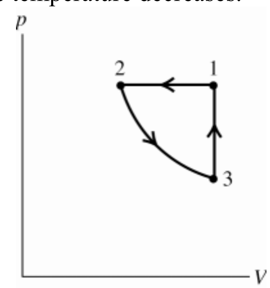


Figure 19.64

19.65. IDENTIFY: Use the appropriate expressions for Q, W and ΔU for each type of process. $\Delta U = Q - W$ can also be used.

SET UP: For N_2, $C_V = 20.76$ J/mol·K and $C_p = 29.07$ J/mol·K.

EXECUTE: **(a)** $W = p\Delta V = nR\Delta T = (0.150 \text{ mol})(8.3145 \text{ J/mol}\cdot\text{K})(-150 \text{ K}) = -187$ J,

$Q = nC_p\Delta T = (0.150 \text{ mol})(29.07 \text{ mol}\cdot\text{K})(-150 \text{ K}) = -654 \text{ J}$, $\Delta U = Q - W = -467$ J.

(b) From Eq. (19.24), using the expression for the temperature found in Problem 19.64,

$W = \dfrac{1}{0.40}(0.150 \text{ mol})(8.3145 \text{ J/mol}\cdot\text{K})(150 \text{ K})(1-(1/2^{0.40})) = 113$ J. $Q = 0$ for an adiabatic process, and

$\Delta U = Q - W = -W = -113$ J.

(c) $\Delta V = 0$, so $W = 0$. Using the temperature change as found in Problem 19.64 and part (b),

$Q = nC_V\Delta T = (0.150 \text{ mol})(20.76 \text{ J/mol}\cdot\text{K})(300 \text{ K} - 113.7 \text{ K}) = 580$ J and $\Delta U = Q - W = Q = 580$ J.

EVALUATE: For each process we could also use $\Delta U = nC_V\Delta T$ to calculate ΔU.

19.66. IDENTIFY: Use the appropriate expression for W for each type of process.

SET UP: For a monatomic ideal gas, $\gamma = 5/3$ and $C_V = 3R/2$.

EXECUTE: **(a)** $W = nRT\ln(V_2/V_1) = nRT\ln(3) = 3.29\times10^3$ J.

(b) $Q = 0$ so $W = -\Delta U = -nC_V\Delta T$. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ gives $T_2 = T_1(1/3)^{2/3}$. Then

$W = nC_VT_1(1-(1/3)^{2/3}) = 2.33\times10^3$ J.

(c) $V_2 = 3V_1$, so $W = p\Delta V = 2pV_1 = 2nRT_1 = 6.00\times10^3$ J.

(d) Each process is shown in Figure 19.66. The most work done is in the isobaric process, as the pressure is maintained at its original value. The least work is done in the adiabatic process.

(e) The isobaric process involves the most work and the largest temperature increase, and so requires the most heat. Adiabatic processes involve no heat transfer, and so the magnitude is zero.

(f) The isobaric process doubles the Kelvin temperature, and so has the largest change in internal energy. The isothermal process necessarily involves no change in internal energy.

EVALUATE: The work done is the area under the path for the process in the pV-diagram. Figure 19.66 shows that the work done is greatest in the isobaric process and least in the adiabatic process.

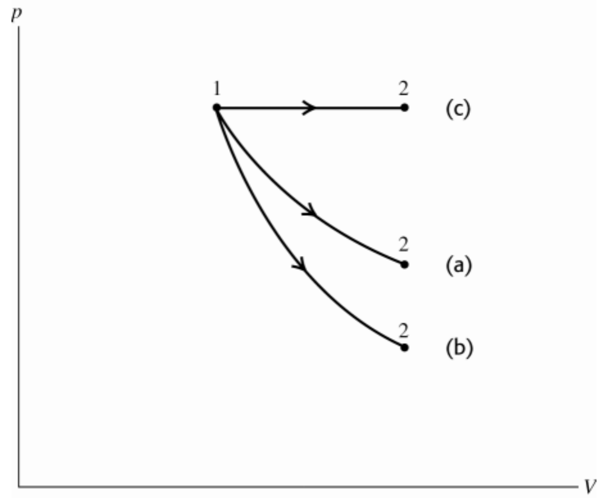


Figure 19.66

19.67. **IDENTIFY:** Assume the compression is adiabatic. Apply $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ and $pV = nRT$.

SET UP: For N_2, $\gamma = 1.40$. $V_1 = 3.00$ L, $p = 1.00\text{ atm} = 1.013\times10^5$ Pa, $T = 273.15$ K. $V_2 = V_1/2 = 1.50$ L.

EXECUTE: **(a)** $T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = (273.15\text{ K})\left(\frac{V_1}{V_1/2}\right)^{0.40} = (273.15\text{ K})(2)^{0.4} = 360.4\text{ K} = 87.3°\text{C}$. $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$.

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right) = (1.00\text{ atm})\left(\frac{V_1}{V_1/2}\right)\left(\frac{360.4\text{ K}}{273.15\text{ K}}\right) = 2.64\text{ atm}.$$

(b) p is constant, so $\frac{V}{T} = \frac{nR}{T} = \text{constant}$ and $\frac{V_2}{T_2} = \frac{V_3}{T_3}$. $V_3 = V_2\left(\frac{T_3}{T_2}\right) = (1.50\text{ L})\left(\frac{273.15\text{ K}}{360.4\text{ K}}\right) = 1.14$ L.

EVALUATE: In an adiabatic compression the temperature increases.

19.68. **IDENTIFY:** At equilibrium the net upward force of the gas on the piston equals the weight of the piston. When the piston moves upward the gas expands, the pressure of the gas drops and there is a net downward force on the piston. For simple harmonic motion the net force has the form $F_y = -ky$, for a displacement y from equilibrium, and $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$.

SET UP: $pV = nRT$. T is constant.

(a) The difference between the pressure, inside and outside the cylinder, multiplied by the area of the piston, must be the weight of the piston. The pressure in the trapped gas is $p_0 + \frac{mg}{A} = p_0 + \frac{mg}{\pi r^2}$.

(b) When the piston is a distance $h + y$ above the cylinder, the pressure in the trapped gas is $\left(p_0 + \frac{mg}{\pi r^2}\right)\left(\frac{h}{h+y}\right)$ and for values of y small compared to h, $\frac{h}{h+y} = \left(1 + \frac{y}{h}\right)^{-1} \sim 1 - \frac{y}{h}$. The net force, taking the positive direction to be upward, is the then $F_y = \left[\left(p_0 + \frac{mg}{\pi r^2}\right)\left(1 - \frac{y}{h}\right) - p_0\right](\pi r^2) - mg = -\left(\frac{y}{h}\right)(p_0\pi r^2 + mg)$.

This form shows that for positive h, the net force is down; the trapped gas is at a lower pressure than the equilibrium pressure, and so the net force tends to restore the piston to equilibrium.

(c) The angular frequency of small oscillations would be given by $\omega^2 = \frac{(p_0\pi r^2 + mg)/h}{m} = \frac{g}{h}\left(1 + \frac{p_0\pi r^2}{mg}\right)$.

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{h}\left(1 + \frac{p_0\pi r^2}{mg}\right)^{1/2}}.$$

If the displacements are not small, the motion is not simple harmonic. This can be seen be considering what happens if $y \sim -h$; the gas is compressed to a very small volume, and the force due to the pressure of the gas would become unboundedly large for a finite displacement, which is not characteristic of simple harmonic motion. If $y >> h$ (but not so large that the piston leaves the cylinder), the force due to the pressure of the gas becomes small, and the restoring force due to the atmosphere and the weight would tend toward a constant, and this is not characteristic of simple harmonic motion.

EVALUATE: The assumption of small oscillations was made when $\dfrac{h}{h+y}$ was replaced by $1 - y/h$; this is accurate only when y/h is small.

19.69. IDENTIFY: $W = \int_{V_1}^{V_2} p\,dV$.

SET UP: For an isothermal process of an ideal gas, $W = nRT\ln(V_2/V_1)$.

EXECUTE: **(a)** Solving for p as a function of V and T and integrating with respect to V,

$$p = \frac{nRT}{V-nb} - \frac{an^2}{V^2} \text{ and } W = \int_{V_1}^{V_2} p\,dV = nRT\ln\left[\frac{V_2 - nb}{V_1 - nb}\right] + an^2\left[\frac{1}{V_2} - \frac{1}{V_1}\right].$$

When $a = b = 0$, $W = nRT\ln(V_2/V_1)$, as expected.

(b) (i) Using the expression found in part (a),

$$W = (1.80\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})(300\text{ K})$$
$$\times\ \ln\left[\frac{(4.00\times10^{-3}\text{ m}^3) - (1.80\text{ mol})(6.38\times10^{-5}\text{ m}^2/\text{mol})}{(2.00\times10^{-3}\text{ m}^3) - (1.80\text{ mol})(6.38\times10^{-5}\text{ m}^2/\text{mol})}\right]$$
$$+ (0.554\text{ J}\cdot\text{m}^3/\text{mol}^2)(1.80\text{ mol})^2\left[\frac{1}{4.00\times10^{-3}\text{ m}^3} - \frac{1}{2.00\times10^{-3}\text{ m}^3}\right]$$

$W = 2.80\times10^3$ J.

(ii) $W = nRT\ln(2) = 3.11\times10^3$ J.

(c) The work for the ideal gas is larger by about 300 J. For this case, the difference due to nonzero a is more than that due to nonzero b. The presence of a nonzero a indicates that the molecules are attracted to each other and so do not do as much work in the expansion.

EVALUATE: The difference in the two results for W is about 10%, which can be considered to be important.

THE SECOND LAW OF THERMODYNAMICS

20

20.1. **IDENTIFY:** For a heat engine, $W = |Q_H| - |Q_C|$. $e = \frac{W}{Q_H}$. $Q_H > 0$, $Q_C < 0$.

SET UP: $W = 2200\text{ J}$. $|Q_C| = 4300\text{ J}$.

EXECUTE: **(a)** $Q_H = W + |Q_C| = 6500\text{ J}$.

(b) $e = \frac{2200\text{ J}}{6500\text{ J}} = 0.34 = 34\%$.

EVALUATE: Since the engine operates on a cycle, the net Q equal the net W. But to calculate the efficiency we use the heat energy input, Q_H.

20.2. **IDENTIFY:** For a heat engine, $W = |Q_H| - |Q_C|$. $e = \frac{W}{Q_H}$. $Q_H > 0$, $Q_C < 0$.

SET UP: $|Q_H| = 9000\text{ J}$. $|Q_C| = 6400\text{ J}$.

EXECUTE: **(a)** $W = 9000\text{ J} - 6400\text{ J} = 2600\text{ J}$.

(b) $e = \frac{W}{Q_H} = \frac{2600\text{ J}}{9000\text{ J}} = 0.29 = 29\%$.

EVALUATE: Since the engine operates on a cycle, the net Q equal the net W. But to calculate the efficiency we use the heat energy input, Q_H.

20.3. **IDENTIFY** and **SET UP:** The problem deals with a heat engine. $W = +3700\text{ W}$ and $Q_H = +16{,}100\text{ J}$. Use Eq.(20.4) to calculate the efficiency e and Eq.(20.2) to calculate $|Q_C|$. Power $= W/t$.

EXECUTE: **(a)** $e = \frac{\text{work output}}{\text{heat energy input}} = \frac{W}{Q_H} = \frac{3700\text{ J}}{16{,}100\text{ J}} = 0.23 = 23\%$.

(b) $W = Q = |Q_H| - |Q_C|$

Heat discarded is $|Q_C| = |Q_H| - W = 16{,}100\text{ J} - 3700\text{ J} = 12{,}400\text{ J}$.

(c) Q_H is supplied by burning fuel; $Q_H = mL_c$ where L_c is the heat of combustion.

$m = \frac{Q_H}{L_c} = \frac{16{,}100\text{ J}}{4.60\times 10^4\text{ J/g}} = 0.350\text{ g}$.

(d) $W = 3700\text{ J}$ per cycle

In $t = 1.00\text{ s}$ the engine goes through 60.0 cycles.

$P = W/t = 60.0(3700\text{ J})/1.00\text{ s} = 222\text{ kW}$

$P = (2.22\times 10^5\text{ W})(1\text{ hp}/746\text{ W}) = 298\text{ hp}$

EVALUATE: $Q_C = -12{,}400\text{ J}$. In one cycle $Q_{tot} = Q_C + Q_H = 3700\text{ J}$. This equals W_{tot} for one cycle.

20.4. **IDENTIFY:** $W = |Q_H| - |Q_C|$. $e = \frac{W}{Q_H}$. $Q_H > 0$, $Q_C < 0$.

SET UP: For 1.00 s, $W = 180\times 10^3\text{ J}$.

EXECUTE: **(a)** $Q_H = \frac{W}{e} = \frac{180\times 10^3\text{ J}}{0.280} = 6.43\times 10^5\text{ J}$.

(b) $|Q_C| = |Q_H| - W = 6.43\times 10^5\text{ J} - 1.80\times 10^5\text{ J} = 4.63\times 10^5\text{ J}$.

EVALUATE: Of the $6.43\times 10^5\text{ J}$ of heat energy supplied to the engine each second, $1.80\times 10^5\text{ J}$ is converted to mechanical work and the remaining $4.63\times 10^5\text{ J}$ is discarded into the low temperature reservoir.

20.5. IDENTIFY: $W = |Q_H| - |Q_C|$. $e = \dfrac{W}{Q_H}$. $Q_H > 0$, $Q_C < 0$. Dividing by t gives equivalent equations for the rate of heat flows and power output.

SET UP: $W/t = 330\text{ MW}$. $|Q_H|/t = 1300\text{ MW}$.

EXECUTE: **(a)** $e = \dfrac{W}{Q_H} = \dfrac{W/t}{Q_H/t} = \dfrac{330\text{ MW}}{1300\text{ MW}} = 0.25 = 25\%$.

(b) $|Q_C| = |Q_H| - W$ so $|Q_C|/t = |Q_H|/t - W/t = 1300\text{ MW} - 330\text{ MW} = 970\text{ MW}$.

EVALUATE: The equations for e and W have the same form when written in terms of power output and rate of heat flow.

20.6. IDENTIFY: Apply $e = 1 - \dfrac{1}{r^{\gamma-1}}$. $e = 1 - \dfrac{|Q_C|}{|Q_H|}$.

SET UP: In part (b), $Q_H = 10{,}000\text{ J}$. The heat discarded is $|Q_C|$.

EXECUTE: **(a)** $e = 1 - \dfrac{1}{9.50^{0.40}} = 0.594 = 59.4\%$.

(b) $|Q_C| = |Q_H|(1-e) = (10{,}000\text{ J})(1-0.594) = 4060\text{ J}$.

EVALUATE: The work output of the engine is $W = |Q_H| - |Q_C| = 10{,}000\text{ J} - 4060\text{ J} = 5940\text{ J}$

20.7. IDENTIFY: $e = 1 - \dfrac{1}{r^{\gamma-1}}$.

SET UP: $\gamma = 1.40$ and $e = 0.650$.

EXECUTE: $\dfrac{1}{r^{\gamma-1}} = 1 - e = 0.350$. $r^{0.40} = \dfrac{1}{0.350}$ and $r = 13.8$.

EVALUATE: e increases when r increases.

20.8. IDENTIFY: $e = 1 - r^{1-\gamma}$

SET UP: r is the compression ratio.

EXECUTE: **(a)** $e = 1 - (8.8)^{-0.40} = 0.581$, which rounds to 58%.

(b) $e = 1 - (9.6)^{-0.40} = 0.595$ an increase of 1.4%.

EVALUATE: An increase in r gives an increase in e.

20.9. IDENTIFY and SET UP: For the refrigerator $K = 2.10$ and $Q_C = +3.4\times10^4\text{ J}$. Use Eq.(20.9) to calculate $|W|$ and then Eq.(20.2) to calculate Q_H.

(a) EXECUTE: Performance coefficient $K = Q_C/|W|$ (Eq.20.9)

$|W| = Q_C/K = 3.40\times10^4\text{ J}/2.10 = 1.62\times10^4\text{ J}$

(b) SET UP: The operation of the device is illustrated in Figure 20.9

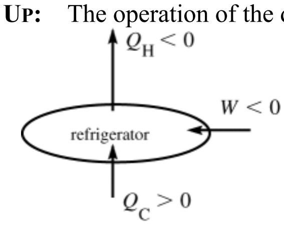


Figure 20.9

EXECUTE:

$W = Q_C + Q_H$

$Q_H = W - Q_C$

$Q_H = -1.62\times10^4\text{ J} - 3.40\times10^4\text{ J} = -5.02\times10^4\text{ J}$

(negative because heat goes out of the system)

EVALUATE $|Q_H| = |W| + |Q_C|$. The heat $|Q_H|$ delivered to the high temperature reservoir is greater than the heat taken in from the low temperature reservoir.

20.10. IDENTIFY: $K = \dfrac{|Q_C|}{|W|}$ and $|Q_H| = |Q_C| + |W|$.

SET UP: The heat removed from the room is $|Q_C|$ and the heat delivered to the hot outside is $|Q_H|$.

$|W| = (850\text{ J/s})(60.0\text{ s}) = 5.10\times10^4\text{ J}$.

EXECUTE: **(a)** $|Q_C| = K|W| = (2.9)(5.10\times10^4\text{ J}) = 1.48\times10^5\text{ J}$

(b) $|Q_H| = |Q_C| + |W| = 1.48\times10^5\text{ J} + 5.10\times10^4\text{ J} = 1.99\times10^5\text{ J}$.

EVALUATE: **(c)** $|Q_H| = |Q_C| + |W|$, so $|Q_H| > |Q_C|$.

20.11. **IDENTIFY** and **SET UP:** Apply Eq.(20.2) to the cycle and calculate $|W|$ and then $P = |W|/t$. Section 20.4 shows that $\text{EER} = (3.413)K$.

(a) The operation of the device is illustrated in Figure 20.11.

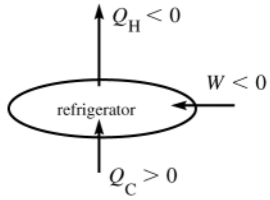


Figure 20.11

EXECUTE:

$Q_C = +9.80 \times 10^4 \text{ J}$

$Q_H = -1.44 \times 10^5 \text{ J}$

$W = Q_C + Q_H = +9.80 \times 10^4 \text{ J} - 1.44 \times 10^5 \text{ J} = -4.60 \times 10^4 \text{ J}$

$P = W/t = -4.60 \times 10^4 \text{ J}/60.0 \text{ s} = -767 \text{ W}$

(b) $\text{EER} = (3.413)K$

$K = |Q_C|/|W| = 9.80 \times 10^4 \text{ J}/4.60 \times 10^4 \text{ J} = 2.13$

$\text{EER} = (3.413)(2.13) = 7.27$

EVALUATE: W negative means power is consumed, not produced, by the device. $|Q_H| = |W| + |Q_C|$.

20.12. **IDENTIFY:** $|Q_H| = |Q_C| + |W|$. $K = \dfrac{|Q_C|}{W}$.

SET UP: For water, $c_w = 4190 \text{ J/kg}\cdot\text{K}$ and $L_f = 3.34 \times 10^5 \text{ J/kg}$. For ice, $c_{ice} = 2010 \text{ J/kg}\cdot\text{K}$.

EXECUTE: **(a)** $Q = mc_{ice}\Delta T_{ice} - mL_f + mc_w\Delta T_w$.

$Q = (1.80 \text{ kg})([2010 \text{ J/kg}\cdot\text{K}][-5.0 \text{ C}°] - 3.34 \times 10^5 \text{ J/kg} + [4190 \text{ J/kg}\cdot\text{K}][-25.0 \text{ C}°]) = -8.08 \times 10^5 \text{ J}$

$Q = -8.08 \times 10^5$ J. Q is negative for the water since heat is removed from it.

(b) $|Q_C| = 8.08 \times 10^5$ J. $W = \dfrac{|Q_C|}{K} = \dfrac{8.08 \times 10^5 \text{ J}}{2.40} = 3.37 \times 10^5$ J.

(c) $|Q_H| = 8.08 \times 10^5 \text{ J} + 3.37 \times 10^5 \text{ J} = 1.14 \times 10^6$ J.

EVALUATE: For this device, $Q_C > 0$ and $Q_H < 0$. More heat is rejected to the room than is removed from the water.

20.13. **IDENTIFY:** Use Eq.(20.2) to calculate $|W|$. Since it is a Carnot device we can use Eq.(20.13) to relate the heat flows out of the reservoirs. The reservoir temperatures can be used in Eq.(20.14) to calculate e.

(a) SET UP: The operation of the device is sketched in Figure 20.13.

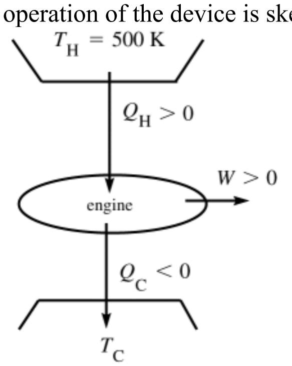


Figure 20.13

EXECUTE:

$W = Q_C + Q_H$

$W = -335 \text{ J} + 550 \text{ J} = 215 \text{ J}$

(b) For a Carnot cycle, $\dfrac{|Q_C|}{|Q_H|} = \dfrac{T_C}{T_H}$ (Eq.20.13)

$$T_C = T_H \frac{|Q_C|}{|Q_H|} = 620 \text{ K}\left(\frac{335 \text{ J}}{550 \text{ J}}\right) = 378 \text{ K}$$

(c) $e(\text{Carnot}) = 1 - T_C/T_H = 1 - 378 \text{ K}/620 \text{ K} = 0.390 = 39.0\%$

EVALUATE: We could use the underlying definition of e (Eq.20.4):

$e = W/Q_H = (215 \text{ J})/(550 \text{ J}) = 39\%$, which checks.

20.14. **IDENTIFY:** $|W| = |Q_H| - |Q_C|$. $Q_C < 0$, $Q_H > 0$. $e = \frac{W}{Q_H}$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$.

SET UP: $T_C = 300$ K, $T_H = 520$ K. $|Q_H| = 6.45 \times 10^3$ J.

EXECUTE: **(a)** $Q_C = -Q_H\left(\frac{T_C}{T_H}\right) = -(6.45\times 10^3 \text{ J})\left(\frac{300 \text{ K}}{520 \text{ K}}\right) = -3.72\times 10^3$ J.

(b) $|W| = |Q_H| - |Q_C| = 6.45\times 10^3 \text{ J} - 3.72\times 10^3 \text{ J} = 2.73\times 10^3$ J

(c) $e = \frac{W}{Q_H} = \frac{2.73\times 10^3 \text{ J}}{6.45\times 10^3 \text{ J}} = 0.423 = 42.3\%$.

EVALUATE: We can verify that $e = 1 - T_C/T_H$ also gives $e = 42.3\%$.

20.15. **IDENTIFY:** $e = \frac{W}{Q_H}$ for any engine. For the Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$.

SET UP: $T_C = 20.0°\text{C} + 273.15 \text{ K} = 293.15$ K

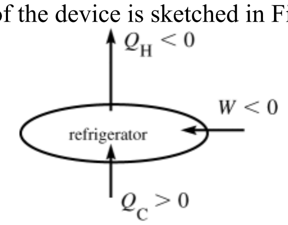
EXECUTE: **(a)** $Q_H = \frac{W}{e} = \frac{2.5\times 10^4 \text{ J}}{0.59} = 4.24\times 10^4$ J

(b) $W = Q_H + Q_C$ so $Q_C = W - Q_H = 2.5\times 10^4 \text{ J} - 4.24\times 10^4 \text{ J} = -1.74\times 10^4$ J.

$T_H = -T_C\frac{Q_H}{Q_C} = -(293.15 \text{ K})\left(\frac{4.24\times 10^4 \text{ J}}{-1.74\times 10^4 \text{ J}}\right) = 714 \text{ K} = 441°\text{C}$.

EVALUATE: For a heat engine, $W > 0$, $Q_H > 0$ and $Q_C < 0$.

20.16. **IDENTIFY** and **SET UP:** The device is a Carnot refrigerator.
We can use Eqs.(20.2) and (20.13).
(a) The operation of the device is sketched in Figure 20.16.



$T_H = 24.0°\text{C} = 297$ K
$T_C = 0.0°\text{C} = 273$ K

Figure 20.16

The amount of heat taken out of the water to make the liquid $\to$ solid phase change is $Q = -mL_f = -(85.0 \text{ kg})(334\times 10^3 \text{ J/kg}) = -2.84\times 10^7$ J. This amount of heat must go into the working substance of the refrigerator, so $Q_C = +2.84\times 10^7$ J. For Carnot cycle $|Q_C|/|Q_H| = T_C/T_H$

EXECUTE: $|Q_H| = |Q_C|(T_H/T_C) = 2.84\times 10^7 \text{ J}(297 \text{ K}/273 \text{ K}) = 3.09\times 10^7$ J

(b) $W = Q_C + Q_H = +2.84\times 10^7 \text{ J} - 3.09\times 10^7 \text{ J} = -2.5\times 10^6$ J

EVALUATE: W is negative because this much energy must be supplied to the refrigerator rather than obtained from it. Note that in Eq.(20.13) we must use Kelvin temperatures.

20.17. **IDENTIFY:** $|Q_H| = |W| + |Q_C|$. $Q_H < 0$, $Q_C > 0$. $K = \frac{|Q_C|}{|W|}$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$.

SET UP: $T_C = 270$ K, $T_H = 320$ K. $|Q_C| = 415$ J.

EXECUTE: **(a)** $Q_H = -\left(\frac{T_H}{T_C}\right)Q_C = -\left(\frac{320 \text{ K}}{270 \text{ K}}\right)(415 \text{ J}) = -492$ J.

(b) For one cycle, $|W| = |Q_H| - |Q_C| = 492 \text{ J} - 415 \text{ J} = 77$ J. $P = \frac{(165)(77 \text{ J})}{60 \text{ s}} = 212$ W.

(c) $K = \frac{|Q_C|}{|W|} = \frac{415 \text{ J}}{77 \text{ J}} = 5.4$.

EVALUATE: The amount of heat energy $|Q_H|$ delivered to the high-temperature reservoir is greater than the amount of heat energy $|Q_C|$ removed from the low-temperature reservoir.

20.18. **IDENTIFY:** $|W| = |Q_H| - |Q_C|$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$, where the temperatures must be in kelvins.

SET UP: $-10.0°C = 263.15$ K, $25.0°C = 298.15$ K, $0.0°C = 273.15$ K and $-25.0°C = 248.15$ K.

EXECUTE: **(a)** The heat is discarded at a higher temperature, and a refrigerator is required. $|Q_H| = |Q_C|(T_H/T_C)$ and $|W| = |Q_C|((T_H/T_C) - 1) = (5.00\times10^3\text{ J})((298.15\text{ K}/263.15\text{ K}) - 1) = 665\text{ J}$.

(b) Again, the device is a refrigerator, and $|W| = (5.00\times10^3\text{ J})((273.15\text{ K}/263.15\text{ K}) - 1) = 190\text{ J}$.

(c) The device is an engine; the heat is taken from the hot reservoir, and the work done by the engine is $|W| = (5.00\times10^3\text{ J})(1 - (248.15\text{ K}/263.15\text{ K})) = 285\text{ J}$.

EVALUATE: For a refrigerator work must be supplied to the device. For a heat engine, there is mechanical work output from the device.

20.19. **IDENTIFY:** The theoretical maximum performance coefficient is $K_{\text{Carnot}} = \frac{T_C}{T_H - T_C}$. $K = \frac{|Q_C|}{|W|}$. $|Q_C|$ is the heat removed from the water to convert it to ice. For the water, $|Q| = mc_w\Delta T + mL_f$.

SET UP: $T_C = -5.0°C = 268$ K. $T_H = 20.0°C = 293$ K. $c_w = 4190\text{ J/kg}\cdot\text{K}$ and $L_f = 334\times10^3$ J/kg.

EXECUTE: **(a)** In one year the freezer operates $(5\text{ h/day})(365\text{ days}) = 1825\text{ h}$.

$$P = \frac{730\text{ kWh}}{1825\text{ h}} = 0.400\text{ kW} = 400\text{ W}.$$

(b) $K_{\text{Carnot}} = \frac{268\text{ K}}{293\text{ K} - 268\text{ K}} = 10.7$

(c) $|W| = Pt = (400\text{ W})(3600\text{ s}) = 1.44\times10^6$ J. $|Q_C| = K|W| = 1.54\times10^7$ J. $|Q| = mc_w\Delta T + mL_f$ gives

$$m = \frac{|Q_C|}{c_w\Delta T + L_f} = \frac{1.54\times10^7\text{ J}}{(4190\text{ J/kg}\cdot\text{K})(20.0\text{ K}) + 334\times10^3\text{ J/kg}} = 36.9\text{ kg}.$$

EVALUATE: For any actual device, $K < K_{\text{Carnot}}$, $|Q_C|$ is less than we calculated and the freezer makes less ice in one hour than the mass we calculated in part (c).

20.20. **IDENTIFY:** The total work that must be done is $W_{\text{tot}} = mg\Delta y$. $|W| = |Q_H| - |Q_C|$. $Q_H > 0$, $W > 0$ and $Q_C < 0$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$,

SET UP: $T_C = 373$ K, $T_H = 773$ K. $|Q_H| = 250$ J.

EXECUTE: $Q_C = -Q_H\left(\frac{T_C}{T_H}\right) = -(250\text{ J})\left(\frac{373\text{ K}}{773\text{ K}}\right) = -121\text{ J}$. $|W| = 250\text{ J} - 121\text{ J} = 129\text{ J}$. This is the work done in one cycle. $W_{\text{tot}} = (500\text{ kg})(9.80\text{ m/s}^2)(100\text{ m}) = 4.90\times10^5$ J. The number of cycles required is

$$\frac{W_{\text{tot}}}{|W|} = \frac{4.90\times10^5\text{ J}}{129\text{ J/cycle}} = 3.80\times10^3\text{ cycles}.$$

EVALUATE: In $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$, the temperatures must be in kelvins.

20.21. **IDENTIFY:** $e = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ and $e = 1 - \frac{T_C}{T_H}$.

SET UP: $T_H = 800$ K. $Q_C = -3000$ J.

EXECUTE: For a heat engine, $Q_H = -Q_C/(1-e) = -(-3000\text{ J})/(1-0.600) = 7500\text{ J}$, and then $W = eQ_H = (0.600)(7500\text{ J}) = 4500\text{ J}$.

EVALUATE: This does not make use of the given value of T_H. If T_H is used, then $T_C = T_H(1-e) = (800\text{ K})(1-0.600) = 320\text{ K}$ and $Q_H = -Q_C T_H/T_C$, which gives the same result.

20.22. **IDENTIFY:** $W = Q_C + Q_H$. For a Carnot cycle, $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$. For the ice to liquid water phase transition, $Q = mL_f$.

SET UP: For water, $L_f = 334\times10^3$ J/kg

EXECUTE: $Q_C = -mL_f = -(0.0400\text{ kg})(334\times10^3\text{ J/kg}) = -1.336\times10^4\text{ J}$. $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ gives

$Q_H = -(T_H/T_C)Q_C = -(-1.336\times10^4\text{ J})[(373.15\text{ K})/(273.15\text{ K})] = +1.825\times10^4\text{ J}$. $W = Q_C + Q_H = 4.89\times10^3\text{ J}$.

EVALUATE: For a heat engine, Q_C is negative and Q_H is positive. The heat that comes out of the engine ($Q<0$) goes into the ice ($Q>0$).

20.23. **IDENTIFY:** The power output is $P = \frac{W}{t}$. The theoretical maximum efficiency is $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$. $e = \frac{W}{Q_H}$.

SET UP: $Q_H = 1.50\times10^4\text{ J}$. $T_C = 350\text{ K}$. $T_H = 650\text{ K}$. $1\text{ hp} = 746\text{ W}$.

EXECUTE: $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{350\text{ K}}{650\text{ K}} = 0.4615$. $W = eQ_H = (0.4615)(1.50\times10^4\text{ J}) = 6.923\times10^3\text{ J}$; this is the work output in one cycle. $P = \frac{W}{t} = \frac{(240)(6.923\times10^3\text{ J})}{60.0\text{ s}} = 2.77\times10^4\text{ W} = 37.1\text{ hp}$.

EVALUATE: We could also use $\frac{Q_C}{Q_H} = -\frac{T_C}{T_H}$ to calculate $Q_C = -\left(\frac{T_C}{T_H}\right)Q_H = -\left(\frac{350\text{ K}}{650\text{ K}}\right)(1.50\times10^4\text{ J}) = -8.08\times10^3\text{ J}$. Then $W = Q_C + Q_H = 6.92\times10^3\text{ J}$, the same as previously calculated.

20.24. **IDENTIFY** and **SET UP:** $e_{\text{Carnot}} = 1 - \frac{T_C}{T_H}$. $K_{\text{Carnot}} = \frac{T_C}{T_H - T_C}$.

EXECUTE: **(a)** $T_C = T_H(1-e)$. $K = \frac{T_H(1-e)}{T_H - T_H(1-e)} = \frac{1-e}{e}$.

EVALUATE: **(b)** When $e \to 1$, $K \to 0$. When $e \to 0$, $K \to \infty$.
$e \to 1$ when $|Q_C| << |Q_H|$. $|Q_C|$ is small in this limit. That is good for an engine since $|Q_C|$ is wasted. But it is bad for a refrigerator since $|Q_C|$ is what is useful. $e \to 0$ when $|Q_C| \to |Q_H|$ and $|W|$ is very small. That is bad for an engine but good for a refrigerator.

20.25. **IDENTIFY:** $\Delta S = \frac{Q}{T}$ for each object, where T must be in kelvins. The temperature of each object remains constant.

SET UP: For water, $L_f = 3.34\times10^5\text{ J/kg}$.

EXECUTE: **(a)** The heat flow into the ice is $Q = mL_f = (0.350\text{ kg})(3.34\times10^5\text{ J/kg}) = 1.17\times10^5\text{ J}$. The heat flow occurs at $T = 273\text{ K}$, so $\Delta S = \frac{Q}{T} = \frac{1.17\times10^5\text{ J}}{273\text{ K}} = 429\text{ J/K}$. Q is positive and ΔS is positive.

(b) $Q = -1.17\times10^5\text{ J}$ flows out of the heat source, at $T = 298\text{ K}$. $\Delta S = \frac{Q}{T} = \frac{-1.17\times10^5\text{ J}}{298\text{ K}} = -393\text{ J/K}$. Q is negative and ΔS is negative.

(c) $\Delta S_{\text{tot}} = 429\text{ J/K} + (-393\text{ J/K}) = +36\text{ J/K}$.

EVALUATE: For the total isolated system, $\Delta S > 0$ and the process is irreversible.

20.26. **IDENTIFY:** Apply $Q_{\text{system}} = 0$ to calculate the final temperature. $Q = mc\Delta T$. Example 20.6 shows that $\Delta S = mc\ln(T_2/T_1)$ when an object undergoes a temperature change.

SET UP: For water $c = 4190\text{ J/kg}\cdot\text{K}$. Boiling water has $T = 100.0°\text{C} = 373\text{ K}$.

EXECUTE: **(a)** The heat transfer between 100°C water and 30°C water occurs over a finite temperature difference and the process is irreversible.

(b) $(270\text{ kg})c(T_2 - 30.0°\text{C}) + (5.00\text{ kg})c(T_2 - 100°\text{C}) = 0$. $T_2 = 31.27\text{ °C} = 304.42\text{ K}$.

(c) $\Delta S = (270\text{ kg})(4190\text{ J/kg}\cdot\text{K})\ln\left(\frac{304.42\text{ K}}{303.15\text{ K}}\right) + (5.00\text{ kg})(4190\text{ J/kg}\cdot\text{K})\ln\left(\frac{304.42\text{ K}}{373.15\text{ K}}\right)$.

$\Delta S = 4730\text{ J/K} + (-4265\text{ J/K}) = +470\text{ J/K}$.

EVALUATE: $\Delta S_{\text{system}} > 0$, as it should for an irreversible process.

20.27. **IDENTIFY:** Both the ice and the room are at a constant temperature, so $\Delta S = \frac{Q}{T}$. For the melting phase transition, $Q = mL_f$. Conservation of energy requires that the quantity of heat that goes into the ice is the amount of heat that comes out of the room.

SET UP: For ice, $L_f = 334\times10^3$ J/kg. When heat flows into an object, $Q > 0$, and when heat flows out of an object, $Q < 0$.

EXECUTE: **(a)** Irreversible because heat will not spontaneously flow out of 15 kg of water into a warm room to freeze the water.

(b) $\Delta S = \Delta S_{\text{ice}} + \Delta S_{\text{room}} = \dfrac{mL_f}{T_{\text{ice}}} + \dfrac{-mL_f}{T_{\text{room}}} = \dfrac{(15.0\text{ kg})(334\times10^3\text{ J/kg})}{273\text{ K}} + \dfrac{-(15.0\text{ kg})(334\times10^3\text{ J/kg})}{293\text{ K}}$. $\Delta S = +1250$ J/K.

EVALUATE: This result is consistent with the answer in (a) because $\Delta S > 0$ for irreversible processes.

20.28. **IDENTIFY:** $Q = mc\Delta T$ for the water. Example 20.6 shows that $\Delta S = mc\ln(T_2/T_1)$ when an object undergoes a temperature change. $\Delta S = Q/T$ for an isothermal process.

SET UP: For water, $c = 4190\text{ J/kg}\cdot\text{K}$. $85.0°\text{C} = 358.2$ K. $20.0°\text{C} = 293.2$ K.

EXECUTE: **(a)** $\Delta S = mc\ln\left(\dfrac{T_2}{T_1}\right) = (0.250\text{ kg})(4190\text{ J/kg}\cdot\text{K})\ln\left(\dfrac{293.2\text{ K}}{358.2\text{ K}}\right) = -210$ J/K. Heat comes out of the water and its entropy decreases.

(b) $Q = mc\Delta T = (0.250\text{ kg})(4190\text{ J/kg}\cdot\text{K})(-65.0\text{ K}) = -6.81\times10^4$ J. The amount of heat that goes into the air is $+6.81\times10^4$ J. For the air, $\Delta S = \dfrac{Q}{T} = \dfrac{+6.81\times10^4\text{ J}}{293.1\text{ K}} = +232$ J/K. $\Delta S_{\text{system}} = -210\text{ J/K} + 232\text{ J/K} = +22$ J/K.

EVALUATE: $\Delta S_{\text{system}} > 0$ and the process is irreversible.

20.29. **IDENTIFY:** The process is at constant temperature, so $\Delta S = \dfrac{Q}{T}$. $\Delta U = Q - W$.

SET UP: For an isothermal process of an ideal gas, $\Delta U = 0$ and $Q = W$. For a compression, $\Delta V < 0$ and $W < 0$.

EXECUTE: $Q = W = -1850$ J. $\Delta S = \dfrac{-1850\text{ J}}{293\text{ K}} = -6.31$ J/K.

EVALUATE: The entropy change of the gas is negative. Heat must be removed from the gas during the compression to keep its temperature constant and therefore the gas is not an isolated system.

20.30. **IDENTIFY** and **SET UP:** The initial and final states are at the same temperature, at the normal boiling point of 4.216 K. Calculate the entropy change for the irreversible process by considering a reversible isothermal process that connects the same two states, since ΔS is path independent and depends only on the initial and final states. For the reversible isothermal process we can use Eq.(20.18).

The heat flow for the helium is $Q = -mL_v$, negative since in condensation heat flows out of the helium. The heat of vaporization L_v is given in Table 17.4 and is $L_v = 20.9\times10^3$ J/kg.

EXECUTE: $Q = -mL_v = -(0.130\text{ kg})(20.9\times10^3\text{ J/kg}) = -2717$ J

$\Delta S = Q/T = -2717\text{ J}/4.216\text{ K} = -644$ J/K.

EVALUATE: The system we considered is the 0.130 kg of helium; ΔS is the entropy change of the helium. This is not an isolated system since heat must flow out of it into some other material. Our result that $\Delta S < 0$ doesn't violate the 2nd law since it is not an isolated system. The material that receives the heat that flows out of the helium would have a positive entropy change and the total entropy change would be positive.

20.31. **IDENTIFY:** Each phase transition occurs at constant temperature and $\Delta S = \dfrac{Q}{T}$. $Q = mL_v$.

SET UP: For vaporization of water, $L_v = 2256\times10^3$ J/kg.

EXECUTE: **(a)** $\Delta S = \dfrac{Q}{T} = \dfrac{mL_v}{T} = \dfrac{(1.00\text{ kg})(2256\times10^3\text{ J/kg})}{(373.15\text{ K})} = 6.05\times10^3$ J/K. Note that this is the change of entropy of the water as it changes to steam.

(b) The magnitude of the entropy change is roughly five times the value found in Example 20.5.

EVALUATE: Water is less ordered (more random) than ice, but water is far less random than steam; a consideration of the density changes indicates why this should be so.

20.32. **IDENTIFY:** The phase transition occurs at constant temperature and $\Delta S = \dfrac{Q}{T}$. $Q = mL_v$. The mass of one mole is the molecular mass M.

SET UP: For water, $L_v = 2256\times10^3$ J/kg. For N_2, $M = 28.0\times10^{-3}$ kg/mol, the boiling point is 77.34 K and $L_v = 201\times10^3$ J/kg. For silver (Ag), $M = 107.9\times10^{-3}$ kg/mol, the boiling point is 2466 K and $L_v = 2336\times10^3$ J/kg. For mercury (Hg), $M = 200.6\times10^{-3}$ kg/mol, the boiling point is 630 K and $L_v = 272\times10^3$ J/kg.

EXECUTE: **(a)** $\Delta S = \frac{Q}{T} = \frac{mL_v}{T} = \frac{(18.0\times10^{-3}\text{kg})(2256\times10^3\text{ J/kg})}{(373.15\text{ K})} = 109\text{ J/K}.$

(b) N_2: $\frac{(28.0\times10^{-3}\text{ kg})(201\times10^3\text{ J/kg})}{(77.34\text{ K})} = 72.8\text{ J/K}.$ Ag: $\frac{(107.9\times10^{-3}\text{ kg})(2336\times10^3\text{ J/kg})}{(2466\text{ K})} = 102.2\text{ J/K}.$

Hg: $\frac{(200.6\times10^{-3}\text{ kg})(272\times10^3\text{ J/kg})}{(630\text{ K})} = 86.6\text{ J/K}$

(c) The results are the same order or magnitude, all around 100 J/K.

EVALUATE: The entropy change is a measure of the increase in randomness when a certain number (one mole) goes from the liquid to the vapor state. The entropy per particle for any substance in a vapor state is expected to be roughly the same, and since the randomness is much higher in the vapor state (see Exercise 20.31), the entropy change per molecule is roughly the same for these substances.

20.33. **IDENTIFY:** During the phase transition the gallium is at a constant temperature equal to the melting point of gallium. Your hand is at a constant temperature of $98.6°\text{F} = 37.0°\text{C} = 310.1\text{ K}$. Heat $|Q| = mL_f$ flows out of your hand and into the gallium. For heat flow at constant temperature, $\Delta S = \frac{Q}{T}$.

SET UP: For gallium, $L_f = 8.04\times10^4$ J/kg and the melting point is $29.8°\text{C} = 303.0\text{ K}$.

EXECUTE: $|Q| = mL_f = (25.0\times10^{-3}\text{ kg})(8.04\times10^4\text{ J/kg}) = 2.01\times10^3\text{ J}$. For your hand,

$\Delta S = \frac{Q}{T} = \frac{-2.01\times10^3\text{ J}}{310.1\text{ K}} = -6.48\text{ J/K}$. Heat flows out of your hand, Q is negative, and ΔS is negative. For the gallium, $\Delta S = \frac{Q}{303.0\text{ K}}$. The temperature of the gallium is less than that of your hand and $|Q|$ is the same, so the magnitude of the entropy change of the gallium is greater than the magnitude of the entropy change of your hand.

EVALUATE: For the gallium, $\Delta S > 0$, so $\Delta S_{\text{system}} > 0$ and the process is irreversible.

20.34. **IDENTIFY:** Apply Eq.(20.23) and follow the procedure used in Example 20.11.

SET UP: After the partition is punctured each molecule has equal probability of being on each side of the box. The probability of two independent events occurring simultaneously is the product of the probabilities of each separate event.

EXECUTE: **(a)** On the average, each half of the box will contain half of each type of molecule, 250 of nitrogen and 50 of oxygen.

(b) See Example 20.11. The total change in entropy is

$\Delta S = kN_1\ln(2) + kN_2\ln(2) = (N_1+N_2)k\ \ln(2) = (600)(1.381\times10^{-23}\text{ J/K})\ln(2) = 5.74\times10^{-21}\text{ J/K}.$

(c) The probability is $(1/2)^{500}\times(1/2)^{100} = (1/2)^{600} = 2.4\times10^{-181}$, and is not likely to happen. The numerical result for part (c) above may not be obtained directly on some standard calculators. For such calculators, the result may be found by taking the log base ten of 0.5 and multiplying by 600, then adding 181 and then finding 10 to the power of the sum. The result is then $10^{-181}\times10^{0.87} = 2.4\times10^{-181}$.

EVALUATE: The contents of the box constitutes an isolated system. $\Delta S > 0$ and the process is irreversible.

20.35. **(a) IDENTIFY** and **SET UP:** The velocity distribution of Eq.(18.32) depends only on *T*, so in an isothermal process it does not change.

(b) EXECUTE: Calculate the change in the number of available microscopic states and apply Eq.(20.23). Following the reasoning of Example 20.11, the number of possible positions available to each molecule is altered by a factor of 3 (becomes larger). Hence the number of microscopic states the gas occupies at volume $3V$ is $w_2 = (3)^N w_1$, where N is the number of molecules and w_1 is the number of possible microscopic states at the start of the process, where the volume is V. Then, by Eq.(20.23),

$\Delta S = k\ln(w_2/w_1) = k\ln(3)^N = Nk\ln(3) = nN_Ak\ln(3) = nR\ln(3)$

$\Delta S = (2.00\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})\ln(3) = +18.3\text{ J/K}$

(c) IDENTIFY and **SET UP:** For an isothermal reversible process $\Delta S = Q/T$.

EXECUTE: Calculate W and then use the first law to calculate Q.

$\Delta T = 0$ implies $\Delta U = 0$, since system is an ideal gas.

Then by $\Delta U = Q - W$, $Q = W$.

For an isothermal process, $W = \int_{V_1}^{V_2} p\ dV = \int_{V_1}^{V_2}(nRT/V)\ dV = nRT\ln(V_2/V_1)$

Thus $Q = nRT\ln(V_2/V_1)$ and $\Delta S = Q/T = nR\ln(V_2/V_1)$

$\Delta S = (2.00\text{ mol})(8.3145\text{ J/mol}\cdot\text{K})\ln(3V_1/V_1) = +18.3\text{ J/K}$

EVALUATE: This is the same result as obtained in part (b).

20.36. **IDENTIFY:** Example 20.8 shows that for a free expansion, $\Delta S = nR\ln(V_2/V_1)$.

SET UP: $V_1 = 2.40\ \text{L} = 2.40\times10^{-3}\ \text{m}^3$

EXECUTE: $\Delta S = (0.100\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})\ln\left(\dfrac{425\ \text{m}^3}{2.40\times10^{-3}\ \text{m}^3}\right) = 10.0\ \text{J/K}$

EVALUATE: $\Delta S_{\text{system}} > 0$ and the free expansion is irreversible.

20.37. **IDENTIFY:** $e_{\text{Carnot}} = 1 - \dfrac{T_{\text{C}}}{T_{\text{H}}}$. $W = Q_{\text{H}} + Q_{\text{C}}$. $e = \dfrac{W}{Q_{\text{H}}}$.

SET UP: $pV = nRT$; the 300 K isotherm lies below the 400 K isotherm in the pV-diagram.

EXECUTE: **(a)** $e_{\text{Carnot}} = 1 - \dfrac{400\ \text{K}}{500\ \text{K}} = 0.200 = 20.0\%$.

(b) $Q_{\text{H}} = \dfrac{W}{e} = \dfrac{2000\ \text{J}}{0.200} = 10{,}000\ \text{J}$. $|Q_{\text{C}}| = |Q_{\text{H}}| - |W| = 10{,}000\ \text{J} - 2000\ \text{J} = 8000\ \text{J}$.

(c) The 500 K and 400 K isotherms and the Carnot cycle operating between those isotherms are sketched in Figure 20.37.

(d) The 300 K isotherm and the Carnot cycle operating between the 500 K and 300 K isotherms are also sketched in Figure 20.37.

(e) The cycle with $T_{\text{C}} = 300\ \text{K}$ encloses more area than the cycle with $T_{\text{C}} = 400\ \text{K}$.

(f) Less work is done on the gas during the compression at lower temperature, so less heat is ejected to keep the internal energy and temperature constant.

EVALUATE: For $T_{\text{C}} = 300\ \text{K}$, $e_{\text{Carnot}} = 0.400$. $W = eQ_{\text{H}} = (0.400)(10{,}000\ \text{J}) = 4000\ \text{J}$. $|Q_{\text{C}}| = 6000\ \text{J}$.

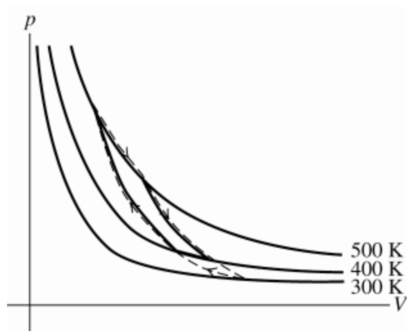


Figure 20.37

20.38. **IDENTIFY:** $W = Q_{\text{C}} + Q_{\text{H}}$. Since it is a Carnot cycle, $\dfrac{Q_{\text{C}}}{Q_{\text{H}}} = -\dfrac{T_{\text{C}}}{T_{\text{H}}}$. The heat required to melt the ice is $Q = mL_{\text{f}}$.

SET UP: For water, $L_{\text{f}} = 334\times10^3\ \text{J/kg}$. $Q_{\text{H}} > 0$, $Q_{\text{C}} < 0$. $Q_{\text{C}} = -mL_{\text{f}}$. $T_{\text{H}} = 527°\text{C} = 800.15\ \text{K}$.

EXECUTE: **(a)** $Q_{\text{H}} = +400\ \text{J}$, $W = +300\ \text{J}$. $Q_{\text{C}} = W - Q_{\text{H}} = -100\ \text{J}$.

$T_{\text{C}} = -T_{\text{H}}(Q_{\text{C}}/Q_{\text{H}}) = -(800.15\ \text{K})[(-100\ \text{J})/(400\ \text{J})] = +200\ \text{K} = -73°\text{C}$

(b) The total Q_{C} required is $-mL_{\text{f}} = -(10.0\ \text{kg})(334\times10^3\ \text{J/kg}) = -3.34\times10^6\ \text{J}$. Q_{C} for one cycle is $-100\ \text{J}$, so the number of cycles required is $\dfrac{-3.34\times10^6\ \text{J}}{-100\ \text{J/cycle}} = 3.34\times10^4$ cycles.

EVALUATE: The results depend only on the maximum temperature of the gas, not on the number of moles or the maximum pressure.

20.39. **IDENTIFY:** $e_{\text{Carnot}} = 1 - \dfrac{T_{\text{C}}}{T_{\text{H}}}$, where T_{C} and T_{H} must be in kelvins.

SET UP: $T_{\text{C}} = -90.0°\text{C} = 183\ \text{K}$.

EXECUTE: **(a)** $T_{\text{H}} = \dfrac{T_{\text{C}}}{1-e}$. For $e = 0.400$, $T_{\text{H}} = \dfrac{183\ \text{K}}{1-0.400} = 305\ \text{K}$. For $e = 0.450$, $T_{\text{H}} = \dfrac{183\ \text{K}}{1-0.450} = 333\ \text{K}$. T_{H} must be increased $28\ \text{K} = 28\ \text{C}°$.

(b) $T_{\text{C}} = (1-e)T_{\text{H}} = (1-0.450)(305\ \text{K}) = 168\ \text{K}$. T_{C} must be decreased $15\ \text{K} = 15\ \text{C}°$.

EVALUATE: A Kelvin degree is the same size as a Celsius degree, so a temperature change ΔT has the same numerical value whether it is expressed in K or in $\text{C}°$.

20.40. **IDENTIFY:** Use the ideal gas law to calculate p and V for each state. Use the first law and specific expressions for Q, W, and ΔU for each process. Use Eq.(20.4) to calculate e. Q_{H} is the net heat flow into the gas.

SET UP: $\gamma = 1.40$

$C_V = R/(\gamma - 1) = 20.79\ \text{J/mol}\cdot\text{K}$; $C_p = C_V + R = 29.10\ \text{J/mol}\cdot\text{K}$. The cycle is sketched in Figure 20.40.

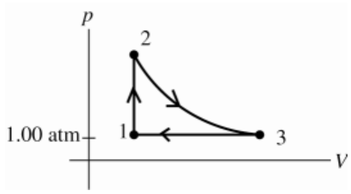


Figure 20.40

$T_1 = 300\ \text{K}$
$T_2 = 600\ \text{K}$
$T_3 = 492\ \text{K}$

EXECUTE: **(a)** point 1

$p_1 = 1.00\ \text{atm} = 1.013\times10^5\ \text{Pa}$ (given); $pV = nRT$;

$$V_1 = \frac{nRT_1}{p_1} = \frac{(0.350\ \text{mol})(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})}{1.013\times10^5\ \text{Pa}} = 8.62\times10^{-3}\ \text{m}^3$$

point 2

process $1\to2$ at constant volume so $V_2 = V_1 = 8.62\times10^{-3}\ \text{m}^3$

$pV = nRT$ and n, R, V constant implies $p_1/T_1 = p_2/T_2$

$p_2 = p_1(T_2/T_1) = (1.00\ \text{atm})(600\ \text{K}/300\ \text{K}) = 2.00\ \text{atm} = 2.03\times10^5\ \text{Pa}$

point 3

Consider the process $3\to1$, since it is simpler than $2\to3$.

Process $3\to1$ is at constant pressure so $p_3 = p_1 = 1.00\ \text{atm} = 1.013\times10^5\ \text{Pa}$

$pV = nRT$ and n, R, p constant implies $V_1/T_1 = V_3/T_3$

$V_3 = V_1(T_3/T_1) = (8.62\times10^{-3}\ \text{m}^3)(492\ \text{K}/300\ \text{K}) = 14.1\times10^{-3}\ \text{m}^3$

(b) process $1\to2$

constant volume $(\Delta V = 0)$

$Q = nC_V\Delta T = (0.350\ \text{mol})(20.79\ \text{J/mol}\cdot\text{K})(600\ \text{K} - 300\ \text{K}) = 2180\ \text{J}$

$\Delta V = 0$ and $W = 0$. Then $\Delta U = Q - W = 2180\ \text{J}$

process $2\to3$

Adiabatic means $Q = 0$.

$\Delta U = nC_V\Delta T$ (any process), so

$\Delta U = (0.350\ \text{mol})(20.79\ \text{J/mol}\cdot\text{K})(492\ \text{K} - 600\ \text{K}) = -780\ \text{J}$

Then $\Delta U = Q - W$ gives $W = Q - \Delta U = +780\ \text{J}$. (It is correct for W to be positive since ΔV is positive.)

process $3\to1$

For constant pressure

$W = p\Delta V = (1.013\times10^5\ \text{Pa})(8.62\times10^{-3}\ \text{m}^3 - 14.1\times10^{-3}\ \text{m}^3) = -560\ \text{J}$

or $W = nR\Delta T = (0.350\ \text{mol})(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K} - 492\ \text{K}) = -560\ \text{J}$, which checks. (It is correct for W to be negative, since ΔV is negative for this process.)

$Q = nC_p\Delta T = (0.350\ \text{mol})(29.10\ \text{J/mol}\cdot\text{K})(300\ \text{K} - 492\ \text{K}) = -1960\ \text{J}$

$\Delta U = Q - W = -1960\ \text{J} - (-560\ \text{K}) = -1400\ \text{J}$

or $\Delta U = nC_V\Delta T = (0.350\ \text{mol})(20.79\ \text{J/mol}\cdot\text{K})(300\ \text{K} - 492\ \text{K}) = -1400\ \text{J}$, which checks

(c) $W_{\text{net}} = W_{1\to2} + W_{2\to3} + W_{3\to1} = 0 + 780\ \text{J} - 560\ \text{J} = +220\ \text{J}$

(d) $Q_{\text{net}} = Q_{1\to2} + Q_{2\to3} + Q_{3\to1} = 2180\ \text{J} + 0 - 1960\ \text{J} = +220\ \text{J}$

(e) $$e = \frac{\text{work output}}{\text{heat energy input}} = \frac{W}{Q_{\text{H}}} = \frac{220\ \text{J}}{2180\ \text{J}} = 0.101 = 10.1\%.$$

$e(\text{Carnot}) = 1 - T_{\text{C}}/T_{\text{H}} = 1 - 300\ \text{K}/600\ \text{K} = 0.500.$

EVALUATE: For a cycle $\Delta U = 0$, so by $\Delta U = Q - W$ it must be that $Q_{\text{net}} = W_{\text{net}}$ for a cycle. We can also check that $\Delta U_{\text{net}} = 0$: $\Delta U_{\text{net}} = \Delta U_{1\to2} + \Delta U_{2\to3} + \Delta U_{3\to1} = 2180\text{ J} - 1050\text{ J} - 1130\text{ J} = 0$

$e < e(\text{Carnot})$, as it must.

20.41. **IDENTIFY:** $pV = nRT$, so pV is constant when T is constant. Use the appropriate expression to calculate Q and W for each process in the cycle. $e = \dfrac{W}{Q_{\text{H}}}$.

SET UP: For an ideal diatomic gas, $C_V = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$.

EXECUTE: **(a)** $p_aV_a = 2.0\times10^3$ J. $p_bV_b = 2.0\times10^3$ J. $pV = nRT$ so $p_aV_a = p_bV_b$ says $T_a = T_b$.

(b) For an isothermal process, $Q = W = nRT\ln(V_2/V_1)$. ab is a compression, with $V_b < V_a$, so $Q < 0$ and heat is rejected. bc is at constant pressure, so $Q = nC_p\Delta T = \dfrac{C_p}{R}p\Delta V$. ΔV is positive, so $Q > 0$ and heat is absorbed. cd is at constant volume, so $Q = nC_V\Delta T = \dfrac{C_V}{R}V\Delta p$. Δp is negative, so $Q < 0$ and heat is rejected.

(c) $T_a = \dfrac{p_aV_a}{nR} = \dfrac{2.0\times10^3\text{ J}}{(1.00)(8.314\text{ J/mol}\cdot\text{K})} = 241\text{ K}$. $T_b = \dfrac{p_bV_b}{nR} = T_a = 241\text{ K}$.

$$T_c = \frac{p_cV_c}{nR} = \frac{4.0\times10^3\text{ J}}{(1.00)(8.314\text{ J/mol}\cdot\text{K})} = 481\text{ K}.$$

(d) $Q_{ab} = nRT\ln\left(\dfrac{V_b}{V_a}\right) = (1.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(241\text{ K})\ln\left(\dfrac{0.0050\text{ m}^3}{0.010\text{ m}^3}\right) = -1.39\times10^3\text{ J}$.

$Q_{bc} = nC_p\Delta T = (1.00)(\frac{7}{2})(8.314\text{ J/mol}\cdot\text{K})(241\text{ K}) = 7.01\times10^3\text{ J}$.

$Q_{ca} = nC_V\Delta T = (1.00)(\frac{5}{2})(8.314\text{ J/mol}\cdot\text{K})(-241\text{ K}) = -5.01\times10^3\text{ J}$. $Q_{\text{net}} = Q_{ab} + Q_{bc} + Q_{ca} = 610\text{ J}$.

$W_{\text{net}} = Q_{\text{net}} = 610\text{ J}$.

(e) $e = \dfrac{W}{Q_{\text{H}}} = \dfrac{610\text{ J}}{7.01\times10^3\text{ J}} = 0.087 = 8.7\%$

EVALUATE: We can calculate W for each process in the cycle. $W_{ab} = Q_{ab} = -1.39\times10^3$ J.

$W_{bc} = p\Delta V = (4.0\times10^5\text{ Pa})(0.0050\text{ m}^3) = 2.00\times10^3\text{ J}$. $W_{ca} = 0$. $W_{\text{net}} = W_{ab} + W_{bc} + W_{ca} = 610\text{ J}$, which does equal Q_{net}.

20.42. **(a) IDENTIFY** and **SET UP:** Combine Eqs.(20.13) and (20.2) to eliminate Q_{C} and obtain an expression for Q_{H} in terms of W, T_{C}, and T_{H}.

$W = 1.00\text{ J}$, $T_{\text{C}} = 268.15\text{ K}$, $T_{\text{H}} = 290.15\text{ K}$

For the heat pump $Q_{\text{C}} > 0$ and $Q_{\text{H}} < 0$

EXECUTE: $W = Q_{\text{C}} + Q_{\text{H}}$; combining this with $\dfrac{Q_{\text{C}}}{Q_{\text{H}}} = -\dfrac{T_{\text{C}}}{T_{\text{H}}}$ gives $Q_{\text{H}} = \dfrac{W}{1 - T_{\text{C}}/T_{\text{H}}} = \dfrac{1.00\text{ J}}{1-(268.15/290.15)} = 13.2\text{ J}$

(b) Electrical energy is converted directly into heat, so an electrical energy input of 13.2 J would be required.

(c) EVALUATE: From part (a), $Q_{\text{H}} = \dfrac{W}{1 - T_{\text{C}}/T_{\text{H}}}$. Q_{H} decreases as T_{C} decreases. The heat pump is less efficient as the temperature difference through which the heat has to be "pumped" increases. In an engine, heat flows from T_{H} to T_{C} and work is extracted. The engine is more efficient the larger the temperature difference through which the heat flows.

20.43. **IDENTIFY:** $T_b = T_c$ and is equal to the maximum temperature. Use the ideal gas law to calculate T_a. Apply the appropriate expression to calculate Q for each process. $e = \dfrac{W}{Q_{\text{H}}}$. $\Delta U = 0$ for a complete cycle and for an isothermal process of an ideal gas.

SET UP: For helium, $C_V = 3R/2$ and $C_p = 5R/2$. The maximum efficiency is for a Carnot cycle, and $e_{\text{Carnot}} = 1 - T_{\text{C}}/T_{\text{H}}$.

EXECUTE: **(a)** $Q_{in} = Q_{ab} + Q_{bc}$. $Q_{out} = Q_{ca}$. $T_{max} = T_b = T_c = 327°C = 600\text{ K}$.

$$\frac{p_a V_a}{T_a} = \frac{p_b V_b}{T_b} \rightarrow T_a = \frac{p_a}{p_b} T_b = \frac{1}{3}(600\text{ K}) = 200\text{ K}.$$

$$p_b V_b = nRT_b \rightarrow V_b = \frac{nRT_b}{p_b} = \frac{(2\text{ moles})(8.31\text{ J/mol}\cdot\text{K})(600\text{ K})}{3.0\times10^5\text{ Pa}} = 0.0332\text{ m}^3.$$

$$\frac{p_b V_b}{T_b} = \frac{p_c V_c}{T_c} \rightarrow V_c = V_b \frac{p_b}{p_c} = (0.0332\text{ m}^3)\left(\frac{3}{1}\right) = 0.0997\text{ m}^3 = V_a.$$

$$Q_{ab} = nC_V \Delta T_{ab} = (2\text{ mol})\left(\frac{3}{2}\right)(8.31\text{ J/mol}\cdot\text{K})(400\text{ K}) = 9.97\times10^3\text{ J}$$

$$Q_{bc} = W_{bc} = \int_b^c p\,dV = \int_b^c \frac{nRT_b}{V} dV = nRT_b \ln\frac{V_c}{V_b} = nRT_b \ln 3.$$

$Q_{bc} = (2.00\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(600\text{ K})\ln 3 = 1.10\times10^4\text{ J}$. $Q_{in} = Q_{ab} + Q_{bc} = 2.10\times10^4\text{ J}$.

$$Q_{out} = Q_{ca} = nC_p \Delta T_{ca} = (2.00\text{ mol})\left(\frac{5}{2}\right)(8.31\text{ J/mol}\cdot\text{K})(400\text{ K}) = 1.66\times10^4\text{ J}.$$

(b) $Q = \Delta U + W = 0 + W \rightarrow W = Q_{in} - Q_{out} = 2.10\times10^4\text{ J} - 1.66\times10^4\text{ J} = 4.4\times10^3\text{ J}$.

$$e = W/Q_{in} = \frac{4.4\times10^3\text{ J}}{2.10\times10^4\text{ J}} = 0.21 = 21\%.$$

(c) $e_{max} = e_{Carnot} = 1 - \dfrac{T_C}{T_H} = 1 - \dfrac{200\text{ K}}{600\text{ K}} = 0.67 = 67\%$

EVALUATE: The thermal efficiency of this cycle is about one-third of the efficiency of a Carnot cycle that operates between the same two temperatures.

20.44. **IDENTIFY:** For a Carnot engine, $\dfrac{Q_C}{Q_H} = -\dfrac{T_C}{T_H}$. $e_{Carnot} = 1 - \dfrac{T_C}{T_H}$. $|W| = |Q_H| - |Q_C|$. $Q_H > 0$, $Q_C < 0$. $pV = nRT$.

SET UP: The work done by the engine each cycle is $mg\Delta y$, with $m = 15.0\text{ kg}$ and $\Delta y = 2.00\text{ m}$. $T_H = 773\text{ K}$. $Q_H = 500\text{ J}$.

EXECUTE: **(a)** The pV diagram is sketched in Figure 20.44.

(b) $W = mg\Delta y = (15.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = 294\text{ J}$. $|Q_C| = |Q_H| - |W| = 500\text{ J} - 294\text{ J} = 206\text{ J}$, and $Q_C = -206\text{ J}$.

$$T_C = -T_H\left(\frac{Q_C}{Q_H}\right) = -(773\text{ K})\left(\frac{-206\text{ J}}{500\text{ J}}\right) = 318\text{ K} = 45°\text{C}.$$

(c) $e = 1 - \dfrac{T_C}{T_H} = 1 - \dfrac{318\text{ K}}{773\text{ K}} = 0.589 = 58.9\%$.

(d) $|Q_C| = 206\text{ J}$.

(e) The maximum pressure is for state a. This is also where the volume is a minimum, so $V_a = 5.00\text{ L} = 5.00\times10^{-3}\text{ m}^3$. $T_a = T_H = 773\text{ K}$. $p_a = \dfrac{nRT_a}{V_a} = \dfrac{(2.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(773\text{ K})}{5.00\times10^{-3}\text{ m}^3} = 2.57\times10^6\text{ Pa}$.

EVALUATE: We can verify that $e = \dfrac{W}{Q_H}$ gives the same value for e as calculated in part (c).

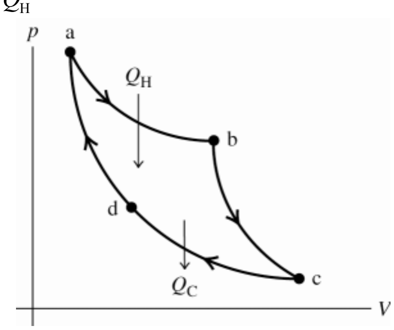


Figure 20.44

20.45. **IDENTIFY:** $e_{\max} = e_{\text{Carnot}} = 1 - T_C/T_H$. $e = \dfrac{W}{Q_H} = \dfrac{W/t}{Q_H/t}$. $W = Q_H + Q_C$ so $\dfrac{W}{t} = \dfrac{Q_C}{t} + \dfrac{Q_H}{t}$. For a temperature change $Q = mc\Delta T$.

SET UP: $T_H = 300.15$ K, $T_C = 279.15$ K. For water, $\rho = 1000\ \text{kg/m}^3$, so a mass of 1 kg has a volume of 1 L. For water, $c = 4190\ \text{J/kg}\cdot\text{K}$.

EXECUTE: **(a)** $e = 1 - \dfrac{279.15\ \text{K}}{300.15\ \text{K}} = 7.0\%$.

(b) $\dfrac{Q_H}{t} = \dfrac{P_{\text{out}}}{e} = \dfrac{210\ \text{kW}}{0.070} = 3.0\ \text{MW}$. $\dfrac{Q_C}{t} = \dfrac{Q_H}{t} - \dfrac{W}{t} = 3.0\ \text{MW} - 210\ \text{kW} = 2.8\ \text{MW}$.

(c) $\dfrac{m}{t} = \dfrac{|Q_C|/t}{c\Delta T} = \dfrac{(2.8\times 10^6\ \text{W})\,(3600\ \text{s/h})}{(4190\ \text{J/kg}\cdot\text{K})\,(4\ \text{K})} = 6\times 10^5\ \text{kg/h} = 6\times 10^5\ \text{L/h}$.

EVALUATE: The efficiency is small since T_C and T_H don't differ greatly.

20.46. **IDENTIFY:** Use Eq.(20.4) to calculate e.

SET UP: The cycle is sketched in Figure 20.46.

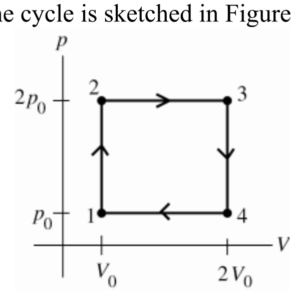


Figure 20.46

$C_V = 5R/2$

for an ideal gas $C_p = C_V + R = 7R/2$

SET UP: Calculate Q and W for each process.

<u>process $1\to 2$</u>

$\Delta V = 0$ implies $W = 0$

$\Delta V = 0$ implies $Q = nC_V\Delta T = nC_V(T_2 - T_1)$

But $pV = nRT$ and V constant says $p_1V = nRT_1$ and $p_2V = nRT_2$.

Thus $(p_2 - p_1)V = nR(T_2 - T_1)$; $V\Delta p = nR\Delta T$ (true when V is constant).

Then $Q = nC_V\Delta T = nC_V(V\Delta p/nR) = (C_V/R)V\Delta p = (C_V/R)V_0(2p_0 - p_0) = (C_V/R)p_0V_0$. $Q > 0$; heat is absorbed by the gas.)

<u>process $2\to 3$</u>

$\Delta p = 0$ so $W = p\Delta V = p(V_3 - V_2) = 2p_0(2V_0 - V_0) = 2p_0V_0$ (W is positive since V increases.)

$\Delta p = 0$ implies $Q = nC_p\Delta T = nC_p(T_2 - T_1)$

But $pV = nRT$ and p constant says $pV_1 = nRT_1$ and $pV_2 = nRT_2$.

Thus $p(V_2 - V_1) = nR(T_2 - T_1)$; $p\Delta V = nR\Delta T$ (true when p is constant).

Then $Q = nC_p\Delta T = nC_p(p\Delta V/nR) = (C_p/R)p\Delta V = (C_p/R)2p_0(2V_0 - V_0) = (C_p/R)2p_0V_0$. ($Q > 0$; heat is absorbed by the gas.)

<u>process $3\to 4$</u>

$\Delta V = 0$ implies $W = 0$

$\Delta V = 0$ so

$Q = nC_V\Delta T = nC_V(V\Delta p/nR) = (C_V/R)(2V_0)(p_0 - 2p_0) = -2(C_V/R)p_0V_0$

($Q < 0$ so heat is rejected by the gas.)

<u>process $4\to 1$</u>

$\Delta p = 0$ so $W = p\Delta V = p(V_1 - V_4) = p_0(V_0 - 2V_0) = -p_0V_0$ (W is negative since V decreases)

$\Delta p = 0$ so $Q = nC_p\Delta T = nC_p(p\Delta V/nR) = (C_p/R)p\Delta V = (C_p/R)p_0(V_0 - 2V_0) = -(C_p/R)p_0V_0$ ($Q < 0$ so heat is rejected by the gas.)

total work performed by the gas during the cycle:

$W_{tot} = W_{1\to 2} + W_{2\to 3} + W_{3\to 4} + W_{4\to 1} = 0 + 2p_0V_0 + 0 - p_0V_0 = p_0V_0$

(Note that W_{tot} equals the area enclosed by the cycle in the pV-diagram.)

total heat absorbed by the gas during the cycle (Q_H):

Heat is absorbed in processes $1\to 2$ and $2\to 3$.

$$Q_H = Q_{1\to 2} + Q_{2\to 3} = \frac{C_V}{R}p_0V_0 + 2\frac{C_p}{R}p_0V_0 = \left(\frac{C_V + 2C_p}{R}\right)p_0V_0$$

But $C_p = C_V + R$ so $Q_H = \frac{C_V + 2(C_V + R)}{R}p_0V_0 = \left(\frac{3C_V + 2R}{R}\right)p_0V_0.$

total heat rejected by the gas during the cycle (Q_C):

Heat is rejected in processes $3\to 4$ and $4\to 1$.

$$Q_C = Q_{3\to 4} + Q_{4\to 1} = -2\frac{C_V}{R}p_0V_0 - \frac{C_p}{R}p_0V_0 = -\left(\frac{2C_V + C_p}{R}\right)p_0V_0$$

But $C_p = C_V + R$ so $Q_C = -\frac{2C_V + (C_V + R)}{R}p_0V_0 = -\left(\frac{3C_V + R}{R}\right)p_0V_0.$

efficiency

$$e = \frac{W}{Q_H} = \frac{p_0V_0}{([3C_V + 2R]/R)(p_0V_0)} = \frac{R}{3C_V + 2R} = \frac{R}{3(5R/2) + 2R} = \frac{2}{19}.$$

$e = 0.105 = 10.5\%$

EVALUATE: As a check on the calculations note that $Q_C + Q_H = -\left(\frac{3C_V + R}{R}\right)p_0V_0 + \left(\frac{3C_V + 2R}{R}\right)p_0V_0 = p_0V_0 = W$, as it should.

20.47. IDENTIFY: Use $pV = nRT$. Apply the expressions for Q and W that apply to each type of process. $e = \frac{W}{Q_H}$.

SET UP: For O_2, $C_V = 20.85\ \text{J/mol}\cdot\text{K}$ and $C_p = 29.17\ \text{J/mol}\cdot\text{K}$.

EXECUTE: **(a)** $p_1 = 2.00$ atm, $V_1 = 4.00$ L, $T_1 = 300$ K.

$p_2 = 2.00$ atm. $\frac{V_1}{T_1} = \frac{V_2}{T_2}$. $V_2 = \left(\frac{T_2}{T_1}\right)V_1 = \left(\frac{450\ \text{K}}{300\ \text{K}}\right)(4.00\ \text{L}) = 6.00\ \text{L}.$

$V_3 = 6.00$ L. $\frac{p_2}{T_2} = \frac{p_3}{T_3}$. $p_3 = \left(\frac{T_3}{T_2}\right)p_2 = \left(\frac{250\ \text{K}}{450\ \text{K}}\right)(2.00\ \text{atm}) = 1.11\ \text{atm}$

$V_4 = 4.00$ L. $p_3V_3 = p_4V_4$. $p_4 = p_3\left(\frac{V_3}{V_4}\right) = (1.11\ \text{atm})\left(\frac{6.00\ \text{L}}{4.00\ \text{L}}\right) = 1.67\ \text{atm}.$

These processes are shown in Figure 20.47.

(b) $n = \frac{p_1V_1}{RT_1} = \frac{(2.00\ \text{atm})(4.00\ \text{L})}{(0.08206\ \text{L}\cdot\text{atm/mol}\cdot\text{K})(300\ \text{K})} = 0.325\ \text{mol}$

process $1\to 2$: $W = p\Delta V = nR\Delta T = (0.325\ \text{mol})(8.315\ \text{J/mol}\cdot\text{K})(150\ \text{K}) = 405\ \text{J}.$

$Q = nC_p\Delta T = (0.325\ \text{mol})(29.17\ \text{J/mol}\cdot\text{K})(150\ \text{K}) = 1422\ \text{J}.$

process $2\to 3$: $W = 0$. $Q = nC_V\Delta T = (0.325\ \text{mol})(20.85\ \text{J/mol}\cdot\text{K})(-200\ \text{K}) = -1355\ \text{J}.$

process $3\to 4$: $\Delta U = 0$ and $Q = W = nRT_3\ln\left(\frac{V_4}{V_3}\right) = (0.325\ \text{mol})(8.315\ \text{J/mol}\cdot\text{K})(250\ \text{K})\ln\left(\frac{4.00\ \text{L}}{6.00\ \text{L}}\right) = -274\ \text{J}.$

process $4\to 1$: $W = 0$. $Q = nC_V\Delta T = (0.325\ \text{mol})(20.85\ \text{J/mol}\cdot\text{K})(50\ \text{K}) = 339\ \text{J}.$

(c) $W = 405\ \text{J} - 274\ \text{J} = 131\ \text{J}$

(d) $e = \frac{W}{Q_H} = \frac{131\ \text{J}}{1422\ \text{J} + 339\ \text{J}} = 0.0744 = 7.44\%.$

$e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = 1 - \frac{250\ \text{K}}{450\ \text{K}} = 0.444 = 44.4\%$; e_{Carnot} is much larger.

EVALUATE: $Q_{\text{tot}} = +1422\text{ J} + (-1355\text{ J}) + (-274\text{ J}) + 339\text{ J} = 132\text{ J}.$ This is equal to W_{tot}, apart from a slight difference due to rounding. For a cycle, $W_{\text{tot}} = Q_{\text{tot}}$, since $\Delta U = 0$.

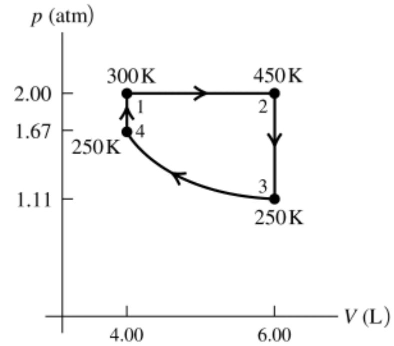


Figure 20.47

20.48. **IDENTIFY** and **SET UP:** For the constant pressure processes *ab* and *cd* calculate *W* and use the first law to calculate *Q*. Calculate Q_{tot} and use that $W_{\text{tot}} = Q_{\text{tot}}$ for a cycle. The coefficient of performance is given by Eq.(20.9); Q_C is the net heat that goes into the system. The cycle is sketched in Figure 20.48.

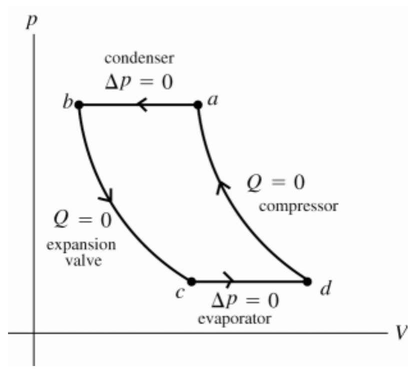


Figure 20.48

EXECUTE: **(a)** process $c \to d$

$\Delta U = U_d - U_c = 1657\times10^3\text{ J} - 1005\times10^3\text{ J} = 6.52\times10^5\text{ J}$

$W = \int_{V_c}^{V_d} p\,dV = p\Delta V$ (since is a constant pressure process)

$W = (363\times10^3\text{ Pa})(0.4513\text{ m}^3 - 0.2202\text{ m}^3) = +8.39\times10^4\text{ J}$ (positive since process is an expansion)

$\Delta U = Q - W$ so $Q = \Delta U + W = 6.52\times10^5\text{ J} + 8.39\times10^4\text{ J} = 7.36\times10^5\text{ J}.$

(Q positive so heat goes into the coolant)

(b) process $a \to b$

$\Delta U = U_b - U_a = 1171\times10^3\text{ J} - 1969\times10^3\text{ J} = -7.98\times10^5\text{ J}$

$W = p\Delta V = (2305\times10^3\text{ Pa})(0.00946\text{ m}^3 - 0.0682\text{ m}^3) = -1.35\times10^5\text{ J}$

(negative since $\Delta V < 0$ for the process)

$Q = \Delta U + W = -7.98\times10^5\text{ J} - 1.35\times10^5\text{ J} = -9.33\times10^5\text{ J}$

(negative so heat comes out of coolant).

(c) The coolant cannot be treated as an ideal gas, so we can't calculate *W* for the adiabatic processes. But $\Delta U = 0$ (for cycle) so $W_{\text{net}} = Q_{\text{net}}$.

$Q = 0$ for the two adiabatic processes, so $Q_{\text{net}} = Q_{cd} + Q_{ab} = 7.36\times10^5\text{ J} - 9.33\times10^5\text{ J} = -1.97\times10^5\text{ J}$

Thus $W_{\text{net}} = -1.97\times10^5\text{ J}$ (negative since work is done on the coolant, the working substance).

(d) $K = Q_C/|W| = (+7.36\times10^5\text{ J})/(+1.97\times10^5\text{ J}) = 3.74.$

EVALUATE: $W_{\text{net}} < 0$ when the cycle is taken in the counterclockwise direction, as is the case here.

20.49. **IDENTIFY:** Use $\Delta U = Q - W$ and the appropriate expressions for Q, W and ΔU for each type of process. $pV = nRT$ relates ΔT to p and V values. $e = \dfrac{W}{Q_H}$, where Q_H is the heat that enters the gas during the cycle.

SET UP: For a monatomic ideal gas, $C_P = \frac{5}{2}R$ and $C_V = \frac{3}{2}R$.

(a) ab: The temperature changes by the same factor as the volume, and so

$$Q = nC_P\Delta T = \frac{C_P}{R}p_a(V_a - V_b) = (2.5)(3.00\times10^5\,\text{Pa})(0.300\text{ m}^3) = 2.25\times10^5\,\text{J}.$$

The work $p\Delta V$ is the same except for the factor of $\frac{5}{2}$, so $W = 0.90\times10^5$ J.

$\Delta U = Q - W = 1.35\times10^5$ J.

bc: The temperature now changes in proportion to the pressure change, and

$Q = \frac{3}{2}(p_c - p_b)V_b = (1.5)(-2.00\times10^5\text{ Pa})(0.800\text{ m}^3) = -2.40\times10^5$ J, and the work is zero

$(\Delta V = 0)$. $\Delta U = Q - W = -2.40\times10^5$ J.

ca: The easiest way to do this is to find the work done first; W will be the negative of area in the p-V plane bounded by the line representing the process ca and the verticals from points a and c. The area of this trapezoid is $\frac{1}{2}(3.00\times10^5\text{ Pa} + 1.00\times10^5\text{ Pa})(0.800\text{ m}^3 - 0.500\text{ m}^3) = 6.00\times10^4$ J and so the work is -0.60×10^5 J. ΔU must be 1.05×10^5 J (since $\Delta U = 0$ for the cycle, anticipating part (b)), and so Q must be $\Delta U + W = 0.45\times10^5$ J.

(b) See above; $Q = W = 0.30\times10^5$ J, $\Delta U = 0$.

(c) The heat added, during process ab and ca, is $2.25\times10^5\text{ J} + 0.45\times10^5\text{ J} = 2.70\times10^5$ J and the efficiency is

$$e = \frac{W}{Q_H} = \frac{0.30\times10^5}{2.70\times10^5} = 0.111 = 11.1\%.$$

EVALUATE: For any cycle, $\Delta U = 0$ and $Q = W$.

20.50. **IDENTIFY:** Use the appropriate expressions for Q, W and ΔU for each process. $e = W/Q_H$ and $e_{\text{Carnot}} = 1 - T_C/T_H$.

SET UP: For this cycle, $T_H = T_2$ and $T_C = T_1$

EXECUTE: **(a)** ab: For the isothermal process, $\Delta T = 0$ and $\Delta U = 0$.

$W = nRT_1\ln(V_b/V_a) = nRT_1\ln(1/r) = -nRT_1\ln(r)$ and $Q = W = -nRT_1\ln(r)$.

bc: For the isochoric process, $\Delta V = 0$ and $W = 0$. $Q = \Delta U = nC_V\Delta T = nC_V(T_2 - T_1)$.

cd: As in the process ab, $\Delta U = 0$ and $W = Q = nRT_2\ln(r)$.

da: As in process bc, $\Delta V = 0$ and $W = 0$; $\Delta U = Q = nC_V(T_1 - T_2)$.

(b) The values of Q for the processes are the negatives of each other.

(c) The net work for one cycle is $W_{\text{net}} = nR(T_2 - T_1)\ln(r)$, and the heat added (neglecting the heat exchanged during the isochoric expansion and compression, as mentioned in part (b)) is $Q_{cd} = nRT_2\ln(r)$, and the efficiency is

$e = \dfrac{W_{\text{net}}}{Q_{cd}} = 1 - (T_1/T_2)$. This is the same as the efficiency of a Carnot-cycle engine operating between the two temperatures.

EVALUATE: For a Carnot cycle two steps in the cycle are isothermal and two are adiabatic and all the heat flow occurs in the isothermal processes. For the Stirling cycle all the heat flow is also in the isothermal steps, since the net heat flow in the two constant volume steps is zero.

20.51. **IDENTIFY:** The efficiency of the composite engine is $e_{12} = \dfrac{W_1 + W_2}{Q_{H1}}$, where Q_{H1} is the heat input to the first engine and W_1 and W_2 are the work outputs of the two engines. For any heat engine, $W = Q_C + Q_H$, and for a Carnot engine, $\dfrac{Q_{\text{low}}}{Q_{\text{high}}} = -\dfrac{T_{\text{low}}}{T_{\text{high}}}$, where Q_{low} and Q_{high} are the heat flows at the two reservoirs that have temperatures T_{low} and T_{high}.

SET UP: $Q_{\text{high},2} = -Q_{\text{low},1}$. $T_{\text{low},1} = T'$, $T_{\text{high},1} = T_H$, $T_{\text{low},2} = T_C$ and $T_{\text{high},2} = T'$.

EXECUTE: $e_{12} = \dfrac{W_1 + W_2}{Q_{H1}} = \dfrac{Q_{\text{high},1} + Q_{\text{low},1} + Q_{\text{high},2} + Q_{\text{low},2}}{Q_{\text{high},1}}$. Since $Q_{\text{high},2} = -Q_{\text{low},1}$, this reduces to $e_{12} = 1 + \dfrac{Q_{\text{low},2}}{Q_{\text{high},1}}$.

$Q_{\text{low},2} = -Q_{\text{high},2}\dfrac{T_{\text{low},2}}{T_{\text{high},2}} = Q_{\text{low},1}\dfrac{T_C}{T'} = -Q_{\text{high},1}\left(\dfrac{T_{\text{low},1}}{T_{\text{high},1}}\right)\dfrac{T_C}{T'} = -Q_{\text{high},1}\left(\dfrac{T'}{T_H}\right)\dfrac{T_C}{T'}$. This gives $e_{12} = 1 - \dfrac{T_C}{T_H}$. The efficiency of the composite system is the same as that of the original engine.

EVALUATE: The overall efficiency is independent of the value of the intermediate temperature T'.

20.52. **IDENTIFY:** $e = \dfrac{W}{Q_H}$. 1 day $= 8.64\times10^4$ s. For the river water, $Q = mc\Delta T$, where the heat that goes into the water is the heat Q_C rejected by the engine. The density of water is $1000\ \text{kg/m}^3$. When an object undergoes a temperature change, $\Delta S = mc\ln(T_2/T_1)$.

SET UP: $18.0°\text{C} = 291.1$ K. $18.5°\text{C} = 291.6$ K.

EXECUTE: **(a)** $Q_H = \dfrac{W}{e}$ so $P_H = \dfrac{P_W}{e} = \dfrac{1000\ \text{MW}}{0.40} = 2.50\times10^3$ MW.

(b) The heat input in one day is $(2.50\times10^9\ \text{W})(8.64\times10^4\ \text{s}) = 2.16\times10^{14}$ J. The mass of coal used per day is $\dfrac{2.16\times10^{14}\ \text{J}}{2.65\times10^7\ \text{J/kg}} = 8.15\times10^6$ kg.

(c) $|Q_H| = |W| + |Q_C|$. $|Q_C| = |Q_H| - |W|$. $P_C = P_H - P_W = 2.50\times10^3\ \text{MW} - 1000\ \text{MW} = 1.50\times10^3$ MW.

(d) The heat input to the river is 1.50×10^9 J/s. $Q = mc\Delta T$ and $\Delta T = 0.5\ \text{C}°$ gives

$$m = \frac{Q}{c\Delta T} = \frac{1.50\times10^9\ \text{J}}{(4190\ \text{J/kg}\cdot\text{K})(0.5\ \text{K})} = 7.16\times10^5\ \text{kg}.\quad V = \frac{m}{\rho} = 716\ \text{m}^3.$$ The river flow rate must be $716\ \text{m}^3/\text{s}$.

(e) In one second, 7.16×10^5 kg of water goes from 291.1 K to 291.6 K.

$$\Delta S = mc\ln\left(\frac{T_2}{T_1}\right) = (7.16\times10^5\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})\ln\left(\frac{291.6\ \text{K}}{291.1\ \text{K}}\right) = 5.1\times10^6\ \text{J/K}.$$

EVALUATE: The entropy of the river increases because heat flows into it. The mass of coal used per second is huge.

20.53. **(a) IDENTIFY** and **SET UP:** Calcualte e from Eq.(20.6), Q_C from Eq.(20.4) and then W from Eq.(20.2).

EXECUTE: $e = 1 - 1/(r^{\gamma-1}) = 1 - 1/(10.6^{0.4}) = 0.6111$

$e = (Q_H + Q_C)/Q_H$ and we are given $Q_H = 200$ J; calculate Q_C.

$Q_C = (e-1)Q_H = (0.6111-1)(200\ \text{J}) = -78$ J (negative since corresponds to heat leaving)

Then $W = Q_C + Q_H = -78\ \text{J} + 200\ \text{J} = 122$ J. (Positive, in agreement with Fig. 20.6.)

EVALUATE: Q_H, $W > 0$, and $Q_C < 0$ for an engine cycle.

(b) IDENTIFY and **SET UP:** The stoke times the bore equals the change in volume. The initial volume is the final volume V times the compression ratio r. Combining these two expressions gives an equation for V. For each cylinder of area $A = \pi(d/2)^2$ the piston moves 0.864 m and the volume changes from rV to V, as shown in Figure 20.53a.

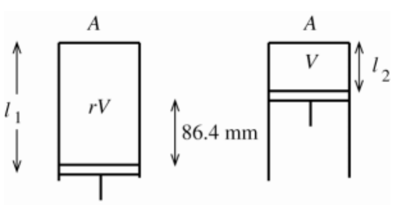


Figure 20.53a

$l_1A = rV$
$l_2A = V$
and
$l_1 - l_2 = 86.4\times10^{-3}$ m

EXECUTE: $l_1A - l_2A = rV - V$ and $(l_1 - l_2)A = (r-1)V$

$$V = \frac{(l_1-l_2)A}{r-1} = \frac{(86.4\times10^{-3}\ \text{m})\pi(41.25\times10^{-3}\ \text{m})^2}{10.6-1} = 4.811\times10^{-5}\ \text{m}^3$$

At point a the volume is $rV = 10.6(4.811\times10^{-5}\ \text{m}^3) = 5.10\times10^{-4}\ \text{m}^3$.

(c) IDENTIFY and **SET UP:** The processes in the Otto cycle are either constant volume or adiabatic. Use the Q_H that is given to calculate ΔT for process bc. Use Eq.(19.22) and $pV = nRT$ to relate p, V and T for the adiabatic processes ab and cd.

EXECUTE: point a: $T_a = 300$ K, $p_a = 8.50\times10^4$ Pa, and $V_a = 5.10\times10^{-4}\ \text{m}^3$

point b: $V_b = V_a/r = 4.81\times10^{-5}\ \text{m}^3$. Process $a \to b$ is adiabatic, so $T_aV_a^{\gamma-1} = T_bV_b^{\gamma-1}$.

$T_a(rV)^{\gamma-1} = T_bV^{\gamma-1}$

$T_b = T_ar^{\gamma-1} = 300\ \text{K}(10.6)^{0.4} = 771$ K

$pV = nRT$ so $pV/T = nR = \text{constant}$, so $p_aV_a/T_a = p_bV_b/T_b$

$p_b = p_a(V_a/V_b)(T_b/T_a) = (8.50\times10^4\ \text{Pa})(rV/V)(771\ \text{K}/300\ \text{K}) = 2.32\times10^6$ Pa

point c: Process $b \to c$ is at constant volume, so $V_c = V_b = 4.81\times10^{-5}\ \text{m}^3$

$Q_\text{H} = nC_V\Delta T = nC_V(T_c - T_b)$. The problem specifies $Q_\text{H} = 200\ \text{J}$; use to calculate T_c. First use the p, V, T values at point a to calculate the number of moles n.

$$n = \frac{pV}{RT} = \frac{(8.50\times10^4\ \text{Pa})(5.10\times10^{-4}\ \text{m}^3)}{(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})} = 0.01738\ \text{mol}$$

Then $T_c - T_b = \dfrac{Q_\text{H}}{nC_V} = \dfrac{200\ \text{J}}{(0.01738\ \text{mol})(20.5\ \text{J/mol}\cdot\text{K})} = 561.3\ \text{K}$, and $T_c = T_b + 561.3\ \text{K} = 771\ \text{K} + 561\ \text{K} = 1332\ \text{K}$

$p/T = nR/V = \text{constant}$ so $p_b/T_b = p_c/T_c$

$p_c = p_b(T_c/T_b) = (2.32\times10^6\ \text{Pa})(1332\ \text{K}/771\ \text{K}) = 4.01\times10^6\ \text{Pa}$

point d: $V_d = V_a = 5.10\times10^{-4}\ \text{m}^3$

process $c \to d$ is adiabatic, so $T_dV_d^{\gamma-1} = T_cV_c^{\gamma-1}$

$T_d(rV)^{\gamma-1} = T_cV^{\gamma-1}$

$T_d = T_c/r^{\gamma-1} = 1332\ \text{K}/10.6^{0.4} = 518\ \text{K}$

$p_cV_c/T_c = p_dV_d/T_d$

$p_d = p_c(V_c/V_d)(T_d/T_c) = (4.01\times10^6\ \text{Pa})(V/rV)(518\ \text{K}/1332\ \text{K}) = 1.47\times10^5\ \text{Pa}$

EVALUATE: Can look at process $d \to a$ as a check.

$Q_\text{C} = nC_V(T_a - T_d) = (0.01738\ \text{mol})(20.5\ \text{J/mol}\cdot\text{K})(300\ \text{K} - 518\ \text{K}) = -78\ \text{J}$, which agrees with part (a). The cycle is sketched in Figure 20.53b.

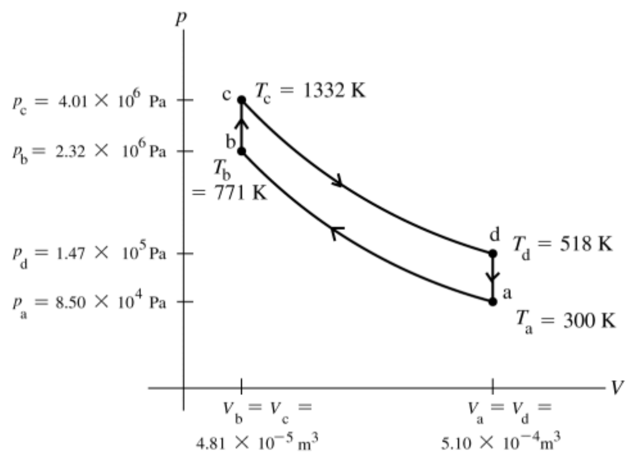


Figure 20.53b

(d) IDENTIFY and SET UP: The Carnot efficiency is given by Eq.(20.14). T_H is the highest temperature reached in the cycle and T_C is the lowest.

EXECUTE: From part (a) the efficiency of this Otto cycle is $e = 0.611 = 61.1\%$.
The efficiency of a Carnot cycle operating between 1332 K and 300 K is
$e(\text{Carnot}) = 1 - T_\text{C}/T_\text{H} = 1 - 300\ \text{K}/1332\ \text{K} = 0.775 = 77.5\%$, which is larger.

EVALUATE: The 2nd law requires that $e \le e(\text{Carnot})$, and our result obeys this law.

20.54. IDENTIFY: $K = \dfrac{|Q_\text{C}|}{|W|}$. $|Q_\text{H}| = |Q_\text{C}| + |W|$. The heat flows for the inside and outside air occur at constant T, so $\Delta S = Q/T$.

SET UP: $21.0°\text{C} = 294.1\ \text{K}$. $35.0°\text{C} = 308.1\ \text{K}$.

EXECUTE: **(a)** $|Q_\text{C}| = K|W|$. $P_\text{C} = KP_\text{W} = (2.80)(800\ \text{W}) = 2.24\times10^3\ \text{W}$.

(b) $P_\text{H} = P_\text{C} + P_\text{W} = 2.24\times10^3\ \text{W} + 800\ \text{W} = 3.04\times10^3\ \text{W}$.

(c) In $1\ \text{h} = 3600\ \text{s}$, $Q_\text{H} = P_\text{H}t = 1.094\times10^7\ \text{J}$. $\Delta S_\text{out} = \dfrac{Q_\text{H}}{T_\text{H}} = \dfrac{1.094\times10^7\ \text{J}}{308.1\ \text{K}} = 3.55\times10^4\ \text{J/K}$.

(d) $Q_C = P_C t = 8.064\times10^6$ J. Heat Q_C is removed from the inside air.

$\Delta S_{in} = \dfrac{-Q_C}{T_C} = \dfrac{-8.064\times10^6\text{ J}}{294.1\text{ K}} = -2.74\times10^4$ J/K. $\Delta S_{net} = \Delta S_{out} + \Delta S_{in} = 8.1\times10^3$ J/K.

EVALUATE: The increase in the entropy of the outside air is greater than the entropy decrease of the air in the room.

20.55. **IDENTIFY** and **SET UP:** Use Eq.(20.13) for an infinitesimal heat flow dQ_H from the hot reservoir and use that expression with Eq.(20.19) to relate ΔS_H, the entropy change of the hot reservoir, to $|Q_C|$

(a) EXECUTE: Consider an infinitesimal heat flow dQ_H that occurs when the temperature of the hot reservoir is T':

$dQ_C = -(T_C/T')dQ_H$

$\int dQ_C = -T_C \int \dfrac{dQ_H}{T'}$

$|Q_C| = T_C \left|\int \dfrac{dQ_H}{T'}\right| = T_C|\Delta S_H|$

(b) The 1.00 kg of water (the high-temperature reservoir) goes from 373 K to 273 K.

$Q_H = mc\Delta T = (1.00\text{ kg})(4190\text{ J/kg}\cdot\text{K})(100\text{ K}) = 4.19\times10^5$ J

$\Delta S_H = mc\ln(T_2/T_1) = (1.00\text{ kg})(4190\text{ J/kg}\cdot\text{K})\ln(273/373) = -1308$ J/K

The result of part (a) gives $|Q_C| = (273\text{ K})(1308\text{ J/K}) = 3.57\times10^5$ J

Q_C comes out of the engine, so $Q_C = -3.57\times10^5$ J

Then $W = Q_C + Q_H = -3.57\times10^5\text{ J} + 4.19\times10^5\text{ J} = 6.2\times10^4$ J.

(c) 2.00 kg of water goes from 323 K to 273 K

$Q_H = -mc\Delta T = (2.00\text{ kg})(4190\text{ J/kg}\cdot\text{K})(50\text{ K}) = 4.19\times10^5$ J

$\Delta S_H = mc\ln(T_2/T_1) = (2.00\text{ kg})(4190\text{ J/kg}\cdot\text{K})\ln(272/323) = -1.41\times10^3$ J/K

$Q_C = -T_C|\Delta S_H| = -3.85\times10^5$ J

$W = Q_C + Q_H = 3.4\times10^4$ J

(d) EVALUATE: More work can be extracted from 1.00 kg of water at 373 K than from 2.00 kg of water at 323 K even though the energy that comes out of the water as it cools to 273 K is the same in both cases. The energy in the 323 K water is less available for conversion into mechanical work.

20.56. **IDENTIFY:** The maximum power that can be extracted is the total kinetic energy *K* of the mass of air that passes over the turbine blades in time *t*.

SET UP: The volume of a cylinder of diameter *d* and length *L* is $(\pi d^2/4)L$. Kinetic energy is $\frac{1}{2}mv^2$.

EXECUTE: **(a)** The cylinder described contains a mass of air $m = \rho(\pi d^2/4)L,$ and so the total kinetic energy is $K = \rho(\pi/8)d^2Lv^2$. This mass of air will pass by the turbine in a time $t = L/v$, and so the maximum power is $P = \dfrac{K}{t} = \rho(\pi/8)d^2v^3$. Numerically, the product $\rho_{air}(\pi/8) \approx 0.5\text{ kg/m}^3 = 0.5\text{ W}\cdot\text{s}^3/\text{m}^5$. This completes the proof.

(b) $v = \left(\dfrac{P/e}{kd^2}\right)^{1/3} = \left(\dfrac{(3.2\times10^6\text{ W})/(0.25)}{(0.5\text{ W}\cdot\text{s}^3/\text{m}^5)(97\text{ m})^2}\right)^{1/3} = 14\text{ m/s} = 50\text{ km/h}.$

(c) Wind speeds tend to be higher in mountain passes.

EVALUATE: The maximum power is proportional to v^3, so increases rapidly with increase in wind speed.

20.57. **IDENTIFY:** For a Carnot device, $\dfrac{T_C}{T_H} = -\dfrac{Q_C}{Q_H}$. $W = Q_H + Q_C$.

SET UP: $Q_C = 1000$ J. $10.0°\text{C} = 283.1$ K. $35.0°\text{C} = 308.1$ K. $15.0°\text{C} = 288.1$ K.

EXECUTE: **(a)** $Q_H = -\left(\dfrac{T_H}{T_C}\right)Q_C = -\left(\dfrac{308.1\text{ K}}{283.1\text{ K}}\right)(1000\text{ J}) = -1.088\times10^3$ J. $W = 1000\text{ J} + (-1.088\times10^3\text{ J}) = -88$ J.

(b) Now $Q_H = -\left(\dfrac{288.1\text{ K}}{283.1\text{ K}}\right)(1000\text{ J}) = -1.018\times10^3$ J. $W = 1000\text{ J} + (-1.018\times10^3\text{ J}) = -18$ J.

(c) The *pV*-diagrams for the two Carnot cycles are sketched in Figure 20.57.

EVALUATE: More work must be done to move the heat energy through a greater temperature difference.

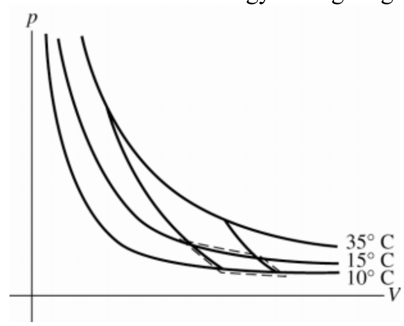


Figure 20.57

20.58. **IDENTIFY** and **SET UP:** First use the methods of Chapter 17 to calculate the final temperature T of the system.

EXECUTE: 0.600 kg of water (cools from 45.0°C to T)

$Q = mc\Delta T = (0.600\text{ kg})(4190\text{ J/kg}\cdot\text{K})(T - 45.0°\text{C}) = (2514\text{ J/K})T - 1.1313\times10^5\text{ J}$

0.0500 kg of ice (warms to 0°C, melts, and water warms from 0°C to T)

$Q = mc_{\text{ice}}(0°\text{C} - (-15.0°\text{C})) + mL_\text{f} + mc_{\text{water}}(T - 0°\text{C})$

$Q = 0.0500\text{ kg}\left[(2100\text{ J/kg}\cdot\text{K})(15.0°\text{C}) + 334\times10^3\text{ J/kg} + (4190\text{ J/kg}\cdot\text{K})(T - 0°\text{C})\right]$

$Q = 1575\text{ J} + 1.67\times10^4\text{ J} + (209.5\text{ J/K})T = 1.828\times10^4\text{ J} + (209.5\text{ J/K})T$

$Q_{\text{system}} = 0$ gives $(2514\text{ J/K})T - 1.1313\times10^5\text{ J} + 1.828\times10^4\text{ J} + (209.5\text{ J/K})T = 0$

$(2.724\times10^3\text{ J/K})T = 9.485\times10^4\text{ J}$

$T = (9.485\times10^4\text{ J})/(2.724\times10^3\text{ J/K}) = 34.83°\text{C} = 308\text{ K}$

EVALUATE: The final temperature must lie between –15.0°C and 45.0°C. A final temperature of 34.8°C is consistent with only liquid water being present at equilibrium.

IDENTIFY and **SET UP:** Now we can calculate the entropy changes. Use $\Delta S = Q/T$ for phase changes and the method of Example 20.6 to calculate ΔS for temperature changes.

EXECUTE: ice: The process takes ice at –15°C and produces water at 34.8°C. Calculate ΔS for a reversible process between these two states, in which heat is added very slowly. ΔS is path independent, so ΔS for a reversible process is the same as ΔS for the actual (irreversible) process as long as the initial and final states are the same.

$\Delta S = \int_1^2 dQ/T$, where T must be in kelvins

For a temperature change $dQ = mc\,dT$ so $\Delta S = \int_{T_1}^{T_2}(mc/T)\,dT = mc\ln(T_2/T_1)$.

For a phase change, since it occurs at constant T,

$\Delta S = \int_1^2 dQ/T = Q/T = \pm mL/T$.

Therefore $\Delta S_{\text{ice}} = mc_{\text{ice}}\ln(273\text{ K}/258\text{ K}) + mL_\text{f}/273\text{ K} + mc_{\text{water}}\ln(308\text{ K}/273\text{ K})$

$\Delta S_{\text{ice}} = (0.0500\text{ kg})[(2100\text{ J/kg}\cdot\text{K})\ln(273\text{ K}/258\text{ K}) + (334\times10^3\text{ J/kg})/273\text{ K} + (4190\text{ J/kg}\cdot\text{K})\ln(308\text{ K}/273\text{ K})]$

$\Delta S_{\text{ice}} = 5.93\text{ J/K} + 61.17\text{ J/K} + 25.27\text{ J/K} = 92.4\text{ J/K}$

water: $\Delta S_{\text{water}} = mc\ln(T_2/T_1) = (0.600\text{ kg})(4190\text{ J/kg}\cdot\text{K})\ln(308\text{ K}/318\text{ K}) = -80.3\text{ J/K}$

For the system, $\Delta S = \Delta S_{\text{ice}} + \Delta S_{\text{water}} = 92.4\text{ J/K} - 80.3\text{ J/K} = +12\text{ J/K}$

EVALUATE: Our calculation gives $\Delta S > 0$, as it must for an irreversible process of an isolated system.

20.59. **IDENTIFY:** Apply Eq.(20.19). From the derivation of Eq. (20.6), $T_b = r^{\gamma-1}T_a$ and $T_c = r^{\gamma-1}T_d$.

SET UP: For a constant volume process, $dQ = nC_V dT$.

EXECUTE: **(a)** For a constant-volume process for an ideal gas, where the temperature changes from T_1 to T_2,

$\Delta S = nC_V\int_{T_1}^{T_2}\frac{dT}{T} = nC_V\ln\left(\frac{T_2}{T_1}\right)$. The entropy changes are $nC_V\ln(T_c/T_b)$ and $nC_V\ln(T_a/T_d)$.

(b) The total entropy change for one cycle is the sum of the entropy changes found in part (a); the other processes in the cycle are adiabatic, with $Q = 0$ and $\Delta S = 0$. The total is then

$\Delta S = nC_V\ln\frac{T_c}{T_b} + nC_V\ln\frac{T_a}{T_d} = nC_V\ln\left(\frac{T_cT_a}{T_bT_d}\right)$. $\frac{T_cT_a}{T_bT_d} = \frac{r^{\gamma-1}T_dT_a}{r^{\gamma-1}T_dT_a} = 1$. $\ln(1) = 0$, so $\Delta S = 0$.

(c) The system is not isolated, and a zero change of entropy for an irreversible system is certainly possible.

EVALUATE: In an irreversible process for an isolated system, $\Delta S > 0$. But the entropy change for some of the components of the system can be negative or zero.

20.60. **IDENTIFY:** For a reversible isothermal process, $\Delta S = \frac{Q}{T}$. For a reversible adiabatic process, $Q = 0$ and $\Delta S = 0$. The Carnot cycle consists of two reversible isothermal processes and two reversible adiabatic processes.

SET UP: Use the results for the Stirling cycle from Problem 20.50.

EXECUTE: **(a)** The graph is given in Figure 20.60.

(b) For a reversible process, $dS = \frac{dQ}{T}$, and so $dQ = T\,dS$, and $Q = \int dQ = \int T\,dS$, which is the area under the curve in the *TS* plane.

(c) Q_H is the area under the rectangle bounded by the horizontal part of the rectangle at T_H and the verticals. $|Q_C|$ is the area bounded by the horizontal part of the rectangle at T_C and the verticals. The net work is then $Q_H - |Q_C|$, the area bounded by the rectangle that represents the process. The ratio of the areas is the ratio of the lengths of the vertical sides of the respective rectangles, and the efficiency is $e = \frac{W}{Q_H} = \frac{T_H - T_C}{T_H}$.

(d) As explained in Problem 20.50, the substance that mediates the heat exchange during the isochoric expansion and compression does not leave the system, and the diagram is the same as in part (a). As found in that problem, the ideal efficiency is the same as for a Carnot-cycle engine.

EVALUATE: The derivation of e_{Carnot} using the concept of entropy is much simpler than the derivation in Section 20.6, but yields the same result.

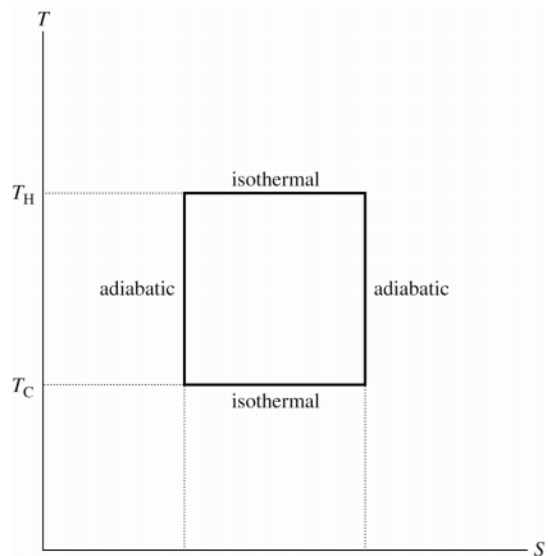


Figure 20.60

20.61. **IDENTIFY:** The temperatures of the ice-water mixture and of the boiling water are constant, so $\Delta S = \frac{Q}{T}$. The heat flow for the melting phase transition of the ice is $Q = +mL_f$.

SET UP: For water, $L_f = 3.34 \times 10^5$ J/kg.

EXECUTE: **(a)** The heat that goes into the ice-water mixture is $Q = mL_f = (0.160\text{ kg})(3.34 \times 10^5\text{ J/kg}) = 5.34 \times 10^4$ J. This is same amount of heat leaves the boiling water, so

$$\Delta S = \frac{Q}{T} = \frac{-5.34 \times 10^4\text{ J}}{373\text{ K}} = -143\text{ J/K}.$$

(b) $\Delta S = \frac{Q}{T} = \frac{5.34 \times 10^4\text{ J}}{273\text{ K}} = +196\text{ J/K}$

(c) For any segment of the rod, the net heat flow is zero, so $\Delta S = 0$.

(d) $\Delta S_{tot} = -143\text{ J/K} + 196\text{ J/K} = +53\text{ J/K}$.

EVALUATE: The heat flow is irreversible, since the system is isolated and the total entropy change is positive.

20.62. **IDENTIFY:** Use the expression derived in Example 20.6 for the entropy change in a temperature change.

SET UP: For water, $c = 4190\text{ J/kg}\cdot\text{K}$. $20°\text{C} = 293.15\text{ K}$, $65°\text{C} = 338.15\text{ K}$ and $120°\text{C} = 393.15\text{ K}$.

EXECUTE: **(a)** $\Delta S = mc\ln(T_2/T_1) = (250 \times 10^{-3}\text{ kg})(4190\text{ J/kg}\cdot\text{K})\ln(338.15\text{ K}/293.15\text{ K}) = 150\text{ J/K}$.

(b) $\Delta S = \frac{-mc\Delta T}{T_{element}} = \frac{-(250 \times 10^{-3}\text{ kg})(4190\text{ J/kg}\cdot\text{K})(338.15\text{ K} - 293.15\text{ K})}{393.15\text{ K}} = -120\text{ J/K}$.

(c) The sum of the result of parts (a) and (b) is $\Delta S_{system} = 30\text{ J/K}$.

EVALUATE: **(d)** Heating a liquid is not reversible. Whatever the energy source for the heating element, heat is being delivered at a higher temperature than that of the water, and the entropy loss of the source will be less in magnitude than the entropy gain of the water. The net entropy change is positive.

20.63. **IDENTIFY:** Use the expression derived in Example 20.6 for the entropy change in a temperature change. For the value of T for which ΔS is a maximum, $d(\Delta S)/dT = 0$.

SET UP: The heat flow for a temperature change is $Q = mc\Delta T$

EXECUTE: **(a)** As in Example 20.10, the entropy change of the first object is $m_1c_1\ln(T/T_1)$ and that of the second is $m_2c_2\ln(T'/T_2)$, and so the net entropy change is as given. Neglecting heat transfer to the surroundings, $Q_1 + Q_2 = 0$, $m_1c_1(T - T_1) + m_2c_2(T' - T_2) = 0$, which is the given expression.

(b) Solving the energy-conservation relation for T' and substituting into the expression for ΔS gives $\Delta S = m_1c_1\ln\left(\frac{T}{T_1}\right) + m_2c_2\ln\left(1 - \frac{m_1c_1}{m_2c_2}\left(\frac{T}{T_2} - \frac{T_1}{T_2}\right)\right)$. Differentiating with respect to T and setting the derivative equal to 0 gives $0 = \frac{m_1c_1}{T} + \frac{(m_2c_2)(m_1c_1/m_2c_2)(-1/T_2)}{\left(1 - (m_1c_1/m_2c_2)\left(\frac{T}{T_2} - \frac{T_1}{T_2}\right)\right)}$. This may be solved for $T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$. Using this value for T in the conservation of energy expression in part (a) and solving for T' gives $T' = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$. Therefore, $T = T'$ when ΔS is a maximum.

EVALUATE: **(c)** The final state of the system will be that for which no further entropy change is possible. If $T < T'$, it is possible for the temperatures to approach each other while increasing the total entropy, but when $T = T'$, no further spontaneous heat exchange is possible.

20.64. **IDENTIFY:** Calculate Q_C and Q_H in terms of p and V at each point. Use the ideal gas law and the pressure-volume relation for adiabatic processes for an ideal gas. $e = 1 - \frac{|Q_C|}{|Q_H|}$.

SET UP: For an ideal gas, $C_p = C_V + R$, and taking air to be diatomic, $C_p = \frac{7}{2}R$, $C_V = \frac{5}{2}R$ and $\gamma = \frac{7}{5}$.

EXECUTE: Referring to Figure 20.7 in the textbook, $Q_H = n\frac{7}{2}R(T_c - T_b) = \frac{7}{2}(p_cV_c - p_bV_b)$. Similarly, $Q_C = n\frac{5}{2}R(p_aV_a - p_dV_d)$. What needs to be done is to find the relations between the product of the pressure and the volume at the four points. For an ideal gas, $\frac{p_cV_c}{T_c} = \frac{p_bV_b}{T_b}$ so $p_cV_c = p_aV_a\left(\frac{T_c}{T_a}\right)$. For a compression ratio r, and given that for the Diesel cycle the process ab is adiabatic, $p_bV_b = p_aV_a\left(\frac{V_a}{V_b}\right)^{\gamma-1} = p_aV_ar^{\gamma-1}$. Similarly, $p_dV_d = p_cV_c\left(\frac{V_c}{V_a}\right)^{\gamma-1}$. Note that the last result uses the fact that process da is isochoric, and $V_d = V_a$; also, $p_c = p_b$ (process bc is isobaric), and so $V_c = V_b\left(\frac{T_c}{T_a}\right)$. Then,

$$\frac{V_c}{V_a} = \frac{T_c}{T_b}\cdot\frac{V_b}{V_a} = \frac{T_b}{T_a}\cdot\frac{T_a}{T_b}\cdot\frac{V_a}{V_b} = \frac{T_c}{T_a}\cdot\left(\frac{T_aV_a^{\gamma-1}}{T_bV_b^{\gamma-1}}\right)\left(\frac{V_a}{V_b}\right)^{-\gamma} = \frac{T_c}{T_a}r^{\gamma}$$

Combining the above results, $p_dV_d = p_aV_a\left(\frac{T_c}{T_a}\right)^{\gamma} r^{\gamma-\gamma^2}$. Substitution of the above results into Eq. (20.4) gives

$$e = 1 - \frac{5}{7}\left[\frac{\left(\frac{T_c}{T_a}\right)^{\gamma} r^{\gamma-\gamma^2} - 1}{\left(\frac{T_c}{T_a}\right) - r^{\gamma-1}}\right].$$

(b) $e = 1 - \frac{1}{1.4}\left[\frac{(5.002)r^{-0.56} - 1}{(3.167) - r^{0.40}}\right]$, where $\frac{T_c}{T_a} = 3.167$ and $\gamma = 1.40$ have been used. Substitution of $r = 21.0$ yields $e = 0.708 = 70.8\%$.

EVALUATE: The efficiency for an Otto cycle with $r = 21.0$ and $\gamma = 1.40$ is $e = 1 - r^{1-\gamma} = 1 - (21.0)^{-0.40} = 70.4\%$. This is very close to the value for the Diesel cycle.

ELECTRIC CHARGE AND ELECTRIC FIELD

21

21.1. **(a) IDENTIFY** and **SET UP:** Use the charge of one electron $(-1.602\times10^{-19}\text{ C})$ to find the number of electrons required to produce the net charge.

EXECUTE: The number of excess electrons needed to produce net charge q is

$$\frac{q}{-e}=\frac{-3.20\times10^{-9}\text{ C}}{-1.602\times10^{-19}\text{ C/electron}}=2.00\times10^{10}\text{ electrons.}$$

(b) IDENTIFY and **SET UP:** Use the atomic mass of lead to find the number of lead atoms in 8.00×10^{-3} kg of lead. From this and the total number of excess electrons, find the number of excess electrons per lead atom.

EXECUTE: The atomic mass of lead is 207×10^{-3} kg/mol, so the number of moles in 8.00×10^{-3} kg is

$n=\dfrac{m_{\text{tot}}}{M}=\dfrac{8.00\times10^{-3}\text{ kg}}{207\times10^{-3}\text{ kg/mol}}=0.03865\text{ mol.}$ N_{A} (Avogadro's number) is the number of atoms in 1 mole, so the number of lead atoms is $N=nN_{\text{A}}=(0.03865\text{ mol})(6.022\times10^{23}\text{ atoms/mol})=2.328\times10^{22}$ atoms. The number of excess electrons per lead atom is $\dfrac{2.00\times10^{10}\text{ electrons}}{2.328\times10^{22}\text{ atoms}}=8.59\times10^{-13}.$

EVALUATE: Even this small net charge corresponds to a large number of excess electrons. But the number of atoms in the sphere is much larger still, so the number of excess electrons per lead atom is very small.

21.2. **IDENTIFY:** The charge that flows is the rate of charge flow times the duration of the time interval.

SET UP: The charge of one electron has magnitude $e=1.60\times10^{-19}$ C.

EXECUTE: The rate of charge flow is 20,000 C/s and $t=100\ \mu\text{s}=1.00\times10^{-4}$ s.

$Q=(20{,}000\text{ C/s})(1.00\times10^{-4}\text{ s})=2.00\text{ C}$. The number of electrons is $n_{\text{e}}=\dfrac{Q}{1.60\times10^{-19}\text{ C}}=1.25\times10^{19}.$

EVALUATE: This is a very large amount of charge and a large number of electrons.

21.3. **IDENTIFY:** From your mass estimate the number of protons in your body. You have an equal number of electrons.

SET UP: Assume a body mass of 70 kg. The charge of one electron is -1.60×10^{-19} C.

EXECUTE: The mass is primarily protons and neutrons of $m=1.67\times10^{-27}$ kg. The total number of protons and neutrons is $n_{\text{p and n}}=\dfrac{70\text{ kg}}{1.67\times10^{-27}\text{ kg}}=4.2\times10^{28}$. About one-half are protons, so $n_{\text{p}}=2.1\times10^{28}=n_{\text{e}}$. The number of electrons is about 2.1×10^{28}. The total charge of these electrons is

$Q=(-1.60\times10^{-19}\text{ C/electron})(2.10\times10^{28}\text{ electrons})=-3.35\times10^{9}\text{ C}.$

EVALUATE: This is a huge amount of negative charge. But your body contains an equal number of protons and your net charge is zero. If you carry a net charge, the number of excess or missing electrons is a very small fraction of the total number of electrons in your body.

21.4. **IDENTIFY:** Use the mass m of the ring and the atomic mass M of gold to calculate the number of gold atoms. Each atom has 79 protons and an equal number of electrons.

SET UP: $N_{\text{A}}=6.02\times10^{23}$ atoms/mol. A proton has charge $+e$.

EXECUTE: The mass of gold is 17.7 g and the atomic weight of gold is 197 g/mol. So the number of atoms is $N_{\text{A}}n=(6.02\times10^{23}\text{ atoms/mol})\left(\dfrac{17.7\text{ g}}{197\text{ g/mol}}\right)=5.41\times10^{22}$ atoms. The number of protons is

$n_{\text{p}}=(79\text{ protons/atom})(5.41\times10^{22}\text{ atoms})=4.27\times10^{24}$ protons. $Q=(n_{\text{p}})(1.60\times10^{-19}\text{ C/proton})=6.83\times10^{5}$ C.

(b) The number of electrons is $n_{\text{e}}=n_{\text{p}}=4.27\times10^{24}.$

EVALUATE: The total amount of positive charge in the ring is very large, but there is an equal amount of negative charge.

21.5. **IDENTIFY:** Apply $F = \dfrac{k|q_1q_2|}{r^2}$ and solve for *r*.

SET UP: $F = 650\ \text{N}$.

EXECUTE: $r = \sqrt{\dfrac{k|q_1q_2|}{F}} = \sqrt{\dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0\ \text{C})^2}{650\ \text{N}}} = 3.7\times10^3\ \text{m} = 3.7\ \text{km}$

EVALUATE: Charged objects typically have net charges much less than 1 C.

21.6. **IDENTIFY:** Apply Coulomb's law and calculate the net charge *q* on each sphere.

SET UP: The magnitude of the charge of an electron is $e = 1.60\times10^{-19}\ \text{C}$.

EXECUTE: $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{r^2}$. This gives $|q| = \sqrt{4\pi\epsilon_0 Fr^2} = \sqrt{4\pi\epsilon_0(4.57\times10^{-21}\ \text{N})(0.200\ \text{m})^2} = 1.43\times10^{-16}\ \text{C}$. And therefore, the total number of electrons required is $n = |q|/e = (1.43\times10^{-16}\ \text{C})/(1.60\times10^{-19}\ \text{C/electron}) = 890$ electrons.

EVALUATE: Each sphere has 890 excess electrons and each sphere has a net negative charge. The two like charges repel.

21.7. **IDENTIFY:** Apply Coulomb's law.
SET UP: Consider the force on one of the spheres.
(a) EXECUTE: $q_1 = q_2 = q$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} = \frac{q^2}{4\pi\epsilon_0 r^2} \text{ so } q = r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = 0.150\ \text{m}\sqrt{\frac{0.220\ \text{N}}{8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 7.42\times10^{-7}\ \text{C (on each)}$$

(b) $q_2 = 4q_1$

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} = \frac{4q_1^2}{4\pi\epsilon_0 r^2} \text{ so } q_1 = r\sqrt{\frac{F}{4(1/4\pi\epsilon_0)}} = \tfrac{1}{2}r\sqrt{\frac{F}{(1/4\pi\epsilon_0)}} = \tfrac{1}{2}(7.42\times10^{-7}\ \text{C}) = 3.71\times10^{-7}\ \text{C}.$$

And then $q_2 = 4q_1 = 1.48\times10^{-6}\ \text{C}$.

EVALUATE: The force on one sphere is the same magnitude as the force on the other sphere, whether the sphere have equal charges or not.

21.8. **IDENTIFY:** Use the mass of a sphere and the atomic mass of aluminum to find the number of aluminum atoms in one sphere. Each atom has 13 electrons. Apply Coulomb's law and calculate the magnitude of charge $|q|$ on each sphere.

SET UP: $N_A = 6.02\times10^{23}$ atoms/mol. $|q| = n'_e e$, where n'_e is the number of electrons removed from one sphere and added to the other.

EXECUTE: **(a)** The total number of electrons on each sphere equals the number of protons.

$$n_e = n_p = (13)(N_A)\left(\frac{0.0250\ \text{kg}}{0.026982\ \text{kg/mol}}\right) = 7.25\times10^{24}\ \text{electrons}.$$

(b) For a force of 1.00×10^4 N to act between the spheres, $F = 1.00\times10^4\ \text{N} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{r^2}$. This gives $|q| = \sqrt{4\pi\epsilon_0(1.00\times10^4\ \text{N})(0.0800\ \text{m})^2} = 8.43\times10^{-4}\ \text{C}$. The number of electrons removed from one sphere and added to the other is $n'_e = |q|/e = 5.27\times10^{15}$ electrons.

(c) $n'_e/n_e = 7.27\times10^{-10}$.

EVALUATE: When ordinary objects receive a net charge the fractional change in the total number of electrons in the object is very small .

21.9. **IDENTIFY:** Apply $F = ma$, with $F = k\dfrac{|q_1q_2|}{r^2}$.

SET UP: $a = 25.0g = 245\ \text{m/s}^2$. An electron has charge $-e = -1.60\times10^{-19}$ C.

EXECUTE: $F = ma = (8.55\times10^{-3}\ \text{kg})(245\ \text{m/s}^2) = 2.09\ \text{N}$. The spheres have equal charges *q*, so $F = k\dfrac{q^2}{r^2}$ and $|q| = r\sqrt{\dfrac{F}{k}} = (0.150\ \text{m})\sqrt{\dfrac{2.09\ \text{N}}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = 2.29\times10^{-6}\ \text{C}$. $N = \dfrac{|q|}{e} = \dfrac{2.29\times10^{-6}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = 1.43\times10^{13}$ electrons. The charges on the spheres have the same sign so the electrical force is repulsive and the spheres accelerate away from each other.

EVALUATE: As the spheres move apart the repulsive force they exert on each other decreases and their acceleration decreases.

21.10. **(a) IDENTIFY:** The electrical attraction of the proton gives the electron an acceleration equal to the acceleration due to gravity on earth.

SET UP: Coulomb's law gives the force and Newton's second law gives the acceleration this force produces. $ma = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$ and $r = \sqrt{\frac{e^2}{4\pi\epsilon_0 ma}}$.

EXECUTE: $r = \sqrt{\frac{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(9.11\times10^{-31}\ \text{kg})(9.80\ \text{m/s}^2)}} = 5.08\ \text{m}$

EVALUATE: The electron needs to be about 5 m from a single proton to have the same acceleration as it receives from the gravity of the entire earth.

(b) IDENTIFY: The force on the electron comes from the electrical attraction of all the protons in the earth.

SET UP: First find the number *n* of protons in the earth, and then find the acceleration of the electron using Newton's second law, as in part (a).

$$n = m_E/m_p = (5.97 \times 10^{24}\ \text{kg})/(1.67 \times 10^{-27}\ \text{kg}) = 3.57 \times 10^{51}\ \text{protons.}$$

$$a = F/m = \frac{\frac{1}{4\pi\epsilon_0}\frac{|q_p q_e|}{R_E^2}}{m_e} = \frac{\frac{1}{4\pi\epsilon_0} ne^2}{m_e R_E^2}.$$

EXECUTE: $a = (9.00 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.57 \times 10^{51})(1.60 \times 10^{-19}\ \text{C})^2/[(9.11 \times 10^{-31}\ \text{kg})(6.38 \times 10^6\ \text{m})^2] = 2.22 \times 10^{40}\ \text{m/s}^2$. One can ignore the gravitation force since it produces an acceleration of only 9.8 m/s^2 and hence is much much less than the electrical force.

EVALUATE: With the electrical force, the acceleration of the electron would nearly 10^{40} times greater than with gravity, which shows how strong the electrical force is.

21.11. **IDENTIFY:** In a space satellite, the only force accelerating the free proton is the electrical repulsion of the other proton.

SET UP: Coulomb's law gives the force, and Newton's second law gives the acceleration: $a = F/m = (1/4\pi\epsilon_0)(e^2/r^2)/m$.

EXECUTE: **(a)** $a = (9.00 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\ \text{C})^2/[(0.00250\ \text{m})^2(1.67 \times 10^{-27}\ \text{kg})] = 2.21 \times 10^4\ \text{m/s}^2$.

(b) The graphs are sketched in Figure 21.11.

EVALUATE: The electrical force of a single stationary proton gives the moving proton an initial acceleration about 20,000 times as great as the acceleration caused by the gravity of the entire earth. As the protons move farther apart, the electrical force gets weaker, so the acceleration decreases. Since the protons continue to repel, the velocity keeps increasing, but at a decreasing rate.

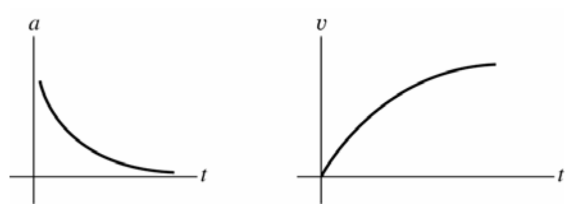


Figure 21.11

21.12. **IDENTIFY:** Apply Coulomb's law.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: **(a)** $F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$. This gives $0.200\ \text{N} = \frac{1}{4\pi\epsilon_0}\frac{(0.550\times10^{-6}\ \text{C})|q_2|}{(0.30\ \text{m})^2}$ and $|q_2| = +3.64\times10^{-6}\ \text{C}$. The force is attractive and $q_1 < 0$, so $q_2 = +3.64\times10^{-6}\ \text{C}$.

(b) $F = 0.200$ N. The force is attractive, so is downward.

EVALUATE: The forces between the two charges obey Newton's third law.

21.13. **IDENTIFY:** Apply Coulomb's law. The two forces on q_3 must have equal magnitudes and opposite directions.

SET UP: Like charges repel and unlike charges attract.

EXECUTE: The force $\vec{F}_2$ that q_2 exerts on q_3 has magnitude $F_2 = k\dfrac{|q_2 q_3|}{r_2^2}$ and is in the +*x* direction. $\vec{F}_1$ must be in the $-x$ direction, so q_1 must be positive. $F_1 = F_2$ gives $k\dfrac{|q_1||q_3|}{r_1^2} = k\dfrac{|q_2||q_3|}{r_2^2}$.

$$|q_1| = |q_2|\left(\frac{r_1}{r_2}\right)^2 = (3.00\text{ nC})\left(\frac{2.00\text{ cm}}{4.00\text{ cm}}\right)^2 = 0.750\text{ nC}.$$

EVALUATE: The result for the magnitude of q_1 doesn't depend on the magnitude of q_2.

21.14. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on *Q*.

SET UP: The force that q_1 exerts on *Q* is repulsive, as in Example 21.4, but now the force that q_2 exerts is attractive.

EXECUTE: The *x*-components cancel. We only need the *y*-components, and each charge contributes equally. $F_{1y} = F_{2y} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{(2.0\times10^{-6}\text{ C})(4.0\times10^{-6}\text{ C})}{(0.500\text{ m})^2}\sin\alpha = -0.173\text{ N}$ (since $\sin\alpha = 0.600$). Therefore, the total force is $2F = 0.35\text{ N}$, in the $-y$-direction.

EVALUATE: If q_1 is $-2.0\ \mu\text{C}$ and q_2 is $+2.0\ \mu\text{C}$, then the net force is in the +*y*-direction.

21.15. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on q_1.

SET UP: Like charges repel and unlike charges attract, so $\vec{F}_2$ and $\vec{F}_3$ are both in the +*x*-direction.

EXECUTE: $F_2 = k\dfrac{|q_1 q_2|}{r_{12}^2} = 6.749\times10^{-5}\text{ N}$, $F_3 = k\dfrac{|q_1 q_3|}{r_{13}^2} = 1.124\times10^{-4}\text{ N}$. $F = F_2 + F_3 = 1.8\times10^{-4}\text{ N}$.

$F = 1.8\times10^{-4}$ N and is in the +*x*-direction.

EVALUATE: Comparing our results to those in Example 21.3, we see that $\vec{F}_{1\text{ on }3} = -\vec{F}_{3\text{ on }1}$, as required by Newton's third law.

21.16. **IDENTIFY:** Apply Coulomb's law and find the vector sum of the two forces on q_2.

SET UP: $\vec{F}_{2\text{ on }1}$ is in the +*y*-direction.

EXECUTE: $F_{2\text{on }1} = \dfrac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.0\times10^{-6}\text{ C})(2.0\times10^{-6}\text{ C})}{(0.60\text{ m})^2} = 0.100\text{ N}$. $\left(F_{2\text{ on }1}\right)_x = 0$ and $\left(F_{2\text{ on }1}\right)_y = +0.100\text{ N}$. $F_{Q\text{ on }1}$ is equal and opposite to $F_{1\text{ on }Q}$ (Example 21.4), so $\left(F_{Q\text{ on }1}\right)_x = -0.23\text{N}$ and $\left(F_{Q\text{ on }1}\right)_y = 0.17\text{ N}$. $F_x = \left(F_{2\text{ on }1}\right)_x + \left(F_{Q\text{ on }1}\right)_x = -0.23\text{ N}$. $F_y = \left(F_{2\text{ on }1}\right)_y + \left(F_{Q\text{ on }1}\right)_y = 0.100\text{ N} + 0.17\text{ N} = 0.27\text{ N}$.

The magnitude of the total force is $F = \sqrt{(0.23\text{ N})^2 + (0.27\text{ N})^2} = 0.35\text{ N}$. $\tan^{-1}\dfrac{0.23}{0.27} = 40°$, so $\vec{F}$ is $40°$ counterclockwise from the +*y* axis, or $130°$ counterclockwise from the +*x* axis.

EVALUATE: Both forces on q_1 are repulsive and are directed away from the charges that exert them.

21.17. **IDENTIFY** and **SET UP:** Apply Coulomb's law to calculate the force exerted by q_2 and q_3 on q_1. Add these forces as vectors to get the net force. The target variable is the *x*-coordinate of q_3.

EXECUTE: $\vec{F}_2$ is in the *x*-direction.

$F_2 = k\dfrac{|q_1 q_2|}{r_{12}^2} = 3.37\text{ N}$, so $F_{2x} = +3.37\text{ N}$

$F_x = F_{2x} + F_{3x}$ and $F_x = -7.00\text{ N}$

$F_{3x} = F_x - F_{2x} = -7.00\text{ N} - 3.37\text{ N} = -10.37\text{ N}$

For F_{3x} to be negative, q_3 must be on the $-x$-axis.

$F_3 = k\dfrac{|q_1 q_3|}{x^2}$, so $|x| = \sqrt{\dfrac{k|q_1 q_3|}{F_3}} = 0.144\text{ m}$, so $x = -0.144\text{ m}$

EVALUATE: q_2 attracts q_1 in the $+x$-direction so q_3 must attract q_1 in the $-x$-direction, and q_3 is at negative *x*.

21.18. **IDENTIFY:** Apply Coulomb's law.

SET UP: Like charges repel and unlike charges attract. Let $\vec{F}_{21}$ be the force that q_2 exerts on q_1 and let $\vec{F}_{31}$ be the force that q_3 exerts on q_1.

EXECUTE: The charge q_3 must be to the right of the origin; otherwise both q_2 and q_3 would exert forces in the $+x$ direction. Calculating the two forces:

$$F_{21} = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r_{12}^2} = \frac{(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\ \text{C})(5.00\times10^{-6}\ \text{C})}{(0.200\ \text{m})^2} = 3.375\ \text{N}\text{, in the } +x \text{ direction.}$$

$$F_{31} = \frac{(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\,(3.00\times10^{-6}\ \text{C})\,(8.00\times10^{-6}\ \text{C})}{r_{13}^2} = \frac{0.216\ \text{N}\cdot\text{m}^2}{r_{13}^2}\text{, in the } -x \text{ direction.}$$

We need $F_x = F_{21} - F_{31} = -7.00\ \text{N}$, so $3.375\ \text{N} - \dfrac{0.216\ \text{N}\cdot\text{m}^2}{r_{13}^2} = -7.00\ \text{N}$. $r_{13} = \sqrt{\dfrac{0.216\ \text{N}\cdot\text{m}^2}{3.375\ \text{N} + 7.00\ \text{N}}} = 0.144\ \text{m}$. q_3 is at $x = 0.144\ \text{m}$.

EVALUATE: $F_{31} = 10.4\ \text{N}$. F_{31} is larger than F_{21}, because $|q_3|$ is larger than $|q_2|$ and also because r_{13} is less than r_{12}.

21.19. **IDENTIFY:** Apply Coulomb's law to calculate the force each of the two charges exerts on the third charge. Add these forces as vectors.

SET UP: The three charges are placed as shown in Figure 21.19a.

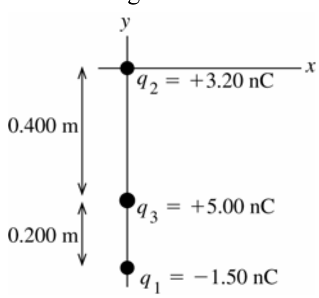
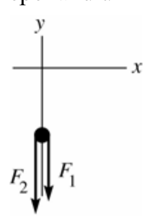


Figure 21.19a

EXECUTE: Like charges repel and unlike attract, so the free-body diagram for q_3 is as shown in Figure 21.19b.



$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_3|}{r_{13}^2}$$

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2 q_3|}{r_{23}^2}$$

Figure 21.19b

$$F_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.50\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 1.685\times10^{-6}\ \text{N}$$

$$F_2 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.20\times10^{-9}\ \text{C})(5.00\times10^{-9}\ \text{C})}{(0.400\ \text{m})^2} = 8.988\times10^{-7}\ \text{N}$$

The resultant force is $\vec{R} = \vec{F}_1 + \vec{F}_2$.

$R_x = 0$.

$R_y = F_1 + F_2 = 1.685\times10^{-6}\ \text{N} + 8.988\times10^{-7}\ \text{N} = 2.58\times10^{-6}\ \text{N}$.

The resultant force has magnitude $2.58\times10^{-6}\ \text{N}$ and is in the $-y$-direction.

EVALUATE: The force between q_1 and q_3 is attractive and the force between q_2 and q_3 is replusive.

21.20. **IDENTIFY:** Apply $F = k\dfrac{|qq'|}{r^2}$ to each pair of charges. The net force is the vector sum of the forces due to q_1 and q_2.

SET UP: Like charges repel and unlike charges attract. The charges and their forces on q_3 are shown in Figure 21.20.

EXECUTE: $F_1 = k\dfrac{|q_1q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(4.00\times10^{-9}\ \text{C})(0.600\times10^{-9}\ \text{C})}{(0.200\ \text{m})^2} = 5.394\times10^{-7}\ \text{N}$.

$F_2 = k\dfrac{|q_2q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(5.00\times10^{-9}\ \text{C})(0.600\times10^{-9}\ \text{C})}{(0.300\ \text{m})^2} = 2.997\times10^{-7}\ \text{N}$.

$F_x = F_{1x} + F_{2x} = +F_1 - F_2 = 2.40\times10^{-7}\ \text{N}$. The net force has magnitude $2.40\times10^{-7}\ \text{N}$ and is in the $+x$ direction.

EVALUATE: Each force is attractive, but the forces are in opposite directions because of the placement of the charges. Since the forces are in opposite directions, the net force is obtained by subtracting their magnitudes.

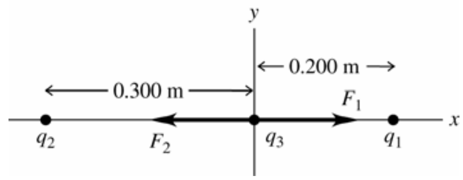


Figure 21.20

21.21. **IDENTIFY:** Apply Coulomb's law to calculate each force on $-Q$.

SET UP: Let $\vec{F}_1$ be the force exerted by the charge at $y = a$ and let $\vec{F}_2$ be the force exerted by the charge at $y = -a$.

EXECUTE: **(a)** The two forces on $-Q$ are shown in Figure 21.21a. $\sin\theta = \dfrac{a}{(a^2+x^2)^{1/2}}$ and $r = (a^2+x^2)^{1/2}$ is the distance between q and $-Q$ and between $-q$ and $-Q$.

(b) $F_x = F_{1x} + F_{2x} = 0$. $F_y = F_{1y} + F_{2y} = 2\dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{(a^2+x^2)}\sin\theta = \dfrac{1}{4\pi\epsilon_0}\dfrac{2qQa}{(a^2+x^2)^{3/2}}$.

(c) At $x = 0$, $F_y = \dfrac{1}{4\pi\epsilon_0}\dfrac{2qQ}{a^2}$, in the $+y$ direction.

(d) The graph of F_y versus x is given in Figure 21.21b.

EVALUATE: $F_x = 0$ for all values of x and $F_y > 0$ for all x.

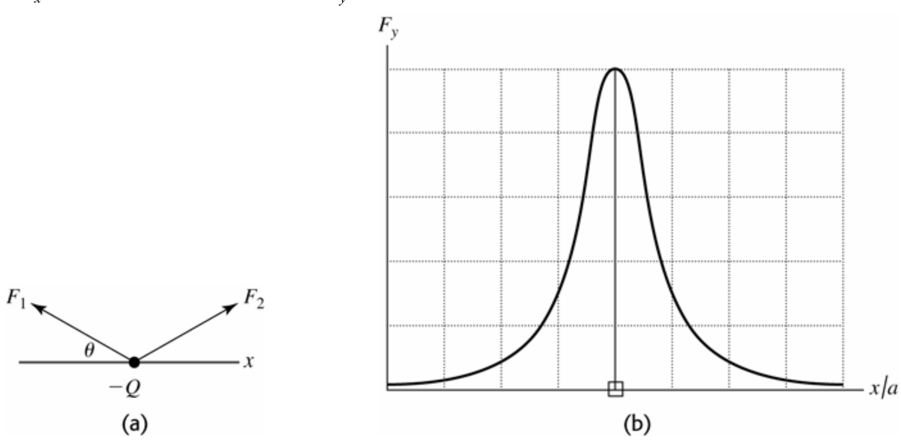


Figure 21.21

21.22. **IDENTIFY:** Apply Coulomb's law to calculate each force on $-Q$.

SET UP: Let $\vec{F}_1$ be the force exerted by the charge at $y = a$ and let $\vec{F}_2$ be the force exerted by the charge at $y = -a$. The distance between each charge q and Q is $r = \left(a^2+x^2\right)^{1/2}$. $\cos\theta = \dfrac{|x|}{\left(a^2+x^2\right)^{1/2}}$.

EXECUTE: **(a)** The two forces on $-Q$ are shown in Figure 21.22a.

(b) When $x > 0$, F_{1x} and F_{2x} are negative. $F_x = F_{1x} + F_{2x} = -2\dfrac{1}{4\pi\epsilon_0}\dfrac{qQ}{(a^2+x^2)}\cos\theta = \dfrac{1}{4\pi\epsilon_0}\dfrac{-2qQx}{(a^2+x^2)^{3/2}}$. When $x < 0$, F_{1x} and F_{2x} are positive and the same expression for F_x applies. $F_y = F_{1y} + F_{2y} = 0$.

(c) At $x = 0$, $F_x = 0$.

(d) The graph of F_x versus x is sketched in Figure 21.22b.

EVALUATE: The direction of the net force on $-Q$ is always toward the origin.

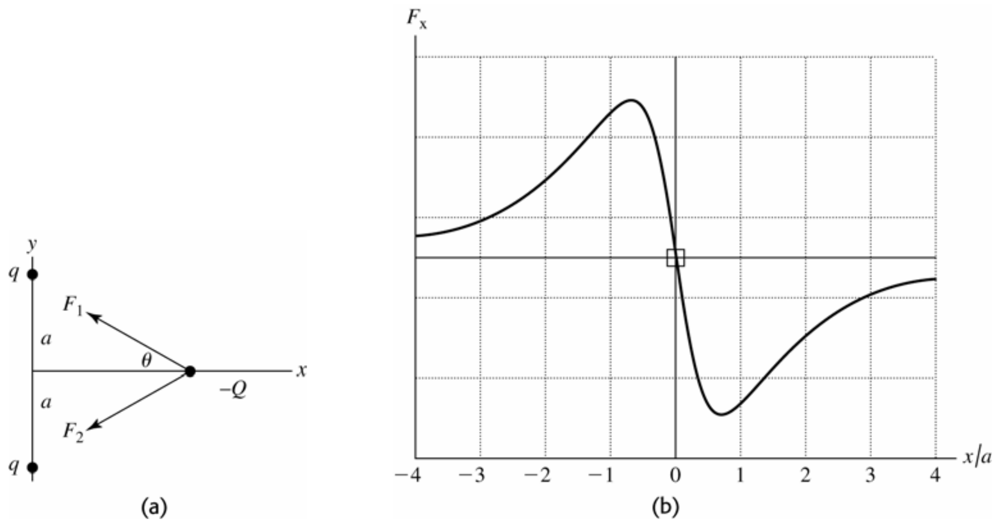
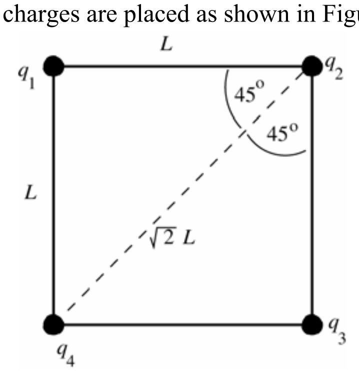


Figure 21.22

21.23. IDENTIFY: Apply Coulomb's law to calculate the force exerted on one of the charges by each of the other three and then add these forces as vectors.

(a) SET UP: The charges are placed as shown in Figure 21.23a.



$q_1 = q_2 = q_3 = q_4 = q$

Figure 21.23a

Consider forces on q_4. The free-body diagram is given in Figure 21.23b. Take the y-axis to be parallel to the diagonal between q_2 and q_4 and let $+y$ be in the direction away from q_2. Then $\vec{F}_2$ is in the $+y$-direction.

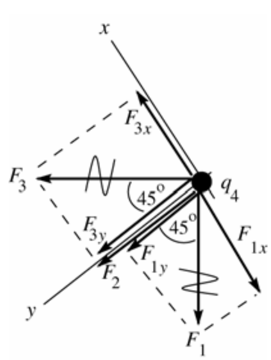


Figure 21.23b

EXECUTE: $F_3 = F_1 = \frac{1}{4\pi\epsilon_0}\frac{q^2}{L^2}$

$$F_2 = \frac{1}{4\pi\epsilon_0}\frac{q^2}{2L^2}$$

$$F_{1x} = -F_1 \sin 45° = -F_1/\sqrt{2}$$

$$F_{1y} = +F_1 \cos 45° = +F_1/\sqrt{2}$$

$$F_{3x} = +F_3 \sin 45° = +F_3/\sqrt{2}$$

$$F_{3y} = +F_3 \cos 45° = +F_3/\sqrt{2}$$

$$F_{2x} = 0,\ F_{2y} = F_2$$

(b) $R_x = F_{1x} + F_{2x} + F_{3x} = 0$

$$R_y = F_{1y} + F_{2y} + F_{3y} = (2/\sqrt{2})\frac{1}{4\pi\epsilon_0}\frac{q^2}{L^2} + \frac{1}{4\pi\epsilon_0}\frac{q^2}{2L^2} = \frac{q^2}{8\pi\epsilon_0 L^2}(1+2\sqrt{2})$$

$R = \frac{q^2}{8\pi\epsilon_0 L^2}(1+2\sqrt{2})$. Same for all four charges.

EVALUATE: In general the resultant force on one of the charges is directed away from the opposite corner. The forces are all repulsive since the charges are all the same. By symmetry the net force on one charge can have no component perpendicular to the diagonal of the square.

21.24. IDENTIFY: Apply $F = \dfrac{k|qq'|}{r^2}$ to find the force of each charge on $+q$. The net force is the vector sum of the individual forces.

SET UP: Let $q_1 = +2.50\ \mu\text{C}$ and $q_2 = -3.50\ \mu\text{C}$. The charge $+q$ must be to the left of q_1 or to the right of q_2 in order for the two forces to be in opposite directions. But for the two forces to have equal magnitudes, $+q$ must be closer to the charge q_1, since this charge has the smaller magnitude. Therefore, the two forces can combine to give zero net force only in the region to the left of q_1. Let $+q$ be a distance *d* to the left of q_1, so it is a distance $d + 0.600\ \text{m}$ from q_2.

EXECUTE: $F_1 = F_2$ gives $\dfrac{kq|q_1|}{d^2} = \dfrac{kq|q_2|}{(d + 0.600\ \text{m})^2}$. $d = \pm\sqrt{\dfrac{|q_1|}{|q_2|}}(d + 0.600\ \text{m}) = \pm(0.8452)(d + 0.600\ \text{m})$. *d* must be positive, so $d = \dfrac{(0.8452)(0.600\ \text{m})}{1 - 0.8452} = 3.27\ \text{m}$. The net force would be zero when $+q$ is at $x = -3.27\ \text{m}$.

EVALUATE: When $+q$ is at $x = -3.27\ \text{m}$, $\vec{F}_1$ is in the $-x$ direction and $\vec{F}_2$ is in the $+x$ direction.

21.25. IDENTIFY: $F = |q|E$. Since the field is uniform, the force and acceleration are constant and we can use a constant acceleration equation to find the final speed.

SET UP: A proton has charge $+e$ and mass $1.67 \times 10^{-27}\ \text{kg}$.

EXECUTE: **(a)** $F = (1.60 \times 10^{-19}\ \text{C})(2.75 \times 10^3\ \text{N/C}) = 4.40 \times 10^{-16}\ \text{N}$

(b) $a = \dfrac{F}{m} = \dfrac{4.40 \times 10^{-16}\ \text{N}}{1.67 \times 10^{-27}\ \text{kg}} = 2.63 \times 10^{11}\ \text{m/s}^2$

(c) $v_x = v_{0x} + a_x t$ gives $v = (2.63 \times 10^{11}\ \text{m/s}^2)(1.00 \times 10^{-6}\ \text{s}) = 2.63 \times 10^5\ \text{m/s}$

EVALUATE: The acceleration is very large and the gravity force on the proton can be ignored.

21.26. IDENTIFY: For a point charge, $E = k\dfrac{|q|}{r^2}$.

SET UP: $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: **(a)** The field is toward the negative charge so is downward.

$$E = (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{3.00 \times 10^{-9}\ \text{C}}{(0.250\ \text{m})^2} = 432\ \text{N/C}.$$

(b) $r = \sqrt{\dfrac{k|q|}{E}} = \sqrt{\dfrac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00 \times 10^{-9}\ \text{C})}{12.0\ \text{N/C}}} = 1.50\ \text{m}$

EVALUATE: At different points the electric field has different directions, but it is always directed toward the negative point charge.

21.27. IDENTIFY: The acceleration that stops the charge is produced by the force that the electric field exerts on it. Since the field and the acceleration are constant, we can use the standard kinematics formulas to find acceleration and time.

(a) SET UP: First use kinematics to find the proton's acceleration. $v_x = 0$ when it stops. Then find the electric field needed to cause this acceleration using the fact that $F = qE$.

EXECUTE: $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$. $0 = (4.50 \times 10^6\ \text{m/s})^2 + 2a(0.0320\ \text{m})$ and $a = 3.16 \times 10^{14}\ \text{m/s}^2$. Now find the electric field, with $q = e$. $eE = ma$ and $E = ma/e = (1.67 \times 10^{-27}\ \text{kg})(3.16 \times 10^{14}\ \text{m/s}^2)/(1.60 \times 10^{-19}\ \text{C}) = 3.30 \times 10^6\ \text{N/C}$, to the left.

(b) SET UP: Kinematics gives $v = v_0 + at$, and $v = 0$ when the electron stops, so $t = v_0/a$.

EXECUTE: $t = v_0/a = (4.50 \times 10^6\ \text{m/s})/(3.16 \times 10^{14}\ \text{m/s}^2) = 1.42 \times 10^{-8}\ \text{s} = 14.2\ \text{ns}$

(c) SET UP: In part (a) we saw that the electric field is proportional to *m*, so we can use the ratio of the electric fields. $E_\text{e}/E_\text{p} = m_\text{e}/m_\text{p}$ and $E_\text{e} = \left(m_\text{e}/m_\text{p}\right)E_\text{p}$.

EXECUTE: $E_\text{e} = [(9.11 \times 10^{-31}\ \text{kg})/(1.67 \times 10^{-27}\ \text{kg})](3.30 \times 10^6\ \text{N/C}) = 1.80 \times 10^3\ \text{N/C}$, to the right

EVALUATE: Even a modest electric field, such as the ones in this situation, can produce enormous accelerations for electrons and protons.

21.28. **IDENTIFY:** Use constant acceleration equations to calculate the upward acceleration a and then apply $\vec{F} = q\vec{E}$ to calculate the electric field.

SET UP: Let $+y$ be upward. An electron has charge $q = -e$.

EXECUTE: **(a)** $v_{0y} = 0$ and $a_y = a$, so $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ gives $y - y_0 = \frac{1}{2}at^2$. Then

$$a = \frac{2(y - y_0)}{t^2} = \frac{2(4.50\text{ m})}{(3.00\times10^{-6}\text{ s})^2} = 1.00\times10^{12}\text{ m/s}^2.\quad E = \frac{F}{q} = \frac{ma}{q} = \frac{(9.11\times10^{-31}\text{ kg})(1.00\times10^{12}\text{ m/s}^2)}{1.60\times10^{-19}\text{ C}} = 5.69\text{ N/C}$$

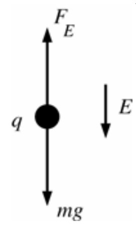
The force is up, so the electric field must be *downward* since the electron has negative charge.

(b) The electron's acceleration is $\sim 10^{11}g$, so gravity must be negligibly small compared to the electrical force.

EVALUATE: Since the electric field is uniform, the force it exerts is constant and the electron moves with constant acceleration.

21.29. **(a) IDENTIFY:** Eq. (21.4) relates the electric field, charge of the particle, and the force on the particle. If the particle is to remain stationary the net force on it must be zero.

SET UP: The free-body diagram for the particle is sketched in Figure 21.29. The weight is *mg,* downward. For the net force to be zero the force exerted by the electric field must be upward. The electric field is downward. Since the electric field and the electric force are in opposite directions the charge of the particle is negative.



$mg = |q|E$

Figure 21.29

EXECUTE: $|q| = \dfrac{mg}{E} = \dfrac{(1.45\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{650\text{ N/C}} = 2.19\times10^{-5}\text{ C}$ and $q = -21.9\ \mu\text{C}$

(b) SET UP: The electrical force has magnitude $F_E = |q|E = eE.$ The weight of a proton is $w = mg.$ $F_E = w$ so $eE = mg$

EXECUTE: $E = \dfrac{mg}{e} = \dfrac{(1.673\times10^{-27}\text{ kg})(9.80\text{ m/s}^2)}{1.602\times10^{-19}\text{ C}} = 1.02\times10^{-7}\text{ N/C}.$

This is a very small electric field.

EVALUATE: In both cases $|q|E = mg$ and $E = (m/|q|)g.$ In part (b) the $m/|q|$ ratio is much smaller $(\sim 10^{-8})$ than in part (a) $(\sim 10^{-2})$ so E is much smaller in (b). For subatomic particles gravity can usually be ignored compared to electric forces.

21.30. **IDENTIFY:** Apply $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}$.

SET UP: The iron nucleus has charge $+26e$. A proton has charge $+e$.

EXECUTE: **(a)** $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{(26)(1.60\times10^{-19}\text{ C})}{(6.00\times10^{-10}\text{ m})^2} = 1.04\times10^{11}\text{ N/C}.$

(b) $E_{\text{proton}} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(1.60\times10^{-19}\text{ C})}{(5.29\times10^{-11}\text{ m})^2} = 5.15\times10^{11}\text{ N/C}.$

EVALUATE: These electric fields are very large. In each case the charge is positive and the electric fields are directed away from the nucleus or proton.

21.31. **IDENTIFY:** For a point charge, $E = k\dfrac{|q|}{r^2}$. The net field is the vector sum of the fields produced by each charge. A charge q in an electric field $\vec{E}$ experiences a force $\vec{F} = q\vec{E}$.

SET UP: The electric field of a negative charge is directed toward the charge. Point A is 0.100 m from q_2 and 0.150 m from q_1. Point B is 0.100 m from q_1 and 0.350 m from q_2.

EXECUTE: **(a)** The electric fields due to the charges at point A are shown in Figure 21.31a.

$$E_1 = k\frac{|q_1|}{r_{A1}^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{6.25\times10^{-9}\text{ C}}{(0.150\text{ m})^2} = 2.50\times10^3\text{ N/C}$$

$$E_2 = k\frac{|q_2|}{r_{A2}^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{12.5\times10^{-9}\text{ C}}{(0.100\text{ m})^2} = 1.124\times10^4\text{ N/C}$$

Since the two fields are in opposite directions, we subtract their magnitudes to find the net field.
$E = E_2 - E_1 = 8.74\times10^3\ \text{N/C}$, to the right.

(b) The electric fields at points B are shown in Figure 21.31b.

$$E_1 = k\frac{|q_1|}{r_{B1}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.25\times10^{-9}\ \text{C}}{(0.100\ \text{m})^2} = 5.619\times10^3\ \text{N/C}$$

$$E_2 = k\frac{|q_2|}{r_{B2}^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{12.5\times10^{-9}\ \text{C}}{(0.350\ \text{m})^2} = 9.17\times10^2\ \text{N/C}$$

Since the fields are in the same direction, we add their magnitudes to find the net field. $E = E_1 + E_2 = 6.54\times10^3$ N/C, to the right.

(c) At A, $E = 8.74\times10^3$ N/C, to the right. The force on a proton placed at this point would be
$F = qE = (1.60\times10^{-19}\ \text{C})(8.74\times10^3\ \text{N/C}) = 1.40\times10^{-15}\ \text{N}$, to the right.

EVALUATE: A proton has positive charge so the force that an electric field exerts on it is in the same direction as the field.

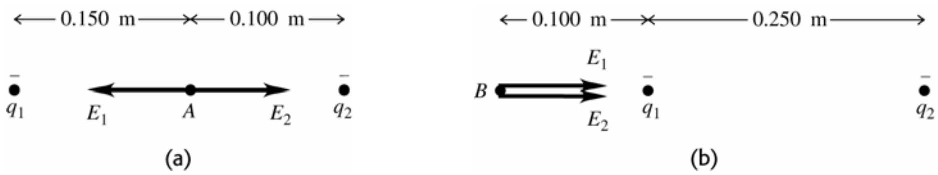


Figure 21.31

21.32. **IDENTIFY:** The electric force is $\vec{F} = q\vec{E}$.

SET UP: The gravity force (weight) has magnitude $w = mg$ and is downward.

EXECUTE: **(a)** To balance the weight the electric force must be upward. The electric field is downward, so for an upward force the charge q of the person must be negative. $w = F$ gives $mg = |q|E$ and

$$|q| = \frac{mg}{E} = \frac{(60\ \text{kg})(9.80\ \text{m/s}^2)}{150\ \text{N/C}} = 3.9\ \text{C}.$$

(b) $F = k\dfrac{|qq'|}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(3.9\ \text{C})^2}{(100\ \text{m})^2} = 1.4\times10^7\ \text{N}$. The repulsive force is immense and this is not a feasible means of flight.

EVALUATE: The net charge of charged objects is typically much less than 1 C.

21.33. **IDENTIFY:** Eq. (21.3) gives the force on the particle in terms of its charge and the electric field between the plates. The force is constant and produces a constant acceleration. The motion is similar to projectile motion; use constant acceleration equations for the horizontal and vertical components of the motion.

(a) SET UP: The motion is sketched in Figure 21.33a.

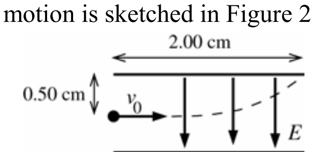


Figure 21.33a

For an electron $q = -e$.

$\vec{F} = q\vec{E}$ and q negative gives that $\vec{F}$ and $\vec{E}$ are in opposite directions, so $\vec{F}$ is upward. The free-body diagram for the electron is given in Figure 21.33b.

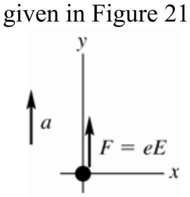


Figure 21.33b

EXECUTE: $\sum F_y = ma_y$

$eE = ma$

Solve the kinematics to find the acceleration of the electron: Just misses upper plate says that $x - x_0 = 2.00$ cm when $y - y_0 = +0.500$ cm.

<u>x-component</u>

$v_{0x} = v_0 = 1.60\times10^6$ m/s, $a_x = 0$, $x - x_0 = 0.0200$ m, $t = ?$

$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2$

$t = \dfrac{x - x_0}{v_{0x}} = \dfrac{0.0200\text{ m}}{1.60\times10^6\text{ m/s}} = 1.25\times10^{-8}\text{ s}$

In this same time t the electron travels 0.0050 m vertically:

<u>y-component</u>

$t = 1.25\times10^{-8}\text{s},\ v_{0y} = 0,\ y - y_0 = +0.0050\text{ m},\ a_y = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

$a_y = \dfrac{2(y - y_0)}{t^2} = \dfrac{2(0.0050\text{ m})}{(1.25\times10^{-8}\text{ s})^2} = 6.40\times10^{13}\text{ m/s}^2$

(This analysis is very similar to that used in Chapter 3 for projectile motion, except that here the acceleration is upward rather than downward.) This acceleration must be produced by the electric-field force: $eE = ma$

$$E = \frac{ma}{e} = \frac{(9.109\times10^{-31}\text{ kg})(6.40\times10^{13}\text{ m/s}^2)}{1.602\times10^{-19}\text{ C}} = 364\text{ N/C}$$

Note that the acceleration produced by the electric field is <u>much</u> larger than g, the acceleration produced by gravity, so it is perfectly ok to neglect the gravity force on the elctron in this problem.

(b) $a = \dfrac{eE}{m_p} = \dfrac{(1.602\times10^{-19}\text{ C})(364\text{ N/C})}{1.673\times10^{-27}\text{ kg}} = 3.49\times10^{10}\text{ m/s}^2$

This is much less than the acceleration of the electron in part (a) so the vertical deflection is less and the proton won't hit the plates. The proton has the same initial speed, so the proton takes the same time $t = 1.25\times10^{-8}\text{ s}$ to travel horizontally the length of the plates. The force on the proton is downward (in the same direction as $\vec{E}$, since q is positive), so the acceleration is downward and $a_y = -3.49\times10^{10}\text{ m/s}^2$.

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.49\times10^{10}\text{ m/s}^2)(1.25\times10^{-8}\text{ s})^2 = -2.73\times10^{-6}\text{ m}$. The displacement is $2.73\times10^{-6}\text{ m}$, downward.

(c) EVALUATE: The displacements are in opposite directions because the electron has negative charge and the proton has positive charge. The electron and proton have the same magnitude of charge, so the force the electric field exerts has the same magnitude for each charge. But the proton has a mass larger by a factor of 1836 so its acceleration and its vertical displacement are smaller by this factor.

21.34. **IDENTIFY:** Apply Eq.(21.7) to calculate the electric field due to each charge and add the two field vectors to find the resultant field.

SET UP: For q_1, $\hat{r} = \hat{j}$. For q_2, $\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$, where θ is the angle between $\vec{E}_2$ and the $+x$-axis.

EXECUTE: **(a)** $\vec{E}_1 = \dfrac{q_1}{4\pi\epsilon_0 r_1^2}\hat{j} = \dfrac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(-5.00\times10^{-9}\text{ C})}{(0.0400\text{ m})^2} = (-2.813\times10^4\text{ N/C})\,\hat{j}$.

$\left|\vec{E}_2\right| = \dfrac{q_2}{4\pi\epsilon_0 r_2^2} = \dfrac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-9}\text{ C})}{(0.0300\text{ m})^2 + (0.0400\text{ m})^2} = 1.080\times10^4\text{ N/C}$. The angle of $\vec{E}_2$, measured from the x-axis, is $180° - \tan^{-1}\left(\dfrac{4.00\text{ cm}}{3.00\text{ cm}}\right) = 126.9°$ Thus

$\vec{E}_2 = (1.080\times10^4\text{ N/C})(\hat{i}\cos126.9° + \hat{j}\sin126.9°) = (-6.485\times10^3\text{ N/C})\hat{i} + (8.64\times10^3\text{ N/C})\hat{j}$

(b) The resultant field is $\vec{E}_1 + \vec{E}_2 = (-6.485\times10^3\text{ N/C})\hat{i} + (-2.813\times10^4\text{ N/C} + 8.64\times10^3\text{ N/C})\hat{j}$.

$$\vec{E}_1 + \vec{E}_2 = (-6.485\times10^3\text{ N/C})\hat{i} - (1.95\times10^4\text{ N/C})\hat{j}.$$

EVALUATE: $\vec{E}_1$ is toward q_1 since q_1 is negative. $\vec{E}_2$ is directed away from q_2, since q_2 is positive.

21.35. **IDENTIFY:** Apply constant acceleration equations to the motion of the electron.

SET UP: Let $+x$ be to the right and let $+y$ be downward. The electron moves 2.00 cm to the right and 0.50 cm downward.

EXECUTE: Use the horizontal motion to find the time when the electron emerges from the field.

$x - x_0 = 0.0200\text{ m},\ a_x = 0,\ v_{0x} = 1.60\times10^6\text{ m/s}$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = 1.25\times10^{-8}\text{ s}$. Since $a_x = 0$, $v_x = 1.60\times10^6\text{ m/s}$. $y - y_0 = 0.0050\text{ m},\ v_{0y} = 0,\ t = 1.25\times10^{-8}\text{ s}$. $y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $v_y = 8.00\times10^5\text{ m/s}$.

Then $v = \sqrt{v_x^2 + v_y^2} = 1.79\times10^6\text{ m/s}$.

EVALUATE: $v_y = v_{0y} + a_y t$ gives $a_y = 6.4\times10^{13}\text{ m/s}^2$. The electric field between the plates is

$E = \dfrac{ma_y}{e} = \dfrac{(9.11\times10^{-31}\text{ kg})(6.4\times10^{13}\text{ m/s}^2)}{1.60\times10^{-19}\text{ C}} = 364\text{ V/m}$. This is not a very large field.

21.36. **IDENTIFY:** Use the components of $\vec{E}$ from Example 21.6 to calculate the magnitude and direction of $\vec{E}$. Use $\vec{F} = q\vec{E}$ to calculate the force on the -2.5 nC charge and use Newton's third law for the force on the -8.0 nC charge.

SET UP: From Example 21.6, $\vec{E} = (-11 \text{ N/C})\hat{i} + (14 \text{ N/C})\hat{j}$.

EXECUTE: **(a)** $E = \sqrt{E_x^2 + E_y^2} = \sqrt{(-11 \text{ N/C})^2 + (14 \text{ N/C})^2} = 17.8 \text{ N/C}$. $\tan^{-1}\left(\dfrac{|E_y|}{|E_x|}\right) = \tan^{-1}(14/11) = 51.8°$, so $\theta = 128°$ counterclockwise from the +x-axis.

(b) (i) $\vec{F} = \vec{E}q$ so $F = (17.8 \text{ N/C})(2.5\times10^{-9} \text{ C}) = 4.45\times10^{-8} \text{ N}$, at $52°$ below the +x-axis.

(ii) 4.45×10^{-8} N at $128°$ counterclockwise from the +x-axis.

EVALUATE: The forces in part (b) are repulsive so they are along the line connecting the two charges and in each case the force is directed away from the charge that exerts it.

21.37. **IDENTIFY** and **SET UP:** The electric force is given by Eq. (21.3). The gravitational force is $w_e = m_e g$. Compare these forces.

(a) EXECUTE: $w_e = (9.109\times10^{-31} \text{ kg})(9.80 \text{ m/s}^2) = 8.93\times10^{-30} \text{ N}$

In Examples 21.7 and 21.8, $E = 1.00\times10^4$ N/C, so the electric force on the electron has magnitude $F_E = |q|E = eE = (1.602\times10^{-19} \text{ C})(1.00\times10^4 \text{ N/C}) = 1.602\times10^{-15} \text{ N}$.

$$\frac{w_e}{F_E} = \frac{8.93\times10^{-30} \text{ N}}{1.602\times10^{-15} \text{ N}} = 5.57\times10^{-15}$$

The gravitational force is much smaller than the electric force and can be neglected.

(b) $mg = |q|E$

$m = |q|E/g = (1.602\times10^{-19} \text{ C})(1.00\times10^4 \text{ N/C})/(9.80 \text{ m/s}^2) = 1.63\times10^{-16} \text{ kg}$

$$\frac{m}{m_e} = \frac{1.63\times10^{-16} \text{ kg}}{9.109\times10^{-31} \text{ kg}} = 1.79\times10^{14};\ m = 1.79\times10^{14} m_e.$$

EVALUATE: m is much larger than m_e. We found in part (a) that if $m = m_e$ the gravitational force is much smaller than the electric force. $|q|$ is the same so the electric force remains the same. To get w large enough to equal F_E, the mass must be made much larger.

(c) The electric field in the region between the plates is uniform so the force it exerts on the charged object is independent of where between the plates the object is placed.

21.38. **IDENTIFY:** Apply constant acceleration equations to the motion of the proton. $E = F/|q|$.

SET UP: A proton has mass $m_p = 1.67\times10^{-27}$ kg and charge $+e$. Let +x be in the direction of motion of the proton.

EXECUTE: **(a)** $v_{0x} = 0$. $a = \dfrac{eE}{m_p}$. $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $x - x_0 = \dfrac{1}{2}a_x t^2 = \dfrac{1}{2}\dfrac{eE}{m_p}t^2$. Solving for E gives

$$E = \frac{2(0.0160 \text{ m})(1.67\times10^{-27} \text{ kg})}{(1.60\times10^{-19} \text{ C})(1.50\times10^{-6} \text{ s})^2} = 148 \text{ N/C}.$$

(b) $v_x = v_{0x} + a_x t = \dfrac{eE}{m_p}t = 2.13\times10^4 \text{ m/s}.$

EVALUATE: The electric field is directed from the positively charged plate toward the negatively charged plate and the force on the proton is also in this direction.

21.39. **IDENTIFY:** Find the angle θ that $\hat{r}$ makes with the +x-axis. Then $\hat{r} = (\cos\theta)\hat{i} + (\sin\theta)\hat{j}$.

SET UP: $\tan\theta = y/x$

EXECUTE: **(a)** $\tan^{-1}\left(\dfrac{-1.35}{0}\right) = -\dfrac{\pi}{2}$ rad. $\hat{r} = -\hat{j}$.

(b) $\tan^{-1}\left(\dfrac{12}{12}\right) = \dfrac{\pi}{4}$ rad. $\hat{r} = \dfrac{\sqrt{2}}{2}\hat{i} + \dfrac{\sqrt{2}}{2}\hat{j}$.

(c) $\tan^{-1}\left(\dfrac{2.6}{+1.10}\right) = 1.97 \text{ rad} = 112.9°$. $\hat{r} = -0.39\hat{i} + 0.92\hat{j}$ (Second quadrant).

EVALUATE: In each case we can verify that $\hat{r}$ is a unit vector, because $\hat{r}\cdot\hat{r} = 1$.

21.40. **IDENTIFY:** The net force on each charge must be zero.

SET UP: The force diagram for the $-6.50\ \mu\text{C}$ charge is given in Figure 21.40. F_E is the force exerted on the charge by the uniform electric field. The charge is negative and the field is to the right, so the force exerted by the field is to the left. F_q is the force exerted by the other point charge. The two charges have opposite signs, so the force is attractive. Take the $+x$ axis to be to the right, as shown in the figure.

EXECUTE: **(a)** $F = |q|E = (6.50\times10^{-6}\ \text{C})(1.85\times10^{8}\ \text{N/C}) = 1.20\times10^{3}\ \text{N}$

$$F_q = k\frac{|q_1q_2|}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(6.50\times10^{-6}\ \text{C})(8.75\times10^{-6}\ \text{C})}{(0.0250\ \text{m})^2} = 8.18\times10^2\ \text{N}$$

$\sum F_x = 0$ gives $T + F_q - F_E = 0$ and $T = F_E - F_q = 382\ \text{N}$.

(b) Now F_q is to the left, since like charges repel.

$\sum F_x = 0$ gives $T - F_q - F_E = 0$ and $T = F_E + F_q = 2.02\times10^{3}\ \text{N}$.

EVALUATE: The tension is much larger when both charges have the same sign, so the force one charge exerts on the other is repulsive.

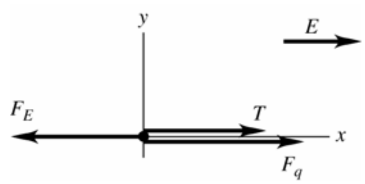


Figure 21.40

21.41. **IDENTIFY** and **SET UP:** Use $\vec{E}$ in Eq. (21.3) to calculate $\vec{F}$, $\vec{F} = m\vec{a}$ to calculate $\vec{a}$, and a constant acceleration equation to calculate the final velocity. Let $+x$ be east.

(a) EXECUTE: $F_x = |q|E = (1.602\times10^{-19}\ \text{C})(1.50\ \text{N/C}) = 2.403\times10^{-19}\ \text{N}$

$a_x = F_x/m = (2.403\times10^{-19}\ \text{N})/(9.109\times10^{-31}\ \text{kg}) = +2.638\times10^{11}\ \text{m/s}^2$

$v_{0x} = +4.50\times10^{5}\ \text{m/s},\ a_x = +2.638\times10^{11}\ \text{m/s}^2,\ x - x_0 = 0.375\ \text{m},\ v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 6.33\times10^{5}\ \text{m/s}$

EVALUATE: $\vec{E}$ is west and q is negative, so $\vec{F}$ is east and the electron speeds up.

(b) EXECUTE: $F_x = -|q|E = -(1.602\times10^{-19}\ \text{C})(1.50\ \text{N/C}) = -2.403\times10^{-19}\ \text{N}$

$a_x = F_x/m = (-2.403\times10^{-19}\ \text{N})/(1.673\times10^{-27}\ \text{kg}) = -1.436\times10^{8}\ \text{m/s}^2$

$v_{0x} = +1.90\times10^{4}\text{m/s},\ a_x = -1.436\times10^{8}\ \text{m/s}^2,\ x - x_0 = 0.375\ \text{m},\ v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 1.59\times10^{4}\ \text{m/s}$

EVALUATE: $q > 0$ so $\vec{F}$ is west and the proton slows down.

21.42. **IDENTIFY:** Coulomb's law for a single point-charge gives the electric field.

(a) SET UP: Coulomb's law for a point-charge is $E = (1/4\pi\epsilon_0)q/r^2$.

EXECUTE: $E = (9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})/(1.50\times10^{-15}\ \text{m})^2 = 6.40\times10^{20}\ \text{N/C}$

(b) Taking the ratio of the electric fields gives

$$E/E_{\text{plates}} = (6.40\times10^{20}\ \text{N/C})/(1.00\times10^{4}\ \text{N/C}) = 6.40\times10^{16}\ \text{times as strong}$$

EVALUATE: The electric field within the nucleus is huge compared to typical laboratory fields!

21.43. **IDENTIFY:** Calculate the electric field due to each charge and find the vector sum of these two fields.

SET UP: At points on the x-axis only the x component of each field is nonzero. The electric field of a point charge points away from the charge if it is positive and toward it if it is negative.

EXECUTE: **(a)** Halfway between the two charges, $E = 0$.

(b) For $|x| < a$, $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right) = -\dfrac{4q}{4\pi\epsilon_0}\dfrac{ax}{(x^2-a^2)^2}$.

For $x > a$, $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right) = \dfrac{2q}{4\pi\epsilon_0}\dfrac{x^2+a^2}{(x^2-a^2)^2}$.

For $x < -a$, $E_x = \dfrac{-1}{4\pi\epsilon_0}\left(\dfrac{q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right) = -\dfrac{2q}{4\pi\epsilon_0}\dfrac{x^2+a^2}{(x^2-a^2)^2}$.

The graph of E_x versus x is sketched in Figure 21.43.

EVALUATE: The magnitude of the field approaches infinity at the location of one of the point charges.

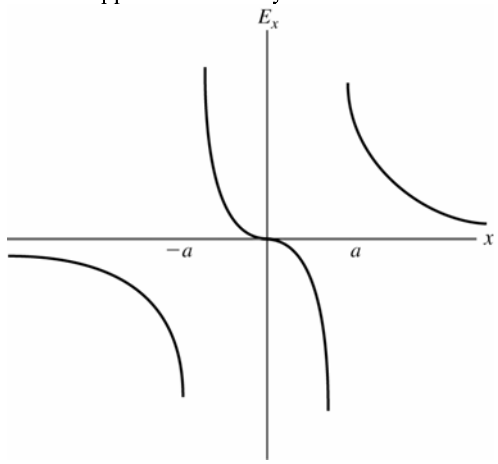


Figure 21.43

21.44. **IDENTIFY:** For a point charge, $E = k\frac{|q|}{r^2}$. For the net electric field to be zero, $\vec{E}_1$ and $\vec{E}_2$ must have equal magnitudes and opposite directions.

SET UP: Let $q_1 = +0.500$ nC and $q_2 = +8.00$ nC. $\vec{E}$ is toward a negative charge and away from a positive charge.

EXECUTE: The two charges and the directions of their electric fields in three regions are shown in Figure 21.44. Only in region II are the two electric fields in opposite directions. Consider a point a distance x from q_1 so a distance $1.20\text{ m} - x$ from q_2. $E_1 = E_2$ gives $k\frac{0.500\text{ nC}}{x^2} = k\frac{8.00\text{ nC}}{(1.20 - x)^2}$. $16x^2 = (1.20\text{ m} - x)^2$. $4x = \pm(1.20\text{ m} - x)$ and $x = 0.24$ m is the positive solution. The electric field is zero at a point between the two charges, 0.24 m from the 0.500 nC charge and 0.96 m from the 8.00 nC charge.

EVALUATE: There is only one point along the line connecting the two charges where the net electric field is zero. This point is closer to the charge that has the smaller magnitude.

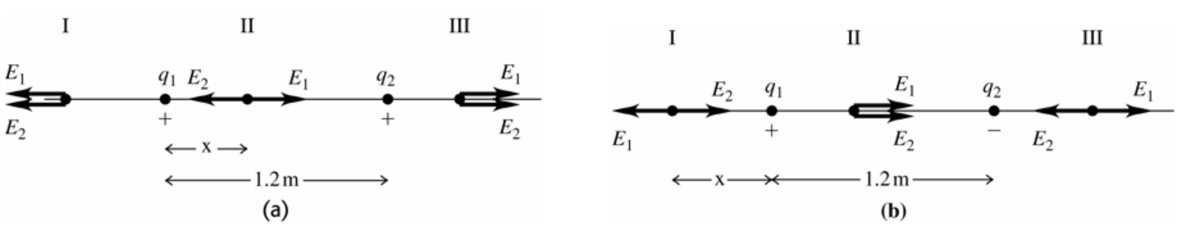


Figure 21.44

21.45. **IDENTIFY:** Eq.(21.7) gives the electric field of each point charge. Use the principle of superposition and add the electric field vectors. In part (b) use Eq.(21.3) to calculate the force, using the electric field calculated in part (a).

(a) SET UP: The placement of charges is sketched in Figure 21.45a.

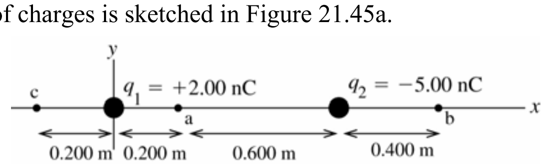


Figure 21.45a

The electric field of a point charge is directed away from the point charge if the charge is positive and toward the point charge if the charge is negative. The magnitude of the electric field is $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$, where r is the distance between the point where the field is calculated and the point charge.

(i) At point a the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.45b.

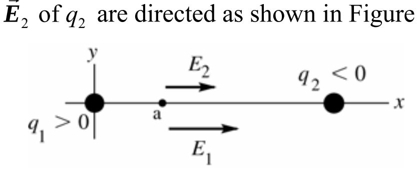


Figure 21.45b

EXECUTE: $E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 449.4\ \text{N/C}$

$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{5.00\times10^{-9}\ \text{C}}{(0.600\ \text{m})^2} = 124.8\ \text{N/C}$

$E_{1x} = 449.4\ \text{N/C},\ E_{1y} = 0$

$E_{2x} = 124.8\ \text{N/C},\ E_{2y} = 0$

$E_x = E_{1x} + E_{2x} = +449.4\ \text{N/C} + 124.8\ \text{N/C} = +574.2\ \text{N/C}$

$E_y = E_{1y} + E_{2y} = 0$

The resultant field at point a has magnitude 574 N/C and is in the $+x$-direction.

(ii) **SET UP:** At point b the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.45c.

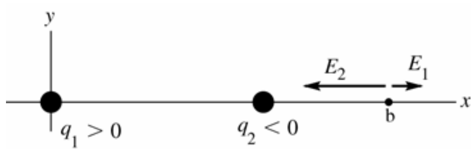


Figure 21.45c

EXECUTE: $E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(1.20\ \text{m})^2} = 12.5\ \text{N/C}$

$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{5.00\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2} = 280.9\ \text{N/C}$

$E_{1x} = 12.5\ \text{N/C},\ E_{1y} = 0$

$E_{2x} = -280.9\ \text{N/C},\ E_{2y} = 0$

$E_x = E_{1x} + E_{2x} = +12.5\ \text{N/C} - 280.9\ \text{N/C} = -268.4\ \text{N/C}$

$E_y = E_{1y} + E_{2y} = 0$

The resultant field at point b has magnitude 268 N/C and is in the $-x$-direction.

(iii) **SET UP:** At point c the fields $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2 are directed as shown in Figure 21.45d.

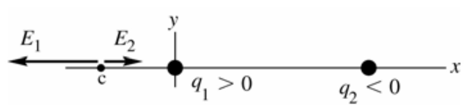


Figure 21.45d

EXECUTE: $E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} = 449.4\ \text{N/C}$

$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{5.00\times10^{-9}\ \text{C}}{(1.00\ \text{m})^2} = 44.9\ \text{N/C}$

$E_{1x} = -449.4\ \text{N/C},\ E_{1y} = 0$

$E_{2x} = +44.9\ \text{N/C},\ E_{2y} = 0$

$E_x = E_{1x} + E_{2x} = -449.4\ \text{N/C} + 44.9\ \text{N/C} = -404.5\ \text{N/C}$

$E_y = E_{1y} + E_{2y} = 0$

The resultant field at point b has magnitude 404 N/C and is in the $-x$-direction.

(b) SET UP: Since we have calculated $\vec{E}$ at each point the simplest way to get the force is to use $\vec{F} = -e\vec{E}$.

EXECUTE: (i) $F = (1.602\times10^{-19}\ \text{C})(574.2\ \text{N/C}) = 9.20\times10^{-17}\ \text{N}$, $-x$-direction

(ii) $F = (1.602\times10^{-19}\ \text{C})(268.4\ \text{N/C}) = 4.30\times10^{-17}\ \text{N}$, $+x$-direction

(iii) $F = (1.602\times10^{-19}\ \text{C})(404.5\ \text{N/C}) = 6.48\times10^{-17}\ \text{N}$, $+x$-direction

EVALUATE: The general rule for electric field direction is away from positive charge and toward negative charge. Whether the field is in the $+x$- or $-x$-direction depends on where the field point is relative to the charge that produces the field. In part (a) the field magnitudes were added because the fields were in the same direction and in (b) and (c) the field magnitudes were subtracted because the two fields were in opposite directions. In part (b) we could have used Coulomb's law to find the forces on the electron due to the two charges and then added these force vectors, but using the resultant electric field is much easier.

21.46. **IDENTIFY:** Apply Eq.(21.7) to calculate the field due to each charge and then require that the vector sum of the two fields to be zero.

SET UP: The field of each charge is directed toward the charge if it is negative and away from the charge if it is positive .

EXECUTE: The point where the two fields cancel each other will have to be closer to the negative charge, because it is smaller. Also, it can't be between the two charges, since the two fields would then act in the same direction. We could use Coulomb's law to calculate the actual values, but a simpler way is to note that the 8.00 nC charge is twice as large as the -4.00 nC charge. The zero point will therefore have to be a factor of $\sqrt{2}$ farther from the 8.00 nC charge for the two fields to have equal magnitude. Calling x the distance from the –4.00 nC charge: $1.20 + x = \sqrt{2}x$ and $x = 2.90\text{ m}$.

EVALUATE: This point is 4.10 m from the 8.00 nC charge. The two fields at this point are in opposite directions and have equal magnitudes.

21.47. **IDENTIFY:** $E = k\dfrac{|q|}{r^2}$. The net field is the vector sum of the fields due to each charge.

SET UP: The electric field of a negative charge is directed toward the charge. Label the charges q_1, q_2 and q_3, as shown in Figure 21.47a. This figure also shows additional distances and angles. The electric fields at point P are shown in Figure 21.47b. This figure also shows the xy coordinates we will use and the x and y components of the fields $\vec{E}_1$, $\vec{E}_2$ and $\vec{E}_3$.

EXECUTE: $E_1 = E_3 = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{5.00\times10^{-6}\text{ C}}{(0.100\text{ m})^2} = 4.49\times10^6\text{ N/C}$

$$E_2 = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{2.00\times10^{-6}\text{ C}}{(0.0600\text{ m})^2} = 4.99\times10^6\text{ N/C}$$

$$E_y = E_{1y} + E_{2y} + E_{3y} = 0 \text{ and } E_x = E_{1x} + E_{2x} + E_{3x} = E_2 + 2E_1\cos53.1° = 1.04\times10^7\text{ N/C}$$

$E = 1.04\times10^7\text{ N/C}$, toward the $-2.00\ \mu\text{C}$ charge.

EVALUATE: The x-components of the fields of all three charges are in the same direction.

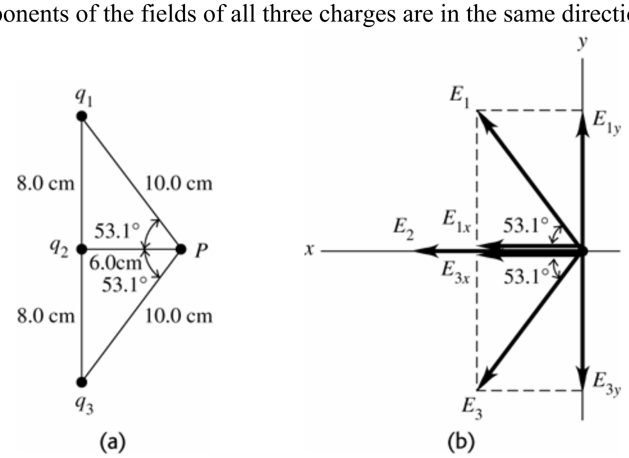


Figure 21.47

21.48. **IDENTIFY:** A positive and negative charge, of equal magnitude q, are on the x-axis, a distance a from the origin. Apply Eq.(21.7) to calculate the field due to each charge and then calculate the vector sum of these fields.

SET UP: $\vec{E}$ due to a point charge is directed away from the charge if it is positive and directed toward the charge if it is negative.

EXECUTE: **(a)** Halfway between the charges, both fields are in the $-x$-direction and $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{2q}{a^2}$, in the $-x$-direction .

(b) $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{-q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right)$ for $|x| < a$. $E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{-q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right)$ for $x > a$.

$E_x = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{-q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right)$ for $x < -a$. E_x is graphed in Figure 21.48.

EVALUATE: At points on the x axis and between the charges, E_x is in the $-x$-direction because the fields from both charges are in this direction. For $x < -a$ and $x > +a$, the fields from the two charges are in opposite directions and the field from the closer charge is larger in magnitude.

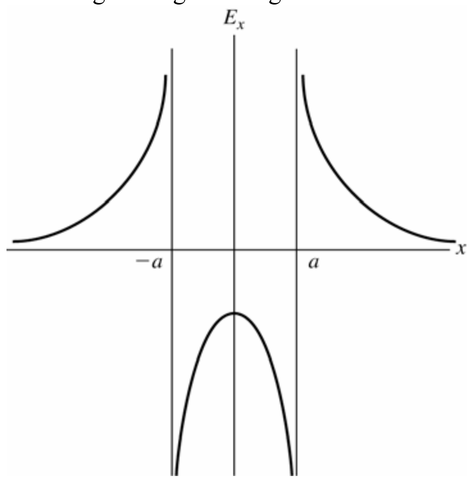


Figure 21.48

21.49. IDENTIFY: The electric field of a positive charge is directed radially outward from the charge and has magnitude $E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$. The resultant electric field is the vector sum of the fields of the individual charges.

SET UP: The placement of the charges is shown in Figure 21.49a.

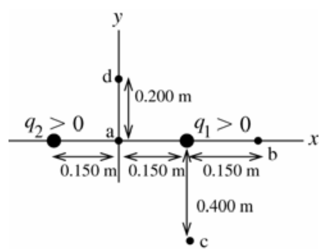


Figure 21.49a

EXECUTE: **(a)** The directions of the two fields are shown in Figure 21.49b.

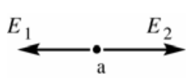


Figure 21.49b

$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$ with $r = 0.150$ m.

$E = E_2 - E_1 = 0;\ E_x = 0,\ E_y = 0$

(b) The two fields have the directions shown in Figure 21.49c.

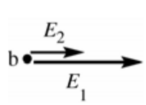


Figure 21.49c

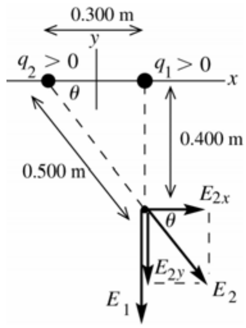
$E = E_1 + E_2$, in the $+x$-direction

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.150\ \text{m})^2} = 2396.8\ \text{N/C}$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.450\ \text{m})^2} = 266.3\ \text{N/C}$$

$$E = E_1 + E_2 = 2396.8\ \text{N/C} + 266.3\ \text{N/C} = 2660\ \text{N/C};\ E_x = +2260\ \text{N/C},\ E_y = 0$$

(c) The two fields have the directions shown in Figure 21.49d.



$$\sin\theta = \frac{0.400\text{ m}}{0.500\text{ m}} = 0.800$$

$$\cos\theta = \frac{0.300\text{ m}}{0.500\text{ m}} = 0.600$$

Figure 21.49d

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2}$$

$$E_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.400\text{ m})^2} = 337.1\text{ N/C}$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2}$$

$$E_2 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.500\text{ m})^2} = 215.7\text{ N/C}$$

$$E_{1x} = 0,\ E_{1y} = -E_1 = -337.1\text{ N/C}$$

$$E_{2\text{x}} = +E_2\cos\theta = +(215.7\text{ N/C})(0.600) = +129.4\text{ N/C}$$

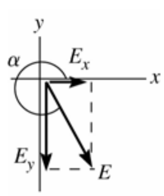
$$E_{2\text{y}} = -E_2\sin\theta = -(215.7\text{ N/C})(0.800) = -172.6\text{ N/C}$$

$$E_\text{x} = E_{1x} + E_{2x} = +129\text{ N/C}$$

$$E_\text{y} = E_{1y} + E_{2y} = -337.1\text{ N/C } -172.6\text{ N/C } = -510\text{ N/C}$$

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(129\text{ N/C})^2 + (-510\text{ N/C})^2} = 526\text{ N/C}$$

$\vec{E}$ and its components are shown in Figure 21.49e.



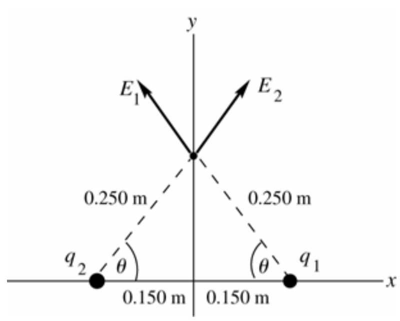
$$\tan\alpha = \frac{E_y}{E_x}$$

$$\tan\alpha = \frac{-510\text{ N/C}}{+129\text{ N/C}} = -3.953$$

$\alpha = 284°\text{C}$, counterclockwise from $+x$-axis

Figure 21.49e

(d) The two fields have the directions shown in Figure 21.49f.



$$\sin\theta = \frac{0.200\text{ m}}{0.250\text{ m}} = 0.800$$

Figure 21.49f

The components of the two fields are shown in Figure 21.49g.

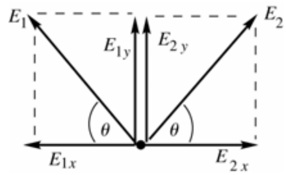


Figure 21.49g

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$$

$$E_1 = (8.988\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{6.00\times10^{-9}\ \mathrm{C}}{(0.250\ \mathrm{m})^2}$$

$$E_1 = E_2 = 862.8\ \mathrm{N/C}$$

$E_{1x} = -E_1\cos\theta,\ E_{2x} = +E_2\cos\theta$

$E_x = E_{1x} + E_{2x} = 0$

$E_{1y} = +E_1\sin\theta,\ E_{2y} = +E_2\sin\theta$

$E_y = E_{1y} + E_{2y} = 2E_{1y} = 2E_1\sin\theta = 2(862.8\ \mathrm{N/C})(0.800) = 1380\ \mathrm{N/C}$

$E = 1380$ N/C, in the $+y$-direction.

EVALUATE: Point a is symmetrically placed between identical charges, so symmetry tells us the electric field must be zero. Point b is to the right of both charges and both electric fields are in the $+x$-direction and the resultant field is in this direction. At point c both fields have a downward component and the field of q_2 has a component to the right, so the net $\vec{E}$ is in the 4th quadrant. At point d both fields have an upward component but by symmetry they have equal and opposite x-components so the net field is in the $+y$-direction. We can use this sort of reasoning to deduce the general direction of the net field before doing any calculations.

21.50. **IDENTIFY:** Apply Eq.(21.7) to calculate the field due to each charge and then calculate the vector sum of those fields.

SET UP: The fields due to q_1 and to q_2 are sketched in Figure 21.50.

EXECUTE: $\vec{E}_2 = \frac{1}{4\pi\epsilon_0}\frac{(6.00\times10^{-9}\ \mathrm{C})}{(0.6\ \mathrm{m})^2}(-\hat{i}) = -150\hat{i}$ N/C.

$$\vec{E}_1 = \frac{1}{4\pi\epsilon_0}(4.00\times10^{-9}\ \mathrm{C})\left(\frac{1}{(1.00\ \mathrm{m})^2}(0.600)\hat{i} + \frac{1}{(1.00\ \mathrm{m})^2}(0.800)\hat{j}\right) = (21.6\hat{i} + 28.8\hat{j})\ \mathrm{N/C}.$$

$\vec{E} = \vec{E}_1 + \vec{E}_2 = (-128.4\ \mathrm{N/C})\hat{i} + (28.8\ \mathrm{N/C})\hat{j}$. $E = \sqrt{(128.4\ \mathrm{N/C})^2 + (28.8\ \mathrm{N/C})^2} = 131.6\ \mathrm{N/C}$ at

$\theta = \tan^{-1}\left(\frac{28.8}{128.4}\right) = 12.6°$ above the $-x$ axis and therefore $196.2°$ counterclockwise from the $+x$ axis.

EVALUATE: $\vec{E}_1$ is directed toward q_1 because q_1 is negative and $\vec{E}_2$ is directed away from q_2 because q_2 is positive.

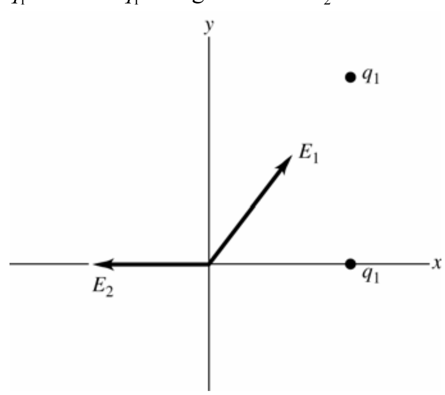


Figure 21.50

21.51. **IDENTIFY:** The resultant electric field is the vector sum of the field $\vec{E}_1$ of q_1 and $\vec{E}_2$ of q_2.

SET UP: The placement of the charges is shown in Figure 21.51a.

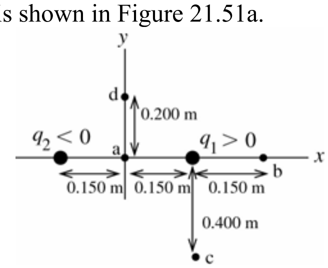


Figure 21.51a

EXECUTE: **(a)** The directions of the two fields are shown in Figure 21.51b.

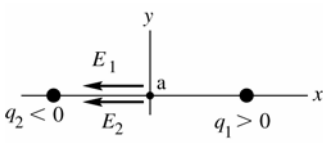


Figure 21.51b

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2}$$

$$E_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.150\ \text{m})^2}$$

$$E_1 = E_2 = 2397\ \text{N/C}$$

$E_{1x} = -2397$ N/C, $E_{1y} = 0$ $E_{2x} = -2397$ N/C, $E_{2y} = 0$

$E_x = E_{1x} + E_{2x} = 2(-2397\ \text{N/C}) = -4790\ \text{N/C}$

$E_y = E_{1y} + E_{2y} = 0$

The resultant electric field at point a in the sketch has magnitude 4790 N/C and is in the $-x$-direction.

(b) The directions of the two fields are shown in Figure 21.51c.

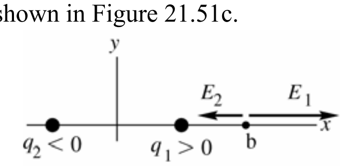


Figure 21.51c

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.150\ \text{m})^2} = 2397\ \text{N/C}$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.450\ \text{m})^2} = 266\ \text{N/C}$$

$E_{1x} = +2397$ N/C, $E_{1y} = 0$ $E_{2x} = -266$ N/C, $E_{2y} = 0$

$E_x = E_{1x} + E_{2x} = +2397\ \text{N/C} - 266\ \text{N/C} = +2130\ \text{N/C}$

$E_y = E_{1y} + E_{2y} = 0$

The resultant electric field at point b in the sketch has magnitude 2130 N/C and is in the $+x$-direction.

(c) The placement of the charges is shown in Figure 21.51d.

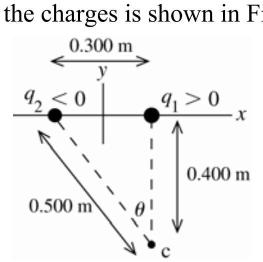


Figure 21.51d

$$\sin\theta = \frac{0.300\ \text{m}}{0.500\ \text{m}} = 0.600$$

$$\cos\theta = \frac{0.400\ \text{m}}{0.500\ \text{m}} = 0.800$$

The directions of the two fields are shown in Figure 21.51e.

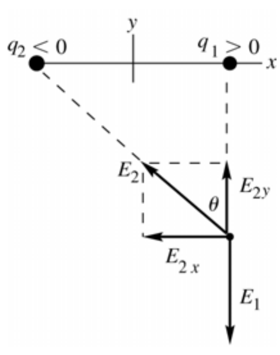


Figure 21.51e

$$E_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1|}{r_1^2}$$

$$E_1 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2}$$

$$E_1 = 337.0\ \text{N/C}$$

$$E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q_2|}{r_2^2}$$

$$E_2 = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9}\ \text{C}}{(0.500\ \text{m})^2}$$

$$E_2 = 215.7\ \text{N/C}$$

$E_{1x} = 0$, $E_{1y} = -E_1 = -337.0$ N/C

$E_{2x} = -E_2\sin\theta = -(215.7\ \text{N/C})(0.600) = -129.4\ \text{N/C}$

$E_{2y} = +E_2\cos\theta = +(215.7\ \text{N/C})(0.800) = +172.6\ \text{N/C}$

$E_x = E_{1x} + E_{2x} = -129 \text{ N/C}$

$E_y = E_{1y} + E_{2y} = -337.0 \text{ N/C } +172.6 \text{ N/C} = -164 \text{ N/C}$

$E = \sqrt{E_x^2 + E_y^2} = 209 \text{ N/C}$

The field $\vec{E}$ and its components are shown in Figure 21.51f.

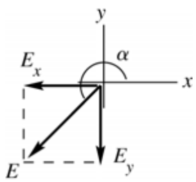


Figure 21.51f

$$\tan\alpha = \frac{E_y}{E_x}$$

$$\tan\alpha = \frac{-164 \text{ N/C}}{-129 \text{ N/C}} = +1.271$$

$\alpha = 232°$, counterclockwise from $+x$-axis

(d) The placement of the charges is shown in Figure 21.51g.

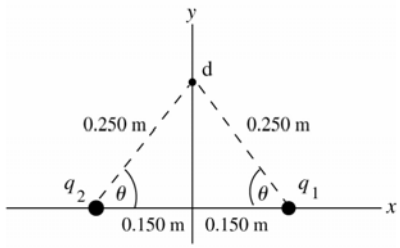


Figure 21.51g

$$\sin\theta = \frac{0.200 \text{ m}}{0.250 \text{ m}} = 0.800$$

$$\cos\theta = \frac{0.150 \text{ m}}{0.250 \text{ m}} = 0.600$$

The directions of the two fields are shown in Figure 21.51h.

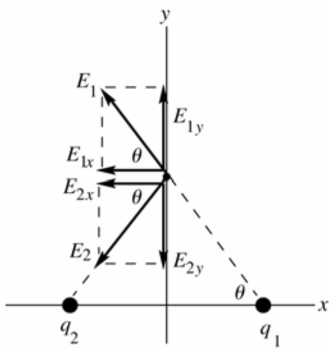


Figure 21.51h

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}$$

$$E_1 = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{6.00\times10^{-9} \text{ C}}{(0.250 \text{ m})^2}$$

$E_1 = 862.8 \text{ N/C}$

$E_2 = E_1 = 862.8 \text{ N/C}$

$E_{1x} = -E_1\cos\theta,\ E_{2x} = -E_2\cos\theta$

$E_x = E_{1x} + E_{2x} = -2(862.8 \text{ N/C})(0.600) = -1040 \text{ N/C}$

$E_{1y} = +E_1\sin\theta,\ E_{2y} = -E_2\sin\theta$

$E_y = E_{1y} + E_{2y} = 0$

$E = 1040$ N/C, in the $-x$-direction.

EVALUATE: The electric field produced by a charge is toward a negative charge and away from a positive charge. As in Exercise 21.45, we can use this rule to deduce the direction of the resultant field at each point before doing any calculations.

21.52. IDENTIFY: For a long straight wire, $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$.

SET UP: $\dfrac{1}{2\pi\epsilon_0} = 4.49\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2$.

EXECUTE: $r = \dfrac{1.5\times10^{-10} \text{ C/m}}{2\pi\epsilon_0(2.50 \text{ N/C})} = 1.08 \text{ m}$

EVALUATE: For a point charge, E is proportional to $1/r^2$. For a long straight line of charge, E is proportional to $1/r$.

21.53. **IDENTIFY:** Apply Eq.(21.10) for the finite line of charge and $E = \dfrac{\lambda}{2\pi\epsilon_0}$ for the infinite line of charge.

SET UP: For the infinite line of positive charge, $\vec{E}$ is in the $+x$ direction.

EXECUTE: **(a)** For a line of charge of length $2a$ centered at the origin and lying along the y-axis, the electric field is given by Eq.(21.10): $\vec{E} = \dfrac{1}{2\pi\epsilon_0}\dfrac{\lambda}{x\sqrt{x^2/a^2+1}}\hat{i}$.

(b) For an infinite line of charge: $\vec{E} = \dfrac{\lambda}{2\pi\epsilon_0 x}\hat{i}$. Graphs of electric field versus position for both distributions of charge are shown in Figure 21.53.

EVALUATE: For small x, close to the line of charge, the field due to the finite line approaches that of the infinite line of charge. As x increases, the field due to the infinite line falls off more slowly and is larger than the field of the finite line.

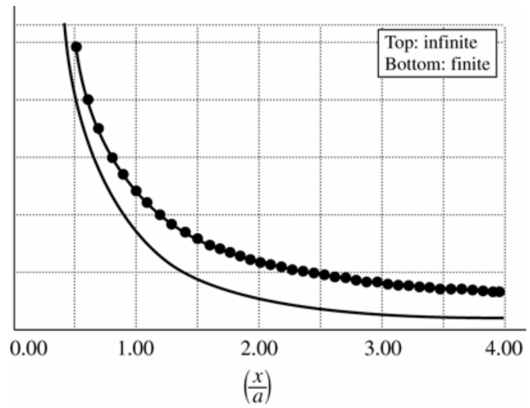


Figure 21.53

21.54. **(a) IDENTIFY:** The field is caused by a finite uniformly charged wire.

SET UP: The field for such a wire a distance x from its midpoint is

$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{x\sqrt{(x/a)^2+1}} = 2\left(\frac{1}{4\pi\epsilon_0}\right)\frac{\lambda}{x\sqrt{(x/a)^2+1}}.$$

EXECUTE: $E = \dfrac{(18.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(175\times10^{-9}\ \text{C/m})}{(0.0600\ \text{m})\sqrt{\left(\dfrac{6.00\ \text{cm}}{4.25\ \text{cm}}\right)^2+1}}$ = 3.03 × 10^4 N/C, directed upward.

(b) IDENTIFY: The field is caused by a uniformly charged circular wire.

SET UP: The field for such a wire a distance x from its midpoint is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$. We first find the radius of the circle using $2\pi r = l$.

EXECUTE: Solving for r gives $r = l/2\pi = (8.50\ \text{cm})/2\pi = 1.353$ cm

The charge on this circle is $Q = \lambda l = (175\ \text{nC/m})(0.0850\ \text{m}) = 14.88$ nC

The electric field is

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{3/2}} = \frac{(9.00\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(14.88\times10^{-9}\,\text{C/m})(0.0600\,\text{m})}{\left[(0.0600\,\text{m})^2+(0.01353\,\text{m})^2\right]^{3/2}}$$

$$E = 3.45 \times 10^4\ \text{N/C, upward.}$$

EVALUATE: In both cases, the fields are of the same order of magnitude, but the values are different because the charge has been bent into different shapes.

21.55. **IDENTIFY:** For a ring of charge, the electric field is given by Eq. (21.8). $\vec{F} = q\vec{E}$. In part (b) use Newton's third law to relate the force on the ring to the force exerted by the ring.

SET UP: $Q = 0.125\times10^{-9}$ C, $a = 0.025$ m and $x = 0.400$ m .

EXECUTE: **(a)** $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}\hat{i} = (7.0\ \text{N/C})\hat{i}$.

(b) $\vec{F}_{\text{on ring}} = -\vec{F}_{\text{on q}} = -q\vec{E} = -(-2.50\times10^{-6}\text{C})\,(7.0\ \text{N/C})\hat{i} = (1.75\times10^{-5}\ \text{N})\hat{i}$

EVALUATE: Charges q and Q have opposite sign, so the force that q exerts on the ring is attractive.

21.56. IDENTIFY: We must use the appropriate electric field formula: a uniform disk in (a), a ring in (b) because all the charge is along the rim of the disk, and a point-charge in (c).

(a) SET UP: First find the surface charge density (Q/A), then use the formula for the field due to a disk of charge, $E_x = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R/x)^2+1}}\right]$.

EXECUTE: The surface charge density is $\sigma = \frac{Q}{A} = \frac{Q}{\pi r^2} = \frac{6.50\times 10^{-9}\text{ C}}{\pi(0.0125\text{ m})^2} = 1.324 \times 10^{-5}\text{ C/m}^2$.

The electric field is

$$E_x = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{(R/x)^2+1}}\right] = \frac{1.324\times 10^{-5}\text{ C/m}^2}{2(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\left[1 - \frac{1}{\sqrt{\left(\frac{1.25\text{ cm}}{2.00\text{ cm}}\right)^2+1}}\right]$$

$E_x = 1.14 \times 10^5$ N/C, toward the center of the disk.

(b) SET UP: For a ring of charge, the field is $E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{3/2}}$.

EXECUTE: Substituting into the electric field formula gives

$$E = \frac{1}{4\pi\epsilon_0}\frac{Qx}{\left(x^2+a^2\right)^{3/2}} = \frac{(9.00\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(6.50\times 10^{-9}\text{ C})(0.0200\text{ m})}{\left[(0.0200\text{ m})^2+(0.0125\text{ m})^2\right]^{3/2}}$$

$E = 8.92 \times 10^4$ N/C, toward the center of the disk.

(c) SET UP: For a point charge, $E = \left(1/4\pi\epsilon_0\right)q/r^2$.

EXECUTE: $E = (9.00 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(6.50 \times 10^{-9}\text{ C})/(0.0200\text{ m})^2 = 1.46 \times 10^5$ N/C

(d) EVALUATE: With the ring, more of the charge is farther from P than with the disk. Also with the ring the component of the electric field parallel to the plane of the ring is greater than with the disk, and this component cancels. With the point charge in (c), all the field vectors add with no cancellation, and all the charge is closer to point P than in the other two cases.

21.57. IDENTIFY: By superposition we can add the electric fields from two parallel sheets of charge.

SET UP: The field due to each sheet of charge has magnitude $\sigma/2\epsilon_0$ and is directed toward a sheet of negative charge and away from a sheet of positive charge.

(a) The two fields are in opposite directions and $E = 0$.

(b) The two fields are in opposite directions and $E = 0$.

(c) The fields of both sheets are downward and $E = 2\frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$, directed downward.

EVALUATE: The field produced by an infinite sheet of charge is uniform, independent of distance from the sheet.

21.58. IDENTIFY and SET UP: The electric field produced by an infinite sheet of charge with charge density σ has magnitude $E = \frac{|\sigma|}{2\epsilon_0}$. The field is directed toward the sheet if it has negative charge and is away from the sheet if it has positive charge.

EXECUTE: (a) The field lines are sketched in Figure 21.58a.

(b) The field lines are sketched in Figure 21.58b.

EVALUATE: The spacing of the field lines indicates the strength of the field. In part (a) the two fields add between the sheets and subtract in the regions to the left of A and to the right of B. In part (b) the opposite is true.

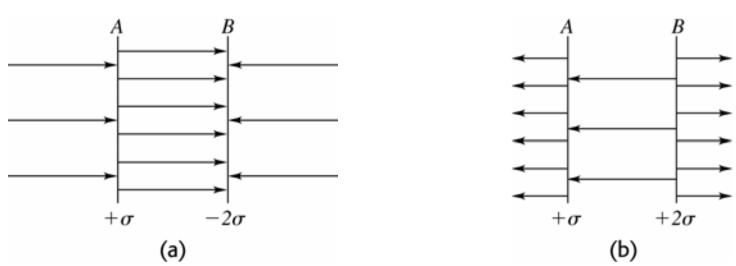


Figure 21.58

21.59. **IDENTIFY:** The force on the particle at any point is always tangent to the electric field line at that point.
SET UP: The instantaneous velocity determines the path of the particle.
EXECUTE: In Fig.21.29a the field lines are straight lines so the force is always in a straight line and velocity and acceleration are always in the same direction. The particle moves in a straight line along a field line, with increasing speed. In Fig.21.29b the field lines are curved. As the particle moves its velocity and acceleration are not in the same direction and the trajectory does not follow a field line.
EVALUATE: In two-dimensional motion the velocity is always tangent to the trajectory but the velocity is not always in the direction of the net force on the particle.

21.60. **IDENTIFY:** The field appears like that of a point charge a long way from the disk and an infinite sheet close to the disk's center. The field is symmetrical on the right and left.
SET UP: For a positive point charge, E is proportional to $1/r^2$ and is directed radially outward. For an infinite sheet of positive charge, the field is uniform and is directed away from the sheet.
EXECUTE: The field is sketched in Figure 21.60.
EVALUATE: Near the disk the field lines are parallel and equally spaced, which corresponds to a uniform field. Far from the disk the field lines are getting farther apart, corresponding to the $1/r^2$ dependence for a point charge.

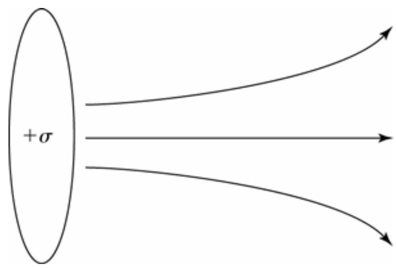


Figure 21.60

21.61. **IDENTIFY:** Use symmetry to deduce the nature of the field lines.
(a) SET UP: The only distinguishable direction is toward the line or away from the line, so the electric field lines are perpendicular to the line of charge, as shown in Figure 21.61a.

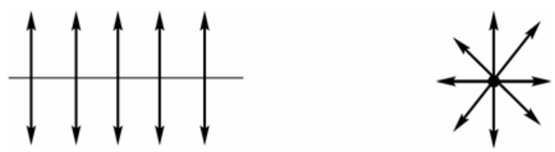


Figure 21.61a

(b) EXECUTE and EVALUATE: The magnitude of the electric field is inversely proportional to the spacing of the field lines. Consider a circle of radius r with the line of charge passing through the center, as shown in Figure 21.61b.

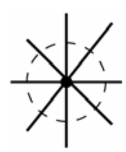


Figure 21.61b

The spacing of field lines is the same all around the circle, and in the direction perpendicular to the plane of the circle the lines are equally spaced, so E depends only on the distance r. The number of field lines passing out through the circle is independent of the radius of the circle, so the spacing of the field lines is proportional to the reciprocal of the circumference $2\pi r$ of the circle. Hence E is proportional to $1/r$.

21.62. **IDENTIFY:** Field lines are directed away from a positive charge and toward a negative charge. The density of field lines is proportional to the magnitude of the electric field.
SET UP: The field lines represent the resultant field at each point, the net field that is the vector sum of the fields due to each of the three charges.
EXECUTE: **(a)** Since field lines pass from positive charges and toward negative charges, we can deduce that the top charge is positive, middle is negative, and bottom is positive.
(b) The electric field is the smallest on the horizontal line through the middle charge, at two positions on either side where the field lines are least dense. Here the y-components of the field are cancelled between the positive charges and the negative charge cancels the x-component of the field from the two positive charges.
EVALUATE: Far from all three charges the field is the same as the field of a point charge equal to the algebraic sum of the three charges.

21.63. **(a) IDENTIFY and SET UP:** Use Eq.(21.14) to relate the dipole moment to the charge magnitude and the separation d of the two charges. The direction is from the negative charge toward the positive charge.

EXECUTE: $p = qd = (4.5\times10^{-9}\ \text{C})(3.1\times10^{-3}\ \text{m}) = 1.4\times10^{-11}\ \text{C}\cdot\text{m}$; The direction of $\vec{p}$ is from q_1 toward q_2.

(b) IDENTIFY and SET UP: Use Eq. (21.15) to relate the magnitudes of the torque and field.

EXECUTE: $\tau = pE\sin\phi$, with ϕ as defined in Figure 21.63, so

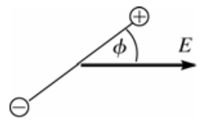


Figure 21.63

$$E = \frac{\tau}{p\sin\phi}$$

$$E = \frac{7.2\times10^{-9}\ \text{N}\cdot\text{m}}{(1.4\times10^{-11}\ \text{C}\cdot\text{m})\sin 36.9^\circ} = 860\ \text{N/C}$$

EVALUATE: Eq.(21.15) gives the torque about an axis through the center of the dipole. But the forces on the two charges form a couple (Problem 11.53) and the torque is the same for any axis parallel to this one. The force on each charge is $|q|E$ and the maximum moment arm for an axis at the center is $d/2$, so the maximum torque is $2(|q|E)(d/2) = 1.2\times10^{-8}\ \text{N}\cdot\text{m}$. The torque for the orientation of the dipole in the problem is less than this maximum.

21.64. **(a) IDENTIFY:** The potential energy is given by Eq.(21.17).

SET UP: $U(\phi) = -\vec{p}\cdot\vec{E} = -pE\cos\phi$, where ϕ is the angle between $\vec{p}$ and $\vec{E}$.

EXECUTE: parallel: $\phi = 0$ and $U(0^\circ) = -pE$

perpendicular: $\phi = 90^\circ$ and $U(90^\circ) = 0$

$$\Delta U = U(90^\circ) - U(0^\circ) = pE = (5.0\times10^{-30}\ \text{C}\cdot\text{m})(1.6\times10^{6}\ \text{N/C}) = 8.0\times10^{-24}\ \text{J}.$$

(b) $\frac{3}{2}kT = \Delta U$ so $T = \dfrac{2\Delta U}{3k} = \dfrac{2(8.0\times10^{-24}\ \text{J})}{3(1.381\times10^{-23}\ \text{J/K})} = 0.39\ \text{K}$

EVALUATE: Only at very low temperatures are the dipoles of the molecules aligned by a field of this strength. A much larger field would be required for alignment at room temperature.

21.65. **IDENTIFY:** Follow the procedure specified in part (a) of the problem.

SET UP: Use that $y >> d$.

EXECUTE: **(a)** $\dfrac{1}{(y-d/2)^2} - \dfrac{1}{(y+d/2)^2} = \dfrac{(y+d/2)^2-(y-d/2)^2}{(y^2-d^2/4)^2} = \dfrac{2yd}{(y^2-d^2/4)^2}$. This gives

$E_y = \dfrac{q}{4\pi\epsilon_0}\dfrac{2yd}{(y^2-d^2/4)^2} = \dfrac{qd}{2\pi\epsilon_0}\dfrac{y}{(y^2-d^2/4)^2}$. Since $y^2 >> d^2/4$, $E_y \approx \dfrac{p}{2\pi\epsilon_0 y^3}$.

(b) For points on the $-y$-axis, $\vec{E}_-$ is in the $+y$ direction and $\vec{E}_+$ is in the $-y$ direction. The field point is closer to $-q$, so the net field is upward. A similar derivation gives $E_y \approx \dfrac{p}{2\pi\epsilon_0 y^3}$. E_y has the same magnitude and direction at points where $y >> d$ as where $y << -d$.

EVALUATE: E falls off like $1/r^3$ for a dipole, which is faster than the $1/r^2$ for a point charge. The total charge of the dipole is zero.

21.66. **IDENTIFY:** Calculate the electric field due to the dipole and then apply $\vec{F} = q\vec{E}$.

SET UP: From Example 21.15, $E_{\text{dipole}}(x) = \dfrac{p}{2\pi\epsilon_0 x^3}$.

EXECUTE: $E_{\text{dipole}} = \dfrac{6.17\times10^{-30}\ \text{C}\cdot\text{m}}{2\pi\epsilon_0(3.0\times10^{-9}\ \text{m})^3} = 4.11\times10^{6}\ \text{N/C}$. The electric force is $F = qE =$ $(1.60\times10^{-19}\ \text{C})(4.11\times10^{6}\ \text{N/C}) = 6.58\times10^{-13}\ \text{N}$ and is toward the water molecule (negative x-direction).

EVALUATE: $\vec{E}_{\text{dipole}}$ is in the direction of $\vec{p}$, so is in the $+x$ direction. The charge q of the ion is negative, so $\vec{F}$ is directed opposite to $\vec{E}$ and is therefore in the $-x$ direction.

21.67. **IDENTIFY:** Like charges repel and unlike charges attract. The force increases as the distance between the charges decreases.

SET UP: The forces on the dipole that is between the slanted dipoles are sketched in Figure 21.67a.

EXECUTE: The forces are attractive because the + and − charges of the two dipoles are closest. The forces are toward the slanted dipoles so have a net upward component. In Figure 21.67b, adjacent dipoles charges of opposite sign are closer than charges of the same sign so the attractive forces are larger than the repulsive forces and the dipoles attract.

EVALUATE: Each dipole has zero net charge, but because of the charge separation there is a non-zero force between dipoles.

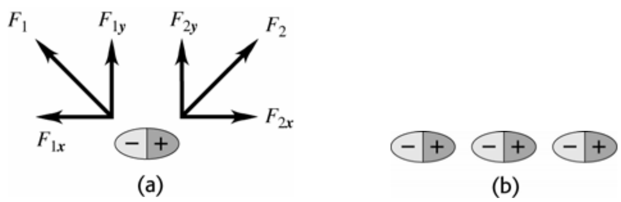


Figure 21.67

21.68. IDENTIFY: Find the vector sum of the fields due to each charge in the dipole.

SET UP: A point on the x-axis with coordinate x is a distance $r=\sqrt{(d/2)^2+x^2}$ from each charge.

EXECUTE: **(a)** The magnitude of the field the due to each charge is $E=\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}=\dfrac{q}{4\pi\epsilon_0}\left(\dfrac{1}{(d/2)^2+x^2}\right)$,

where d is the distance between the two charges. The x-components of the forces due to the two charges are equal and oppositely directed and so cancel each other. The two fields have equal y-components,

so $E=2E_y=\dfrac{2q}{4\pi\epsilon_0}\left(\dfrac{1}{(d/2)^2+x^2}\right)\sin\theta$, where θ is the angle below the x-axis for both fields. $\sin\theta=\dfrac{d/2}{\sqrt{(d/2)^2+x^2}}$

and $E_{\text{dipole}}=\left(\dfrac{2q}{4\pi\epsilon_0}\right)\left(\dfrac{1}{(d/2)^2+x^2}\right)\left(\dfrac{d/2}{\sqrt{(d/2)^2+x^2}}\right)=\dfrac{qd}{4\pi\epsilon_0((d/2)^2+x^2)^{3/2}}$. The field is the $-y$ direction.

(b) At large x, $x^2 >> (d/2)^2$, so the expression in part (a) reduces to the approximation $E_{\text{dipole}}\approx\dfrac{qd}{4\pi\epsilon_0 x^3}$.

EVALUATE: Example 21.15 shows that at points on the $+y$ axis far from the dipole, $E_{\text{dipole}}\approx\dfrac{qd}{2\pi\epsilon_0 y^3}$. The expression in part (b) for points on the x axis has a similar form.

21.69. IDENTIFY: The torque on a dipole in an electric field is given by $\vec{\tau}=\vec{p}\times\vec{E}$.

SET UP: $\tau=pE\sin\phi$, where ϕ is the angle between the direction of $\vec{p}$ and the direction of $\vec{E}$.

EXECUTE: **(a)** The torque is zero when $\vec{p}$ is aligned either in the *same* direction as $\vec{E}$ or in the *opposite* direction, as shown in Figure 21.69a.

(b) The stable orientation is when $\vec{p}$ is aligned in the *same* direction as $\vec{E}$. In this case a small rotation of the dipole results in a torque directed so as to bring $\vec{p}$ back into alignment with $\vec{E}$. When $\vec{p}$ is directed opposite to $\vec{E}$, a small displacement results in a torque that takes $\vec{p}$ farther from alignment with $\vec{E}$.

(c) Field lines for E_{dipole} in the stable orientation are sketched in Figure 21.69b.

EVALUATE: The field of the dipole is directed from the + charge toward the − charge.

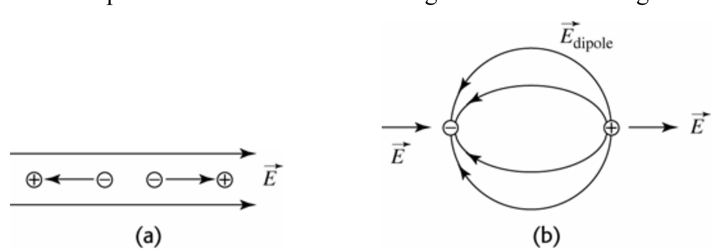


Figure 21.69

21.70. IDENTIFY: The plates produce a uniform electric field in the space between them. This field exerts torque on a dipole and gives it potential energy.

SET UP: The electric field between the plates is given by $E=\sigma/\epsilon_0$, and the dipole moment is $p=ed$. The potential energy of the dipole due to the field is $U=-\vec{p}\cdot\vec{E}=-pE\cos\phi$, and the torque the field exerts on it is $\tau=pE\sin\phi$.

EXECUTE: (a) The potential energy, $U = -\vec{p}\cdot\vec{E} = -pE\cos\phi$, is a maximum when $\phi = 180°$. The field between the plates is $E = \sigma/\epsilon_0$, giving

$$U_{\text{max}} = (1.60\times10^{-19}\ \text{C})(220\times10^{-9}\ \text{m})(125\times10^{-6}\ \text{C/m}^2)/(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 4.97\times10^{-19}\ \text{J}$$

The orientation is parallel to the electric field (perpendicular to the plates) with the positive charge of the dipole toward the positive plate.

(b) The torque, $\tau = pE\sin\phi$, is a maximum when $\phi = 90°$ or 270°. In this case

$$\tau_{\text{max}} = pE = p\sigma/\epsilon_0 = ed\sigma/\epsilon_0$$

$$\tau_{\text{max}} = \left(1.60\times10^{-19}\ \text{C}\right)\left(220\times10^{-9}\ \text{m}\right)\left(125\times10^{-6}\ \text{C/m}^2\right)\Big/\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)$$

$$\tau_{\text{max}} = 4.97\times10^{-19}\ \text{N}\cdot\text{m}$$

The dipole is oriented perpendicular to the electric field (parallel to the plates).

(c) $F = 0$.

EVALUATE: When the potential energy is a maximum, the torque is zero. In both cases, the net force on the dipole is zero because the forces on the charges are equal but opposite (which would not be true in a nonuniform electric field).

21.71. (a) IDENTIFY: Use Coulomb's law to calculate each force and then add them as vectors to obtain the net force. Torque is force times moment arm.

SET UP: The two forces on each charge in the dipole are shown in Figure 21.71a.

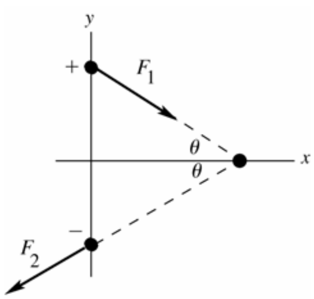


Figure 21.71a

$\sin\theta = 1.50/2.00$ so $\theta = 48.6°$

Opposite charges attract and like charges repel.

$F_x = F_{1x} + F_{2x} = 0$

EXECUTE: $F_1 = k\dfrac{|qq'|}{r^2} = k\dfrac{(5.00\times10^{-6}\ \text{C})(10.0\times10^{-6}\ \text{C})}{(0.0200\ \text{m})^2} = 1.124\times10^3\ \text{N}$

$F_{1y} = -F_1\sin\theta = -842.6\ \text{N}$

$F_{2y} = -842.6\ \text{N}$ so $F_y = F_{1y} + F_{2y} = -1680\ \text{N}$ (in the direction from the $+5.00\text{-}\mu\text{C}$ charge toward the $-5.00\text{-}\mu\text{C}$ charge).

EVALUATE: The *x*-components cancel and the *y*-components add.

(b) SET UP: Refer to Figure 21.71b.

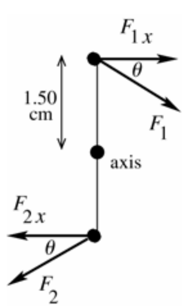


Figure 21.71b

The *y*-components have zero moment arm and therefore zero torque.

F_{1x} and F_{2x} both produce clockwise torques.

EXECUTE: $F_{1X} = F_1\cos\theta = 743.1\ \text{N}$

$\tau = 2(F_{1x})(0.0150\ \text{m}) = 22.3\ \text{N}\cdot\text{m}$, clockwise

EVALUATE: The electric field produced by the $-10.00\mu\text{C}$ charge is not uniform so Eq. (21.15) does not apply.

21.72. IDENTIFY: Apply $F = k\dfrac{|qq'|}{r^2}$ for each pair of charges and find the vector sum of the forces that q_1 and q_2 exert on q_3.

SET UP: Like charges repel and unlike charges attract. The three charges and the forces on q_3 are shown in Figure 21.72.

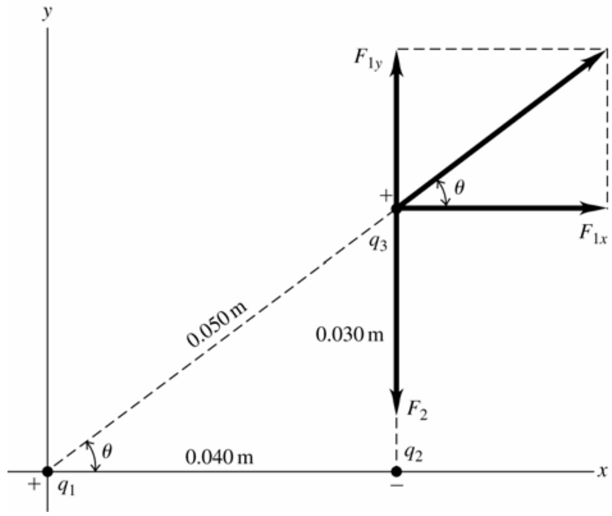


Figure 21.72

EXECUTE: **(a)** $F_1 = k\dfrac{|q_1q_3|}{r_1^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(5.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.0500\ \text{m})^2} = 1.079\times10^{-4}\ \text{C}$.

$\theta = 36.9°$. $F_{1x} = +F_1\cos\theta = 8.63\times10^{-5}\ \text{N}$. $F_{1y} = +F_1\sin\theta = 6.48\times10^{-5}\ \text{N}$.

$F_2 = k\dfrac{|q_2q_3|}{r_2^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2.00\times10^{-9}\ \text{C})(6.00\times10^{-9}\ \text{C})}{(0.0300\ \text{m})^2} = 1.20\times10^{-4}\ \text{C}$.

$F_{2x} = 0$, $F_{2y} = -F_2 = -1.20\times10^{-4}\ \text{N}$. $F_x = F_{1x} + F_{2x} = 8.63\times10^{-5}\ \text{N}$.

$F_y = F_{1y} + F_{2y} = 6.48\times10^{-5}\ \text{N} + (-1.20\times10^{-4}\ \text{N}) = -5.52\times10^{-5}\ \text{N}$.

(b) $F = \sqrt{F_x^2 + F_y^2} = 1.02\times10^{-4}\ \text{N}$. $\tan\phi = \left|\dfrac{F_y}{F_x}\right| = 0.640$. $\phi = 32.6°$, below the $+x$ axis.

EVALUATE: The individual forces on q_3 are computed from Coulomb's law and then added as vectors, using components.

21.73. (a) IDENTIFY: Use Coulomb's law to calculate the force exerted by each Q on q and add these forces as vectors to find the resultant force. Make the approximation $x >> a$ and compare the net force to $F = -kx$ to deduce k and then $f = (1/2\pi)\sqrt{k/m}$.

SET UP: The placement of the charges is shown in Figure 21.73a.

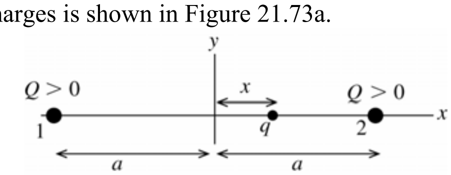


Figure 21.73a

EXECUTE: Find the net force on q.

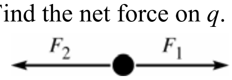


Figure 21.73b

$F_x = F_{1x} + F_{2x}$ and $F_{1x} = +F_1,\ F_{2x} = -F_2$

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{qQ}{(a+x)^2},\quad F_2 = \frac{1}{4\pi\epsilon_0}\frac{qQ}{(a-x)^2}$$

$$F_x = F_1 - F_2 = \frac{qQ}{4\pi\epsilon_0}\left[\frac{1}{(a+x)^2} - \frac{1}{(a-x)^2}\right]$$

$$F_x = \frac{qQ}{4\pi\epsilon_0 a^2}\left[+\left(1+\frac{x}{a}\right)^{-2} - \left(1-\frac{x}{a}\right)^{-2}\right]$$

Since $x << a$ we can use the binomial expansion for $(1-x/a)^{-2}$ and $(1+x/a)^{-2}$ and keep only the first two terms: $(1+z)^n \approx 1+nz$. For $(1-x/a)^{-2}$, $z=-x/a$ and $n=-2$ so $(1-x/a)^{-2} \approx 1+2x/a$. For $(1+x/a)^{-2}$, $z=+x/a$ and $n=-2$ so $(1+x/a)^{-2} \approx 1-2x/a$. Then $F \approx \frac{qQ}{4\pi\epsilon_0 a^2}\left[\left(1-\frac{2x}{a}\right)-\left(1+\frac{2x}{a}\right)\right] = -\left(\frac{qQ}{\pi\epsilon_0 a^3}\right)x$. For simple harmonic motion $F=-kx$ and the frequency of oscillation is $f=(1/2\pi)\sqrt{k/m}$. The net force here is of this form, with $k = qQ/\pi\epsilon_0 a^3$. Thus $f = \frac{1}{2\pi}\sqrt{\frac{qQ}{\pi\epsilon_0 m a^3}}$.

(b) The forces and their components are shown in Figure 21.73c.

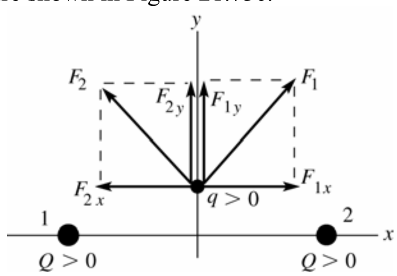


Figure 21.73c

The x-components of the forces exerted by the two charges cancel, the y-components add, and the net force is in the $+y$-direction when $y > 0$ and in the $-y$-direction when $y < 0$. The charge moves away from the origin on the y-axis and never returns.

EVALUATE: The directions of the forces and of the net force depend on where q is located relative to the other two charges. In part (a), $F = 0$ at $x = 0$ and when the charge q is displaced in the $+x$- or $-x$-direction the net force is a restoring force, directed to return q to $x = 0$. The charge oscillates back and forth, similar to a mass on a spring.

21.74. **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to one of the spheres.

SET UP: The free-body diagram is sketched in Figure 21.74. F_e is the repulsive Coulomb force between the spheres. For small θ, $\sin\theta \approx \tan\theta$.

EXECUTE: $\sum F_x = T\sin\theta - F_e = 0$ and $\sum F_y = T\cos\theta - mg = 0$. So $\frac{mg\sin\theta}{\cos\theta} = F_e = \frac{kq^2}{d^2}$. But $\tan\theta \approx \sin\theta = \frac{d}{2L}$, so $d^3 = \frac{2kq^2L}{mg}$ and $d = \left(\frac{q^2L}{2\pi\epsilon_0 mg}\right)^{1/3}$.

EVALUATE: d increases when q increases.

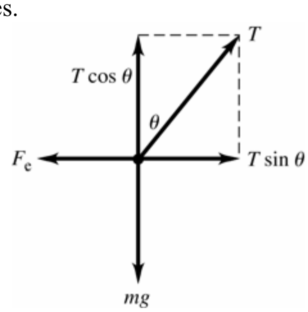


Figure 21.74

21.75. **IDENTIFY:** Use Coulomb's law for the force that one sphere exerts on the other and apply the 1st condition of equilibrium to one of the spheres.

(a) SET UP: The placement of the spheres is sketched in Figure 21.75a.

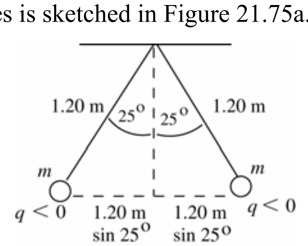


Figure 21.75a

The free-body diagrams for each sphere are given in Figure 21.75b.

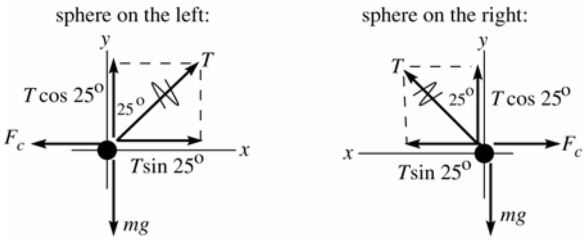


Figure 21.75b

F_c is the repulsive Coulomb force exerted by one sphere on the other.

(b) EXECUTE: From either force diagram in part (a): $\sum F_y = ma_y$

$T\cos 25.0° - mg = 0$ and $T = \dfrac{mg}{\cos 25.0°}$

$\sum F_x = ma_x$

$T\sin 25.0° - F_c = 0$ and $F_c = T\sin 25.0°$

Use the first equation to eliminate T in the second: $F_c = (mg/\cos 25.0°)(\sin 25.0°) = mg\tan 25.0°$

$$F_c = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{[2(1.20\text{ m})\sin 25.0°]^2}$$

Combine this with $F_c = mg\tan 25.0°$ and get $mg\tan 25.0° = \dfrac{1}{4\pi\epsilon_0}\dfrac{q^2}{[2(1.20\text{ m})\sin 25.0°]^2}$

$$q = (2.40\text{ m})\sin 25.0°\sqrt{\frac{mg\tan 25.0°}{(1/4\pi\epsilon_0)}}$$

$$q = (2.40\text{ m})\sin 25.0°\sqrt{\frac{(15.0\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)\tan 25.0°}{8.988\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2}} = 2.80\times10^{-6}\text{ C}$$

(c) The separation between the two spheres is given by $2L\sin\theta$. $q = 2.80\,\mu\text{C}$ as found in part (b).

$F_c = (1/4\pi\epsilon_0)q^2/(2L\sin\theta)^2$ and $F_c = mg\tan\theta$. Thus $(1/4\pi\epsilon_0)q^2/(2L\sin\theta)^2 = mg\tan\theta$.

$$(\sin\theta)^2\tan\theta = \frac{1}{4\pi\epsilon_0}\frac{q^2}{4L^2mg} = (8.988\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(2.80\times10^{-6}\text{ C})^2}{4(0.600\text{ m})^2(15.0\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)} = 0.3328.$$

Solve this equation by trial and error. This will go quicker if we can make a good estimate of the value of θ that solves the equation. For θ small, $\tan\theta \approx \sin\theta$. With this approximation the equation becomes $\sin^3\theta = 0.3328$ and $\sin\theta = 0.6930$, so $\theta = 43.9°$. Now refine this guess:

θ	$\sin^2\theta\tan\theta$	
45.0°	0.5000	
40.0°	0.3467	
39.6°	0.3361	
39.5°	0.3335	
39.4°	0.3309	so $\theta = 39.5°$

EVALUATE: The expression in part (c) says $\theta \to 0$ as $L \to \infty$ and $\theta \to 90°$ as $L \to 0$. When L is decreased from the value in part (a), θ increases.

21.76. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to each sphere.

SET UP: **(a)** Free body diagrams are given in Figure 21.76. F_e is the repulsive electric force that one sphere exerts on the other.

EXECUTE: **(b)** $T = mg/\cos 20° = 0.0834\text{ N}$, so $F_e = T\sin 20° = 0.0285\text{ N} = \dfrac{kq_1q_2}{r_1^2}$. (Note: $r_1 = 2(0.500\text{ m})\sin 20° = 0.342\text{ m}$.)

(c) From part (b), $q_1q_2 = 3.71\times10^{-13}\text{ C}^2$.

(d) The charges on the spheres are made equal by connecting them with a wire, but we still have $F_e = mg\tan\theta = 0.0453\text{ N} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q^2}{r_2^2}$, where $Q = \dfrac{q_1+q_2}{2}$. But the separation r_2 is known:

$r_2 = 2(0.500\text{ m})\sin 30° = 0.500\text{ m}$. Hence: $Q = \dfrac{q_1+q_2}{2} = \sqrt{4\pi\epsilon_0 F_e r_2^2} = 1.12\times10^{-6}\text{ C}$. This equation, along with that from part (c), gives us two equations in q_1 and q_2: $q_1+q_2 = 2.24\times10^{-6}\text{ C}$ and $q_1q_2 = 3.71\times10^{-13}\text{ C}^2$. By elimination, substitution and after solving the resulting quadratic equation, we find: $q_1 = 2.06\times10^{-6}\text{ C}$ and $q_2 = 1.80\times10^{-7}\text{ C}$.

EVALUATE: After the spheres are connected by the wire, the charge on sphere 1 decreases and the charge on sphere 2 increases. The product of the charges on the sphere increases and the thread makes a larger angle with the vertical.

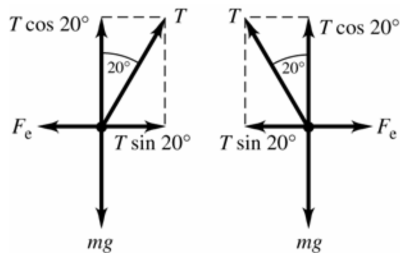


Figure 21.76

21.77. **IDENTIFY** and **SET UP:** Use Avogadro's number to find the number of Na^+ and Cl^- ions and the total positive and negative charge. Use Coulomb's law to calculate the electric force and $\vec{F} = m\vec{a}$ to calculate the acceleration.

(a) EXECUTE: The number of Na^+ ions in 0.100 mol of NaCl is $N = nN_A$. The charge of one ion is $+e$, so the total charge is $q_1 = nN_Ae =$ (0.100 mol)$(6.022\times10^{23}$ ions/mol)$(1.602\times10^{-19}$C/ion$) = 9.647\times10^3$ C

There are the same number of Cl^- ions and each has charge $-e$, so $q_2 = -9.647\times10^3$ C.

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1q_2|}{r^2} = (8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(9.647\times10^3\text{ C})^2}{(0.0200\text{ m})^2} = 2.09\times10^{21}\text{ N}$$

(b) $a = F/m$. Need the mass of 0.100 mol of Cl^- ions. For Cl, $M = 35.453\times10^{-3}$ kg/mol, so

$m = (0.100\text{ mol})(35.453\times10^{-3}\text{ kg/mol}) = 35.45\times10^{-4}\text{ kg}$. Then $a = \dfrac{F}{m} = \dfrac{2.09\times10^{21}\text{ N}}{35.45\times10^{-4}\text{ kg}} = 5.90\times10^{23}\text{ m/s}^2$.

(c) EVALUATE: Is is not reasonable to have such a huge force. The net charges of objects are rarely larger than 1 μC; a charge of 10^4 C is immense. A small amount of material contains huge amounts of positive and negative charges.

21.78. **IDENTIFY:** For the acceleration (and hence the force) on Q to be upward, as indicated, the forces due to q_1 and q_2 must have equal strengths, so q_1 and q_2 must have equal magnitudes. Furthermore, for the force to be upward, q_1 must be positive and q_2 must be negative.

SET UP: Since we know the acceleration of Q, Newton's second law gives us the magnitude of the force on it. We can then add the force components using $F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$. The electrical force on Q is given by Coulomb's law, $F_{Qq_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qq_1}{r^2}$ (for q_1) and likewise for q_2.

EXECUTE: First find the net force: $F = ma = (0.00500\text{ kg})(324\text{ m/s}^2) = 1.62$ N. Now add the force components, calling θ the angle between the line connecting q_1 and q_2 and the line connecting q_1 and Q.

$F = F_{Qq_1}\cos\theta + F_{Qq_2}\cos\theta = 2F_{Qq_1}\cos\theta$ and $F_{Qq_1} = \dfrac{F}{2\cos\theta} = \dfrac{1.62\text{ N}}{2\left(\dfrac{2.25\text{ cm}}{3.00\text{ cm}}\right)} = 1.08$ N. Now find the charges by solving for q_1 in Coulomb's law and use the fact that q_1 and q_2 have equal magnitudes but opposite signs.

$$F_{Qq_1} = \frac{1}{4\pi\epsilon_0}\frac{Qq_1}{r^2} \text{ and } q_1 = \frac{r^2F_{Qq_1}}{\dfrac{1}{4\pi\epsilon_0}Q} = \frac{(0.0300\text{ m})^2(1.08\text{ N})}{(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.75\times10^{-6}\text{C})} = 6.17\times10^{-8}\text{ C}.$$

$q_2 = -q_1 = -6.17\times10^{-8}$ C.

EVALUATE: Simple reasoning allows us first to conclude that q_1 and q_2 must have equal magnitudes but opposite signs, which makes the equations much easier to set up than if we had tried to solve the problem in the general case. As Q accelerates and hence moves upward, the magnitude of the acceleration vector will change in a complicated way.

21.79. IDENTIFY: Use Coulomb's law to calculate the forces between pairs of charges and sum these forces as vectors to find the net charge.

(a) SET UP: The forces are sketched in Figure 21.79a.

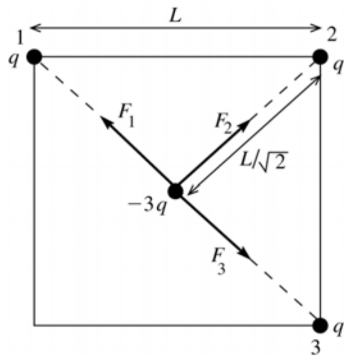


Figure 21.79a

EXECUTE: $\vec{F}_1 + \vec{F}_3 = \mathbf{0}$, so the net force is $\vec{F} = \vec{F}_2$.

$$F = \frac{1}{4\pi\epsilon_0}\frac{q(3q)}{(L/\sqrt{2})^2} = \frac{6q^2}{4\pi\epsilon_0 L^2},$$ away from the vacant corner.

(b) SET UP: The forces are sketched in Figure 21.79b.

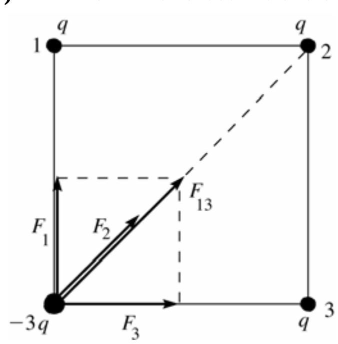


Figure 21.79b

EXECUTE: $$F_2 = \frac{1}{4\pi\epsilon_0}\frac{q(3q)}{\left(\sqrt{2}L\right)^2} = \frac{3q^2}{4\pi\epsilon_0\left(2L^2\right)}$$

$$F_1 = F_3 = \frac{1}{4\pi\epsilon_0}\frac{q(3q)}{L^2} = \frac{3q^2}{4\pi\epsilon_0 L^2}$$

The vector sum of F_1 and F_3 is $F_{13} = \sqrt{F_1^2 + F_3^2}$.

$F_{13} = \sqrt{2}F_1 = \dfrac{3\sqrt{2}q^2}{4\pi\epsilon_0 L^2}$; $\vec{F}_{13}$ and $\vec{F}_2$ are in the same direction.

$F = F_{13} + F_2 = \dfrac{3q^2}{4\pi\epsilon_0 L^2}\left(\sqrt{2} + \dfrac{1}{2}\right)$, and is directed toward the center of the square.

EVALUATE: By symmetry the net force is along the diagonal of the square. The net force is only slightly larger when the $-3q$ charge is at the center. Here it is closer to the charge at point 2 but the other two forces cancel.

21.80. IDENTIFY: Use Eq.(21.7) for the electric field produced by each point charge. Apply the principle of superposition and add the fields as vectors to find the net field.

(a) SET UP: The fields due to each charge are shown in Figure 21.80a.

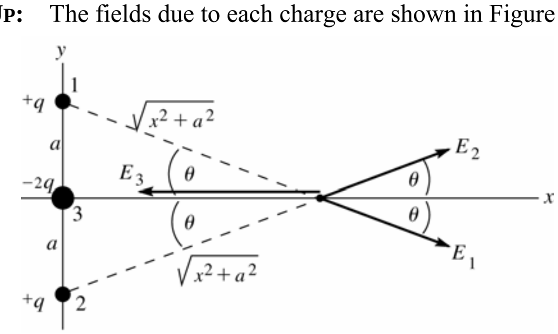


Figure 21.80a

$$\cos\theta = \frac{x}{\sqrt{x^2 + a^2}}$$

EXECUTE: The components of the fields are given in Figure 21.80b.

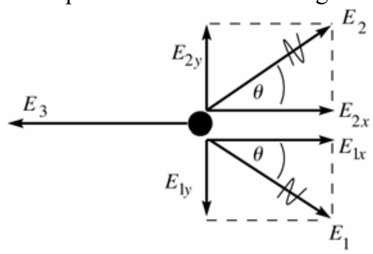


Figure 21.80b

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{a^2+x^2}\right)$$
$$E_3 = \frac{1}{4\pi\epsilon_0}\left(\frac{2q}{x^2}\right)$$

$E_{1y} = -E_1\sin\theta,\ E_{2y} = +E_2\sin\theta$ so $E_y = E_{1y} + E_{2y} = 0.$

$$E_{1x} = E_{2x} = +E_1\cos\theta = \frac{1}{4\pi\epsilon_0}\left(\frac{q}{a^2+x^2}\right)\left(\frac{x}{\sqrt{x^2+a^2}}\right),\ E_{3x} = -E_3$$

$$E_x = E_{1x} + E_{2x} + E_{3x} = 2\left(\frac{1}{4\pi\epsilon_0}\left(\frac{q}{a^2+x^2}\right)\left(\frac{x}{\sqrt{x^2+a^2}}\right)\right) - \frac{2q}{4\pi\epsilon_0 x^2}$$

$$E_x = -\frac{2q}{4\pi\epsilon_0}\left(\frac{1}{x^2} - \frac{x}{(a^2+x^2)^{3/2}}\right) = -\frac{2q}{4\pi\epsilon_0 x^2}\left(1 - \frac{1}{(1+a^2/x^2)^{3/2}}\right)$$

Thus $E = \dfrac{2q}{4\pi\epsilon_0 x^2}\left(1 - \dfrac{1}{(1+a^2/x^2)^{3/2}}\right)$, in the $-x$-direction.

(b) $x >> a$ implies $a^2/x^2 << 1$ and $(1+a^2/x^2)^{-3/2} \approx 1 - 3a^2/2x^2.$

Thus $E \approx \dfrac{2q}{4\pi\epsilon_0 x^2}\left(1 - \left(1 - \dfrac{3a^2}{2x^2}\right)\right) = \dfrac{3qa^2}{4\pi\epsilon_0 x^4}.$

EVALUATE: $E \sim 1/x^4$. For a point charge $E \sim 1/x^2$ and for a dipole $E \sim 1/x^3$. The total charge is zero so at large distances the electric field should decrease faster with distance than for a point charge. By symmetry $\vec{E}$ must lie along the x-axis, which is the result we found in part (a).

21.81. **IDENTIFY:** The small bags of protons behave like point-masses and point-charges since they are extremely far apart.

SET UP: For point-particles, we use Newton's formula for universal gravitation ($F = Gm_1m_2/r^2$) and Coulomb's law. The number of protons is the mass of protons in the bag divided by the mass of a single proton.

EXECUTE: **(a)** $(0.0010\text{ kg})/(1.67\times10^{-27}\text{ kg}) = 6.0\times10^{23}$ protons

(b) Using Coulomb's law, where the separation is twice is the radius of the earth, we have

$F_{\text{electrical}} = (9.00 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(6.0 \times 10^{23} \times 1.60 \times 10^{-19}\ \text{C})^2/(2 \times 6.38 \times 10^6\ \text{m})^2 = 5.1 \times 10^5$ N

$F_{\text{grav}} = (6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(0.0010\ \text{kg})^2/(2 \times 6.38 \times 10^6\ \text{m})^2 = 4.1 \times 10^{-31}$ N

(c) EVALUATE: The electrical force (≈200,000 lb!) is certainly large enough to feel, but the gravitational force clearly is not since it is about 10^{36} times weaker.

21.82. **IDENTIFY:** We can treat the protons as point-charges and use Coulomb's law.

SET UP: (a) Coulomb's law is $F = (1/4\pi\epsilon_0)|q_1q_2|/r^2$.

EXECUTE: $F = (9.00 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\ \text{C})^2/(2.0 \times 10^{-15}\ \text{m}) = 58$ N = 13 lb, which is certainly large enough to feel.

(b) EVALUATE: Something must be holding the nucleus together by opposing this enormous repulsion. This is the strong nuclear force.

21.83. **IDENTIFY:** Estimate the number of protons in the textbook and from this find the net charge of the textbook. Apply Coulomb's law to find the force and use $F_{\text{net}} = ma$ to find the acceleration.

SET UP: With the mass of the book about 1.0 kg, most of which is protons and neutrons, we find that the number of protons is $\frac{1}{2}(1.0\text{ kg})/(1.67\times10^{-27}\text{ kg}) = 3.0\times10^{26}$.

EXECUTE: **(a)** The charge difference present if the electron's charge was 99.999% of the proton's is

$\Delta q = (3.0\times10^{26})(0.00001)(1.6\times10^{-19}\text{ C}) = 480\text{ C}$.

(b) $F = k(\Delta q)^2/r^2 = k(480\text{ C})^2/(5.0\text{ m})^2 = 8.3\times10^{13}$ N, and is repulsive.

$a = F/m = (8.3\times10^{13}\text{ N})/(1\text{ kg}) = 8.3\times10^{13}\text{ m/s}^2$.

EXECUTE: **(c)** Even the slightest charge imbalance in matter would lead to explosive repulsion!

21.84. **IDENTIFY:** The electric field exerts equal and opposite forces on the two balls, causing them to swing away from each other. When the balls hang stationary, they are in equilibrium so the forces on them (electrical, gravitational, and tension in the strings) must balance.

SET UP: (a) The force on the left ball is in the direction of the electric field, so it must be positive, while the force on the right ball is opposite to the electric field, so it must be negative.

(b) Balancing horizontal and vertical forces gives $qE = T \sin \theta/2$ and $mg = T \cos \theta/2$.

EXECUTE: Solving for the angle θ gives: $\theta = 2 \arctan(qE/mg)$.

(c) As $E \to \infty$, $\theta \to 2 \arctan(\infty) = 2 (\pi/2) = \pi = 180°$

EVALUATE: If the field were large enough, the gravitational force would not be important, so the strings would be horizontal.

21.85. **IDENTIFY** and **SET UP:** Use the density of copper to calculate the number of moles and then the number of atoms. Calculate the net charge and then use Coulomb's law to calculate the force.

EXECUTE: **(a)** $m = \rho V = \rho\left(\frac{4}{3}\pi r^3\right) = \left(8.9\times10^3 \text{ kg/m}^3\right)\left(\frac{4}{3}\pi\right)\left(1.00\times10^{-3} \text{ m}\right)^3 = 3.728\times10^{-5} \text{ kg}$

$n = m/M = \left(3.728\times10^{-5} \text{ kg}\right)/\left(63.546\times10^{-3} \text{ kg/mol}\right) = 5.867\times10^{-4} \text{ mol}$

$N = nN_A = 3.5\times10^{20}$ atoms

(b) $N_e = (29)\left(3.5\times10^{20}\right) = 1.015\times10^{22}$ electrons and protons

$q_{\text{net}} = eN_e - (0.99900)eN_e = \left(0.100\times10^{-2}\right)\left(1.602\times10^{-19} \text{ C}\right)\left(1.015\times10^{22}\right) = 1.6 \text{ C}$

$$F = k\frac{q^2}{r^2} = k\frac{(1.6 \text{ C})^2}{(1.00 \text{ m})^2} = 2.3\times10^{10} \text{ N}$$

EVALUATE: The amount of positive and negative charge in even small objects is immense. If the charge of an electron and a proton weren't exactly equal, objects would have large net charges.

21.86. **IDENTIFY:** Apply constant acceleration equations to a drop to find the acceleration. Then use $F = ma$ to find the force and $F = |q|E$ to find $|q|$.

SET UP: Let $D = 2.0$ cm be the horizontal distance the drop travels and $d = 0.30$ mm be its vertical displacement. Let $+x$ be horizontal and in the direction from the nozzle toward the paper and let $+y$ be vertical, in the direction of the deflection of the drop. $a_x = 0$ and $a_y = a$.

EXECUTE: First, the mass of the drop: $m = \rho V = (1000 \text{ kg/m}^3)\left(\frac{4\pi(15.0\times10^{-6} \text{ m})^3}{3}\right) = 1.41\times10^{-11} \text{ kg}$. Next, the time of flight: $t = D/v = (0.020 \text{ m})/(20 \text{ m/s}) = 0.00100 \text{ s}$. $d = \frac{1}{2}at^2$. $a = \frac{2d}{t^2} = \frac{2(3.00\times10^{-4} \text{ m})}{(0.001 \text{ s})^2} = 600 \text{ m/s}^2$.

Then $a = F/m = qE/m$ gives $q = ma/E = \frac{(1.41\times10^{-11} \text{ kg})(600 \text{ m/s}^2)}{8.00\times10^4 \text{ N/C}} = 1.06\times10^{-13} \text{ C}$.

EVALUATE: Since q is positive the vertical deflection is in the direction of the electric field.

21.87. **IDENTIFY:** Eq. (21.3) gives the force exerted by the electric field. This force is constant since the electric field is uniform and gives the proton a constant acceleration. Apply the constant acceleration equations for the x- and y-components of the motion, just as for projectile motion.

(a) SET UP: The electric field is upward so the electric force on the positively charged proton is upward and has magnitude $F = eE$. Use coordinates where positive y is downward. Then applying $\sum \vec{F} = m\vec{a}$ to the proton gives that $a_x = 0$ and $a_y = -eE/m$. In these coordinates the initial velocity has components $v_x = +v_0 \cos\alpha$ and $v_y = +v_0 \sin\alpha$, as shown in Figure 21.87a.

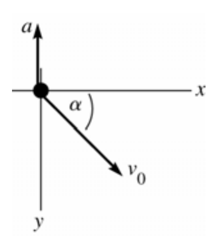


Figure 21.87a

EXECUTE: Finding h_{max}: At $y = h_{max}$ the y-component of the velocity is zero.

$v_y = 0,\ v_{0y} = v_0 \sin\alpha,\ a_y = -eE/m,\ y - y_0 = h_{max} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

$$y - y_0 = \frac{v_y^2 - v_{0y}^2}{2a_y}$$

$$h_{max} = \frac{-v_0^2 \sin^2\alpha}{2(-eE/m)} = \frac{mv_0^2 \sin^2\alpha}{2eE}$$

(b) Use the vertical motion to find the time t: $y - y_0 = 0,\ v_{0y} = v_0 \sin\alpha,\ a_y = -eE/m,\ t = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

With $y - y_0 = 0$ this gives $t = -\dfrac{2v_{0y}}{a_y} = -\dfrac{2(v_0 \sin\alpha)}{-eE/m} = \dfrac{2mv_0 \sin\alpha}{eE}$

Then use the x-component motion to find d: $a_x = 0,\ v_{0x} = v_0 \cos\alpha,\ t = 2mv_0 \sin\alpha / eE,\ x - x_0 = d = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $d = v_0 \cos\alpha\left(\dfrac{2mv_0 \sin\alpha}{eE}\right) = \dfrac{mv_0^2 2\sin\alpha\cos\alpha}{eE} = \dfrac{mv_0^2 \sin 2\alpha}{eE}$

(c) The trajectory of the proton is sketched in Figure 21.87b.

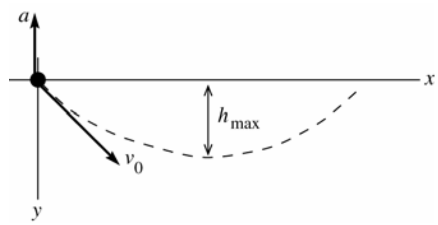


Figure 21.87b

(d) Use the expression in part (a): $h_{max} = \dfrac{[(4.00 \times 10^5 \text{ m/s})(\sin 30.0°)]^2 (1.673 \times 10^{-27} \text{ kg})}{2(1.602 \times 10^{-19} \text{ C})(500 \text{ N/C})} = 0.418 \text{ m}$

Use the expression in part (b): $d = \dfrac{(1.673 \times 10^{-27} \text{ kg})(4.00 \times 10^5 \text{ m/s})^2 \sin 60.0°}{(1.602 \times 10^{-19} \text{ C})(500 \text{ N/C})} = 2.89 \text{ m}$

EVALUATE: In part (a), $a_y = -eE/m = -4.8 \times 10^{10} \text{ m/s}^2$. This is much larger in magnitude than g, the acceleration due to gravity, so it is reasonable to ignore gravity. The motion is just like projectile motion, except that the acceleration is upward rather than downward and has a much different magnitude. h_{max} and d increase when α or v_0 increase and decrease when E increases.

21.88. IDENTIFY: $E_x = E_{1x} + E_{2x}$. Use Eq.(21.7) for the electric field due to each point charge.

SET UP: $\vec{E}$ is directed away from positive charges and toward negative charges.

EXECUTE: (a) $E_x = +50.0 \text{ N/C}$. $E_{1x} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1|}{r_1^2} = (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{4.00 \times 10^{-9} \text{ C}}{(0.60 \text{ m})^2} = +99.9 \text{ N/C}$.

$E_x = E_{1x} + E_{2x}$, so $E_{2x} = E_x - E_{1x} = +50.0 \text{ N/C} - 99.9 \text{ N/C} = -49.9 \text{ N/C}$. Since E_{2x} is negative, q_2 must be negative. $|q_2| = \dfrac{|E_{2x}|r_2^2}{(1/4\pi\epsilon_0)} = \dfrac{(49.9 \text{ N/C})(1.20 \text{ m})^2}{8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 7.99 \times 10^{-9} \text{ C}$. $q_2 = -7.99 \times 10^{-9} \text{ C}$

(b) $E_x = -50.0 \text{ N/C}$. $E_{1x} = +99.9 \text{ N/C}$, as in part (a). $E_{2x} = E_x - E_{1x} = -149.9 \text{ N/C}$. q_2 is negative.

$|q_2| = \dfrac{|E_{2x}|r_2^2}{(1/4\pi\epsilon_0)} = \dfrac{(149.9 \text{ N/C})(1.20 \text{ m})^2}{8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2} = 2.40 \times 10^{-8} \text{ C}$. $q_2 = -2.40 \times 10^{-8} \text{ C}$.

EVALUATE: q_2 would be positive if E_{2x} were positive.

21.89. **IDENTIFY:** Divide the charge distribution into infinitesimal segments of length dx . Calculate $E_x$and E_u due to a segment and integrate to find the total field.

SET UP: The charge dQ of a segment of length dx is $dQ=(Q/a)dx$. The distance between a segment at x and the charge q is $a+r-x$. $(1-y)^{-1}\approx 1+y$ when $|y|<<1$.

EXECUTE: **(a)** $dE_x=\frac{1}{4\pi\epsilon_0}\frac{dQ}{(a+r-x)^2}$ so $E_x=\frac{1}{4\pi\epsilon_0}\int_0^a\frac{Qdx}{a(a+r-x)^2}=\frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{1}{r}-\frac{1}{a+r}\right)$.

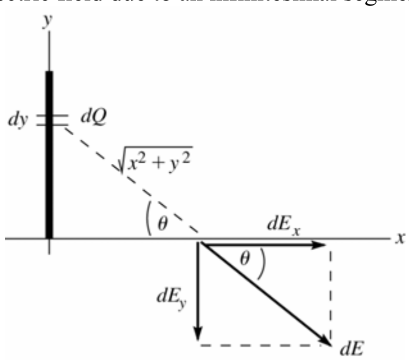
$a+r=x$, so $E_x=\frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{1}{x-a}-\frac{1}{x}\right)$. $E_y=0$.

(b) $\vec{F}=q\vec{E}=\frac{1}{4\pi\epsilon_0}\frac{qQ}{a}\left(\frac{1}{x-a}-\frac{1}{x}\right)\hat{i}$.

EVALUATE: **(c)** For $x>>a$, $F=\frac{kqQ}{ax}((1-a/x)^{-1}-1)=\frac{kqQ}{ax}(1+a/x+\cdots-1)\approx\frac{kqQ}{x^2}\approx\frac{1}{4\pi\epsilon_0}\frac{qQ}{r^2}$. (Note that for $x>>a$, $r=x-a\approx x$.) The charge distribution looks like a point charge from far away, so the force takes the form of the force between a pair of point charges.

21.90. **IDENTIFY:** Use Eq. (21.7) to calculate the electric field due to a small slice of the line of charge and integrate as in Example 21.11. Use Eq. (21.3) to calculate $\vec{F}$.

SET UP: The electric field due to an infinitesimal segment of the line of charge is sketched in Figure 21.90.



$\sin\theta=\frac{y}{\sqrt{x^2+y^2}}$

$\cos\theta=\frac{x}{\sqrt{x^2+y^2}}$

Figure 21.90

Slice the charge distribution up into small pieces of length dy. The charge dQ in each slice is $dQ=Q(dy/a)$. The electric field this produces at a distance x along the x-axis is dE. Calculate the components of $d\vec{E}$ and then integrate over the charge distribution to find the components of the total field.

EXECUTE: $dE=\frac{1}{4\pi\epsilon_0}\left(\frac{dQ}{x^2+y^2}\right)=\frac{Q}{4\pi\epsilon_0 a}\left(\frac{dy}{x^2+y^2}\right)$

$$dE_x=dE\cos\theta=\frac{Qx}{4\pi\epsilon_0 a}\left(\frac{dy}{\left(x^2+y^2\right)^{3/2}}\right)$$

$$dE_y=-dE\sin\theta=-\frac{Q}{4\pi\epsilon_0 a}\left(\frac{ydy}{\left(x^2+y^2\right)^{3/2}}\right)$$

$$E_x=\int dE_x=-\frac{Qx}{4\pi\epsilon_0 a}\int_0^a\frac{dy}{(x^2+y^2)^{3/2}}=\frac{Qx}{4\pi\epsilon_0 a}\left[\frac{1}{x^2}\frac{y}{\sqrt{x^2+y^2}}\right]_0^a=\frac{Q}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}}$$

$$E_y=\int dEy=-\frac{Q}{4\pi\epsilon_0 a}\int_0^a\frac{ydy}{(x^2+y^2)^{3/2}}=-\frac{Q}{4\pi\epsilon_0 a}\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^a=-\frac{Q}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right)$$

(b) $\vec{F}=q_0\vec{E}$

$$F_x=-qE_x=\frac{-qQ}{4\pi\epsilon_0 x}\frac{1}{\sqrt{x^2+a^2}};\ F_y=-qE_y=\frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{\sqrt{x^2+a^2}}\right)$$

(c) For $x>>a$, $\frac{1}{\sqrt{x^2+a^2}}=\frac{1}{x}\left(1+\frac{a^2}{x^2}\right)^{-1/2}=\frac{1}{x}\left(1-\frac{a^2}{2x^2}\right)=\frac{1}{x}-\frac{a^2}{2x^3}$

$$F_x\approx-\frac{qQ}{4\pi\epsilon_0 x^2},\ F_y\approx\frac{qQ}{4\pi\epsilon_0 a}\left(\frac{1}{x}-\frac{1}{x}+\frac{a^2}{2x^3}\right)=\frac{qQa}{8\pi\epsilon_0 x^3}$$

EVALUATE: For $x >> a$, $F_y << F_x$ and $F \approx |F_x| = \dfrac{qQ}{4\pi\epsilon_0 x^2}$ and $\vec{F}$ is in the $-x$-direction. For $x >> a$ the charge distribution Q acts like a point charge.

21.91. IDENTIFY: Apply Eq.(21.9) from Example 21.11.

SET UP: $a = 2.50$ cm. Replace Q by $|Q|$. Since Q is negative, $\vec{E}$ is toward the line of charge and $\vec{E} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|Q|}{x\sqrt{x^2+a^2}}\hat{i}$.

EXECUTE: $\vec{E} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|Q|}{x\sqrt{x^2+a^2}}\hat{i} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{9.00\times10^{-9}\text{ C}}{(0.100\text{ m})\sqrt{(0.100\text{ m})^2+(0.025\text{ m})^2}}\hat{i} = (-7850\text{ N/C})\hat{i}$.

(b) The electric field is less than that at the same distance from a point charge (8100 N/C). For large x, $(x+a)^{-1/2} = \dfrac{1}{x}(1+a^2/x^2)^{-1/2} \approx \dfrac{1}{x}\left(1-\dfrac{a^2}{2x^2}\right)$. $E_{x\to\infty} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{x^2}\left(1-\dfrac{a^2}{2x^2}+\cdots\right)$. The first correction term to the point charge result is negative.

(c) For a 1% difference, we need the first term in the expansion beyond the point charge result to be less than 0.010: $\dfrac{a^2}{2x^2} \approx 0.010 \Rightarrow x \approx a\sqrt{1/(2(0.010))} = 0.025\sqrt{1/0.020} \Rightarrow x \approx 0.177$ m.

EVALUATE: At $x = 10.0$ cm (part b), the exact result for the line of charge is 3.1% smaller than for a point charge. It is sensible, therefore, that the difference is 1.0% at a somewhat larger distance, 17.7 cm.

21.92. IDENTIFY: The electrical force has magnitude $F = \dfrac{kQ^2}{r^2}$ and is attractive. Apply $\sum\vec{F} = m\vec{a}$ to the earth.

SET UP: For a circular orbit, $a = \dfrac{v^2}{r}$. The period T is $\dfrac{2\pi r}{v}$. The mass of the earth is $m_E = 5.97\times10^{24}$ kg, the orbit radius of the earth is 1.50×10^{11} m and its orbital period is 3.146×10^7 s.

EXECUTE: $F = ma$ gives $\dfrac{kQ}{r^2} = m_E\dfrac{v^2}{r}$. $v^2 = \dfrac{4\pi^2 r^2}{T^2}$, so

$$Q = \sqrt{\frac{m_E 4\pi^2 r^3}{kT^2}} = \sqrt{\frac{(5.97\times10^{24}\text{ kg})(4)(\pi^2)(1.50\times10^{11}\text{ m})^3}{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(3.146\times10^7\text{ s})^2}} = 2.99\times10^{17}\text{ C}.$$

EVALUATE: A very large net charge would be required.

21.93. IDENTIFY: Apply Eq.(21.11).

SET UP: $\sigma = Q/A = Q/\pi R^2$. $(1+y^2)^{-1/2} \approx 1 - y^2/2$, when $y^2 << 1$.

EXECUTE: **(a)** $E = \dfrac{\sigma}{2\epsilon_0}\left[1-\left(R^2/x^2+1\right)^{-1/2}\right]$.

$$E = \frac{4.00\text{ pC}/\pi(0.025\text{ m})^2}{2\epsilon_0}\left[1-\left(\frac{(0.025\text{ m})^2}{(0.200\text{ m})^2}+1\right)^{-1/2}\right] = 0.89\text{ N/C},$$ in the $+x$ direction.

(b) For $x >> R$ $E = \dfrac{\sigma}{2\epsilon_0}[1-(1-R^2/2x^2+\cdots)] \approx \dfrac{\sigma}{2\epsilon_0}\dfrac{R^2}{2x^2} = \dfrac{\sigma\pi R^2}{4\pi\epsilon_0 x^2} = \dfrac{Q}{4\pi\epsilon_0 x^2}$.

(c) The electric field of (a) is less than that of the point charge (0.90 N/C) since the first correction term to the point charge result is negative.

(d) For $x = 0.200$ m, the percent difference is $\dfrac{(0.90-0.89)}{0.89} = 0.01 = 1\%$. For $x = 0.100$ m, $E_{\text{disk}} = 3.43$ N/C and $E_{\text{point}} = 3.60$ N/C, so the percent difference is $\dfrac{(3.60-3.43)}{3.60} = 0.047 \approx 5\%$.

EVALUATE: The field of a disk becomes closer to the field of a point charge as the distance from the disk increases. At $x = 10.0$ cm, $R/x = 25\%$ and the percent difference between the field of the disk and the field of a point charge is 5%.

21.94. IDENTIFY: Apply the procedure specified in the problem.

SET UP: $\int_{x_1}^{x_2} f(x)dx = -\int_{x_2}^{x_1} f(x)dx$.

EXECUTE: **(a)** For $f(x) = f(-x)$, $\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx = \int_{0}^{-a} f(-x)d(-x) + \int_{0}^{a} f(x)dx$. Now replace $-x$ with *y*. This gives $\int_{-a}^{a} f(x)dx = \int_{0}^{a} f(y)dy + \int_{0}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$.

(b) For $g(x) = -g(-x)$, $\int_{-a}^{a} g(x)dx = \int_{-a}^{0} g(x)dx + \int_{0}^{a} g(x)dx = -\int_{0}^{-a} -g(-x)(-d(-x)) + \int_{0}^{a} g(x)dx$. Now replace $-x$ with *y*. This gives $\int_{-a}^{a} g(x)dx = -\int_{0}^{a} g(y)dy + \int_{0}^{a} g(x)dx = 0$.

(c) The integrand in E_y for Example 21.11 is odd, so E_y =0.

EVALUATE: In Example 21.11, $E_y = 0$ because for each infinitesimal segment in the upper half of the line of charge, there is a corresponding infinitesimal segment in the bottom half of the line that has E_y in the opposite direction.

21.95. **IDENTIFY:** Find the resultant electric field due to the two point charges. Then use $\vec{F} = q\vec{E}$ to calculate the force on the point charge.

SET UP: Use the results of Problems 21.90 and 21.89.

EXECUTE: **(a)** The *y*-components of the electric field cancel, and the *x*-component from both charges, as given in Problem 21.90, is $E_x = \frac{1}{4\pi\epsilon_0}\frac{-2Q}{a}\left(\frac{1}{y} - \frac{1}{(y^2+a^2)^{1/2}}\right)$. Therefore, $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{-2Qq}{a}\left(\frac{1}{y} - \frac{1}{(y^2+a^2)^{1/2}}\right)\hat{i}$. If $y >> a$

$\vec{F} \approx \frac{1}{4\pi\epsilon_0}\frac{-2Qq}{ay}(1-(1-a^2/2y^2+\cdots))\hat{i} = -\frac{1}{4\pi\epsilon_0}\frac{Qqa}{y^3}\hat{i}$.

(b) If the point charge is now on the *x*-axis the two halves of the charge distribution provide different forces, though still along the *x*-axis, as given in Problem 21.89: $\vec{F}_+ = q\vec{E}_+ = \frac{1}{4\pi\epsilon_0}\frac{Qq}{a}\left(\frac{1}{x-a} - \frac{1}{x}\right)\hat{i}$

and $\vec{F}_- = q\,\vec{E}_- = -\frac{1}{4\pi\epsilon_0}\frac{Qq}{a}\left(\frac{1}{x} - \frac{1}{x+a}\right)\hat{i}$. Therefore, $\vec{F} = \vec{F}_+ + \vec{F}_- = \frac{1}{4\pi\epsilon_0}\frac{Qq}{a}\left(\frac{1}{x-a} - \frac{2}{x} + \frac{1}{x+a}\right)\hat{i}$. For $x >> a$,

$\vec{F} \approx \frac{1}{4\pi\epsilon_0}\frac{Qq}{ax}\left(\left(1+\frac{a}{x}+\frac{a^2}{x^2}+\ldots\right) - 2 + \left(1-\frac{a}{x}+\frac{a^2}{x^2}-\ldots\right)\right)\hat{i} = \frac{1}{4\pi\epsilon_0}\frac{2Qqa}{x^3}\hat{i}$.

EVALUATE: If the charge distributed along the *x*-axis were all positive or all negative, the force would be proportional to $1/y^2$ in part (a) and to $1/x^2$ in part (b), when *y* or *x* is very large.

21.96. **IDENTIFY:** Divide the semicircle into infinitesimal segments. Find the electric field $d\vec{E}$ due to each segment and integrate over the semicircle to find the total electric field.

SET UP: The electric fields along the *x*-direction from the left and right halves of the semicircle cancel. The remaining *y*-component points in the negative *y*-direction. The charge per unit length of the semicircle is $\lambda = \frac{Q}{\pi a}$ and $dE = \frac{k\lambda\, dl}{a^2} = \frac{k\lambda\, d\theta}{a}$.

EXECUTE: $dE_y = dE\sin\theta = \frac{k\lambda\sin\theta\, d\theta}{a}$. Therefore, $E_y = \frac{2k\lambda}{a}\int_0^{\pi/2}\sin\theta\, d\theta = \frac{2k\lambda}{a}[-\cos\theta]_0^{\pi/2} = \frac{2k\lambda}{a} = \frac{2kQ}{\pi a^2}$, in the $-y$-direction.

EVALUATE: For a full circle of charge the electric field at the center would be zero. For a quarter-circle of charge, in the first quadrant, the electric field at the center of curvature would have nonzero *x* and *y* components. The calculation for the semicircle is particularly simple, because all the charge is the same distance from point *P*.

21.97. **IDENTIFY:** Divide the charge distribution into small segments, use the point charge formula for the electric field due to each small segment and integrate over the charge distribution to find the *x* and *y* components of the total field.

SET UP: Consider the small segment shown in Figure 21.97a.

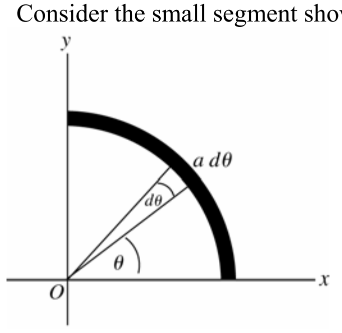


Figure 21.97a

EXECUTE: A small segment that subtends angle $d\theta$ has length $a\, d\theta$ and contains charge $dQ = \left(\frac{a d\theta}{\frac{1}{2}\pi a}\right)Q = \frac{2Q}{\pi}d\theta.$ ($\frac{1}{2}\pi a$ is the total length of the charge distribution.)

The charge is negative, so the field at the origin is directed toward the small segment. The small segment is located at angle θ as shown in the sketch. The electric field due to dQ is shown in Figure 21.97b, along with its components.

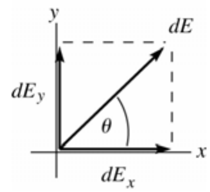


Figure 21.97b

$$dE = \frac{1}{4\pi\epsilon_0}\frac{|dQ|}{a^2}$$

$$dE = \frac{Q}{2\pi^2\epsilon_0 a^2}d\theta$$

$$dE_x = dE\cos\theta = \left(Q/2\pi^2\epsilon_0 a^2\right)\cos\theta d\theta$$

$$E_x = \int dE_x = \frac{Q}{2\pi^2\epsilon_0 a^2}\int_0^{\pi/2}\cos\theta d\theta = \frac{Q}{2\pi^2\epsilon_0 a^2}\left(\sin\theta\Big|_0^{\pi/2}\right) = \frac{Q}{2\pi^2\epsilon_0 a^2}$$

$$dE_y = dE\sin\theta = \left(Q/2\pi^2\epsilon_0 a^2\right)\sin\theta d\theta$$

$$E_y = \int dE_y = \frac{Q}{2\pi^2\epsilon_0 a^2}\int_0^{\pi/2}\sin\theta d\theta = \frac{Q}{2\pi^2\epsilon_0 a^2}\left(-\cos\theta\Big|_0^{\pi/2}\right) = \frac{Q}{2\pi^2\epsilon_0 a^2}$$

EVALUATE: Note that $E_x = E_y$, as expected from symmetry.

21.98. IDENTIFY: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere, with x horizontal and y vertical.

SET UP: The free-body diagram for the sphere is given in Figure 21.98. The electric field $\vec{E}$ of the sheet is directed away from the sheet and has magnitude $E = \frac{\sigma}{2\epsilon_0}$ (Eq.21.12).

EXECUTE: $\sum F_y = 0$ gives $T\cos\alpha = mg$ and $T = \frac{mg}{\cos\alpha}$. $\sum F_x = 0$ gives $T\sin\alpha = \frac{q\sigma}{2\epsilon_0}$ and $T = \frac{q\sigma}{2\epsilon_0\sin\alpha}$.

Combining these two equations we have $\frac{mg}{\cos\alpha} = \frac{q\sigma}{2\epsilon_0\sin\alpha}$ and $\tan\alpha = \frac{q\sigma}{2\epsilon_0 mg}$. Therefore, $\alpha = \arctan\left(\frac{q\sigma}{2\epsilon_0 mg}\right)$.

EVALUATE: The electric field of the sheet, and hence the force it exerts on the sphere, is independent of the distance of the sphere from the sheet.

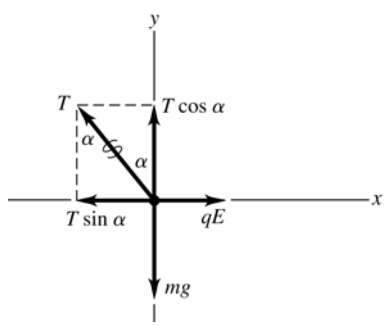


Figure 21.98

21.99. IDENTIFY: Each wire produces an electric field at P due to a finite wire. These fields add by vector addition.

SET UP: Each field has magnitude $\frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}$. The field due to the negative wire points to the left, while the field due to the positive wire points downward, making the two fields perpendicular to each other and of equal magnitude. The net field is the vector sum of these two, which is $E_{net} = 2E_1\cos 45° = 2\frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\cos 45°$. In part (b), the electrical force on an electron at P is eE.

EXECUTE: **(a)** The net field is $E_{net} = 2\frac{1}{4\pi\epsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\cos 45°$.

$$E_{net} = \frac{2\left(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.50\times10^{-6}\ \text{C}\right)\cos 45°}{(0.600\ \text{m})\sqrt{(0.600\ \text{m})^2+(0.600\ \text{m})^2}} = 6.25\times10^4\ \text{N/C}.$$

The direction is 225° counterclockwise from an axis pointing to the right through the positive wire.

(b) $F = eE = (1.60 \times 10^{-19}\ \text{C})(6.25 \times 10^{4}\ \text{N/C}) = 1.00 \times 10^{-14}\ \text{N}$, opposite to the direction of the electric field, since the electron has negative charge.

EVALUATE: Since the electric fields due to the two wires have equal magnitudes and are perpendicular to each other, we only have to calculate one of them in the solution.

21.100. **IDENTIFY:** Each sheet produces an electric field that is independent of the distance from the sheet. The net field is the vector sum of the two fields.

SET UP: The formula for each field is $E = \sigma/2\epsilon_0$, and the net field is the vector sum of these,

$E_{\text{net}} = \dfrac{\sigma_B}{2\epsilon_0} \pm \dfrac{\sigma_A}{2\epsilon_0} = \dfrac{\sigma_B \pm \sigma_A}{2\epsilon_0}$, where we use the + or – sign depending on whether the fields are in the same or opposite directions and σ_B and σ_A are the magnitudes of the surface charges.

EXECUTE: **(a)** The two fields oppose and the field of B is stronger than that of A, so

$$E_{\text{net}} = \frac{\sigma_B}{2\epsilon_0} - \frac{\sigma_A}{2\epsilon_0} = \frac{\sigma_B - \sigma_A}{2\epsilon_0} = \frac{11.6\,\mu\text{C/m}^2 - 9.50\,\mu\text{C/m}^2}{2\left(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2\right)} = 1.19 \times 10^5\ \text{N/C, to the right.}$$

(b) The fields are now in the same direction, so their magnitudes add.

$$E_{\text{net}} = (11.6\ \mu\text{C/m}^2 + 9.50\ \mu\text{C/m}^2)/2\,\epsilon_0 = 1.19 \times 10^6\ \text{N/C, to the right}$$

(c) The fields add but now point to the left, so $E_{\text{net}} = 1.19 \times 10^6$ N/C, to the left.

EVALUATE: We can simplify the calculations by sketching the fields and doing an algebraic solution first.

21.101. **IDENTIFY:** Each sheet produces an electric field that is independent of the distance from the sheet. The net field is the vector sum of the two fields.

SET UP: The formula for each field is $E = \sigma/2\epsilon_0$, and the net field is the vector sum of these,

$E_{\text{net}} = \dfrac{\sigma_B}{2\epsilon_0} \pm \dfrac{\sigma_A}{2\epsilon_0} = \dfrac{\sigma_B \pm \sigma_A}{2\epsilon_0}$, where we use the + or – sign depending on whether the fields are in the same or opposite directions and σ_B and σ_A are the magnitudes of the surface charges.

EXECUTE: **(a)** The fields add and point to the left, giving $E_{\text{net}} = 1.19 \times 10^6$ N/C.

(b) The fields oppose and point to the left, so $E_{\text{net}} = 1.19 \times 10^5$ N/C.

(c) The fields oppose but now point to the right, giving $E_{\text{net}} = 1.19 \times 10^5$ N/C.

EVALUATE: We can simplify the calculations by sketching the fields and doing an algebraic solution first.

21.102. **IDENTIFY:** The sheets produce an electric field in the region between them which is the vector sum of the fields from the two sheets.

SET UP: The force on the negative oil droplet must be upward to balance gravity. The net electric field between the sheets is $E = \sigma/\epsilon_0$, and the electrical force on the droplet must balance gravity, so $qE = mg$.

EXECUTE: **(a)** The electrical force on the drop must be upward, so the field should point downward since the drop is negative.

(b) The charge of the drop is $5e$, so $qE = mg$. $(5e)(\sigma/\epsilon_0) = mg$ and

$$\sigma = \frac{mg\epsilon_0}{5e} = \frac{\left(324\times10^{-9}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)}{5\left(1.60\times10^{-19}\ \text{C}\right)} = 35.1\ \text{C/m}^2$$

EVALUATE: Balancing oil droplets between plates was the basis of the Milliken Oil-Drop Experiment which produced the first measurement of the mass of an electron.

21.103. **IDENTIFY** and **SET UP:** Example 21.12 gives the electric field due to one infinite sheet. Add the two fields as vectors.

EXECUTE: The electric field due to the first sheet, which is in the xy-plane, is $\vec{E}_1 = (\sigma/2\epsilon_0)\hat{k}$ for $z > 0$ and $\vec{E}_1 = -(\sigma/2\epsilon_0)\hat{k}$ for $z < 0$. We can write this as $\vec{E}_1 = (\sigma/2\epsilon_0)(z/|z|)\hat{k}$, since $z/|z| = +1$ for $z > 0$ and $z/|z| = -z/z = -1$ for $z < 0$. Similarly, we can write the electric field due to the second sheet as $\vec{E}_2 = -(\sigma/2\epsilon_0)(x/|x|)\hat{i}$, since its charge density is $-\sigma$. The net field is $\vec{E} = \vec{E}_1 + \vec{E}_2 = (\sigma/2\epsilon_0)\left(-(x/|x|)\hat{i} + (z/|z|)\hat{k}\right)$.

EVALUATE: The electric field is independent of the y-component of the field point since displacement in the $\pm y$- direction is parallel to both planes. The field depends on which side of each plane the field is located.

21.104. **IDENTIFY:** Apply Eq.(21.11) for the electric field of a disk. The hole can be described by adding a disk of charge density $-\sigma$ and radius R_1 to a solid disk of charge density $+\sigma$ and radius R_2.

SET UP: The area of the annulus is $\pi(R_2^2 - R_1^2)\sigma$. The electric field of a disk, Eq.(21.11) is $E = \dfrac{\sigma}{2\epsilon_0}\left[1 - 1\big/\sqrt{(R/x)^2 + 1}\right]$.

EXECUTE: **(a)** $Q = A\sigma = \pi(R_2^2 - R_1^2)\sigma$

(b) $\vec{E}(x) = \dfrac{\sigma}{2\epsilon_0}\left(\left[1 - 1/\sqrt{(R_2/x)^2+1}\right] - \left[1 - 1/\sqrt{(R_1/x)^2+1}\right]\right)\dfrac{|x|}{x}\hat{i}$. $\vec{E}(x) = \dfrac{-\sigma}{2\epsilon_0}\left(1/\sqrt{(R_1/x)^2+1} - 1/\sqrt{(R_2/x)^2+1}\right)\dfrac{|x|}{x}\hat{i}$.

The electric field is in the $+x$ direction at points above the disk and in the $-x$ direction at points below the disk, and the factor $\dfrac{|x|}{x}\hat{i}$ specifies these directions.

(c) Note that $1\big/\sqrt{(R_1/x)^2+1} = \dfrac{|x|}{R_1}(1 + (x/R_1)^2)^{-1/2} \approx \dfrac{|x|}{R_1}$. This gives $\vec{E}(x) = \dfrac{\sigma}{2\epsilon_0}\left(\dfrac{x}{R_1} - \dfrac{x}{R_2}\right)\dfrac{|x|^2}{x}\hat{i} = \dfrac{\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)x\hat{i}$.

Sufficiently close means that $(x/R_1)^2 << 1$.

(d) $F_x = qE_x = -\dfrac{q\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)x$. The force is in the form of Hooke's law: $F_x = -kx$, with $k = \dfrac{q\sigma}{2\epsilon_0}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$.

$f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}} = \dfrac{1}{2\pi}\sqrt{\dfrac{q\sigma}{2\epsilon_0 m}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)}$.

EVALUATE: The frequency is independent of the initial position of the particle, so long as this position is sufficiently close to the center of the annulus for $(x/R_1)^2$ to be small.

21.105. IDENTIFY: Apply Coulomb's law to calculate the forces that q_1 and q_2 exert on q_3, and add these force vectors to get the net force.

SET UP: Like charges repel and unlike charges attract. Let $+x$ be to the right and $+y$ be toward the top of the page.

EXECUTE: **(a)** The four possible force diagrams are sketched in Figure 21.105a.

Only the last picture can result in a net force in the $-x$-direction.

(b) $q_1 = -2.00\ \mu$C, $q_3 = +4.00\ \mu$C, and $q_2 > 0$.

(c) The forces $\vec{F}_1$ and $\vec{F}_2$ and their components are sketched in Figure 21.105b.

$F_y = 0 = -\dfrac{1}{4\pi\epsilon_0}\dfrac{|q_1||q_3|}{(0.0400\ \text{m})^2}\sin\theta_1 + \dfrac{1}{4\pi\epsilon_0}\dfrac{|q_2||q_3|}{(0.0300\ \text{m})^2}\sin\theta_2$. This gives

$q_2 = \dfrac{9}{16}|q_1|\dfrac{\sin\theta_1}{\sin\theta_2} = \dfrac{9}{16}\ |q_1|\dfrac{3/5}{4/5} = \dfrac{27}{64}|q_1| = 0.843\ \mu\text{C}$.

(d) $F_x = F_{1x} + F_{2x}$ and $F_y = 0$, so $F = |q_3|\dfrac{1}{4\pi\epsilon_0}\left(\dfrac{|q_1|}{(0.0400\ \text{m})^2}\dfrac{4}{5} + \dfrac{|q_2|}{(0.0300\ \text{m})^2}\dfrac{3}{5}\right) = 56.2\ \text{N}$.

EVALUATE: The net force $\vec{F}$ on q_3 is in the same direction as the resultant electric field at the location of q_3 due to q_1 and q_2.

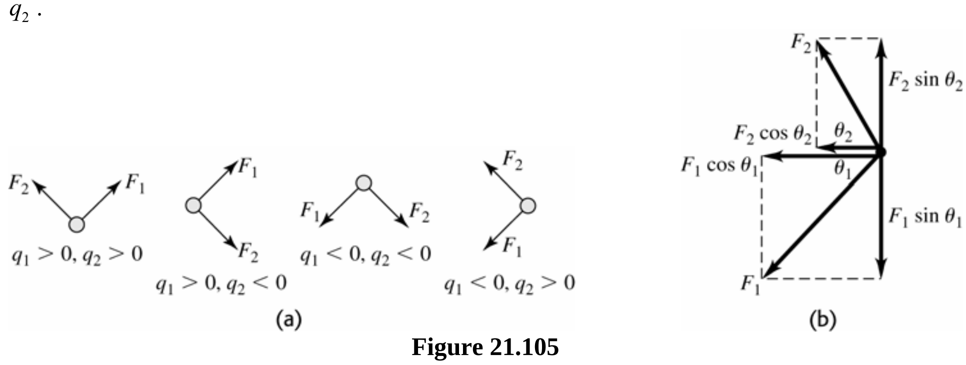


Figure 21.105

21.106. IDENTIFY: Calculate the electric field at P due to each charge and add these field vectors to get the net field.

SET UP: The electric field of a point charge is directed away from a positive charge and toward a negative charge. Let $+x$ be to the right and let $+y$ be toward the top of the page.

EXECUTE: **(a)** The four possible diagrams are sketched in Figure 21.106a.

The first diagram is the only one in which the electric field must point in the negative y-direction.

(b) $q_1 = -3.00\ \mu$C, and $q_2 < 0$.

(c) The electric fields $\vec{E}_1$ and $\vec{E}_2$ and their components are sketched in Figure 24.106b. $\cos\theta_1 = \frac{5}{13}$, $\sin\theta_1 = \frac{12}{13}$, $\cos\theta_2 = \frac{12}{13}$ and $\sin\theta_2 = \frac{5}{13}$. $E_x = 0 = -\frac{k|q_1|}{(0.050\text{ m})^2}\frac{5}{13} + \frac{k|q_2|}{(0.120\text{ m})^2}\frac{12}{13}$. This gives $\frac{k|q_2|}{(0.120\text{ m})^2} = \frac{k|q_1|}{(0.050\text{ m})^2}\frac{5}{12}$. Solving for $|q_2|$ gives $|q_2| = 7.2\ \mu\text{C}$, so $q_2 = -7.2\ \mu\text{C}$. Then

$$E_y = -\frac{k|q_1|}{(0.050\text{ m})^2}\frac{12}{13} - \frac{kq_2}{(0.120\text{ m})^2}\frac{5}{13} = -1.17\times10^7\text{ N/C}.\ E = 1.17\times10^7\text{ N/C}.$$

EVALUATE: With q_1 known, specifying the direction of $\vec{E}$ determines both q_2 and E.

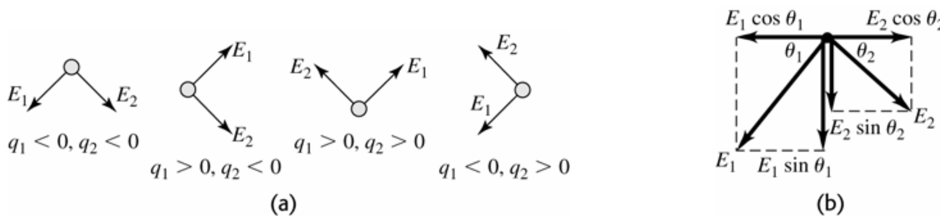


Figure 21.106

21.107. IDENTIFY: To find the electric field due to the second rod, divide that rod into infinitesimal segments of length *dx*, calculate the field *dE* due to each segment and integrate over the length of the rod to find the total field due to the rod. Use $d\vec{F} = dq\,\vec{E}$ to find the force the electric field of the second rod exerts on each infinitesimal segment of the first rod.

SET UP: An infinitesimal segment of the second rod is sketched in Figure 21.107. $dQ = (Q/L)dx'$.

EXECUTE: **(a)** $dE = \frac{k\,dQ}{(x + a/2 + L - x')^2} = \frac{kQ}{L}\frac{dx'}{(x + a/2 + L - x')^2}$.

$$E_x = \int_0^L dE_x = \frac{kQ}{L}\int_0^L \frac{dx'}{(x + a/2 + L - x')^2} = \frac{kQ}{L}\left[\frac{1}{x + a/2 + L - x'}\right]_0^L = \frac{kQ}{L}\left(\frac{1}{x + a/2} - \frac{1}{x + a/2 + L}\right).$$

$$E_x = \frac{2kQ}{L}\left(\frac{1}{2x + a} - \frac{1}{2L + 2x + a}\right).$$

(b) Now consider the force that the field of the second rod exerts on an infinitesimal segment *dq* of the first rod. This force is in the +*x*-direction. $dF = dq\,E$.

$$F = \int E\,dq = \int_{a/2}^{L+a/2}\frac{EQ}{L}\,dx = \frac{2kQ^2}{L^2}\int_{a/2}^{L+a/2}\left(\frac{1}{2x + a} - \frac{1}{2L + 2x + a}\right)dx.$$

$$F = \frac{2kQ^2}{L^2}\frac{1}{2}\left([\ln(a + 2x)]_{a/2}^{L+a/2} - [\ln(2L + 2x + a)]_{a/2}^{L+a/2}\right) = \frac{kQ^2}{L^2}\ln\left(\left(\frac{a + 2L + a}{2a}\right)\left(\frac{2L + 2a}{4L + 2a}\right)\right).$$

$$F = \frac{kQ^2}{L^2}\ln\left(\frac{(a + L)^2}{a(a + 2L)}\right).$$

(c) For $a >> L$, $F = \frac{kQ^2}{L^2}\ln\left(\frac{a^2(1 + L/a)^2}{a^2(1 + 2L/a)}\right) = \frac{kQ^2}{L^2}(2\ln(1 + L/a) - \ln(1 + 2L/a))$.

For small *z*, $\ln(1 + z) \approx z - \frac{z^2}{2}$. Therefore, for $a >> L$, $F \approx \frac{kQ^2}{L^2}\left(2\left(\frac{L}{a} - \frac{L^2}{2a^2} + \cdots\right) - \left(\frac{2L}{a} - \frac{2L^2}{a^2} + \cdots\right)\right) \approx \frac{kQ^2}{a^2}$.

EVALUATE: The distance between adjacent ends of the rods is *a*. When $a >> L$ the distance between the rods is much greater than their lengths and they interact as point charges.

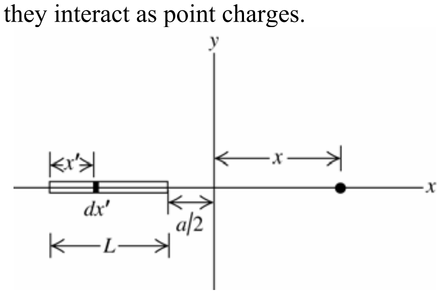


Figure 21.107

GAUSS'S LAW

22

22.1. **(a) IDENTIFY** and **SET UP:** $\Phi_E = \int E\cos\phi dA,$ where ϕ is the angle between the normal to the sheet $\hat{\boldsymbol{n}}$ and the electric field $\vec{\boldsymbol{E}}$.

EXECUTE: In this problem E and $\cos\phi$ are constant over the surface so

$\Phi_E = E\cos\phi\int dA = E\cos\phi A = (14\text{ N/C})(\cos 60°)(0.250\text{ m}^2) = 1.8\text{ N}\cdot\text{m}^2/\text{C}.$

(b) EVALUATE: Φ_E is independent of the shape of the sheet as long as ϕ and E are constant at all points on the sheet.

(c) EXECUTE: (i) $\Phi_E = E\cos\phi A.$ Φ_E is largest for $\phi = 0°,$ so $\cos\phi = 1$ and $\Phi_E = EA.$

(ii) Φ_E is smallest for $\phi = 90°,$ so $\cos\phi = 0$ and $\Phi_E = 0.$

EVALUATE: Φ_E is 0 when the surface is parallel to the field so no electric field lines pass through the surface.

22.2. **IDENTIFY:** The field is uniform and the surface is flat, so use $\Phi_E = EA\cos\phi$.

SET UP: ϕ is the angle between the normal to the surface and the direction of $\vec{\boldsymbol{E}}$, so $\phi = 70°$.

EXECUTE: $\Phi_E = (75.0\text{ N/C})(0.400\text{ m})(0.600\text{ m})\cos 70° = 6.16\text{ N}\cdot\text{m}^2/\text{C}$

EVALUATE: If the field were perpendicular to the surface the flux would be $\Phi_E = EA = 18.0\text{ N}\cdot\text{m}^2/\text{C}.$ The flux in this problem is much less than this because only the component of $\vec{\boldsymbol{E}}$ perpendicular to the surface contributes to the flux.

22.3. **IDENTIFY:** The electric flux through an area is defined as the product of the component of the electric field perpendicular to the area times the area.

(a) SET UP: In this case, the electric field is perpendicular to the surface of the sphere, so $\Phi_E = EA = E(4\pi r^2)$.

EXECUTE: Substituting in the numbers gives

$$\Phi_E = (1.25\times10^6\text{ N/C})4\pi(0.150\text{ m})^2 = 3.53\times10^5\text{ N}\cdot\text{m}^2/\text{C}$$

(b) IDENTIFY: We use the electric field due to a point charge.

SET UP: $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$

EXECUTE: Solving for q and substituting the numbers gives

$$q = 4\pi\epsilon_0 r^2 E = \frac{1}{9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}(0.150\text{ m})^2(1.25\times10^6\text{ N/C}) = 3.13\times10^{-6}\text{ C}$$

EVALUATE: The flux would be the same no matter how large the circle, since the area is proportional to r^2 while the electric field is proportional to $1/r^2$.

22.4. **IDENTIFY:** Use Eq.(22.3) to calculate the flux for each surface. Use Eq.(22.8) to calculate the total enclosed charge.

SET UP: $\vec{\boldsymbol{E}} = (-5.00\text{ N/C}\cdot\text{m})x\hat{\boldsymbol{i}} + (3.00\text{ N/C}\cdot\text{m})z\hat{\boldsymbol{k}}$. The area of each face is L^2, where $L = 0.300\text{ m}$.

EXECUTE: $\hat{\boldsymbol{n}}_{S_1} = -\hat{\boldsymbol{j}} \Rightarrow \Phi_1 = \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}_{S_1}A = 0$.

$\hat{\boldsymbol{n}}_{S_2} = +\hat{\boldsymbol{k}} \Rightarrow \Phi_2 = \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}_{S_2}A = (3.00\text{ N/C}\cdot\text{m})(0.300\text{ m})^2 z = (0.27\text{ (N/C)}\cdot\text{m})z$.

$\Phi_2 = (0.27\text{ (N/C)m})(0.300\text{ m}) = 0.081\text{ (N/C)}\cdot\text{m}^2$.

$\hat{\boldsymbol{n}}_{S_3} = +\hat{\boldsymbol{j}} \Rightarrow \Phi_3 = \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}_{S_3}A = 0$.

$\hat{\boldsymbol{n}}_{S_4} = -\hat{\boldsymbol{k}} \Rightarrow \Phi_4 = \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}_{S_4}A = -(0.27\text{ (N/C)}\cdot\text{m})z = 0$ (since $z = 0$).

$\hat{\boldsymbol{n}}_{S_5} = +\hat{\boldsymbol{i}} \Rightarrow \Phi_5 = \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}_{S_5}A = (-5.00\text{ N/C}\cdot\text{m})(0.300\text{ m})^2 x = -(0.45\text{ (N/C)}\cdot\text{m})x.$

$\Phi_5 = -(0.45\text{ (N/C)}\cdot\text{m})(0.300\text{ m}) = -(0.135\text{ (N/C)}\cdot\text{m}^2).$

$\hat{\boldsymbol{n}}_{S_6} = -\hat{\boldsymbol{i}} \Rightarrow \Phi_6 = \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{n}}_{S_6}A = +(0.45\text{ (N/C)}\cdot\text{m})x = 0$ (since $x = 0$).

(b) Total flux: $\Phi = \Phi_2 + \Phi_5 = (0.081 - 0.135)(\text{N/C}) \cdot \text{m}^2 = -0.054\ \text{N} \cdot \text{m}^2/\text{C}$. Therefore, $q = \epsilon_0 \Phi = -4.78 \times 10^{-13}$ C.

EVALUATE: Flux is positive when $\vec{E}$ is directed out of the volume and negative when it is directed into the volume.

22.5. **IDENTIFY:** The flux through the curved upper half of the hemisphere is the same as the flux through the flat circle defined by the bottom of the hemisphere because every electric field line that passes through the flat circle also must pass through the curved surface of the hemisphere.

SET UP: The electric field is perpendicular to the flat circle, so the flux is simply the product of E and the area of the flat circle of radius r.

EXECUTE: $\Phi_E = EA = E(\pi r^2) = \pi r^2 E$

EVALUATE: The flux would be the same if the hemisphere were replaced by any other surface bounded by the flat circle.

22.6. **IDENTIFY:** Use Eq.(22.3) to calculate the flux for each surface.

SET UP: $\Phi = \vec{E} \cdot \vec{A} = EA\cos\phi$ where $\vec{A} = A\hat{n}$.

EXECUTE: **(a)** $\hat{n}_{S_1} = -\hat{j}$ (left). $\Phi_{S_1} = -(4 \times 10^3\ \text{N/C})(0.10\ \text{m})^2 \cos(90° - 36.9°) = -24\ \text{N} \cdot \text{m}^2/\text{C}$.

$\hat{n}_{S_2} = +\hat{k}$ (top). $\Phi_{S_2} = -(4 \times 10^3\ \text{N/C})(0.10\ \text{m})^2 \cos 90° = 0$.

$\hat{n}_{S_3} = +\hat{j}$ (right). $\Phi_{S_3} = +(4 \times 10^3\ \text{N/C})(0.10\ \text{m})^2 \cos(90° - 36.9°) = +24\ \text{N} \cdot \text{m}^2/\text{C}$.

$\hat{n}_{S_4} = -\hat{k}$ (bottom). $\Phi_{S_4} = (4 \times 10^3\ \text{N/C})(0.10\ \text{m})^2 \cos 90° = 0$.

$\hat{n}_{S_5} = +\hat{i}$ (front). $\Phi_{S_5} = +(4 \times 10^3\ \text{N/C})(0.10\ \text{m})^2 \cos 36.9° = 32\ \text{N} \cdot \text{m}^2/\text{C}$.

$\hat{n}_{S_6} = -\hat{i}$ (back). $\Phi_{S_6} = -(4 \times 10^3\ \text{N/C})(0.10\ \text{m})^2 \cos 36.9° = -32\ \text{N} \cdot \text{m}^2/\text{C}$.

EVALUATE: **(b)** The total flux through the cube must be zero; any flux entering the cube must also leave it, since the field is uniform. Our calculation gives the result; the sum of the fluxes calculated in part (a) is zero.

22.7. **(a) IDENTIFY:** Use Eq.(22.5) to calculate the flux through the surface of the cylinder.

SET UP: The line of charge and the cylinder are sketched in Figure 22.7.

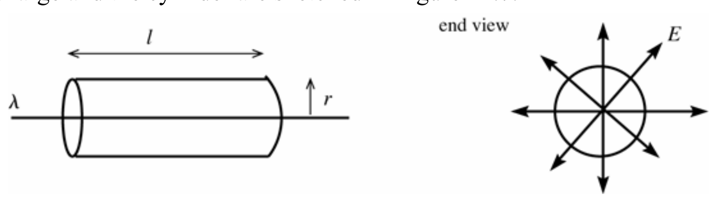


Figure 22.7

EXECUTE: The area of the curved part of the cylinder is $A = 2\pi rl$.

The electric field is parallel to the end caps of the cylinder, so $\vec{E} \cdot \vec{A} = 0$ for the ends and the flux through the cylinder end caps is zero.

The electric field is normal to the curved surface of the cylinder and has the same magnitude $E = \lambda / 2\pi\epsilon_0 r$ at all points on this surface. Thus $\phi = 0°$ and

$$\Phi_E = EA\cos\phi = EA = (\lambda / 2\pi\epsilon_0 r)(2\pi rl) = \frac{\lambda l}{\epsilon_0} = \frac{(6.00 \times 10^{-6}\ \text{C/m})(0.400\ \text{m})}{8.854 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2} = 2.71 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}$$

(b) In the calculation in part (a) the radius r of the cylinder divided out, so the flux remains the same, $\Phi_E = 2.71 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}$.

(c) $\Phi_E = \dfrac{\lambda l}{\epsilon_0} = \dfrac{(6.00 \times 10^{-6}\ \text{C/m})(0.800\ \text{m})}{8.854 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2} = 5.42 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}$ (twice the flux calculated in parts (b) and (c)).

EVALUATE: The flux depends on the number of field lines that pass through the surface of the cylinder.

22.8. **IDENTIFY:** Apply Gauss's law to each surface.

SET UP: Q_{encl} is the algebraic sum of the charges enclosed by each surface. Flux out of the volume is positive and flux into the enclosed volume is negative.

EXECUTE: **(a)** $\Phi_{S_1} = q_1/\epsilon_0 = (4.00 \times 10^{-9}\ \text{C})/\epsilon_0 = 452\ \text{N} \cdot \text{m}^2/\text{C}$.

(b) $\Phi_{S_2} = q_2/\epsilon_0 = (-7.80 \times 10^{-9}\ \text{C})/\epsilon_0 = -881\ \text{N} \cdot \text{m}^2/\text{C}$.

(c) $\Phi_{S_3} = (q_1 + q_2)/\epsilon_0 = ((4.00 - 7.80) \times 10^{-9}\ \text{C})/\epsilon_0 = -429\ \text{N} \cdot \text{m}^2/\text{C}$.

(d) $\Phi_{S_4} = (q_1 + q_3)/\epsilon_0 = ((4.00 + 2.40) \times 10^{-9}\ \text{C})/\epsilon_0 = 723\ \text{N} \cdot \text{m}^2/\text{C}$.

(e) $\Phi_{S_5} = (q_1 + q_2 + q_3)/\epsilon_0 = ((4.00 - 7.80 + 2.40) \times 10^{-9}\ \text{C})/\epsilon_0 = -158\ \text{N} \cdot \text{m}^2/\text{C}$.

EVALUATE: **(f)** All that matters for Gauss's law is the total amount of charge enclosed by the surface, not its distribution within the surface.

22.9. **IDENTIFY:** Apply the results in Example 21.10 for the field of a spherical shell of charge.

SET UP: Example 22.10 shows that $E = 0$ inside a uniform spherical shell and that $E = k\dfrac{|q|}{r^2}$ outside the shell.

EXECUTE: **(a)** $E = 0$

(b) $r = 0.060\text{ m}$ and $E = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{15.0\times10^{-6}\text{ C}}{(0.060\text{ m})^2} = 3.75\times10^7\text{ N/C}$

(c) $r = 0.110\text{ m}$ and $E = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{15.0\times10^{-6}\text{ C}}{(0.110\text{ m})^2} = 1.11\times10^7\text{ N/C}$

EVALUATE: Outside the shell the electric field is the same as if all the charge were concentrated at the center of the shell. But inside the shell the field is not the same as for a point charge at the center of the shell, inside the shell the electric field is zero.

22.10. **IDENTIFY:** Apply Gauss's law to the spherical surface.

SET UP: Q_{encl} is the algebraic sum of the charges enclosed by the sphere.

EXECUTE: **(a)** No charge enclosed so $\Phi = 0$.

(b) $\Phi = \dfrac{q_2}{\epsilon_0} = \dfrac{-6.00\times10^{-9}\text{ C}}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = -678\text{ N}\cdot\text{m}^2/\text{C}.$

(c) $\Phi = \dfrac{q_1+q_2}{\epsilon_0} = \dfrac{(4.00-6.00)\times10^{-9}\text{ C}}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = -226\text{ N}\cdot\text{m}^2/\text{C}.$

EVALUATE: Negative flux corresponds to flux directed into the enclosed volume. The net flux depends only on the net charge enclosed by the surface and is not affected by any charges outside the enclosed volume.

22.11. **IDENTIFY:** Apply Gauss's law.

SET UP: In each case consider a small Gaussian surface in the region of interest.

EXECUTE: **(a)** Since $\vec{E}$ is uniform, the flux through a closed surface must be zero. That is:

$\Phi = \oint \vec{E}\cdot d\vec{A} = \dfrac{q}{\epsilon_0} = \dfrac{1}{\epsilon_0}\int \rho dV = 0 \Rightarrow \int \rho dV = 0.$ But because we can choose any volume we want, ρ must be zero if the integral equals zero.

(b) If there is no charge in a region of space, that does NOT mean that the electric field is uniform. Consider a closed volume close to, but not including, a point charge. The field diverges there, but there is no charge in that region.

EVALUATE: The electric field within a region can depend on charges located outside the region. But the flux through a closed surface depends only on the net charge contained within that surface.

22.12. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a small Gaussian surface located in the region of question.

EXECUTE: **(a)** If $\rho > 0$ and uniform, then q inside any closed surface is greater than zero. This implies $\Phi > 0$, so $\oint \vec{E}\cdot d\vec{A} > 0$ and so the electric field cannot be uniform. That is, since an arbitrary surface of our choice encloses a non-zero amount of charge, E must depend on position.

(b) However, inside a small bubble of zero charge density within the material with density ρ, the field can be uniform. All that is important is that there be zero flux through the surface of the bubble (since it encloses no charge). (See Problem 22.61.)

EVALUATE: In a region of uniform field, the flux through any closed surface is zero.

22.13. **(a) IDENTIFY** and **SET UP:** It is rather difficult to calculate the flux directly from $\Phi = \oint \vec{E}\cdot d\vec{A}$ since the magnitude of $\vec{E}$ and its angle with $d\vec{A}$ varies over the surface of the cube. A much easier approach is to use Gauss's law to calculate the total flux through the cube. Let the cube be the Gaussian surface. The charge enclosed is the point charge.

EXECUTE: $\Phi_E = Q_{\text{encl}}/\epsilon_0 = \dfrac{9.60\times10^{-6}\text{ C}}{8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 1.084\times10^6\text{ N}\cdot\text{m}^2/\text{C}.$

By symmetry the flux is the same through each of the six faces, so the flux through one face is $\frac{1}{6}\left(1.084\times10^6\text{ N}\cdot\text{m}^2/\text{C}\right) = 1.81\times10^5\text{ N}\cdot\text{m}^2/\text{C}.$

(b) EVALUATE: In part (a) the size of the cube did not enter into the calculations. The flux through one face depends only on the amount of charge at the center of the cube. So the answer to (a) would not change if the size of the cube were changed.

22.14. **IDENTIFY:** Apply the results of Examples 22.9 and 22.10.

SET UP: $E = k\dfrac{|q|}{r^2}$ outside the sphere. A proton has charge $+e$.

EXECUTE: **(a)** $E = k\dfrac{|q|}{r^2} = (8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{92(1.60\times10^{-19}\ \text{C})}{(7.4\times10^{-15}\ \text{m})^2} = 2.4\times10^{21}\ \text{N/C}$

(b) For $r = 1.0\times10^{-10}\ \text{m}$, $E = (2.4\times10^{21}\ \text{N/C})\left(\dfrac{7.4\times10^{-15}\ \text{m}}{1.0\times10^{-10}\ \text{m}}\right)^2 = 1.3\times10^{13}\ \text{N/C}$

(c) $E = 0$, inside a spherical shell.

EVALUATE: The electric field in an atom is very large.

22.15. **IDENTIFY:** The electric fields are produced by point charges.

SET UP: We use Coulomb's law, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$, to calculate the electric fields.

EXECUTE: **(a)** $E = \left(9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\dfrac{5.00\times10^{-6}\ \text{C}}{(1.00\ \text{m})^2} = 4.50\times10^{4}\ \text{N/C}$

(b) $E = \left(9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\dfrac{5.00\times10^{-6}\ \text{C}}{(7.00\ \text{m})^2} = 9.18\times10^{2}\ \text{N/C}$

(c) Every field line that enters the sphere on one side leaves it on the other side, so the net flux through the surface is zero.

EVALUATE: The flux would be zero no matter what shape the surface had, providing that no charge was inside the surface.

22.16. **IDENTIFY:** Apply the results of Example 22.5.

SET UP: At a point 0.100 m outside the surface, $r = 0.550\ \text{m}$.

EXECUTE: **(a)** $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(2.50\times10^{-10}\ \text{C})}{(0.550\ \text{m})^2} = 7.44\ \text{N/C}.$

(b) $E = 0$ inside of a conductor or else free charges would move under the influence of forces, violating our electrostatic assumptions (i.e., that charges aren't moving).

EVALUATE: Outside the sphere its electric field is the same as would be produced by a point charge at its center, with the same charge.

22.17. **IDENTIFY:** The electric field required to produce a spark 6 in. long is 6 times as strong as the field needed to produce a spark 1 in. long.

SET UP: By Gauss's law, $q = \epsilon_0 EA$ and the electric field is the same as for a point-charge, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$.

EXECUTE: **(a)** The electric field for 6-inch sparks is $E = 6\times2.00\times10^{4}\ \text{N/C} = 1.20\times10^{5}\ \text{N/C}$

The charge to produce this field is

$q = \epsilon_0 EA = \epsilon_0 E(4\pi r^2) = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.20\times10^{5}\ \text{N/C})(4\pi)(0.15\,\text{m})^2 = 3.00\times10^{-7}\ \text{C}$.

(b) Using Coulomb's law gives $E = (9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{3.00\times10^{-7}\ \text{C}}{(0.150\ \text{m})^2} = 1.20\times10^{5}\ \text{N/C}$.

EVALUATE: It takes only about $0.3\ \mu\text{C}$ to produce a field this strong.

22.18. **IDENTIFY:** According to Exercise 21.32, the Earth's electric field points towards its center. Since Mars's electric field is similar to that of Earth, we assume it points toward the center of Mars. Therefore the charge on Mars must be negative. We use Gauss's law to relate the electric flux to the charge causing it.

SET UP: Gauss's law is $\Phi_E = \dfrac{q}{\epsilon_0}$ and the electric flux is $\Phi_E = EA$.

EXECUTE: **(a)** Solving Gauss's law for q, putting in the numbers, and recalling that q is negative, gives

$q = -\epsilon_0\Phi_E = -(3.63\times10^{16}\ \text{N}\cdot\text{m}^2/\text{C})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = -3.21\times10^{5}\ \text{C}$.

(b) Use the definition of electric flux to find the electric field. The area to use is the surface area of Mars.

$$E = \frac{\Phi_E}{A} = \frac{3.63\times10^{16}\ \text{N}\cdot\text{m}^2/\text{C}}{4\pi(3.40\times10^{6}\ \text{m})^2} = 2.50\times10^{2}\ \text{N/C}$$

(c) The surface charge density on Mars is therefore $\sigma = \dfrac{q}{A_{\text{Mars}}} = \dfrac{-3.21\times10^{5}\ \text{C}}{4\pi(3.40\times10^{6}\ \text{m})^2} = -2.21\times10^{-9}\ \text{C/m}^2$

EVALUATE: Even though the charge on Mars is very large, it is spread over a large area, giving a small surface charge density.

22.19. **IDENTIFY** and **SET UP:** Example 22.5 derived that the electric field just outside the surface of a spherical conductor that has net charge q is $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{R^2}$. Calculate q and from this the number of excess electrons.

EXECUTE: $q = \dfrac{R^2 E}{(1/4\pi\epsilon_0)} = \dfrac{(0.160\text{ m})^2(1150\text{ N/C})}{8.988\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 3.275\times 10^{-9}\text{ C}.$

Each electron has a charge of magnitude $e = 1.602\times 10^{-19}\text{ C},$ so the number of excess electrons needed is $\dfrac{3.275\times 10^{-9}\text{ C}}{1.602\times 10^{-19}\text{ C}} = 2.04\times 10^{10}.$

EVALUATE: The result we obtained for q is a typical value for the charge of an object. Such net charges correspond to a large number of excess electrons since the charge of each electron is very small.

22.20. **IDENTIFY:** Apply Gauss's law.

SET UP: Draw a cylindrical Gaussian surface with the line of charge as its axis. The cylinder has radius 0.400 m and is 0.0200 m long. The electric field is then 840 N/C at every point on the cylindrical surface and is directed perpendicular to the surface.

EXECUTE: $\oint \vec{E}\cdot d\vec{A} = EA_{\text{cylinder}} = E(2\pi rL) = (840\text{ N/C})(2\pi)(0.400\text{ m})(0.0200\text{ m}) = 42.2\text{ N}\cdot\text{m}^2/\text{C}.$

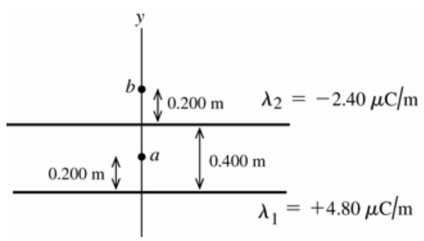
The field is parallel to the end caps of the cylinder, so for them $\oint \vec{E}\cdot d\vec{A} = 0.$ From Gauss's law,

$q = \epsilon_0\Phi_E = (8.854\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(42.2\text{ N}\cdot\text{m}^2/\text{C}) = 3.74\times 10^{-10}\text{ C}.$

EVALUATE: We could have applied the result in Example 22.6 and solved for λ. Then $q = \lambda L.$

22.21. **IDENTIFY:** Add the vector electric fields due to each line of charge. $E(r)$ for a line of charge is given by Example 22.6 and is directed toward a negative line of chage and away from a positive line.

SET UP: The two lines of charge are shown in Figure 22.21.



$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

Figure 22.21

EXECUTE: **(a)** At point a, $\vec{E}_1$ and $\vec{E}_2$ are in the $+y$-direction (toward negative charge, away from positive charge).

$E_1 = (1/2\pi\epsilon_0)\left[(4.80\times 10^{-6}\text{ C/m})/(0.200\text{ m})\right] = 4.314\times 10^5\text{ N/C}$

$E_2 = (1/2\pi\epsilon_0)\left[(2.40\times 10^{-6}\text{ C/m})/(0.200\text{ m})\right] = 2.157\times 10^5\text{ N/C}$

$E = E_1 + E_2 = 6.47\times 10^5\text{ N/C},$ in the y-direction.

(b) At point b, $\vec{E}_1$ is in the $+y$-direction and $\vec{E}_2$ is in the $-y$-direction.

$E_1 = (1/2\pi\epsilon_0)\left[(4.80\times 10^{-6}\text{ C/m})/(0.600\text{ m})\right] = 1.438\times 10^5\text{ N/C}$

$E_2 = (1/2\pi\epsilon_0)\left[(2.40\times 10^{-6}\text{ C/m})/(0.200\text{ m})\right] = 2.157\times 10^5\text{ N/C}$

$E = E_2 - E_1 = 7.2\times 10^4\text{ N/C},$ in the $-y$-direction.

EVALUATION: At point a the two fields are in the same direction and the magnitudes add. At point b the two fields are in opposite directions and the magnitudes subtract.

22.22. **IDENTIFY:** Apply the results of Examples 22.5, 22.6 and 22.7.

SET UP: Gauss's law can be used to show that the field outside a long conducting cylinder is the same as for a line of charge along the axis of the cylinder.

EXECUTE: **(a)** For points outside a uniform spherical charge distribution, all the charge can be considered to be concentrated at the center of the sphere. The field outside the sphere is thus inversely proportional to the square of the distance from the center. In this case,

$$E = (480\text{ N/C})\left(\frac{0.200\text{ cm}}{0.600\text{ cm}}\right)^2 = 53\text{ N/C}$$

(b) For points outside a long cylindrically symmetrical charge distribution, the field is identical to that of a long line of charge: $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$, that is, inversely proportional to the distance from the axis of the cylinder. In this case

$$E = (480 \text{ N/C})\left(\frac{0.200 \text{ cm}}{0.600 \text{ cm}}\right) = 160 \text{ N/C}$$

(c) The field of an infinite sheet of charge is $E = \sigma/2\epsilon_0$; i.e., it is independent of the distance from the sheet. Thus in this case $E = 480$ N/C.

EVALUATE: For each of these three distributions of charge the electric field has a different dependence on distance.

22.23. **IDENTIFY:** The electric field inside the conductor is zero, and all of its initial charge lies on its outer surface. The introduction of charge into the cavity induces charge onto the surface of the cavity, which induces an equal but opposite charge on the outer surface of the conductor. The net charge on the outer surface of the conductor is the sum of the positive charge initially there and the additional negative charge due to the introduction of the negative charge into the cavity.

(a) SET UP: First find the initial positive charge on the outer surface of the conductor using $q_i = \sigma A$, where A is the area of its outer surface. Then find the net charge on the surface after the negative charge has been introduced into the cavity. Finally use the definition of surface charge density.

EXECUTE: The original positive charge on the outer surface is

$$q_i = \sigma A = \sigma(4\pi r^2) = (6.37\times10^{-6} \text{ C/m}^2)4\pi(0.250 \text{ m}^2) = 5.00\times10^{-6} \text{ C/m}^2$$

After the introduction of $-0.500\ \mu\text{C}$ into the cavity, the outer charge is now

$$5.00\ \mu\text{C} - 0.500\ \mu\text{C} = 4.50\ \mu\text{C}$$

The surface charge density is now $\sigma = \dfrac{q}{A} = \dfrac{q}{4\pi r^2} = \dfrac{4.50\times10^{-6} \text{ C}}{4\pi(0.250 \text{ m})^2} = 5.73\times10^{-6} \text{ C/m}^2$

(b) SET UP: Using Gauss's law, the electric field is $E = \dfrac{\Phi_E}{A} = \dfrac{q}{\epsilon_0 A} = \dfrac{q}{\epsilon_0 4\pi r^2}$

EXECUTE: Substituting numbers gives

$$E = \frac{4.50\times10^{-6} \text{ C}}{(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(4\pi)(0.250 \text{ m})^2} = 6.47\times10^5 \text{ N/C}.$$

(c) SET UP: We use Gauss's law again to find the flux. $\Phi_E = \dfrac{q}{\epsilon_0}$.

EXECUTE: Substituting numbers gives

$$\Phi_E = \frac{-0.500\times10^{-6} \text{ C}}{8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2} = -5.65\times10^4 \text{ N}\cdot\text{m}^2/\text{C}^2.$$

EVALUATE: The excess charge on the conductor is still $+5.00\ \mu\text{C}$, as it originally was. The introduction of the $-0.500\ \mu\text{C}$ inside the cavity merely induced equal but opposite charges (for a net of zero) on the surfaces of the conductor.

22.24. **IDENTIFY:** We apply Gauss's law, taking the Gaussian surface beyond the cavity but inside the solid.

SET UP: Because of the symmetry of the charge, Gauss's law gives us $E = \dfrac{q_{\text{total}}}{\epsilon_0 A}$, where A is the surface area of a sphere of radius R = 9.50 cm centered on the point-charge, and q_{total} is the total charge contained within that sphere. This charge is the sum of the $-2.00\ \mu\text{C}$ point charge at the center of the cavity plus the charge within the solid between r = 6.50 cm and R = 9.50 cm. The charge within the solid is $q_{\text{solid}} = \rho V = \rho([4/3]\pi R^3 - [4/3]\pi r^3) = ([4\pi/3]\rho)(R^3 - r^3)$

EXECUTE: First find the charge within the solid between r = 6.50 cm and R = 9.50 cm:

$$q_{\text{solid}} = \frac{4\pi}{3}(7.35\times10^{-4} \text{ C/m}^3)\left[(0.0950 \text{ m})^3 - (0.0650 \text{ m})^3\right] = 1.794\times10^{-6} \text{ C},$$

Now find the total charge within the Gaussian surface:

$$q_{\text{total}} = q_{\text{solid}} + q_{\text{point}} = -2.00\ \mu\text{C} + 1.794\ \mu\text{C} = -0.2059\ \mu\text{C}$$

Now find the magnitude of the electric field from Gauss's law:

$$E = \frac{q}{\epsilon_0 A} = \frac{q}{\epsilon_0 4\pi r^2} = \frac{0.2059\times10^{-6} \text{ C}}{(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(4\pi)(0.0950 \text{ m})^2} = 2.05\times10^5 \text{ N/C}.$$

The fact that the charge is negative means that the electric field points radially inward.

EVALUATE: Because of the uniformity of the charge distribution, the charge beyond 9.50 cm does not contribute to the electric field.

22.25. **IDENTIFY:** The magnitude of the electric field is constant at any given distance from the center because the charge density is uniform inside the sphere. We can use Gauss's law to relate the field to the charge causing it.

(a) SET UP: Gauss's law tells us that $EA = \dfrac{q}{\epsilon_0}$, and the charge density is given by $\rho = \dfrac{q}{V} = \dfrac{q}{(4/3)\pi R^3}$.

EXECUTE: Solving for q and substituting numbers gives
$q = EA\epsilon_0 = E(4\pi r^2)\epsilon_0 = (1750\text{ N/C})(4\pi)(0.500\text{ m})^2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = 4.866\times10^{-8}\text{ C}$. Using the formula for charge density we get $\rho = \dfrac{q}{V} = \dfrac{q}{(4/3)\pi R^3} = \dfrac{4.866\times10^{-8}\text{ C}}{(4/3)\pi(0.355\text{ m})^3} = 2.60\times10^{-7}\text{ C/m}^3$.

(b) SET UP: Take a Gaussian surface of radius $r = 0.200$ m, concentric with the insulating sphere. The charge enclosed within this surface is $q_{\text{encl}} = \rho V = \rho\left(\dfrac{4}{3}\pi r^3\right)$, and we can treat this charge as a point-charge, using Coulomb's law $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_{\text{encl}}}{r^2}$. The charge beyond $r = 0.200$ m makes no contribution to the electric field.

EXECUTE: First find the enclosed charge:

$$q_{\text{encl}} = \rho\left(\frac{4}{3}\pi r^3\right) = \left(2.60\times10^{-7}\text{ C/m}^3\right)\left[\frac{4}{3}\pi\left(0.200\text{ m}\right)^3\right] = 8.70\times10^{-9}\text{ C}$$

Now treat this charge as a point-charge and use Coulomb's law to find the field:

$$E = \left(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{8.70\times10^{-9}\text{ C}}{\left(0.200\text{ m}\right)^2} = 1.96\times10^3\text{ N/C}$$

EVALUATE: Outside this sphere, it behaves like a point-charge located at its center. Inside of it, at a distance r from the center, the field is due only to the charge between the center and r.

22.26. **IDENTIFY:** Apply Gauss's law and conservation of charge.
SET UP: Use a Gaussian surface that lies wholly within he conducting material.
EXECUTE: **(a)** Positive charge is attracted to the inner surface of the conductor by the charge in the cavity. Its magnitude is the same as the cavity charge: $q_{inner} = +6.00\text{ nC}$, since $E = 0$ inside a conductor and a Gaussian surface that lies wholly within the conductor must enclose zero net charge.
(b) On the outer surface the charge is a combination of the net charge on the conductor and the charge "left behind" when the $+6.00\text{ nC}$ moved to the inner surface:

$$q_{\text{tot}} = q_{\text{inner}} + q_{\text{outer}} \Rightarrow q_{\text{outer}} = q_{\text{tot}} - q_{\text{inner}} = 5.00\text{ nC} - 6.00\text{ nC} = -1.00\text{ nC}.$$

EVALUATE: The electric field outside the conductor is due to the charge on its surface.

22.27. **IDENTIFY:** Apply Gauss's law to each surface.
SET UP: The field is zero within the plates. By symmetry the field is perpendicular to the plates outside the plates and can depend only on the distance from the plates. Flux into the enclosed volume is positive.
EXECUTE: S_2 and S_3 enclose no charge, so the flux is zero, and electric field outside the plates is zero. Between the plates, S_4 shows that $-EA = -q/\epsilon_0 = -\sigma A/\epsilon_0$ and $E = \sigma/\epsilon_0$.
EVALUATE: Our result for the field between the plates agrees with the result stated in Example 22.8.

22.28. **IDENTIFY:** Close to a finite sheet the field is the same as for an infinite sheet. Very far from a finite sheet the field is that of a point charge.

SET UP: For an infinite sheet, $E = \dfrac{\sigma}{2\epsilon_0}$. For a point charge, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2}$.

EXECUTE: **(a)** At a distance of 0.1 mm from the center, the sheet appears "infinite," so

$$E \approx \frac{q}{2\epsilon_0 A} = \frac{7.50\times10^{-9}\text{ C}}{2\epsilon_0(0.800\text{ m})^2} = 662\text{ N/C}.$$

(b) At a distance of 100 m from the center, the sheet looks like a point, so:

$$E \approx \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{(7.50\times10^{-9}\text{ C})}{(100\text{ m})^2} = 6.75\times10^{-3}\text{ N/C}.$$

(c) There would be no difference if the sheet was a conductor. The charge would automatically spread out evenly over both faces, giving it half the charge density on either face as the insulator but the same electric field. Far away, they both look like points with the same charge.
EVALUATE: The sheet can be treated as infinite at points where the distance to the sheet is much less than the distance to the edge of the sheet. The sheet can be treated as a point charge at points for which the distance to the sheet is much greater than the dimensions of the sheet.

22.29. **IDENTIFY:** Apply Gauss's law to a Gaussian surface and calculate *E*.

(a) SET UP: Consider the charge on a length *l* of the cylinder. This can be expressed as $q = \lambda l$. But since the surface area is $2\pi Rl$ it can also be expressed as $q = \sigma 2\pi Rl$. These two expressions must be equal, so $\lambda l = \sigma 2\pi Rl$ and $\lambda = 2\pi R\sigma$.

(b) Apply Gauss's law to a Gaussian surface that is a cylinder of length *l*, radius *r*, and whose axis coincides with the axis of the charge distribution, as shown in Figure 22.29.

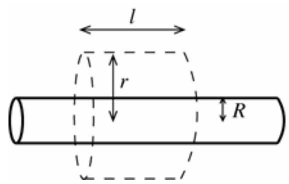


Figure 22.29

EXECUTE:

$Q_{\text{encl}} = \sigma(2\pi Rl)$

$\Phi_E = 2\pi rlE$

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $2\pi rlE = \dfrac{\sigma(2\pi Rl)}{\epsilon_0}$

$$E = \frac{\sigma R}{\epsilon_0 r}$$

(c) EVALUATE: Example 22.6 shows that the electric field of an infinite line of charge is $E = \lambda/2\pi\epsilon_0 r$. $\sigma = \dfrac{\lambda}{2\pi R}$, so $E = \dfrac{\sigma R}{\epsilon_0 r} = \dfrac{R}{\epsilon_0 r}\left(\dfrac{\lambda}{2\pi R}\right) = \dfrac{\lambda}{2\pi\epsilon_0 r}$, the same as for an infinite line of charge that is along the axis of the cylinder.

22.30. **IDENTIFY:** The net electric field is the vector sum of the fields due to each of the four sheets of charge.

SET UP: The electric field of a large sheet of charge is $E = \sigma/2\epsilon_0$. The field is directed away from a positive sheet and toward a negative sheet.

EXECUTE: **(a)** At A: $E_A = \dfrac{|\sigma_2|}{2\epsilon_0} + \dfrac{|\sigma_3|}{2\epsilon_0} + \dfrac{|\sigma_4|}{2\epsilon_0} - \dfrac{|\sigma_1|}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(|\sigma_2| + |\sigma_3| + |\sigma_4| - |\sigma_1|)$.

$$E_A = \frac{1}{2\epsilon_0}(5\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 6\ \mu\text{C/m}^2) = 2.82 \times 10^5\ \text{N/C to the left.}$$

(b) $E_B = \dfrac{|\sigma_1|}{2\epsilon_0} + \dfrac{|\sigma_3|}{2\epsilon_0} + \dfrac{|\sigma_4|}{2\epsilon_0} - \dfrac{|\sigma_2|}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(|\sigma_1| + |\sigma_3| + |\sigma_4| - |\sigma_2|)$.

$$E_B = \frac{1}{2\epsilon_0}(6\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2) = 3.95 \times 10^5\ \text{N/C to the left.}$$

(c) $E_C = \dfrac{|\sigma_4|}{2\epsilon_0} + \dfrac{|\sigma_1|}{2\epsilon_0} - \dfrac{|\sigma_2|}{2\epsilon_0} - \dfrac{|\sigma_3|}{2\epsilon_0} = \dfrac{1}{2\epsilon_0}(|\sigma_2| + |\sigma_3| - |\sigma_4| - |\sigma_1|)$.

$$E_C = \frac{1}{2\epsilon_0}(4\ \mu\text{C/m}^2 + 6\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2 - 2\ \mu\text{C/m}^2) = 1.69 \times 10^5\ \text{N/C to the left}$$

EVALUATE: The field at *C* is not zero. The pieces of plastic are not conductors.

22.31. **IDENTIFY:** Apply Gauss's law and conservation of charge.

SET UP: $E = 0$ in a conducting material.

EXECUTE: **(a)** Gauss's law says $+Q$ on inner surface, so $E = 0$ inside metal.

(b) The outside surface of the sphere is grounded, so no excess charge.

(c) Consider a Gaussian sphere with the $-Q$ charge at its center and radius less than the inner radius of the metal. This sphere encloses net charge $-Q$ so there is an electric field flux through it; there is electric field in the cavity.

(d) In an electrostatic situation $E = 0$ inside a conductor. A Gaussian sphere with the $-Q$ charge at its center and radius greater than the outer radius of the metal encloses zero net charge (the $-Q$ charge and the $+Q$ on the inner surface of the metal) so there is no flux through it and $E = 0$ outside the metal.

(e) No, $E = 0$ there. Yes, the charge has been shielded by the grounded conductor. There is nothing like positive and negative mass (the gravity force is always attractive), so this cannot be done for gravity.

EVALUATE: Field lines within the cavity terminate on the charges induced on the inner surface.

22.32. **IDENTIFY** and **SET UP:** Eq.(22.3) to calculate the flux. Identify the direction of the normal unit vector $\hat{\boldsymbol{n}}$ for each surface.

EXECUTE: **(a)** $\vec{E} = -B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}};\ A = L^2$

face S_1: $\hat{\boldsymbol{n}} = -\hat{\boldsymbol{j}}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{\boldsymbol{n}}) = (-B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}})\cdot(-A\hat{\boldsymbol{j}}) = -CL^2.$

face S_2: $\hat{\boldsymbol{n}} = +\hat{\boldsymbol{k}}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{\boldsymbol{n}}) = (-B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}})\cdot(A\hat{\boldsymbol{k}}) = -DL^2.$

face S_3: $\hat{\boldsymbol{n}} = +\hat{\boldsymbol{j}}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{\boldsymbol{n}}) = (-B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}})\cdot(A\hat{\boldsymbol{j}}) = +CL^2.$

face S_4: $\hat{\boldsymbol{n}} = -\hat{\boldsymbol{k}}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{\boldsymbol{n}}) = (-B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}})\cdot(-A\hat{\boldsymbol{k}}) = +DL^2.$

face S_5: $\hat{\boldsymbol{n}} = +\hat{\boldsymbol{i}}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{\boldsymbol{n}}) = (-B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}})\cdot(A\hat{\boldsymbol{i}}) = -BL^2.$

face S_6: $\hat{\boldsymbol{n}} = -\hat{\boldsymbol{i}}$

$\Phi_E = \vec{E}\cdot\vec{A} = \vec{E}\cdot(A\hat{\boldsymbol{n}}) = (-B\hat{\boldsymbol{i}} + C\hat{\boldsymbol{j}} - D\hat{\boldsymbol{k}})\cdot(-A\hat{\boldsymbol{i}}) = +BL^2.$

(b) Add the flux through each of the six faces: $\Phi_E = -CL^2 - DL^2 + CL^2 + DL^2 - BL^2 + BL^2 = 0$

The total electric flux through all sides is zero.

EVALUATE: All electric field lines that enter one face of the cube leave through another face. No electric field lines terminate inside the cube and the net flux is zero.

22.33. **IDENTIFY:** Use Eq.(22.3) to calculate the flux through each surface and use Gauss's law to relate the net flux to the enclosed charge.

SET UP: Flux into the enclosed volume is negative and flux out of the volume is positive.

EXECUTE: **(a)** $\Phi = EA = (125\ \text{N/C})(6.0\ \text{m}^2) = 750\ \text{N}\cdot\text{m}^2/\text{C}.$

(b) Since the field is parallel to the surface, $\Phi = 0.$

(c) Choose the Gaussian surface to equal the volume's surface. Then $750\ \text{N}\cdot\text{m}^2/\text{C} - EA = q/\epsilon_0$ and

$E = \dfrac{1}{6.0\ \text{m}^2}(2.40\times10^{-8}\ \text{C}/\epsilon_0 + 750\ \text{N}\cdot\text{m}^2/\text{C}) = 577\ \text{N/C}$, in the positive *x*-direction. Since $q < 0$ we must have some net flux flowing *in* so the flux is $-|E\,A|$ on second face.

EVALUATE: **(d)** $q < 0$ but we have E pointing *away* from face I. This is due to an external field that does not affect the flux but affects the value of E.

22.34. **IDENTIFY:** Apply Gauss's law to a cube centered at the origin and with side length $2L$.

SET UP: The total surface area of a cube with side length $2L$ is $6(2L)^2 = 24L^2$.

EXECUTE: **(a)** The square is sketched in Figure 22.34.

(b) Imagine a charge q at the center of a cube of edge length $2L$. Then: $\Phi = q/\epsilon_0$. Here the square is one 24th of the surface area of the imaginary cube, so it intercepts 1/24 of the flux. That is, $\Phi = q/24\epsilon_0$.

EVALUATE: Calculating the flux directly from Eq.(22.5) would involve a complicated integral. Using Gauss's law and symmetry considerations is much simpler.

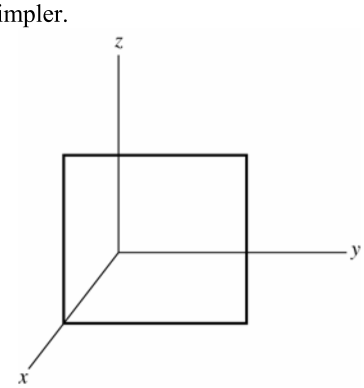


Figure 22.34

22.35. **(a) IDENTIFY:** Find the net flux through the parallelepiped surface and then use that in Gauss's law to find the net charge within. Flux out of the surface is positive and flux into the surface is negative.

SET UP: $\vec{E}_1$ gives flux out of the surface. See Figure 22.35a.

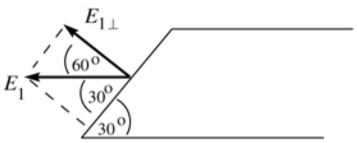


Figure 22.35a

EXECUTE: $\Phi_1 = +E_{1\perp}A$

$A = (0.0600\text{ m})(0.0500\text{ m}) = 3.00\times10^{-3}\text{ m}^2$

$E_{1\perp} = E_1\cos60° = (2.50\times10^4\text{ N/C})\cos60°$

$E_{1\perp} = 1.25\times10^4\text{ N/C}$

$\Phi_{E_1} = +E_{1\perp}A = +(1.25\times10^4\text{ N/C})(3.00\times10^{-3}\text{ m}^2) = 37.5\text{ N}\cdot\text{m}^2/\text{C}$

SET UP: $\vec{E}_2$ gives flux into the surface. See Figure 22.35b.

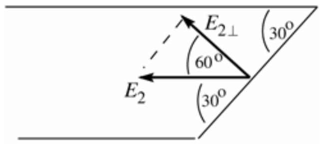


Figure 22.35b

EXECUTE: $\Phi_2 = -E_{2\perp}A$

$A = (0.0600\text{ m})(0.0500\text{ m}) = 3.00\times10^{-3}\text{m}^2$

$E_{2\perp} = E_2\cos60° = (7.00\times10^4\text{ N/C})\cos60°$

$E_{2\perp} = 3.50\times10^4\text{ N/C}$

$\Phi_{E_2} = -E_{2\perp}A = -(3.50\times10^4\text{ N/C})(3.00\times10^{-3}\text{ m}^2) = -105.0\text{ N}\cdot\text{m}^2/\text{C}$

The net flux is $\Phi_E = \Phi_{E_1} + \Phi_{E_2} = +37.5\text{ N}\cdot\text{m}^2/\text{C} - 105.0\text{ N}\cdot\text{m}^2/\text{C} = -67.5\text{ N}\cdot\text{m}^2/\text{C}$.

The net flux is negative (inward), so the net charge enclosed is negative.

Apply Gauss's law: $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$

$Q_{\text{encl}} = \Phi_E\epsilon_0 = (-67.5\text{ N}\cdot\text{m}^2/\text{C})(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2) = -5.98\times10^{-10}\text{ C}$.

(b) EVALUATE: If there were no charge within the parallelpiped the net flux would be zero. This is not the case, so there is charge inside. The electric field lines that pass out through the surface of the parallelpiped must terminate on charges, so there also must be charges outside the parallelpiped.

22.36. **IDENTIFY:** The α particle feels no force where the net electric field due to the two distributions of charge is zero.

SET UP: The fields can cancel only in the regions A and B shown in Figure 22.36, because only in these two regions are the two fields in opposite directions.

EXECUTE: $E_{\text{line}} = E_{\text{sheet}}$ gives $\dfrac{\lambda}{2\pi\epsilon_0 r} = \dfrac{\sigma}{2\epsilon_0}$ and $r = \lambda/\pi\sigma = \dfrac{50\ \mu\text{C/m}}{\pi(100\ \mu\text{C/m}^2)} = 0.16\text{ m} = 16\text{ cm}$.

The fields cancel 16 cm from the line in regions A and B.

EVALUATE: The result is independent of the distance between the line and the sheet. The electric field of an infinite sheet of charge is uniform, independent of the distance from the sheet.

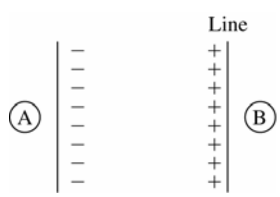


Figure 22.36

22.37. **(a) IDENTIFY:** Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $a < r < b$, and calculate E on the surface of the cylinder.

SET UP: The Gaussian surface is sketched in Figure 22.37a.

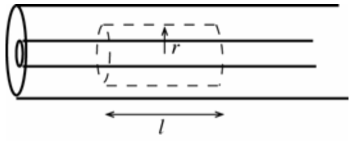


Figure 22.37a

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface).

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(2\pi rl) = \dfrac{\lambda l}{\epsilon_0}$

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(b) SET UP: Apply Gauss's law to a Gaussian cylinder of length l and radius r, where $r > c$, as shown in Figure 22.37b.

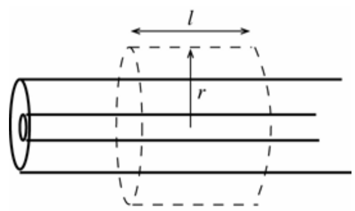


Figure 22.37b

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \lambda l$ (the charge on the length l of the inner conductor that is inside the Gaussian surface; the outer conductor carries no net charge).

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(2\pi rl) = \dfrac{\lambda l}{\epsilon_0}$

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(c) $E = 0$ within a conductor. Thus $E = 0$ for $r < a$;

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $a < r < b$; $E = 0$ for $b < r < c$;

$E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $r > c$. The graph of E versus r is sketched in Figure 22.37c.

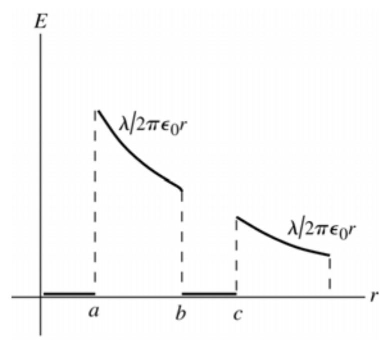


Figure 22.37c

EVALUATE: Inside either conductor $E = 0$. Between the conductors and outside both conductors the electric field is the same as for a line of charge with linear charge density λ lying along the axis of the inner conductor.

(d) IDENTIFY and **SET UP:** <u>inner surface:</u> Apply Gauss's law to a Gaussian cylinder with radius r, where $b < r < c$. We know E on this surface; calculate Q_{encl}.

EXECUTE: This surface lies within the conductor of the outer cylinder, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses charge λl on the inner conductor, so it must enclose charge $-\lambda l$ on the inner surface of the outer conductor. The charge per unit length on the inner surface of the outer cylinder is $-\lambda$.

<u>outer surface:</u> The outer cylinder carries no net charge. So if there is charge per unit length $-\lambda$ on its inner surface there must be charge per until length $+\lambda$ on the outer surface.

EVALUATE: The electric field lines between the conductors originate on the surface charge on the outer surface of the inner conductor and terminate on the surface charges on the inner surface of the outer conductor. These surface charges are equal in magnitude (per unit length) and opposite in sign. The electric field lines outside the outer conductor originate from the surface charge on the outer surface of the outer conductor.

22.38. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a cylinder of radius r, length l and that has the line of charge along its axis. The charge on a length l of the line of charge or of the tube is $q = \alpha l$.

EXECUTE: **(a)** (i) For $r < a$, Gauss's law gives $E(2\pi rl) = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{\alpha l}{\epsilon_0}$ and $E = \dfrac{\alpha}{2\pi\epsilon_0 r}$.

(ii) The electric field is zero because these points are within the conducting material.

(iii) For $r > b$, Gauss's law gives $E(2\pi rl) = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{2\alpha l}{\epsilon_0}$ and $E = \dfrac{\alpha}{\pi\epsilon_0 r}$.

The graph of E versus r is sketched in Figure 22.38.

(b) (i) The Gaussian cylinder with radius *r*, for $a < r < b$, must enclose zero net charge, so the charge per unit length on the inner surface is $-\alpha$. (ii) Since the net charge per length for the tube is $+\alpha$ and there is $-\alpha$ on the inner surface, the charge per unit length on the outer surface must be $+2\alpha$.

EVALUATE: For $r > b$ the electric field is due to the charge on the outer surface of the tube.

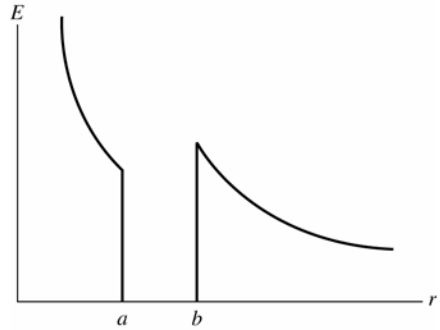


Figure 22.38

22.39. (a) IDENTIFY: Use Gauss's law to calculate $E(r)$.

(i) **SET UP:** <u>$r < a$:</u> Apply Gauss's law to a cylindrical Gaussian surface of length *l* and radius *r*, where $r < a$, as sketched in Figure 22.39a.

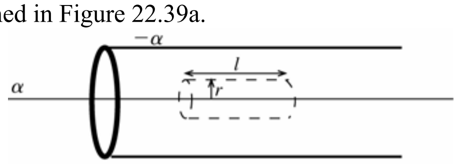


Figure 22.39a

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \alpha l$ (the charge on the length *l* of the line of charge)

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(2\pi rl) = \dfrac{\alpha l}{\epsilon_0}$

$E = \dfrac{\alpha}{2\pi\epsilon_0 r}$. The enclosed charge is positive so the direction of $\vec{E}$ is radially outward.

(ii) <u>$a < r < b$:</u> Points in this region are within the conducting tube, so $E = 0$.

(iii) **SET UP:** <u>$r > b$:</u> Apply Gauss's law to a cylindrical Gaussian surface of length *l* and radius *r*, where $r > b$, as sketched in Figure 22.39b.

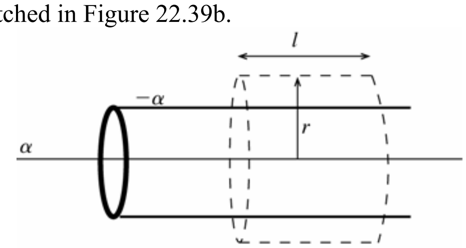


Figure 22.39b

EXECUTE: $\Phi_E = E(2\pi rl)$

$Q_{\text{encl}} = \alpha l$ (the charge on length *l* of the line of charge) $-\alpha l$ (the charge on length *l* of the tube) Thus $Q_{\text{encl}} = 0$.

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(2\pi rl) = 0$ and $E = 0$. The graph of *E* versus *r* is sketched in Figure 22.39c.

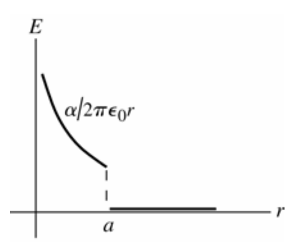


Figure 22.39c

(b) IDENTIFY: Apply Gauss's law to cylindrical surfaces that lie just outside the inner and outer surfaces of the tube. We know *E* so can calculate Q_{encl}.

(i) **SET UP:** <u>inner surface</u>

Apply Gauss's law to a cylindrical Gaussian surface of length *l* and radius *r*, where $a < r < b$.

EXECUTE: This surface lies within the conductor of the tube, where $E = 0$, so $\Phi_E = 0$. Then by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses charge αl on the line of charge so must enclose charge $-\alpha l$ on the inner surface of the tube. The charge per unit length on the inner surface of the tube is $-\alpha$.

(ii) outer surface

The net charge per unit length on the tube is $-\alpha$. We have shown in part (i) that this must all reside on the inner surface, so there is no net charge on the outer surface of the tube.

EVALUATE: For $r < a$ the electric field is due only to the line of charge. For $r > b$ the electric field of the tube is the same as for a line of charge along its axis. The fields of the line of charge and of the tube are equal in magnitude and opposite in direction and sum to zero. For $r < a$ the electric field lines originate on the line of charge and terminate on the surface charge on the inner surface of the tube. There is no electric field outside the tube and no surface charge on the outer surface of the tube.

22.40. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a cylinder of radius r and length l, and that is coaxial with the cylindrical charge distributions. The volume of the Gaussian cylinder is $\pi r^2 l$ and the area of its curved surface is $2\pi rl$. The charge on a length l of the charge distribution is $q = \lambda l$, where $\lambda = \rho\pi R^2$.

EXECUTE: **(a)** For $r < R$, $Q_{\text{encl}} = \rho\pi r^2 l$ and Gauss's law gives $E(2\pi rl) = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{\rho\pi r^2 l}{\epsilon_0}$ and $E = \dfrac{\rho r}{2\epsilon_0}$, radially outward.

(b) For $r > R$, $Q_{\text{encl}} = \lambda l = \rho\pi R^2 l$ and Gauss's law gives $E(2\pi rl) = \dfrac{q}{\epsilon_0} = \dfrac{\rho\pi R^2 l}{\epsilon_0}$ and $E = \dfrac{\rho R^2}{2\epsilon_0 r} = \dfrac{\lambda}{2\pi\epsilon_0 r}$, radially outward.

(c) At $r = R$, the electric field for BOTH regions is $E = \dfrac{\rho R}{2\epsilon_0}$, so they are consistent.

(d) The graph of E versus r is sketched in Figure 22.40.

EVALUATE: For $r > R$ the field is the same as for a line of charge along the axis of the cylinder.

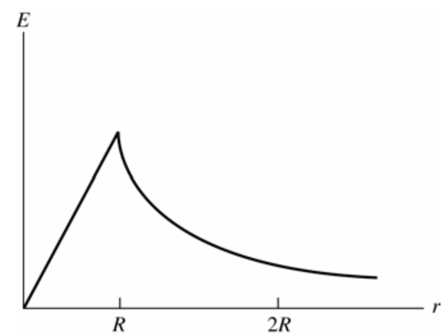


Figure 22.40

22.41. **IDENTIFY:** First make a free-body diagram of the sphere. The electric force acts to the left on it since the electric field due to the sheet is horizontal. Since it hangs at rest, the sphere is in equilibrium so the forces on it add to zero, by Newton's first law. Balance horizontal and vertical force components separately.

SET UP: Call T the tension in the thread and E the electric field. Balancing horizontal forces gives $T \sin \theta = qE$. Balancing vertical forces we get $T \cos \theta = mg$. Combining these equations gives $\tan \theta = qE/mg$, which means that $\theta = \arctan(qE/mg)$. The electric field for a sheet of charge is $E = \sigma 2\epsilon_0$.

EXECUTE: Substituting the numbers gives us $E = \dfrac{\sigma}{2\epsilon_0} = \dfrac{2.50\times10^{-7}\ \text{C/m}^2}{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = 1.41\times10^4\ \text{N/C}$. Then

$$\theta = \arctan\left[\frac{\left(5.00\times10^{-8}\ \text{C}\right)\left(1.41\times10^4\ \text{N/C}\right)}{\left(2.00\times10^{-2}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}\right] = 19.8°$$

EVALUATE: Increasing the field, or decreasing the mass of the sphere, would cause the sphere to hang at a larger angle.

22.42. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the conducting spheres.

EXECUTE: **(a)** For $r < a$, $E = 0$, since these points are within the conducting material.

For $a < r < b$, $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$, since there is $+q$ inside a radius r.

For $b < r < c$, $E = 0$, since since these points are within the conducting material

For $r > c$, $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$, since again the total charge enclosed is $+q$.

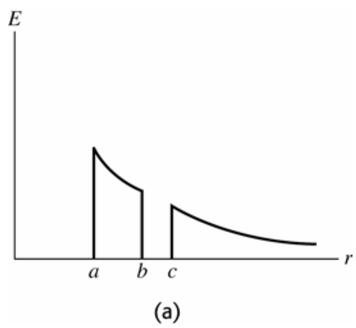
(b) The graph of E versus r is sketched in Figure 22.42a.

(c) Since the Gaussian sphere of radius r, for $b < r < c$, must enclose zero net charge, the charge on inner shell surface is $-q$.

(d) Since the hollow sphere has no net charge and has charge $-q$ on its inner surface, the charge on outer shell surface is $+q$.

(e) The field lines are sketched in Figure 22.42b. Where the field is nonzero, it is radially outward.

EVALUATE: The net charge on the inner solid conducting sphere is on the surface of that sphere. The presence of the hollow sphere does not affect the electric field in the region $r < b$.



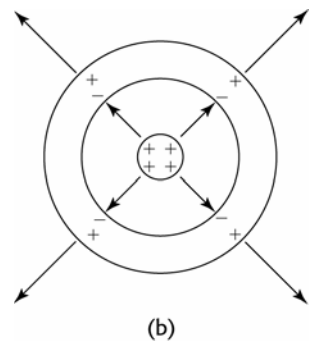


Figure 22.42

22.43. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the charge distributions.

EXECUTE: **(a)** For $r < R$, $E = 0$, since these points are within the conducting material. For $R < r < 2R$, $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$, since the charge enclosed is Q. For $r > 2R$, $E = \frac{1}{4\pi\epsilon_0}\frac{2Q}{r^2}$ since the charge enclosed is $2Q$.

(b) The graph of E versus r is sketched in Figure 22.43.

EVALUATE: For $r < 2R$ the electric field is unaffected by the presence of the charged shell.

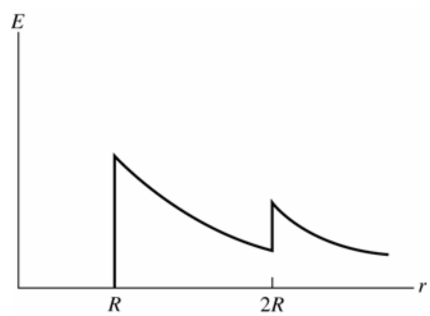


Figure 22.43

22.44. **IDENTIFY:** Apply Gauss's law and conservation of charge.

SET UP: Use a Gaussian surface that is a sphere of radius r and that has the point charge at its center.

EXECUTE: **(a)** For $r < a$, $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$, radially outward, since the charge enclosed is Q, the charge of the point charge. For $a < r < b$, $E = 0$ since these points are within the conducting material. For $r > b$, $E = \frac{1}{4\pi\epsilon_0}\frac{2Q}{r^2}$, radially inward, since the total enclosed charge is $-2Q$.

(b) Since a Gaussian surface with radius r, for $a < r < b$, must enclose zero net charge, the total charge on the inner surface is $-Q$ and the surface charge density on inner surface is $\sigma = -\dfrac{Q}{4\pi a^2}$.

(c) Since the net charge on the shell is $-3Q$ and there is $-Q$ on the inner surface, there must be $-2Q$ on the outer surface. The surface charge density on the outer surface is $\sigma = -\dfrac{2Q}{4\pi b^2}$.

(d) The field lines and the locations of the charges are sketched in Figure 22.44a.

(e) The graph of E versus r is sketched in Figure 22.44b.

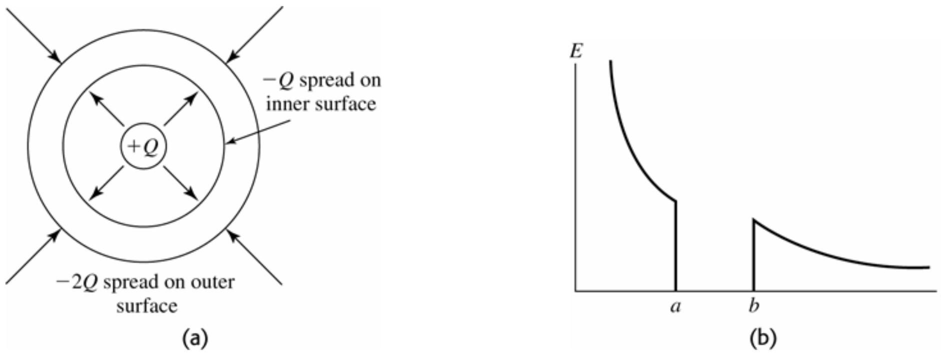


Figure 22.44

EVALUATE: For $r < a$ the electric field is due solely to the point charge Q. For $r > b$ the electric field is due to the charge $-2Q$ that is on the outer surface of the shell.

22.45. **IDENTIFY:** Apply Gauss's law to a spherical Gaussian surface with radius r. Calculate the electric field at the surface of the Gaussian sphere.

(a) SET UP: (i) $r < a$: The Gaussian surface is sketched in Figure 22.45a.



Figure 22.45a

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

$Q_{\text{encl}} = 0$; no charge is enclosed

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ says $E(4\pi r^2) = 0$ and $E = 0$.

(ii) $a < r < b$: Points in this region are in the conductor of the small shell, so $E = 0$.

(iii) **SET UP:** $b < r < c$: The Gaussian surface is sketched in Figure 22.45b.

Apply Gauss's law to a spherical Gaussian surface with radius $b < r < c$.

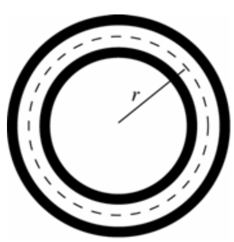


Figure 22.45b

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

The Gaussian surface encloses all of the small shell and none of the large shell, so $Q_{\text{encl}} = +2q$.

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{2q}{\epsilon_0}$ so $E = \dfrac{2q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

(iv) $c < r < d$: Points in this region are in the conductor of the large shell, so $E = 0$.

(v) **SET UP:** $r > d$: Apply Gauss's law to a spherical Gaussian surface with radius $r > d$, as shown in Figure 22.45c.

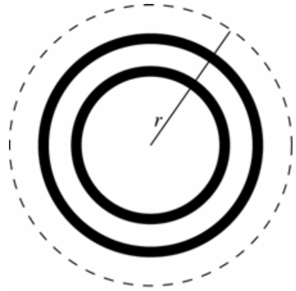


Figure 22.45c

EXECUTE: $\Phi_E = EA = E(4\pi r^2)$

The Gaussian surface encloses all of the small shell and all of the large shell, so $Q_{\text{encl}} = +2q + 4q = 6q$.

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{6q}{\epsilon_0}$

$E = \dfrac{6q}{4\pi\epsilon_0 r^2}$. Since the enclosed charge is positive the electric field is radially outward.

The graph of E versus r is sketched in Figure 22.45d.

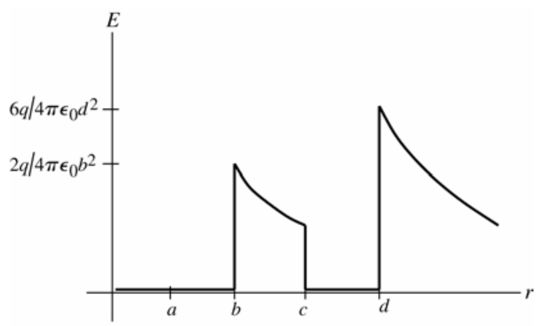


Figure 22.45d

(b) IDENTIFY and **SET UP:** Apply Gauss's law to a sphere that lies outside the surface of the shell for which we want to find the surface charge.

EXECUTE: (i) charge on inner surface of the small shell: Apply Gauss's law to a spherical Gaussian surface with radius $a < r < b$. This surface lies within the conductor of the small shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$, so there is zero charge on the inner surface of the small shell.

(ii) charge on outer surface of the small shell: The total charge on the small shell is $+2q$. We found in part (i) that there is zero charge on the inner surface of the shell, so all $+2q$ must reside on the outer surface.

(iii) charge on inner surface of large shell: Apply Gauss's law to a spherical Gaussian surface with radius $c < r < d$. The surface lies within the conductor of the large shell, where $E = 0$, so $\Phi_E = 0$. Thus by Gauss's law $Q_{\text{encl}} = 0$. The surface encloses the $+2q$ on the small shell so there must be charge $-2q$ on the inner surface of the large shell to make the total enclosed charge zero.

(iv) charge on outer surface of large shell: The total charge on the large shell is $+4q$. We showed in part (iii) that the charge on the inner surface is $-2q$, so there must be $+6q$ on the outer surface.

EVALUATE: The electric field lines for $b < r < c$ originate from the surface charge on the outer surface of the inner shell and all terminate on the surface charge on the inner surface of the outer shell. These surface charges have equal magnitude and opposite sign. The electric field lines for $r > d$ originate from the surface charge on the outer surface of the outer sphere.

22.46. IDENTIFY: Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r and that is concentric with the charged shells.

EXECUTE: **(a)** (i) For $r < a$, $E = 0$, since the charge enclosed is zero. (ii) For $a < r < b, E = 0$, since the points are within the conducting material. (iii) For $b < r < c, E = \dfrac{1}{4\pi\epsilon_0}\dfrac{2q}{r^2}$, outward, since charge enclosed is $+2q$.

(iv) For $c < r < d$, $E = 0$, since the points are within the conducting material. (v) For $r > d$, $E = 0$, since the net charge enclosed is zero. The graph of E versus r is sketched in Figure 22.46.

(b) (i) small shell inner surface: Since a Gaussian surface with radius *r*, for $a < r < b$, must enclose zero net charge, the charge on this surface is zero. (ii) small shell outer surface: $+2q$. (iii) large shell inner surface: Since a Gaussian surface with radius *r*, for $c < r < d$, must enclose zero net charge, the charge on this surface is $-2q$. (iv) large shell outer surface: Since there is $-2q$ on the inner surface and the total charge on this conductor is $-2q$, the charge on this surface is zero.

EVALUATE: The outer shell has no effect on the electric field for $r < c$. For $r > d$ the electric field is due only to the charge on the outer surface of the larger shell.

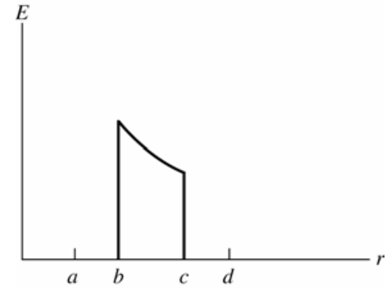


Figure 22.46

22.47. IDENTIFY: Apply Gauss's law

SET UP: Use a Gaussian surface that is a sphere of radius *r* and that is concentric with the charged shells.

EXECUTE: **(a)** (i) For $r < a$, $E = 0$, since charge enclosed is zero. (ii) $a < r < b$, $E = 0$, since the points are within the conducting material. (iii) For $b < r < c$, $E = \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}$, outward, since charge enclosed is $+2q$.

(iv) For $c < r < d$, $E = 0$, since the points are within the conducting material. (v) For $r > d$, $E = \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}$, inward, since charge enclosed is $-2q$. The graph of the radial component of the electric field versus *r* is sketched in Figure 22.47, where we use the convention that outward field is positive and inward field is negative.

(b) (i) small shell inner surface: Since a Gaussian surface with radius *r*, for $a < r < b$, must enclose zero net charge, the charge on this surface is zero. (ii) small shell outer surface: $+2q$. (iii) large shell inner surface: Since a Gaussian surface with radius *r*, for $c < r < d$, must enclose zero net charge, the charge on this surface is $-2q$. (iv) large shell outer surface: Since there is $-2q$ on the inner surface and the total charge on this conductor is $-4q$, the charge on this surface is $-2q$.

EVALUATE: The outer shell has no effect on the electric field for $r < c$. For $r > d$ the electric field is due only to the charge on the outer surface of the larger shell.

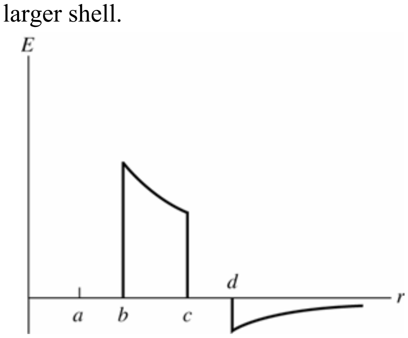


Figure 22.47

22.48. IDENTIFY: Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius *r* and that is concentric with the sphere and shell. The volume of the insulating shell is $V = \frac{4}{3}\pi\left([2R]^3 - R^3\right) = \frac{28\pi}{3}R^3$.

EXECUTE: **(a)** Zero net charge requires that $-Q = \frac{28\pi\rho R^3}{3}$, so $\rho = -\frac{3Q}{28\pi R^3}$.

(b) For $r < R$, $E = 0$ since this region is within the conducting sphere. For $r > 2R$, $E = 0$, since the net charge enclosed by the Gaussian surface with this radius is zero. For $R < r < 2R$, Gauss's law gives $E(4\pi r^2) = \frac{Q}{\epsilon_0} + \frac{4\pi\rho}{3\epsilon_0}(r^3 - R^3)$ and $E = \frac{Q}{4\pi\epsilon_0 r^2} + \frac{\rho}{3\epsilon_0 r^2}(r^3 - R^3)$. Substituting ρ from part (a) gives $E = \frac{2}{7\pi\epsilon_0}\frac{Q}{r^2} - \frac{Qr}{28\pi\epsilon_0 R^3}$. The net enclosed charge for each *r* in this range is positive and the electric field is outward.

(c) The graph is sketched in Figure 22.48. We see a discontinuity in going from the conducting sphere to the insulator due to the thin surface charge of the conducting sphere. But we see a smooth transition from the uniform insulator to the surrounding space.

EVALUATE: The expression for E within the insulator gives $E = 0$ at $r = 2R$.

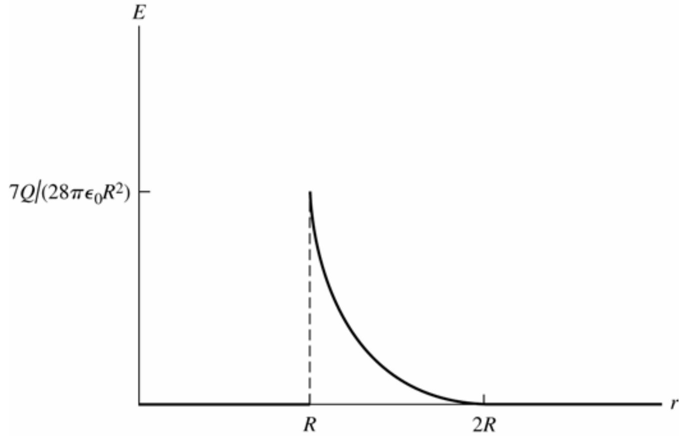


Figure 22.48

22.49. **IDENTIFY:** Use Gauss's law to find the electric field $\vec{E}$ produced by the shell for $r < R$ and $r > R$ and then use $\vec{F} = q\vec{E}$ to find the force the shell exerts on the point charge.

(a) SET UP: Apply Gauss's law to a spherical Gaussian surface that has radius $r > R$ and that is concentric with the shell, as sketched in Figure 22.49a.

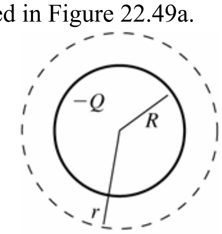


Figure 22.49a

EXECUTE: $\Phi_E = E(4\pi r^2)$

$Q_{\text{encl}} = -Q$

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = \dfrac{-Q}{\epsilon_0}$

The magnitude of the field is $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$ and it is directed toward the center of the shell. Then $F = qE = \dfrac{qQ}{4\pi\epsilon_0 r^2}$, directed toward the center of the shell. (Since q is positive, $\vec{E}$ and $\vec{F}$ are in the same direction.)

(b) SET UP: Apply Gauss's law to a spherical Gaussian surface that has radius $r < R$ and that is concentric with the shell, as sketched in Figure 22.49b.

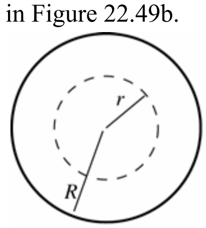


Figure 22.49b

EXECUTE: $\Phi_E = E(4\pi r^2)$

$Q_{\text{encl}} = 0$

$\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $E(4\pi r^2) = 0$

Then $E = 0$ so $F = 0$.

EVALUATE: Outside the shell the electric field and the force it exerts is the same as for a point charge $-Q$ located at the center of the shell. Inside the shell $E = 0$ and there is no force.

22.50. **IDENTIFY:** The method of Example 22.9 shows that the electric field outside the sphere is the same as for a point charge of the same charge located at the center of the sphere.

SET UP: The charge of an electron has magnitude $e = 1.60 \times 10^{-19}\,\text{C}$.

EXECUTE: **(a)** $E = k\dfrac{|q|}{r^2}$. For $r = R = 0.150\text{ m}$, $E = 1150\text{ N/C}$ so $|q| = \dfrac{Er^2}{k} = \dfrac{(1150\text{ N/C})(0.150\text{ m})^2}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2} = 2.88\times10^{-9}\text{ C}$.

The number of excess electrons is $\dfrac{2.88\times10^{-9}\text{C}}{1.60\times10^{-19}\text{C/electron}} = 1.80\times10^{10}\text{ electrons}$.

(b) $r = R + 0.100\text{ m} = 0.250\text{ m}$. $E = k\dfrac{|q|}{r^2} = (8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{2.88\times10^{-9}\text{ C}}{(0.250\text{ m})^2} = 414\text{ N/C}$.

EVALUATE: The magnitude of the electric field decreases according to the square of the distance from the center of the sphere.

22.51. **IDENTIFY:** The net electric field is the vector sum of the fields due to the sheet of charge on each surface of the plate.

SET UP: The electric field due to the sheet of charge on each surface is $E = \sigma/2\epsilon_0$ and is directed away from the surface.

EXECUTE: **(a)** For the conductor the charge sheet on each surface produces fields of magnitude $\sigma/2\epsilon_0$ and in the same direction, so the total field is twice this, or σ/ϵ_0.

(b) At points inside the plate the fields of the sheets of charge on each surface are equal in magnitude and opposite in direction, so their vector sum is zero. At points outside the plate, on either side, the fields of the two sheets of charge are in the same direction so their magnitudes add, giving $E = \sigma/\epsilon_0$.

EVALUATE: Gauss's law can also be used directly to determine the fields in these regions.

22.52. **IDENTIFY:** Example 22.9 gives the expression for the electric field both inside and outside a uniformly charged sphere. Use $\vec{F} = -e\vec{E}$ to calculate the force on the electron.

SET UP: The sphere has charge $Q = +e$.

EXECUTE: **(a)** Only at $r = 0$ is $E = 0$ for the uniformly charged sphere.

(b) At points inside the sphere, $E_r = \dfrac{er}{4\pi\epsilon_0 R^3}$. The field is radially outward. $F_r = -eE = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2 r}{R^3}$. The minus sign denotes that F_r is radially inward. For simple harmonic motion, $F_r = -kr = -m\omega^2 r$, where $\omega = \sqrt{k/m} = 2\pi f$.

$F_r = -m\omega^2 r = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2 r}{R^3}$ so $\omega = \sqrt{\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{mR^3}}$ and $f = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{mR^3}}$.

(c) If $f = 4.57\times10^{14}\text{ Hz} = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{mR^3}}$ then $R = \sqrt[3]{\dfrac{1}{4\pi\epsilon_0}\dfrac{(1.60\times10^{-19}\text{ C})^2}{4\pi^2(9.11\times10^{-31}\text{ kg})(4.57\times10^{14}\text{ Hz})^2}} = 3.13\times10^{-10}\text{ m}$.

The atom radius in this model is the correct order of magnitude.

(d) If $r > R$, $E_r = \dfrac{e}{4\pi\epsilon_0 r^2}$ and $F_r = -\dfrac{e^2}{4\pi\epsilon_0 r^2}$. The electron would still oscillate because the force is directed toward the equilibrium position at $r = 0$. But the motion would not be simple harmonic, since F_r is proportional to $1/r^2$ and simple harmonic motion requires that the restoring force be proportional to the displacement from equilibrium.

EVALUATE: As long as the initial displacement is less than R the frequency of the motion is independent of the initial displacement.

22.53. **IDENTIFY:** There is a force on each electron due to the other electron and a force due to the sphere of charge. Use Coulomb's law for the force between the electrons. Example 22.9 gives E inside a uniform sphere and Eq.(21.3) gives the force.

SET UP: The positions of the electrons are sketched in Figure 22.53a.

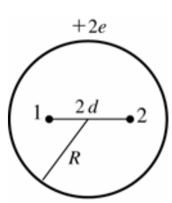


Figure 22.53a

If the electrons are in equilibrium the net force on each one is zero.

EXECUTE: Consider the forces on electron 2. There is a repulsive force F_1 due to the other electron, electron 1.

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{(2d)^2}$$

The electric field inside the uniform distribution of positive charge is $E = \frac{Qr}{4\pi\epsilon_0 R^3}$ (Example 22.9), where $Q = +2e$.

At the position of electron 2, $r = d$. The force F_{cd} exerted by the positive charge distribution is $F_{cd} = eE = \frac{e(2e)d}{4\pi\epsilon_0 R^3}$ and is attractive.

The force diagram for electron 2 is given in Figure 22.53b.

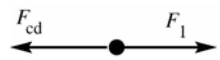


Figure 22.53b

Net force equals zero implies $F_1 = F_{cd}$ and $\frac{1}{4\pi\epsilon_0}\frac{e^2}{4d^2} = \frac{2e^2 d}{4\pi\epsilon_0 R^3}$

Thus $(1/4d^2) = 2d/R^3$, so $d^3 = R^3/8$ and $d = R/2$.

EVALUATE: The electric field of the sphere is radially outward; it is zero at the center of the sphere and increases with distance from the center. The force this field exerts on one of the electrons is radially inward and increases as the electron is farther from the center. The force from the other electron is radially outward, is infinite when $d = 0$ and decreases as d increases. It is reasonable therefore for there to be a value of d for which these forces balance.

22.54. IDENTIFY: Use Gauss's law to find the electric field both inside and outside the slab.

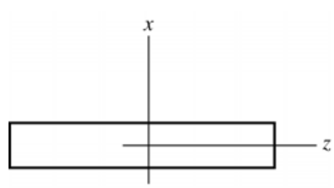
SET UP: Use a Gaussian surface that has one face of area A in the yz plane at $x = 0$, and the other face at a general value x. The volume enclosed by such a Gaussian surface is Ax.

EXECUTE: **(a)** The electric field of the slab must be zero by symmetry. There is no preferred direction in the yz plane, so the electric field can only point in the x-direction. But at the origin, neither the positive nor negative x-directions should be singled out as special, and so the field must be zero.

(b) For $|x| \le d$, Gauss's law gives $EA = \frac{Q_{encl}}{\epsilon_0} = \frac{\rho A|x|}{\epsilon_0}$ and $E = \frac{\rho|x|}{\epsilon_0}$, with direction given by $\frac{x}{|x|}\hat{\boldsymbol{i}}$ (away from the center of the slab). Note that this expression does give $E = 0$ at $x = 0$. Outside the slab, the enclosed charge does not depend on x and is equal to ρAd. For $|x| \ge d$, Gauss's law gives $EA = \frac{Q_{encl}}{\epsilon_0} = \frac{\rho Ad}{\epsilon_0}$ and $E = \frac{\rho d}{\epsilon_0}$, again with direction given by $\frac{x}{|x|}\hat{\boldsymbol{i}}$.

EVALUATE: At the surfaces of the slab, $x = \pm d$. For these values of x the two expressions for E (for inside and outside the slab) give the same result. The charge per unit area σ of the slab is given by $\sigma A = \rho A(2d)$ and $\rho d = \sigma/2$. The result for E outside the slab can therefore be written as $E = \sigma/2\epsilon_0$ and is the same as for a thin sheet of charge.

22.55. (a) IDENTIFY and SET UP: Consider the direction of the field for x slightly greater than and slightly less than zero. The slab is sketched in Figure 22.55a.



$$\rho(x) = \rho_0 (x/d)^2$$

Figure 22.55a

EXECUTE: The charge distribution is symmetric about $x = 0$, so by symmetry $E(x) = E(-x)$. But for $x > 0$ the field is in the $+x$ direction and for $x < 0$ the field is in the $-x$ direction. At $x = 0$ the field can't be both in the $+x$ and $-x$ directions so must be zero. That is, $E_x(x) = -E_x(-x)$. At point $x = 0$ this gives $E_x(0) = -E_x(0)$ and this equation is satisfied only for $E_x(0) = 0$.

(b) IDENTIFY and SET UP: $|x| > d$ (outside the slab)

Apply Gauss's law to a cylindrical Gaussian surface whose axis is perpendicular to the slab and whose end caps have area A and are the same distance $|x| > d$ from $x = 0$, as shown in Figure 22.55b.

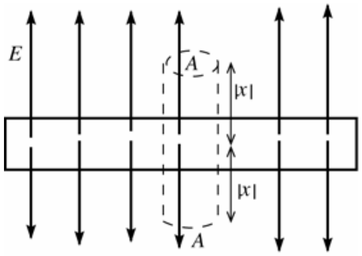


Figure 22.55b

EXECUTE: $\Phi_E = 2EA$

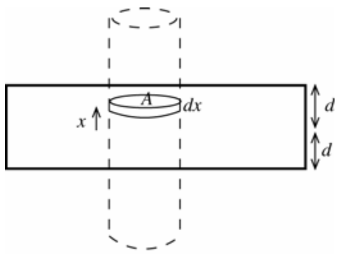


Figure 22.55c

To find Q_{encl} consider a thin disk at coordinate x and with thickness dx, as shown in Figure 22.55c. The charge within this disk is $dq = \rho dV = \rho A dx = \left(\rho_0 A/d^2\right)x^2 dx.$

The total charge enclosed by the Gaussian cylinder is

$$Q_{\text{encl}} = 2\int_0^d dq = \left(2\rho_0 A/d^2\right)\int_0^d x^2 dx = \left(2\rho_0 A/d^2\right)\left(d^3/3\right) = \tfrac{2}{3}\rho_0 A d.$$

Then $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $2EA = 2\rho_0 A d/3\epsilon_0$.

$$E = \rho_0 d/3\epsilon_0$$

$\vec{E}$ is directed away from $x = 0$, so $\vec{E} = \left(\rho_0 d/3\epsilon_0\right)\left(x/|x|\right)\hat{\boldsymbol{i}}$.

IDENTIFY and SET UP: $|x| < d$ (inside the slab)

Apply Gauss's law to a cylindrical Gaussian surface whose axis is perpendicular to the slab and whose end caps have area A and are the same distance $|x| < d$ from $x = 0$, as shown in Figure 22.55d.

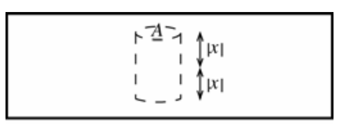


Figure 22.55d

EXECUTE: $\Phi_E = 2EA$

Q_{encl} is found as above, but now the integral on dx is only from 0 to x instead of 0 to d.

$$Q_{\text{encl}} = 2\int_0^x dq = \left(2\rho_0 A/d^2\right)\int_0^x x^2 dx = \left(2\rho_0 A/d^2\right)\left(x^3/3\right).$$

Then $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$ gives $2EA = 2\rho_0 A x^3/3\epsilon_0 d^2$.

$$E = \rho_0 x^3/3\epsilon_0 d^2$$

$\vec{E}$ is directed away from $x = 0$, so $\vec{E} = \left(\rho_0 x^3/3\epsilon_0 d^2\right)\hat{\boldsymbol{i}}$.

EVALUATE: Note that $E = 0$ at $x = 0$ as stated in part (a). Note also that the expressions for $|x| > d$ and $|x| < d$ agree for $x = d$.

22.56. **IDENTIFY:** Apply $\vec{F} = q\vec{E}$ to relate the force on q to the electric field at the location of q.

SET UP: Flux is negative if the electric field is directed into the enclosed volume.

EXECUTE: **(a)** We could place two charges $+Q$ on either side of the charge $+q$, as shown in Figure 22.56.

(b) In order for the charge to be stable, the electric field in a neighborhood around it must always point back to the equilibrium position.

(c) If q is moved to infinity and we require there to be an electric field always pointing in to the region where q had been, we could draw a small Gaussian surface there. We would find that we need a negative flux into the surface. That is, there has to be a negative charge in that region. However, there is none, and so we cannot get such a stable equilibrium.

(d) For a negative charge to be in stable equilibrium, we need the electric field to always point away from the charge position. The argument in (c) carries through again, this time implying that a positive charge must be in the space where the negative charge was if stable equilibrium is to be attained.

EVALUATE: If q is displaced to the left or right in Figure 22.56, the net force is directed back toward the equilibrium position. But if q is displaced slightly in a direction perpendicular to the line connecting the two charges Q, then the net force on q is directed away from the equilibrium position and the equilibrium is not stable for such a displacement.

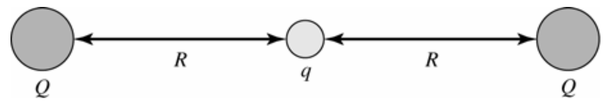


Figure 22.56

22.57. $\rho(r)=\rho_0(1-r/R)$ for $r \le R$ where $\rho_0 = 3Q/\pi R^3$. $\rho(r)=0$ for $r \ge R$

(a) IDENTIFY: The charge density varies with r inside the spherical volume. Divide the volume up into thin concentric shells, of radius r and thickness dr. Find the charge dq in each shell and integrate to find the total charge.

SET UP: The thin shell is sketched in Figure 22.57a.

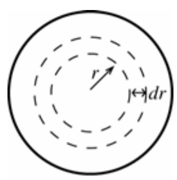


Figure 22.57a

EXECUTE: The volume of such a shell is $dV = 4\pi r^2 dr$

The charge contained within the shell is

$dq = \rho(r)dV = 4\pi r^2 \rho_0 (1-r/R)dr$

The total charge Q in the charge distribution is obtained by integrating dq over all such shells into which the sphere can be subdivided:

$$Q=\int dq=\int_0^R 4\pi r^2\rho_0(1-r/R)dr = 4\pi\rho_0\int_0^R (r^2 - r^3/R)dr$$

$Q = 4\pi\rho_0\left[\dfrac{r^3}{3}-\dfrac{r^4}{4R}\right]_0^R = 4\pi\rho_0\left(\dfrac{R^3}{3}-\dfrac{R^4}{4R}\right) = 4\pi\rho_0(R^3/12) = 4\pi(3Q/\pi R^3)(R^3/12) = Q,$ as was to be shown.

(b) IDENTIFY: Apply Gauss's law to a spherical surface of radius r, where $r > R$.

SET UP: The Gaussian surface is shown in Figure 22.57b.

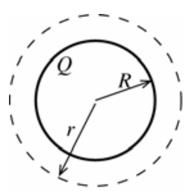


Figure 22.57b

EXECUTE: $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$

$E(4\pi r^2) = \dfrac{Q}{\epsilon_0}$

$E = \dfrac{Q}{4\pi\epsilon_0 r^2}$; same as for point charge of charge Q.

(c) IDENTIFY: Apply Gauss's law to a spherical surface of radius r, where $r < R$:

SET UP: The Gaussian surface is shown in Figure 22.57c.

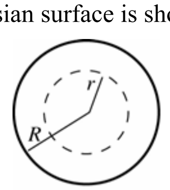


Figure 22.57c

EXECUTE: $\Phi_E = \dfrac{Q_{\text{encl}}}{\epsilon_0}$

$\Phi_E = E(4\pi r^2)$

The calculate the enclosed charge Q_{encl} use the same technique as in part (a), except integrate *dq* out to *r* rather than *R*. (We want the charge that is inside radius *r*.)

$$Q_{\text{encl}} = \int_0^r 4\pi r'^2 \rho_0 \left(1 - \frac{r'}{R}\right) dr' = 4\pi\rho_0 \int_0^r \left(r'^2 - \frac{r'^3}{R}\right) dr'$$

$$Q_{\text{encl}} = 4\pi\rho_0 \left[\frac{r'^3}{3} - \frac{r'^4}{4R}\right]_0^r = 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R}\right) = 4\pi\rho_0 r^3 \left(\frac{1}{3} - \frac{r}{4R}\right)$$

$\rho_0 = \dfrac{3Q}{\pi R^3}$ so $Q_{\text{encl}} = 12Q\dfrac{r^3}{R^3}\left(\dfrac{1}{3} - \dfrac{r}{4R}\right) = Q\left(\dfrac{r^3}{R^3}\right)\left(4 - 3\dfrac{r}{R}\right)$.

Thus Gauss's law gives $E\left(4\pi r^2\right) = \dfrac{Q}{\epsilon_0}\left(\dfrac{r^3}{R^3}\right)\left(4 - 3\dfrac{r}{R}\right)$

$$E = \frac{Qr}{4\pi\epsilon_0 R^3}\left(4 - \frac{3r}{R}\right),\ r \le R$$

(d) The graph of *E* versus *r* is sketched in Figure 22.57d.

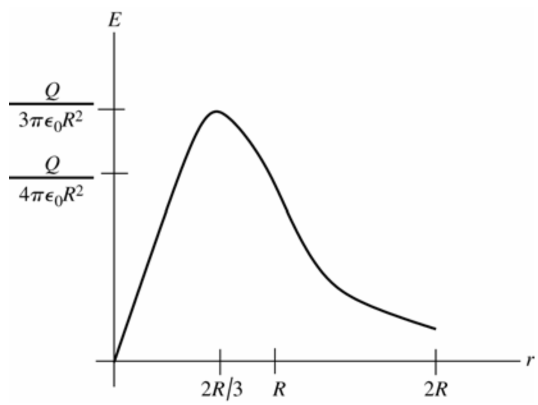


Figure 22.57d

(e) Where the electric field is a maximum, $\dfrac{dE}{dr} = 0$. Thus $\dfrac{d}{dr}\left(4r - \dfrac{3r^2}{R}\right) = 0$ so $4 - 6r/R = 0$ and $r = 2R/3$.

At this value of *r*, $E = \dfrac{Q}{4\pi\epsilon_0 R^3}\left(\dfrac{2R}{3}\right)\left(4 - \dfrac{3}{R}\dfrac{2R}{3}\right) = \dfrac{Q}{3\pi\epsilon_0 R^2}$

EVALUATE: Our expressions for $E(r)$ for $r < R$ and for $r > R$ agree at $r = R$. The results of part (e) for the value of *r* where $E(r)$ is a maximum agrees with the graph in part (d).

22.58. **IDENTIFY:** Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius *r* and that is concentric with the spherical distribution of charge. The volume of a thin spherical shell of radius *r* and thickness *dr* is $dV = 4\pi r^2 dr$.

EXECUTE: **(a)** $Q = \int \rho(r) dV = 4\pi \int_0^\infty \rho(r) r^2 dr = 4\pi\rho_0 \int_0^R \left(1 - \dfrac{4r}{3R}\right) r^2 dr = 4\pi\rho_0 \left[\int_0^R r^2 dr - \dfrac{4}{3R}\int_0^R r^3 dr\right]$

$Q = 4\pi\rho_0 \left[\dfrac{R^3}{3} - \dfrac{4}{3R}\dfrac{R^4}{4}\right] = 0$. The total charge is zero.

(b) For $r \ge R$, $\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\epsilon_0} = 0$, so $E = 0$.

(c) For $r \le R$, $\oint \vec{E} \cdot d\vec{A} = \dfrac{Q_{\text{encl}}}{\epsilon_0} = \dfrac{4\pi}{\epsilon_0}\int_0^r \rho(r') r'^2 dr'$. $E 4\pi r^2 = \dfrac{4\pi\rho_0}{\epsilon_0}\left[\int_0^r r'^2 dr' - \dfrac{4}{3R}\int_0^r r'^3 dr'\right]$ and

$$E = \frac{\rho_0}{\epsilon_0}\frac{1}{r^2}\left[\frac{r^3}{3} - \frac{r^4}{3R}\right] = \frac{\rho_0}{3\epsilon_0} r \left[1 - \frac{r}{R}\right].$$

(d) The graph of *E* versus *r* is sketched in Figure 22.58.

(e) Where *E* is a maximum, $\dfrac{dE}{dr} = 0$. This gives $\dfrac{\rho_0}{3\epsilon_0} - \dfrac{2\rho_0 r_{\max}}{3\epsilon_0 R} = 0$ and $r_{\max} = \dfrac{R}{2}$. At this *r*, $E = \dfrac{\rho_0}{3\epsilon_0}\dfrac{R}{2}\left[1 - \dfrac{1}{2}\right] = \dfrac{\rho_0 R}{12\epsilon_0}$.

EVALUATE: The result in part (b) for $r \le R$ gives $E = 0$ at $r = R$; the field is continuous at the surface of the charge distribution.

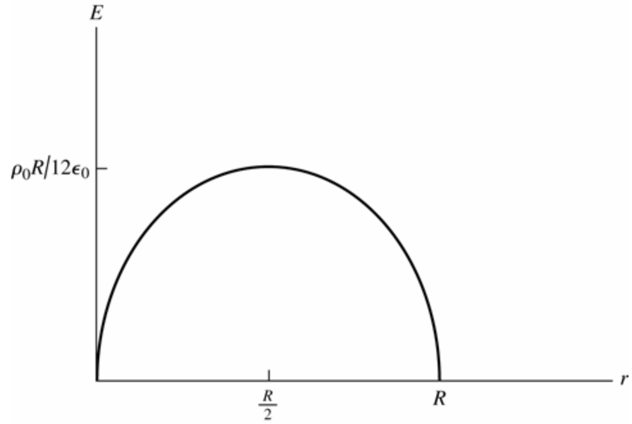


Figure 22.58

22.59. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: In spherical polar coordinates $d\vec{A} = r^2 \sin\theta\, d\theta\, d\phi\, \hat{r}$. $\oint \sin\theta\, d\theta\, d\phi = 4\pi$.

EXECUTE: **(a)** $\Phi_g = \oint \vec{g} \cdot d\vec{A} = -Gm \oint \frac{r^2 \sin\theta\, d\theta\, d\phi}{r^2} = -4\pi Gm.$

(b) For any closed surface, mass OUTSIDE the surface contributes zero to the flux passing through the surface. Thus the formula above holds for any situation where m is the mass enclosed by the Gaussian surface.

That is, $\Phi_g = \oint \vec{g} \cdot d\vec{A} = -4\pi G M_{\text{encl}}$.

EVALUATE: The minus sign in the expression for the flux signifies that the flux is directed inward.

22.60. **IDENTIFY:** Apply $\oint \vec{g} \cdot d\vec{A} = -4\pi G M_{\text{encl}}$.

SET UP: Use a Gaussian surface that is a sphere of radius r, concentric with the mass distribution. Let Φ_g denote $\oint \vec{g} \cdot d\vec{A}$

EXECUTE: **(a)** Use a Gaussian sphere with radius $r > R$, where R is the radius of the mass distribution. g is constant on this surface and the flux is inward. The enclosed mass is M. Therefore, $\Phi_g = -g 4\pi r^2 = -4\pi GM$ and $g = \frac{GM}{r^2}$, which is the same as for a point mass.

(b) For a Gaussian sphere of radius $r < R$, where R is the radius of the shell, $M_{\text{encl}} = 0$, so $g = 0$.

(c) Use a Gaussian sphere of radius $r < R$, where R is the radius of the planet. Then $M_{\text{encl}} = \rho\left(\frac{4}{3}\pi r^3\right) = Mr^3/R^3$.

This gives $\Phi_g = -g 4\pi r^2 = -4\pi G M_{\text{encl}} = -4\pi G \left(M \frac{r^3}{R^3}\right)$ and $g = \frac{GMr}{R^3}$, which is linear in r.

EVALUATE: The spherically synmetric distribution of mass results in an acceleration due to gravity $\vec{g}$ that is radical and that depends only on r, the distance from the center of the mass distribution.

22.61. **(a) IDENTIFY:** Use $\vec{E}(\vec{r})$ from Example (22.9) (inside the sphere) and relate the position vector of a point inside the sphere measured from the origin to that measured from the center of the sphere.

SET UP: For an insulating sphere of uniform charge density ρ and centered at the origin, the electric field inside the sphere is given by $E = Qr'/4\pi\epsilon_0 R^3$ (Example 22.9), where $\vec{r}\,'$ is the vector from the center of the sphere to the point where E is calculated.

But $\rho = 3Q/4\pi R^3$ so this may be written as $E = \rho r/3\epsilon_0$. And $\vec{E}$ is radially outward, in the direction of $\vec{r}\,'$, so $\vec{E} = \rho \vec{r}\,'/3\epsilon_0$.

For a sphere whose center is located by vector $\vec{b}$, a point inside the sphere and located by $\vec{r}$ is located by the vector $\vec{r}' = \vec{r} - \vec{b}$ relative to the center of the sphere, as shown in Figure 22.61.

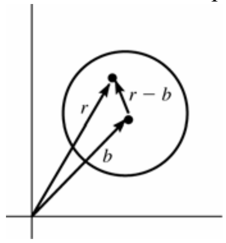


Figure 22.61

EXECUTE: Thus $\vec{E} = \dfrac{\rho(\vec{r} - \vec{b})}{3\epsilon_0}$

EVALUATE: When $b = 0$ this reduces to the result of Example 22.9. When $\vec{r} = \vec{b}$, this gives $E = 0$, which is correct since we know that $E = 0$ at the center of the sphere.

(b) IDENTIFY: The charge distribution can be represented as a uniform sphere with charge density ρ and centered at the origin added to a uniform sphere with charge density $-\rho$ and centered at $\vec{r} = \vec{b}$.

SET UP: $\vec{E} = \vec{E}_{\text{uniform}} + \vec{E}_{\text{hole}}$, where $\vec{E}_{\text{uniform}}$ is the field of a uniformly charged sphere with charge density ρ and $\vec{E}_{\text{hole}}$ is the field of a sphere located at the hole and with charge density $-\rho$. (Within the spherical hole the net charge density is $+\rho - \rho = 0$.)

EXECUTE: $\vec{E}_{\text{uniform}} = \dfrac{\rho\vec{r}}{3\epsilon_0}$, where $\vec{r}$ is a vector from the center of the sphere.

$\vec{E}_{\text{hole}} = \dfrac{-\rho(\vec{r} - \vec{b})}{3\epsilon_0}$, at points inside the hole.

Then $\vec{E} = \dfrac{\rho\vec{r}}{3\epsilon_0} + \left(\dfrac{-\rho(\vec{r} - \vec{b})}{3\epsilon_0}\right) = \dfrac{\rho\vec{b}}{3\epsilon_0}$.

EVALUATE: $\vec{E}$ is independent of $\vec{r}$ so is uniform inside the hole. The direction of $\vec{E}$ inside the hole is in the direction of the vector $\vec{b}$, the direction from the center of the insulating sphere to the center of the hole.

22.62. IDENTIFY: We first find the field of a cylinder off-axis, then the electric field in a hole in a cylinder is the difference between two electric fields: that of a solid cylinder on-axis, and one off-axis, at the location of the hole.

SET UP: Let $\vec{r}$ locate a point within the hole, relative to the axis of the cylinder and let $\vec{r}'$ locate this point relative to the axis of the hole. Let $\vec{b}$ locate the axis of the hole relative to the axis of the cylinder. As shown in Figure 22.62, $\vec{r}' = \vec{r} - \vec{b}$. Problem 23.48 shows that at points within a long insulating cylinder, $\vec{E} = \dfrac{\rho\vec{r}}{2\epsilon_0}$.

EXECUTE: $\vec{E}_{\text{off-axis}} = \dfrac{\rho\vec{r}'}{2\epsilon_0} = \dfrac{\rho(\vec{r} - \vec{b})}{2\epsilon_0}$. $\vec{E}_{\text{hole}} = \vec{E}_{\text{cylinder}} - \vec{E}_{\text{off-axis}} = \dfrac{\rho\,\vec{r}}{2\epsilon_0} - \dfrac{\rho(\vec{r} - \vec{b})}{2\epsilon_0} = \dfrac{\rho\,\vec{b}}{2\epsilon_0}$.

Note that $\vec{E}$ is uniform.

EVALUATE: If the hole is coaxial with the cylinder, $b = 0$ and $E_{\text{hole}} = 0$.

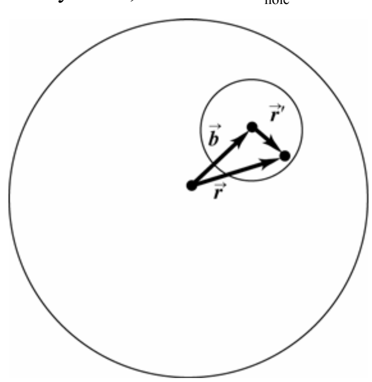


Figure 22.62

22.63. **IDENTIFY:** The electric field at each point is the vector sum of the fields of the two charge distributions.

SET UP: Inside a sphere of uniform positive charge, $E = \dfrac{\rho r}{3\epsilon_0}$.

$\rho = \dfrac{Q}{\frac{4}{3}\pi R^3} = \dfrac{3Q}{4\pi R^3}$ so $E = \dfrac{Qr}{4\pi\epsilon_0 R^3}$, directed away from the center of the sphere. Outside a sphere of uniform positive charge, $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$, directed away from the center of the sphere.

EXECUTE: **(a)** $x = 0$. This point is inside sphere 1 and outside sphere 2. The fields are shown in Figure 22.63a.

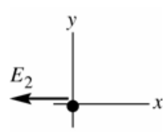


Figure 22.63a

$$E_1 = \frac{Qr}{4\pi\epsilon_0 R^3} = 0, \text{ since } r = 0.$$

$E_2 = \dfrac{Q}{4\pi\epsilon_0 r^2}$ with $r = 2R$ so $E_2 = \dfrac{Q}{16\pi\epsilon_0 R^2}$, in the $-x$-direction.

Thus $\vec{E} = \vec{E}_1 + \vec{E}_2 = \dfrac{Q}{16\pi\epsilon_0 R^2}\hat{i}$.

(b) $x = R/2$. This point is inside sphere 1 and outside sphere 2. Each field is directed away from the center of the sphere that produces it. The fields are shown in Figure 22.63b.

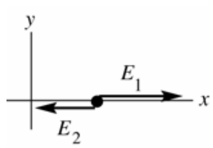


Figure 22.63b

$E_1 = \dfrac{Qr}{4\pi\epsilon_0 R^3}$ with $r = R/2$ so

$E_1 = \dfrac{Q}{8\pi\epsilon_0 R^2}$

$E_2 = \dfrac{Q}{4\pi\epsilon_0 r^2}$ with $r = 3R/2$ so $E_2 = \dfrac{Q}{9\pi\epsilon_0 R^2}$

$E = E_1 - E_2 = \dfrac{Q}{72\pi\epsilon_0 R^2}$, in the $+x$-direction and $\vec{E} = \dfrac{Q}{72\pi\epsilon_0 R^2}\hat{i}$

(c) $x = R$. This point is at the surface of each sphere. The fields have equal magnitudes and opposite directions, so $E = 0$.

(d) $x = 3R$. This point is outside both spheres. Each field is directed away from the center of the sphere that produces it. The fields are shown in Figure 22.63c.

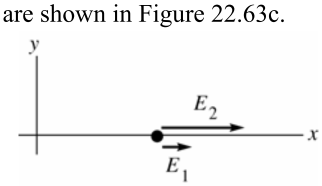


Figure 22.63c

$E_1 = \dfrac{Q}{4\pi\epsilon_0 r^2}$ with $r = 3R$ so

$E_1 = \dfrac{Q}{36\pi\epsilon_0 R^2}$

$E_2 = \dfrac{Q}{4\pi\epsilon_0 r^2}$ with $r = R$ so $E_2 = \dfrac{Q}{4\pi\epsilon_0 R^2}$

$E = E_1 + E_2 = \dfrac{5Q}{18\pi\epsilon_0 R^2}$, in the $+x$-direction and $\vec{E} = \dfrac{5Q}{18\pi\epsilon_0 R^2}\hat{i}$

EVALUATE: The field of each sphere is radially outward from the center of the sphere. We must use the correct expression for $E(r)$ for each sphere, depending on whether the field point is inside or outside that sphere.

22.64. **IDENTIFY:** The net electric field at any point is the vector sum of the fields at each sphere.

SET UP: Example 22.9 gives the electric field inside and outside a uniformly charged sphere. For the positively charged sphere the field is radially outward and for the negatively charged sphere the electric field is radially inward.

EXECUTE: **(a)** At this point the field of the left-hand sphere is zero and the field of the right-hand sphere is toward the center of that sphere, in the +*x*-direction. This point is outside the right-hand sphere, a distance $r = 2R$ from its center. $\vec{E} = +\frac{1}{4\pi\epsilon_0}\frac{Q}{4R^2}\hat{i}$.

(b) This point is inside the left-hand sphere, at $r = R/2$, and is outside the right-hand sphere, at $r = 3R/2$. Both fields are in the +*x*-direction.

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{Q(R/2)}{R^3} + \frac{Q}{(3R/2)^2}\right]\hat{i} = \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{2R^2} + \frac{4Q}{9R^2}\right]\hat{i} = \frac{1}{4\pi\epsilon_0}\frac{17Q}{18R^2}\hat{i}.$$

(c) This point is outside both spheres, at a distance $r = R$ from their centers. Both fields are in the +*x*-direction.
$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{R^2} + \frac{Q}{R^2}\right]\hat{i} = \frac{Q}{2\pi\epsilon_0 R^2}\hat{i}$.

(d) This point is outside both spheres, a distance $r = 3R$ from the center of the left-hand sphere and a distance $r = R$ from the center of the right-hand sphere. The field of the left-hand sphere is in the +*x*-direction and the field of the right-hand sphere is in the $-x$-direction. $\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{(3R)^2} - \frac{Q}{R^2}\right]\hat{i} = \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{9R^2} - \frac{Q}{R^2}\right]\hat{i} = \frac{-1}{4\pi\epsilon_0}\frac{8Q}{9R^2}\hat{i}$.

EVALUATE: At all points on the *x*-axis the net field is parallel to the *x*-axis.

22.65. **IDENTIFY:** Let $-dQ$ be the electron charge contained within a spherical shell of radius r' and thickness dr'. Integrate r' from 0 to *r* to find the electron charge within a sphere of radius *r*. Apply Gauss's law to a sphere of radius *r* to find the electric field $E(r)$.

SET UP: The volume of the spherical shell is $dV = 4\pi r'^2\, dr'$.

EXECUTE: **(a)** $Q(r) = Q - \int \rho dV = Q - \frac{Q4\pi}{\pi a_0^3}\int e^{-2r/a_0} r^2 dr = Q - \frac{4Q}{a_0^3}\int_0^r r'^2 e^{-2r'/a_0}\, dr'$.

$$Q(r) = Q - \frac{4Qe^{-\alpha r}}{a_0^3\alpha^3}(2e^{\alpha r} - \alpha^2 r^2 - 2\alpha r - 2) = Qe^{-2r/a_0}[2(r/a_0)^2 + 2(r/a_0) + 1].$$

Note if $r \to \infty$, $Q(r) \to 0$; the total net charge of the atom is zero.

(b) The electric field is radially outward. Gauss's law gives $E(4\pi r^2) = \frac{Q(r)}{\epsilon_0}$ and

$E = \frac{kQe^{-2r/a_0}}{r^2}(2(r/a_0)^2 + 2(r/a_0) + 1)$.

(c) The graph of *E* versus *r* is sketched in Figure 22.65. What is plotted is the scaled *E*, equal to $E/E_{\text{pt charge}}$, versus scaled *r*, equal to r/a_0. $E_{\text{pt charge}} = \frac{kQ}{r^2}$ is the field of a point charge.

EVALUATE: As $r \to 0$, the field approaches that of the positive point charge (the proton). For increasing *r* the electric field falls to zero more rapidly than the $1/r^2$ dependence for a point charge.

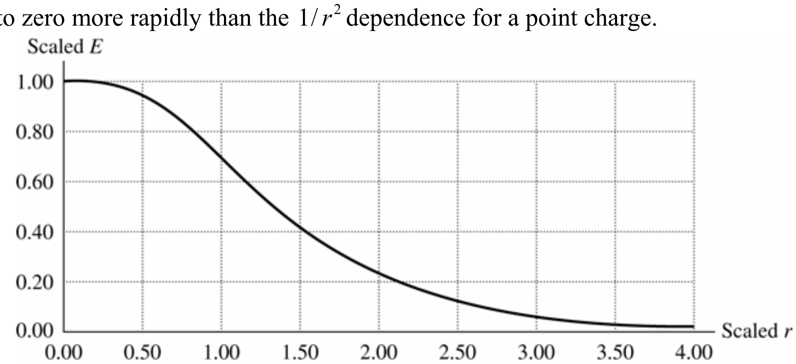


Figure 22.65

22.66. **IDENTIFY:** The charge in a spherical shell of radius *r* and thickness *dr* is $dQ = \rho(r)4\pi r^2\, dr$. Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius *r*. Let Q_i be the charge in the region $r \le R/2$ and let Q_0 be the charge in the region where $R/2 \le r \le R$.

EXECUTE: **(a)** The total charge is $Q = Q_i + Q_0$, where $Q_i = \alpha \dfrac{4\pi(R/2)^3}{3} = \dfrac{\alpha\pi R^3}{6}$ and

$Q_0 = 4\pi(2\alpha)\int_{R/2}^{R}(r^2 - r^3/R)dr = 8\alpha\pi\left(\dfrac{(R^3 - R^3/8)}{3} - \dfrac{(R^4 - R^4/16)}{4R}\right) = \dfrac{11\alpha\pi R^3}{24}$. Therefore, $Q = \dfrac{15\alpha\pi R^3}{24}$ and $\alpha = \dfrac{8Q}{5\pi R^3}$.

(b) For $r \le R/2$, Gauss's law gives $E4\pi r^2 = \dfrac{\alpha 4\pi r^3}{3\epsilon_0}$ and $E = \dfrac{\alpha r}{3\epsilon_0} = \dfrac{8Qr}{15\pi\epsilon_0 R^3}$. For $R/2 \le r \le R$,

$E4\pi r^2 = \dfrac{Q_i}{\epsilon_0} + \dfrac{1}{\epsilon_0}\left(8\alpha\pi\left(\dfrac{(r^3 - R^3/8)}{3} - \dfrac{(r^4 - R^4/16)}{4R}\right)\right)$ and

$E = \dfrac{\alpha\pi R^3}{24\epsilon_0(4\pi r^2)}(64(r/R)^3 - 48(r/R)^4 - 1) = \dfrac{kQ}{15r^2}(64(r/R)^3 - 48(r/R)^4 - 1).$

For $r \ge R$, $E(4\pi r^2) = \dfrac{Q}{\epsilon_0}$ and $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$.

(c) $\dfrac{Q_i}{Q} = \dfrac{(4Q/15)}{Q} = \dfrac{4}{15} = 0.267.$

(d) For $r \le R/2$, $F_r = -eE = -\dfrac{8eQ}{15\pi\epsilon_0 R^3}r$, so the restoring force depends upon displacement to the first power, and we have simple harmonic motion.

(e) Comparing to $F = -kr$, $k = \dfrac{8eQ}{15\pi\epsilon_0 R^3}$. Then $\omega = \sqrt{\dfrac{k}{m_e}} = \sqrt{\dfrac{8eQ}{15\pi\epsilon_0 R^3 m_e}}$ and $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{\dfrac{15\pi\epsilon_0 R^3 m_e}{8eQ}}.$

EVALUATE: **(f)** If the amplitude of oscillation is greater than $R/2$, the force is no longer linear in r, and is thus no longer simple harmonic.

22.67. **IDENTIFY:** The charge in a spherical shell of radius r and thickness dr is $dQ = \rho(r)4\pi r^2\,dr$. Apply Gauss's law.

SET UP: Use a Gaussian surface that is a sphere of radius r. Let Q_i be the charge in the region $r \le R/2$ and let Q_0 be the charge in the region where $R/2 \le r \le R$.

EXECUTE: **(a)** The total charge is $Q = Q_i + Q_0$, where $Q_i = 4\pi\int_0^{R/2}\dfrac{3\alpha r^3}{2R}dr = \dfrac{6\pi\alpha}{R}\dfrac{1}{4}\dfrac{R^4}{16} = \dfrac{3}{32}\pi\alpha R^3$ and

$Q_0 = 4\pi\alpha\int_{R/2}^{R}(1-(r/R)^2)r^2\,dr = 4\pi\alpha R^3\left(\dfrac{7}{24} - \dfrac{31}{160}\right) = \dfrac{47}{120}\pi\alpha R^3.$ Therefore, $Q = \left(\dfrac{3}{32} + \dfrac{47}{120}\right)\pi\alpha R^3 = \dfrac{233}{480}\pi\alpha R^3$ and

$\alpha = \dfrac{480Q}{233\pi R^3}$.

(b) For $r \le R/2$, Gauss's law gives $E4\pi r^2 = \dfrac{4\pi}{\epsilon_0}\int_0^r\dfrac{3\alpha r'^3}{2R}dr' = \dfrac{3\pi\alpha r^4}{2\epsilon_0 R}$ and $E = \dfrac{6\alpha r^2}{16\epsilon_0 R} = \dfrac{180Qr^2}{233\pi\epsilon_0 R^4}$. For $R/2 \le r \le R$,

$E4\pi r^2 = \dfrac{Q_i}{\epsilon_0} + \dfrac{4\pi\alpha}{\epsilon_0}\int_{R/2}^{r}(1-(r'/R)^2)r'^2\,dr' = \dfrac{Q_i}{\epsilon_0} + \dfrac{4\pi\alpha}{\epsilon_0}\left(\dfrac{r^3}{3} - \dfrac{R^3}{24} - \dfrac{r^5}{5R^2} + \dfrac{R^3}{160}\right).$

$E4\pi r^2 = \dfrac{3}{128}\dfrac{4\pi\alpha R^3}{\epsilon_0} + \dfrac{4\pi\alpha R^3}{\epsilon_0}\left(\dfrac{1}{3}\left(\dfrac{r}{R}\right)^3 - \dfrac{1}{5}\left(\dfrac{r}{R}\right)^5 - \dfrac{17}{480}\right)$ and $E = \dfrac{480Q}{233\pi\epsilon_0 r^2}\left(\dfrac{1}{3}\left(\dfrac{r}{R}\right)^3 - \dfrac{1}{5}\left(\dfrac{r}{R}\right)^5 - \dfrac{23}{1920}\right)$. For $r \ge R$,

$E = \dfrac{Q}{4\pi\epsilon_0 r^2}$, since all the charge is enclosed.

(c) The fraction of Q between $R/2 \le r \le R$ is $\dfrac{Q_0}{Q} = \dfrac{47}{120}\dfrac{480}{233} = 0.807.$

(d) $E = \dfrac{180}{233}\dfrac{Q}{4\pi\epsilon_0 R^2}$ using either of the electric field expressions above, evaluated at $r = R/2$.

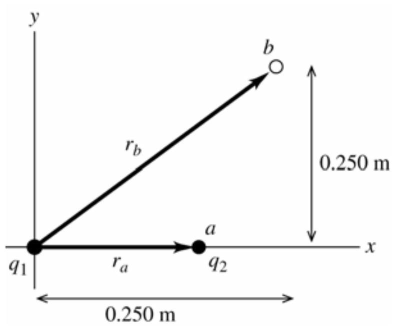
EVALUATE: **(e)** The force an electron would feel never is proportional to $-r$ which is necessary for simple harmonic oscillations. It is oscillatory since the force is always attractive, but it has the wrong power of r to be simple harmonic motion.

23

ELECTRIC POTENTIAL

23.1. **IDENTIFY:** Apply Eq.(23.2) to calculate the work. The electric potential energy of a pair of point charges is given by Eq.(23.9).

SET UP: Let the initial position of q_2 be point *a* and the final position be point *b*, as shown in Figure 23.1.



$r_a = 0.150$ m

$r_b = \sqrt{(0.250 \text{ m})^2 + (0.250 \text{ m})^2}$

$r_b = 0.3536$ m

Figure 23.1

EXECUTE: $W_{a\to b} = U_a - U_b$

$$U_a = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_a} = (8.988\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+2.40\times 10^{-6}\text{ C})(-4.30\times 10^{-6}\text{ C})}{0.150\text{ m}}$$

$$U_a = -0.6184 \text{ J}$$

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_b} = (8.988\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(+2.40\times 10^{-6}\text{ C})(-4.30\times 10^{-6}\text{ C})}{0.3536\text{ m}}$$

$$U_b = -0.2623 \text{ J}$$

$$W_{a\to b} = U_a - U_b = -0.6184 \text{ J} - (-0.2623 \text{ J}) = -0.356 \text{ J}$$

EVALUATE: The attractive force on q_2 is toward the origin, so it does negative work on q_2 when q_2 moves to larger *r*.

23.2. **IDENTIFY:** Apply $W_{a\to b} = U_a - U_b$.

SET UP: $U_a = +5.4\times 10^{-8}$ J. Solve for U_b.

EXECUTE: $W_{a\to b} = -1.9\times 10^{-8} \text{ J} = U_a - U_b$. $U_b = U_a - W_{a\to b} = 1.9\times 10^{-8} \text{ J} - (-5.4\times 10^{-8} \text{ J}) = 7.3\times 10^{-8}$ J.

EVALUATE: When the electric force does negative work the electrical potential energy increases.

23.3. **IDENTIFY:** The work needed to assemble the nucleus is the sum of the electrical potential energies of the protons in the nucleus, relative to infinity.

SET UP: The total potential energy is the scalar sum of all the individual potential energies, where each potential energy is $U = (1/4\pi\epsilon_0)(qq_0/r)$. Each charge is *e* and the charges are equidistant from each other, so the total potential energy is $U = \frac{1}{4\pi\epsilon_0}\left(\frac{e^2}{r} + \frac{e^2}{r} + \frac{e^2}{r}\right) = \frac{3e^2}{4\pi\epsilon_0 r}$.

EXECUTE: Adding the potential energies gives

$$U = \frac{3e^2}{4\pi\epsilon_0 r} = \frac{3(1.60\times 10^{-19}\text{ C})^2(9.00\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)}{2.00\times 10^{-15}\text{ m}} = 3.46\times 10^{-13} \text{ J} = 2.16 \text{ MeV}$$

EVALUATE: This is a small amount of energy on a macroscopic scale, but on the scale of atoms, 2 MeV is quite a lot of energy.

23.4. **IDENTIFY:** The work required is the change in electrical potential energy. The protons gain speed after being released because their potential energy is converted into kinetic energy.

(a) SET UP: Using the potential energy of a pair of point charges relative to infinity, $U = (1/4\pi\epsilon_0)(qq_0/r)$. we have

$$W = \Delta U = U_2 - U_1 = \frac{1}{4\pi\epsilon_0}\left(\frac{e^2}{r_2} - \frac{e^2}{r_1}\right).$$

EXECUTE: Factoring out the e^2 and substituting numbers gives

$$W = \left(9.00\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2\left(\frac{1}{3.00\times10^{-15}\ \text{m}} - \frac{1}{2.00\times10^{-15}\ \text{m}}\right) = 7.68\times10^{-14}\ \text{J}$$

(b) SET UP: The protons have equal momentum, and since they have equal masses, they will have equal speeds and hence equal kinetic energy. $\Delta U = K_1 + K_2 = 2K = 2\left(\frac{1}{2}mv^2\right) = mv^2.$

EXECUTE: Solving for v gives $v = \sqrt{\frac{\Delta U}{m}} = \sqrt{\frac{7.68\times10^{-14}\ \text{J}}{1.67\times10^{-27}\ \text{kg}}} = 6.78\times10^6$ m/s

EVALUATE: The potential energy may seem small (compared to macroscopic energies), but it is enough to give each proton a speed of nearly 7 million m/s.

23.5. **(a) IDENTIFY:** Use conservation of energy:

$$K_a + U_a + W_{\text{other}} = K_b + U_b$$

U for the pair of point charges is given by Eq.(23.9).

SET UP:

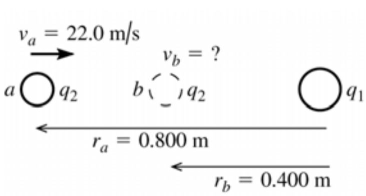


Figure 23.5a

Let point a be where q_2 is 0.800 m from q_1 and point b be where q_2 is 0.400 m from q_1, as shown in Figure 23.5a.

EXECUTE: Only the electric force does work, so $W_{\text{other}} = 0$ and $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$.

$$K_a = \tfrac{1}{2}mv_a^2 = \tfrac{1}{2}(1.50\times10^{-3}\ \text{kg})(22.0\ \text{m/s})^2 = 0.3630\ \text{J}$$

$$U_a = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_a} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{0.800\ \text{m}} = +0.2454\ \text{J}$$

$$K_b = \tfrac{1}{2}mv_b^2$$

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_b} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{0.400\ \text{m}} = +0.4907\ \text{J}$$

The conservation of energy equation then gives $K_b = K_a + (U_a - U_b)$

$$\tfrac{1}{2}mv_b^2 = +0.3630\ \text{J} + (0.2454\ \text{J} - 0.4907\ \text{J}) = 0.1177\ \text{J}$$

$$v_b = \sqrt{\frac{2(0.1177\ \text{J})}{1.50\times10^{-3}\ \text{kg}}} = 12.5\ \text{m/s}$$

EVALUATE: The potential energy increases when the two positively charged spheres get closer together, so the kinetic energy and speed decrease.

(b) IDENTIFY: Let point c be where q_2 has its speed momentarily reduced to zero. Apply conservation of energy to points a and c: $K_a + U_a + W_{\text{other}} = K_c + U_c$.

SET UP: Points a and c are shown in Figure 23.5b.

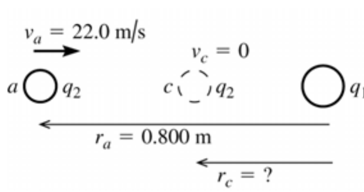


Figure 23.5b

EXECUTE: $K_a = +0.3630\ \text{J}$ (from part (a))

$U_a = +0.2454\ \text{J}$ (from part (a))

$K_c = 0$ (at distance of closest approach the speed is zero)

$$U_c = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_c}$$

Thus conservation of energy $K_a + U_a = U_c$ gives $\frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_c} = +0.3630\ \text{J} + 0.2454\ \text{J} = 0.6084\ \text{J}$

$$r_c = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{0.6084\ \text{J}} = (8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times 10^{-6}\ \text{C})(-7.80\times 10^{-6}\ \text{C})}{+0.6084\ \text{J}} = 0.323\ \text{m}.$$

EVALUATE: $U \to \infty$ as $r \to 0$ so q_2 will stop no matter what its initial speed is.

23.6. IDENTIFY: Apply $U = k\frac{q_1 q_2}{r}$ and solve for r.

SET UP: $q_1 = -7.2\times 10^{-6}\ \text{C}$, $q_2 = +2.3\times 10^{-6}\ \text{C}$

EXECUTE: $r = \frac{kq_1 q_2}{U} = \frac{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-7.20\times 10^{-6}\ \text{C})(+2.30\times 10^{-6}\ \text{C})}{-0.400\ \text{J}} = 0.372\ \text{m}$

EVALUATE: The potential energy U is a scalar and can take positive and negative values.

23.7. (a) IDENTIFY and SET UP: U is given by Eq.(23.9).

EXECUTE: $U = \frac{1}{4\varepsilon\pi_0}\frac{qq'}{r}$

$$U = (8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(+4.60\times 10^{-6}\ \text{C})(+1.20\times 10^{-6}\ \text{C})}{0.250\ \text{m}} = +0.198\ \text{J}$$

EVALUATE: The two charges are both of the same sign so their electric potential energy is positive.

(b) IDENTIFY: Use conservation of energy: $K_a + U_a + W_{\text{other}} = K_b + U_b$

SET UP: Let point a be where q is released and point b be at its final position, as shown in Figure 23.7.

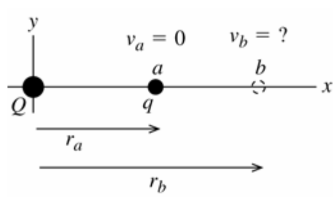


Figure 23.7

EXECUTE: $K_a = 0$ (released from rest)

$U_a = +0.198\ \text{J}$ (from part (a))

$K_b = \frac{1}{2}mv_b^2$

Only the electric force does work, so $W_{\text{other}} = 0$ and $U = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r}$.

(i) $r_b = 0.500\ \text{m}$

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{qQ}{r} = (8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(+4.60\times 10^{-6}\ \text{C})(+1.20\times 10^{-6}\ \text{C})}{0.500\ \text{m}} = +0.0992\ \text{J}$$

Then $K_a + U_a + W_{\text{other}} = K_b + U_b$ gives $K_b = U_a - U_b$ and $\frac{1}{2}mv_b^2 = U_a - U_b$ and

$$v_b = \sqrt{\frac{2(U_a - U_b)}{m}} = \sqrt{\frac{2(+0.198\ \text{J} - 0.0992\ \text{J})}{2.80\times 10^{-4}\ \text{kg}}} = 26.6\ \text{m/s}.$$

(ii) $r_b = 5.00\ \text{m}$ r_b is now ten times larger than in (i) so U_b is ten times smaller: $U_b = +0.0992\ \text{J}/10 = +0.00992\ \text{J}$.

$$v_b = \sqrt{\frac{2(U_a - U_b)}{m}} = \sqrt{\frac{2(+0.198\ \text{J} - 0.00992\ \text{J})}{2.80\times 10^{-4}\ \text{kg}}} = 36.7\ \text{m/s}.$$

(iii) $r_b = 50.0$ m

r_b is now ten times larger than in (ii) so U_b is ten times smaller:

$$U_b = +0.00992 \text{ J}/10 = +0.000992 \text{ J}.$$

$$v_b = \sqrt{\frac{2(U_a - U_b)}{m}} = \sqrt{\frac{2(+0.198 \text{ J} - 0.000992 \text{ J})}{2.80\times 10^{-4} \text{ kg}}} = 37.5 \text{ m/s}.$$

EVALUATE: The force between the two charges is repulsive and provides an acceleration to q. This causes the speed of q to increase as it moves away from Q.

23.8. IDENTIFY: Call the three charges 1, 2 and 3. $U = U_{12} + U_{13} + U_{23}$

SET UP: $U_{12} = U_{23} = U_{13}$ because the charges are equal and each pair of charges has the same separation, 0.500 m.

EXECUTE: $U = \dfrac{3kq^2}{0.500 \text{ m}} = \dfrac{3k(1.2\times 10^{-6} \text{ C})^2}{0.500 \text{ m}} = 0.078$ J.

EVALUATE: When the three charges are brought in from infinity to the corners of the triangle, the repulsive electrical forces between each pair of charges do negative work and electrical potential energy is stored.

23.9. IDENTIFY: $U = k\left(\dfrac{q_1q_2}{r_{12}} + \dfrac{q_1q_3}{r_{13}} + \dfrac{q_2q_3}{r_{23}}\right)$

SET UP: In part (a), $r_{12} = 0.200$ m, $r_{23} = 0.100$ m and $r_{13} = 0.100$ m. In part (b) let particle 3 have coordinate x, so $r_{12} = 0.200$ m, $r_{13} = x$ and $r_{23} = 0.200 - x$.

EXECUTE: **(a)** $U = k\left(\dfrac{(4.00 \text{ nC})(-3.00 \text{ nC})}{(0.200 \text{ m})} + \dfrac{(4.00 \text{ nC})(2.00 \text{ nC})}{(0.100 \text{ m})} + \dfrac{(-3.00 \text{ nC})(2.00 \text{ nC})}{(0.100 \text{ m})}\right) = 3.60\times 10^{-7}$ J

(b) If $U = 0$, then $0 = k\left(\dfrac{q_1q_2}{r_{12}} + \dfrac{q_1q_3}{x} + \dfrac{q_2q_3}{r_{12} - x}\right)$. Solving for x we find:

$0 = -60 + \dfrac{8}{x} - \dfrac{6}{0.2 - x} \Rightarrow 60x^2 - 26x + 1.6 = 0 \Rightarrow x = 0.074 \text{ m}, 0.360 \text{ m}$. Therefore, $x = 0.074$ m since it is the only value between the two charges.

EVALUATE: U_{13} is positive and both U_{23} and U_{12} are negative. If $U = 0$, then $|U_{13}| = |U_{23}| + |U_{12}|$. For $x = 0.074$ m, $U_{13} = +9.7\times 10^{-7}$ J, $U_{23} = -4.3\times 10^{-7}$ J and $U_{12} = -5.4\times 10^{-7}$ J. It is true that $U = 0$ at this x.

23.10. IDENTIFY: The work done on the alpha particle is equal to the difference in its potential energy when it is moved from the midpoint of the square to the midpoint of one of the sides.

SET UP: We apply the formula $W_{a\to b} = U_a - U_b$. In this case, a is the center of the square and b is the midpoint of one of the sides. Therefore $W_{\text{center}\to\text{side}} = U_{\text{center}} - U_{\text{side}}$.

There are 4 electrons, so the potential energy at the center of the square is 4 times the potential energy of a single alpha-electron pair. At the center of the square, the alpha particle is a distance $r_1 = \sqrt{50}$ nm from each electron. At the midpoint of the side, the alpha is a distance $r_2 = 5.00$ nm from the two nearest electrons and a distance $r_2 = \sqrt{125}$ nm from the two most distant electrons. Using the formula for the potential energy (relative to infinity) of two point charges, $U = (1/4\pi\epsilon_0)(qq_0/r)$, the total work is

$$W_{\text{center}\to\text{side}} = U_{\text{center}} - U_{\text{side}} = 4\frac{1}{4\pi\epsilon_0}\frac{q_\alpha q_e}{r_1} - \left(2\frac{1}{4\pi\epsilon_0}\frac{q_\alpha q_e}{r_2} + 2\frac{1}{4\pi\epsilon_0}\frac{q_\alpha q_e}{r_3}\right)$$

Substituting $q_e = e$ and $q_\alpha = 2e$ and simplifying gives

$$W_{\text{center}\to\text{side}} = -4e^2\frac{1}{4\pi\epsilon_0}\left[\frac{2}{r_1} - \left(\frac{1}{r_2} + \frac{1}{r_3}\right)\right]$$

EXECUTE: Substituting the numerical values into the equation for the work gives

$$W = -4\left(1.60\times 10^{-19} \text{ C}\right)^2\left[\frac{2}{\sqrt{50} \text{ m}} - \left(\frac{1}{5.00 \text{ nm}} + \frac{1}{\sqrt{125} \text{ nm}}\right)\right] = 6.08\times 10^{-21} \text{ J ???}$$

EVALUATE: Since the work is positive, the system has more potential energy with the alpha particle at the center of the square than it does with it at the midpoint of a side.

23.11. **IDENTIFY:** Apply Eq.(23.2). The net work to bring the charges in from infinity is equal to the change in potential energy. The total potential energy is the sum of the potential energies of each pair of charges, calculated from Eq.(23.9).

SET UP: Let 1 be where all the charges are infinitely far apart. Let 2 be where the charges are at the corners of the triangle, as shown in Figure 23.11.

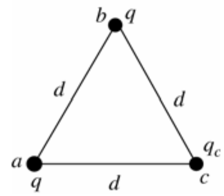


Figure 23.11

Let q_c be the third, unknown charge.

EXECUTE: $W = -\Delta U = -(U_2 - U_1)$

$$U_1 = 0$$

$$U_2 = U_{ab} + U_{ac} + U_{bc} = \frac{1}{4\pi\epsilon_0 d}(q^2 + 2qq_c)$$

Want $W = 0$, so $W = -(U_2 - U_1)$ gives $0 = -U_2$

$$0 = \frac{1}{4\pi\epsilon_0 d}(q^2 + 2qq_c)$$

$q^2 + 2qq_c = 0$ and $q_c = -q/2$.

EVALUATE: The potential energy for the two charges q is positive and for each q with q_c it is negative. There are two of the q, q_c terms so must have $q_c < q$.

23.12. **IDENTIFY:** Use conservation of energy $U_a + K_a = U_b + K_b$ to find the distance of closest approach r_b. The maximum force is at the distance of closest approach, $F = k\dfrac{|q_1 q_2|}{r_b^2}$.

SET UP: $K_b = 0$. Initially the two protons are far apart, so $U_a = 0$. A proton has mass 1.67×10^{-27} kg and charge $q = +e = +1.60\times 10^{-19}$ C.

EXECUTE: $K_a = U_b$. $2(\tfrac{1}{2}mv_a^2) = k\dfrac{q_1 q_2}{r_b}$. $mv_a^2 = k\dfrac{e^2}{r_b}$ and

$$r_b = \frac{ke^2}{mv_a^2} = \frac{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19}\ \text{C})^2}{(1.67\times 10^{-27}\ \text{kg})(1.00\times 10^6\ \text{m/s})^2} = 1.38\times 10^{-13}\ \text{m}.$$

$$F = k\frac{e^2}{r_b^2} = (8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60\times 10^{-19}\ \text{C})^2}{(1.38\times 10^{-13}\ \text{C})^2} = 0.012\ \text{N}.$$

EVALUATE: The acceleration $a = F/m$ of each proton produced by this force is extremely large.

23.13. **IDENTIFY:** $\vec{E}$ points from high potential to low potential. $\dfrac{W_{a\to b}}{q_0} = V_a - V_b$.

SET UP: The force on a positive test charge is in the direction of $\vec{E}$.

EXECUTE: V decreases in the eastward direction. A is east of B, so $V_B > V_A$. C is east of A, so $V_C < V_A$. The force on a positive test charge is east, so no work is done on it by the electric force when it moves due south (the force and displacement are perpendicular), and $V_D = V_A$.

EVALUATE: The electric potential is constant in a direction perpendicular to the electric field.

23.14. **IDENTIFY:** $\dfrac{W_{a\to b}}{q_0} = V_a - V_b$. For a point charge, $V = \dfrac{kq}{r}$.

SET UP: Each vacant corner is the same distance, 0.200 m, from each point charge.

EXECUTE: Taking the origin at the center of the square, the symmetry means that the potential is the same at the two corners not occupied by the $+5.00\ \mu\text{C}$ charges. This means that no net work is done is moving from one corner to the other.

EVALUATE: If the charge q_0 moves along a diagonal of the square, the electrical force does positive work for part of the path and negative work for another part of the path, but the net work done is zero.

23.15. **IDENTIFY** and **SET UP:** Apply conservation of energy to points *A* and *B*.

EXECUTE: $K_A + U_A = K_B + U_B$

$U = qV$, so $K_A + qV_A = K_B + qV_B$

$K_B = K_A + q(V_A - V_B) = 0.00250\text{ J} + (-5.00\times10^{-6}\text{ C})(200\text{ V} - 800\text{ V}) = 0.00550\text{ J}$

$v_B = \sqrt{2K_B/m} = 7.42\text{ m/s}$

EVALUATE: It is faster at *B;* a negative charge gains speed when it moves to higher potential.

23.16. **IDENTIFY:** The work-energy theorem says $W_{a\to b} = K_b - K_a$. $\dfrac{W_{a\to b}}{q} = V_a - V_b$.

SET UP: Point *a* is the starting point and point *b* is the ending point. Since the field is uniform, $W_{a\to b} = Fs\cos\phi = E|q|s\cos\phi$. The field is to the left so the force on the positive charge is to the left. The particle moves to the left so $\phi = 0°$ and the work $W_{a\to b}$ is positive.

EXECUTE: **(a)** $W_{a\to b} = K_b - K_a = 1.50\times10^{-6}\text{ J} - 0 = 1.50\times10^{-6}\text{ J}$

(b) $V_a - V_b = \dfrac{W_{a\to b}}{q} = \dfrac{1.50\times10^{-6}\text{ J}}{4.20\times10^{-9}\text{ C}} = 357\text{ V}$. Point *a* is at higher potential than point *b*.

(c) $E|q|s = W_{a\to b}$, so $E = \dfrac{W_{a\to b}}{|q|s} = \dfrac{V_a - V_b}{s} = \dfrac{357\text{ V}}{6.00\times10^{-2}\text{ m}} = 5.95\times10^{3}\text{ V/m}$.

EVALUATE: A positive charge gains kinetic energy when it moves to lower potential; $V_b < V_a$.

23.17. **IDENTIFY:** Apply the equation that precedes Eq.(23.17): $W_{a\to b} = q'\int_a^b \vec{E}\cdot d\vec{l}$.

SET UP: Use coordinates where $+y$ is upward and $+x$ is to the right. Then $\vec{E} = E\hat{j}$ with $E = 4.00\times10^{4}$ N/C.

(a) The path is sketched in Figure 23.17a.

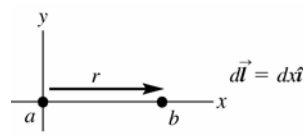


Figure 23.17a

EXECUTE: $\vec{E}\cdot d\vec{l} = (E\hat{j})\cdot(dx\hat{i}) = 0$ so $W_{a\to b} = q'\int_a^b \vec{E}\cdot d\vec{l} = 0$.

EVALUATE: The electric force on the positive charge is upward (in the direction of the electric field) and does no work for a horizontal displacement of the charge.

(b) SET UP: The path is sketched in Figure 23.17b.

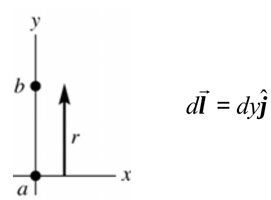


Figure 23.17b

EXECUTE: $\vec{E}\cdot d\vec{l} = (E\hat{j})\cdot(dy\hat{j}) = E\,dy$

$$W_{a\to b} = q'\int_a^b \vec{E}\cdot d\vec{l} = q'E\int_a^b dy = q'E(y_b - y_a)$$

$y_b - y_a = +0.670$ m, positive since the displacement is upward and we have taken $+y$ to be upward.

$$W_{a\to b} = q'E(y_b - y_a) = (+28.0\times10^{-9}\text{ C})(4.00\times10^{4}\text{ N/C})(+0.670\text{ m}) = +7.50\times10^{-4}\text{ J}.$$

EVALUATE: The electric force on the positive charge is upward so it does positive work for an upward displacement of the charge.

(c) SET UP: The path is sketched in Figure 23.17c.

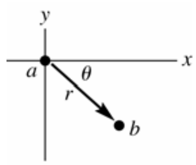


Figure 23.17c

$y_a = 0$

$y_b = -r\sin\theta = -(2.60\text{ m})\sin 45° = -1.838\text{ m}$

The vertical component of the 2.60 m displacement is 1.838 m downward.

EXECUTE: $d\vec{l} = dx\hat{i} + dy\hat{j}$ (The displacement has both horizontal and vertical components.)

$\vec{E}\cdot d\vec{l} = (E\hat{j})\cdot(dx\hat{i} + dy\hat{j}) = E\,dy$ (Only the vertical component of the displacement contributes to the work.)

$$W_{a\to b} = q'\int_a^b \vec{E}\cdot d\vec{l} = q'E\int_a^b dy = q'E(y_b - y_a)$$

$$W_{a\to b} = q'E(y_b - y_a) = (+28.0\times10^{-9}\text{ C})(4.00\times10^{4}\text{ N/C})(-1.838\text{ m}) = -2.06\times10^{-3}\text{ J}.$$

EVALUATE: The electric force on the positive charge is upward so it does negative work for a displacement of the charge that has a downward component.

23.18. IDENTIFY: Apply $K_a + U_a = K_b + U_b$.

SET UP: Let $q_1 = +3.00\text{ nC}$ and $q_2 = +2.00\text{ nC}$. At point *a*, $r_{1a} = r_{2a} = 0.250\text{ m}$. At point *b*, $r_{1b} = 0.100\text{ m}$ and $r_{2b} = 0.400\text{ m}$. The electron has $q = -e$ and $m_e = 9.11\times10^{-31}\text{ kg}$. $K_a = 0$ since the electron is released from rest.

EXECUTE: $-\dfrac{keq_1}{r_{1a}} - \dfrac{keq_2}{r_{2a}} = -\dfrac{keq_1}{r_{1b}} - \dfrac{keq_2}{r_{2b}} + \dfrac{1}{2}m_e v_b^2$.

$$E_a = K_a + U_a = k(-1.60\times10^{-19}\text{ C})\left(\frac{(3.00\times10^{-9}\text{ C})}{0.250\text{ m}} + \frac{(2.00\times10^{-9}\text{ C})}{0.250\text{ m}}\right) = -2.88\times10^{-17}\text{ J}.$$

$$E_b = K_b + U_b = k(-1.60\times10^{-19}\text{ C})\left(\frac{(3.00\times10^{-9}\text{ C})}{0.100\text{ m}} + \frac{(2.00\times10^{-9}\text{ C}}{0.400\text{ m}}\right) + \frac{1}{2}m_e v_b^2 = -5.04\times10^{-17}\text{ J} + \frac{1}{2}m_e v_b^2$$

Setting $E_a = E_b$ gives $v_b = \sqrt{\dfrac{2}{9.11\times10^{-31}\text{ kg}}(5.04\times10^{-17}\text{ J} - 2.88\times10^{-17}\text{ J})} = 6.89\times10^{6}\text{ m/s}$.

EVALUATE: $V_a = V_{1a} + V_{2a} = 180\text{ V}$. $V_b = V_{1b} + V_{2b} = 315\text{ V}$. $V_b > V_a$. The negatively charged electron gains kinetic energy when it moves to higher potential.

23.19. IDENTIFY and SET UP: For a point charge $V = \dfrac{kq}{r}$. Solve for *r*.

EXECUTE: (a) $r = \dfrac{kq}{V} = \dfrac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(2.50\times10^{-11}\text{ C})}{90.0\text{ V}} = 2.50\times10^{-3}\text{ m} = 2.50\text{ mm}$

(b) $Vr = kq = \text{constant}$ so $V_1r_1 = V_2r_2$. $r_2 = r_1\left(\dfrac{V_1}{V_2}\right) = (2.50\text{ mm})\left(\dfrac{90.0\text{ V}}{30.0\text{ V}}\right) = 7.50\text{ mm}$.

EVALUATE: The potential of a positive charge is positive and decreases as the distance from the point charge increases.

23.20. IDENTIFY: The total potential is the *scalar* sum of the individual potentials, but the net electric field is the *vector* sum of the two fields.

SET UP: The net potential can only be zero if one charge is positive and the other is negative, since it is a scalar. The electric field can only be zero if the two fields point in opposite directions.

EXECUTE: (a) (i) Since both charges have the same sign, there are no points for which the potential is zero.

(ii) The two electric fields are in opposite directions only between the two charges, and midway between them the fields have equal magnitudes. So $E = 0$ midway between the charges, but V is never zero.

(b) (i) The two potentials have equal magnitude but opposite sign midway between the charges, so $V = 0$ midway between the charges, but $E \neq 0$ there since the fields point in the same direction.

(ii) Between the two charges, the fields point in the same direction, so E cannot be zero there. In the other two regions, the field due to the nearer charge is always greater than the field due to the more distant charge, so they cannot cancel. Hence E is not zero anywhere.

EVALUATE: It does *not* follow that the electric field is zero where the potential is zero, or that the potential is zero where the electric field is zero.

23.21. **IDENTIFY:** $V = \dfrac{1}{4\pi\epsilon_0}\sum_i \dfrac{q_i}{r_i}$

SET UP: The locations of the changes and points *A* and *B* are sketched in Figure 23.21.

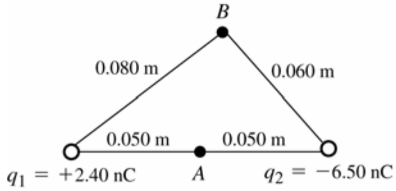


Figure 23.21

EXECUTE: **(a)** $V_A = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_{A1}} + \dfrac{q_2}{r_{A2}}\right)$

$$V_A = (8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.40\times 10^{-9}\ \text{C}}{0.050\ \text{m}} + \frac{-6.50\times 10^{-9}\ \text{C}}{0.050\ \text{m}}\right) = -737\ \text{V}$$

(b) $V_B = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q_1}{r_{B1}} + \dfrac{q_2}{r_{B2}}\right)$

$$V_B = (8.988\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{+2.40\times 10^{-9}\ \text{C}}{0.080\ \text{m}} + \frac{-6.50\times 10^{-9}\ \text{C}}{0.060\ \text{m}}\right) = -704\ \text{V}$$

(c) IDENTIFY and **SET UP:** Use Eq.(23.13) and the results of parts (a) and (b) to calculate *W*.

EXECUTE: $W_{B\to A} = q'(V_B - V_A) = (2.50\times 10^{-9}\ \text{C})(-704\ \text{V} - (-737\ \text{V})) = +8.2\times 10^{-8}\ \text{J}$

EVALUATE: The electric force does positive work on the positive charge when it moves from higher potential (point *B*) to lower potential (point *A*).

23.22. **IDENTIFY:** For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: **(a)** The positions of the two charges are shown in Figure 23.22a. $r = \sqrt{a^2 + x^2}$.

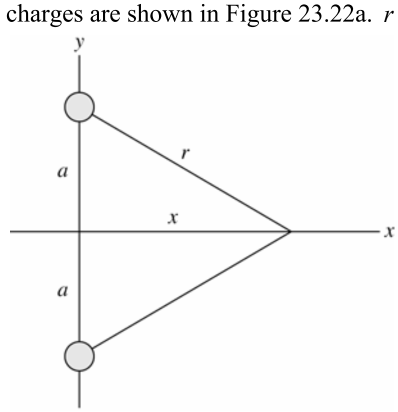


Figure 23.22a

EXECUTE: **(b)** $V_0 = 2\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{a}$.

(c) $V(x) = 2\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r} = 2\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{\sqrt{a^2 + x^2}}$

(d) The graph of V versus x is sketched in Figure 23.22b.

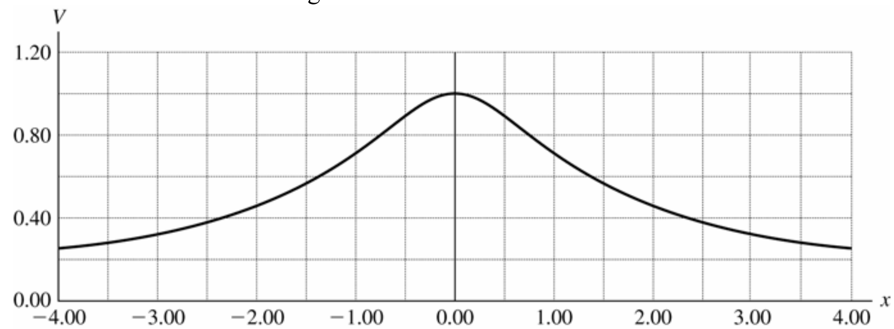


Figure 23.22b

EVALUATE: **(e)** When $x >> a,\ V = \dfrac{1}{4\pi\epsilon_0}\dfrac{2q}{x}$, just like a point charge of charge $+2q$. At distances from the charges much greater than their separation, the two charges act like a single point charge.

23.23. **IDENTIFY:** For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: **(a)** The positions of the two charges are shown in Figure 23.23.

EXECUTE: **(b)** $V = \dfrac{kq}{r} + \dfrac{k(-q)}{r} = 0.$

(c) The potential along the x-axis is always zero, so a graph would be flat.

(d) If the two charges are interchanged, then the results of (b) and (c) still hold. The potential is zero.

EVALUATE: The potential is zero at any point on the x-axis because any point on the x-axis is equidistant from the two charges.

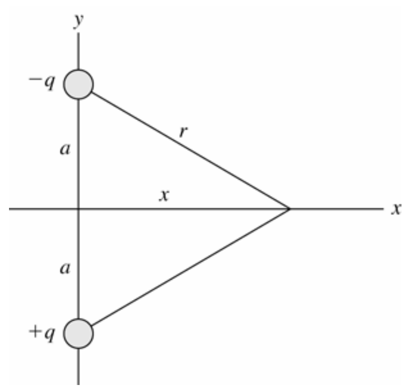


Figure 23.23

23.24. **IDENTIFY:** For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: Consider the distances from the point on the y-axis to each charge for the three regions $-a \le y \le a$ (between the two charges), $y > a$ (above both charges) and $y < -a$ (below both charges).

EXECUTE: **(a)** $|y| < a: V = \dfrac{kq}{(a+y)} - \dfrac{kq}{(a-y)} = \dfrac{2kqy}{y^2 - a^2}$. $y > a: V = \dfrac{kq}{(a+y)} - \dfrac{kq}{y-a} = \dfrac{-2kqa}{y^2-a^2}$.

$y < -a: V = \dfrac{-kq}{(a+y)} - \dfrac{kq}{(-y+a)} = \dfrac{2kqa}{y^2-a^2}$.

A general expression valid for any y is $V = k\left(\dfrac{-q}{|y-a|} + \dfrac{q}{|y+a|}\right)$.

(b) The graph of V versus y is sketched in Figure 23.24.

(c) $y >> a: V = \dfrac{-2kqa}{y^2-a^2} \approx \dfrac{-2kqa}{y^2}$.

(d) If the charges are interchanged, then the potential is of the opposite sign.

EVALUATE: $V = 0$ at $y = 0$. $V \to +\infty$ as the positive charge is approached and $V \to -\infty$ as the negative charge is approached.

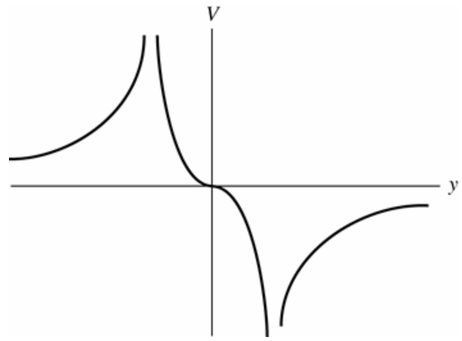


Figure 23.24

23.25. IDENTIFY: For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: **(a)** The positions of the two charges are shown in Figure 23.25a.

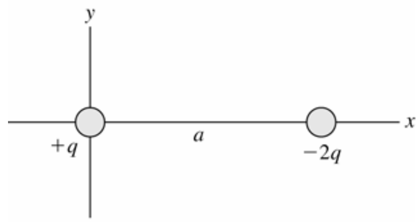


Figure 23.25a

(b) $x > a: V = \dfrac{kq}{x} - \dfrac{2kq}{x-a} = \dfrac{-kq(x+a)}{x(x-a)}$. $0 < x < a: V = \dfrac{kq}{x} - \dfrac{2kq}{a-x} = \dfrac{kq(3x-a)}{x(x-a)}$.

$x < 0: V = \dfrac{-kq}{x} + \dfrac{2kq}{x-a} = \dfrac{kq(x+a)}{x(x-a)}$. A general expression valid for any *y* is $V = k\left(\dfrac{q}{|x|} - \dfrac{2q}{|x-a|}\right)$.

(c) The potential is zero at $x = -a$ and $a/3$.

(d) The graph of *V* versus *x* is sketched in Figure 23.25b.

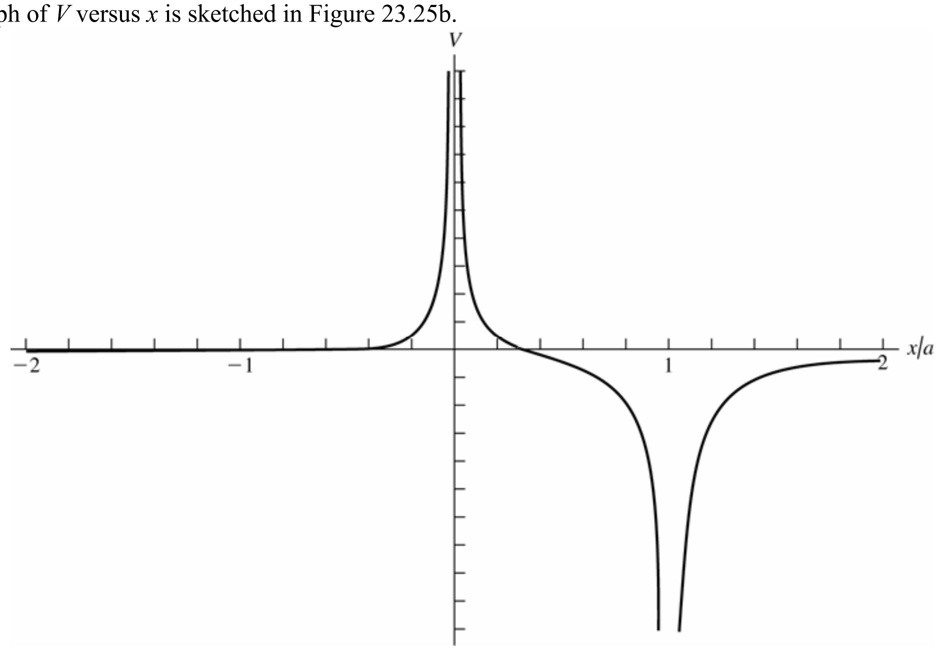


Figure 23.25b

EVALUATE: **(e)** For $x >> a: V \approx \dfrac{-kqx}{x^2} = \dfrac{-kq}{x}$, which is the same as the potential of a point charge –*q*. Far from the two charges they appear to be a point charge with a charge that is the algebraic sum of their two charges.

23.26. IDENTIFY: For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges.

SET UP: The distance of a point with coordinate y from the positive charge is $|y|$ and the distance from the negative charge is $r = \sqrt{a^2 + y^2}$.

EXECUTE: **(a)** $V = \dfrac{kq}{|y|} - \dfrac{2kq}{r} = kq\left(\dfrac{1}{|y|} - \dfrac{2}{\sqrt{a^2 + y^2}}\right).$

(b) $V = 0$, when $y^2 = \dfrac{a^2 + y^2}{4} \Rightarrow 3y^2 = a^2 \Rightarrow y = \pm\dfrac{a}{\sqrt{3}}.$

(c) The graph of V versus y is sketched in Figure 23.26. $V \to \infty$ as the positive charge at the origin is approached.

EVALUATE: **(d)** $y >> a : V \approx kq\left(\dfrac{1}{y} - \dfrac{2}{y}\right) = -\dfrac{kq}{y},$ which is the potential of a point charge $-q$. Far from the two charges they appear to be a point charge with a charge that is the algebraic sum of their two charges.

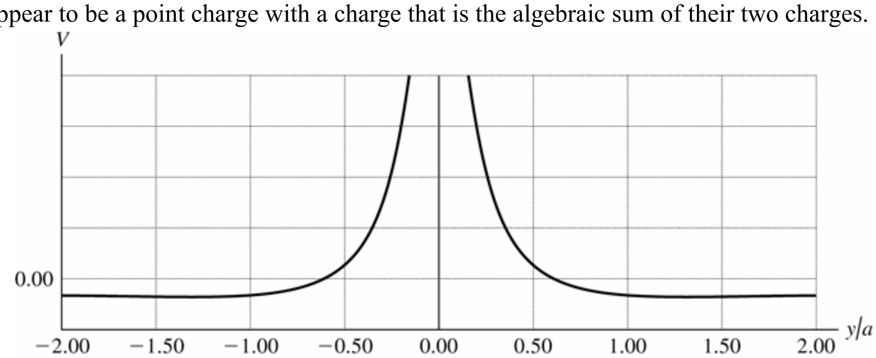


Figure 23.26

23.27. IDENTIFY: $K_a + qV_a = K_b + qV_b$.

SET UP: Let point a be at the cathode and let point b be at the anode. $K_a = 0$. $V_b - V_a = 295\ \text{V}$. An electron has $q = -e$ and $m = 9.11\times10^{-31}\ \text{kg}$.

EXECUTE: $K_b = q(V_a - V_b) = -(1.60\times10^{-19}\ \text{C})(-295\ \text{V}) = 4.72\times10^{-17}\ \text{J}$. $K_b = \dfrac{1}{2}mv_b^2$, so

$v_b = \sqrt{\dfrac{2(4.72\times10^{-17}\ \text{J})}{9.11\times10^{-31}\ \text{kg}}} = 1.01\times10^7\ \text{m/s}.$

EVALUATE: The negatively charged electron gains kinetic energy when it moves to higher potential.

23.28. IDENTIFY: For a point charge, $E = \dfrac{k|q|}{r^2}$ and $V = \dfrac{kq}{r}$.

SET UP: The electric field is directed toward a negative charge and away from a positive charge.

EXECUTE: **(a)** $V > 0$ so $q > 0$. $\dfrac{V}{E} = \dfrac{kq/r}{k|q|/r^2} = \left(\dfrac{kq}{r}\right)\left(\dfrac{r^2}{kq}\right) = r$. $r = \dfrac{4.98\ \text{V}}{12.0\ \text{V/m}} = 0.415\ \text{m}$.

(b) $q = \dfrac{rV}{k} = \dfrac{(0.415\ \text{m})(4.98\ \text{V})}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 2.30\times10^{-10}\ \text{C}$

(c) $q > 0$, so the electric field is directed away from the charge.

EVALUATE: The ratio of V to E due to a point charge increases as the distance r from the charge increases, because E falls off as $1/r^2$ and V falls off as $1/r$.

23.29. (a) IDENTIFY and **SET UP:** The direction of $\vec{E}$ is always from high potential to low potential so point b is at higher potential.

(b) Apply Eq.(23.17) to relate $V_b - V_a$ to E.

EXECUTE: $V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\,dx = E(x_b - x_a).$

$$E = \frac{V_b - V_a}{x_b - x_a} = \frac{+240\ \text{V}}{0.90\ \text{m} - 0.60\ \text{m}} = 800\ \text{V/m}$$

(c) $W_{b\to a} = q(V_b - V_a) = (-0.200\times10^{-6}\ \text{C})(+240\ \text{V}) = -4.80\times10^{-5}\ \text{J}$.

EVALUATE: The electric force does negative work on a negative charge when the negative charge moves from high potential (point *b*) to low potential (point *a*).

23.30. IDENTIFY: For a point charge, $V = \dfrac{kq}{r}$. The total potential at any point is the algebraic sum of the potentials of the two charges. For a point charge, $E = \dfrac{k|q|}{r^2}$. The net electric field is the vector sum of the electric fields of the two charges.

SET UP: $\vec{E}$ produced by a point charge is directed away from the point charge if it is positive and toward the charge if it is negative.

EXECUTE: **(a)** $V = V_Q + V_{2Q} > 0$, so *V* is zero nowhere except for infinitely far from the charges. The fields can cancel only between the charges, because only there are the fields of the two charges in opposite directions. Consider a point a distance *x* from *Q* and $d - x$ from 2*Q*, as shown in Figure 23.30a. $E_Q = E_{2Q} \to \dfrac{kQ}{x^2} = \dfrac{k(2Q)}{(d-x)^2} \to (d-x)^2 = 2x^2$.

$x = \dfrac{d}{1+\sqrt{2}}$. The other root, $x = \dfrac{d}{1-\sqrt{2}}$, does not lie between the charges.

(b) *V* can be zero in 2 places, *A* and *B*, as shown in Figure 23.30b. Point *A* is a distance *x* from $-Q$ and $d - x$ from 2*Q*. *B* is a distance *y* from $-Q$ and $d + y$ from 2*Q*. $At\ A: \dfrac{k(-Q)}{x} + \dfrac{k(2Q)}{d-x} = 0 \to x = d/3$.

$At\ B: \dfrac{k(-Q)}{y} + \dfrac{k(2Q)}{d+y} = 0 \to y = d$.

The two electric fields are in opposite directions to the left of $-Q$ or to the right of 2*Q* in Figure 23.30c. But for the magnitudes to be equal, the point must be closer to the charge with smaller magnitude of charge. This can be the case only in the region to the left of $-Q$. $E_Q = E_{2Q}$ gives $\dfrac{kQ}{x^2} = \dfrac{k(2Q)}{(d+x)^2}$ and $x = \dfrac{d}{\sqrt{2}-1}$.

EVALUATE: **(d)** *E* and *V* are not zero at the same places. $\vec{E}$ is a vector and *V* is a scalar. *E* is proportional to $1/r^2$ and *V* is proportional to $1/r$. $\vec{E}$ is related to the force on a test charge and ΔV is related to the work done on a test charge when it moves from one point to another.



Figure 23.30

23.31. IDENTIFY and **SET UP:** Apply conservation of energy, Eq.(23.3). Use Eq.(23.12) to express *U* in terms of *V*.

(a) EXECUTE: $K_1 + qV_1 = K_2 + qV_2$

$q(V_1 - V_2) = K_2 - K_1$; $\quad q = -1.602\times10^{-19}\ \text{C}$

$K_1 = \frac{1}{2}m_e v_1^2 = 4.099\times10^{-18}\ \text{J}$; $\quad K_2 = \frac{1}{2}m_e v_2^2 = 2.915\times10^{-17}\ \text{J}$

$V_1 - V_2 = \dfrac{K_2 - K_1}{q} = -156\ \text{V}$

EVALUATE: The electron gains kinetic energy when it moves to higher potential.

(b) EXECUTE: Now $K_1 = 2.915\times10^{-17}\ \text{J}, K_2 = 0$

$$V_1 - V_2 = \frac{K_2 - K_1}{q} = +182\ \text{V}$$

EVALUATE: The electron loses kinetic energy when it moves to lower potential.

23.32. IDENTIFY and **SET UP:** Expressions for the electric potential inside and outside a solid conducting sphere are derived in Example 23.8.

EXECUTE: **(a)** This is outside the sphere, so $V = \dfrac{kq}{r} = \dfrac{k(3.50\times10^{-9}\ \text{C})}{0.480\ \text{m}} = 65.6\ \text{V}$.

(b) This is at the surface of the sphere, so $V = \dfrac{k(3.50\times10^{-9}\ \text{C})}{0.240\ \text{m}} = 131\ \text{V}$.

(c) This is inside the sphere. The potential has the same value as at the surface, 131 V.

EVALUATE: All points of a conductor are at the same potential.

23.33. **(a) IDENTIFY and SET UP:** The electric field on the ring's axis is calculated in Example 21.10. The force on the electron exerted by this field is given by Eq.(21.3).

EXECUTE: When the electron is on either side of the center of the ring, the ring exerts an attractive force directed toward the center of the ring. This restoring force produces oscillatory motion of the electron along the axis of the ring, with amplitude 30.0 cm. The force on the electron is *not* of the form $F = -kx$ so the oscillatory motion is not simple harmonic motion.

(b) IDENTIFY: Apply conservation of energy to the motion of the electron.

SET UP: $K_a + U_a = K_b + U_b$ with *a* at the initial position of the electron and *b* at the center of the ring. From Example 23.11, $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{\sqrt{x^2+R^2}}$, where *R* is the radius of the ring.

EXECUTE: $x_a = 30.0 \text{ cm}, x_b = 0.$

$K_a = 0$ (released from rest), $K_b = \frac{1}{2}mv^2$

Thus $\frac{1}{2}mv^2 = U_a - U_b$

And $U = qV = -eV$ so $v = \sqrt{\dfrac{2e(V_b - V_a)}{m}}.$

$$V_a = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x_a^2+R^2}} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{24.0\times10^{-9}\text{ C}}{\sqrt{(0.300\text{ m})^2+(0.150\text{ m})^2}}$$

$$V_a = 643 \text{ V}$$

$$V_b = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x_b^2+R^2}} = (8.988\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\frac{24.0\times10^{-9}\text{ C}}{0.150\text{ m}} = 1438 \text{ V}$$

$$v = \sqrt{\frac{2e(V_b-V_a)}{m}} = \sqrt{\frac{2(1.602\times10^{-19}\text{ C})(1438\text{ V}-643\text{ V})}{9.109\times10^{-31}\text{ kg}}} = 1.67\times10^7 \text{ m/s}$$

EVALUATE: The positively charged ring attracts the negatively charged electron and accelerates it. The electron has its maximum speed at this point. When the electron moves past the center of the ring the force on it is opposite to its motion and it slows down.

23.34. **IDENTIFY:** Example 23.10 shows that for a line of charge, $V_a - V_b = \dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$. Apply conservation of energy to the motion of the proton.

SET UP: Let point *a* be 18.0 cm from the line and let point *b* be at the distance of closest approach, where $K_b = 0$.

EXECUTE: **(a)** $K_a = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\text{ kg})(1.50\times10^3\text{ m/s})^2 = 1.88\times10^{-21}\text{ J}$.

(b) $K_a + qV_a = K_b + qV_b$. $V_a - V_b = \dfrac{K_b - K_a}{q} = \dfrac{-1.88\times10^{-21}\text{ J}}{1.60\times10^{-19}\text{ C}} = -0.01175\text{ V}$. $\ln(r_b/r_a) = \left(\dfrac{2\pi\epsilon_0}{\lambda}\right)(-0.01175\text{ V})$.

$r_b = r_a\exp\left(\dfrac{2\pi\epsilon_0(-0.01175\text{ V})}{\lambda}\right) = (0.180\text{ m})\exp\left(-\dfrac{2\pi\epsilon_0(0.01175\text{ V})}{5.00\times10^{-12}\text{ C/m}}\right) = 0.158\text{ m}$.

EVALUATE: The potential increases with decreasing distance from the line of charge. As the positively charged proton approaches the line of charge it gains electrical potential energy and loses kinetic energy.

23.35. **IDENTIFY:** The voltmeter measures the potential difference between the two points. We must relate this quantity to the linear charge density on the wire.

SET UP: For a very long (infinite) wire, the potential difference between two points is $\Delta V = \dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$.

EXECUTE: **(a)** Solving for λ gives

$$\lambda = \frac{(\Delta V)2\pi\epsilon_0}{\ln(r_b/r_a)} = \frac{575\text{ V}}{(18\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\ln\left(\dfrac{3.50\text{ cm}}{2.50\text{ cm}}\right)} = 9.49\times10^{-8}\text{ C/m}$$

(b) The meter will read less than 575 V because the electric field is weaker over this 1.00-cm distance than it was over the 1.00-cm distance in part (a).

(c) The potential difference is zero because both probes are at the same distance from the wire, and hence at the same potential.

EVALUATE: Since a voltmeter measures potential difference, we are actually given ΔV, even though that is not stated explicitly in the problem. We must also be careful when using the formula for the potential difference because each *r* is the distance from the *center* of the cylinder, not from the surface.

23.36. **IDENTIFY:** The voltmeter reads the potential difference between the two points where the probes are placed. Therefore we must relate the potential difference to the distances of these points from the center of the cylinder. For points outside the cylinder, its electric field behaves like that of a line of charge.

SET UP: Using $\Delta V = \dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$ and solving for r_b, we have $r_b = r_a e^{2\pi\epsilon_0 \Delta V/\lambda}$.

EXECUTE: The exponent is $\dfrac{\left(\dfrac{1}{2\times 9.00\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}\right)(175\ \text{V})}{15.0\times 10^{-9}\ \text{C/m}} = 0.648$, which gives

$$r_b = (2.50\ \text{cm})\, e^{0.648} = 4.78\ \text{cm}.$$

The distance above the *surface* is 4.78 cm – 2.50 cm = 2.28 cm.

EVALUATE: Since a voltmeter measures potential difference, we are actually given ΔV, even though that is not stated explicitly in the problem. We must also be careful when using the formula for the potential difference because each r is the distance from the *center* of the cylinder, not from the surface.

23.37. **IDENTIFY:** For points outside the cylinder, its electric field behaves like that of a line of charge. Since a voltmeter reads potential difference, that is what we need to calculate.

SET UP: The potential difference is $\Delta V = \dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$.

EXECUTE: **(a)** Substituting numbers gives

$$\Delta V = \frac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a) = \left(8.50\times 10^{-6}\ \text{C/m}\right)\left(2\times 9.00\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\ln\left(\frac{10.0\ \text{cm}}{6.00\ \text{cm}}\right)$$

$$\Delta V = 7.82\times 10^4\ \text{V} = 78{,}200\ \text{V} = 78.2\ \text{kV}$$

(b) $E = 0$ inside the cylinder, so the potential is constant there, meaning that the voltmeter reads zero.

EVALUATE: Caution! The fact that the voltmeter reads zero in part (b) does not mean that $V = 0$ inside the cylinder. The electric field is zero, but the potential is constant and equal to the potential at the surface.

23.38. **IDENTIFY:** The work required is equal to the change in the electrical potential energy of the charge-ring system. We need only look at the beginning and ending points, since the potential difference is independent of path for a conservative field.

SET UP: **(a)** $W = \Delta U = q\Delta V = q\left(V_{\text{center}} - V_\infty\right) = q\left(\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{a} - 0\right)$

EXECUTE: Substituting numbers gives

$$\Delta U = (3.00\times 10^{-6}\ \text{C})(9.00\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times 10^{-6}\ \text{C})/(0.0400\ \text{m}) = 3.38\ \text{J}$$

(b) We can take any path since the potential is independent of path.

(c) SET UP: The net force is away from the ring, so the ball will accelerate away. Energy conservation gives $U_0 = K_{\text{max}} = \frac{1}{2}mv^2$.

EXECUTE: Solving for v gives

$$v = \sqrt{\frac{2U_0}{m}} = \sqrt{\frac{2(3.38\ \text{J})}{0.00150\ \text{kg}}} = 67.1\ \text{m/s}$$

EVALUATE: Direct calculation of the work from the electric field would be extremely difficult, and we would need to know the path followed by the charge. But, since the electric field is conservative, we can bypass all this calculation just by looking at the end points (infinity and the center of the ring) using the potential.

23.39. **IDENTIFY:** The electric field is zero everywhere except between the plates, and in this region it is uniform and points from the positive to the negative plate (to the left in Figure 23.32).

SET UP: Since the field is uniform between the plates, the potential increases linearly as we go from left to right starting at b.

EXECUTE: Since the potential is taken to be zero at the left surface of the negative plate (a in Figure 23.32), it is zero everywhere to the left of b. It increases linearly from b to c, and remains constant (since $E = 0$) past c. The graph is sketched in Figure 23.39.

EVALUATE: When the electric field is zero, the potential remains constant but not necessarily zero (as to the right of c). When the electric field is constant, the potential is linear.

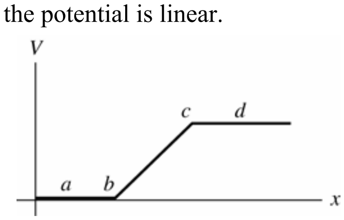


Figure 23.39

23.40. **IDENTIFY** and **SET UP:** For oppositely charged parallel plates, $E = \sigma / \epsilon_0$ between the plates and the potential difference between the plates is $V = Ed$.

EXECUTE: **(a)** $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{47.0\times10^{-9}\ \text{C/m}^2}{\epsilon_0} = 5310\ \text{N/C}.$

(b) $V = Ed = (5310\ \text{N/C})(0.0220\ \text{m}) = 117\ \text{V}.$

(c) The electric field stays the same if the separation of the plates doubles. The potential difference between the plates doubles.

EVALUATE: The electric field of an infinite sheet of charge is uniform, independent of distance from the sheet. The force on a test charge between the two plates is constant because the electric field is constant. The potential difference is the work per unit charge on a test charge when it moves from one plate to the other. When the distance doubles the work, which is force times distance, doubles and the potential difference doubles.

23.41. **IDENTIFY** and **SET UP:** Use the result of Example 23.9 to relate the electric field between the plates to the potential difference between them and their separation. The force this field exerts on the particle is given by Eq.(21.3). Use the equation that precedes Eq.(23.17) to calculate the work.

EXECUTE: **(a)** From Example 23.9, $E = \dfrac{V_{ab}}{d} = \dfrac{360\ \text{V}}{0.0450\ \text{m}} = 8000\ \text{V/m}$

(b) $F = |q|E = (2.40\times10^{-9}\ \text{C})(8000\ \text{V/m}) = +1.92\times10^{-5}\ \text{N}$

(c) The electric field between the plates is shown in Figure 23.41.

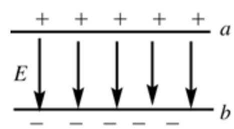


Figure 23.41

The plate with positive charge (plate *a*) is at higher potential. The electric field is directed from high potential toward low potential (or, $\vec{E}$ is from + charge toward − charge), so $\vec{E}$ points from *a* to *b*. Hence the force that $\vec{E}$ exerts on the positive charge is from *a* to *b*, so it does positive work.

$W = \int_a^b \vec{F}\cdot d\vec{l} = Fd$, where *d* is the separation between the plates.

$W = Fd = (1.92\times10^{-5}\ \text{N})(0.0450\ \text{m}) = +8.64\times10^{-7}\ \text{J}$

(d) $V_a - V_b = +360\ \text{V}$ (plate *a* is at higher potential)

$\Delta U = U_b - U_a = q(V_b - V_a) = (2.40\times10^{-9}\ \text{C})(-360\ \text{V}) = -8.64\times10^{-7}\ \text{J}.$

EVALUATE: We see that $W_{a\to b} = -(U_b - U_a) = U_a - U_b$.

23.42. **IDENTIFY:** The electric field is zero inside the sphere, so the potential is constant there. Thus the potential at the center must be the same as at the surface, where it is equivalent to that of a point-charge.

SET UP: At the surface, and hence also at the center of the sphere, the field is that of a point-charge, $E = Q/(4\pi\epsilon_0 R).$

EXECUTE: **(a)** Solving for *Q* and substituting the numbers gives

$$Q = 4\pi\epsilon_0 RV = (0.125\ \text{m})(1500\ \text{V})/(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2) = 2.08\times10^{-8}\ \text{C} = 20.8\ \text{nC}$$

(b) Since the potential is constant inside the sphere, its value at the surface must be the same as at the center, 1.50 kV.

EVALUATE: The electric field inside the sphere is zero, so the potential is constant but is not zero.

23.43. **IDENTIFY** and **SET UP:** Consider the electric field outside and inside the shell and use that to deduce the potential.

EXECUTE: **(a)** The electric field outside the shell is the same as for a point charge at the center of the shell, so the potential outside the shell is the same as for a point charge:

$$V = \frac{q}{4\pi\epsilon_0 r}\ \text{for}\ r > R.$$

The electric field is zero inside the shell, so no work is done on a test charge as it moves inside the shell and all points inside the shell are at the same potential as the surface of the shell: $V = \dfrac{q}{4\pi\epsilon_0 R}$ for $r \le R$.

(b) $V = \dfrac{kq}{R}$ so $q = \dfrac{RV}{k} = \dfrac{(0.15\ \text{m})(-1200\ \text{V})}{k} = -20\ \text{nC}$

(c) EVALUATE: No, the amount of charge on the sphere is very small. Since $U = qV$ the total amount of electric energy stored on the balloon is only $(20\ \text{nC})(1200\ \text{V}) = 2.4\times10^{-5}\ \text{J}.$

23.44. IDENTIFY: Example 23.8 shows that the potential of a solid conducting sphere is the same at every point inside the sphere and is equal to its value $V = q/2\pi\epsilon_0 R$ at the surface. Use the given value of E to find q.

SET UP: For negative charge the electric field is directed toward the charge.
For points outside this spherical charge distribution the field is the same as if all the charge were concentrated at the center.

EXECUTE: $E = \dfrac{q}{4\pi\epsilon_0 r^2}$ and $q = 4\pi\epsilon_0 E r^2 = \dfrac{(3800\ \text{N/C})(0.200\ \text{m})^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 1.69\times10^{-8}\ \text{C}$.

Since the field is directed inward, the charge must be negative. The potential of a point charge, taking ∞ as zero, is $V = \dfrac{q}{4\pi\epsilon_0 r} = \dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-1.69\times10^{-8}\ \text{C})}{0.200\ \text{m}} = -760\ \text{V}$ at the surface of the sphere. Since the charge all resides on the surface of a conductor, the field inside the sphere due to this symmetrical distribution is zero. No work is therefore done in moving a test charge from just inside the surface to the center, and the potential at the center must also be -760 V.

EVALUATE: Inside the sphere the electric field is zero and the potential is constant.

23.45. IDENTIFY: Example 23.9 shows that $V(y) = Ey$, where y is the distance from the negatively charged plate, whose potential is zero. The electric field between the plates is uniform and perpendicular to the plates.

SET UP: V increases toward the positively charged plate. $\vec{E}$ is directed from the positively charged plated toward the negatively charged plate.

EXECUTE: **(a)** $E = \dfrac{V}{d} = \dfrac{480\ \text{V}}{0.0170\ \text{m}} = 2.82\times10^4\ \text{V/m}$ and $y = \dfrac{V}{E}$. $V = 0$ at $y = 0$, $V = 120$ V at $y = 0.43$ cm, $V = 240$ V at $y = 0.85$ cm, $V = 360$ V at $y = 1.28$ cm and $V = 480$ V at $y = 1.70$ cm. The equipotential surfaces are sketched in Figure 23.45. The surfaces are planes parallel to the plates.

(b) The electric field lines are also shown in Figure 23.45. The field lines are perpendicular to the plates and the equipotential lines are parallel to the plates, so the electric field lines and the equipotential lines are mutually perpendicular.

EVALUATE: Only differences in potential have physical significance. Letting $V = 0$ at the negative plate is a choice we are free to make.

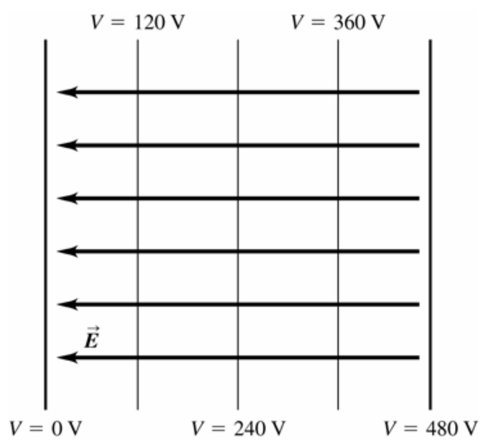


Figure 23.45

23.46. IDENTIFY: By the definition of electric potential, if a positive charge gains potential along a path, then the potential along that path must have increased. The electric field produced by a very large sheet of charge is uniform and is independent of the distance from the sheet.

(a) SET UP: No matter what the reference point, we must do work on a positive charge to move it away from the negative sheet.

EXECUTE: Since we must do work on the positive charge, it gains potential energy, so the potential increases.

(b) SET UP: Since the electric field is uniform and is equal to $\sigma/2\varepsilon_0$, we have $\Delta V = Ed = \dfrac{\sigma}{2\epsilon_0} d$.

EXECUTE: Solving for d gives

$$d = \frac{2\epsilon_0 \Delta V}{\sigma} = \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(1.00\,\text{V})}{6.00\times10^{-9}\ \text{C/m}^2} = 0.00295\ \text{m} = 2.95\ \text{mm}$$

EVALUATE: Since the spacing of the equipotential surfaces (d = 2.95 mm) is independent of the distance from the sheet, the equipotential surfaces are planes parallel to the sheet and spaced 2.95 mm apart.

23.47 **IDENTIFY** and **SET UP:** Use Eq.(23.19) to calculate the components of $\vec{E}$.

EXECUTE: $V = Axy - Bx^2 + Cy$

(a) $E_x = -\dfrac{\partial V}{\partial x} = -Ay + 2Bx$

$Ey = -\dfrac{\partial V}{\partial y} = -Ax - C$

$E_z = -\dfrac{\partial V}{\partial z} = 0$

(b) $E = 0$ requires that $E_x = E_y = E_z = 0$.

$E_z = 0$ everywhere.

$E_y = 0$ at $x = -C/A$.

And E_x is also equal zero for this *x,* any value of *z,* and $y = 2Bx/A = (2B/A)(-C/A) = -2BC/A^2$.

EVALUATE: *V* doesn't depend on *z* so $E_z = 0$ everywhere.

23.48. **IDENTIFY:** Apply Eq.(21.19).

SET UP: Eq.(21.7) says $\vec{E} = \dfrac{1}{4\pi\epsilon_0}\dfrac{|q|}{r^2}\hat{\boldsymbol{r}}$ is the electric field due to a point charge *q*.

EXECUTE: **(a)** $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\partial}{\partial x}\left(\dfrac{kQ}{\sqrt{x^2+y^2+z^2}}\right) = \dfrac{kQx}{(x^2+y^2+z^2)^{3/2}} = \dfrac{kQx}{r^3}$.

Similarly, $E_y = \dfrac{kQy}{r^3}$ and $E_z = \dfrac{kQz}{r^3}$.

(b) From part (a), $E = \dfrac{kQ}{r^2}\left(\dfrac{x\hat{\boldsymbol{i}}}{r} + \dfrac{y\hat{\boldsymbol{j}}}{r} + \dfrac{z\hat{\boldsymbol{k}}}{r}\right) = \dfrac{kQ}{r^2}\hat{\boldsymbol{r}}$, which agrees with Equation (21.7).

EVALUATE: *V* is a scalar. $\vec{E}$ is a vector and has components.

23.49. **IDENTIFY** and **SET UP:** For a solid metal sphere or for a spherical shell, $V = \dfrac{kq}{r}$ outside the sphere and $V = \dfrac{kq}{R}$ at all points inside the sphere, where *R* is the radius of the sphere. When the electric field is radial, $E = -\dfrac{\partial V}{\partial r}$.

EXECUTE: **(a)** (i) $r < r_a$: This region is inside both spheres. $V = \dfrac{kq}{r_a} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$.

(ii) $r_a < r < r_b$: This region is outside the inner shell and inside the outer shell. $V = \dfrac{kq}{r} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right)$.

(iii) $r > r_b$: This region is outside both spheres and $V = 0$ since outside a sphere the potential is the same as for point charge. Therefore the potential is the same as for two oppositely charged point charges at the same location. These potentials cancel.

(b) $V_a = \dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q}{r_a} - \dfrac{q}{r_b}\right)$ and $V_b = 0$, so $V_{ab} = \dfrac{1}{4\pi\epsilon_0}q\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$.

(c) Between the spheres $r_a < r < r_b$ and $V = kq\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right)$. $E = -\dfrac{\partial V}{\partial r} = -\dfrac{q}{4\pi\epsilon_0}\dfrac{\partial}{\partial r}\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right) = +\dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r^2} = \dfrac{V_{ab}}{\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)}\dfrac{1}{r^2}$.

(d) From Equation (23.23): $E = 0$, since *V* is constant (zero) outside the spheres.

(e) If the outer charge is different, then outside the outer sphere the potential is no longer zero but is $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{r} - \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r} = \dfrac{1}{4\pi\epsilon_0}\dfrac{(q-Q)}{r}$. All potentials inside the outer shell are just shifted by an amount $V = -\dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r_b}$. Therefore relative potentials within the shells are not affected. Thus (b) and (c) do not change.

However, now that the potential does vary outside the spheres, there is an electric field there:

$E = -\dfrac{\partial V}{\partial r} = -\dfrac{\partial}{\partial r}\left(\dfrac{kq}{r} + \dfrac{-kQ}{r}\right) = \dfrac{kq}{r^2}\left(1 - \dfrac{Q}{q}\right) = \dfrac{k}{r^2}(q-Q)$.

EVALUATE: In part (a) the potential is greater than zero for all $r < r_b$.

23.50. **IDENTIFY:** Exercise 23.49 shows that $V = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$ for $r < r_a$, $V = kq\left(\frac{1}{r} - \frac{1}{r_b}\right)$ for $r_a < r < r_b$ and $V_{ab} = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$.

SET UP: $E = \frac{kq}{r^2}$, radially outward, for $r_a \le r \le r_b$

EXECUTE: **(a)** $V_{ab} = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = 500\text{ V}$ gives $q = \frac{500\text{ V}}{k\left(\frac{1}{0.012\text{ m}} - \frac{1}{0.096\text{ m}}\right)} = 7.62\times10^{-10}\text{ C}$.

(b) $V_b = 0$ so $V_a = 500\text{ V}$. The inner metal sphere is an equipotential with $V = 500\text{ V}$. $\frac{1}{r} = \frac{1}{r_a} + \frac{V}{kq}$. $V = 400\text{ V}$ at $r = 1.45\text{ cm}$, $V = 300\text{ V}$ at $r = 1.85\text{ cm}$, $V = 200\text{ V}$ at $r = 2.53\text{ cm}$, $V = 100\text{ V}$ at $r = 4.00\text{ cm}$, $V = 0$ at $r = 9.60\text{ cm}$. The equipotential surfaces are sketched in Figure 23.50.

EVALUATE: **(c)** The equipotential surfaces are concentric spheres and the electric field lines are radial, so the field lines and equipotential surfaces are mutually perpendicular. The equipotentials are closest at smaller *r*, where the electric field is largest.

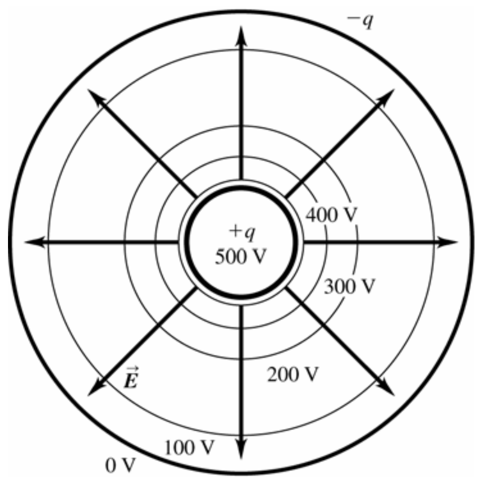


Figure 23.50

23.51. **IDENTIFY:** Outside the cylinder it is equivalent to a line of charge at its center.

SET UP: The difference in potential between the surface of the cylinder (a distance *R* from the central axis) and a general point a distance *r* from the central axis is given by $\Delta V = \frac{\lambda}{2\pi\epsilon_0}\ln(r/R)$.

EXECUTE: **(a)** The potential difference depends only on *r*, and not direction. Therefore all points at the same value of *r* will be at the same potential. Thus the equipotential surfaces are cylinders coaxial with the given cylinder.

(b) Solving $\Delta V = \frac{\lambda}{2\pi\epsilon_0}\ln(r/R)$ for *r*, gives $r = R\,e^{2\pi\epsilon_0\Delta V/\lambda}$.

For 10 V, the exponent is $(10\text{ V})/[(2 \times 9.00 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.50 \times 10^{-9}\text{ C/m})] = 0.370$, which gives $r = (2.00\text{ cm})\,e^{0.370} = 2.90$ cm. Likewise, the other radii are 4.20 cm (for 20 V) and 6.08 cm (for 30 V).

(c) $\Delta r_1 = 2.90\text{ cm} - 2.00\text{ cm} = 0.90\text{ cm}$; $\Delta r_2 = 4.20\text{ cm} - 2.90\text{ cm} = 1.30\text{ cm}$; $\Delta r_3 = 6.08\text{ cm} - 4.20\text{ cm} = 1.88\text{ cm}$

EVALUATE: As we can see, Δr increases, so the surfaces get farther apart. This is very different from a sheet of charge, where the surfaces are equally spaced planes.

23.52. **IDENTIFY:** The electric field is the negative gradient of the potential.

SET UP: $E_x = -\frac{\partial V}{\partial x}$, so E_x is the negative slope of the graph of *V* as a function of *x*.

EXECUTE: The graph is sketched in Figure 23.52. Up to *a*, *V* is constant, so $E_x = 0$. From *a* to *b*, *V* is linear with a positive slope, so E_x is a negative constant. Past *b*, the *V*-*x* graph has a decreasing positive slope which approaches zero, so E_x is negative and approaches zero.

EVALUATE: Notice that an increasing potential does not necessarily mean that the electric field is increasing.

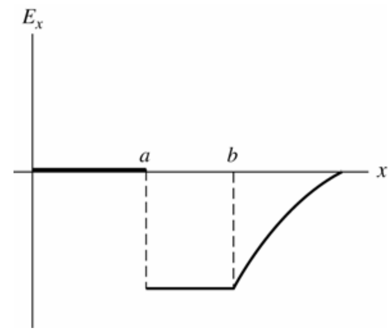


Figure 23.52

23.53. **(a) IDENTIFY:** Apply the work-energy theorem, Eq.(6.6).

SET UP: Points *a* and *b* are shown in Figure 23.53a.

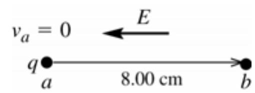


Figure 23.53a

EXECUTE: $W_{\text{tot}} = \Delta K = K_b - K_a = K_b = 4.35\times10^{-5}\ \text{J}$

The electric force F_E and the additional force F both do work, so that $W_{\text{tot}} = W_{F_E} + W_F$.

$$W_{F_E} = W_{\text{tot}} - W_F = 4.35\times10^{-5}\,\text{J} - 6.50\times10^{-5}\ \text{J} = -2.15\times10^{-5}\ \text{J}$$

EVALUATE: The forces on the charged particle are shown in Figure 23.53b.

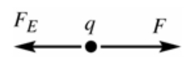


Figure 23.53b

The electric force is to the left (in the direction of the electric field since the particle has positive charge). The displacement is to the right, so the electric force does negative work. The additional force F is in the direction of the displacement, so it does positive work.

(b) IDENTIFY and **SET UP:** For the work done by the electric force, $W_{a\to b} = q(V_a - V_b)$

EXECUTE: $V_a - V_b = \dfrac{W_{a\to b}}{q} = \dfrac{-2.15\times10^{-5}\ \text{J}}{7.60\times10^{-9}\ \text{C}} = -2.83\times10^{3}\ \text{V}.$

EVALUATE: The starting point (point *a*) is at $2.83\times10^3\ \text{V}$ lower potential than the ending point (point *b*). We know that $V_b > V_a$ because the electric field always points from high potential toward low potential.

(c) IDENTIFY: Calculate *E* from $V_a - V_b$ and the separation *d* between the two points.

SET UP: Since the electric field is uniform and directed opposite to the displacement $W_{a\to b} = -F_E d = -qEd$, where $d = 8.00\ \text{cm}$ is the displacement of the particle.

EXECUTE: $E = -\dfrac{W_{a\to b}}{qd} = -\dfrac{V_a - V_b}{d} = \dfrac{-2.83\times10^3\ \text{V}}{0.0800\ \text{m}} = 3.54\times10^4\ \text{V/m}.$

EVALUATE: In part (a), W_{tot} is the total work done by both forces. In parts (b) and (c) $W_{a\to b}$ is the work done just by the electric force.

23.54. **IDENTIFY:** The electric force between the electron and proton is attractive and has magnitude $F = \dfrac{ke^2}{r^2}$. For circular motion the acceleration is $a_{\text{rad}} = v^2/r$. $U = -k\dfrac{e^2}{r}$.

SET UP: $e = 1.60\times10^{-19}\ \text{C}$. $1\ \text{eV} = 1.60\times10^{-19}\ \text{J}$.

EXECUTE: **(a)** $\dfrac{mv^2}{r} = \dfrac{ke^2}{r^2}$ and $v = \sqrt{\dfrac{ke^2}{mr}}$.

(b) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\dfrac{ke^2}{r} = -\dfrac{1}{2}U$

(c) $E = K + U = \frac{1}{2}U = -\frac{1}{2}\frac{ke^2}{r} = -\frac{1}{2}\frac{k(1.60\times10^{-19}\text{ C})^2}{5.29\times10^{-11}\text{ m}} = -2.17\times10^{-18}\text{ J} = -13.6\text{ eV}$.

EVALUATE: The total energy is negative, so the electron is bound to the proton. Work must be done on the electron to take it far from the proton.

23.55. **IDENTIFY** and **SET UP:** Calculate the components of $\vec{E}$ from Eq.(23.19). Eq.(21.3) gives $\vec{F}$ from $\vec{E}$.

EXECUTE: **(a)** $V = Cx^{4/3}$

$$C = V/x^{4/3} = 240\text{ V}/(13.0\times10^{-3}\text{ m})^{4/3} = 7.85\times10^{4}\text{ V/m}^{4/3}$$

(b) $E_x = -\frac{\partial V}{\partial x} = -\frac{4}{3}Cx^{1/3} = -(1.05\times10^{5}\text{ V/m}^{4/3})x^{1/3}$

The minus sign means that E_x is in the $-x$-direction, which says that $\vec{E}$ points from the positive anode toward the negative cathode.

(c) $\vec{F} = q\vec{E}$ so $F_x = -eE_x = \frac{4}{3}eCx^{1/3}$

Halfway between the electrodes means $x = 6.50\times10^{-3}$ m.

$$F_x = \tfrac{4}{3}(1.602\times10^{-19}\text{ C})(7.85\times10^{4}\text{ V/m}^{4/3})(6.50\times10^{-3}\text{ m})^{1/3} = 3.13\times10^{-15}\text{ N}$$

F_x is positive, so the force is directed toward the positive anode.

EVALUATE: V depends only on x, so $E_y = E_z = 0$. $\vec{E}$ is directed from high potential (anode) to low potential (cathode). The electron has negative charge, so the force on it is directed opposite to the electric field.

23.56. **IDENTIFY:** At each point (a and b), the potential is the sum of the potentials due to *both* spheres. The voltmeter reads the difference between these two potentials. The spheres behave like a point-charges since the meter is connected to the surface of each one.

SET UP: **(a)** Call a the point on the surface of one sphere and b the point on the surface of the other sphere, call r the radius of each sphere, and call d the center-to-center distance between the spheres. The potential difference V_{ab} between points a and b is then

$$V_b - V_a = V_{ab} = \frac{1}{4\pi\epsilon_0}\left[\frac{-q}{r} + \frac{q}{d-r} - \left(\frac{q}{r} + \frac{-q}{d-r}\right)\right] = \frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{d-r} - \frac{1}{r}\right)$$

EXECUTE: Substituting the numbers gives

$$V_b - V_a = 2(175\text{ }\mu\text{C})\left(9.00\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1}{0.750\text{ m}} - \frac{1}{0.250\text{ m}}\right) = -8.40\times10^{6}\text{ V}$$

The meter reads 8.40 MV.

(b) Since $V_b - V_a$ is negative, $V_a > V_b$, so point a is at the higher potential.

EVALUATE: An easy way to see that the potential at a is higher than the potential at b is that it would take positive work to move a positive test charge from b to a since this charge would be attracted by the negative sphere and repelled by the positive sphere.

23.57. **IDENTIFY:** $U = \frac{kq_1q_2}{r}$

SET UP: Eight charges means there are $8(8-1)/2 = 28$ pairs. There are 12 pairs of q and $-q$ separated by d, 12 pairs of equal charges separated by $\sqrt{2}d$ and 4 pairs of q and $-q$ separated by $\sqrt{3}d$.

EXECUTE: **(a)** $U = kq^2\left(-\frac{12}{d} + \frac{12}{\sqrt{2}d} - \frac{4}{\sqrt{3}d}\right) = -\frac{12kq^2}{d}\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right) = -1.46q^2/\pi\epsilon_0 d$

EVALUATE: **(b)** The fact that the electric potential energy is less than zero means that it is energetically favorable for the crystal ions to be together.

23.58. **IDENTIFY:** For two small spheres, $U = \frac{kq_1q_2}{r}$. For part (b) apply conservation of energy.

SET UP: Let $q_1 = 2.00\text{ }\mu\text{C}$ and $q_2 = -3.50\text{ }\mu\text{C}$. Let $r_a = 0.250$ m and $r_b \to \infty$.

EXECUTE: **(a)** $U = \frac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(2.00\times10^{-6}\text{ C})(-3.50\times10^{-6}\text{ C})}{0.250\text{ m}} = -0.252\text{ J}$

(b) $K_b = 0$. $U_b = 0$. $U_a = -0.252$ J. $K_a + U_a = K_b + U_b$ gives $K_a = 0.252$ J. $K_a = \frac{1}{2}mv_a^2$, so

$v_a = \sqrt{\frac{2K_a}{m}} = \sqrt{\frac{2(0.252\text{ J})}{1.50\times10^{-3}\text{ kg}}} = 18.3\text{ m/s}$

EVALUATE: As the sphere moves away, the attractive electrical force exerted by the other sphere does negative work and removes all the kinetic energy it initially had. Note that it doesn't matter which sphere is held fixed and which is shot away; the answer to part (b) is unaffected.

23.59. **(a) IDENTIFY:** Use Eq.(23.10) for the electron and each proton.

SET UP: The positions of the particles are shown in Figure 23.59a.



$r = (1.07\times10^{-10}\ \text{m})/2 = 0.535\times10^{-10}\ \text{m}$

Figure 23.59a

EXECUTE: The potential energy of interaction of the electron with each proton is

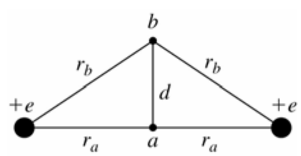
$U = \dfrac{1}{4\pi\epsilon_0}\dfrac{(-e^2)}{r}$, so the total potential energy is

$$U = -\frac{2e^2}{4\pi\epsilon_0 r} = -\frac{2(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{0.535\times10^{-10}\ \text{m}} = -8.60\times10^{-18}\ \text{J}$$

$$U = -8.60\times10^{-18}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = -53.7\ \text{eV}$$

EVALUATE: The electron and proton have charges of opposite signs, so the potential energy of the system is negative.

(b) IDENTIFY and **SET UP:** The positions of the protons and points *a* and *b* are shown in Figure 23.59b.



$r_b = \sqrt{r_a^2 + d^2}$

$r_a = r = 0.535\times10^{-10}\ \text{m}$

Figure 23.59b

Apply $K_a + U_a + W_{\text{other}} = K_b + U_b$ with point *a* midway between the protons and point *b* where the electron instantaneously has $v = 0$ (at its maximum displacement *d* from point *a*).

EXECUTE: Only the Coulomb force does work, so $W_{\text{other}} = 0$.

$U_a = -8.60\times10^{-18}\ \text{J}$ (from part (a))

$K_a = \frac{1}{2}mv^2 = \frac{1}{2}(9.109\times10^{-31}\ \text{kg})(1.50\times10^6\ \text{m/s})^2 = 1.025\times10^{-18}\ \text{J}$

$K_b = 0$

$U_b = -2ke^2/r_b$

Then $U_b = K_a + U_a - K_b = 1.025\times10^{-18}\ \text{J} - 8.60\times10^{-18}\ \text{J} = -7.575\times10^{-18}\ \text{J}$.

$$r_b = -\frac{2ke^2}{U_b} = -\frac{2(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{-7.575\times10^{-18}\ \text{J}} = 6.075\times10^{-11}\ \text{m}$$

Then $d = \sqrt{r_b^2 - r_a^2} = \sqrt{(6.075\times10^{-11}\ \text{m})^2 - (5.35\times10^{-11}\ \text{m})^2} = 2.88\times10^{-11}\ \text{m}$.

EVALUATE: The force on the electron pulls it back toward the midpoint. The transverse distance the electron moves is about 0.27 times the separation of the protons.

23.60. **IDENTIFY:** Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere. The electric force on the sphere is $F_e = qE$. The potential difference between the plates is $V = Ed$.

SET UP: The free-body diagram for the sphere is given in Figure 23.56.

EXECUTE: $T\cos\theta = mg$ and $T\sin\theta = F_e$ gives $F_e = mg\tan\theta = (1.50\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)\tan(30°) = 0.0085\ \text{N}$.

$F_e = Eq = \dfrac{Vq}{d}$ and $V = \dfrac{Fd}{q} = \dfrac{(0.0085\ \text{N})(0.0500\ \text{m})}{8.90\times10^{-6}\ \text{C}} = 47.8\ \text{V}$.

EVALUATE: $E = V/d = 956\ \text{V/m}$. $E = \sigma/\epsilon_0$ and $\sigma = E\epsilon_0 = 8.46\times10^{-9}\ \text{C/m}^2$.

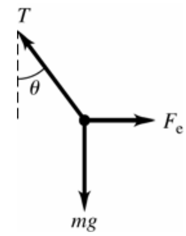


Figure 23.60

23.61. **(a) IDENTIFY:** The potential at any point is the sum of the potentials due to each of the two charged conductors.
SET UP: From Example 23.10, for a conducting cylinder with charge per unit length λ the potential outside the cylinder is given by $V = (\lambda/2\pi\epsilon_0)\ln(r_0/r)$ where r is the distance from the cylinder axis and r_0 is the distance from the axis for which we take $V = 0$. Inside the cylinder the potential has the same value as on the cylinder surface. The electric field is the same for a solid conducting cylinder or for a hollow conducting tube so this expression for V applies to both. This problem says to take $r_0 = b$.

EXECUTE: For the hollow tube of radius b and charge per unit length $-\lambda$: outside $V = -(\lambda/2\pi\epsilon_0)\ln(b/r)$; inside $V = 0$ since $V = 0$ at $r = b$.
For the metal cylinder of radius a and charge per unit length λ:
outside $V = (\lambda/2\pi\epsilon_0)\ln(b/r)$, inside $V = (\lambda/2\pi\epsilon_0)\ln(b/a)$, the value at $r = a$.
(i) $r < a$; inside both $V = (\lambda/2\pi\epsilon_0)\ln(b/a)$
(ii) $a < r < b$; outside cylinder, inside tube $V = (\lambda/2\pi\epsilon_0)\ln(b/r)$
(iii) $r > b$; outside both the potentials are equal in magnitude and opposite in sign so $V = 0$.
(b) For $r = a$, $V_a = (\lambda/2\pi\epsilon_0)\ln(b/a)$.
For $r = b$, $V_b = 0$.
Thus $V_{ab} = V_a - V_b = (\lambda/2\pi\epsilon_0)\ln(b/a)$.
(c) IDENTIFY and **SET UP:** Use Eq.(23.23) to calculate E.

EXECUTE: $E = -\dfrac{\partial V}{\partial r} = -\dfrac{\lambda}{2\pi\epsilon_0}\dfrac{\partial}{\partial r}\ln\left(\dfrac{b}{r}\right) = -\dfrac{\lambda}{2\pi\epsilon_0}\left(\dfrac{r}{b}\right)\left(-\dfrac{b}{r^2}\right) = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$.

(d) The electric field between the cylinders is due only to the inner cylinder, so V_{ab} is not changed, $V_{ab} = (\lambda/2\pi\epsilon_0)\ln(b/a)$.

EVALUATE: The electric field is not uniform between the cylinders, so $V_{ab} \neq E(b-a)$.

23.62. **IDENTIFY:** The wire and hollow cylinder form coaxial cylinders. Problem 23.61 gives $E(r) = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$.

SET UP: $a = 145\times10^{-6}$ m, $b = 0.0180$ m.

EXECUTE: $E = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$ and $V_{ab} = E\ln(b/a)r = (2.00\times10^4\ \text{N/C})(\ln(0.018\ \text{m}/145\times10^{-6}\ \text{m}))0.012\ \text{m} = 1157\ \text{V}$.

EVALUATE: The electric field at any r is directly proportional to the potential difference between the wire and the cylinder.

23.63. **IDENTIFY** and **SET UP:** Use Eq.(21.3) to calculate $\vec{F}$ and then $\vec{F} = m\vec{a}$ gives $\vec{a}$.

EXECUTE: **(a)** $\vec{F}_E = q\vec{E}$. Since $q = -e$ is negative $\vec{F}_E$ and $\vec{E}$ are in opposite directions; $\vec{E}$ is upward so $\vec{F}_E$ is downward. The magnitude of F_E is

$$F_E = |q|E = eE = (1.602\times10^{-19}\ \text{C})(1.10\times10^3\ \text{N/C}) = 1.76\times10^{-16}\ \text{N}.$$

(b) Calculate the acceleration of the electron produced by the electric force:

$$a = \frac{F}{m} = \frac{1.76\times10^{-16}\ \text{N}}{9.109\times10^{-31}\ \text{kg}} = 1.93\times10^{14}\ \text{m/s}^2$$

EVALUATE: This is much larger than $g = 9.80\ \text{m/s}^2$, so the gravity force on the electron can be neglected. $\vec{F}_E$ is downward, so $\vec{a}$ is downward.
(c) IDENTIFY and **SET UP:** The acceleration is constant and downward, so the motion is like that of a projectile. Use the horizontal motion to find the time and then use the time to find the vertical displacement.

EXECUTE: <u>x-component</u>

$v_{0x} = 6.50\times10^6$ m/s; $a_x = 0$; $x - x_0 = 0.060$ m; $t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ and the a_x term is zero, so

$$t = \frac{x - x_0}{v_{0x}} = \frac{0.060\text{ m}}{6.50\times10^6\text{ m/s}} = 9.231\times10^{-9}\text{ s}$$

<u>y-component</u>

$v_{0y} = 0$; $a_y = 1.93\times10^{14}$ m/s^2; $t = 9.231\times10^{-9}$ m/s; $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$

$y - y_0 = \frac{1}{2}(1.93\times10^{14}\text{ m/s}^2)(9.231\times10^{-9}\text{ s})^2 = 0.00822\text{ m} = 0.822\text{ cm}$

(d) The velocity and its components as the electron leaves the plates are sketched in Figure 23.63.

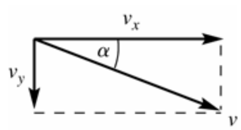


Figure 23.63

$v_x = v_{0x} = 6.50\times10^6$ m/s (since $a_x = 0$)

$v_y = v_{0y} + a_y t$

$v_y = 0 + (1.93\times10^{14}\text{ m/s}^2)(9.231\times10^{-9}\text{ s})$

$v_y = 1.782\times10^6$ m/s

$$\tan\alpha = \frac{v_y}{v_x} = \frac{1.782\times10^6\text{ m/s}}{6.50\times10^6\text{ m/s}} = 0.2742 \text{ so } \alpha = 15.3°.$$

EVALUATE: The greater the electric field or the smaller the initial speed the greater the downward deflection.

(e) IDENTIFY and SET UP: Consider the motion of the electron after it leaves the region between the plates. Outside the plates there is no electric field, so $a = 0$. (Gravity can still be neglected since the electron is traveling at such high speed and the times are small.) Use the horizontal motion to find the time it takes the electron to travel 0.120 m horizontally to the screen. From this time find the distance downward that the electron travels.

EXECUTE: <u>x-component</u>

$v_{0x} = 6.50\times10^6$ m/s; $a_x = 0$; $x - x_0 = 0.120$ m; $t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ and the a_x term is term is zero, so

$$t = \frac{x - x_0}{v_{0x}} = \frac{0.120\text{ m}}{6.50\times10^6\text{ m/s}} = 1.846\times10^{-8}\text{ s}$$

<u>y-component</u>

$v_{0y} = 1.782\times10^6$ m/s (from part (b)); $a_y = 0$; $t = 1.846\times10^{-8}$ m/s; $y - y_0 = ?$

$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2 = (1.782\times10^6\text{ m/s})(1.846\times10^{-8}\text{ s}) = 0.0329\text{ m} = 3.29\text{ cm}$

EVALUATE: The electron travels downward a distance 0.822 cm while it is between the plates and a distance 3.29 cm while traveling from the edge of the plates to the screen. The total downward deflection is $0.822\text{ cm} + 3.29\text{ cm} = 4.11\text{ cm}$.

The horizontal distance between the plates is half the horizontal distance the electron travels after it leaves the plates. And the vertical velocity of the electron increases as it travels between the plates, so it makes sense for it to have greater downward displacement during the motion after it leaves the plates.

23.64. IDENTIFY: The charge on the plates and the electric field between them depend on the potential difference across the plates. Since we do not know the numerical potential, we shall call this potential V and find the answers in terms of V.

(a) SET UP: For two parallel plates, the potential difference between them is $V = Ed = \frac{\sigma}{\epsilon_0}d = \frac{Qd}{\epsilon_0 A}$.

EXECUTE: Solving for Q gives

$$Q = \epsilon_0 AV/d = (8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.030\text{ m})^2 V/(0.0050\text{ m})$$

$$Q = 1.59V\times10^{-12}\text{ C} = 1.59V\text{ pC, when } V \text{ is in volts.}$$

(b) $E = V/d = V/(0.0050\text{ m}) = 200V$ V/m, with V in volts.

(c) SET UP: Energy conservation gives $\frac{1}{2}mv^2 = eV$.

EXECUTE: Solving for v gives

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})V}{9.11\times10^{-31}\text{ kg}}} = 5.93\times10^5 V^{1/2}\text{ m/s, with } V \text{ in volts}$$

EVALUATE: Typical voltages in student laboratory work run up to around 25 V, so the charge on the plates is typically about around 40 pC, the electric field is about 5000 V/m, and the electron speed would be about 3 million m/s.

23.65. **(a) IDENTIFY** and **SET UP:** Problem 23.61 derived that $E = \dfrac{V_{ab}}{\ln(b/a)}\dfrac{1}{r}$, where *a* is the radius of the inner cylinder (wire) and *b* is the radius of the outer hollow cylinder. The potential difference between the two cylinders is V_{ab}. Use this expression to calculate *E* at the specified *r*.

EXECUTE: Midway between the wire and the cylinder wall is at a radius of $r = (a+b)/2 = (90.0\times10^{-6}\ \text{m} + 0.140\ \text{m})/2 = 0.07004\ \text{m}.$

$$E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r} = \frac{50.0\times10^{3}\ \text{V}}{\ln(0.140\ \text{m}/90.0\times10^{-6}\ \text{m})(0.07004\ \text{m})} = 9.71\times10^{4}\ \text{V/m}$$

(b) IDENTIFY and **SET UP:** The electric force is given by Eq.(21.3). Set this equal to ten times the weight of the particle and solve for $|q|$, the magnitude of the charge on the particle.

EXECUTE: $F_E = 10mg$

$$|q|E = 10mg \text{ and } |q| = \frac{10mg}{E} = \frac{10(30.0\times10^{-9}\ \text{kg})(9.80\ \text{m/s}^2)}{9.71\times10^{4}\ \text{V/m}} = 3.03\times10^{-11}\ \text{C}$$

EVALUATE: It requires only this modest net charge for the electric force to be much larger than the weight.

23.66. **(a) IDENTIFY:** Calculate the potential due to each thin ring and integrate over the disk to find the potential. *V* is a scalar so no components are involved.

SET UP: Consider a thin ring of radius *y* and width *dy*. The ring has area $2\pi y\,dy$ so the charge on the ring is $dq = \sigma(2\pi y\,dy)$.

EXECUTE: The result of Example 23.11 then says that the potential due to this thin ring at the point on the axis at a distance *x* from the ring is

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{\sqrt{x^2+y^2}} = \frac{2\pi\sigma}{4\pi\epsilon_0}\frac{y\,dy}{\sqrt{x^2+y^2}}$$

$$V = \int dV = \frac{\sigma}{2\epsilon_0}\int_0^R \frac{y\,dy}{\sqrt{x^2+y^2}} = \frac{\sigma}{2\epsilon_0}\left[\sqrt{x^2+y^2}\right]_0^R = \frac{\sigma}{2\epsilon_0}(\sqrt{x^2+R^2} - x)$$

EVALUATE: For $x \gg R$ this result should reduce to the potential of a point charge with $Q = \sigma\pi R^2$.

$$\sqrt{x^2+R^2} = x(1+R^2/x^2)^{1/2} \approx x(1+R^2/2x^2) \text{ so } \sqrt{x^2+R^2} - x \approx R^2/2x$$

Then $V \approx \dfrac{\sigma}{2\epsilon_0}\dfrac{R^2}{2x} = \dfrac{\sigma\pi R^2}{4\pi\epsilon_0 x} = \dfrac{Q}{4\pi\epsilon_0 x}$, as expected.

(b) IDENTIFY and **SET UP:** Use Eq.(23.19) to calculate E_x.

EXECUTE: $E_x = -\dfrac{\partial V}{\partial x} = -\dfrac{\sigma}{2\epsilon_0}\left(\dfrac{x}{\sqrt{x^2+R^2}} - 1\right) = \dfrac{\sigma x}{2\epsilon_0}\left(\dfrac{1}{x} - \dfrac{1}{\sqrt{x^2+R^2}}\right).$

EVALUATE: Our result agrees with Eq.(21.11) in Example 21.12.

23.67. **(a) IDENTIFY:** Use $V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}.$

SET UP: From Problem 22.48, $E(r) = \dfrac{\lambda r}{2\pi\epsilon_0 R^2}$ for $r \le R$ (inside the cylindrical charge distribution) and $E(r) = \dfrac{\lambda r}{2\pi\epsilon_0 r}$ for $r \ge R$. Let $V = 0$ at $r = R$ (at the surface of the cylinder).

EXECUTE: $r > R$

Take point *a* to be at *R* and point *b* to be at *r*, where $r > R$. Let $d\vec{l} = d\vec{r}$. $\vec{E}$ and $d\vec{r}$ are both radially outward, so $\vec{E}\cdot d\vec{r} = E\,dr$. Thus $V_R - V_r = \int_R^r E\,dr$. Then $V_R = 0$ gives $V_r = -\int_R^r E\,dr$. In this interval $(r > R)$, $E(r) = \lambda/2\pi\epsilon_0 r$, so

$$V_r = -\int_R^r \frac{\lambda}{2\pi\epsilon_0 r}dr = -\frac{\lambda}{2\pi\epsilon_0}\int_R^r \frac{dr}{r} = -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r}{R}\right).$$

EVALUATE: This expression gives $V_r = 0$ when $r = R$ and the potential decreases (becomes a negative number of larger magnitude) with increasing distance from the cylinder.

EXECUTE: $r < R$

Take point a at r, where $r < R$, and point b at R. $\vec{E}\cdot d\vec{r} = E\,dr$ as before. Thus $V_r - V_R = \int_r^R E\,dr$. Then $V_R = 0$ gives $V_r = \int_r^R E\,dr$. In this interval $(r < R)$, $E(r) = \lambda r/2\pi\epsilon_0 R^2$, so

$$V_r = \int_r^R \frac{\lambda}{2\pi\epsilon_0 R^2}dr = \frac{\lambda}{2\pi\epsilon_0 R^2}\int_r^R r\,dr = \frac{\lambda}{2\pi\epsilon_0 R^2}\left(\frac{R^2}{2} - \frac{r^2}{2}\right).$$

$$V_r = \frac{\lambda}{4\pi\epsilon_0}\left(1 - \left(\frac{r}{R}\right)^2\right).$$

EVALUATE: This expression also gives $V_r = 0$ when $r = R$. The potential is $\lambda/4\pi\epsilon_0$ at $r = 0$ and decreases with increasing r.

(b) EXECUTE: Graphs of V and E as functions of r are sketched in Figure 23.67.

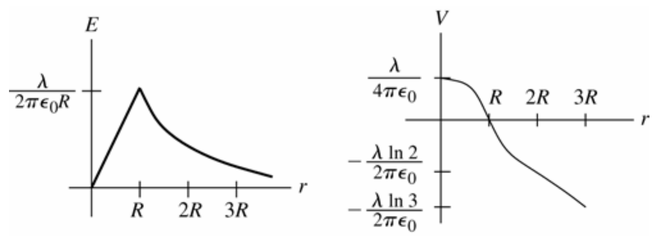


Figure 23.67

EVALUATE: E at any r is the negative of the slope of $V(r)$ at that r (Eq.23.23).

23.68. **IDENTIFY:** The alpha particles start out with kinetic energy and wind up with electrical potential energy at closest approach to the nucleus.

SET UP: **(a)** The energy of the system is conserved, with $U = (1/4\pi\epsilon_0)(qq_0/r)$ being the electric potential energy. With the charge of the alpha particle being $2e$ and that of the gold nucleus being Ze, we have

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\epsilon_0}\frac{2Ze^2}{R}$$

EXECUTE: Solving for v and using $Z = 79$ for gold gives

$$v = \sqrt{\left(\frac{1}{4\pi\epsilon_0}\right)\frac{4Ze^2}{mR}} = \sqrt{\frac{\left(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(4)(79)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(6.7\times10^{-27}\ \text{kg}\right)\left(5.6\times10^{-15}\ \text{m}\right)}} = 4.4\times10^7\ \text{m/s}$$

We have neglected any relativistic effects.

(b) Outside the atom, it is neutral. Inside the atom, we can model the 79 electrons as a uniform spherical shell, which produces no electric field inside of itself, so the only electric field is that of the nucleus.

EVALUATE: Neglecting relativistic effects was not such a good idea since the speed in part (a) is over 10% the speed of light. Modeling 79 electrons as a uniform spherical shell is reasonable, but we would not want to do this with small atoms.

23.69. **IDENTIFY:** $V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$.

SET UP: From Example 21.10, we have: $E_x = \frac{1}{4\pi\epsilon_0}\frac{Qx}{(x^2+a^2)^{3/2}}$. $\vec{E}\cdot d\vec{l} = E_x dx$. Let $a = \infty$ so $V_a = 0$.

EXECUTE: $V = -\frac{Q}{4\pi\epsilon_0}\int_\infty^x \frac{x'}{(x'^2+a^2)^{3/2}}dx' = \frac{Q}{4\pi\epsilon_0}u^{-1/2}\Big|_{u=\infty}^{u=x^2+a^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\sqrt{x^2+a^2}}$.

EVALUATE: Our result agrees with Eq.(23.16) in Example 23.11.

23.70. **IDENTIFY:** Divide the rod into infinitesimal segments with charge dq. The potential dV due to the segment is $dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r}$. Integrate over the rod to find the total potential.

SET UP: $dq = \lambda dl$, with $\lambda = Q/\pi a$ and $dl = a\,d\theta$.

EXECUTE: $dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\,dl}{a} = \frac{1}{4\pi\epsilon_0}\frac{Q}{\pi a}\frac{dl}{a} = \frac{1}{4\pi\epsilon_0}\frac{Q\,d\theta}{\pi a}$. $V = \frac{1}{4\pi\epsilon_0}\int_0^\pi \frac{Q\,d\theta}{\pi a} = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}$.

EVALUATE: All the charge of the ring is the same distance a from the center of curvature.

23.71. IDENTIFY: We must integrate to find the total energy because the energy to bring in more charge depends on the charge already present.

SET UP: If ρ is the uniform volume charge density, the charge of a spherical shell or radius r and thickness dr is $dq = \rho\, 4\pi r^2\, dr$, and $\rho = Q/(4/3\ \pi R^3)$. The charge already present in a sphere of radius r is $q = \rho(4/3\ \pi r^3)$. The energy to bring the charge dq to the surface of the charge q is Vdq, where V is the potential due to q, which is $q/4\pi\epsilon_0 r$.

EXECUTE: The total energy to assemble the entire sphere of radius R and charge Q is sum (integral) of the tiny increments of energy.

$$U = \int V dq = \int \frac{q}{4\pi\epsilon_0 r} dq = \int_0^R \frac{\rho \frac{4}{3}\pi r^3}{4\pi\epsilon_0 r}\left(\rho 4\pi r^2 dr\right) = \frac{3}{5}\left(\frac{1}{4\pi\epsilon_0}\frac{Q^2}{R}\right)$$

where we have substituted $\rho = Q/(4/3\ \pi R^3)$ and simplified the result.

EVALUATE: For a point-charge, $R \to 0$ so $U \to \infty$, which means that a point-charge should have infinite self-energy. This suggests that either point-charges are impossible, or that our present treatment of physics is not adequate at the extremely small scale, or both.

23.72. IDENTIFY: $V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$. The electric field is radially outward, so $\vec{E}\cdot d\vec{l} = E\, dr$.

SET UP: Let $a = \infty$, so $V_a = 0$.

EXECUTE: From Example 22.9, we have the following. For $r > R: E = \frac{kQ}{r^2}$ and $V = -kQ\int_\infty^r \frac{dr'}{r'^2} = \frac{kQ}{r}$.

For $r < R: E = \frac{kQr}{R^3}$ and $V = -\int_\infty^R \vec{E}\cdot d\vec{r}' - \int_R^r \vec{E}\cdot d\vec{r}' = \frac{kQ}{R} - \frac{kQ}{R^3}\int_R^r r'dr' = \frac{kQ}{R} - \frac{kQ}{R^3}\frac{1}{2}r'^2\Big|_R^r = \frac{kQ}{R} + \frac{kQ}{2R} - \frac{kQr^2}{2R^3} = \frac{kQ}{2R}\left[3 - \frac{r^2}{R^2}\right]$.

(b) The graphs of V and E versus r are sketched in Figure 23.72.

EVALUATE: For $r < R$ the potential depends on the electric field in the region r to ∞.

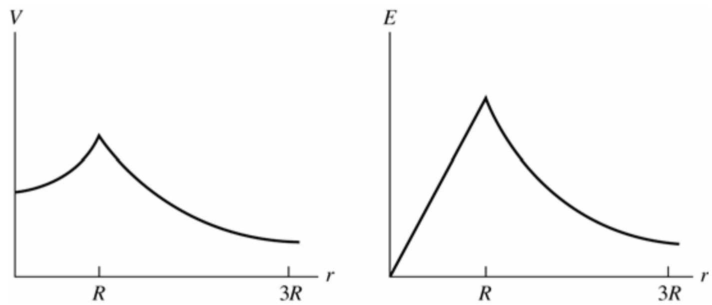


Figure 23.72

23.73. IDENTIFY: Problem 23.70 shows that $V_r = \frac{Q}{8\pi\epsilon_0 R}(3 - r^2/R^2)$ for $r \le R$ and $V_r = \frac{Q}{4\pi\epsilon_0 r}$ for $r \ge R$.

SET UP: $V_0 = \frac{3Q}{8\pi\epsilon_0 R}, V_R = \frac{Q}{4\pi\epsilon_0 R}$

EXECUTE: **(a)** $V_0 - V_R = \frac{Q}{8\pi\epsilon_0 R}$

(b) If $Q > 0$, V is higher at the center. If $Q < 0$, V is higher at the surface.

EVALUATE: For $Q > 0$ the electric field is radially outward, $\vec{E}$ is directed toward lower potential, so V is higher at the center. If $Q < 0$, the electric field is directed radially inward and V is higher at the surface.

23.74. IDENTIFY: For $r < c$, $E = 0$ and the potential is constant. For $r > c$, E is the same as for a point charge and $V = \frac{kq}{r}$.

SET UP: $V_\infty = 0$

EXECUTE: **(a)** Points $a, b,$ and c are all at the same potential, so $V_a - V_b = V_b - V_c = V_a - V_c = 0$.

$$V_c - V_\infty = \frac{kq}{R} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(150\times10^{-6}\ \text{C})}{0.60\ \text{m}} = 2.25\times10^6\ \text{V}$$

(b) They are all at the same potential.

(c) Only $V_c - V_\infty$ would change; it would be -2.25×10^6 V.

EVALUATE: The voltmeter reads the potential difference between the two points to which it is connected.

23.75. **IDENTIFY and SET UP:** Apply $F_r = -dU/dr$ and Newton's third law.

EXECUTE: **(a)** The electrical potential energy for a spherical shell with uniform surface charge density and a point charge q outside the shell is the same as if the shell is replaced by a point charge at its center. Since $F_r = -dU/dr$, this means the force the shell exerts on the point charge is the same as if the shell were replaced by a point charge at its center. But by Newton's 3rd law, the force q exerts on the shell is the same as if the shell were a point charge. But q can be replaced by a spherical shell with uniform surface charge and the force is the same, so the force between the shells is the same as if they were both replaced by point charges at their centers. And since the force is the same as for point charges, the electrical potential energy for the pair of spheres is the same as for a pair of point charges.
(b) The potential for solid insulating spheres with uniform charge density is the same outside of the sphere as for a spherical shell, so the same result holds.
(c) The result doesn't hold for conducting spheres or shells because when two charged conductors are brought close together, the forces between them causes the charges to redistribute and the charges are no longer distributed uniformly over the surfaces.

EVALUATE: For the insulating shells or spheres, $F = k\dfrac{|q_1 q_2|}{r^2}$ and $U = \dfrac{kq_1 q_2}{r}$, where q_1 and q_2 are the charges of the objects and r is the distance between their centers.

23.76. **IDENTIFY:** Apply Newton's second law to calculate the acceleration. Apply conservation of energy and conservation of momentum to the motions of the spheres.

SET UP: Problem 23.75 shows that $F = k\dfrac{|q_1 q_2|}{r^2}$ and $U = \dfrac{kq_1 q_2}{r}$, where q_1 and q_2 are the charges of the objects and r is the distance between their centers.

EXECUTE: Maximum speed occurs when the spheres are very far apart. Energy conservation gives $\dfrac{kq_1 q_2}{r} = \dfrac{1}{2}m_{50}v_{50}^2 + \dfrac{1}{2}m_{150}v_{150}^2$. Momentum conservation gives $m_{50}v_{50} = m_{150}v_{150}$ and $v_{50} = 3v_{150}$. $r = 0.50\text{ m}$. Solve for v_{50} and v_{150}: $v_{50} = 12.7\text{ m/s}$, $v_{150} = 4.24\text{ m/s}$. Maximum acceleration occurs just after spheres are released. $\sum F = ma$ gives $\dfrac{kq_1 q_2}{r^2} = m_{150}a_{150}$. $\dfrac{(9\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(10^{-5}\text{ C})(3\times 10^{-5}\text{ C})}{(0.50\text{ m})^2} = (0.15\text{ kg})a_{150}$. $a_{150} = 72.0\text{ m/s}^2$ and $a_{50} = 3a_{150} = 216\text{ m/s}^2$.

EVALUATE: The more massive sphere has a smaller acceleration and a smaller final speed.

23.77. **IDENTIFY:** Use Eq.(23.17) to calculate V_{ab}.

SET UP: From Problem 22.43, for $R \le r \le 2R$ (between the sphere and the shell) $E = Q/4\pi\epsilon_0 r^2$

Take a at R and b at $2R$.

EXECUTE: $V_{ab} = V_a - V_b = \int_R^{2R} E\,dr = \dfrac{Q}{4\pi\epsilon_0}\int_R^{2R}\dfrac{dr}{r^2} = \dfrac{Q}{4\pi\epsilon_0}\left[-\dfrac{1}{r}\right]_R^{2R} = \dfrac{Q}{4\pi\epsilon_0}\left(\dfrac{1}{R} - \dfrac{1}{2R}\right)$

$$V_{ab} = \frac{Q}{8\pi\epsilon_0 R}$$

EVALUATE: The electric field is radially outward and points in the direction of decreasing potential, so the sphere is at higher potential than the shell.

23.78. **IDENTIFY:** $V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$

SET UP: $\vec{E}$ is radially outward, so $\vec{E}\cdot d\vec{l} = E\,dr$. Problem 22.42 shows that $E(r) = 0$ for $r \le a$, $E(r) = kq/r^2$ for $a < r < b$, $E(r) = 0$ for $b < r < c$ and $E(r) = kq/r^2$ for $r > c$.

EXECUTE: **(a)** At $r = c$: $V_c = -\int_\infty^c \dfrac{kq}{r^2}dr = \dfrac{kq}{c}$.

(b) At $r = b$: $V_b = -\int_\infty^c \vec{E}\cdot d\vec{r} - \int_c^b \vec{E}\cdot d\vec{r} = \dfrac{kq}{c} - 0 = \dfrac{kq}{c}$.

(c) At $r = a$: $V_a = -\int_\infty^c \vec{E}\cdot d\vec{r} - \int_c^b \vec{E}\cdot d\vec{r} - \int_b^a \vec{E}\cdot d\vec{r} = \dfrac{kq}{c} - kq\int_b^a \dfrac{dr}{r^2} = kq\left[\dfrac{1}{c} - \dfrac{1}{b} + \dfrac{1}{a}\right]$

(d) At $r = 0$: $V_0 = kq\left[\dfrac{1}{c} - \dfrac{1}{b} + \dfrac{1}{a}\right]$ since it is inside a metal sphere, and thus at the same potential as its surface.

EVALUATE: The potential difference between the two conductors is $V_a - V_b = kq\left[\dfrac{1}{a} - \dfrac{1}{b}\right]$.

23.79. **IDENTIFY:** Slice the rod into thin slices and use Eq.(23.14) to calculate the potential due to each slice. Integrate over the length of the rod to find the total potential at each point.

(a) SET UP: An infinitesimal slice of the rod and its distance from point *P* are shown in Figure 23.79a.

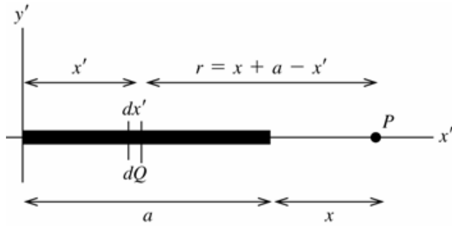


Figure 23.79a

Use coordinates with the origin at the left-hand end of the rod and one axis along the rod. Call the axes x' and y' so as not to confuse them with the distance *x* given in the problem.

EXECUTE: Slice the charged rod up into thin slices of width dx'. Each slice has charge $dQ = Q(dx'/a)$ and a distance $r = x + a - x'$ from point *P*. The potential at *P* due to the small slice *dQ* is

$$dV = \frac{1}{4\pi\epsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\left(\frac{dx'}{x+a-x'}\right).$$

Compute the total *V* at *P* due to the entire rod by integrating *dV* over the length of the rod $(x' = 0$ to $x' = a)$:

$$V = \int dV = \frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{dx'}{(x+a-x')} = \frac{Q}{4\pi\epsilon_0 a}[-\ln(x+a-x')]_0^a = \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{x+a}{x}\right).$$

EVALUATE: As $x \to \infty$, $V \to \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{x}{x}\right) = 0.$

(b) SET UP: An infinitesimal slice of the rod and its distance from point *R* are shown in Figure 23.79b.

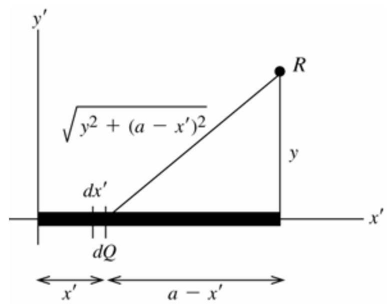


Figure 23.79b

$dQ = (Q/a)dx'$ as in part (a)

Each slice *dQ* is a distance $r = \sqrt{y^2 + (a-x')^2}$ from point *R*.

EXECUTE: The potential *dV* at *R* due to the small slice *dQ* is

$$dV = \frac{1}{4\pi\epsilon_0}\left(\frac{dQ}{r}\right) = \frac{1}{4\pi\epsilon_0}\frac{Q}{a}\frac{dx'}{\sqrt{y^2+(a-x')^2}}.$$

$$V = \int dV = \frac{Q}{4\pi\epsilon_0 a}\int_0^a \frac{dx'}{\sqrt{y^2+(a-x')^2}}.$$

In the integral make the change of variable $u = a - x'$; $du = -dx'$

$$V = -\frac{Q}{4\pi\epsilon_0 a}\int_a^0 \frac{du}{\sqrt{y^2+u^2}} = -\frac{Q}{4\pi\epsilon_0 a}\left[\ln(u+\sqrt{y^2+u^2})\right]_a^0$$

$$V = -\frac{Q}{4\pi\epsilon_0 a}[\ln y - \ln(a+\sqrt{y^2+a^2})] = \frac{Q}{4\pi\epsilon_0 a}\left[\ln\left(\frac{a+\sqrt{a^2+y^2}}{y}\right)\right].$$

(The expression for the integral was found in appendix B.)

EVALUATE: As $y \to \infty$, $V \to \frac{Q}{4\pi\epsilon_0 a}\ln\left(\frac{y}{y}\right) = 0.$

(c) SET UP: *part (a):* $V = \dfrac{Q}{4\pi\epsilon_0 a}\ln\left(\dfrac{x+a}{x}\right) = \dfrac{Q}{4\pi\epsilon_0 a}\ln\left(1+\dfrac{a}{x}\right)$.

From Appendix B, $\ln(1+u) = u - u^2/2\ldots$, so $\ln(1+a/x) = a/x - a^2/2x^2$ and this becomes a/x when x is large.

EXECUTE: Thus $V \to \dfrac{Q}{4\pi\epsilon_0 a}\left(\dfrac{a}{x}\right) = \dfrac{Q}{4\pi\epsilon_0 a}$. For large *x*, *V* becomes the potential of a point charge.

part (b): $V = \dfrac{Q}{4\pi\epsilon_0 a}\left[\ln\left(\dfrac{a+\sqrt{a^2+y^2}}{y}\right)\right] = \dfrac{Q}{4\pi\epsilon_0 a}\ln\left(\dfrac{a}{y}+\sqrt{1+\dfrac{a^2}{y^2}}\right)$.

From Appendix B, $\sqrt{1+a^2/y^2} = (1+a^2/y^2)^{1/2} = 1 + a^2/2y^2 + \ldots$

Thus $a/y + \sqrt{1+a^2/y^2} \to 1 + a/y + a^2/2y^2 + \ldots \to 1 + a/y$. And then using $\ln(1+u) \approx u$ gives

$$V \to \frac{Q}{4\pi\epsilon_0 a}\ln(1+a/y) \to \frac{Q}{4\pi\epsilon_0 a}\left(\frac{a}{y}\right) = \frac{Q}{4\pi\epsilon_0 y}.$$

EVALUATE: For large *y, V* becomes the potential of a point charge.

23.80. IDENTIFY: The potential at the surface of a uniformly charged sphere is $V = \dfrac{kQ}{R}$.

SET UP: For a sphere, $V = \dfrac{4}{3}\pi R^3$. When the raindrops merge, the total charge and volume is conserved.

EXECUTE: **(a)** $V = \dfrac{kQ}{R} = \dfrac{k(-1.20\times10^{-12}\text{ C})}{6.50\times10^{-4}\text{ m}} = -16.6\text{ V}$.

(b) The volume doubles, so the radius increases by the cube root of two: $R_{\text{new}} = \sqrt[3]{2}\,R = 8.19\times10^{-4}\text{ m}$ and the new charge is $Q_{\text{new}} = 2Q = -2.40\times10^{-12}\text{ C}$. The new potential is $V_{\text{new}} = \dfrac{kQ_{\text{new}}}{R_{\text{new}}} = \dfrac{k(-2.40\times10^{-12}\text{ C})}{8.19\times10^{-4}\text{ m}} = -26.4\text{ V}$.

EVALUATE: The charge doubles but the radius also increases and the potential at the surface increases by only a factor of $\dfrac{2}{2^{1/3}} = 2^{2/3}$.

23.81. (a) IDENTIFY and **SET UP:** The potential at the surface of a charged conducting sphere is given by Example 23.8: $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{q}{R}$. For spheres *A* and *B* this gives

$$V_A = \frac{Q_A}{4\pi\epsilon_0 R_A} \text{ and } V_B = \frac{Q_B}{4\pi\epsilon_0 R_B}.$$

EXECUTE: $V_A = V_B$ gives $Q_A/4\pi\epsilon_0 R_A = Q_B/4\pi\epsilon_0 R_B$ and $Q_B/Q_A = R_B/R_A$. And then $R_A = 3R_B$ implies $Q_B/Q_A = 1/3$.

(b) IDENTIFY and **SET UP:** The electric field at the surface of a charged conducting sphere is given in Example 22.5:

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{R^2}.$$

EXECUTE: For spheres *A* and *B* this gives

$$E_A = \frac{|Q_A|}{4\pi\epsilon_0 R_A^2} \text{ and } E_B = \frac{|Q_B|}{4\pi\epsilon_0 R_B^2}.$$

$$\frac{E_B}{E_A} = \left(\frac{|Q_B|}{4\pi\epsilon_0 R^2{}_B}\right)\left(\frac{4\pi\epsilon_0 R_A^2}{|Q_A|}\right) = |Q_B/Q_A|(R_A/R_B)^2 = (1/3)(3)^2 = 3.$$

EVALUATE: The sphere with the larger radius needs more net charge to produce the same potential. We can write $E = V/R$ for a sphere, so with equal potentials the sphere with the smaller *R* has the larger *V.*

23.82. IDENTIFY: Apply conservation of energy, $K_a + U_a = K_b + U_b$.

SET UP: Assume the particles initially are far apart, so $U_a = 0$, The alpha particle has zero speed at the distance of closest approach, so $K_b = 0$. $1\text{ eV} = 1.60\times10^{-19}\text{ J}$. The alpha particle has charge $+2e$ and the lead nucleus has charge $+82e$.

EXECUTE: Set the alpha particle's kinetic energy equal to its potential energy: $K_a = U_b$ gives

$11.0\text{ MeV} = \dfrac{k(2e)(82e)}{r}$ and $r = \dfrac{k(164)(1.60\times10^{-19}\text{ C})^2}{(11.0\times10^6\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 2.15\times10^{-14}\text{ m}$.

EVALUATE: The calculation assumes that at the distance of closest approach the alpha particle is outside the radius of the lead nucleus.

23.83. **IDENTIFY** and **SET UP:** The potential at the surface is given by Example 23.8 and the electric field at the surface is given by Example 22.5. The charge initially on sphere 1 spreads between the two spheres such as to bring them to the same potential.

EXECUTE: **(a)** $E_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_1}{R_1^2}$, $V_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_1}{R_1} = R_1 E_1$

(b) Two conditions must be met:

1) Let $q_1$and q_2 be the final potentials of each sphere. Then $q_1 + q_2 = Q_1$ (charge conservation)

2) Let V_1 and V_2 be the final potentials of each sphere. All points of a conductor are at the same potential, so $V_1 = V_2$.

$V_1 = V_2$ requires that $\dfrac{1}{4\pi\epsilon_0}\dfrac{q_1}{R_1} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_2}{R_2}$ and then $q_1/R_1 = q_2/R_2$

$$q_1 R_2 = q_2 R_1 = (Q_1 - q_1)R_1$$

This gives $q_1 = (R_1/[R_1+R_2])Q_1$ and $q_2 = Q_1 - q_1 = Q_1(1 - R_1/[R_1+R_2]) = Q_1(R_2/[R_1+R_2])$

(c) $V_1 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1}{R_1} = \dfrac{Q_1}{4\pi\epsilon_0(R_1+R_2)}$ and $V_2 = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_2}{R_2} = \dfrac{Q_1}{4\pi\epsilon_0(R_1+R_2)}$, which equals V_1 as it should.

(d) $E_1 = \dfrac{V_1}{R_1} = \dfrac{Q_1}{4\pi\epsilon_0 R_1(R_1+R_2)}$. $E_2 = \dfrac{V_2}{R_2} = \dfrac{Q_1}{4\pi\epsilon_0 R_2(R_1+R_2)}$.

EVALUATE: Part (a) says $q_2 = q_1(R_2/R_1)$. The sphere with the larger radius needs more charge to produce the same potential at its surface. When $R_1 = R_2$, $q_1 = q_2 = Q_1/2$. The sphere with the larger radius has the smaller electric field at its surface.

23.84. **IDENTIFY:** Apply $V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$

SET UP: From Problem 22.57, for $r \ge R$, $E = \dfrac{kQ}{r^2}$. For $r \le R$, $E = \dfrac{kQ}{r^2}\left[4\dfrac{r^3}{R^3} - 3\dfrac{r^4}{R^4}\right]$.

EXECUTE: **(a)** $r \ge R$: $E = \dfrac{kQ}{r^2} \Rightarrow V = -\int_\infty^r \dfrac{kQ}{r'^2}dr' = \dfrac{kQ}{r}$, which is the potential of a point charge.

(b) $r \le R$: $E = \dfrac{kQ}{r^2}\left[4\dfrac{r^3}{R^3} - 3\dfrac{r^4}{R^4}\right]$ and $V = -\int_\infty^R E dr' - \int_R^r E dr' = \dfrac{kQ}{R}\left[1 - 2\dfrac{r^2}{R^2} + 2\dfrac{R^2}{R^2} + \dfrac{r^3}{R^3} - \dfrac{R^3}{R^3}\right] = \dfrac{kQ}{R}\left[\dfrac{r^3}{R^3} - 2\dfrac{r^2}{R^2} + 2\right]$.

EVALUATE: At $r = R$, $V = \dfrac{kQ}{R}$. At $r = 0$, $V = \dfrac{2kQ}{R}$. The electric field is radially outward and V increases as r decreases.

23.85. **IDENTIFY:** Apply conservation of energy: $E_\text{i} = E_\text{f}$.

SET UP: In the collision the initial kinetic energy of the two particles is converted into potential energy at the distance of closest approach.

EXECUTE: **(a)** The two protons must approach to a distance of $2r_\text{p}$, where r_p is the radius of a proton.

$E_\text{i} = E_\text{f}$ gives $2\left[\dfrac{1}{2}m_\text{p}v^2\right] = \dfrac{ke^2}{2r_\text{p}}$ and $v = \sqrt{\dfrac{k(1.60\times10^{-19}\text{ C})^2}{2(1.2\times10^{-15}\text{ m})(1.67\times10^{-27}\text{ kg})}} = 7.58\times10^6\text{ m/s}$.

(b) For a helium-helium collision, the charges and masses change from (a) and

$v = \sqrt{\dfrac{k(2(1.60\times10^{-19}\text{ C}))^2}{(3.5\times10^{-15}\text{ m})(2.99)(1.67\times10^{-27}\text{ kg})}} = 7.26\times10^6\text{ m/s}$.

(c) $K = \dfrac{3kT}{2} = \dfrac{mv^2}{2}$. $T_\text{p} = \dfrac{m_\text{p}v^2}{3k} = \dfrac{(1.67\times10^{-27}\text{ kg})(7.58\times10^6\text{ m/s})^2}{3(1.38\times10^{-23}\text{ J/K})} = 2.3\times10^9\text{ K}$.

$$T_\text{He} = \frac{m_\text{He}v^2}{3k} = \frac{(2.99)(1.67\times10^{-27}\text{ kg})(7.26\times10^6\text{ m/s})^2}{3(1.38\times10^{-23}\text{ J/K})} = 6.4\times10^9\text{ K}.$$

(d) These calculations were based on the particles' average speed. The distribution of speeds ensures that there are always a certain percentage with a speed greater than the average speed, and these particles can undergo the necessary reactions in the sun's core.

EVALUATE: The kinetic energies required for fusion correspond to very high temperatures.

23.86. IDENTIFY and **SET UP:** Apply Eq.(23.20). $\frac{W_{a\to b}}{q_0} = V_a - V_b$ and $V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l}$.

EXECUTE: **(a)** $\vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} = -2Ax\hat{i} + 6Ay\hat{j} - 2Az\hat{k}$

(b) A charge is moved in along the z-axis. The work done is given by $W = q\int_{z_0}^{0} \vec{E}\cdot\hat{k}dz = q\int_{z_0}^{0}(-2Az)dz = +(Aq)z_0^2$.

Therefore, $A = \frac{W_{a\to b}}{qz_0^2} = \frac{6.00\times10^{-5}\text{ J}}{(1.5\times10^{-6}\text{ C})(0.250\text{ m})^2} = 640\text{ V/m}^2$.

(c) $\vec{E}(0,0,0.250) = -2(640\text{ V/m}^2)(0.250\text{ m})\hat{k} = -(320\text{ V/m})\hat{k}$.

(d) In every plane parallel to the xz-plane, y is constant, so $V(x,y,z) = Ax^2 + Az^2 - C$, where $C = 3Ay^2$.

$x^2 + z^2 = \frac{V+C}{A} = R^2$, which is the equation for a circle since R is constant as long as we have constant potential on those planes.

(e) $V = 1280\text{ V}$ and $y = 2.00\text{ m}$, so $x^2 + z^2 = \frac{1280\text{ V} + 3(640\text{ V/m}^2)(2.00\text{ m})^2}{640\text{ V/m}^2} = 14.0\text{ m}^2$ and the radius of the circle is 3.74 m.

EVALUATE: In any plane parallel to the xz-plane, $\vec{E}$ projected onto the plane is radial and hence perpendicular to the equipotential circles.

23.87. IDENTIFY: Apply conservation of energy to the motion of the daughter nuclei.

SET UP: Problem 23.73 shows that the electrical potential energy of the two nuclei is the same as if all their charge was concentrated at their centers.

EXECUTE: **(a)** The two daughter nuclei have half the volume of the original uranium nucleus, so their radii are smaller by a factor of the cube root of 2: $r = \frac{7.4\times10^{-15}\text{ m}}{\sqrt[3]{2}} = 5.9\times10^{-15}\text{ m}$.

(b) $U = \frac{k(46e)^2}{2r} = \frac{k(46)^2(1.60\times10^{-19}\text{ C})^2}{1.18\times10^{-14}\text{ m}} = 4.14\times10^{-11}\text{ J}$. $U = 2K$, where K is the final kinetic energy of each nucleus. $K = U/2 = (4.14\times10^{-11}\text{ J})/2 = 2.07\times10^{-11}\text{ J}$.

(c) If we have 10.0 kg of uranium, then the number of nuclei is $n = \frac{10.0\text{ kg}}{(236\text{ u})(1.66\times10^{-27}\text{ kg/u})} = 2.55\times10^{25}$ nuclei .

And each releases energy U, so $E = nU = (2.55\times10^{25})(4.14\times10^{-11}\text{ J}) = 1.06\times10^{15}\text{ J} = 253$ kilotons of TNT .

(d) We could call an atomic bomb an "electric" bomb since the electric potential energy provides the kinetic energy of the particles.

EVALUATE: This simple model considers only the electrical force between the daughter nuclei and neglects the nuclear force.

23.88. IDENTIFY and **SET UP:** In part (a) apply $E = -\frac{\partial V}{\partial r}$. In part (b) apply Gauss's law.

EXECUTE: **(a)** For $r \le a$, $E = -\frac{\partial V}{\partial r} = -\frac{\rho_0 a^2}{18\epsilon_0}\left[-6\frac{r}{a^2} + 6\frac{r^2}{a^3}\right] = \frac{\rho_0 a}{3\epsilon_0}\left[\frac{r}{a} - \frac{r^2}{a^2}\right]$. For $r \ge a$, $E = -\frac{\partial V}{\partial r} = 0$. $\vec{E}$ has only a radial component because V depends only on r.

(b) For $r \le a$, Gauss's law gives $E_r 4\pi r^2 = \frac{Q_r}{\epsilon_0} = \frac{\rho_0 a}{3\epsilon_0}\left[\frac{r}{a} - \frac{r^2}{a^2}\right]4\pi r^2$ and

$E_{r+dr}4\pi(r^2 + 2rdr) = \frac{Q_{r+dr}}{\epsilon_0} = \frac{\rho_0 a}{3\epsilon_0}\left[\frac{r+dr}{a} - \frac{(r^2+2rdr)}{a^2}\right]4\pi(r^2+2rdr)$. Therefore,

$\frac{Q_{r+dr} - Q_r}{\epsilon_0} = \frac{\rho(r)4\pi r^2\,dr}{\epsilon_0} \approx \frac{\rho_0 a 4\pi r^2 dr}{3\epsilon_0}\left[-\frac{2r}{a^2} + \frac{2}{a} - \frac{2r}{a^2} + \frac{1}{a}\right]$ and $\rho(r) = \frac{\rho_0}{3}\left[3 - \frac{4r}{a}\right] = \rho_0\left[1 - \frac{4r}{3a}\right]$.

(c) For $r \ge a$, $\rho(r) = 0$, so the total charge enclosed will be given by

$$Q = 4\pi\int_0^a \rho(r) r^2 dr = 4\pi\rho_0 \int_0^a \left[r^2 - \frac{4r^3}{3a}\right] dr = 4\pi\rho_0 \left[\frac{1}{3}r^3 - \frac{r^4}{3a}\right]_0^a = 0.$$

EVALUATE: Apply Gauss's law to a sphere of radius $r > R$. The result of part (c) says that $Q_{\text{encl}} = 0$, so $E = 0$. This agrees with the result we calculated in part (a).

23.89. IDENTIFY: Angular momentum and energy must be conserved.

SET UP: At the distance of closest approach the speed is not zero. $E = K + U$. $q_1 = 2e$, $q_2 = 82e$.

EXECUTE: $mv_1 b = mv_2 r_2$. $E_1 = E_2$ gives $E_1 = \frac{1}{2}mv_2^2 + \frac{kq_1q_2}{r_2}$. $E_1 = 11\text{ MeV} = 1.76\times10^{-12}$ J. r_2 is the distance of closest approach. Substituting in for $v_2 = v_1\left(\frac{b}{r_2}\right)$ we find $E_1 = E_1\frac{b^2}{r_2^2} + \frac{kq_1q_2}{r_2}$.

$(E_1)r_2^2 - (kq_1q_2)r_2 - E_1 b^2 = 0$. For $b = 10^{-12}$ m, $r_2 = 1.01\times10^{-12}$ m. For $b = 10^{-13}$ m, $r_2 = 1.11\times10^{-13}$ m. And for $b = 10^{-14}$ m, $r_2 = 2.54\times10^{-14}$ m.

EVALUATE: As b decreases the collision is closer to being head-on and the distance of closest approach decreases. Problem 23.82 shows that the distance of closest approach is 2.15×10^{-14} m when $b = 0$.

23.90. IDENTIFY: Consider the potential due to an infinitesimal slice of the cylinder and integrate over the length of the cylinder to find the total potential. The electric field is along the axis of the tube and is given by $E = -\frac{\partial V}{\partial x}$.

SET UP: Use the expression from Example 23.11 for the potential due to each infinitesimal slice. Let the slice be at coordinate z along the x-axis, relative to the center of the tube.

EXECUTE: **(a)** For an infinitesimal slice of the finite cylinder, we have the potential

$$dV = \frac{k\,dQ}{\sqrt{(x-z)^2 + R^2}} = \frac{kQ}{L}\frac{dz}{\sqrt{(x-z)^2 + R^2}}.$$ Integrating gives

$$V = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{dz}{\sqrt{(x-z)^2 + R^2}} = \frac{kQ}{L}\int_{-L/2-x}^{L/2-x}\frac{du}{\sqrt{u^2 + R^2}}$$ where $u = x - z$. Therefore,

$$V = \frac{kQ}{L}\ln\left[\frac{\sqrt{(L/2-x)^2 + R^2} + (L/2-x)}{\sqrt{(L/2+x)^2 + R^2} - L/2 - x}\right]$$ on the cylinder axis.

(b) For $L << R$, $V \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{(L/2-x)^2 + R^2} + L/2 - x}{\sqrt{(L/2+x)^2 + R^2} - L/2 - x}\right] \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{x^2 - xL + R^2} + L/2 - x}{\sqrt{x^2 + xL + R^2} - L/2 - x}\right].$

$$V \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{1 - xL/(R^2+x^2)} + (L/2-x)/\sqrt{R^2+x^2}}{\sqrt{1 + xL/(R^2+x^2)} + (-L/2-x)/\sqrt{R^2+x^2}}\right] = \frac{kQ}{L}\ln\left[\frac{1 - xL/2(R^2+x^2) + (L/2-x)/\sqrt{R^2+x^2}}{1 + xL/2(R^2+x^2) + (-L/2-x)/\sqrt{R^2+x^2}}\right].$$

$$V \approx \frac{kQ}{L}\ln\left[\frac{1 + L/2\sqrt{R^2+x^2}}{1 - L/2\sqrt{R^2+x^2}}\right] = \frac{kQ}{L}\left(\ln\left[1 + \frac{L}{2\sqrt{R^2+x^2}}\right] - \ln\left[1 - \frac{L}{2\sqrt{R^2+x^2}}\right]\right).$$

$$V \approx \frac{kQ}{L}\frac{2L}{2\sqrt{x^2+R^2}} = \frac{kQ}{\sqrt{x^2+R^2}},$$ which is the same as for a ring.

(c) $$E_x = -\frac{\partial V}{\partial x} = \frac{2kQ\left(\sqrt{(L-2x)^2 + 4R^2} - \sqrt{(L+2x)^2 + 4R^2}\right)}{\sqrt{(L-2x)^2 + 4R^2}\sqrt{(L+2x)^2 + 4R^2}}$$

EVALUATE: For $L << R$ the expression for E_x reduces to that for a ring of charge, as given in Example 23.14.

23.91. IDENTIFY: When the oil drop is at rest, the upward force $|q|E$ from the electric field equals the downward weight of the drop. When the drop is falling at its terminal speed, the upward viscous force equals the downward weight of the drop.

SET UP: The volume of the drop is related to its radius r by $V = \frac{4}{3}\pi r^3$.

EXECUTE: **(a)** $F_g = mg = \frac{4\pi r^3}{3}\rho g$. $F_e = |q|E = |q|V_{AB}/d$. $F_e = F_g$ gives $|q| = \frac{4\pi}{3}\frac{\rho r^3 gd}{V_{AB}}$.

(b) $\frac{4\pi r^3}{3}\rho g = 6\pi\eta r v_t$ gives $r = \sqrt{\frac{9\eta v_t}{2\rho g}}$. Using this result to replace *r* in the expression in part (a) gives

$$|q| = \frac{4\pi}{3}\frac{\rho g d}{V_{AB}}\left[\sqrt{\frac{9\eta v_t}{2\rho g}}\right]^3 = 18\pi \frac{d}{V_{AB}}\sqrt{\frac{\eta^3 v_t^3}{2\rho g}}.$$

(c) $|q| = 18\pi \frac{10^{-3}\text{ m}}{9.16\text{ V}}\sqrt{\frac{(1.81\times10^{-5}\text{ N}\cdot\text{s/m}^2)^3(1.00\times10^{-3}\text{ m/39.3 s})^3}{2(824\text{ kg/m}^3)(9.80\text{ m/s}^2)}} = 4.80\times10^{-19}\text{ C} = 3e$. The drop has acquired three excess electrons.

$$r = \sqrt{\frac{9(1.81\times10^{-5}\text{ N}\cdot\text{s/m}^2)(1.00\times10^{-3}\text{ m/39.3 s})}{2(824\text{ kg/m}^3)(9.80\text{ m/s}^2)}} = 5.07\times10^{-7}\text{ m} = 0.507\ \mu\text{m}.$$

EVALUATE: The weight of the drop is $\left(\frac{4\pi r^3}{3}\right)\rho g = 4.4\times10^{-15}\text{ N}$. The density of air at room temperature is 1.2 kg/m^3, so the buoyancy force is $\rho_{\text{air}}Vg = 6.4\times10^{-18}\text{ N}$ and can be neglected.

23.92. IDENTIFY: $v_{\text{cm}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

SET UP: $E = K_1 + K_2 + U$, where $U = \frac{kq_1q_2}{r}$.

EXECUTE: **(a)** $v_{\text{cm}} = \frac{(6\times10^{-5}\text{ kg})(400\text{ m/s}) + (3\times10^{-5}\text{ kg})(1300\text{ m/s})}{6.0\times10^{-5}\text{ kg} + 3.0\times10^{-5}\text{ kg}} = 700\text{ m/s}$

(b) $E_{\text{rel}} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{kq_1q_2}{r} - \frac{1}{2}(m_1 + m_2)v_{\text{cm}}^2$. After expanding the center of mass velocity and collecting like terms $E_{\text{rel}} = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}[v_1^2 + v_2^2 - 2v_1 v_2] + \frac{kq_1q_2}{r} = \frac{1}{2}\mu(v_1 - v_2)^2 + \frac{kq_1q_2}{r}$.

(c) $E_{\text{rel}} = \frac{1}{2}(2.0\times10^{-5}\text{ kg})(900\text{ m/s})^2 + \frac{k(2.0\times10^{-6}\text{ C})(-5.0\times10^{-6}\text{ C})}{0.0090\text{ m}} = -1.9\text{ J}$

(d) Since the energy is less than zero, the system is "bound."

(e) The maximum separation is when the velocity is zero: $-1.9\text{ J} = \frac{kq_1q_2}{r}$ gives

$$r = \frac{k(2.0\times10^{-6}\text{ C})(-5.0\times10^{-6}\text{ C})}{-1.9\text{ J}} = 0.047\text{ m}.$$

(f) Now using $v_1 = 400\text{ m/s}$ and $v_2 = 1800\text{ m/s}$, we find $E_{\text{rel}} = +9.6\text{ J}$. The particles do escape, and the final relative velocity is $|v_1 - v_2| = \sqrt{\frac{2E_{\text{rel}}}{\mu}} = \sqrt{\frac{2(9.6\text{ J})}{2.0\times10^{-5}\text{ kg}}} = 980\text{ m/s}$.

EVALUATE: For an isolated system the velocity of the center of mass is constant and the system must retain the kinetic energy associated with the motion of the center of mass.

24

CAPACITANCE AND DIELECTRICS

24.1. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$

SET UP: $1\ \mu\text{F} = 10^{-6}\ \text{F}$

EXECUTE: $Q = CV_{ab} = (7.28\times10^{-6}\ \text{F})(25.0\ \text{V}) = 1.82\times10^{-4}\ \text{C} = 182\ \mu\text{C}$

EVALUATE: One plate has charge $+Q$ and the other has charge $-Q$.

24.2. **IDENTIFY** and **SET UP:** $C = \dfrac{\epsilon_0 A}{d}$, $C = \dfrac{Q}{V}$ and $V = Ed$.

(a) $C = \epsilon_0 \dfrac{A}{d} = \epsilon_0 \dfrac{0.00122\ \text{m}^2}{0.00328\ \text{m}} = 3.29\ \text{pF}$

(b) $V = \dfrac{Q}{C} = \dfrac{4.35\times10^{-8}\ \text{C}}{3.29\times10^{-12}\ \text{F}} = 13.2\ \text{kV}$

(c) $E = \dfrac{V}{d} = \dfrac{13.2\times10^{3}\ \text{V}}{0.00328\ \text{m}} = 4.02\times10^{6}\ \text{V/m}$

EVALUATE: The electric field is uniform between the plates, at points that aren't close to the edges.

24.3. **IDENTIFY** and **SET UP:** It is a parallel-plate air capacitor, so we can apply the equations of Sections 24.1.

EXECUTE: **(a)** $C = \dfrac{Q}{V_{ab}}$ so $V_{ab} = \dfrac{Q}{C} = \dfrac{0.148\times10^{-6}\ \text{C}}{245\times10^{-12}\ \text{F}} = 604\ \text{V}$

(b) $C = \dfrac{\epsilon_0 A}{d}$ so $A = \dfrac{Cd}{\epsilon_0} = \dfrac{(245\times10^{-12}\ \text{F})(0.328\times10^{-3}\ \text{m})}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 9.08\times10^{-3}\ \text{m}^2 = 90.8\ \text{cm}^2$

(c) $V_{ab} = Ed$ so $E = \dfrac{V_{ab}}{d} = \dfrac{604\ \text{V}}{0.328\times10^{-3}\ \text{m}} = 1.84\times10^{6}\ \text{V/m}$

(d) $E = \dfrac{\sigma}{\epsilon_0}$ so $\sigma = E\epsilon_0 = (1.84\times10^{6}\ \text{V/m})(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2) = 1.63\times10^{-5}\ \text{C/m}^2$

EVALUATE: We could also calculate σ directly as Q/A. $\sigma = \dfrac{Q}{A} = \dfrac{0.148\times10^{-6}\ \text{C}}{9.08\times10^{-3}\ \text{m}^2} = 1.63\times10^{-5}\ \text{C/m}^2$, which checks.

24.4. **IDENTIFY:** $C = \epsilon_0 \dfrac{A}{d}$ when there is air between the plates.

SET UP: $A = (3.0\times10^{-2}\ \text{m})^2$ is the area of each plate.

EXECUTE: $C = \dfrac{(8.854\times10^{-12}\ \text{F/m})(3.0\times10^{-2}\ \text{m})^2}{5.0\times10^{-3}\ \text{m}} = 1.59\times10^{-12}\ \text{F} = 1.59\ \text{pF}$

EVALUATE: C increases when A increases and C increases when d decreases.

24.5. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\epsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, $V_{ab} = 12.0\ \text{V}$.

EXECUTE: **(a)** $Q = CV_{ab} = (10.0\times10^{-6}\ \text{F})(12.0\ \text{V}) = 1.20\times10^{-4}\ \text{C} = 120\ \mu\text{C}$

(b) When d is doubled C is halved, so Q is halved. $Q = 60\ \mu\text{C}$.

(c) If r is doubled, A increases by a factor of 4. C increases by a factor of 4 and Q increases by a factor of 4. $Q = 480\ \mu\text{C}$.

EVALUATE: When the plates are moved apart, less charge on the plates is required to produce the same potential difference. With the separation of the plates constant, the electric field must remain constant to produce the same potential difference. The electric field depends on the surface charge density, σ. To produce the same σ, more charge is required when the area increases.

24.6. **IDENTIFY:** $C = \dfrac{Q}{V_{ab}}$. $C = \dfrac{\epsilon_0 A}{d}$.

SET UP: When the capacitor is connected to the battery, enough charge flows onto the plates to make $V_{ab} = 12.0\text{ V}$.

EXECUTE: **(a)** 12.0 V

(b) (i) When d is doubled, C is halved. $V_{ab} = \dfrac{Q}{C}$ and Q is constant, so V doubles. $V = 24.0\text{ V}$.

(ii) When r is doubled, A increases by a factor of 4. V decreases by a factor of 4 and $V = 3.0\text{ V}$.

EVALUATE: The electric field between the plates is $E = Q/\epsilon_0 A$. $V_{ab} = Ed$. When d is doubled E is unchanged and V doubles. When A is increased by a factor of 4, E decreases by a factor of 4 so V decreases by a factor of 4.

24.7. **IDENTIFY:** $C = \dfrac{\epsilon_0 A}{d}$. Solve for d.

SET UP: Estimate $r = 1.0\text{ cm}$. $A = \pi r^2$.

EXECUTE: $C = \dfrac{\epsilon_0 A}{d}$ so $d = \dfrac{\epsilon_0 \pi r^2}{C} = \dfrac{\epsilon_0 \pi (0.010\text{ m})^2}{1.00\times 10^{-12}\text{ F}} = 2.8\text{ mm}$.

EVALUATE: The separation between the pennies is nearly a factor of 10 smaller than the diameter of a penny, so it is a reasonable approximation to treat them as infinite sheets.

24.8. **INCREASE:** $C = \dfrac{Q}{V_{ab}}$. $V_{ab} = Ed$. $C = \dfrac{\epsilon_0 A}{d}$.

SET UP: We want $E = 1.00\times 10^4\text{ N/C}$ when $V = 100\text{ V}$.

EXECUTE: **(a)** $d = \dfrac{V_{ab}}{E} = \dfrac{1.00\times 10^2\text{ V}}{1.00\times 10^4\text{ N/C}} = 1.00\times 10^{-2}\text{ m} = 1.00\text{ cm}$.

$A = \dfrac{Cd}{\epsilon_0} = \dfrac{(5.00\times 10^{-12}\text{ F})(1.00\times 10^{-2}\text{ m})}{8.854\times 10^{-12}\text{ C}^2/(\text{N}\cdot\text{m}^2)} = 5.65\times 10^{-3}\text{ m}^2$. $A = \pi r^2$ so $r = \sqrt{\dfrac{A}{\pi}} = 4.24\times 10^{-2}\text{ m} = 4.24\text{ cm}$.

(b) $Q = CV_{ab} = (5.00\times 10^{-12}\text{ F})(1.00\times 10^2\text{ V}) = 5.00\times 10^{-10}\text{ C} = 500\text{ pC}$

EVALUATE: $C = \dfrac{\epsilon_0 A}{d}$. We could have a larger d, along with a larger A, and still achieve the required C without exceeding the maximum allowed E.

24.9. **IDENTIFY:** Apply the results of Example 24.4. $C = Q/V$.

SET UP: $r_a = 0.50\text{ mm}$, $r_b = 5.00\text{ mm}$

EXECUTE: **(a)** $C = \dfrac{L 2\pi\epsilon_0}{\ln(r_b/r_a)} = \dfrac{(0.180\text{ m})2\pi\epsilon_0}{\ln(5.00/0.50)} = 4.35\times 10^{-12}\text{ F}$.

(b) $V = Q/C = (10.0\times 10^{-12}\text{ C})/(4.35\times 10^{-12}\text{ F}) = 2.30\text{ V}$

EVALUATE: $\dfrac{C}{L} = 24.2\text{ pF}$. This value is similar to those in Example 24.4. The capacitance is determined entirely by the dimensions of the cylinders.

24.10. **IDENTIFY:** Capacitance depends on the geometry of the object.

(a) SET UP: The capacitance of a cylindrical capacitor is $C = \dfrac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$. Solving for r_b gives $r_b = r_a e^{2\pi\epsilon_0 L/C}$.

EXECUTE: Substituting in the numbers for the exponent gives

$$\frac{2\pi\left(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)(0.120\text{ m})}{3.67\times 10^{-11}\text{ F}} = 0.182$$

Now use this value to calculate r_b: $r_b = r_a e^{0.182} = (0.250\text{ cm})e^{0.182} = 0.300\text{ cm}$

(b) SET UP: For any capacitor, $C = Q/V$ and $\lambda = Q/L$. Combining these equations and substituting the numbers gives $\lambda = Q/L = CV/L$.

EXECUTE: Numerically we get

$$\lambda = \frac{CV}{L} = \frac{\left(3.67\times 10^{-11}\text{ F}\right)\left(125\text{ V}\right)}{0.120\text{ m}} = 3.82\times 10^{-8}\text{ C/m} = 38.2\text{ nC/m}$$

EVALUATE: The distance between the surfaces of the two cylinders would be only 0.050 cm, which is just 0.50 mm. These cylinders would have to be carefully constructed.

24.11. **IDENTIFY** and **SET UP:** Use the expression for *C*/*L* derived in Example 24.4. Then use Eq.(24.1) to calculate *Q*.

EXECUTE: **(a)** From Example 24.4, $\frac{C}{L} = \frac{2\pi\epsilon_0}{\ln(r_b/r_a)}$

$$\frac{C}{L} = \frac{2\pi\left(8.854\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)}{\ln(3.5\ \mathrm{mm}/1.5\ \mathrm{mm})} = 6.57\times10^{-11}\ \mathrm{F/m} = 66\ \mathrm{pF/m}$$

(b) $C = \left(6.57\times10^{-11}\ \mathrm{F/m}\right)(2.8\ \mathrm{m}) = 1.84\times10^{-10}\ \mathrm{F}$.

$$Q = CV = \left(1.84\times10^{-10}\ \mathrm{F}\right)\left(350\times10^{-3}\ \mathrm{V}\right) = 6.4\times10^{-11}\ \mathrm{C} = 64\ \mathrm{pC}$$

The conductor at higher potential has the positive charge, so there is +64 pC on the inner conductor and -64 pC on the outer conductor.

EVALUATE: *C* depends only on the dimensions of the capacitor. *Q* and *V* are proportional.

24.12. **IDENTIFY:** Apply the results of Example 24.3. $C = Q/V$.

SET UP: $r_a = 15.0$ cm. Solve for r_b.

EXECUTE: **(a)** For two concentric spherical shells, the capacitance is $C = \frac{1}{k}\left(\frac{r_a r_b}{r_b - r_a}\right)$. $kCr_b - kCr_a = r_a r_b$ and

$$r_b = \frac{kCr_a}{kC - r_a} = \frac{k(116\times10^{-12}\ \mathrm{F})(0.150\ \mathrm{m})}{k(116\times10^{-12}\ \mathrm{F}) - 0.150\ \mathrm{m}} = 0.175\ \mathrm{m}.$$

(b) $V = 220$ V and $Q = CV = (116\times10^{-12}\ \mathrm{F})(220\ \mathrm{V}) = 2.55\times10^{-8}\ \mathrm{C}$.

EVALUATE: A parallel-plate capacitor with $A = 4\pi r_a r_b = 0.33\ \mathrm{m}^2$ and $d = r_b - r_a = 2.5\times10^{-2}$ m has $C = \frac{\epsilon_0 A}{d} = 117$ pF, in excellent agreement with the value of *C* for the spherical capacitor.

24.13. **IDENTIFY:** We can use the definition of capacitance to find the capacitance of the capacitor, and then relate the capacitance to geometry to find the inner radius.

(a) SET UP: By the definition of capacitance, *C* = *Q*/*V*.

EXECUTE: $C = \frac{Q}{V} = \frac{3.30\times10^{-9}\ \mathrm{C}}{2.20\times10^{2}\ \mathrm{V}} = 1.50\times10^{-11}\ \mathrm{F} = 15.0\ \mathrm{pF}$

(b) SET UP: The capacitance of a spherical capacitor is $C = 4\pi\epsilon_0 \frac{r_a r_b}{r_b - r_a}$.

EXECUTE: Solve for r_a and evaluate using *C* = 15.0 pF and r_b = 4.00 cm, giving r_a = 3.09 cm.

(c) SET UP: We can treat the inner sphere as a point-charge located at its center and use Coulomb's law, $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$.

EXECUTE: $E = \frac{\left(9.00\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(3.30\times10^{-9}\ \mathrm{C}\right)}{(0.0309\ \mathrm{m})^2} = 3.12\times10^{4}\ \mathrm{N/C}$

EVALUATE: Outside the capacitor, the electric field is zero because the charges on the spheres are equal in magnitude but opposite in sign.

24.14. **IDENTIFY:** The capacitors between *b* and *c* are in parallel. This combination is in series with the 15 pF capacitor.

SET UP: Let $C_1 = 15$ pF, $C_2 = 9.0$ pF and $C_3 = 11$ pF.

EXECUTE: **(a)** For capacitors in parallel, $C_{\mathrm{eq}} = C_1 + C_2 + \cdots$ so $C_{23} = C_2 + C_3 = 20$ pF

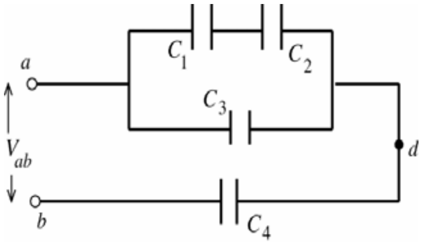
(b) $C_1 = 15$ pF is in series with $C_{23} = 20$ pF. For capacitors in series, $\frac{1}{C_{\mathrm{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$ so $\frac{1}{C_{123}} = \frac{1}{C_1} + \frac{1}{C_{23}}$ and

$$C_{123} = \frac{C_1 C_{23}}{C_1 + C_{23}} = \frac{(15\ \mathrm{pF})(20\ \mathrm{pF})}{15\ \mathrm{pF} + 20\ \mathrm{pF}} = 8.6\ \mathrm{pF}.$$

EVALUATE: For capacitors in parallel the equivalent capacitance is larger than any of the individual capacitors. For capacitors in series the equivalent capacitance is smaller than any of the individual capacitors.

24.15. **IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

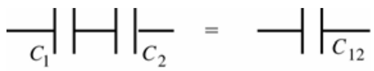
SET UP: Do parts **(a)** and **(b)** together. The capacitor network is drawn in Figure 24.15a.



$$C_1 = C_2 = C_3 = C_4 = 400\ \mu\text{F}$$
$$V_{ab} = 28.0\ \text{V}$$

Figure 24.15a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_1 and C_2 are in series and are equivalent to C_{12} (Figure 24.15b).

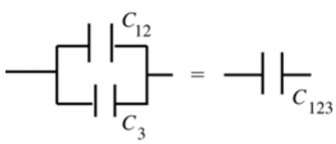


$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Figure 24.15b

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(4.00\times10^{-6}\ \text{F})(4.00\times10^{-6}\ \text{F})}{4.00\times10^{-6}\ \text{F} + 4.00\times10^{-6}\ \text{F}} = 2.00\times10^{-6}\ \text{F}$$

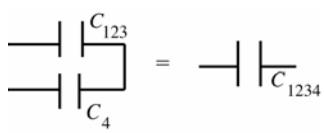
C_{12} and C_3 are in parallel and are equivalent to C_{123} (Figure 24.15c).



$$C_{123} = C_{12} + C_3$$
$$C_{123} = 2.00\times10^{-6}\ \text{F} + 4.00\times10^{-6}\ \text{F}$$
$$C_{123} = 6.00\times10^{-6}\ \text{F}$$

Figure 24.15c

C_{123} and C_4 are in series and are equivalent to C_{1234} (Figure 24.15d).

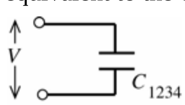


$$\frac{1}{C_{1234}} = \frac{1}{C_{123}} + \frac{1}{C_4}$$

Figure 24.15d

$$C_{1234} = \frac{C_{123} C_4}{C_{123} + C_4} = \frac{(6.00\times10^{-6}\ \text{F})(4.00\times10^{-6}\ \text{F})}{6.00\times10^{-6}\ \text{F} + 4.00\times10^{-6}\ \text{F}} = 2.40\times10^{-6}\ \text{F}$$

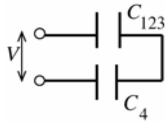
The circuit is equivalent to the circuit shown in Figure 24.15e.



$$V_{1234} = V = 28.0\ \text{V}$$
$$Q_{1234} = C_{1234} V = (2.40\times10^{-6}\ \text{F})(28.0\ \text{V}) = 67.2\ \mu\text{C}$$

Figure 24.15e

Now build back up the original circuit, step by step. C_{1234} represents C_{123} and C_4 in series (Figure 24.15f).



$$Q_{123} = Q_4 = Q_{1234} = 67.2\ \mu\text{C}$$
(charge same for capacitors in series)

Figure 24.15f

Then $V_{123} = \dfrac{Q_{123}}{C_{123}} = \dfrac{67.2\ \mu\text{C}}{6.00\ \mu\text{F}} = 11.2\ \text{V}$

$$V_4 = \frac{Q_4}{C_4} = \frac{67.2\ \mu\text{C}}{4.00\ \mu\text{F}} = 16.8\ \text{V}$$

Note that $V_4 + V_{123} = 16.8\ \text{V} + 11.2\ \text{V} = 28.0\ \text{V}$, as it should.

Next consider the circuit as written in Figure 24.15g.

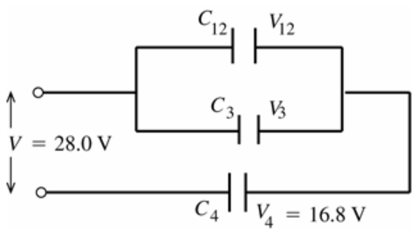


Figure 24.15g

$V_3 = V_{12} = 28.0\text{ V} - V_4$

$V_3 = 11.2\text{ V}$

$Q_3 = C_3V_3 = (4.00\ \mu\text{F})(11.2\text{ V})$

$Q_3 = 44.8\ \mu\text{C}$

$Q_{12} - C_{12}V_{12} = (2.00\ \mu\text{F})(11.2\text{ V})$

$Q_{12} = 22.4\ \mu\text{C}$

Finally, consider the original circuit, as shown in Figure 24.15h.

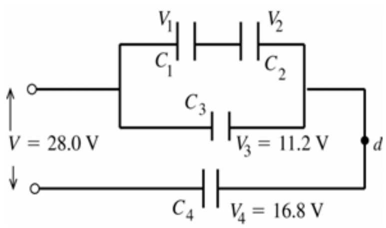


Figure 24.15h

$Q_1 = Q_2 = Q_{12} = 22.4\ \mu\text{C}$

(charge same for capacitors in series)

$$V_1 = \frac{Q_1}{C_1} = \frac{22.4\ \mu\text{C}}{4.00\ \mu\text{F}} = 5.6\text{ V}$$

$$V_2 = \frac{Q_2}{C_2} = \frac{22.4\ \mu\text{C}}{4.00\ \mu\text{F}} = 5.6\text{ V}$$

Note that $V_1 + V_2 = 11.2\text{ V}$, which equals V_3 as it should.

Summary: $Q_1 = 22.4\ \mu\text{C},\ V_1 = 5.6\text{ V}$

$Q_2 = 22.4\ \mu\text{C},\ V_2 = 5.6\text{ V}$

$Q_3 = 44.8\ \mu\text{C},\ V_3 = 11.2\text{ V}$

$Q_4 = 67.2\ \mu\text{C},\ V_4 = 16.8\text{ V}$

(c) $V_{ad} = V_3 = 11.2\text{ V}$

EVALUATE: $V_1 + V_2 + V_4 = V$, or $V_3 + V_4 = V$. $Q_1 = Q_2$, $Q_1 + Q_3 = Q_4$ and $Q_4 = Q_{1234}$.

24.16. **IDENTIFY:** The two capacitors are in series. The equivalent capacitance is given by $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$.

SET UP: For capacitors in series the charges are the same and the potentials add to give the potential across the network.

EXECUTE: **(a)** $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{(3.0\times10^{-6}\text{ F})} + \frac{1}{(5.0\times10^{-6}\text{ F})} = 5.33\times10^{5}\text{ F}^{-1}$. $C_{\text{eq}} = 1.88\times10^{-6}\text{ F}$. Then

$Q = VC_{\text{eq}} = (52.0\text{ V})(1.88\times10^{-6}\text{ F}) = 9.75\times10^{-5}\text{ C}$. Each capacitor has charge $9.75\times10^{-5}\text{ C}$.

(b) $V_1 = Q/C_1 = 9.75\times10^{-5}\text{ C}/3.0\times10^{-6}\text{ F} = 32.5\text{ V}$.

$V_2 = Q/C_2 = 9.75\times10^{-5}\text{ C}/5.0\times10^{-6}\text{ F} = 19.5\text{ V}$.

EVALUATE: $V_1 + V_2 = 52.0\text{ V}$, which is equal to the applied potential V_{ab}. The capacitor with the smaller *C* has the larger *V*.

24.17. **IDENTIFY:** The two capacitors are in parallel so the voltage is the same on each, and equal to the applied voltage V_{ab}.

SET UP: Do parts **(a)** and **(b)** together. The network is sketched in Figure 24.17.

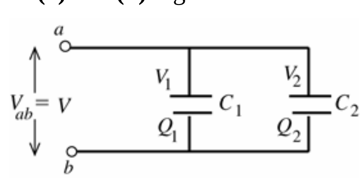


Figure 24.17

EXECUTE: $V_1 = V_2 = V$

$V_1 = 52.0\text{ V}$

$V_2 = 52.0\text{ V}$

$C = Q/V$ so $Q = CV$

$Q_1 = C_1V_1 = (3.00\ \mu\text{F})(52.0\text{ V}) = 156\ \mu\text{C}$. $Q_2 = C_2V_2 = (5.00\ \mu\text{F})(52.0\text{ V}) = 260\ \mu\text{C}$.

EVALUATE: To produce the same potential difference, the capacitor with the larger *C* has the larger *Q*.

24.18. **IDENTIFY:** For capacitors in parallel the voltages are the same and the charges add. For capacitors in series, the charges are the same and the voltages add. $C = Q/V$.

SET UP: C_1 and C_2 are in parallel and C_3 is in series with the parallel combination of C_1 and C_2.

EXECUTE: **(a)** C_1 and C_2 are in parallel and so have the same potential across them:

$V_1 = V_2 = \dfrac{Q_2}{C_2} = \dfrac{40.0\times10^{-6}\ \text{C}}{3.00\times10^{-6}\ \text{F}} = 13.33\ \text{V}$. Therefore, $Q_1 = V_1C_1 = (13.33\ \text{V})(3.00\times10^{-6}\ \text{F}) = 80.0\times10^{-6}\ \text{C}$. Since C_3 is in series with the parallel combination of C_1 and C_2, its charge must be equal to their combined charge: $C_3 = 40.0\times10^{-6}\ \text{C} + 80.0\times10^{-6}\ \text{C} = 120.0\times10^{-6}\ \text{C}$.

(b) The total capacitance is found from $\dfrac{1}{C_{\text{tot}}} = \dfrac{1}{C_{12}} + \dfrac{1}{C_3} = \dfrac{1}{9.00\times10^{-6}\ \text{F}} + \dfrac{1}{5.00\times10^{-6}\ \text{F}}$ and $C_{\text{tot}} = 3.21\ \mu\text{F}$.

$V_{ab} = \dfrac{Q_{\text{tot}}}{C_{\text{tot}}} = \dfrac{120.0\times10^{-6}\ \text{C}}{3.21\times10^{-6}\ \text{F}} = 37.4\ \text{V}$.

EVALUATE: $V_3 = \dfrac{Q_3}{C_3} = \dfrac{120.0\times10^{-6}\ \text{C}}{5.00\times10^{-6}\ \text{F}} = 24.0\ \text{V}$. $V_{ab} = V_1 + V_3$.

24.19. **IDENTIFY** and **SET UP:** Use the rules for *V* for capacitors in series and parallel: for capacitors in parallel the voltages are the same and for capacitors in series the voltages add.

EXECUTE: $V_1 = Q_1/C_1 = (150\ \mu\text{C})/(3.00\ \mu\text{F}) = 50\ \text{V}$

C_1 and C_2 are in parallel, so $V_2 = 50\ \text{V}$

$V_3 = 120\ \text{V} - V_1 = 70\ \text{V}$

EVALUATE: Now that we know the voltages, we could also calculate *Q* for the other two capacitors.

24.20. **IDENTIFY** and **SET UP:** $C = \dfrac{\epsilon_0 A}{d}$. For two capacitors in series, $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

EXECUTE: $C_{\text{eq}} = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)^{-1} = \left(\dfrac{d_1}{\epsilon_0 A} + \dfrac{d_2}{\epsilon_0 A}\right)^{-1} = \dfrac{\epsilon_0 A}{d_1 + d_2}$. This shows that the combined capacitance for two capacitors in series is the same as that for a capacitor of area *A* and separation $(d_1 + d_2)$.

EVALUATE: C_{eq} is smaller than either C_1 or C_2.

24.21. **IDENTIFY** and **SET UP:** $C = \dfrac{\epsilon_0 A}{d}$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$.

EXECUTE: $C_{\text{eq}} = C_1 + C_2 = \dfrac{\epsilon_0 A_1}{d} + \dfrac{\epsilon_0 A_2}{d} = \dfrac{\epsilon_0 (A_1 + A_2)}{d}$. So the combined capacitance for two capacitors in parallel is that of a single capacitor of their combined area $(A_1 + A_2)$ and common plate separation *d*.

EVALUATE: C_{eq} is larger than either C_1 or C_2.

24.22. **IDENTIFY:** Simplify the network by replacing series and parallel combinations of capacitors by their equivalents.

SET UP: For capacitors in series the voltages add and the charges are the same; $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots$ For capacitors in parallel the voltages are the same and the charges add; $C_{\text{eq}} = C_1 + C_2 + \cdots$ $C = \dfrac{Q}{V}$.

EXECUTE: **(a)** The equivalent capacitance of the $5.0\ \mu\text{F}$ and $8.0\ \mu\text{F}$ capacitors in parallel is $13.0\ \mu\text{F}$. When these two capacitors are replaced by their equivalent we get the network sketched in Figure 24.22. The equivalent capacitance of these three capacitors in series is $3.47\ \mu\text{F}$.

(b) $Q_{\text{tot}} = C_{\text{tot}}V = (3.47\ \mu\text{F})(50.0\ \text{V}) = 174\ \mu\text{C}$

(c) Q_{tot} is the same as *Q* for each of the capacitors in the series combination shown in Figure 24.22, so *Q* for each of the capacitors is $174\ \mu\text{C}$.

EVALUATE: The voltages across each capacitor in Figure 24.22 are $V_{10} = \dfrac{Q_{\text{tot}}}{C_{10}} = 17.4\ \text{V}$, $V_{13} = \dfrac{Q_{\text{tot}}}{C_{13}} = 13.4\ \text{V}$ and $V_9 = \dfrac{Q_{\text{tot}}}{C_9} = 19.3\ \text{V}$. $V_{10} + V_{13} + V_9 = 17.4\ \text{V} + 13.4\ \text{V} + 19.3\ \text{V} = 50.1\ \text{V}$. The sum of the voltages equals the applied voltage, apart from a small difference due to rounding.

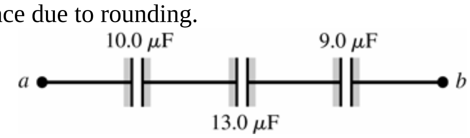


Figure 24.22

24.23. **IDENTIFY:** Refer to Figure 24.10b in the textbook. For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + \cdots$. For capacitors in series, $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots$.

SET UP: The $11\ \mu\text{F}$, $4\ \mu\text{F}$ and replacement capacitor are in parallel and this combination is in series with the $9.0\ \mu\text{F}$ capacitor.

EXECUTE: $\frac{1}{C_{\text{eq}}} = \frac{1}{8.0\ \mu\text{F}} = \left(\frac{1}{(11 + 4.0 + x)\ \mu\text{F}} + \frac{1}{9.0\ \mu\text{F}}\right)$. $(15 + x)\ \mu\text{F} = 72\ \mu\text{F}$ and $x = 57\ \mu\text{F}$.

EVALUATE: Increasing the capacitance of the one capacitor by a large amount makes a small increase in the equivalent capacitance of the network.

24.24. **IDENTIFY:** Apply $C = Q/V$. $C = \frac{\epsilon_0 A}{d}$. The work done to double the separation equals the change in the stored energy.

SET UP: $U = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$.

EXECUTE: **(a)** $V = Q/C = (2.55\ \mu\text{C})\big/(920 \times 10^{-12}\ \text{F}) = 2770\ \text{V}$

(b) $C = \frac{\epsilon_0 A}{d}$ says that since the charge is kept constant while the separation doubles, that means that the capacitance halves and the voltage doubles to 5540 V.

(c) $U = \frac{Q^2}{2C} = \frac{(2.55 \times 10^{-6}\ \text{C})^2}{2(920 \times 10^{-12}\ \text{F})} = 3.53 \times 10^{-3}\ \text{J}$. When if the separation is doubled while Q stays the same, the capacitance halves, and the energy stored doubles. So the amount of work done to move the plates equals the difference in energy stored in the capacitor, which is 3.53×10^{-3} J.

EVALUATE: The oppositely charged plates attract each other and positive work must be done by an external force to pull them farther apart.

24.25. **IDENTIFY** and **SET UP:** The energy density is given by Eq.(24.11): $u = \frac{1}{2}\epsilon_0 E^2$. Use $V = Ed$ to solve for E.

EXECUTE: Calculate $E : E = \frac{V}{d} = \frac{400\ \text{V}}{5.00 \times 10^{-3}\ \text{m}} = 8.00 \times 10^4\ \text{V/m}$.

Then $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\left(8.854 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2\right)\left(8.00 \times 10^4\ \text{V/m}\right)^2 = 0.0283\ \text{J/m}^3$

EVALUATE: E is smaller than the value in Example 24.8 by about a factor of 6 so u is smaller by about a factor of $6^2 = 36$.

24.26. **IDENTIFY:** $C = \frac{Q}{V_{ab}}$. $C = \frac{\epsilon_0 A}{d}$. $V_{ab} = Ed$. The stored energy is $\frac{1}{2}QV$.

SET UP: $d = 1.50 \times 10^{-3}\ \text{m}$. $1\ \mu\text{C} = 10^{-6}\ \text{C}$

EXECUTE: **(a)** $C = \frac{0.0180 \times 10^{-6}\ \text{C}}{200\ \text{V}} = 9.00 \times 10^{-11}\ \text{F} = 90.0\ \text{pF}$

(b) $C = \frac{\epsilon_0 A}{d}$ so $A = \frac{Cd}{\epsilon_0} = \frac{(9.00 \times 10^{-11}\ \text{F})(1.50 \times 10^{-3}\ \text{m})}{8.854 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2)} = 0.0152\ \text{m}^2$.

(c) $V = Ed = (3.0 \times 10^6\ \text{V/m})(1.50 \times 10^{-3}\ \text{m}) = 4.5 \times 10^3\ \text{V}$

(d) $\text{Energy} = \frac{1}{2}QV = \frac{1}{2}(0.0180 \times 10^{-6}\ \text{C})(200\ \text{V}) = 1.80 \times 10^{-6}\ \text{J} = 1.80\ \mu\text{J}$

EVALUATE: We could also calculate the stored energy as $\frac{Q^2}{2C} = \frac{(0.0180 \times 10^{-6}\ \text{C})^2}{2(9.00 \times 10^{-11}\ \text{F})} = 1.80\ \mu\text{J}$.

24.27. **IDENTIFY:** The energy stored in a charged capacitor is $\frac{1}{2}CV^2$.

SET UP: $1\ \mu\text{F} = 10^{-6}\ \text{F}$

EXECUTE: $\frac{1}{2}CV^2 = \frac{1}{2}(450 \times 10^{-6}\ \text{F})(295\ \text{V})^2 = 19.6\ \text{J}$

EVALUATE: Thermal energy is generated in the wire at the rate $I^2 R$, where I is the current in the wire. When the capacitor discharges there is a flow of charge that corresponds to current in the wire.

24.28. **IDENTIFY:** After the two capacitors are connected they must have equal potential difference, and their combined charge must add up to the original charge.

SET UP: $C = Q/V$. The stored energy is $U = \dfrac{Q^2}{2C} = \dfrac{1}{2}CV^2$

EXECUTE: **(a)** $Q = CV_0$.

(b) $V = \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2}$ and also $Q_1 + Q_2 = Q = CV_0$. $C_1 = C$ and $C_2 = \dfrac{C}{2}$ so $\dfrac{Q_1}{C} = \dfrac{Q_2}{(C/2)}$ and $Q_2 = \dfrac{Q_1}{2}$. $Q = \dfrac{3}{2}Q_1$.

$Q_1 = \dfrac{2}{3}Q$ and $V = \dfrac{Q_1}{C} = \dfrac{2}{3}\dfrac{Q}{C} = \dfrac{2}{3}V_0$.

(c) $U = \dfrac{1}{2}\left(\dfrac{Q_1^2}{C_1} + \dfrac{Q_2^2}{C_2}\right) = \dfrac{1}{2}\left[\dfrac{(\frac{2}{3}Q)^2}{C} + \dfrac{2(\frac{1}{3}Q)^2}{C}\right] = \dfrac{1}{3}\dfrac{Q^2}{C} = \dfrac{1}{3}CV_0^2$

(d) The original U was $U = \frac{1}{2}CV_0^2$, so $\Delta U = -\dfrac{1}{6}CV_0^2$.

(e) Thermal energy of capacitor, wires, etc., and electromagnetic radiation.

EVALUATE: The original charge of the charged capacitor must distribute between the two capacitors to make the potential the same across each capacitor. The voltage V for each after they are connected is less than the original voltage V_0 of the charged capacitor.

24.29. **IDENTIFY** and **SET UP:** Combine Eqs. (24.9) and (24.2) to write the stored energy in terms of the separation between the plates.

EXECUTE: **(a)** $U = \dfrac{Q^2}{2C}$; $C = \dfrac{\epsilon_0 A}{x}$ so $U = \dfrac{xQ^2}{2\epsilon_0 A}$

(b) $x \to x + dx$ gives $U = \dfrac{(x+dx)Q^2}{2\epsilon_0 A}$

$$dU = \frac{(x+dx)Q^2}{2\epsilon_0 A} - \frac{xQ^2}{2\epsilon_0 A} = \left(\frac{Q^2}{2\epsilon_0 A}\right)dx$$

(c) $dW = F\ dx = dU$, so $F = \dfrac{Q^2}{2\epsilon_0 A}$

(d) **EVALUATE:** The capacitor plates and the field between the plates are shown in Figure 24.29a.

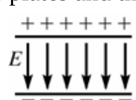


Figure 24.29a

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$
$$F = \tfrac{1}{2}QE, \text{ not } QE$$

The reason for the difference is that E is the field due to <u>both</u> plates. If we consider the positive plate only and calculate its electric field using Gauss's law (Figure 24.29b):

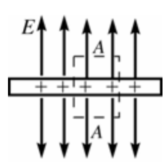


Figure 24.29b

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$
$$2EA = \frac{\sigma A}{\epsilon_0}$$
$$E = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2\epsilon_0 A}$$

The force this field exerts on the other plate, that has charge $-Q$, is $F = \dfrac{Q^2}{2\epsilon_0 A}$.

24.30. **IDENTIFY:** $C = \dfrac{\varepsilon_0 A}{d}$. The stored energy can be expressed either as $\dfrac{Q^2}{2C}$ or as $\dfrac{CV^2}{2}$, whichever is more convenient for the calculation.

SET UP: Since d is halved, C doubles.

EXECUTE: **(a)** If the separation distance is halved while the charge is kept fixed, then the capacitance increases and the stored energy, which was 8.38 J, decreases since $U = Q^2/2C$. Therefore the new energy is 4.19 J.

(b) If the voltage is kept fixed while the separation is decreased by one half, then the doubling of the capacitance leads to a doubling of the stored energy to 16.8 J, using $U = CV^2/2$, when V is held constant throughout.

EVALUATE: When the capacitor is disconnected, the stored energy decreases because of the positive work done by the attractive force between the plates. When the capacitor remains connected to the battery, $Q = CV$ tells us that the charge on the plates increases. The increased stored energy comes from the battery when it puts more charge onto the plates.

24.31. **IDENTIFY** and **SET UP:** $C = \dfrac{Q}{V}$. $U = \frac{1}{2}CV^2$.

EXECUTE: **(a)** $Q = CV = (5.0\ \mu\text{F})(1.5\ \text{V}) = 7.5\ \mu\text{C}$. $U = \frac{1}{2}CV^2 = \frac{1}{2}(5.0\ \mu\text{F})(1.5\ \text{V})^2 = 5.62\ \mu\text{J}$

(b) $U = \frac{1}{2}CV^2 = \frac{1}{2}C(Q/C)^2 = Q^2/2C$. $Q = \sqrt{2CU} = \sqrt{2(5.0\times10^{-6}\ \text{F})(1.0\ \text{J})} = 3.2\times10^{-3}\ \text{C}$.

$V = \dfrac{Q}{C} = \dfrac{3.2\times10^{-3}\ \text{C}}{5.0\times10^{-6}\ \text{F}} = 640\ \text{V}$.

EVALUATE: The stored energy is proportional to Q^2 and to V^2.

24.32. **IDENTIFY:** The two capacitors are in series. $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots$ $C = \dfrac{Q}{V}$. $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same.

EXECUTE: **(a)** $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$ so $C_{\text{eq}} = \dfrac{C_1C_2}{C_1 + C_2} = \dfrac{(150\ \text{nF})(120\ \text{nF})}{150\ \text{nF} + 120\ \text{nF}} = 66.7\ \text{nF}$.

$Q = CV = (66.7\ \text{nF})(36\ \text{V}) = 2.4\times10^{-6}\ \text{C} = 2.4\ \mu\text{C}$

(b) $Q = 2.4\ \mu\text{C}$ for each capacitor.

(c) $U = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(66.7\times10^{-9}\ \text{F})(36\ \text{V})^2 = 43.2\ \mu\text{J}$

(d) We know *C* and *Q* for each capacitor so rewrite *U* in terms of these quantities. $U = \frac{1}{2}CV^2 = \frac{1}{2}C(Q/C)^2 = Q^2/2C$

150 nF: $U = \dfrac{(2.4\times10^{-6}\ \text{C})^2}{2(150\times10^{-9}\ \text{F})} = 19.2\ \mu\text{J}$; 120 nF: $U = \dfrac{(2.4\times10^{-6}\ \text{C})^2}{2(120\times10^{-9}\ \text{F})} = 24.0\ \mu\text{J}$

Note that $19.2\ \mu\text{J} + 24.0\ \mu\text{J} = 43.2\ \mu\text{J}$, the total stored energy calculated in part (c).

(e) 150 nF: $V = \dfrac{Q}{C} = \dfrac{2.4\times10^{-6}\ \text{C}}{150\times10^{-9}\ \text{F}} = 16\ \text{V}$; 120 nF: $V = \dfrac{Q}{C} = \dfrac{2.4\times10^{-6}\ \text{C}}{120\times10^{-9}\ \text{F}} = 20\ \text{V}$

Note that these two voltages sum to 36 V, the voltage applied across the network.

EVALUATE: Since *Q* is the same the capacitor with smaller *C* stores more energy ($U = Q^2/2C$) and has a larger voltage ($V = Q/C$).

24.33. **IDENTIFY:** The two capacitors are in parallel. $C_{\text{eq}} = C_1 + C_2$. $C = \dfrac{Q}{V}$. $U = \frac{1}{2}CV^2$.

SET UP: For capacitors in parallel, the voltages are the same and the charges add.

EXECUTE: **(a)** $C_{\text{eq}} = C_1 + C_2 = 35\ \text{nF} + 75\ \text{nF} = 110\ \text{nF}$. $Q_{\text{tot}} = C_{\text{eq}}V = (110\times10^{-9}\ \text{F})(220\ \text{V}) = 24.2\ \mu\text{C}$

(b) $V = 220\ \text{V}$ for each capacitor.

35 nF: $Q_{35} = C_{35}V = (35\times10^{-9}\ \text{F})(220\ \text{V}) = 7.7\ \mu\text{C}$; 75 nF: $Q_{75} = C_{75}V = (75\times10^{-9}\ \text{F})(220\ \text{V}) = 16.5\ \mu\text{C}$. Note that $Q_{35} + Q_{75} = Q_{\text{tot}}$.

(c) $U_{\text{tot}} = \frac{1}{2}C_{\text{eq}}V^2 = \frac{1}{2}(110\times10^{-9}\ \text{F})(220\ \text{V})^2 = 2.66\ \text{mJ}$

(d) 35 nF: $U_{35} = \frac{1}{2}C_{35}V^2 = \frac{1}{2}(35\times10^{-9}\ \text{F})(220\ \text{V})^2 = 0.85\ \text{mJ}$;

75 nF: $U_{75} = \frac{1}{2}C_{75}V^2 = \frac{1}{2}(75\times10^{-9}\ \text{F})(220\ \text{V})^2 = 1.81\ \text{mJ}$. Since *V* is the same the capacitor with larger *C* stores more energy.

(e) 220 V for each capacitor.

EVALUATE: The capacitor with the larger *C* has the larger *Q*.

24.34. **IDENTIFY:** Capacitance depends on the geometry of the object.

(a) SET UP: The potential difference between the core and tube is $V = \dfrac{\lambda}{2\pi\epsilon_0}\ln(r_b/r_a)$. Solving for the linear charge density gives $\lambda = \dfrac{2\pi\epsilon_0 V}{\ln(r_b/r_a)} = \dfrac{4\pi\epsilon_0 V}{2\ln(r_b/r_a)}$.

EXECUTE: Using the given values gives $\lambda = \dfrac{6.00\,\text{V}}{2\left(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\ln\left(\dfrac{2.00}{1.20}\right)} = 6.53\times10^{-10}\ \text{C/m}$

(b) SET UP: $Q = \lambda L$

EXECUTE: $Q = (6.53\times10^{-10}\ \text{C/m})(0.350\ \text{m}) = 2.29\times10^{-10}\ \text{C}$

(c) SET UP: The definition of capacitance is $C = Q/V$.

EXECUTE: $C = \dfrac{2.29\times10^{10}\ \text{C}}{6.00\ \text{V}} = 3.81\times10^{-11}\ \text{F}$

(d) SET UP: The energy stored in a capacitor is $U = \frac{1}{2}CV^2$.

EXECUTE: $U = \frac{1}{2}(3.81\times10^{-11}\ \text{F})(6.00\ \text{V})^2 = 6.85\times10^{-10}\ \text{J}$

EVALUATE: The stored energy could be converted to heat or other forms of energy.

24.35. IDENTIFY: $U = \frac{1}{2}QV$. Solve for *Q*. $C = Q/V$.

SET UP: Example 24.4 shows that for a cylindrical capacitor, $\dfrac{C}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$.

EXECUTE: **(a)** $U = \frac{1}{2}QV$ gives $Q = \dfrac{2U}{V} = \dfrac{2(3.20\times10^{-9}\ \text{J})}{4.00\ \text{V}} = 1.60\times10^{-9}\ \text{C}$.

(b) $\dfrac{C}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$. $\dfrac{r_b}{r_a} = \exp(2\pi\epsilon_0 L/C) = \exp(2\pi\epsilon_0 LV/Q) = \exp(2\pi\epsilon_0(15.0\ \text{m})(4.00\ \text{V})\big/(1.60\times10^{-9}\ \text{C})) = 8.05$.

The radius of the outer conductor is 8.05 times the radius of the inner conductor.

EVALUATE: When the ratio r_b/r_a increases, C/L decreases and less charge is stored for a given potential difference.

24.36. IDENTIFY: Apply Eq.(24.11).

SET UP: Example 24.3 shows that $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$ between the conducting shells and that $\dfrac{Q}{4\pi\epsilon_0} = \left(\dfrac{r_a r_b}{r_b - r_a}\right)V_{ab}$.

EXECUTE: $E = \left(\dfrac{r_a r_b}{r_b - r_a}\right)\dfrac{V_{ab}}{r^2} = \left(\dfrac{[0.125\ \text{m}][0.148\ \text{m}]}{0.148\ \text{m} - 0.125\ \text{m}}\right)\dfrac{120\ \text{V}}{r^2} = \dfrac{96.5\ \text{V}\cdot\text{m}}{r^2}$

(a) For $r = 0.126\ \text{m}$, $E = 6.08\times10^3\ \text{V/m}$. $u = \frac{1}{2}\epsilon_0 E^2 = 1.64\times10^{-4}\ \text{J/m}^3$.

(b) For $r = 0.147\ \text{m}$, $E = 4.47\times10^3\ \text{V/m}$. $u = \frac{1}{2}\epsilon_0 E^2 = 8.85\times10^{-5}\ \text{J/m}^3$.

EVALUATE: **(c)** No, the results of parts (a) and (b) show that the energy density is not uniform in the region between the plates. *E* decreases as *r* increases, so *u* decreases also.

24.37. IDENTIFY: Use the rules for series and for parallel capacitors to express the voltage for each capacitor in terms of the applied voltage. Express *U*, *Q*, and *E* in terms of the capacitor voltage.

SET UP: Le the applied voltage be *V*. Let each capacitor have capacitance *C*. $U = \frac{1}{2}CV^2$ for a single capacitor with voltage *V*.

EXECUTE: **(a) series**

Voltage across each capacitor is $V/2$. The total energy stored is $U_s = 2\left(\frac{1}{2}C[V/2]^2\right) = \frac{1}{4}CV^2$

parallel

Voltage across each capacitor is *V*. The total energy stored is $U_p = 2\left(\frac{1}{2}CV^2\right) = CV^2$

$U_p = 4U_s$

(b) $Q = CV$ for a single capacitor with voltage *V*. $Q_s = 2\left(C[V/2]\right) = CV$; $Q_p = 2(CV) = 2CV$; $Q_p = 2Q_s$

(c) $E = V/d$ for a capacitor with voltage *V*. $E_s = V/2d$; $E_p = V/d$; $E_p = 2E_s$

EVALUATE: The parallel combination stores more energy and more charge since the voltage for each capacitor is larger for parallel. More energy stored and larger voltage for parallel means larger electric field in the parallel case.

24.38. IDENTIFY: $V = Ed$ and $C = Q/V$. With the dielectric present, $C = KC_0$.

SET UP: $V = Ed$ holds both with and without the dielectric.

EXECUTE: **(a)** $V = Ed = (3.00\times10^4\ \text{V/m})(1.50\times10^{-3}\ \text{m}) = 45.0\ \text{V}$.

$Q = C_0 V = (5.00\times10^{-12}\ \text{F})(45.0\ \text{V}) = 2.25\times10^{-10}\ \text{C}$.

(b) With the dielectric, $C = KC_0 = (2.70)(5.00\ \text{pF}) = 13.5\ \text{pF}$. *V* is still 45.0 V, so

$Q = CV = (13.5\times10^{-12}\ \text{F})(45.0\ \text{V}) = 6.08\times10^{-10}\ \text{C}$.

EVALUATE: The presence of the dielectric increases the amount of charge that can be stored for a given potential difference and electric field between the plates. *Q* increases by a factor of *K*.

24.39. **IDENTIFY** and **SET UP:** *Q* is constant so we can apply Eq.(24.14). The charge density on each surface of the dielectric is given by Eq.(24.16).

EXECUTE: $E=\dfrac{E_0}{K}$ so $K=\dfrac{E_0}{E}=\dfrac{3.20\times10^5\ \text{V/m}}{2.50\times10^5\ \text{V/m}}=1.28$

(a) $\sigma_i=\sigma(1-1/K)$

$\sigma=\epsilon_0E_0=(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.20\times10^5\ \text{N/C})=2.833\times10^{-6}\ \text{C/m}^2$

$\sigma_i=(2.833\times10^{-6}\ \text{C/m}^2)(1-1/1.28)=6.20\times10^{-7}\ \text{C/m}^2$

(b) As calculated above, $K=1.28$.

EVALUATE: The surface charges on the dielectric produce an electric field that partially cancels the electric field produced by the charges on the capacitor plates.

24.40. **IDENTIFY:** Capacitance depends on geometry, and the introduction of a dielectric increases the capacitance.

SET UP: For a parallel-plate capacitor, $C=K\epsilon_0A/d$.

EXECUTE: **(a)** Solving for *d* gives

$$d=\frac{K\epsilon_0A}{C}=\frac{(3.0)(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(0.22\ \text{m})(0.28\ \text{m})}{1.0\times10^{-9}\ \text{F}}=1.64\times10^{-3}\ \text{m}=1.64\ \text{mm}\,.$$

Dividing this result by the thickness of a sheet of paper gives $\dfrac{1.64\ \text{mm}}{0.20\ \text{mm/sheet}}\approx8\ \text{sheets}$.

(b) Solving for the area of the plates gives $A=\dfrac{Cd}{K\epsilon_0}=\dfrac{(1.0\times10^{-9}\ \text{F})(0.012\ \text{m})}{(3.0)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}=0.45\ \text{m}^2$.

(c) Teflon has a smaller dielectric constant (2.1) than the posterboard, so she will need more area to achieve the same capacitance.

EVALUATE: The use of dielectric makes it possible to construct reasonable-sized capacitors since the dielectric increases the capacitance by a factor of *K*.

24.41. **IDENTIFY** and **SET UP:** For a parallel-plate capacitor with a dielectric we can use the equation $C=K\epsilon_0A/d$. Minimum *A* means smallest possible *d*. *d* is limited by the requirement that *E* be less than $1.60\times10^7\ \text{V/m}$ when *V* is as large as 5500 V.

EXECUTE: $V=Ed$ so $d=\dfrac{V}{E}=\dfrac{5500\ \text{V}}{1.60\times10^7\ \text{V/m}}=3.44\times10^{-4}\ \text{m}$

Then $A=\dfrac{Cd}{K\epsilon_0}=\dfrac{(1.25\times10^{-9}\ \text{F})(3.44\times10^{-4}\ \text{m})}{(3.60)(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}=0.0135\ \text{m}^2.$

EVALUATE: The relation $V=Ed$ applies with or without a dielectric present. *A* would have to be larger if there were no dielectric.

24.42. **IDENTIFY** and **SET UP:** Adapt the derivation of Eq.(24.1) to the situation where a dielectric is present.

EXECUTE: Placing a dielectric between the plates just results in the replacement of ϵ for ϵ_0 in the derivation of Equation (24.20). One can follow exactly the procedure as shown for Equation (24.11).

EVALUATE: The presence of the dielectric increases the energy density for a given electric field.

24.43. **IDENTIFY:** The permittivity ϵ of a material is related to its dielectric constant by $\epsilon=K\epsilon_0$. The maximum voltage is related to the maximum possible electric field before dielectric breakdown by $V_{\text{max}}=E_{\text{max}}d$. $E=\dfrac{E_0}{K}=\dfrac{\sigma}{K\epsilon_0}$, where σ is the surface charge density on each plate. The induced surface charge density on the surface of the dielectric is given by $\sigma_\text{i}=\sigma(1-1/K)$.

SET UP: From Table 24.2, for polystyrene $K=2.6$ and the dielectric strength (maximum allowed electric field) is $2\times10^7\ \text{V/m}$.

EXECUTE: **(a)** $\epsilon=K\epsilon_0=(2.6)\epsilon_0=2.3\times10^{-11}\ \text{C}^2/\text{N}\cdot\text{m}^2$

(b) $V_{\text{max}}=E_{\text{max}}d=(2.0\times10^7\ \text{V/m})(2.0\times10^{-3}\ \text{m})=4.0\times10^4\ \text{V}$

(c) $E=\dfrac{\sigma}{K\epsilon_0}$ and $\sigma=\epsilon E=(2.3\times10^{-11}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.0\times10^7\ \text{V/m})=0.46\times10^{-3}\ \text{C/m}^2.$

$$\sigma_\text{i}=\sigma\left(1-\frac{1}{K}\right)=(0.46\times10^{-3}\ \text{C/m}^2)(1-1/2.6)=2.8\times10^{-4}\ \text{C/m}^2.$$

EVALUATE: The net surface charge density is $\sigma_{\text{net}}=\sigma-\sigma_\text{i}=1.8\times10^{-4}\ \text{C/m}^2$ and the electric field between the plates is $E=\sigma_{\text{net}}/\epsilon_0$.

24.44. **IDENTIFY:** $C = Q/V$. $C = KC_0$. $V = Ed$.

SET UP: Table 24.1 gives $K = 3.1$ for mylar.

EXECUTE: **(a)** $\Delta Q = Q - Q_0 = (K-1)Q_0 = (K-1)C_0V_0 = (2.1)(2.5\times10^{-7}\text{ F})(12\text{ V}) = 6.3\times10^{-6}\text{ C}$.

(b) $\sigma_i = \sigma(1-1/K)$ so $Q_i = Q(1-1/K) = (9.3\times10^{-6}\text{ C})(1-1/3.1) = 6.3\times10^{-6}\text{ C}$.

(c) The addition of the mylar doesn't affect the electric field since the induced charge cancels the additional charge drawn to the plates.

EVALUATE: $E = V/d$ and V is constant so E doesn't change when the dielectric is inserted.

24.45. **(a) IDENTIFY** and **SET UP:** Since the capacitor remains connected to the power supply the potential difference doesn't change when the dielectric is inserted. Use Eq.(24.9) to calculate V and combine it with Eq.(24.12) to obtain a relation between the stored energies and the dielectric constant and use this to calculate K.

EXECUTE: Before the dielectric is inserted $U_0 = \frac{1}{2}C_0V^2$ so $V = \sqrt{\dfrac{2U_0}{C_0}} = \sqrt{\dfrac{2\left(1.85\times10^{-5}\text{ J}\right)}{360\times10^{-9}\text{ F}}} = 10.1\text{ V}$

(b) $K = C/C_0$

$U_0 = \frac{1}{2}C_0V^2,\ U = \frac{1}{2}CV^2$ so $C/C_0 = U/U_0$

$$K = \frac{U}{U_0} = \frac{1.85\times10^{-5}\text{ J} + 2.32\times10^{-5}\text{ J}}{1.85\times10^{-5}\text{ J}} = 2.25$$

EVALUATE: K increases the capacitance and then from $U = \frac{1}{2}CV^2$, with V constant an increase in C gives an increase in U.

24.46. **IDENTIFY:** $C = KC_0$. $C = Q/V$. $V = Ed$.

SET UP: Since the capacitor remains connected to the battery the potential between the plates of the capacitor doesn't change.

EXECUTE: **(a)** The capacitance changes by a factor of K when the dielectric is inserted. Since V is unchanged (the battery is still connected), $\dfrac{C_{\text{after}}}{C_{\text{before}}} = \dfrac{Q_{\text{after}}}{Q_{\text{before}}} = \dfrac{45.0\text{ pC}}{25.0\text{ pC}} = K = 1.80$.

(b) The area of the plates is $\pi r^2 = \pi(0.0300\text{ m})^2 = 2.827\times10^{-3}\text{m}^2$ and the separation between them is thus $d = \dfrac{\epsilon_0 A}{C} = \dfrac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\text{ m}^2)}{12.5\times10^{-12}\text{ F}} = 2.00\times10^{-3}\text{ m}$. Before the dielectric is inserted, $C = \dfrac{\epsilon_0 A}{d} = \dfrac{Q}{V}$ and $V = \dfrac{Qd}{\epsilon_0 A} = \dfrac{(25.0\times10^{-12}\text{ C})(2.00\times10^{-3}\text{ m})}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\text{ m}^2)} = 2.00\text{ V}$. The battery remains connected, so the potential difference is unchanged after the dielectric is inserted.

(c) Before the dielectric is inserted, $E = \dfrac{Q}{\epsilon_0 A} = \dfrac{25.0\times10^{-12}\text{ C}}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\text{ m}^2)} = 1000\text{ N/C}$

Again, since the voltage is unchanged after the dielectric is inserted, the electric field is also unchanged.

EVALUATE: $E = \dfrac{V}{d} = \dfrac{2.00\text{ V}}{2.00\times10^{-3}\text{ m}} = 1000\text{ N/C}$, whether or not the dielectric is present. This agrees with the result in part (c). The electric field has this value at any point between the plates. We need d to calculate E because V is the potential difference between points separated by distance d.

24.47. **IDENTIFY:** $C = KC_0$. $U = \frac{1}{2}CV^2$.

SET UP: $C_0 = 12.5\ \mu\text{F}$ is the value of the capacitance without the dielectric present.

EXECUTE: **(a)** With the dielectric, $C = (3.75)(12.5\ \mu\text{F}) = 46.9\ \mu\text{F}$.

before: $U = \frac{1}{2}C_0V^2 = \frac{1}{2}(12.5\times10^{-6}\text{ F})(24.0\text{ V})^2 = 3.60\text{ mJ}$

after: $U = \frac{1}{2}CV^2 = \frac{1}{2}(46.9\times10^{-6}\text{ F})(24.0\text{ V})^2 = 13.5\text{ mJ}$

(b) $\Delta U = 13.5\text{ mJ} - 3.6\text{ mJ} = 9.9\text{ mJ}$. The energy increased.

EVALUATE: The power supply must put additional charge on the plates to maintain the same potential difference when the dielectric is inserted. $U = \frac{1}{2}QV$, so the stored energy increases.

24.48. **IDENTIFY:** Gauss's law in dielectrics has the same form as in vacuum except that the electric field is multiplied by a factor of K and the charge enclosed by the Gaussian surface is the free charge. The capacitance of an object depends on its geometry.

(a) SET UP: The capacitance of a parallel-plate capacitor is $C = K\epsilon_0 A/d$ and the charge on its plates is $Q = CV$.

EXECUTE: First find the capacitance:

$$C = \frac{K\epsilon_0 A}{d} = \frac{(2.1)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)}{1.00\times10^{-3}\ \text{m}} = 4.18\times10^{-10}\ \text{F}.$$

Now find the charge on the plates: $Q = CV = (4.18\times10^{-10}\ \text{F})(12.0\ \text{V}) = 5.02\times10^{-9}\ \text{C}$.

(b) SET UP: Gauss's law within the dielectric gives $KEA = Q_{\text{free}}/\epsilon_0$.

EXECUTE: Solving for E gives

$$E = \frac{Q_{\text{free}}}{KA\epsilon_0} = \frac{5.02\times10^{-9}\ \text{C}}{(2.1)(0.0225\ \text{m}^2)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 1.20\times10^4\ \text{N/C}$$

(c) SET UP: Without the Teflon and the voltage source, the charge is unchanged but the potential increases, so $C = \epsilon_0 A/d$ and Gauss's law now gives $EA = Q/\epsilon_0$.

EXECUTE: First find the capacitance:

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)}{1.00\times10^{-3}\ \text{m}} = 1.99\times10^{-10}\ \text{F}.$$

The potential difference is $V = \dfrac{Q}{C} = \dfrac{5.02\times10^{-9}\ \text{C}}{1.99\times10^{-10}\ \text{F}} = 25.2\ \text{V}$. From Gauss's law, the electric field is

$$E = \frac{Q}{\epsilon_0 A} = \frac{5.02\times10^{-9}\ \text{C}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0225\ \text{m}^2)} = 2.52\times10^4\ \text{N/C}.$$

EVALUATE: The dielectric reduces the electric field inside the capacitor because the electric field due to the dipoles of the dielectric is opposite to the external field due to the free charge on the plates.

24.49. **IDENTIFY:** Apply Eq.(24.23) to calculate *E*. $V = Ed$ and $C = Q/V$ apply whether there is a dielectric between the plates or not.

(a) SET UP: Apply Eq.(24.23) to the dashed surface in Figure 24.49:

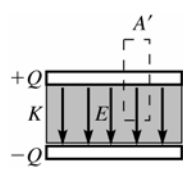


Figure 24.49

EXECUTE: $\oint K\vec{E}\cdot d\vec{A} = \dfrac{Q_{\text{encl-free}}}{\epsilon_0}$

$\oint K\vec{E}\cdot d\vec{A} = KEA'$

since $E = 0$ outside the plates

$Q_{\text{encl-free}} = \sigma A' = (Q/A)A'$

Thus $KEA' = \dfrac{(Q/A)A'}{\epsilon_0}$ and $E = \dfrac{Q}{\epsilon_0 AK}$

(b) $V = Ed = \dfrac{Qd}{\epsilon_0 AK}$

(c) $C = \dfrac{Q}{V} = \dfrac{Q}{(Qd/\epsilon_0 AK)} = K\dfrac{\epsilon_0 A}{d} = KC_0.$

EVALUATE: Our result shows that $K = C/C_0$, which is Eq.(24.12).

24.50. **IDENTIFY:** $C = \dfrac{\epsilon_0 A}{d}$. $C = Q/V$. $V = Ed$. $U = \frac{1}{2}CV^2$.

SET UP: With the battery disconnected, *Q* is constant. When the separation *d* is doubled, *C* is halved.

EXECUTE: **(a)** $C = \dfrac{\epsilon_0 A}{d} = \dfrac{\epsilon_0 (0.16\ \text{m})^2}{4.7\times10^{-3}\ \text{m}} = 4.8\times10^{-11}\ \text{F}$

(b) $Q = CV = (4.8\times10^{-11}\ \text{F})(12\ \text{V}) = 0.58\times10^{-9}\ \text{C}$

(c) $E = V/d = (12\ \text{V})/(4.7\times10^{-3}\ \text{m}) = 2550\ \text{V/m}$

(d) $U = \frac{1}{2}CV^2 = \frac{1}{2}(4.8\times10^{-11}\ \text{F})(12\ \text{V})^2 = 3.46\times10^{-9}\ \text{J}$

(e) If the battery is disconnected, so the charge remains constant, and the plates are pulled further apart to 0.0094 m, then the calculations above can be carried out just as before, and we find: (a) $C = 2.41\times10^{-11}\ \text{F}$ (b) $Q = 0.58\times10^{-9}\ \text{C}$

(c) $E = 2550\ \text{V/m}$ (d) $U = \dfrac{Q^2}{2C} = \dfrac{(0.58\times10^{-9}\ \text{C})^2}{2(2.41\times10^{-11}\ \text{F})} = 6.91\times10^{-9}\ \text{J}$

EVALUATE: *Q* is unchanged. $E = \dfrac{Q}{\epsilon_0 A}$ so *E* is unchanged. *U* doubles because *C* is halved. The additional stored energy comes from the work done by the force that pulled the plates apart.

24.51. **IDENTIFY** and **SET UP:** If the capacitor remains connected to the battery, the battery keeps the potential difference between the plates constant by changing the charge on the plates.

EXECUTE: **(a)** $C = \dfrac{\epsilon_0 A}{d}$

$$C = \frac{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.16\ \text{m})^2}{9.4\times10^{-3}\ \text{m}} = 2.4\times10^{-11}\ \text{F} = 24\ \text{pF}$$

(b) Remains connected to the battery says that V stays 12 V. $Q = CV = (2.4\times10^{-11}\ \text{F})(12\ \text{V}) = 2.9\times10^{-10}\ \text{C}$

(c) $E = \dfrac{V}{d} = \dfrac{12\ \text{V}}{9.4\times10^{-3}\ \text{m}} = 1.3\times10^{3}\ \text{V/m}$

(d) $U = \frac{1}{2}QV = \frac{1}{2}(2.9\times10^{-10}\ \text{C})(12.0\ \text{V}) = 1.7\times10^{-9}\ \text{J}$

EVALUATE: Increasing the separation decreases C. With V constant, this means that Q decreases and U decreases. Q decreases and $E = Q/\epsilon_0 A$ so E decreases. We come to the same conclusion from $E = V/d$.

24.52. **IDENTIFY:** $C = KC_0 = K\epsilon_0 \dfrac{A}{d}$. $V = Ed$ for a parallel plate capacitor; this equation applies whether or not a dielectric is present.

SET UP: $A = 1.0\ \text{cm}^2 = 1.0\times10^{-4}\ \text{m}^2$.

EXECUTE: **(a)** $C = (10)\dfrac{(8.85\times10^{-12}\ \text{F/m})(1.0\times10^{-4}\ \text{m}^2)}{7.5\times10^{-9}\ \text{m}} = 1.18\ \mu\text{F}$ per cm^2.

(b) $E = \dfrac{V}{K} = \dfrac{85\ \text{mV}}{7.5\times10^{-9}\ \text{m}} = 1.13\times10^{7}\ \text{V/m}$.

EVALUATE: The dielectric material increases the capacitance. If the dielectric were not present, the same charge density on the faces of the membrane would produce a larger potential difference across the membrane.

24.53. **IDENTIFY:** $P = E/t$, where E is the total light energy output. The energy stored in the capacitor is $U = \frac{1}{2}CV^2$.

SET UP: $E = 0.95U$

EXECUTE: **(a)** The power output is 600 W, and 95% of the original energy is converted, so

$E = Pt = (2.70\times10^{5}\ \text{W})(1.48\times10^{-3}\ \text{s}) = 400\ \text{J}$. $E_0 = \dfrac{400\ \text{J}}{0.95} = 421\ \text{J}$.

(b) $U = \frac{1}{2}CV^2$ so $C = \dfrac{2U}{V^2} = \dfrac{2(421\ \text{J})}{(125\ \text{V})^2} = 0.054\ \text{F}$.

EVALUATE: For a given V, the stored energy increases linearly with C.

24.54. **IDENTIFY:** $C = \dfrac{\epsilon_0 A}{d}$

SET UP: $A = 4.2\times10^{-5}\ \text{m}^2$. The original separation between the plates is $d = 0.700\times10^{-3}\ \text{m}$. d' is the separation between the plates at the new value of C.

EXECUTE: $C_0 = \dfrac{A\epsilon_0}{d} = \dfrac{(4.20\times10^{-5}\ \text{m}^2)\epsilon_0}{7.00\times10^{-4}\ \text{m}} = 5.31\times10^{-13}\ \text{F}$. The new value of C is $C = C_0 + 0.25\ \text{pF} = 7.81\times10^{-13}\ \text{F}$.

But $C = \dfrac{A\epsilon_0}{d'}$, so $d' = \dfrac{A\epsilon_0}{C} = \dfrac{(4.20\times10^{-5}\ \text{m}^2)\epsilon_0}{7.81\times10^{-13}\ \text{F}} = 4.76\times10^{-4}\text{m}$. Therefore the key must be depressed by a distance of $7.00\times10^{-4}\ \text{m} - 4.76\times10^{-4}\ \text{m} = 0.224\ \text{mm}$.

EVALUATE: When the key is depressed, d decreases and C increases.

24.55. **IDENTIFY:** Example 24.4 shows that $C = \dfrac{2\pi\epsilon_0 L}{\ln(r_b/r_a)}$ for a cylindrical capacitor.

SET UP: $\ln(1+x) \approx x$ when x is small. The area of each conductor is approximately $A = 2\pi r_a L$.

EXECUTE: **(a)** $d << r_a$: $C = \dfrac{2\pi\epsilon_0 L}{\ln(r_b/r_a)} = \dfrac{2\pi\epsilon_0 L}{\ln((d+r_a)/r_a)} = \dfrac{2\pi\epsilon_0 L}{\ln(1+d/r_a)} \approx \dfrac{2\pi r_a L\epsilon_0}{d} = \dfrac{\epsilon_0 A}{d}$

EVALUATE: **(b)** At the scale of part (a) the cylinders appear to be flat, and so the capacitance should appear like that of flat plates.

24.56. **IDENTIFY:** Initially the capacitors are connected in parallel to the source and we can calculate the charges Q_1 and Q_2 on each. After they are reconnected to each other the total charge is $Q = Q_2 - Q_1$. $U = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C}$.

SET UP: After they are reconnected, the charges add and the voltages are the same, so $C_{\text{eq}} = C_1 + C_2$, as for capacitors in parallel.

EXECUTE: Originally $Q_1 = C_1V_1 = (9.0\ \mu\text{F})\,(28\ \text{V}) = 2.52\times10^{-4}\ \text{C}$ and $Q_2 = C_2V_2 = (4.0\ \mu\text{F})(28\ \text{V}) = 1.12\times10^{-4}\ \text{C}$. $C_{\text{eq}} = C_1 + C_2 = 13.0\ \mu\text{F}$. The original energy stored is $U = \tfrac{1}{2}C_{\text{eq}}V^2 = \tfrac{1}{2}(13.0\times10^{-6}\ \text{F})(28\ \text{V})^2 = 5.10\times10^{-3}\ \text{J}$. Disconnect and flip the capacitors, so now the total charge is $Q = Q_2 - Q_1 = 1.4\times10^{-4}$ C and the equivalent capacitance is still the same, $C_{\text{eq}} = 13.0\ \mu\text{F}$. The new energy stored is $U = \dfrac{Q^2}{2C_{\text{eq}}} = \dfrac{(1.4\times10^{-4}\ \text{C})^2}{2(13.0\times10^{-6}\ \text{F})} = 7.54\times10^{-4}\ \text{J}$. The change in stored energy is $\Delta U = 7.45\times10^{-4}\ \text{J} - 5.10\times10^{-3}\ \text{J} = -4.35\times10^{-3}\ \text{J}$.

EVALUATE: When they are reconnected, charge flows and thermal energy is generated and energy is radiated as electromagnetic waves.

24.57. IDENTIFY: Simplify the network by replacing series and parallel combinations by their equivalent. The stored energy in a capacitor is $U = \tfrac{1}{2}CV^2$.

SET UP: For capacitors in series the voltages add and the charges are the same; $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots$. For capacitors in parallel the voltages are the same and the charges add; $C_{\text{eq}} = C_1 + C_2 + \cdots$ $C = \dfrac{Q}{V}$. $U = \tfrac{1}{2}CV^2$.

EXECUTE: **(a)** Find C_{eq} for the network by replacing each series or parallel combination by its equivalent. The successive simplified circuits are shown in Figure 24.57a–c.

$U_{\text{tot}} = \tfrac{1}{2}C_{\text{eq}}V^2 = \tfrac{1}{2}(2.19\times10^{-6}\ \text{F})(12.0\ \text{V})^2 = 1.58\times10^{-4}\ \text{J} = 158\ \mu\text{J}$

(b) From Figure 24.57c, $Q_{\text{tot}} = C_{\text{eq}}V = (2.19\times10^{-6}\ \text{F})(12.0\ \text{V}) = 2.63\times10^{-5}\ \text{C}$. From Figure 24.57b, $Q_{\text{tot}} = 2.63\times10^{-5}\ \text{C}$.

$V_{4.8} = \dfrac{Q_{4.8}}{C_{4.8}} = \dfrac{2.63\times10^{-5}\ \text{C}}{4.80\times10^{-6}\ \text{F}} = 5.48\ \text{V}$. $U_{4.8} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(4.80\times10^{-6}\ \text{F})(5.48\ \text{V})^2 = 7.21\times10^{-5}\ \text{J} = 72.1\ \mu\text{J}$

This one capacitor stores nearly half the total stored energy.

EVALUATE: $U = \dfrac{Q^2}{2C}$. For capacitors in series the capacitor with the smallest C stores the greatest amount of energy.

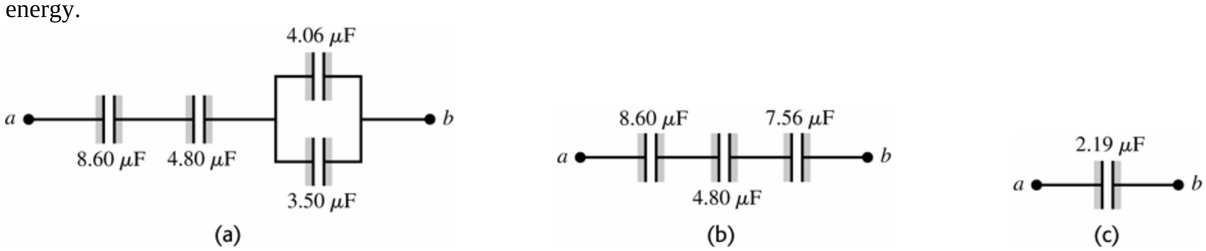


Figure 24.57

24.58. IDENTIFY: Apply the rules for combining capacitors in series and parallel. For capacitors in series the voltages add and in parallel the voltages are the same.

SET UP: When a capacitor is a moderately good conductor it can be replaced by a wire and the potential across it is zero.

EXECUTE: **(a)** A network that has the desired properties is sketched in Figure 24.58a. $C_{\text{eq}} = \dfrac{C}{2} + \dfrac{C}{2} = C$. The total capacitance is the same as each individual capacitor, and the voltage is spilt over each so that $V = 480\ \text{V}$.

(b) If one capacitor is a moderately good conductor, then it can be treated as a "short" and thus removed from the circuit, and one capacitor will have greater than 600 V across it.

EVALUATE: An alternative solution is two in parallel in series with two in parallel, as sketched in Figure 24.58b.

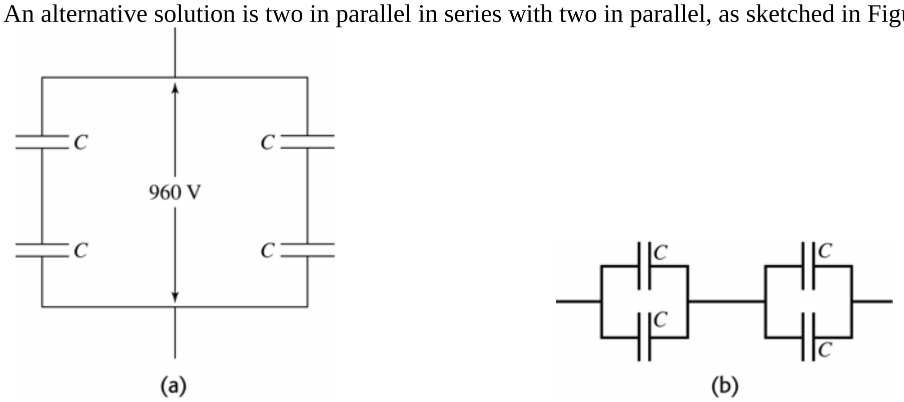
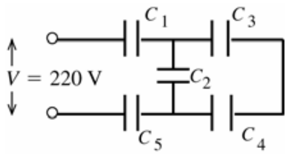


Figure 24.58

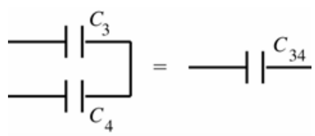
24.59. **(a) IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents.
SET UP: The network is sketched in Figure 24.59a.



$C_1 = C_5 = 8.4\ \mu\text{F}$
$C_2 = C_3 = C_4 = 4.2\ \mu\text{F}$

Figure 24.59a

EXECUTE: Simplify the circuit by replacing the capacitor combinations by their equivalents: C_3 and C_4 are in series and can be replaced by C_{34} (Figure 24.59b):

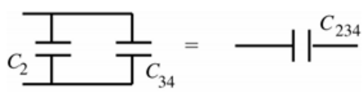


$$\frac{1}{C_{34}} = \frac{1}{C_3} + \frac{1}{C_4}$$
$$\frac{1}{C_{34}} = \frac{C_3 + C_4}{C_3 C_4}$$

Figure 24.59b

$$C_{34} = \frac{C_3 C_4}{C_3 + C_4} = \frac{(4.2\ \mu\text{F})(4.2\ \mu\text{F})}{4.2\ \mu\text{F} + 4.2\ \mu\text{F}} = 2.1\ \mu\text{F}$$

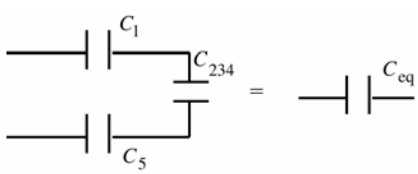
C_2 and C_{34} are in parallel and can be replaced by their equivalent (Figure 24.59c):



$$C_{234} = C_2 + C_{34}$$
$$C_{234} = 4.2\ \mu\text{F} + 2.1\ \mu\text{F}$$
$$C_{234} = 6.3\ \mu\text{F}$$

Figure 24.59c

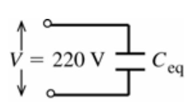
C_1, C_5 and C_{234} are in series and can be replaced by C_{eq} (Figure 24.59d):



$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_5} + \frac{1}{C_{234}}$$
$$\frac{1}{C_{\text{eq}}} = \frac{2}{8.4\ \mu\text{F}} + \frac{1}{6.3\ \mu\text{F}}$$
$$C_{\text{eq}} = 2.5\ \mu\text{F}$$

Figure 24.59d

EVALUATE: For capacitors in series the equivalent capacitor is smaller than any of those in series. For capacitors in parallel the equivalent capacitance is larger than any of those in parallel.
(b) IDENTIFY and **SET UP:** In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.
EXECUTE: The equivalent circuit is drawn in Figure 24.59e.



$$Q_{\text{eq}} = C_{\text{eq}} V$$
$$Q_{\text{eq}} = (2.5\ \mu\text{F})(220\ \text{V}) = 550\ \mu\text{C}$$

Figure 24.59e

$Q_1 = Q_5 = Q_{234} = 550\ \mu\text{C}$ (capacitors in series have same charge)

$$V_1 = \frac{Q_1}{C_1} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}$$

$$V_5 = \frac{Q_5}{C_5} = \frac{550\ \mu\text{C}}{8.4\ \mu\text{F}} = 65\ \text{V}$$

$$V_{234} = \frac{Q_{234}}{C_{234}} = \frac{550\ \mu\text{C}}{6.3\ \mu\text{F}} = 87\ \text{V}$$

Now draw the network as in Figure 24.59f.

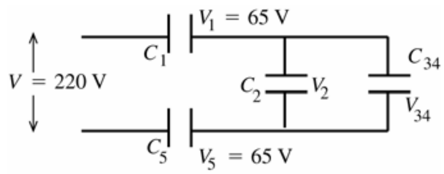


Figure 24.59f

$V_2 = V_{34} = V_{234} = 87\text{ V}$

capacitors in parallel have the same potential

$Q_2 = C_2V_2 = (4.2\ \mu\text{F})(87\text{ V}) = 370\ \mu\text{C}$

$Q_{34} = C_{34}V_{34} = (2.1\ \mu\text{F})(87\text{ V}) = 180\ \mu\text{C}$

Finally, consider the original circuit (Figure 24.59g).

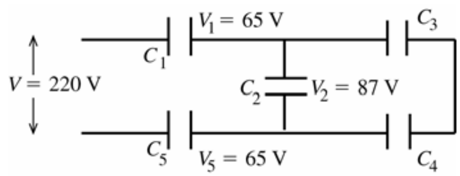


Figure 24.59g

$Q_3 = Q_4 = Q_{34} = 180\ \mu\text{C}$

capacitors in series have the same charge

$V_3 = \dfrac{Q_3}{C_3} = \dfrac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\text{ V}$

$V_4 = \dfrac{Q_4}{C_4} = \dfrac{180\ \mu\text{C}}{4.2\ \mu\text{F}} = 43\text{ V}$

Summary: $Q_1 = 550\ \mu\text{C},\ V_1 = 65\text{ V}$

$Q_2 = 370\ \mu\text{C},\ V_2 = 87\text{ V}$

$Q_3 = 180\ \mu\text{C},\ V_3 = 43\text{ V}$

$Q_4 = 180\ \mu\text{C},\ V_4 = 43\text{ V}$

$Q_5 = 550\ \mu\text{C},\ V_5 = 65\text{ V}$

EVALUATE: $V_3 + V_4 = V_2$ and $V_1 + V_2 + V_5 = 220\text{ V}$ (apart from some small rounding error)

$Q_1 = Q_2 + Q_3$ and $Q_5 = Q_2 + Q_4$

24.60. **IDENTIFY:** Apply the rules for combining capacitors in series and in parallel.

SET UP: With the switch open each pair of $3.00\ \mu\text{F}$ and $6.00\ \mu\text{F}$ capacitors are in series with each other and each pair is in parallel with the other pair. When the switch is closed each pair of $3.00\ \mu\text{F}$ and $6.00\ \mu\text{F}$ capacitors are in parallel with each other and the two pairs are in series.

EXECUTE: **(a)** With the switch open $C_{\text{eq}} = \left(\left(\dfrac{1}{3\ \mu\text{F}} + \dfrac{1}{6\ \mu\text{F}}\right)^{-1} + \left(\dfrac{1}{3\ \mu\text{F}} + \dfrac{1}{6\ \mu\text{F}}\right)^{-1}\right) = 4.00\ \mu\text{F}$.

$Q_{\text{total}} = C_{\text{eq}}V = (4.00\ \mu\text{F})\ (210\text{ V}) = 8.40\times10^{-4}\text{ C}$. By symmetry, each capacitor carries 4.20×10^{-4} C. The voltages are then calculated via $V = Q/C$. This gives $V_{ad} = Q/C_3 = 140\text{ V}$ and $V_{ac} = Q/C_6 = 70\text{ V}$.

$V_{cd} = V_{ad} - V_{ac} = 70\text{ V}$.

(b) When the switch is closed, the points *c* and *d* must be at the same potential, so the equivalent capacitance is

$C_{\text{eq}} = \left(\dfrac{1}{(3.00+6.00)\ \mu\text{F}} + \dfrac{1}{(3.00+6.00)\ \mu\text{F}}\right)^{-1} = 4.5\ \mu\text{F}$. $Q_{\text{total}} = C_{\text{eq}}V = (4.50\ \mu\text{F})(210\text{ V}) = 9.5\times10^{-4}\text{ C}$, and each capacitor has the same potential difference of 105 V (again, by symmetry).

(c) The only way for the sum of the positive charge on one plate of C_2 and the negative charge on one plate of C_1 to change is for charge to flow through the switch. That is, the quantity of charge that flows through the switch is equal to the change in $Q_2 - Q_1$. With the switch open, $Q_1 = Q_2$ and $Q_2 - Q_1 = 0$. After the switch is closed, $Q_2 - Q_1 = 315\ \mu\text{C}$, so $315\ \mu\text{C}$ of charge flowed through the switch.

EVALUATE: When the switch is closed the charge must redistribute to make points *c* and *d* be at the same potential.

24.61. **(a) IDENTIFY:** Replace the three capacitors in series by their equivalent. The charge on the equivalent capacitor equals the charge on each of the original capacitors.

SET UP: The three capacitors can be replaced by their equivalent as shown in Figure 24.61a.

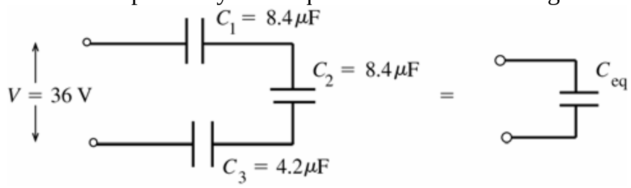


Figure 24.61a

EXECUTE: $C_3 = C_1/2$ so $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} = \dfrac{4}{8.4\ \mu\text{F}}$ and $C_{eq} = 8.4\ \mu\text{F}/4 = 2.1\ \mu\text{F}$

$Q = C_{eq}V = (2.1\ \mu\text{F})(36\ \text{V}) = 76\ \mu\text{C}$

The three capacitors are in series so they each have the same charge: $Q_1 = Q_2 = Q_3 = 76\ \mu\text{C}$

EVALUATE: The equivalent capacitance for capacitors in series is smaller than each of the original capacitors.

(b) IDENTIFY and **SET UP:** Use $U = \frac{1}{2}QV$. We know each Q and we know that $V_1 + V_2 + V_3 = 36$ V.

EXECUTE: $U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$

But $Q_1 = Q_2 = Q_3 = Q$ so $U = \frac{1}{2}Q(V_1 + V_2 + V_3)$

But also $V_1 + V_2 + V_3 = V = 36$ V, so $U = \frac{1}{2}QV = \frac{1}{2}(76\ \mu\text{C})(36\ \text{V}) = 1.4\times10^{-3}$ J.

EVALUATE: We could also use $U = Q^2/2C$ and calculate U for each capacitor.

(c) IDENTIFY: The charges on the plates redistribute to make the potentials across each capacitor the same.

SET UP: The capacitors before and after they are connected are sketched in Figure 24.61b.

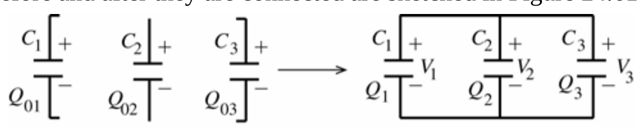


Figure 24.61b

EXECUTE: The total positive charge that is available to be distributed on the upper plates of the three capacitors is $Q_0 = Q_{01} + Q_{02} + Q_{03} = 3(76\ \mu\text{C}) = 228\ \mu\text{C}$. Thus $Q_1 + Q_2 + Q_3 = 228\ \mu\text{C}$. After the circuit is completed the charge distributes to make $V_1 = V_2 = V_3$. $V = Q/C$ and $V_1 = V_2$ so $Q_1/C_1 = Q_2/C_2$ and then $C_1 = C_2$ says $Q_1 = Q_2$. $V_1 = V_3$ says $Q_1/C_1 = Q_3/C_3$ and $Q_1 = Q_3(C_1/C_3) = Q_3(8.4\ \mu\text{F}/4.2\ \mu\text{F}) = 2Q_3$

Using $Q_2 = Q_1$ and $Q_1 = 2Q_3$ in the above equation gives $2Q_3 + 2Q_3 + Q_3 = 228\ \mu\text{C}$.

$5Q_3 = 228\ \mu\text{C}$ and $Q_3 = 45.6\ \mu\text{C}$, $Q_1 = Q_2 = 91.2\ \mu\text{C}$

Then $V_1 = \dfrac{Q_1}{C_1} = \dfrac{91.2\ \mu\text{C}}{8.4\ \mu\text{F}} = 11$ V, $V_2 = \dfrac{Q_2}{C_2} = \dfrac{91.2\ \mu\text{C}}{8.4\ \mu\text{F}} = 11$ V, and $V_3 = \dfrac{Q_3}{C_3} = \dfrac{45.6\ \mu\text{C}}{4.2\ \mu\text{F}} = 11$ V.

The voltage across each capacitor in the parallel combination is 11 V.

(d) $U = \frac{1}{2}Q_1V_1 + \frac{1}{2}Q_2V_2 + \frac{1}{2}Q_3V_3$.

But $V_1 = V_2 = V_3$ so $U = \frac{1}{2}V_1(Q_1 + Q_2 + Q_3) = \frac{1}{2}(11\ \text{V})(228\ \mu\text{C}) = 1.3\times10^{-3}$ J.

EVALUATE: This is less than the original energy of 1.4×10^{-3} J. The stored energy has decreased, as in Example 24.7.

24.62. **IDENTIFY:** $C = \dfrac{\epsilon_0 A}{d}$. $C = \dfrac{Q}{V}$. $V = Ed$. $U = \frac{1}{2}QV$.

SET UP: $d = 3.0\times10^3$ m. $A = \pi r^2$, with $r = 1.0\times10^3$ m.

EXECUTE: **(a)** $C = \dfrac{\epsilon_0 A}{d} = \dfrac{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\pi(1.0\times10^3\ \text{m})^2}{3.0\times10^3\ \text{m}} = 9.3\times10^{-9}$ F.

(b) $V = \dfrac{Q}{C} = \dfrac{20\ \text{C}}{9.3\times10^{-9}\ \text{F}} = 2.2\times10^9$ V

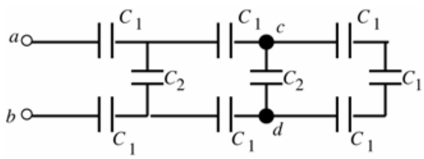
(c) $E = \dfrac{V}{d} = \dfrac{2.2\times10^9\ \text{V}}{3.0\times10^3\ \text{m}} = 7.3\times10^5$ V/m

(d) $U = \frac{1}{2}QV = \frac{1}{2}(20\ \text{C})(2.2\times10^9\ \text{V}) = 2.2\times10^{10}$ J

EVALUATE: Thunderclouds involve very large potential differences and large amounts of stored energy.

24.63. **IDENTIFY:** Replace series and parallel combinations of capacitors by their equivalents. In each equivalent network apply the rules for Q and V for capacitors in series and parallel; start with the simplest network and work back to the original circuit.

(a) SET UP: The network is sketched in Figure 24.63a.

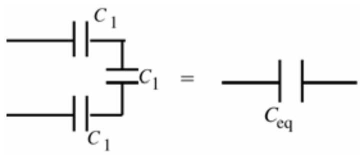


$C_1 = 6.9\ \mu\text{F}$

$C_2 = 4.6\ \mu\text{F}$

Figure 24.63a

EXECUTE: Simplify the network by replacing the capacitor combinations by their equivalents. Make the replacement shown in Figure 24.63b.

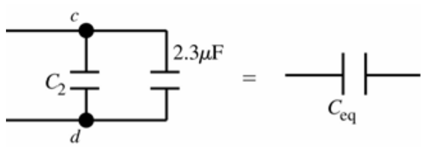


$$\frac{1}{C_{\text{eq}}} = \frac{3}{C_1}$$

$$C_{\text{eq}} = \frac{C_1}{3} = \frac{6.9\ \mu\text{F}}{3} = 2.3\ \mu\text{F}$$

Figure 24.63b

Next make the replacement shown in Figure 24.63c.

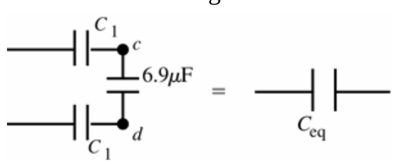


$$C_{\text{eq}} = 2.3\ \mu\text{F} + C_2$$

$$C_{\text{eq}} = 2.3\ \mu\text{F} + 4.6\ \mu\text{F} = 6.9\ \mu\text{F}$$

Figure 24.63c

Make the replacement shown in Figure 24.63d.

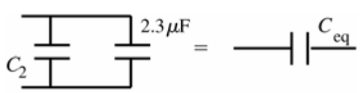


$$\frac{1}{C_{\text{eq}}} = \frac{2}{C_1} + \frac{1}{6.9\ \mu\text{F}} = \frac{3}{6.9\ \mu\text{F}}$$

$$C_{\text{eq}} = 2.3\ \mu\text{F}$$

Figure 24.63d

Make the replacement shown in Figure 24.63e.

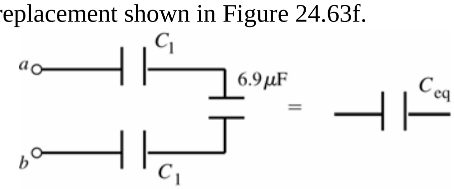


$$C_{\text{eq}} = C_2 + 2.3\ \mu\text{F} = 4.6\ \mu\text{F} + 2.3\ \mu\text{F}$$

$$C_{\text{eq}} = 6.9\ \mu\text{F}$$

Figure 24.63e

Make the replacement shown in Figure 24.63f.



$$\frac{1}{C_{\text{eq}}} = \frac{2}{C_1} + \frac{1}{6.9\ \mu\text{F}} = \frac{3}{6.9\ \mu\text{F}}$$

$$C_{\text{eq}} = 2.3\ \mu\text{F}$$

Figure 24.63f

(b) Consider the network as drawn in Figure 24.63g.

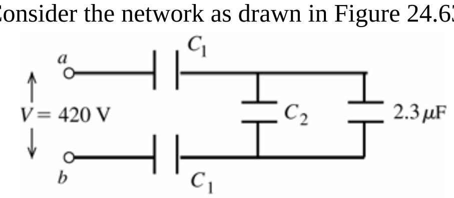
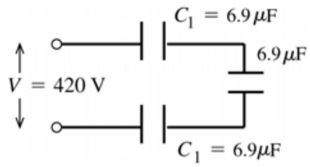


Figure 24.63g

From part (a) $2.3\ \mu\text{F}$ is the equivalent capacitance of the rest of the network.

The equivalent network is shown in Figure 24.63h.

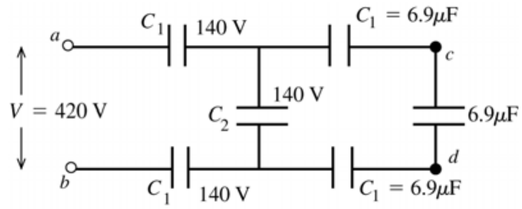


The capacitors are in series, so all three capacitors have the same *Q*.

Figure 24.63h

But here all three have the same *C*, so by *V* = *Q/C* all three must have the same *V*. The three voltages must add to 420 V, so each capacitor has *V* = 140 V. The $6.9\ \mu\text{F}$ to the right is the equivalent of C_2 and the $2.3\ \mu\text{F}$ capacitor in parallel, so $V_2 = 140\ \text{V}$. (Capacitors in parallel have the same potential difference.) Hence $Q_1 = C_1V_1 = (6.9\ \mu\text{F})(140\ \text{V}) = 9.7\times10^{-4}\ \text{C}$ and $Q_2 = C_2V_2 = (4.6\ \mu\text{F})(140\ \text{V}) = 6.4\times10^{-4}\ \text{C}$.

(c) From the potentials deduced in part (b) we have the situation shown in Figure 24.63i.



From part (a) $6.9\ \mu\text{F}$ is the equivalent capacitance of the rest of the network.

Figure 24.63i

The three right-most capacitors are in series and therefore have the same charge. But their capacitances are also equal, so by *V* = *Q/C* they each have the same potential difference. Their potentials must sum to 140 V, so the potential across each is 47 V and $V_{cd} = 47\ \text{V}$.

EVALUATE: In each capacitor network the rules for combining *V* for capacitors in series and parallel are obeyed. Note that $V_{cd} < V$, in fact $V - 2(140\ \text{V}) - 2(47\ \text{V}) = V_{cd}$.

24.64. IDENTIFY: Find the total charge on the capacitor network when it is connected to the battery. This is the amount of charge that flows through the signal device when the switch is closed.

SET UP: For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + C_3 + \cdots$

EXECUTE: $C_{\text{equiv}} = C_1 + C_2 + C_3 = 60.0\ \mu\text{F}$. $Q = CV = (60.0\ \mu\text{F})(120\ \text{V}) = 7200\ \mu\text{C}$.

EVALUATE: More charge is stored by the three capacitors in parallel than would be stored in each capacitor used alone.

24.65. (a) IDENTIFY and **SET UP:** *Q* is constant. $C = KC_0$; use Eq.(24.1) to relate the dielectric constant *K* to the ratio of the voltages without and with the dielectric.

EXECUTE: With the dielectric: $V = Q/C = Q/(KC_0)$

without the dielectric: $V_0 = Q/C_0$

$V_0/V = K$, so $K = (45.0\ \text{V})/(11.5\ \text{V}) = 3.91$

EVALUATE: Our analysis agrees with Eq.(24.13).

(b) IDENTIFY: The capacitor can be treated as equivalent to two capacitors C_1 and C_2 in parallel, one with area 2*A*/3 and air between the plates and one with area *A*/3 and dielectric between the plates.

SET UP: The equivalent network is shown in Figure 24.65.

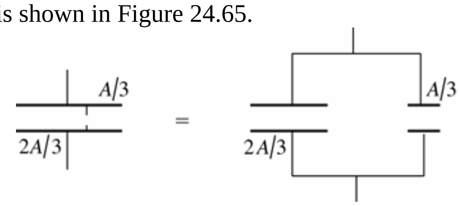


Figure 24.65

EXECUTE: Let $C_0 = \epsilon_0 A/d$ be the capacitance with only air between the plates. $C_1 = KC_0/3$, $C_2 = 2C_0/3$; $C_{\text{eq}} = C_1 + C_2 = (C_0/3)(K+2)$

$$V = \frac{Q}{C_{\text{eq}}} = \frac{Q}{C_0}\left(\frac{3}{K+2}\right) = V_0\left(\frac{3}{K+2}\right) = (45.0\ \text{V})\left(\frac{3}{5.91}\right) = 22.8\ \text{V}$$

EVALUATE: The voltage is reduced by the dielectric. The voltage reduction is less when the dielectric doesn't completely fill the volume between the plates.

24.66. **IDENTIFY:** This situation is analogous to having two capacitors C_1 in series, each with separation $\frac{1}{2}(d-a)$.

SET UP: For capacitors in series, $\dfrac{1}{C_{\text{eq}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

EXECUTE: **(a)** $C = \left(\dfrac{1}{C_1} + \dfrac{1}{C_1}\right)^{-1} = \frac{1}{2}C_1 = \frac{1}{2}\dfrac{\epsilon_0 A}{(d-a)/2} = \dfrac{\epsilon_0 A}{d-a}$

(b) $C = \dfrac{\epsilon_0 A}{d-a} = \dfrac{\epsilon_0 A}{d}\dfrac{d}{d-a} = C_0 \dfrac{d}{d-a}$

(c) As $a \to 0$, $C \to C_0$. The metal slab has no effect if it is very thin. And as $a \to d$, $C \to \infty$. $V = Q/C$. $V = Ey$ is the potential difference between two points separated by a distance *y* parallel to a uniform electric field. When the distance is very small, it takes a very large field and hence a large *Q* on the plates for a given potential difference. Since $Q = CV$ this corresponds to a very large *C*.

24.67. **(a) IDENTIFY:** The conductor can be at some potential *V*, where *V* = 0 far from the conductor. This potential depends on the charge *Q* on the conductor so we can define *C = Q/V* where *C* will not depend on *V* or *Q*.
(b) SET UP: Use the expression for the potential at the surface of the sphere in the analysis in part (a).
EXECUTE: For any point on a solid conducting sphere $V = Q/4\pi\epsilon_0 R$ if $V = 0$ at $r \to \infty$.

$$C = \frac{Q}{V} = Q\left(\frac{4\pi\epsilon_0 R}{Q}\right) = 4\pi\epsilon_0 R$$

(c) $C = 4\pi\epsilon_0 R = 4\pi\left(8.854\times10^{-12}\ \text{F/m}\right)\left(6.38\times10^{6}\ \text{m}\right) = 7.10\times10^{-4}\ \text{F} = 710\ \mu\text{F}.$

EVALUATE: The capacitance of the earth is about seven times larger than the largest capacitances in this range. The capacitance of the earth is quite small, in view of its large size.

24.68. **IDENTIFY:** The electric field energy density is $\frac{1}{2}\epsilon_0 E^2$. For a capacitor $U = \dfrac{Q^2}{2C}$.

SET UP: For a solid conducting sphere of radius *R*, $E = 0$ for $r < R$ and $E = \dfrac{Q}{4\pi\epsilon_0 r^2}$ for $r > R$.

EXECUTE: **(a)** $r < R$: $u = \frac{1}{2}\epsilon_0 E^2 = 0.$

(b) $r > R$: $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\dfrac{Q}{4\pi\epsilon_0 r^2}\right)^2 = \dfrac{Q^2}{32\pi^2\epsilon_0 r^4}.$

(c) $U = \int u\,dV = 4\pi\int_R^\infty r^2 u\,dr = \dfrac{Q^2}{8\pi\epsilon_0}\int_R^\infty \dfrac{dr}{r^2} = \dfrac{Q^2}{8\pi\epsilon_0 R}$.

(d) This energy is equal to $\dfrac{1}{2}\dfrac{Q^2}{4\pi\epsilon_0 R}$ which is just the energy required to assemble all the charge into a spherical distribution. (Note that being aware of double counting gives the factor of 1/2 in front of the familiar potential energy formula for a charge *Q* a distance *R* from another charge *Q*.)

EVALUATE: **(e)** From Equation (24.9), $U = \dfrac{Q^2}{2C}$. $U = \dfrac{Q^2}{8\pi\epsilon_0 R}$ from part (c), $C = 4\pi\epsilon_0 R$, as in Problem (24.67).

24.69. **IDENTIFY:** We model the earth as a spherical capacitor.

SET UP: The capacitance of the earth is $C = 4\pi\epsilon_0\dfrac{r_a r_b}{r_b - r_a}$ and, the charge on it is *Q = CV*, and its stored energy is $U = \frac{1}{2}CV^2$.

EXECUTE: **(a)** $C = \dfrac{1}{9.00\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}\dfrac{\left(6.38\times10^{6}\ \text{m}\right)\left(6.45\times10^{6}\ \text{m}\right)}{6.45\times10^{6}\ \text{m} - 6.38\times10^{6}\ \text{m}} = 6.5\times10^{-2}\ \text{F}$

(b) $Q = CV = \left(6.54\times10^{-2}\ \text{F}\right)(350{,}000\ \text{V}) = 2.3\times10^{4}\ \text{C}$

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}\left(6.54\times10^{-2}\ \text{F}\right)(350{,}000\ \text{V})^2 = 4.0\times10^{9}\ \text{J}$

EVALUATE: While the capacitance of the earth is larger than ordinary laboratory capacitors, capacitors much larger than this, such as 1 F, are readily available.

24.70. **IDENTIFY:** The electric field energy density is $u = \frac{1}{2}\epsilon_0 E^2$. $U = \dfrac{Q^2}{2C}$.

SET UP: For this charge distribution, $E = 0$ for $r < r_a$, $E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for $r_a < r < r_b$ and $E = 0$ for $r > r_b$.

Example 24.4 shows that $\dfrac{U}{L} = \dfrac{2\pi\epsilon_0}{\ln(r_b/r_a)}$ for a cylindrical capacitor.

EXECUTE: **(a)** $u = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\dfrac{\lambda}{2\pi\epsilon_0 r}\right)^2 = \dfrac{\lambda^2}{8\pi^2\epsilon_0 r^2}$

(b) $U = \int u\,dV = 2\pi L \int u r\,dr = \dfrac{L\lambda^2}{4\pi\epsilon_0}\displaystyle\int_{r_a}^{r_b}\dfrac{dr}{r}$ and $\dfrac{U}{L} = \dfrac{\lambda^2}{4\pi\epsilon_0}\ln(r_b/r_a)$.

(c) Using Equation (24.9), $U = \dfrac{Q^2}{2C} = \dfrac{Q^2}{4\pi\epsilon_0 L}\ln(r_b/r_a) = \dfrac{\lambda^2 L}{4\pi\epsilon_0}\ln(r_b/r_a)$. This agrees with the result of part (b).

EVALUATE: We could have used the results of part (b) and $U = \dfrac{Q^2}{2C}$ to calculate U/L and would obtain the same result as in Example 24.4.

24.71. **IDENTIFY:** $C = Q/V$, so we need to calculate the effect of the dielectrics on the potential difference between the plates.

SET UP: Let the potential of the positive plate be V_a, the potential of the negative plate be V_c, and the potential midway between the plates where the dielectrics meet be V_b, as shown in Figure 24.71.

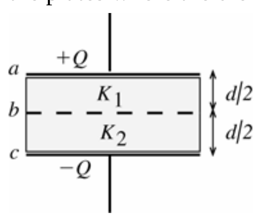


Figure 24.71

$$C = \frac{Q}{V_a - V_c} = \frac{Q}{V_{ac}}.$$
$$V_{ac} = V_{ab} + V_{bc}.$$

EXECUTE: The electric field in the absence of any dielectric is $E_0 = \dfrac{Q}{\epsilon_0 A}$. In the first dielectric the electric field is reduced to $E_1 = \dfrac{E_0}{K_1} = \dfrac{Q}{K_1\epsilon_0 A}$ and $V_{ab} = E_1\left(\dfrac{d}{2}\right) = \dfrac{Qd}{K_1 2\epsilon_0 A}$. In the second dielectric the electric field is reduced to $E_2 = \dfrac{E_0}{K_2} = \dfrac{Q}{K_2\epsilon_0 A}$ and $V_{bc} = E_2\left(\dfrac{d}{2}\right) = \dfrac{Qd}{K_2 2\epsilon_0 A}$. Thus $V_{ac} = V_{ab} + V_{bc} = \dfrac{Qd}{K_1 2\epsilon_0 A} + \dfrac{Qd}{K_2 2\epsilon_0 A} = \dfrac{Qd}{2\epsilon_0 A}\left(\dfrac{1}{K_1} + \dfrac{1}{K_2}\right)$.

$V_{ac} = \dfrac{Qd}{2\epsilon_0 A}\left(\dfrac{K_1 + K_2}{K_1 K_2}\right)$. This gives $C = \dfrac{Q}{V_{ac}} = Q\left(\dfrac{2\epsilon_0 A}{Qd}\right)\left(\dfrac{K_1 K_2}{K_1 + K_2}\right) = \dfrac{2\epsilon_0 A}{d}\left(\dfrac{K_1 K_2}{K_1 + K_2}\right)$.

EVALUATE: An equivalent way to calculate C is to consider the capacitor to be two in series, one with dielectric constant K_1 and the other with dielectric constant K_2 and both with plate separation $d/2$. (Can imagine inserting a thin conducting plate between the dielectric slabs.)

$$C_1 = K_1\frac{\epsilon_0 A}{d/2} = 2K_1\frac{\epsilon_0 A}{d}$$
$$C_2 = K_2\frac{\epsilon_0 A}{d/2} = 2K_2\frac{\epsilon_0 A}{d}$$

Since they are in series the total capacitance C is given by $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$ so $C = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{2\epsilon_0 A}{d}\left(\dfrac{K_1 K_2}{K_1 + K_2}\right)$

24.72. **IDENTIFY:** This situation is analogous to having two capacitors in parallel, each with an area $A/2$.

SET UP: For capacitors in parallel, $C_{eq} = C_1 + C_2$. For a parallel-plate capacitor with plates of area $A/2$, $C = \dfrac{\epsilon_0(A/2)}{d}$.

EXECUTE: $C_{eq} = C_1 + C_2 = \dfrac{\epsilon_0 A/2}{d} + \dfrac{\epsilon_0 A/2}{d} = \dfrac{\epsilon_0 A}{2d}(K_1 + K_2)$

EVALUATE: If $K_1 = K_2$, $C_{eq} = K\dfrac{\epsilon_0 A}{d}$, which is Eq.(24.19).

24.73. IDENTIFY and **SET UP:** Show the transformation from one circuit to the other:

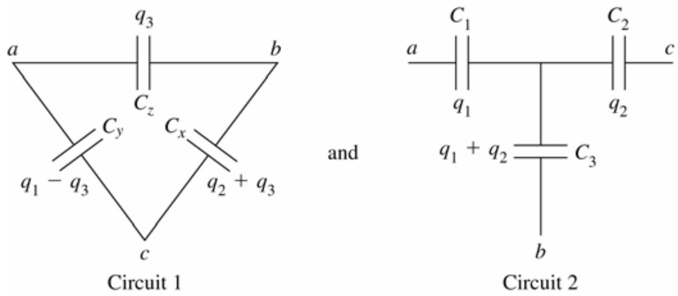


Figure 24.73a

EXECUTE: **(a)** Consider the two networks shown in Figure 24.73a. From Circuit 1: $V_{ac} = \dfrac{q_1 - q_3}{C_y}$ and $V_{bc} = \dfrac{q_2 + q_3}{C_x}$.

q_3 is derived from V_{ab}: $V_{ab} = \dfrac{q_3}{C_z} = \dfrac{q_1 - q_3}{C_y} = \dfrac{q_2 - q_3}{C_x}$. This gives $q_3 = \dfrac{C_x C_y C_z}{C_x + C_y + C_z}\left(\dfrac{q_1}{C_y} - \dfrac{q_2}{C_x}\right) \equiv K\left(\dfrac{q_1}{C_y} - \dfrac{q_2}{C_x}\right)$.

From Circuit 2: $V_{ac} = \dfrac{q_1}{C_1} + \dfrac{q_1 + q_2}{C_3} = q_1\left(\dfrac{1}{C_1} + \dfrac{1}{C_3}\right) + q_2\dfrac{1}{C_3}$ and $V_{bc} = \dfrac{q_2}{C_2} + \dfrac{q_1 + q_2}{C_3} = q_1\dfrac{1}{C_3} + q_2\left(\dfrac{1}{C_2} + \dfrac{1}{C_3}\right)$. Setting the coefficients of the charges equal to each other in matching potential equations from the two circuits results in three independent equations relating the two sets of capacitances. The set of equations are $\dfrac{1}{C_1} = \dfrac{1}{C_y}\left(1 - \dfrac{1}{KC_y} - \dfrac{1}{KC_x}\right)$,

$\dfrac{1}{C_2} = \dfrac{1}{C_x}\left(1 - \dfrac{1}{KC_y} - \dfrac{1}{KC_x}\right)$ and $\dfrac{1}{C_3} = \dfrac{1}{KC_y C_x}$. From these, subbing in the expression for K, we get

$C_1 = (C_x C_y + C_y C_z + C_z C_x)/C_x$, $C_2 = (C_x C_y + C_y C_z + C_z C_x)/C_y$ and $C_3 = (C_x C_y + C_y C_z + C_z C_x)/C_z$.

(b) Using the transformation of part (a) we have the equivalent networks shown in Figure 24.73b:

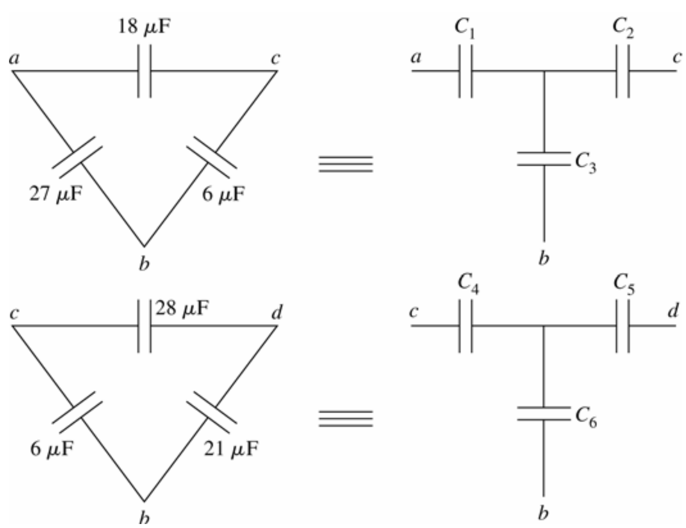


Figure 24.73b

$C_1 = 126\ \mu\text{F}$, $C_2 = 28\ \mu\text{F}$, $C_3 = 42\ \mu\text{F}$, $C_4 = 42\ \mu\text{F}$, $C_5 = 147\ \mu\text{F}$ and $C_6 = 32\ \mu\text{F}$. The total equivalent capacitance is $C_{\text{eq}} = \left(\dfrac{1}{72\ \mu\text{F}} + \dfrac{1}{126\ \mu\text{F}} + \dfrac{1}{34.8\ \mu\text{F}} + \dfrac{1}{147\ \mu\text{F}} + \dfrac{1}{72\ \mu\text{F}}\right)^{-1} = 14.0\ \mu\text{F}$, where the $34.8\ \mu\text{F}$ comes from

$$34.8\ \mu\text{F} = \left(\left(\dfrac{1}{42\ \mu\text{F}} + \dfrac{1}{32\ \mu\text{F}}\right)^{-1} + \left(\dfrac{1}{28\ \mu\text{F}} + \dfrac{1}{42\ \mu\text{F}}\right)^{-1}\right).$$

(c) The circuit diagram can be redrawn as shown in Figure 25.73c. The overall charge is given by $Q = C_{\text{eq}}V = (14.0\ \mu\text{F})(36\text{ V}) = 5.04\times10^{-4}\text{ C}$. And this is also the charge on the $72\ \mu\text{F}$ capacitors, so $V_{72} = \dfrac{5.04\times10^{-4}\text{ C}}{72\times10^{-6}\text{ F}} = 7.0\text{ V}$.

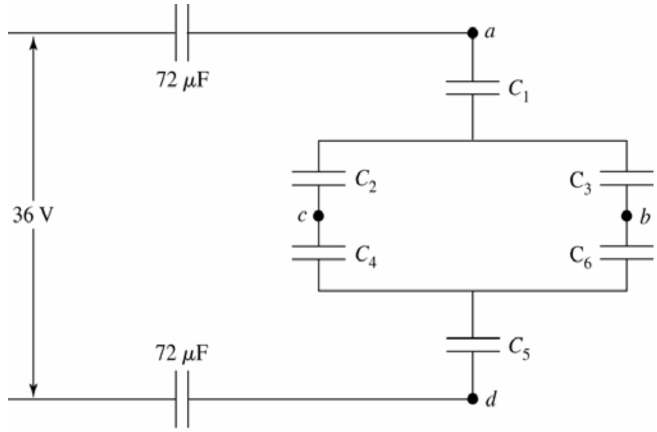


Figure 24.73c

Next we will find the voltage over the numbered capacitors, and their associated voltages. Then those voltages will be changed back into voltage of the original capacitors, and then their charges. $Q_{C_1} = Q_{C_5} = Q_{72} = 5.04\times10^{-4}\text{ C}$.

$V_{C_5} = \dfrac{5.04\times10^{-4}\text{ C}}{147\times10^{-6}\text{ F}} = 3.43\text{ V}$ and $V_{C_1} = \dfrac{5.04\times10^{-4}\text{C}}{126\times10^{-6}\text{ F}} = 4.00\text{ V}$. Therefore,

$V_{C_2C_4} = V_{C_3C_8} = (36.0 - 7.00 - 7.00 - 4.00 - 3.43)\text{ V} = 14.6\text{ V}$. But $C_{\text{eq}}(C_2C_4) = \left(\dfrac{1}{C_2} + \dfrac{1}{C_4}\right)^{-1} = 16.8\ \mu\text{F}$ and

$C_{\text{eq}}(C_3C_6) = \left(\dfrac{1}{C_3} + \dfrac{1}{C_6}\right)^{-1} = 18.2\ \mu\text{F}$, so $Q_{C_2} = Q_{C_4} = V_{C_2C_4}C_{\text{eq}(C_2C_4)} = 2.45\times10^{-4}\text{ C}$ and

$Q_{C_3} = Q_{C_6} = V_{C_3C_6}C_{\text{eq}(C_3C_6)} = 2.64\times10^{-4}\text{ C}$. Then $V_{C_2} = \dfrac{Q_{C_2}}{C_2} = 8.8\text{ V}$, $V_{C_3} = \dfrac{Q_{C_3}}{C_3} = 6.3\text{ V}$, $V_{C_4} = \dfrac{Q_{C_4}}{C_4} = 5.8\text{ V}$ and

$V_{C_6} = \dfrac{Q_{C_6}}{C_6} = 8.3\text{ V}$. $V_{ac} = V_{C_1} + V_{C_2} = V_{18} = 13\text{ V}$ and $Q_{18} = C_{18}V_{18} = 2.3\times10^{-4}\text{ C}$. $V_{ab} = V_{C_1} + V_{C_3} = V_{27} = 10\text{ V}$ and $Q_{27} = C_{27}V_{27} = 2.8\times10^{-4}\text{ C}$. $V_{cd} = V_{C_4} + V_{C_5} = V_{28} = 9\text{ V}$ and $Q_{28} = C_{28}V_{28} = 2.6\times10^{-4}\text{ C}$. $V_{bd} = V_{C_5} + V_{C_6} = V_{21} = 12\text{ V}$ and $Q_{21} = C_{21}V_{21} = 2.5\times10^{-4}\text{ C}$. $V_{bc} = V_{C_3} - V_{C_2} = V_6 = 2.5\text{ V}$ and $Q_6 = C_6V_6 = 1.5\times10^{-5}\text{ C}$.

EVALUATE: Note that $2V_{72} + V_{18} + V_{28} = 2(7.0\text{ V}) + 13\text{ V} + 9\text{ V} = 36\text{ V}$, as it should.

24.74. **IDENTIFY:** The force on one plate is due to the electric field of the other plate. The electrostatic force must be balanced by the forces from the springs.

SET UP: The electric field due to one plate is $E = \dfrac{\sigma}{2\epsilon_0}$. The force exerted by a spring compressed a distance $z_0 - z$ from equilibrium is $k(z_0 - z)$.

EXECUTE: **(a)** The force between the two parallel plates is $F = qE = \dfrac{q\sigma}{2\epsilon_0} = \dfrac{q^2}{2\epsilon_0 A} = \dfrac{(CV)^2}{2\epsilon_0 A} = \dfrac{\epsilon_0^2A^2}{z^2}\dfrac{V^2}{2\epsilon_0 A} = \dfrac{\epsilon_0 AV^2}{2z^2}$.

(b) When $V = 0$, the separation is just z_0. When $V \neq 0$, the total force from the four springs must equal the electrostatic force calculated in part (a). $F_{4\text{ springs}} = 4k(z_0 - z) = \dfrac{\epsilon_0 AV^2}{2z^2}$ and $2z^3 - 2z^3z_0 + \dfrac{\epsilon_0 AV^2}{4k} = 0$.

(c) For $A = 0.300\text{ m}^2$, $z_0 = 1.2\times10^{-3}\text{ m}$, $k = 25\text{ N/m}$ and $V = 120\text{ V}$, so $2z^3 - (2.4\times10^{-3}\text{ m})z^2 + 3.82\times10^{-10}\text{ m}^3 = 0$. The physical solutions to this equation are $z = 0.537$ mm and 1.014 mm.

EVALUATE: **(d)** Stable equilibrium occurs if a slight displacement from equilibrium yields a force back toward the equilibrium point. If one evaluates the forces at small displacements from the equilibrium positions above, the 1.014 mm separation is seen to be stable, but not the 0.537 mm separation.

24.75. **IDENTIFY:** The system can be considered to be two capacitors in parallel, one with plate area $L(L-x)$ and air between the plates and one with area Lx and dielectric filling the space between the plates.

SET UP: $C=\dfrac{K\epsilon_0 A}{d}$ for a parallel-plate capacitor with plate area A.

EXECUTE: **(a)** $C=\dfrac{\epsilon_0}{D}((L-x)L+xKL)=\dfrac{\epsilon_0 L}{D}(L+(K-1)x)$

(b) $dU=\tfrac{1}{2}(dC)V^2$, where $C=C_0+\dfrac{\epsilon_0 L}{D}(-dx+dxK)$, with $C_0=\dfrac{\epsilon_0 L}{D}(L+(K-1)x)$. This gives $dU=\tfrac{1}{2}\left(\dfrac{\epsilon_0 L dx}{D}(K-1)\right)V^2=\dfrac{(K-1)\epsilon_0 V^2 L}{2D}dx$.

(c) If the charge is kept constant on the plates, then $Q=\dfrac{\epsilon_0 LV}{D}(L+(K-1)x)$ and $U=\tfrac{1}{2}CV^2=\tfrac{1}{2}C_0V^2\left(\dfrac{C}{C_0}\right)$. $U\approx\dfrac{C_0V^2}{2}\left(1-\dfrac{\epsilon_0 L}{DC_0}(K-1)dx\right)$ and $\Delta U=U-U_0=-\dfrac{(K-1)\epsilon_0 V^2 L}{2D}dx$.

(d) Since $dU=-Fdx=-\dfrac{(K-1)\epsilon_0 V^2 L}{2D}dx$, the force is in the opposite direction to the motion dx, meaning that the slab feels a force pushing it out.

EVALUATE: **(e)** When the plates are connected to the battery, the plates plus slab are not an isolated system. In addition to the work done on the slab by the charges on the plates, energy is also transferred between the battery and the plates. Comparing the results for dU in part (c) to $dU=-Fdx$ gives $F=\dfrac{(K-1)\epsilon_0 V^2 L}{2D}$.

24.76. **IDENTIFY:** $C=Q/V$. Apply Gauss's law and the relation between potential difference and electric field.

SET UP: Each conductor is an equipotential surface. $V_a-V_b=\int_{r_0}^{r_b}\vec{\boldsymbol{E}}_{\text{U}}\cdot d\vec{\boldsymbol{r}}=\int_{r_a}^{r_b}\vec{\boldsymbol{E}}_{\text{L}}\cdot d\vec{\boldsymbol{r}}$, so $E_{\text{U}}=E_{\text{L}}$, where these are the fields between the upper and lower hemispheres. The electric field is the same in the air space as in the dielectric.

EXECUTE: **(a)** For a normal spherical capacitor with air between the plates, $C_0=4\pi\epsilon_0\left(\dfrac{r_ar_b}{r_b-r_a}\right)$. The capacitor in this problem is equivalent to two parallel capacitors, C_{L} and C_{U}, each with half the plate area of the normal capacitor. $C_{\text{L}}=\dfrac{KC_0}{2}=2\pi K\epsilon_0\left(\dfrac{r_ar_b}{r_b-r_a}\right)$ and $C_{\text{U}}=\dfrac{C_0}{2}=2\pi\epsilon_0\left(\dfrac{r_ar_b}{r_b-r_a}\right)$. $C=C_{\text{U}}+C_{\text{L}}=2\pi\epsilon_0(1+K)\left(\dfrac{r_ar_b}{r_b-r_a}\right)$.

(b) Using a hemispherical Gaussian surface for each respective half, $E_{\text{L}}\dfrac{4\pi r^2}{2}=\dfrac{Q_{\text{L}}}{K\epsilon_0}$, so $E_{\text{L}}=\dfrac{Q_{\text{L}}}{2\pi K\epsilon_0 r^2}$, and $E_{\text{U}}\dfrac{4\pi r^2}{2}=\dfrac{Q_{\text{U}}}{\epsilon_0}$, so $E_{\text{U}}=\dfrac{Q_{\text{U}}}{2\pi\epsilon_0 r^2}$. But $Q_{\text{L}}=VC_{\text{L}}$ and $Q_{\text{U}}=VC_{\text{U}}$. Also, $Q_{\text{L}}+Q_{\text{U}}=Q$. Therefore, $Q_{\text{L}}=\dfrac{VC_0K}{2}=KQ_{\text{U}}$ and $Q_{\text{U}}=\dfrac{Q}{1+K}$, $Q_{\text{L}}=\dfrac{KQ}{1+K}$. This gives $E_{\text{L}}=\dfrac{KQ}{1+K}\dfrac{1}{2\pi K\epsilon_0 r^2}=\dfrac{2}{1+K}\dfrac{Q}{4\pi\epsilon_0 r^2}$ and $E_{\text{U}}=\dfrac{Q}{1+K}\dfrac{1}{2\pi K\epsilon_0 r^2}=\dfrac{2}{1+K}\dfrac{Q}{4\pi K\epsilon_0 r^2}$. We do find that $E_{\text{U}}=E_{\text{L}}$.

(c) The free charge density on upper and lower hemispheres are: $(\sigma_{\text{f},r_a})_{\text{U}}=\dfrac{Q_{\text{U}}}{2\pi r_a^{\,2}}=\dfrac{Q}{2\pi r_a^{\,2}(1+K)}$ and $(\sigma_{\text{f},r_b})_{\text{U}}=\dfrac{Q_{\text{U}}}{2\pi r_b^{\,2}}=\dfrac{Q}{2\pi r_b^{\,2}(1+K)}$; $(\sigma_{\text{f},r_a})_{\text{L}}=\dfrac{Q_{\text{L}}}{2\pi r_a^{\,2}}=\dfrac{KQ}{2\pi r_a^{\,2}(1+K)}$ and $(\sigma_{\text{f},r_b})_{\text{L}}=\dfrac{Q_{\text{L}}}{2\pi r_b^{\,2}}=\dfrac{KQ}{2\pi r_b^{\,2}(1+K)}$.

(d) $\sigma_{\text{i},r_a}=\sigma_{\text{f},r_a}(1-1/K)=\left(\dfrac{(K-1)}{K}\right)\dfrac{Q}{2\pi r_a^{\,2}}\left(\dfrac{K}{K+1}\right)=\left(\dfrac{K-1}{K+1}\right)\dfrac{Q}{2\pi r_a^{\,2}}$

$\sigma_{\text{i},r_b}=\sigma_{\text{f},r_b}(1-1/K)=\left(\dfrac{(K-1)}{K}\right)\dfrac{Q}{2\pi r_b^2}\left(\dfrac{K}{K+1}\right)=\left(\dfrac{K-1}{K+1}\right)\dfrac{Q}{2\pi r_b^2}$

(e) There is zero bound charge on the flat surface of the dielectric-air interface, or else that would imply a circumferential electric field, or that the electric field changed as we went around the sphere.

EVALUATE: The charge is not equally distributed over the surface of each conductor. There must be more charge on the lower half, by a factor of K, because the polarization of the dielectric means more free charge is needed on the lower half to produce the same electric field.

24.77. **IDENTIFY:** The object is equivalent to two identical capacitors in parallel, where each has the same area *A*, plate separation *d* and dielectric with dielectric constant *K*.

SET UP: For each capacitor in the parallel combination, $C = \frac{\epsilon_0 A}{d}$.

EXECUTE: **(a)** The charge distribution on the plates is shown in Figure 24.77.

(b) $C = 2\left(\frac{\epsilon_0 A}{d}\right) = \frac{2(4.2)\epsilon_0 (0.120\text{ m})^2}{4.5\times 10^{-4}\text{ m}} = 2.38\times 10^{-9}\text{ F}$.

EVALUATE: If two of the plates are separated by both sheets of paper to form a capacitor, $C = \frac{\epsilon_0 A}{2d} = \frac{2.38\times 10^{-9}\text{ F}}{4}$, smaller by a factor of 4 compared to the capacitor in the problem.

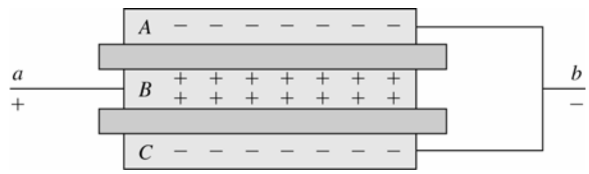


Figure 24.77

24.78. **IDENTIFY:** As in Problem 24.72, the system is equivalent to two capacitors in parallel. One of the capacitors has plate separation *d*, plate area $w(L-h)$ and air between the plates. The other has the same plate separation *d*, plate area *wh* and dielectric constant *K*.

SET UP: Define K_{eff} by $C_{\text{eq}} = \frac{K_{\text{eff}}\epsilon_0 A}{d}$, where $A = wL$. For two capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$.

EXECUTE: **(a)** The capacitors are in parallel, so $C = \frac{\epsilon_0 w(L-h)}{d} + \frac{K\epsilon_0 wh}{d} = \frac{\epsilon_0 wL}{d}\left(1 + \frac{Kh}{L} - \frac{h}{L}\right)$. This gives

$$K_{\text{eff}} = \left(1 + \frac{Kh}{L} - \frac{h}{L}\right).$$

(b) For gasoline, with $K = 1.95$: $\frac{1}{4}$ full: $K_{\text{eff}}\left(h = \frac{L}{4}\right) = 1.24$; $\frac{1}{2}$ full: $K_{\text{eff}}\left(h = \frac{L}{2}\right) = 1.48$;

$\frac{3}{4}$ full: $K_{\text{eff}}\left(h = \frac{3L}{4}\right) = 1.71$.

(c) For methanol, with $K = 33$: $\frac{1}{4}$ full: $K_{\text{eff}}\left(h = \frac{L}{4}\right) = 9$; $\frac{1}{2}$ full: $K_{\text{eff}}\left(h = \frac{L}{2}\right) = 17$; $\frac{3}{4}$ full: $K_{\text{eff}}\left(h = \frac{3L}{4}\right) = 25$.

(d) This kind of fuel tank sensor will work best for methanol since it has the greater range of K_{eff} values.

EVALUATE: When $h = 0$, $K_{\text{eff}} = 1$. When $h = L$, $K_{\text{eff}} = K$.

25

CURRENT, RESISTANCE, AND ELECTROMOTIVE FORCE

25.1. **IDENTIFY:** $I = Q/t$.

SET UP: $1.0\text{ h} = 3600\text{ s}$

EXECUTE: $Q = It = (3.6\text{ A})(3.0)(3600\text{ s}) = 3.89\times10^4\text{ C}$.

EVALUATE: Compared to typical charges of objects in electrostatics, this is a huge amount of charge.

25.2. **IDENTIFY:** $I = Q/t$. Use $I = n|q|v_d A$ to calculate the drift velocity v_d.

SET UP: $n = 5.8\times10^{28}\text{ m}^{-3}$. $|q| = 1.60\times10^{-19}\text{ C}$.

EXECUTE: **(a)** $I = \dfrac{Q}{t} = \dfrac{420\text{ C}}{80(60\text{ s})} = 8.75\times10^{-2}\text{ A}$.

(b) $I = n|q|v_d A$. This gives $v_d = \dfrac{I}{nqA} = \dfrac{8.75\times10^{-2}\text{ A}}{(5.8\times10^{28})(1.60\times10^{-19}\text{ C})(\pi(1.3\times10^{-3}\text{ m})^2)} = 1.78\times10^{-6}\text{ m/s}$.

EVALUATE: v_d is smaller than in Example 25.1, because I is smaller in this problem.

25.3. **IDENTIFY:** $I = Q/t$. $J = I/A$. $J = n|q|v_d$

SET UP: $A = (\pi/4)D^2$, with $D = 2.05\times10^{-3}\text{ m}$. The charge of an electron has magnitude $+e = 1.60\times10^{-19}\text{ C}$.

EXECUTE: **(a)** $Q = It = (5.00\text{ A})(1.00\text{ s}) = 5.00\text{ C}$. The number of electrons is $\dfrac{Q}{e} = 3.12\times10^{19}$.

(b) $J = \dfrac{I}{(\pi/4)D^2} = \dfrac{5.00\text{ A}}{(\pi/4)(2.05\times10^{-3}\text{ m})^2} = 1.51\times10^6\text{ A/m}^2$.

(c) $v_d = \dfrac{J}{n|q|} = \dfrac{1.51\times10^6\text{ A/m}^2}{(8.5\times10^{28}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})} = 1.11\times10^{-4}\text{ m/s} = 0.111\text{ mm/s}$.

EVALUATE: **(a)** If I is the same, $J = I/A$ would decrease and v_d would decrease. The number of electrons passing through the light bulb in 1.00 s would not change.

25.4. **(a) IDENTIFY:** By definition, $J = I/A$ and radius is one-half the diameter.

SET UP: Solve for the current: $I = JA = J\pi(D/2)^2$

EXECUTE: $I = (1.50\times10^6\text{ A/m}^2)(\pi)[(0.00102\text{ m})/2]^2 = 1.23\text{ A}$

EVALUATE: This is a realistic current.

(b) IDENTIFY: The current density is $J = nqv_d$

SET UP: Solve for the drift velocity: $v_d = J/nq$

EXECUTE: Since most laboratory wire is copper, we use the value of n for copper, giving

$v_d = (1.50\times10^6\text{ A/m}^2)/[(8.5\times10^{28}\text{ el/m}^3)(1.60\times10^{-19}\text{ C}) = 1.1\times10^{-4}\text{ m/s} = 0.11\text{ mm/s}$

EVALUATE: This is a typical drift velocity for ordinary currents and wires.

25.5. **IDENTIFY** and **SET UP:** Use Eq. (25.3) to calculate the drift speed and then use that to find the time to travel the length of the wire.

EXECUTE: **(a)** Calculate the drift speed v_d:

$$J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{4.85\text{ A}}{\pi\left(1.025\times10^{-3}\text{ m}\right)^2} = 1.469\times10^6\text{ A/m}^2$$

$$v_d = \frac{J}{n|q|} = \frac{1.469\times10^6\text{ A/m}^2}{\left(8.5\times10^{28}/\text{m}^3\right)\left(1.602\times10^{-19}\text{ C}\right)} = 1.079\times10^{-4}\text{ m/s}$$

$$t = \frac{L}{v_d} = \frac{0.710\text{ m}}{1.079\times10^{-4}\text{ m/s}} = 6.58\times10^3\text{ s} = 110\text{ min.}$$

(b) $v_d = \dfrac{I}{\pi r^2 n|q|}$

$t = \dfrac{L}{v_d} = \dfrac{\pi r^2 n|q|L}{I}$

t is proportional to r^2 and hence to d^2 where $d = 2r$ is the wire diameter.

$t = (6.58\times 10^3\ \text{s})\left(\dfrac{4.12\ \text{mm}}{2.05\ \text{mm}}\right)^2 = 2.66\times 10^4\ \text{s} = 440\ \text{min}.$

(c) EVALUATE: The drift speed is proportional to the current density and therefore it is inversely proportional to the square of the diameter of the wire. Increasing the diameter by some factor decreases the drift speed by the square of that factor.

25.6. **IDENTIFY:** The number of moles of copper atoms is the mass of 1.00 m^3 divided by the atomic mass of copper. There are $N_A = 6.023\times 10^{23}$ atoms per mole.

SET UP: The atomic mass of copper is $63.55\ \text{g/mole}$, and its density is $8.96\ \text{g/cm}^3$. Example 25.1 says there are 8.5×10^{28} free electrons per m^3.

EXECUTE: The number of copper atoms in $1.00\ \text{m}^3$ is

$$\frac{(8.96\ \text{g/cm}^3)(1.00\times 10^6\ \text{cm}^3/\text{m}^3)(6.023\times 10^{23}\ \text{atoms/mole})}{63.55\ \text{g/mole}} = 8.49\times 10^{28}\ \text{atoms/m}^3.$$

EVALUATE: Since there are the same number of free electrons/m^3 as there are atoms of copper/m^3, the number of free electrons per copper atom is one.

25.7. **IDENTIFY** and **SET UP:** Apply Eq. (25.1) to find the charge dQ in time dt. Integrate to find the total charge in the whole time interval.

EXECUTE: **(a)** $dQ = I\ dt$

$Q = \int_0^{8.0\ \text{s}} \left(55\ \text{A} - (0.65\ \text{A/s}^2)t^2\right)dt = \left[(55\ \text{A})t - (0.217\ \text{A/s}^2)t^3\right]_0^{8.0\ \text{s}}$

$Q = (55\ \text{A})(8.0\ \text{s}) - (0.217\ \text{A/s}^2)(8.0\ \text{s})^3 = 330\ \text{C}$

(b) $I = \dfrac{Q}{t} = \dfrac{330\ \text{C}}{8.0\ \text{s}} = 41\ \text{A}$

EVALUATE: The current decreases from 55 A to 13.4 A during the interval. The decrease is not linear and the average current is not equal to (55A + 13.4 A) / 2.

25.8. **IDENTIFY:** $I = Q/t$. Positive charge flowing in one direction is equivalent to negative charge flowing in the opposite direction, so the two currents due to Cl^- and Na^+ are in the same direction and add.

SET UP: Na^+ and Cl^- each have magnitude of charge $|q| = +e$

EXECUTE: **(a)** $Q_{\text{total}} = (n_{\text{Cl}} + n_{\text{Na}})e = (3.92\times 10^{16} + 2.68\times 10^{16})(1.60\times 10^{-19}\ \text{C}) = 0.0106\ \text{C}$. Then

$I = \dfrac{Q_{\text{total}}}{t} = \dfrac{0.0106\ \text{C}}{1.00\ \text{s}} = 0.0106\ \text{A} = 10.6\ \text{mA}.$

(b) Current flows, by convention, in the direction of positive charge. Thus, current flows with Na^+ toward the negative electrode.

EVALUATE: The Cl^- ions have negative charge and move in the direction opposite to the conventional current direction.

25.9. **IDENTIFY:** The number of moles of silver atoms is the mass of 1.00 m^3 divided by the atomic mass of silver. There are $N_A = 6.023\times 10^{23}$ atoms per mole.

SET UP: For silver, $\text{density} = 10.5\times 10^3\ \text{kg/m}^3$ and the atomic mass is $M = 107.868\times 10^{-3}\ \text{kg/mol}$.

EXECUTE: Consider 1 m^3 of silver. $m = (\text{density})V = 10.5\times 10^3\ \text{kg}$. $n = m/M = 9.734\times 10^4$ mol and the number of atoms is $N = nN_A = 5.86\times 10^{28}$ atoms. If there is one free electron per atom, there are 5.86×10^{28} free electrons/m^3. This agrees with the value given in Exercise 25.2.

EVALUATE: Our result verifies that for silver there is approximately one free electron per atom. Exercise 25.6 showed that for copper there is also one free electron per atom.

25.10. **(a) IDENTIFY:** Start with the definition of resisitivity and solve for E.

SET UP: $E = \rho J = \rho I/\pi r^2$

EXECUTE: $E = (1.72\times 10^{-8}\ \Omega\cdot\text{m})(2.75\ \text{A})/[\pi(0.001025\ \text{m})^2] = 1.43\times 10^{-2}\ \text{V/m}$

EVALUATE: The field is quite weak, since the potential would drop only a volt in 70 m of wire.

(b) IDENTIFY: Take the ratio of the field in silver to the field in copper.

SET UP: Take the ratio and solve for the field in silver: $E_S = E_C(\rho_S/\rho_C)$

EXECUTE: $E_S = (0.0143\text{ V/m})[(1.47)/(1.72)] = 1.22 \times 10^{-2}\text{ V/m}$

EVALUATE: Since silver is a better conductor than copper, the field in silver is smaller than the field in copper.

25.11. **IDENTIFY:** First use Ohm's law to find the resistance at 20.0°C; then calculate the resistivity from the resistance. Finally use the dependence of resistance on temperature to calculate the temperature coefficient of resistance.

SET UP: Ohm's law is $R = V/I$, $R = \rho L/A$, $R = R_0[1 + \alpha(T - T_0)]$, and the radius is one-half the diameter.

EXECUTE: **(a)** At 20.0°C, $R = V/I = (15.0\text{ V})/(18.5\text{ A}) = 0.811\ \Omega$. Using $R = \rho L/A$ and solving for ρ gives $\rho = RA/L = R\pi(D/2)^2/L = (0.811\ \Omega)\pi[(0.00500\text{ m})/2]^2/(1.50\text{ m}) = 1.06 \times 10^{-6}\ \Omega\cdot\text{m}$.

(b) At 92.0°C, $R = V/I = (15.0\text{ V})/(17.2\text{ A}) = 0.872\ \Omega$. Using $R = R_0[1 + \alpha(T - T_0)]$ with T_0 taken as 20.0°C, we have $0.872\ \Omega = (0.811\ \Omega)[1 + \alpha(92.0°\text{C} - 20.0°\text{C})]$. This gives $\alpha = 0.00105\ (\text{C}°)^{-1}$

EVALUATE: The results are typical of ordinary metals.

25.12. **IDENTIFY:** $E = \rho J$, where $J = I/A$. The drift velocity is given by $I = n|q|v_d A$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $n = 8.5\times10^{28}/\text{m}^3$.

EXECUTE: **(a)** $J = \dfrac{I}{A} = \dfrac{3.6\text{ A}}{(2.3\times10^{-3}\text{ m})^2} = 6.81\times10^5\text{ A/m}^2$.

(b) $E = \rho J = (1.72\times10^{-8}\ \Omega\cdot\text{m})(6.81\times10^5\text{ A/m}^2) = 0.012\text{ V/m}$.

(c) The time to travel the wire's length l is

$$t = \frac{l}{v_d} = \frac{ln|q|A}{I} = \frac{(4.0\text{ m})(8.5\times10^{28}/\text{m}^3)(1.6\times10^{-19}\text{ C})(2.3\times10^{-3}\text{ m})^2}{3.6\text{ A}} = 8.0\times10^4\text{ s}.$$

$t = 1333\text{ min} \approx 22$ hrs!

EVALUATE: The currents propagate very quickly along the wire but the individual electrons travel very slowly.

25.13. **IDENTIFY:** $E = \rho J$, where $J = I/A$.

SET UP: For tungsten $\rho = 5.25\times10^{-8}\ \Omega\cdot\text{m}$ and for aluminum $\rho = 2.75\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** tungsten: $E = \rho J = \dfrac{\rho I}{A} = \dfrac{(5.25\times10^{-8}\ \Omega\cdot\text{m})(0.820\text{ A})}{(\pi/4)(3.26\times10^{-3}\text{ m})^2} = 5.16\times10^{-3}\text{ V/m}$.

(b) aluminum: $E = \rho J = \dfrac{\rho I}{A} = \dfrac{(2.75\times10^{-8}\ \Omega\cdot\text{m})(0.820\text{ A})}{(\pi/4)(3.26\times10^{-3}\text{ m})^2} = 2.70\times10^{-3}\text{ V/m}$.

EVALUATE: A larger electric field is required for tungsten, because it has a larger resistivity.

25.14. **IDENTIFY:** The resistivity of the wire should identify what the material is.

SET UP: $R = \rho L/A$ and the radius of the wire is half its diameter.

EXECUTE: Solve for ρ and substitute the numerical values.

$$\rho = AR/L = \pi(D/2)^2 R/L = \frac{\pi\left([0.00205\text{ m}]/2\right)^2(0.0290\ \Omega)}{6.50\text{ m}} = 1.47\times10^{-8}\ \Omega\cdot\text{m}$$

EVALUATE: This result is the same as the resistivity of silver, which implies that the material is silver.

25.15. **(a) IDENTIFY:** Start with the definition of resistivity and use its dependence on temperature to find the electric field.

SET UP: $E = \rho J = \rho_{20}[1 + \alpha(T - T_0)]\dfrac{I}{\pi r^2}$

EXECUTE: $E = (5.25\times10^{-8}\ \Omega\cdot\text{m})[1 + (0.0045/\text{C}°)(120°\text{C} - 20°\text{C})](12.5\text{ A})/[\pi(0.000500\text{ m})^2] = 1.21\text{ V/m}$.

(Note that the resistivity at 120°C turns out to be $7.61\times10^{-8}\ \Omega\cdot\text{m}$.)

EVALUATE: This result is fairly large because tungsten has a larger resisitivity than copper.

(b) IDENTIFY: Relate resistance and resistivity.

SET UP: $R = \rho L/A = \rho L/\pi r^2$

EXECUTE: $R = (7.61\times10^{-8}\ \Omega\cdot\text{m})(0.150\text{ m})/[\pi(0.000500\text{ m})^2] = 0.0145\ \Omega$

EVALUATE: Most metals have very low resistance.

(c) IDENTIFY: The potential difference is proportional to the length of wire.

SET UP: $V = EL$

EXECUTE: $V = (1.21\text{ V/m})(0.150\text{ m}) = 0.182\text{ V}$

EVALUATE: We could also calculate $V = IR = (12.5\text{ A})(0.0145\ \Omega) = 0.181\text{ V}$, in agreement with part (c).

25.16. **IDENTIFY:** Apply $R=\dfrac{\rho L}{A}$ and solve for L.

SET UP: $A=\pi D^2/4$, where $D=0.462$ mm.

EXECUTE: $L=\dfrac{RA}{\rho}=\dfrac{(1.00\ \Omega)(\pi/4)(0.462\times10^{-3}\ \text{m})^2}{1.72\times10^{-8}\ \Omega\cdot\text{m}}=9.75\ \text{m}.$

EVALUATE: The resistance is proportional to the length of the wire.

25.17. **IDENTIFY:** $R=\dfrac{\rho L}{A}$.

SET UP: For copper, $\rho=1.72\times10^{-8}\ \Omega\cdot\text{m}$. $A=\pi r^2$.

EXECUTE: $R=\dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(24.0\ \text{m})}{\pi(1.025\times10^{-3}\ \text{m})^2}=0.125\ \Omega$

EVALUATE: The resistance is proportional to the length of the piece of wire.

25.18. **IDENTIFY:** $R=\dfrac{\rho L}{A}=\dfrac{\rho L}{\pi d^2/4}$.

SET UP: For aluminum, $\rho_{\text{al}}=2.63\times10^{-8}\ \Omega\cdot\text{m}$. For copper, $\rho_{\text{c}}=1.72\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: $\dfrac{\rho}{d^2}=\dfrac{R\pi}{4L}=$ constant, so $\dfrac{\rho_{\text{al}}}{d_{\text{al}}^2}=\dfrac{\rho_{\text{c}}}{d_{\text{c}}^2}$. $d_{\text{c}}=d_{\text{al}}\sqrt{\dfrac{\rho_{\text{c}}}{\rho_{\text{al}}}}=(3.26\ \text{mm})\sqrt{\dfrac{1.72\times10^{-8}\ \Omega\cdot\text{m}}{2.63\times10^{-8}\ \Omega\cdot\text{m}}}=2.64\ \text{mm}.$

EVALUATE: Copper has a smaller resistivity, so the copper wire has a smaller diameter in order to have the same resistance as the aluminum wire.

25.19. **IDENTIFY** and **SET UP:** Use Eq. (25.10) to calculate A. Find the volume of the wire and use the density to calculate the mass.

EXECUTE: Find the volume of one of the wires:

$R=\dfrac{\rho L}{A}$ so $A=\dfrac{\rho L}{R}$ and

$\text{volume}=AL=\dfrac{\rho L^2}{R}=\dfrac{\left(1.72\times10^{-8}\Omega\cdot\ \text{m}\right)\left(3.50\ \text{m}\right)^2}{0.125\ \Omega}=1.686\times10^{-6}\ \text{m}^3$

$m=\left(\text{density}\right)V=\left(8.9\times10^3\ \text{kg/m}^3\right)\left(1.686\times10^{-6}\ \text{m}^3\right)=15\ \text{g}$

EVALUATE: The mass we calculated is reasonable for a wire.

25.20. **IDENTIFY:** $R=\dfrac{\rho L}{A}$.

SET UP: The length of the wire in the spring is the circumference πd of each coil times the number of coils.

EXECUTE: $L=(75)\pi d=(75)\pi(3.50\times10^{-2}\ \text{m})=8.25\ \text{m}.$

$A=\pi r^2=\pi d^2/4=\pi(3.25\times10^{-3}\ \text{m})^2/4=8.30\times10^{-6}\ \text{m}^2.$

$\rho=\dfrac{RA}{L}=\dfrac{(1.74\ \Omega)(8.30\times10^{-6}\ \text{m}^2)}{8.25\ \text{m}}=1.75\times10^{-6}\ \Omega\cdot\text{m}.$

EVALUATE: The value of ρ we calculated is about a factor of 100 times larger than ρ for copper. The metal of the spring is not a very good conductor.

25.21. **IDENTIFY:** $R=\dfrac{\rho L}{A}$.

SET UP: $L=1.80$ m, the length of one side of the cube. $A=L^2$.

EXECUTE: $R=\dfrac{\rho L}{A}=\dfrac{\rho L}{L^2}=\dfrac{\rho}{L}=\dfrac{2.75\times10^{-8}\ \Omega\cdot\text{m}}{1.80\ \text{m}}=1.53\times10^{-8}\ \Omega$

EVALUATE: The resistance is very small because A is very much larger than the typical value for a wire.

25.22. **IDENTIFY:** Apply $R_T=R_0(1+\alpha(T-T_0))$.

SET UP: Since $V=IR$ and V is the same, $\dfrac{R_T}{R_{20}}=\dfrac{I_{20}}{I_T}$. For tungsten, $\alpha=4.5\times10^{-3}\ (\text{C}°)^{-1}$.

EXECUTE: The ratio of the current at $20°\text{C}$ to that at the higher temperature is $(0.860\ \text{A})/(0.220\ \text{A}) = 3.909.$

$\frac{R_T}{R_{20}} = 1+\alpha(T-T_0) = 3.909$, where $T_0 = 20°\text{C}.$

$$T = T_0 + \frac{R_T/R_{20}-1}{\alpha} = 20°\text{C} + \frac{3.909-1}{4.5\times10^{-3}\ (\text{C}°)^{-1}} = 666°\text{C}.$$

EVALUATE: As the temperature increases, the resistance increases and for constant applied voltage the current decreases. The resistance increases by nearly a factor of four.

25.23. **IDENTIFY:** Relate resistance to resistivity.

SET UP: $R = \rho L/A$

EXECUTE: **(a)** $R = \rho L/A = (0.60\ \Omega\cdot\text{m})(0.25\ \text{m})/(0.12\ \text{m})^2 = 10.4\ \Omega$

(b) $R = \rho L/A = (0.60\ \Omega\cdot\text{m})(0.12\ \text{m})/(0.12\ \text{m})(0.25\ \text{m}) = 2.4\ \Omega$

EVALUATE: The resistance is greater for the faces that are farther apart.

25.24. **IDENTIFY:** Apply $R = \frac{\rho L}{A}$ and $V = IR$.

SET UP: $A = \pi r^2$

EXECUTE: $\rho = \frac{RA}{L} = \frac{VA}{IL} = \frac{(4.50\ \text{V})\pi(6.54\times10^{-4}\ \text{m})^2}{(17.6\ \text{A})(2.50\ \text{m})} = 1.37\times10^{-7}\ \Omega\cdot\text{m}.$

EVALUATE: Our result for ρ shows that the wire is made of a metal with resistivity greater than that of good metallic conductors such as copper and aluminum.

25.25. **IDENTIFY** and **SET UP:** Eq. (25.5) relates the electric field that is given to the current density. $V = EL$ gives the potential difference across a length L of wire and Eq. (25.11) allows us to calculate R.

EXECUTE: **(a)** Eq. (25.5): $\rho = E/J$ so $J = E/\rho$

From Table 25.1 the resistivity for gold is $2.44\times10^{-8}\ \Omega\cdot\text{m}.$

$$J = \frac{E}{\rho} = \frac{0.49\ \text{V/m}}{2.44\times10^{-8}\ \Omega\cdot\text{m}} = 2.008\times10^7\ \text{A/m}^2$$

$$I = JA = J\pi r^2 = \left(2.008\times10^7\ \text{A/m}^2\right)\pi\left(0.41\times10^{-3}\ \text{m}\right)^2 = 11\ \text{A}$$

(b) $V = EL = (0.49\ \text{V/m})(6.4\ \text{m}) = 3.1\ \text{V}$

(c) We can use Ohm's law (Eq. (25.11)): $V = IR.$

$$R = \frac{V}{I} = \frac{3.1\ \text{V}}{11\ \text{A}} = 0.28\ \Omega$$

EVALUATE: We can also calculate R from the resistivity and the dimensions of the wire (Eq. 25.10):

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = \frac{\left(2.44\times10^{-8}\ \Omega\cdot\text{m}\right)(6.4\ \text{m})}{\pi\left(0.42\times10^{-3}\ \text{m}\right)^2} = 0.28\ \Omega,\ \text{which checks.}$$

25.26. **IDENTIFY** and **SET UP:** Use $V = EL$ to calculate E and then $\rho = E/J$ to calculate ρ.

EXECUTE: **(a)** $E = \frac{V}{L} = \frac{0.938\ \text{V}}{0.750\ \text{m}} = 1.25\ \text{V/m}$

(b) $E = \rho J$ so $\rho = \frac{E}{J} = \frac{1.25\ \text{V/m}}{4.40\times10^7\ \text{A/m}^2} = 2.84\times10^{-8}\ \Omega\cdot\text{m}$

EVALUATE: This value of ρ is similar to that for the good metallic conductors in Table 25.1.

25.27. **IDENTIFY:** Apply $R = R_0\left[1+\alpha(T-T_0)\right]$ to calculate the resistance at the second temperature.

(a) SET UP: $\alpha = 0.0004\ (\text{C}°)^{-1}$ (Table 25.1). Let T_0 be 0.0°C and T be 11.5°C.

EXECUTE: $R_0 = \frac{R}{1+\alpha(T-T_0)} = \frac{100.0\ \Omega}{1+\left(0.0004\ (\text{C}°)^{-1}(11.5\ \text{C}°)\right)} = 99.54\ \Omega$

(b) SET UP: $\alpha = -0.0005\ (\text{C}°)^{-1}$ (Table 25.2). Let $T_0 = 0.0°\text{C}$ and $T = 25.8°\text{C}.$

EXECUTE: $R = R_0\left[1+\alpha(T-T_0)\right] = 0.0160\ \Omega\left[1+\left(-0.0005\ (\text{C}°)^{-1}\right)(25.8\ \text{C}°)\right] = 0.0158\ \Omega$

EVALUATE: Nichrome, like most metallic conductors, has a positive α and its resistance increases with temperature. For carbon, α is negative and its resistance decreases as T increases.

25.28. **IDENTIFY:** $R_T = R_0[1+\alpha(T-T_0)]$

SET UP: $R_0 = 217.3\ \Omega$. $R_T = 215.8\ \Omega$. For carbon, $\alpha = -0.00050\ (\text{C}°)^{-1}$.

EXECUTE: $T - T_0 = \dfrac{(R_T/R_0)-1}{\alpha} = \dfrac{(215.8\ \Omega/217.3\ \Omega)-1}{-0.00050\ (\text{C}°)^{-1}} = 13.8\ \text{C}°$. $T = 13.8\ \text{C}° + 4.0°\text{C} = 17.8°\text{C}$.

EVALUATE: For carbon, α is negative so R decreases as T increases.

25.29. **IDENTIFY** and **SET UP:** Apply $R = \dfrac{\rho L}{A}$ to determine the effect of increasing A and L.

EXECUTE: **(a)** If 120 strands of wire are placed side by side, we are effectively increasing the area of the current carrier by 120. So the resistance is smaller by that factor: $R = (5.60\times10^{-6}\ \Omega)/120 = 4.67\times10^{-8}\ \Omega$.

(b) If 120 strands of wire are placed end to end, we are effectively increasing the length of the wire by 120, and so $R = (5.60\times10^{-6}\ \Omega)120 = 6.72\times10^{-4}\ \Omega$.

EVALUATE: Placing the strands side by side decreases the resistance and placing them end to end increases the resistance.

25.30. **IDENTIFY:** When the ohmmeter is connected between the opposite faces, the current flows along its length, but when the meter is connected between the inner and outer surfaces, the current flows radially outward.

(a) SET UP: For a hollow cylinder, $R = \rho L/A$, where $A = \pi(b^2 - a^2)$.

EXECUTE: $R = \rho L/A = \dfrac{\rho L}{\pi\left(b^2-a^2\right)} = \dfrac{\left(2.75\times10^{-8}\ \Omega\cdot\text{m}\right)(2.50\ \text{m})}{\pi\left[(0.0460\ \text{m})^2-(0.0320\ \text{m})^2\right]} = 2.00\times10^{-5}\ \Omega$

(b) SET UP: For radial current flow from $r = a$ to $r = b$, $R = (\rho/2\pi L)\ln(b/a)$ (Example 25.4)

EXECUTE: $R = \dfrac{\rho}{2\pi L}\ln(b/a) = \dfrac{2.75\times10^{-8}\ \Omega\cdot\text{m}}{2\pi(2.50\ \text{m})}\ln\left(\dfrac{4.60\ \text{cm}}{3.20\ \text{cm}}\right) = 6.35\times10^{-10}\ \Omega$

EVALUATE: The resistance is much smaller for the radial flow because the current flows through a much smaller distance and the area through which it flows is much larger.

25.31. **IDENTIFY:** Use $R = \dfrac{\rho L}{A}$ to calculate R and then apply $V = IR$. $P = VI$ and energy $= Pt$

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. $A = \pi r^2$, where $r = 0.050$ m.

EXECUTE: **(a)** $R = \dfrac{\rho L}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(100\times10^3\text{m})}{\pi(0.050\ \text{m})^2} = 0.219\ \Omega$. $V = IR = (125\ \text{A})(0.219\ \Omega) = 27.4\ \text{V}$.

(b) $P = VI = (27.4\ \text{V})(125\ \text{A}) = 3422\ \text{W} = 3422\ \text{J/s}$ and energy $= Pt = (3422\ \text{J/s})(3600\ \text{s}) = 1.23\times10^7\ \text{J}$.

EVALUATE: The rate of electrical energy loss in the cable is large, over 3 kW.

25.32. **IDENTIFY:** When current passes through a battery in the direction from the − terminal toward the + terminal, the terminal voltage V_{ab} of the battery is $V_{ab} = \mathcal{E} - Ir$. Also, $V_{ab} = IR$, the potential across the circuit resistor.

SET UP: $\mathcal{E} = 24.0\ \text{V}$. $I = 4.00\ \text{A}$.

EXECUTE: **(a)** $V_{ab} = \mathcal{E} - Ir$ gives $r = \dfrac{\mathcal{E} - V_{ab}}{I} = \dfrac{24.0\ \text{V} - 21.2\ \text{V}}{4.00\ \text{A}} = 0.700\ \Omega$.

(b) $V_{ab} - IR = 0$ so $R = \dfrac{V_{ab}}{I} = \dfrac{21.2\ \text{V}}{4.00\ \text{A}} = 5.30\ \Omega$.

EVALUATE: The voltage drop across the internal resistance of the battery causes the terminal voltage of the battery to be less than its emf. The total resistance in the circuit is $R + r = 6.00\ \Omega$. $I = \dfrac{24.0\ \text{V}}{6.00\ \Omega} = 4.00\ \text{A}$, which agrees with the value specified in the problem.

25.33. **IDENTIFY:** $V = \mathcal{E} - Ir$.

SET UP: The graph gives $V = 9.0\ \text{V}$ when $I = 0$ and $I = 2.0\ \text{A}$ when $V = 0$.

EXECUTE: **(a)** $\mathcal{E}$ is equal to the terminal voltage when the current is zero. From the graph, this is 9.0 V.

(b) When the terminal voltage is zero, the potential drop across the internal resistance is just equal in magnitude to the internal emf, so $rI = \mathcal{E}$, which gives $r = \mathcal{E}/I = (9.0\ \text{V})/(2.0\ \text{A}) = 4.5\ \Omega$.

EVALUATE: The terminal voltage decreases as the current through the battery increases.

25.34. **(a) IDENTIFY:** The *idealized* ammeter has no resistance so there is no potential drop across it. Therefore it acts like a short circuit across the terminals of the battery and removes the 4.00-Ω resistor from the circuit. Thus the only resistance in the circuit is the 2.00-Ω internal resistance of the battery.

SET UP: Use Ohm's law: $I = \mathcal{E}/r$.

EXECUTE: $I = (10.0\ \text{V})/(2.00\ \Omega) = 5.00\ \text{A}$.

(b) The zero-resistance ammeter is in parallel with the 4.00-Ω resistor, so all the current goes through the ammeter. If no current goes through the 4.00-Ω resistor, the potential drop across it must be zero.

(c) The terminal voltage is zero since there is no potential drop across the ammeter.

EVALUATE: An ammeter should *never* be connected this way because it would seriously alter the circuit!

25.35. **IDENTIFY:** The terminal voltage of the battery is $V_{ab} = \mathcal{E} - Ir$. The voltmeter reads the potential difference between its terminals.

SET UP: An ideal voltmeter has infinite resistance.

EXECUTE: **(a)** Since an ideal voltmeter has infinite resistance, so there would be NO current through the 2.0 Ω resistor.

(b) $V_{ab} = \mathcal{E} = 5.0\text{ V}$; since there is no current there is no voltage lost over the internal resistance.

(c) The voltmeter reading is therefore 5.0 V since with no current flowing there is no voltage drop across either resistor.

EVALUATE: This not the proper way to connect a voltmeter. If we wish to measure the terminal voltage of the battery in a circuit that does not include the voltmeter, then connect the voltmeter across the terminals of the battery.

25.36. **IDENTIFY:** The sum of the potential changes around the circuit loop is zero. Potential decreases by *IR* when going through a resistor in the direction of the current and increases by $\mathcal{E}$ when passing through an emf in the direction from the $-$ to + terminal.

SET UP: The current is counterclockwise, because the 16 V battery determines the direction of current flow.

EXECUTE: $+16.0\text{ V} - 8.0\text{ V} - I(1.6\ \Omega + 5.0\ \Omega + 1.4\ \Omega + 9.0\ \Omega) = 0$

$$I = \frac{16.0\text{ V} - 8.0\text{ V}}{1.6\ \Omega + 5.0\ \Omega + 1.4\ \Omega + 9.0\ \Omega} = 0.47\text{ A}$$

(b) $V_b + 16.0\text{ V} - I(1.6\ \Omega) = V_a$, so $V_a - V_b = V_{ab} = 16.0\text{ V} - (1.6\ \Omega)(0.47\text{ A}) = 15.2\text{ V}$.

(c) $V_c + 8.0\text{ V} + I(1.4\ \Omega + 5.0\ \Omega) = V_a$ so $V_{ac} = (5.0\ \Omega)(0.47\text{ A}) + (1.4\ \Omega)(0.47\text{ A}) + 8.0\text{ V} = 11.0\text{ V}$.

(d) The graph is sketched in Figure 25.36.

EVALUATE: $V_{cb} = (0.47\text{ A})(9.0\ \Omega) = 4.2\text{ V}$. The potential at point *b* is 15.2 V below the potential at point *a* and the potential at point *c* is 11.0 V below the potential at point *a*, so the potential of point *c* is $15.2\text{ V} - 11.0\text{ V} = 4.2\text{ V}$ above the potential of point *b*.

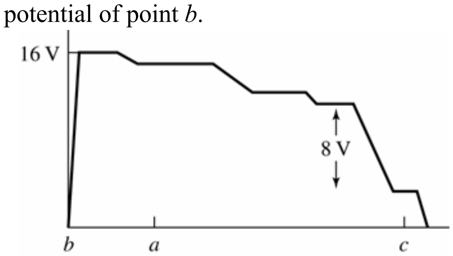


Figure 25.36

25.37. **IDENTIFY:** The voltmeter reads the potential difference V_{ab} between the terminals of the battery.

SET UP: open circuit $I = 0$. The circuit is sketched in Figure 25.37a.

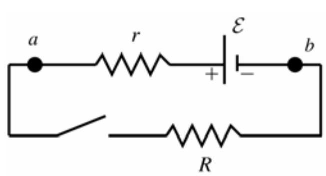


Figure 25.37a

EXECUTE: $V_{ab} = \mathcal{E} = 3.08\text{ V}$

SET UP: switch closed The circuit is sketched in Figure 35.37b.

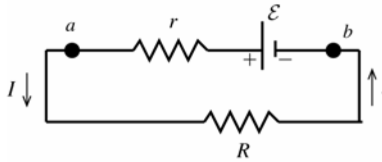


Figure 25.37b

EXECUTE:

$V_{ab} = \mathcal{E} - Ir = 2.97\text{ V}$

$$r = \frac{\mathcal{E} - 2.97\text{ V}}{I}$$

$$r = \frac{3.08\text{ V} - 2.97\text{ V}}{1.65\text{ A}} = 0.067\ \Omega$$

And $V_{ab} = IR$ so $R = \dfrac{V_{ab}}{I} = \dfrac{2.97\text{ V}}{1.65\text{ A}} = 1.80\ \Omega.$

EVALUATE: When current flows through the battery there is a voltage drop across its internal resistance and its terminal voltage *V* is less than its emf.

25.38. **IDENTIFY:** The sum of the potential changes around the loop is zero.

SET UP: The voltmeter reads the *IR* voltage across the $9.0\ \Omega$ resistor. The current in the circuit is counterclockwise because the 16 V battery determines the direction of the current flow.

EXECUTE: **(a)** $V_{bc} = 1.9\text{ V}$ gives $I = V_{bc}/R_{bc} = 1.9\text{ V}/9.0\ \Omega = 0.21\text{ A}.$

(b) $16.0\text{ V} - 8.0\text{ V} = (1.6\ \Omega + 9.0\ \Omega + 1.4\ \Omega + R)(0.21\text{ A})$ and $R = \dfrac{5.48\text{ V}}{0.21\text{ A}} = 26.1\ \Omega.$

(c) The graph is sketched in Figure 25.38.

EVALUATE: In Exercise 25.36 the current is 0.47 A. When the $5.0\ \Omega$ resistor is replaced by the $26.1\ \Omega$ resistor the current decreases to 0.21 A.

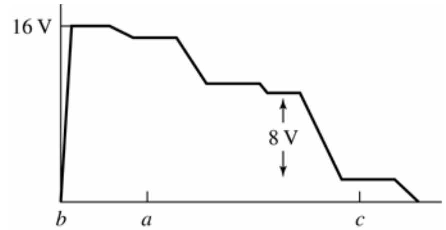


Figure 25.38

25.39. **(a) IDENTIFY** and **SET UP:** Assume that the current is clockwise. The circuit is sketched in Figure 25.39a.

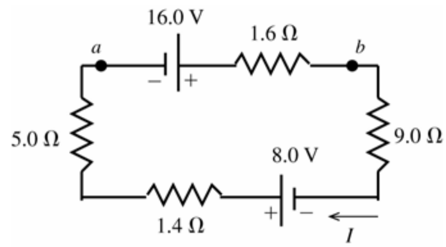


Figure 25.39a

Add up the potential rises and drops as travel clockwise around the circuit.

EXECUTE: $16.0\text{ V} - I(1.6\ \Omega) - I(9.0\ \Omega) + 8.0\text{ V} - I(1.4\ \Omega) - I(5.0\ \Omega) = 0$

$$I = \frac{16.0\text{ V} + 8.0\text{ V}}{9.0\ \Omega + 1.4\ \Omega + 5.0\ \Omega + 1.6\ \Omega} = \frac{24.0\text{ V}}{17.0\ \Omega} = 1.41\text{ A, clockwise}$$

EVALUATE: The 16.0 V battery drives the current clockwise more strongly than the 8.0 V battery does in the opposite direction.

(b) IDENTIFY and **SET UP:** Start at point *a* and travel through the battery to point *b*, keeping track of the potential changes. At point *b* the potential is V_b.

EXECUTE: $V_a + 16.0\text{ V} - I(1.6\ \Omega) = V_b$

$V_a - V_b = -16.0\text{ V} + (1.41\text{ A})(1.6\ \Omega)$

$V_{ab} = -16.0\text{ V} + 2.3\text{ V} = -13.7\text{ V}$ (point *a* is at lower potential; it is the negative terminal)

EVALUATE: Could also go counterclockwise from *a* to *b:*

$V_a + (1.41\text{ A})(5.0\ \Omega) + (1.41\text{ A})(1.4\ \Omega) - 8.0\text{ V} + (1.41\text{ A})(9.0\ \Omega) = V_b$

$V_{ab} = -13.7\text{ V},$ which checks.

(c) IDENTIFY and **SET UP:** State at point *a* and travel through the battery to point *c*, keeping track of the potential changes.

EXECUTE: $V_a + 16.0\text{ V} - I(1.6\ \Omega) - I(9.0\ \Omega) = V_c$

$V_a - V_c = -16.0\text{ V} + (1.41\text{ A})(1.6\ \Omega + 9.0\ \Omega)$

$V_{ac} = -16.0\text{ V} + 15.0\text{ V} = -1.0\text{ V}$ (point *a* is at lower potential than point *c*)

EVALUATE: Could also go counterclockwise from *a* to *c*:

$V_a + (1.41\text{ A})(5.0\ \Omega) + (1.41\text{ A})(1.4\ \Omega) - 8.0\text{ V} = V_c$

$V_{ac} = -1.0\text{ V},$ which checks.

(d) Call the potential zero at point *a*. Travel clockwise around the circuit. The graph is sketched in Figure 25.39b.

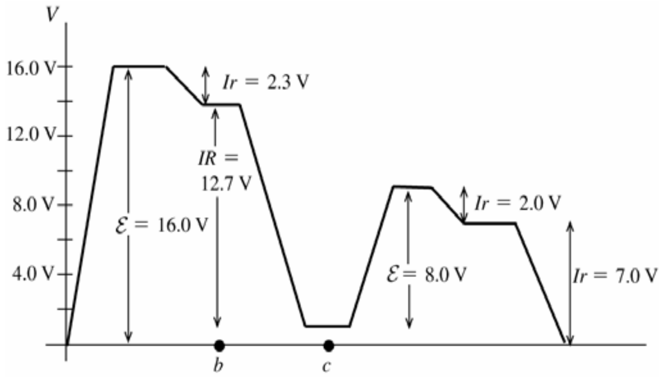


Figure 25.39b

25.40. **IDENTIFY:** Ohm's law says $R=\dfrac{V_{ab}}{I}$ is a constant.

SET UP: **(a)** The graph is given in Figure 25.40a.

EXECUTE: **(b)** No. The graph of V_{ab} versus I is not a straight line so Thyrite does not obey Ohm's law.

(c) The graph of R versus I is given in Figure 25.40b. R is not constant; it decreases as I increases.

EVALUATE: Not all materials obey Ohm's law.

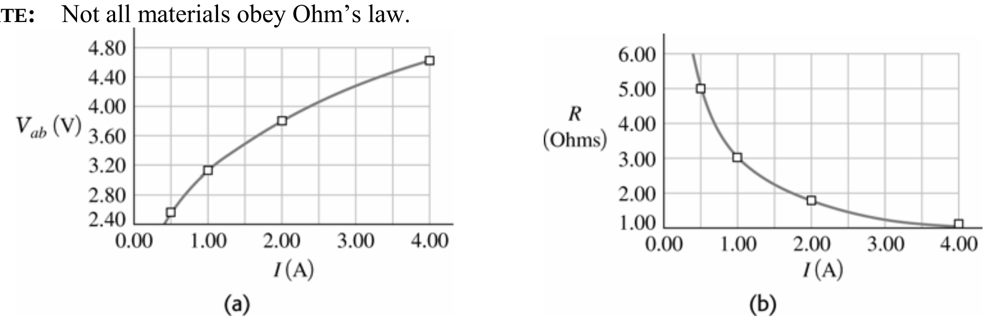


Figure 25.40

25.41. **IDENTIFY:** Ohm's law says $R=\dfrac{V_{ab}}{I}$ is a constant.

SET UP: **(a)** The graph is given in Figure 25.41.

EXECUTE: **(b)** The graph of V_{ab} versus I is a straight line so Nichrome obeys Ohm's law.

(c) R is the slope of the graph in part (a). $R=\dfrac{15.52\text{ V}-1.94\text{ V}}{4.00\text{ A}-0.50\text{ A}}=3.88\ \Omega.$

EVALUATE: V_{ab}/I for every I gives the same result for R, $R=3.88\ \Omega.$

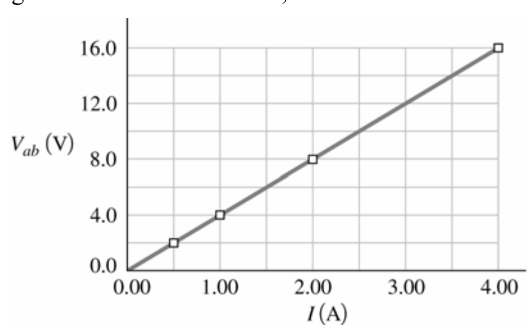


Figure 25.41

25.42. **IDENTIFY** and **SET UP:** For a resistor, $P=VI=V^2/R$ and $V=IR$.

EXECUTE: **(a)** $R=\dfrac{V^2}{P}=\dfrac{(15.0\text{ V})^2}{327\text{ W}}=0.688\ \Omega$

(b) $I=\dfrac{V}{R}=\dfrac{15.0\text{ V}}{0.688\ \Omega}=21.8\text{ A}$

EVALUATE: We could also write $P=VI$ to calculate $I=\dfrac{P}{V}=\dfrac{327\text{ W}}{15.0\text{ V}}=21.8\text{ A}.$

25.43. **IDENTIFY:** The bulbs are each connected across a 120-V potential difference.
SET UP: Use $P = V^2/R$ to solve for R and Ohm's law ($I = V/R$) to find the current.
EXECUTE: **(a)** $R = V^2/P = (120\text{ V})^2/(100\text{ W}) = 144\ \Omega$.
(b) $R = V^2/P = (120\text{ V})^2/(60\text{ W}) = 240\ \Omega$
(c) For the 100-W bulb: $I = V/R = (120\text{ V})/(144\ \Omega) = 0.833\text{ A}$
For the 60-W bulb: $I = (120\text{ V})/(240\ \Omega) = 0.500\text{ A}$
EVALUATE: The 60-W bulb has *more* resistance than the 100-W bulb, so it draws less current.

25.44. **IDENTIFY:** Across 120 V, a 75-W bulb dissipates 75 W. Use this fact to find its resistance, and then find the power the bulb dissipates across 220 V.
SET UP: $P = V^2/R$, so $R = V^2/P$
EXECUTE: Across 120 V: $R = (120\text{ V})^2/(75\text{ W}) = 192\ \Omega$. Across a 220-V line, its power will be $P = V^2/R = (220\text{ V})^2/(192\ \Omega) = 252\text{ W}$.
EVALUATE: The bulb dissipates much more power across 220 V, so it would likely blow out at the higher voltage. An alternative solution to the problem is to take the ratio of the powers.

$$\frac{P_{220}}{P_{120}} = \frac{V_{220}^2/R}{V_{120}^2/R} = \left(\frac{V_{220}}{V_{120}}\right)^2 = \left(\frac{220}{120}\right)^2.\ \text{This gives } P_{220} = (75\text{ W})\left(\frac{220}{120}\right)^2 = 252\text{ W}.$$

25.45. **IDENTIFY:** A "100-W" European bulb dissipates 100 W when used across 220 V.
(a) SET UP: Take the ratio of the power in the US to the power in Europe, as in the alternative method for problem 25.44, using $P = V^2/R$.

EXECUTE: $$\frac{P_{\text{US}}}{P_{\text{E}}} = \frac{V_{\text{US}}^2/R}{V_{\text{E}}^2/R} = \left(\frac{V_{\text{US}}}{V_{\text{E}}}\right)^2 = \left(\frac{120\text{ V}}{220\text{ V}}\right)^2.\ \text{This gives } P_{\text{US}} = (100\text{ W})\left(\frac{120\text{ V}}{220\text{ V}}\right)^2 = 29.8\text{ W}.$$

(b) SET UP: Use $P = IV$ to find the current.
EXECUTE: $I = P/V = (29.8\text{ W})/(120\text{ V}) = 0.248\text{ A}$
EVALUATE: The bulb draws considerably less power in the U.S., so it would be much dimmer than in Europe.

25.46. **IDENTIFY:** $P = VI$. Energy $= Pt$.
SET UP: $P = (9.0\text{ V})(0.13\text{ A}) = 1.17\text{ W}$
EXECUTE: $\text{Energy} = (1.17\text{ W})(1.5\text{ h})(3600\text{ s/h}) = 6320\text{ J}$
EVALUATE: The energy consumed is proportional to the voltage, to the current and to the time.

25.47. **IDENTIFY** and **SET UP:** By definition $p = \dfrac{P}{LA}$. Use $P = VI$, $E = VL$ and $I = JA$ to rewrite this expression in terms of the specified variables.

EXECUTE: **(a)** E is related to V and J is related to I, so use $P = VI$. This gives $p = \dfrac{VI}{LA}$

$\dfrac{V}{L} = E$ and $\dfrac{I}{A} = J$ so $p = EJ$

(b) J is related to I and ρ is related to R, so use $P = IR^2$. This gives $p = \dfrac{I^2R}{LA}$.

$I = JA$ and $R = \dfrac{\rho L}{A}$ so $p = \dfrac{J^2A^2\rho L}{LA^2}\rho J^2$

(c) E is related to V and ρ is related to R, so use $P = V^2/R$. This gives $p = \dfrac{V^2}{RLA}$.

$V = EL$ and $R = \dfrac{\rho L}{A}$ so $p = \dfrac{E^2L^2}{LA}\left(\dfrac{A}{\rho L}\right) = \dfrac{E^2}{\rho}$.

EVALUATE: For a given material (ρ constant), p is proportional to J^2 or to E^2.

25.48. **IDENTIFY:** Calculate the current in the circuit. The power output of a battery is its terminal voltage times the current through it. The power dissipated in a resistor is I^2R.
SET UP: The sum of the potential changes around the circuit is zero.

EXECUTE: **(a)** $I = \dfrac{8.0\text{ V}}{17\ \Omega} = 0.47\text{ A}$. Then $P_{9\Omega} = I^2R = (0.47\text{ A})^2(5.0\ \Omega) = 1.1\text{ W}$ and

$P_{9\Omega} = I^2R = (0.47\text{ A})^2(9.0\ \Omega) = 2.0\text{ W}$.

(b) $P_{16\text{V}} = \mathcal{E}I - I^2r = (16\text{ V})(0.47\text{ A}) - (0.47\text{ A})^2(1.6\ \Omega) = 7.2\text{ W}$.

(c) $P_{8\text{V}} = \mathcal{E}I + Ir^2 = (8.0\text{ V})(0.47\text{ A}) + (0.47\text{ A})^2(1.4\ \Omega) = 4.1\text{ W}$.

EVALUATE: **(d)** $(\text{b}) = (\text{a}) + (\text{c})$. The rate at which the 16.0 V battery delivers electrical energy to the circuit equals the rate at which it is consumed in the 8.0 V battery and the $5.0\ \Omega$ and $9.0\ \Omega$ resistors.

25.49. **(a) IDENTIFY** and **SET UP:** $P = VI$ and energy = (power) × (time).

EXECUTE: $P = VI = (12\text{ V})(60\text{ A}) = 720\text{ W}$

The battery can provide this for 1.0 h, so the energy the battery has stored is

$U = Pt = (720\text{ W})(3600\text{ s}) = 2.6\times10^6\text{ J}$

(b) IDENTIFY and **SET UP:** For gasoline the heat of combustion is $L_c = 46\times10^6$ J/kg. Solve for the mass *m* required to supply the energy calculated in part (a) and use density $\rho = m/V$ to calculate *V*.

EXECUTE: The mass of gasoline that supplies 2.6×10^6 J is $m = \dfrac{2.6\times10^6\text{ J}}{46\times10^6\text{ J/kg}} = 0.0565\text{ kg.}$

The volume of this mass of gasoline is

$$V = \frac{m}{\rho} = \frac{0.0565\text{ kg}}{900\text{ kg/m}^3} = 6.3\times10^{-5}\text{ m}^3\left(\frac{1000\text{ L}}{1\text{ m}^3}\right) = 0.063\text{ L}$$

(c) IDENTIFY and **SET UP:** Energy = (power) × (time); the energy is that calculated in part (a).

EXECUTE: $U = Pt,\ t = \dfrac{U}{P} = \dfrac{2.6\times10^6\text{ J}}{450\text{ W}} = 5800\text{ s} = 97\text{ min} = 1.6\text{ h.}$

EVALUATE: The battery discharges at a rate of 720 W (for 60 A) and is charged at a rate of 450 W, so it takes longer to charge than to discharge.

25.50. **IDENTIFY:** The rate of conversion of chemical to electrical energy in an emf is $\mathcal{E}I$. The rate of dissipation of electrical energy in a resistor *R* is I^2R.

SET UP: Example 25.10 finds that $I = 1.2$ A for this circuit. In Example 25.9, $\mathcal{E}I = 24$ W and $I^2r = 8$ W. In Example 25.10, $I^2R = 12$ W, or 11.5 W if expressed to three significant figures.

EXECUTE: **(a)** $P = \mathcal{E}I = (12\text{ V})(1.2\text{ A}) = 14.4\text{ W}$. This is less than the previous value of 24 W.

(b) The energy dissipated in the battery is $P = I^2r = (1.2\text{ A})^2(2.0\ \Omega) = 2.9$ W. This is less than 8 W, the amount found in Example (25.9).

(c) The net power output of the battery is $14.4\text{ W} - 2.9\text{ W} = 11.5\text{ W}$. This is the same as the power dissipated in the $8.0\ \Omega$ resistor.

EVALUATE: With the larger circuit resistance the current is less and the power input and power consumption are less.

25.51. **IDENTIFY:** Some of the power generated by the internal emf of the battery is dissipated across the battery's internal resistance, so it is not available to the bulb.

SET UP: Use $P = I^2R$ and take the ratio of the power dissipated in the internal resistance *r* to the total power.

EXECUTE: $\dfrac{P_r}{P_{\text{Total}}} = \dfrac{I^2r}{I^2(r+R)} = \dfrac{r}{r+R} = \dfrac{3.5\ \Omega}{28.5\ \Omega} = 0.123 = 12.3\%$

EVALUATE: About 88% of the power of the battery goes to the bulb. The rest appears as heat in the internal resistance.

25.52. **IDENTIFY:** The voltmeter reads the terminal voltage of the battery, which is the potential difference across the appliance. The terminal voltage is less than 15.0 V because some potential is lost across the internal resistance of the battery.

(a) SET UP: $P = V^2/R$ gives the power dissipated by the appliance.

EXECUTE: $P = (11.3\text{ V})^2/(75.0\ \Omega) = 1.70$ W

(b) SET UP: The drop in terminal voltage ($\mathcal{E} - V_{ab}$) is due to the potential drop across the internal resistance *r*. Use $Ir = \mathcal{E} - V_{ab}$ to find the internal resistance *r*, but first find the current using $P = IV$.

EXECUTE: $I = P/V = (1.70\text{ W})/(11.3\text{ V}) = 0.151$ A. Then $Ir = \mathcal{E} - V_{ab}$ gives $(0.151\text{ A})r = 15.0\text{ V} - 11.3\text{ V}$ and $r = 24.6\ \Omega$.

EVALUATE: The full 15.0 V of the battery would be available only when no current (or a very small current) is flowing in the circuit. This would be the base if the appliance had a resistance much greater than 24.6 Ω.

25.53. **IDENTIFY:** Solve for the current *I* in the circuit. Apply Eq. (25.17) to the specified circuit elements to find the rates of energy conversion.

SET UP: The circuit is sketched in Figure 25.53.

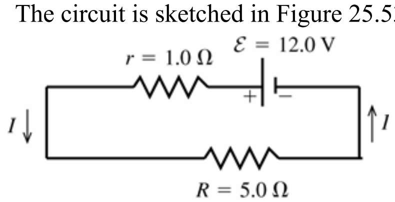


Figure 25.53

EXECUTE: Compute *I*:

$\mathcal{E} - Ir - IR = 0$

$$I = \frac{\mathcal{E}}{r+R} = \frac{12.0\text{ V}}{1.0\ \Omega + 5.0\ \Omega} = 2.00\text{ A}$$

(a) The rate of conversion of chemical energy to electrical energy in the emf of the battery is $P = \mathcal{E}I = (12.0\text{ V})(2.00\text{ A}) = 24.0\text{ W}$.

(b) The rate of dissipation of electrical energy in the internal resistance of the battery is $P = I^2 r = (2.00\text{ A})^2 (1.0\ \Omega) = 4.0\text{ W}$.

(c) The rate of dissipation of electrical energy in the external resistor R is $P = I^2 R = (2.00\text{ A})^2 (5.0\ \Omega) = 20.0\text{ W}$.

EVALUATE: The rate of production of electrical energy in the circuit is 24.0 W. The total rate of consumption of electrical energy in the circuit is 4.00 W + 20.0 W = 24.0 W. Equal rate of production and consumption of electrical energy are required by energy conservation.

25.54. **IDENTIFY:** The power delivered to the bulb is $I^2 R$. Energy $= Pt$.

SET UP: The circuit is sketched in Figure 25.54. r_{total} is the combined internal resistance of both batteries.

EXECUTE: **(a)** $r_{\text{total}} = 0$. The sum of the potential changes around the circuit is zero, so $1.5\text{ V} + 1.5\text{ V} - I(17\ \Omega) = 0$. $I = 0.1765\text{ A}$. $P = I^2 R = (0.1765\text{ A})^2 (17\ \Omega) = 0.530\text{ W}$. This is also $(3.0\text{ V})(0.1765\text{ A})$.

(b) Energy $= (0.530\text{ W})(5.0\text{ h})(3600\text{ s/h}) = 9540\text{ J}$

(c) $P = \dfrac{0.530\text{ W}}{2} = 0.265\text{ W}$. $P = I^2 R$ so $I = \sqrt{\dfrac{P}{R}} = \sqrt{\dfrac{0.265\text{ W}}{17\ \Omega}} = 0.125\text{ A}$.

The sum of the potential changes around the circuit is zero, so $1.5\text{ V} + 1.5\text{ V} - IR - Ir_{\text{total}} = 0$.

$$r_{\text{total}} = \frac{3.0\text{ V} - (0.125\text{ A})(17\ \Omega)}{0.125\text{ A}} = 7.0\ \Omega.$$

EVALUATE: When the power to the bulb has decreased to half its initial value, the total internal resistance of the two batteries is nearly half the resistance of the bulb. Compared to a single battery, using two identical batteries in series doubles the emf but also doubles the total internal resistance.

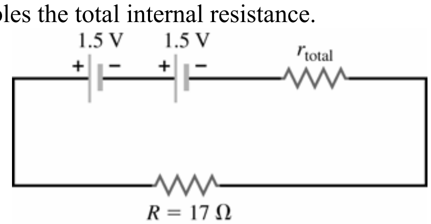


Figure 25.54

25.55. **IDENTIFY:** $P = I^2 R = \dfrac{V^2}{R} = VI$. $V = IR$.

SET UP: The heater consumes 540 W when $V = 120\text{ V}$. Energy $= Pt$.

EXECUTE: **(a)** $P = \dfrac{V^2}{R}$ so $R = \dfrac{V^2}{P} = \dfrac{(120\text{ V})^2}{540\text{ W}} = 26.7\ \Omega$

(b) $P = VI$ so $I = \dfrac{P}{V} = \dfrac{540\text{ W}}{120\text{ V}} = 4.50\text{ A}$

(c) Assuming that R remains $26.7\ \Omega$, $P = \dfrac{V^2}{R} = \dfrac{(110\text{ V})^2}{26.7\ \Omega} = 453\text{ W}$. P is smaller by a factor of $(110/120)^2$.

EVALUATE: **(d)** With the lower line voltage the current will decrease and the operating temperature will decrease. R will be less than $26.7\ \Omega$ and the power consumed will be greater than the value calculated in part (c).

25.56. **IDENTIFY:** From Eq. (25.24), $\rho = \dfrac{m}{ne^2\tau}$.

SET UP: For silicon, $\rho = 2300\ \Omega \cdot \text{m}$.

EXECUTE: **(a)** $\tau = \dfrac{m}{ne^2\rho} = \dfrac{9.11\times10^{-31}\text{ kg}}{(1.0\times10^{16}\text{ m}^{-3})(1.60\times10^{-19}\text{ C})^2(2300\ \Omega\cdot\text{m})} = 1.55\times10^{-12}\text{ s}$.

EVALUATE: **(b)** The number of free electrons in copper $(8.5\times10^{28}\text{ m}^{-3})$ is much larger than in pure silicon $(1.0\times10^{16}\text{ m}^{-3})$. A smaller density of current carriers means a higher resistivity.

25.57. **(a) IDENTIFY** and **SET UP:** Use $R = \dfrac{\rho L}{A}$.

EXECUTE: $\rho = \dfrac{RA}{L} = \dfrac{(0.104\ \Omega)\pi(1.25\times10^{-3}\text{ m})^2}{14.0\text{ m}} = 3.65\times10^{-8}\ \Omega\cdot\text{m}$

EVALUATE: This value is similar to that for good metallic conductors in Table 25.1.
(b) IDENTIFY and SET UP: Use $V = EL$ to calculate E and then Ohm's law gives I.
EXECUTE: $V = EL = (1.28 \text{ V/m})(14.0 \text{ m}) = 17.9 \text{ V}$

$$I = \frac{V}{R} = \frac{17.9 \text{ V}}{0.104\ \Omega} = 172 \text{ A}$$

EVALUATE: We could do the calculation another way:

$$E = \rho J \text{ so } J = \frac{E}{\rho} = \frac{1.28 \text{ V/m}}{3.65\times10^{-8}\ \Omega\cdot\text{m}} = 3.51\times10^{7} \text{ A/m}^2$$

$$I = JA = \left(3.51\times10^{7} \text{ A/m}^2\right)\pi\left(1.25\times10^{-3} \text{ m}\right)^2 = 172 \text{ A, which checks}$$

(c) IDENTIFY and SET UP: Calculate $J = I/A$ or $J = E/\rho$ and then use Eq. (25.3) for the target variable v_d.

EXECUTE: $J = n|q|v_\text{d} = nev_\text{d}$

$$v_\text{d} = \frac{J}{ne} = \frac{3.51\times10^{7} \text{ A/m}^2}{\left(8.5\times10^{28} \text{ m}^{-3}\right)\left(1.602\times10^{-19} \text{ C}\right)} = 2.58\times10^{-3} \text{ m/s} = 2.58 \text{ mm/s}$$

EVALUATE: Even for this very large current the drift speed is small.

25.58. IDENTIFY: Use $R = \dfrac{\rho L}{A}$ to calculate the resistance of the silver tube. Then $I = V/R$.

SET UP: For silver, $\rho = 1.47\times10^{-8}\ \Omega\cdot\text{m}$. The silver tube is sketched in Figure 25.58. Since the thickness $T = 0.100$ mm is much smaller than the radius, $r = 2.00$ cm, the cross section area of the silver is $2\pi rT$. The length of the tube is $l = 25.0$ m.

EXECUTE: $I = \dfrac{V}{R} = \dfrac{V}{\rho l/A} = \dfrac{VA}{\rho l} = \dfrac{V(2\pi rT)}{\rho l} = \dfrac{(12 \text{ V})(2\pi)(2.00\times10^{-2} \text{ m})(0.100\times10^{-3} \text{ m})}{(1.47\times10^{-8}\ \Omega\cdot\text{m})(25.0 \text{ m})} = 410 \text{ A}$

EVALUATE: The resistance is small, $R = 0.0292\ \Omega$, so 12.0 V produces a large current.

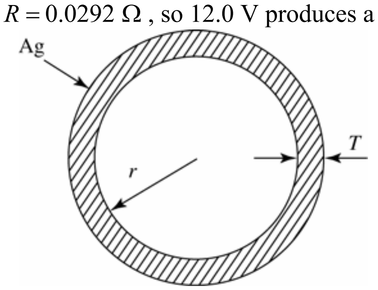


Figure 25.58

25.59. IDENTIFY and SET UP: With the voltmeter connected across the terminals of the battery there is no current through the battery and the voltmeter reading is the battery emf; $\mathcal{E} = 12.6$ V.
With a wire of resistance R connected to the battery current I flows and $\mathcal{E} - Ir - IR = 0$, where r is the internal resistance of the battery. Apply this equation to each piece of wire to get two equations in the two unknowns.
EXECUTE: Call the resistance of the 20.0-m piece R_1; then the resistance of the 40.0-m piece is $R_2 = 2R_1$.

$$\mathcal{E} - I_1 r - I_1 R_1 = 0; \qquad 12.6 \text{ V} - (7.00 \text{ A})r - (7.00 \text{ A})R_1 = 0$$

$$\mathcal{E} - I_2 r - I_2(2R_1) = 0; \qquad 12.6 \text{ V} - (4.20 \text{ A})r - (4.20 \text{ A})(2R_1) = 0$$

Solving these two equations in two unknowns gives $R_1 = 1.20\ \Omega$. This is the resistance of 20.0 m, so the resistance of one meter is $\left[1.20\ \Omega/(20.0 \text{ m})\right](1.00 \text{ m}) = 0.060\ \Omega$

EVALUATE: We can also solve for r and we get $r = 0.600\ \Omega$. When measuring small resistances, the internal resistance of the battery has a large effect.

25.60. IDENTIFY: Conservation of charge requires that the current is the same in both sections. The voltage drops across each section add, so $R = R_\text{Cu} + R_\text{Ag}$. The total resistance is the sum of the resistances of each section.

$E = \rho J = \dfrac{\rho I}{A}$, so $E = \dfrac{IR}{L}$, where R is the resistance of a section and L is its length.

SET UP: For copper, $\rho_\text{Cu} = 1.72\times10^{-8}\ \Omega\cdot\text{m}$. For silver, $\rho_\text{Ag} = 1.47\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** $I = \dfrac{V}{R} = \dfrac{V}{R_{\text{Cu}} + R_{\text{Ag}}}$. $R_{\text{Cu}} = \dfrac{\rho_{\text{Cu}} L_{\text{Cu}}}{A_{\text{Cu}}} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(0.8\ \text{m})}{(\pi/4)(6.0\times10^{-4}\text{m})^2} = 0.049\ \Omega$ and

$R_{\text{Ag}} = \dfrac{\rho_{\text{Ag}} L_{\text{Ag}}}{A_{\text{Ag}}} = \dfrac{(1.47\times10^{-8}\ \Omega\cdot\text{m})(1.2\ \text{m})}{(\pi/4)(6.0\times10^{-4}\ \text{m})^2} = 0.062\ \Omega$. This gives $I = \dfrac{5.0\ \text{V}}{0.049\ \Omega + 0.062\ \Omega} = 45\ \text{A}$.

The current in the copper wire is 45 A.

(b) The current in the silver wire is 45 A, the same as that in the copper wire or else charge would build up at their interface.

(c) $E_{\text{Cu}} = J\rho_{\text{Cu}} = \dfrac{IR_{\text{Cu}}}{L_{\text{Cu}}} = \dfrac{(45\ \text{A})(0.049\ \Omega)}{0.8\ \text{m}} = 2.76\ \text{V/m}$.

(d) $E_{\text{Ag}} = J\rho_{\text{Ag}} = \dfrac{IR_{\text{Ag}}}{L_{\text{Ag}}} = \dfrac{(45\ \text{A})(0.062\ \Omega)}{1.2\ \text{m}} = 2.33\ \text{V/m}$.

(e) $V_{\text{Ag}} = IR_{\text{Ag}} = (45\ \text{A})(0.062\ \Omega) = 2.79\ \text{V}$.

EVALUATE: For the copper section, $V_{\text{Cu}} = IR_{\text{Cu}} = 2.21\ \text{V}$. Note that $V_{\text{Cu}} + V_{\text{Ag}} = 5.0\ \text{V}$, the voltage applied across the ends of the composite wire.

25.61. **IDENTIFY:** Conservation of charge requires that the current be the same in both sections of the wire. $E = \rho J = \dfrac{\rho I}{A}$. For each section, $V = IR = JAR = \left(\dfrac{EA}{\rho}\right)\left(\dfrac{\rho L}{A}\right) = EL$. The voltages across each section add.

SET UP: $A = (\pi/4)D^2$, where D is the diameter.

EXECUTE: **(a)** The current must be the same in both sections of the wire, so the current in the thin end is 2.5 mA.

(b) $E_{1.6\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(2.5\times10^{-3}\ \text{A})}{(\pi/4)(1.6\times10^{-3}\ \text{m})^2} = 2.14\times10^{-5}\ \text{V/m}$.

(c) $E_{0.8\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(2.5\times10^{-3}\ \text{A})}{(\pi/4)(0.80\times10^{-3}\ \text{m})^2} = 8.55\times10^{-5}\ \text{V/m}$. This is $4E_{1.6\text{mm}}$.

(d) $V = E_{1.6\text{mm}}L_{1.6\ \text{mm}} + E_{0.8\ \text{mm}}L_{0.8\ \text{mm}}$. $V = (2.14\times10^{-5}\ \text{V/m})(1.20\ \text{m}) + (8.55\times10^{-5}\ \text{V/m})(1.80\ \text{m}) = 1.80\times10^{-4}\ \text{V}$.

EVALUATE: The currents are the same but the current density is larger in the thinner section and the electric field is larger there.

25.62. **IDENTIFY:** $I = JA$.

SET UP: From Example 25.1, an 18-gauge wire has $A = 8.17\times10^{-3}\ \text{cm}^2$.

EXECUTE: **(a)** $I = JA = (1.0\times10^{5}\ \text{A/cm}^2)(8.17\times10^{-3}\ \text{cm}^2) = 820\ \text{A}$

(b) $A = I/J = (1000\ \text{A})/(1.0\times10^{6}\ \text{A/cm}^2) = 1.0\times10^{-3}\ \text{cm}^2$. $A = \pi r^2$ so

$r = \sqrt{A/\pi} = \sqrt{(1.0\times10^{-3}\ \text{cm}^2)/\pi} = 0.0178\ \text{cm}$ and $d = 2r = 0.36\ \text{mm}$.

EVALUATE: These wires can carry very large currents.

25.63. **(a) IDENTIFY:** Apply Eq. (25.10) to calculate the resistance of each thin disk and then integrate over the truncated cone to find the total resistance.

SET UP:

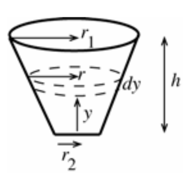


Figure 25.63

EXECUTE: The radius of a truncated cone a distance y above the bottom is given by $r = r_2 + (y/h)(r_1 - r_2) = r_2 + y\beta$ with $\beta = (r_1 - r_2)/h$

Consider a thin slice a distance y above the bottom. The slice has thickness dy and radius r. The resistance of the slice is

$$dR = \frac{\rho dy}{A} = \frac{\rho dy}{\pi r^2} = \frac{\rho dy}{\pi(r_2 + \beta y)^2}$$

The total resistance of the cone if obtained by integrating over these thin slices:

$$R = \int dR = \frac{\rho}{\pi}\int_0^h \frac{dy}{(r_2 + \beta y)^2} = \frac{\rho}{\pi}\left[-\frac{1}{\beta}(r_2 + y\beta)^{-1}\right]_0^h = -\frac{\rho}{\pi\beta}\left[\frac{1}{r_2 + h\beta} - \frac{1}{r_2}\right]$$

But $r_2 + h\beta = r_1$

$$R = \frac{\rho}{\pi\beta}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] = \frac{\rho}{\pi}\left(\frac{h}{r_1 - r_2}\right)\left(\frac{r_1 - r_2}{r_1 r_2}\right) = \frac{\rho h}{\pi r_1 r_2}$$

(b) EVALUATE: Let $r_1 = r_2 = r$. Then $R = \rho h/\pi r^2 = \rho L/A$ where $A = \pi r^2$ and $L = h$. This agrees with Eq. (25.10).

25.64. IDENTIFY: Divide the region into thin spherical shells of radius r and thickness dr. The total resistance is the sum of the resistances of the thin shells and can be obtained by integration.

SET UP: $I = V/R$ and $J = I/4\pi r^2$, where $4\pi r^2$ is the surface area of a shell of radius r.

EXECUTE: **(a)** $dR = \dfrac{\rho dr}{4\pi r^2} \Rightarrow R = \dfrac{\rho}{4\pi}\displaystyle\int_a^b \dfrac{dr}{r^2} = -\dfrac{\rho}{4\pi}\dfrac{1}{r}\Big|_a^b = \dfrac{\rho}{4\pi}\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = \dfrac{\rho}{4\pi}\left(\dfrac{b-a}{ab}\right)$.

(b) $I = \dfrac{V_{ab}}{R} = \dfrac{V_{ab} 4\pi ab}{\rho(b-a)}$ and $J = \dfrac{I}{A} = \dfrac{V_{ab} 4\pi ab}{\rho(b-a)4\pi r^2} = \dfrac{V_{ab} ab}{\rho(b-a) r^2}$.

(c) If the thickness of the shells is small, then $4\pi ab \approx 4\pi a^2$ is the surface area of the conducting material.

$R = \dfrac{\rho}{4\pi}\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = \dfrac{\rho(b-a)}{4\pi ab} \approx \dfrac{\rho L}{4\pi a^2} = \dfrac{\rho L}{A}$, where $L = b - a$.

EVALUATE: The current density in the material is proportional to $1/r^2$.

25.65. IDENTIFY and SET UP: Use $E = \rho J$ to calculate the current density between the plates. Let A be the area of each plate; then $I = JA$.

EXECUTE: $J = \dfrac{E}{\rho}$ and $E = \dfrac{\sigma}{K\epsilon_0} = \dfrac{Q}{KA\epsilon_0}$

Thus $J = \dfrac{Q}{KA\epsilon_0 \rho}$ and $I = JA = \dfrac{Q}{K\epsilon_0 \rho}$, as was to be shown.

EVALUATE: $C = K\epsilon_0 A/d$ and $V = Q/C = Qd/K\epsilon_0 A$ so the result can also be written as $I = VA/d\rho$. The resistance of the dielectric is $R = V/I = d\rho/A$, which agrees with Eq. (25.10).

25.66. IDENTIFY: As the resistance R varies, the current in the circuit also varies, which causes the potential drop across the internal resistance of the battery to vary.

SET UP: The largest current will occur when $R = 0$, and the smallest current will occur when $R \to \infty$. The largest terminal voltage will occur when the current is zero ($R \to \infty$) and the smallest terminal voltage will be when the current is a maximum ($R = 0$).

EXECUTE: **(a)** As $R \to \infty$, $I \to 0$, so $V_{ab} \to \mathcal{E} = 15.0$ V, which is the largest reading of the voltmeter. When $R = 0$, the current is largest at (15.0 V)/(4.00 Ω) = 3.75 A, so the smallest terminal voltage is $V_{ab} = \mathcal{E} - rI = 15.0\text{ V} - (4.00\ \Omega)(3.75\text{ A}) = 0$.

(b) From part (a), the maximum current is 3.75 A when $R = 0$, and the minimum current is 0.00 A when $R \to \infty$.

(c) The graphs are sketched in Figure 25.66.

EVALUATE: Increasing the resistance R increases the terminal voltage, but at the same time it decreases the current in the circuit.

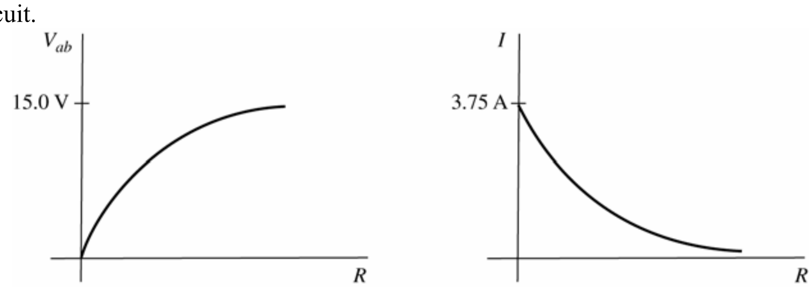


Figure 25.66

25.67. IDENTIFY: Apply $R = \dfrac{\rho L}{A}$.

SET UP: For mercury at 20°C, $\rho = 9.5 \times 10^{-7}\ \Omega \cdot \text{m}$, $\alpha = 0.00088\ (\text{C}°)^{-1}$ and $\beta = 18 \times 10^{-5}\ (\text{C}°)^{-1}$.

EXECUTE: **(a)** $R = \dfrac{\rho L}{A} = \dfrac{(9.5 \times 10^{-7}\ \Omega \cdot \text{m})(0.12\text{ m})}{(\pi/4)(0.0016\text{ m})^2} = 0.057\ \Omega$.

(b) $\rho(T)=\rho_0(1+\alpha\Delta T)$ gives $\rho(60°\text{ C})=(9.5\times10^{-7}\ \Omega\cdot\text{m})(1+(0.00088\ (\text{C}°)^{-1})(40\ \text{C}°)=9.83\times10^{-7}\ \Omega\cdot\text{m}$, so $\Delta\rho=3.34\times10^{-8}\ \Omega\cdot\text{m}$.

(c) $\Delta V=\beta V_0\Delta T$ gives $A\Delta L=A\ (\beta L_0\Delta T)$. Therefore $\Delta L=\beta L_0\Delta T=(18\times10^{-5}\ (\text{C}°)^{-1})(0.12\ \text{m})(40\ \text{C}°)=8.64\times10^{-4}\ \text{m}=0.86\ \text{mm}$. The cross sectional area of the mercury remains constant because the diameter of the glass tube doesn't change. All of the change in volume of the mercury must be accommodated by a change in length of the mercury column.

(d) $R=\dfrac{\rho L}{A}$ gives $\Delta R=\dfrac{L\Delta\rho}{A}+\dfrac{\rho\Delta L}{A}$.

$$\Delta R=\frac{(3.34\times10^{-8}\ \Omega\cdot\text{m})(0.12\ \text{m})}{(\pi/4)(0.0016\ \text{m})^2}+\frac{(95\times10^{-8}\ \Omega\cdot\text{m})(0.86\times10^{-3}\ \text{m})}{(\pi/4)(0.0016\ \text{m})^2}=2.40\times10^{-3}\ \Omega.$$

EVALUATE: **(e)** From Equation (25.12),

$$\alpha=\frac{1}{\Delta T}\left(\frac{R}{R_0}-1\right)=\frac{1}{40\ \text{C}°}\left(\frac{(0.057\ \Omega+2.40\times10^{-3}\ \Omega)}{0.057\ \Omega}-1\right)=1.1\times10^{-3}\ (\text{C}°)^{-1}.$$

This value is 25% greater than the temperature coefficient of resistivity and the length increase is important.

25.68. **IDENTIFY:** Consider the potential changes around the circuit. For a complete loop the sum of the potential changes is zero.

SET UP: There is a potential drop of IR when you pass through a resistor in the direction of the current.

EXECUTE: **(a)** $I=\dfrac{8.0\ \text{V}-4.0\ \text{V}}{24.0\ \Omega}=0.167\ \text{A}$. $V_d+8.00\ \text{V}-I(0.50\ \Omega+8.00\ \Omega)=V_a$, so

$V_{ad}=8.00\ \text{V}-(0.167\ \text{A})\,(8.50\ \Omega)=6.58\ \text{V}$.

(b) The terminal voltage is $V_{bc}=V_b-V_c$. $V_c+4.00\ \text{V}+I(0.50\ \Omega)=V_b$ and

$V_{bc}=+\,4.00\ \text{V}+(0.167\ \text{A})\,(0.50\ \Omega)=+\,4.08\ \text{V}$.

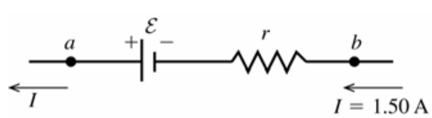
(c) Adding another battery at point d in the opposite sense to the 8.0 V battery produces a counterclockwise current with magnitude $I=\dfrac{10.3\ \text{V}-8.0\ \text{V}+4.0\ \text{V}}{24.5\ \Omega}=0.257\ \text{A}$. Then $V_c+4.00\ \text{V}-I(0.50\ \Omega)=V_b$ and

$V_{bc}=4.00\ \text{V}-(0.257\ \text{A})\,(0.50\ \Omega)=3.87\ \text{V}$.

EVALUATE: When current enters the battery at its negative terminal, as in part (c), the terminal voltage is less than its emf. When current enters the battery at the positive terminal, as in part (b), the terminal voltage is greater than its emf.

25.69. **IDENTIFY:** In each case write the terminal voltage in terms of $\mathcal{E}$, I, and r. Since I is known, this gives two equations in the two unknowns $\mathcal{E}$ and r.

SET UP: The battery with the 1.50 A current is sketched in Figure 25.69a.



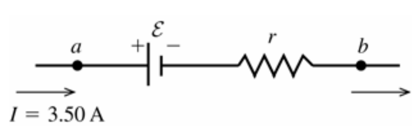
$V_{ab}=8.4\ \text{V}$

$V_{ab}=\mathcal{E}-Ir$

$\mathcal{E}-(1.50\ \text{A})r=8.4\ \text{V}$

Figure 25.69a

The battery with the 3.50 A current is sketched in Figure 25.69b.



$V_{ab}=9.4\ \text{V}$

$V_{ab}=\mathcal{E}+Ir$

$\mathcal{E}+(3.5\ \text{A})r=9.4\ \text{V}$

Figure 25.69b

EXECUTE: **(a)** Solve the first equation for $\mathcal{E}$ and use that result in the second equation:

$\mathcal{E}=8.4\ \text{V}+(1.50\ \text{A})r$

$8.4\ \text{V}+(1.50\ \text{A})r+(3.50\ \text{A})r=9.4\ \text{V}$

$(5.00\ \text{A})r=1.0\ \text{V}$ so $r=\dfrac{1.0\ \text{V}}{5.00\ \text{A}}=0.20\ \Omega$

(b) Then $\mathcal{E} = 8.4\text{ V} + (1.50\text{ A})r = 8.4\text{ V} + (1.50\text{ A})(0.20\ \Omega) = 8.7\text{ V}$

EVALUATE: When the current passes through the emf in the direction from $-$ to $+$, the terminal voltage is less than the emf and when it passes through from $+$ to $-$, the terminal voltage is greater than the emf.

25.70. IDENTIFY: $V = IR$. $P = I^2R$.

SET UP: The total resistance is the resistance of the person plus the internal resistance of the power supply.

EXECUTE: **(a)** $I = \dfrac{V}{R_{\text{tot}}} = \dfrac{14\times10^3\text{ V}}{10\times10^3\ \Omega + 2000\ \Omega} = 1.17\text{ A}$

(b) $P = I^2R = (1.17\text{ A})^2(10\times10^3\ \Omega) = 1.37\times10^4\text{ J} = 13.7\text{ kJ}$

(c) $R_{\text{tot}} = \dfrac{V}{I} = \dfrac{14\times10^3\text{ V}}{1.00\times10^{-3}\text{ A}} = 14\times10^6\ \Omega.$ The resistance of the power supply would need to be $14\times10^6\ \Omega - 10\times10^3\ \Omega = 14\times10^6\ \Omega = 14\text{ M}\Omega.$

EVALUATE: The current through the body in part (a) is large enough to be fatal.

25.71. IDENTIFY: $R = \dfrac{\rho L}{A}$. $V = IR$. $P = I^2R$.

SET UP: The area of the end of a cylinder of radius *r* is πr^2.

EXECUTE: **(a)** $R = \dfrac{(5.0\ \Omega\cdot\text{m})(1.6\text{ m})}{\pi(0.050\text{ m})^2} = 1.0\times10^3\ \Omega$

(b) $V = IR = (100\times10^{-3}\text{ A})(1.0\times10^3\ \Omega) = 100\text{ V}$

(c) $P = I^2R = (100\times10^{-3}\text{ A})^2(1.0\times10^3\ \Omega) = 10\text{ W}$

EVALUATE: The resistance between the hands when the skin is wet is about a factor of ten less than when the skin is dry (Problem 25.70).

25.72. IDENTIFY: The cost of operating an appliance is proportional to the amount of energy consumed. The energy depends on the power the item consumes and the length of time for which it is operated.

SET UP: At a constant power, the energy is equal to *Pt*, and the total cost is the cost per kilowatt-hour (kWh) times the time the energy (in kWh).

EXECUTE: **(a)** Use the fact that 1.00 kWh = (1000 J/s)(3600 s) = 3.60×10^6 J, and one year contains 3.156×10^7 s.

$$(75\text{ J/s})\left(\frac{3.156\times10^7\text{ s}}{1\text{ yr}}\right)\left(\frac{\$0.120}{3.60\times10^6\text{ J}}\right) = \$78.90$$

(b) At 8 h/day, the refrigerator runs for 1/3 of a year. Using the same procedure as above gives

$$(400\text{ J/s})\left(\frac{1}{3}\right)\left(\frac{3.156\times10^7\text{ s}}{1\text{ yr}}\right)\left(\frac{\$0.120}{3.60\times10^6\text{ J}}\right) = \$140.27$$

EVALUATE: Electric lights can be a substantial part of the cost of electricity in the home if they are left on for a long time!

25.73. IDENTIFY: Set the sum of the potential rises and drops around the circuit equal to zero and solve for *I*.

SET UP: The circuit is sketched in Figure 25.73.

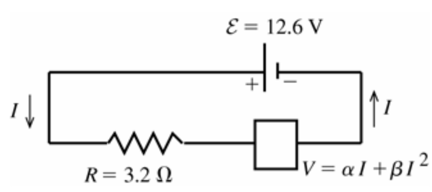


Figure 25.73

EXECUTE:

$\mathcal{E} - IR - V = 0$

$\mathcal{E} - IR - \alpha I - \beta I^2 = 0$

$\beta I^2 + (R+\alpha)I - \mathcal{E} = 0$

The quadratic formula gives $I = (1/2\beta)\left[-(R+\alpha) \pm \sqrt{(R+\alpha)^2 + 4\beta\mathcal{E}}\right]$

I must be positive, so take the + sign

$I = (1/2\beta)\left[-(R+\alpha) + \sqrt{(R+\alpha)^2 + 4\beta\mathcal{E}}\right]$

$I = -2.692\text{ A} + 4.116\text{ A} = 1.42\text{ A}$

EVALUATE: For this *I* the voltage across the thermistor is 8.0 V. The voltage across the resistor must then be $12.6\text{ V} - 8.0\text{ V} = 4.6\text{ V},$ and this agrees with Ohm's law for the resistor.

25.74. **(a) IDENTIFY:** The rate of heating (power) in the cable depends on the potential difference across the cable and the resistance of the cable.

SET UP: The power is $P = V^2/R$ and the resistance is $R = \rho L/A$. The diameter D of the cable is twice its radius. $P = \dfrac{V^2}{R} = \dfrac{V^2}{(\rho L/A)} = \dfrac{AV^2}{\rho L} = \dfrac{\pi r^2 V^2}{\rho L}$. The electric field in the cable is equal to the potential difference across its ends divided by the length of the cable: $E = V/L$.

EXECUTE: Solving for r and using the resistivity of copper gives

$$r = \sqrt{\frac{P\rho L}{\pi V^2}} = \sqrt{\frac{(50.0\text{ W})(1.72\times10^{-8}\ \Omega\cdot\text{m})(1500\text{ m})}{\pi(220.0\text{ V})^2}} = 9.21\times10^{-5}\text{ m.}\ \ D = 2r = 0.184\text{ mm}$$

(b) SET UP: $E = V/L$

EXECUTE: $E = (220\text{ V})/(1500\text{ m}) = 0.147\text{ V/m}$

EVALUATE: This would be an extremely thin (and hence fragile) cable.

25.75. **IDENTIFY:** The ammeter acts as a resistance in the circuit loop. Set the sum of the potential rises and drops around the circuit equal to zero.

(a) SET UP: The circuit with the ammeter is sketched in Figure 25.75a.

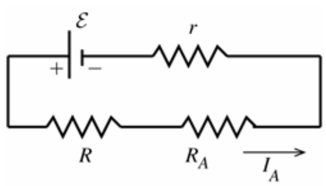


Figure 25.75a

EXECUTE:

$$I_A = \frac{\mathcal{E}}{r + R + R_A}$$

$$\mathcal{E} = I_A(r + R + R_A)$$

SET UP: The circuit with the ammeter removed is sketched in Figure 25.75b.

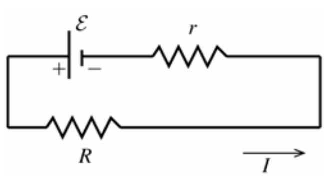


Figure 25.75b

EXECUTE:

$$I = \frac{\mathcal{E}}{R + r}$$

Combining the two equations gives

$$I = \left(\frac{1}{R+r}\right) I_A(r + R + R_A) = I_A\left(1 + \frac{R_A}{r+R}\right)$$

(b) Want $I_A = 0.990I$. Use this in the result for part (a).

$$I = 0.990I\left(1 + \frac{R_A}{r+R}\right)$$

$$0.010 = 0.990\left(\frac{R_A}{r+R}\right)$$

$$R_A = (r+R)(0.010/0.990) = (0.45\ \Omega + 3.80\ \Omega)(0.010/0.990) = 0.0429\ \Omega$$

(c) $I - I_A = \dfrac{\mathcal{E}}{r+R} - \dfrac{\mathcal{E}}{r+R+R_A}$

$$I - I_A = \mathcal{E}\left(\frac{r+R+R_A-r-R}{(r+R)(r+R+R_A)}\right) = \frac{\mathcal{E}R_A}{(r+R)(r+R+R_A)}.$$

EVALUATE: The difference between I and I_A increases as R_A increases. If R_A is larger than the value calculated in part (b) then I_A differs from I by more than 1.0%.

25.76. **IDENTIFY:** Since the resistivity is a function of the position along the length of the cylinder, we must integrate to find the resistance.

(a) SET UP: The resistance of a cross-section of thickness dx is $dR = \rho dx/A$.

EXECUTE: Using the given function for the resistivity and integrating gives

$$R = \int \frac{\rho dx}{A} = \int_0^L \frac{(a + bx^2)dx}{\pi r^2} = \frac{aL + bL^3/3}{\pi r^2}.$$

Now get the constants a and b: $\rho(0) = a = 2.25 \times 10^{-8}\ \Omega\cdot\text{m}$ and

$\rho(L) = a + bL^2$ gives $8.50 \times 10^{-8}\ \Omega\cdot\text{m} = 2.25 \times 10^{-8}\ \Omega\cdot\text{m} + b(1.50\ \text{m})^2$

which gives $b = 2.78 \times 10^{-8}\ \Omega/\text{m}$. Now use the above result to find R.

$$R = \frac{\left(2.25\times10^{-8}\ \Omega\cdot\text{m}\right)(1.50\ \text{m}) + \left(2.78\times10^{-8}\ \Omega/\text{m}\right)(1.50\ \text{m})^3/3}{\pi(0.0110\ \text{m})^2} = 1.71 \times 10^{-4}\ \Omega = 171\ \mu\Omega$$

(b) IDENTIFY: Use the definition of resistivity to find the electric field at the midpoint of the cylinder, where $x = L/2$.

SET UP: $E = \rho J$. Evaluate the resistivity, using the given formula, for $x = L/2$.

EXECUTE: At the midpoint, $x = L/2$, giving $E = \dfrac{\rho I}{\pi r^2} = \dfrac{\left[a + b\left(L/2\right)^2\right]I}{\pi r^2}$.

$$E = \frac{\left[2.25\times10^{-8}\ \Omega\cdot\text{m} + \left(2.78\times10^{-8}\ \Omega/\text{m}\right)(0.750\ \text{m})^2\right](1.75\ \text{A})}{\pi(0.0110\ \text{m})^2} = 1.76\times10^{-4}\ \text{V/m}$$

(c) IDENTIFY: For the first segment, the result is the same as in part (a) except that the upper limit of the integral is $L/2$ instead of L.

SET UP: Integrating using the upper limit of $L/2$ gives $R_1 = \dfrac{a\left(L/2\right) + (b/3)\left(L^3/8\right)}{\pi r^2}$.

EXECUTE: Substituting the numbers gives

$$R_1 = \frac{\left(2.25\times10^{-8}\ \Omega\cdot\text{m}\right)(0.750\ \text{m}) + (2.78\times10^{-8}\ \Omega/\text{m})/3\left((1.50\ \text{m})^3/8\right)}{\pi(0.0110\ \text{m})^2} = 5.47\times10^{-5}\ \Omega$$

The resistance R_2 of the second half is equal to the total resistance minus the resistance of the first half.

$R_2 = R - R_1 = 1.71 \times 10^{-4}\ \Omega - 5.47\times10^{-5}\ \Omega = 1.16 \times 10^{-4}\ \Omega$

EVALUATE: The second half has a greater resistance than the first half because the resistance increases with distance along the cylinder.

25.77. **IDENTIFY:** The power supplied to the house is $P = VI$. The rate at which electrical energy is dissipated in the wires is I^2R, where $R = \dfrac{\rho L}{A}$.

SET UP: For copper, $\rho = 1.72\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** The line voltage, current to be drawn, and wire diameter are what must be considered in household wiring.

(b) $P = VI$ gives $I = \dfrac{P}{V} = \dfrac{4200\ \text{W}}{120\ \text{V}} = 35\ \text{A}$, so the 8-gauge wire is necessary, since it can carry up to 40 A.

(c) $P = I^2R = \dfrac{I^2\rho L}{A} = \dfrac{(35\ \text{A})^2\ (1.72\times10^{-8}\ \Omega\cdot\text{m})\ (42.0\ \text{m})}{(\pi/4)\ (0.00326\ \text{m})^2} = 106\ \text{W}$.

(d) If 6-gauge wire is used, $P = \dfrac{I^2\rho L}{A} = \dfrac{(35\ \text{A})^2\ (1.72\times10^{-8}\ \Omega\cdot\text{m})\ (42\ \text{m})}{(\pi/4)\ )\ (0.00412\ \text{m})^2} = 66\ \text{W}$. The decrease in energy consumption is $\Delta E = \Delta P t = (40\ \text{W})\ (365\ \text{days/yr})\ (12\ \text{h/day}) = 175\ \text{kWh/yr}$ and the savings is $(175\ \text{kWh/yr})\ (\$0.11/\text{kWh}) = \$19.25$ per year.

EVALUATE: The cost of the 4200 W used by the appliances is \$2020. The savings is about 1%.

25.78. **IDENTIFY:** $R_T = R_0(1 + \alpha[T - T_0])$. $R = \dfrac{V}{I}$. $P = VI$.

SET UP: When the temperature increases the resistance increases and the current decreases.

EXECUTE: **(a)** $\dfrac{V}{I_T} = \dfrac{V}{I_0}(1 + \alpha[T - T_0])$. $I_0 = I_T(1 + \alpha[T - T_0])$.

$$T - T_0 = \frac{I_0 - I_T}{\alpha I_T} = \frac{1.35\ \text{A} - 1.23\ \text{A}}{(1.23\ \text{A})(4.5\times10^{-4}\ (\text{C}°)^{-1})} = 217\ \text{C}°.\quad T = 20°\text{C} + 217°\text{C} = 237°\text{C}$$

(b) (i) $P = VI = (120\ \text{V})(1.35\ \text{A}) = 162\ \text{W}$ (ii) $P = (120\ \text{V})(1.23\ \text{A}) = 148\ \text{W}$

EVALUATE: $P = V^2/R$ shows that the power dissipated decreases when the resistance increases.

25.79. **(a) IDENTIFY:** Set the sum of the potential rises and drops around the circuit equal to zero and solve for the resulting equation for the current I. Apply Eq. (25.17) to each circuit element to find the power associated with it.

SET UP: The circuit is sketched in Figure 25.79.

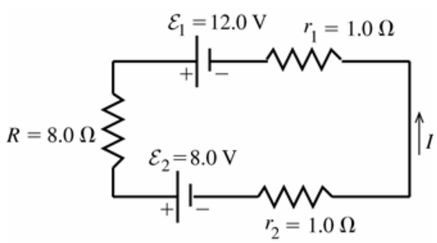


Figure 25.79

EXECUTE:

$$\mathcal{E}_1 - \mathcal{E}_2 - I(r_1 + r_2 + R) = 0$$

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{r_1 + r_2 + R}$$

$$I = \frac{12.0\ \text{V} - 8.0\ \text{V}}{1.0\ \Omega + 1.0\ \Omega + 8.0\ \Omega}$$

$$I = 0.40\ \text{A}$$

(b) $P = I^2R + I^2r_1 + I^2r_2 = I^2(R + r_1 + r_2) = (0.40\ \text{A})^2(8.0\ \Omega + 1.0\ \Omega + 1.0\ \Omega)$

$P = 1.6$ W

(c) Chemical energy is converted to electrical energy in a battery when the current goes through the battery from the negative to the positive terminal, so the electrical energy of the charges increases as the current passes through. This happens in the 12.0 V battery, and the rate of production of electrical energy is

$P = \mathcal{E}_1 I = (12.0\ \text{V})(0.40\ \text{A}) = 4.8$ W.

(d) Electrical energy is converted to chemical energy in a battery when the current goes through the battery from the positive to the negative terminal, so the electrical energy of the charges decreases as the current passes through. This happens in the 8.0 V battery, and the rate of consumption of electrical energy is

$P = \mathcal{E}_2 I = (8.0\ \text{V})(0.40\ \text{V}) = 3.2$ W.

(e) EVALUATE: Total rate of production of electrical energy = 4.8 W. Total rate of consumption of electrical energy = 1.6 W + 3.2 W = 4.8 W, which equals the rate of production, as it must.

25.80. IDENTIFY: Apply $R = \dfrac{\rho L}{A}$ for each material. The total resistance is the sum of the resistances of the rod and the wire. The rate at which energy is dissipated is I^2R.

SET UP: For steel, $\rho = 2.0 \times 10^{-7}\ \Omega\cdot\text{m}$. For copper, $\rho = 1.72 \times 10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: **(a)** $R_{\text{steel}} = \dfrac{\rho L}{A} = \dfrac{(2.0\times 10^{-7}\,\Omega\cdot\text{m})\ (2.0\ \text{m})}{(\pi/4)\ (0.018\ \text{m})^2} = 1.57\times 10^{-3}\ \Omega$ and

$R_{\text{Cu}} = \dfrac{\rho L}{A} = \dfrac{(1.72\times 10^{-8}\ \Omega\cdot\text{m})\ (35\ \text{m})}{(\pi/4)\ (0.008\ \text{m})^2} = 0.012\ \Omega$. This gives

$V = IR = I\ (R_{\text{steel}} + R_{\text{Cu}}) = (15000\ \text{A})\ (1.57\times 10^{-3}\ \Omega + 0.012\ \Omega) = 204$ V.

(b) $E = Pt = I^2Rt = (15000\ \text{A})^2 (0.0136\ \Omega)\ (65\times 10^{-6}\ \text{s}) = 199$ J.

EVALUATE: I^2R is large but *t* is very small, so the energy deposited is small. The wire and rod each have a mass of about 1 kg, so their temperature rise due to the deposited energy will be small.

25.81. IDENTIFY and **SET UP:** The terminal voltage is $V_{ab} = \mathcal{E} - Ir = IR$, where R is the resistance connected to the battery. During the charging the terminal voltage is $V_{ab} = \mathcal{E} + Ir$. $P = VI$ and energy is $E = Pt$. I^2r is the rate at which energy is dissipated in the internal resistance of the battery.

EXECUTE: **(a)** $V_{ab} = \mathcal{E} + Ir = 12.0\ \text{V} + (10.0\ \text{A})\ (0.24\ \Omega) = 14.4$ V.

(b) $E = Pt = IVt = (10\ \text{A})\ (14.4\ \text{V})\ (5)\ (3600\ \text{s}) = 2.59\times 10^6$ J.

(c) $E_{\text{diss}} = P_{\text{diss}}t = I^2rt = (10\ \text{A})^2\ (0.24\ \Omega)\ (5)\ (3600\ \text{s}) = 4.32\times 10^5$ J.

(d) Discharged at 10 A: $I = \dfrac{\mathcal{E}}{r + R} \Rightarrow R = \dfrac{\mathcal{E} - Ir}{I} = \dfrac{12.0\ \text{V} - (10\ \text{A})\ (0.24\ \Omega)}{10\ \text{A}} = 0.96\ \Omega.$

(e) $E = Pt = IVt = (10\ \text{A})\ (9.6\ \text{V})\ (5)\ (3600\ \text{s}) = 1.73\times 10^6$ J.

(f) Since the current through the internal resistance is the same as before, there is the same energy dissipated as in (c): $E_{\text{diss}} = 4.32\times 10^5$ J.

(g) Part of the energy originally supplied was stored in the battery and part was lost in the internal resistance. So the stored energy was less than what was supplied during charging. Then when discharging, even more energy is lost in the internal resistance, and only what is left is dissipated by the external resistor.

25.82. **IDENTIFY** and **SET UP:** The terminal voltage is $V_{ab} = \mathcal{E} - Ir = IR$, where R is the resistance connected to the battery. During the charging the terminal voltage is $V_{ab} = \mathcal{E} + Ir$. $P = VI$ and energy is $E = Pt$. $I^2 r$ is the rate at which energy is dissipated in the internal resistance of the battery.

EXECUTE: **(a)** $V_{ab} = \mathcal{E} + Ir = 12.0\text{ V} + (30\text{ A})(0.24\,\Omega) = 19.2\text{ V}$.

(b) $E = Pt = IVt = (30\text{ A})(19.2\text{ V})(1.7)(3600\text{ s}) = 3.53 \times 10^6\text{ J}$.

(c) $E_{\text{diss}} = P_{\text{diss}} t = I^2 Rt = (30\text{ A})^2\,(0.24\,\Omega)(1.7)(3600\text{ s}) = 1.32 \times 10^6\text{ J}$.

(d) Discharged at 30 A: $I = \dfrac{\mathcal{E}}{r + R}$ gives $R = \dfrac{\mathcal{E} - Ir}{I} = \dfrac{12.0\text{ V} - (30\text{ A})(0.24\,\Omega)}{30\text{ A}} = 0.16\,\Omega$.

(e) $E = Pt = I^2 Rt = (30\text{ A})^2\,(0.16\,\Omega)(1.7)(3600\text{ s}) = 8.81 \times 10^5\text{ J}$.

(f) Since the current through the internal resistance is the same as before, there is the same energy dissipated as in (c): $E_{\text{diss}} = 1.32 \times 10^6\text{ J}$.

(g) Again, part of the energy originally supplied was stored in the battery and part was lost in the internal resistance. So the stored energy was less than what was supplied during charging. Then when discharging, even more energy is lost in the internal resistance, and what is left is dissipated over the external resistor. This time, at a higher current, much more energy is lost in the internal resistance. Slow charging and discharging is more energy efficient.

25.83. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** $\Sigma F = ma = |q|E$ gives $\dfrac{|q|}{m} = \dfrac{a}{E}$.

(b) If the electric field is constant, $V_{bc} = EL$ and $\dfrac{|q|}{m} = \dfrac{aL}{V_{bc}}$

(c) The free charges are "left behind" so the left end of the rod is negatively charged, while the right end is positively charged. Thus the right end, point *c*, is at the higher potential.

(d) $a = \dfrac{V_{bc}|q|}{mL} = \dfrac{(1.0 \times 10^{-3}\text{ V})(1.6 \times 10^{-19}\text{ C})}{(9.11 \times 10^{-31}\text{ kg})(0.50\text{ m})} = 3.5 \times 10^8\text{ m/s}^2$.

EVALUATE: **(e)** Performing the experiment in a rotational way enables one to keep the experimental apparatus in a localized area—whereas an acceleration like that obtained in (d), if linear, would quickly have the apparatus moving at high speeds and large distances. Also, the rotating spool of thin wire can have many turns of wire and the total potential is the sum of the potentials in each turn, the potential in each turn times the number of turns.

25.84. **IDENTIFY:** $\mathcal{E} - IR - V = 0$

SET UP: With $T = 293\text{ K}$, $\dfrac{e}{kT} = 39.6\text{ V}^{-1}$.

EXECUTE: **(a)** $\mathcal{E} = IR + V$ gives $2.00\text{ V} = I(1.0\,\Omega) + V$. Dropping units and using the expression given in the problem for *I*, this becomes $2.00 = I_S[\exp(eV/kT) - 1] + V$.

(b) For $I_S = 1.50 \times 10^{-3}$ A and $T = 293$ K, $1333 = \exp[39.6\,V] - 1 + 667V$. Trial and error shows that the right-hand side (rhs) above, for specific *V* values, equals 1333 V when $V = 0.179$ V. The current then is just $I = I_S[\exp(39.6\,V) - 1] = (1.5 \times 10^{-3}\text{ A})\left(\exp([39.6][0.179]) - 1]\right) = 1.80\text{ A}$.

EVALUATE: The voltage across the resistor *R* is 1.80 V. The diode does not obey Ohm's law.

25.85. **IDENTIFY:** Apply $R = \dfrac{\rho L}{A}$ to find the resistance of a thin slice of the rod and integrate to find the total *R*. $V = IR$. Also find $R(x)$, the resistance of a length *x* of the rod.

SET UP: $E(x) = \rho(x)J$

EXECUTE: **(a)** $dR = \dfrac{\rho\,dx}{A} = \dfrac{\rho_0\,\exp[-x/L]\,dx}{A}$ so

$R = \dfrac{\rho_0}{A}\int_0^L \exp[-x/L]\,dx = \dfrac{\rho_0}{A}[-L\exp[-x/L]]_0^L = \dfrac{\rho_0 L}{A}(1 - e^{-1})$ and $I = \dfrac{V_0}{R} = \dfrac{V_0 A}{\rho_0 L(1 - e^{-1})}$. With an upper limit of *x* rather than *L* in the integration, $R(x) = \dfrac{\rho_0 L}{A}\left(1 - e^{-x/L}\right)$.

(b) $E(x) = \rho(x)J = \dfrac{I\,\rho_0 e^{-x/L}}{A} = \dfrac{V_0 e^{-x/L}}{L\left(1 - e^{-1}\right)}$.

(c) $V = V_0 - IR(x)$. $V = V_0 - \left(\dfrac{V_0 A}{\rho_0 L[1-e^{-1}]}\right)\left(\dfrac{\rho_0 L}{A}\right)\left(1-e^{-x/L}\right) = V_0 \dfrac{(e^{-x/L}-e^{-1})}{(1-e^{-1})}$

(d) Graphs of resistivity, electric field and potential from $x = 0$ to L are given in Figure 25.85. Each quantity is given in terms of the indicated unit.

EVALUATE: The current is the same at all points in the rod. Where the resistivity is larger the electric field must be larger, in order to produce the same current density.

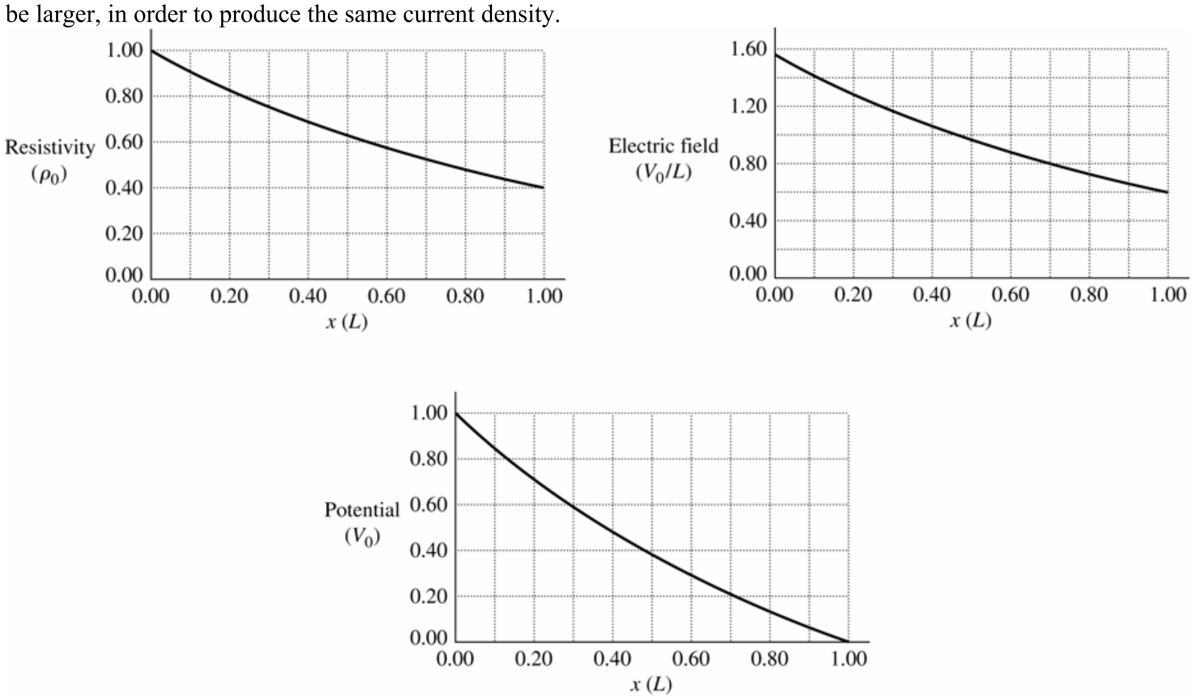


Figure 25.85

25.86. **IDENTIFY:** The power output of the source is $VI = (\mathcal{E} - Ir)I$.

SET UP: The short-circuit current is $I_{\text{short circuit}} = \mathcal{E}/r$.

EXECUTE: **(a)** $P = \mathcal{E}I - I^2 r$, so $\dfrac{dP}{dI} = \mathcal{E} - 2Ir = 0$ for maximum power output and $I_{P\,\max} = \dfrac{1}{2}\dfrac{\mathcal{E}}{r} = \dfrac{1}{2} I_{\text{short circuit}}$.

(b) For the maximum power output of part (a), $I = \dfrac{\mathcal{E}}{r+R} = \dfrac{1}{2}\dfrac{\mathcal{E}}{r}$. $r + R = 2r$ and $R = r$.

Then, $P = I^2 R = \left(\dfrac{\mathcal{E}}{2r}\right)^2 r = \dfrac{\mathcal{E}^2}{4r}$.

EVALUATE: When R is smaller than r, I is large and the $I^2 r$ losses in the battery are large. When R is larger than r, I is small and the power output $\mathcal{E}I$ of the battery emf is small.

25.87. **IDENTIFY:** Use $\alpha = -n/T$ in $\alpha = \dfrac{1}{\rho}\dfrac{d\rho}{dT}$ to get a separable differential equation that can be integrated.

SET UP: For carbon, $\rho = 3.5\times10^{-5}\ \Omega\cdot\text{m}$ and $\alpha = -5\times10^{-4}\ (\text{K})^{-1}$.

EXECUTE: **(a)** $\alpha = \dfrac{1}{\rho}\left(\dfrac{d\rho}{dT}\right) = -\dfrac{n}{T} \Rightarrow \dfrac{ndT}{T} = \dfrac{d\rho}{\rho} \Rightarrow \ln(T^{-n}) = \ln(\rho) \Rightarrow \rho = \dfrac{a}{T^n}$.

(b) $n = -\alpha T = -(-5\times10^{-4}\ (\text{K})^{-1})(293\ \text{K}) = 0.15$.

$\rho = \dfrac{a}{T^n} \Rightarrow a = \rho T^n = (3.5\times10^{-5}\ \Omega\cdot\text{m})(293\ \text{K})^{0.15} = 8.0\times10^{-5}\ \Omega\cdot\text{m}\cdot\text{K}^{0.15}$.

(c) $T = -196°\text{C} = 77\ \text{K}$: $\rho = \dfrac{8.0\times10^{-5}}{(77\ \text{K})^{0.15}} = 4.3\times10^{-5}\ \Omega\cdot\text{m}$.

$T = -300°\text{C} = 573\ \text{K}$: $\rho = \dfrac{8.0\times10^{-5}}{(573\ \text{K})^{0.15}} = 3.2\times10^{-5}\ \Omega\cdot\text{m}$.

EVALUATE: α is negative and decreases as T decreases, so ρ changes more rapidly with temperature at lower temperatures.

DIRECT-CURRENT CIRCUITS

26

26.1. **IDENTIFY:** The newly-formed wire is a combination of series and parallel resistors.
SET UP: Each of the three linear segments has resistance $R/3$. The circle is two $R/6$ resistors in parallel.
EXECUTE: The resistance of the circle is $R/12$ since it consists of two $R/6$ resistors in parallel. The equivalent resistance is two $R/3$ resistors in series with an $R/6$ resistor, giving $R_{\text{equiv}} = R/3 + R/3 + R/12 = 3R/4$.
EVALUATE: The equivalent resistance of the original wire has been reduced because the circle's resistance is less than it was as a linear wire.

26.2. **IDENTIFY:** It may appear that the meter measures X directly. But note that X is in parallel with three other resistors, so the meter measures the equivalent parallel resistance between ab.
SET UP: We use the formula for resistors in parallel.
EXECUTE: $1/(2.00\ \Omega) = 1/X + 1/(15.0\ \Omega) + 1/(5.0\ \Omega) + 1/(10.0\ \Omega)$, so $X = 7.5\ \Omega$.
EVALUATE: X is *greater* than the equivalent parallel resistance of $2.00\ \Omega$.

26.3. **(a) IDENTIFY:** Suppose we have two resistors in parallel, with $R_1 < R_2$.

SET UP: The equivalent resistance is $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

EXECUTE: It is always true that $\dfrac{1}{R_1} + \dfrac{1}{R_2} > \dfrac{1}{R_1}$. Therefore $\dfrac{1}{R_{\text{eq}}} > \dfrac{1}{R_1}$ and $R_{\text{eq}} < R_1$.

EVALUATE: The equivalent resistance is always less than that of the smallest resistor.
(b) IDENTIFY: Suppose we have N resistors in parallel, with $R_1 < R_2 < \cdots < R_N$.

SET UP: The equivalent resistance is $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_N}$

EXECUTE: It is always true that $\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_N} > \dfrac{1}{R_1}$. Therefore $\dfrac{1}{R_{\text{eq}}} > \dfrac{1}{R_1}$ and $R_{\text{eq}} < R_1$.

EVALUATE: The equivalent resistance is always less than that of the smallest resistor.

26.4. **IDENTIFY:** For resistors in parallel the voltages are the same and equal to the voltage across the equivalent resistance.

SET UP: $V = IR$. $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$.

EXECUTE: **(a)** $R_{\text{eq}} = \left(\dfrac{1}{32\ \Omega} + \dfrac{1}{20\ \Omega}\right)^{-1} = 12.3\ \Omega.$

(b) $I = \dfrac{V}{R_{\text{eq}}} = \dfrac{240\text{ V}}{12.3\ \Omega} = 19.5\text{ A}.$

(c) $I_{32\Omega} = \dfrac{V}{R} = \dfrac{240\text{ V}}{32\ \Omega} = 7.5\text{ A}; I_{20\Omega} = \dfrac{V}{R} = \dfrac{240\text{ V}}{20\ \Omega} = 12\text{ A}.$

EVALUATE: More current flows through the resistor that has the smaller R.

26.5. **IDENTIFY:** The equivalent resistance will vary for the different connections because the series-parallel combinations vary, and hence the current will vary.
SET UP: First calculate the equivalent resistance using the series-parallel formulas, then use Ohm's law ($V = RI$) to find the current.
EXECUTE: **(a)** $1/R = 1/(15.0\ \Omega) + 1/(30.0\ \Omega)$ gives $R = 10.0\ \Omega$. $I = V/R = (35.0\text{ V})/(10.0\ \Omega) = 3.50\text{ A}$.
(b) $1/R = 1/(10.0\ \Omega) + 1/(35.0\ \Omega)$ gives $R = 7.78\ \Omega$. $I = (35.0\text{ V})/(7.78\ \Omega) = 4.50\text{ A}$
(c) $1/R = 1/(20.0\ \Omega) + 1/(25.0\ \Omega)$ gives $R = 11.11\ \Omega$, so $I = (35.0\text{ V})/(11.11\ \Omega) = 3.15\text{ A}$.

(d) From part (b), the resistance of the triangle alone is 7.78 Ω. Adding the 3.00-Ω internal resistance of the battery gives an equivalent resistance for the circuit of 10.78 Ω. Therefore the current is I = (35.0 V)/(10.78 Ω) = 3.25 A

EVALUATE: It makes a big difference how the triangle is connected to the battery.

26.6. **IDENTIFY:** The potential drop is the same across the resistors in parallel, and the current into the parallel combination is the same as the current through the 45.0-Ω resistor.

(a) SET UP: Apply Ohm's law in the parallel branch to find the current through the 45.0-Ω resistor. Then apply Ohm's law to the 45.0-Ω resistor to find the potential drop across it.

EXECUTE: The potential drop across the 25.0-Ω resistor is V_{25} = (25.0 Ω)(1.25 A) = 31.25 V. The potential drop across each of the parallel branches is 31.25 V. For the 15.0-Ω resistor: I_{15} = (31.25 V)/(15.0 Ω) = 2.083 A. The resistance of the 10.0-Ω + 15.0 Ω combination is 25.0 Ω, so the current through it must be the same as the current through the upper 25.0 Ω resistor: I_{10+15} = 1.25 A. The sum of currents in the parallel branch will be the current through the 45.0-Ω resistor.

$$I_{\text{Total}} = 1.25\text{ A} + 2.083\text{ A} + 1.25\text{ A} = 4.58\text{ A}$$

Apply Ohm's law to the 45.0 Ω resistor: V_{45} = (4.58 A)(45.0 Ω) = 206 V

(b) SET UP: First find the equivalent resistance of the circuit and then apply Ohm's law to it.

EXECUTE: The resistance of the parallel branch is 1/R = 1/(25.0 Ω) + 1/(15.0 Ω) + 1/(25.0 Ω), so R = 6.82 Ω. The equivalent resistance of the circuit is 6.82 Ω + 45.0 Ω + 35.00 Ω = 86.82 Ω. Ohm's law gives V_{Bat} = (86.62 Ω)(4.58 A) = 398 V.

EVALUATE: The emf of the battery is the sum of the potential drops across each of the three segments (parallel branch and two series resistors).

26.7. **IDENTIFY:** First do as much series-parallel reduction as possible.

SET UP: The 45.0-Ω and 15.0-Ω resistors are in parallel, so first reduce them to a single equivalent resistance. Then find the equivalent series resistance of the circuit.

EXECUTE: 1/R_p = 1/(45.0 Ω) + 1/(15.0 Ω) and R_p = 11.25 Ω. The total equivalent resistance is 18.0 Ω + 11.25 Ω + 3.26 Ω = 32.5 Ω. Ohm's law gives I = (25.0 V)/(32.5 Ω) = 0.769 A.

EVALUATE: The circuit appears complicated until we realize that the 45.0-Ω and 15.0-Ω resistors are in parallel.

26.8. **IDENTIFY:** Eq.(26.2) gives the equivalent resistance of the three resistors in parallel. For resistors in parallel, the voltages are the same and the currents add.

(a) SET UP: The circuit is sketched in Figure 26.8a.

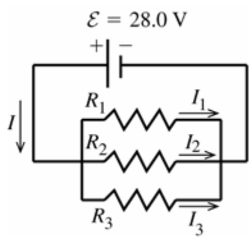


Figure 26.8a

EXECUTE: parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{1.60\ \Omega} + \frac{1}{2.40\ \Omega} + \frac{1}{4.80\ \Omega}$$

$$R_{eq} = 0.800\ \Omega$$

(b) For resistors in parallel the voltage is the same across each and equal to the applied voltage;

$$V_1 = V_2 = V_3 = \mathcal{E} = 28.0\text{ V}$$

$$V = IR \text{ so } I_1 = \frac{V_1}{R_1} = \frac{28.0\text{ V}}{1.60\ \Omega} = 17.5\text{ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{28.0\text{ V}}{2.40\ \Omega} = 11.7\text{ A and } I_3 = \frac{V_3}{R_3} = \frac{28.0\text{ V}}{4.8\ \Omega} = 5.8\text{ A}$$

(c) The currents through the resistors add to give the current through the battery:

$$I = I_1 + I_2 + I_3 = 17.5\text{ A} + 11.7\text{ A} + 5.8\text{ A} = 35.0\text{ A}$$

EVALUATE: Alternatively, we can use the equivalent resistance R_{eq} as shown in Figure 26.8b.

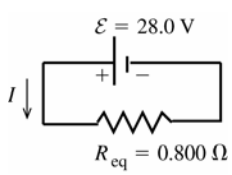


Figure 26.8b

$$\mathcal{E} - IR_{eq} = 0$$

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{28.0\text{ V}}{0.800\ \Omega} = 35.0\text{ A},$$

which checks

(d) As shown in part (b), the voltage across each resistor is 28.0 V.

(e) IDENTIFY and **SET UP:** We can use any of the three expressions for P: $P = VI = I^2R = V^2/R$. They will all give the same results, if we keep enough significant figures in intermediate calculations.

EXECUTE: Using $P = V^2/R$, $P_1 = V_1^2/R_1 = \dfrac{(28.0\ \text{V})^2}{1.60\ \Omega} = 490\ \text{W}$, $P_2 = V_2^2/R_2 = \dfrac{(28.0\ \text{V})^2}{2.40\ \Omega} = 327\ \text{W}$, and

$P_3 = V_3^2/R_3 = \dfrac{(28.0\ \text{V})^2}{4.80\ \Omega} = 163\ \text{W}$

EVALUATE: The total power dissipated is $P_{\text{out}} = P_1 + P_2 + P_3 = 980\ \text{W}$. This is the same as the power $P_{\text{in}} = \mathcal{E}I = (2.80\ \text{V})(35.0\ \text{A}) = 980\ \text{W}$ delivered by the battery.

(f) $P = V^2/R$. The resistors in parallel each have the same voltage, so the power P is largest for the one with the least resistance.

26.9. **IDENTIFY:** For a series network, the current is the same in each resistor and the sum of voltages for each resistor equals the battery voltage. The equivalent resistance is $R_{\text{eq}} = R_1 + R_2 + R_3$. $P = I^2R$.

SET UP: Let $R_1 = 1.60\ \Omega$, $R_2 = 2.40\ \Omega$, $R_3 = 4.80\ \Omega$.

EXECUTE: **(a)** $R_{\text{eq}} = 1.60\ \Omega + 2.40\ \Omega + 4.80\ \Omega = 8.80\ \Omega$

(b) $I = \dfrac{V}{R_{\text{eq}}} = \dfrac{28.0\ \text{V}}{8.80\ \Omega} = 3.18\ \text{A}$

(c) $I = 3.18\ \text{A}$, the same as for each resistor.

(d) $V_1 = IR_1 = (3.18\ \text{A})(1.60\ \Omega) = 5.09\ \text{V}$. $V_2 = IR_2 = (3.18\ \text{A})(2.40\ \Omega) = 7.63\ \text{V}$.

$V_3 = IR_3 = (3.18\ \text{A})(4.80\ \Omega) = 15.3\ \text{V}$. Note that $V_1 + V_2 + V_3 = 28.0\ \text{V}$.

(e) $P_1 = I^2R_1 = (3.18\ \text{A})^2(1.60\ \Omega) = 16.2\ \text{W}$. $P_2 = I^2R_2 = (3.18\ \text{A})^2(2.40\ \Omega) = 24.3\ \text{W}$.

$P_3 = I^2R_3 = (3.18\ \text{A})^2(4.80\ \Omega) = 48.5\ \text{W}$.

(f) Since $P = I^2R$ and the current is the same for each resistor, the resistor with the greatest R dissipates the greatest power.

EVALUATE: When resistors are connected in parallel, the resistor with the smallest R dissipates the greatest power.

26.10. **(a) IDENTIFY:** The current, and hence the power, depends on the potential difference across the resistor.

SET UP: $P = V^2/R$

EXECUTE: **(a)** $V = \sqrt{PR} = \sqrt{(5.0\ \text{W})(15{,}000\ \Omega)} = 274\ \text{V}$

(b) $P = V^2/R = (120\ \text{V})^2/(9{,}000\ \Omega) = 1.6\ \text{W}$

SET UP: **(c)** If the larger resistor generates 2.00 W, the smaller one will generate less and hence will be safe. Therefore the maximum power in the larger resistor must be 2.00 W. Use $P = I^2R$ to find the maximum current through the series combination and use Ohm's law to find the potential difference across the combination.

EXECUTE: $P = I^2R$ gives $I = P/R = (2.00\ \text{W})/(150\ \Omega) = 0.0133\ \text{A}$. The same current flows through both resistors, and their equivalent resistance is 250 Ω. Ohm's law gives $V = IR = (0.0133\ \text{A})(250\ \Omega) = 3.33\ \text{V}$. Therefore $P_{150} = 2.00\ \text{W}$ and $P_{100} = I^2R = (0.0133\ \text{A})^2(100\ \Omega) = 0.0177\ \text{W}$.

EVALUATE: If the resistors in a series combination all have the same power rating, it is the *largest* resistance that limits the amount of current.

26.11. **IDENTIFY:** For resistors in parallel, the voltages are the same and the currents add. $\dfrac{1}{R_{\text{eq}}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$ so $R_{\text{eq}} = \dfrac{R_1R_2}{R_1 + R_2}$,

For resistors in series, the currents are the same and the voltages add. $R_{\text{eq}} = R_1 + R_2$.

SET UP: The rules for combining resistors in series and parallel lead to the sequences of equivalent circuits shown in Figure 26.11.

EXECUTE: $R_{\text{eq}} = 5.00\ \Omega$. In Figure 26.11c, $I = \dfrac{60.0\ \text{V}}{5.00\ \Omega} = 12.0\ \text{A}$. This is the current through each of the resistors in Figure 26.11b. $V_{12} = IR_{12} = (12.0\ \text{A})(2.00\ \Omega) = 24.0\ \text{V}$. $V_{34} = IR_{34} = (12.0\ \text{A})(3.00\ \Omega) = 36.0\ \text{V}$. Note that $V_{12} + V_{34} = 60.0\ \text{V}$. V_{12} is the voltage across R_1 and across R_2, so $I_1 = \dfrac{V_{12}}{R_1} = \dfrac{24.0\ \text{V}}{3.00\ \Omega} = 8.00\ \text{A}$ and

$I_2 = \dfrac{V_{12}}{R_2} = \dfrac{24.0\ \text{V}}{6.00\ \Omega} = 4.00\ \text{A}$. V_{34} is the voltage across R_3 and across R_4, so $I_3 = \dfrac{V_{34}}{R_3} = \dfrac{36.0\ \text{V}}{12.0\ \Omega} = 3.00\ \text{A}$ and

$I_4 = \dfrac{V_{34}}{R_4} = \dfrac{36.0\ \text{V}}{4.00\ \Omega} = 9.00\ \text{A}$.

EVALUATE: Note that $I_1 + I_2 = I_3 + I_4$.

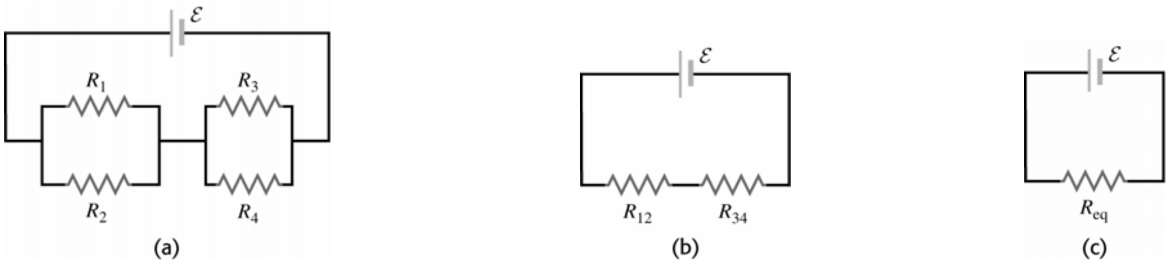


Figure 26.11

26.12. **IDENTIFY:** Replace the series combinations of resistors by their equivalents. In the resulting parallel network the battery voltage is the voltage across each resistor.

SET UP: The circuit is sketched in Figure 26.12a.

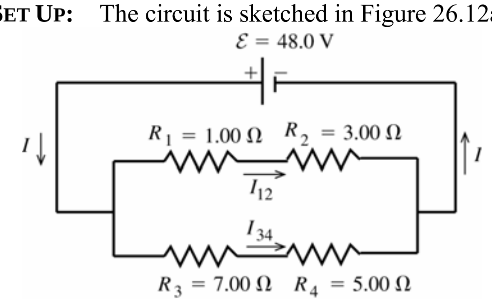


Figure 26.12a

EXECUTE: R_1 and R_2 in series have an equivalent resistance of $R_{12} = R_1 + R_2 = 4.00\ \Omega$

R_3 and R_4 in series have an equivalent resistance of $R_{34} = R_3 + R_4 = 12.0\ \Omega$

The circuit is equivalent to the circuit sketched in Figure 26.12b.

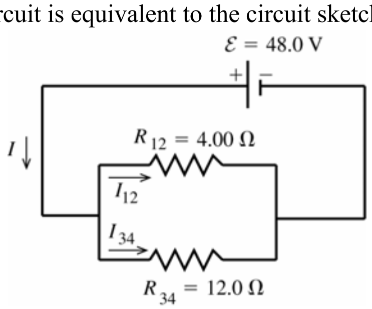


Figure 26.12b

R_{12} and R_{34} in parallel are equivalent to R_{eq} given by $\dfrac{1}{R_{eq}} = \dfrac{1}{R_{12}} + \dfrac{1}{R_{34}} = \dfrac{R_{12} + R_{34}}{R_{12}R_{34}}$

$$R_{eq} = \frac{R_{12}R_{34}}{R_{12} + R_{34}}$$

$$R_{eq} = \frac{(4.00\ \Omega)(12.0\ \Omega)}{4.00\ \Omega + 12.0\ \Omega} = 3.00\ \Omega$$

The voltage across each branch of the parallel combination is $\mathcal{E}$, so $\mathcal{E} - I_{12}R_{12} = 0$.

$$I_{12} = \frac{\mathcal{E}}{R_{12}} = \frac{48.0\ \text{V}}{4.00\ \Omega} = 12.0\ \text{A}$$

$$\mathcal{E} - I_{34}R_{34} = 0 \text{ so } I_{34} = \frac{\mathcal{E}}{R_{34}} = \frac{48.0\ \text{V}}{12.0\ \Omega} = 4.0\ \text{A}$$

The current is 12.0 A through the 1.00 Ω and 3.00 Ω resistors, and it is 4.0 A through the 7.00 Ω and 5.00 Ω resistors.

EVALUATE: The current through the battery is $I = I_{12} + I_{34} = 12.0\ \text{A} + 4.0\ \text{A} = 16.0\ \text{A}$, and this is equal to $\mathcal{E}/R_{eq} - 48.0\ \text{V}/3.00\ \Omega = 16.0\ \text{A}$.

26.13. **IDENTIFY:** In both circuits, with and without R_4, replace series and parallel combinations of resistors by their equivalents. Calculate the currents and voltages in the equivalent circuit and infer from this the currents and voltages in the original circuit. Use $P = I^2R$ to calculate the power dissipated in each bulb.

(a) SET UP: The circuit is sketched in Figure 26.13a.

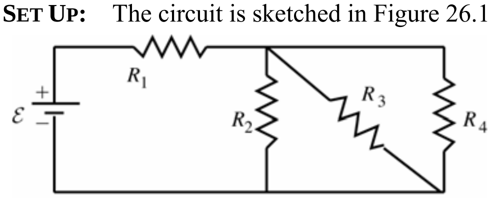


Figure 26.13a

EXECUTE: R_2, R_3, and R_4 are in parallel, so their equivalent resistance R_{eq} is given by $\dfrac{1}{R_{eq}} = \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4}$

$$\frac{1}{R_{eq}} = \frac{3}{4.50\ \Omega} \text{ and } R_{eq} = 1.50\ \Omega.$$

The equivalent circuit is drawn in Figure 26.13b.

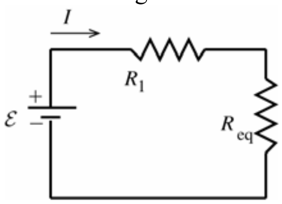


Figure 26.13b

$$\mathcal{E} - I\left(R_1 + R_{eq}\right) = 0$$

$$I = \frac{\mathcal{E}}{R_1 + R_{eq}}$$

$I = \dfrac{9.00\text{ V}}{4.50\ \Omega + 1.50\ \Omega} = 1.50\text{ A}$ and $I_1 = 1.50\text{ A}$

Then $V_1 = I_1 R_1 = (1.50\text{ A})(4.50\ \Omega) = 6.75\text{ V}$

$I_{eq} = 1.50\text{ A}$, $V_{eq} = I_{eq} R_{eq} = (1.50\text{ A})(1.50\ \Omega) = 2.25\text{ V}$

For resistors in parallel the voltages are equal and are the same as the voltage across the equivalent resistor, so $V_2 = V_3 = V_4 = 2.25\text{ V}$.

$I_2 = \dfrac{V_2}{R_2} = \dfrac{2.25\text{ V}}{4.50\ \Omega} = 0.500\text{ A}$, $I_3 = \dfrac{V_3}{R_3} = 0.500\text{ A}$, $I_4 = \dfrac{V_4}{R_4} = 0.500\text{ A}$

EVALUATE: Note that $I_2 + I_3 + I_4 = 1.50\text{ A}$, which is I_{eq}. For resistors in parallel the currents add and their sum is the current through the equivalent resistor.

(b) SET UP: $P = I^2 R$

EXECUTE: $P_1 = (1.50\text{ A})^2 (4.50\ \Omega) = 10.1\text{ W}$

$P_2 = P_3 = P_4 = (0.500\text{ A})^2 (4.50\ \Omega) = 1.125\text{ W}$, which rounds to 1.12 W. R_1 glows brightest.

EVALUATE: Note that $P_2 + P_3 + P_4 = 3.37\text{ W}$. This equals $P_{eq} = I_{eq}^2 R_{eq} = (1.50\text{ A})^2 (1.50\ \Omega) = 3.37\text{ W}$, the power dissipated in the equivalent resistor.

(c) SET UP: With R_4 removed the circuit becomes the circuit in Figure 26.13c.

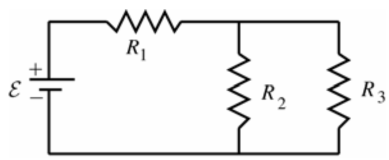


Figure 26.13c

EXECUTE: R_2 and R_3 are in parallel and their equivalent resistance R_{eq} is given by

$\dfrac{1}{R_{eq}} = \dfrac{1}{R_2} + \dfrac{1}{R_3} = \dfrac{2}{4.50\ \Omega}$ and $R_{eq} = 2.25\ \Omega$

The equivalent circuit is shown in Figure 26.13d.

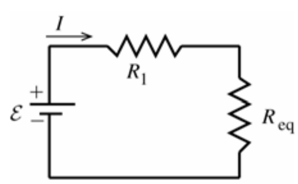


Figure 26.13d

$$\mathcal{E} - I\left(R_1 + R_{eq}\right) = 0$$

$$I = \frac{\mathcal{E}}{R_1 + R_{eq}}$$

$$I = \frac{9.00\text{ V}}{4.50\ \Omega + 2.25\ \Omega} = 1.333\text{ A}$$

$I_1 = 1.33\text{ A}$, $V_1 = I_1 R_1 = (1.333\text{ A})(4.50\ \Omega) = 6.00\text{ V}$

$I_{eq} = 1.33\text{ A}$, $V_{eq} = I_{eq} R_{eq} = (1.333\text{ A})(2.25\ \Omega) = 3.00\text{ V}$ and $V_2 = V_3 = 3.00\text{ V}$.

$I_2 = \dfrac{V_2}{R_2} = \dfrac{3.00\text{ V}}{4.50\ \Omega} = 0.667\text{ A}$, $I_3 = \dfrac{V_3}{R_3} = 0.667\text{ A}$

(d) SET UP: $P = I^2 R$

EXECUTE: $P_1 = (1.333\text{ A})^2 (4.50\ \Omega) = 8.00\text{ W}$

$P_2 = P_3 = (0.667\text{ A})^2 (4.50\ \Omega) = 2.00\text{ W}$.

(e) EVALUATE: When R_4 is removed, P_1 decreases and P_2 and P_3 increase. Bulb R_1 glows less brightly and bulbs R_2 and R_3 glow more brightly. When R_4 is removed the equivalent resistance of the circuit increases and the current through R_1 decreases. But in the parallel combination this current divides into two equal currents rather than three, so the currents through R_2 and R_3 increase. Can also see this by noting that with R_4 removed and less current through R_1 the voltage drop across R_1 is less so the voltage drop across R_2 and across R_3 must become larger.

26.14. **IDENTIFY:** Apply Ohm's law to each resistor.

SET UP: For resistors in parallel the voltages are the same and the currents add. For resistors in series the currents are the same and the voltages add.

EXECUTE: From Ohm's law, the voltage drop across the 6.00 Ω resistor is $V = IR = (4.00\text{ A})(6.00\ \Omega) = 24.0\text{ V}$. The voltage drop across the 8.00 Ω resistor is the same, since these two resistors are wired in parallel. The current through the 8.00 Ω resistor is then $I = V/R = 24.0\text{ V}/8.00\ \Omega = 3.00\text{ A}$. The current through the 25.0 Ω resistor is the sum of these two currents: 7.00 A. The voltage drop across the 25.0 Ω resistor is $V = IR = (7.00\text{ A})(25.0\ \Omega) = 175\text{ V}$, and total voltage drop across the top branch of the circuit is 175 V + 24.0 V = 199 V, which is also the voltage drop across the 20.0 Ω resistor. The current through the 20.0 Ω resistor is then $I = V/R = 199\text{ V}/20\ \Omega = 9.95\text{ A}$.

EVALUATE: The total current through the battery is $7.00\text{ A} + 9.95\text{ A} = 16.95\text{ A}$. Note that we did not need to calculate the emf of the battery.

26.15. **IDENTIFY:** Apply Ohm's law to each resistor.

SET UP: For resistors in parallel the voltages are the same and the currents add. For resistors in series the currents are the same and the voltages add.

EXECUTE: The current through 2.00-Ω resistor is 6.00 A. Current through 1.00-Ω resistor also is 6.00 A and the voltage is 6.00 V. Voltage across the 6.00-Ω resistor is 12.0 V + 6.0 V = 18.0 V. Current through the 6.00-Ω resistor is (18.0 V)/(6.00 Ω) = 3.00 A. The battery emf is 18.0 V.

EVALUATE: The current through the battery is 6.00 A + 3.00 A = 9.00 A. The equivalent resistor of the resistor network is 2.00 Ω, and this equals (18.0 V)/(9.00 A).

26.16. **IDENTIFY:** The filaments must be connected such that the current can flow through each separately, and also through both in parallel, yielding three possible current flows. The parallel situation always has less resistance than any of the individual members, so it will give the highest power output of 180 W, while the other two must give power outputs of 60 W and 120 W.

SET UP: $P = V^2/R$, where R is the equivalent resistance.

EXECUTE: **(a)** $60\text{ W} = \dfrac{V^2}{R_1}$ gives $R_1 = \dfrac{(120\text{ V})^2}{60\text{ W}} = 240\ \Omega$. $120\text{ W} = \dfrac{V^2}{R_2}$ gives $R_2 = \dfrac{(120\text{ V})^2}{120\text{ W}} = 120\ \Omega$. For these two resistors in parallel, $R_{eq} = \dfrac{R_1R_2}{R_1 + R_2} = 80\ \Omega$ and $P = \dfrac{V^2}{R_{eq}} = \dfrac{(120\text{ V})^2}{80\ \Omega} = 180\text{ W}$, which is the desired value.

(b) If R_1 burns out, the 120 W setting stays the same, the 60 W setting does not work and the 180 W setting goes to 120 W: brightnesses of zero, medium and medium.

(c) If R_2 burns out, the 60 W setting stays the same, the 120 W setting does not work, and the 180 W setting is now 60 W: brightnesses of low, zero and low.

EVALUATE: Since in each case 120 V is supplied to each filament network, the lowest resistance dissipates the greatest power.

26.17. **IDENTIFY:** For resistors in series, the voltages add and the current is the same. For resistors in parallel, the voltages are the same and the currents add. $P = I^2R$.

(a) SET UP: The circuit is sketched in Figure 26.17a.

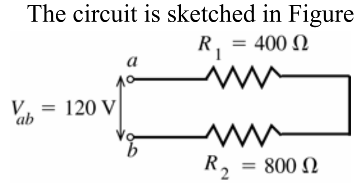


Figure 26.17a

For resistors in series the current is the same through each.

EXECUTE: $R_{eq} = R_1 + R_2 = 1200\ \Omega$. $I = \dfrac{V}{R_{eq}} = \dfrac{120\text{ V}}{1200\ \Omega} = 0.100\text{ A}$. This is the current drawn from the line.

(b) $P_1 = I_1^2R_1 = (0.100\text{ A})^2(400\ \Omega) = 4.0\text{ W}$

$P_2 = I_2^2R_2 = (0.100\text{ A})^2(800\ \Omega) = 8.0\text{ W}$

(c) $P_{out} = P_1 + P_2 = 12.0\text{ W}$, the total power dissipated in both bulbs. Note that $P_{in} = V_{ab}I = (120\text{ V})(0.100\text{ A}) = 12.0\text{ W}$, the power delivered by the potential source, equals $P_{out.}$

(d) SET UP: The circuit is sketched in Figure 26.17b.

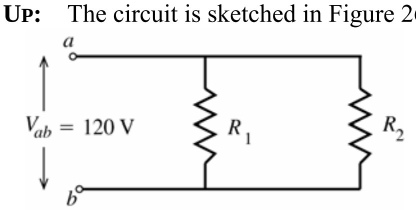


Figure 26.17b

For resistors in parallel the voltage across each resistor is the same.

EXECUTE: $I_1 = \dfrac{V_1}{R_1} = \dfrac{120\text{ V}}{400\ \Omega} = 0.300\text{ A},\ I_2 = \dfrac{V_2}{R_2} = \dfrac{120\text{ V}}{800\ \Omega} = 0.150\text{ A}$

EVALUATE: Note that each current is larger than the current when the resistors are connected in series.

(e) EXECUTE: $P_1 = I_1^2 R_1 = (0.300\text{ A})^2 (400\ \Omega) = 36.0\text{ W}$

$P_2 = I_2^2 R_2 = (0.150\text{ A})^2 (800\ \Omega) = 18.0\text{ W}$

(f) $P_{\text{out}} = P_1 + P_2 = 54.0\text{ W}$

EVALUATE: Note that the total current drawn from the line is $I = I_1 + I_2 = 0.450\text{ A}$. The power input from the line is $P_{\text{in}} = V_{ab} I = (120\text{ V})(0.450\text{ A}) = 54.0\text{ W}$, which equals the total power dissipated by the bulbs.

(g) The bulb that is dissipating the most power glows most brightly. For the series connection the currents are the same and by $P = I^2 R$ the bulb with the larger *R* has the larger *P*; the $800\ \Omega$ bulb glows more brightly. For the parallel combination the voltages are the same and by $P = V^2/R$ the bulb with the smaller *R* has the larger *P*; the $400\ \Omega$ bulb glows more brightly.

(h) The total power output P_{out} equals $P_{\text{in}} = V_{ab} I$, so P_{out} is larger for the parallel connection where the current drawn from the line is larger (because the equivalent resistance is smaller.)

26.18. **IDENTIFY:** Use $P = V^2/R$ with $V = 120\text{ V}$ and the wattage for each bulb to calculate the resistance of each bulb. When connected in series the voltage across each bulb will not be 120 V and the power for each bulb will be different.

SET UP: For resistors in series the currents are the same and $R_{\text{eq}} = R_1 + R_2$.

EXECUTE: **(a)** $R_{60\text{W}} = \dfrac{V^2}{P} = \dfrac{(120\text{ V})^2}{60\text{ W}} = 240\ \Omega$; $R_{200\text{W}} = \dfrac{V^2}{P} = \dfrac{(120\text{ V})^2}{200\text{ W}} = 72\ \Omega.$

Therefore, $I_{60\text{W}} = I_{200\text{W}} = \dfrac{\mathcal{E}}{R} = \dfrac{240\text{ V}}{(240\ \Omega + 72\ \Omega)} = 0.769\text{ A}.$

(b) $P_{60\text{W}} = I^2 R = (0.769\text{ A})^2 (240\ \Omega) = 142\text{ W}$; $P_{200\text{W}} = I^2 R = (0.769\text{ A})^2 (72\ \Omega) = 42.6\text{ W}.$

(c) The 60 W bulb burns out quickly because the power it delivers (142 W) is 2.4 times its rated value.

EVALUATE: In series the largest resistance dissipates the greatest power.

26.19. **IDENTIFY** and **SET UP:** Replace series and parallel combinations of resistors by their equivalents until the circuit is reduced to a single loop. Use the loop equation to find the current through the $20.0\ \Omega$ resistor. Set $P = I^2 R$ for the $20.0\ \Omega$ resistor equal to the rate Q/t at which heat goes into the water and set $Q = mc\Delta T$.

EXECUTE: Replace the network by the equivalent resistor, as shown in Figure 26.19.

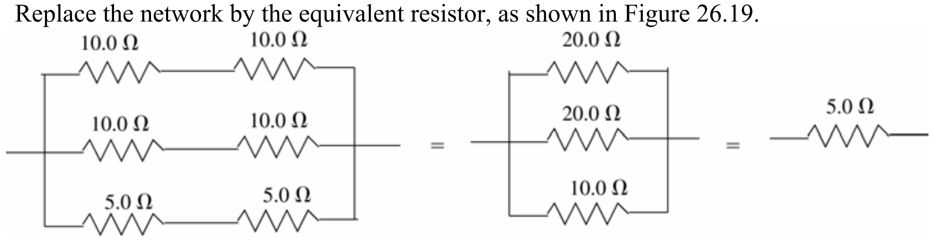


Figure 26.19

$30.0\text{ V} - I(20.0\ \Omega + 5.0\ \Omega + 5.0\ \Omega) = 0;\ I = 1.00\text{ A}$

For the $20.0\text{-}\Omega$ resistor thermal energy is generated at the rate $P = I^2 R = 20.0\text{ W}$. $Q = Pt$ and $Q = mc\Delta T$ gives

$t = \dfrac{mc\Delta T}{P} = \dfrac{(0.100\text{ kg})(4190\text{ J/kg}\cdot\text{ K})(48.0\text{ C}°)}{20.0\text{ W}} = 1.01 \times 10^3\text{ s}$

EVALUATE: The battery is supplying heat at the rate $P = \mathcal{E}I = 30.0\text{ W}$. In the series circuit, more energy is dissipated in the larger resistor $(20.0\ \Omega)$ than in the smaller ones $(5.00\ \Omega)$.

26.20. **IDENTIFY:** $P = I^2 R$ determines R_1. R_1, R_2 and the $10.0\ \Omega$ resistor are all in parallel so have the same voltage. Apply the junction rule to find the current through R_2.

SET UP: $P = I^2 R$ for a resistor and $P = \mathcal{E}I$ for an emf. The emf inputs electrical energy into the circuit and electrical energy is removed in the resistors.

EXECUTE: **(a)** $P_1 = I_1^2 R_1$. $20\text{ W} = (2\text{ A})^2 R_1$ and $R_1 = 5.00\ \Omega$. R_1 and $10\ \Omega$ are in parallel, so $(10\ \Omega) I_{10} = (5\ \Omega)(2\text{ A})$ and $I_{10} = 1\text{ A}$. So $I_2 = 3.50\text{ A} - I_1 - I_{10} = 0.50\text{ A}$. R_1 and R_2 are in parallel, so $(0.50\text{ A}) R_2 = (2\text{ A})(5\ \Omega)$ and $R_2 = 20.0\ \Omega$.

(b) $\mathcal{E} = V_1 = (2.00\text{ A})(5.00\ \Omega) = 10.0\text{ V}$

(c) From part (a), $I_2 = 0.500\text{ A},\ I_{10} = 1.00\text{ A}$

(d) $P_1 = 20.0\text{ W}$ (given). $P_2 = I_2^2 R_2 = (0.50\text{ A})^2(20\ \Omega) = 5.00\text{ W}$. $P_{10} = I_{10}^2 R_{10} = (1.0\text{ A})^2(10\ \Omega) = 10.0\text{ W}$. The total rate at which the resistors remove electrical energy is $P_{\text{Resist}} = 20\text{ W} + 5\text{ W} + 10\text{ W} = 35.0\text{ W}$. The total rate at which the battery inputs electrical energy is $P_{\text{Battery}} = I\mathcal{E} = (3.50\text{ A})(10.0\text{ V}) = 35.0\text{ W}$. $P_{\text{Resist}} = P_{\text{Battery}}$, which agrees with conservation of energy.

EVALUATE: The three resistors are in parallel, so the voltage for each is the battery voltage, 10.0 V. The currents in the three resistors add to give the current in the battery.

26.21. **IDENTIFY:** Apply Kirchhoff's point rule at point *a* to find the current through *R*. Apply Kirchhoff's loop rule to loops (1) and (2) shown in Figure 26.21a to calculate *R* and $\mathcal{E}$. Travel around each loop in the direction shown.

(a) SET UP:

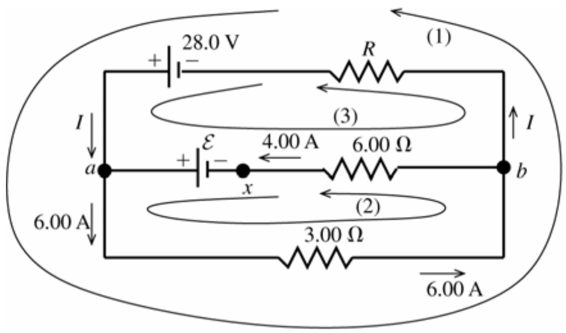


Figure 26.21a

EXECUTE: Apply Kirchhoff's point rule to point *a*: $\sum I = 0$ so $I + 4.00\text{ A} - 6.00\text{ A} = 0$

$I = 2.00$ A (in the direction shown in the diagram).

(b) Apply Kirchhoff's loop rule to loop (1): $-(6.00\text{ A})(3.00\ \Omega) - (2.00\text{ A})R + 28.0\text{ V} = 0$

$-18.0\text{ V} - (2.00\ \Omega)R + 28.0\text{ V} = 0$

$$R = \frac{28.0\text{ V} - 18.0\text{ V}}{2.00\text{ A}} = 5.00\ \Omega$$

(c) Apply Kirchhoff's loop rule to loop (2): $-(6.00\text{ A})(3.00\ \Omega) - (4.00\text{ A})(6.00\ \Omega) + \mathcal{E} = 0$

$\mathcal{E} = 18.0\text{ V} + 24.0\text{ V} = 42.0\text{ V}$

EVALUATE: Can check that the loop rule is satisfied for loop (3), as a check of our work:

$28.0\text{ V} - \mathcal{E} + (4.00\text{ A})(6.00\ \Omega) - (2.00\text{ A})R = 0$

$28.0\text{ V} - 42.0\text{ V} + 24.0\text{ V} - (2.00\text{ A})(5.00\ \Omega) = 0$

$52.0\text{ V} = 42.0\text{ V} + 10.0\text{ V}$

$52.0\text{ V} = 52.0\text{ V}$, so the loop rule is satisfied for this loop.

(d) IDENTIFY: If the circuit is broken at point *x* there can be no current in the $6.00\ \Omega$ resistor. There is now only a single current path and we can apply the loop rule to this path.

SET UP: The circuit is sketched in Figure 26.21b.

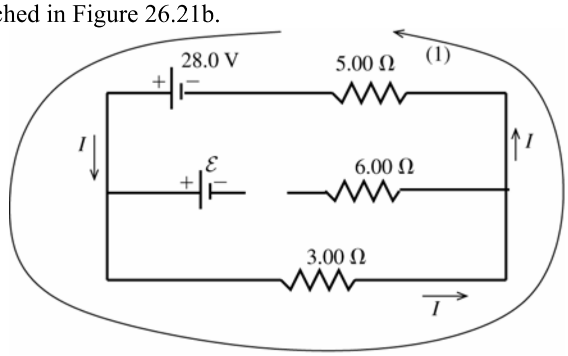


Figure 26.21b

EXECUTE: $+28.0\text{ V} - (3.00\ \Omega)I - (5.00\ \Omega)I = 0$

$$I = \frac{28.0\text{ V}}{8.00\ \Omega} = 3.50\text{ A}$$

EVALUATE: Breaking the circuit at *x* removes the 42.0 V emf from the circuit and the current through the $3.00\ \Omega$ resistor is reduced.

26.22. **IDENTIFY:** Apply the loop rule and junction rule.

SET UP: The circuit diagram is given in Figure 26.22. The junction rule has been used to find the magnitude and direction of the current in the middle branch of the circuit. There are no remaining unknown currents.

EXECUTE: The loop rule applied to loop (1) gives:

$+20.0\text{ V}-(1.00\text{ A})(1.00\ \Omega)+(1.00\text{ A})(4.00\ \Omega)+(1.00\text{ A})(1.00\ \Omega)-\mathcal{E}_1-(1.00\text{ A})(6.00\ \Omega)=0$

$\mathcal{E}_1=20.0\text{ V}-1.00\text{ V}+4.00\text{ V}+1.00\text{ V}-6.00\text{ V}=18.0\text{ V}$. The loop rule applied to loop (2) gives:

$+20.0\text{ V}-(1.00\text{ A})(1.00\ \Omega)-(2.00\text{ A})(1.00\ \Omega)-\mathcal{E}_2-(2.00\text{ A})(2.00\ \Omega)-(1.00\text{ A})(6.00\ \Omega)=0$

$\mathcal{E}_2=20.0\text{ V}-1.00\text{ V}-2.00\text{ V}-4.00\text{ V}-6.00\text{ V}=7.0\text{ V}$. Going from b to a along the lower branch,

$V_b+(2.00\text{ A})(2.00\ \Omega)+7.0\text{ V}+(2.00\text{ A})(1.00\ \Omega)=V_a$. $V_b-V_a=-13.0\text{ V}$; point b is at 13.0 V lower potential than point a.

EVALUATE: We can also calculate V_b-V_a by going from b to a along the upper branch of the circuit.

$V_b-(1.00\text{ A})(6.00\ \Omega)+20.0\text{ V}-(1.00\text{ A})(1.00\ \Omega)=V_a$ and $V_b-V_a=-13.0\text{ V}$. This agrees with V_b-V_a calculated along a different path between b and a.

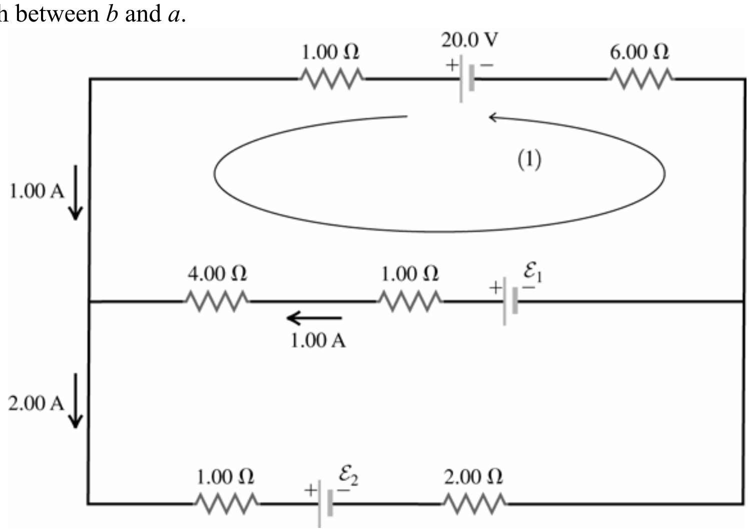


Figure 26.22

26.23. **IDENTIFY:** Apply the junction rule at points *a, b, c* and *d* to calculate the unknown currents. Then apply the loop rule to three loops to calculate $\mathcal{E}_1$, $\mathcal{E}_2$ and R.

(a) SET UP: The circuit is sketched in Figure 26.23.

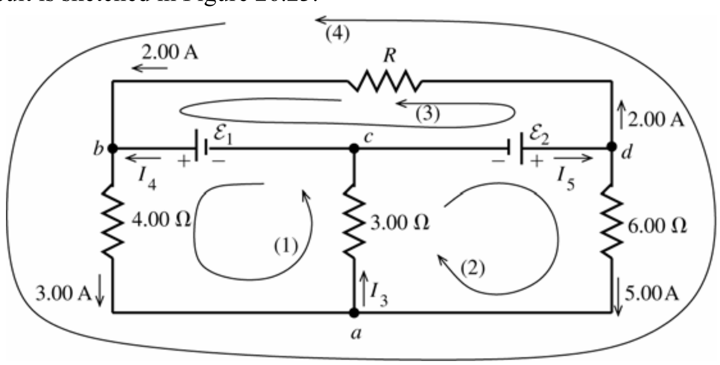


Figure 26.23

EXECUTE: Apply the junction rule to point a: $3.00\text{ A}+5.00\text{ A}-I_3=0$

$I_3=8.00\text{ A}$

Apply the junction rule to point b: $2.00\text{ A}+I_4-3.00\text{ A}=0$

$I_4=1.00\text{ A}$

Apply the junction rule to point c: $I_3-I_4-I_5=0$

$I_5=I_3-I_4=8.00\text{ A}-1.00\text{ A}=7.00\text{ A}$

EVALUATE: As a check, apply the junction rule to point d: $I_5-2.00\text{ A}-5.00\text{ A}=0$

$I_5=7.00\text{ A}$

(b) EXECUTE: Apply the loop rule to loop (1): $\mathcal{E}_1 - (3.00\text{ A})(4.00\ \Omega) - I_3(3.00\ \Omega) = 0$

$\mathcal{E}_1 = 12.0\text{ V} + (8.00\text{ A})(3.00\ \Omega) = 36.0\text{ V}$

Apply the loop rule to loop (2): $\mathcal{E}_2 - (5.00\text{ A})(6.00\ \Omega) - I_3(3.00\ \Omega) = 0$

$\mathcal{E}_2 = 30.0\text{ V} + (8.00\text{ A})(3.00\ \Omega) = 54.0\text{ V}$

(c) Apply the loop rule to loop (3): $-(2.00\text{ A})R - \mathcal{E}_1 + \mathcal{E}_2 = 0$

$$R = \frac{\mathcal{E}_2 - \mathcal{E}_1}{2.00\text{ A}} = \frac{54.0\text{ V} - 36.0\text{ V}}{2.00\text{ A}} = 9.00\ \Omega$$

EVALUATE: Apply the loop rule to loop (4) as a check of our calculations:

$-(2.00\text{ A})R - (3.00\text{ A})(4.00\ \Omega) + (5.00\text{ A})(6.00\ \Omega) = 0$

$-(2.00\text{ A})(9.00\ \Omega) - 12.0\text{ V} + 30.0\text{ V} = 0$

$-18.0\text{ V} + 18.0\text{ V} = 0$

26.24. IDENTIFY: Use Kirchhoff's Rules to find the currents.

SET UP: Since the 1.0 V battery has the larger voltage, assume I_1 is to the left through the 10 V battery, I_2 is to the right through the 5 V battery, and I_3 is to the right through the $10\ \Omega$ resistor. Go around each loop in the counterclockwise direction.

EXECUTE: Upper loop: $10.0\text{ V} - (2.00\ \Omega + 3.00\ \Omega)I_1 - (1.00\ \Omega + 4.00\ \Omega)I_2 - 5.00\text{ V} = 0$. This gives $5.0\text{ V} - (5.00\ \Omega)I_1 - (5.00\ \Omega)I_2 = 0$, and $\Rightarrow I_1 + I_2 = 1.00\text{ A}$.

Lower loop: $5.00\text{ V} + (1.00\ \Omega + 4.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0$. This gives $5.00\text{ V} + (5.00\ \Omega)I_2 - (10.0\ \Omega)I_3 = 0$, and $I_2 - 2I_3 = -1.00\text{ A}$

Along with $I_1 = I_2 + I_3$, we can solve for the three currents and find:

$$I_1 = 0.800\text{ A}, I_2 = 0.200\text{ A}, I_3 = 0.600\text{ A}.$$

(b) $V_{ab} = -(0.200\text{ A})(4.00\ \Omega) - (0.800\text{ A})(3.00\ \Omega) = -3.20\text{ V}.$

EVALUATE: Traveling from *b* to *a* through the $4.00\ \Omega$ and $3.00\ \Omega$ resistors you pass through the resistors in the direction of the current and the potential decreases; point *b* is at higher potential than point *a*.

26.25. IDENTIFY: Apply the junction rule to reduce the number of unknown currents. Apply the loop rule to two loops to obtain two equations for the unknown currents I_1 and I_2

(a) SET UP: The circuit is sketched in Figure 26.25.

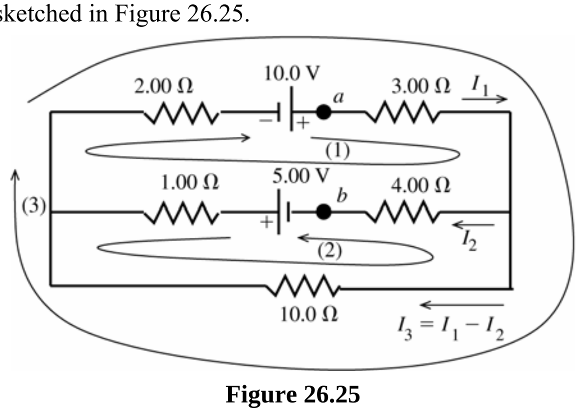


Figure 26.25

Let I_1 be the current in the $3.00\ \Omega$ resistor and I_2 be the current in the $4.00\ \Omega$ resistor and assume that these currents are in the directions shown. Then the current in the $10.0\ \Omega$ resistor is $I_3 = I_1 - I_2$, in the direction shown, where we have used Kirchhoff's point rule to relate I_3 to I_1 and I_2. If we get a negative answer for any of these currents we know the current is actually in the opposite direction to what we have assumed. Three loops and directions to travel around the loops are shown in the circiut diagram. Apply Kirchhoff's loop rule to each loop.

EXECUTE: <u>loop (1)</u>

$+10.0\text{ V} - I_1(3.00\ \Omega) - I_2(4.00\ \Omega) + 5.00\text{ V} - I_2(1.00\ \Omega) - I_1(2.00\ \Omega) = 0$

$15.00\text{ V} - (5.00\ \Omega)I_1 - (5.00\ \Omega)I_2 = 0$

$3.00\text{ A} - I_1 - I_2 = 0$

loop (2)

$+5.00\text{ V}-I_2(1.00\ \Omega)+(I_1-I_2)10.0\ \Omega-I_2(4.00\ \Omega)=0$

$5.00\text{ V}+(10.0\ \Omega)I_1-(15.0\ \Omega)I_2=0$

$1.00\text{ A}+2.00I_1-3.00I_2=0$

The first equation says $I_2=3.00\text{ A}-I_1$.

Use this in the second equation: $1.00\text{ A}+2.00I_1-9.00\text{ A}+3.00I_1=0$

$5.00I_1=8.00\text{ A},\ I_1=1.60\text{ A}$

Then $I_2=3.00\text{ A}-I_1=3.00\text{ A}-1.60\text{ A}=1.40\text{ A}$.

$I_3=I_1-I_2=1.60\text{ A}-1.40\text{ A}=0.20\text{ A}$

EVALUATE: Loop (3) can be used as a check.

$+10.0\text{ V}-(1.60\text{ A})(3.00\ \Omega)-(0.20\text{ A})(10.00\ \Omega)-(1.60\text{ A})(2.00\ \Omega)=0$

$10.0\text{ V}=4.8\text{ V}+2.0\text{ V}+3.2\text{ V}$

$10.0\text{ V}=10.0\text{ V}$

We find that with our calculated currents the loop rule is satisfied for loop (3). Also, all the currents came out to be positive, so the current directions in the circuit diagram are correct.

(b) IDENTIFY and **SET UP:** To find $V_{ab}=V_a-V_b$ start at point *b* and travel to point *a*. Many different routes can be taken from *b* to *a* and all must yield the same result for V_{ab}.

EXECUTE: Travel through the $4.00\ \Omega$ resistor and then through the $3.00\ \Omega$ resistor:

$V_b+I_2(4.00\ \Omega)+I_1(3.00\ \Omega)=V_a$

$V_a-V_b=(1.40\text{ A})(4.00\ \Omega)+(1.60\text{ A})(3.00\ \Omega)=5.60\text{ V}+4.8\text{ V}=10.4\text{ V}$ (point *a* is at higher potential than point *b*)

EVALUATE: Alternatively, travel through the 5.00 V emf, the $1.00\ \Omega$ resistor, the $2.00\ \Omega$ resistor, and the 10.0 V emf.

$V_b+5.00\text{ V}-I_2(1.00\ \Omega)-I_1(2.00\ \Omega)+10.0\text{ V}=V_a$

$V_a-V_b=15.0\text{ V}-(1.40\text{ A})(1.00\ \Omega)-(1.60\text{ A})(2.00\ \Omega)=15.0\text{ V}-1.40\text{ V}-3.20\text{ V}=10.4\text{ V}$, the same as before.

26.26. **IDENTIFY:** Use Kirchhoff's rules to find the currents

SET UP: Since the 20.0 V battery has the largest voltage, assume I_1 is to the right through the 10.0 V battery, I_2 is to the left through the 20.0 V battery, and I_3 is to the right through the $10\ \Omega$ resistor. Go around each loop in the counterclockwise direction.

EXECUTE: Upper loop: $10.0\text{ V}+(2.00\ \Omega+3.00\ \Omega)I_1+(1.00\ \Omega+4.00\ \Omega)I_2-20.00\text{ V}=0$.

$-10.0\text{ V}+(5.00\ \Omega)I_1+(5.00\ \Omega)I_2=0$, so $I_1+I_2=+2.00\text{ A}$.

Lower loop: $20.00\text{ V}-(1.00\ \Omega+4.00\ \Omega)I_2-(10.0\ \Omega)I_3=0$.

$20.00\text{ V}-(5.00\ \Omega)I_2-(10.0\ \Omega)I_3=0$, so $I_2+2I_3=4.00\text{ A}$.

Along with $I_2=I_1+I_3$, we can solve for the three currents and find $I_1=+0.4\text{ A},\ I_2=+1.6\text{ A},\ I_3=+1.2\text{ A}$.

(b) $V_{ab}=I_2(4\ \Omega)+I_1(3\ \Omega)=(1.6\text{ A})(4\ \Omega)+(0.4\text{ A})(3\ \Omega)=7.6\text{ V}$

EVALUATE: Traveling from *b* to *a* through the $4.00\ \Omega$ and $3.00\ \Omega$ resistors you pass through each resistor opposite to the direction of the current and the potential increases; point *a* is at higher potential than point *b.*

26.27. **(a) IDENTIFY:** With the switch open, the circuit can be solved using series-parallel reduction.

SET UP: Find the current through the unknown battery using Ohm's law. Then use the equivalent resistance of the circuit to find the emf of the battery.

EXECUTE: The 30.0-Ω and 50.0-Ω resistors are in series, and hence have the same current. Using Ohm's law I_{50} = (15.0 V)/(50.0 Ω) = 0.300 A = I_{30}. The potential drop across the 75.0-Ω resistor is the same as the potential drop across the 80.0-Ω series combination. We can use this fact to find the current through the 75.0-Ω resistor using Ohm's law: V_{75} = V_{80} = (0.300 A)(80.0 Ω) = 24.0 V and I_{75} = (24.0 V)/(75.0 Ω) = 0.320 A.

The current through the unknown battery is the sum of the two currents we just found:

$$I_{\text{Total}} = 0.300\text{ A} + 0.320\text{ A} = 0.620\text{ A}$$

The equivalent resistance of the resistors in parallel is $1/R_p$ = 1/(75.0 Ω) + 1/(80.0 Ω). This gives R_p = 38.7 Ω. The equivalent resistance "seen" by the battery is R_{equiv} = 20.0 Ω + 38.7 Ω = 58.7 Ω.

Applying Ohm's law to the battery gives $\mathcal{E} = R_{\text{equiv}}I_{\text{Total}}$ = (58.7 Ω)(0.620 A) = 36.4 V

(b) IDENTIFY: With the switch closed, the 25.0-V battery is connected across the 50.0-Ω resistor.

SET UP: Taking a loop around the right part of the circuit.

EXECUTE: Ohm's law gives I = (25.0 V)/(50.0 Ω) = 0.500 A

EVALUATE: The current through the 50.0-Ω resistor, and the rest of the circuit, depends on whether or not the switch is open.

26.28. **IDENTIFY:** We need to use Kirchhoff's rules.

SET UP: Take a loop around the outside of the circuit, use the current at the upper junction, and then take a loop around the right side of the circuit.

EXECUTE: The outside loop gives 75.0 V – (12.0 Ω)(1.50 A) – (48.0 Ω)I_{48} = 0 , so I_{48} = 1.188 A. At a junction we have 1.50 A = I_ε + 1.188 A , and I_ε = 0.313 A. A loop around the right part of the circuit gives $\mathcal{E}$ – (48 Ω)(1.188 A) + (15.0 Ω)(0.313 A). $\mathcal{E}$ = 52.3 V, with the polarity shown in the figure in the problem.

EVALUATE: The unknown battery has a smaller emf than the known one, so the current through it goes against its polarity.

26.29. **(a) IDENTIFY:** With the switch open, we have a series circuit with two batteries.

SET UP: Take a loop to find the current, then use Ohm's law to find the potential difference between *a* and *b*.

EXECUTE: Taking the loop: I = (40.0 V)/(175 Ω) = 0.229 A. The potential difference between *a* and *b* is $V_b - V_a$ = +15.0 V – (75.0 Ω)(0.229 A) = – 2.14 V.

EVALUATE: The minus sign means that *a* is at a higher potential than *b*.

(b) IDENTIFY: With the switch closed, the ammeter part of the circuit divides the original circuit into two circuits. We can apply Kirchhoff's rules to both parts.

SET UP: Take loops around the left and right parts of the circuit, and then look at the current at the junction.

EXECUTE: The left-hand loop gives I_{100} = (25.0 V)/(100.0 Ω) = 0.250 A. The right-hand loop gives I_{75} = (15.0 V)/(75.0 Ω) = 0.200 A. At the junction just above the switch we have I_{100} = 0.250 A (in) and I_{75} = 0.200 A (out) , so I_A = 0.250 A – 0.200 A = 0.050 A, downward. The voltmeter reads zero because the potential difference across it is zero with the switch closed.

EVALUATE: The ideal ammeter acts like a short circuit, making *a* and *b* at the same potential. Hence the voltmeter reads zero.

26.30. **IDENTIFY:** The circuit is sketched in Figure 26.30a. Since all the external resistors are equal, the current must be symmetrical through them. That is, there can be no current through the resistor *R* for that would imply an imbalance in currents through the other resistors. With no current going through *R*, the circuit is like that shown in Figure 26.30b.

SET UP: For resistors in series, the equivalent resistance is $R_s = R_1 + R_2$. For resistors in parallel, the equivalent resistance is $\dfrac{1}{R_p} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$

EXECUTE: The equivalent resistance of the circuit is $R_{eq} = \left(\dfrac{1}{2\ \Omega} + \dfrac{1}{2\ \Omega}\right)^{-1} = 1\ \Omega$ and $I_{total} = \dfrac{13\ \text{V}}{1\ \Omega} = 13\ \text{A}.$ The two parallel branches have the same resistance, so $I_{\text{each branch}} = \dfrac{1}{2} I_{total} = 6.5\ \text{A}.$ The current through each 1 Ω resistor is 6.5 A and no current passes through *R*.

(b) As worked out above, $R_{eq} = 1\ \Omega$.

(c) $V_{ab} = 0,$ since no current flows through *R*.

EVALUATE: **(d)** *R* plays no role since no current flows through it and the voltage across it is zero.

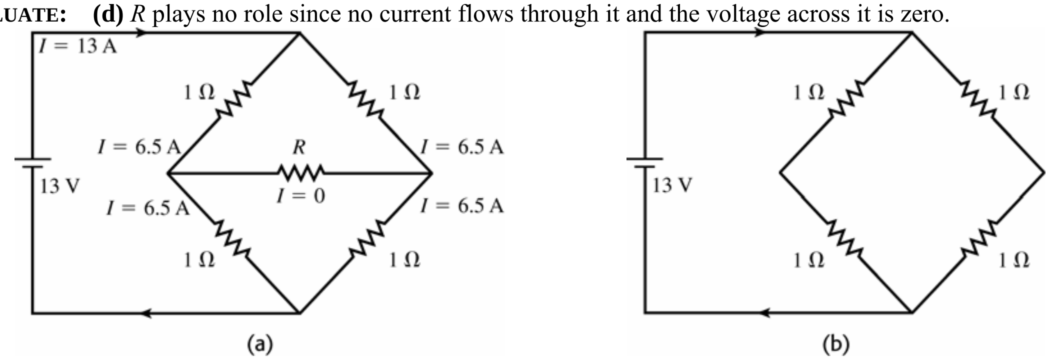


Figure 26.30

26.31. **IDENTIFY:** To construct an ammeter, add a shunt resistor in parallel with the galvanometer coil. To construct a voltmeter, add a resistor in series with the galvanometer coil.

SET UP: The full-scale deflection current is 500 μA and the coil resistance is 25.0 Ω .

EXECUTE: **(a)** For a 20-mA ammeter, the two resistances are in parallel and the voltages across each are the same. $V_c = V_s$ gives $I_c R_c = I_s R_s$. $(500\times10^{-6}\ \text{A})(25.0\ \Omega) = (20\times10^{-3}\ \text{A} - 500\times10^{-6}\ \text{A})R_s$ and $R_s = 0.641\ \Omega$.

(b) For a 500-mV voltmeter, the resistances are in series and the current is the same through each: $V_{ab} = I\left(R_c + R_s\right)$ and $R_s = \dfrac{V_{ab}}{I} - R_c = \dfrac{500\times10^{-3}\text{ V}}{500\times10^{-6}\text{ A}} - 25.0\ \Omega = 975\ \Omega.$

EVALUATE: The equivalent resistance of the voltmeter is $R_{eq} = R_s + R_c = 1000\ \Omega$. The equivalent resistance of the ammeter is given by $\dfrac{1}{R_{eq}} = \dfrac{1}{R_{sh}} + \dfrac{1}{R_c}$ and $R_{eq} = 0.625\ \Omega$. The voltmeter is a high-resistance device and the ammeter is a low-resistance device.

26.32. IDENTIFY: The galvanometer is represented in the circuit as a resistance R_c. Use the junction rule to relate the current through the galvanometer and the current through the shunt resistor. The voltage drop across each parallel path is the same; use this to write an equation for the resistance *R*.

SET UP: The circuit is sketched in Figure 26.32.

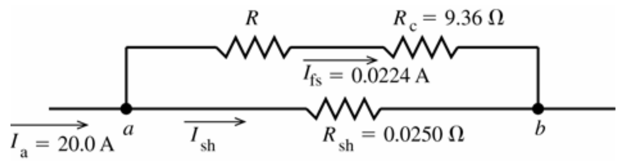


Figure 26.32

We want that $I_a = 20.0$ A in the external circuit to produce $I_{fs} = 0.0224$ A through the galvanometer coil.

EXECUTE: Applying the junction rule to point *a* gives $I_a - I_{fs} - I_{sh} = 0$

$I_{sh} = I_a - I_{fs} = 20.0\text{ A} - 0.0224\text{ A} = 19.98\text{ A}$

The potential difference V_{ab} between points *a* and *b* must be the same for both paths between these two points:

$I_{fs}\left(R + R_c\right) = I_{sh}R_{sh}$

$$R = \frac{I_{sh}R_{sh}}{I_{fs}} - R_c = \frac{(19.98\text{ A})(0.0250\ \Omega)}{0.0224\text{ A}} - 9.36\ \Omega = 22.30\ \Omega - 9.36\ \Omega = 12.9\ \Omega$$

EVALUATE: $R_{sh} << R + R_c$; most of the current goes through the shunt. Adding *R* decreases the fraction of the current that goes through R_c.

26.33. IDENTIFY: The meter introduces resistance into the circuit, which affects the current through the 5.00-kΩ resistor and hence the potential drop across it.

SET UP: Use Ohm's law to find the current through the 5.00-kΩ resistor and then the potential drop across it.

EXECUTE: **(a)** The parallel resistance with the voltmeter is 3.33 kΩ, so the total equivalent resistance across the battery is 9.33 kΩ, giving I = (50.0 V)/(9.33 kΩ) = 5.36 mA. Ohm's law gives the potential drop across the 5.00-kΩ resistor: $V_{5\text{ k}\Omega}$ = (3.33 kΩ)(5.36 mA) = 17.9 V

(b) The current in the circuit is now I = (50.0 V)/(11.0 kΩ) = 4.55 mA. $V_{5\text{ k}\Omega}$ = (5.00 kΩ)(4.55 mA) = 22.7 V.

(c) % error = (22.7 V – 17.9 V)/(22.7 V) = 0.214 = 21.4%. (We carried extra decimal places for accuracy since we had to subtract our answers.)

EVALUATE: The presence of the meter made a very large percent error in the reading of the "true" potential across the resistor.

26.34. IDENTIFY: The resistance of the galvanometer can alter the resistance in a circuit.

SET UP: The shunt is in parallel with the galvanometer, so we find the parallel resistance of the ammeter. Then use Ohm's law to find the current in the circuit.

EXECUTE: **(a)** The resistance of the ammeter is given by $1/R_A$ = 1/(1.00 Ω) + 1/(25.0 Ω), so R_A = 0.962 Ω. The current through the ammeter, and hence the current it measures, is $I = V/R$ = (25.0 V)/(15.96 Ω) = 1.57 A.

(b) Now there is no meter in the circuit, so the total resistance is only 15.0 Ω. I = (25.0 V)/(15.0 Ω) = 1.67 A

(c) (1.67 A – 1.57 A)/(1.67 A) = 0.060 = 6.0%

EVALUATE: A 1-Ω shunt can introduce noticeable error in the measurement of an ammeter.

26.35. IDENTIFY: When the galvanometer reading is zero $\mathcal{E}_2 = IR_{cb}$ and $\mathcal{E}_1 = IR_{ab}$.

SET UP: R_{cb} is proportional to *x* and R_{ab} is proportional to *l*.

EXECUTE: **(a)** $\mathcal{E}_2 = \mathcal{E}_1\dfrac{R_{cb}}{R_{ab}} = \mathcal{E}_1\dfrac{x}{l}.$

(b) The value of the galvanometer's resistance is unimportant since no current flows through it.

(c) $\mathcal{E}_2 = \mathcal{E}_1\dfrac{x}{l} = (9.15\text{ V})\dfrac{0.365\text{ m}}{1.000\text{ m}} = 3.34\text{ V}$

EVALUATE: The potentiometer measures the emf $\mathcal{E}_2$ of the source directly, unaffected by the internal resistance of the source, since the measurement is made with no current through $\mathcal{E}_2$.

26.36. **IDENTIFY:** A half-scale reading occurs with $R = 600\ \Omega$, so the current through the galvanometer is half the full-scale current.

SET UP: The resistors R_s, R_c and R are in series, so the total resistance of the circuit is $R_{\text{total}} = R_s + R_c + R$.

EXECUTE: $\mathcal{E} = IR_{\text{total}}$. $1.50\ \text{V} = \left(\dfrac{3.60\times10^{-3}\ \text{A}}{2}\right)(15.0\ \Omega + 600\ \Omega + R_s)$ and $R_s = 218\ \Omega$.

EVALUATE: We have assumed that the device is linear, in the sense that the deflection is proportional to the current through the meter.

26.37. **IDENTIFY:** Apply $\mathcal{E} = IR_{\text{total}}$ to relate the resistance R_x to the current in the circuit.

SET UP: R, R_x and the meter are in series, so $R_{\text{total}} = R + R_x + R_M$, where $R_M = 65.0\ \Omega$ is the resistance of the meter. $I_{\text{fsd}} = 2.50\ \text{mA}$ is the current required for full-scale deflection.

EXECUTE: **(a)** When the wires are shorted, the full-scale deflection current is obtained: $\mathcal{E} = IR_{\text{total}}$.

$1.52\ \text{V} = (2.50\times10^{-3}\ \text{A})(65.0\ \Omega + R)$ and $R = 543\ \Omega$.

(b) If the resistance $R_x = 200\ \Omega$: $I = \dfrac{V}{R_{\text{total}}} = \dfrac{1.52\ \text{V}}{65.0\ \Omega + 543\ \Omega + R_x} = 1.88\ \text{mA}.$

(c) $I_x = \dfrac{\mathcal{E}}{R_{\text{total}}} = \dfrac{1.52\ \text{V}}{65.0\ \Omega + 543\ \Omega + R_x}$ and $R_x = \dfrac{1.52\ \text{V}}{I_x} - 608\ \Omega$. For each value of I_x we have:

For $I_x = \frac{1}{4} I_{\text{fsd}} = 6.25\times10^{-4}\ \text{A}$, $R_x = \dfrac{1.52\ \text{V}}{6.25\times10^{-4}\ \text{A}} - 608\ \Omega = 1824\ \Omega$.

For $I_x = \frac{1}{2} I_{\text{fsd}} = 1.25\times10^{-3}\ \text{A}$, $R_x = \dfrac{1.52\ \text{V}}{1.25\times10^{-3}\ \text{A}} - 608\ \Omega = 608\ \Omega.$

For $I_x = \frac{3}{4} I_{\text{fsd}} = 1.875\times10^{-3}\ \text{A}$, $R_x = \dfrac{1.52\ \text{V}}{1.875\times10^{-3}\ \text{A}} - 608\ \Omega = 203\ \Omega$.

EVALUATE: The deflection of the meter increases when the resistance R_x decreases.

26.38. **IDENTIFY:** An uncharged capacitor is placed into a circuit. Apply the loop rule at each time.

SET UP: The voltage across a capacitor is $V_C = q/C$.

EXECUTE: **(a)** At the instant the circuit is completed, there is no voltage over the capacitor, since it has no charge stored.

(b) Since $V_C = 0$, the full battery voltage appears across the resistor $V_R = \mathcal{E} = 125\ \text{V}$.

(c) There is no charge on the capacitor.

(d) The current through the resistor is $i = \dfrac{\mathcal{E}}{R_{\text{total}}} = \dfrac{125\ \text{V}}{7500\ \Omega} = 0.0167\ \text{A}.$

(e) After a long time has passed the full battery voltage is across the capacitor and $i = 0$. The voltage across the capacitor balances the emf: $V_C = 125\ \text{V}$. The voltage across the resister is zero. The capacitor's charge is $q = CV_C = (4.60\times10^{-6}\ \text{F})(125\ \text{V}) = 5.75\times10^{-4}\ \text{C}.$ The current in the circuit is zero.

EVALUATE: The current in the circuit starts at 0.0167 A and decays to zero. The charge on the capacitor starts at zero and rises to $q = 5.75\times10^{-4}\ \text{C}$.

26.39. **IDENTIFY:** The capacitor discharges exponentially through the voltmeter. Since the potential difference across the capacitor is directly proportional to the charge on the plates, the voltage across the plates decreases exponentially with the same time constant as the charge.

SET UP: The reading of the voltmeter obeys the equation $V = V_0 e^{-t/RC}$, where RC is the time constant.

EXECUTE: **(a)** Solving for C and evaluating the result when $t = 4.00$ s gives

$$C = \frac{t}{R\ln(V/V_0)} = \frac{4.00\ \text{s}}{(3.40\times10^6\ \Omega)\ln\left(\dfrac{12.0\ \text{V}}{3.00\ \text{V}}\right)} = 8.49\times10^{-7}\ \text{F}$$

(b) $\tau = RC = (3.40\times10^6\ \Omega)(8.49\times10^{-7}\ \text{F}) = 2.89\ \text{s}$

EVALUATE: In most laboratory circuits, time constants are much shorter than this one.

26.40. **IDENTIFY:** For a charging capacitor $q(t) = C\mathcal{E}(1 - e^{-t/\tau})$ and $i(t) = \dfrac{\mathcal{E}}{R} e^{-t/\tau}$.

SET UP: The time constant is $RC = (0.895\times10^6\ \Omega)(12.4\times10^{-6}\ \text{F}) = 11.1\ \text{s}.$

EXECUTE: **(a)** At $t = 0$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = 0.$

At $t = 5$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(5.0\ \text{s})/(11.1\ \text{s})}) = 2.70\times10^{-4}\ \text{C}$.

At $t = 10$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(10.0\ \text{s})/(11.1\ \text{s})}) = 4.42\times10^{-4}\ \text{C}$.

At $t = 20$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(20.0\ \text{s})/(11.1\ \text{s})}) = 6.21\times10^{-4}\ \text{C}$.

At $t = 100$ s: $q = C\mathcal{E}(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})(60.0\ \text{V})(1 - e^{-(100\ \text{s})/(11.1\ \text{s})}) = 7.44\times10^{-4}\ \text{C}$.

(b) The current at time t is given by: $i = \frac{\mathcal{E}}{R}e^{-t/RC}$.

At $t = 0$ s: $i = \frac{60.0\ \text{V}}{8.95\times10^{5}\ \Omega}e^{-0/11.1} = 6.70\times10^{-5}\ \text{A}$.

At $t = 5$ s: $i = \frac{60.0\ \text{V}}{8.95\times10^{5}\ \Omega}e^{-5/11.1} = 4.27\times10^{-5}\ \text{A}$.

At $t = 10$ s: $i = \frac{60.0\ \text{V}}{8.95\times10^{5}\ \Omega}e^{-10/11.1} = 2.27\times10^{-5}\ \text{A}$.

At $t = 20$ s: $i = \frac{60.0\ \text{V}}{8.95\times10^{5}\ \Omega}e^{-20/11.1} = 1.11\times10^{-5}\ \text{A}$.

At $t = 100$ s: $i = \frac{60.0\ \text{V}}{8.95\times10^{5}\ \Omega}e^{-100/11.1} = 8.20\times10^{-9}\ \text{A}$.

(c) The graphs of $q(t)$ and $i(t)$ are given in Figure 26.40a and 26.40b

EVALUATE: The charge on the capacitor increases in time as the current decreases.

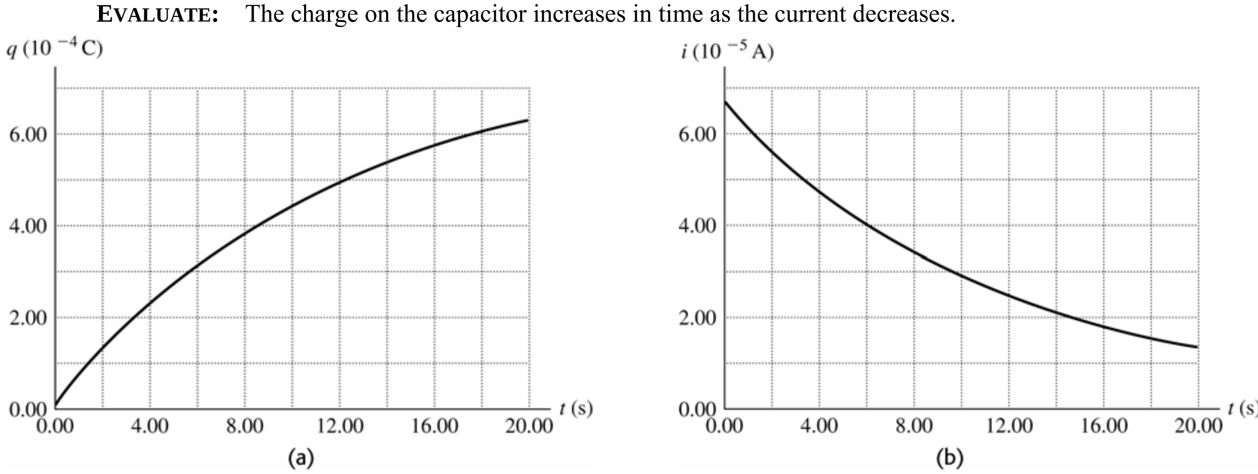


Figure 26.40

26.41. **IDENTIFY:** The capacitors, which are in parallel, will discharge exponentially through the resistors.

SET UP: Since V is proportional to Q, V must obey the same exponential equation as Q, $V = V_0\, e^{-t/RC}$. The current is $I = (V_0/R)\, e^{-t/RC}$.

EXECUTE: **(a)** Solve for time when the potential across each capacitor is 10.0 V:

$$t = -RC\ln(V/V_0) = -(80.0\ \Omega)(35.0\ \mu\text{F})\ln(10/45) = 4210\ \mu\text{s} = 4.21\ \text{ms}$$

(b) $I = (V_0/R)\, e^{-t/RC}$. Using the above values, with $V_0 = 45.0$ V, gives $I = 0.125$ A.

EVALUATE: Since the current and the potential both obey the same exponential equation, they are both reduced by the same factor (0.222) in 4.21 ms.

26.42. **IDENTIFY:** In $\tau = RC$ use the equivalent capacitance of the two capacitors.

SET UP: For capacitors in series, $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$. For capacitors in parallel, $C_{\text{eq}} = C_1 + C_2$. Originally, $\tau = RC = 0.870$ s.

EXECUTE: **(a)** The combined capacitance of the two identical capacitors in series is given by $\frac{1}{C_{\text{eq}}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$, so $C_{\text{eq}} = \frac{C}{2}$. The new time constant is thus $R(C/2) = \frac{0.870\ \text{s}}{2} = 0.435\ \text{s}$.

(b) With the two capacitors in parallel the new total capacitance is simply $2C$. Thus the time constant is $R(2C) = 2(0.870\ \text{s}) = 1.74\ \text{s}$.

EVALUATE: The time constant is proportional to C_{eq}. For capacitors in series the capacitance is decreased and for capacitors in parallel the capacitance is increased.

26.43. **IDENTIFY** and **SET UP:** Apply the loop rule. The voltage across the resistor depends on the current through it and the voltage across the capacitor depends on the charge on its plates.

EXECUTE: $\mathcal{E} - V_R - V_C = 0$

$\mathcal{E} = 120\text{ V}$, $V_R = IR = (0.900\text{ A})(80.0\ \Omega) = 72\text{ V}$, so $V_C = 48\text{ V}$

$Q = CV = (4.00\times 10^{-6}\text{ F})(48\text{ V}) = 192\ \mu\text{C}$

EVALUATE: The initial charge is zero and the final charge is $C\mathcal{E} = 480\ \mu\text{C}$. Since current is flowing at the instant considered in the problem the capacitor is still being charged and its charge has not reached its final value.

26.44. **IDENTIFY:** The charge is increasing while the current is decreasing. Both obey exponential equations, but they are not the same equation.

SET UP: The charge obeys the equation $Q = Q_{max}(1 - e^{-t/RC})$, but the equation for the current is $I = I_{max}e^{-t/RC}$.

EXECUTE: When the charge has reached $\frac{1}{4}$ of its maximum value, we have $Q_{max}/4 = Q_{max}(1 - e^{-t/RC})$, which says that the exponential term has the value $e^{-t/RC} = \frac{3}{4}$. The current at this time is $I = I_{max}e^{-t/RC} = I_{max}(3/4) = (3/4)[(10.0\text{ V})/(12.0\ \Omega)] = 0.625\text{ A}$

EVALUATE: Notice that the current will be $\frac{3}{4}$, not $\frac{1}{4}$, of its maximum value when the charge is $\frac{1}{4}$ of its maximum. Although current and charge both obey exponential equations, the equations have different forms for a charging capacitor.

26.45. **IDENTIFY:** The stored energy is proportional to the square of the charge on the capacitor, so it will obey an exponential equation, but not the same equation as the charge.

SET UP: The energy stored in the capacitor is $U = Q^2/2C$ and the charge on the plates is $Q_0\, e^{-t/RC}$. The current is $I = I_0\, e^{-t/RC}$.

EXECUTE: $U = Q^2/2C = (Q_0\, e^{-t/RC})^2/2C = U_0\, e^{-2t/RC}$

When the capacitor has lost 80% of its stored energy, the energy is 20% of the initial energy, which is $U_0/5$. $U_0/5 = U_0\, e^{-2t/RC}$ gives $t = (RC/2)\ln 5 = (25.0\ \Omega)(4.62\text{ pF})(\ln 5)/2 = 92.9\text{ ps}$.

At this time, the current is $I = I_0\, e^{-t/RC} = (Q_0/RC)\, e^{-t/RC}$, so

$$I = (3.5\text{ nC})/[(25.0\ \Omega)(4.62\text{ pF})]\ e^{-(92.9\text{ ps})/[(25.0\ \Omega)(4.62\text{ pF})]} = 13.6\text{ A}.$$

EVALUATE: When the energy reduced by 80%, neither the current nor the charge are reduced by that percent.

26.46. **IDENTIFY:** Both the charge and energy decay exponentially, but not with the same time constant since the energy is proportional to the *square* of the charge.

SET UP: The charge obeys the equation $Q = Q_0\, e^{-t/RC}$ but the energy obeys the equation $U = Q^2/2C = (Q_0\, e^{-t/RC})/2C = U_0\, e^{-2t/RC}$.

EXECUTE: **(a)** The charge is reduced by half: $Q_0/2 = Q_0\, e^{-t/RC}$. This gives

$$t = RC\ln 2 = (175\ \Omega)(12.0\ \mu\text{F})(\ln 2) = 1.456\text{ ms} = 1.46\text{ ms}.$$

(b) The energy is reduced by half: $U_0/2 = U_0\, e^{-2t/RC}$. This gives

$$t = (RC\ln 2)/2 = (1.456\text{ ms})/2 = 0.728\text{ ms}.$$

EVALUATE: The energy decreases faster than the charge because it is proportional to the square of the charge.

26.47. **IDENTIFY:** In both cases, simplify the complicated circuit by eliminating the appropriate circuit elements. The potential across an uncharged capacitor is initially zero, so it behaves like a short circuit. A fully charged capacitor allows no current to flow through it.

(a) SET UP: Just after closing the switch, the uncharged capacitors all behave like short circuits, so any resistors in parallel with them are eliminated from the circuit.

EXECUTE: The equivalent circuit consists of 50 Ω and 25 Ω in parallel, with this combination in series with 75 Ω, 15 Ω, and the 100-V battery. The equivalent resistance is 90 Ω + 16.7 Ω = 106.7 Ω, which gives $I = (100\text{ V})/(106.7\ \Omega) = 0.937\text{ A}$.

(b) SET UP: Long after closing the switch, the capacitors are essentially charged up and behave like open circuits since no charge can flow through them. They effectively eliminate any resistors in series with them since no current can flow through these resistors.

EXECUTE: The equivalent circuit consists of resistances of 75 Ω, 15 Ω, and three 25-Ω resistors, all in series with the 100-V battery, for a total resistance of 165 Ω. Therefore $I = (100\text{ V})/(165\ \Omega) = 0.606\text{ A}$

EVALUATE: The initial and final behavior of the circuit can be calculated quite easily using simple series-parallel circuit analysis. Intermediate times would require much more difficult calculations!

26.48. **IDENTIFY:** When the capacitor is fully charged the voltage V across the capacitor equals the battery emf and $Q = CV$. For a charging capacitor, $q = Q(1 - e^{-t/RC})$.

SET UP: $\ln e^x = x$

EXECUTE: **(a)** $Q = CV = (5.90\times10^{-6}\text{ F})(28.0\text{ V}) = 1.65\times10^{-4}\text{ C}$.

(b) $q = Q(1-e^{-t/RC})$, so $e^{-t/RC} = 1-\dfrac{q}{Q}$ and $R = \dfrac{-t}{C\ln(1-q/Q)}$. After

$t = 3\times10^{-3}\text{ s}$: $R = \dfrac{-3\times10^{-3}\text{ s}}{(5.90\times10^{-6}\text{ F})(\ln(1-110/165))} = 463\ \Omega.$

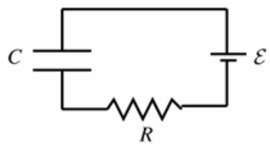
(c) If the charge is to be 99% of final value: $\dfrac{q}{Q} = (1-e^{-t/RC})$ gives

$t = -RC\ln(1-q/Q) = -(463\ \Omega)(5.90\times10^{-6}\text{ F})\ln(0.01) = 0.0126\text{ s}.$

EVALUATE: The time constant is $\tau = RC = 2.73\text{ ms}$. The time in part (b) is a bit more than one time constant and the time in part (c) is about 4.6 time constants.

26.49. IDENTIFY: For each circuit apply the loop rule to relate the voltages across the circuit elements.

(a) SET UP: With the switch in position 2 the circuit is the charging circuit shown in Figure 26.49a.



At $t = 0$, $q = 0$.

Figure 26.49a

EXECUTE: The charge q on the capacitor is given as a function of time by Eq.(26.12):

$q = C\mathcal{E}\left(1-e^{-t/RC}\right)$

$Q_f = C\mathcal{E} = \left(1.50\times10^{-5}\text{ F}\right)\left(18.0\text{ V}\right) = 2.70\times10^{-4}\text{ C}.$

$RC = \left(980\ \Omega\right)\left(1.50\times10^{-5}\text{ F}\right) = 0.0147\text{ s}$

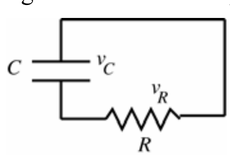
Thus, at $t = 0.0100\text{ s}$, $q = \left(2.70\times10^{-4}\text{ C}\right)\left(1-e^{-(0.0100\text{ s})/(0.0147\text{ s})}\right) = 133\ \mu\text{C}.$

(b) $v_C = \dfrac{q}{C} = \dfrac{133\ \mu\text{C}}{1.50\times10^{-5}\text{ F}} = 8.87\text{ V}$

The loop rule says $\mathcal{E} - v_C - v_R = 0$

$v_R = \mathcal{E} - v_C = 18.0\text{ V} - 8.87\text{ V} = 9.13\text{ V}$

(c) SET UP: Throwing the switch back to position 1 produces the discharging circuit shown in Figure 26.49b.



The initial charge Q_0 is the charge calculated in part (b), $Q_0 = 133\ \mu\text{C}$.

Figure 26.49b

EXECUTE: $v_C = \dfrac{q}{C} = \dfrac{133\ \mu\text{C}}{1.50\times10^{-5}\text{ F}} = 8.87\text{ V}$, the same as just before the switch is thrown. But now $v_C - v_R = 0$, so $v_R = v_C = 8.87\text{ V}$.

(d) SET UP: In the discharging circuit the charge on the capacitor as a function of time is given by Eq.(26.16): $q = Q_0 e^{-t/RC}$.

EXECUTE: $RC = 0.0147\text{ s}$, the same as in part (a). Thus at $t = 0.0100\text{ s}$, $q = (133\ \mu\text{C})e^{-((0.0100\text{ s})/(0.0147\text{ s}))} = 67.4\ \mu\text{C}$.

EVALUATE: $t = 10.0\text{ ms}$ is less than one time constant, so at the instant described in part (a) the capacitor is not fully charged; its voltage (8.87 V) is less than the emf. There is a charging current and a voltage drop across the resistor. In the discharging circuit the voltage across the capacitor starts at 8.87 V and decreases. After $t = 10.0\text{ ms}$ it has decreased to $v_C = q/C = 4.49\text{ V}$.

26.50. IDENTIFY: $P = VI = I^2R$

SET UP: Problem 25.77 says that for 12-gauge wire the maximum safe current is 2.5 A.

EXECUTE: **(a)** $I = \dfrac{P}{V} = \dfrac{4100\text{ W}}{240\text{ V}} = 17.1\text{ A}$. So we need at least 14-gauge wire (good up to 18 A). 12 gauge is also ok (good up to 25 A).

(b) $P = \frac{V^2}{R}$ and $R = \frac{V^2}{P} = \frac{(240\ \text{V})^2}{4100\ \text{W}} = 14\ \Omega$.

(c) At 11¢ per kWH, for 1 hour the cost is $(11¢/\text{kWh})(1\ \text{h})(4.1\ \text{kW}) = 45¢$.

EVALUATE: The cost to operate the device is proportional to its power consumption.

26.51. **IDENTIFY** and **SET UP:** The heater and hair dryer are in parallel so the voltage across each is 120 V and the current through the fuse is the sum of the currents through each appliance. As the power consumed by the dryer increases the current through it increases. The maximum power setting is the highest one for which the current through the fuse is less than 20 A.

EXECUTE: Find the current through the heater. $P = VI$ so $I = P/V = 1500\ \text{W}/120\ \text{V} = 12.5\ \text{A}$. The maximum total current allowed is 20 A, so the current through the dryer must be less than $20\ \text{A} - 12.5\ \text{A} = 7.5\ \text{A}$. The power dissipated by the dryer if the current has this value is $P = VI = (120\ \text{V})(7.5\ \text{A}) = 900\ \text{W}$. For *P* at this value or larger the circuit breaker trips.

EVALUATE: $P = V^2/R$ and for the dryer *V* is a constant 120 V. The higher power settings correspond to a smaller resistance *R* and larger current through the device.

26.52. **IDENTIFY:** The current gets split evenly between all the parallel bulbs.

SET UP: A single bulb will draw $I = \frac{P}{V} = \frac{90\ \text{W}}{120\ \text{V}} = 0.75\ \text{A}$.

EXECUTE: Number of bulbs $\le \frac{20\ \text{A}}{0.75\ \text{A}} = 26.7$. So you can attach 26 bulbs safely.

EVALUATE: In parallel the voltage across each bulb is the circuit voltage.

26.53. **IDENTIFY** and **SET UP:** Ohm's law and Eq.(25.18) can be used to calculate *I* and *P* given *V* and *R*. Use Eq.(25.12) to calculate the resistance at the higher temperature.

(a) EXECUTE: When the heater element is first turned on it is at room temperature and has resistance $R = 20\ \Omega$.

$$I = \frac{V}{R} = \frac{120\ \text{V}}{20\ \Omega} = 6.0\ \text{A}$$

$$P = \frac{V^2}{R} = \frac{(120\ \text{V})^2}{20\ \Omega} = 720\ \text{W}$$

(b) Find the resistance *R*(*T*) of the element at the operating temperature of 280°C.

Take $T_0 = 23.0°\text{C}$ and $R_0 = 20\ \Omega$. Eq.(25.12) gives

$$R(T) = R_0\left(1 + \alpha(T - T_0)\right) = 20\ \Omega\left(1 + \left(2.8\times10^{-3}\,(\text{C}°)^{-1}\right)(280°\text{C} - 23.0°\text{C})\right) = 34.4\ \Omega.$$

$$I = \frac{V}{R} = \frac{120\ \text{V}}{34.4\ \Omega} = 3.5\ \text{A}$$

$$P = \frac{V^2}{R} = \frac{(120\ \text{V})^2}{34.4\ \Omega} = 420\ \text{W}$$

EVALUATE: When the temperature increases, *R* increases and *I* and *P* decrease. The changes are substantial.

26.54. **(a) IDENTIFY:** Two of the resistors in series would each dissipate one-half the total, or 1.2 W, which is ok. But the series combination would have an equivalent resistance of $800\ \Omega$, not the $400\ \Omega$ that is required. Resistors in parallel have an equivalent resistance that is less than that of the individual resistors, so a solution is two in series in parallel with another two in series.

SET UP: The network can be simplified as shown in Figure 26.54a.

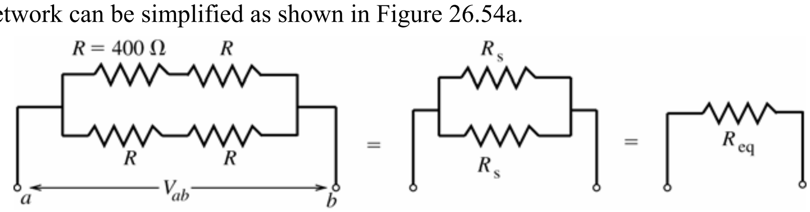


Figure 26.54a

EXECUTE: R_s is the resistance equivalent to two of the $400\ \Omega$ resistors in series. $R_s = R + R = 800\ \Omega$. R_{eq} is the resistance equivalent to the two $R_s = 800\ \Omega$ resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_s} + \frac{1}{R_s} = \frac{2}{R_s}$; $R_{eq} = \frac{800\ \Omega}{2} = 400\ \Omega$.

EVALUATE: This combination does have the required $400\ \Omega$ equivalent resistance. It will be shown in part (b) that a total of 2.4 W can be dissipated without exceeding the power rating of each individual resistor.

IDENTIFY: Another solution is two resistors in parallel in series with two more in parallel.
SET UP: The network can be simplified as shown in Figure 26.54b.

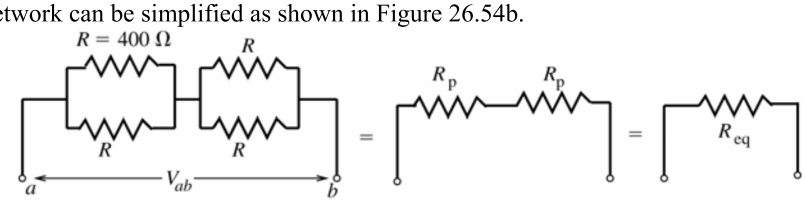


Figure 26.54b

EXECUTE: $\dfrac{1}{R_p}=\dfrac{1}{R}+\dfrac{1}{R}=\dfrac{2}{400\ \Omega}$; $R_p = 200\ \Omega$**;** $R_{eq}=R_p+R_p=400\ \Omega$

EVALUATE: This combination has the required $400\ \Omega$ equivalent resistance It will be shown in part (b) that a total of 2.4 W can be dissipated without exceeding the power rating of each individual resistor.

(b) IDENTIFY and **SET UP:** Find the applied voltage V_{ab} such that a total of 2.4 W is dissipated and then for this V_{ab} find the power dissipated by each resistor.

EXECUTE: For a combination with equivalent resistance $R_{eq}=400\ \Omega$ to dissipate 2.4 W the voltage V_{ab} applied to the network must be given by $P=V_{ab}^2/R_{eq}$ so $V_{ab}=\sqrt{PR_{eq}}=\sqrt{(2.4\text{ W})(400\ \Omega)}=31.0\text{ V}$ and the current through the equivalent resistance is $I=V_{ab}/R=31.0\text{ V}/400\ \Omega=0.0775\text{ A}$. For the first combination this means 31.0 V across each parallel branch and $\frac{1}{2}(31.0\text{ V})=15.5\text{ V}$ across each 400 Ω resistor. The power dissipated by each individual resistor is then $P=V^2/R=(15.5\text{ V})^2/400\ \Omega=0.60\text{ W}$, which is less than the maximum allowed value of 1.20 W. For the second combination this means a voltage of $IR_p=(0.0775\text{ A})(200\ \Omega)=15.5\text{ V}$ across each parallel combination and hence across each separate resistor. The power dissipated by each resistor is again $P=V^2/R=(15.5\text{ V})^2/400\ \Omega=0.60\text{ W}$, which is less than the maximum allowed value of 1.20 W.

EVALUATE: The symmetry of each network says that each resistor in the network dissipates the same power. So, for a total of 2.4 W dissipated by the network, each resistor dissipates $(2.4\text{ W})/4=0.60\text{ W}$, which agrees with the above analysis.

26.55. **IDENTIFY:** The Cu and Ni cables are in parallel. For each, $R=\dfrac{\rho L}{A}$.

SET UP: The composite cable is sketched in Figure 26.55. The cross sectional area of the nickel segment is πa^2 and the area of the copper portion is $\pi(b^2-a^2)$. For nickel $\rho=7.8\times10^{-8}\ \Omega\cdot\text{m}$ and for copper $\rho=1.72\times10^{-8}\ \Omega\cdot\text{m}$.

EXECUTE: $\dfrac{1}{R_{\text{Cable}}}=\dfrac{1}{R_{\text{Ni}}}+\dfrac{1}{R_{\text{Cu}}}$. $R_{\text{Ni}}=\rho_{\text{Ni}}L/A=\rho_{\text{Ni}}\dfrac{L}{\pi a^2}$ and $R_{\text{Cu}}=\rho_{\text{Cu}}\ L/A=\rho_{\text{Cu}}\dfrac{L}{\pi(b^2-a^2)}$**.** Therefore,

$$\frac{1}{R_{\text{cable}}}=\frac{\pi a^2}{\rho_{\text{Ni}}L}+\frac{\pi(b^2-a^2)}{\rho_{\text{Cu}}L}.$$

$$\frac{1}{R_{\text{cable}}}=\frac{\pi}{L}\left(\frac{a^2}{\rho_{\text{Ni}}}+\frac{b^2-a^2}{\rho_{\text{Cu}}}\right)=\frac{\pi}{20\text{ m}}\left[\frac{(0.050\text{ m})^2}{7.8\times10^{-8}\ \Omega\cdot\text{m}}+\frac{(0.100\text{ m})^2-(0.050\text{ m})^2}{1.72\times10^{-8}\ \Omega\cdot\text{m}}\right]$$ and $R_{\text{Cable}}=13.6\times10^{-6}\ \Omega=13.6\ \mu\Omega$.

(b) $R=\rho_{\text{eff}}\dfrac{L}{A}=\rho_{\text{eff}}\dfrac{L}{\pi b^2}$. This gives $\rho_{\text{eff}}=\dfrac{\pi b^2 R}{L}=\dfrac{\pi(0.10\text{ m})^2\,(13.6\times10^{-6}\ \Omega)}{20\text{ m}}=2.14\times10^{-8}\ \Omega\cdot\text{m}$

EVALUATE: The effective resistivity of the cable is about 25% larger than the resistivity of copper. If nickel had infinite resitivity and only the copper portion conducted, the resistance of the cable would be $14.6\ \mu\Omega$, which is not much larger than the resistance calculated in part (a).

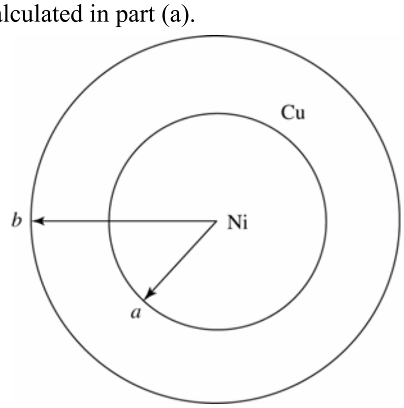


Figure 26.55

26.56. **IDENTIFY** and **SET UP:** Let $R = 1.00\ \Omega$, the resistance of one wire. Each half of the wire has $R_h = R/2 = 0.500\ \Omega$. The combined wires are the same as a resistor network. Use the rules for equivalent resistance for resistors in series and parallel to find the resistance of the network, as shown in Figure 26.56.

EXECUTE:

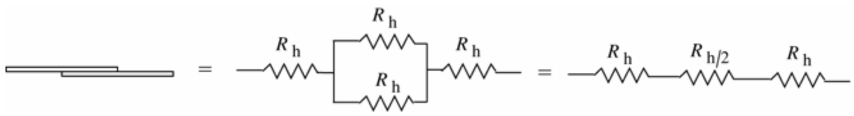


Figure 26.56

The equivalent resistance is $R_h + R_h/2 + R_h = 5R_h/2 = \frac{5}{2}(0.500\ \Omega) = 1.25\ \Omega$

EVALUATE: If the two wires were connected end-to-end, the total resistance would be $2.00\ \Omega$. If they were joined side-by-side, the total resistance would be $0.500\ \Omega$. Our answer is between these two limiting values.

26.57. **IDENTIFY:** The terminal voltage of the battery depends on the current through it and therefore on the equivalent resistance connected to it. The power delivered to each bulb is $P = I^2R$, where I is the current through it.

SET UP: The terminal voltage of the source is $\mathcal{E} - Ir$.

EXECUTE: **(a)** The equivalent resistance of the two bulbs is $1.0\ \Omega$. This equivalent resistance is in series with the internal resistance of the source, so the current through the battery is $I = \dfrac{V}{R_{total}} = \dfrac{8.0\ \text{V}}{1.0\ \Omega + 0.80\ \Omega} = 4.4\ \text{A}$ and the current through each bulb is 2.2 A. The voltage applied to each bulb is $\mathcal{E} - Ir = 8.0\ \text{V} - (4.4\ \text{A})(0.80\ \Omega) = 4.4\ \text{V}$. Therefore, $P_{bulb} = I^2R = (2.2\ \text{A})^2(2.0\ \Omega) = 9.7\ \text{W}$.

(b) If one bulb burns out, then $I = \dfrac{V}{R_{total}} = \dfrac{8.0\ \text{V}}{2.0\ \Omega + 0.80\ \Omega} = 2.9\ \text{A}$. . The current through the remaining bulb is 2.9 A, and $P = I^2R = (2.9\ \text{A})^2\ (2.0\ \Omega) = 16.3\ \text{W}$. The remaining bulb is brighter than before, because it is consuming more power.

EVALUATE: In Example 26.2 the internal resistance of the source is negligible and the brightness of the remaining bulb doesn't change when one burns out.

26.58. **IDENTIFY:** Half the current flows through each parallel resistor and the full current flows through the third resistor, that is in series with the parallel combination. Therefore, only the series resistor will be at its maximum power.

SET UP: $P = I^2R$

EXECUTE: The maximum allowed power is when the total current is the maximum allowed value of $I = \sqrt{P/R} = \sqrt{36\ \text{W}/2.4\ \Omega} = 3.9\ \text{A}$. Then half the current flows through the parallel resistors and the maximum power is $P_{max} = (I/2)^2R + (I/2)^2R + I^2R = \frac{3}{2}I^2R = \frac{3}{2}(3.9\ \text{A})^2(2.4\ \Omega) = 54\ \text{W}$.

EVALUATE: If all three resistors were in series or all three were in parallel, then the maximum power would be $3(36\ \text{W}) = 108\ \text{W}$. For the network in this problem, the maximum power is half this value.

26.59. **IDENTIFY:** The ohmmeter reads the equivalent resistance between points a and b. Replace series and parallel combinations by their equivalent.

SET UP: For resistors in parallel, $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$. For resistors in series, $R_{eq} = R_1 + R_2$

EXECUTE: Circuit (a): The $75.0\ \Omega$ and $40.0\ \Omega$ resistors are in parallel and have equivalent resistance $26.09\ \Omega$. The $25.0\ \Omega$ and $50.0\ \Omega$ resistors are in parallel and have an equivalent resistance of $16.67\ \Omega$. The equivalent network is given in Figure 26.59a. $\dfrac{1}{R_{eq}} = \dfrac{1}{100.0\ \Omega} + \dfrac{1}{23.05\ \Omega}$, so $R_{eq} = 18.7\ \Omega$.

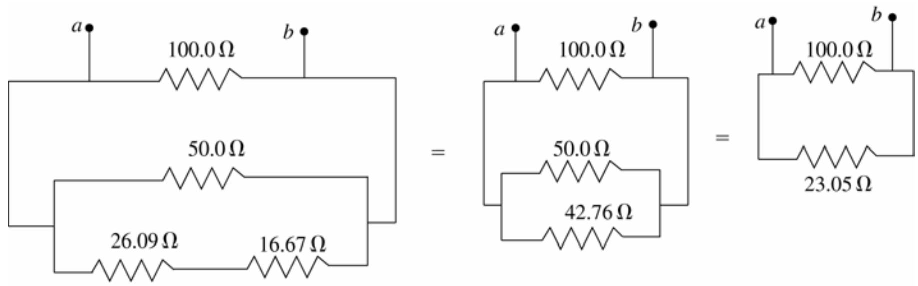


Figure 26.59a

Circuit (b): The $30.0\ \Omega$ and $45.0\ \Omega$ resistors are in parallel and have equivalent resistance $18.0\ \Omega$. The equivalent network is given in Figure 26.59b. $\frac{1}{R_{\text{eq}}} = \frac{1}{10.0\ \Omega} + \frac{1}{30.3\ \Omega}$, so $R_{\text{eq}} = 7.5\ \Omega$.

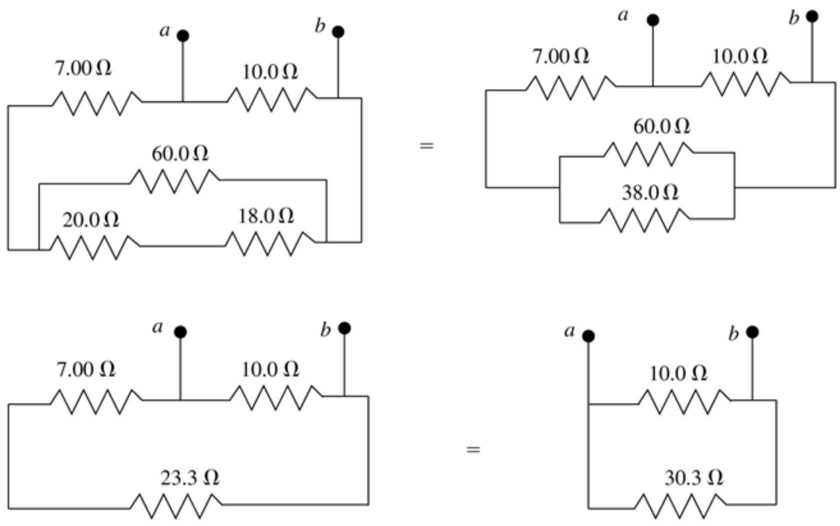


Figure 26.59b

EVALUATE: In circuit (a) the resistance along one path between *a* and *b* is $100.0\ \Omega$, but that is not the equivalent resistance between these points. A similar comment can be made about circuit (b).

26.60. **IDENTIFY:** Heat, which is generated in the resistor, melts the ice.

SET UP: Find the rate at which heat is generated in the 20.0-Ω resistor using $P = V^2/R$. Then use the heat of fusion of ice to find the rate at which the ice melts. The heat dH to melt a mass of ice dm is $dH = L_F\,dm$, where L_F is the latent heat of fusion. The rate at which heat enters the ice, dH/dt, is the power P in the resistor, so $P = L_F\,dm/dt$. Therefore the rate of melting of the ice is $dm/dt = P/L_F$.

EXECUTE: The equivalent resistance of the parallel branch is 5.00 Ω, so the total resistance in the circuit is 35.0 Ω. Therefore the total current in the circuit is $I_{\text{Total}} = (45.0\text{ V})/(35.0\ \Omega) = 1.286$ A. The potential difference across the 20.0-Ω resistor in the ice is the same as the potential difference across the parallel branch: $V_{\text{ice}} = I_{\text{Total}}R_p = (1.286\text{ A})(5.00\ \Omega) = 6.429$ V. The rate of heating of the ice is $P_{\text{ice}} = V_{\text{ice}}^2/R = (6.429\text{ V})^2/(20.0\ \Omega) = 2.066$ W. This power goes into to heat to melt the ice, so

$$dm/dt = P/L_F = (2.066\text{ W})/(3.34 \times 10^5\text{ J/kg}) = 6.19 \times 10^{-6}\text{ kg/s} = 6.19 \times 10^{-3}\text{ g/s}$$

EVALUATE: The melt rate is about 6 mg/s, which is not much. It would take 1000 s to melt just 6 g of ice.

26.61. **IDENTIFY:** Apply the junction rule to express the currents through the $5.00\ \Omega$ and $8.00\ \Omega$ resistors in terms of I_1, I_2 and I_3. Apply the loop rule to three loops to get three equations in the three unknown currents.

SET UP: The circuit is sketched in Figure 26.61.

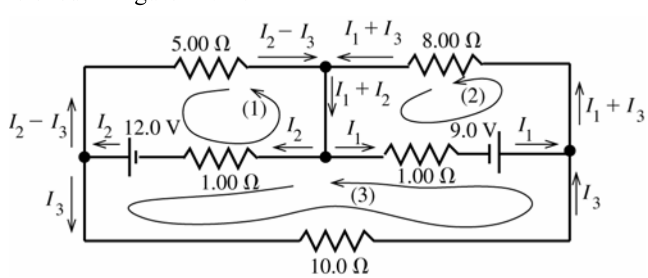


Figure 26.61

The current in each branch has been written in terms of I_1, I_2 and I_3 such that the junction rule is satisfied at each junction point.

EXECUTE: Apply the loop rule to loop (1).

$-12.0\text{ V} + I_2(1.00\ \Omega) + (I_2 - I_3)(5.00\ \Omega) = 0$

$I_2(6.00\ \Omega) - I_3(5.00\ \Omega) = 12.0\text{ V}$ eq.(1)

Apply the loop rule to loop (2).

$-I_1(1.00\ \Omega)+9.00\ \text{V}-(I_1+I_3)(8.00\ \Omega)=0$

$I_1(9.00\ \Omega)+I_3(8.00\ \Omega)=9.00\ \text{V}$ eq.(2)

Apply the loop rule to loop (3).

$-I_3(10.0\ \Omega)-9.00\ \text{V}+I_1(1.00\ \Omega)-I_2(1.00\ \Omega)+12.0\ \text{V}=0$

$-I_1(1.00\ \Omega)+I_2(1.00\ \Omega)+I_3(10.0\ \Omega)=3.00\ \text{V}$ eq.(3)

Eq.(1) gives $I_2=2.00\ \text{A}+\frac{5}{6}I_3$; eq.(2) gives $I_1=1.00\ \text{A}-\frac{8}{9}I_3$

Using these results in eq.(3) gives $-(1.00\ \text{A}-\frac{8}{9}I_3)(1.00\ \Omega)+(2.00\ \text{A}+\frac{5}{6}I_3)(1.00\ \Omega)+I_3(10.0\ \Omega)=3.00\ \text{V}$

$\left(\frac{16+15+180}{18}\right)I_3=2.00\ \text{A};\ I_3=\frac{18}{211}(2.00\ \text{A})=0.171\ \text{A}$

Then $I_2=2.00\ \text{A}+\frac{5}{6}I_3=2.00\ \text{A}+\frac{5}{6}(0.171\ \text{A})=2.14\ \text{A}$ and $I_1=1.00\ \text{A}-\frac{8}{9}I_3=1.00\ \text{A}-\frac{8}{9}(0.171\ \text{A})=0.848\ \text{A}$.

EVALUATE: We could check that the loop rule is satisfied for a loop that goes through the $5.00\ \Omega$, $8.00\ \Omega$ and $10.0\ \Omega$ resistors. Going around the loop clockwise: $-(I_2-I_3)(5.00\ \Omega)+(I_1+I_3)(8.00\ \Omega)+I_3(10.0\ \Omega)=$ $-9.85\ \text{V}+8.15\ \text{V}+1.71\ \text{V}$, which does equal zero, apart from rounding.

26.62. **IDENTIFY:** Apply the junction rule and the loop rule to the circuit.

SET UP: Because of the polarity of each emf, the current in the $7.00\ \Omega$ resistor must be in the direction shown in Figure 26.62a. Let I be the current in the 24.0 V battery.

EXECUTE: The loop rule applied to loop (1) gives: $+24.0\ \text{V}-(1.80\ \text{A})(7.00\ \Omega)-I(3.00\ \Omega)=0$. $I=3.80\ \text{A}$. The junction rule then says that the current in the middle branch is 2.00 A, as shown in Figure 26.62b. The loop rule applied to loop (2) gives: $+\mathcal{E}-(1.80\ \text{A})(7.00\ \Omega)+(2.00\ \text{A})(2.00\ \Omega)=0$ and $\mathcal{E}=8.6\ \text{V}$.

EVALUATE: We can check our results by applying the loop rule to loop (3) in Figure 26.62b: $+24.0\ \text{V}-\mathcal{E}-(2.00\ \text{A})(2.00\ \Omega)-(3.80\ \text{A})(3.00\ \Omega)=0$ and $\mathcal{E}=24.0\ \text{V}-4.0\ \text{V}-11.4\ \text{V}=8.6\ \text{V}$, which agrees with our result from loop (2).

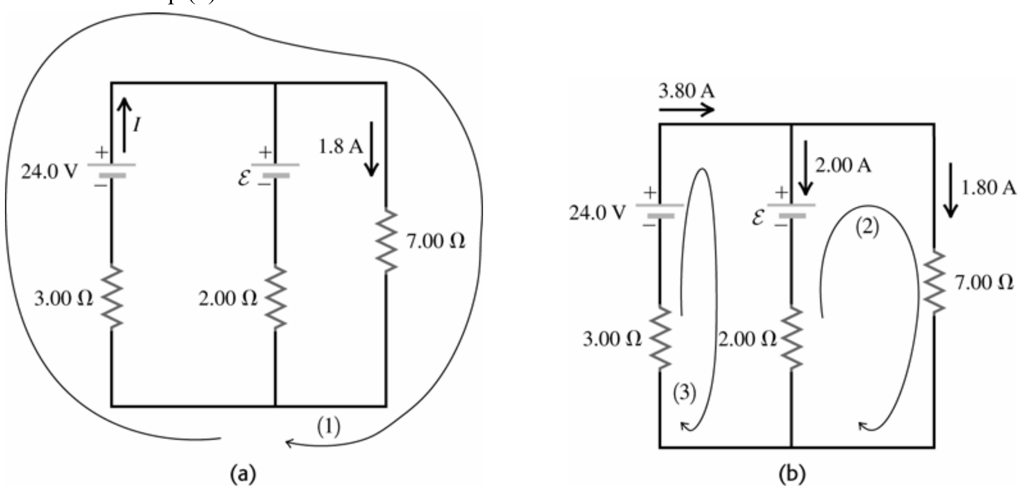


Figure 26.62

26.63. **IDENTIFY** and **SET UP:** The circuit is sketched in Figure 26.63.

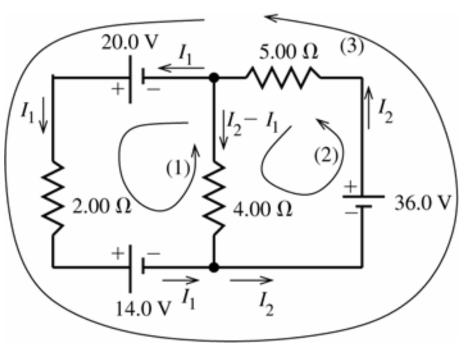


Figure 26.63

Two unknown currents I_1 (through the $2.00\ \Omega$ resistor) and I_2 (through the $5.00\ \Omega$ resistor) are labeled on the circuit diagram. The current through the $4.00\ \Omega$ resistor has been written as I_2-I_1 using the junction rule.

Apply the loop rule to loops (1) and (2) to get two equations for the unknown currents, I_1 and I_2. Loop (3) can then be used to check the results.

EXECUTE: loop (1): $+20.0\text{ V}-I_1(2.00\ \Omega)-14.0\text{ V}+(I_2-I_1)(4.00\ \Omega)=0$

$6.00I_1-4.00I_2=6.00\text{ A}$

$3.00I_1-2.00I_2=3.00\text{ A}$ eq.(1)

loop (2): $+36.0\text{ V}-I_2(5.00\ \Omega)-(I_2-I_1)(4.00\ \Omega)=0$

$-4.00I_1+9.00I_2=36.0\text{ A}$ eq.(2)

Solving eq. (1) for I_1 gives $I_1=1.00\text{ A}+\frac{2}{3}I_2$

Using this in eq.(2) gives $-4.00\left(1.00\text{ A}+\frac{2}{3}I_2\right)+9.00I_2=36.0\text{ A}$

$\left(-\frac{8}{3}+9.00\right)I_2=40.0\text{ A}$ and $I_2=6.32\text{ A}$.

Then $I_1=1.00\text{ A}+\frac{2}{3}I_2=1.00\text{ A}+\frac{2}{3}(6.32\text{ A})=5.21\text{ A}$.

In summary then

Current through the $2.00\ \Omega$ resistor: $I_1=5.21\text{ A}$.

Current through the $5.00\ \Omega$ resistor: $I_2=6.32\text{ A}$.

Current through the $4.00\ \Omega$ resistor: $I_2-I_1=6.32\text{ A}-5.21\text{ A}=1.11\text{ A}$.

EVALUATE: Use loop (3) to check. $+20.0\text{ V}-I_1(2.00\ \Omega)-14.0\text{ V}+36.0\text{ V}-I_2(5.00\ \Omega)=0$

$(5.21\text{ A})(2.00\ \Omega)+(6.32\text{ A})(5.00\ \Omega)=42.0\text{ V}$

$10.4\text{ V}+31.6\text{ V}=42.0\text{ V}$, so the loop rule is satisfied for this loop.

26.64. IDENTIFY: Apply the loop and junction rules.

SET UP: Use the currents as defined on the circuit diagram in Figure 26.64 and obtain three equations to solve for the currents.

EXECUTE: Left loop: $14-I_1-2(I_1-I_2)=0$ and $3I_1-2I_2=14$.

Top loop: $-2(I-I_1)+I_2+I_1=0$ and $-2I+3I_1+I_2=0$.

Bottom loop: $-(I-I_1+I_2)+2(I_1-I_2)-I_2=0$ and $-I+3I_1-4I_2=0$.

Solving these equations for the currents we find: $I=I_{\text{battery}}=10.0\text{ A}$; $I_1=I_{R_1}=6.0\text{ A}$; $I_2=I_{R_3}=2.0\text{ A}$.

So the other currents are: $I_{R_2}=I-I_1=4.0\text{ A}$; $I_{R_4}=I_1-I_2=4.0\text{ A}$; $I_{R_5}=I-I_1+I_2=6.0\text{ A}$.

(b) $R_{\text{eq}}=\dfrac{V}{I}=\dfrac{14.0\text{ V}}{10.0\text{ A}}=1.40\ \Omega$.

EVALUATE: It isn't possible to simplify the resistor network using the rules for resistors in series and parallel. But the equivalent resistance is still defined by $V=IR_{\text{eq}}$.

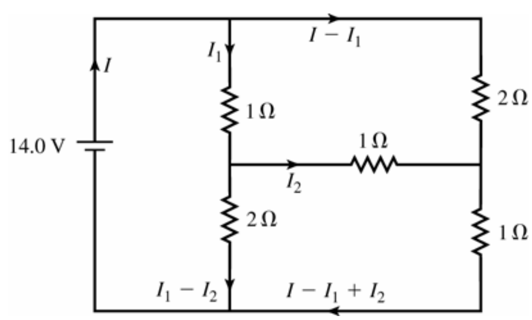


Figure 26.64

26.65. (a) IDENTIFY: Break the circuit between points *a* and *b* means no current in the middle branch that contains the $3.00\ \Omega$ resistor and the 10.0 V battery. The circuit therefore has a single current path. Find the current, so that potential drops across the resistors can be calculated. Calculate V_{ab} by traveling from *a* to *b*, keeping track of the potential changes along the path taken.

SET UP: The circuit is sketched in Figure 26.65a.

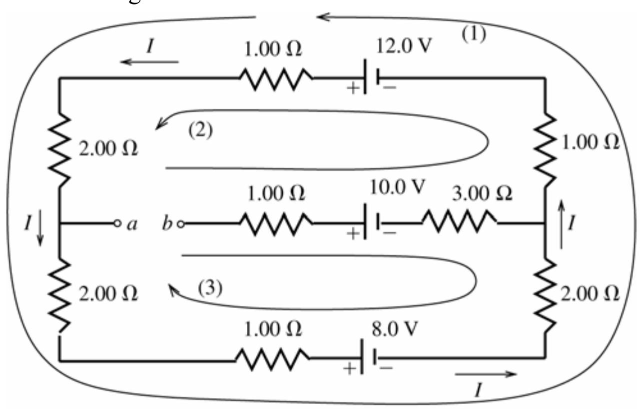


Figure 26.65a

EXECUTE: Apply the loop rule to loop (1).

$$+12.0\text{ V} - I(1.00\ \Omega + 2.00\ \Omega + 2.00\ \Omega + 1.00\ \Omega) - 8.0\text{ V} - I(2.00\ \Omega + 1.00\ \Omega) = 0$$

$$I = \frac{12.0\text{ V} - 8.0\text{ V}}{9.00\ \Omega} = 0.4444\text{ A.}$$

To find V_{ab} start at point b and travel to a, adding up the potential rises and drops. Travel on path (2) shown on the diagram. The $1.00\ \Omega$ and $3.00\ \Omega$ resistors in the middle branch have no current through them and hence no voltage across them. Therefore, $V_b - 10.0\text{ V} + 12.0\text{ V} - I(1.00\ \Omega + 1.00\ \Omega + 2.00\ \Omega) = V_a$; thus

$V_a - V_b = 2.0\text{ V} - (0.4444\text{ A})(4.00\ \Omega) = +0.22\text{ V}$ (point a is at higher potential)

EVALUATE: As a check on this calculation we also compute V_{ab} by traveling from b to a on path (3).

$$V_b - 10.0\text{ V} + 8.0\text{ V} + I(2.00\ \Omega + 1.00\ \Omega + 2.00\ \Omega) = V_a$$

$V_{ab} = -2.00\text{ V} + (0.4444\text{ A})(5.00\ \Omega) = +0.22\text{ V}$, which checks.

(b) IDENTIFY and SET UP: With points a and b connected by a wire there are three current branches, as shown in Figure 26.65b.

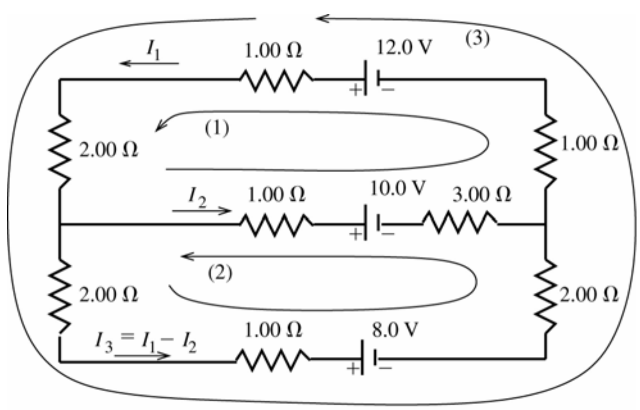


Figure 26.65b

The junction rule has been used to write the third current (in the 8.0 V battery) in terms of the other currents. Apply the loop rule to loops (1) and (2) to obtain two equations for the two unknowns I_1 and I_2.

EXECUTE: Apply the loop rule to loop (1).

$$12.0\text{ V} - I_1(1.00\ \Omega) - I_1(2.00\ \Omega) - I_2(1.00\ \Omega) - 10.0\text{ V} - I_2(3.00\ \Omega) - I_1(1.00\ \Omega) = 0$$

$$2.0\text{ V} - I_1(4.00\ \Omega) - I_2(4.00\ \Omega) = 0$$

$$(2.00\ \Omega)I_1 + (2.00\ \Omega)I_2 = 1.0\text{ V} \qquad \text{eq.(1)}$$

Apply the loop rule to loop (2).

$$-(I_1 - I_2)(2.00\ \Omega) - (I_1 - I_2)(1.00\ \Omega) - 8.0\text{ V} - (I_1 - I_2)(2.00\ \Omega) + I_2(3.00\ \Omega) + 10.0\text{ V} + I_2(1.00\ \Omega) = 0$$

$$2.0\text{ V} - (5.00\ \Omega)I_1 + (9.00\ \Omega)I_2 = 0 \qquad \text{eq.(2)}$$

Solve eq.(1) for I_2 and use this to replace I_2 in eq.(2).

$I_2 = 0.50\text{ A} - I_1$

$2.0\text{ V} - (5.00\ \Omega)I_1 + (9.00\ \Omega)(0.50\text{ A} - I_1) = 0$

$(14.0\ \Omega)I_1 = 6.50\text{ V}$ so $I_1 = (6.50\text{ V})/(14.0\ \Omega) = 0.464\text{ A}$

$I_2 = 0.500\text{ A} - 0.464\text{ A} = 0.036\text{ A}.$

The current in the 12.0 V battery is $I_1 = 0.464\text{ A}.$

EVALUATE: We can apply the loop rule to loop (3) as a check.

$+12.0\text{ V} - I_1(1.00\ \Omega + 2.00\ \Omega + 1.00\ \Omega) - (I_1 - I_2)(2.00\ \Omega + 1.00\ \Omega + 2.00\ \Omega) - 8.0\text{ V} = 4.0\text{ V} - 1.86\text{ V} - 2.14\text{ V} = 0,$ as it should.

26.66. IDENTIFY: Simplify the resistor networks as much as possible using the rule for series and parallel combinations of resistors. Then apply Kirchhoff's laws.

SET UP: First do the series/parallel reduction. This gives the circuit in Figure 26.66. The rate at which the $10.0\ \Omega$ resistor generates thermal energy is $P = I^2R$.

EXECUTE: Apply Kirchhoff's laws and solve for $\mathcal{E}$. $\Delta V_{\text{adefa}} = 0: -(20\ \Omega)(2\text{ A}) - 5\text{ V} - (20\ \Omega)I_2 = 0$.

This gives $I_2 = -2.25\text{ A}$. Then $I_1 + I_2 = 2\text{ A}$ gives $I_1 = 2\text{ A} - (-2.25\text{ A}) = 4.25\text{ A}$.

$\Delta V_{\text{abcdefa}} = 0$: $(15\ \Omega)(4.25\text{ A}) + \mathcal{E} - (20\ \Omega)(-2.25\text{ A}) = 0$. This gives $\mathcal{E} = -109\text{ V}$. Since $\mathcal{E}$ is calculated to be negative, its polarity should be reversed.

(b) The parallel network that contains the $10.0\ \Omega$ resistor in one branch has an equivalent resistance of $10\ \Omega$. The voltage across each branch of the parallel network is $V_{\text{par}} = RI = (10\ \Omega)(2\text{A}) = 20\text{ V}$. The current in the upper branch is $I = \frac{V}{R} = \frac{20\text{ V}}{30\ \Omega} = \frac{2}{3}\text{ A}$. $Pt = E$, so $I^2\ Rt = E$, where $E = 60.0\text{ J}$. $\left(\frac{2}{3}\text{A}\right)^2(10\ \Omega)t = 60\text{ J}$, and $t = 13.5\text{ s}$.

EVALUATE: For the $10.0\ \Omega$ resistor, $P = I^2R = 4.44\text{ W}$. The total rate at which electrical energy is inputed to the circuit in the emf is $(5.0\text{ V})(2.0\text{ A}) + (109\text{ V})(4.25\text{ A}) = 473\text{ J}$. Only a small fraction of the energy is dissipated in the $10.0\ \Omega$ resistor.

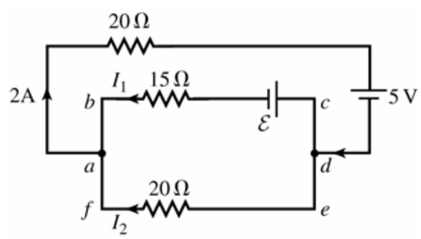


Figure 26.66

26.67. IDENTIFY: In Figure 26.67, points *a* and *c* are at the same potential and points *d* and *b* are at the same potential, so we can calculate V_{ab} by calculating V_{cd}. We know the current through the resistor that is between points *c* and *d*. We thus can calculate the terminal voltage of the 24.0 V battery without calculating the current through it.

SET UP:

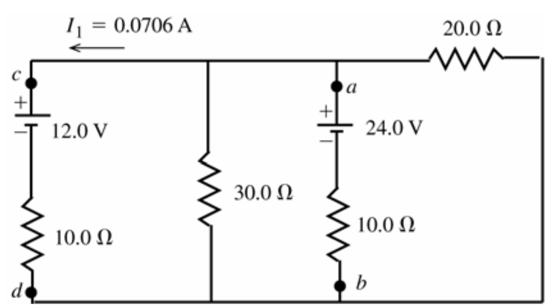


Figure 26.67

EXECUTE: $V_d + I_1(10.0\ \Omega) + 12.0\text{ V} = V_c$

$V_c - V_d = 12.7\text{ V};\ V_a - V_b = V_c - V_d = 12.7\text{ V}$

EVALUATE: The voltage across each parallel branch must be the same. The current through the 24.0 V battery must be $(24.0\text{ V} - 12.7\text{ V})/(10.0\ \Omega) = 1.13\text{ A}$ in the direction *b* to *a*.

26.68. **IDENTIFY:** The current through the $40.0\ \Omega$ resistor equals the current through the emf, and the current through each of the other resistors is less than or equal to this current. So, set $P_{40} = 1.00$ W and use this to solve for the current I through the emf. If $P_{40} = 1.00\text{ W}$, then P for each of the other resistors is less than 1.00 W.

SET UP: Use the equivalent resistance for series and parallel combinations to simplify the circuit.

EXECUTE: $I^2R = P$ gives $I^2(40\ \Omega) = 1\text{ W}$, and $I = 0.158\text{ A}$. Now use series / parallel reduction to simplify the circuit. The upper parallel branch is $6.38\ \Omega$ and the lower one is $25\ \Omega$. The series sum is now $126\ \Omega$. Ohm's law gives $\mathcal{E} = (126\ \Omega)(0.158\text{ A}) = 19.9\text{ V}$.

EVALUATE: The power input from the emf is $\mathcal{E}I = 3.14\text{ W}$, so nearly one-third of the total power is dissipated in the $40.0\ \Omega$ resistor.

26.69. **IDENTIFY** and **SET UP:** Simplify the circuit by replacing the parallel networks of resistors by their equivalents. In this simplified circuit apply the loop and junction rules to find the current in each branch.

EXECUTE: The $20.0\text{-}\Omega$ and $30.0\text{-}\Omega$ resistors are in parallel and have equivalent resistance $12.0\ \Omega$. The two resistors R are in parallel and have equivalent resistance $R/2$. The circuit is equivalent to the circuit sketched in Figure 26.69.

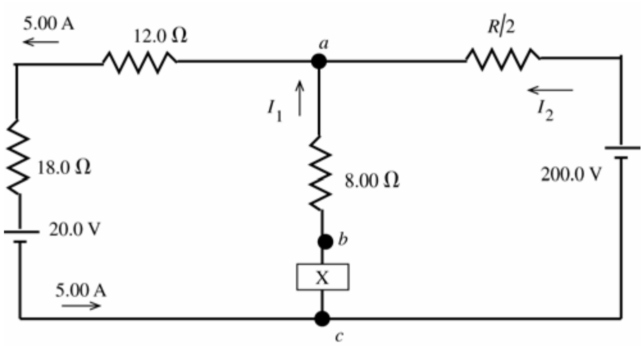


Figure 26.69

(a) Calculate V_{ca} by traveling along the branch that contains the 20.0 V battery, since we know the current in that branch.

$V_a - (5.00\text{ A})(12.0\ \Omega) - (5.00\text{ A})(18.0\ \Omega) - 20.0\text{ V} = V_c$

$V_a - V_c = 20.0\text{ V} + 90.0\text{ V} + 60.0\text{ V} = 170.0\text{ V}$

$V_b - V_a = V_{ab} = 16.0\text{ V}$

$X - V_{ba} = 170.0\text{ V so } X = 186.0\text{ V},$ with the upper terminal +

(b) $I_1 = (16.0\text{ V})/(8.0\ \Omega) = 2.00\text{ A}$

The junction rule applied to point a gives $I_2 + I_1 = 5.00\text{ A}$, so $I_2 = 3.00\text{ A}$. The current through the 200.0 V battery is in the direction from the − to the + terminal, as shown in the diagram.

(c) $200.0\text{ V} - I_2(R/2) = 170.0\text{ V}$

$(3.00\text{ A})(R/2) = 30.0\text{ V so } R = 20.0\ \Omega$

EVALUATE: We can check the loop rule by going clockwise around the outer circuit loop. This gives $+20.0\text{ V} + (5.00\text{ A})(18.0\ \Omega + 12.0\ \Omega) + (3.00\text{ A})(10.0\ \Omega) - 200.0\text{ V} = 20.0\text{ V} + 150.0\text{ V} + 30.0\text{ V} - 200.0\text{ V},$ which does equal zero.

26.70. **IDENTIFY:** $P_{\text{tot}} = \dfrac{V^2}{R_{\text{eq}}}$.

SET UP: Let R be the resistance of each resistor.

EXECUTE: When the resistors are in series, $R_{\text{eq}} = 3R$ and $P_{\text{s}} = \dfrac{V^2}{3R}$. When the resistors are in parallel, $R_{\text{eq}} = R/3$.

$$P_{\text{p}} = \frac{V^2}{R/3} = 3\frac{V^2}{R} = 9P_{\text{s}} = 9(27\text{ W}) = 243\text{ W}.$$

EVALUATE: In parallel, the voltage across each resistor is the full applied voltage V. In series, the voltage across each resistor is $V/3$ and each resistor dissipates less power.

26.71. **IDENTIFY** and **SET UP:** For part (a) use that the full emf is across each resistor. In part (b), calculate the power dissipated by the equivalent resistance, and in this expression express R_1 and R_2 in terms of P_1, P_2 and $\mathcal{E}$.

EXECUTE: $P_1 = \mathcal{E}^2/R_1$ so $R_1 = \mathcal{E}^2/P_1$

$P_2 = \mathcal{E}^2/R_2$ so $R_2 = \mathcal{E}^2/P_2$

(a) When the resistors are connected in parallel to the emf, the voltage across each resistor is $\mathcal{E}$ and the power dissipated by each resistor is the same as if only the one resistor were connected. $P_{tot} = P_1 + P_2$

(b) When the resistors are connected in series the equivalent resistance is $R_{eq} = R_1 + R_2$

$$P_{tot} = \frac{\mathcal{E}^2}{R_1 + R_2} = \frac{\mathcal{E}^2}{\mathcal{E}^2/P_1 + \mathcal{E}^2/P_2} = \frac{P_1 P_2}{P_1 + P_2}$$

EVALUATE: The result in part (b) can be written as $\frac{1}{P_{tot}} = \frac{1}{P_1} + \frac{1}{P_2}$. Our results are that for parallel the powers add and that for series the reciprocals of the power add. This is opposite the result for combining resistance. Since $P = \mathcal{E}^2/R$ tells us that P is proportional to $1/R$, this makes sense.

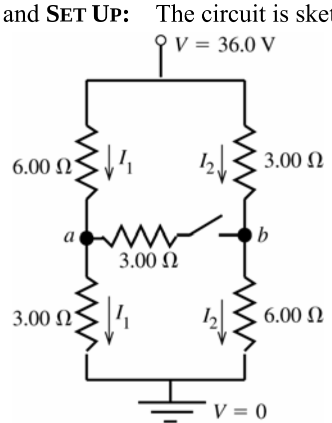
26.72. **IDENTIFY** and **SET UP:** Just after the switch is closed the charge on the capacitor is zero, the voltage across the capacitor is zero and the capacitor can be replaced by a wire in analyzing the circuit. After a long time the current to the capacitor is zero, so the current through R_3 is zero. After a long time the capacitor can be replaced by a break in the circuit.

EXECUTE: **(a)** Ignoring the capacitor for the moment, the equivalent resistance of the two parallel resistors is $\frac{1}{R_{eq}} = \frac{1}{6.00\ \Omega} + \frac{1}{3.00\ \Omega} = \frac{3}{6.00\ \Omega}$; $R_{eq} = 2.00\ \Omega$. In the absence of the capacitor, the total current in the circuit (the current through the $8.00\ \Omega$ resistor) would be $i = \frac{\mathcal{E}}{R} = \frac{42.0\text{ V}}{8.00\ \Omega + 2.00\ \Omega} = 4.20\text{ A}$, of which $2/3$, or 2.80 A, would go through the $3.00\ \Omega$ resistor and $1/3$, or 1.40 A, would go through the $6.00\ \Omega$ resistor. Since the current through the capacitor is given by $i = \frac{V}{R}e^{-t/RC}$, at the instant $t = 0$ the circuit behaves as through the capacitor were not present, so the currents through the various resistors are as calculated above.

(b) Once the capacitor is fully charged, no current flows through that part of the circuit. The $8.00\ \Omega$ and the $6.00\ \Omega$ resistors are now in series, and the current through them is $i = \mathcal{E}/R = (42.0\text{ V})/(8.00\ \Omega + 6.00\ \Omega) = 3.00\text{ A}$. The voltage drop across both the $6.00\ \Omega$ resistor and the capacitor is thus $V = iR = (3.00\text{ A})(6.00\ \Omega) = 18.0\text{ V}$. (There is no current through the $3.00\ \Omega$ resistor and so no voltage drop across it.) The charge on the capacitor is $Q = CV = (4.00\times10^{-6}\text{ F})(18.0\text{ V}) = 7.2\times10^{-5}\text{ C}$.

EVALUATE: The equivalent resistance of R_2 and R_3 in parallel is less than R_3, so initially the current through R_1 is larger than its value after a long time has elapsed.

26.73. **(a) IDENTIFY** and **SET UP:** The circuit is sketched in Figure 26.73a.



With the switch open there is no current through it and there are only the two currents I_1 and I_2 indicated in the sketch.

Figure 26.73a

The potential drop across each parallel branch is 36.0 V. Use this fact to calculate I_1 and I_2. Then travel from point a to point b and keep track of the potential rises and drops in order to calculate V_{ab}.

EXECUTE: $-I_1(6.00\ \Omega + 3.00\ \Omega) + 36.0\ \text{V} = 0$

$$I_1 = \frac{36.0\ \text{V}}{6.00\ \Omega + 3.00\ \Omega} = 4.00\ \text{A}$$

$-I_2(3.00\ \Omega + 6.00\ \Omega) + 36.0\ \text{V} = 0$

$$I_2 = \frac{36.0\ \text{V}}{3.00\ \Omega + 6.00\ \Omega} = 4.00\ \text{A}$$

To calculate $V_{ab} = V_a - V_b$ start at point *b* and travel to point *a*, adding up all the potential rises and drops along the way. We can do this by going from *b* up through the $3.00\ \Omega$ resistor:

$V_b + I_2(3.00\ \Omega) - I_1(6.00\ \Omega) = V_a$

$V_a - V_b = (4.00\ \text{A})(3.00\ \Omega) - (4.00\ \text{A})(6.00\ \Omega) = 12.0\ \text{V} - 24.0\ \text{V} = -12.0\ \text{V}$

$V_{ab} = -12.0\ \text{V}$ (point *a* is 12.0 V lower in potential than point *b*)

EVALUATE: Alternatively, we can go from point *b* down through the $6.00\ \Omega$ resistor.

$V_b - I_2(6.00\ \Omega) + I_1(3.00\ \Omega) = V_a$

$V_a - V_b = -(4.00\ \text{A})(6.00\ \Omega) + (4.00\ \text{A})(3.00\ \Omega) = -24.0\ \text{V} + 12.0\ \text{V} = -12.0\ \text{V},$ which checks.

(b) IDENTIFY: Now there are multiple current paths, as shown in Figure 26.73b. Use junction rule to write the current in each branch in terms of three unknown currents I_1, I_2, and I_3. Apply the loop rule to three loops to get three equations for the three unknowns. The target variable is I_3, the current through the switch. R_{eq} is calculated from $V = IR_{eq}$, where *I* is the total current that passes through the network.

SET UP:

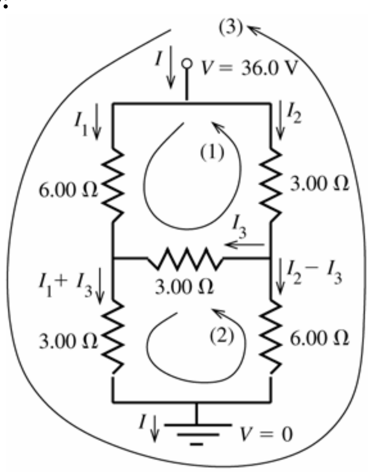


Figure 26.73b

The three unknown currents $I_1, I_2,$ and I_3 are labeled on Figure 26.73b.

EXECUTE: Apply the loop rule to loops (1), (2), and (3).

<u>loop (1):</u> $-I_1(6.00\ \Omega) + I_3(3.00\ \Omega) + I_2(3.00\ \Omega) = 0$

$I_2 = 2I_1 - I_3$ eq.(1)

<u>loop (2):</u> $-(I_1 + I_3)(3.00\ \Omega) + (I_2 - I_3)(6.00\ \Omega) - I_3(3.00\ \Omega) = 0$

$6I_2 - 12I_3 - 3I_1 = 0$ so $2I_2 - 4I_3 - I_1 = 0$

Use eq(1) to replace I_2:

$4I_1 - 2I_3 - 4I_3 - I_1 = 0$

$3I_1 = 6I_3$ and $I_1 = 2I_3$ eq.(2)

<u>loop (3)</u> (This loop is completed through the battery [not shown], in the direction from the $-$ to the $+$ terminal.):

$-I_1(6.00\ \Omega) - (I_1 + I_3)(3.00\ \Omega) + 36.0\ \text{V} = 0$

$9I_1 + 3I_3 = 36.0\ \text{A}$ and $3I_1 + I_3 = 12.0\ \text{A}$ eq.(3)

Use eq.(2) in eq.(3) to replace I_1:

$3(2I_3) + I_3 = 12.0\ \text{A}$

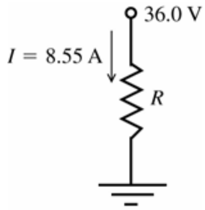
$I_3 = 12.0\ \text{A}/7 = 1.71\ \text{A}$

$I_1 = 2I_3 = 3.42\ \text{A}$

$I_2 = 2I_1 - I_3 = 2(3.42\ \text{A}) - 1.71\ \text{A} = 5.13\ \text{A}$

The current through the switch is $I_3 = 1.71\ \text{A}$.

(c) From the results in part (a) the current through the battery is $I = I_1 + I_2 = 3.42\text{ A} + 5.13\text{ A} = 8.55\text{ A}$. The equivalent circuit is a single resistor that produces the same current through the 36.0 V battery, as shown in Figure 26.73c.



$$-IR + 36.0\text{ V} = 0$$
$$R = \frac{36.0\text{ V}}{I} = \frac{36.0\text{ V}}{8.55\text{ A}} = 4.21\ \Omega$$

Figure 26.73c

EVALUATE: With the switch open (part a), point *b* is at higher potential than point *a*, so when the switch is closed the current flows in the direction from *b* to *a*. With the switch closed the circuit cannot be simplified using series and parallel combinations but there is still an equivalent resistance that represents the network.

26.74. **IDENTIFY:** With *S* open and after equilibrium has been reached, no current flows and the voltage across each capacitor is 18.0 V. When *S* is closed, current *I* flows through the $6.00\ \Omega$ and $3.00\ \Omega$ resistors.

SET UP: With the switch closed, *a* and *b* are at the same potential and the voltage across the $6.00\ \Omega$ resistor equals the voltage across the $6.00\ \mu\text{F}$ capacitor and the voltage is the same across the $3.00\ \mu\text{F}$ capacitor and $3.00\ \Omega$ resistor.

EXECUTE: **(a)** With an open switch: $V_{ab} = \mathcal{E} = 18.0\text{ V}$.

(b) Point *a* is at a higher potential since it is directly connected to the positive terminal of the battery.

(c) When the switch is closed $18.0\text{ V} = I(6.00\ \Omega + 3.00\ \Omega)$. $I = 2.00\text{ A}$ and $V_b = (2.00\text{ A})(3.00\ \Omega) = 6.00\text{ V}$.

(d) Initially the capacitor's charges were $Q_3 = CV = (3.00\times10^{-6}\text{ F})(18.0\text{ V}) = 5.40\times10^{-5}\text{ C}$ and

$Q_6 = CV = (6.00\times10^{-6}\text{ F})(18.0\text{ V}) = 1.08\times10^{-4}\text{ C}$. After the switch is closed

$Q_3 = CV = (3.00\times10^{-6}\text{ F})(18.0\text{ V} - 12.0\text{ V}) = 1.80\times10^{-5}\text{ C}$ and

$Q_6 = CV = (6.00\times10^{-6}\text{ F})(18.0\text{ V} - 6.0\text{ V}) = 7.20\times10^{-5}\text{ C}$. Both capacitors lose $3.60\times10^{-5}\text{ C}$.

EVALUATE: The voltage across each capacitor decreases when the switch is closed, because there is then current through each resistor and therefore a potential drop across each resistor.

26.75. **IDENTIFY:** The current through the galvanometer for full-scale deflection is 0.0200 A. For each connection, there are two parallel branches and the voltage across each is the same.

SET UP: The sum of the two currents in the parallel branches for each connection equals the current into the meter for that connection.

EXECUTE: From the circuit we can derive three equations:

(i) $(R_1 + R_2 + R_3)(0.100\text{ A} - 0.0200\text{ A}) = (48.0\ \Omega)(0.0200\text{ A})$ and $R_1 + R_2 + R_3 = 12.0\ \Omega$.

(ii) $(R_1 + R_2)(1.00\text{ A} - 0.0200\text{ A}) = (48.0\ \Omega + R_3)(0.0200\text{ A})$ and $R_1 + R_2 - 0.0204R_3 = 0.980\ \Omega$.

(iii) $R_1(10.0\text{ A} - 0.0200\text{ A}) = (48.0\ \Omega + R_2 + R_3)(0.0200\text{ A})$ and $R_1 - 0.002R_2 - 0.002R_3 = 0.096\ \Omega$.

From (i) and (ii), $R_3 = 10.8\ \Omega$. From (ii) and (iii), $R_2 = 1.08\ \Omega$. Therefore, $R_1 = 0.12\ \Omega$.

EVALUATE: For the 0.100 A setting the circuit consists of $48.0\ \Omega$ and $R_1 + R_2 + R_3 = 12.0\ \Omega$ in parallel and the equivalent resistance of the meter is $9.6\ \Omega$. For each of the other two settings the equivalent resistance of the meter is less than $9.6\ \Omega$.

26.76. **IDENTIFY:** In each case the sum of the voltage drops across the resistors in the circuit must equal the full-scale voltage reading. The resistors are in series so the total resistance is the sum of the resistances in the circuit.

SET UP: For each range setting the circuit has the form shown in Figure 26.76.

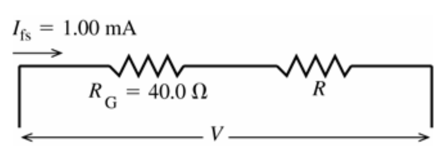


Figure 26.76

EXECUTE: <u>3.00 V</u>

For $V = 3.00\text{ V}$, $R = R_1$ and the total meter resistance R_m is $R_m = R_G + R_1$.

$$V = I_{fs}R_m \text{ so } R_m = \frac{V}{I_{fs}} = \frac{3.00\text{ V}}{1.00\times10^{-3}\text{ A}} = 3.00\times10^3\ \Omega.$$

$R_m = R_G + R_1$ so $R_1 = R_m - R_G = 3.00\times10^3\ \Omega - 40.0\ \Omega = 2960\ \Omega$

15.0 V

For $V = 15.0\text{ V}$, $R = R_1 + R_2$ and the total meter resistance is $R_m = R_G + R_1 + R_2$.

$V = I_{fs}R_m$ so $R_m = \dfrac{V}{I_{fs}} = \dfrac{15.0\text{ V}}{1.00\times10^{-3}\text{ A}} = 1.50\times10^4\ \Omega$.

$R_2 = R_m - R_G - R_1 = 1.50\times10^4\ \Omega - 40.0\ \Omega - 2960\ \Omega = 1.20\times10^4\ \Omega$

150 V

For $V = 150\text{ V}$, $R = R_1 + R_2 + R_3$ and the total meter resistance is $R_m = R_G + R_1 + R_2 + R_3$.

$V = I_{fs}R_m$ so $R_m = \dfrac{V}{I_{fs}} = \dfrac{150\text{ V}}{1.00\times10^{-3}\text{ A}} = 1.50\times10^5\ \Omega$.

$R_3 = R_m - R_G - R_1 - R_2 = 1.50\times10^5\ \Omega - 40.0\ \Omega - 2960\ \Omega - 1.20\times10^4\ \Omega = 1.35\times10^5\ \Omega$.

EVALUATE: The greater the total resistance in series inside the meter the greater the potential difference between the two connections to the meter when the same 1.00 mA current flows through it.

26.77. **IDENTIFY:** Connecting the voltmeter between point *b* and ground gives a resistor network and we can solve for the current through each resistor. The voltmeter reading equals the potential drop across the $200\text{ k}\Omega$ resistor.

SET UP: For resistors in parallel, $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2}$. For resistors in series, $R_{eq} = R_1 + R_2$.

EXECUTE: **(a)** $R_{eq} = 100\text{ k}\Omega + \left(\dfrac{1}{200\text{ k}\Omega} + \dfrac{1}{50\text{ k}\Omega}\right)^{-1} = 140\text{ k}\Omega$. The total current is $I = \dfrac{0.400\text{ kV}}{140\text{ k}\Omega} = 2.86\times10^{-3}\text{ A}$.

The voltage across the $200\text{ k}\Omega$ resistor is $V_{200\text{k}\Omega} = IR = (2.86\times10^{-3}\text{ A})\left(\dfrac{1}{200\text{ k}\Omega} + \dfrac{1}{50\text{ k}\Omega}\right)^{-1} = 114.4\text{ V}$.

(b) If $V_R = 5.00\times10^6\ \Omega$, then we carry out the same calculations as above to find $R_{eq} = 292\text{ k}\Omega$, $I = 1.37\times10^{-3}\text{ A}$ and $V_{200\text{k}\Omega} = 263\text{ V}$.

(c) If $V_R = \infty$, then we find $R_{eq} = 300\text{ k}\Omega$, $I = 1.33\times10^{-3}\text{ A}$ and $V_{200\text{k}\Omega} = 266\text{ V}$.

EVALUATE: When a voltmeter of finite resistance is connected to a circuit, current flows through the voltmeter and the presence of the voltmeter alters the currents and voltages in the original circuit. The effect of the voltmeter on the circuit decreases as the resistance of the voltmeter increases.

26.78. **IDENTIFY:** The circuit consists of two resistors in series with 110 V applied across the series combination.

SET UP: The circuit resistance is $30\text{ k}\Omega + R$. The voltmeter reading of 68 V is the potential across the voltmeter terminals, equal to $I(30\text{ k}\Omega)$.

EXECUTE: $I = \dfrac{110\text{ V}}{(30\text{ k}\Omega + R)}$. $I(30\text{ k}\Omega) = 68\text{ V}$ gives $(68\text{ V})(30\text{ k}\Omega + R) = (110\text{ V})30\text{ k}\Omega$ and $R = 18.5\text{ k}\Omega$. .

EVALUATE: This is a method for measuring large resistances.

26.79. **IDENTIFY** and **SET UP:** Zero current through the galvanometer means the current I_1 through *N* is also the current through *M* and the current I_2 through *P* is the same as the current through *X*. And it means that points *b* and *c* are at the same potential, so $I_1N = I_2P$.

EXECUTE: **(a)** The voltage between points *a* and *d* is $\mathcal{E}$, so $I_1 = \dfrac{\mathcal{E}}{N+M}$ and $I_2 = \dfrac{\mathcal{E}}{P+X}$. Using these expressions in $I_1N = I_2P$ gives $\dfrac{\mathcal{E}}{N+M}N = \dfrac{\mathcal{E}}{P+X}P$. $N(P+X) = P(N+M)$. $NX = PM$ and $X = MP/N$.

(b) $X = \dfrac{MP}{N} = \dfrac{(850.0\ \Omega)(33.48\ \Omega)}{15.00\ \Omega} = 1897\ \Omega$

EVALUATE: The measurement of *X* does not require that we know the value of the emf.

26.80. **IDENTIFY:** Add resistors in series and parallel with the second galvanometer, so that the equivalent resistance is $65.0\ \Omega$ and so that for a current of 1.50 mA into the device the current through the galvanometer is $3.60\ \mu\text{A}$.

SET UP: In order for the second galvanometer to give the same full-scale deflection and to have the same resistance as the first, we need two additional resistances as shown in Figure 26.80.

EXECUTE: For $3.60\ \mu\text{A}$ through *R* the current through R_1 is 1.496 mA. *R* and R_1 are in parallel so have equal voltages: $(3.6\ \mu\text{A})(38.0\ \Omega) = (1.496\text{ mA})R_1$ and $R_1 = 91.4\text{ m}\Omega$. And for the total resistance to be $65.0\ \Omega$:

$$65.0\ \Omega = R_2 + \left(\frac{1}{38.0\ \Omega} + \frac{1}{0.0914\ \Omega}\right)^{-1} \text{ and } R_2 = 64.9\ \Omega.$$

EVALUATE: Adding R_1 in parallel lowers the equivalent resistance so R_2 must be added in series to raise the equivalent resistance to $65.0\ \Omega$.

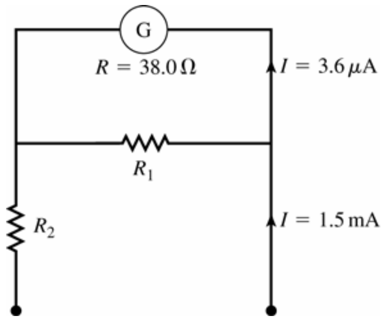


Figure 26.80

26.81. IDENTIFY and SET UP: Without the meter, the circuit consists of the two resistors in series. When the meter is connected, its resistance is added to the circuit in parallel with the resistor it is connected across.

(a) EXECUTE: $I = I_1 = I_2$

$$I = \frac{90.0\ \text{V}}{R_1 + R_2} = \frac{90.0\ \text{V}}{224\ \Omega + 589\ \Omega} = 0.1107\ \text{A}$$

$$V_1 = I_1 R_1 = (0.1107\ \text{A})(224\ \Omega) = 24.8\ \text{V};\ V_2 = I_2 R_2 = (0.1107\ \text{A})(589\ \Omega) = 65.2\ \text{V}$$

(b) SET UP: The resistor network is sketched in Figure 26.81a.

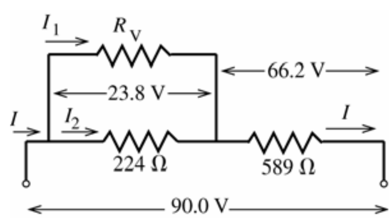


Figure 26.81c

The voltmeter reads the potential difference across its terminals, which is 23.8 V. If we can find the current I_1 through the voltmeter then we can use Ohm's law to find its resistance.

EXECUTE: The voltage drop across the $589\ \Omega$ resistor is $90.0\ \text{V} - 23.8\ \text{V} = 66.2\ \text{V}$, so

$I = \dfrac{V}{R} = \dfrac{66.2\ \text{V}}{589\ \Omega} = 0.1124\ \text{A}$. The voltage drop across the $224\ \Omega$ resistor is 23.8 V, so $I_2 = \dfrac{V}{R} = \dfrac{23.8\ \text{V}}{224\ \Omega} = 0.1062\ \text{A}$.

Then $I = I_1 + I_2$ gives $I_1 = I - I_2 = 0.1124\ \text{A} - 0.1062\ \text{A} = 0.0062\ \text{A}$. $R_V = \dfrac{V}{I_1} = \dfrac{23.8\ \text{V}}{0.0062\ \text{A}} = 3840\ \Omega$

(c) SET UP: The circuit with the voltmeter connected is sketched in Figure 26.81b.

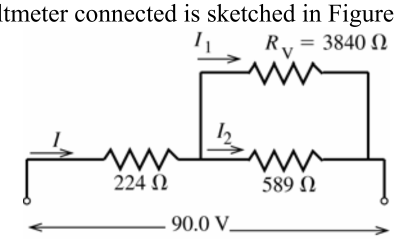


Figure 26.81b

EXECUTE: Replace the two resistors in parallel by their equivalent, as shown in Figure 26.81c.

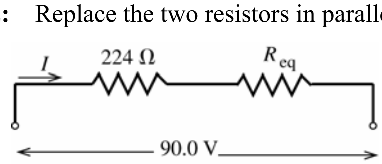


Figure 26.81c

$$\frac{1}{R_{eq}} = \frac{1}{3840\ \Omega} + \frac{1}{589\ \Omega};$$

$$R_{eq} = \frac{(3840\ \Omega)(589\ \Omega)}{3840\ \Omega + 589\ \Omega} = 510.7\ \Omega$$

$$I = \frac{90.0\ \text{V}}{224\ \Omega + 510.7\ \Omega} = 0.1225\ \text{A}$$

The potential drop across the $224\ \Omega$ resistor then is $IR = (0.1225\ \text{A})(224\ \Omega) = 27.4\ \text{V}$, so the potential drop across the $589\ \Omega$ resistor and across the voltmeter (what the voltmeter reads) is $90.0\ \text{V} - 27.4\ \text{V} = 62.6\ \text{V}$.

(d) EVALUATE: No, any real voltmeter will draw some current and thereby reduce the current through the resistance whose voltage is being measured. Thus the presence of the voltmeter connected in parallel with the resistance lowers the voltage drop across that resistance. The resistance of the voltmeter is only about a factor of ten larger than the resistances in the circuit, so the voltmeter has a noticeable effect on the circuit.

26.82. **IDENTIFY:** Just after the connection is made, $q = 0$ and the voltage across the capacitor is zero. After a long time $i = 0$.

SET UP: The rate at which the resistor dissipates electrical energy is $P_R = V^2/R$, where V is the voltage across the resistor. The energy stored in the capacitor is $q^2/2C$. The power output of the source is $P_\varepsilon = \mathcal{E}i$.

EXECUTE: **(a)** (i) $P_R = \dfrac{V^2}{R} = \dfrac{(120\text{ V})^2}{4.26\,\Omega} = 3380\text{ W}$. (ii) $P_C = \dfrac{dU}{dt} = \dfrac{1}{2C}\dfrac{d(q^2)}{dt} = \dfrac{iq}{C} = 0.$

(iii) $P_\varepsilon = \mathcal{E}I = (120\text{ V})\dfrac{120\text{ V}}{4.26\,\Omega} = 3380\text{ W}$.

(b) After a long time, $i = 0$, so $P_R = 0$, $P_C = 0$, $P_\varepsilon = 0$.

EVALUATE: Initially all the power output of the source is dissipated in the resistor. After a long time energy is stored in the capacitor but the amount stored isn't changing.

26.83. **IDENTIFY:** Apply the loop rule to the circuit. The initial current determines R. We can then use the time constant to calculate C.

SET UP: The circuit is sketched in Figure 26.83.

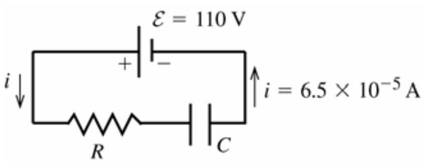


Figure 26.83

Initially, the charge of the capacitor is zero, so by $v = q/C$ the voltage across the capacitor is zero.

EXECUTE: The loop rule therefore gives $\mathcal{E} - iR = 0$ and $R = \dfrac{\mathcal{E}}{i} = \dfrac{110\text{ V}}{6.5\times10^{-5}\text{ A}} = 1.7\times10^{6}\ \Omega$

The time constant is given by $\tau = RC$ (Eq.26.14), so $C = \dfrac{\tau}{R} = \dfrac{6.2\text{ s}}{1.7\times10^{6}\ \Omega} = 3.6\ \mu\text{F}.$

EVALUATE: The resistance is large so the initial current is small and the time constant is large.

26.84. **IDENTIFY:** The energy stored in a capacitor is $U = q^2/2C$. The electrical power dissipated in the resistor is $P = i^2R$.

SET UP: For a discharging capacitor, $i = -\dfrac{q}{RC}$.

EXECUTE: **(a)** $U_0 = \dfrac{Q_0^2}{2C} = \dfrac{(0.0081\text{ C})^2}{2(4.62\times10^{-6}\text{ F})} = 7.10\text{ J}.$

(b) $P_0 = I_0^2R = \left(\dfrac{Q_0}{RC}\right)^2 R = \dfrac{(0.0081\text{ C})^2}{(850\,\Omega)(4.62\times10^{-6}\text{ F})^2} = 3616\text{ W}$

(c) When $U = \dfrac{1}{2}U_0 = \dfrac{1}{2}\dfrac{Q_0^2}{2C}$, $Q = \dfrac{Q_0}{\sqrt{2}}$. This gives $P = \left(\dfrac{Q}{RC}\right)^2 R = \dfrac{1}{2}\left(\dfrac{Q_0}{RC}\right)^2 R = \dfrac{1}{2}P_0 = 1808\text{ W}.$

EVALUATE: All the energy originally stored in the capacitor is dissipated as current flow through the resistor.

26.85. **IDENTIFY:** $q = Q_0 e^{-t/RC}$. The time constant is $\tau = RC$.

SET UP: The charge of one electron has magnitude $e = 1.60\times10^{-19}\text{ C}$.

EXECUTE: **(a)** We will say that a capacitor is discharged if its charge is less than that of one electron. The time this takes is then given by $q = Q_0 e^{-t/RC}$, so $t = RC\ln(Q_0/e) = (6.7\times10^5\ \Omega)(9.2\times10^{-7}\text{ F})\ln(7.0\times10^{-6}\text{ C}/1.6\times10^{-19}\text{ C}) = 19.36\text{ s}$, or 31.4 time constants.

EVALUATE: **(b)** As shown in part (a), $t = \tau\ln(Q_0/q)$ and so the number of time constants required to discharge the capacitor is independent of R and C, and depends only on the initial charge.

26.86. **IDENTIFY:** The energy changes exponentially, but it does not obey exactly the same equation as the charge since it is proportional to the *square* of the charge.

(a) SET UP: For charging, $U = Q^2/2C = (Q_0\, e^{-t/RC})^2/2C = U_0\, e^{-2t/RC}$.

EXECUTE: To reduce the energy to $1/e$ of its initial value:

$$U_0/e = U_0 e^{-2t/RC}$$

$$t = RC/2$$

(b) SET UP: For discharging, $U = Q^2/2C = [Q_0(1-e^{-t/RC})]^2/2C = U_{\max}(1-e^{-t/RC})^2$

EXECUTE: To reach $1/e$ of the maximum energy, $U_{\max}/e = U_{\max}(1-e^{-t/RC})^2$ and $t = -RC\ln\left(1-\dfrac{1}{\sqrt{e}}\right)$.

EVALUATE: The time to reach $1/e$ of the maximum energy is not the same as the time to discharge to $1/e$ of the maximum energy.

26.87. IDENTIFY and **SET UP:** For parts (a) and (b) evaluate the integrals as specified in the problem. The current as a function of time is given by Eq.(26.13) $i = \dfrac{\mathcal{E}}{R}e^{-t/RC}$. The energy stored in the capacitor is given by $Q^2/2C$.

EXECUTE: **(a)** $P = \mathcal{E}i$

The total energy supplied by the battery is $\int_0^\infty Pdt = \int_0^\infty \mathcal{E}idt = \left(\mathcal{E}^2/R\right)\int_0^\infty e^{-t/RC}dt = \left(\mathcal{E}^2/R\right)\left[-RCe^{-t/RC}\right]_0^\infty = C\mathcal{E}^2.$

(b) $P = i^2R$

The total energy dissipated in the resistor is

$\int_0^\infty Pdt = \int_0^\infty i^2Rdt = \left(\mathcal{E}^2/R\right)\int_0^\infty e^{-2t/RC}dt = \left(\mathcal{E}^2/R\right)\left[-\left(RC/2\right)e^{-2t/RC}\right]_0^\infty = \tfrac{1}{2}C\mathcal{E}^2.$

(c) The final charge on the capacitor is $Q = C\mathcal{E}$. The energy stored is $U = Q^2/(2C) = \tfrac{1}{2}C\mathcal{E}^2$. The final energy stored in the capacitor $\left(\tfrac{1}{2}C\mathcal{E}^2\right) =$ total energy supplied by the battery $\left(C\mathcal{E}^2\right) -$ energy dissipated in the resistor $\left(\tfrac{1}{2}C\mathcal{E}^2\right)$

(d) EVALUATE: $\tfrac{1}{2}$ of the energy supplied by the battery is stored in the capacitor. This fraction is independent of R. The other $\tfrac{1}{2}$ of the energy supplied by the battery is dissipated in the resistor. When R is small the current initially is large but dies away quickly. When R is large the current initially is small but lasts longer.

26.88. IDENTIFY: $E = \int_0^\infty Pdt$. The energy stored in a capacitor is $U = q^2/2C$.

SET UP: $i = -\dfrac{Q_0}{RC}e^{-t/RC}$

EXECUTE: $i = -\dfrac{Q_0}{RC}e^{-t/RC}$ gives $P = i^2R = \dfrac{Q_0{}^2}{RC^2}e^{-2t/RC}$ and $E = \dfrac{Q_0{}^2}{RC^2}\int_0^\infty e^{-2t/RC}dt = \dfrac{Q_0{}^2}{RC^2}\dfrac{RC}{2} = \dfrac{Q_0{}^2}{2C} = U_0$.

EVALUATE: Increasing the energy stored in the capacitor increases current through the resistor as the capacitor discharges.

26.89. IDENTIFY and **SET UP:**

EXECUTE: **(a)** Using Kirchhoff's Rules on the circuit we find:

Left loop: $92 - 140I_1 - 210I_2 + 55 = 0 \Rightarrow 147 - 140I_1 - 210I_2 = 0$.

Right loop: $57 - 35I_3 - 210I_2 + 55 = 0 \Rightarrow 112 - 210I_2 - 35I_3 = 0$.

Junction rule: $I_1 - I_2 + I_3 = 0$.

Solving for the three currents we have: $I_1 = 0.300\text{ A}$, $I_2 = 0.500\text{ A}$, $I_3 = 0.200\text{ A}$.

(b) Leaving only the 92-V battery in the circuit:

Left loop: $92 - 140I_1 - 210I_2 = 0$. Right loop: $-35I_3 - 210I_2 = 0$.

Junction rule: $I_1 - I_2 + I_3 = 0$. Solving for the three currents:

$$I_1 = 0.541\text{ A},\qquad I_2 = 0.077\text{ A},\qquad I_3 = -0.464\text{ A}.$$

(c) Leaving only the 57-V battery in the circuit:

Left loop: $140I_1 + 210I_2 = 0$. Right loop: $57 - 35I_3 - 210I_2 = 0$.

Junction rule: $I_1 - I_2 + I_3 = 0$. Solving for the three currents:

$$I_1 = -0.287\text{ A},\qquad I_2 = 0.192\text{ A},\qquad I_3 = 0.480\text{ A}.$$

(d) Leaving only the 55-V battery in the circuit:

Left loop: $55 - 140I_1 - 210I_2 = 0$. Right loop: $55 - 35I_3 - 210I_2 = 0$.

Junction rule: $I_1 - I_2 + I_3 = 0$. Solving for the three currents:

$$I_1 = 0.046\text{ A},\qquad I_2 = 0.231\text{ A},\qquad I_3 = 0.185\text{ A}.$$

(e) If we sum the currents from the previous three parts we find:

$$I_1 = 0.300\text{ A},\qquad I_2 = 0.500\text{ A},\qquad I_3 = 0.200\text{ A},\ \text{just as in part (a).}$$

(f) Changing the 57-V battery for an 80-V battery just affects the calculation in part (c). It changes to: Left loop: $140I_1 + 210I_2 = 0.$ Right loop: $80 - 35I_3 - 210I_2 = 0.$

Junction rule: $I_1 - I_2 + I_3 = 0.$ Solving for the three currents:

$$I_1 = -0.403 \text{ A}, \qquad I_2 = 0.269 \text{ A}, \qquad I_3 = 0.672 \text{ A}.$$

The total current for the full circuit is the sum of (b), (d) and (f) above:

$$I_1 = 0.184 \text{ A}, \qquad I_2 = 0.576 \text{ A}, \qquad I_3 = 0.392 \text{ A}.$$

EVALUATE: This problem presents an alternative means of solving for currents in multiloop circuits.

26.90. **IDENTIFY** and **SET UP:** When *C* changes after the capacitor is charged, the voltage across the capacitor changes. Current flows through the resistor until the voltage across the capacitor again equals the emf.

EXECUTE: **(a)** Fully charged: $Q = CV = (10.0\times10^{-12}\text{ F})(1000\text{ V}) = 1.00\times10^{-8}\text{ C}.$

(b) The initial current just after the capacitor is charged is $I_0 = \dfrac{\mathcal{E} - V_{C'}}{R} = \dfrac{\mathcal{E}}{R} - \dfrac{q}{RC'}$. This gives $i(t) = \left(\dfrac{\mathcal{E}}{R} - \dfrac{q}{RC'}\right)e^{-t/RC'}$, where $C' = 1.1C$.

(c) We need a resistance such that the current will be greater than 1 μA for longer than 200 μs. This requires that at $t = 200\ \mu\text{s}$, $i = 1.0\times10^{-6}\text{ A} = \dfrac{1}{R}\left(1000\text{ V} - \dfrac{1.0\times10^{-8}\text{ C}}{1.1(1.0\times10^{-11}\text{ F})}\right)e^{-(2.0\times10^{-4}\text{ s})/R(11\times10^{-12}\text{ F})}$. This says

$1.0\times10^{-6}\text{ A} = \dfrac{1}{R}(90.9)e^{-(1.8\times10^{7}\,\Omega)/R}$ and $18.3R - R\ln R - 1.8\times10^{7} = 0.$ Solving for R numerically we find $7.15\times10^{6}\ \Omega \le R \le 7.01\times10^{7}\ \Omega.$

EVALUATE: If the resistance is too small, then the capacitor discharges too quickly, and if the resistance is too large, the current is not large enough.

26.91. **IDENTIFY:** Consider one segment of the network attached to the rest of the network.

SET UP: We can re-draw the circuit as shown in Figure 26.91.

EXECUTE: $R_T = 2R_1 + \left(\dfrac{1}{R_2} + \dfrac{1}{R_T}\right)^{-1} = 2R_1 + \dfrac{R_2R_T}{R_2 + R_T}$. $R_T^{\,2} - 2R_1R_T - 2R_1R_2 = 0$. $R_T = R_1 \pm \sqrt{R_1^{\,2} + 2R_1R_2}$. $R_T > 0$, so $R_T = R_1 + \sqrt{R_1^{\,2} + 2R_1R_2}$.

EVALUATE: Even though there are an infinite number of resistors, the equivalent resistance of the network is finite.

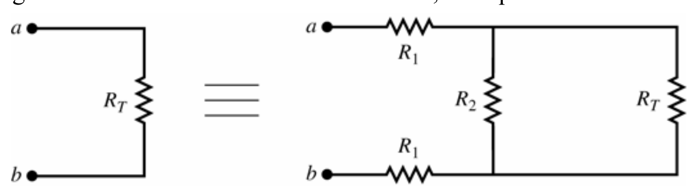


Figure 26.91

26.92. **IDENTIFY:** Assume a voltage *V* applied between points *a* and *b* and consider the currents that flow along each path between *a* and *b*.

SET UP: The currents are shown in Figure 26.92.

EXECUTE: Let current *I* enter at *a* and exit at *b*. At *a* there are three equivalent branches, so current is $I/3$ in each. At the next junction point there are two equivalent branches so each gets current $I/6$. Then at b there are three equivalent branches with current $I/3$ in each. The voltage drop from a to b then is

$V = \left(\dfrac{I}{3}\right)R + \left(\dfrac{I}{6}\right)R + \left(\dfrac{I}{3}\right)R = \frac{5}{6}IR.$ This must be the same as $V = IR_{\text{eq}}$, so $R_{\text{eq}} = \frac{5}{6}R.$

EVALUATE: The equivalent resistance is less than *R*, even though there are 12 resistors in the network.

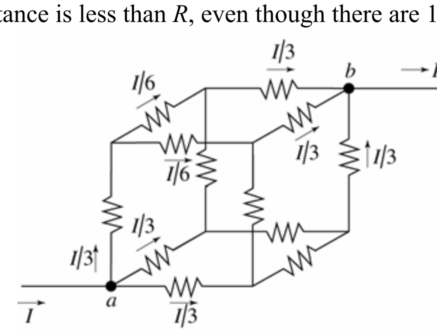


Figure 26.92

26.93. **IDENTIFY:** The network is the same as the one in Challenge Problem 26.91, and that problem shows that the equivalent resistance of the network is $R_T = \sqrt{R_1^2 + 2R_1R_2}$.

SET UP: The circuit can be redrawn as shown in Figure 26.93.

EXECUTE: **(a)** $V_{cd} = V_{ab}\dfrac{R_{\text{eq}}}{2R_1 + R_{\text{eq}}} = V_{ab}\dfrac{1}{2R_1/R_{\text{eq}} + 1}$ and $R_{\text{eq}} = \dfrac{R_2R_T}{R_2 + R_T}$. But $\beta = \dfrac{2R_1(R_T + R_2)}{R_TR_2} = \dfrac{2R_1}{R_{\text{eq}}}$, so

$V_{cd} = V_{ab}\dfrac{1}{1+\beta}$.

(b) $V_1 = \dfrac{V_0}{(1+\beta)} \Rightarrow V_2 = \dfrac{V_1}{(1+\beta)} = \dfrac{V_0}{(1+\beta)^2} \Rightarrow V_n = \dfrac{V_{n-1}}{(1+\beta)} = \dfrac{V_0}{(1+\beta)^n}$.

If $R_1 = R_2$, then $R_T = R_1 + \sqrt{R_1^{\,2} + 2R_1R_1} = R_1(1+\sqrt{3})$ and $\beta = \dfrac{2(2+\sqrt{3})}{1+\sqrt{3}} = 2.73$. So, for the nth segment to have 1% of the original voltage, we need: $\dfrac{1}{(1+\beta)^n} = \dfrac{1}{(1+2.73)^n} \le 0.01$. This says $n = 4$, and then $V_4 = 0.005V_0$.

(c) $R_T = R_1 + \sqrt{R_1^{\,2} + 2R_1R_2}$ gives $R_T = 6400\ \Omega + \sqrt{(6400\ \Omega)^2 + 2(6400\ \Omega)(8.0\times10^8\ \Omega)} = 3.2\times10^6\ \Omega$ and

$\beta = \dfrac{2(6400\ \Omega)(3.2\times10^6\ \Omega + 8.0\times10^8\ \Omega)}{(3.2\times10^6\ \Omega)(8.0\times10^8\ \Omega)} = 4.0\times10^{-3}$.

(d) Along a length of 2.0 mm of axon, there are 2000 segments each 1.0 μm long. The voltage therefore attenuates by $V_{2000} = \dfrac{V_0}{(1+\beta)^{2000}}$, so $\dfrac{V_{2000}}{V_0} = \dfrac{1}{(1+4.0\times10^{-3})^{2000}} = 3.4\times10^{-4}$.

(e) If $R_2 = 3.3\times10^{12}\ \Omega$, then $R_T = 2.1\times10^8\ \Omega$ and $\beta = 6.2\times10^{-5}$. This gives

$\dfrac{V_{2000}}{V_0} = \dfrac{1}{(1+6.2\times10^{-5})^{2000}} = 0.88.$

EVALUATE: As R_2 increases, β decreases and the potential difference decrease from one section to the next is less.

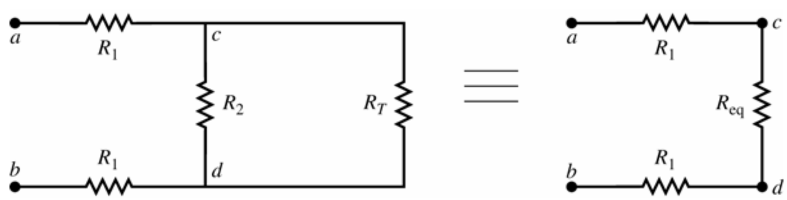


Figure 26.93

27

MAGNETIC FIELD AND MAGNETIC FORCES

27.1. **IDENTIFY** and **SET UP:** Apply Eq.(27.2) to calculate $\vec{F}$. Use the cross products of unit vectors from Section 1.10.

EXECUTE: $\vec{v} = (+4.19\times10^4 \text{ m/s})\hat{i} + (-3.85\times10^4 \text{ m/s})\hat{j}$

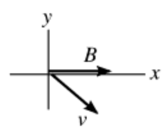
(a) $\vec{B} = (1.40 \text{ T})\hat{i}$

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8} \text{ C})(1.40 \text{ T})\left[(4.19\times10^4 \text{ m/s})\hat{i}\times\hat{i} - (3.85\times10^4 \text{ m/s})\hat{j}\times\hat{i}\right]$

$\hat{i}\times\hat{i} = 0,\ \hat{j}\times\hat{i} = -\hat{k}$

$\vec{F} = (-1.24\times10^{-8} \text{ C})(1.40 \text{ T})(-3.85\times10^4 \text{ m/s})(-\hat{k}) = (-6.68\times10^{-4} \text{ N})\hat{k}$

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1a.



The right-hand rule gives that $\vec{v}\times\vec{B}$ is directed out of the paper (+z-direction). The charge is negative so $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$;

Figure 27.1a

$\vec{F}$ is in the $-z$- direction. This agrees with the direction calculated with unit vectors.

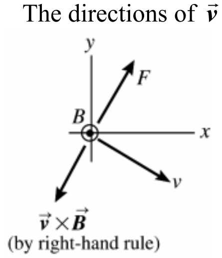
(b) EXECUTE: $\vec{B} = (1.40 \text{ T})\hat{k}$

$\vec{F} = q\vec{v}\times\vec{B} = (-1.24\times10^{-8} \text{ C})(1.40 \text{ T})\left[(+4.19\times10^4 \text{ m/s})\hat{i}\times\hat{k} - (3.85\times10^4 \text{ m/s})\hat{j}\times\hat{k}\right]$

$\hat{i}\times\hat{k} = -\hat{j},\ \hat{j}\times\hat{k} = \hat{i}$

$\vec{F} = (-7.27\times10^{-4} \text{ N})(-\hat{j}) + (6.68\times10^{-4} \text{ N})\hat{i} = \left[(6.68\times10^{-4} \text{ N})\hat{i} + (7.27\times10^{-4} \text{ N})\hat{j}\right]$

EVALUATE: The directions of $\vec{v}$ and $\vec{B}$ are shown in Figure 27.1b.



The direction of $\vec{F}$ is opposite to $\vec{v}\times\vec{B}$ since q is negative. The direction of $\vec{F}$ computed from the right-hand rule agrees qualitatively with the direction calculated with unit vectors.

Figure 27.1b

27.2. **IDENTIFY:** The net force must be zero, so the magnetic and gravity forces must be equal in magnitude and opposite in direction.

SET UP: The gravity force is downward so the force from the magnetic field must be upward. The charge's velocity and the forces are shown in Figure 27.2. Since the charge is negative, the magnetic force is opposite to the right-hand rule direction. The minimum magnetic field is when the field is perpendicular to $\vec{v}$. The force is also perpendicular to $\vec{B}$, so $\vec{B}$ is either eastward or westward.

EXECUTE: If $\vec{B}$ is eastward, the right-hand rule direction is into the page and $\vec{F}_B$ is out of the page, as required. Therefore, $\vec{B}$ is eastward. $mg = |q|vB\sin\phi$. $\phi = 90°$ and $B = \dfrac{mg}{v|q|} = \dfrac{(0.195\times10^{-3} \text{ kg})(9.80 \text{ m/s}^2)}{(4.00\times10^4 \text{ m/s})(2.50\times10^{-8} \text{ C})} = 1.91 \text{ T}$.

EVALUATE: The magnetic field could also have a component along the north-south direction, that would not contribute to the force, but then the field wouldn't have minimum magnitude.

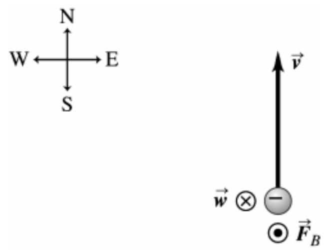


Figure 27.2

27.3. **IDENTIFY:** The force $\vec{F}$ on the particle is in the direction of the deflection of the particle. Apply the right-hand rule to the directions of $\vec{v}$ and $\vec{B}$. See if your thumb is in the direction of $\vec{F}$, or opposite to that direction. Use $F = |q|vB\sin\phi$ with $\phi = 90°$ to calculate *F*.

SET UP: The directions of $\vec{v}$, $\vec{B}$ and $\vec{F}$ are shown in Figure 27.3.

EXECUTE: **(a)** When you apply the right-hand rule to $\vec{v}$ and $\vec{B}$, your thumb points east. $\vec{F}$ is in this direction, so the charge is positive.

(b) $F = |q|vB\sin\phi = (8.50\times10^{-6}\ \text{C})(4.75\times10^{3}\ \text{m/s})(1.25\ \text{T})\sin 90° = 0.0505\ \text{N}$

EVALUATE: If the particle had negative charge and $\vec{v}$ and $\vec{B}$ are unchanged, the particle would be deflected toward the west.

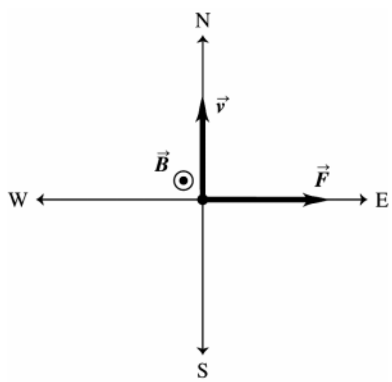


Figure 27.3

27.4. **IDENTIFY:** Apply Newton's second law, with the force being the magnetic force.

SET UP: $\hat{\boldsymbol{j}}\times\hat{\boldsymbol{i}} = -\hat{\boldsymbol{k}}$

EXECUTE: $\vec{F} = m\vec{a} = q\vec{v}\times\vec{B}$ gives $\vec{a} = \dfrac{q\vec{v}\times\vec{B}}{m}$ and

$$\vec{a} = \frac{(1.22\times10^{-8}\ \text{C})(3.0\times10^{4}\ \text{m/s})(1.63\ \text{T})(\hat{\boldsymbol{j}}\times\hat{\boldsymbol{i}})}{1.81\times10^{-3}\ \text{kg}} = -(0.330\ \text{m/s}^2)\hat{\boldsymbol{k}}.$$

EVALUATE: The acceleration is in the $-z$-direction and is perpendicular to both $\vec{v}$ and $\vec{B}$.

27.5. **IDENTIFY:** Apply $F = |q|vB\sin\phi$ and solve for *v*.

SET UP: An electron has $q = -1.60\times10^{-19}\ \text{C}$.

EXECUTE: $v = \dfrac{F}{|q|B\sin\phi} = \dfrac{4.60\times10^{-15}\ \text{N}}{(1.6\times10^{-19}\ \text{C})(3.5\times10^{-3}\ \text{T})\sin 60°} = 9.49\times10^{6}\ \text{m/s}$

EVALUATE: Only the component $B\sin\phi$ of the magnetic field perpendicular to the velocity contributes to the force.

27.6. **IDENTIFY:** Apply Newton's second law and $F = |q|vB\sin\phi$.

SET UP: ϕ is the angle between the direction of $\vec{v}$ and the direction of $\vec{B}$.

EXECUTE: **(a)** The smallest possible acceleration is zero, when the motion is parallel to the magnetic field. The greatest acceleration is when the velocity and magnetic field are at right angles:

$$a = \frac{qvB}{m} = \frac{(1.6\times10^{-19}\ \text{C})(2.50\times10^{6}\ \text{m/s})(7.4\times10^{-2}\ \text{T})}{(9.11\times10^{-31}\ \text{kg})} = 3.25\times10^{16}\ \text{m/s}^2.$$

(b) If $a = \frac{1}{4}(3.25\times10^{16}\ \text{m/s}^2) = \dfrac{qvB\sin\phi}{m}$, then $\sin\phi = 0.25$ and $\phi = 14.5°$.

EVALUATE: The force and acceleration decrease as the angle ϕ approaches zero.

27.7. **IDENTIFY:** Apply $\vec{F} = q\vec{v}\times\vec{B}$.

SET UP: $\vec{v} = v_y\hat{j}$, with $v_y = -3.80\times10^3\ \text{m/s}$. $F_x = +7.60\times10^{-3}\ \text{N}$, $F_y = 0$, and $F_z = -5.20\times10^{-3}\ \text{N}$.

EXECUTE: **(a)** $F_x = q(v_yB_z - v_zB_y) = qv_yB_z$.

$B_z = F_x/qv_y = (7.60\times10^{-3}\ \text{N})/([7.80\times10^{-6}\ \text{C})(-3.80\times10^3\ \text{m/s})] = -0.256\ \text{T}$

$F_y = q(v_zB_x - v_xB_z) = 0$, which is consistent with $\vec{F}$ as given in the problem. There is no force component along the direction of the velocity.

$F_z = q(v_xB_y - v_yB_x) = -qv_yB_x$. $B_x = -F_z/qv_y = -0.175\ \text{T}$.

(b) B_y is not determined. No force due to this component of $\vec{B}$ along $\vec{v}$; measurement of the force tells us nothing about B_y.

(c) $\vec{B}\cdot\vec{F} = B_xF_x + B_yF_y + B_zF_z = (-0.175\ \text{T})(+7.60\times10^{-3}\ \text{N}) + (-0.256\ \text{T})(-5.20\times10^{-3}\ \text{N})$

$\vec{B}\cdot\vec{F} = 0$. $\vec{B}$ and $\vec{F}$ are perpendicular (angle is $90°$).

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$, so $\vec{v}\cdot\vec{F}$ is also zero.

27.8. **IDENTIFY and SET UP:** $\vec{F} = q\vec{v}\times\vec{B} = qB_z[v_x(\hat{i}\times\hat{k}) + v_y(\hat{j}\times\hat{k}) + v_z(\hat{k}\times\hat{k})] = qB_z[v_x(-\hat{j}) + v_y(\hat{i})]$.

EXECUTE: **(a)** Set the expression for $\vec{F}$ equal to the given value of $\vec{F}$ to obtain:

$$v_x = \frac{F_y}{-qB_z} = \frac{(7.40\times10^{-7}\ \text{N})}{-(-5.60\times10^{-9}\ \text{C})(-1.25\ \text{T})} = -106\ \text{m/s}$$

$$v_y = \frac{F_x}{qB_z} = \frac{-(3.40\times10^{-7}\ \text{N})}{(-5.60\times10^{-9}\ \text{C})(-1.25\ \text{T})} = -48.6\ \text{m/s}.$$

(b) v_z does not contribute to the force, so is not determined by a measurement of $\vec{F}$.

(c) $\vec{v}\cdot\vec{F} = v_xF_x + v_yF_y + v_zF_z = \dfrac{F_y}{-qB_z}F_x + \dfrac{F_x}{qB_z}F_y = 0$; $\theta = 90°$.

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$, so $\vec{B}\cdot\vec{F}$ is also zero.

27.9. **IDENTIFY:** Apply $\vec{F} = q\vec{v}\times\vec{B}$ to the force on the proton and to the force on the electron. Solve for the components of $\vec{B}$.

SET UP: $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. Since the force on the proton is in the +*y*-direction, $B_y = 0$ and $\vec{B} = B_x\hat{i} + B_z\hat{k}$. For the proton, $\vec{v} = (1.50\ \text{km/s})\hat{i}$.

EXECUTE: **(a)** For the proton, $\vec{F} = q(1.50\times10^3\ \text{m/s})\hat{i}\times(B_x\hat{i} + B_z\hat{k}) = q(1.50\times10^3\ \text{m/s})B_z(-\hat{j})$. $\vec{F} = (2.25\times10^{-16}\ \text{N})\hat{j}$, so $B_z = -\dfrac{2.25\times10^{-16}\ \text{N}}{(1.60\times10^{-19}\ \text{C})(1.50\times10^3\ \text{m/s})} = -0.938\ \text{T}$. The force on the proton is independent of B_x. For the electron, $\vec{v} = (4.75\ \text{km/s})(-\hat{k})$. $\vec{F} = q\vec{v}\times\vec{B} = (-e)(4.75\times10^3\ \text{m/s})(-\hat{k})\times(B_x\hat{i} + B_z\hat{k}) = +e(4.75\times10^3\ \text{m/s})B_x\hat{j}$.

The magnitude of the force is $F = e(4.75\times10^3\ \text{m/s})|B_x|$. Since $F = 8.50\times10^{-16}\ \text{N}$,

$|B_x| = \dfrac{8.50\times10^{-16}\ \text{N}}{(1.60\times10^{-19}\ \text{C})(4.75\times10^3\ \text{m/s})} = 1.12\ \text{T}$. $B_x = \pm1.12\ \text{T}$. The sign of B_x is not determined by measuring the magnitude of the force on the electron. $B = \sqrt{B_x^2 + B_z^2} = \sqrt{(\pm1.12\ \text{T})\ + (-0.938\ \text{T})^2} = 1.46\ \text{T}$.

$\tan\theta = \dfrac{B_z}{B_x} = \dfrac{-0.938\ \text{T}}{\pm1.12\ \text{T}}$. $\theta = \pm40°$. $\vec{B}$ is in the *xz*-plane and is either at $40°$ from the +*x*-direction toward the $-z$-direction or $40°$ from the $-x$-direction toward the $-z$-direction.

(b) $\vec{B} = B_x\hat{i} + B_z\hat{k}$. $\vec{v} = (3.2 \text{ km/s})(-\hat{j})$.

$\vec{F} = q\vec{v}\times\vec{B} = (-e)(3.2 \text{ km/s})(-\hat{j})\times(B_x\hat{i} + B_z\hat{k}) = e(3.2\times10^3 \text{ m/s})(B_x(-\hat{k}) + B_z\hat{i})$.

$\vec{F} = e(3.2\times10^3 \text{ m/s})(-[\pm1.12 \text{ T}]\hat{k} - [0.938 \text{ T}]\hat{i}) = -(4.80\times10^{-16} \text{ N})\hat{i} \pm (5.73\times10^{-16} \text{ N})\hat{k}$

$F = \sqrt{F_x^2 + F_z^2} = 7.47\times10^{-16}$ N. $\tan\theta = \dfrac{F_z}{F_x} = \dfrac{\pm5.73\times10^{-16} \text{ N}}{-4.80\times10^{-16} \text{ N}}$. $\theta = \pm50.0°$. The force is in the *xz*-plane and is directed at $50.0°$ from the $-x$-axis toward either the $+z$ or $-z$ axis, depending on the sign of B_x.

EVALUATE: If the direction of the force on the first electron were measured, then the sign of B_x would be determined.

27.10. IDENTIFY: Magnetic field lines are closed loops, so the net flux through any closed surface is zero.

SET UP: Let magnetic field directed out of the enclosed volume correspond to positive flux and magnetic field directed into the volume correspond to negative flux.

EXECUTE: **(a)** The total flux must be zero, so the flux through the remaining surfaces must be -0.120 Wb.

(b) The shape of the surface is unimportant, just that it is closed.

(c) One possibility is sketched in Figure 27.10.

EVALUATE: In Figure 27.10 all the field lines that enter the cube also exit through the surface of the cube.

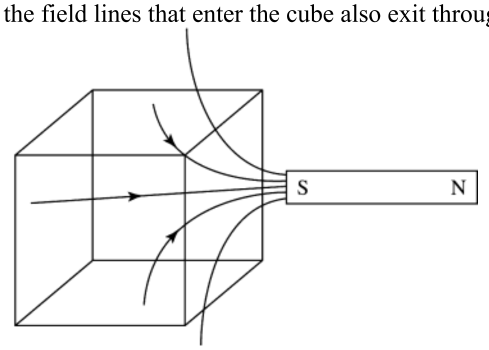


Figure 27.10

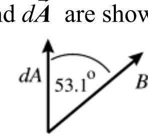
27.11. IDENTIFY and SET UP: $\Phi_B = \int \vec{B}\cdot d\vec{A}$

Circular area in the *xy*-plane, so $A = \pi r^2 = \pi(0.0650 \text{ m})^2 = 0.01327 \text{ m}^2$ and $d\vec{A}$ is in the *z*-direction. Use Eq.(1.18) to calculate the scalar product.

EXECUTE: **(a)** $\vec{B} = (0.230 \text{ T})\hat{k}$; $\vec{B}$ and $d\vec{A}$ are parallel $(\phi = 0°)$ so $\vec{B}\cdot d\vec{A} = B\,dA$.

B is constant over the circular area so $\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\,dA = B\int dA = BA = (0.230 \text{ T})(0.01327 \text{ m}^2) = 3.05\times10^{-3}$ Wb

(b) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11a.



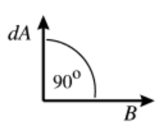
$\vec{B}\cdot d\vec{A} = B\cos\phi\,dA$
with $\phi = 53.1°$

Figure 27.11a

B and ϕ are constant over the circular area so $\Phi_B = \int \vec{B}\cdot d\vec{A} = \int B\cos\phi\,dA = B\cos\phi\int dA = B\cos\phi A$

$\Phi_B = (0.230 \text{ T})\cos 53.1°(0.01327 \text{ m}^2) = 1.83\times10^{-3}$ Wb

(c) The directions of $\vec{B}$ and $d\vec{A}$ are shown in Figure 27.11b.



$\vec{B}\cdot d\vec{A} = 0$ since $d\vec{A}$ and $\vec{B}$ are perpendicular $(\phi = 90°)$

$\Phi_B = \int \vec{B}\cdot d\vec{A} = 0.$

Figure 27.11b

EVALUATE: Magnetic flux is a measure of how many magnetic field lines pass through the surface. It is maximum when $\vec{B}$ is perpendicular to the plane of the loop (part a) and is zero when $\vec{B}$ is parallel to the plane of the loop (part c).

27.12. IDENTIFY: When $\vec{B}$ is uniform across the surface, $\Phi_B = \vec{B}\cdot\vec{A} = BA\cos\phi$.

SET UP: $\vec{A}$ is normal to the surface and is directed outward from the enclosed volume. For surface *abcd*, $\vec{A} = -A\hat{i}$. For surface *befc*, $\vec{A} = -A\hat{k}$. For surface *aefd*, $\cos\phi = 3/5$ and the flux is positive.

EXECUTE: **(a)** $\Phi_B(abcd) = \vec{B}\cdot\vec{A} = 0.$

(b) $\Phi_B(befc) = \vec{B}\cdot\vec{A} = -(0.128\text{ T})(0.300\text{ m})(0.300\text{ m}) = -0.0115\text{ Wb}.$

(c) $\Phi_B(aefd) = \vec{B}\cdot\vec{A} = BA\cos\phi = \frac{3}{5}(0.128\text{ T})(0.500\text{ m})(0.300\text{ m}) = +0.0115\text{ Wb}.$

(d) The net flux through the rest of the surfaces is zero since they are parallel to the x-axis. The total flux is the sum of all parts above, which is zero.

EVALUATE: The total flux through any closed surface, that encloses a volume, is zero.

27.13. **IDENTIFY:** The total flux through the bottle is zero because it is a closed surface.

SET UP: The total flux through the bottle is the flux through the plastic plus the flux through the open cap, so the sum of these must be zero. $\Phi_{\text{plastic}} + \Phi_{\text{cap}} = 0.$

$$\Phi_{\text{plastic}} = -\Phi_{\text{cap}} = -BA\cos\Phi = -B(\pi r^2)\cos\Phi$$

EXECUTE: Substituting the numbers gives $\Phi_{\text{plastic}} = -(1.75\text{ T})\pi(0.0125\text{ m})^2\cos 25° = -7.8\times 10^{-4}\text{ Wb}$

EVALUATE: It would be impossible to calculate the flux through the plastic directly because of the complicated shape of the bottle, but with a little thought we can find this flux through a simple calculation.

27.14. **IDENTIFY:** $p = mv$ and $L = Rp$, since the velocity and linear momentum are tangent to the circular path.

SET UP: $|q|vB = mv^2/R$.

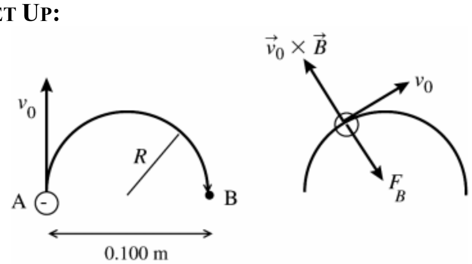
EXECUTE: **(a)** $p = mv = m\left(\dfrac{RqB}{m}\right) = RqB = (4.68\times10^{-3}\text{ m})(6.4\times10^{-19}\text{ C})(1.65\text{ T}) = 4.94\times10^{-21}\text{ kg m/s}.$

(b) $L = Rp = R^2qB = (4.68\times10^{-3}\text{ m})^2(6.4\times10^{-19}\text{ C})(1.65\text{ T}) = 2.31\times10^{-23}\text{ kg}\cdot\text{m}^2/\text{s}.$

EVALUATE: $\vec{p}$ is tangent to the orbit and $\vec{L}$ is perpendicular to the orbit plane.

27.15. **(a) IDENTIFY:** Apply Eq.(27.2) to relate the magnetic force $\vec{F}$ to the directions of $\vec{v}$ and $\vec{B}$. The electron has negative charge so $\vec{F}$ is opposite to the direction of $\vec{v}\times\vec{B}$. For motion in an arc of a circle the acceleration is toward the center of the arc so $\vec{F}$ must be in this direction. $a = v^2/R$.

SET UP:



As the electron moves in the semicircle, its velocity is tangent to the circular path. The direction of $\vec{v}_0\times\vec{B}$ at a point along the path is shown in Figure 27.15.

Figure 27.15

EXECUTE: For circular motion the acceleration of the electron $\vec{a}_{\text{rad}}$ is directed in toward the center of the circle. Thus the force $\vec{F}_B$ exerted by the magnetic field, since it is the only force on the electron, must be radially inward. Since q is negative, $\vec{F}_B$ is opposite to the direction given by the right-hand rule for $\vec{v}_0\times\vec{B}$. Thus $\vec{B}$ is directed into the page. Apply Newton's 2nd law to calculate the magnitude of $\vec{B}$: $\sum\vec{F} = m\vec{a}$ gives $\sum F_{\text{rad}} = ma$

$$F_B = m(v^2/R)$$

$$F_B = |q|vB\sin\phi = |q|vB,\ \text{so}\ |q|vB = m(v^2/R)$$

$$B = \frac{mv}{|q|R} = \frac{(9.109\times10^{-31}\text{ kg})(1.41\times10^6\text{ m/s})}{(1.602\times10^{-19}\text{ C})(0.050\text{ m})} = 1.60\times10^{-4}\text{ T}$$

(b) IDENTIFY and **SET UP:** The speed of the electron as it moves along the path is constant. ($\vec{F}_B$ changes the direction of $\vec{v}$ but not its magnitude.) The time is given by the distance divided by v_0.

EXECUTE: The distance along the semicircular path is πR, so $t = \dfrac{\pi R}{v_0} = \dfrac{\pi(0.050\text{ m})}{1.41\times10^6\text{ m/s}} = 1.11\times10^{-7}\text{ s}$

EVALUATE: The magnetic field required increases when v increases or R decreases and also depends on the mass to charge ratio of the particle.

27.16. **IDENTIFY:** Newton's second law gives $|q|vB = mv^2/R$. The speed v is constant and equals v_0. The direction of the magnetic force must be in the direction of the acceleration and is toward the center of the semicircular path.

SET UP: A proton has $q = +1.60\times10^{-19}\text{ C}$ and $m = 1.67\times10^{-27}\text{ kg}$. The direction of the magnetic force is given by the right-hand rule.

EXECUTE: **(a)** $B = \dfrac{mv}{qR} = \dfrac{(1.67\times10^{-27}\text{ kg})(1.41\times10^{6}\text{ m/s})}{(1.60\times\ 10^{-19}\text{ C})(0.0500\text{ m})} = 0.294\text{ T}$

The direction of the magnetic field is out of the page (the charge is positive), in order for $\vec{F}$ to be directed to the right at point A.

(b) The time to complete half a circle is $t = \pi R/v_0 = 1.11\times10^{-7}$ s.

EVALUATE: The magnetic field required to produce this path for a proton has a different magnitude (because of the different mass) and opposite direction (because of opposite sign of the charge) than the field required to produce the path for an electron.

27.17. **IDENTIFY** and **SET UP:** Use conservation of energy to find the speed of the ball when it reaches the bottom of the shaft. The right-hand rule gives the direction of $\vec{F}$ and Eq.(27.1) gives its magnitude. The number of excess electrons determines the charge of the ball.

EXECUTE: $q = \left(4.00\times10^{8}\right)\left(-1.602\times10^{-19}\text{ C}\right) = -6.408\times10^{-11}\text{ C}$

speed at bottom of shaft: $\frac{1}{2}mv^2 = mgy;\ v = \sqrt{2gy} = 49.5\text{ m/s}$

$\vec{v}$ is downward and $\vec{B}$ is west, so $\vec{v}\times\vec{B}$ is north. Since $q < 0$, $\vec{F}$ is south.

$F = |q|vB\sin\theta = \left(6.408\times10^{-11}\text{ C}\right)\left(49.5\text{ m/s}\right)\left(0.250\text{ T}\right)\sin 90° = 7.93\times10^{-10}\text{ N}$

EVALUATE: Both the charge and speed of the ball are relatively small so the magnetic force is small, much less than the gravity force of 1.5 N.

27.18. **IDENTIFY:** Since the particle moves perpendicular to the uniform magnetic field, the radius of its path is $R = \dfrac{mv}{|q|B}$. The magnetic force is perpendicular to both $\vec{v}$ and $\vec{B}$.

SET UP: The alpha particle has charge $q = +2e = 3.20\times10^{-19}\text{ C}$.

EXECUTE: **(a)** $R = \dfrac{(6.64\times10^{-27}\text{ kg})(35.6\times10^{3}\text{ m/s})}{(3.20\times10^{-19}\text{ C})(1.10\text{ T})} = 6.73\times10^{-4}\text{ m} = 0.673\text{ mm}$. The alpha particle moves in a circular arc of diameter $2R = 1.35\text{ mm}$.

(b) For a very short time interval the displacement of the particle is in the direction of the velocity. The magnetic force is always perpendicular to this direction so it does no work. The work-energy theorem therefore says that the kinetic energy of the particle, and hence its speed, is constant.

(c) The acceleration is $a = \dfrac{F_B}{m} = \dfrac{|q|vB\sin\phi}{m} = \dfrac{(3.20\times10^{-19}\text{ C})(35.6\times10^{3}\text{ m/s})(1.10\text{ T})\sin 90°}{6.64\times10^{-27}\text{ kg}} = 1.88\times10^{12}\text{ m/s}^2$. We can also use $a = \dfrac{v^2}{R}$ and the result of part (a) to calculate $a = \dfrac{(35.6\times10^{3}\text{ m/s})^2}{6.73\times10^{-4}\text{ m}} = 1.88\times10^{12}\text{ m/s}^2$, the same result. The acceleration is perpendicular to $\vec{v}$ and $\vec{B}$ and so is horizontal, toward the center of curvature of the particle's path.

EVALUATE: **(d)** The unbalanced force ($\vec{F}_B$) is perpendicular to $\vec{v}$, so it changes the direction of $\vec{v}$ but not its magnitude, which is the speed.

27.19. **IDENTIFY:** In part (a), apply conservation of energy to the motion of the two nuclei. In part (b) apply $|q|vB = mv^2/R$.

SET UP: In part (a), let point 1 be when the two nuclei are far apart and let point 2 be when they are at their closest separation.

EXECUTE: **(a)** $K_1 + U_1 = K_2 + U_2$. $U_1 = K_2 = 0$, so $K_1 = U_2$ and $\frac{1}{2}mv^2 = ke^2/r$.

$$v = e\sqrt{\frac{2k}{mr}} = (1.602\times10^{-19}\text{ C})\sqrt{\frac{2k}{(3.34\times10^{-27}\text{ kg})(1.0\times10^{-15}\text{ m})}} = 1.2\times10^{7}\text{ m/s}$$

(b) $\sum\vec{F} = m\vec{a}$ gives $qvB = mv^2/r$. $B = \dfrac{mv}{qr} = \dfrac{(3.34\times10^{-27}\text{ kg})(1.2\times10^{7}\text{m/s})}{(1.602\times10^{-19}\text{ C})(2.50\text{ m})} = 0.10\text{ T}$.

EVALUATE: The speed calculated in part (a) is large, 4% of the speed of light.

27.20. **IDENTIFY:** $F = |q|vB\sin\phi$. The direction of $\vec{F}$ is given by the right-hand rule.

SET UP: An electron has $q = -e$.

EXECUTE: **(a)** $F = |q|vB\sin\phi$. $B = \dfrac{F}{|q|v\sin\phi} = \dfrac{0.00320\times10^{-9}\text{ N}}{8(1.60\times10^{-19}\text{ C})(500{,}000\text{ m/s})\sin 90°} = 5.00\text{ T}$. If the angle ϕ is less than $90°$, a larger field is needed to produce the same force. The direction of the field must be toward the south so that $\vec{v}\times\vec{B}$ is downward.

(b) $F = |q|vB\sin\phi$. $v = \dfrac{F}{|q|B\sin\phi} = \dfrac{4.60\times10^{-12}\text{ N}}{(1.60\times10^{-19}\text{ C})(2.10\text{ T})\sin 90°} = 1.37\times10^{7}\text{ m/s}$. If ϕ is less than 90°, the speed would have to be larger to have the same force. The force is upward, so $\vec{v}\times\vec{B}$ must be downward since the electron is negative, and the velocity must be toward the south.

EVALUATE: The component of $\vec{B}$ along the direction of $\vec{v}$ produces no force and the component of $\vec{v}$ along the direction of $\vec{B}$ produces no force.

27.21. **(a) IDENTIFY** and **SET UP:** Apply Newton's 2nd law, with $a = v^2/R$ since the path of the particle is circular.

EXECUTE: $\sum\vec{F} = m\vec{a}$ says $|q|vB = m\left(v^2/R\right)$

$$v = \frac{|q|BR}{m} = \frac{\left(1.602\times10^{-19}\text{ C}\right)\left(2.50\text{ T}\right)\left(6.96\times10^{-3}\text{ m}\right)}{3.34\times10^{-27}\text{ kg}} = 8.35\times10^{5}\text{ m/s}$$

(b) IDENTIFY and **SET UP:** The speed is constant so t = distance/v.

EXECUTE: $t = \dfrac{\pi R}{v} = \dfrac{\pi\left(6.96\times10^{-3}\text{ m}\right)}{8.35\times10^{5}\text{ m/s}} = 2.62\times10^{-8}\text{ s}$

(c) IDENTIFY and **SET UP:** kinetic energy gained = electric potential energy lost

EXECUTE: $\frac{1}{2}mv^2 = |q|V$

$$V = \frac{mv^2}{2|q|} = \frac{\left(3.34\times10^{-27}\text{ kg}\right)\left(8.35\times10^{5}\text{ m/s}\right)^2}{2\left(1.602\times10^{-19}\text{ C}\right)} = 7.27\times10^{3}\text{ V} = 7.27\text{ kV}$$

EVALUATE: The deutron has a much larger mass to charge ratio than an electron so a much larger B is required for the same v and R. The deutron has positive charge so gains kinetic energy when it goes from high potential to low potential.

27.22. **IDENTIFY:** For motion in an arc of a circle, $a = \dfrac{v^2}{R}$ and the net force is radially inward, toward the center of the circle.

SET UP: The direction of the force is shown in Figure 27.22. The mass of a proton is 1.67×10^{-27} kg.

EXECUTE: **(a)** $\vec{F}$ is opposite to the right-hand rule direction, so the charge is negative. $\vec{F} = m\vec{a}$ gives $|q|vB\sin\phi = m\dfrac{v^2}{R}$. $\phi = 90°$ and $v = \dfrac{|q|BR}{m} = \dfrac{3(1.60\times10^{-19}\text{ C})(0.250\text{ T})(0.475\text{ m})}{12(1.67\times10^{-27}\text{ kg})} = 2.84\times10^{6}\text{ m/s}$.

(b) $F_B = |q|vB\sin\phi = 3(1.60\times10^{-19}\text{ C})(2.84\times10^{6}\text{ m/s})(0.250\text{ T})\sin 90° = 3.41\times10^{-13}\text{ N}$.

$w = mg = 12(1.67\times10^{-27}\text{ kg})(9.80\text{ m/s}^2) = 1.96\times10^{-25}\text{ N}$. The magnetic force is much larger than the weight of the particle, so it is a very good approximation to neglect gravity.

EVALUATE: **(c)** The magnetic force is always perpendicular to the path and does no work. The particles move with constant speed.

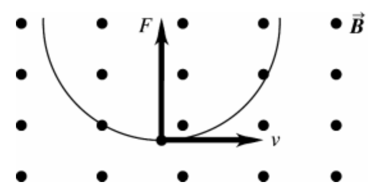


Figure 27.22

27.23. **IDENTIFY:** Example 27.3 shows that $B = \dfrac{m2\pi f}{|q|}$, where f is the frequency, in Hz, of the electromagnetic waves that are produced.

SET UP: An electron has charge $q = -e$ and mass $m = 9.11\times10^{-31}$ kg. A proton has charge $q = +e$ and mass $m = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $B = \dfrac{m2\pi f}{|q|} = \dfrac{(9.11\times10^{-31}\text{ kg})2\pi(3.00\times10^{12}\text{ Hz})}{(1.60\times10^{-19}\text{ C})} = 107\text{ T}$. This is about 2.4 times the greatest magnitude of magnetic field yet obtained on earth.

(b) Protons have a greater mass than the electrons, so a greater magnetic field would be required to accelerate them with the same frequency and there would be no advantage in using them.

EVALUATE: Electromagnetic waves with frequency $f = 3.0$ THz have a wavelength in air of $\lambda = \dfrac{v}{f} = 3.0\times10^{-4}$ m. The shorter the wavelength the greater the frequency and the greater the magnetic field that is required. B depends only on f and on the mass-to-charge ratio of the particle that moves in the circular path.

27.24. IDENTIFY: The magnetic force on the beam bends it through a quarter circle.

SET UP: The distance that particles in the beam travel is $s = R\theta$, and the radius of the quarter circle is $R = mv/qB$.

EXECUTE: Solving for R gives $R = s/\theta = s/(\pi/2) = 1.18$ cm$/(\pi/2) = 0.751$ cm. Solving for the magnetic field: $B = mv/qR = (1.67\times10^{-27}\text{ kg})(1200\text{ m/s})/[(1.60\times10^{-19}\text{ C})(0.00751\text{ m})] = 1.67\times10^{-3}$ T

EVALUATE: This field is about 10 times stronger than the Earth's magnetic field, but much weaker than many laboratory fields.

27.25. IDENTIFY: When a particle of charge $-e$ is accelerated through a potential difference of magnitude V, it gains kinetic energy eV. When it moves in a circular path of radius R, its acceleration is $\dfrac{v^2}{R}$.

SET UP: An electron has charge $q = -e = -1.60\times10^{-19}$ C and mass 9.11×10^{-31} kg.

EXECUTE: $\frac{1}{2}mv^2 = eV$ and $v = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2(1.60\times10^{-19}\text{ C})(2.00\times10^{3}\text{ V})}{9.11\times10^{-31}\text{ kg}}} = 2.65\times10^{7}$ m/s. $\vec{F} = m\vec{a}$ gives $|q|vB\sin\phi = m\dfrac{v^2}{R}$. $\phi = 90°$ and $B = \dfrac{mv}{|q|R} = \dfrac{(9.11\times10^{-31}\text{ kg})(2.65\times10^{7}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.180\text{ m})} = 8.38\times10^{-4}$ T.

EVALUATE: The smaller the radius of the circular path, the larger the magnitude of the magnetic field that is required.

27.26. IDENTIFY: After being accelerated through a potential difference V the ion has kinetic energy qV. The acceleration in the circular path is v^2/R.

SET UP: The ion has charge $q = +e$.

EXECUTE: $K = qV = +eV$. $\frac{1}{2}mv^2 = eV$ and $v = \sqrt{\dfrac{2eV}{m}} = \sqrt{\dfrac{2(1.60\times10^{-19}\text{ C})(220\text{ V})}{1.16\times10^{-26}\text{ kg}}} = 7.79\times10^{4}$ m/s. $F_B = |q|vB\sin\phi$. $\phi = 90°$. $\vec{F} = m\vec{a}$ gives $|q|vB = m\dfrac{v^2}{R}$. $R = \dfrac{mv}{|q|B} = \dfrac{(1.16\times10^{-26}\text{ kg})(7.79\times10^{4}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.723\text{ T})} = 7.81\times10^{-3}$ m $= 7.81$ mm.

EVALUATE: The larger the accelerating voltage, the larger the speed of the particle and the larger the radius of its path in the magnetic field.

27.27. **(a) IDENTIFY and SET UP:** Eq.(27.4) gives the total force on the proton. At $t = 0$,

$\vec{F} = q\vec{v}\times\vec{B} = q\left(v_x\hat{i} + v_z\hat{k}\right)\times B_x\hat{i} = qv_zB_x\hat{j}$. $\vec{F} = \left(1.60\times10^{-19}\text{ C}\right)\left(2.00\times10^{5}\text{ m/s}\right)\left(0.500\text{ T}\right)\hat{j} = \left(1.60\times10^{-14}\text{ N}\right)\hat{j}$.

(b) Yes. The electric field exerts a force in the direction of the electric field, since the charge of the proton is positive and there is a component of acceleration in this direction.

(c) EXECUTE: In the plane perpendicular to $\vec{B}$ (the yz-plane) the motion is circular. But there is a velocity component in the direction of $\vec{B}$, so the motion is a helix. The electric field in the $+\hat{i}$ direction exerts a force in the $+\hat{i}$ direction. This force produces an acceleration in the $+\hat{i}$ direction and this causes the pitch of the helix to vary. The force does not affect the circular motion in the yz-plane, so the electric field does not affect the radius of the helix.

(d) IDENTIFY and SET UP: Eq.(27.12) and $T = 2\pi/\omega$ to calculate the period of the motion. Calculate a_x produced by the electric force and use a constant acceleration equation to calculate the displacement in the x-direction in time $T/2$.

EXECUTE: Calculate the period T: $\omega = |q|B/m$

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{|q|B} = \frac{2\pi\left(1.67\times10^{-27}\text{ kg}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.500\text{ T}\right)} = 1.312\times10^{-7}\text{ s. Then } t = T/2 = 6.56\times10^{-8}\text{ s. } v_{0x} = 1.50\times10^{5}\text{ m/s}$$

$$a_x = \frac{F_x}{m} = \frac{\left(1.60\times10^{-19}\text{ C}\right)\left(2.00\times10^{4}\text{ V/m}\right)}{1.67\times10^{-27}\text{ kg}} = +1.916\times10^{12}\text{ m/s}^2$$

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_xt^2$$

$$x - x_0 = \left(1.50\times10^{5}\text{ m/s}\right)\left(6.56\times10^{-8}\text{ s}\right) + \tfrac{1}{2}\left(1.916\times10^{12}\text{ m/s}^2\right)\left(6.56\times10^{-8}\text{ s}\right)^2 = 1.40\text{ cm}$$

EVALUATE: The electric and magnetic fields are in the same direction but produce forces that are in perpendicular directions to each other.

27.28. **IDENTIFY:** For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

SET UP: $v = E/B$ for no deflection. With only the magnetic force, $|q|vB = mv^2/R$

EXECUTE: **(a)** $v = E/B = (1.56\times10^4\ \text{V/m})/(4.62\times10^{-3}\ \text{T}) = 3.38\times10^6\ \text{m/s}$.

(b) The directions of the three vectors $\vec{v}$, $\vec{E}$ and $\vec{B}$ are sketched in Figure 27.28.

(c) $R = \dfrac{mv}{|q|B} = \dfrac{(9.11\times10^{-31}\ \text{kg})(3.38\times10^6\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(4.62\times10^{-3}\ \text{T})} = 4.17\times10^{-3}\ \text{m}$.

$$T = \frac{2\pi m}{|q|B} = \frac{2\pi R}{v} = \frac{2\pi(4.17\times10^{-3}\ \text{m})}{(3.38\times10^6\ \text{m/s})} = 7.74\times10^{-9}\ \text{s}.$$

EVALUATE: For the field directions shown in Figure 27.28, the electric force is toward the top of the page and the magnetic force is toward the bottom of the page.

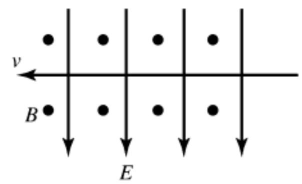


Figure 27.28

27.29. **IDENTIFY:** For the alpha particles to emerge from the plates undeflected, the magnetic force on them must exactly cancel the electric force. The battery produces an electric field between the plates, which acts on the alpha particles.

SET UP: First use energy conservation to find the speed of the alpha particles as they enter the plates: $qV = 1/2\ mv^2$. The electric field between the plates due to the battery is $E = V_b d$. For the alpha particles not to be deflected, the magnetic force must cancel the electric force, so $qvB = qE$, giving $B = E/v$.

EXECUTE: Solve for the speed of the alpha particles just as they enter the region between the plates. Their charge is $2e$.

$$v_\alpha = \sqrt{\frac{2(2e)V}{m}} = \sqrt{\frac{4\left(1.60\times10^{-19}\ \text{C}\right)(1750\ \text{V})}{6.64\times10^{-27}\ \text{kg}}} = 4.11\times10^5\ \text{m/s}$$

The electric field between the plates, produced by the battery, is

$$E = V_b/d = (150\ \text{V})/(0.00820\ \text{m}) = 18{,}300\ \text{V}$$

The magnetic force must cancel the electric force:

$$B = E/v_\alpha = (18{,}300\ \text{V})/(4.11\times10^5\ \text{m/s}) = 0.0445\ \text{T}$$

The magnetic field is perpendicular to the electric field. If the charges are moving to the right and the electric field points upward, the magnetic field is out of the page.

EVALUATE: The sign of the charge of the alpha particle does not enter the problem, so negative charges of the same magnitude would also not be deflected.

27.30. **IDENTIFY:** For no deflection the magnetic and electric forces must be equal in magnitude and opposite in direction.

SET UP: $v = E/B$ for no deflection.

EXECUTE: To pass undeflected in both cases, $E = vB = (5.85\times10^3\ \text{m/s})(1.35\ \text{T}) = 7898\ \text{N/C}$.

(a) If $q = 0.640\times10^{-9}$ C, the electric field direction is given by $-(\hat{\boldsymbol{j}}\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since it must point in the opposite direction to the magnetic force.

(b) If $q = -0.320\times10^{-9}$ C, the electric field direction is given by $((-\hat{\boldsymbol{j}})\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since the electric force must point in the opposite direction as the magnetic force. Since the particle has negative charge, the electric force is opposite to the direction of the electric field and the magnetic force is opposite to the direction it has in part (a).

EVALUATE: The same configuration of electric and magnetic fields works as a velocity selector for both positively and negatively charged particles.

27.31. **IDENTIFY** and **SET UP:** Use the fields in the velocity selector to find the speed v of the particles that pass through. Apply Newton's 2nd law with $a = v^2/R$ to the circular motion in the second region of the spectrometer. Solve for the mass m of the ion.

EXECUTE: In the velocity selector $|q|E = |q|vB$.

$$v = \frac{E}{B} = \frac{1.12\times10^5\ \text{V/m}}{0.540\ \text{T}} = 2.074\times10^5\ \text{m/s}$$

In the region of the circular path $\sum \vec{F} = m\vec{a}$ gives $|q|vB = m(v^2/R)$ so $m = |q|RB/v$

Singly charged ion, so $|q| = +e = 1.602\times10^{-19}$ C

$$m = \frac{(1.602\times10^{-19}\ \text{C})(0.310\ \text{m})(0.540\ \text{T})}{2.074\times10^5\ \text{m/s}} = 1.29\times10^{-25}\ \text{kg}$$

Mass number = mass in atomic mass units, so is $\dfrac{1.29\times10^{-25}\ \text{kg}}{1.66\times10^{-27}\ \text{kg}} = 78.$

EVALUATE: Appendix D gives the average atomic mass of selenium to be 78.96. One of its isotopes has atomic mass 78.

27.32. **IDENTIFY** and **SET UP:** For a velocity selector, $E = vB$. For parallel plates with opposite charge, $V = Ed$.

EXECUTE: **(a)** $E = vB = (1.82\times10^6\ \text{m/s})(0.650\ \text{T}) = 1.18\times10^6\ \text{V/m}$.

(b) $V = Ed = (1.18\times10^6\ \text{V/m})(5.20\times10^{-3}\ \text{m}) = 6.14\ \text{kV}$.

EVALUATE: Any charged particle with $v = 1.82\times10^6\ \text{m/s}$ will pass through undeflected, regardless of the sign and magnitude of its charge.

27.33. **IDENTIFY:** The magnetic force is $F = IlB\sin\phi$. For the wire to be completely supported by the field requires that $F = mg$ and that $\vec{F}$ and $\vec{w}$ are in opposite directions.

SET UP: The magnetic force is maximum when $\phi = 90°$. The gravity force is downward.

EXECUTE: **(a)** $IlB = mg$. $I = \dfrac{mg}{lB} = \dfrac{(0.150\ \text{kg})(9.80\ \text{m/s}^2)}{(2.00\ \text{m})(0.55\times10^{-4}\ \text{T})} = 1.34\times10^4$ A. This is a very large current and ohmic heating due to the resistance of the wire would be severe; such a current isn't feasible.

(b) The magnetic force must be upward. The directions of I, $\vec{B}$ and $\vec{F}$ are shown in Figure 27.33, where we have assumed that $\vec{B}$ is south to north. To produce an upward magnetic force, the current must be to the east. The wire must be horizontal and perpendicular to the earth's magnetic field.

EVALUATE: The magnetic force is perpendicular to both the direction of I and the direction of $\vec{B}$.

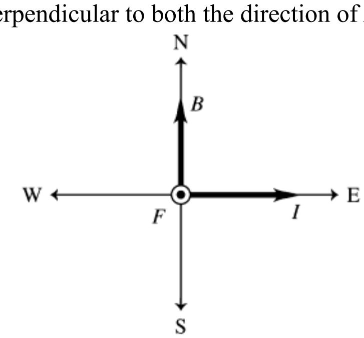


Figure 27.33

27.34. **IDENTIFY:** Apply $F = IlB\sin\phi$.

SET UP: $l = 0.0500$ m is the length of wire in the magnetic field. Since the wire is perpendicular to $\vec{B}$, $\phi = 90°$.

EXECUTE: $F = IlB = (10.8\ \text{A})(0.0500\ \text{m})(0.550\ \text{T}) = 0.297\ \text{N}$.

EVALUATE: The force per unit length of wire is proportional to both B and I.

27.35. **IDENTIFY:** Apply $F = IlB\sin\phi$.

SET UP: Label the three segments in the field as a, b, and c. Let x be the length of segment a. Segment b has length 0.300 m and segment c has length $0.600\ \text{cm} - x$. Figure 27.35a shows the direction of the force on each segment. For each segment, $\phi = 90°$. The total force on the wire is the vector sum of the forces on each segment.

EXECUTE: $F_a = IlB = (4.50\ \text{A})x(0.240\ \text{T})$. $F_c = (4.50\ \text{A})(0.600\ \text{m} - x)(0.240\ \text{T})$. Since $\vec{F}_a$ and $\vec{F}_c$ are in the same direction their vector sum has magnitude $F_{ac} = F_a + F_c = (4.50\ \text{A})(0.600\ \text{m})(0.240\ \text{T}) = 0.648\ \text{N}$ and is directed toward the bottom of the page in Figure 27.35a. $F_b = (4.50\ \text{A})(0.300\ \text{m})(0.240\ \text{T}) = 0.324\ \text{N}$ and is directed to the right. The vector addition diagram for $\vec{F}_{ac}$ and $\vec{F}_b$ is given in Figure 27.35b.

$F = \sqrt{F_{ac}^2 + F_b^2} = \sqrt{(0.648\ \text{N})^2 + (0.324\ \text{N})^2} = 0.724\ \text{N}$. $\tan\theta = \dfrac{F_{ac}}{F_b} = \dfrac{0.648\ \text{N}}{0.324\ \text{N}}$ and $\theta = 63.4°$. The net force has magnitude 0.724 N and its direction is specified by $\theta = 63.4°$ in Figure 27.35b.

EVALUATE: All three current segments are perpendicular to the magnetic field, so $\phi = 90°$ for each in the force equation. The direction of the force on a segment depends on the direction of the current for that segment.

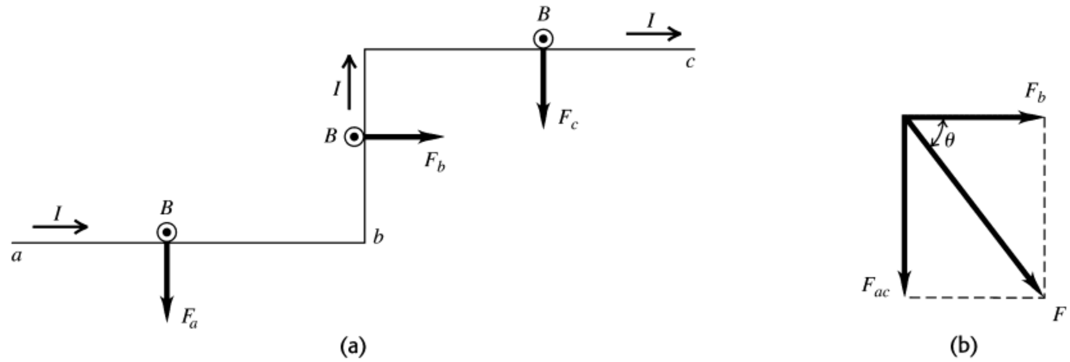


Figure 27.35

27.36. **IDENTIFY** and **SET UP:** $F = IlB\sin\phi$. The direction of $\vec{F}$ is given by applying the right-hand rule to the directions of I and $\vec{B}$.

EXECUTE: **(a)** The current and field directions are shown in Figure 27.36a. The right-hand rule gives that $\vec{F}$ is directed to the south, as shown. $\phi = 90°$ and $F = (1.20\text{A})(1.00\times10^{-2}\ \text{m})(0.588\ \text{T}) = 7.06\times10^{-3}\ \text{N}$.

(b) The right-hand rule gives that $\vec{F}$ is directed to the west, as shown in Figure 27.36b. $\phi = 90°$ and $F = 7.06\times10^{-3}\ \text{N}$, the same as in part (a).

(c) The current and field directions are shown in Figure 27.36c. The right-hand rule gives that $\vec{F}$ is $60.0°$ north of west. $\phi = 90°$ so $F = 7.06\times10^{-3}\ \text{N}$, the same as in part (a).

EVALUATE: In each case the current direction is perpendicular to the magnetic field. The magnitude of the magnetic force is the same in each case but its direction depends on the direction of the magnetic field.

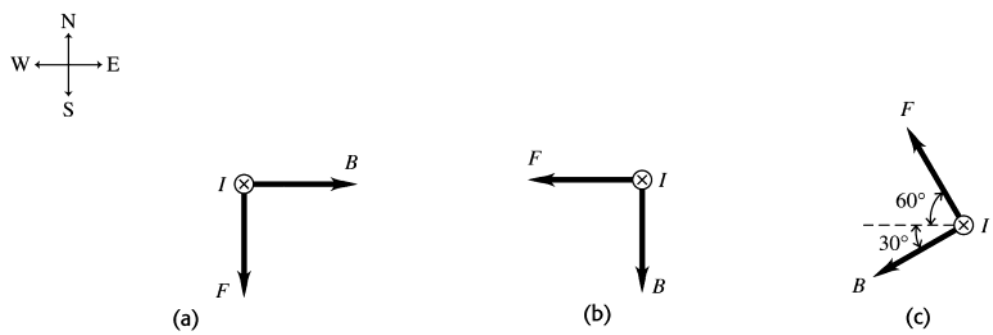


Figure 27.36

27.37. **IDENTIFY:** $F = IlB\sin\phi$.

SET UP: Since the field is perpendicular to the rod it is perpendicular to the current and $\phi = 90°$.

EXECUTE: $I = \dfrac{F}{lB} = \dfrac{0.13\ \text{N}}{(0.200\ \text{m})(0.067\ \text{T})} = 9.7\ \text{A}$

EVALUATE: The force and current are proportional. We have assumed that the entire 0.200 m length of the rod is in the magnetic field.

27.38. **IDENTIFY:** Apply $\vec{F} = I\vec{l}\times\vec{B}$.

SET UP: The magnetic field of a bar magnet points away from the north pole and toward the south pole.

EXECUTE: Between the poles of the magnet, the magnetic field points to the right. Using the fingertips of your right hand, rotate the current vector by $90°$ into the direction of the magnetic field vector. Your thumb points downward—which is the direction of the magnetic force.

EVALUATE If the two magnets had their poles interchanged, then the force would be upward.

27.39. **IDENTIFY** and **SET UP:** The magnetic force is given by Eq.(27.19). $F_I = mg$ when the bar is just ready to levitate. When I becomes larger, $F_I > mg$ and $F_I - mg$ is the net force that accelerates the bar upward. Use Newton's 2nd law to find the acceleration.

(a) EXECUTE: $IlB = mg,\ I = \dfrac{mg}{lB} = \dfrac{(0.750\text{ kg})(9.80\text{ m/s}^2)}{(0.500\text{ m})(0.450\text{ T})} = 32.67\text{ A}$

$\mathcal{E} = IR = (32.67\text{ A})(25.0\ \Omega) = 817\text{ V}$

(b) $R = 2.0\ \Omega,\ I = \mathcal{E}/R = (816.7\text{ V})/(2.0\ \Omega) = 408\text{ A}$

$F_I = IlB = 92\text{ N}$

$a = (F_I - mg)/m = 113\text{ m/s}^2$

EVALUATE: I increases by over an order of magnitude when R changes to $F_I >> mg$ and a is an order of magnitude larger than g.

27.40. IDENTIFY: The magnetic force $\vec{F}_B$ must be upward and equal to mg. The direction of $\vec{F}_B$ is determined by the direction of I in the circuit.

SET UP: $F_B = IlB\sin\phi$, with $\phi = 90°$. $I = \dfrac{V}{R}$, where V is the battery voltage.

EXECUTE: (a) The forces are shown in Figure 27.40. The current I in the bar must be to the right to produce $\vec{F}_B$ upward. To produce current in this direction, point a must be the positive terminal of the battery.

(b) $F_B = mg$. $IlB = mg$. $m = \dfrac{IlB}{g} = \dfrac{VlB}{Rg} = \dfrac{(175\text{ V})(0.600\text{ m})(1.50\text{ T})}{(5.00\ \Omega)(9.80\text{ m/s}^2)} = 3.21\text{ kg}$.

EVALUATE: If the battery had opposite polarity, with point a as the negative terminal, then the current would be clockwise and the magnetic force would be downward.

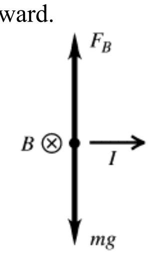


Figure 27.40

27.41. IDENTIFY: Apply $\vec{F} = I\vec{l} \times \vec{B}$ to each segment of the conductor: the straight section parallel to the x axis, the semicircular section and the straight section that is perpendicular to the plane of the figure in Example 27.8.

SET UP: $\vec{B} = B_x\hat{i}$. The force is zero when the current is along the direction of $\vec{B}$.

EXECUTE: (a) The force on the straight section along the $-x$-axis is zero. For the half of the semicircle at negative x the force is out of the page. For the half of the semicircle at positive x the force is into the page. The net force on the semicircular section is zero. The force on the straight section that is perpendicular to the plane of the figure is in the $-y$-direction and has magnitude $F = ILB$. The total magnetic force on the conductor is ILB, in the $-y$-direction.

EVALUATE: (b) If the semicircular section is replaced by a straight section along the x-axis, then the magnetic force on that straight section would be zero, the same as it is for the semicircle.

27.42. IDENTIFY: $\tau = IAB\sin\phi$. The magnetic moment of the loop is $\mu = IA$.

SET UP: Since the plane of the loop is parallel to the field, the field is perpendicular to the normal to the loop and $\phi = 90°$.

EXECUTE: (a) $\tau = IAB = (6.2\text{ A})(0.050\text{ m})(0.080\text{ m})(0.19\text{ T}) = 4.7\times10^{-3}\text{ N}\cdot\text{m}$

(b) $\mu = IA = (6.2\text{ A})(0.050\text{ m})(0.080\text{ m}) = 0.025\text{ A}\cdot\text{m}^2$

EVALUATE: The torque is a maximum when the field is in the plane of the loop and $\phi = 90°$.

27.43. IDENTIFY: The period is $T = 2\pi r/v$, the current is Q/t and the magnetic moment is $\mu = IA$

SET UP: The electron has charge $-e$. The area enclosed by the orbit is πr^2.

EXECUTE: (a) $T = 2\pi r/v = 1.5\times10^{-16}\text{ s}$

(b) Charge $-e$ passes a point on the orbit once during each period, so $I = Q/t = e/t = 1.1\text{ mA}$.

(c) $\mu = IA = I\pi r^2 = 9.3\times10^{-24}\text{ A}\cdot\text{m}^2$

EVALUATE: Since the electron has negative charge, the direction of the current is opposite to the direction of motion of the electron.

27.44. **IDENTIFY:** $\tau = IAB\sin\phi$, where ϕ is the angle between $\vec{B}$ and the normal to the loop.

SET UP: The coil as viewed along the axis of rotation is shown in Figure 27.44a for its original position and in Figure 27.44b after it has rotated $30.0°$.

EXECUTE: **(a)** The forces on each side of the coil are shown in Figure 27.44a. $\vec{F}_1 + \vec{F}_2 = 0$ and $\vec{F}_3 + \vec{F}_4 = 0$. The net force on the coil is zero. $\phi = 0°$ and $\sin\phi = 0$, so $\tau = 0$. The forces on the coil produce no torque.

(b) The net force is still zero. $\phi = 30.0°$ and the net torque is

$\tau = (1)(1.40\text{ A})(0.220\text{ m})(0.350\text{ m})(1.50\text{ T})\sin 30.0° = 0.0808\text{ N}\cdot\text{m}$. The net torque is clockwise in Figure 27.44b and is directed so as to increase the angle ϕ.

EVALUATE: For any current loop in a uniform magnetic field the net force on the loop is zero. The torque on the loop depends on the orientation of the plane of the loop relative to the magnetic field direction.

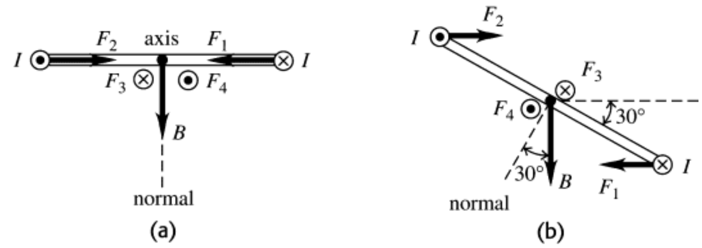


Figure 27.44

27.45. **IDENTIFY:** The magnetic field exerts a torque on the current-carrying coil, which causes it to turn. We can use the rotational form of Newton's second law to find the angular acceleration of the coil.

SET UP: The magnetic torque is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$, and the rotational form of Newton's second law is $\sum \tau = I\alpha$. The magnetic field is parallel to the plane of the loop.

EXECUTE: **(a)** The coil rotates about axis A_2 because the only torque is along top and bottom sides of the coil.

(b) To find the moment of inertia of the coil, treat the two 1.00-m segments as point-masses (since all the points in them are 0.250 m from the rotation axis) and the two 0.500-m segments as thin uniform bars rotated about their centers. Since the coil is uniform, the mass of each segment is proportional to its fraction of the total perimeter of the coil. Each 1.00-m segment is 1/3 of the total perimeter, so its mass is (1/3)(210 g) = 70 g = 0.070 kg. The mass of each 0.500-m segment is half this amount, or 0.035 kg. The result is

$$I = 2(0.070\text{ kg})(0.250\text{ m})^2 + 2\tfrac{1}{12}(0.035\text{ kg})(0.500\text{ m})^2 = 0.0102\text{ kg}\cdot\text{m}^2$$

The torque is

$$|\vec{\tau}| = |\vec{\mu} \times \vec{B}| = IAB\sin 90° = (2.00\text{ A})(0.500\text{ m})(1.00\text{ m})(3.00\text{ T}) = 3.00\text{ N}\cdot\text{m}$$

Using the above values, the rotational form of Newton's second law gives

$$\alpha = \frac{\tau}{I} = 290\text{ rad/s}^2$$

EVALUATE: This angular acceleration will not continue because the torque changes as the coil turns.

27.46. **IDENTIFY:** $\vec{\tau} = \vec{\mu} \times \vec{B}$ and $U = -\mu B\cos\phi$, where $\mu = NIB$. $\tau = \mu B\sin\phi$.

SET UP: ϕ is the angle between $\vec{B}$ and the normal to the plane of the loop.

EXECUTE: **(a)** $\phi = 90°$. $\tau = NIAB\sin(90°) = NIAB$, direction $\hat{k} \times \hat{j} = -\hat{i}$. $U = -\mu B\cos\phi = 0$.

(b) $\phi = 0$. $\tau = NIAB\sin(0) = 0$, no direction. $U = -\mu B\cos\phi = -NIAB$.

(c) $\phi = 90°$. $\tau = NIAB\sin(90°) = NIAB$, direction $-\hat{k} \times \hat{j} = \hat{i}$. $U = -\mu B\cos\phi = 0$.

(d) $\phi = 180°$: $\tau = NIAB\sin(180°) = 0$, no direction, $U = -\mu B\cos(180°) = NIAB$.

EVALUATE: When τ is maximum, $U = 0$. When $|U|$ is maximum, $\tau = 0$.

27.47. **IDENTIFY** and **SET UP:** The potential energy is given by Eq.(27.27): $U = \vec{\mu}\cdot\vec{B}$. The scalar product depends on the angle between $\vec{\mu}$ and $\vec{B}$.

EXECUTE: For $\vec{\mu}$ and $\vec{B}$ parallel, $\phi = 0°$ and $\vec{\mu}\cdot\vec{B} = \mu B\cos\phi = \mu B$. For $\vec{\mu}$ and $\vec{B}$ antiparallel, $\phi = 180°$ and $\vec{\mu}\cdot\vec{B} = \mu B\cos\phi = -\mu B$.

$U_1 = +\mu B,\ U_2 = -\mu B$

$\Delta U = U_2 - U_1 = -2\mu B = -2(1.45\text{ A}\cdot\text{m}^2)(0.835\text{ T}) = -2.42\text{ J}$

EVALUATE: U is maximum when $\vec{\mu}$ and $\vec{B}$ are antiparallel and minimum when they are parallel. When the coil is rotated as specified its magnetic potential energy decreases.

27.48. **IDENTIFY:** Apply Eq.(27.29) in order to calculate *I*. The power drawn from the line is $P_{\text{supplied}} = IV_{ab}$. The mechanical power is the power supplied minus the I^2r electrical power loss in the internal resistance of the motor.

SET UP: $V_{ab} = 120\text{V}$, $\mathcal{E} = 105\text{ V}$, and $r = 3.2\ \Omega$.

EXECUTE: **(a)** $V_{ab} = \mathcal{E} + Ir \Rightarrow I = \dfrac{V_{ab} - \mathcal{E}}{r} = \dfrac{120\text{ V} - 105\text{ V}}{3.2\ \Omega} = 4.7\text{ A}.$

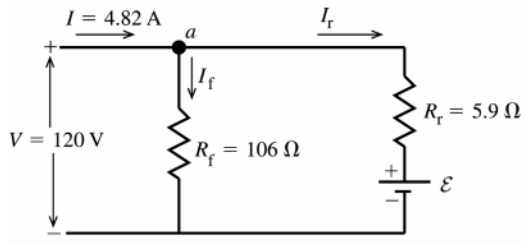
(b) $P_{\text{supplied}} = IV_{ab} = (4.7\text{ A})(120\text{ V}) = 564\text{ W}.$

(c) $P_{\text{mech}} = IV_{ab} - I^2r = 564\text{ W} - (4.7\text{ A})^2(3.2\ \Omega) = 493\text{ W}.$

EVALUATE: If the rotor isn't turning, when the motor is first turned on or if the rotor bearings fail, then $\mathcal{E} = 0$ and $I = \dfrac{120\text{V}}{3.2\ \Omega} = 37.5\text{ A}$. This large current causes large I^2r heating and can trip the circuit breaker.

27.49. **IDENTIFY:** The circuit consists of two parallel branches with the potential difference of 120 V applied across each. One branch is the rotor, represented by a resistance R_{r} and an induced emf that opposes the applied potential. Apply the loop rule to each parallel branch and use the junction rule to relate the currents through the field coil and through the rotor to the 4.82 A supplied to the motor.

SET UP: The circuit is sketched in Figure 27.49.



$\mathcal{E}$ is the induced emf developed by the motor. It is directed so as to oppose the current through the rotor.

Figure 27.49

EXECUTE: **(a)** The field coils and the rotor are in parallel with the applied potential difference V, so $V = I_{\text{f}}R_{\text{f}}$.

$I_{\text{f}} = \dfrac{V}{R_{\text{f}}} = \dfrac{120\text{ V}}{106\ \Omega} = 1.13\text{ A}.$

(b) Applying the junction rule to point *a* in the circuit diagram gives $I - I_{\text{f}} - I_{\text{r}} = 0.$

$I_{\text{r}} = I - I_{\text{f}} = 4.82\text{ A} - 1.13\text{ A} = 3.69\text{ A}.$

(c) The potential drop across the rotor, $I_{\text{r}}R_{\text{r}} + \mathcal{E}$, must equal the applied potential difference $V : V = I_{\text{r}}R_{\text{r}} + \mathcal{E}$

$\mathcal{E} = V - I_{\text{r}}R_{\text{r}} = 120\text{ V} - (3.69\text{ A})(5.9\ \Omega) = 98.2\text{ V}$

(d) The mechanical power output is the electrical power input minus the rate of dissipation of electrical energy in the resistance of the motor:

electrical power input to the motor

$P_{\text{in}} = IV = (4.82\text{ A})(120\text{ V}) = 578\text{ W}$

electrical power loss in the two resistances

$P_{\text{loss}} = I_{\text{f}}^2R_{\text{f}} + I_{\text{f}}^2R = (1.13\text{ A})^2(106\ \Omega) + (3.69\text{ A})^2(5.9\ \Omega) = 216\text{ W}$

mechanical power output

$P_{\text{out}} = P_{\text{in}} - P_{\text{loss}} = 578\text{ W} - 216\text{ W} = 362\text{ W}$

The mechanical power output is the power associated with the induced emf $\mathcal{E}$

$P_{\text{out}} = P_{\mathcal{E}} = \mathcal{E}I_{\text{r}} = (98.2\text{ V})(3.69\text{ A}) = 362\text{ W},$ which agrees with the above calculation.

EVALUATE: The induced emf reduces the amount of current that flows through the rotor. This motor differs from the one described in Example 27.12. In that example the rotor and field coils are connected in series and in this problem they are in parallel.

27.50. **IDENTIFY:** The field and rotor coils are in parallel, so $V_{ab} = I_{\text{f}}R_{\text{f}} = \mathcal{E} + I_{\text{r}}R_{\text{r}}$ and $I = I_{\text{f}} + I_{\text{r}}$, where *I* is the current drawn from the line. The power input to the motor is $P = V_{ab}I$. The power output of the motor is the power input minus the electrical power losses in the resistances and friction losses.

SET UP: $V_{ab} = 120\text{ V}$. $I = 4.82\text{ A}$.

EXECUTE: **(a)** Field current $I_{\text{f}} = \dfrac{120\text{ V}}{218\ \Omega} = 0.550\text{ A}.$

(b) Rotor current $I_{\text{r}} = I_{\text{total}} - I_{\text{f}} = 4.82\text{ A} - 0.550\text{ A} = 4.27\text{ A}.$

(c) $V = \mathcal{E} + I_{\rm r}R_{\rm r}$ and $\mathcal{E} = V - I_{\rm r}R_{\rm r} = 120\text{ V} - (4.27\text{ A})(5.9\ \Omega) = 94.8\text{ V}.$

(d) $P_{\rm f} = I_{\rm f}^2 R_{\rm f} = (0.550\text{ A})^2(218\ \Omega) = 65.9\text{ W}.$

(e) $P_{\rm r} = I_{\rm r}^2 R_{\rm r} = (4.27\text{ A})^2(5.9\ \Omega) = 108\text{ W}.$

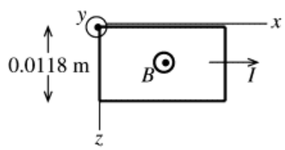
(f) Power input = (120 V) (4.82 A) = 578 W.

(g) Efficiency = $\dfrac{P_{\text{output}}}{P_{\text{input}}} = \dfrac{(578\text{ W} - 65.9\text{ W} - 108\text{ W} - 45\text{ W})}{578\text{ W}} = \dfrac{359\text{ W}}{578\text{ W}} = 0.621.$

EVALUATE: I^2R losses in the resistance of the rotor and field coils are larger than the friction losses for this motor.

27.51. **IDENTIFY:** The drift velocity is related to the current density by Eq.(25.4). The electric field is determined by the requirement that the electric and magnetic forces on the current-carrying charges are equal in magnitude and opposite in direction.

(a) SET UP: The section of the silver ribbon is sketched in Figure 27.51a.



$J_x = n|q|v_{\rm d}$

so $v_{\rm d} = \dfrac{J_x}{n|q|}$

Figure 27.51a

EXECUTE: $J_x = \dfrac{I}{A} = \dfrac{I}{y_1 z_1} = \dfrac{120\text{ A}}{(0.23\times10^{-3}\text{ m})(0.0118\text{ m})} = 4.42\times10^{7}\text{ A/m}^2$

$$v_{\rm d} = \frac{J_x}{n|q|} = \frac{4.42\times10^{7}\text{ A/m}^2}{\left(5.85\times10^{28}/\text{m}^3\right)\left(1.602\times10^{-19}\text{ C}\right)} = 4.7\times10^{-3}\text{ m/s} = 4.7\text{ mm/s}$$

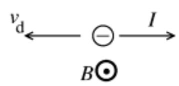
(b) magnitude of $\vec{E}$

$|q|E_z = |q|v_{\rm d}B_y$

$E_z = v_{\rm d}B_y = (4.7\times10^{-3}\text{ m/s})(0.95\text{ T}) = 4.5\times10^{-3}\text{ V/m}$

direction of $\vec{E}$

The drift velocity of the electrons is in the opposite direction to the current, as shown in Figure 27.51b.



$\vec{v}\times\vec{B}\uparrow$

$\vec{F}_B = q\vec{v}\times\vec{B} = -e\vec{v}\times\vec{B}\downarrow$

Figure 27.51b

The directions of the electric and magnetic forces on an electron in the ribbon are shown in Figure 27.51c.



$\vec{F}_E$ must oppose $\vec{F}_B$ so $\vec{F}_E$ is in the $-z$-direction

Figure 27.51c

$\vec{F}_E = q\vec{E} = -e\vec{E}$ so $\vec{E}$ is opposite to the direction of $\vec{F}_E$ and thus $\vec{E}$ is in the $+z$-direction.

(c) The Hall emf is the potential difference between the two edges of the strip (at $z = 0$ and $z = z_1$) that results from the electric field calculated in part (b). $\mathcal{E}_{\text{Hall}} = Ez_1 = (4.5\times10^{-3}\text{ V/m})(0.0118\text{ m}) = 53\ \mu\text{V}$

EVALUATE: Even though the current is quite large the Hall emf is very small. Our calculated Hall emf is more than an order of magnitude larger than in Example 27.13. In this problem the magnetic field and current density are larger than in the example, and this leads to a larger Hall emf.

27.52. **IDENTIFY:** Apply Eq.(27.30).

SET UP: $A = y_1 z_1.\ \ E = \mathcal{E}/z_1.\ \ |q| = e.$

EXECUTE: $n = \dfrac{J_x B_y}{|q|E_z} = \dfrac{IB_y}{A|q|E_z} = \dfrac{IB_y z_1}{A|q|\mathcal{E}} = \dfrac{IB_y}{y_1|q|\mathcal{E}}$

$$n = \frac{(78.0\text{ A})(2.29\text{ T})}{(2.3\times10^{-4}\text{ m})(1.6\times10^{-19}\text{ C})(1.31\times10^{-4}\text{ V})} = 3.7\times10^{28}\text{ electrons/m}^3$$

EVALUATE: The value of n for this metal is about one-third the value of n calculated in Example 27.12 for copper.

27.53. **(a) IDENTIFY:** Use Eq.(27.2) to relate $\vec{v}$, $\vec{B}$, and $\vec{F}$.

SET UP: The directions of $\vec{v}_1$ and $\vec{F}_1$ are shown in Figure 27.53a.

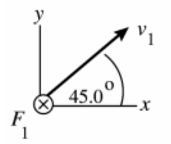


Figure 27.53a

$\vec{F} = q\vec{v} \times \vec{B}$ says that $\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$. The information given here means that $\vec{B}$ can have no z-component.

The directions of $\vec{v}_2$ and $\vec{F}_2$ are shown in Figure 27.53b.

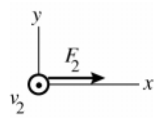


Figure 27.53b

$\vec{F}$ is perpendicular to $\vec{v}$ and $\vec{B}$, so $\vec{B}$ can have no x-component.

Both pieces of information taken together say that $\vec{B}$ is in the y-direction; $\vec{B} = B_y\hat{j}$.

EXECUTE: Use the information given about $\vec{F}_2$ to calculate F_y: $\vec{F}_2 = F_2\hat{i}$, $\vec{v}_2 = v_2\hat{k}$, $\vec{B} = B_y\hat{j}$.

$\vec{F}_2 = q\vec{v}_2 \times \vec{B}$ says $F_2\hat{i} = qv_2B_y\hat{k} \times \hat{j} = qv_2B_y(-\hat{i})$ and $F_2 = -qv_2B_y$

$B_y = -F_2/(qv_2) = -F_2/(qv_1)$. $\vec{B}$ has the maginitude $F_2/(qv_1)$ and is in the $-y$-direction.

(b) $F_1 = qvB\sin\phi = qv_1|B_y|/\sqrt{2} = F_2/\sqrt{2}$

EVALUATE: $v_1 = v_2$. $\vec{v}_2$ is perpendicular to $\vec{B}$ whereas only the component of $\vec{v}_1$ perpendicular to $\vec{B}$ contributes to the force, so it is expected that $F_2 > F_1$, as we found.

27.54. **IDENTIFY:** Apply $\vec{F} = q\vec{v} \times \vec{B}$.

SET UP: $B_x = 0.450$ T, $B_y = 0$ and $B_z = 0$.

EXECUTE: $F_x = q(v_yB_z - v_zB_y) = 0$.

$F_y = q(v_zB_x - v_xB_z) = (9.45 \times 10^{-8}\text{ C})(5.85 \times 10^4\text{ m/s})(0.450\text{ T}) = 2.49 \times 10^{-3}$ N.

$F_z = q(v_xB_y - v_yB_x) = -(9.45 \times 10^{-8}\text{ C})(-3.11 \times 10^4\text{ m/s})(0.450\text{ T}) = 1.32 \times 10^{-3}$ N.

EVALUATE: $\vec{F}$ is perpendicular to both $\vec{v}$ and $\vec{B}$. We can verify that $\vec{F} \cdot \vec{v} = 0$. Since $\vec{B}$ is along the x-axis, v_x does not affect the force components.

27.55. **IDENTIFY:** The sum of the magnetic, electrical, and gravitational forces must be zero to aim at and hit the target.
SET UP: The magnetic field must point to the left when viewed in the direction of the target for no net force. The net force is zero, so $\sum F = F_B - F_E - mg = 0$ and $qvB - qE - mg = 0$.

EXECUTE: Solving for B gives

$$B = \frac{qE + mg}{qv} = \frac{(2500 \times 10^{-6}\text{ C})(27.5\text{ N/C}) + (0.0050\text{ kg})(9.80\text{ m/s}^2)}{(2500 \times 10^{-6}\text{ C})(12.8\text{ m/s})} = 3.7\text{ T}$$

The direction should be perpendicular to the initial velocity of the coin.
EVALUATE: This is a very strong magnetic field, but achievable in some labs.

27.56. **IDENTIFY:** Apply $R = mv/|q|B$. $\omega = v/R$

SET UP: $1\text{ eV} = 1.60 \times 10^{-19}$ J

EXECUTE: **(a)** $K = 2.7\text{ MeV} = (2.7 \times 10^6\text{eV})(1.6 \times 10^{-19}\text{ J/eV}) = 4.32 \times 10^{-13}$ J.

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4.32 \times 10^{-13}\text{ J})}{1.67 \times 10^{-27}\text{ kg}}} = 2.27 \times 10^7\text{ m/s}.$$

$$R = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27}\text{ kg})(2.27 \times 10^7\text{ m/s})}{(1.6 \times 10^{-19}\text{ C})(3.5\text{ T})} = 0.068\text{ m. Also, } \omega = \frac{v}{R} = \frac{2.27 \times 10^7\text{ m/s}}{0.068\text{ m}} = 3.34 \times 10^8\text{ rad/s}.$$

(b) If the energy reaches the final value of 5.4 MeV, the velocity increases by $\sqrt{2}$, as does the radius, to 0.096 m. The angular frequency is unchanged from part (a) so is 3.34×10^8 rad/s.

EVALUATE: $\omega = |q|B/m$, so ω is independent of the energy of the protons. The orbit radius increases when the energy of the proton increases.

27.57. **(a) IDENTIFY** and **SET UP:** The maximum radius of the orbit determines the maximum speed v of the protons. Use Newton's 2nd law and $a_c = v^2/R$ for circular motion to relate the variables. The energy of the particle is the kinetic energy $K = \frac{1}{2}mv^2$.

EXECUTE: $\sum \vec{F} = m\vec{a}$ gives $|q|vB = m(v^2/R)$

$v = \dfrac{|q|BR}{m} = \dfrac{(1.60\times 10^{-19}\text{ C})(0.85\text{ T})(0.40\text{ m})}{1.67\times 10^{-27}\text{ kg}} = 3.257\times 10^7\text{ m/s}.$ The kinetic energy of a proton moving with this speed is $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times 10^{-27}\text{ kg})(3.257\times 10^7\text{ m/s})^2 = 8.9\times 10^{-13}\text{ J} = 5.6\text{ MeV}$

(b) The time for one revolution is the period $T = \dfrac{2\pi R}{v} = \dfrac{2\pi(0.40\text{ m})}{3.257\times 10^7\text{ m/s}} = 7.7\times 10^{-8}\text{ s}$

(c) $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\dfrac{|q|BR}{m}\right)^2 = \frac{1}{2}\dfrac{|q|^2 B^2R^2}{m}$. Or, $B = \dfrac{\sqrt{2Km}}{|q|R}$. B is proportional to $\sqrt{K}$, so if K is increased by a factor of 2 then B must be increased by a factor of $\sqrt{2}$. $B = \sqrt{2}(0.85\text{ T}) = 1.2\text{ T}$.

(d) $v = \dfrac{|q|BR}{m} = \dfrac{(3.20\times 10^{-19}\text{ C})(0.85\text{ T})(0.40\text{ m})}{6.65\times 10^{-27}\text{ kg}} = 1.636\times 10^7\text{ m/s}$

$K = \frac{1}{2}mv^2 = \frac{1}{2}(6.65\times 10^{-27}\text{ kg})(1.636\times 10^7\text{ m/s})^2 = 8.9\times 10^{-13}\text{ J} = 5.5\text{ MeV},$ the same as the maximum energy for protons.

EVALUATE: We can see that the maximum energy must be approximately the same as follows: From part (c), $K = \frac{1}{2}m\left(\dfrac{|q|BR}{m}\right)^2$. For alpha particles $|q|$ is larger by a factor of 2 and m is larger by a factor of 4 (approximately). Thus $|q|^2/m$ is unchanged and K is the same.

27.58. **IDENTIFY:** Apply $\vec{F} = q\vec{v}\times\vec{B}$.

SET UP: $\vec{v} = -v\hat{j}$

EXECUTE: **(a)** $\vec{F} = -qv[B_x(\hat{j}\times\hat{i}) + B_y(\hat{j}\times\hat{j}) + B_z(\hat{j}\times\hat{k})] = qvB_x\hat{k} - qvB_z\hat{i}$

(b) $B_x > 0, B_z < 0,$ sign of B_y doesn't matter.

(c) $\vec{F} = |q|vB_x\hat{i} - |q|vB_x\hat{k}$ and $|\vec{F}| = \sqrt{2}|q|vB_x$.

EVALUATE: $\vec{F}$ is perpendicular to $\vec{v}$, so $\vec{F}$ has no y-component.

27.59. **IDENTIFY:** The contact at a will break if the bar rotates about b. The magnetic field is directed out of the page, so the magnetic torque is counterclockwise, whereas the gravity torque is clockwise in the figure in the problem. The maximum current corresponds to zero net torque, in which case the torque due to gravity is just equal to the torque due to the magnetic field.

SET UP: The magnetic force is perpendicular to the bar and has moment arm $l/2$, where $l = 0.750\text{ m}$ is the length of the bar. The gravity torque is $mg\left(\dfrac{l}{2}\cos 60.0°\right)$

EXECUTE: $\tau_{\text{gravity}} = \tau_B$ and $mg\dfrac{l}{2}\cos 60.0° = IlB\sin 90°\dfrac{l}{2}$. This gives

$I = \dfrac{mg\cos 60.0°}{lB\sin 90°} = \dfrac{(0.458\text{ kg})\left(9.80\text{ m/s}^2\right)(\cos 60.0°)}{(0.750\text{ m})(1.55\text{ T})(1)} = 1.93\text{ A}$

EVALUATE: Once contact is broken, the magnetic torque ceases. The $90.0°$ angle in the expression for τ_B is the angle between the direction of I and the direction of $\vec{B}$.

27.60. **IDENTIFY:** Apply $R = \dfrac{mv}{|q|B}$.

SET UP: Assume $D \ll R$

EXECUTE: **(a)** The path is sketched in Figure 27.60.

(b) Motion is circular: $x^2 + y^2 = R^2 \Rightarrow x = D \Rightarrow y_1 = \sqrt{R^2 - D^2}$ (path of deflected particle)

$y_2 = R$ (equation for tangent to the circle, path of undeflected particle).

$d = y_2 - y_1 = R - \sqrt{R^2 - D^2} = R - R\sqrt{1 - \frac{D^2}{R^2}} = R\left[1 - \sqrt{1 - \frac{D^2}{R^2}}\right]$. If $R >> D$, $d \approx R\left[1 - \left(1 - \frac{1}{2}\frac{D^2}{R^2}\right)\right] = \frac{D^2}{2R}$. For a particle moving in a magnetic field, $R = \frac{mv}{qB}$. But $\frac{1}{2}mv^2 = qV$, so $R = \frac{1}{B}\sqrt{\frac{2mV}{q}}$. Thus, the deflection

$$d \approx \frac{D^2 B}{2}\sqrt{\frac{q}{2mV}} = \frac{D^2 B}{2}\sqrt{\frac{e}{2mV}}.$$

(c) $d = \frac{(0.50\text{ m})^2(5.0\times10^{-5}\text{ T})}{2}\sqrt{\frac{(1.6\times10^{-19}\text{ C})}{2(9.11\times10^{-31}\text{ kg})(750\text{ V})}} = 0.067\text{ m} = 6.7\text{ cm}$. $d \approx 13\%$ of D, which is fairly significant.

EVALUATE: In part (c), $R = \frac{1}{B}\sqrt{\frac{2mV}{e}} = \frac{D^2}{2d} = \left(\frac{D}{2d}\right)D = 3.7D$ and $\left(\frac{R}{D}\right)^2 = 14$, so the approximation made in part (b) is valid.

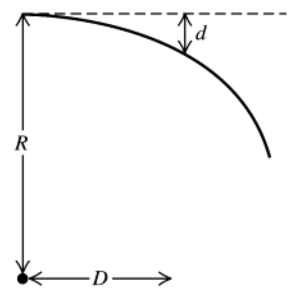


Figure 27.60

27.61. **IDENTIFY** and **SET UP:** Use Eq.(27.2) to relate $q, \vec{v}, \vec{B}$ and $\vec{F}$. The force $\vec{F}$ and $\vec{a}$ are related by Newton's 2nd law.

$\vec{B} = -(0.120\text{ T})\hat{k}, \vec{v} = (1.05\times10^6\text{ m/s})(-3\hat{i} + 4\hat{j} + 12\hat{k}),\ F = 1.25\text{ N}$

(a) EXECUTE: $\vec{F} = q\vec{v}\times\vec{B}$

$\vec{F} = q(-0.120\text{ T})(1.05\times10^6\text{ m/s})(-3\hat{i}\times\hat{k} + 4\hat{j}\times\hat{k} + 12\hat{k}\times\hat{k})$

$\hat{i}\times\hat{k} = -\hat{j}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{k} = 0$

$\vec{F} = -q(1.26\times10^5\text{ N/C})(+3\hat{j} + 4\hat{i}) = -q(1.26\times10^5\text{ N/C})(+4\hat{i} + 3\hat{j})$

The magnitude of the vector $+4\hat{i} + 3\hat{j}$ is $\sqrt{3^2 + 4^2} = 5$. Thus $F = -q(1.26\times10^5\text{ N/C})(5)$.

$$q = -\frac{F}{5(1.26\times10^5\text{ N/C})} = -\frac{1.25\text{ N}}{5(1.26\times10^5\text{ N/C})} = -1.98\times10^{-6}\text{ C}$$

(b) $\sum\vec{F} = m\vec{a}$ so $\vec{a} = \vec{F}/m$

$\vec{F} = -q(1.26\times10^5\text{ N/C})(+4\hat{i} + 3\hat{j}) = -(-1.98\times10^{-6}\text{ C})(1.26\times10^5\text{ N/C})(+4\hat{i} + 3\hat{j}) = +0.250\text{ N}(+4\hat{i} + 3\hat{j})$

Then $\vec{a} = \vec{F}/m = \left(\frac{0.250\text{ N}}{2.58\times10^{-15}\text{ kg}}\right)(+4\hat{i} + 3\hat{j}) = (9.69\times10^{13}\text{ m/s}^2)(+4\hat{i} + 3\hat{j})$

(c) IDENTIFY and **SET UP:** $\vec{F}$ is in the *xy*-plane, so in the *z*-direction the particle moves with constant speed 12.6×10^6 m/s. In the *xy*-plane the force $\vec{F}$ causes the particle to move in a circle, with $\vec{F}$ directed in towards the center of the circle.

EXECUTE: $\sum\vec{F} = m\vec{a}$ gives $F = m(v^2/R)$ and $R = mv^2/F$

$v^2 = v_x^2 + v_y^2 = (-3.15\times10^6\text{ m/s})^2 + (+4.20\times10^6\text{ m/s})^2 = 2.756\times10^{13}\text{ m}^2/\text{s}^2$

$F = \sqrt{F_x^2 + F_y^2} = (0.250\text{ N})\sqrt{4^2 + 3^2} = 1.25\text{ N}$

$$R = \frac{mv^2}{F} = \frac{(2.58\times10^{-15}\text{ kg})(2.756\times10^{13}\text{ m}^2/\text{s}^2)}{1.25\text{ N}} = 0.0569\text{ m} = 5.69\text{ cm}$$

(d) IDENTIFY and **SET UP:** By Eq.(27.12) the cyclotron frequency is $f = \omega/2\pi = v/2\pi R$.

EXECUTE: The circular motion is in the *xy*-plane, so $v = \sqrt{v_x^2 + v_y^2} = 5.25\times10^6$ m/s.

$$f = \frac{v}{2\pi R} = \frac{5.25\times10^6 \text{ m/s}}{2\pi(0.0569 \text{ m})} = 1.47\times10^7 \text{ Hz, and } \omega = 2\pi f = 9.23\times10^7 \text{ rad/s}$$

(e) IDENTIFY and **SET UP** Compare t to the period T of the circular motion in the *xy*-plane to find the x and y coordinates at this t. In the z-direction the particle moves with constant speed, so $z = z_0 + v_z t$.

EXECUTE: The period of the motion in the *xy*-plane is given by $T = \frac{1}{f} = \frac{1}{1.47\times10^7 \text{ Hz}} = 6.80\times10^{-8}$ s

In $t = 2T$ the particle has returned to the same x and y coordinates. The z-component of the motion is motion with a constant velocity of $v_z = +12.6\times10^6$ m/s. Thus $z = z_0 + v_z t = 0 + (12.6\times10^6 \text{ m/s})(2)(6.80\times10^{-8} \text{ s}) = +1.71$ m.

The coordinates at $t = 2T$ are $x = R, y = 0, z = +1.71$ m.

EVALUATE: The circular motion is in the plane perpendicular to $\vec{B}$. The radius of this motion gets smaller when B increases and it gets larger when v increases. There is no magnetic force in the direction of $\vec{B}$ so the particle moves with constant velocity in that direction. The superposition of circular motion in the *xy*-plane and constant speed motion in the z-direction is a helical path.

27.62. IDENTIFY: The net magnetic force on the wire is the vector sum of the force on the straight segment plus the force on the curved section. We must integrate to get the force on the curved section.

SET UP: $\sum F = F_{\text{straight, top}} + F_{\text{curved}} + F_{\text{straight, bottom}}$ and $F_{\text{straight, top}} = F_{\text{straight, bottom}} = iL_{\text{straight}}B$. $F_{\text{curved}, x} = \int_0^{\pi} iRB\sin\theta d\theta = 2iRB$

(the same as if it were a straight segment $2R$ long) and $F_y = 0$ due to symmetry. Therefore, $F = 2iL_{\text{straight}}B + 2iRB$

EXECUTE: Using $L_{\text{straight}} = 0.55$ m, $R = 0.95$ m, $i = 3.40$ A, and $B = 2.20$ T gives $F = 22$ N, to right.

EVALUATE: Notice that the curve has no effect on the force. In other words, the force is the same as if the wire were simply a straight wire 3.00 m long.

27.63. IDENTIFY: $\tau = NIAB\sin\phi$.

SET UP: The area A is related to the diameter D by $A = \frac{1}{4}\pi D^2$.

EXECUTE: $\tau = NI(\frac{1}{4}\pi D^2)B\sin\phi$. τ is proportional to D^2. Increasing D by a factor of 3 increases τ by a factor of $3^2 = 9$.

EVALUATE: The larger diameter means larger length of wire in the loop and also larger moment arms because parts of the loop are farther from the axis.

27.64. IDENTIFY: Apply $\vec{F} = q\vec{v}\times\vec{B}$

SET UP: $\vec{v} = v\hat{k}$

EXECUTE: **(a)** $\vec{F} = -qvB_y\hat{i} + qvB_x\hat{j}$. But $\vec{F} = 3F_0\hat{i} + 4F_0\hat{j}$, so $3F_0 = -qvB_y$ and $4F_0 = qvB_x$

Therefore, $B_y = -\frac{3F_0}{qv}$, $B_x = \frac{4F_0}{qv}$ and B_z is undetermined.

(b) $B = \frac{6F_0}{qv} = \sqrt{B_x^2 + B_y^2 + B_z^2} = \frac{F_0}{qv}\sqrt{9 + 16 + \left(\frac{qv}{F_0}\right)^2 B_z^2} = \frac{F_0}{qv}\sqrt{25 + \left(\frac{qv}{F_0}\right)^2 B_z^2}$, so $B_z = \pm\frac{11F_0}{qv}$.

EVALUATE: The force doesn't depend on B_z, since $\vec{v}$ is along the z-direction.

27.65. IDENTIFY: For the velocity selector, $E = vB$. For the circular motion in the field B', $R = \frac{mv}{|q|B'}$.

SET UP: $B = B' = 0.701$ T.

EXECUTE: $v = \frac{E}{B} = \frac{1.88\times10^4 \text{ N/C}}{0.701 \text{ T}} = 2.68\times10^4$ m/s. $R = \frac{mv}{qB'}$, so

$$R_{82} = \frac{82(1.66\times10^{-27} \text{ kg})(2.68\times10^4 \text{ m/s})}{(1.60\times10^{-19} \text{ C})(0.701 \text{ T})} = 0.0325 \text{ m.}$$

$$R_{84} = \frac{84(1.66\times10^{-27} \text{ kg})(2.68\times10^4 \text{ m/s})}{(1.60\times10^{-19} \text{ C})(0.701 \text{ T})} = 0.0333 \text{ m.}$$

$$R_{86} = \frac{86(1.66\times10^{-27} \text{ kg})(2.68\times10^4 \text{ m/s})}{(1.60\times10^{-19} \text{ C})(0.701 \text{ T})} = 0.0341 \text{ m.}$$

The distance between two adjacent lines is $\Delta R = 1.6$ mm.

EVALUATE: The distance between the ^{82}Kr line and the ^{84}Kr line is 1.6 mm and the distance between the ^{84}Kr line and the ^{86}Kr line is 1.6 mm. Adjacent lines are equally spaced since the ^{82}Kr versus ^{84}Kr and ^{84}Kr versus ^{86}Kr mass differences are the same.

27.66. **IDENTIFY:** Apply conservation of energy to the acceleration of the ions and Newton's second law to their motion in the magnetic field.

SET UP: The singly ionized ions have $q = +e$. A ^{12}C ion has mass 12 u and a ^{14}C ion has mass 14 u, where $1\ \text{u} = 1.66\times10^{-27}$ kg

EXECUTE: **(a)** During acceleration of the ions, $qV = \frac{1}{2}mv^2$ and $v = \sqrt{\frac{2qV}{m}}$. In the magnetic field,

$$R = \frac{mv}{qB} = \frac{m\sqrt{2qV/m}}{qB} \text{ and } m = \frac{qB^2R^2}{2V}.$$

(b) $V = \dfrac{qB^2R^2}{2m} = \dfrac{(1.60\times10^{-19}\ \text{C})(0.150\ \text{T})^2(0.500\ \text{m})^2}{2(12)(1.66\times10^{-27}\ \text{kg})} = 2.26\times10^4\ \text{V}$

(c) The ions are separated by the differences in the diameters of their paths. $D = 2R = 2\sqrt{\dfrac{2Vm}{qB^2}}$, so

$$\Delta D = D_{14} - D_{12} = 2\sqrt{\frac{2Vm}{qB^2}}\bigg|_{14} - 2\sqrt{\frac{2Vm}{qB^2}}\bigg|_{12} = 2\sqrt{\frac{2V(1\ \text{u})}{qB^2}}\left(\sqrt{14}-\sqrt{12}\right).$$

$\Delta D = 2\sqrt{\dfrac{2(2.26\times10^4\ \text{V})(1.66\times10^{-27}\ \text{kg})}{(1.6\times10^{-19}\ \text{C})(0.150\ \text{T})^2}}\left(\sqrt{14}-\sqrt{12}\right) = 8.01\times10^{-2}$ m. This is about 8 cm and is easily distinguishable.

EVALUATE: The speed of the ^{12}C ion is $v = \sqrt{\dfrac{2(1.60\times10^{-19}\ \text{C})(2.26\times10^4\ \text{V})}{12(1.66\times10^{-27}\ \text{kg})}} = 6.0\times10^5$ m/s. This is very fast, but well below the speed of light, so relativistic mechanics is not needed.

27.67. **IDENTIFY:** The force exerted by the magnetic field is given by Eq.(27.19). The net force on the wire must be zero.

SET UP: For the wire to remain at rest the force exerted on it by the magnetic field must have a component directed up the incline. To produce a force in this direction, the current in the wire must be directed from right to left in Figure 27.61 in the textbook. Or, viewing the wire from its left-hand end the directions are shown in Figure 27.67a.

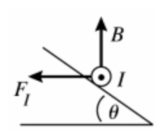


Figure 27.67a

The free-body diagram for the wire is given in Figure 27.67b.

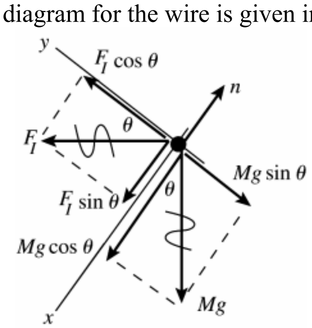


Figure 27.67b

EXECUTE: $\sum F_y = 0$

$F_I \cos\theta - Mg\sin\theta = 0$

$F_I = ILB\sin\phi$

$\phi = 90°$ since $\vec{B}$ is perpendicular to the current direction.

Thus $(ILB)\cos\theta - Mg\sin\theta = 0$ and $I = \dfrac{Mg\tan\theta}{LB}$

EVALUATE: The magnetic and gravitational forces are in perpendicular directions so their components parallel to the incline involve different trig functions. As the tilt angle θ increases there is a larger component of Mg down the incline and the component of F_I up the incline is smaller; I must increase with θ to compensate. As $\theta \to 0$, $I \to 0$ and as $\theta \to 90°, I \to \infty$.

27.68. **IDENTIFY:** The current in the bar is downward, so the magnetic force on it is vertically upwards. The net force on the bar is equal to the magnetic force minus the gravitational force, so Newton's second law gives the acceleration. The bar is in parallel with the 10.0-Ω resistor, so we must use circuit analysis to find the initial current through it.

SET UP: First find the current. The equivalent resistance across the battery is 30.0 Ω, so the total current is 4.00 A, half of which goes through the bar. Applying Newton's second law to the bar gives $\sum F = ma = F_B - mg = iLB - mg.$

EXECUTE: Solving for the acceleration gives

$$a = \frac{iLB - mg}{m} = \frac{(2.0\text{ A})(1.50\text{ m})(1.60\text{ T}) - 3.00\text{ N}}{(3.00\text{ N}/9.80\text{ m/s}^2)} = 5.88\text{ m/s}^2.$$

The direction is upward.

EVALUATE: Once the bar is free of the conducting wires, its acceleration will become 9.8 m/s^2 downward since only gravity will be acting on it.

27.69. **IDENTIFY:** Calculate the acceleration of the ions when they first enter the field and assume this acceleration is constant. Apply conservation of energy to the acceleration of the ions by the potential difference.

SET UP: Assume $\vec{v} = v_x\hat{i}$ and neglect the y-component of $\vec{v}$ that is produced by the magnetic force.

EXECUTE: **(a)** $\frac{1}{2}mv_x{}^2 = qV$, so $v_x = \sqrt{\dfrac{2qV}{m}}$. Also, $a_y = \dfrac{qv_xB}{m}$ and $t = \dfrac{x}{v_x}$.

$$y = \tfrac{1}{2}a_yt^2 = \tfrac{1}{2}a_y\left(\frac{x}{v_x}\right)^2 = \frac{1}{2}\left(\frac{qv_xB}{m}\right)\left(\frac{x}{v_x}\right)^2 = \frac{1}{2}\left(\frac{qBx^2}{m}\right)\left(\frac{m}{2qV}\right)^{1/2} = Bx^2\left(\frac{q}{8mV}\right)^{1/2}.$$

(b) This can be used for isotope separation since the mass in the denominator leads to different locations for different isotopes.

EVALUATE: For $B = 0.1$ T, $v = 1\times10^4$ m/s, $q = +e$ and $m = 12\text{ u} = 2.0\times10^{-26}$ kg, $y = (1.0\text{ m}^{-2})x^2$. The approximation $y << x$ is valid as long as x is on the order of 10 cm or less.

27.70. **IDENTIFY:** Turning the charged loop creates a current, and the external magnetic field exerts a torque on that current.

SET UP: The current is $I = q/T = q/(1/f) = qf = q(\omega/2\pi) = q\omega/2\pi$. The torque is $\tau = \mu B\sin\phi$.

EXECUTE: In this case, $\phi = 90°$ and $\mu = AB$, giving $\tau = IAB$. Combining the results for the torque and current and using $A = \pi r^2$ gives $\tau = \left(\dfrac{q\omega}{2\pi}\right)\pi r^2B = \tfrac{1}{2}q\omega r^2B$

EVALUATE: Any moving charge is a current, so turning the loop creates a current causing a magnetic force.

27.71. **IDENTIFY:** $R = \dfrac{mv}{|q|B}$.

SET UP: After completing one semicircle the separation between the ions is the difference in the diameters of their paths, or $2(R_{13} - R_{12})$. A singly ionized ion has charge $+e$.

EXECUTE: **(a)** $B = \dfrac{mv}{|q|R} = \dfrac{(1.99\times10^{-26}\text{ kg})(8.50\times10^3\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.125\text{ m})} = 8.46\times10^{-3}$ T.

(b) The only difference between the two isotopes is their masses. $\dfrac{R}{m} = \dfrac{v}{|q|B} = \text{constant}$ and $\dfrac{R_{12}}{m_{12}} = \dfrac{R_{13}}{m_{13}}$.

$R_{13} = R_{12}\left(\dfrac{m_{13}}{m_{12}}\right) = (12.5\text{ cm})\left(\dfrac{2.16\times10^{-26}\text{ kg}}{1.99\times10^{-26}\text{ kg}}\right) = 13.6$ cm. The diameter is 27.2 cm.

(c) The separation is $2(R_{13} - R_{12}) = 2(13.6\text{ cm} - 12.5\text{ cm}) = 2.2$ cm. This distance can be easily observed.

EVALUATE: Decreasing the magnetic field increases the separation between the two isotopes at the detector.

27.72. **IDENTIFY:** The force exerted by the magnetic field is $F = ILB\sin\phi$. $a = F/m$ and is constant. Apply a constant acceleration equation to relate v and d.

SET UP: $\phi = 90°$. The direction of $\vec{F}$ is given by the right-hand rule.

EXECUTE: **(a)** $F = ILB$, to the right.

(b) $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v^2 = 2ad$ and $d = \dfrac{v^2}{2a} = \dfrac{v^2m}{2ILB}$.

(c) $d = \dfrac{(1.12\times10^4\text{ m/s})^2(25\text{ kg})}{2(2000\text{ A})(0.50\text{ m})(0.50\text{ T})} = 3.14\times10^6\text{ m} = 3140\text{ km}$

EVALUATE: $a = \dfrac{ILB}{m} = \dfrac{(20\times10^3\text{ A})(0.50\text{ m})(0.50\text{ T})}{25\text{ kg}} = 20\text{ m/s}^2$. The acceleration due to gravity is not negligible.

27.73. **IDENTIFY:** Apply $F = IlB\sin\phi$ to calculate the force on each segment of the wire that is in the magnetic field. The net force is the vector sum of the forces on each segment.

SET UP: The direction of the magnetic force on each current segment in the field is shown in Figure 27.73. By symmetry, $F_a = F_b$. $\vec{F}_a$ and $\vec{F}_b$ are in opposite directions so their vector sum is zero. The net force equals F_c. For F_c, $\phi = 90°$ and $l = 0.450$ m.

EXECUTE: $F_c = IlB = (6.00\text{ A})(0.450\text{ m})(0.666\text{ T}) = 1.80\text{ N}$. The net force is 1.80 N, directed to the left.

EVALUATE: The shape of the region of uniform field doesn't matter, as long as all of segment *c* is in the field and as long as the lengths of the portions of segments *a* and *b* that are in the field are the same.

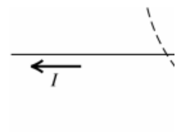


Figure 27.73

27.74. IDENTIFY: Apply $\vec{F} = I\vec{l} \times \vec{B}$.

SET UP: $\vec{l} = l\hat{k}$

EXECUTE: **(a)** $\vec{F} = I(l\hat{k}) \times \vec{B} = Il\left[(-B_y)\hat{i} + (B_x)\hat{j}\right]$. This gives

$F_x = -IlB_y = -(9.00\text{ A})(0.250\text{ m})(-0.985\text{ T}) = 2.22\text{ N}$ and $F_y = IlB_x = (9.00\text{ A})(0.250\text{ m})(-0.242\text{ T}) = -0.545\text{ N}$..

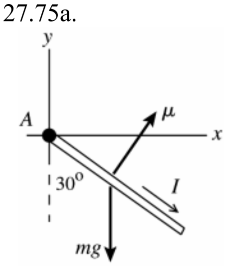
$F_z = 0$, since the wire is in the *z*-direction.

(b) $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(2.22\text{ N})^2 + (0.545\text{ N})^2} = 2.29\text{ N}$.

EVALUATE: $\vec{F}$ must be perpendicular to the current direction, so $\vec{F}$ has no *z* component.

27.75. IDENTIFY: For the loop to be in equilibrium the net torque on it must be zero. Use Eq.(27.26) to calculate the torque due to the magnetic field and use Eq.(10.3) for the torque due to the gravity force.

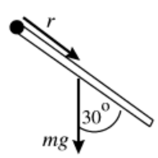
SET UP: See Figure 27.75a.



Use $\sum \tau_A = 0$, where point *A* is at the origin.

Figure 27.75a

EXECUTE: See Figure 27.75b.

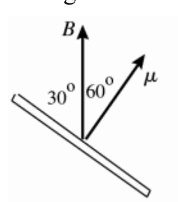


$\tau_{mg} = mgr\sin\phi = mg(0.400\text{ m})\sin 30.0°$

The torque is clockwise; $\vec{\tau}_{mg}$ is directed into the paper.

Figure 27.75b

For the loop to be in equilibrium the torque due to $\vec{B}$ must be counterclockwise (opposite to $\vec{\tau}_{mg}$) and it must be that $\tau_B = \tau_{mg}$. See Figure 27.75c.



$\vec{\tau}_B = \vec{\mu} \times \vec{B}$. For this torque to be counterclockwise ($\vec{\tau}_B$ directed out of the paper), $\vec{B}$ must be in the $+y$-direction.

Figure 27.75c

$\tau_B = \mu B \sin\phi = IAB \sin 60.0°$

$\tau_B = \tau_{mg}$ gives $IAB \sin 60.0° = mg(0.0400\ \text{m}) \sin 30.0°$

$m = (0.15\ \text{g/cm})2(8.00\ \text{cm} + 6.00\ \text{cm}) = 4.2\ \text{g} = 4.2 \times 10^{-3}\ \text{kg}$

$A = (0.800\ \text{m})(0.0600\ \text{m}) = 4.80 \times 10^{-3}\ \text{m}^2$

$$B = \frac{mg(0.0400\ \text{m})(\sin 30.0°)}{IA \sin 60.0°}$$

$$B = \frac{(4.2 \times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)(0.0400\ \text{m}) \sin 30.0°}{(8.2\ \text{A})(4.80 \times 10^{-3}\ \text{m}^2) \sin 60.0°} = 0.024\ \text{T}$$

EVALUATE: As the loop swings up the torque due to $\vec{B}$ decreases to zero and the torque due to mg increases from zero, so there must be an orientation of the loop where the net torque is zero.

27.76. **IDENTIFY:** The torque exerted by the magnetic field is $\vec{\tau} = \vec{\mu} \times \vec{B}$. The torque required to hold the loop in place is $-\vec{\tau}$.

SET UP: $\mu = IA$. $\vec{\mu}$ is normal to the plane of the loop, with a direction given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. $\tau = IAB \sin\phi$, where ϕ is the angle between the normal to the loop and the direction of $\vec{B}$.

EXECUTE: **(a)** $\tau = IAB \sin 60° = (15.0\ \text{A})(0.060\ \text{m})(0.080\ \text{m})(0.48\ \text{T}) \sin 60° = 0.030\ \text{N} \cdot \text{m}$, in the $-\hat{j}$ direction. To keep the loop in place, you must provide a torque in the $+\hat{j}$ direction.

(b) $\tau = IAB \sin 30° = (15.0\ \text{A})(0.60\ \text{m})(0.080\ \text{m})(0.48\ \text{T}) \sin 30° = 0.017\ \text{N} \cdot \text{m}$, in the $+\hat{j}$ direction. You must provide a torque in the $-\hat{j}$ direction to keep the loop in place.

EVALUATE: **(c)** If the loop was pivoted through its center, then there would be a torque on both sides of the loop parallel to the rotation axis. However, the lever arm is only half as large, so the total torque in each case is identical to the values found in parts (a) and (b).

27.77. **IDENTIFY:** Use Eq.(27.20) to calculate the force and then the torque on each small section of the rod and integrate to find the total magnetic torque. At equilibrium the torques from the spring force and from the magnetic force cancel. The spring force depends on the amount x the spring is stretched and then $U = \frac{1}{2}kx^2$ gives the energy stored in the spring.

(a) SET UP:

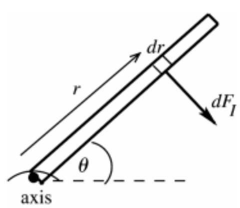


Figure 27.77

Divide the rod into infinitesimal sections of length dr, as shown in Figure 27.77.

EXECUTE: The magnetic force on this section is $dF_I = IBdr$ and is perpendicular to the rod. The torque $d\tau$ due to the force on this section is $d\tau = rdF_I = IBr\,dr$. The total torque is $\int d\tau = IB\int_0^l r\,dr = \frac{1}{2}Il^2B = 0.0442\ \text{N} \cdot \text{m}$, clockwise.

(b) SET UP: F_I produces a clockwise torque so the spring force must produce a counterclockwise torque. The spring force must be to the left; the spring is stretched.

EXECUTE: Find x, the amount the spring is stretched:

$\sum \tau = 0$, axis at hinge, counterclockwise torques positive

$(kx)l \sin 53° - \frac{1}{2}Il^2B = 0$

$$x = \frac{IlB}{2k \sin 53.0°} = \frac{(6.50\ \text{A})(0.200\ \text{m})(0.340\ \text{T})}{2(4.80\ \text{N/m}) \sin 53.0°} = 0.05765\ \text{m}$$

$U = \frac{1}{2}kx^2 = 7.98 \times 10^{-3}\ \text{J}$

EVALUATE: The magnetic torque calculated in part (a) is the same torque calculated from a force diagram in which the total magnetic force $F_I = IlB$ acts at the center of the rod. We didn't include a gravity torque since the problem said the rod had negligible mass.

27.78. **IDENTIFY:** Apply $\vec{F} = I\vec{l} \times \vec{B}$ to calculate the force on each side of the loop.

SET UP: The net force is the vector sum of the forces on each side of the loop.

EXECUTE: **(a)** $F_{PQ} = (5.00 \text{ A})(0.600 \text{ m})(3.00 \text{ T})\sin(0°) = 0 \text{ N}$.

$F_{RP} = (5.00 \text{ A})\,(0.800 \text{ m})\,(3.00 \text{ T})\,\sin(90°) = 12.0 \text{ N}$, into the page.

$F_{QR} = (5.00 \text{ A})(1.00 \text{ m})(3.00 \text{ T})\left(0.800/1.00\right) = 12.0 \text{ N}$, out of the page.

(b) The net force on the triangular loop of wire is zero.

(c) For calculating torque on a straight wire we can assume that the force on a wire is applied at the wire's center. Also, note that we are finding the torque with respect to the *PR*-axis (not about a point), and consequently the lever arm will be the distance from the wire's center to the *x*-axis. $\tau = rF\sin\phi$ gives $\tau_{PQ} = r(0 \text{ N}) = 0$,

$\tau_{RP} = (0 \text{ m})F\sin\phi = 0$ and $\tau_{QR} = (0.300 \text{ m})(12.0 \text{ N})\sin(90°) = 3.60 \text{ N}\cdot\text{m}$. The net torque is $3.60 \text{ N}\cdot\text{m}$.

(d) According to Eq.(27.28), $\tau = NIAB\sin\phi = (1)(5.00 \text{ A})\left(\frac{1}{2}\right)(0.600 \text{ m})(0.800 \text{ m})(3.00 \text{ T})\sin(90°) = 3.60 \text{ N}\cdot\text{m}$, which agrees with part (c).

(e) Since F_{QR} is out of the page and since this is the force that produces the net torque, the point *Q* will be rotated out of the plane of the figure.

EVALUATE: In the expression $\tau = NIAB\sin\phi$, ϕ is the angle between the plane of the loop and the direction of $\vec{B}$. In this problem, $\phi = 90°$.

27.79. **IDENTIFY:** Use Eq.(27.20) to calculate the force on a short segment of the coil and integrate over the entire coil to find the total force.

SET UP: See Figure 27.79a.

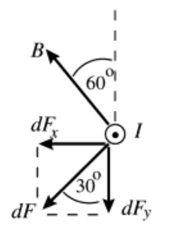


Figure 27.79a

Consider the force $d\vec{F}$ on a short segment *dl* at the left-hand side of the coil, as viewed in Figure 27.69 in the textbook. The current at this point is directed out of the page. $d\vec{F}$ is perpendicular both to $\vec{B}$ and to the direction of *I*.

See Figure 27.79b.

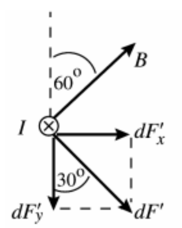


Figure 27.79b

Consider also the force $d\vec{F}'$ on a short segment on the opposite side of the coil, at the right-hand side of the coil in Figure 27.69 in the textbook. The current at this point is directed into the page.

The two sketches show that the *x*-components cancel and that the *y*-components add. This is true for all pairs of short segments on opposite sides of the coil. The net magnetic force on the coil is in the *y*-direction and its magnitude is given by $F = \int dF_y$.

EXECUTE: $dF = Idl\,B\sin\phi$. But $\vec{B}$ is perpendicular to the current direction so $\phi = 90°$.

$dF_y = dF\cos 30.0 = IB\cos 30.0°dl$

$F = \int dF_y = IB\cos 30.0° \int dl$

But $\int dl = N\left(2\pi r\right)$, the total length of wire in the coil.

$F = IB\cos 30.0° N\left(2\pi r\right) = \left(0.950 \text{ A}\right)\left(0.200 \text{ T}\right)\left(\cos 30.0°\right)\left(50\right)2\pi\left(0.0078 \text{ m}\right) = 0.444 \text{ N}$ and $\vec{F} = -\left(0.444 \text{ N}\right)\hat{j}$

EVALUATE: The magnetic field makes a constant angle with the plane of the coil but has a different direction at different points around the circumference of the coil so is not uniform. The net force is proportional to the magnitude of the current and reverses direction when the current reverses direction.

27.80. IDENTIFY: Conservation of energy relates the accelerating potential difference V to the final speed of the ions. In the magnetic field region the ions travel in an arc of a circle that has radius $R = \dfrac{mv}{|q|B}$.

SET UP: The quarter-circle paths of the two ions are shown in Figure 27.80. The separation at the detector is $\Delta r = R_{18} - R_{16}$. Each ion has charge $q = +e$.

EXECUTE: **(a)** Conservation of energy gives $|q|V = \tfrac{1}{2}mv^2$ and $v = \sqrt{\dfrac{2|q|V}{m}}$. $R = \dfrac{m}{|q|B}\sqrt{\dfrac{2|q|V}{m}} = \dfrac{\sqrt{2|q|mV}}{|q|B}$.

$|q| = e$ for each ion. $\Delta r = R_{18} - R_{16} = \dfrac{\sqrt{2eV}}{eB}(\sqrt{m_{18}} - \sqrt{m_{16}})$.

(b) $V = \dfrac{(\Delta reB)^2}{2e\left(\sqrt{m_{18}} - \sqrt{m_{16}}\right)^2} = \dfrac{e(\Delta r)^2B^2}{2\left(\sqrt{m_{18}} - \sqrt{m_{16}}\right)^2} = \dfrac{(1.60\times10^{-19}\ \text{C})(4.00\times10^{-2}\ \text{m})^2(0.050\ \text{T})^2}{2\left(\sqrt{2.99\times10^{-26}\ \text{kg}} - \sqrt{2.66\times10^{-26}\ \text{kg}}\right)^2}$

$V = 3.32\times10^3\ \text{V}$.

EVALUATE: The speed of the ^{16}O ion after it has been accelerated through a potential difference of $V = 3.32\times10^3\ \text{V}$ is $2.00\times10^5\ \text{m/s}$. Increasing the accelerating voltage increases the separation of the two isotopes at the detector. But it does this by increasing the radius of the path for each ion, and this increases the required size of the magnetic field region.

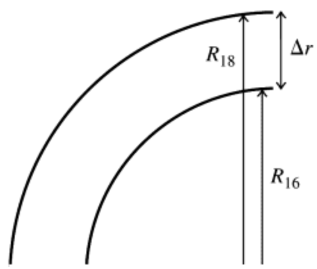


Figure 27.80

27.81. IDENTIFY: Apply $d\vec{F} = Id\vec{l}\times\vec{B}$ to each side of the loop.

SET UP: For each side of the loop, $d\vec{l}$ is parallel to that side of the loop and is in the direction of I. Since the loop is in the xy-plane, $z = 0$ at the loop and $B_y = 0$ at the loop.

EXECUTE: **(a)** The magnetic field lines in the yz-plane are sketched in Figure 27.81.

(b) Side 1, that runs from (0,0) to (0,L): $\vec{F} = \int_0^L Id\vec{l}\times\vec{B} = I\int_0^L \dfrac{B_0 y\,dy}{L}\hat{i} = \tfrac{1}{2}B_0LI\hat{i}$.

Side 2, that runs from (0,L) to (L,L): $\vec{F} = \int_{0,y=L}^{L} Id\vec{l}\times\vec{B} = I\int_{0,y=L}^{L}\dfrac{B_0 y\,dx}{L}\hat{j} = -IB_0L\hat{j}$.

Side 3, that runs from (L,L) to (L,0): $\vec{F} = \int_{L,x=L}^{0} Id\vec{l}\times\vec{B} = I\int_{L,x=L}^{0}\dfrac{B_0 y\,dy}{L}(-\hat{i}) = -\tfrac{1}{2}IB_0L\hat{i}$.

Side 4, that runs from (L,0) to (0,0): $\vec{F} = \int_{L,y=0}^{0} Id\vec{l}\times\vec{B} = I\int_{L,y=0}^{0}\dfrac{B_0 y\,dx}{L}\hat{j} = 0.$

(c) The sum of all forces is $\vec{F}_{\text{total}} = -IB_0L\hat{j}$.

EVALUATE: The net force on sides 1 and 3 is zero. The force on side 4 is zero, since $y = 0$ and $z = 0$ at that side and therefore $B = 0$ there. The net force on the loop equals the force on side 2.

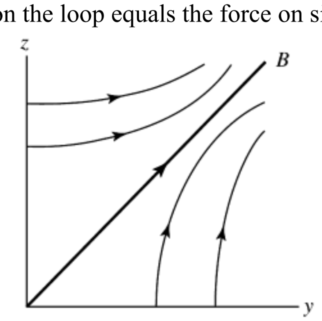


Figure 27.81

27.82. **IDENTIFY:** Apply $d\vec{F} = Id\vec{l} \times \vec{B}$ to each side of the loop. $\vec{\tau} = \vec{r} \times \vec{F}$.

SET UP: For each side of the loop, $d\vec{l}$ is parallel to that side of the loop and is in the direction of I.

EXECUTE: **(a)** The magnetic field lines in the xy-plane are sketched in Figure 27.82.

(b) Side 1, that runs from (0,0) to (0,L): $\vec{F} = \int_0^L Id\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 y\ dy}{L}(-\hat{k}) = -\tfrac{1}{2}B_0 LI\hat{k}$.

Side 2, that runs from (0,L) to (L,L): $\vec{F} = \int_0^L Id\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 x\ dx}{L}\hat{k} = \tfrac{1}{2}IB_0 L\hat{k}$.

Side 3, that runs from (L,L) to (L,0): $\vec{F} = \int_0^L Id\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 ydy}{L}\hat{k} = +\tfrac{1}{2}IB_0 L\hat{k}$

Side 4, that runs from (L,0) to (0,0): $\vec{F} = \int_0^L Id\vec{l} \times \vec{B} = I\int_0^L \frac{B_0 xdx}{L}(-\hat{k}) = -\tfrac{1}{2}IB_0 L\hat{k}$.

(c) If free to rotate about the x-axis, the torques due to the forces on sides 1 and 3 cancel and the torque due to the forces on side 4 is zero. For side 2, $\vec{r} = L\hat{j}$. Therefore, $\vec{\tau} = \vec{r} \times \vec{F} = \frac{IB_0 L^2}{2}\hat{i} = \tfrac{1}{2}IAB_0\hat{i}$.

(d) If free to rotate about the y-axis, the torques due to the forces on sides 2 and 4 cancel and the torque due to the forces on side 1 is zero. For side 3, $\vec{r} = L\hat{i}$. Therefore, $\vec{\tau} = \vec{r} \times \vec{F} = \frac{IB_0 L^2}{2}\hat{j} = -\tfrac{1}{2}IAB_0\hat{j}$.

EVALUATE: **(e)** The equation for the torque $\vec{\tau} = \vec{\mu} \times \vec{B}$ is not appropriate, since the magnetic field is not constant.

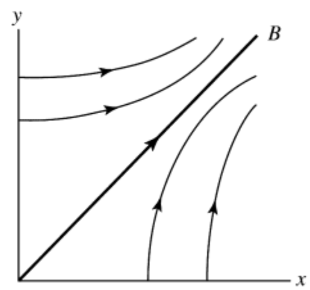


Figure 27.82

27.83. **IDENTIFY:** While the ends of the wire are in contact with the mercury and current flows in the wire, the magnetic field exerts an upward force and the wire has an upward acceleration. After the ends leave the mercury the electrical connection is broken and the wire is in free-fall.

(a) SET UP: After the wire leaves the mercury its acceleration is g, downward. The wire travels upward a total distance of 0.350 m from its initial position. Its ends lose contact with the mercury after the wire has traveled 0.025 m, so the wire travels upward 0.325 m after it leaves the mercury. Consider the motion of the wire after it leaves the mercury. Take $+y$ to be upward and take the origin at the position of the wire as it leaves the mercury.

$a_y = -9.80\ \text{m/s}^2$, $y - y_0 = +0.325$ m, $v_y = 0$ (at maximum height), $v_{0y} = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $v_{0y} = \sqrt{-2a_y(y - y_0)} = \sqrt{-2(-9.80\ \text{m/s}^2)(0.325\ \text{m})} = 2.52\ \text{m/s}$

(b) SET UP: Now consider the motion of the wire while it is in contact with the mercury. Take $+y$ to be upward and the origin at the initial position of the wire. Calculate the acceleration: $y - y_0 = +0.025$ m, $v_{0y} = 0$ (starts from rest), $v_y = +2.52$ m/s (from part (a)), $a_y = ?$

$v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$

EXECUTE: $a_y = \frac{v_y^2}{2(y - y_0)} = \frac{(2.52\ \text{m/s})^2}{2(0.025\ \text{m})} = 127\ \text{m/s}^2$

SET UP: The free-body diagram for the wire is given in Figure 27.83.

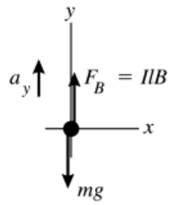


Figure 27.83

EXECUTE: $\sum F_y = ma_y$

$F_B - mg = ma_y$

$IlB = m(g + a_y)$

$I = \dfrac{m(g + a_y)}{lB}$

l is the length of the horizontal section of the wire; $l = 0.150$ m

$$I = \frac{(5.40\times10^{-5}\ \text{kg})(9.80\ \text{m/s}^2 + 127\ \text{m/s}^2)}{(0.150\ \text{m})(0.00650\ \text{T})} = 7.58\ \text{A}$$

(c) IDENTIFY and **SET UP:** Use Ohm's law.

EXECUTE: $V = IR$ so $R = \dfrac{V}{I} = \dfrac{1.50\ \text{V}}{7.58\ \text{A}} = 0.198\ \Omega$

EVALUATE: The current is large and the magnetic force provides a large upward acceleration. During this upward acceleration the wire moves a much shorter distance as it gains speed than the distance is moves while in free-fall with a much smaller acceleration, as it loses the speed it gained. The large current means the resistance of the wire must be small.

27.84. IDENTIFY and **SET UP:** Follow the procedures specified in the problem.

EXECUTE: **(a)** $d\vec{l} = dl\hat{t}$, where $\hat{t}$ is a unit vector in the tangential direction. $d\vec{l} = Rd\theta\left[-\sin\theta\hat{i} + \cos\theta\hat{j}\right]$. Note that this implies that when $\theta = 0$, the line element points in the +y-direction, and when the angle is $90°$, the line element points in the –x-direction. This is in agreement with the diagram.

$d\vec{F} = Id\vec{l}\times\vec{B} = IRd\theta\left[-\sin\theta\hat{i} + \cos\theta\hat{j}\right]\times(B_x\hat{i}) = IB_xRd\theta\left[-\cos\theta\hat{k}\right]$.

(b) $\vec{F} = \int_0^{2\pi} -\cos\theta IB_xR\ d\theta\hat{k} = -IB_xR\int_0^{2\pi}\cos\theta d\theta\hat{k} = 0.$

(c) $d\vec{\tau} = \vec{r}\times d\vec{F} = R(\cos\theta\hat{i} + \sin\theta\hat{j})\times(IB_xR\ d\theta[-\cos\theta\hat{k}]) = -R^2IB_xd\theta(\sin\theta\cos\theta\hat{i} - \cos^2\theta\hat{j})$

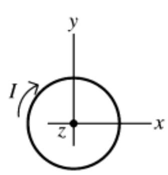
(d) $\vec{\tau} = \int d\vec{\tau} = -R^2IB_x\left(\int_0^{2\pi}\sin\theta\cos\theta d\theta\hat{i} - \int_0^{2\pi}\cos^2\theta d\theta\hat{j}\right) = IR^2B_x\left(\dfrac{\theta}{2} + \dfrac{\sin 2\theta}{4}\right)_0^{2\pi}\hat{j}$. $\vec{\tau} = IR^2B_x\pi\hat{j} = I\pi R^2B_x\hat{j} = IA\hat{k}\times B_x\hat{i}$

and $\vec{\tau} = \vec{\mu}\times\vec{B}$.

EVALUATE: Section 27.7 of the textbook derived $\vec{\tau} = \vec{\mu}\times\vec{B}$ for the case of a rectangular coil. This problem shows that the same result also applies to a circular coil.

27.85. (a) IDENTIFY: Use Eq.(27.27) to relate U, μ and $\vec{B}$ and use Eq.(27.26) to relate $\vec{\tau}$, $\vec{\mu}$ and $\vec{B}$. We also know that $B_0^2 = B_x^2 + B_y^2 + B_z^2$. This gives three equations for the three components of $\vec{B}$.

SET UP: The loop and current are shown in Figure 27.85.



$\vec{\mu}$ is into the plane of the paper, in the –z-direction

Figure 27.85

$\vec{\mu} = -\mu\hat{k} = -IA\hat{k}$

(b) EXECUTE: $\vec{\tau} = D(+4\hat{i} - 3\hat{j})$, where $D > 0$.

$\vec{\mu} = -IA\hat{k}$, $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_y\hat{k}$

$\vec{\tau} = \vec{\mu}\times\vec{B} = (-IA)(B_x\hat{k}\times\hat{i} + B_y\hat{k}\times\hat{j} + B_z\hat{k}\times\hat{k}) = IAB_y\hat{i} - IAB_x\hat{j}$

Compare this to the expression given for $\vec{\tau}$: $IAB_y = 4D$ so $B_y = 4D/IA$ and $-IAB_x = -3D$ so $B_x = 3D/IA$

B_z doesn't contribute to the torque since $\vec{\mu}$ is along the z-direction. But $B = B_0$ and $B_x^2 + B_y^2 + B_z^2 = B_0^2$; with $B_0 = 13D/IA$. Thus $B_z = \pm\sqrt{B_0^2 - B_x^2 - B_y^2} = \pm(D/IA)\sqrt{169 - 9 - 16} = \pm12(D/IA)$

That $U = -\vec{\mu}\cdot\vec{B}$ is negative determines the sign of B_z: $U = -\vec{\mu}\cdot\vec{B} = -(-IA\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) = +IAB_z$

So U negative says that B_z is negative, and thus $B_z = -12D/IA$.

EVALUATE: $\vec{\mu}$ is along the z-axis so only B_x and B_y contribute to the torque. B_x produces a y-component of $\vec{\tau}$ and B_y produces an x-component of $\vec{\tau}$. Only B_z affects U, and U is negative when $\vec{\mu}$ and $\vec{B}_z$ are parallel.

27.86. IDENTIFY: $I = \dfrac{\Delta q}{\Delta t}$ and $\mu = IA$.

SET UP: The direction of $\vec{\mu}$ is given by the right-hand rule that is illustrated in Figure 27.32 in the textbook. I is in the direction of flow of positive charge and opposite to the direction of flow of negative charge.

EXECUTE: **(a)** $I_u = \dfrac{dq}{dt} = \dfrac{\Delta q}{\Delta t} = \dfrac{q_u v}{2\pi r} = \dfrac{ev}{3\pi r}$.

(b) $\mu_u = I_u A = \dfrac{ev}{3\pi r}\pi r^2 = \dfrac{evr}{3}$.

(c) Since there are two down quarks, each of half the charge of the up quark, $\mu_d = \mu_u = \dfrac{evr}{3}$. Therefore, $\mu_{\text{total}} = \dfrac{2evr}{3}$.

(d) $v = \dfrac{3\mu}{2er} = \dfrac{3(9.66\times10^{-27}\ \text{A}\cdot\text{m}^2)}{2(1.60\times10^{-19}\ \text{C})(1.20\times10^{-15}\text{m})} = 7.55\times10^{7}\ \text{m/s}$.

EVALUATE: The speed calculated in part (d) is 25% of the speed of light.

27.87. IDENTIFY: Eq.(27.8) says that the magnetic field through any closed surface is zero.

SET UP: The cylindrical Gaussian surface has its top at $z = L$ and its bottom at $z = 0$. The rest of the surface is the curved portion of the cylinder and has radius r and length L. $B = 0$ at the bottom of the surface, since $z = 0$ there.

EXECUTE: **(a)** $\oint \vec{B}\cdot d\vec{A} = \int_{\text{top}} B_z dA + \int_{\text{curved}} B_r dA = \int_{\text{top}} (\beta L) dA + \int_{\text{curved}} B_r dA = 0$. This gives $0 = \beta L\pi r^2 + B_r 2\pi r L$, and

$B_r(r) = -\dfrac{\beta r}{2}$.

(b) The two diagrams in Figure 27.87 show views of the field lines from the top and side of the Gaussian surface.

EVALUATE: Only a portion of each field line is shown; the field lines are closed loops.

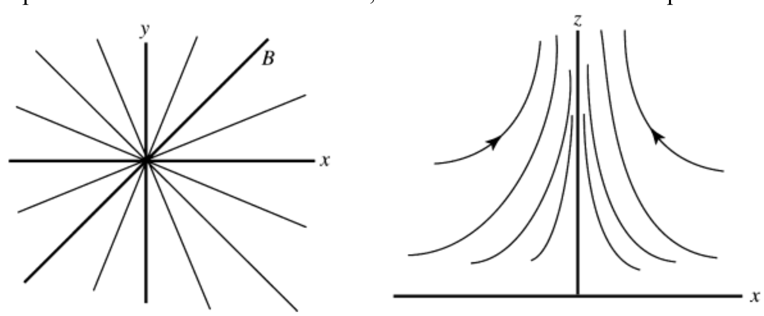


Figure 27.87

27.88. IDENTIFY: $U = -\vec{\mu}\cdot\vec{B}$. In part (b) apply conservation of energy.

SET UP: The kinetic energy of the rotating ring is $K = \tfrac{1}{2}I\omega^2$.

EXECUTE: **(a)** $\Delta U = -(\vec{\mu}_{\text{f}}\cdot\vec{B} - \vec{\mu}_{\text{i}}\cdot\vec{B}) = -(\vec{\mu}_{\text{f}} - \vec{\mu}_{\text{i}})\cdot\vec{B} = \left[-\mu(-\hat{k} - (-0.8\hat{i} + 0.6\hat{j}))\right]\cdot\left[B_0(12\hat{i} + 3\hat{j} - 4\hat{k})\right]$.

$\Delta U = IAB_0[(-0.8)(+12) + (0.6)(+3) + (+1)(-4)] = (12.5\ \text{A})(4.45\times10^{-4}\ \text{m}^2)(0.0115\ \text{T})(-11.8)$.

$\Delta U = -7.55\times10^{-4}\text{J}$.

(b) $\Delta K = \dfrac{1}{2}I\omega^2$. $\omega = \sqrt{\dfrac{2\Delta K}{I}} = \sqrt{\dfrac{2(7.55\times10^{-4}\ \text{J})}{8.50\times10^{-7}\ \text{kg}\cdot\text{m}^2}} = 42.1\ \text{rad/s}$.

EVALUATE: The potential energy of the ring decreases and its kinetic energy increases.

27.89. IDENTIFY and SET UP: In the magnetic field, $R = \dfrac{mv}{qB}$. Once the particle exits the field it travels in a straight line. Throughout the motion the speed of the particle is constant.

EXECUTE: **(a)** $R = \dfrac{mv}{qB} = \dfrac{(3.20\times10^{-11}\ \text{kg})(1.45\times10^{5}\ \text{m/s})}{(2.15\times10^{-6}\ \text{C})(0.420\ \text{T})} = 5.14\ \text{m}$.

(b) See Figure 27.89. The distance along the curve, d, is given by $d = R\theta$. $\sin\theta = \frac{0.35\text{ m}}{5.14\text{ m}}$, so

$\theta = 2.78° = 0.0486$ rad. $d = R\theta = (5.14\text{ m})(0.0486\text{ rad}) = 0.25$ m. And $t = \frac{d}{v} = \frac{0.25\text{ m}}{1.45\times10^{5}\text{ m/s}} = 1.72\times10^{-6}$s.

(c) $\Delta x_1 = d\tan(\theta/2) = (0.25\text{ m})\tan(2.79°/2) = 6.08\times10^{-3}$ m.

(d) $\Delta x = \Delta x_1 + \Delta x_2$, where Δx_2 is the horizontal displacement of the particle from where it exits the field region to where it hits the wall. $\Delta x_2 = (0.50\text{ m})\tan 2.79° = 0.0244$ m. Therefore, $\Delta x = 6.08\times10^{-3}\text{ m} + 0.0244\text{ m} = 0.0305$ m.

EVALUATE: d is much less than R, so the horizontal deflection of the particle is much smaller than the distance it travels in the y-direction.

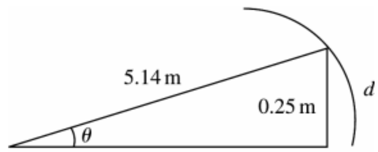


Figure 27.89

27.90. **IDENTIFY:** The current direction is perpendicular to $\vec{B}$, so $F = IlB$. If the liquid doesn't flow, a force $(\Delta p)A$ from the pressure difference must oppose F.

SET UP: $J = I/A$, where $A = hw$.

EXECUTE: **(a)** $\Delta p = F/A = IlB/A = JlB$.

(b) $J = \frac{\Delta p}{lB} = \frac{(1.00\text{ atm})(1.013\times10^{5}\text{ Pa/atm})}{(0.0350\text{ m})(2.20\text{ T})} = 1.32\times10^{6}\text{ A/m}^2$.

EVALUATE: A current of 1 A in a wire with diameter 1 mm corresponds to a current density of $J = 1.36\times10^{6}\text{ A/m}^2$, so the current density calculated in part (c) is a typical value for circuits.

27.91. **IDENTIFY:** The electric and magnetic fields exert forces on the moving charge. The work done by the electric field equals the change in kinetic energy. At the top point, $a_y = \frac{v^2}{R}$ and this acceleration must correspond to the net force.

SET UP: The electric field is uniform so the work it does for a displacement y in the y-direction is $W = Fy = qEy$. At the top point, $\vec{F}_B$ is in the $-y$-direction and $\vec{F}_E$ is in the $+y$-direction.

EXECUTE: **(a)** The maximum speed occurs at the top of the cycloidal path, and hence the radius of curvature is greatest there. Once the motion is beyond the top, the particle is being slowed by the electric field. As it returns to $y = 0$, the speed decreases, leading to a smaller magnetic force, until the particle stops completely. Then the electric field again provides the acceleration in the y-direction of the particle, leading to the repeated motion.

(b) $W = qEy = \frac{1}{2}mv^2$ and $v = \sqrt{\frac{2qEy}{m}}$.

(c) At the top, $F_y = qE - qvB = -\frac{mv^2}{R} = -\frac{m}{2y}\frac{2qEy}{m} = -qE$. $2qE = qvB$ and $v = \frac{2E}{B}$.

EVALUATE: The speed at the top depends on B because B determines the y-displacement and the work done by the electric force depends on the y-displacement.

SOURCES OF MAGNETIC FIELD

28

28.1. IDENTIFY and SET UP: Use Eq.(28.2) to calculate $\vec{B}$ at each point.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}, \text{ since } \hat{r} = \frac{\vec{r}}{r}.$$

$\vec{v} = (8.00\times10^6 \text{ m/s})\hat{j}$ and $\vec{r}$ is the vector from the charge to the point where the field is calculated.

EXECUTE: (a) $\vec{r} = (0.500 \text{ m})\hat{i},\ r = 0.500 \text{ m}$

$$\vec{v}\times\vec{r} = vr\hat{j}\times\hat{i} = -vr\hat{k}$$

$$\vec{B} = -\frac{\mu_0}{4\pi}\frac{qv}{r^2}\hat{k} = -(1\times10^{-7} \text{ T}\cdot\text{m/A})\frac{(6.00\times10^{-6} \text{ C})(8.00\times10^6 \text{ m/s})}{(0.500 \text{ m})^2}\hat{k}$$

$$\vec{B} = -(1.92\times10^{-5} \text{ T})\hat{k}$$

(b) $\vec{r} = -(0.500 \text{ m})\hat{j},\ r = 0.500 \text{ m}$

$\vec{v}\times\vec{r} = -vr\hat{j}\times\hat{j} = 0$ and $\vec{B} = 0$.

(c) $\vec{r} = (0.500 \text{ m})\hat{k},\ r = 0.500 \text{ m}$

$$\vec{v}\times\vec{r} = vr\hat{j}\times\hat{k} = vr\hat{i}$$

$$\vec{B} = (1\times10^{-7} \text{ T}\cdot\text{m/A})\frac{(6.00\times10^{-6} \text{ C})(8.00\times10^6 \text{ m/s})}{(0.500 \text{ m})^2}\hat{i} = +(1.92\times10^{-5} \text{ T})\hat{i}$$

(d) $\vec{r} = -(0.500 \text{ m})\hat{j} + (0.500 \text{ m})\hat{k},\ r = \sqrt{(0.500 \text{ m})^2 + (0.500 \text{ m})^2} = 0.7071 \text{ m}$

$$\vec{v}\times\vec{r} = v(0.500 \text{ m})(-\hat{j}\times\hat{j} + \hat{j}\times\hat{k}) = (4.00\times10^6 \text{ m}^2/\text{s})\hat{i}$$

$$\vec{B} = (1\times10^{-7} \text{ T}\cdot\text{m/A})\frac{(6.00\times10^{-6} \text{ C})(4.00\times10^6 \text{ m/s})}{(0.7071 \text{ m})^3}\hat{i} = +(6.79\times10^{-6} \text{ T})\hat{i}$$

EVALUATE: At each point $\vec{B}$ is perpendicular to both $\vec{v}$ and $\vec{r}$. $B = 0$ along the direction of $\vec{v}$.

28.2. IDENTIFY: A moving charge creates a magnetic field as well as an electric field.

SET UP: The magnetic field caused by a moving charge is $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$, and its electric field is $E = \frac{1}{4\pi\epsilon_0}\frac{e}{r^2}$ since $q = e$.

EXECUTE: Substitute the appropriate numbers into the above equations.

$$B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2} = \frac{4\pi\times10^{-7} \text{ T}\cdot\text{m/A}}{4\pi}\frac{(1.60\times10^{-19}\text{C})(2.2\times10^6\text{m/s})\sin90^\circ}{(5.3\times10^{-11}\text{m})^2} = 13 \text{ T, out of the page.}$$

$$E = \frac{1}{4\pi\epsilon_0}\frac{e}{r^2} = \frac{(9.00\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19} \text{ C})}{(5.3\times10^{-11} \text{ m})^2} = 5.1\times10^{11} \text{ N/C, toward the electron.}$$

EVALUATE: There are enormous fields within the atom!

28.3. IDENTIFY: A moving charge creates a magnetic field.

SET UP: The magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$.

EXECUTE: Substituting numbers into the above equation gives

(a) $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2} = \dfrac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\dfrac{(1.6\times10^{-19}\ \text{C})(3.0\times10^{7}\ \text{m/s})\sin30^\circ}{(2.00\times10^{-6}\ \text{m})^2}.$

$B = 6.00 \times 10^{-8}$ T, out of the paper, and it is the same at point B.

(b) $B = (1.00 \times 10^{-7}\ \text{T}\cdot\text{m/A})(1.60 \times 10^{-19}\ \text{C})(3.00 \times 10^{7}\ \text{m/s})/(2.00 \times 10^{-6}\ \text{m})^2$

$B = 1.20 \times 10^{-7}$ T, out of the page.

(c) $B = 0$ T since $\sin(180^\circ) = 0$.

EVALUATE: Even at high speeds, these charges produce magnetic fields much less than the Earth's magnetic field.

28.4. **IDENTIFY:** Both moving charges produce magnetic fields, and the net field is the vector sum of the two fields.

SET UP: Both fields point out of the paper, so their magnitudes add, giving

$$B = B_{\text{alpha}} + B_{\text{el}} = \frac{\mu_0 v}{4\pi r^2}\left(e\sin40^\circ + 2e\sin140^\circ\right)$$

EXECUTE: Factoring out an e and putting in the numbers gives

$$B = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.60\times10^{-19}\text{C})(2.50\times10^{5}\text{m/s})}{(1.75\times10^{-9}\text{m})^2}\left(\sin40^\circ + 2\sin140^\circ\right)$$

$$B = 2.52\times10^{-3}\ \text{T} = 2.52\ \text{mT, out of the page.}$$

EVALUATE: At distances very close to the charges, the magnetic field is strong enough to be important.

28.5. **IDENTIFY:** Apply $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\vec{r}}{r^3}$.

SET UP: Since the charge is at the origin, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

EXECUTE: **(a)** $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{i}$; $\vec{v}\times\vec{r} = 0$, $B = 0$.

(b) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{j}$; $\vec{v}\times\vec{r} = vr\hat{k}$, $r = 0.500$ m.

$B = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{|q|v}{r^2} = \dfrac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^{5}\ \text{m/s})}{(0.500\ \text{m})^2} = 1.31\times10^{-6}\ \text{T}.$

q is negative, so $\vec{B} = -(1.31\times10^{-6}\ \text{T})\hat{k}$.

(c) $\vec{v} = v\hat{i}$, $\vec{r} = (0.500\ \text{m})(\hat{i} + \hat{j})$; $\vec{v}\times\vec{r} = (0.500\ \text{m})v\hat{k}$, $r = 0.7071$ m.

$B = \left(\dfrac{\mu_0}{4\pi}\right)\left(|q||\vec{v}\times\vec{r}|/r^3\right) = \dfrac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(0.500\ \text{m})(6.80\times10^{5}\ \text{m/s})}{(0.7071\ \text{m})^3}.$

$B = 4.62\times10^{-7}$ T. $\vec{B} = -(4.62\times10^{-7}\ \text{T})\hat{k}$.

(d) $\vec{v} = v\hat{i}$, $\vec{r} = r\hat{k}$; $\vec{v}\times\vec{r} = -vr\hat{j}$, $r = 0.500$ m

$B = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{|q|v}{r^2} = \dfrac{(1.0\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^{5}\ \text{m/s})}{(0.500\ \text{m})^2} = 1.31\times10^{-6}\ \text{T}.$

$\vec{B} = \left(1.31\times10^{-6}\ \text{T}\right)\hat{j}$.

EVALUATE: In each case, $\vec{B}$ is perpendicular to both $\vec{r}$ and $\vec{v}$.

28.6. **IDENTIFY:** Apply $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\vec{r}}{r^3}$. For the magnetic force, apply the results of Example 28.1, except here the two charges and velocities are different.

SET UP: In part (a), $r = d$ and $\vec{r}$ is perpendicular to $\vec{v}$ in each case, so $\dfrac{|\vec{v}\times\vec{r}|}{r^3} = \dfrac{v}{d^2}$. For calculating the force between the charges, $r = 2d$.

EXECUTE: **(a)** $B_{\text{total}} = B + B' = \dfrac{\mu_0}{4\pi}\left(\dfrac{qv}{d^2} + \dfrac{q'v'}{d^2}\right).$

$B = \dfrac{\mu_0}{4\pi}\left(\dfrac{(8.0\times10^{-6}\ \text{C})(4.5\times10^{6}\ \text{m/s})}{(0.120\ \text{m})^2} + \dfrac{(3.0\times10^{-6}\ \text{C})(9.0\times10^{6}\ \text{m/s})}{(0.120\ \text{m})^2}\right) = 4.38\times10^{-4}\ \text{T}.$

The direction of $\vec{B}$ is into the page.

(b) Following Example 28.1 we can find the magnetic force between the charges:

$$F_B = \frac{\mu_0}{4\pi}\frac{qq'vv'}{r^2} = (10^{-7}\ \text{T}\cdot\text{m/A})\frac{(8.00\times10^{-6}\ \text{C})(3.00\times10^{-6}\ \text{C})(4.50\times10^{6}\ \text{m/s})(9.00\times10^{6}\ \text{m/s})}{(0.240\ \text{m})^2}$$

$F_B = 1.69\times10^{-3}$ N. The force on the upper charge points up and the force on the lower charge points down. The Coulomb force between the charges is $F_C = k\frac{q_1q_2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(8.0\times10^{-6}\ \text{C})(3.0\times10^{-6}\ \text{C})}{(0.240\ \text{m})^2} = 3.75\ \text{N}$.

The force on the upper charge points up and the force on the lower charge points down. The ratio of the Coulomb force to the magnetic force is $\frac{F_C}{F_B} = \frac{c^2}{v_1v_2} = \frac{3.75\ \text{N}}{1.69\times10^{-3}\ \text{N}} = 2.22\times10^3$; the Coulomb force is much larger.

(b) The magnetic forces are reversed in direction when the direction of only one velocity is reversed but the magnitude of the force is unchanged.

EVALUATE: When two charges have the same sign and move in opposite directions, the force between them is repulsive. When two charges of the same sign move in the same direction, the force between them is attractive.

28.7. **IDENTIFY:** Apply $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$. For the magnetic force on q', use $\vec{F}_B = q'\vec{v}\times\vec{B}_q$ and for the magnetic force on q use $\vec{F}_B = q\vec{v}\times\vec{B}_{q'}$.

SET UP: In part (a), $r = d$ and $\frac{|\vec{v}\times\vec{r}|}{r^3} = \frac{v}{d^2}$.

EXECUTE: **(a)** $q' = -q$; $B_q = \frac{\mu_0 qv}{4\pi d^2}$, into the page; $B_{q'} = \frac{\mu_0 qv'}{4\pi d^2}$, out of the page.

(i) $v' = \frac{v}{2}$ gives $B = \frac{\mu_0 qv}{4\pi d^2}\left(1-\tfrac{1}{2}\right) = \frac{\mu_0 qv}{4\pi(2d^2)}$, into the page. (ii) $v' = v$ gives $B = 0$.

(iii) $v' = 2v$ gives $B = \frac{\mu_0 qv}{4\pi d^2}$, out of the page.

(b) The force that q exerts on q' is given by $\vec{F} = q'\vec{v}'\times\vec{B}_q$, so $F = \frac{\mu_0 q^2 v'v}{4\pi(2d)^2}$. $\vec{B}_q$ is into the page, so the force on q' is toward q. The force that q' exerts on q is toward q'. The force between the two charges is attractive.

(c) $F_B = \frac{\mu_0 q^2 vv'}{4\pi(2d)^2}$, $F_C = \frac{q^2}{4\pi\epsilon_0(2d)^2}$ so $\frac{F_B}{F_C} = \mu_0\epsilon_0 vv' = \mu_0\epsilon_0(3.00\times10^5\ \text{m/s})^2 = 1.00\times10^{-6}$.

EVALUATE: When charges of opposite sign move in opposite directions, the force between them is attractive. For the values specified in part (c), the magnetic force between the two charges is much smaller in magnitude than the Coulomb force between them.

28.8. **IDENTIFY:** Both moving charges create magnetic fields, and the net field is the vector sum of the two. The magnetic force on a moving charge is $F_{\text{mag}} = qvB\sin\phi$ and the electrical force obeys Coulomb's law.

SET UP: The magnetic field due to a moving charge is $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$.

EXECUTE: **(a)** Both fields are into the page, so their magnitudes add, giving

$$B = B_e + B_p = \frac{\mu_0}{4\pi}\left(\frac{ev}{r_e^2}+\frac{ev}{r_p^2}\right)\sin 90°$$

$$B = \frac{\mu_0}{4\pi}\left(1.60\times10^{-19}\ \text{C}\right)\left(845{,}000\ \text{m/s}\right)\left[\frac{1}{(5.00\times10^{-9}\ \text{m})^2}+\frac{1}{(4.00\times10^{-9}\ \text{m})^2}\right]$$

$B = 1.39 \times 10^{-3}$ T = 1.39 mT, into the page.

(b) Using $B = \frac{\mu_0}{4\pi}\frac{qv\sin\phi}{r^2}$, where $r = \sqrt{41}$ nm and $\phi = 180° - \arctan(5/4) = 128.7°$, we get

$$B = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(1.6\times10^{-19}\ \text{C})(845{,}000\ \text{m/s})\sin 128.7°}{(\sqrt{41}\times10^{-9}\ \text{m})^2} = 2.58\times10^{-4}\ \text{T},\ \text{into the page.}$$

(c) $F_{\text{mag}} = qvB\sin 90° = (1.60\times10^{-19}\ \text{C})(845{,}000\ \text{m/s})(2.58\times10^{-4}\ \text{T}) = 3.48\times10^{-17}\ \text{N}$, in the $+x$ direction.

$F_{\text{elec}} = (1/4\pi\epsilon_0)e^2/r^2 = \frac{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(\sqrt{41}\times10^{-9}\ \text{m})^2} = 5.62\times10^{-12}\ \text{N}$, at 51.3° below the $+x$-axis measured clockwise.

EVALUATE: The electric force is much stronger than the magnetic force.

28.9. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$.

EXECUTE: Applying the law of Biot and Savart gives

(a) $dB = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\frac{(10.0\ \text{A})(0.00110\ \text{m})\sin 90°}{(0.0500\ \text{m})^2}$ = 4.40 × 10^{-7} T, out of the paper.

(b) The same as above, except $r = \sqrt{(5.00\ \text{cm})^2 + (14.0\ \text{cm})^2}$ and ϕ = arctan(5/14) = 19.65°, giving dB = 1.67 × 10^{-8} T, out of the page.

(c) $dB = 0$ since $\phi = 0°$.

EVALUATE: This is a very small field, but it comes from a very small segment of current.

28.10. **IDENTIFY:** Apply $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\vec{r}}{r^3}$.

SET UP: The magnitude of the field due to the current element is $dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$, where ϕ is the angle between $\vec{r}$ and the current direction.

EXECUTE: The magnetic field at the given points is:

$$dB_a = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\ \text{A})(0.000100\ \text{m})}{(0.100\ \text{m})^2} = 2.00\times10^{-6}\ \text{T}.$$

$$dB_b = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\ \text{A})(0.000100\ \text{m})\sin 45°}{2(0.100\ \text{m})^2} = 0.705\times10^{-6}\ \text{T}.$$

$$dB_c = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\ \text{A})(0.000100\ \text{m})}{(0.100\ \text{m})^2} = 2.00\times10^{-6}\ \text{T}.$$

$$dB_d = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{Idl\sin(0°)}{r^2} = 0.$$

$$dB_e = \frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\ \text{A})(0.00100\ \text{m})}{3(0.100\ \text{m})^2}\frac{\sqrt{2}}{\sqrt{3}} = 0.545\times10^{-6}\ \text{T}$$

The field vectors at each point are shown in Figure 28.10.

EVALUATE: In each case $d\vec{B}$ is perpendicular to the current direction.

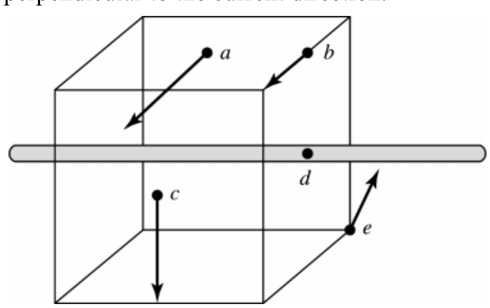


Figure 28.10

28.11. **IDENTIFY and SET UP:** The magnetic field produced by an infinitesimal current element is given by Eq.(28.6).

$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\vec{l}\times\hat{r}}{r^2}$ As in Example 28.2 use this equation for the finite 0.500-mm segment of wire since the $\Delta l = 0.500$ mm length is much smaller than the distances to the field points.

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\Delta\vec{l}\times\vec{r}}{r^3}$$

I is in the $+z$-direction, so $\Delta\vec{l} = (0.500\times10^{-3}\ \text{m})\hat{k}$

EXECUTE: **(a)** Field point is at x = 2.00 m, y = 0, z = 0 so the vector $\vec{r}$ from the source point (at the origin) to the field point is $\vec{r} = (2.00\ \text{m})\hat{i}$.

$$\Delta\vec{l}\times\vec{r} = (0.500\times10^{-3}\ \text{m})(2.00\ \text{m})\hat{k}\times\hat{i} = +(1.00\times10^{-3}\ \text{m}^2)\hat{j}$$

$$\vec{B} = \frac{(1\times10^{-7}\ \text{T}\cdot\text{m/A})(4.00\ \text{A})(1.00\times10^{-3}\ \text{m}^2)}{(2.00\ \text{m})^3}\hat{j} = (5.00\times10^{-11}\ \text{T})\hat{j}$$

(b) $\vec{r}=(2.00\text{ m})\hat{\boldsymbol{j}},\ r=2.00\text{ m}.$

$\Delta\vec{l}\times\vec{r}=(0.500\times10^{-3}\text{ m})(2.00\text{ m})\hat{\boldsymbol{k}}\times\hat{\boldsymbol{j}}=-(1.00\times10^{-3}\text{ m}^2)\hat{\boldsymbol{i}}$

$$\vec{B}=-\frac{(1\times10^{-7}\text{ T}\cdot\text{m/A})(4.00\text{ A})(1.00\times10^{-3}\text{ m}^2)}{(2.00\text{ m})^3}\hat{\boldsymbol{i}}=-(5.00\times10^{-11}\text{ T})\hat{\boldsymbol{i}}$$

(c) $\vec{r}=(2.00\text{ m})(\hat{\boldsymbol{i}}+\hat{\boldsymbol{j}}),\ r=\sqrt{2}(2.00\text{ m}).$

$\Delta\vec{l}\times\vec{r}=(0.500\times10^{-3}\text{ m})(2.00\text{ m})\hat{\boldsymbol{k}}\times(\hat{\boldsymbol{i}}+\hat{\boldsymbol{j}})=(1.00\times10^{-3}\text{ m}^2)(\hat{\boldsymbol{j}}-\hat{\boldsymbol{i}})$

$$\vec{B}=\frac{(1\times10^{-7}\text{ T}\cdot\text{m/A})(4.00\text{ A})(1.00\times10^{-3}\text{ m}^2)}{\left[\sqrt{2}(2.00\text{ m})\right]^3}(\hat{\boldsymbol{j}}-\hat{\boldsymbol{i}})=(-1.77\times10^{-11}\text{ T})(\hat{\boldsymbol{i}}-\hat{\boldsymbol{j}})$$

(d) $\vec{r}=(2.00\text{ m})\hat{\boldsymbol{k}},\ r=2.00\text{ m}.$

$\Delta\vec{l}\times\vec{r}=(0.500\times10^{-3}\text{ m})(2.00\text{ m})\hat{\boldsymbol{k}}\times\hat{\boldsymbol{k}}=0;\ \vec{B}=0.$

EVALUATE: At each point $\vec{B}$ is perpendicular to both $\vec{r}$ and $\Delta\vec{l}$. $B=0$ along the length of the wire.

28.12. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB=\frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$.

Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the law of Biot and Savart for the 12.0-A current gives

$$dB=\frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\frac{(12.0\text{ A})(0.00150\text{ m})\left(\frac{2.50\text{ cm}}{8.00\text{ cm}}\right)}{(0.0800\text{ m})^2}=8.79\times10^{-8}\text{ T}$$

The field from the 24.0-A segment is twice this value, so the total field is 2.64×10^{-7} T, into the page.

EVALUATE: The rest of each wire also produces field at *P*. We have calculated just the field from the two segments that are indicated in the problem.

28.13. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB=\frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$. Both fields are into the page, so their magnitudes add.

EXECUTE: Applying the Biot and Savart law, where $r=\frac{1}{2}\sqrt{(3.00\text{ cm})^2+(3.00\text{ cm})^2}=2.121$ cm, we have

$$dB=2\frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\frac{(28.0\text{ A})(0.00200\text{ m})\sin45.0°}{(0.02121\text{ m})^2}=1.76\times10^{-5}\text{ T, into the paper.}$$

EVALUATE: Even though the two wire segments are at right angles, the magnetic fields they create are in the same direction.

28.14. **IDENTIFY:** A current segment creates a magnetic field.

SET UP: The law of Biot and Savart gives $dB=\frac{\mu_0}{4\pi}\frac{Idl\sin\phi}{r^2}$. All four fields are of equal magnitude and into the page, so their magnitudes add.

EXECUTE: $dB=4\frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{4\pi}\frac{(15.0\text{ A})(0.00120\text{ m})\sin90°}{(0.0500\text{ m})^2}=2.88\times10^{-6}\text{ T, into the page.}$

EVALUATE: A small current element causes a small magnetic field.

28.15. **IDENTIFY:** We can model the lightning bolt and the household current as very long current-carrying wires.

SET UP: The magnetic field produced by a long wire is $B=\frac{\mu_0 I}{2\pi r}$.

EXECUTE: Substituting the numerical values gives

(a) $B=\frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(20{,}000\text{ A})}{2\pi(5.0\text{ m})}=8\times10^{-4}\text{ T}$

(b) $B=\frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(10\text{ A})}{2\pi(0.050\text{ m})}=4.0\times10^{-5}\text{ T.}$

EVALUATE: The field from the lightning bolt is about 20 times as strong as the field from the household current.

28.16. **IDENTIFY:** The long current-carrying wire produces a magnetic field.

SET UP: The magnetic field due to a long wire is $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: First find the current: $I = (3.50 \times 10^{18}\ \text{el/s})(1.60 \times 10^{-19}\ \text{C/el}) = 0.560\ \text{A}$

Now find the magnetic field: $\dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(0.560\ \text{A})}{2\pi(0.0400\ \text{m})} = 2.80 \times 10^{-6}\ \text{T}$

Since electrons are negative, the conventional current runs from east to west, so the magnetic field above the wire points toward the north.

EVALUATE: This magnetic field is much less than that of the Earth, so any experiments involving such a current would have to be shielded from the Earth's magnetic field, or at least would have to take it into consideration.

28.17. **IDENTIFY:** The long current-carrying wire produces a magnetic field.

SET UP: The magnetic field due to a long wire is $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: First solve for the current, then substitute the numbers using the above equation.

(a) Solving for the current gives

$$I = 2\pi r B/\mu_0 = 2\pi(0.0200\ \text{m})(1.00\times10^{-4}\ \text{T})/(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}) = 10.0\ \text{A}$$

(b) The earth's horizontal field points northward, so at all points directly above the wire the field of the wire would point northward.

(c) At all points directly east of the wire, its field would point northward.

EVALUATE: Even though the Earth's magnetic field is rather weak, it requires a fairly large current to cancel this field.

28.18. **IDENTIFY:** For each wire $B = \dfrac{\mu_0 I}{2\pi r}$ (Eq.28.9), and the direction of $\vec{B}$ is given by the right-hand rule (Fig. 28.6 in the textbook). Add the field vectors for each wire to calculate the total field.

(a) SET UP: The two fields at this point have the directions shown in Figure 28.18a.

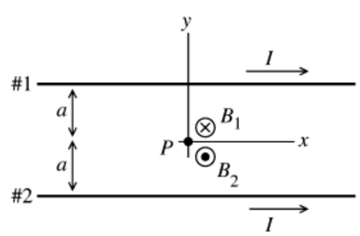


Figure 28.18a

EXECUTE: At point P midway between the two wires the fields $\vec{B}_1$ and $\vec{B}_2$ due to the two currents are in opposite directions, so $B = B_2 - B_1$.

But $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi a}$, so $B = 0$.

(b) SET UP: The two fields at this point have the directions shown in Figure 28.18b.

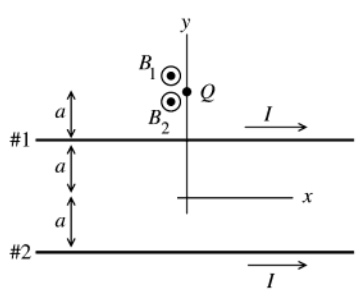


Figure 28.18b

EXECUTE: At point Q above the upper wire $\vec{B}_1$ and $\vec{B}_2$ are both directed out of the page ($+z$-direction), so $B = B_1 + B_2$.

$B_1 = \dfrac{\mu_0 I}{2\pi a},\ B_2 = \dfrac{\mu_0 I}{2\pi(3a)}$

$B = \dfrac{\mu_0 I}{2\pi a}\left(1+\tfrac{1}{3}\right) = \dfrac{2\mu_0 I}{3\pi a};\ \vec{B} = \dfrac{2\mu_0 I}{3\pi a}\hat{k}$

(c) SET UP: The two fields at this point have the directions shown in Figure 28.18c.

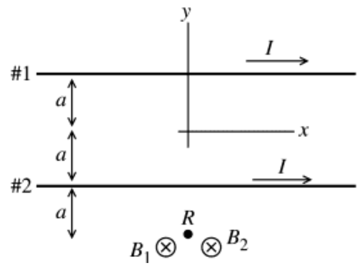


Figure 28.18c

EXECUTE: At point R below the lower wire $\vec{B}_1$ and $\vec{B}_2$ are both directed into the page ($-z$-direction), so $B = B_1 + B_2$.

$$B_1 = \frac{\mu_0 I}{2\pi(3a)},\ B_2 = \frac{\mu_0 I}{2\pi a}$$

$$B_1 = \frac{\mu_0 I}{2\pi a}\left(1+\tfrac{1}{3}\right) = \frac{2\mu_0 I}{3\pi a};\ \vec{B} = -\frac{2\mu_0 I}{3\pi a}\hat{k}$$

EVALUATE: In the figures we have drawn, $\vec{B}$ due to each wire is out of the page at points above the wire and into the page at points below the wire. If the two field vectors are in opposite directions the magnitudes subtract.

28.19. IDENTIFY: The total magnetic field is the vector sum of the constant magnetic field and the wire's magnetic field.

SET UP: For the wire, $B_{\text{wire}} = \dfrac{\mu_0 I}{2\pi r}$ and the direction of B_{wire} is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook. $\vec{B}_0 = (1.50\times10^{-6}\ \text{T})\hat{i}$.

EXECUTE: **(a)** At (0, 0, 1 m), $\vec{B} = \vec{B}_0 - \dfrac{\mu_0 I}{2\pi r}\hat{i} = (1.50\times10^{-6}\ \text{T})\hat{i} - \dfrac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{i} = -(1.0\times10^{-7}\ \text{T})\hat{i}$.

(b) At (1 m, 0, 0), $\vec{B} = \vec{B}_0 + \dfrac{\mu_0 I}{2\pi r}\hat{k} = (1.50\times10^{-6}\ \text{T})\hat{i} + \dfrac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{k}$.

$\vec{B} = (1.50\times10^{-6}\ \text{T})\hat{i} + (1.6\times10^{-6}\ \text{T})\hat{k} = 2.19\times10^{-6}\ \text{T}$, at $\theta = 46.8°$ from x to z.

(c) At (0, 0, –0.25 m), $\vec{B} = \vec{B}_0 + \dfrac{\mu_0 I}{2\pi r}\hat{i} = (1.50\times10^{-6}\ \text{T})\hat{i} + \dfrac{\mu_0(8.00\ \text{A})}{2\pi(0.25\ \text{m})}\hat{i} = (7.9\times10^{-6}\ \text{T})\hat{i}$.

EVALUATE: At point c the two fields are in the same direction and their magnitudes add. At point a they are in opposite directions and their magnitudes subtract. At point b the two fields are perpendicular.

28.20. IDENTIFY and **SET UP:** The magnitude of $\vec{B}$ is given by Eq.(28.9) and the direction is given by the right-hand rule.

(a) EXECUTE: Viewed from above, the current is in the direction shown in Figure 28.20.

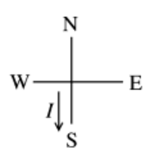


Figure 28.20

Directly below the wire the direction of the magnetic field due to the current in the wire is east.

$$B = \frac{\mu_0 I}{2\pi r} = (2\times10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{800\ \text{A}}{5.50\ \text{m}}\right) = 2.91\times10^{-5}\ \text{T}$$

(b) EVALUATE: B from the current is nearly equal in magnitude to the earth's field, so, yes, the current really is a problem.

28.21. IDENTIFY: $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule in Section 20.7.

SET UP: Call the wires a and b, as indicated in Figure 28.21. The magnetic fields of each wire at points P_1 and P_2 are shown in Figure 28.21a. The fields at point 3 are shown in Figure 28.21b.

EXECUTE: **(a)** At P_1, $B_a = B_b$ and the two fields are in opposite directions, so the net field is zero.

(b) $B_a = \dfrac{\mu_0 I}{2\pi r_a}$. $B_b = \dfrac{\mu_0 I}{2\pi r_b}$. $\vec{B}_a$ and $\vec{B}_b$ are in the same direction so

$$B = B_a + B_b = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r_a}+\frac{1}{r_b}\right) = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(4.00\ \text{A})}{2\pi}\left[\frac{1}{0.300\ \text{m}}+\frac{1}{0.200\ \text{m}}\right] = 6.67\times10^{-6}\ \text{T}$$

$\vec{B}$ has magnitude $6.67\ \mu\text{T}$ and is directed toward the top of the page.

(c) In Figure 28.21b, $\vec{B}_a$ is perpendicular to $\vec{r}_a$ and $\vec{B}_b$ is perpendicular to $\vec{r}_b$. $\tan\theta = \dfrac{5\text{ cm}}{20\text{ cm}}$ and $\theta = 14.04°$.

$r_a = r_b = \sqrt{(0.200\text{ m})^2 + (0.050\text{ m})^2} = 0.206\text{ m}$ and $B_a = B_b$.

$$B = B_a\cos\theta + B_b\cos\theta = 2B_a\cos\theta = 2\left(\frac{\mu_0 I}{2\pi r_a}\right)\cos\theta = \frac{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(4.0\text{ A})\cos14.04°}{2\pi(0.206\text{ m})} = 7.54\ \mu\text{T}$$

B has magnitude $7.53\ \mu\text{T}$ and is directed to the left.

EVALUATE: At points directly to the left of both wires the net field is directed toward the bottom of the page.

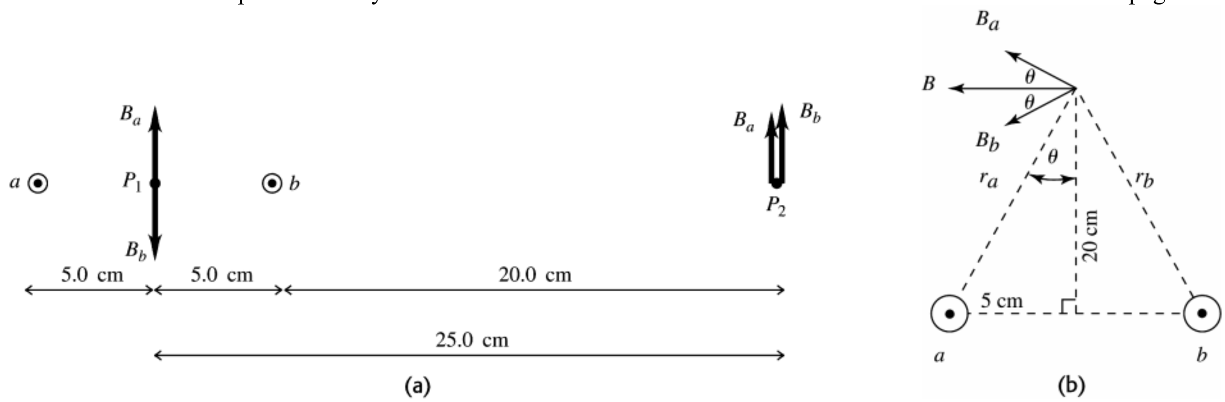


Figure 28.21

28.22. IDENTIFY: Use Eq.(28.9) and the right-hand rule to determine points where the fields of the two wires cancel.

(a) SET UP: The only place where the magnetic fields of the two wires are in opposite directions is between the wires, in the plane of the wires. Consider a point a distance x from the wire carrying $I_2 = 75.0$ A. B_{tot} will be zero where $B_1 = B_2$.

EXECUTE: $\dfrac{\mu_0 I_1}{2\pi(0.400\text{ m} - x)} = \dfrac{\mu_0 I_2}{2\pi x}$

$I_2(0.400\text{ m} - x) = I_1 x;\ I_1 = 25.0\text{ A},\ I_2 = 75.0\text{ A}$

$x = 0.300$ m; $B_{tot} = 0$ along a line 0.300 m from the wire carrying 75.0 A and 0.100 m from the wire carrying current 25.0 A.

(b) SET UP: Let the wire with $I_1 = 25.0$ A be 0.400 m above the wire with $I_2 = 75.0$ A. The magnetic fields of the two wires are in opposite directions in the plane of the wires and at points above both wires or below both wires. But to have $B_1 = B_2$ must be closer to wire #1 since $I_1 < I_2$, so can have $B_{tot} = 0$ only at points above both wires. Consider a point a distance x from the wire carrying $I_1 = 25.0$ A. B_{tot} will be zero where $B_1 = B_2$.

EXECUTE: $\dfrac{\mu_0 I_1}{2\pi x} = \dfrac{\mu_0 I_2}{2\pi(0.400\text{ m} + x)}$

$I_2 x = I_1(0.400\text{ m} + x);\ x = 0.200\text{ m}$

$B_{tot} = 0$ along a line 0.200 m from the wire carrying current 25.0 A and 0.600 m from the wire carrying current $I_2 = 75.0$ A.

EVALUATE: For parts (a) and (b) the locations of zero field are in different regions. In each case the points of zero field are closer to the wire that has the smaller current.

28.23. IDENTIFY: The net magnetic field at the center of the square is the vector sum of the fields due to each wire.

SET UP: For each wire, $B = \dfrac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule that is illustrated in Figure 28.6 in the textbook.

EXECUTE: **(a)** and **(b)** $B = 0$ since the magnetic fields due to currents at opposite corners of the square cancel.

(c) The fields due to each wire are sketched in Figure 28.23.

$$B = B_a\cos45° + B_b\cos45° + B_c\cos45° + B_d\cos45° = 4B_a\cos45° = 4\left(\frac{\mu_0 I}{2\pi r}\right)\cos45°.$$

$r = \sqrt{(10\text{ cm})^2 + (10\text{ cm})^2} = 10\sqrt{2}\text{ cm} = 0.10\sqrt{2}\text{ m}$, so

$$B = 4\frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(100\text{ A})}{2\pi(0.10\sqrt{2}\text{ m})}\cos45° = 4.0\times10^{-4}\text{ T, to the left.}$$

EVALUATE: In part (c), if all four currents are reversed in direction, the net field at the center of the square would be to the right.

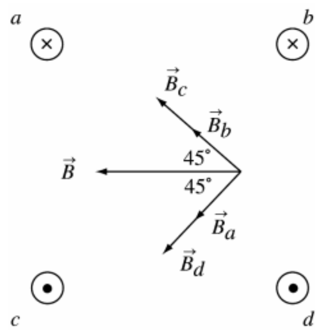
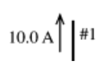


Figure 28.23

28.24. **IDENTIFY:** Use Eq.(28.9) and the right-hand rule to determine the field due to each wire. Set the sum of the four fields equal to zero and use that equation to solve for the field and the current of the fourth wire.
SET UP: The three known currents are shown in Figure 28.24.



$\vec{B}_1 \otimes,\ \vec{B}_2 \otimes,\ \vec{B}_3 \odot$

$B = \dfrac{\mu_0 I}{2\pi r}$; $r = 0.200$ m for each wire

Figure 28.24

EXECUTE: Let $\odot$ be the positive z-direction. $I_1 = 10.0$ A, $I_2 = 8.0$ A, $I_3 = 20.0$ A. Then $B_1 = 1.00\times 10^{-5}$ T, $B_2 = 0.80\times 10^{-5}$ T, and $B_3 = 2.00\times 10^{-5}$ T.

$B_{1z} = -1.00\times 10^{-5}\ \text{T},\ B_{2z} = -0.80\times 10^{-5}\ \text{T},\ B_{3z} = +2.00\times 10^{-5}\ \text{T}$

$B_{1z} + B_{2z} + B_{3z} + B_{4z} = 0$

$B_{4z} = -(B_{1z} + B_{2z} + B_{3z}) = -2.0\times 10^{-6}\ \text{T}$

To give $\vec{B}_4$ in the $\otimes$ direction the current in wire 4 must be toward the bottom of the page.

$$B_4 = \frac{\mu_0 I}{2\pi r}\ \text{so}\ I_4 = \frac{rB_4}{(\mu_0/2\pi)} = \frac{(0.200\ \text{m})(2.0\times 10^{-6}\ \text{T})}{(2\times 10^{-7}\ \text{T}\cdot\text{m/A})} = 2.0\ \text{A}$$

EVALUATE: The fields of wires #2 and #3 are in opposite directions and their net field is the same as due to a current 20.0 A – 8.0 A = 12.0 A in one wire. The field of wire #4 must be in the same direction as that of wire #1, and $10.0\ \text{A} + I_4 = 12.0\ \text{A}$.

28.25. **IDENTIFY:** Apply Eq.(28.11).
SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: **(a)** $F = \dfrac{\mu_0 I_1 I_2 L}{2\pi r} = \dfrac{\mu_0(5.00\ \text{A})(2.00\ \text{A})(1.20\ \text{m})}{2\pi(0.400\ \text{m})} = 6.00\times 10^{-6}\ \text{N}$, and the force is repulsive since the currents are in opposite directions.

(b) Doubling the currents makes the force increase by a factor of four to $F = 2.40\times 10^{-5}$ N.

EVALUATE: Doubling the current in a wire doubles the magnetic field of that wire. For fixed magnetic field, doubling the current in a wire doubles the force that the magnetic field exerts on the wire.

28.26. **IDENTIFY:** Apply Eq.(28.11).
SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: **(a)** $\dfrac{F}{L} = \dfrac{\mu_0 I_1 I_2}{2\pi r}$ gives $I_2 = \dfrac{F}{L}\dfrac{2\pi r}{\mu_0 I_1} = (4.0\times 10^{-5}\ \text{N/m})\dfrac{2\pi(0.0250\ \text{m})}{\mu_0(0.60\ \text{A})} = 8.33\ \text{A}$.

(b) The two wires repel so the currents are in opposite directions.
EVALUATE: The force between the two wires is proportional to the product of the currents in the wires.

28.27. **IDENTIFY:** The lamp cord wires are two parallel current-carrying wires, so they must exert a magnetic force on each other.

SET UP: First find the current in the cord. Since it is connected to a light bulb, the power consumed by the bulb is $P = IV$. Then find the force per unit length using $F/L = \frac{\mu_0}{2\pi}\frac{I'I}{r}$.

EXECUTE: For the light bulb, 100 W = I(120 V) gives I = 0.833 A. The force per unit length is

$$F/L = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{2\pi}\frac{(0.833\ \text{A})^2}{0.003\ \text{m}} = 4.6\times10^{-5}\ \text{N/m}$$

Since the currents are in opposite directions, the force is repulsive.

EVALUATE: This force is too small to have an appreciable effect for an ordinary cord.

28.28. **IDENTIFY:** Apply Eq.(28.11) for the force from each wire.

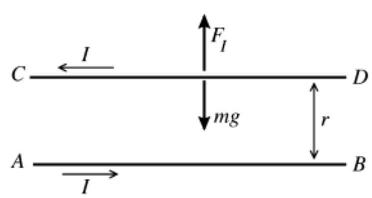
SET UP: Two parallel conductors carrying current in the same direction attract each other. Parallel conductors carrying currents in opposite directions repel each other.

EXECUTE: On the top wire $\frac{F}{L} = \frac{\mu_0 I^2}{2\pi}\left(\frac{1}{d} - \frac{1}{2d}\right) = \frac{\mu_0 I^2}{4\pi d}$, upward. On the middle wire, the magnetic forces cancel so the net force is zero. On the bottom wire $\frac{F}{L} = \frac{\mu_0 I^2}{2\pi}\left(-\frac{1}{d} + \frac{1}{2d}\right) = \frac{\mu_0 I^2}{4\pi d}$, downward.

EVALUATE: The net force on the middle wire is zero because at the location of the middle wire the net magnetic field due to the other two wires is zero.

28.29. **IDENTIFY:** The wire CD rises until the upward force F_I due to the currents balances the downward force of gravity.

SET UP: The forces on wire CD are shown in Figure 28.29.



Currents in opposite directions so the force is repulsive and F_I is upward, as shown.

Figure 28.29

Eq.(28.11) says $F_I = \frac{\mu_0 I^2 L}{2\pi h}$ where L is the length of wire CD and h is the distance between the wires.

EXECUTE: $mg = \lambda L g$

Thus $F_I - mg = 0$ says $\frac{\mu_0 I^2 L}{2\pi h} = \lambda L g$ and $h = \frac{\mu_0 I^2}{2\pi g \lambda}$.

EVALUATE: The larger I is or the smaller λ is, the larger h will be.

28.30. **IDENTIFY:** The magnetic field at the center of a circular loop is $B = \frac{\mu_0 I}{2R}$. By symmetry each segment of the loop that has length Δl contributes equally to the field, so the field at the center of a semicircle is $\frac{1}{2}$ that of a full loop.

SET UP: Since the straight sections produce no field at P, the field at P is $B = \frac{\mu_0 I}{4R}$.

EXECUTE: $B = \frac{\mu_0 I}{4R}$. The direction of $\vec{B}$ is given by the right-hand rule: $\vec{B}$ is directed into the page.

EVALUATE: For a quarter-circle section of wire the magnetic field at its center of curvature is $B = \frac{\mu_0 I}{8R}$.

28.31. **IDENTIFY:** Calculate the magnetic field vector produced by each wire and add these fields to get the total field.

SET UP: First consider the field at P produced by the current I_1 in the upper semicircle of wire. See Figure 28.31a.

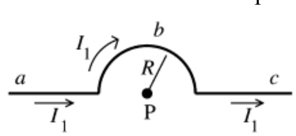
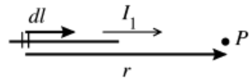


Figure 28.31a

Consider the three parts of this wire
a: long straight section,
b: semicircle
c: long, straight section

Apply the Biot-Savart law $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I d\vec{l}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{I d\vec{l}\times\vec{r}}{r^3}$ to each piece.

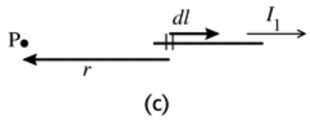
EXECUTE: part *a* See Figure 28.31b.



$d\vec{l} \times \vec{r} = 0$,
so $dB = 0$

Figure 28.31b

The same is true for all the infinitesimal segments that make up this piece of the wire, so $B = 0$ for this piece.
part *c* See Figure 28.31c.



$d\vec{l} \times \vec{r} = 0$,
so $dB = 0$ and $B = 0$ for this piece.

Figure 28.31c

part *b* See Figure 28.31d.

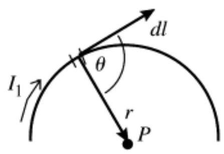


Figure 28.31d

$d\vec{l} \times \vec{r}$ is directed into the paper for all infinitesimal segments that make up this semicircular piece, so $\vec{B}$ is directed into the paper and $B = \int dB$ (the vector sum of the $d\vec{B}$ is obtained by adding their magnitudes since they are in the same direction).

$\left|d\vec{l} \times \vec{r}\right| = r\,dl\sin\theta$. The angle θ between $d\vec{l}$ and $\vec{r}$ is 90° and $r = R$, the radius of the semicircle. Thus $\left|d\vec{l} \times \vec{r}\right| = R\,dl$

$$dB = \frac{\mu_0}{4\pi}\frac{I\left|d\vec{l} \times \vec{r}\right|}{r^3} = \frac{\mu_0 I_1}{4\pi}\frac{R}{R^3}dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)dl$$

$$B = \int dB = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)\int dl = \left(\frac{\mu_0 I_1}{4\pi R^2}\right)(\pi R) = \frac{\mu_0 I_1}{4R}$$

(We used that $\int dl$ is equal to πR, the length of wire in the semicircle.) We have shown that the two straight sections make zero contribution to $\vec{B}$, so $B_1 = \mu_0 I_1/4R$ and is directed into the page.

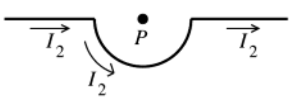


Figure 28.31e

For current in the direction shown in Figure 28.31e, a similar analysis gives $B_2 = \mu_0 I_2/4R$, out of the paper

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions, so the magnitude of the net field at P is $B = \left|B_1 - B_2\right| = \dfrac{\mu_0\left|I_1 - I_2\right|}{4R}$.

EVALUATE: When $I_1 = I_2$, $B = 0$.

28.32. **IDENTIFY:** Apply Eq.(28.16).

SET UP: At the center of the coil, $x = 0$. a is the radius of the coil, 0.0240 m.

EXECUTE: **(a)** $B_x = \mu_0 NI/2a$, so $I = \dfrac{2aB_x}{\mu_0 N} = \dfrac{2(0.024\text{ m})\,(0.0580\text{ T})}{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})\,(800)} = 2.77\text{ A}$

(b) At the center, $B_c = \mu_0 NI/2a$. At a distance x from the center,

$B_x = \dfrac{\mu_0 NIa^2}{2(x^2 + a^2)^{3/2}} = \left(\dfrac{\mu_0 NI}{2a}\right)\left(\dfrac{a^3}{(x^2 + a^2)^{3/2}}\right) = B_c\left(\dfrac{a^3}{(x^2 + a^2)^{3/2}}\right)$. $B_x = \frac{1}{2}B_c$ says $\dfrac{a^3}{(x^2 + a^2)^{3/2}} = \frac{1}{2}$, and $(x^2 + a^2)^3 = 4a^6$.

Since $a = 0.024$ m, $x = 0.0184$ m.

EVALUATE: As shown in Figure 28.41 in the textbook, the field has its largest magnitude at the center of the coil and decreases with distance along the axis from the center.

28.33. **IDENTIFY:** Apply Eq.(28.16).

SET UP: At the center of the coil, $x = 0$. a is the radius of the coil, 0.020 m.

EXECUTE: **(a)** $B_{\text{center}} = \dfrac{\mu_0 NI}{2a} = \dfrac{\mu_0(600)\,(0.500\text{ A})}{2(0.020\text{ m})} = 9.42 \times 10^{-3}\text{ T}$.

(b) $B(x) = \dfrac{\mu_0 NIa^2}{2(x^2 + a^2)^{3/2}}$. $B(0.08\text{ m}) = \dfrac{\mu_0(600)(0.500\text{ A})(0.020\text{ m})^2}{2((0.080\text{ m})^2 + (0.020\text{ m})^2)^{3/2}} = 1.34 \times 10^{-4}\text{ T}$.

EVALUATE: As shown in Figure 28.41 in the textbook, the field has its largest magnitude at the center of the coil and decreases with distance along the axis from the center.

28.34. **IDENTIFY** and **SET UP:** The magnetic field at a point on the axis of N circular loops is given by $B_x = \dfrac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}$. Solve for N and set $x = 0.0600$ m.

EXECUTE: $N = \dfrac{2B_x(x^2+a^2)^{3/2}}{\mu_0 I a^2} = \dfrac{2(6.39\times10^{-4}\ \text{T})[(0.0600\ \text{m})^2+(0.0600\ \text{m})^2]^{3/2}}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.50\ \text{A})(0.0600\ \text{m})^2} = 69.$

EVALUATE: At the center of the coil the field is $B_x = \dfrac{\mu_0 N I}{2a} = 1.8\times10^{-3}\ \text{T}$. The field 6.00 cm from the center is a factor of $1/2^{3/2}$ times smaller.

28.35. **IDENTIFY:** Apply Ampere's law.

SET UP: $\mu_0 = 4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}$

EXECUTE: **(a)** $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} = 3.83\times10^{-4}\ \text{T}\cdot\text{m}$ and $I_{\text{encl}} = 305\ \text{A}$.

(b) $-3.83\times10^{-4}\ \text{T}\cdot\text{m}$ since at each point on the curve the direction of $d\vec{l}$ is reversed.

EVALUATE: The line integral $\oint \vec{B}\cdot d\vec{l}$ around a closed path is proportional to the net current that is enclosed by the path.

28.36. **IDENTIFY:** Apply Ampere's law.

SET UP: From the right-hand rule, when going around the path in a counterclockwise direction currents out of the page are positive and currents into the page are negative.

EXECUTE: Path a: $I_{\text{encl}} = 0 \Rightarrow \oint \vec{B}\cdot d\vec{l} = 0.$

Path b: $I_{\text{encl}} = -I_1 = -4.0\ \text{A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = -\mu_0(4.0\ \text{A}) = -5.03\times10^{-6}\ \text{T}\cdot\text{m}.$

Path c: $I_{\text{encl}} = -I_1 + I_2 = -4.0\ \text{A} + 6.0\ \text{A} = 2.0\ \text{A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0(2.0\ \text{A}) = 2.51\times10^{-6}\ \text{T}\cdot\text{m}$

Path d: $I_{\text{encl}} = -I_1 + I_2 + I_3 = 4.0\ \text{A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = +\mu_0(4.0\ \text{A}) = 5.03\times10^{-6}\ \text{T}\cdot\text{m}.$

EVALUATE: If we instead went around each path in the clockwise direction, the sign of the line integral would be reversed.

28.37. **IDENTIFY:** Apply Ampere's law.

SET UP: To calculate the magnetic field at a distance r from the center of the cable, apply Ampere's law to a circular path of radius r. By symmetry, $\oint \vec{B}\cdot d\vec{l} = B(2\pi r)$ for such a path.

EXECUTE: **(a)** For $a < r < b$, $I_{\text{encl}} = I \Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 I \Rightarrow B2\pi r = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}.$

(b) For $r > c$, the enclosed current is zero, so the magnetic field is also zero.

EVALUATE: A useful property of coaxial cables for many applications is that the current carried by the cable doesn't produce a magnetic field outside the cable.

28.38. **IDENTIFY:** Apply Ampere's law to calculate $\vec{B}$.

(a) SET UP: For $a < r < b$ the end view is shown in Figure 28.38a.

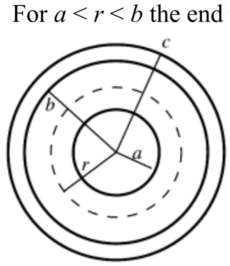


Figure 28.38a

Apply Ampere's law to a circle of radius r, where $a < r < b$. Take currents I_1 and I_2 to be directed into the page. Take this direction to be positive, so go around the integration path in the clockwise direction.

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$

$\oint \vec{B}\cdot d\vec{l} = B(2\pi r)$, $I_{\text{encl}} = I_1$

Thus $B(2\pi r) = \mu_0 I_1$ and $B = \dfrac{\mu_0 I_1}{2\pi r}$

(b) SET UP: <u>$r > c$:</u> See Figure 28.38b.

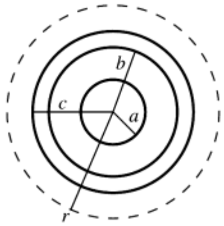


Figure 28.38b

Apply Ampere's law to a circle of radius r, where $r > c$. Both currents are in the positive direction.

EXECUTE: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$

$\oint \vec{B} \cdot d\vec{l} = B(2\pi r),\ I_{\text{encl}} = I_1 + I_2$

Thus $B(2\pi r) = \mu_0(I_1 + I_2)$ and $B = \dfrac{\mu_0(I_1 + I_2)}{2\pi r}$

EVALUATE: For $a < r < b$ the field is due only to the current in the central conductor. For $r > c$ both currents contribute to the total field.

28.39. IDENTIFY: The largest value of the field occurs at the surface of the cylinder. Inside the cylinder, the field increases linearly from zero at the center, and outside the field decreases inversely with distance from the central axis of the cylinder.

SET UP: At the surface of the cylinder, $B = \dfrac{\mu_0 I}{2\pi R}$, inside the cylinder, Eq. 28.21 gives $B = \dfrac{\mu_0 I}{2\pi}\dfrac{r}{R^2}$, and outside the field is $B = \dfrac{\mu_0 I}{2\pi r}$.

EXECUTE: For points inside the cylinder, the field is half its maximum value when $\dfrac{\mu_0 I}{2\pi}\dfrac{r}{R^2} = \dfrac{1}{2}\left(\dfrac{\mu_0 I}{2\pi R}\right)$, which gives $r = R/2$. Outside the cylinder, we have $\dfrac{\mu_0 I}{2\pi r} = \dfrac{1}{2}\left(\dfrac{\mu_0 I}{2\pi R}\right)$, which gives $r = 2R$.

EVALUATE: The field has half its maximum value at all points on cylinders coaxial with the wire but of radius $R/2$ and of radius $2R$.

28.40. IDENTIFY: $B = \mu_0 n I = \dfrac{\mu_0 N I}{L}$

SET UP: $L = 0.150$ m

EXECUTE: $B = \dfrac{\mu_0 (600)\,(8.00\text{ A})}{(0.150\text{ m})} = 0.0402\text{ T}$

EVALUATE: The field near the center of the solenoid is independent of the radius of the solenoid, as long as the radius is much less than the length.

28.41. (a) IDENTIFY and SET UP: The magnetic field near the center of a long solenoid is given by Eq.(28.23), $B = \mu_0 n I$.

EXECUTE: Turns per unit length $n = \dfrac{B}{\mu_0 I} = \dfrac{0.0270\text{ T}}{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(12.0\text{ A})} = 1790\text{ turns/m}$

(b) $N = nL = (1790\text{ turns/m})(0.400\text{ m}) = 716\text{ turns}$

Each turn of radius R has a length $2\pi R$ of wire. The total length of wire required is $N(2\pi R) = (716)(2\pi)(1.40 \times 10^{-2}\text{ m}) = 63.0\text{ m}$.

EVALUATE: A large length of wire is required. Due to the length of wire the solenoid will have appreciable resistance.

28.42. IDENTIFY and SET UP: At the center of a long solenoid $B = \mu_0 n I = \mu_0 \dfrac{N}{L} I$.

EXECUTE: $I = \dfrac{BL}{\mu_0 N} = \dfrac{(0.150\text{ T})(1.40\text{ m})}{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(4000)} = 41.8\text{ A}$

EVALUATE: The magnetic field inside the solenoid is independent of the radius of the solenoid, if the radius is much less than the length, as is the case here.

28.43. **IDENTIFY** and **SET UP:** Use the appropriate expression for the magnetic field produced by each current configuration.

EXECUTE: **(a)** $B=\dfrac{\mu_0 I}{2\pi r}$ so $I=\dfrac{2\pi B}{\mu_0}=\dfrac{2\pi(2.00\times10^{-2}\text{ m})(37.2\text{ T})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}=3.72\times10^{6}\text{ A}$.

(b) $B=\dfrac{N\mu_0 I}{2R}$ so $I=\dfrac{2RB}{N\mu_0}=\dfrac{2(0.210\text{ m})(37.2\text{ T})}{(100)(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})}=1.24\times10^{5}\text{ A}$.

(c) $B=\mu_0\dfrac{N}{L}I$ so $I=\dfrac{BL}{\mu_0 N}=\dfrac{(37.2\text{ T})(0.320\text{ m})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(40{,}000)}=237\text{ A}$.

EVALUATE: Much less current is needed for the solenoid, because of its large number of turns per unit length.

28.44. **IDENTIFY:** Example 28.10 shows that outside a toroidal solenoid there is no magnetic field and inside it the magnetic field is given by $B=\dfrac{\mu_0 NI}{2\pi r}$.

SET UP: The torus extends from $r_1=15.0\text{ cm}$ to $r_2=18.0\text{ cm}$.

EXECUTE: **(a)** $r = 0.12$ m, which is outside the torus, so $B = 0$.

(b) $r = 0.16$ m, so $B=\dfrac{\mu_0 NI}{2\pi r}=\dfrac{\mu_0(250)(8.50\text{ A})}{2\pi(0.160\text{ m})}=2.66\times10^{-3}\text{ T}$.

(c) $r = 0.20$ m, which is outside the torus, so $B = 0$.

EVALUATE: The magnetic field inside the torus is proportional to $1/r$, so it varies somewhat over the cross-section of the torus.

28.45. **IDENTIFY:** Example 28.10 shows that inside a toroidal solenoid, $B=\dfrac{\mu_0 NI}{2\pi r}$.

SET UP: $r=0.070\text{ m}$

EXECUTE: $B=\dfrac{\mu_0 NI}{2\pi r}=\dfrac{\mu_0(600)(0.650\text{ A})}{2\pi(0.070\text{ m})}=1.11\times10^{-3}\text{ T}$.

EVALUATE: If the radial thickness of the torus is small compared to its mean diameter, B is approximately uniform inside its windings.

28.46. **IDENTIFY:** Use Eq.(28.24), with μ_0 replaced by $\mu=K_\text{m}\mu_0$, with $K_\text{m}=80$.

SET UP: The contribution from atomic currents is the difference between B calculated with μ and B calculated with μ_0.

EXECUTE: **(a)** $B=\dfrac{\mu NI}{2\pi r}=\dfrac{K_\text{m}\mu_0 NI}{2\pi r}=\dfrac{\mu_0(80)(400)(0.25\text{ A})}{2\pi(0.060\text{ m})}=0.0267\text{ T}$.

(b) The amount due to atomic currents is $B'=\dfrac{79}{80}B=\dfrac{79}{80}(0.0267\text{ T})=0.0263\text{ T}$.

EVALUATE: The presence of the core greatly enhances the magnetic field produced by the solenoid.

28.47. **IDENTIFY** and **SET UP:** $B=\dfrac{K_\text{m}\mu_0 NI}{2\pi r}$ (Eq.28.24, with μ_0 replaced by $K_\text{m}\mu_0$)

EXECUTE: **(a)** $K_\text{m}=1400$

$$I=\frac{2\pi rB}{\mu_0 K_\text{m}N}=\frac{(2.90\times10^{-2}\text{ m})(0.350\text{ T})}{(2\times10^{-7}\text{ T}\cdot\text{m/A})(1400)(500)}=0.0725\text{ A}$$

(b) $K_\text{m}=5200$

$$I=\frac{2\pi rB}{\mu_0 K_\text{m}N}=\frac{(2.90\times10^{-2}\text{ m})(0.350\text{ T})}{(2\times10^{-7}\text{ T}\cdot\text{m/A})(5200)(500)}=0.0195\text{ A}$$

EVALUATE: If the solenoid were air-filled instead, a much larger current would be required to produce the same magnetic field.

28.48. **IDENTIFY:** Apply $B=\dfrac{K_\text{m}\mu_0 NI}{2\pi r}$.

SET UP: K_m is the relative permeability and $\chi_\text{m}=K_\text{m}-1$ is the magnetic susceptibility.

EXECUTE: **(a)** $K_\text{m}=\dfrac{2\pi rB}{\mu_0 NI}=\dfrac{2\pi(0.2500\text{ m})(1.940\text{ T})}{\mu_0(500)(2.400\text{ A})}=2021$.

(b) $\chi_\text{m}=K_\text{m}-1=2020$.

EVALUATE: Without the magnetic material the magnetic field inside the windings would be $B/2021=9.6\times10^{-4}\text{ T}$. The presence of the magnetic material greatly enhances the magnetic field inside the windings.

28.49. **IDENTIFY:** The magnetic field from the solenoid alone is $B_0 = \mu_0 nI$. The total magnetic field is $B = K_m B_0$. M is given by Eq.(28.29).

SET UP: $n = 6000$ turns/m

EXECUTE: **(a)** (i) $B_0 = \mu_0 nI = \mu_0(6000\ \text{m}^{-1})(0.15\ \text{A}) = 1.13\times10^{-3}\ \text{T}$.

(ii) $M = \dfrac{K_m - 1}{\mu_0}B_0 = \dfrac{5199}{\mu_0}(1.13\times10^{-3}\ \text{T}) = 4.68\times10^{6}\ \text{A/m}$.

(iii) $B = K_m B_0 = (5200)(1.13\times10^{-3}\ \text{T}) = 5.88\ \text{T}$.

(b) The directions of $\vec{B}$, $\vec{B}_0$ and $\vec{M}$ are shown in Figure 28.49. Silicon steel is paramagnetic and $\vec{B}_0$ and $\vec{M}$ are in the same direction.

EVALUATE: The total magnetic field is much larger than the field due to the solenoid current alone.

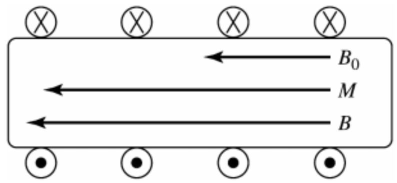


Figure 28.49

28.50. **IDENTIFY:** Curie's law (Eq.28.32) says that $1/M$ is proportional to T, so $1/\chi_m$ is proportional to T.

SET UP: The graph of $1/\chi_m$ versus the Kelvin temperature is given in Figure 28.50.

EXECUTE: The material does obey Curie's law because the graph in Figure 28.50 is a straight line. $M = C\dfrac{B}{T}$ and $M = \dfrac{B - B_0}{\mu_0}$ says that $\chi_m = \dfrac{C\mu_0}{T}$. $1/\chi_m = \dfrac{T}{C\mu_0}$ and the slope of $1/\chi_m$ versus T is $1/(C\mu_0)$. Therefore, from the graph the Curie constant is $C = \dfrac{1}{\mu_0(\text{slope})} = \dfrac{1}{\mu_0(5.13\ \text{K}^{-1})} = 1.55\times10^{5}\ \text{K}\cdot\text{A/T}\cdot\text{m}$.

EVALUATE: For this material Curie's law is valid over a wide temperature range.

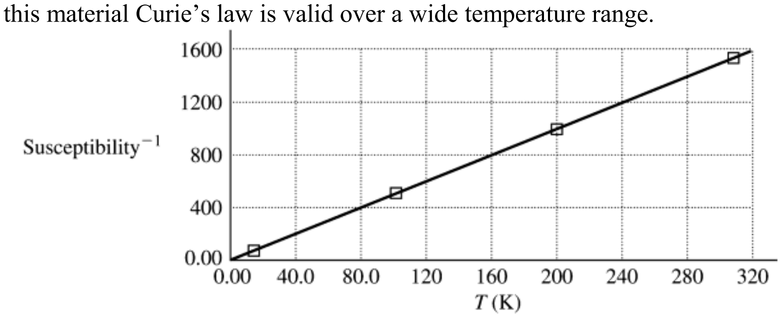


Figure 28.50

28.51. **IDENTIFY:** Moving charges create magnetic fields. The net field is the vector sum of the two fields. A charge moving in an external magnetic field feels a force.

(a) SET UP: The magnetic field due to a moving charge is $B = \dfrac{\mu_0}{4\pi}\dfrac{qv\sin\phi}{r^2}$. Both fields are into the paper, so their magnitudes add, giving $B_{\text{net}} = B + B' = \dfrac{\mu_0}{4\pi}\left(\dfrac{qv\sin\phi}{r^2} + \dfrac{q'v'\sin\phi'}{r'^2}\right)$.

EXECUTE: Substituting numbers gives

$$B_{\text{net}} = \frac{\mu_0}{4\pi}\left[\frac{(8.00\ \mu\text{C})(9.00\times10^4\ \text{m/s})\sin90^\circ}{(0.300\ \text{m})^2} + \frac{(5.00\ \mu\text{C})(6.50\times10^4\ \text{m/s})\sin90^\circ}{(0.400\ \text{m})^2}\right]$$

$B_{\text{net}} = 1.00\times10^{-6}\ \text{T} = 1.00\ \mu\text{T}$, into the paper.

(b) SET UP: The magnetic force on a moving charge is $\vec{F} = q\vec{v}\times\vec{B}$, and the magnetic field of charge q' at the location of charge q is into the page. The force on q is

$$\vec{F} = q\vec{v}\times\vec{B}' = (qv)\hat{i}\times\frac{\mu_0}{4\pi}\frac{q\vec{v}'\times\hat{r}}{r^2} = (qv)\hat{i}\times\left(\frac{\mu_0}{4\pi}\frac{qv'\sin\phi}{r^2}\right)(-\hat{k}) = \left(\frac{\mu_0}{4\pi}\frac{qq'vv'\sin\phi}{r^2}\right)\hat{j}$$

where ϕ is the angle between $\vec{v}'$ and $\hat{r}'$.

EXECUTE: Substituting numbers gives

$$\vec{F} = \frac{\mu_0}{4\pi}\left[\frac{(8.00\times10^{-6}\text{ C})(5.00\times10^{-6}\text{ C})(9.00\times10^{-6}\text{ m/s})(6.50\times10^{-6}\text{ m/s})}{(0.500\text{ m})^2}\left(\frac{0.400}{0.500}\right)\right]\hat{j}$$

$$\vec{F} = (7.49\times10^{-8}\text{ N})\hat{j}.$$

EVALUATE: These are small fields and small forces, but if the charge has small mass, the force can affect its motion.

28.52. **IDENTIFY:** The wire creates a magnetic field near it, and the moving electron feels a force due to this field.

SET UP: The magnetic field due to the wire is $B = \dfrac{\mu_0 I}{2\pi r}$, and the force on a moving charge is $F = qvB\sin\phi$.

EXECUTE: $F = qvB\sin\phi = (ev\mu_0 I\sin\phi)/2\pi r.$ Substituting numbers gives

$F = (1.60\times10^{-19}\text{ C})(6.00\times10^{4}\text{ m/s})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(2.50\text{ A})(\sin 90°)/[2\pi(0.0450\text{ m})]$

$F = 1.07\times10^{-19}$ N

From the right hand rule for the cross product, the direction of $\vec{v}\times\vec{B}$ is opposite to the current, but since the electron is negative, the force is in the same direction as the current.

EVALUATE: This force is small at an everyday level, but it would give the electron an acceleration of about 10^{11} m/s^2.

28.53. **IDENTIFY:** Find the force that the magnetic field of the wire exerts on the electron.

SET UP: The force on a moving charge has magnitude $F = |q|vB\sin\phi$ and direction given by the right-hand rule.

For a long straight wire, $B = \dfrac{\mu_0 I}{2\pi r}$ and the direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** $a = \dfrac{F}{m} = \dfrac{|q|vB\sin\phi}{m} = \dfrac{ev}{m}\left(\dfrac{\mu_0 I}{2\pi r}\right)$

$$a = \frac{(1.6\times10^{-17}\text{C})(2.50\times10^{5}\text{ m/s})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(25.0\text{ A})}{(9.11\times10^{-31}\text{ kg})(2\pi)(0.0200\text{ m})} = 1.1\times10^{13}\text{ m/s}^2,$$

away from the wire.

(b) The electric force must balance the magnetic force. $eE = evB$, and

$E = vB = v\dfrac{\mu_0 I}{2\pi r} = \dfrac{(250,000\text{ m/s})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(25.0\text{ A})}{2\pi(0.0200\text{ m})} = 62.5\text{ N/C}$. The magnetic force is directed away from the wire so the force from the electric field must be toward the wire. Since the charge of the electron is negative, the electric field must be directed away from the wire to produce a force in the desired direction.

EVALUATE: **(c)** $mg = (9.11\times10^{-31}\text{ kg})(9.8\text{ m/s}^2)\approx 10^{-29}\text{ N}.$ $F_{\text{el}} = eE = (1.6\times10^{-19}\text{ C})(62.5\text{ N/C})\approx 10^{-17}\text{ N}.$

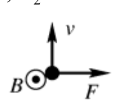
$F_{\text{el}}\approx 10^{12}\,F_{\text{grav}}$, so we can neglect gravity.

28.54. **IDENTIFY:** Use Eq.(28.9) and the right-hand rule to calculate the magnetic field due to each wire. Add these field vectors to calculate the net field and then use Eq.(27.2) to calculate the force.

SET UP: Let the wire connected to the $25.0\ \Omega$ resistor be #2 and the wire connected to the $10.0\ \Omega$ resistor be #1. Both I_1 and I_2 are directed toward the right in the figure, so at the location of the proton B_2 is $\otimes$ and $B_1 = \odot$.

$B_1 = \dfrac{\mu_0 I_1}{2\pi r}$ and $B_2 = \dfrac{\mu_0 I_2}{2\pi r}$, with $r = 0.0250$ m. $I_1 = (100.0\text{ V})/(10.0\ \Omega) = 10.0\text{ A}$ and $I_2 = (100.0\text{ V})/(25.0\ \Omega) = 4.00\text{ A}$

EXECUTE: $B_1 = 8.00\times10^{-5}\text{ T}$, $B_2 = 3.20\times10^{-5}\text{ T}$ and $B = B_1 - B_2 = 4.80\times10^{-5}\text{ T}$ and in the direction $\odot$.



Force is to the right.

Figure 28.54

$F = qvB = (1.602\times10^{-19}\text{ C})(650\times10^{3}\text{ m/s})(4.80\times10^{-5}\text{ T}) = 5.00\times10^{-18}\text{ N}$

EVALUATE: The force is perpendicular to both $\vec{v}$ and $\vec{B}$. The magnetic force is much larger than the gravity force on the proton.

28.55. **IDENTIFY:** Find the net magnetic field due to the two loops at the location of the proton and then find the force these fields exert on the proton.

SET UP: For a circular loop, the field on the axis, a distance x from the center of the loop is $B = \dfrac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$.

$R = 0.200$ m and $x = 0.125$ m.

EXECUTE: The fields add, so $B = B_1 + B_2 = 2B_1 = 2\left[\dfrac{\mu_0 IR^2}{2(R^2 + x^2)^{3/2}}\right]$.

$$B = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1.50\ \text{A})(0.200\ \text{m})^2}{[(0.200\ \text{m})^2 + (0.125\ \text{m})^2]^{3/2}} = 5.75\times 10^{-6}\ \text{T}.$$

$F = |q|vB\sin\phi = (1.6\times 10^{-19}\ \text{C})(2400\ \text{m/s})(5.75\times 10^{-6}\ \text{T})\sin 90° = 2.21\times 10^{-21}\ \text{N}$, perpendicular to the line ab and to the velocity.

EVALUATE: The weight of a proton is $w = mg = 1.6\times 10^{-24}\ \text{N}$, so the force from the loops is much greater than the gravity force on the proton.

28.56. **IDENTIFY:** The net magnetic field is the vector sum of the fields due to each wire.

SET UP: $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** The currents are the same so points where the two fields are equal in magnitude are equidistant from the two wires. The net field is zero along the dashed line shown in Figure 28.56a.

(b) For the magnitudes of the two fields to be the same at a point, the point must be 3 times closer to the wire with the smaller current. The net field is zero along the dashed line shown in Figure 28.56b.

(c) As in (a), the points are equidistant from both wires. The net field is zero along the dashed line shown in Figure 28.56c.

EVALUATE: The lines of zero net field consist of points at which the fields of the two wires have opposite directions and equal magnitudes.

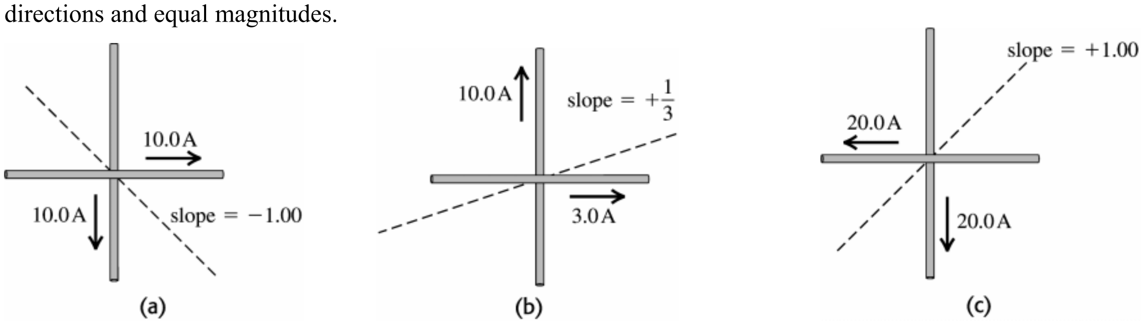


Figure 28.56

28.57. **IDENTIFY:** $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}_0 \times \hat{r}}{r^2}$

SET UP: $\hat{r} = \hat{i}$ and $r = 0.250$ m, so $\vec{v}_0 \times \hat{r} = v_{0z}\hat{j} - v_{0y}\hat{k}$.

EXECUTE: $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q}{r^2}\left(v_{0z}\hat{j} - v_{0y}\hat{k}\right) = \left(6.00\times 10^{-6}\ \text{T}\right)\hat{j}$. $v_{0y} = 0$. $\dfrac{\mu_0}{4\pi}\dfrac{q}{r^2}v_{0z} = 6.00\times 10^{-6}$ T and

$v_{0z} = \dfrac{4\pi(6.00\times 10^{-6}\ \text{T})(0.25\ \text{m})^2}{\mu_0(-7.20\times 10^{-3}\ \text{C})} = -521\ \text{m/s}$. $v_{0x} = \pm\sqrt{v_0^2 - v_{0y}{}^2 - v_{0z}{}^2} = \pm\sqrt{(800\ \text{m/s})^2 - (-521\ \text{m/s})^2} = \pm 607\ \text{m/s}$.

The sign of v_{0x} isn't determined.

(b) Now $\vec{r} = \hat{j}$ and $r = 0.250$ m. $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}_0 \times \hat{r}}{r^2} = \dfrac{\mu_0}{4\pi}\dfrac{q}{r^2}\left(v_{0x}\hat{k} - v_{0z}\hat{i}\right)$.

$$B = \frac{\mu_0}{4\pi}\frac{|q|}{r^2}\sqrt{v_{0x}^2 + v_{0z}^2} = \frac{\mu_0}{4\pi}\frac{|q|}{r^2}v_0 = \frac{\mu_0}{4\pi}\frac{(7.20\times 10^{-3}\ \text{C})}{(0.250\ \text{m})^2}800\ \text{m/s} = 9.20\times 10^{-6}\ \text{T}.$$

EVALUATE: The magnetic field in part (b) doesn't depend on the sign of v_{0x}.

28.58. **IDENTIFY** and **SET UP:** $\vec{B} = B_0(x/a)\hat{i}$

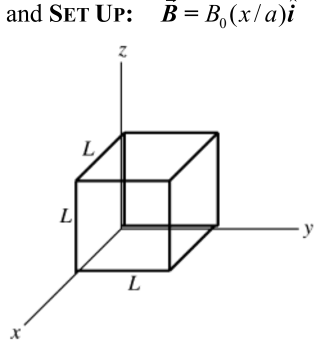


Figure 28.58

Apply Gauss's law for magnetic fields to a cube with side length L, one corner at the origin, and sides parallel to the x, y and z axes, as shown in Figure 28.58.

EXECUTE: Since $\vec{B}$ is parallel to the *x*-axis the only sides that have nonzero flux are the front side (parallel to the *yz*-plane at $x = L$) and the back side (parallel to the *yz*-plane at $x = 0$.)

front $\Phi_B = \int \vec{B} \cdot d\vec{A} = B_0(x/a)\int dA(\hat{i} \cdot \hat{i}) = B_0(x/a)\int dA$

$x = L$ on this face so $\vec{B} \cdot d\vec{A} = B_0(L/a)dA$

$\Phi_B = B_0(L/a)\int dA = B_0(L/a)L^2 = B_0(L^3/a)$

back On the back face $x = 0$ so $B = 0$ and $\Phi_B = 0$. The total flux through the cubical Gaussian surface is $\Phi_B = B_0(L^3/a)$.

EVALUATE: This violates Eq.(27.8), which says that $\Phi_B = 0$ for any closed surface. The claimed $\vec{B}$ is impossible because it has been shown to violate Gauss's law for magnetism.

28.59. **IDENTIFY:** Use Eq.(28.9) and the right-hand rule to calculate the magnitude and direction of the magnetic field at *P* produced by each wire. Add these two field vectors to find the net field.

(a) SET UP: The directions of the fields at point *P* due to the two wires are sketched in Figure 28.59a.

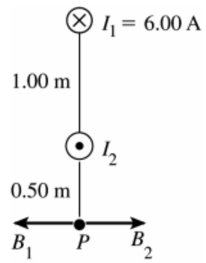


Figure 28.59a

EXECUTE: $\vec{B}_1$ and $\vec{B}_2$ must be equal and opposite for the resultant field at *P* to be zero. $\vec{B}_2$ is to the right so I_2 is out of the page.

$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{\mu_0}{2\pi}\left(\frac{6.00\text{ A}}{1.50\text{ m}}\right)$ $\quad B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0}{2\pi}\left(\frac{I_2}{0.50\text{ m}}\right)$

$B_1 = B_2$ says $\frac{\mu_0}{2\pi}\left(\frac{6.00\text{ A}}{1.50\text{ m}}\right) = \frac{\mu_0}{2\pi}\left(\frac{I_2}{0.50\text{ m}}\right)$

$I_2 = \left(\frac{0.50\text{ m}}{1.50\text{ m}}\right)(6.00\text{ A}) = 2.00\text{ A}$

(b) SET UP: The directions of the fields at point *Q* are sketched in Figure 28.59b.

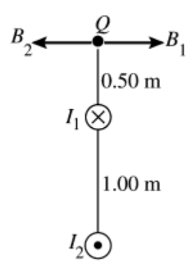


Figure 28.59b

EXECUTE: $B_1 = \frac{\mu_0 I_1}{2\pi r_1}$

$B_1 = (2\times10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{6.00\text{ A}}{0.50\text{ m}}\right) = 2.40\times10^{-6}\text{ T}$

$B_2 = \frac{\mu_0 I_2}{2\pi r_2}$

$B_2 = (2\times10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{2.00\text{ A}}{1.50\text{ m}}\right) = 2.67\times10^{-7}\text{ T}$

$\vec{B}_1$ and $\vec{B}_2$ are in opposite directions and $B_1 > B_2$ so

$B = B_1 - B_2 = 2.40\times10^{-6}\text{ T} - 2.67\times10^{-7}\text{ T} = 2.13\times10^{-6}\text{ T}$, and $\vec{B}$ is to the right.

(c) SET UP: The directions of the fields at point *S* are sketched in Figure 28.59c.

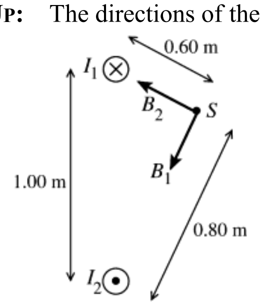


Figure 28.59c

EXECUTE: $B_1 = \frac{\mu_0 I_1}{2\pi r_1}$

$B_1 = (2\times10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{6.00\text{ A}}{0.60\text{ m}}\right) = 2.00\times10^{-6}\text{ T}$

$B_2 = \frac{\mu_0 I_2}{2\pi r_2}$

$B_2 = (2\times10^{-7}\text{ T}\cdot\text{m/A})\left(\frac{2.00\text{ A}}{0.80\text{ m}}\right) = 5.00\times10^{-7}\text{ T}$

$\vec{B}_1$ and $\vec{B}_2$ are right angles to each other, so the magnitude of their resultant is given by

$B = \sqrt{B_1^2 + B_2^2} = \sqrt{(2.00\times10^{-6}\text{ T})^2 + (5.00\times10^{-7}\text{ T})^2} = 2.06\times10^{-6}\text{ T}$

EVALUATE: The magnetic field lines for a long, straight wire are concentric circles with the wire at the center. The magnetic field at each point is tangent to the field line, so $\vec{B}$ is perpendicular to the line from the wire to the point where the field is calculated.

28.60. IDENTIFY: Find the vector sum of the magnetic fields due to each wire.

SET UP: For a long straight wire $B = \dfrac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule and is perpendicular to the line from the wire to the point where then field is calculated.

EXECUTE: **(a)** The magnetic field vectors are shown in Figure 28.60a.

(b) At a position on the x-axis $B_{\text{net}} = 2\dfrac{\mu_0 I}{2\pi r}\sin\theta = \dfrac{\mu_0 I}{\pi\sqrt{x^2+a^2}}\dfrac{a}{\sqrt{x^2+a^2}} = \dfrac{\mu_0 Ia}{\pi(x^2+a^2)}$, in the positive x-direction.

(c) The graph of B versus x/a is given in Figure 28.60b.

EVALUATE: **(d)** The magnetic field is a maximum at the origin, $x = 0$.

(e) When $x >> a$, $B \approx \dfrac{\mu_0 Ia}{\pi x^2}$.

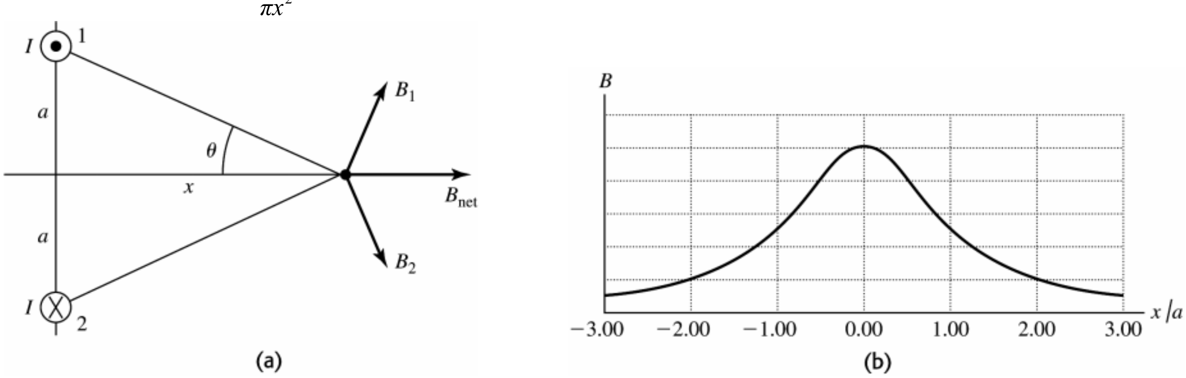


Figure 28.60

28.61. IDENTIFY: Apply $F = IlB\sin\phi$, with the magnetic field at point P that is calculated in problem 28.60.

SET UP: The net field of the first two wires at the location of the third wire is $B = \dfrac{\mu_0 Ia}{\pi(x^2+a^2)}$, in the $+x$-direction.

EXECUTE: **(a)** Wire is carrying current into the page, so it feels a force in the $-y$-direction .

$$\frac{F}{L} = IB = I\left(\frac{\mu_0 Ia}{\pi(x^2+a^2)}\right) = \frac{\mu_0(6.00\text{ A})^2(0.400\text{ m})}{\pi\left((0.600\text{ m})^2+(0.400\text{ m})^2\right)} = 1.11\times10^{-5}\text{ N/m}.$$

(b) If the wire carries current out of the page then the force felt will be in the opposite direction as in part (a). Thus the force will be $1.11\times10^{-5}\text{ N/m}$, in the $+y$-direction.

EVALUATE: We could also calculate the force exerted by each of the first two wires and find the vector sum of the two forces.

28.62. IDENTIFY: The wires repel each other since they carry currents in opposite directions, so the wires will move away from each other until the magnetic force is just balanced by the force due to the spring.

SET UP: The force of the spring is kx and the magnetic force on each wire is $F_{\text{mag}} = \dfrac{\mu_0}{2\pi}\dfrac{I^2L}{x}$.

EXECUTE: Call x the distance the springs each stretch. On each wire, $F_{\text{spr}} = F_{\text{mag}}$, and there are two spring forces on each wire. Therefore $2kx = \dfrac{\mu_0}{2\pi}\dfrac{I^2L}{x}$, which gives $x = \sqrt{\dfrac{\mu_0}{2\pi}\dfrac{I^2L}{k}}$.

EVALUATE: Since $\mu_0/2\pi$ is small, x will likely be much less than the length of the wires.

28.63. IDENTIFY: Apply $\sum\vec{F} = \mathbf{0}$ to one of the wires. The force one wire exerts on the other depends on I so $\sum\vec{F} = \mathbf{0}$ gives two equations for the two unknowns T and I.

SET UP: The force diagram for one of the wires is given in Figure 28.63.

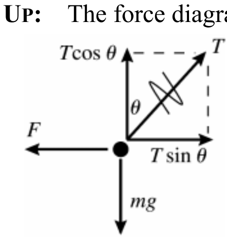


Figure 28.63

The force one wire exerts on the other is $F = \left(\dfrac{\mu_0 I^2}{2\pi r}\right)L$, where $r = 2(0.040\text{ m})\sin\theta = 8.362\times10^{-3}\text{ m}$ is the distance between the two wires.

EXECUTE: $\sum F_y = 0$ gives $T\cos\theta = mg$ and $T = mg/\cos\theta$

$\sum F_x = 0$ gives $F = T\sin\theta = (mg/\cos\theta)\sin\theta = mg\tan\theta$

And $m = \lambda L,$ so $F = \lambda Lg\tan\theta$

$$\left(\frac{\mu_0 I^2}{2\pi r}\right)L = \lambda Lg\tan\theta$$

$$I = \sqrt{\frac{\lambda gr\tan\theta}{(\mu_0/2\pi)}}$$

$$I = \sqrt{\frac{(0.0125\text{ kg/m})(9.80\text{ m/s})^2(\tan\ 6.00°)(8.362\times 10^{-3}\text{ m})}{2\times 10^{-7}\text{T}\cdot\text{m/A}}} = 23.2\text{ A}$$

EVALUATE: Since the currents are in opposite directions the wires repel. When I is increased, the angle θ from the vertical increases; a large current is required even for the small displacement specified in this problem.

28.64. **IDENTIFY:** Consider the forces on each side of the loop.

SET UP: The forces on the left and right sides cancel. The forces on the top and bottom segments of the loop are in opposite directions, so the magnitudes subtract.

EXECUTE: $F = F_\text{t} - F_\text{b} = \left(\frac{\mu_0 I_\text{wire}}{2\pi}\right)\left(\frac{Il}{r_\text{t}} - \frac{Il}{r_\text{b}}\right) = \frac{\mu_0 IlI_\text{wire}}{2\pi}\left(\frac{1}{r_\text{r}} - \frac{1}{r_\text{l}}\right).$

$F = \frac{\mu_0(5.00\text{ A})(0.200\text{ m})(14.0\text{ A})}{2\pi}\left(\frac{1}{0.100\text{ m}} - \frac{1}{0.026\text{ m}}\right) = 7.97\times 10^{-5}\text{ N}$. The force on the top segment is away from the wire, so the net force is away from the wire.

EVALUATE: The net force on a current loop in a uniform magnetic field is zero, but the magnetic field of the wire is not uniform, it is stronger closer to the wire.

28.65. **IDENTIFY:** Find the magnetic field of the first loop at the location of the second loop and apply $\tau = |\vec{\boldsymbol{\mu}}\times\vec{\boldsymbol{B}}|$ and $U = -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}}$ to find μ and U.

SET UP: Since x is much larger than a', assume B is uniform over the second loop and equal to its value on the axis of the first loop.

EXECUTE: **(a)** $x >> a \Rightarrow B = \frac{N\mu_0 Ia^2}{2(x^2+a^2)^{3/2}} \approx \frac{N\mu_0 Ia^2}{2x^3}.$

$$\tau = |\vec{\boldsymbol{\mu}}\times\vec{\boldsymbol{B}}| = \mu B\sin\theta = (N'I'A')\left(\frac{N\mu_0 Ia^2}{2x^3}\right)\sin\theta = \frac{NN'\mu_0\pi I\ I'a^2a'^2\sin\theta}{2x^3}$$

(b) $U = -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}} = -\mu B\cos\theta = -(N'I'\pi a'^2)\left(\frac{N\mu_0 Ia^2}{2x^3}\right)\cos\theta = -\frac{NN'\mu_0\pi II'a^2a'^2\cos\theta}{2x^3}.$

EVALUATE: **(c)** Having $x >> a$ allows us to simplify the form of the magnetic field, whereas assuming $x >> a'$ means we can assume that the magnetic field from the first loop is constant over the second loop.

28.66. **IDENTIFY:** Apply $d\vec{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{Id\vec{\boldsymbol{l}}\times\hat{\boldsymbol{r}}}{r^2}.$

SET UP: The two straight segments produce zero field at P. The field at the center of a circular loop of radius R is $B = \frac{\mu_0 I}{2R}$, so the field at the center of curvature of a semicircular loop is $B = \frac{\mu_0 I}{4R}.$

EXECUTE: The semicircular loop of radius a produces field out of the page at P and the semicircular loop of radius b produces field into the page. Therefore, $B = B_a - B_b = \frac{1}{2}\left(\frac{\mu_0 I}{2}\right)\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\mu_0 I}{4a}\left(1 - \frac{a}{b}\right)$, out of page.

EVALUATE: If $a = b$, $B = 0$.

28.67. **IDENTIFY:** Find the vector sum of the fields due to each loop.

SET UP: For a single loop $B = \frac{\mu_0 Ia^2}{2(x^2+a^2)^{3/2}}$. Here we have two loops, each of N turns, and measuring the field along the x-axis from between them means that the "x" in the formula is different for each case:

EXECUTE:

$$\text{Left coil: } x \to x + \frac{a}{2} \Rightarrow B_\text{l} = \frac{\mu_0 NIa^2}{2((x+a/2)^2+a^2)^{3/2}}.$$

$$\text{Right coil: } x \to x - \frac{a}{2} \Rightarrow B_\text{r} = \frac{\mu_0 NIa^2}{2((x-a/2)^2+a^2)^{3/2}}.$$

So, the total field at a point a distance x from the point between them is

$$B=\frac{\mu_0 N I a^2}{2}\left(\frac{1}{((x+a/2)^2+a^2)^{3/2}}+\frac{1}{((x-a/2)^2+a^2)^{3/2}}\right).$$

(b) B versus x is graphed in Figure 28.67. Figure 28.67a is the total field and Figure 27.67b is the field from the right-hand coil.

(c) At point P, $x=0$ and $B=\frac{\mu_0 N I a^2}{2}\left(\frac{1}{((a/2)^2+a^2)^{3/2}}+\frac{1}{((-a/2)^2+a^2)^{3/2}}\right)=\frac{\mu_0 N I a^2}{(5a^2/4)^{3/2}}=\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 N I}{a}$

(d) $B=\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0 N I}{a}=\left(\frac{4}{5}\right)^{3/2}\frac{\mu_0(300)(6.00\text{ A})}{(0.080\text{ m})}=0.0202\text{ T}.$

(e) $\frac{dB}{dx}=\frac{\mu_0 N I a^2}{2}\left(\frac{-3(x+a/2)}{((x+a/2)^2+a^2)^{5/2}}+\frac{-3(x-a/2)}{((x-a/2)^2+a^2)^{5/2}}\right)$. At $x=0$,

$$\left.\frac{dB}{dx}\right|_{x=0}=\frac{\mu_0 N I a^2}{2}\left(\frac{-3(a/2)}{((a/2)^2+a^2)^{5/2}}+\frac{-3(-a/2)}{((-a/2)^2+a^2)^{5/2}}\right)=0\ .$$

$$\frac{d^2B}{dx^2}=\frac{\mu_0 N I a^2}{2}\left(\frac{-3}{((x+a/2)^2+a^2)^{5/2}}+\frac{6(x+a/2)^2(5/2)}{((x+a/2)^2+a^2)^{7/2}}+\frac{-3}{((x-a/2)^2+a^2)^{5/2}}+\frac{6(x-a/2)^2(5/2)}{((x-a/2)^2+a^2)^{7/2}}\right)$$

At $x=0$, $\left.\frac{d^2B}{dx^2}\right|_{x=0}=\frac{\mu_0 N I a^2}{2}\left(\frac{-3}{((a/2)^2+a^2)^{5/2}}+\frac{6(a/2)^2(5/2)}{((a/2)^2+a^2)^{7/2}}+\frac{-3}{((a/2)^2+a^2)^{5/2}}+\frac{6(-a/2)^2(5/2)}{((a/2)^2+a^2)^{7/2}}\right)=0.$

EVALUATE: Since both first and second derivatives are zero, the field can only be changing very slowly.

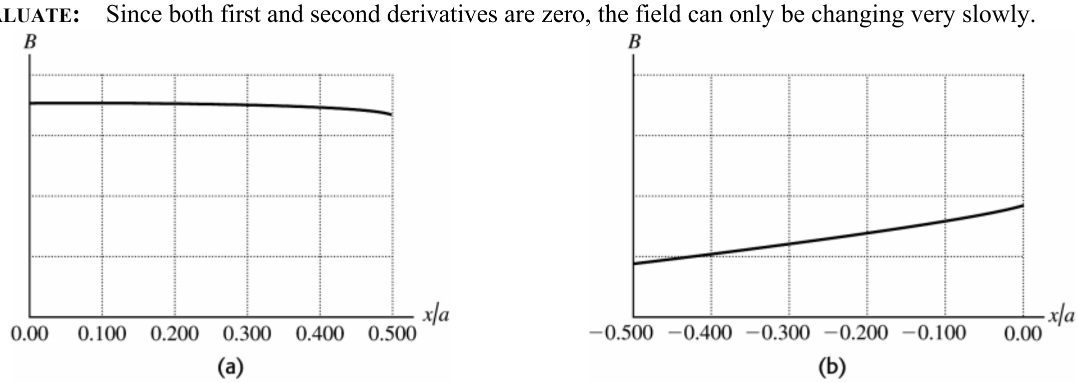


Figure 28.67

28.68. **IDENTIFY:** A current-carrying wire produces a magnetic field, but the strength of the field depends on the shape of the wire.

SET UP: The magnetic field at the center of a circular wire of radius a is $B=\mu_0 I/2a$, and the field a distance x from the center of a straight wire of length $2a$ is $B=\frac{\mu_0 I}{4\pi}\frac{2a}{x\sqrt{x^2+a^2}}$.

EXECUTE: **(a)** Since the diameter $D=2a$, we have $B=\mu_0 I/2a=\mu_0 I/D$.

(b) In this case, the length of the wire is equal to the diameter of the circle, so $2a=\pi D$, giving $a=\pi D/2$, and $x=D/2$. Therefore $B=\frac{\mu_0 I}{4\pi}\frac{2(\pi D/2)}{(D/2)\sqrt{D^2/4+\pi^2 D^2/4}}=\frac{\mu_0 I}{D\sqrt{1+\pi^2}}$.

EVALUATE: The field in part (a) is greater by a factor of $\sqrt{1+\pi^2}$. It is reasonable that the field due to the circular wire is greater than the field due to the straight wire because more of the current is close to point A for the circular wire than it is for the straight wire.

28.69. **IDENTIFY:** Apply $d\vec{B}=\frac{\mu_0}{4\pi}\frac{Id\vec{l}\times\hat{r}}{r^2}$.

SET UP: The contribution from the straight segments is zero since $d\vec{l}\times\vec{r}=0$. The magnetic field from the curved wire is just one quarter of a full loop.

EXECUTE: $B=\frac{1}{4}\left(\frac{\mu_0 I}{2R}\right)=\frac{\mu_0 I}{8R}$ and is directed out of the page.

EVALUATE: It is very simple to calculate B at point P but it would be much more difficult to calculate B at other points.

28.70. **IDENTIFY:** Apply $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{Id\vec{l} \times \hat{r}}{r^2}$.

SET UP: The horizontal wire yields zero magnetic field since $d\vec{l} \times \vec{r} = 0$. The vertical current provides the magnetic field of half of an infinite wire. (The contributions from all infinitesimal pieces of the wire point in the same direction, so there is no vector addition or components to worry about.)

EXECUTE: $B = \frac{1}{2}\left(\dfrac{\mu_0 I}{2\pi R}\right) = \dfrac{\mu_0 I}{4\pi R}$ and is directed out of the page.

EVALUATE: In the equation preceding Eq.(28.8) the limits on the integration are 0 to a rather than $-a$ to a and this introduces a factor of $\frac{1}{2}$ into the expression for B.

28.71. **(a) IDENTIFY:** Consider current density J for a small concentric ring and integrate to find the total current in terms of α and R.

SET UP: We can't say $I = JA = J\pi R^2$, since J varies across the cross section.

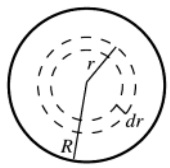


Figure 28.71

To integrate J over the cross section of the wire divide the wire cross section up into thin concentric rings of radius r and width dr, as shown in Figure 28.71.

EXECUTE: The area of such a ring is dA, and the current through it is $dI = J\,dA$; $dA = 2\pi r\,dr$ and

$$dI = J\,dA = \alpha r(2\pi r\,dr) = 2\pi\alpha r^2 dr$$

$$I = \int dI = 2\pi\alpha\int_0^R r^2 dr = 2\pi\alpha(R^3/3) \text{ so } \alpha = \frac{3I}{2\pi R^3}$$

(b) IDENTIFY and SET UP: (i) $r \le R$

Apply Ampere's law to a circle of radius $r < R$. Use the method of part (a) to find the current enclosed by the Ampere's law path.

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \oint B\,dl = B\oint dl = B(2\pi r)$, by the symmetry and direction of $\vec{B}$. The current passing through the path is $I_{\text{encl}} = \int dl$, where the integration is from 0 to r. $I_{\text{encl}} = 2\pi\alpha\int_0^r r^2 dr = \dfrac{2\pi\alpha r^3}{3} = \dfrac{2\pi}{3}\left(\dfrac{3I}{2\pi R^3}\right)r^3 = \dfrac{Ir^3}{R^3}$. Thus $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives $B(2\pi r) = \mu_0\left(\dfrac{Ir^3}{R^3}\right)$ and $B = \dfrac{\mu_0 I r^2}{2\pi R^3}$

(ii) **IDENTIFY and SET UP:** $r \ge R$

Apply Ampere's law to a circle of radius $r > R$.

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \oint B\,dl = B\oint dl = B(2\pi r)$

$I_{\text{encl}} = I$; all the current in the wire passes through this path. Thus $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives $B(2\pi r) = \mu_0 I$ and $B = \dfrac{\mu_0 I}{2\pi r}$

EVALUATE: Note that at $r = R$ the expression in (i) (for $r \le R$) gives $B = \dfrac{\mu_0 I}{2\pi R}$. At $r = R$ the expression in (ii) (for $r \ge R$) gives $B = \dfrac{\mu_0 I}{2\pi R}$, which is the same.

28.72. **IDENTIFY:** Apply Ampere's law to a circle of radius r in each case.

SET UP: Assume that the currents are uniform over the cross sections of the conductors.

EXECUTE: **(a)** $r < a \Rightarrow I_{\text{encl}} = I\left(\dfrac{A_r}{A_a}\right) = I\left(\dfrac{r^2}{a^2}\right)$. $\oint \vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I\left(\dfrac{r^2}{a^2}\right)$ and $B = \dfrac{\mu_0 I r}{2\pi a^2}$. When $r = a$, $B = \dfrac{\mu_0 I}{2\pi a}$, which is just what was found in part (a) of Exercise 28.37.

(b) $b < r < c \Rightarrow I_{\text{encl}} = I - I\left(\frac{A_{b\to r}}{A_{b\to c}}\right) = I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)$. $\oint \vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) = \mu_0 I\left(\frac{c^2 - r^2}{c^2 - b^2}\right)$ and $B = \frac{\mu_0 I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right)$. When $r = b$, $B = \frac{\mu_0 I}{2\pi b}$, just as in part (a) of Exercise 28.37 and when $r = c$, $B = 0$, just as in part (b) of Exercise 28.37.

EVALUATE: Unlike *E*, *B* is not zero within the conductors. *B* varies across the cross section of each conductor.

28.73. **IDENTIFY:** Apply $\oint \vec{B}\cdot d\vec{A} = 0$.

SET UP: Take the closed gaussian surface to be a cylinder whose axis coincides with the wire.

EXECUTE: If there is a magnetic field component in the *z*-direction, it must be constant because of the symmetry of the wire. Therefore the contribution to a surface integral over a closed cylinder, encompassing a long straight wire will be zero: no flux through the barrel of the cylinder, and equal but opposite flux through the ends. The radial field will have no contribution through the ends, but through the barrel:

$0 = \oint \vec{B}\cdot d\vec{A} = \oint \vec{B}_r \cdot d\vec{A} = \int_{\text{barrel}} \vec{B}_r \cdot d\vec{A} = \int_{\text{barrel}} B_r dA = B_r A_{\text{barrel}} = 0$. Therefore, $B_r = 0$.

EVALUATE: The magnetic field of a long straight wire is everywhere tangent to a circular area whose plane is perpendicular to the wire, with the wire passing through the center of the circular area. This field produces zero flux through the cylindrical gaussian surface.

28.74. **IDENTIFY:** Apply Ampere's law to a circular path of radius *r*.

SET UP: Assume the current is uniform over the cross section of the conductor.

EXECUTE: **(a)** $r < a \Rightarrow I_{\text{encl}} = 0 \Rightarrow B = 0$.

(b) $a < r < b \Rightarrow I_{\text{encl}} = I\left(\frac{A_{a\to r}}{A_{a\to b}}\right) = I\left(\frac{\pi(r^2 - a^2)}{\pi(b^2 - a^2)}\right) = I\frac{(r^2 - a^2)}{(b^2 - a^2)}$. $\oint \vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I\frac{(r^2 - a^2)}{(b^2 - a^2)}$ and $B = \frac{\mu_0 I}{2\pi r}\frac{(r^2 - a^2)}{(b^2 - a^2)}$.

(c) $r > b \Rightarrow I_{\text{encl}} = I$. $\oint \vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I$ and $B = \frac{\mu_0 I}{2\pi r}$.

EVALUATE: The expression in part (b) gives $B = 0$ at $r = a$ and this agrees with the result of part (a). The expression in part (b) gives $B = \frac{\mu_0 I}{2\pi b}$ at $r = b$ and this agrees with the result of part (c).

28.75. **IDENTIFY:** Use Ampere's law to find the magnetic field at $r = 2a$ from the axis. The analysis of Example 28.9 shows that the field outside the cylinder is the same as for a long, straight wire along the axis of the cylinder.

SET UP:

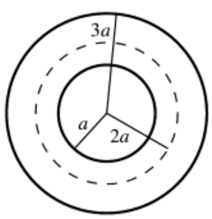


Figure 28.75

EXECUTE: Apply Ampere's law to a circular path of radius 2*a*, as shown in Figure 28.75.

$B(2\pi) = \mu_0 I_{\text{encl}}$

$I_{\text{encl}} = I\left(\frac{(2a)^2 - a^2}{(3a)^2 - a^2}\right) = 3I/8$

$B = \frac{3}{16}\frac{\mu_0 I}{2\pi a}$; this is the magnetic field inside the metal at a distance of 2*a* from the cylinder axis. Outside the cylinder, $B = \frac{\mu_0 I}{2\pi r}$. The value of *r* where these two fields are equal is given by $1/r = 3/(16a)$ and $r = 16a/3$.

EVALUATE: For $r < 3a$, as *r* increases the magnetic field increases from zero at $r = 0$ to $\mu_0 I/(2\pi(3a))$ at $r = 3a$. For $r > 3a$ the field decreases as *r* increases so it is reasonable for there to be a $r > 3a$ where the field is the same as at $r = 2a$.

28.76. **IDENTIFY:** The net field is the vector sum of the fields due to the circular loop and to the long straight wire.

SET UP: For the long wire, $B = \frac{\mu_0 I_1}{2\pi D}$, and for the loop, $B = \frac{\mu_0 I_2}{2R}$.

EXECUTE: At the center of the circular loop the current I_2 generates a magnetic field that is into the page, so the current I_1 must point to the right. For complete cancellation the two fields must have the same magnitude:

$\frac{\mu_0 I_1}{2\pi D} = \frac{\mu_0 I_2}{2R}$. Thus, $I_1 = \frac{\pi D}{R} I_2$.

EVALUATE: If I_1 is to the left the two fields add.

28.77. **IDENTIFY:** Use the current density J to find dI through a concentric ring and integrate over the appropriate cross section to find the current through that cross section. Then use Ampere's law to find $\vec{B}$ at the specified distance from the center of the wire.

(a) SET UP:

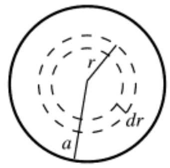


Figure 28.77a

Divide the cross section of the cylinder into thin concentric rings of radius r and width dr, as shown in Figure 28.77a. The current through each ring is $dI = J\,dA = J2\pi r\,dr$.

EXECUTE: $dI = \dfrac{2I_0}{\pi a^2}\left[1-(r/a)^2\right]2\pi r\;dr = \dfrac{4I_0}{a^2}\left[1-(r/a)^2\right]r\;dr$. The total current I is obtained by integrating dI over the cross section $I = \int_0^a dI = \left(\dfrac{4I_0}{a^2}\right)\int_0^a\left(1-r^2/a^2\right)r\;dr = \left(\dfrac{4I_0}{a^2}\right)\left[\dfrac{1}{2}r^2-\dfrac{1}{4}r^4/a^2\right]_0^a = I_0$, as was to be shown.

(b) SET UP: Apply Ampere's law to a path that is a circle of radius $r > a$, as shown in Figure 28.77b.

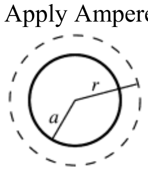


Figure 28.77b

$\oint \vec{B}\cdot d\vec{l} = B(2\pi r)$

$I_{\text{encl}} = I_0$ (the path encloses the entire cylinder)

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0 I_0$ and $B = \dfrac{\mu_0 I_0}{2\pi r}$.

(c) SET UP:

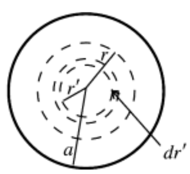


Figure 28.77c

Divide the cross section of the cylinder into concentric rings of radius r' and width dr', as was done in part (a). See Figure 28.77c. The current dI through each ring is $dI = \dfrac{4I_0}{a^2}\left[1-\left(\dfrac{r'}{a}\right)^2\right]r'\;dr'$

EXECUTE: The current I is obtained by integrating dI from $r' = 0$ to $r' = r$:

$$I = \int dI = \frac{4I_0}{a^2}\int_0^r\left[1-\left(\frac{r'}{a}\right)^2\right]r'\;dr' = \frac{4I_0}{a^2}\left[\tfrac{1}{2}(r')^2-\tfrac{1}{4}(r')^4/a^2\right]_0^r$$

$$I = \frac{4I_0}{a^2}(r^2/2 - r^4/4a^2) = \frac{I_0 r^2}{a^2}\left(2-\frac{r^2}{a^2}\right)$$

(d) SET UP: Apply Ampere's law to a path that is a circle of radius $r < a$, as shown in Figure 28.77d.

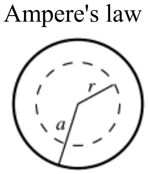


Figure 28.77d

$\oint \vec{B}\cdot d\vec{l} = B(2\pi r)$

$I_{\text{encl}} = \dfrac{I_0 r^2}{a^2}\left(2-\dfrac{r^2}{a^2}\right)$ (from part (c))

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}}$ says $B(2\pi r) = \mu_0\dfrac{I_0 r^2}{a^2}(2-r^2/a^2)$ and $B = \dfrac{\mu_0 I_0}{2\pi}\dfrac{r}{a^2}(2-r^2/a^2)$

EVALUATE: Result in part (b) evaluated at $r = a$: $B = \dfrac{\mu_0 I_0}{2\pi a}$. Result in part (d) evaluated at $r = a$: $B = \dfrac{\mu_0 I_0}{2\pi}\dfrac{a}{a^2}(2-a^2/a^2) = \dfrac{\mu_0 I_0}{2\pi a}$. The two results, one for $r > a$ and the other for $r < a$, agree at $r = a$.

28.78. **IDENTIFY:** Apply Ampere's law to a circle of radius r.

SET UP: The current within a radius r is $I = \int \vec{J}\cdot d\vec{A}$, where the integration is over a disk of radius r.

EXECUTE: **(a)** $I_0 = \int \vec{J}\cdot d\vec{A} = \int \left(\frac{b}{r}e^{(r-a)/\delta}\right) r dr d\theta = 2\pi b\int_0^a e^{(r-a)/\delta}dr = 2\pi b\delta\; e^{(r-a)/\delta}\Big|_0^a = 2\pi b\delta\left(1-e^{-a/\delta}\right).$

$I_0 = 2\pi(600\ \text{A/m})\ (0.025\ \text{m})\ (1-e^{(0.050/0.025)}) = 81.5\ \text{A}.$

(b) For $r \ge a$, $\oint \vec{B}\cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0$ and $B = \dfrac{\mu_0 I_0}{2\pi r}.$

(c) For $r \le a$, $I(r) = \int \vec{J}\cdot d\vec{A} = \int \left(\frac{b}{r'}e^{(r'-a)/\delta}\right) r'dr'd\theta = 2\pi b\int_0^r e^{(r-a)/\delta}dr = 2\pi b\delta e^{(r'-a)/\delta}\Big|_0^r.$

$I(r) = 2\pi b\delta(e^{(r'-a)/\delta} - e^{-a/\delta}) = 2\pi b\delta e^{-a/\delta}(e^{r/\delta}-1)$ and $I(r) = I_0\dfrac{(e^{r/\delta}-1)}{(e^{a/\delta}-1)}.$

(d) For $r \le a$, $\oint \vec{B}\cdot d\vec{l} = B(r)2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0\dfrac{(e^{r/\delta}-1)}{(e^{a/\delta}-1)}$ and $B = \dfrac{\mu_0 I_0(e^{r/\delta}-1)}{2\pi r(e^{a/\delta}-1)}.$

(e) At $r = \delta = 0.025$ m, $B = \dfrac{\mu_0 I_0 (e-1)}{2\pi\delta(e^{a/\delta}-1)} = \dfrac{\mu_0(81.5\ \text{A})}{2\pi(0.025\ \text{m})}\dfrac{(e-1)}{(e^{0.050/0.025}-1)} = 1.75\times 10^{-4}$ T.

At $r = a = 0.050$ m, $B = \dfrac{\mu_0 I_0}{2\pi a}\dfrac{(e^{a/\delta}-1)}{(e^{a/\delta}-1)} = \dfrac{\mu_0(81.5\ \text{A})}{2\pi(0.050\ \text{m})} = 3.26\times 10^{-4}$ T.

At $r = 2a = 0.100$ m, $B = \dfrac{\mu_0 I_0}{2\pi r} = \dfrac{\mu_0(81.5\ \text{A})}{2\pi(0.100\ \text{m})} = 1.63\times 10^{-4}$ T.

EVALUATE: At points outside the cylinder, the magnetic field is the same as that due to a long wire running along the axis of the cylinder.

28.79. **IDENTIFY:** Evaluate the integral as specified in the problem.

SET UP: Eq.(28.15) says $B_x = \dfrac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$.

EXECUTE: $\int_{-\infty}^{\infty} B_x dx = \int_{-\infty}^{\infty}\dfrac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}dx = \dfrac{\mu_0 I}{2}\int_{-\infty}^{\infty}\dfrac{1}{((x/a)^2+1)^{3/2}}d(x/a).$

$$B = \frac{\mu_0 I}{2}\int_{-\infty}^{\infty}\frac{dz}{(z^2+1)^{3/2}} \Rightarrow \int_{-\infty}^{\infty} B_x dx = \frac{\mu_0 I}{2}\int_{-\pi/2}^{\pi/2}\cos\theta d\theta = \frac{\mu_0 I}{2}(\sin\theta)\Big|_{-\pi/2}^{\pi/2} = \mu_0 I,$$

where we used the substitution $z = \tan\theta$ to go from the first to second line.

EVALUATE: This is just what Ampere's Law tells us to expect if we imagine the loop runs along the x-axis closing on itself at infinity: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I.$

28.80. **IDENTIFY:** Follow the procedure specified in the problem.

SET UP: The field and integration path are sketched in Figure 28.80.

EXECUTE: $\oint \vec{B}\cdot d\vec{l} = 0$ (no currents in the region). Using the figure, let $\vec{B} = B_0\hat{i}$ for $y < 0$ and $B = 0$ for $y > 0$.

Then $\int_{abcde} \vec{B}\cdot d\vec{l} = B_{ab}L - B_{cd}L = 0.$ $B_{cd} = 0$, so $B_{ab}L = 0$. But we have assumed that $B_{ab} \ne 0$. This is a contradiction and violates Ampere's Law.

EVALUATE: It is often convenient to approximate B as confined to a particular region of space, but this result tells us that the boundary of such a region isn't sharp.

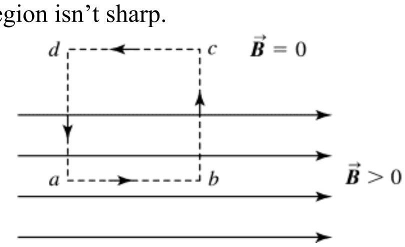


Figure 28.80

28.81. **IDENTIFY:** Use what we know about the magnetic field of a long, straight conductor to deduce the symmetry of the magnetic field. Then apply Ampere's law to calculate the magnetic field at a distance a above and below the current sheet.

SET UP: Do parts (a) and (b) together.

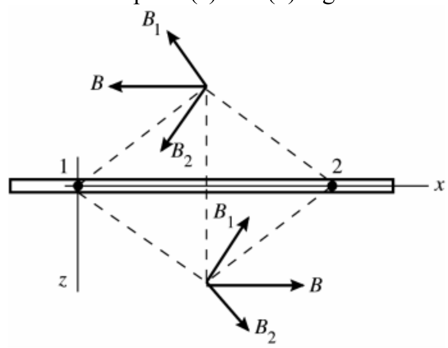


Figure 28.81a

Consider the individual currents in pairs, where the currents in each pair are equidistant on either side of the point where $\vec{B}$ is being calculated. Figure 28.81a shows that for each pair the z-components cancel, and that above the sheet the field is in the $-x$-direction and that below the sheet it is in the $+x$-direction.

Also, by symmetry the magnitude of $\vec{B}$ a distance a above the sheet must equal the magnitude of $\vec{B}$ a distance a below the sheet. Now that we have deduced the symmetry of $\vec{B}$, apply Ampere's law. Use a path that is a rectangle, as shown in Figure 28.81b.

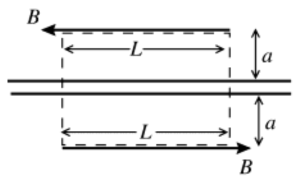


Figure 28.81b

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

I is directed out of the page, so for I to be positive the integral around the path is taken in the counterclockwise direction.

EXECUTE: Since $\vec{B}$ is parallel to the sheet, on the sides of the rectangle that have length $2a$, $\oint \vec{B} \cdot d\vec{l} = 0$. On the long sides of length L, $\vec{B}$ is parallel to the side, in the direction we are integrating around the path, and has the same magnitude, B, on each side. Thus $\oint \vec{B} \cdot d\vec{l} = 2BL$. n conductors per unit length and current I out of the page in each conductor gives $I_{\text{encl}} = InL$. Ampere's law then gives $2BL = \mu_0 InL$ and $B = \frac{1}{2}\mu_0 In$.

EVALUATE: Note that B is independent of the distance a from the sheet. Compare this result to the electric field due to an infinite sheet of charge (Example 22.7).

28.82. **IDENTIFY:** Find the vector sum of the fields due to each sheet.

SET UP: Problem 28.81 shows that for an infinite sheet $B = \frac{1}{2}\mu_0 In$. If I is out of the page, $\vec{B}$ is to the left above the sheet and to the right below the sheet. If I is into the page, $\vec{B}$ is to the right above the sheet and to the left below the sheet. B is independent of the distance from the sheet. The directions of the two fields at points P, R and S are shown in Figure 28.82.

EXECUTE: **(a)** Above the two sheets, the fields cancel (since there is no dependence upon the distance from the sheets).

(b) In between the sheets the two fields add up to yield $B = \mu_0 nI$, to the right.

(c) Below the two sheets, their fields again cancel (since there is no dependence upon the distance from the sheets).

EVALUATE: The two sheets with currents in opposite directions produce a uniform field between the sheets and zero field outside the two sheets. This is analogous to the electric field produced by large parallel sheets of charge of opposite sign.

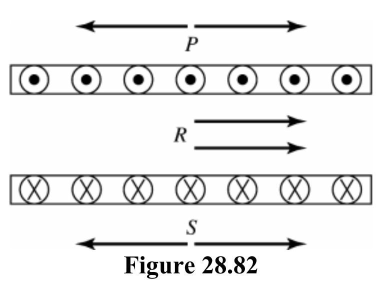


Figure 28.82

28.83. **IDENTIFY** and **SET UP:** Use Eq.(28.28) to calculate the total magnetic moment of a volume V of the iron. Use the density and atomic mass of iron to find the number of atoms in this volume and use that to find the magnetic dipole moment per atom.

EXECUTE: $M = \frac{\mu_{\text{total}}}{V}$, so $\mu_{\text{total}} = MV$ The average magnetic moment per atom is $\mu_{\text{atom}} = \mu_{\text{total}}/N = MV/N$, where N is the number of atoms in volume V. The mass of volume V is $m = \rho V$, where ρ is the density. ($\rho_{\text{iron}} = 7.8\times10^3\ \text{kg/m}^3$). The number of moles of iron in volume V is

$n = \frac{m}{55.847\times10^{-3}\ \text{kg/mol}} = \frac{\rho V}{55.847\times10^{-3}\ \text{kg/mol}}$, where $55.847\times10^{-3}\ \text{kg/mol}$ is the atomic mass of iron from appendix D. $N = nN_{\text{A}}$, where $N_{\text{A}} = 6.022\times10^{23}$ atoms/mol is Avogadro's number. Thus

$$N = nN_{\text{A}} = \frac{\rho V N_{\text{A}}}{55.847\times10^{-3}\ \text{kg/mol}}.$$

$$\mu_{\text{atom}} = \frac{MV}{N} = MV\left(\frac{55.847\times10^{-3}\ \text{kg/mol}}{\rho V N_{\text{A}}}\right) = \frac{M(55.847\times10^{-3}\ \text{kg/mol})}{\rho N_{\text{A}}}.$$

$$\mu_{\text{atom}} = \frac{(6.50\times10^4\ \text{A/m})(55.847\times10^{-3}\ \text{kg/mol})}{(7.8\times10^3\ \text{kg/m}^3)(6.022\times10^{23}\ \text{atoms/mol})}$$

$$\mu_{\text{atom}} = 7.73\times10^{-25}\ \text{A}\cdot\text{m}^2 = 7.73\times10^{-25}\ \text{J/T}$$

$\mu_{\text{B}} = 9.274\times10^{-24}\ \text{A}\cdot\text{m}^2$, so $\mu_{\text{atom}} = 0.0834\mu_{\text{B}}$.

EVALUATE: The magnetic moment per atom is much less than one Bohr magneton. The magnetic moments of each electron in the iron must be in different directions and mostly cancel each other.

28.84. IDENTIFY: The force on the cube of iron must equal the weight of the iron cube. The weight is proportional to the density and the magnetic force is proportional to μ, which is in turn proportional to K_{m}.

SET UP: The densities if iron, aluminum and silver are $\rho_{\text{Fe}} = 7.8\times10^3\ \text{kg/m}^3$, $\rho_{\text{Al}} = 2.7\times10^3\ \text{kg/m}^3$ and $\rho_{\text{Ag}} = 10.5\times10^3\ \text{kg/m}^3$. The relative permeabilities of iron, aluminum and silver are $K_{\text{Fe}} = 1400$, $K_{\text{Al}} = 1.00022$ and $K_{\text{Ag}} = 1.00 - 2.6\times10^{-5}$.

EXECUTE: **(a)** The microscopic magnetic moments of an initially unmagnetized ferromagnetic material experience torques from a magnet that aligns the magnetic domains with the external field, so they are attracted to the magnet. For a paramagnetic material, the same attraction occurs because the magnetic moments align themselves parallel to the external field. For a diamagnetic material, the magnetic moments align antiparallel to the external field so it is like two magnets repelling each other.

(b) The magnet can just pick up the iron cube so the force it exerts is $F_{\text{Fe}} = m_{\text{Fe}}g = \rho_{\text{Fe}}a^3g = (7.8\times10^3\ \text{kg/m}^3)(0.020\ \text{m})^3(9.8\ \text{m/s}^2) = 0.612\ \text{N}$. If the magnet tries to lift the aluminum cube of the same dimensions as the iron block, then the upward force felt by the cube is $F_{\text{Al}} = \frac{K_{\text{Al}}}{K_{\text{Fe}}}(0.612\ \text{N}) = \frac{1.000022}{1400}(0.612\ \text{N}) = 4.37\times10^{-4}\ \text{N}$. The weight of the aluminum cube is $W_{\text{Al}} = m_{\text{Al}}g = \rho_{\text{Al}}a^3g = (2.7\times10^3\ \text{kg/m}^3)(0.020\ \text{m})^3(9.8\ \text{m/s}^2) = 0.212\ \text{N}$. Therefore, the ratio of the magnetic force on the aluminum cube to the weight of the cube is $\frac{4.37\times10^{-4}\ \text{N}}{0.212\ \text{N}} = 2.1\times10^{-3}$ and the magnet cannot lift it.

(c) If the magnet tries to lift a silver cube of the same dimensions as the iron block, then the downward force felt by the cube is $F_{\text{Al}} = \frac{K_{\text{Ag}}}{K_{\text{Fe}}}(0.612\ \text{N}) = \frac{(1.00 - 2.6\times10^{-5})}{1400}(0.612\ \text{N}) = 4.37\times10^{-4}\ \text{N}$. But the weight of the silver cube is $W_{\text{Ag}} = m_{\text{Ag}}g = \rho_{\text{Ag}}a^3g = (10.5\times10^3\ \text{kg/m}^3)(0.020\ \text{m})^3(9.8\ \text{m/s}^2) = 0.823\ \text{N}$. So the ratio of the magnetic force on the silver cube to the weight of the cube is $\frac{4.37\times10^{-4}\ \text{N}}{0.823\ \text{N}} = 5.3\times10^{-4}$ and the magnet's effect would not be noticeable.

EVALUATE: Silver is diamagnetic and is repelled by the magnet. Aluminum is paramagnetic and is attracted by the magnet. But for both these materials the force is much less that the force on a similar cube of ferromagnetic iron.

28.85. IDENTIFY: The current-carrying wires repel each other magnetically, causing them to accelerate horizontally. Since gravity is vertical, it plays no initial role.

SET UP: The magnetic force per unit length is $\frac{F}{L} = \frac{\mu_0}{2\pi}\frac{I^2}{d}$, and the acceleration obeys the equation $F/L = m/L\ a$.

The rms current over a short discharge time is $I_0/\sqrt{2}$.

EXECUTE: **(a)** First get the force per unit length:

$$\frac{F}{L}=\frac{\mu_0}{2\pi}\frac{I^2}{d}=\frac{\mu_0}{2\pi d}\left(\frac{I_0}{\sqrt{2}}\right)^2=\frac{\mu_0}{4\pi d}\left(\frac{V}{R}\right)^2=\frac{\mu_0}{4\pi d}\left(\frac{Q_0}{RC}\right)^2$$

Now apply Newton's second law using the result above: $\frac{F}{L}=\frac{m}{L}a=\lambda a=\frac{\mu_0}{4\pi d}\left(\frac{Q_0}{RC}\right)^2$. Solving for a gives

$a=\frac{\mu_0 Q_0^2}{4\pi\lambda R^2C^2d}$. From the kinematics equation $v_x=v_{0x}+a_xt$, we have $v_0=at=aRC=\frac{\mu_0 Q_0^2}{4\pi\lambda RCd}$

(b) Conservation of energy gives $\frac{1}{2}mv_0^2=mgh$ and $h=\frac{v_0^2}{2g}=\frac{\left(\frac{\mu_0 Q_0^2}{4\pi\lambda RCd}\right)^2}{2g}=\frac{1}{2g}\left(\frac{\mu_0 Q_0^2}{4\pi\lambda RCd}\right)^2$.

EVALUATE: Once the wires have swung apart, we would have to consider gravity in applying Newton's second law.

28.86. **IDENTIFY:** Approximate the moving belt as an infinite current sheet.

SET UP: Problem 28.81 shows that $B=\frac{1}{2}\mu_0 In$ for an infinite current sheet. Let L be the width of the sheet, so $n=I/L$.

EXECUTE: The amount of charge on a length Δx of the belt is $\Delta Q=L\Delta x\sigma$, so $I=\frac{\Delta Q}{\Delta t}=L\frac{\Delta x}{\Delta t}\sigma=Lv\sigma.$

Approximating the belt as an infinite sheet $B=\frac{\mu_0 I}{2L}=\frac{\mu_0 v\sigma}{2}$. $\vec{B}$ is directed out of the page, as shown in Figure 28.86.

EVALUATE: The field is uniform above the sheet, for points close enough to the sheet for it to be considered infinite.

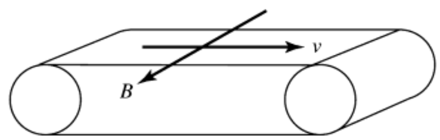


Figure 28.86

28.87. **IDENTIFY:** The rotating disk produces concentric rings of current. Calculate the field due to each ring and integrate over the surface of the disk to find the total field.

SET UP: At the center of a circular ring carrying current I, $B=\frac{\mu_0 I}{2r}$.

EXECUTE: The charge on a ring of radius r is $q=\sigma A=\sigma 2\pi r dr=\frac{2Qrdr}{a^2}$. If the disk rotates at n turns per second, then the current from that ring is $dI=\frac{dq}{dt}=ndq=\frac{2Qnrdr}{a^2}$. Therefore, $dB=\frac{\mu_0 I}{2r}=\frac{\mu_0}{2r}\frac{2Qnrdr}{a^2}=\frac{\mu_0 nQdr}{a^2}$.

We integrate out from the center to the edge of the disk and find $B=\int_0^a dB=\int_0^a\frac{\mu_0 nQdr}{a^2}=\frac{\mu_0 nQ}{a}$.

EVALUATE: The magnetic field is proportional to the total charge on the disk and to its rotation rate.

28.88. **IDENTIFY:** There are two parts to the magnetic field: that from the half loop and that from the straight wire segment running from $-a$ to a.

SET UP: Apply Eq.(28.14). Let the ϕ be the angle that locates dl around the ring.

EXECUTE: $B_x(ring)=\frac{1}{2}B_{\text{loop}}=-\frac{\mu_0 Ia^2}{4(x^2+a^2)^{3/2}}$.

$dB_y(ring)=dB\sin\theta\sin\phi=\frac{\mu_0 I}{4\pi}\frac{dl}{(x^2+a^2)}\frac{x}{(x^2+a^2)^{1/2}}\sin\phi=\frac{\mu_0 Iax\sin\phi\,d\phi}{4\pi(x^2+a^2)^{3/2}}$ and

$$B_y(ring)=\int_0^\pi dB_y(ring)=\int_0^\pi\frac{\mu_0 Iax\sin\phi d\phi}{4\pi(x^2+a^2)^{3/2}}=\frac{\mu_0 Iax}{4\pi(x^2+a^2)^{3/2}}\cos\phi\Big|_0^\pi=-\frac{\mu_0 Iax}{2\pi(x^2+a^2)^{3/2}}.$$

$B_y(rod)=\frac{\mu_0 Ia}{2\pi x(x^2+a^2)^{1/2}}$, using Eq. (28.8). The total field components are:

$$B_x=-\frac{\mu_0 Ia^2}{4(x^2+a^2)^{3/2}}\text{ and }B_y=\frac{\mu_0 Ia}{2\pi x(x^2+a^2)^{1/2}}\left(1-\frac{x^2}{x^2+a^2}\right)=\frac{\mu_0 Ia^3}{2\pi x(x^2+a^2)^{3/2}}.$$

EVALUATE: $B_y=-\frac{2}{\pi}\frac{a}{x}B_x$. B_y decreases faster than B_x as x increases. For very small x, $B_x=-\frac{\mu_0 I}{4a}$ and $B_y=\frac{\mu_0 I}{2\pi a}$.

In this limit B_x is the field at the center of curvature of a semicircle and B_y is the field of a long straight wire.

29

ELECTROMAGNETIC INDUCTION

29.1. **IDENTIFY:** Altering the orientation of a coil relative to a magnetic field changes the magnetic flux through the coil. This change then induces an emf in the coil.
SET UP: The flux through a coil of N turns is $\Phi = NBA\cos\phi$, and by Faraday's law the magnitude of the induced emf is $\mathcal{E} = d\Phi/dt$.
EXECUTE: **(a)** $\Delta\Phi = NBA = (50)(1.20\text{ T})(0.250\text{ m})(0.300\text{ m}) = 4.50\text{ Wb}$
(b) $\mathcal{E} = d\Phi/dt = (4.50\text{ Wb})/(0.222\text{ s}) = 20.3\text{ V}$
EVALUATE: This induced potential is certainly large enough to be easily detectable.

29.2. **IDENTIFY:** $\mathcal{E} = \left|\frac{\Delta\Phi_B}{\Delta t}\right|$. $\Phi_B = BA\cos\phi$. Φ_B is the flux through each turn of the coil.
SET UP: $\phi_i = 0°$. $\phi_f = 90°$.
EXECUTE: **(a)** $\Phi_{B,i} = BA\cos 0° = (6.0\times10^{-5}\text{ T})(12\times10^{-4}\text{ m}^2)(1) = 7.2\times10^{-8}\text{ Wb}$. The total flux through the coil is $N\Phi_{B,i} = (200)(7.2\times10^{-8}\text{ Wb}) = 1.44\times10^{-5}\text{ Wb}$. $\Phi_{B,f} = BA\cos 90° = 0$.
(b) $\mathcal{E} = \left|\frac{N\Phi_i - N\Phi_f}{\Delta t}\right| = \frac{1.44\times10^{-5}\text{ Wb}}{0.040\text{ s}} = 3.6\times10^{-4}\text{ V} = 0.36\text{ mV}$.
EVALUATE: The average induced emf depends on how rapidly the flux changes.

29.3. **IDENTIFY** and **SET UP:** Use Faraday's law to calculate the average induced emf and apply Ohm's law to the coil to calculate the average induced current and charge that flows.
(a) EXECUTE: The magnitude of the average emf induced in the coil is $|\mathcal{E}_{av}| = N\left|\frac{\Delta\Phi I_B}{\Delta t}\right|$. Initially,

$\Phi_{Bi} = BA\cos\phi = BA$. The final flux is zero, so $|\mathcal{E}_{av}| = N\frac{|\Phi_{Bf} - \Phi_{Bi}|}{\Delta t} = \frac{NBA}{\Delta t}$. The average induced current is

$I = \frac{|\mathcal{E}_{av}|}{R} = \frac{NBA}{R\Delta t}$. The total charge that flows through the coil is $Q = I\Delta t = \left(\frac{NBA}{R\Delta t}\right)\Delta t = \frac{NBA}{R}$.

EVALUATE: The charge that flows is proportional to the magnetic field but does not depend on the time Δt.
(b) The magnetic stripe consists of a pattern of magnetic fields. The pattern of charges that flow in the reader coil tell the card reader the magnetic field pattern and hence the digital information coded onto the card.
(c) According to the result in part (a) the charge that flows depends only on the change in the magnetic flux and it does not depend on the rate at which this flux changes.

29.4. **IDENTIFY** and **SET UP:** Apply the result derived in Exercise 29.3: $Q = NBA/R$. In the present exercise the flux changes from its maximum value of $\Phi_B = BA$ to zero, so this equation applies. R is the total resistance so here $R = 60.0\ \Omega + 45.0\ \Omega = 105.0\ \Omega$.
EXECUTE: $Q = \frac{NBA}{R}$ says $B = \frac{QR}{NA} = \frac{(3.56\times10^{-5}\text{ C})(105.0\ \Omega)}{120(3.20\times10^{-4}\text{ m}^2)} = 0.0973\text{ T}$.
EVALUATE: A field of this magnitude is easily produced.

29.5. **IDENTIFY:** Apply Faraday's law.
SET UP: Let $+z$ be the positive direction for $\vec{A}$. Therefore, the initial flux is positive and the final flux is zero.
EXECUTE: **(a)** and **(b)** $\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{0 - (1.5\text{ T})\pi(0.120\text{ m})^2}{2.0\times10^{-3}\text{ s}} = +34\text{ V}$. Since $\mathcal{E}$ is positive and $\vec{A}$ is toward us, the induced current is counterclockwise.
EVALUATE: The shorter the removal time, the larger the average induced emf.

29.6. **IDENTIFY:** Apply Eq.(29.4). $I = \mathcal{E}/R$.

SET UP: $d\Phi_B/dt = AdB/dt$.

EXECUTE: **(a)** $\mathcal{E} = \frac{Nd\Phi_B}{dt} = NA\frac{d}{dt}(B) = NA\frac{d}{dt}\left((0.012\text{ T/s})t + (3.00\times10^{-5}\text{ T/s}^4)t^4\right)$

$\mathcal{E} = NA\left((0.012\text{ T/s}) + (1.2\times10^{-4}\text{ T/s}^4)t^3\right) = 0.0302\text{ V} + (3.02\times10^{-4}\text{ V/s}^3)t^3$.

(b) At $t = 5.00$ s, $\mathcal{E} = 0.0302\text{ V} + (3.02\times10^{-4}\text{ V/s}^3)(5.00\text{ s})^3 = 0.0680\text{ V}$. $I = \frac{\mathcal{E}}{R} = \frac{0.0680\text{ V}}{600\ \Omega} = 1.13\times10^{-4}$ A.

EVALUATE: The rate of change of the flux is increasing in time, so the induced current is not constant but rather increases in time.

29.7. **IDENTIFY:** Calculate the flux through the loop and apply Faraday's law.

SET UP: To find the total flux integrate $d\Phi_B$ over the width of the loop. The magnetic field of a long straight wire, at distance r from the wire, is $B = \frac{\mu_0 I}{2\pi r}$. The direction of $\vec{B}$ is given by the right-hand rule.

EXECUTE: **(a)** When $B = \frac{\mu_0 i}{2\pi r}$, into the page.

(b) $d\Phi_B = BdA = \frac{\mu_0 i}{2\pi r}Ldr$.

(c) $\Phi_B = \int_a^b d\Phi_B = \frac{\mu_0 iL}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 iL}{2\pi}\ln(b/a)$.

(d) $\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{\mu_0 L}{2\pi}\ln(b/a)\frac{di}{dt}$.

(e) $\mathcal{E} = \frac{\mu_0(0.240\text{ m})}{2\pi}\ln(0.360/0.120)(9.60\text{ A/s}) = 5.06\times10^{-7}$ V.

EVALUATE: The induced emf is proportional to the rate at which the current in the long straight wire is changing

29.8. **IDENTIFY:** Apply Faraday's law.

SET UP: Let $\vec{A}$ be upward in Figure 29.28 in the textbook.

EXECUTE: **(a)** $|\mathcal{E}_{\text{ind}}| = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}(B_\perp A)\right|$

$|\mathcal{E}_{\text{ind}}| = A\sin 60°\left|\frac{dB}{dt}\right| = A\sin 60°\left|\frac{d}{dt}\left((1.4\text{ T})e^{-(0.057\text{s}^{-1})t}\right)\right| = (\pi r^2)(\sin 60°)(1.4\text{ T})(0.057\text{ s}^{-1})e^{-(0.057\text{s}^{-1})t}$

$|\mathcal{E}_{\text{ind}}| = \pi(0.75\text{ m})^2(\sin 60°)(1.4\text{ T})(0.057\text{ s}^{-1})e^{-(0.057\text{s}^{-1})t} = (0.12\text{ V})\ e^{-(0.057\text{ s}^{-1})t}$.

(b) $\mathcal{E} = \frac{1}{10}\mathcal{E}_0 = \frac{1}{10}(0.12\text{ V})$. $\frac{1}{10}(0.12\text{ V}) = (0.12\text{ V})\ e^{-(0.057\text{ s}^{-1})t}$. $\ln(1/10) = -(0.057\text{ s}^{-1})t$ and $t = 40.4$ s.

(c) $\vec{B}$ is in the direction of $\vec{A}$ so Φ_B is positive. B is getting weaker, so the magnitude of the flux is decreasing and $d\Phi_B/dt < 0$. Faraday's law therefore says $\mathcal{E} > 0$. Since $\mathcal{E} > 0$, the induced current must flow *counterclockwise* as viewed from above.

EVALUATE: The flux changes because the magnitude of the magnetic field is changing.

29.9. **IDENTIFY** and **SET UP:** Use Faraday's law to calculate the emf (magnitude and direction). The direction of the induced current is the same as the direction of the emf. The flux changes because the area of the loop is changing; relate dA/dt to dc/dt, where c is the circumference of the loop.

(a) EXECUTE: $c = 2\pi r$ and $A = \pi r^2$ so $A = c^2/4\pi$

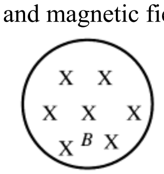
$\Phi_B = BA = (B/4\pi)c^2$

$|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = \left(\frac{B}{2\pi}\right)c\left|\frac{dc}{dt}\right|$

At $t = 9.0$ s, $c = 1.650\text{ m} - (9.0\text{ s})(0.120\text{ m/s}) = 0.570$ m

$|\mathcal{E}| = (0.500\text{ T})(1/2\pi)(0.570\text{ m})(0.120\text{ m/s}) = 5.44$ mV

(b) SET UP: The loop and magnetic field are sketched in Figure 29.9.



Take into the page to be the positive direction for $\vec{A}$. Then the magnetic flux is positive.

Figure 29.9

EXECUTE: The positive flux is decreasing in magnitude; $d\Phi_B/dt$ is negative and $\mathcal{E}$ is positive. By the right-hand rule, for $\vec{A}$ into the page, positive $\mathcal{E}$ is clockwise.

EVALUATE: Even though the circumference is changing at a constant rate, dA/dt is not constant and $|\mathcal{E}|$ is not constant. Flux ⊗ is decreasing so the flux of the induced current is ⊗ and this means that I is clockwise, which checks.

29.10. **IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

SET UP: The flux through a coil is $\Phi = NBA\cos\phi$ and the induced emf is $\mathcal{E} = d\Phi/dt$.

EXECUTE: **(a)** and **(c)** The magnetic flux is constant, so the induced emf is zero.

(b) The area inside the field is changing. If we let x be the length (along the 30.0-cm side) in the field, then $A = (0.400\text{ m})x$. $\Phi_B = BA = (0.400\text{ m})x$

$$\mathcal{E} = d\Phi/dt = B\,d[(0.400\text{ m})x]/dt = B(0.400\text{ m})dx/dt = B(0.400\text{ m})v$$

$$\mathcal{E} = (1.25\text{ T})(0.400\text{ m})(0.0200\text{ m/s}) = 0.0100\text{ V}$$

EVALUATE: It is not a large *flux* that induces an emf, but rather a large *rate of change* of the flux. The induced emf in part (b) is small enough to be ignored in many instances.

29.11. **IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

SET UP: The flux through a coil is $\Phi = NBA\cos\phi$ and the induced emf is $\mathcal{E} = d\Phi/dt$.

EXECUTE: **(a)** $\mathcal{E} = d\Phi/dt = d[A(B_0 + bx)]/dt = bA\,dx/dt = bAv$

(b) clockwise

(c) Same answers except the current is counterclockwise.

EVALUATE: Even though the coil remains within the magnetic field, the flux through it increases because the strength of the field is increasing.

29.12. **IDENTIFY:** Use the results of Example 29.5.

SET UP: $\mathcal{E}_{max} = NBA\omega$. $\mathcal{E}_{av} = \dfrac{2}{\pi}\mathcal{E}_{max}$. $\omega = (440\text{ rev/min})\left(\dfrac{2\pi\text{ rad/rev}}{60\text{ s/min}}\right) = 46.1\text{ rad/s}$.

EXECUTE: **(a)** $\mathcal{E}_{max} = NBA\omega = (150)(0.060\text{ T})\pi(0.025\text{ m})^2(46.1\text{ rad/s}) = 0.814\text{ V}$

(b) $\mathcal{E}_{av} = \dfrac{2}{\pi}\mathcal{E}_{max} = \dfrac{2}{\pi}(0.815\text{ V}) = 0.519\text{ V}$

EVALUATE: In $\mathcal{E}_{max} = NBA\omega$, ω must be in rad/s.

29.13. **IDENTIFY:** Apply the results of Example 29.5.

SET UP: $\mathcal{E}_{max} = NBA\omega$

EXECUTE: $\omega = \dfrac{\mathcal{E}_{max}}{NBA} = \dfrac{2.40\times10^{-2}\text{ V}}{(120)(0.0750\text{ T})(0.016\text{ m})^2} = 10.4\text{ rad/s}$

EVALUATE: We may also express ω as 99.3 rev/min or 1.66 rev/s.

29.14. **IDENTIFY:** A change in magnetic flux through a coil induces an emf in the coil.

SET UP: The flux through a coil is $\Phi = NBA\cos\phi$ and the induced emf is $\mathcal{E} = d\Phi/dt$.

EXECUTE: The flux is constant in each case, so the induced emf is zero in all cases.

EVALUATE: Even though the coil is moving within the magnetic field and has flux through it, this flux is not *changing*, so no emf is induced in the coil.

29.15. **IDENTIFY** and **SET UP:** The field of the induced current is directed to oppose the change in flux.

EXECUTE: **(a)** The field is into the page and is increasing so the flux is increasing. The field of the induced current is out of the page. To produce field out of the page the induced current is counterclockwise.

(b) The field is into the page and is decreasing so the flux is decreasing. The field of the induced current is into the page. To produce field into the page the induced current is clockwise.

(c) The field is constant so the flux is constant and there is no induced emf and no induced current.

EVALUATE: The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

29.16. **IDENTIFY:** By Lenz's law, the induced current flows to oppose the flux change that caused it.

SET UP and **EXECUTE:** The magnetic field is outward through the round coil and is decreasing, so the magnetic field due to the induced current must also point outward to oppose this decrease. Therefore the induced current is counterclockwise.

EVALUATE: Careful! Lenz's law does not say that the induced current flows to oppose the magnetic flux. Instead it says that the current flows to oppose the *change* in flux.

29.17. **IDENTIFY** and **SET UP:** Apply Lenz's law, in the form that states that the flux of the induced current tends to oppose the change in flux.

EXECUTE: **(a)** With the switch closed the magnetic field of coil A is to the right at the location of coil B. When the switch is opened the magnetic field of coil A goes away. Hence by Lenz's law the field of the current induced in coil B is to the right, to oppose the decrease in the flux in this direction. To produce magnetic field that is to the right the current in the circuit with coil B must flow through the resistor in the direction a to b.

(b) With the switch closed the magnetic field of coil A is to the right at the location of coil B. This field is stronger at points closer to coil A so when coil B is brought closer the flux through coil B increases. By Lenz's law the field of the induced current in coil B is to the left, to oppose the increase in flux to the right. To produce magnetic field that is to the left the current in the circuit with coil B must flow through the resistor in the direction *b* to *a*.
(c) With the switch closed the magnetic field of coil A is to the right at the location of coil B. The current in the circuit that includes coil A increases when R is decreased and the magnetic field of coil A increases when the current through the coil increases. By Lenz's law the field of the induced current in coil B is to the left, to oppose the increase in flux to the right. To produce magnetic field that is to the left the current in the circuit with coil B must flow through the resistor in the direction *b* to *a*.
EVALUATE: In parts (b) and (c) the change in the circuit causes the flux through circuit B to increase and in part (a) it causes the flux to decrease. Therefore, the direction of the induced current is the same in parts (b) and (c) and opposite in part (a).

29.18. **IDENTIFY:** Apply Lenz's law.
SET UP: The field of the induced current is directed to oppose the change in flux in the primary circuit.
EXECUTE: **(a)** The magnetic field in A is to the left and is increasing. The flux is increasing so the field due to the induced current in B is to the right. To produce magnetic field to the right, the induced current flows through R from right to left.
(b) The magnetic field in A is to the right and is decreasing. The flux is decreasing so the field due to the induced current in B is to the right. To produce magnetic field to the right the induced current flows through R from right to left.
(c) The magnetic field in A is to the right and is increasing. The flux is increasing so the field due to the induced current in B is to the left. To produce magnetic field to the left the induced current flows through R from left to right.
EVALUATE: The direction of the induced current depends on the direction of the external magnetic field and whether the flux due to this field is increasing or decreasing.

29.19. **IDENTIFY** and **SET UP:** Lenz's law requires that the flux of the induced current opposes the change in flux.
EXECUTE: **(a)** Φ_B is $\odot$ and increasing so the flux Φ_{ind} of the induced current is $\otimes$ and the induced current is clockwise.
(b) The current reaches a constant value so Φ_B is constant. $d\Phi_B/dt = 0$ and there is no induced current.
(c) Φ_B is $\odot$ and decreasing, so Φ_{ind} is $\odot$ and current is counterclockwise.
EVALUATE: Only a change in flux produces an induced current. The induced current is in one direction when the current in the outer ring is increasing and is in the opposite direction when that current is decreasing.

29.20. **IDENTIFY:** Use the results of Example 29.6. Use the three approaches specified in the problem for determining the direction of the induced current. $I = \mathcal{E}/R$.
SET UP: Let $\vec{A}$ be directed into the figure, so a clockwise emf is positive.
EXECUTE: **(a)** $\mathcal{E} = vBl = (5.0\text{ m/s})(0.750\text{ T})(1.50\text{ m}) = 5.6\text{ V}$
(b) (i) Let q be a positive charge in the moving bar, as shown in Figure 29.20a. The magnetic force on this charge is $\vec{F} = q\vec{v} \times \vec{B}$, which points *upward*. This force pushes the current in a *counterclockwise* direction through the circuit.
(ii) Φ_B is positive and is increasing in magnitude, so $d\Phi_B/dt > 0$. Then by Faraday's law $\mathcal{E} < 0$ and the emf and induced current are counterclockwise.
(iii) The flux through the circuit is increasing, so the induced current must cause a magnetic field out of the paper to oppose this increase. Hence this current must flow in a *counterclockwise* sense, as shown in Figure 29.20b.
(c) $\mathcal{E} = RI$. $I = \dfrac{\mathcal{E}}{R} = \dfrac{5.6\text{ V}}{25\ \Omega} = 0.22\text{ A}$.
EVALUATE: All three methods agree on the direction of the induced current.

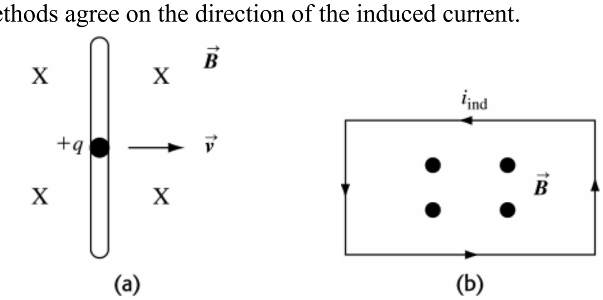


Figure 29.20

29.21. **IDENTIFY:** A conductor moving in a magnetic field may have a potential difference induced across it, depending on how it is moving.
SET UP: The induced emf is $\mathcal{E} = vBL \sin \phi$, where ϕ is the angle between the velocity and the magnetic field.

EXECUTE: **(a)** $\mathcal{E} = vBL\sin\phi = (5.00\text{ m/s})(0.450\text{ T})(0.300\text{ m})(\sin 90°) = 0.675\text{ V}$

(b) The positive charges are moved to end b, so b is at the higher potential.

(c) $E = V/L = (0.675\text{ V})/(0.300\text{ m}) = 2.25\text{ V/m}$. The direction of $\vec{E}$ is from, b to a.

(d) The positive charge are pushed to b, so b has an excess of positive charge.

(e) (i) If the rod has no appreciable thickness, $L = 0$, so the emf is zero. (ii) The emf is zero because no magnetic force acts on the charges in the rod since it moves parallel to the magnetic field.

EVALUATE: The motional emf is large enough to have noticeable effects in some cases.

29.22. **IDENTIFY:** The moving bar has a motional emf induced across its ends, so it causes a current to flow.

SET UP: The induced potential is $\mathcal{E} = vBL$ and Ohm's law is $\mathcal{E} = IR$.

EXECUTE: (a) $\mathcal{E} = vBL = (5.0\text{ m/s})(0.750\text{ T})(1.50\text{ m}) = 5.6\text{ V}$

(b) $I = \mathcal{E}/R = (5.6\text{ V})/(25\ \Omega) = 0.23\text{ A}$

EVALUATE: Both the induced potential and the current are large enough to have noticeable effects.

29.23. **IDENTIFY:** $\mathcal{E} = vBL$

SET UP: $L = 5.00\times10^{-2}$ m. 1 mph = 0.4470 m/s.

EXECUTE: $v = \dfrac{\mathcal{E}}{BL} = \dfrac{1.50\text{ V}}{(0.650\text{ T})(5.00\times10^{-2}\text{ m})} = 46.2\text{ m/s} = 103\text{ mph}.$

EVALUATE: This is a large speed and not practical. It is also difficult to produce a 5.00 cm wide region of 0.650 T magnetic field.

29.24. **IDENTIFY:** $\mathcal{E} = vBL$.

SET UP: 1 mph = 0.4470 m/s. 1 G = 10^{-4} T.

EXECUTE: **(a)** $\mathcal{E} = (180\text{ mph})\left(\dfrac{0.4470\text{ m/s}}{1\text{ mph}}\right)(0.50\times10^{-4}\text{ T})(1.5\text{ m}) = 6.0\text{ mV}$. This is much too small to be noticeable.

(b) $\mathcal{E} = (565\text{ mph})\left(\dfrac{0.4470\text{ m/s}}{1\text{ mph}}\right)(0.50\times10^{-4}\text{ T})(64.4\text{ m}) = 0.813\text{ mV}$. This is too small to be noticeable.

EVALUATE: Even though the speeds and values of L are large, the earth's field is small and motional emfs due to the earth's field are not important in these situations.

29.25. **IDENTIFY** and **SET UP:** $\mathcal{E} = vBL$. Use Lenz's law to determine the direction of the induced current. The force F_{ext} required to maintain constant speed is equal and opposite to the force F_I that the magnetic field exerts on the rod because of the current in the rod.

EXECUTE: **(a)** $\mathcal{E} = vBL = (7.50\text{ m/s})(0.800\text{ T})(0.500\text{ m}) = 3.00\text{ V}$

(b) $\vec{B}$ is into the page. The flux increases as the bar moves to the right, so the magnetic field of the induced current is out of the page inside the circuit. To produce magnetic field in this direction the induced current must be counterclockwise, so from b to a in the rod.

(c) $I = \dfrac{\mathcal{E}}{R} = \dfrac{3.00\text{ V}}{1.50\ \Omega} = 2.00\text{ A}$. $F_I = ILB\sin\phi = (2.00\text{ A})(0.500\text{ m})(0.800\text{ T})\sin 90° = 0.800\text{ N}$. $\vec{F}_I$ is to the left. To keep the bar moving to the right at constant speed an external force with magnitude $F_{ext} = 0.800$ N and directed to the right must be applied to the bar.

(d) The rate at which work is done by the force F_{ext} is $F_{ext}v = (0.800\text{ N})(7.50\text{ m/s}) = 6.00\text{ W}$. The rate at which thermal energy is developed in the circuit is $I^2R = (2.00\text{ A})(1.50\ \Omega) = 6.00\text{ W}$. These two rates are equal, as is required by conservation of energy.

EVALUATE: The force on the rod due to the induced current is directed to oppose the motion of the rod. This agrees with Lenz's law.

29.26. **IDENTIFY:** Use Faraday's law to calculate the induced emf. Ohm's law applied to the loop gives I. Use Eq.(27.19) to calculate the force exerted on each side of the loop.

SET UP: The loop before it starts to enter the magnetic field region is sketched in Figure 29.26a.

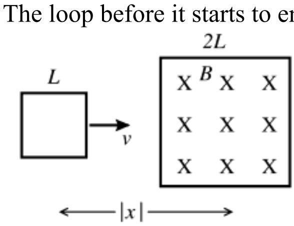


Figure 29.26a

EXECUTE: For $x < -3L/2$ or $x > 3L/2$ the loop is completely outside the field region. $\Phi_B = 0$, and $\dfrac{d\Phi_B}{dt} = 0$.

Thus $\mathcal{E} = 0$ and $I = 0$, so there is no force from the magnetic field and the external force F necessary to maintain constant velocity is zero.

SET UP: The loop when it is completely inside the field region is sketched in Figure 29.26b.

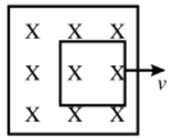


Figure 29.26b

EXECUTE: For $-L/2 < x < L/2$ the loop is completely inside the field region and $\Phi_B = BL^2$.

But $\dfrac{d\Phi_B}{dt} = 0$ so $\mathcal{E} = 0$ and $I = 0$. There is no force $\vec{F} = I\vec{l} \times \vec{B}$ from the magnetic field and the external force F necessary to maintain constant velocity is zero.

SET UP: The loop as it enters the magnetic field region is sketched in Figure 29.26c.

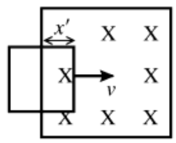


Figure 29.26c

EXECUTE: For $-3L/2 < x < -L/2$ the loop is entering the field region. Let x' be the length of the loop that is within the field.

Then $|\Phi_B| = BLx'$ and $\left|\dfrac{d\Phi_B}{dt}\right| = Blv$. The magnitude of the induced emf is $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = BLv$ and the induced current is $I = \dfrac{|\mathcal{E}|}{R} = \dfrac{BLv}{R}$. Direction of I: Let $\vec{A}$ be directed into the plane of the figure. Then Φ_B is positive. The flux is positive and increasing in magnitude, so $\dfrac{d\Phi_B}{dt}$ is positive. Then by Faraday's law $\mathcal{E}$ is negative, and with our choice for direction of $\vec{A}$ a negative $\mathcal{E}$ is counterclockwise. The current induced in the loop is counterclockwise.

SET UP: The induced current and magnetic force on the loop are shown in Figure 29.26d, for the situation where the loop is entering the field.

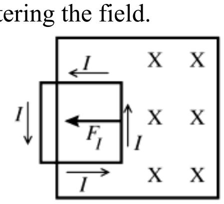


Figure 29.26d

EXECUTE: $\vec{F}_I = I\vec{l} \times \vec{B}$ gives that the force $\vec{F}_I$ exerted on the loop by the magnetic field is to the left and has magnitude $F_I = ILB = \left(\dfrac{BLv}{R}\right)LB = \dfrac{B^2L^2v}{R}$.

The external force $\vec{F}$ needed to move the loop at constant speed is equal in magnitude and opposite in direction to $\vec{F}_I$ so is to the right and has this same magnitude.

SET UP: The loop as it leaves the magnetic field region is sketched in Figure 29.26e.

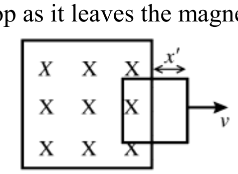


Figure 29.26e

EXECUTE: For $L/2 < x < 3L/2$ the loop is leaving the field region. Let x' be the length of the loop that is outside the field.

Then $|\Phi_B| = BL(L - x')$ and $\left|\dfrac{d\Phi_B}{dt}\right| = BLv$. The magnitude of the induced emf is $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = BLv$ and the induced current is $I = \dfrac{|\mathcal{E}|}{R} = \dfrac{BLv}{R}$. Direction of I: Again let $\vec{A}$ be directed into the plane of the figure. Then Φ_B is positive and decreasing in magnitude, so $\dfrac{d\Phi_B}{dt}$ is negative. Then by Faraday's law $\mathcal{E}$ is positive, and with our choice for direction of $\vec{A}$ a positive $\mathcal{E}$ is clockwise. The current induced in the loop is clockwise.

SET UP: The induced current and magnetic force on the loop are shown in Figure 29.26f, for the situation where the loop is leaving the field.

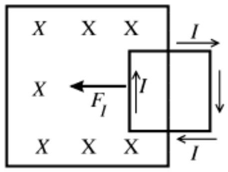


Figure 29.26f

EXECUTE: $\vec{F}_I = I\vec{l} \times \vec{B}$ gives that the force $\vec{F}_I$ exerted on the loop by the magnetic field is to the left and has magnitude $F_I = ILB = \left(\frac{BLv}{R}\right)LB = \frac{B^2L^2v}{R}$.

The external force $\vec{F}$ needed to move the loop at constant speed is equal in magnitude and opposite in direction to $\vec{F}_I$ so is to the right and has this same magnitude.

(a) The graph of F versus x is given in Figure 29.26g.

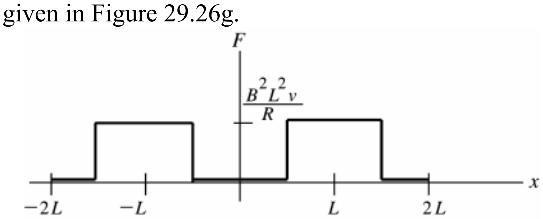


Figure 29.26g

(b) The graph of the induced current I versus x is given in Figure 29.26h.

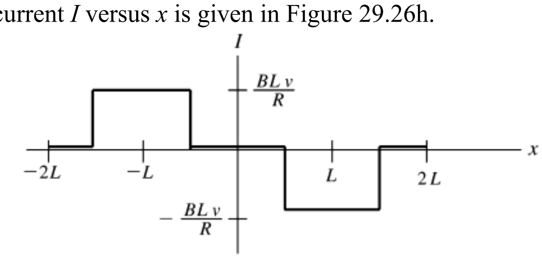


Figure 29.26h

EVALUATE: When the loop is either totally outside or totally inside the magnetic field region the flux isn't changing, there is no induced current, and no external force is needed for the loop to maintain constant speed. When the loop is entering the field the external force required is directed so as to pull the loop in and when the loop is leaving the field the external force required is directed so as to pull the loop out of the field. These directions agree with Lenz's law: the force on the induced current (opposite in direction to the required external force) is directed so as to oppose the loop entering or leaving the field.

29.27. **IDENTIFY:** A bar moving in a magnetic field has an emf induced across its ends.

SET UP: The induced potential is $\mathcal{E} = vBL \sin \phi$.

EXECUTE: Note that $\phi = 90°$ in all these cases because the bar moved perpendicular to the magnetic field. But the effective length of the bar, $L \sin \theta$, is different in each case.

(a) $\mathcal{E} = vBL \sin \theta = (2.50 \text{ m/s})(1.20 \text{ T})(1.41 \text{ m}) \sin (37.0°) = 2.55$ V, with a at the higher potential because positive charges are pushed toward that end.

(b) Same as (a) except $\theta = 53.0°$, giving 3.38 V, with a at the higher potential.

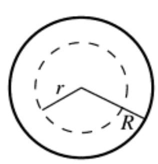
(c) Zero, since the velocity is parallel to the magnetic field.

(d) The bar must move perpendicular to its length, for which the emf is 4.23 V. For $V_b > V_a$, it must move upward and to the left (toward the second quadrant) perpendicular to its length.

EVALUATE: The orientation of the bar affects the potential induced across its ends.

29.28. **IDENTIFY:** Use Eq.(29.10) to calculate the induced electric field E at a distance r from the center of the solenoid. Away from the ends of the solenoid, $B = \mu_0 nI$ inside and $B = 0$ outside.

(a) SET UP: The end view of the solenoid is sketched in Figure 29.28.



Let R be the radius of the solenoid.

Figure 29.28

Apply $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ to an integration path that is a circle of radius r, where $r < R$. We need to calculate just the magnitude of E so we can take absolute values.

EXECUTE: $\left|\oint \vec{E}\cdot d\vec{l}\right| = E(2\pi r)$

$\Phi_B = B\pi r^2,\ \left|-\dfrac{d\Phi_B}{dt}\right| = \pi r^2\left|\dfrac{dB}{dt}\right|$

$\left|\oint \vec{E}\cdot d\vec{l}\right| = \left|-\dfrac{d\Phi_B}{dt}\right|$ implies $E(2\pi r) = \pi r^2\left|\dfrac{dB}{dt}\right|$

$E = \frac{1}{2}r\left|\dfrac{dB}{dt}\right|$

$B = \mu_0 nI$, so $\dfrac{dB}{dt} = \mu_0 n\dfrac{dI}{dt}$

Thus $E = \frac{1}{2}r\mu_0 n\dfrac{dI}{dt} = \frac{1}{2}(0.00500\text{ m})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(900\text{ m}^{-1})(60.0\text{ A/s}) = 1.70\times10^{-4}\text{ V/m}$

(b) $r = 0.0100$ cm is still inside the solenoid so the expression in part (a) applies.

$E = \frac{1}{2}r\mu_0 n\dfrac{dI}{dt} = \frac{1}{2}(0.0100\text{ m})(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(900\text{ m}^{-1})(60.0\text{ A/s}) = 3.39\times10^{-4}\text{ V/m}$

EVALUATE: Inside the solenoid E is proportional to r, so E doubles when r doubles.

29.29. **IDENTIFY:** Apply Eqs.(29.9) and (29.10).

SET UP: Evaluate the integral if Eq.(29.10) for a path which is a circle of radius r and concentric with the solenoid. The magnetic field of the solenoid is confined to the region inside the solenoid, so $B(r) = 0$ for $r > R$

EXECUTE: **(a)** $\dfrac{d\Phi_B}{dt} = A\dfrac{dB}{dt} = \pi r_1^2\dfrac{dB}{dt}$.

(b) $E = \dfrac{1}{2\pi r_1}\dfrac{d\Phi_B}{dt} = \dfrac{\pi r_1^2}{2\pi r_1}\dfrac{dB}{dt} = \dfrac{r_1}{2}\dfrac{dB}{dt}$. The direction of $\vec{E}$ is shown in Figure 29.29a.

(c) All the flux is within $r < R$, so outside the solenoid $E = \dfrac{1}{2\pi r_2}\dfrac{d\Phi_B}{dt} = \dfrac{\pi R^2}{2\pi r_2}\dfrac{dB}{dt} = \dfrac{R^2}{2r_2}\dfrac{dB}{dt}$.

(d) The graph is sketched in Figure 29.29b.

(e) At $r = R/2$, $\mathcal{E} = \dfrac{d\Phi_B}{dt} = \pi(R/2)^2\dfrac{dB}{dt} = \dfrac{\pi R^2}{4}\dfrac{dB}{dt}$.

(f) At $r = R$, $\mathcal{E} = \dfrac{d\Phi_B}{dt} = \pi R^2\dfrac{dB}{dt}$.

(g) At $r = 2R$, $\mathcal{E} = \dfrac{d\Phi_B}{dt} = \pi R^2\dfrac{dB}{dt}$.

EVALUATE: The emf is independent of the distance from the center of the cylinder at all points outside it. Even though the magnetic field is zero for $r > R$, the induced electric field is nonzero outside the solenoid and a nonzero emf is induced in a circular turn that has $r > R$.

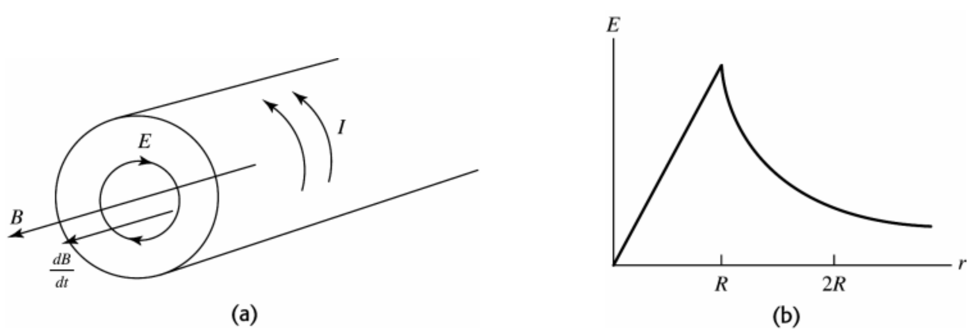


Figure 29.29

29.30. **IDENTIFY:** Use Eq.(29.10) to calculate the induced electric field E and use this E in Eq.(29.9) to calculate $\mathcal{E}$ between two points.

(a) SET UP: Because of the axial symmetry and the absence of any electric charge, the field lines are concentric circles.

(b) See Figure 29.30.

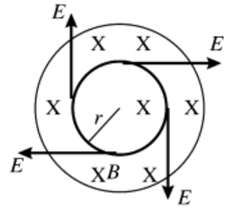


Figure 29.30

$\vec{E}$ is tangent to the ring. The direction of $\vec{E}$ (clockwise or counterclockwise) is the direction in which current will be induced in the ring.

EXECUTE: Use the sign convention for Faraday's law to deduce this direction. Let $\vec{A}$ be into the paper. Then Φ_B is positive. B decreasing then means $\frac{d\Phi_B}{dt}$ is negative, so by $\mathcal{E} = -\frac{d\Phi_B}{dt}$, $\mathcal{E}$ is positive and therefore clockwise. Thus $\vec{E}$ is clockwise around the ring. To calculate E apply $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$ to a circular path that coincides with the ring.

$\oint \vec{E} \cdot d\vec{l} = E(2\pi r)$

$\Phi_B = B\pi r^2$; $\left|\frac{d\Phi_B}{dt}\right| = \pi r^2 \left|\frac{dB}{dt}\right|$

$E(2\pi r) = \pi r^2 \left|\frac{dB}{dt}\right|$ and $E = \frac{1}{2} r \left|\frac{dB}{dt}\right| = \frac{1}{2}(0.100\text{ m})(0.0350\text{ T/s}) = 1.75 \times 10^{-3}\text{ V/m}$

(c) The induced emf has magnitude $\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = E(2\pi r) = (1.75 \times 10^{-3}\text{ V/m})(2\pi)(0.100\text{ m}) = 1.100 \times 10^{-3}\text{ V}$. Then $I = \frac{\mathcal{E}}{R} = \frac{1.100 \times 10^{-3}\text{ V}}{4.00\ \Omega} = 2.75 \times 10^{-4}\text{ A}$.

(d) Points a and b are separated by a distance around the ring of πr so $\mathcal{E} = E(\pi r) = (1.75 \times 10^{-3}\text{ V/m})(\pi)(0.100\text{ m}) = 5.50 \times 10^{-4}\text{ V}$

(e) The ends are separated by a distance around the ring of $2\pi r$ so $\mathcal{E} = 1.10 \times 10^{-3}\text{ V}$ as calculated in part (c).

EVALUATE: The induced emf, calculated from Faraday's law and used to calculate the induced current, is associated with the induced electric field integrated around the total circumference of the ring.

29.31. IDENTIFY: Apply Eq.(29.1) with $\Phi_B = \mu_0 niA$.

SET UP: $A = \pi r^2$, where $r = 0.0110\text{ m}$. In Eq.(29.11), $r = 0.0350\text{ m}$.

EXECUTE: $|\mathcal{E}| = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}(BA)\right| = \left|\frac{d}{dt}(\mu_0 niA)\right| = \mu_0 nA \left|\frac{di}{dt}\right|$ and $|\mathcal{E}| = E(2\pi r)$. Therefore, $\left|\frac{di}{dt}\right| = \frac{E 2\pi r}{\mu_0 nA}$.

$\left|\frac{di}{dt}\right| = \frac{(8.00 \times 10^{-6}\text{ V/m})2\pi(0.0350\text{ m})}{\mu_0(400\text{ m}^{-1})\pi(0.0110\text{ m})^2} = 9.21\text{ A/s}$.

EVALUATE: Outside the solenoid the induced electric field decreases with increasing distance from the axis of the solenoid.

29.32. IDENTIFY: A changing magnetic flux through a coil induces an emf in that coil, which means that an electric field is induced in the material of the coil.

SET UP: According to Faraday's law, the induced electric field obeys the equation $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$.

EXECUTE: **(a)** For the magnitude of the induced electric field, Faraday's law gives

$$E 2\pi r = d(B\pi r^2)/dt = \pi r^2\, dB/dt$$

$$E = \frac{r}{2}\frac{dB}{dt} = \frac{0.0225\text{ m}}{2}(0.250\text{ T/s}) = 2.81 \times 10^{-3}\text{ V/m}$$

(b) The field points toward the south pole of the magnet and is decreasing, so the induced current is counterclockwise.

EVALUATE: This is a very small electric field compared to most others found in laboratory equipment.

29.33. IDENTIFY: Apply Faraday's law in the form $|\mathcal{E}_{av}| = N\left|\frac{\Delta\Phi_B}{\Delta t}\right|$.

SET UP: The magnetic field of a large straight solenoid is $B = \mu_0 nI$ inside the solenoid and zero outside. $\Phi_B = BA$, where A is 8.00 cm^2, the cross-sectional area of the long straight solenoid.

EXECUTE: $|\mathcal{E}_{av}| = N\left|\frac{\Delta\Phi_B}{\Delta t}\right| = \left|\frac{NA(B_f - B_i)}{\Delta t}\right| = \frac{NA\mu_o nI}{\Delta t}.$

$$\mathcal{E}_{av} = \frac{\mu_0(12)(8.00\times10^{-4}\,\text{m}^2)(9000\ \text{m}^{-1})(0.350\ \text{A})}{0.0400\ \text{s}} = 9.50\times10^{-4}\ \text{V}.$$

EVALUATE: An emf is induced in the second winding even though the magnetic field of the solenoid is zero at the location of the second winding. The changing magnetic field induces an electric field outside the solenoid and that induced electric field produces the emf.

29.34. **IDENTIFY:** Apply Eq.(29.14).

SET UP: $\epsilon = 3.5\times10^{-11}$ F/m

EXECUTE: $i_D = \epsilon\frac{d\Phi_E}{dt} = (3.5\times10^{-11}\ \text{F/m})(24.0\times10^3\ \text{V}\cdot\text{m/s}^3)t^2$. $i_D = 21\times10^{-6}$ A gives $t = 5.0$ s.

EVALUATE: i_D depends on the rate at which Φ_E is changing.

29.35. **IDENTIFY:** Apply Eq.(29.14), where $\epsilon = K\epsilon_0$.

SET UP: $d\Phi_E/dt = 4(8.76\times10^3\ \text{V}\cdot\text{m/s}^4)t^3$. $\epsilon_0 = 8.854\times10^{-12}$ F/m.

EXECUTE: $\epsilon = \frac{i_D}{(d\Phi_E/dt)} = \frac{12.9\times10^{-12}\ \text{A}}{4(8.76\times10^3\ \text{V}\cdot\text{m/s}^4)(26.1\times10^{-3}\ \text{s})^3} = 2.07\times10^{-11}$ F/m. The dielectric constant is

$K = \frac{\epsilon}{\epsilon_0} = 2.34.$

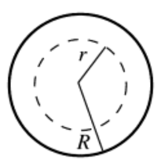
EVALUATE: The larger the dielectric constant, the larger is the displacement current for a given $d\Phi_E/dt$.

29.36. **IDENTIFY** and **SET UP:** Eqs.(29.13) and (29.14) show that $i_C = i_D$ and also relate i_D to the rate of change of the electric field flux between the plates. Use this to calculate dE/dt and apply the generalized form of Ampere's law (Eq.29.15) to calculate B.

(a) EXECUTE: $i_C = i_D$, so $j_D = \frac{i_D}{A} = \frac{i_C}{A} = \frac{0.280\ \text{A}}{\pi r^2} = \frac{0.280\ \text{A}}{\pi(0.0400\ \text{m})^2} = 55.7\ \text{A/m}^2$

(b) $j_D = \epsilon_0\frac{dE}{dt}$ so $\frac{dE}{dt} = \frac{j_D}{\epsilon_0} = \frac{55.7\ \text{A/m}^2}{8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 6.29\times10^{12}\ \text{V/m}\cdot\text{s}$

(c) SET UP: Apply Ampere's law $\oint\vec{B}\cdot d\vec{l} = \mu_0(i_C + i_D)_{encl}$ (Eq.(28.20)) to a circular path with radius $r = 0.0200$ m. An end view of the solenoid is given in Figure 29.36.



By symmetry the magnetic field is tangent to the path and constant around it.

Figure 29.36

EXECUTE: Thus $\oint\vec{B}\cdot d\vec{l} = \oint B\,dl = B\int dl = B(2\pi r)$.

$i_C = 0$ (no conduction current flows through the air space between the plates)

The displacement current enclosed by the path is $j_D\pi r^2$.

Thus $B(2\pi r) = \mu_0(j_D\pi r^2)$ and $B = \frac{1}{2}\mu_0 j_D r = \frac{1}{2}(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(55.7\ \text{A/m}^2)(0.0200\ \text{m}) = 7.00\times10^{-7}$ T

(d) $B = \frac{1}{2}\mu_0 j_D r$. Now r is $\frac{1}{2}$ the value in (c), so B is $\frac{1}{2}$ also: $B = \frac{1}{2}(7.00\times10^{-7}\ \text{T}) = 3.50\times10^{-7}$ T

EVALUATE: The definition of displacement current allows the current to be continuous at the capacitor. The magnetic field between the plates is zero on the axis ($r = 0$) and increases as r increases.

29.37. **IDENTIFY:** $q = CV$. For a parallel-plate capacitor, $C = \frac{\epsilon A}{d}$, where $\epsilon = K\epsilon_0$. $i_C = dq/dt$. $j_D = \epsilon\frac{E}{dt}$.

SET UP: $E = q/\epsilon A$ so $dE/dt = i_C/\epsilon A$.

EXECUTE: **(a)** $q = CV = \left(\frac{\epsilon A}{d}\right)V = \frac{(4.70)\epsilon_0(3.00\times10^{-4}\ \text{m}^2)(120\ \text{V})}{2.50\times10^{-3}\ \text{m}} = 5.99\times10^{-10}$ C.

(b) $\dfrac{dq}{dt} = i_C = 6.00\times10^{-3}$ A.

(c) $j_D = \epsilon \dfrac{dE}{dt} = K\epsilon_0 \dfrac{i_C}{K\epsilon_0 A} = \dfrac{i_C}{A} = j_C$, so $i_D = i_C = 6.00\times10^{-3}$ A.

EVALUATE: $i_D = i_C$, so Kirchhoff's junction rule is satisfied where the wire connects to each capacitor plate.

29.38. **IDENTIFY** and **SET UP:** Use $i_C = q/t$ to calculate the charge q that the current has carried to the plates in time t. The two equations preceeding Eq.(24.2) relate q to the electric field E and the potential difference between the plates. The displacement current density is defined by Eq.(29.16).

EXECUTE: **(a)** $i_C = 1.80\times10^{-3}$ A

$q = 0$ at $t = 0$

The amount of charge brought to the plates by the charging current in time t is

$q = i_C t = (1.80\times10^{-3}\ \text{A})(0.500\times10^{-6}\ \text{s}) = 9.00\times10^{-10}$ C

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{\epsilon_0 A} = \frac{9.00\times10^{-10}\ \text{C}}{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.00\times10^{-4}\ \text{m}^2)} = 2.03\times10^5\ \text{V/m}$$

$V = Ed = (2.03\times10^5\ \text{V/m})(2.00\times10^{-3}\ \text{m}) = 406$ V

(b) $E = q/\epsilon_0 A$

$$\frac{dE}{dt} = \frac{dq/dt}{\epsilon_0 A} = \frac{i_C}{\epsilon_0 A} = \frac{1.80\times10^{-3}\ \text{A}}{(8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.00\times10^{-4}\ \text{m}^2)} = 4.07\times10^{11}\ \text{V/m}\cdot\text{s}$$

Since i_C is constant dE/dt does not vary in time.

(c) $j_D = \epsilon_0 \dfrac{dE}{dt}$ (Eq.(29.16)), with ϵ replaced by ϵ_0 since there is vacuum between the plates.)

$j_D = (8.854\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(4.07\times10^{11}\ \text{V/m}\cdot\text{s}) = 3.60\ \text{A/m}^2$

$i_D = j_D A = (3.60\ \text{A/m}^2)(5.00\times10^{-4}\ \text{m}^2) = 1.80\times10^{-3}\ \text{A};\ i_D = i_C$

EVALUATE: $i_C = i_D$. The constant conduction current means the charge q on the plates and the electric field between them both increase linearly with time and i_D is constant.

29.39. **IDENTIFY:** Ohm's law relates the current in the wire to the electric field in the wire. $j_D = \epsilon \dfrac{dE}{dt}$. Use Eq.(29.15) to calculate the magnetic fields.

SET UP: Ohm's law says $E = \rho J$. Apply Ohm's law to a circular path of radius r.

EXECUTE: **(a)** $E = \rho J = \dfrac{\rho I}{A} = \dfrac{(2.0\times10^{-8}\ \Omega\cdot\text{m})(16\ \text{A})}{2.1\times10^{-6}\ \text{m}^2} = 0.15$ V/m.

(b) $\dfrac{dE}{dt} = \dfrac{d}{dt}\left(\dfrac{\rho I}{A}\right) = \dfrac{\rho}{A}\dfrac{dI}{dt} = \dfrac{2.0\times10^{-8}\ \Omega\cdot\text{m}}{2.1\times10^{-6}\ \text{m}^2}(4000\ \text{A/s}) = 38\ \text{V/m}\cdot\text{s}.$

(c) $j_D = \epsilon_0 \dfrac{dE}{dt} = \epsilon_0(38\ \text{V/m}\cdot\text{s}) = 3.4\times10^{-10}\ \text{A/m}^2.$

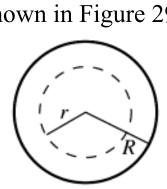
(d) $i_D = j_D A = (3.4\times10^{-10}\ \text{A/m}^2)(2.1\times10^{-6}\ \text{m}^2) = 7.14\times10^{-16}$ A. Eq.(29.15) applied to a circular path of radius r gives $B_D = \dfrac{\mu_0 I_D}{2\pi r} = \dfrac{\mu_0(7.14\times10^{-16}\ \text{A})}{2\pi(0.060\ \text{m})} = 2.38\times10^{-21}$ T, and this is a negligible contribution.

$B_C = \dfrac{\mu_0 I_C}{2\pi r} = \dfrac{\mu_0}{2\pi}\dfrac{(16\ \text{A})}{(0.060\ \text{m})} = 5.33\times10^{-5}$ T.

EVALUATE: In this situation the displacement current is much less than the conduction current.

29.40. **IDENTIFY:** Apply Ampere's law to a circular path of radius $r < R$, where R is the radius of the wire.

SET UP: The path is shown in Figure 29.40.



$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(I_C + \epsilon_0 \frac{d\Phi_E}{dt}\right)$$

Figure 29.40

EXECUTE: There is no displacement current, so $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_C$

The magnetic field inside the superconducting material is zero, so $\oint \vec{B} \cdot d\vec{l} = 0.$ But then Ampere's law says that $I_C = 0;$ there can be no conduction current through the path. This same argument applies to any circular path with $r < R$, so all the current must be at the surface of the wire.

EVALUATE: If the current were uniformly spread over the wire's cross section, the magnetic field would be like that calculated in Example 28.9.

29.41. IDENTIFY: A superconducting region has zero resistance.

SET UP: If the superconducting and normal regions each lie along the length of the cylinder, they provide parallel conducting paths.

EXECUTE: Unless some of the regions with resistance completely fill a cross-sectional area of a long type-II superconducting wire, there will still be no total resistance. The regions of no resistance provide the path for the current.

EVALUATE: The situation here is like two resistors in parallel, where one has zero resistance and the other is non-zero. The equivalent resistance is zero.

29.42. IDENTIFY: Apply Eq.(28.29): $\vec{B} = \vec{B}_0 + \mu_0 \vec{M}$.

SET UP: For magnetic fields less than the critical field, there is no internal magnetic field. For fields greater than the critical field, $\vec{B}$ is very nearly equal to $\vec{B}_0$.

EXECUTE: **(a)** The external field is less than the critical field, so inside the superconductor $\vec{B} = 0$ and $\vec{M} = -\dfrac{\vec{B}_0}{\mu_0} = -\dfrac{(0.130\ T)\hat{i}}{\mu_0} = -(1.03 \times 10^5\ \text{A/m})\hat{i}$. Outside the superconductor, $\vec{B} = \vec{B}_0 = (0.130\ T)\hat{i}$ and $\vec{M} = 0$.

(b) The field is greater than the critical field and $\vec{B} = \vec{B}_0 = (0.260\ \text{T})\hat{i}$, both inside and outside the superconductor.

EVALUATE: Below the critical field the external field is expelled from the superconducting material.

29.43. IDENTIFY: Apply $\vec{B} = \vec{B}_0 + \mu_0 \vec{M}$.

SET UP: When the magnetic flux is expelled from the material the magnetic field $\vec{B}$ in the material is zero. When the material is completely normal, the magnetization is close to zero.

EXECUTE: **(a)** When $\vec{B}_0$ is just under $\vec{B}_{c1}$ (threshold of superconducting phase), the magnetic field in the material must be zero, and $\vec{M} = -\dfrac{\vec{B}_{c1}}{\mu_0} = -\dfrac{(55 \times 10^{-3}\ \text{T})\hat{i}}{\mu_0} = -(4.38 \times 10^4\ \text{A/m})\hat{i}$.

(b) When $\vec{B}_0$ is just over $\vec{B}_{c2}$ (threshold of normal phase), there is zero magnetization, and $\vec{B} = \vec{B}_{c2} = (15.0\ \text{T})\hat{i}$.

EVALUATE: Between B_{c1} and B_{c2} there are filaments of normal phase material and there is magnetic field along these filaments.

29.44. IDENTIFY and **SET UP:** Use Faraday's law to calculate the magnitude of the induced emf and Lenz's law to determine its direction. Apply Ohm's law to calculate I. Use Eq.(25.10) to calculate the resistance of the coil.

(a) EXECUTE: The angle ϕ between the normal to the coil and the direction of $\vec{B}$ is $30.0°$.

$|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = (N\pi r^2)(dB/dt)$ and $I = |\mathcal{E}|/R$.

For $t < 0$ and $t > 1.00$ s, $dB/dt = 0\,|\mathcal{E}| = 0$ and $I = 0$.

For $0 \le t \le 1.00$ s, $dB/dt = (0.120\ \text{T})\,\pi \sin \pi t$

$|\mathcal{E}| = (N\pi r^2)\pi(0.120\ \text{T})\sin \pi t = (0.9475\ \text{V})\sin \pi t$

R for wire: $R_w = \dfrac{\rho L}{A} = \dfrac{\rho L}{\pi r^2}$; $\rho = 1.72 \times 10^{-8}\ \Omega \cdot \text{m}$, $r = 0.0150 \times 10^{-3}$ m

$L = Nc = N2\pi r = (500)(2\pi)(0.0400\ \text{m}) = 125.7$ m

$R_w = 3058\ \Omega$ and the total resistance of the circuit is $R = 3058\ \Omega + 600\ \Omega = 3658\ \Omega$

$I = |\mathcal{E}|/R = (0.259\ \text{mA})\sin \pi t$. The graph of I versus t is sketched in Figure 29.44a.

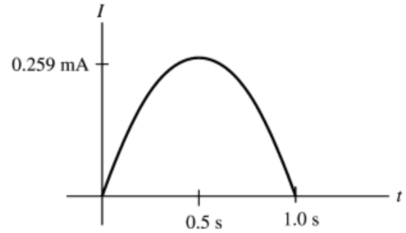
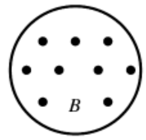


Figure 29.44a

(b) The coil and the magnetic field are shown in Figure 29.44b.



B increasing so Φ_B is $\odot$
and increasing. Φ_B is $\otimes$
so I is clockwise

Figure 29.44b

EVALUATE: The long length of small diameter wire used to make the coil has a rather large resistance, larger than the resistance of the 600-Ω resistor connected to it in the circuit. The flux has a cosine time dependence so the rate of change of flux and the current have a sine time dependence. There is no induced current for $t < 0$ or $t > 1.00$ s.

29.45. IDENTIFY: Apply Faraday's law and Lenz's law.

SET UP: For a discharging RC circuit, $i(t) = \frac{V_0}{R}e^{-t/RC}$, where V_0 is the initial voltage across the capacitor. The resistance of the small loop is $(25)(0.600\text{ m})(1.0\ \Omega/\text{m}) = 15.0\ \Omega$.

EXECUTE: **(a)** The large circuit is an RC circuit with a time constant of $\tau = RC = (10\ \Omega)(20\times10^{-6}\text{ F}) = 200\ \mu\text{s}$. Thus, the current as a function of time is $i = \left((100\text{ V})/(10\ \Omega)\right)e^{-t/200\ \mu\text{s}}$. At $t = 200\ \mu\text{s}$, we obtain $i = (10\text{ A})(e^{-1}) = 3.7\text{ A}$.

(b) Assuming that only the long wire nearest the small loop produces an appreciable magnetic flux through the small loop and referring to the solution of Exercise 29.7 we obtain $\Phi_B = \int_c^{c+a}\frac{\mu_0 ib}{2\pi r}dr = \frac{\mu_0 ib}{2\pi}\ln\left(1+\frac{a}{c}\right)$. Therefore, the emf induced in the small loop at $t = 200\mu\text{s}$ is $\mathcal{E} = -\frac{d\Phi}{dt} = -\frac{\mu_0 b}{2\pi}\ln\left(1+\frac{a}{c}\right)\frac{di}{dt}$.

$\mathcal{E} = -\frac{(4\pi\times10^{-7}\text{ Wb/A}\cdot\text{m}^2)(0.200\text{ m})}{2\pi}\ln(3.0)\left(-\frac{3.7\text{ A}}{200\times10^{-6}\text{s}}\right) = +0.81\text{ mV}$. Thus, the induced current in the small loop is $i' = \frac{\mathcal{E}}{R} = \frac{0.81\text{ mV}}{15.0\ \Omega} = 54\mu\text{A}$.

(c) The magnetic field from the large loop is directed out of the page within the small loop.The induced current will act to oppose the decrease in flux from the large loop. Thus, the induced current flows counterclockwise.

EVALUATE: **(d)** Three of the wires in the large loop are too far away to make a significant contribution to the flux in the small loop–as can be seen by comparing the distance c to the dimensions of the large loop.

29.46. IDENTIFY: A changing magnetic field causes a changing flux through a coil and therefore induces an emf in the coil.

SET UP: Faraday's law says that the induced emf is $\mathcal{E} = -\frac{d\Phi_B}{dt}$ and the magnetic flux through a coil is defined as $\Phi_B = BA\cos\phi$.

EXECUTE: In this case, $\Phi_B = BA$, where A is constant. So the emf is proportional to the *negative* slope of the magnetic field. The result is shown in Figure 29.46.

EVALUATE: It is the rate at which the magnetic field is *changing*, not the field's magnitude, that determines the induced emf. When the field is constant, even though it may have a large value, the induced emf is zero.

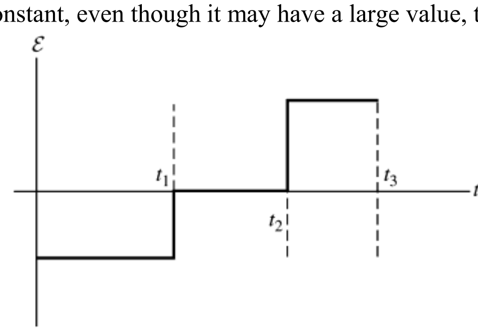


Figure 29.46

29.47. IDENTIFY: Follow the steps specified in the problem.

SET UP: Let the flux through the loop due to the current be positive.

EXECUTE: **(a)** $\Phi_B = BA = \frac{\mu_0 i}{2a}\pi a^2 = \frac{\mu_0 i\pi a}{2}$.

(b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = iR \Rightarrow -\dfrac{d}{dt}\left(\dfrac{\mu_0 i\pi a}{2}\right) = -\dfrac{\mu_0 \pi a}{2}\dfrac{di}{dt} = iR \Rightarrow \dfrac{di}{dt} = -i\dfrac{2R}{\mu_0 \pi a}$

(c) Solving $\dfrac{di}{i} = -dt\dfrac{2R}{\mu_0 \pi a}$ for $i(t)$ yields $i(t) = i_0 e^{-t(2R/\mu_0 \pi a)}$.

(d) We want $i(t) = i_0(0.010) = i_0 e^{-t(2R/\mu_0 \pi a)}$, so $\ln(0.010) = -t(2R/\mu_0 \pi a)$ and

$t = -\dfrac{\mu_0 \pi a}{2R}\ln(0.010) = -\dfrac{\mu_0 \pi (0.50 \text{ m})}{2(0.10\ \Omega)}\ln(0.010) = 4.55 \times 10^{-5}$ s.

EVALUATE: (e) We can ignore the self-induced currents because it takes only a very short time for them to die out.

29.48. IDENTIFY: A changing magnetic field causes a changing flux through a coil and therefore induces an emf in the coil.

SET UP: Faraday's law says that the induced emf is $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$ and the magnetic flux through a coil is defined as $\Phi_B = BA\cos\phi$.

EXECUTE: In this case, $\Phi_B = BA$, where A is constant. So the emf is proportional to the *negative* slope of the magnetic field. The result is shown in Figure 29.48.

EVALUATE: It is the rate at which the magnetic field is *changing*, not the field's magnitude, that determines the induced emf. When the field is constant, even though it may have a large value, the induced emf is zero.

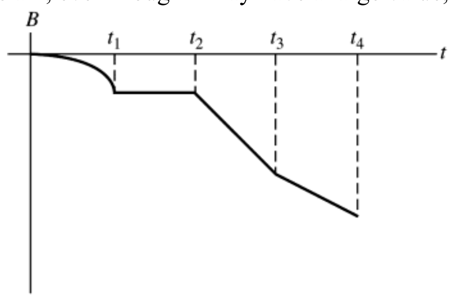


Figure 29.48

29.49. (a) IDENTIFY: (i) $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right|$. The flux is changing because the magnitude of the magnetic field of the wire decreases with distance from the wire. Find the flux through a narrow strip of area and integrate over the loop to find the total flux.

SET UP:

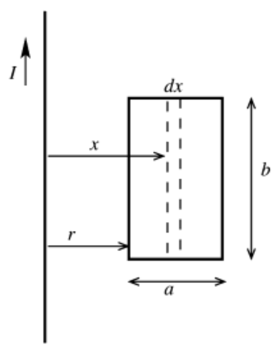


Figure 29.49a

Consider a narrow strip of width dx and a distance x from the long wire, as shown in Figure 29.49a. The magnetic field of the wire at the strip is $B = \mu_0 I/2\pi x$. The flux through the strip is $d\Phi_B = Bb\ dx = (\mu_0 Ib/2\pi)(dx/x)$

EXECUTE: The total flux through the loop is $\Phi_B = \int d\Phi_B = \left(\dfrac{\mu_0 Ib}{2\pi}\right)\int_r^{r+a}\dfrac{dx}{x}$

$$\Phi_B = \left(\frac{\mu_0 Ib}{2\pi}\right)\ln\left(\frac{r+a}{r}\right)$$

$$\frac{d\Phi_B}{dt} = \frac{d\Phi_B}{dt}\frac{dr}{dt} = \frac{\mu_0 Ib}{2\pi}\left(-\frac{a}{r(r+a)}\right)v$$

$$|\mathcal{E}| = \frac{\mu_0 Iabv}{2\pi r(r+a)}$$

(ii) **IDENTIFY:** $\mathcal{E} = Bvl$ for a bar of length l moving at speed v perpendicular to a magnetic field B. Calculate the induced emf in each side of the loop, and combine the emfs according to their polarity.

SET UP: The four segments of the loop are shown in Figure 29.49b.

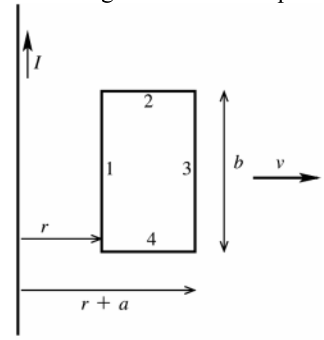


Figure 29.49b

EXECUTE: The emf in each side of the loop is $\mathcal{E}_1 = \left(\frac{\mu_0 I}{2\pi r}\right) vb$,

$\mathcal{E}_2 = \left(\frac{\mu_0 I}{2\pi r(r+a)}\right) vb, \quad \mathcal{E}_2 = \mathcal{E}_4 = 0$

Both emfs $\mathcal{E}_1$ and $\mathcal{E}_2$ are directed toward the top of the loop so oppose each other. The net emf is

$$\mathcal{E} = \mathcal{E}_1 - \mathcal{E}_2 = \frac{\mu_0 I v b}{2\pi}\left(\frac{1}{r} - \frac{1}{r+a}\right) = \frac{\mu_0 I a b v}{2\pi r(r+a)}$$

This expression agrees with what was obtained in (i) using Faraday's law.

(b) (i) **IDENTIFY** and **SET UP:** The flux of the induced current opposes the change in flux.

EXECUTE: $\vec{B}$ is $\otimes$. Φ_B is $\otimes$ and decreasing, so the flux Φ_{ind} of the induced current is $\otimes$ and the current is clockwise.

(ii) **IDENTIFY** and **SET UP:** Use the right-hand rule to find the force on the positive charges in each side of the loop. The forces on positive charges in segments 1 and 2 of the loop are shown in Figure 29.49c.

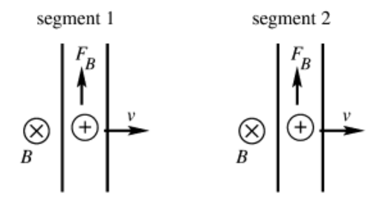


Figure 29.49c

EXECUTE: B is larger at segment 1 since it is closer to the long wire, so F_B is larger in segment 1 and the induced current in the loop is clockwise. This agrees with the direction deduced in (i) using Lenz's law.

(c) EVALUATE: When $v = 0$ the induced emf should be zero; the expression in part (a) gives this. When $a \to 0$ the flux goes to zero and the emf should approach zero; the expression in part (a) gives this. When $r \to \infty$ the magnetic field through the loop goes to zero and the emf should go to zero; the expression in part (a) gives this.

29.50. IDENTIFY: Apply Faraday's law.

SET UP: For rotation about the y-axis the situation is the same as in Examples 29.4 and 29.5 and we can apply the results from those examples.

EXECUTE: **(a)** Rotating about the y-axis: the flux is given by $\Phi_B = BA\cos\phi$ and

$$\mathcal{E}_{\max} = \left|\frac{d\Phi_B}{dt}\right| = \omega BA = (35.0 \text{ rad/s})(0.450 \text{ T})(6.00 \times 10^{-2} \text{ m}) = 0.945 \text{ V}.$$

(b) Rotating about the x-axis: $\frac{d\Phi_\text{B}}{dt} = 0$ and $\mathcal{E} = 0$.

(c) Rotating about the z-axis: the flux is given by $\Phi_B = BA\cos\phi$ and

$$\mathcal{E}_{\max} = \left|\frac{d\Phi_B}{dt}\right| = \omega BA = (35.0 \text{ rad/s})(0.450 \text{ T})(6.00 \times 10^{-2} \text{ m}) = 0.945 \text{ V}.$$

EVALUATE: The maximum emf is the same if the loop is rotated about an edge parallel to the z-axis as it is when it is rotated about the z-axis.

29.51. IDENTIFY: Apply the results of Example 29.4, so $\mathcal{E}_{\max} = N\omega BA$ for N loops.

SET UP: For the minimum ω, let the rotating loop have an area equal to the area of the uniform magnetic field, so $A = (0.100 \text{ m})^2$.

EXECUTE: $N = 400$, $B = 1.5 \text{ T}$, $A = (0.100 \text{ m})^2$ and $\mathcal{E}_{\max} = 120 \text{ V}$ gives

$\omega = \mathcal{E}_{\max}/NBA = (20 \text{ rad/s})(1 \text{ rev}/2\pi \text{ rad})(60 \text{ s}/1 \text{ min}) = 190 \text{ rpm}.$

EVALUATE: In $\mathcal{E}_{\max} = \omega BA$, ω is in rad/s.

29.52. **IDENTIFY:** Apply the results of Example 29.4, generalized to N loops: $\mathcal{E}_{\max} = N\omega BA$. $v = r\omega$.

SET UP: In the expression for $\mathcal{E}_{\max}$, ω must be in rad/s. 30 rpm = 3.14 rad/s

EXECUTE: **(a)** Solving for A we obtain $A = \dfrac{\mathcal{E}_0}{\omega NB} = \dfrac{9.0\text{ V}}{(3.14\text{ rad/s})(2000\text{ turns})(8.0\times 10^{-5}\text{ T})} = 18\text{ m}^2.$

(b) Assuming a point on the coil at maximum distance from the axis of rotation we have

$v = r\omega = \sqrt{\dfrac{A}{\pi}}\omega = \sqrt{\dfrac{18\text{ m}^2}{\pi}}(3.14\text{ rad/s}) = 7.5\text{ m/s}.$

EVALUATE: The device is not very feasible. The coil would need a rigid frame and the effects of air resistance would be appreciable.

29.53. **IDENTIFY:** Apply Faraday's law in the form $\mathcal{E}_{\text{av}} = -N\dfrac{\Delta\Phi_B}{\Delta t}$ to calculate the average emf. Apply Lenz's law to calculate the direction of the induced current.

SET UP: $\Phi_B = BA$. The flux changes because the area of the loop changes.

EXECUTE: **(a)** $\mathcal{E}_{\text{av}} = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right| = B\left|\dfrac{\Delta A}{\Delta t}\right| = B\dfrac{\pi r^2}{\Delta t} = (0.950\text{ T})\dfrac{\pi(0.0650/2\text{ m})^2}{0.250\text{ s}} = 0.0126\text{ V}.$

(b) Since the magnetic field is directed into the page and the magnitude of the flux through the loop is decreasing, the induced current must produce a field that goes into the page. Therefore the current flows from point a through the resistor to point b.

EVALUATE: Faraday's law can be used to find the direction of the induced current. Let $\vec{A}$ be into the page. Then Φ_B is positive and decreasing in magnitude, so $d\Phi_B/dt < 0$. Therefore $\mathcal{E} > 0$ and the induced current is clockwise around the loop.

29.54. **IDENTIFY:** By Lenz's law, the induced current flows to oppose the flux change that caused it.

SET UP: When the switch is suddenly closed with an uncharged capacitor, the current in the outer circuit immediately increases from zero to its maximum value. As the capacitor gets charged, the current in the outer circuit gradually decreases to zero.

EXECUTE: **(a)** (i) The current in the outer circuit is suddenly increasing and is in a counterclockwise direction. The magnetic field through the inner circuit is out of the paper and increasing. The magnetic flux through the inner circuit is increasing, so the induced current in the inner circuit is clockwise (*a* to *b*) to oppose the flux increase. (ii) The current in the outer circuit is still counterclockwise but is now decreasing, so the magnetic field through the inner circuit is out of the page but decreasing. The flux through the inner circuit is now decreasing, so the induced current is counterclockwise (*b* to *a*) to oppose the flux decrease.

(b) The graph is sketched in Figure 29.54.

EVALUATE: Even though the current in the outer circuit does not change direction, the current in the inner circuit does as the flux through it changes from increasing to decreasing.

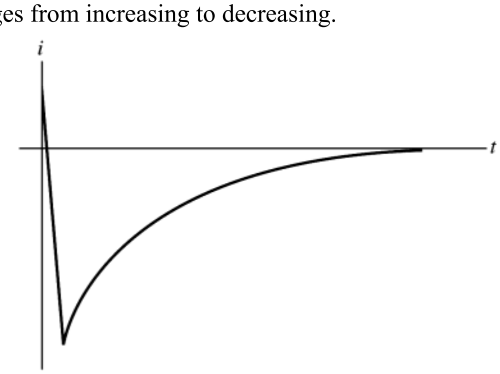


Figure 29.54

29.55. **IDENTIFY:** Use Faraday's law to calculate the induced emf and Ohm's law to find the induced current. Use Eq.(27.19) to calculate the magnetic force F_I on the induced current. Use the net force $F - F_I$ in Newton's 2nd law to calculate the acceleration of the rod and use that to describe its motion.

(a) SET UP: The forces in the rod are shown in Figure 29.55a.

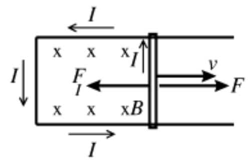


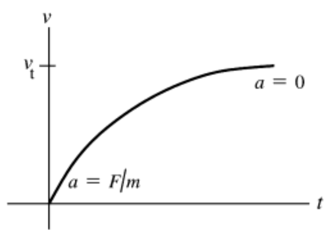
Figure 29.55a

EXECUTE: $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right| = BLv$

$I = \dfrac{BLv}{R}$

Use $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$ to find the direction of I: Let $\vec{A}$ be into the page. Then $\Phi_B > 0$. The area of the circuit is increasing, so $\dfrac{d\Phi_B}{dt} > 0$. Then $\mathcal{E} < 0$ and with our direction for $\vec{A}$ this means that $\mathcal{E}$ and I are counterclockwise, as shown in the sketch. The force F_I on the rod due to the induced current is given by $\vec{F}_I = I\vec{l} \times \vec{B}$. This gives $\vec{F}_I$ to the left with magnitude $F_I = ILB = (BLv/R)LB = B^2L^2v/R$. Note that $\vec{F}_I$ is directed to oppose the motion of the rod, as required by Lenz's law.

EVALUATE: The net force on the rod is $F - F_I$, so its acceleration is $a = (F - F_I)/m = (F - B^2L^2v/R)/m$. The rod starts with $v = 0$ and $a = F/m$. As the speed v increases the acceleration a decreases. When $a = 0$ the rod has reached its terminal speed v_t. The graph of v versus t is sketched in Figure 29.55b.



(Recall that a is the slope of the tangent to the v versus t curve.)

Figure 29.55b

(b) EXECUTE: $v = v_t$ when $a = 0$ so $\dfrac{F - B^2L^2v_t/R}{m} = 0$ and $v_t = \dfrac{RF}{B^2L^2}$.

EVALUATE: A large F produces a large v_t. If B is larger, or R is smaller, the induced current is larger at a given v so F_I is larger and the terminal speed is less.

29.56. IDENTIFY: Apply Newton's 2nd law to the bar. The bar will experience a magnetic force due to the induced current in the loop. Use $a = dv/dt$ to solve for v. At the terminal speed, $a = 0$.

SET UP: The induced emf in the loop has a magnitude BLv. The induced emf is counterclockwise, so it opposes the voltage of the battery, $\mathcal{E}$.

EXECUTE: **(a)** The net current in the loop is $I = \dfrac{\mathcal{E} - BLv}{R}$. The acceleration of the bar is $a = \dfrac{F}{m} = \dfrac{ILB\sin(90°)}{m} = \dfrac{(\mathcal{E} - BLv)LB}{mR}$. To find $v(t)$, set $\dfrac{dv}{dt} = a = \dfrac{(\mathcal{E} - BLv)LB}{mR}$ and solve for v using the method of separation of variables:

$$\int_0^v \frac{dv}{(\mathcal{E} - BLv)} = \int_0^t \frac{LB}{mR}dt \rightarrow v = \frac{\mathcal{E}}{BL}(1 - e^{-B^2L^2t/mR}) = (10\text{ m/s})(1 - e^{-t/3.1\text{ s}})$$

The graph of v versus t is sketched in Figure 29.56. Note that the graph of this function is similar in appearance to that of a charging capacitor.

(b) Just after the switch is closed, $v = 0$ and $I = \mathcal{E}/R = 2.4$ A, $F = ILB = 2.88$ N and $a = F/m = 3.2\text{ m/s}^2$.

(c) When $v = 2.0$ m/s, $a = \dfrac{[12\text{ V} - (1.5\text{ T})(0.8\text{ m})(2.0\text{ m/s})](0.8\text{ m})(1.5\text{ T})}{(0.90\text{ kg})(5.0\ \Omega)} = 2.6\text{ m/s}^2$.

(d) Note that as the speed increases, the acceleration decreases. The speed will asymptotically approach the terminal speed $\dfrac{\mathcal{E}}{BL} = \dfrac{12\text{ V}}{(1.5\text{ T})(0.8\text{ m})} = 10$ m/s, which makes the acceleration zero.

EVALUATE: The current in the circuit is counterclockwise and the magnetic force on the bar is to the right. The energy that appears as kinetic energy of the moving bar is supplied by the battery.

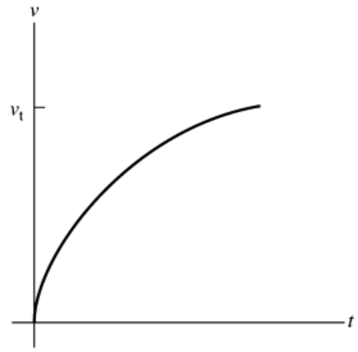


Figure 29.56

29.57. **IDENTIFY:** Apply $\mathcal{E} = BvL$. Use $\sum \vec{F} = m\vec{a}$ applied to the satellite motion to find the speed v of the satellite.

SET UP: The gravitational force on the satellite is $F_g = G\dfrac{mm_E}{r^2}$, where m is the mass of the satellite and r is the radius of its orbit.

EXECUTE: $B = 8.0\times10^{-5}$ T, $L = 2.0$ m. $G\dfrac{mm_E}{r^2} = m\dfrac{v^2}{r}$ and $r = 400\times10^3\ \text{m} + R_E$ gives $v = \sqrt{\dfrac{Gm_E}{r}} = 7.665\times10^3$ m/s.

Using this v in $\mathcal{E} = vBL$ gives $\mathcal{E} = (8.0\times10^{-5}\ \text{T})(7.665\times10^3\ \text{m/s})(2.0\ \text{m}) = 1.2$ V.

EVALUATE: The induced emf is large enough to be measured easily.

29.58. **IDENTIFY:** The induced emf is $\mathcal{E} = BvL$, where L is measured in a direction that is perpendicular to both the magnetic field and the velocity of the bar.

SET UP: The magnetic force pushed positive charge toward the high potential end of the bullet.

EXECUTE: **(a)** $\mathcal{E} = BLv = (8\times10^{-5}\ \text{T})(0.004\ \text{m})(300\ \text{m/s}) = 96\ \mu\text{V}$. Since a positive charge moving to the east would be deflected upward, the top of the bullet will be at a higher potential.

(b) For a bullet that travels south, $\vec{v}$ and $\vec{B}$ are along the same line, there is no magnetic force and the induced emf is zero.

(c) If $\vec{v}$ is horizontal, the magnetic force on positive charges in the bullet is either upward or downward, perpendicular to the line between the front and back of the bullet. There is no emf induced between the front and back of the bullet.

EVALUATE: Since the velocity of a bullet is always in the direction from the back to the front of the bullet, and since the magnetic force is perpendicular to the velocity, there is never an induced emf between the front and back of the bullet, no matter what the direction of the magnetic field is.

29.59. **IDENTIFY:** Find the magnetic field at a distance r from the center of the wire. Divide the rectangle into narrow strips of width dr, find the flux through each strip and integrate to find the total flux.

SET UP: Example 28.8 uses Ampere's law to show that the magnetic field inside the wire, a distance r from the axis, is $B(r) = \mu_0 Ir/2\pi R^2$.

EXECUTE: Consider a small strip of length W and width dr that is a distance r from the axis of the wire, as shown in Figure 29.59. The flux through the strip is $d\Phi_B = B(r)W\,dr = \dfrac{\mu_0 IW}{2\pi R^2} r\,dr$. The total flux through the rectangle is

$$\Phi_B = \int d\Phi_B = \left(\frac{\mu_0 IW}{2\pi R^2}\right)\int_0^R r\,dr = \frac{\mu_0 IW}{4\pi}.$$

EVALUATE: Note that the result is independent of the radius R of the wire.

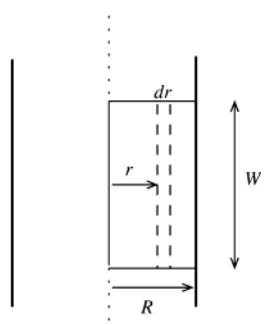


Figure 29.59

29.60. **IDENTIFY:** Apply Faraday's law to calculate the magnitude and direction of the induced emf.

SET UP: Let $\vec{A}$ be directed out of the page in Figure 29.50 in the textbook. This means that counterclockwise emf is positive.

EXECUTE: **(a)** $\Phi_B = BA = B_0\pi r_0^2(1-3(t/t_0)^2 + 2(t/t_0)^3)$.

(b) $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = -B_0\pi r_0^2\dfrac{d}{dt}(1-3(t/t_0)^2+2(t/t_0)^3) = -\dfrac{B_0\pi r_0^2}{t_0}(-6(t/t_0)+6(t/t_0)^2)$. $\mathcal{E} = -\dfrac{6\,B_0\pi r_0^2}{t_0} = \left(\left(\dfrac{t}{t_0}\right)^2 - \left(\dfrac{t}{t_0}\right)\right)$. At $t = 5.0\times10^{-3}\text{s}$, $\mathcal{E} = -\dfrac{6B_0\pi(0.0420\text{ m})^2}{0.010\text{ s}}\left(\left(\dfrac{5.0\times10^{-3}\text{ s}}{0.010\text{ s}}\right)^2 - \left(\dfrac{5.0\times10^{-3}\text{ s}}{0.010\text{ s}}\right)\right) = 0.0665\text{ V}$. $\mathcal{E}$ is positive so it is counterclockwise.

(c) $I = \dfrac{\mathcal{E}}{R_{\text{total}}} \Rightarrow R_{\text{total}} = r + R = \dfrac{\mathcal{E}}{I} \Rightarrow r = \dfrac{0.0665\text{ V}}{3.0\times10^{-3}\text{ A}} - 12\ \Omega = 10.2\ \Omega.$

(d) Evaluating the emf at $t = 1.21\times10^{-2}$ s and using the equations of part (b), $\mathcal{E} = -0.0676$ V, and the current flows clockwise, from b to a through the resistor.

(e) $\mathcal{E} = 0$ when $0 = \left(\left(\dfrac{t}{t_0}\right)^2 - \left(\dfrac{t}{t_0}\right)\right)$. $1 = \dfrac{t}{t_0}$ and $t = t_0 = 0.010$ s.

EVALUATE: At $t = t_0$, $B = 0$. At $t = 5.00\times10^{-3}$ s, $\vec{B}$ is in the $+\hat{k}$ direction and is decreasing in magnitude. Lenz's law therefore says $\mathcal{E}$ is counterclockwise. At $t = 0.0121$ s, $\vec{B}$ is in the $+\hat{k}$ direction and is increasing in magnitude. Lenz's law therefore says $\mathcal{E}$ is clockwise. These results for the direction of $\mathcal{E}$ agree with the results we obtained from Faraday's law.

29.61. **(a)** and **(b)** **IDENTIFY** and **Set Up:**

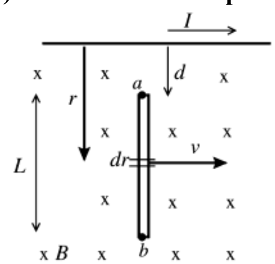


Figure 29.61a

The magnetic field of the wire is given by $B = \dfrac{\mu_0 I}{2\pi r}$ and varies along the length of the bar. At every point along the bar $\vec{B}$ has direction into the page. Divide the bar up into thin slices, as shown in Figure 29.61a.

EXECUTE: The emf $d\mathcal{E}$ induced in each slice is given by $d\mathcal{E} = \vec{v}\times\vec{B}\cdot d\vec{l}$. $\vec{v}\times\vec{B}$ is directed toward the wire, so $d\mathcal{E} = -vB\,dr = -v\left(\dfrac{\mu_0 I}{2\pi r}\right)dr$. The total emf induced in the bar is

$$V_{ba} = \int_a^b d\mathcal{E} = -\int_d^{d+L}\left(\frac{\mu_0 I v}{2\pi r}\right)dr = -\frac{\mu_0 I v}{2\pi}\int_d^{d+L}\frac{dr}{r} = -\frac{\mu_0 I v}{2\pi}\left[\ln(r)\right]_d^{d+L}$$

$$V_{ba} = -\frac{\mu_0 I v}{2\pi}(\ln(d+L) - \ln(d)) = -\frac{\mu_0 I v}{2\pi}\ln(1+L/d)$$

EVALUATE: The minus sign means that V_{ba} is negative, point a is at higher potential than point b. (The force $\vec{F} = q\vec{v}\times\vec{B}$ on positive charge carriers in the bar is towards a, so a is at higher potential.) The potential difference increases when I or v increase, or d decreases.

(c) **IDENTIFY:** Use Faraday's law to calculate the induced emf.

SET UP: The wire and loop are sketched in Figure 29.61b.

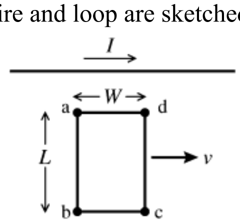


Figure 29.61b

EXECUTE: As the loop moves to the right the magnetic flux through it doesn't change. Thus $\mathcal{E} = -\dfrac{d\Phi_B}{dt} = 0$ and $I = 0$.

EVALUATE: This result can also be understood as follows. The induced emf in section ab puts point a at higher potential; the induced emf in section dc puts point d at higher potential. If you travel around the loop then these two induced emf's sum to zero. There is no emf in the loop and hence no current.

29.62. **IDENTIFY:** $\mathcal{E} = vBL$, where v is the component of velocity perpendicular to the field direction and perpendicular to the bar.

SET UP: Wires A and C have a length of 0.500 m and wire D has a length of $\sqrt{2(0.500\text{ m})^2} = 0.707$ m.

EXECUTE: Wire A: $\vec{v}$ is parallel to $\vec{B}$, so the induced emf is zero.

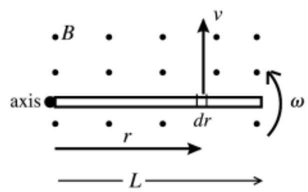
Wire C: $\vec{v}$ is perpendicular to $\vec{B}$. The component of $\vec{v}$ perpendicular to the bar is $v\cos 45°$.
$\mathcal{E} = (0.350\text{ m/s})(\cos 45°)(0.120\text{ T})(0.500\text{ m}) = 0.0148\text{ V}$.

Wire D: $\vec{v}$ is perpendicular to $\vec{B}$. The component of $\vec{v}$ perpendicular to the bar is $v\cos 45°$.
$\mathcal{E} = (0.350\text{ m/s})(\cos 45°)(0.120\text{ T})(0.707\text{ m}) = 0.0210\text{ V}$.

EVALUATE: The induced emf depends on the angle between $\vec{v}$ and $\vec{B}$ and also on the angle between $\vec{v}$ and the bar.

29.63. **(a) IDENTIFY:** Use the expression for motional emf to calculate the emf induced in the rod.

SET UP: The rotating rod is shown in Figure 29.63a.



The emf induced in a thin slice is $d\mathcal{E} = \vec{v} \times \vec{B} \cdot d\vec{l}$.

Figure 29.63a

EXECUTE: Assume that $\vec{B}$ is directed out of the page. Then $\vec{v} \times \vec{B}$ is directed radially outward and $dl = dr$, so $\vec{v} \times \vec{B} \cdot d\vec{l} = vB\,dr$

$v = r\omega$ so $d\mathcal{E} = \omega Br\,dr$.

The $d\mathcal{E}$ for all the thin slices that make up the rod are in series so they add:

$$\mathcal{E} = \int d\mathcal{E} = \int_0^L \omega Br\,dr = \tfrac{1}{2}\omega BL^2 = \tfrac{1}{2}(8.80\text{ rad/s})(0.650\text{ T})(0.240\text{ m})^2 = 0.165\text{ V}$$

EVALUATE: $\mathcal{E}$ increases with ω, B or L^2.

(b) No current flows so there is no IR drop in potential. Thus the potential difference between the ends equals the emf of 0.165 V calculated in part (a).

(c) SET UP: The rotating rod is shown in Figure 29.63b.

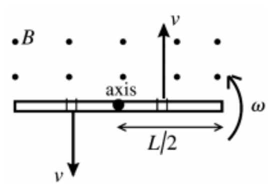


Figure 29.63b

EXECUTE: The emf between the center of the rod and each end is $\mathcal{E} = \frac{1}{2}\omega B(L/2)^2 = \frac{1}{4}(0.165\text{ V}) = 0.0412\text{ V}$, with the direction of the emf from the center of the rod toward each end. The emfs in each half of the rod thus oppose each other and there is no net emf between the ends of the rod.

EVALUATE: ω and B are the same as in part (a) but L of each half is $\frac{1}{2}L$ for the whole rod. $\mathcal{E}$ is proportional to L^2, so is smaller by a factor of $\frac{1}{4}$.

29.64. **IDENTIFY:** The power applied by the person in moving the bar equals the rate at which the electrical energy is dissipated in the resistance.

SET UP: From Example 29.7, the power required to keep the bar moving at a constant velocity is $P = \dfrac{(BLv)^2}{R}$.

EXECUTE: **(a)** $R = \dfrac{(BLv)^2}{P} = \dfrac{[(0.25\text{ T})(3.0\text{ m})(2.0\text{ m/s})]^2}{25\text{ W}} = 0.090\ \Omega$.

(b) For a 50 W power dissipation we would require that the resistance be decreased to half the previous value.

(c) Using the resistance from part (a) and a bar length of 0.20 m,

$$P = \frac{(BLv)^2}{R} = \frac{[(0.25\text{ T})(0.20\text{ m})(2.0\text{ m/s})]^2}{0.090\ \Omega} = 0.11\text{ W}.$$

EVALUATE: When the bar is moving to the right the magnetic force on the bar is to the left and an applied force directed to the right is required to maintain constant speed. When the bar is moving to the left the magnetic force on the bar is to the right and an applied force directed to the left is required to maintain constant speed.

29.65. **(a) IDENTIFY:** Use Faraday's law to calculate the induced emf, Ohm's law to calculate I, and Eq.(27.19) to calculate the force on the rod due to the induced current.

SET UP: The force on the wire is shown in Figure 29.65.

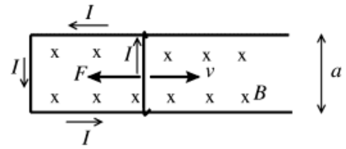


Figure 29.65

EXECUTE: When the wire has speed v the induced emf is $\mathcal{E} = Bva$ and the induced current is $I = \mathcal{E}/R = \dfrac{Bva}{R}$

The induced current flows upward in the wire as shown, so the force $\vec{F} = I\vec{l} \times \vec{B}$ exerted by the magnetic field on the induced current is to the left. $\vec{F}$ opposes the motion of the wire, as it must by Lenz's law. The magnitude of the force is $F = IaB = B^2a^2v/R.$

(b) Apply $\sum \vec{F} = m\vec{a}$ to the wire. Take $+x$ to be toward the right and let the origin be at the location of the wire at $t = 0$, so $x_0 = 0$.

$\sum F_x = ma_x$ says $-F = ma_x$

$$a_x = -\frac{F}{m} = -\frac{B^2a^2v}{mR}$$

Use this expression to solve for $v(t)$:

$$a_x = \frac{dv}{dt} = -\frac{B^2a^2v}{mR} \text{ and } \frac{dv}{v} = -\frac{B^2a^2}{mR}dt$$

$$\int_{v_0}^{v}\frac{dv'}{v'} = -\frac{B^2a^2}{mR}\int_0^t dt'$$

$$\ln(v) - \ln(v_0) = -\frac{B^2a^2t}{mR}$$

$$\ln\left(\frac{v}{v_0}\right) = -\frac{B^2a^2t}{mR} \text{ and } v = v_0e^{-B^2a^2t/mR}$$

Note: At $t = 0$, $v = v_0$ and $v \to 0$ when $t \to \infty$

Now solve for $x(t)$:

$$v = \frac{dx}{dt} = v_0e^{-B^2a^2t/mR} \text{ so } dx = v_0e^{-B^2a^2t/mR}dt$$

$$\int_0^x dx' = \int_0^t v_0e^{-B^2a^2t/mR}dt'$$

$$x = v_0\left(-\frac{mR}{B^2a^2}\right)\left[e^{-B^2a^2t'/mR}\right]_0^t = \frac{mRv_0}{B^2a^2}\left(1 - e^{-B^2a^2t/mR}\right)$$

Comes to rest implies $v = 0$. This happens when $t \to \infty$.

$t \to \infty$ gives $x = \dfrac{mRv_0}{B^2a^2}$. Thus this is the distance the wire travels before coming to rest.

EVALUATE: The motion of the slide wire causes an induced emf and current. The magnetic force on the induced current opposes the motion of the wire and eventually brings it to rest. The force and acceleration depend on v and are constant. If the acceleration were constant, not changing from its initial value of $a_x = -B^2a^2v_0/mR$, then the stopping distance would be $x = -v_0^2/2a_x = mRv_0/2B^2a^2$. The actual stopping distance is twice this.

29.66. **IDENTIFY:** Since the bar is straight and the magnetic field is uniform, integrating $d\mathcal{E} = \vec{v} \times \vec{B} \cdot d\vec{l}$ along the length of the bar gives $\mathcal{E} = (\vec{v} \times \vec{B}) \cdot \vec{L}$

SET UP: $\vec{v} = (4.20 \text{ m/s})\hat{i}$. $\vec{L} = (0.250 \text{ m})(\cos 36.9°\hat{i} + \sin 36.9°\hat{j})$.

EXECUTE: **(a)** $\mathcal{E} = (\vec{v} \times \vec{B}) \cdot \vec{L} = (4.20 \text{ m/s})\hat{i} \times ((0.120 \text{ T})\hat{i} - (0.220 \text{ T})\hat{j} - (0.0900 \text{ T})\hat{k}) \cdot \vec{L}$.

$$\mathcal{E} = \left((0.378 \text{ V/m})\hat{j} - (0.924 \text{ V/m})\hat{k}\right) \cdot \left((0.250 \text{ m})(\cos 36.9°\hat{i} + \sin 36.9°\hat{j})\right).$$

$$\mathcal{E} = (0.378 \text{ V/m})(0.250 \text{ m})\sin 36.9° = 0.0567 \text{ V}.$$

(b) The higher potential end is the end to which positive charges in the rod are pushed by the magnetic force. $\vec{v} \times \vec{B}$ has a positive y-component, so the end of the rod marked + in Figure 29.66 is at higher potential.

EVALUATE: Since $\vec{v}\times\vec{B}$ has nonzero $\hat{j}$ and $\hat{k}$ components, and $\vec{L}$ has nonzero $\hat{i}$ and $\hat{j}$ components, only the $\hat{k}$ component of $\vec{B}$ contributes to $\mathcal{E}$. In fact, $|\mathcal{E}| = |v_x B_z L_y| = (4.20\text{ m/s})(0.0900\text{ T})(0.250\text{ m})\sin 36.9° = 0.0567\text{ V}$.

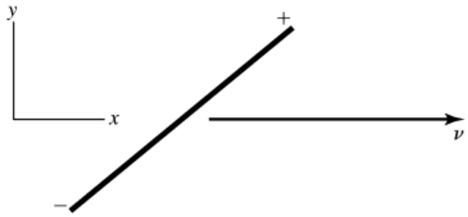


Figure 29.66

29.67. **IDENTIFY:** Use Eq.(29.10) to calculate the induced electric field at each point and then use $\vec{F} = q\vec{E}$.

SET UP:

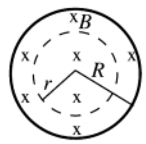


Figure 29.67a

Apply $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$ to a concentric circle of radius r, as shown in Figure 29.67a. Take $\vec{A}$ to be into the page, in the direction of $\vec{B}$.

EXECUTE: B increasing then gives $\dfrac{d\Phi_B}{dt} > 0$, so $\oint \vec{E}\cdot d\vec{l}$ is negative. This means that E is tangent to the circle in the counterclockwise direction, as shown in Figure 29.67b.

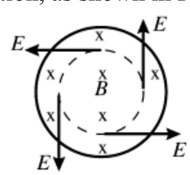


Figure 29.67b

$$\oint \vec{E}\cdot d\vec{l} = -E(2\pi r)$$

$$\frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt}$$

$-E(2\pi r) = -\pi r^2 \dfrac{dB}{dt}$ so $E = \tfrac{1}{2} r \dfrac{dB}{dt}$

<u>point *a*</u> The induced electric field and the force on q are shown in Figure 29.67c.

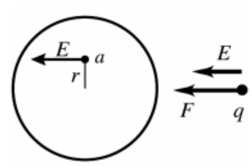


Figure 29.67c

$F = qE = \tfrac{1}{2} qr \dfrac{dB}{dt}$

$\vec{F}$ is to the left

($\vec{F}$ is in the same direction as $\vec{E}$ since q is positive.)

<u>point *b*</u> The induced electric field and the force on q are shown in Figure 29.67d.

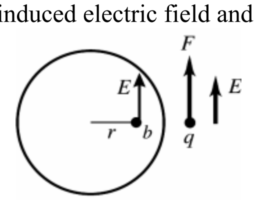


Figure 29.67d

$F = qE = \tfrac{1}{2} qr \dfrac{dB}{dt}$

$\vec{F}$ is toward the top of the page.

<u>point *c*</u> $r = 0$ here, so $E = 0$ and $F = 0$.

EVALUATE: If there were a concentric conducting ring of radius r in the magnetic field region, Lenz's law tells us that the increasing magnetic field would induce a counterclockwise current in the ring. This agrees with the direction of the force we calculated for the individual positive point charges.

29.68. **IDENTIFY:** A bar moving in a magnetic field has an emf induced across its ends. The propeller acts as such a bar.

SET UP: Different parts of the propeller are moving at different speeds, so we must integrate to get the total induced emf. The potential induced across an element of length dx is $d\mathcal{E} = vBdx$, where B is uniform.

EXECUTE: **(a)** Call x the distance from the center to an element of length dx, and L the length of the propeller. The speed of dx is $x\omega$, giving $d\mathcal{E} = vBdx = x\omega Bdx.$ $\mathcal{E} = \int_0^{L/2} x\omega Bdx = \omega BL^2/8.$

(b) The potential difference is zero since the potential is the same at both ends of the propeller.

(c) $\mathcal{E} = 2\pi\left(\dfrac{220\text{ rev}}{60\text{ s}}\right)(0.50\times10^{-4}\text{ T})\dfrac{(2.0\text{ m})^3}{8} = 5.8\times10^{-4}\text{ V} = 0.58\text{ mV}$

EVALUATE: A potential difference of about $\frac{1}{2}$ mV is not large enough to be concerned about in a propeller.

29.69. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: The electric field region is sketched in Figure 29.69.

EXECUTE: $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$. If $\vec{B}$ is constant then $\dfrac{d\Phi_B}{dt} = 0,$ so $\oint \vec{E}\cdot d\vec{l} = 0.$ $\int_{abcda} \vec{E}\cdot d\vec{l} = E_{ab}L - E_{cd}L = 0.$ But $E_{cd} = 0$, so $E_{ab}L = 0.$ But since we assumed $E_{ab} \neq 0,$ this contradicts Faraday's law. Thus, we can't have a uniform electric field abruptly drop to zero in a region in which the magnetic field is constant.

EVALUATE: If the magnetic field in the region is constant, then the integral $\oint \vec{E}\cdot d\vec{l}$ must be zero.

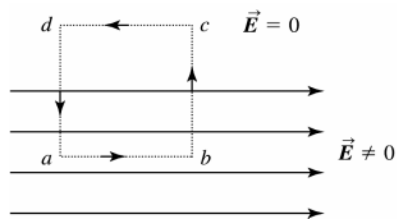


Figure 26.69

29.70. **IDENTIFY** and **SET UP:** At the terminal speed v_t, the upward force F_I exerted on the loop due to the induced current equals the downward force of gravity: $F_I = mg.$ Use Eq.(29.6) to find the induced emf in the side of the loop that is totally within the magnetic field. There is no induced emf in the other sides of the loop.

EXECUTE: $\mathcal{E} = Bvs,\ I = Bvs/R$ and $F_I = IsB - B^2s^2v/R$

$\dfrac{B^2s^2v_t}{R} = mg$ and $v_t = \dfrac{mgR}{B^2s^2}$

$m = \rho_m V = \rho_m(4s)\pi(d/2)^2 = \rho_m \pi d^2$

$R = \dfrac{\rho L}{A} = \dfrac{\rho_R 4s}{\frac{1}{4}\pi d^2} = \dfrac{16\rho_R s}{\pi d^2}$

Using these expressions for m and R gives $v_t = 16\rho_m\rho_R g/B^2$

EVALUATE: We know $\rho_m = 8900\text{ kg/m}^3$ (Table 14.1) and $\rho_R = 1.72\times10^{-8}\ \Omega\cdot\text{m}$ (Table 25.1). Taking B = 0.5 T gives $v_t = 9.6$ cm/s.

29.71. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: **(a)** The magnetic field region is sketched in Figure 29.71.

EXECUTE: **(b)** $\oint \vec{B}\cdot d\vec{l} = 0$ (no currents in the region). Using the figure, let $\vec{B} = B_0\hat{i}$ for $y < 0$ and $B = 0$ for $y > 0.$

$\int_{abcde} \vec{B}\cdot d\vec{l} = B_{ab}L - B_{cd}L = 0$ but $B_{cd} = 0.$ $B_{ab}L = 0,$ but $B_{ab} \neq 0.$ This is a contradiction and violates Ampere's Law.

EVALUATE: We often describe a magnetic field as being confined to a region, but this result shows that the edges of such a region can't be sharp.

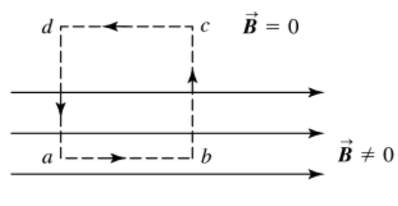


Figure 29.71

29.72. **IDENTIFY** and **SET UP:** Apply Ohm's law to the dielectric to relate the current in the dielectric to the charge on the plates. Use Eq.(25.1) for the current and obtain a differential equation for $q(t)$. Integrate this equation to obtain $q(t)$ and $i(t)$. Use $E = q/\epsilon A$ and Eq.(29.16) to calculate j_D.

EXECUTE: **(a)** Apply Ohm's law to the dielectric: The capacitor is sketched in Figure 29.72.

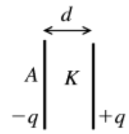


Figure 29.72

$$i(t) = \frac{v(t)}{R}$$

$$v(t) = \frac{q(t)}{C} \text{ and } C = K\frac{\epsilon_0 A}{d}$$

$$v(t) = \left(\frac{d}{K\epsilon_0 A}\right)q(t)$$

The resistance R of the dielectric slab is $R = \rho d/A$. Thus $i(t) = \dfrac{v(t)}{R} = \left(\dfrac{q(t)d}{K\epsilon_0 A}\right)\left(\dfrac{A}{\rho d}\right) = \dfrac{q(t)}{K\epsilon_0\rho}$. But the current $i(t)$ in the dielectric is related to the rate of change dq/dt of the charge $q(t)$ on the plates by $i(t) = -dq/dt$ (a positive i in the direction from the + to the – plate of the capacitor corresponds to a decrease in the charge). Using this in the above gives $-\dfrac{dq}{dt} = \left(\dfrac{1}{K\rho\epsilon_0}\right)q(t)$. $\dfrac{dq}{q} = -\dfrac{dt}{K\rho\epsilon_0}$. Integrate both sides of this equation from $t = 0$, where $q = Q_0$, to a later time t when the charge is $q(t)$. $\int_{Q_0}^{q}\dfrac{dq}{q} = -\left(\dfrac{1}{K\rho\epsilon_0}\right)\int_0^t dt$. $\ln\left(\dfrac{q}{Q_0}\right) = -\dfrac{t}{K\rho\epsilon_0}$ and $q(t) = Q_0 e^{-t/K\rho\epsilon_0}$. Then

$i(t) = -\dfrac{dq}{dt} = \left(\dfrac{Q_0}{K\rho\epsilon_0}\right)e^{-t/K\rho\epsilon_0}$ and $j_\text{C} = \dfrac{i(t)}{A} = \left(\dfrac{Q_0}{AK\rho\epsilon_0}\right)e^{-t/K\rho\epsilon_0}$. The conduction current flows from the positive to the negative plate of the capacitor.

(b) $E(t) = \dfrac{q(t)}{\epsilon A} = \dfrac{q(t)}{K\epsilon_0 A}$

$$j_\text{D}(t) = \epsilon\frac{dE}{dt} = K\epsilon_0\frac{dE}{dt} = K\epsilon_0\frac{dq(t)/dt}{K\epsilon_0 A} = -\frac{i_\text{C}(t)}{A} = -j_\text{C}(t)$$

The minus sign means that $j_\text{D}(t)$ is directed from the negative to the positive plate. $\vec{E}$ is from + to – but dE/dt is negative (E decreases) so $j_\text{D}(t)$ is from – to +.

EVALUATE: There is no conduction current to and from the plates so the concept of displacement current, with $\vec{j}_\text{D} = -\vec{j}_\text{C}$ in the dielectric, allows the current to be continuous at the capacitor.

29.73. **IDENTIFY:** The conduction current density is related to the electric field by Ohm's law. The displacement current density is related to the rate of change of the electric field by Eq.(29.16).

SET UP: $dE/dt = \omega E_0 \cos\omega t$

EXECUTE: **(a)** $j_\text{C}(\text{max}) = \dfrac{E_0}{\rho} = \dfrac{0.450 \text{ V/m}}{2300\ \Omega\cdot\text{m}} = 1.96\times10^{-4}\ \text{A/m}^2$

(b) $j_\text{D}(\text{max}) = \epsilon_0\left(\dfrac{dE}{dt}\right)_\text{max} = \epsilon_0\omega E_0 = 2\pi\epsilon_0 f E_0 = 2\pi\epsilon_0(120\text{ Hz})(0.450\text{ V/m}) = 3.00\times10^{-9}\ \text{A/m}^2$

(c) If $j_\text{C} = j_\text{D}$ then $\dfrac{E_0}{\rho} = \omega\epsilon_0 E_0$ and $\omega = \dfrac{1}{\rho\epsilon_0} = 4.91\times10^7$ rad/s

$$f = \frac{\omega}{2\pi} = \frac{4.91\times10^7 \text{ rad/s}}{2\pi} = 7.82\times10^6 \text{ Hz.}$$

EVALUATE: **(d)** The two current densities are out of phase by $90°$ because one has a sine function and the other has a cosine, so the displacement current leads the conduction current by $90°$.

29.74. **IDENTIFY:** A current is induced in the loop because of its motion and because of this current the magnetic field exerts a torque on the loop.

SET UP: Each side of the loop has mass $m/4$ and the center of mass of each side is at the center of each side. The flux through the loop is $\Phi_B = BA\cos\phi$.

EXECUTE: **(a)** $\vec{\tau}_g = \sum \vec{r}_{cm} \times m\vec{g}$ summed over each leg.

$$\tau_g = \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g\sin(90° - \phi) + \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g\sin(90° - \phi) + (L)\left(\frac{m}{4}\right)g\sin(90° - \phi)$$

$\tau_g = \frac{mgL}{2}\cos\phi$ (clockwise).

$\tau_B = \left|\vec{\tau} \times \vec{B}\right| = IAB\sin\phi$ (counterclockwise).

$I = \frac{\mathcal{E}}{R} = \frac{BA}{R}\frac{d}{dt}\cos\phi = -\frac{BA}{R}\frac{d\phi}{dt}\sin\phi = \frac{BA\omega}{R}\sin\phi$. The current is going counterclockwise looking to the $-\hat{k}$ direction.

Therefore, $\tau_B = \frac{B^2A^2\omega}{R}\sin^2\phi = \frac{B^2L^4\omega}{R}\sin^2\phi$. The net torque is $\tau = \frac{mgL}{2}\cos\phi - \frac{B^2L^4\omega}{R}\sin^2\phi$, opposite to the direction of the rotation.

(b) $\tau = I\alpha$ (I being the moment of inertia). About this axis $I = \frac{5}{12}mL^2$. Therefore,

$$\alpha = \frac{12}{5}\frac{1}{mL^2}\left[\frac{mgL}{2}\cos\phi - \frac{B^2L^4\omega}{R}\sin^2\phi\right] = \frac{6g}{5L}\cos\phi - \frac{12B^2L^2\omega}{5mR}\sin^2\phi.$$

EVALUATE: **(c)** The magnetic torque slows down the fall (since it opposes the gravitational torque).

(d) Some energy is lost through heat from the resistance of the loop.

29.75. **IDENTIFY:** Apply Eq.(29.10).

SET UP: Use an integration path that is a circle of radius r. By symmetry the induced electric field is tangent to this path and constant in magnitude at all points on the path.

EXECUTE: **(a)** The induced electric field at these points is shown in Figure 29.75a.

(b) To work out the amount of the electric field that is in the direction of the loop at a general position, we will use the geometry shown in Figure 29.75b. $E_{loop} = E\cos\theta$ but $E = \frac{\mathcal{E}}{2\pi r} = \frac{\mathcal{E}}{2\pi(a/\cos\theta)} = \frac{\mathcal{E}\cos\theta}{2\pi a}$. Therefore,

$E_{loop} = \frac{\mathcal{E}\cos^2\theta}{2\pi a}$. But $\mathcal{E} = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi r^2\frac{dB}{dt} = \frac{\pi a^2}{\cos^2\theta}\frac{dB}{dt}$, so $E_{loop} = \frac{\pi a^2}{2\pi a}\frac{dB}{dt} = \frac{a}{2}\frac{dB}{dt}$. This is exactly the value for a ring, obtained in Exercise 29.30, and has no dependence on the part of the loop we pick.

(c) $I = \frac{\mathcal{E}}{R} = \frac{A}{R}\frac{dB}{dt} = \frac{L^2}{R}\frac{dB}{dt} = \frac{(0.20\text{ m})^2(0.0350\text{ T/s})}{1.90\ \Omega} = 7.37\times10^{-4}$ A.

(d) $\mathcal{E}_{ab} = \frac{1}{8}\mathcal{E} = \frac{1}{8}L^2\frac{dB}{dt} = \frac{(0.20\text{ m})^2(0.0350\text{ T/s})}{8} = 1.75\times10^{-4}$ V. But there is potential drop $V = IR = -1.75\times10^{-4}$ V, so the potential difference is zero.

EVALUATE: The magnitude of the induced emf between any two points equals the magnitudes of the potential drop due to the current through the resistance of that portion of the loop.

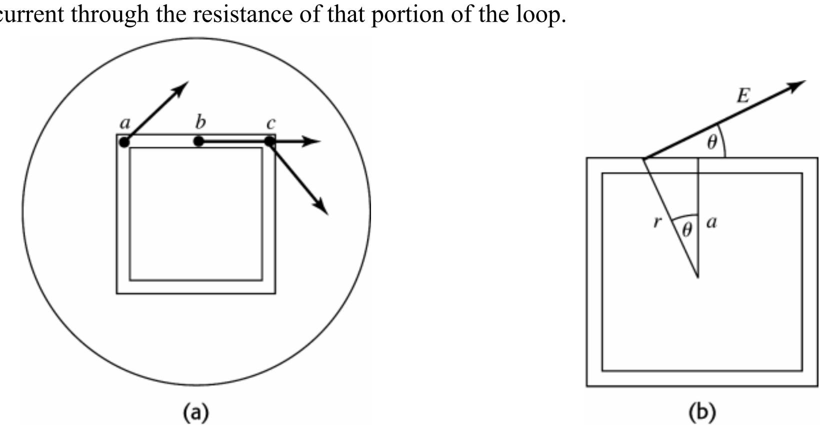


Figure 29.75

29.76. **IDENTIFY:** Apply Eq.(29.10).

SET UP: Use an integration path that is a circle of radius r. By symmetry the induced electric field is tangent to this path and constant in magnitude at all points on the path.

EXECUTE: **(a)** The induced emf at these points is shown in Figure 29.76.

(b) The induced emf on the side ac is zero, because the electric field is always perpendicular to the line ac.

(c) To calculate the total emf in the loop, $\mathcal{E} = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = L^2\frac{dB}{dt}$. $\mathcal{E} = (0.20\text{ m})^2(0.035\text{ T/s}) = 1.40\times10^{-3}$ V.

(d) $I = \frac{\mathcal{E}}{R} = \frac{1.40\times10^{-3}\text{ V}}{1.90\,\Omega} = 7.37\times10^{-4}\text{ A}$

(e) Since the loop is uniform, the resistance in length *ac* is one quarter of the total resistance. Therefore the potential difference between *a* and *c* is $V_{ac} = IR_{ac} = (7.37\times10^{-4}\text{ A})(1.90\,\Omega/4) = 3.50\times10^{-4}\text{ V}$ and the point *a* is at a higher potential since the current is flowing from *a* to *c*.

EVALUATE: This loop has the same resistance as the loop in Challenge Problem 29.75 and the induced current is the same.

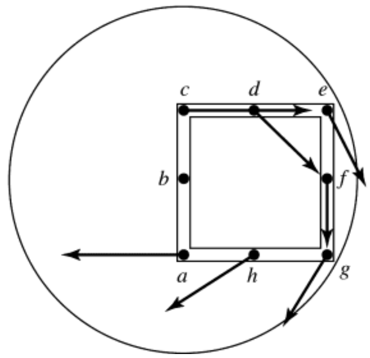


Figure 29.76

29.77. **IDENTIFY:** The motion of the bar produces an induced current and that results in a magnetic force on the bar.

SET UP: $\vec{F}_B$ is perpendicular to $\vec{B}$, so is horizontal. The vertical component of the normal force equals $mg\cos\phi$, so the horizontal component of the normal force equals $mg\tan\phi$.

EXECUTE: **(a)** As the bar starts to slide, the flux is decreasing, so the current flows to increase the flux, which means it flows from *a* to *b*. $F_B = iLB = \frac{LB}{R}\mathcal{E} = \frac{LB}{R}\frac{d\Phi_B}{dt} = \frac{LB}{R}B\frac{dA}{dt} = \frac{LB^2}{R}(vL\cos\phi) = \frac{vL^2B^2}{R}\cos\phi.$ At the terminal speed the horizontal forces balance, so $mg\tan\phi = \frac{v_tL^2B^2}{R}\cos\phi$ and $v_t = \frac{Rmg\tan\phi}{L^2B^2\cos\phi}$.

(c) $i = \frac{\mathcal{E}}{R} = \frac{1}{R}\frac{d\Phi_B}{dt} = \frac{1}{R}B\frac{dA}{dt} = \frac{B}{R}(vL\cos\phi) = \frac{vLB\cos\phi}{R} = \frac{mg\tan\phi}{LB}.$

(d) $P = i^2R = \frac{Rm^2g^2\tan^2\phi}{L^2B^2}.$

(e) $P_g = Fv\cos(90° - \phi) = mg\left(\frac{Rmg\tan\phi}{L^2B^2\cos\phi}\right)\sin\phi$ and $P_g = \frac{Rm^2g^2\tan^2\phi}{L^2B^2}.$

EVALUATE: The power in part (e) equals that in part (d), as is required by conservation of energy.

29.78. **IDENTIFY:** Follow the steps indicated in the problem.

SET UP: The primary assumption throughout the problem is that the square patch is small enough so that the velocity is constant over its whole area, that is, $v = \omega r \approx \omega d$.

EXECUTE: **(a)** $\omega \to$ clockwise, $B \to$ into page. $\mathcal{E} = vBL = \omega dBL$. $I = \frac{\mathcal{E}}{R} = \frac{\mathcal{E}A}{\rho L} = \frac{\omega dBA}{\rho}$. Since $\vec{v}\times\vec{B}$ points outward, A is just the cross-sectional area tL. Therefore, $I = \frac{\omega dBLt}{\rho}$ flowing radially outward since $\vec{v}\times\vec{B}$ points outward.

(b) $\vec{\tau} = \vec{d}\times\vec{F}$ and $\vec{F}_B = I\vec{L}\times\vec{B} = ILB$ pointing counterclockwise. So $\tau = \frac{\omega d^2B^2L^2t}{\rho}$ pointing out of the page (a counterclockwise torque opposing the clockwise rotation).

(c) If $\omega \to$ counterclockwise and $B \to$ into page, then $I \to$ inward radially since $\vec{v}\times\vec{B}$ points inward. $\tau \to$ clockwise (again opposing the motion). If $\omega \to$ counterclockwise and $B \to$ out of the page, then $I \to$ radially outward. $\tau \to$ clockwise (opposing the motion)

The magnitudes of I and τ are the same as in part (a).

EVALUATE: In each case the magnetic torque due to the induced current opposes the rotation of the disk, as is required by conservation of energy.

30

INDUCTANCE

30.1. **IDENTIFY and SET UP:** Apply Eq.(30.4).

EXECUTE: **(a)** $|\mathcal{E}_2| = M\left|\frac{di_1}{dt}\right| = (3.25\times10^{-4}\text{ H})(830\text{ A/s}) = 0.270\text{ V}$; yes, it is constant.

(b) $|\mathcal{E}_1| = M\left|\frac{di_2}{dt}\right|$; M is a property of the pair of coils so is the same as in part (a). Thus $|\mathcal{E}_1| = 0.270\text{ V}$.

EVALUATE: The induced emf is the same in either case. A constant di/dt produces a constant emf.

30.2. **IDENTIFY:** $\mathcal{E}_1 = M\left|\frac{\Delta i_2}{\Delta t}\right|$ and $\mathcal{E}_2 = M\left|\frac{\Delta i_1}{\Delta t}\right|$. $M = \left|\frac{N_2\Phi_{B2}}{i_1}\right|$, where Φ_{B2} is the flux through one turn of the second coil.

SET UP: M is the same whether we consider an emf induced in coil 1 or in coil 2.

EXECUTE: **(a)** $M = \frac{\mathcal{E}_2}{|\Delta i_1/\Delta t|} = \frac{1.65\times10^{-3}\text{ V}}{0.242\text{ A/s}} = 6.82\times10^{-3}\text{ H} = 6.82\text{ mH}$

(b) $\Phi_{B2} = \frac{Mi_1}{N_2} = \frac{(6.82\times10^{-3}\text{ H})(1.20\text{ A})}{25} = 3.27\times10^{-4}\text{ Wb}$

(c) $\mathcal{E}_1 = M\left|\frac{\Delta i_2}{\Delta t}\right| = (6.82\times10^{-3}\text{ H})(0.360\text{ A/s}) = 2.46\times10^{-3}\text{ V} = 2.46\text{ mV}$

EVALUATE: We can express M either in terms of the total flux through one coil produced by a current in the other coil, or in terms of the emf induced in one coil by a changing current in the other coil.

30.3. **IDENTIFY:** Replace units of Wb, A and Ω by their equivalents.

SET UP: $1\text{ Wb} = 1\text{ T}\cdot\text{m}^2$. $1\text{ T} = 1\text{ N/(A}\cdot\text{m)}$. $1\text{ N}\cdot\text{m} = 1\text{ J}$. $1\text{ A} = 1\text{ C/s}$. $1\text{ V} = 1\text{ J/C}$. $1\text{ V/A} = 1\ \Omega$.

EXECUTE: $1\text{ H} = 1\text{ Wb/A} = 1\text{ T}\cdot\text{m}^2/\text{A} = 1\text{ N}\cdot\text{m/A}^2 = 1\text{ J/A}^2 = 1\text{ (J/[A}\cdot\text{C])s} = 1\text{ (V/A)s} = 1\ \Omega\cdot\text{s}$.

EVALUATE: We may use whichever equivalent unit is the most convenient in a particular problem.

30.4. **IDENTIFY:** Changing flux from one object induces an emf in another object.

(a) SET UP: The magnetic field due to a solenoid is $B = \mu_0 nI$.

EXECUTE: The above formula gives

$$B_1 = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(300)(0.120\text{ A})}{0.250\text{ m}} = 1.81\times10^{-4}\text{ T}$$

The average flux through each turn of the inner solenoid is therefore

$$\Phi_B = B_1 A = \left(1.81\times10^{-4}\text{ T}\right)\pi(0.0100\text{ m})^2 = 5.68\times10^{-8}\text{ Wb}$$

(b) SET UP: The flux is the same through each turn of both solenoids due to the geometry, so

$$M = \frac{N_2\Phi_{B,2}}{i_1} = \frac{N_2\Phi_{B,1}}{i_1}$$

EXECUTE: $M = \frac{(25)\left(5.68\times10^{-8}\text{ Wb}\right)}{0.120\text{ A}} = 1.18\times10^{-5}\text{ H}$

(c) SET UP: The induced emf is $\mathcal{E}_2 = -M\frac{di_1}{dt}$.

EXECUTE: $\mathcal{E}_2 = -\left(1.18\times10^{-5}\text{ H}\right)(1750\text{ A/s}) = -0.0207\text{ V}$

EVALUATE: A mutual inductance around 10^{-5} H is not unreasonable.

30.5. **IDENTIFY** and **SET UP:** Apply Eq.(30.5).

EXECUTE: **(a)** $M = \frac{N_2\Phi_{B2}}{i_1} = \frac{400(0.0320\text{ Wb})}{6.52\text{ A}} = 1.96\text{ H}$

(b) $M = \frac{N_1\Phi_{B1}}{i_2}$ so $\Phi_{B1} = \frac{Mi_2}{N_1} = \frac{(1.96\text{ H})(2.54\text{ A})}{700} = 7.11\times10^{-3}\text{ Wb}$

EVALUATE: M relates the current in one coil to the flux through the other coil. Eq.(30.5) shows that M is the same for a pair of coils, no matter which one has the current and which one has the flux.

30.6. **IDENTIFY:** A changing current in an inductor induces an emf in it.

(a) SET UP: The self-inductance of a toroidal solenoid is $L = \frac{\mu_0 N^2 A}{2\pi r}$.

EXECUTE: $L = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(500)^2(6.25\times10^{-4}\text{ m}^2)}{2\pi(0.0400\text{ m})} = 7.81\times10^{-4}\text{ H}$

(b) SET UP: The magnitude of the induced emf is $\mathcal{E} = L\frac{di}{dt}$.

EXECUTE: $\mathcal{E} = (7.81\times10^{-4}\text{ H})\left(\frac{5.00\text{ A} - 2.00\text{ A}}{3.00\times10^{-3}\text{ s}}\right) = 0.781\text{ V}$

(c) The current is decreasing, so the induced emf will be in the same direction as the current, which is from a to b, making b at a higher potential than a.

EVALUATE: This is a reasonable value for self-inductance, in the range of a mH.

30.7. **IDENTIFY:** $\mathcal{E} = L\left|\frac{\Delta i}{\Delta t}\right|$ and $L = \frac{N\Phi_B}{i}$.

SET UP: $\frac{\Delta i}{\Delta t} = 0.0640\text{ A/s}$

EXECUTE: **(a)** $L = \frac{\mathcal{E}}{|\Delta i/\Delta t|} = \frac{0.0160\text{ V}}{0.0640\text{ A/s}} = 0.250\text{ H}$

(b) The average flux through each turn is $\Phi_B = \frac{Li}{N} = \frac{(0.250\text{ H})(0.720\text{ A})}{400} = 4.50\times10^{-4}\text{ Wb}$.

EVALUATE: The self-induced emf depends on the rate of change of flux and therefore on the rate of change of the current, not on the value of the current.

30.8. **IDENTIFY:** Combine the two expressions for L: $L = N\Phi_B/i$ and $L = \mathcal{E}/(di/dt)$.

SET UP: Φ_B is the average flux through one turn of the solenoid.

EXECUTE: Solving for N we have $N = \mathcal{E}i/\Phi_B(di/dt) = \frac{(12.6\times10^{-3}\text{ V})(1.40\text{ A})}{(0.00285\text{ Wb})(0.0260\text{ A/s})} = 238$ turns.

EVALUATE: The induced emf depends on the time rate of change of the total flux through the solenoid.

30.9. **IDENTIFY** and **SET UP:** Apply $|\mathcal{E}| = L|di/dt|$. Apply Lenz's law to determine the direction of the induced emf in the coil.

EXECUTE: **(a)** $|\mathcal{E}| = L(di/dt) = (0.260\text{ H})(0.0180\text{ A/s}) = 4.68\times10^{-3}\text{ V}$

(b) Terminal a is at a higher potential since the coil pushes current through from b to a and if replaced by a battery it would have the $+$ terminal at a.

EVALUATE: The induced emf is directed so as to oppose the decrease in the current.

30.10. **IDENTIFY:** Apply $\mathcal{E} = -L\frac{di}{dt}$.

SET UP: The induced emf points from low potential to high potential across the inductor.

EXECUTE: **(a)** The induced emf points from b to a, in the direction of the current. Therefore, the current is decreasing and the induced emf is directed to oppose this decrease.

(b) $|\mathcal{E}| = L|\Delta i/\Delta t|$, so $|\Delta i/\Delta t| = V_{ab}/L = (1.04\text{ V})/(0.260\text{ H}) = 4.00\text{ A/s}$. In 2.00 s the decrease in i is 8.00 A and the current at 2.00 s is $12.0\text{ A} - 8.0\text{ A} = 4.0\text{ A}$.

EVALUATE: When the current is decreasing the end of the inductor where the current enters is at the lower potential. This agrees with our result and with Figure 30.6d in the textbook.

30.11. **IDENTIFY** and **SET UP:** Use Eq.(30.6) to relate L to the flux through each turn of the solenoid. Use Eq.(28.23) for the magnetic field through the solenoid.

EXECUTE: $L=\dfrac{N\Phi_B}{i}$. If the magnetic field is uniform inside the solenoid $\Phi_B = BA$. From Eq.(28.23), $B=\mu_0 ni=\mu_0\left(\dfrac{N}{l}\right)i$ so $\Phi_B=\dfrac{\mu_0 NiA}{l}$. Then $L=\dfrac{N}{i}\left(\dfrac{\mu_0 NiA}{l}\right)=\dfrac{\mu_0 N^2 A}{l}$.

EVALUATE: Our result is the same as L for a torodial solenoid calculated in Example 30.3, except that the average circumference $2\pi r$ of the toroid is replaced by the length l of the straight solenoid.

30.12. **IDENTIFY** and **SET UP:** The stored energy is $U=\frac{1}{2}LI^2$. The rate at which thermal energy is developed is $P=I^2R$.

EXECUTE: **(a)** $U=\frac{1}{2}LI^2=\frac{1}{2}(12.0\text{ H})(0.300\text{ A})^2=0.540\text{ J}$

(b) $P=I^2R=(0.300\text{ A})^2(180\ \Omega)=16.2\text{ W}=16.2\text{ J/s}$

EVALUATE: **(c)** No. If I is constant then the stored energy U is constant. The energy being consumed by the resistance of the inductor comes from the emf source that maintains the current; it does not come from the energy stored in the inductor.

30.13. **IDENTIFY** and **SET UP:** Use Eq.(30.9) to relate the energy stored to the inductance. Example 30.3 gives the inductance of a toroidal solenoid to be $L=\dfrac{\mu_0 N^2 A}{2\pi r}$, so once we know L we can solve for N.

EXECUTE: $U=\frac{1}{2}LI^2$ so $L=\dfrac{2U}{I^2}=\dfrac{2(0.390\text{ J})}{(12.0\text{ A})^2}=5.417\times10^{-3}\text{ H}$

$$N=\sqrt{\frac{2\pi rL}{\mu_0 A}}=\sqrt{\frac{2\pi(0.150\text{ m})(5.417\times10^{-3}\text{ H})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(5.00\times10^{-4}\text{ m}^2)}}=2850.$$

EVALUATE: L and hence U increase according to the square of N.

30.14. **IDENTIFY:** A current-carrying inductor has a magnetic field inside of itself and hence stores magnetic energy.

(a) SET UP: The magnetic field inside a toroidal solenoid is $B=\dfrac{\mu_0 NI}{2\pi r}$.

EXECUTE: $B=\dfrac{\mu_0(300)(5.00\text{ A})}{2\pi(0.120\text{ m})}=2.50\times10^{-3}\text{ T}=2.50\text{ mT}$

(b) SET UP: The self-inductance of a toroidal solenoid is $L=\dfrac{\mu_0 N^2 A}{2\pi r}$.

EXECUTE: $L=\dfrac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(300)^2(4.00\times10^{-4}\text{ m}^2)}{2\pi(0.0120\text{ m})}=6.00\times10^{-5}\text{ H}$

(c) SET UP: The energy stored in an inductor is $U_L=\frac{1}{2}LI^2$.

EXECUTE: $U_L=\frac{1}{2}(6.00\times10^{-5}\text{ H})(5.00\text{ A})^2=7.50\times10^{-4}\text{ J}$

(d) SET UP: The energy density in a magnetic field is $u=\dfrac{B^2}{2\mu_0}$.

EXECUTE: $u=\dfrac{(2.50\times10^{-3}\text{ T})^2}{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})}=2.49\text{ J/m}^3$

(e) $u=\dfrac{\text{energy}}{\text{volume}}=\dfrac{\text{energy}}{2\pi rA}=\dfrac{7.50\times10^{-4}\text{ J}}{2\pi(0.120\text{ m})(4.00\times10^{-4}\text{ m}^2)}=2.49\text{ J/m}^3$

EVALUATE: An inductor stores its energy in the magnetic field inside of it.

30.15. **IDENTIFY:** A current-carrying inductor has a magnetic field inside of itself and hence stores magnetic energy.

(a) SET UP: The magnetic field inside a solenoid is $B=\mu_0 nI$.

EXECUTE: $B=\dfrac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(400)(80.0\text{ A})}{0.250\text{ m}}=0.161\text{ T}$

(b) SET UP: The energy density in a magnetic field is $u=\dfrac{B^2}{2\mu_0}$.

EXECUTE: $u=\dfrac{(0.161\text{ T})^2}{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})}=1.03\times10^4\text{ J/m}^3$

(c) SET UP: The total stored energy is $U=uV$.

EXECUTE: $U=uV=u(lA)=(1.03\times10^4\text{ J/m}^3)(0.250\text{ m})(0.500\times10^{-4}\text{ m}^2)=0.129\text{ J}$

(d) SET UP: The energy stored in an inductor is $U=\frac{1}{2}LI^2$.

EXECUTE: Solving for L and putting in the numbers gives

$$L=\frac{2U}{I^2}=\frac{2(0.129\text{ J})}{(80.0\text{ A})^2}=4.02\times10^{-5}\text{ H}$$

EVALUATE: An inductor stores its energy in the magnetic field inside of it.

30.16. **IDENTIFY:** Energy $= Pt$. $U = \frac{1}{2}LI^2$.

SET UP: $P = 200\text{ W} = 200\text{ J/s}$

EXECUTE: **(a)** Energy $= (200\text{ W})(24\text{ h})(3600\text{ s/h}) = 1.73\times 10^7\text{ J}$

(b) $L = \dfrac{2U}{I^2} = \dfrac{2(1.73\times 10^7\text{ J})}{(80.0\text{ A})^2} = 5.41\times 10^3\text{ H}$

EVALUATE: A large value of L and a large current would be required, just for one light bulb. Also, the resistance of the inductor would have to be very small, to avoid a large $P = I^2R$ rate of electrical energy loss.

30.17. **IDENTIFY** and **SET UP:** Starting with Eq. (30.9), follow exactly the same steps as in the text except that the magnetic permeability μ is used in place of μ_0.

EXECUTE: Using $L = \dfrac{\mu N^2 A}{2\pi r}$ and $B = \dfrac{\mu NI}{2\pi r}$ gives $u = \dfrac{B^2}{2\mu}$.

EVALUATE: For a given value of B, the energy density is less when μ is larger than μ_0.

30.18. **IDENTIFY** and **SET UP:** The energy density (energy per unit volume) in a magnetic field (in vacuum) is given by $u = \dfrac{U}{V} = \dfrac{B^2}{2\mu_0}$ (Eq.30.10).

EXECUTE: **(a)** $V = \dfrac{2\mu_0 U}{B^2} = \dfrac{2(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(3.60\times 10^6\text{ J})}{(0.600\text{ T})^2} = 25.1\text{ m}^3$.

(b) $u = \dfrac{U}{V} = \dfrac{B^2}{2\mu_0}$

$$B = \sqrt{\frac{2\mu_0 U}{V}} = \sqrt{\frac{2(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A})(3.60\times 10^6\text{ J})}{(0.400\text{ m})^3}} = 11.9\text{ T}$$

EVALUATE: Large-scale energy storage in a magnetic field is not practical. The volume in part (a) is quite large and the field in part (b) would be very difficult to achieve.

30.19. **IDENTIFY:** Apply Kirchhoff's loop rule to the circuit. $i(t)$ is given by Eq.(30.14).

SET UP: The circuit is sketched in Figure 30.19.

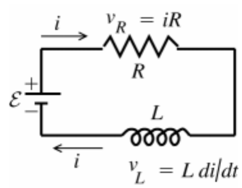


Figure 30.19

$\dfrac{di}{dt}$ is positive as the current increases from its initial value of zero.

EXECUTE: $\mathcal{E} - v_R - v_L = 0$

$\mathcal{E} - iR - L\dfrac{di}{dt} = 0$ so $i = \dfrac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right)$

(a) Initially ($t = 0$), $i = 0$ so $\mathcal{E} - L\dfrac{di}{dt} = 0$

$\dfrac{di}{dt} = \dfrac{\mathcal{E}}{L} = \dfrac{6.00\text{ V}}{2.50\text{ H}} = 2.40\text{ A/s}$

(b) $\mathcal{E} - iR - L\dfrac{di}{dt} = 0$ (Use this equation rather than Eq.(30.15) since i rather than t is given.)

Thus $\dfrac{di}{dt} = \dfrac{\mathcal{E} - iR}{L} = \dfrac{6.00\text{ V} - (0.500\text{ A})(8.00\ \Omega)}{2.50\text{ H}} = 0.800\text{ A/s}$

(c) $i = \dfrac{\mathcal{E}}{R}\left(1 - e^{-(R/L)t}\right) = \left(\dfrac{6.00\text{ V}}{8.00\ \Omega}\right)\left(1 - e^{-(8.00\ \Omega/2.50\text{ H})(0.250\text{ s})}\right) = 0.750\text{ A}(1 - e^{-0.800}) = 0.413\text{ A}$

(d) Final steady state means $t \to \infty$ and $\dfrac{di}{dt} \to 0$, so $\mathcal{E} - iR = 0$.

$i = \dfrac{\mathcal{E}}{R} = \dfrac{6.00\text{ V}}{8.00\ \Omega} = 0.750\text{ A}$

EVALUATE: Our results agree with Fig.30.12 in the textbook. The current is initially zero and increases to its final value of $\mathcal{E}/R$. The slope of the current in the figure, which is di/dt, decreases with t.

30.20. **IDENTIFY:** The current decays exponentially.

SET UP: After opening the switch, the current is $i = I_0 e^{-tR/L}$, and the time constant is $\tau = L/R$.

EXECUTE: **(a)** The initial current is $I_0 = (6.30\text{ V})/(15.0\ \Omega) = 0.420$ A. Now solve for L and put in the numbers.

$$L = \frac{-tR}{\ln(i/I_0)} = \frac{-(2.00\text{ ms})(15.0\ \Omega)}{\ln\left(\dfrac{0.210\text{ A}}{0.420\text{ A}}\right)} = 43.3\text{ mH}$$

(b) $\tau = L/R = (43.3\text{ mH})/(15.0\ \Omega) = 2.89$ ms

(c) Solve $i = I_0 e^{-t/\tau}$ for t, giving $t = -\tau\ln(i/I_0) = -(2.89\text{ ms})\ln(0.0100) = 13.3$ ms.

EVALUATE: In less than 5 time constants, the current is only 1% of its initial value.

30.21. **IDENTIFY:** $i = \mathcal{E}/R(1 - e^{-t/\tau})$, with $\tau = L/R$. The energy stored in the inductor is $U = \frac{1}{2}Li^2$.

SET UP: The maximum current occurs after a long time and is equal to $\mathcal{E}/R$.

EXECUTE: **(a)** $i_{max} = \mathcal{E}/R$ so $i = i_{max}/2$ when $(1 - e^{-t/\tau}) = \frac{1}{2}$ and $e^{-t/\tau} = \frac{1}{2}$. $-t/\tau = \ln\left(\frac{1}{2}\right)$.

$$t = \frac{L\ln 2}{R} = \frac{(\ln 2)(1.25\times 10^{-3}\text{ H})}{50.0\ \Omega} = 17.3\ \mu\text{s}$$

(b) $U = \frac{1}{2}U_{max}$ when $i = i_{max}/\sqrt{2}$. $1 - e^{-t/\tau} = 1/\sqrt{2}$, so $e^{-t/\tau} = 1 - 1/\sqrt{2} = 0.2929$. $t = -L\ln(0.2929)/R = 30.7\ \mu$s.

EVALUATE: $\tau = L/R = 2.50\times 10^{-5}$ s $= 25.0\ \mu$s. The time in part (a) is 0.692τ and the time in part (b) is 1.23τ.

30.22. **IDENTIFY:** With S_1 closed and S_2 open, $i(t)$ is given by Eq.(30.14). With S_1 open and S_2 closed, $i(t)$ is given by Eq.(30.18).

SET UP: $U = \frac{1}{2}Li^2$. After S_1 has been closed a long time, i has reached its final value of $I = \mathcal{E}/R$.

EXECUTE: **(a)** $U = \frac{1}{2}LI^2$ and $I = \sqrt{\dfrac{2U}{L}} = \sqrt{\dfrac{2(0.260\text{ J})}{0.115\text{ H}}} = 2.13$ A. $\mathcal{E} = IR = (2.13\text{ A})(120\ \Omega) = 256$ V.

(b) $i = Ie^{-(R/L)t}$ and $U = \frac{1}{2}Li^2 = \frac{1}{2}LI^2e^{-2(R/L)t} = \frac{1}{2}U_0 = \frac{1}{2}\left(\frac{1}{2}LI^2\right)$. $e^{-2(R/L)t} = \frac{1}{2}$, so

$$t = -\frac{L}{2R}\ln\left(\tfrac{1}{2}\right) = -\frac{0.115\text{ H}}{2(120\ \Omega)}\ln\left(\tfrac{1}{2}\right) = 3.32\times 10^{-4}\text{ s.}$$

EVALUATE: $\tau = L/R = 9.58\times 10^{-4}$ s. The time in part (b) is $\tau\ln(2)/2 = 0.347\tau$.

30.23. **IDENTIFY:** L has units of H and R has units of Ω.

SET UP: $1\text{ H} = 1\ \Omega\cdot\text{s}$

EXECUTE: Units of $L/R = \text{H}/\Omega = (\Omega\cdot\text{s})/\Omega = \text{s} =$ units of time.

EVALUATE: $Rt/L = t/\tau$ is dimensionless.

30.24. **IDENTIFY:** Apply the loop rule.

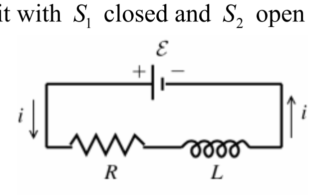
SET UP: In applying the loop rule, go around the circuit in the direction of the current. The voltage across the inductor is $-Ldi/dt$.

EXECUTE: $-Ldi/dt - iR = 0$. $\dfrac{di}{dt} = -i\dfrac{R}{L}$ gives $\displaystyle\int_{I_0}^{i}\frac{di'}{i'} = -\frac{R}{L}\int_0^t dt'$ and $\ln(i/I_0) = -\dfrac{R}{L}t$. $i = I_0 e^{-t(R/L)}$.

EVALUATE: di/dt is negative, so there is a potential rise across the inductor; point c is at higher potential than point b. There is a potential drop across the resistor.

30.25. **IDENTIFY:** Apply the concepts of current decay in an *R*-*L* circuit. Apply the loop rule to the circuit. $i(t)$ is given by Eq.(30.18). The voltage across the resistor depends on i and the voltage across the inductor depends on di/dt.

SET UP: The circuit with S_1 closed and S_2 open is sketched in Figure 30.25a.



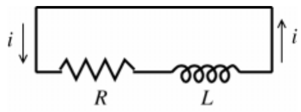
$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

Figure 30.25a

Constant current established means $\dfrac{di}{dt} = 0$.

EXECUTE: $i = \dfrac{\mathcal{E}}{R} = \dfrac{60.0\text{ V}}{240\ \Omega} = 0.250$ A

(a) SET UP: The circuit with S_2 closed and S_1 open is shown in Figure 30.25b.



$i = I_0 e^{-(R/L)t}$

At $t = 0$, $i = I_0 = 0.250$ A

Figure 30.25b

The inductor prevents an instantaneous change in the current; the current in the inductor just after S_2 is closed and S_1 is opened equals the current in the inductor just before this is done.

(b) EXECUTE: $i = I_0 e^{-(R/L)t} = (0.250 \text{ A})e^{-(240\ \Omega/0.160\ \text{H})(4.00\times10^{-4}\ \text{s})} = (0.250 \text{ A})e^{-0.600} = 0.137$ A

(c) SET UP: See Figure 30.25c.

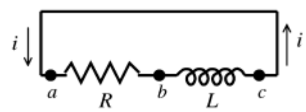


Figure 30.25c

EXECUTE: If we trace around the loop in the direction of the current the potential falls as we travel through the resistor so it must rise as we pass through the inductor: $v_{ab} > 0$ and $v_{bc} < 0$. So point *c* is at higher potential than point *b*.

$v_{ab} + v_{bc} = 0$ and $v_{bc} = -v_{ab}$

Or, $v_{cb} = v_{ab} = iR = (0.137 \text{ A})(240\ \Omega) = 32.9$ V

(d) $i = I_0 e^{-(R/L)t}$

$i = \frac{1}{2}I_0$ says $\frac{1}{2}I_0 = I_0 e^{-(R/L)t}$ and $\frac{1}{2} = e^{-(R/L)t}$

Taking natural logs of both sides of this equation gives $\ln(\frac{1}{2}) = -Rt/L$

$$t = \left(\frac{0.160 \text{ H}}{240\ \Omega}\right)\ln 2 = 4.62\times10^{-4} \text{ s}$$

EVALUATE: The current decays, as shown in Fig. 30.13 in the textbook. The time constant is $\tau = L/R = 6.67\times10^{-4}$ s. The values of *t* in the problem are less than one time constant. At any instant the potential drop across the resistor (in the direction of the current) equals the potential rise across the inductor.

30.26. IDENTIFY: Apply Eq.(30.14).

SET UP: $v_{ab} = iR$. $v_{bc} = L\dfrac{di}{dt}$. The current is increasing, so di/dt is positive.

EXECUTE: **(a)** At $t = 0$, $i = 0$. $v_{ab} = 0$ and $v_{bc} = 60$ V.

(b) As $t \to \infty$, $i \to \mathcal{E}/R$ and $di/dt \to 0$. $v_{ab} \to 60$ V and $v_{bc} \to 0$.

(c) When $i = 0.150$ A, $v_{ab} = iR = 36.0$ V and $v_{bc} = 60.0 \text{ V} - 36.0 \text{ V} = 24.0$ V.

EVALUATE: At all times, $\mathcal{E} = v_{ab} + v_{bc}$, as required by the loop rule.

30.27. IDENTIFY: $i(t)$ is given by Eq.(30.14).

SET UP: The power input from the battery is $\mathcal{E}i$. The rate of dissipation of energy in the resistance is i^2R. The voltage across the inductor has magnitude Ldi/dt, so the rate at which energy is being stored in the inductor is $iLdi/dt$.

EXECUTE: **(a)** $P = \mathcal{E}i = \mathcal{E}I_0(1 - e^{-(R/L)t}) = \dfrac{\mathcal{E}^2}{R}(1 - e^{-(R/L)t}) = \dfrac{(6.00 \text{ V})^2}{8.00\ \Omega}(1 - e^{-(8.00\ \Omega/2.50\ \text{H})t})$.

$P = (4.50 \text{ W})(1 - e^{-(3.20\ \text{s}^{-1})t})$.

(b) $P_R = i^2R = \dfrac{\mathcal{E}^2}{R}(1 - e^{-(R/L)t})^2 = \dfrac{(6.00 \text{ V})^2}{8.00\ \Omega}(1 - e^{-(8.00\ \Omega/2.50\ \text{H})t})^2 = (4.50 \text{ W})(1 - e^{-(3.20\ \text{s}^{-1})t})^2$

(c) $P_L = iL\dfrac{di}{dt} = \dfrac{\mathcal{E}}{R}(1 - e^{-(R/L)t})L\left(\dfrac{\mathcal{E}}{L}e^{-(R/L)t}\right) = \dfrac{\mathcal{E}^2}{R}(e^{-(R/L)t} - e^{-2(R/L)t})$

$P_L = (4.50 \text{ W})(e^{-(3.20\ \text{s}^{-1})t} - e^{-(6.40\ \text{s}^{-1})t})$.

EVALUATE: **(d)** Note that if we expand the square in part (b), then parts (b) and (c) add to give part (a), and the total power delivered is dissipated in the resistor and inductor. Conservation of energy requires that this be so.

30.28. **IDENTIFY:** An *L-C* circuit oscillates, with the energy going back and forth between the inductor and capacitor.

(a) SET UP: The frequency is $f = \dfrac{\omega}{2\pi}$ and $\omega = \dfrac{1}{\sqrt{LC}}$, giving $f = \dfrac{1}{2\pi\sqrt{LC}}$.

EXECUTE: $f = \dfrac{1}{2\pi\sqrt{(0.280\times10^{-3}\text{ H})(20.0\times10^{-6}\text{ F})}} = 2.13\times10^{3}\text{ Hz} = 2.13\text{ kHz}$

(b) SET UP: The energy stored in a capacitor is $U = \tfrac{1}{2}CV^2$.

EXECUTE: $U = \tfrac{1}{2}(20.0\times10^{-6}\text{ F})(150.0\text{ V})^2 = 0.225\text{ J}$

(c) SET UP: The current in the circuit is $i = -\omega Q\sin\omega t$, and the energy stored in the inductor is $U = \tfrac{1}{2}Li^2$.

EXECUTE: First find ω and Q. $\omega = 2\pi f = 1.336\times10^4$ rad/s.

$$Q = CV = (20.0\times10^{-6}\text{ F})(150.0\text{ V}) = 3.00\times10^{-3}\text{ C}$$

Now calculate the current:

$$i = -(1.336\times10^4\text{ rad/s})(3.00\times10^{-3}\text{ C})\sin[(1.336\times10^4\text{ rad/s})(1.30\times10^{-3}\text{ s})]$$

Notice that the argument of the sine is in *radians*, so convert it to degrees if necessary. The result is $i = -39.92$ A

Now find the energy in the inductor: $U = \tfrac{1}{2}Li^2 = \tfrac{1}{2}(0.280\times10^{-3}\text{ H})(-39.92\text{ A})^2 = 0.223\text{ J}$

EVALUATE: At the end of 1.30 ms, nearly all the energy is now in the inductor, leaving very little in the capacitor.

30.29. **IDENTIFY:** The energy moves back and forth between the inductor and capacitor.

(a) SET UP: The period is $T = \dfrac{1}{f} = \dfrac{1}{\omega/2\pi} = \dfrac{2\pi}{\omega} = 2\pi\sqrt{LC}$.

EXECUTE: Solving for L gives

$$L = \frac{T^2}{4\pi^2 C} = \frac{(8.60\times10^{-5}\text{ s})^2}{4\pi^2(7.50\times10^{-9}\text{ C})} = 2.50\times10^{-2}\text{ H} = 25.0\text{ mH}$$

(b) SET UP: The charge on a capacitor is $Q = CV$.

EXECUTE: $Q = CV = (7.50\times10^{-9}\text{ F})(12.0\text{ V}) = 9.00\times10^{-8}\text{ C}$

(c) SET UP: The stored energy is $U = Q^2/2C$.

EXECUTE: $U = \dfrac{(9.00\times10^{-8}\text{ C})^2}{2(7.50\times10^{-9}\text{ F})} = 5.40\times10^{-7}\text{ J}$

(d) SET UP: The maximum current occurs when the capacitor is discharged, so the inductor has all the initial energy. $U_L + U_C = U_{\text{Total}}$. $\tfrac{1}{2}LI^2 + 0 = U_{\text{Total}}$.

EXECUTE: Solve for the current:

$$I = \sqrt{\frac{2U_{\text{Total}}}{L}} = \sqrt{\frac{2(5.40\times10^{-7}\text{ J})}{2.50\times10^{-2}\text{ H}}} = 6.58\times10^{-3}\text{ A} = 6.58\text{ mA}$$

EVALUATE: The energy oscillates back and forth forever. However if there is any resistance in the circuit, no matter how small, all this energy will eventually be dissipated as heat in the resistor.

30.30. **IDENTIFY:** The circuit is described in Figure 30.14 of the textbook.

SET UP: The energy stored in the inductor is $U_L = \tfrac{1}{2}Li^2$ and the energy stored in the capacitor is $U_C = q^2/2C$. Initially, $U_C = \tfrac{1}{2}CV^2$, with $V = 12.0$ V. The period of oscillation is $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(12.0\times10^{-3}\text{ H})(18.0\times10^{-6}\text{ F})} = 2.92\text{ ms}$.

EXECUTE: **(a)** Energy conservation says $U_L(\text{max}) = U_C(\text{max})$, and $\tfrac{1}{2}Li_{\text{max}}^2 = \tfrac{1}{2}CV^2$.

$i_{\text{max}} = V\sqrt{C/L} = (22.5\text{ V})\sqrt{\dfrac{18\times10^{-6}\text{ F}}{12\times10^{-3}\text{ H}}} = 0.871\text{ A}$. The charge on the capacitor is zero because all the energy is in the inductor.

(b) From Figure 30.14 in the textbook, $q = 0$ at $t = T/4 = 0.730$ ms and at $t = 3T/4 = 2.19$ ms.

(c) $q_0 = CV = (18\ \mu\text{F})(22.5\text{ V}) = 405\ \mu\text{C}$ is the maximum charge on the plates. The graphs are sketched in Figure 30.30. q refers to the charge on one plate and the sign of i indicates the direction of the current.

EVALUATE: If the capacitor is fully charged at $t = 0$ it is fully charged again at $t = T/2$, but with the opposite polarity.

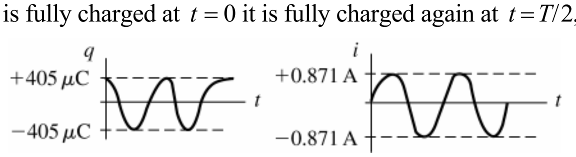
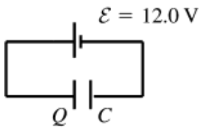


Figure 30.30

30.31. **IDENTIFY** and **SET UP:** The angular frequency is given by Eq.(30.22). $q(t)$ and $i(t)$ are given by Eqs.(30.21) and (30.23). The energy stored in the capacitor is $U_C = \frac{1}{2}CV^2 = q^2/2C$. The energy stored in the inductor is $U_L = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(1.50 \text{ H})(6.00\times10^{-5} \text{ F})}} = 105.4 \text{ rad/s},$ which rounds to 105 rad/s. The period is given by $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{105.4 \text{ rad/s}} = 0.0596 \text{ s}$

(b) The circuit containing the battery and capacitor is sketched in Figure 30.31.



$$\mathcal{E} - \frac{Q}{C} = 0$$

$$Q = \mathcal{E}C = (12.0 \text{ V})(6.00\times10^{-5} \text{ F}) = 7.20\times10^{-4} \text{ C}$$

Figure 30.31

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2}(6.00\times10^{-5} \text{ F})(12.0 \text{ V})^2 = 4.32\times10^{-3} \text{ J}$

(d) $q = Q\cos(\omega t + \phi)$ (Eq.30.21)

$q = Q$ at $t = 0$ so $\phi = 0$

$q = Q\cos\omega t = (7.20\times10^{-4} \text{ C})\cos([105.4 \text{ rad/s}][0.0230 \text{ s}]) = -5.42\times10^{-4} \text{ C}$

The minus sign means that the capacitor has discharged fully and then partially charged again by the current maintained by the inductor; the plate that initially had positive charge now has negative charge and the plate that initially had negative charge now has positive charge.

(e) $i = -\omega Q\sin(\omega t + \phi)$ (Eq.30.23)

$i = -(105 \text{ rad/s})(7.20\times10^{-4} \text{ C})\sin([105.4 \text{ rad/s}][0.0230 \text{ s}]) = -0.050 \text{ A}$

The negative sign means the current is counterclockwise in Figure 30.15 in the textbook.

or

$\frac{1}{2}Li^2 + \dfrac{q^2}{2C} = \dfrac{Q^2}{2C}$ gives $i = \pm\sqrt{\dfrac{1}{LC}}\sqrt{Q^2 - q^2}$ (Eq.30.26)

$i = \pm(105 \text{ rad/s})\sqrt{(7.20\times10^{-4} \text{ C})^2 - (-5.42\times10^{-4} \text{ C})^2} = \pm0.050$ A, which checks.

(f) $U_C = \dfrac{q^2}{2C} = \dfrac{(-5.42\times10^{-4} \text{ C})^2}{2(6.00\times10^{-5} \text{ F})} = 2.45\times10^{-3} \text{ J}$

$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(1.50 \text{ H})(0.050 \text{ A})^2 = 1.87\times10^{-3} \text{ J}$

EVALUATE: Note that $U_C + U_L = 2.45\times10^{-3} \text{ J} + 1.87\times10^{-3} \text{ J} = 4.32\times10^{-3} \text{ J}.$

This agrees with the total energy initially stored in the capacitor, $U = \dfrac{Q^2}{2C} = \dfrac{(7.20\times10^{-4} \text{ C})^2}{2(6.00\times10^{-5} \text{ F})} = 4.32\times10^{-3} \text{ J}.$

Energy is conserved. At some times there is energy stored in both the capacitor and the inductor. When $i = 0$ all the energy is stored in the capacitor and when $q = 0$ all the energy is stored in the inductor. But at all times the total energy stored is the same.

30.32. **IDENTIFY:** $\omega = \dfrac{1}{\sqrt{LC}} = 2\pi f$

SET UP: ω is the angular frequency in rad/s and f is the corresponding frequency in Hz.

EXECUTE: **(a)** $L = \dfrac{1}{4\pi^2 f^2 C} = \dfrac{1}{4\pi^2(1.6\times10^6 \text{ Hz})^2(4.18\times10^{-12} \text{ F})} = 2.37\times10^{-3} \text{ H}.$

(b) The maximum capacitance corresponds to the minimum frequency.

$$C_{\max} = \frac{1}{4\pi^2 f_{\min}^2 L} = \frac{1}{4\pi^2(5.40\times10^5 \text{ Hz})^2(2.37\times10^{-3} \text{ H})} = 3.67\times10^{-11} \text{ F} = 36.7 \text{ pF}$$

EVALUATE: To vary f by a factor of three (approximately the range in this problem), C must be varied by a factor of nine.

30.33. **IDENTIFY:** Apply energy conservation and Eqs. (30.22) and (30.23).

SET UP: If I is the maximum current, $\frac{1}{2}LI^2 = \dfrac{Q^2}{2C}$. For the inductor, $U_L = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $\frac{1}{2}LI^2 = \dfrac{Q^2}{2C}$ gives $Q = i\sqrt{LC} = (0.750 \text{ A})\sqrt{(0.0800 \text{ H})(1.25\times10^{-9} \text{ F})} = 7.50\times10^{-6} \text{ C}$.

(b) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.0800 \text{ H})(1.25\times10^{-9} \text{ F})}} = 1.00\times10^5 \text{ rad/s}$. $f = \dfrac{\omega}{2\pi} = 1.59\times10^4 \text{ Hz}$.

(c) $q = Q$ at $t = 0$ means $\phi = 0$. $i = -\omega Q \sin(\omega t)$, so

$i = -(1.00\times10^5 \text{ rad/s})(7.50\times10^{-6} \text{ C})\sin([1.00\times10^5 \text{ rad/s}][2.50\times10^{-3} \text{ s}]) = -0.7279 \text{ A}$.

$U_L = \frac{1}{2}Li^2 = \frac{1}{2}(0.0800 \text{ H})(-0.7279 \text{ A})^2 = 0.0212 \text{ J}$.

EVALUATE: The total energy of the system is $\frac{1}{2}LI^2 = 0.0225 \text{ J}$. At $t = 2.50 \text{ ms}$, the current is close to its maximum value and most of the system's energy is stored in the inductor.

30.34. **IDENTIFY:** Apply Eq.(30.25).

SET UP: $q = Q$ when $i = 0$. $i = i_{\max}$ when $q = 0$. $1/\sqrt{LC} = 1917 \text{ s}^{-1}$.

EXECUTE: **(a)** $\frac{1}{2}Li_{\max}^2 = \dfrac{Q^2}{2C}$. $Q = i_{\max}\sqrt{LC} = (0.850\times10^{-3} \text{ A})\sqrt{(0.0850 \text{ H})(3.20\times10^{-6} \text{ F})} = 4.43\times10^{-7} \text{ C}$

(b) $q = \sqrt{Q^2 - LCi^2} = \sqrt{(4.43\times10^{-7}\text{C})^2 - \left(\dfrac{5.00\times10^{-4}\text{A}}{1917\text{s}^{-1}}\right)^2} = 3.58\times10^{-7}\text{C}$.

EVALUATE: The value of q calculated in part (b) is less than the maximum value Q calculated in part (a).

30.35. **IDENTIFY:** $q = Q\cos(\omega t + \phi)$ and $i = -\omega Q \sin(\omega + \phi)$

SET UP: $U_C = \dfrac{q^2}{2C}$. $U_L = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $U_C = \frac{1}{2}\dfrac{q^2}{C} = \frac{1}{2}\dfrac{Q^2\cos^2(\omega t + \phi)}{C}$.

$U_L = \frac{1}{2}Li^2 = \frac{1}{2}L\omega^2 Q^2 \sin^2(\omega t + \phi) = \frac{1}{2}\dfrac{Q^2 \sin^2(\omega t + \phi)}{C}$, since $\omega^2 = \dfrac{1}{\sqrt{LC}}$.

(b) $U_{\text{Total}} = U_C + U_L = \dfrac{1}{2}\dfrac{Q^2}{C}\cos^2(\omega t + \phi) + \dfrac{1}{2}L\omega^2 Q^2 \sin^2(\omega t + \phi)$

$$U_{\text{total}} = \frac{1}{2}\frac{Q^2}{C}\cos^2(\omega t + \phi) + \frac{1}{2}L\left(\frac{1}{LC}\right)Q^2\sin^2(\omega t + \phi) = \frac{1}{2}\frac{Q^2}{C}(\cos^2(\omega t + \phi) + \sin^2(\omega t + \phi)) = \frac{1}{2}\frac{Q^2}{C}$$

U_{Total} is a constant.

EVALUATE: Eqs.(30.21) and (30.23) are consistent with conservation of energy in the *L-C* circuit.

30.36. **IDENTIFY:** Evaluate $\dfrac{d^2q}{dt^2}$ and insert into Eq.(20.20).

SET UP: Equation (30.20) is $\dfrac{d^2q}{dt^2} + \dfrac{1}{LC}q = 0.$

EXECUTE: $q = Q\cos(\omega t + \phi) \Rightarrow \dfrac{dq}{dt} = -\omega Q \sin(\omega t + \phi) \Rightarrow \dfrac{d^2q}{dt^2} = -\omega^2 Q\cos(\omega t + \phi).$

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = -\omega^2 Q \cos(\omega t + \phi) + \frac{Q}{LC}\cos(\omega t + \phi) = 0 \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}.$$

EVALUATE: The value of ϕ depends on the initial conditions, the value of q at $t = 0$.

30.37. **IDENTIFY:** The unit of L is H and the unit of C is F.

SET UP: $C = q/V_{ab}$ says $1 \text{ F} = 1 \text{ C/V}$. $1 \text{ H} = 1 \text{ V}\cdot\text{s/A} = 1 \text{ V}\cdot\text{s}^2\text{/C}$.

EXECUTE: $1 \text{ H}\cdot\text{F} = (1 \text{ V}\cdot\text{s}^2\text{/C})(1 \text{ C/V}) = 1 \text{ s}^2$. Therefore, LC has units of s^2 and $\sqrt{LC}$ has units of s.

EVALUATE: Our result shows that ωt is dimensionless, since $\omega = 1/\sqrt{LC}$.

30.38. **IDENTIFY:** The presence of resistance in an *L-R-C* circuit affects the frequency of oscillation and causes the amplitude of the oscillations to decrease over time.

(a) SET UP: The frequency of damped oscillations is $\omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$.

EXECUTE: $\omega' = \sqrt{\dfrac{1}{(22\times10^{-3}\text{ H})(15.0\times10^{-9}\text{ F})} - \dfrac{(75.0\ \Omega)^2}{4(22\times10^{-3}\text{ H})^2}} = 5.5\times10^4 \text{ rad/s}$

The frequency f is $f = \dfrac{\omega}{2\pi} = \dfrac{5.50\times10^4 \text{ rad/s}}{2\pi} = 8.76\times10^3 \text{ Hz} = 8.76 \text{ kHz}$.

(b) SET UP: The amplitude decreases as $A(t) = A_0\, e^{-(R/2L)t}$.

EXECUTE: Solving for t and putting in the numbers gives:

$$t = \frac{-2L\ \ln(A/A_0)}{R} = \frac{-2(22.0\times10^{-3}\text{ H})\ln(0.100)}{75.0\ \Omega} = 1.35\times10^{-3}\text{ s} = 1.35 \text{ ms}$$

(c) SET UP: At critical damping, $R = \sqrt{4L/C}$.

EXECUTE: $R = \sqrt{\dfrac{4\left(22.0\times10^{-3}\text{ H}\right)}{15.0\times10^{-9}\text{ F}}} = 2420\ \Omega$

EVALUATE: The frequency with damping is almost the same as the resonance frequency of this circuit ($1/\sqrt{LC}$), which is plausible because the 75-Ω resistance is considerably less than the 2420 Ω required for critical damping.

30.39. IDENTIFY: Follow the procedure specified in the problem.

SET UP: Make the substitutions $x \to q,\ m \to L,\ b \to R,\ k \to \dfrac{1}{C}$.

EXECUTE: **(a)** Eq. (13.41): $\dfrac{d^2x}{dt^2} + \dfrac{b}{m}\dfrac{dx}{dt} + \dfrac{kx}{m} = 0$. This becomes $\dfrac{d^2q}{dt^2} + \dfrac{R}{L}\dfrac{dq}{dt} + \dfrac{q}{LC} = 0$, which is Eq.(30.27).

(b) Eq. (13.43): $\omega' = \sqrt{\dfrac{k}{m} - \dfrac{b^2}{4m^2}}$. This becomes $\omega' = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{4L^2}}$, which is Eq.(30.29).

(c) Eq. (13.42): $x = Ae^{-(b/2m)t}\cos(\omega' t + \phi)$. This becomes $q = Ae^{-(R/2L)t}\cos(\omega' t + \phi)$, which is Eq.(30.28).

EVALUATE: Equations for the *L-R-C* circuit and for a damped harmonic oscillator have the same form.

30.40. IDENTIFY: For part (a), evaluate the derivatives as specified in the problem. For part (b) set $q = Q$ in Eq.(30.28) and set $dq/dt = 0$ in the expression for dq/dt.

SET UP: In terms of ω', Eq.(30.28) is $q(t) = Ae^{-(R/2L)t}\cos(\omega' t + \phi)$.

EXECUTE: **(a)** $q = Ae^{-(R/2L)t}\cos(\omega' t + \phi)$. $\dfrac{dq}{dt} = -A\dfrac{R}{2L}e^{-(R/2L)t}\cos(\omega' t + \phi) - \omega' Ae^{-(R/2L)t}\sin(\omega' t + \phi)$.

$$\frac{d^2q}{dt^2} = A\left(\frac{R}{2L}\right)^2 e^{-(R/2L)t}\cos(\omega' t + \phi) + 2\omega' A\frac{R}{2L}e^{-(R/2L)t}\sin(\omega' t + \phi) - \omega'^2 Ae^{-(R/2L)t}\cos(\omega' t + \phi)$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = q\left(\left(\frac{R}{2L}\right)^2 - \omega'^2 - \frac{R^2}{2L^2} + \frac{1}{LC}\right) = 0\text{, so } \omega'^2 = \frac{1}{LC} - \frac{R^2}{4L^2}.$$

(b) At $t = 0, q = Q, i = \dfrac{dq}{dt} = 0$, so $q = A\cos\phi = Q$ and $\dfrac{dq}{dt} = -\dfrac{R}{2L}A\cos\phi - \omega' A\sin\phi = 0$. This gives $A = \dfrac{Q}{\cos\phi}$ and

$$\tan\phi = -\frac{R}{2L\omega'} = -\frac{R}{2L\sqrt{1/LC - R^2/4L^2}}.$$

EVALUATE: If $R = 0$, then $A = Q$ and $\phi = 0$.

30.41. IDENTIFY: Evaluate Eq.(30.29).

SET UP: The angular frequency of the circuit is ω'.

EXECUTE: **(a)** When $R = 0,\ \omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.450\text{ H})(2.50\times10^{-5}\text{ F})}} = 298\text{ rad/s}.$

(b) We want $\dfrac{\omega}{\omega_0} = 0.95$, so $\dfrac{(1/LC - R^2/4L^2)}{1/LC} = 1 - \dfrac{R^2C}{4L} = (0.95)^2$. This gives

$$R = \sqrt{\frac{4L}{C}(1-(0.95)^2)} = \sqrt{\frac{4(0.450\text{ H})(0.0975)}{(2.50\times10^{-5}\text{ F})}} = 83.8\ \Omega.$$

EVALUATE: When *R* increases, the angular frequency decreases and approaches zero as $R \to 2\sqrt{L/C}$.

30.42. IDENTIFY: *L* has units of H and *C* has units of F.

SET UP: $1\text{ H} = 1\ \Omega\cdot\text{s}$. $C = q/V$ says $1\text{ F} = 1\text{ C/V}$. $V = IR$ says $1\text{ V/A} = 1\ \Omega$.

EXECUTE: The units of L/C are $\dfrac{\text{H}}{\text{F}} = \dfrac{\Omega\cdot\text{s}}{\text{C/V}} = \dfrac{\Omega\cdot\text{V}}{\text{A}} = \Omega^2$. Therefore, the unit of $\sqrt{L/C}$ is Ω.

EVALUATE: For Eq.(30.28) to be valid, $\dfrac{1}{LC}$ and $\dfrac{R^2}{4L^2}$ must have the same units, so *R* and $\sqrt{L/C}$ must have the same units, and we have shown that this is indeed the case.

30.43. IDENTIFY: The emf $\mathcal{E}_2$ in solenoid 2 produced by changing current i_1 in solenoid 1 is given by $\mathcal{E}_2 = M\left|\dfrac{\Delta i_1}{\Delta t}\right|$. The mutual inductance of two solenoids is derived in Example 30.1. For the two solenoids in this problem $M = \dfrac{\mu_0 A N_1 N_2}{l}$, where *A* is the cross-sectional area of the inner solenoid and *l* is the length of the outer solenoid.

Set Up: $\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}$. Let the outer solenoid be solenoid 1.

Execute: **(a)** $M = \dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\pi(6.00\times 10^{-4}\ \text{m})^2(6750)(15)}{0.500\ \text{m}} = 2.88\times 10^{-7}\ \text{H} = 0.288\ \mu\text{H}$

(b) $\mathcal{E}_2 = \left|\dfrac{\Delta i_1}{\Delta t}\right| = (2.88\times 10^{-7}\ \text{H})(37.5\ \text{A/s}) = 1.08\times 10^{-5}\ \text{V}$

Evaluate: If current in the inner solenoid changed at 37.5 A/s, the emf induced in the outer solenoid would be 1.08×10^{-5} V.

30.44. **Identify:** Apply $\mathcal{E} = -L\dfrac{di}{dt}$ and $Li = N\Phi_B$.

Set Up: Φ_B is the flux through one turn.

Execute: **(a)** $\mathcal{E} = -L\dfrac{di}{dt} = -(3.50\times 10^{-3}\ \text{H})\dfrac{d}{dt}((0.680\ \text{A})\cos(\pi t/[0.0250\ \text{s}]))$.

$\mathcal{E} = (3.50\times 10^{-3}\ \text{H})(0.680\ \text{A})\dfrac{\pi}{0.0250\ \text{s}}\sin(\pi t/[0.0250\ \text{s}])$. Therefore,

$\mathcal{E}_{\text{max}} = (3.50\times 10^{-3}\ \text{H})(0.680\ \text{A})\dfrac{\pi}{0.0250\ \text{s}} = 0.299\ \text{V}.$

(b) $\Phi_{B\text{max}} = \dfrac{Li_{\text{max}}}{N} = \dfrac{(3.50\times 10^{-3}\ \text{H})(0.680\ \text{A})}{400} = 5.95\times 10^{-6}\ \text{Wb}.$

(c) $\mathcal{E}(t) = -L\dfrac{di}{dt} = -(3.50\times 10^{-3}\text{H})(0.680\ \text{A})(\pi/0.0250\ \text{s})\sin(\pi t/0.0250\ \text{s}).$

$\mathcal{E}(t) = -(0.299\ \text{V})\sin((125.6\ \text{s}^{-1})t)$. Therefore, at $t = 0.0180$ s,

$\mathcal{E}(0.0180\ \text{s}) = -(0.299\ \text{V})\sin((125.6\ \text{s}^{-1})(0.0180\ \text{s})) = 0.230\ \text{V}$. The magnitude of the induced emf is 0.230 V.

Evaluate: The maximum emf is when $i = 0$ and at this instant $\Phi_B = 0$.

30.45. **Identify:** $\mathcal{E} = -L\dfrac{di}{dt}$.

Set Up: During an interval in which the graph of *i* versus *t* is a straight line, di/dt is constant and equal to the slope of that line.
Execute: **(a)** The pattern on the oscilloscope is sketched in Figure 30.45.
Evaluate: **(b)** Since the voltage is determined by the derivative of the current, the *V* versus *t* graph is indeed proportional to the derivative of the current graph.

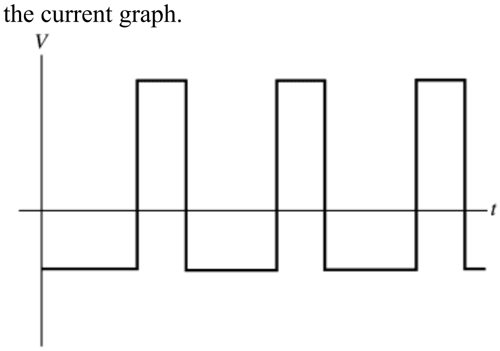


Figure 30.45

30.46. **Identify:** Apply $\mathcal{E} = -L\dfrac{di}{dt}$.

Set Up: $\dfrac{d}{dt}\cos(\omega t) = -\omega\sin(\omega t)$

Execute: **(a)** $\mathcal{E} = -L\dfrac{di}{dt} = -L\dfrac{d}{dt}((0.124\ \text{A})\cos[(240\ \pi/\text{s})t]$.

$$\mathcal{E} = +(0.250\ \text{H})\ (0.124\ \text{A})\ (240\ \pi/\text{s})\sin((240\pi/\text{s})t) = +(23.4\ \text{V})\sin((240\pi/\text{s})t).$$

The graphs are given in Figure 30.46.
(b) $\mathcal{E}_{\text{max}} = 23.4$ V; $i = 0$, since the emf and current are $90°$ out of phase.
(c) $i_{\text{max}} = 0.124$ A; $\mathcal{E} = 0$, since the emf and current are $90°$ out of phase.

EVALUATE: The induced emf depends on the rate at which the current is changing.

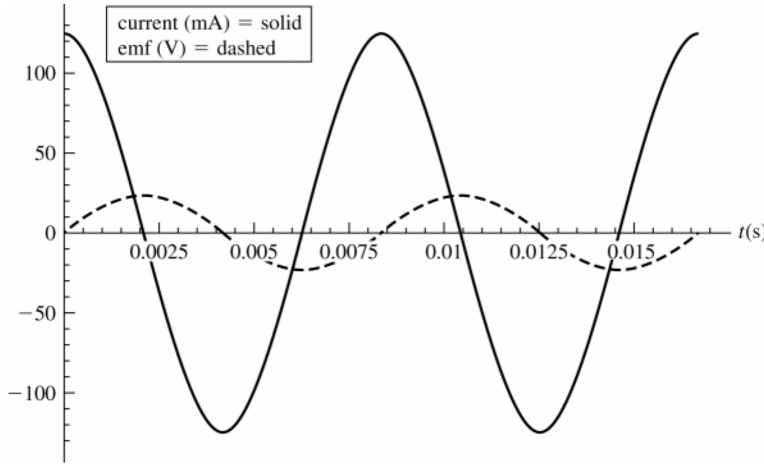


Figure 30.46

30.47. **IDENTIFY:** Apply $\mathcal{E} = -L\dfrac{di}{dt}$ to the series and parallel combinations.

SET UP: In series, $i_1 = i_2$ and the voltages add. In parallel the voltages are the same and the currents add.

EXECUTE: **(a)** Series: $L_1\dfrac{di_1}{dt} + L_2\dfrac{di_2}{dt} = L_{\text{eq}}\dfrac{di}{dt}$, but $i_1 = i_2 = i$ for series components so $\dfrac{di_1}{dt} = \dfrac{di_2}{dt} = \dfrac{di}{dt}$ and $L_1 + L_2 = L_{\text{eq}}$.

(b) Parallel: Now $L_1\dfrac{di_1}{dt} = L_2\dfrac{di_2}{dt} = L_{\text{eq}}\dfrac{di}{dt}$, where $i = i_1 + i_2$. Therefore, $\dfrac{di}{dt} = \dfrac{di_1}{dt} + \dfrac{di_2}{dt}$. But $\dfrac{di_1}{dt} = \dfrac{L_{\text{eq}}}{L_1}\dfrac{di}{dt}$ and $\dfrac{di_2}{dt} = \dfrac{L_{\text{eq}}}{L_2}\dfrac{di}{dt}$. $\dfrac{di}{dt} = \dfrac{L_{\text{eq}}}{L_1}\dfrac{di}{dt} + \dfrac{L_{\text{eq}}}{L_2}\dfrac{di}{dt}$ and $L_{\text{eq}} = \left(\dfrac{1}{L_1} + \dfrac{1}{L_2}\right)^{-1}$.

EVALUATE: Inductors in series and parallel combine in the same way as resistors.

30.48. **IDENTIFY:** Follow the steps outlined in the problem.

SET UP: The energy stored is $U = \frac{1}{2}Li^2$.

EXECUTE: **(a)** $\oint \vec{\mathbf{B}} \cdot d\vec{\boldsymbol{l}} = \mu_0 I_{\text{encl}} \Rightarrow B2\pi r = \mu_0 i \Rightarrow B = \dfrac{\mu_0 i}{2\pi r}$.

(b) $d\Phi_B = BdA = \dfrac{\mu_0 i}{2\pi r} l dr$.

(c) $\Phi_B = \displaystyle\int_a^b d\Phi_B = \frac{\mu_0 il}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 il}{2\pi}\ln(b/a)$.

(d) $L = \dfrac{N\Phi_B}{i} = l\dfrac{\mu_0}{2\pi}\ln(b/a)$.

(e) $U = \dfrac{1}{2}Li^2 = \dfrac{1}{2}l\dfrac{\mu_0}{2\pi}\ln(b/a)i^2 = \dfrac{\mu_0 li^2}{4\pi}\ln(b/a)$.

EVALUATE: The magnetic field between the conductors is due only to the current in the inner conductor.

30.49. **(a) IDENTIFY** and **SET UP:** An end view is shown in Figure 30.49.

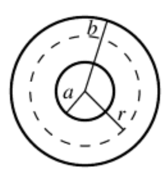


Figure 30.49

Apply Ampere's law to a circular path of radius *r*.

$\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_0 I_{\text{encl}}$

EXECUTE: $\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = B(2\pi r)$

$I_{\text{encl}} = i$, the current in the inner conductor

Thus $B(2\pi r) = \mu_0 i$ and $B = \dfrac{\mu_0 i}{2\pi r}$.

(b) IDENTIFY and SET UP: Follow the procedure specified in the problem.

EXECUTE: $u = \dfrac{B^2}{2\mu_0}$

$dU = u\,dV$, where $dV = 2\pi rl\,dr$

$$dU = \frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2\pi r}\right)^2 (2\pi rl)\,dr = \frac{\mu_0 i^2 l}{4\pi r}dr$$

(c) $U = \int dU = \dfrac{\mu_0 i^2 l}{4\pi}\int_a^b \dfrac{dr}{r} = \dfrac{\mu_0 i^2 l}{4\pi}[\ln r]_a^b$

$$U = \frac{\mu_0 i^2 l}{4\pi}(\ln b - \ln a) = \frac{\mu_0 i^2 l}{4\pi}\ln\left(\frac{b}{a}\right)$$

(d) Eq.(30.9): $U = \frac{1}{2}Li^2$

Part (c): $U = \dfrac{\mu_0 i^2 l}{4\pi}\ln\left(\dfrac{b}{a}\right)$

$$\tfrac{1}{2}Li^2 = \frac{\mu_0 i^2 l}{4\pi}\ln\left(\frac{b}{a}\right)$$

$$L = \frac{\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right).$$

EVALUATE: The value of L we obtain from these energy considerations agrees with L calculated in part (d) of Problem 30.48 by considering flux and Eq.(30.6)

30.50. **IDENTIFY:** Apply $L = \dfrac{N\Phi_B}{i}$ to each solenoid, as in Example 30.3. Use $M = \dfrac{N_2\Phi_{B2}}{i_1}$ to calculate the mutual inductance M.

SET UP: The magnetic field produced by solenoid 1 is confined to the space within its windings and is equal to $B_1 = \dfrac{\mu_0 N_1 i_1}{2\pi r}$.

EXECUTE: **(a)** $L_1 = \dfrac{N_1\Phi_{B_1}}{i_1} = \dfrac{N_1 A}{i_1}\left(\dfrac{\mu_0 N_1 i_1}{2\pi r}\right) = \dfrac{\mu_0 {N_1}^2 A}{2\pi r}$, $L_2 = \dfrac{N_2\Phi_{B_2}}{i_2} = \dfrac{N_2 A}{i_2}\left(\dfrac{\mu_0 N_2 i_2}{2\pi r}\right) = \dfrac{\mu_0 {N_2}^2 A}{2\pi r}$.

(b) $M = \dfrac{N_2 A B_1}{i_1} = \dfrac{\mu_0 N_1 N_2 A}{2\pi r}$. $M^2 = \left(\dfrac{\mu_0 N_1 N_2 A}{2\pi r}\right)^2 = \dfrac{\mu_0 {N_1}^2 A}{2\pi r}\dfrac{\mu_0 {N_2}^2 A}{2\pi r} = L_1 L_2$.

EVALUATE: If the two solenoids are identical, so that $N_1 = N_2$, then $M = L$.

30.51. **IDENTIFY:** $U = \frac{1}{2}LI^2$. The self-inductance of a solenoid is found in Exercise 30.11 to be $L = \dfrac{\mu_0 A N^2}{l}$.

SET UP: The length l of the solenoid is the number of turns divided by the turns per unit length.

EXECUTE: **(a)** $L = \dfrac{2U}{I^2} = \dfrac{2(10.0\text{ J})}{(1.50\text{ A})^2} = 8.89\text{ H}$

(b) $L = \dfrac{\mu_0 A N^2}{l}$. If α is the number of turns per unit length, then $N = \alpha l$ and $L = \mu_0 A\alpha^2 l$. For this coil $\alpha = 10\text{ coils/mm} = 10\times10^3\text{ coils/m}$. $l = \dfrac{L}{\mu_0 A\alpha^2} = \dfrac{8.89\text{ H}}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})\pi(0.0200\text{ m})^2(10\times10^3\text{ coils/m})^2} = 56.3\text{ m}$.

This is not a practical length for laboratory use.

EVALUATE: The number of turns is $N = (56.3\text{ m})(10\times10^3\text{ coils/m}) = 5.63\times10^5\text{ turns}$. The length of wire in the solenoid is the circumference C of one turn times the number of turns. $C = \pi d = \pi(4.00\times10^{-2}\text{ m}) = 0.126\text{ m}$. The length of wire is $(0.126\text{ m})(5.63\times10^5) = 7.1\times10^4\text{ m} = 71\text{ km}$. This length of wire will have a large resistance and I^2R electrical energy loses will be very large.

30.52. **IDENTIFY:** This is an R-L circuit and $i(t)$ is given by Eq.(30.14).

SET UP: When $t \to \infty$, $i \to i_\text{f} = V/R$.

EXECUTE: **(a)** $R = \dfrac{V}{i_\text{f}} = \dfrac{12.0\text{ V}}{6.45\times10^{-3}\text{ A}} = 1860\ \Omega.$

(b) $i = i_\text{f}(1 - e^{-(R/L)t})$ so $\dfrac{Rt}{L} = -\ln(1 - i/i_\text{f})$ and $L = \dfrac{-Rt}{\ln(1 - i/i_\text{f})} = \dfrac{-(1860\ \Omega)(7.25\times10^{-4}\text{s})}{\ln(1-(4.86/6.45))} = 0.963\text{ H}$.

EVALUATE: The current after a long time depends only on R and is independent of L. The value of R/L determines how rapidly the final value of i is reached.

30.53. **IDENTIFY** and **SET UP:** Follow the procedure specified in the problem. $L = 2.50\text{ H}$, $R = 8.00\ \Omega$, $\mathcal{E} = 6.00\text{ V}$. $i = (\mathcal{E}/R)(1 - e^{-t/\tau})$, $\tau = L/R$

EXECUTE: (a) Eq.(30.9): $U_L = \tfrac{1}{2}Li^2$

$t = \tau$ so $i = (\mathcal{E}/R)(1 - e^{-1}) = (6.00\text{ V}/8.00\ \Omega)(1 - e^{-1}) = 0.474\text{ A}$

Then $U_L = \tfrac{1}{2}Li^2 = \tfrac{1}{2}(2.50\text{ H})(0.474\text{ A})^2 = 0.281\text{ J}$

Exercise 30.27 (c): $P_L = \dfrac{dU_L}{dt} = Li\dfrac{di}{dt}$

$$i = \left(\frac{\mathcal{E}}{R}\right)(1 - e^{-t/\tau});\ \frac{di}{dt} = \left(\frac{\mathcal{E}}{L}\right)e^{-(R/L)t} = \frac{\mathcal{E}}{L}e^{-t/\tau}$$

$$P_L = L\left(\frac{\mathcal{E}}{R}(1 - e^{-t/\tau})\right)\left(\frac{\mathcal{E}}{L}e^{-t/\tau}\right) = \frac{\mathcal{E}^2}{R}(e^{-t/\tau} - e^{-2\pm/\tau})$$

$$U_L = \int_0^\tau P_L dt = \frac{\mathcal{E}^2}{R}\int_0^\tau (e^{-t/\tau} - e^{-2t/\tau})dt = \frac{\mathcal{E}^2}{R}\left[-\tau e^{-t/\tau} + \frac{\tau}{2}e^{-2t/\tau}\right]_0^\tau$$

$$U_L = -\frac{\mathcal{E}^2}{R}\tau\left[e^{-t/\tau} - \tfrac{1}{2}e^{-2t/\tau}\right]_0^\tau = \frac{\mathcal{E}^2}{R}\tau\left[1 - \tfrac{1}{2} - e^{-1} + \tfrac{1}{2}e^{-2}\right]$$

$$U_L = \left(\frac{\mathcal{E}^2}{2R}\right)\left(\frac{L}{R}\right)(1 - 2e^{-1} + e^{-2}) = \tfrac{1}{2}\left(\frac{\mathcal{E}}{R}\right)^2 L(1 - 2e^{-1} + e^{-2})$$

$$U_L = \tfrac{1}{2}\left(\frac{6.00\text{ V}}{8.00\ \Omega}\right)^2(2.50\text{ H})(0.3996) = 0.281\text{ J, which checks.}$$

(b) Exercise 30.27(a): The rate at which the battery supplies energy is $P_{\mathcal{E}} = \mathcal{E}i = \mathcal{E}\left(\dfrac{\mathcal{E}}{R}(1 - e^{-t/\tau})\right) = \dfrac{\mathcal{E}^2}{R}(1 - e^{-t/\tau})$

$$U_{\mathcal{E}} = \int_0^\tau P_{\mathcal{E}} dt = \frac{\mathcal{E}^2}{R}\int_0^\tau (1 - e^{-t/\tau})dt = \frac{\mathcal{E}^2}{R}\left[t + \tau e^{-t/\tau}\right]_0^\tau = \left(\frac{\mathcal{E}^2}{R}\right)(\tau + \tau e^{-1} - \tau)$$

$$U_{\mathcal{E}} = \left(\frac{\mathcal{E}^2}{R}\right)\tau e^{-1} = \left(\frac{\mathcal{E}^2}{R}\right)\left(\frac{L}{R}\right)e^{-1} = \left(\frac{\mathcal{E}}{R}\right)^2 Le^{-1}$$

$$U_{\mathcal{E}} = \left(\frac{6.00\text{ V}}{8.00\ \Omega}\right)^2(2.50\text{ H})(0.3679) = 0.517\text{ J}$$

(c) $P_R = i^2R = \left(\dfrac{\mathcal{E}^2}{R}\right)(1 - e^{-t/\tau})^2 = \dfrac{\mathcal{E}^2}{R}(1 - 2e^{-t/\tau} + e^{-2t/\tau})$

$$U_R = \int_0^\tau P_R dt = \frac{\mathcal{E}^2}{R}\int_0^\tau (1 - 2e^{-t/\tau} + e^{-2t/\tau})dt = \frac{\mathcal{E}^2}{R}\left[t + 2\tau e^{-t/\tau} - \frac{\tau}{2}e^{-2t/\tau}\right]_0^\tau$$

$$U_R = \frac{\mathcal{E}^2}{R}\left[\tau + 2\tau e^{-1} - \frac{\tau}{2}e^{-2} - 2\tau + \frac{\tau}{2}\right] = \frac{\mathcal{E}^2}{R}\left[-\frac{\tau}{2} + 2\tau e^{-1} - \frac{\tau}{2}e^{-2}\right]$$

$$U_R = \left(\frac{\mathcal{E}^2}{2R}\right)\left(\frac{L}{R}\right)\left[-1 + 4e^{-1} - e^{-2}\right]$$

$$U_R = \left(\frac{\mathcal{E}}{R}\right)^2(\tfrac{1}{2}L)\left[-1 + 4e^{-1} - e^{-2}\right] = \left(\frac{6.00\text{ V}}{8.00\ \Omega}\right)^2\tfrac{1}{2}(2.50\text{ H})(0.3362) = 0.236\text{ J}$$

(d) EVALUATE: $U_{\mathcal{E}} = U_R + U_L$. $(0.517\text{ J} = 0.236\text{ J} + 0.281\text{ J})$

The energy supplied by the battery equals the sum of the energy stored in the magnetic field of the inductor and the energy dissipated in the resistance of the inductor.

30.54. **IDENTIFY:** This is a decaying *R-L* circuit with $I_0 = \mathcal{E}/R$. $i(t) = I_0 e^{-(R/L)t}$.

SET UP: $\mathcal{E} = 60.0\text{ V}$, $R = 240\ \Omega$ and $L = 0.160\text{ H}$. The rate at which energy stored in the inductor is decreasing is $iLdi/dt$.

EXECUTE: **(a)** $U = \dfrac{1}{2}LI_0^{\,2} = \dfrac{1}{2}L\left(\dfrac{\mathcal{E}}{R}\right)^2 = \dfrac{1}{2}(0.160\text{ H})\left(\dfrac{60\text{ V}}{240\ \Omega}\right)^2 = 5.00\times10^{-3}\text{ J}.$

(b) $i=\frac{\mathcal{E}}{R}e^{-(R/L)t}\Rightarrow\frac{di}{dt}=-\frac{R}{L}i\Rightarrow\frac{dU_L}{dt}=iL\frac{di}{dt}=-Ri^2=\frac{\mathcal{E}^2}{R}e^{-2(R/L)t}$. $\frac{dU_L}{dt}=-\frac{(60\text{ V})^2}{240\ \Omega}e^{-2(240/0.160)(4.00\times10^{-4})}=-4.52\text{ W}.$

(c) In the resistor, $P_R=\frac{dU_R}{dt}=i^2R=\frac{\mathcal{E}^2}{R}e^{-2(R/L)t}=\frac{(60\text{ V})^2}{240\ \Omega}e^{-2(240/0.160)(4.00\times10^{-4})}=4.52\text{ W}$.

(d) $P_R(t)=i^2R=\frac{\mathcal{E}^2}{R}e^{-2(R/L)t}$. $U_R=\frac{\mathcal{E}^2}{R}\int_0^\infty e^{-2(R/L)t}=\frac{\mathcal{E}^2}{R}\frac{L}{2R}=\frac{(60\text{ V})^2(0.160\text{ H})}{2(240\ \Omega)^2}=5.00\times10^{-3}$ J, which is the same as part (a).

EVALUATE: During the decay of the current all the electrical energy originally stored in the inductor is dissipated in the resistor.

30.55. **IDENTIFY** and **SET UP:** Follow the procedure specified in the problem. $\frac{1}{2}Li^2$ is the energy stored in the inductor and $q^2/2C$ is the energy stored in the capacitor. The equation is $-iR-L\frac{di}{dt}-\frac{q}{C}=0.$

EXECUTE: Multiplying by $-i$ gives $i^2R+Li\frac{di}{dt}+\frac{qi}{C}=0.$ $\frac{d}{dt}U_L=\frac{d}{dt}\left(\frac{1}{2}Li^2\right)=\frac{1}{2}L\frac{d}{dt}\left(i^2\right)=\frac{1}{2}L\left(2i\frac{di}{dt}\right)=Li\frac{di}{dt},$ the second term. $\frac{d}{dt}U_C=\frac{d}{dt}\left(\frac{q^2}{2C}\right)=\frac{1}{2C}\frac{d}{dt}(q^2)=\frac{1}{2C}(2q)\frac{dq}{dt}=\frac{qi}{C},$ the third term. $i^2R=P_R,$ the rate at which electrical energy is dissipated in the resistance. $\frac{d}{dt}U_L=P_L,$ the rate at which the amount of energy stored in the inductor is changing. $\frac{d}{dt}U_C=P_C,$ the rate at which the amount of energy stored in the capacitor is changing.

EVALUATE: The equation says that $P_R+P_L+P_C=0;$ the net rate of change of energy in the circuit is zero. Note that at any given time one of P_C or P_L is negative. If the current and U_L are increasing the charge on the capacitor and U_C are decreasing, and vice versa.

30.56. **IDENTIFY:** The energy stored in a capacitor is $U_C=\frac{1}{2}Cv^2$. The energy stored in an inductor is $U_L=\frac{1}{2}Li^2$. Energy conservation requires that the total stored energy be constant.

SET UP: The current is a maximum when the charge on the capacitor is zero and the energy stored in the capacitor is zero.

EXECUTE: **(a)** Initially $v=16.0$ V and $i=0$. $U_L=0$ and $U_C=\frac{1}{2}Cv^2=\frac{1}{2}(5.00\times10^{-6}\text{ F})(16.0\text{ V})^2=6.40\times10^{-4}$ J . The total energy stored is 0.640 mJ .

(b) The current is maximum when $q=0$ and $U_C=0$. $U_C+U_L=6.40\times10^{-4}$ J so $U_L=6.40\times10^{-4}$ J .

$\frac{1}{2}Li_{\text{max}}^2=6.40\times10^{-4}$ J and $i_{\text{max}}=\sqrt{\frac{2(6.40\times10^{-4}\text{ J})}{3.75\times10^{-3}\text{ H}}}=0.584$ A .

EVALUATE: The maximum charge on the capacitor is $Q=CV=80.0\ \mu$C .

30.57. **IDENTIFY** and **SET UP:** Use $U_C=\frac{1}{2}CV_C^2$ (energy stored in a capacitor) to solve for *C*. Then use Eq.(30.22) and $\omega=2\pi f$ to solve for the *L* that gives the desired current oscillation frequency.

EXECUTE: $V_C=12.0$ V; $U_C=\frac{1}{2}CV_C^2$ so $C=2U_C/V_C^2=2(0.0160\text{ J})/(12.0\text{ V})^2=222\ \mu$F

$f=\frac{1}{2\pi\sqrt{LC}}$ so $L=\frac{1}{(2\pi f)^2C}$

$f=3500$ Hz gives $L=9.31\ \mu$H

EVALUATE: *f* is in Hz and ω is in rad/s; we must be careful not to confuse the two.

30.58. **IDENTIFY:** Apply energy conservation to the circuit.

SET UP: For a capacitor $V=q/C$ and $U=q^2/2C$. For an inductor $U=\frac{1}{2}Li^2$

EXECUTE: **(a)** $V_{\text{max}}=\frac{Q}{C}=\frac{6.00\times10^{-6}\text{ C}}{2.50\times10^{-4}\text{ F}}=0.0240$ V.

(b) $\frac{1}{2}Li_{\text{max}}^2=\frac{Q^2}{2C}$, so $i_{\text{max}}=\frac{Q}{\sqrt{LC}}=\frac{6.00\times10^{-6}\text{ C}}{\sqrt{(0.0600\text{ H})(2.50\times10^{-4}\text{ F})}}=1.55\times10^{-3}$A

(c) $U_{\text{max}}=\frac{1}{2}Li_{\text{max}}^2=\frac{1}{2}(0.0600\text{ H})(1.55\times10^{-3}\text{A})^2=7.21\times10^{-8}$ J.

(d) If $i = \frac{1}{2}i_{max}$ then $U_L = \frac{1}{4}U_{max} = 1.80\times10^{-8}$ J and $U_C = \frac{3}{4}U_{max} = \frac{\left(\sqrt{(3/4)}Q\right)^2}{2C} = \frac{q^2}{2C}$. This gives $q = \sqrt{\frac{3}{4}}Q = 5.20\times10^{-6}$ C.

EVALUATE: $U_{max} = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C}$ for all times.

30.59. **IDENTIFY:** Set $U_B = K$, where $K = \frac{1}{2}mv^2$.

SET UP: The energy density in the magnetic field is $u_B = B^2/2\mu_0$. Consider volume $V = 1\ \text{m}^3$ of sunspot material.

EXECUTE: The energy density in the sunspot is $u_B = B^2/2\mu_0 = 6.366\times10^4\ \text{J/m}^3$. The total energy stored in volume V of the sunspot is $U_B = u_B V$. The mass of the material in volume V of the sunspot is $m = \rho V$.

$K = U_B$ so $\frac{1}{2}mv^2 = U_B$. $\frac{1}{2}\rho V v^2 = u_B V$. The volume divides out, and $v = \sqrt{2u_B/\rho} = 2\times10^4\ \text{m/s}$.

EVALUATE: The speed we calculated is about 30 times smaller than the escape speed.

30.60. **IDENTIFY:** $i(t)$ is given by Eq.(30.14).

SET UP: The graph shows $V = 0$ at $t = 0$ and V approaches the constant value of 25 V at large times.

EXECUTE: **(a)** The voltage behaves the same as the current. Since V_R is proportional to i, the scope must be across the $150\ \Omega$ resistor.

(b) From the graph, as $t \to \infty, V_R \to 25$ V, so there is no voltage drop across the inductor, so its internal resistance must be zero. $V_R = V_{max}(1 - e^{-t/\tau})$. When $t = \tau$, $V_R = V_{max}\left(1 - \frac{1}{e}\right) \approx 0.63V_{max}$. From the graph, $V = 0.63V_{max} = 16$ V at $t \approx 0.5$ ms. Therefore $\tau = 0.5$ ms. $L/R = 0.5$ ms gives $L = (0.5\ \text{ms})(150\ \Omega) = 0.075$ H.

(c) The graph if the scope is across the inductor is sketched in Figure 30.60.

EVALUATE: At all times $V_R + V_L = 25.0$ V. At $t = 0$ all the battery voltage appears across the inductor since $i = 0$. At $t \to \infty$ all the battery voltage is across the resistance, since $di/dt = 0$.

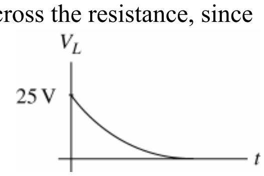


Figure 30.60

30.61. **IDENTIFY** and **SET UP:** The current grows in the circuit as given by Eq.(30.14). In an *R*-*L* circuit the full emf initially is across the inductance and after a long time is totally across the resistance. A solenoid in a circuit is represented as a resistance in series with an inductance. Apply the loop rule to the circuit; the voltage across a resistance is given by Ohm's law.

EXECUTE: **(a)** In the *R*-*L* circuit the voltage across the resistor starts at zero and increases to the battery voltage. The voltage across the solenoid (inductor) starts at the battery voltage and decreases to zero. In the graph, the voltage drops, so the oscilloscope is across the solenoid.

(b) At $t \to \infty$ the current in the circuit approaches its final, constant value. The voltage doesn't go to zero because the solenoid has some resistance R_L. The final voltage across the solenoid is IR_L, where I is the final current in the circuit.

(c) The emf of the battery is the initial voltage across the inductor, 50 V. Just after the switch is closed, the current is zero and there is no voltage drop across any of the resistance in the circuit.

(d) As $t \to \infty$, $\mathcal{E} - IR - IR_L = 0$

$\mathcal{E} = 50$ V and from the graph $IR_L = 15$ V (the final voltage across the inductor), so $IR = 35$ V and $I = (35\ \text{V})/R = 3.5$ A

(e) $IR_L = 15$ V, so $R_L = (15\ \text{V})/(3.5\ \text{A}) = 4.3\ \Omega$

$\mathcal{E} - V_L - iR = 0$, where V_L includes the voltage across the resistance of the solenoid.

$V_L = \mathcal{E} - iR$, $i = \frac{\mathcal{E}}{R_{tot}}\left(1 - e^{-t/\tau}\right)$, so $V_L = \mathcal{E}\left[1 - \frac{R}{R_{tot}}(1 - e^{-t/\tau})\right]$

$\mathcal{E} = 50$ V, $R = 10\ \Omega$, $R_{tot} = 14.3\ \Omega$, so when $t = \tau$, $V_L = 27.9$ *V*. From the graph, V_L has this value when $t = 3.0$ ms (read approximately from the graph), so $\tau = L/R_{tot} = 3.0$ ms. Then $L = (3.0\ \text{ms})(14.3\ \Omega) = 43$ mH.

EVALUATE: At $t = 0$ there is no current and the 50 V measured by the oscilloscope is the induced emf due to the inductance of the solenoid. As the current grows, there are voltage drops across the two resistances in the circuit.

We derived an equation for V_L, the voltage across the solenoid. At $t = 0$ it gives $V_L = \mathcal{E}$ and at $t \to \infty$ it gives $V_L = \mathcal{E}R/R_{\text{tot}} = iR$.

30.62. **IDENTIFY:** At $t = 0$, $i = 0$ through each inductor. At $t \to \infty$, the voltage is zero across each inductor.

SET UP: In each case redraw the circuit. At $t = 0$ replace each inductor by a break in the circuit and at $t \to \infty$ replace each inductor by a wire.

EXECUTE: **(a)** Initially the inductor blocks current through it, so the simplified equivalent circuit is shown in Figure 30.62a. $i = \dfrac{\mathcal{E}}{R} = \dfrac{50\text{ V}}{150\ \Omega} = 0.333\text{ A}$. $V_1 = (100\ \Omega)(0.333\text{ A}) = 33.3\text{ V}$. $V_4 = (50\ \Omega)(0.333\text{ A}) = 16.7\text{ V}$. $V_3 = 0$ since no current flows through it. $V_2 = V_4 = 16.7$ V, since the inductor is in parallel with the $50\ \Omega$ resistor. $A_1 = A_3 = 0.333$ A, $A_2 = 0$.

(b) Long after S is closed, steady state is reached, so the inductor has no potential drop across it. The simplified circuit is sketched in Figure 30.62b. $i = \mathcal{E}/R = \dfrac{50\text{ V}}{130\ \Omega} = 0.385\text{ A}$. $V_1 = (100\ \Omega)(0.385\text{ A}) = 38.5\text{ V}$; $V_2 = 0$; $V_3 = V_4 = 50\text{ V} - 38.5\text{ V} = 11.5\text{ V}$. $i_1 = 0.385$ A; $i_2 = \dfrac{11.5\text{ V}}{75\ \Omega} = 0.153$ A; $i_3 = \dfrac{11.5\text{ V}}{50\ \Omega} = 0.230\text{ A}$.

EVALUATE: Just after the switch is closed the current through the battery is 0.333 A. After a long time the current through the battery is 0.385 A. After a long time there is an additional current path, the equivalent resistance of the circuit is decreased and the current has increased.

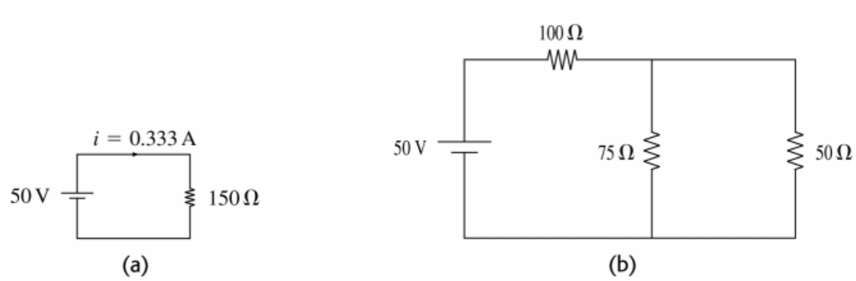


Figure 30.62

30.63. **IDENTIFY** and **SET UP:** Just after the switch is closed, the current in each branch containing an inductor is zero and the voltage across any capacitor is zero. The inductors can be treated as breaks in the circuit and the capacitors can be replaced by wires. After a long time there is no voltage across each inductor and no current in any branch containing a capacitor. The inductors can be replaced by wires and the capacitors by breaks in the circuit.

EXECUTE: **(a)** Just after the switch is closed the voltage V_5 across the capacitor is zero and there is also no current through the inductor, so $V_3 = 0$. $V_2 + V_3 = V_4 = V_5$, and since $V_5 = 0$ and $V_3 = 0$, V_4 and V_2 are also zero.

$V_4 = 0$ means V_3 reads zero. V_1 then must equal 40.0 V, and this means the current read by A_1 is $(40.0\text{ V})/(50.0\ \Omega) = 0.800$ A. $A_2 + A_3 + A_4 = A_1$, but $A_2 = A_3 = 0$ so $A_4 = A_1 = 0.800$ A. $A_1 = A_4 = 0.800$ A; all other ammeters read zero. $V_1 = 40.0$ V and all other voltmeters read zero.

(b) After a long time the capacitor is fully charged so $A_4 = 0$. The current through the inductor isn't changing, so $V_2 = 0$. The currents can be calculated from the equivalent circuit that replaces the inductor by a short circuit, as shown in Figure 30.63a.

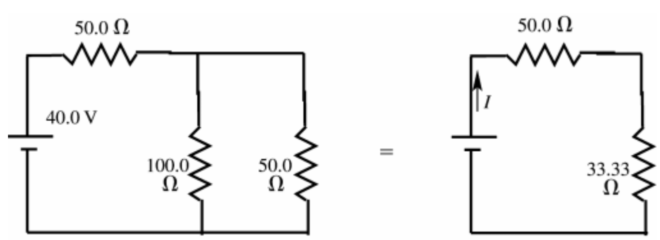


Figure 30.63a

$I = (40.0\text{ V})/(83.33\ \Omega) = 0.480$ A; A_1 reads 0.480 A

$V_1 = I(50.0\ \Omega) = 24.0$ V

The voltage across each parallel branch is 40.0 V – 24.0 V = 16.0 V

$V_2 = 0$, $V_3 = V_4 = V_5 = 16.0$ V

$V_3 = 16.0$ V means A_2 reads 0.160 A. $V_4 = 16.0$ V means A_3 reads 0.320 A. A_4 reads zero. Note that $A_2 + A_3 = A_1$.

(c) $V_5 = 16.0$ V so $Q = CV = (12.0\ \mu\text{F})(16.0\text{ V}) = 192\ \mu\text{C}$

(d) At $t = 0$ and $t \to \infty$, $V_2 = 0$. As the current in this branch increases from zero to 0.160 A the voltage V_2 reflects the rate of change of the current. The graph is sketched in Figure 30.63b.

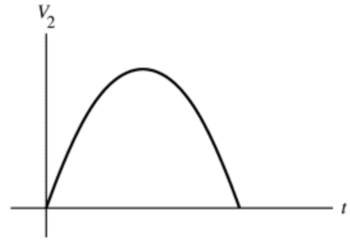


Figure 30.63b

EVALUATE: This reduction of the circuit to resistor networks only apply at $t = 0$ and $t \to \infty$. At intermediate times the analysis is complicated.

30.64. **IDENTIFY:** At all times $v_1 + v_2 = 25.0$ V. The voltage across the resistor depends on the current through it and the voltage across the inductor depends on the rate at which the current through it is changing.

SET UP: Immediately after closing the switch the current thorough the inductor is zero. After a long time the current is no longer changing.

EXECUTE: **(a)** $i = 0$ so $v_1 = 0$ and $v_2 = 25.0$ V. The ammeter reading is $A = 0$.

(b) After a long time, $v_2 = 0$ and $v_1 = 25.0$ V. $v_1 = iR$ and $i = \dfrac{v_1}{R} = \dfrac{25.0\text{ V}}{15.0\ \Omega} = 1.67$ A. The ammeter reading is $A = 1.67$A.

(c) None of the answers in (a) and (b) depend on L so none of them would change.

EVALUATE: The inductance L of the circuit affects the rate at which current reaches its final value. But after a long time the inductor doesn't affect the circuit and the final current does not depend on L.

30.65. **IDENTIFY:** At $t = 0$, $i = 0$ through each inductor. At $t \to \infty$, the voltage is zero across each inductor.

SET UP: In each case redraw the circuit. At $t = 0$ replace each inductor by a break in the circuit and at $t \to \infty$ replace each inductor by a wire.

EXECUTE: **(a)** Just after the switch is closed there is no current through either inductor and they act like breaks in the circuit. The current is the same through the $40.0\ \Omega$ and $15.0\ \Omega$ resistors and is equal to $(25.0\text{ V})/(40.0\ \Omega + 15.0\ \Omega) = 0.455$ A. $A_1 = A_4 = 0.455$ A; $A_2 = A_3 = 0$.

(b) After a long time the currents are constant, there is no voltage across either inductor, and each inductor can be treated as a short-circuit . The circuit is equivalent to the circuit sketched in Figure 30.65.

$I = (25.0\text{ V})/(42.73\ \Omega) = 0.585$ A. A_1 reads 0.585 A. The voltage across each parallel branch is $25.0\text{ V} - (0.585\text{ A})(40.0\ \Omega) = 1.60$ V. A_2 reads $(1.60\text{ V})/(5.0\ \Omega) = 0.320$ A. A_3 reads $(1.60\text{ V})/(10.0\ \Omega) = 0.160$ A. A_4 reads $(1.60\text{ V})/(15.0\ \Omega) = 0.107$ A.

EVALUATE: Just after the switch is closed the current through the battery is 0.455 A. After a long time the current through the battery is 0.585 A. After a long time there are additional current paths, the equivalent resistance of the circuit is decreased and the current has increased.

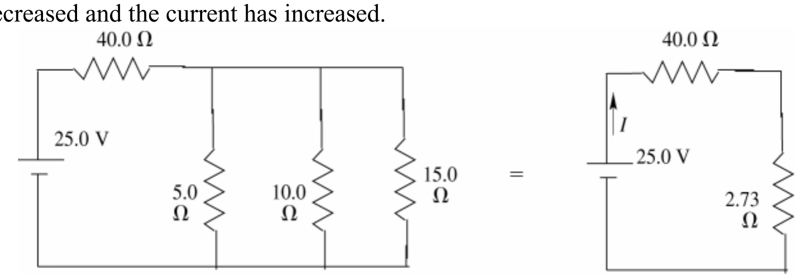


Figure 30.65

30.66. **IDENTIFY:** Closing S_2 and simultaneously opening S_1 produces an L-C circuit with initial current through the inductor of 3.50 A. When the current is a maximum the charge q on the capacitor is zero and when the charge q is a maximum the current is zero. Conservation of energy says that the maximum energy $\frac{1}{2}Li_{max}^2$ stored in the inductor equals the maximum energy $\frac{1}{2}\dfrac{q_{max}^2}{C}$ stored in the capacitor.

SET UP: $i_{max} = 3.50$ A, the current in the inductor just after the switch is closed.

EXECUTE: **(a)** $\frac{1}{2}Li_{max}^2 = \frac{1}{2}\frac{q_{max}^2}{C}$.

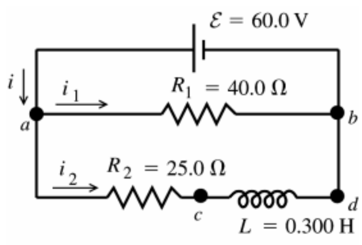
$q_{max} = (\sqrt{LC})i_{max} = \sqrt{(2.0\times10^{-3}\text{ H})(5.0\times10^{-6}\text{ F})}(3.50\text{ A}) = 3.50\times10^{-4}\text{ C} = 0.350\text{ mC}$.

(b) When q is maximum, $i = 0$.

EVALUATE: In the final circuit the current will oscillate.

30.67. **IDENTIFY:** Apply the loop rule to each parallel branch. The voltage across a resistor is given by iR and the voltage across an inductor is given by $L|di/dt|$. The rate of change of current through the inductor is limited.

SET UP: With S closed the circuit is sketched in Figure 30.67a.



The rate of change of the current through the inductor is limited by the induced emf. Just after the switch is closed the current in the inductor has not had time to increase from zero, so $i_2 = 0$.

Figure 30.67a

EXECUTE: (a) $\mathcal{E} - v_{ab} = 0$, so $v_{ab} = 60.0$ V

(b) The voltage drops across R, as we travel through the resistor in the direction of the current, so point a is at higher potential.

(c) $i_2 = 0$ so $v_{R_2} = i_2R_2 = 0$

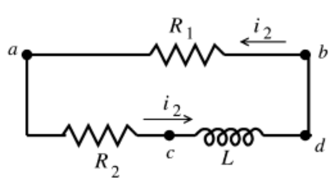
$\mathcal{E} - v_{R_2} - v_L = 0$ so $v_L = \mathcal{E} = 60.0$ V

(d) The voltage rises when we go from b to a through the emf, so it must drop when we go from a to b through the inductor. Point c must be at higher potential than point d.

(e) After the switch has been closed a long time, $\frac{di_2}{dt} \to 0$ so $v_L = 0$. Then $\mathcal{E} - v_{R_2} = 0$ and $i_2R_2 = \mathcal{E}$

so $i_2 = \frac{\mathcal{E}}{R_2} = \frac{60.0\text{ V}}{25.0\ \Omega} = 2.40$ A.

SET UP: The rate of change of the current through the inductor is limited by the induced emf. Just after the switch is opened again the current through the inductor hasn't had time to change and is still $i_2 = 2.40$ A. The circuit is sketched in Figure 30.67b.



EXECUTE: The current through R_1 is $i_2 = 2.40$ A, in the direction b to a. Thus

$v_{ab} = -i_2R_1 = -(2.40\text{ A})(40.0\ \Omega)$

$v_{ab} = -96.0$ V

Figure 30.67b

(f) Point where current enters resistor is at higher potential; point b is at higher potential.

(g) $v_L - v_{R_1} - v_{R_2} = 0$

$v_L = v_{R_1} + v_{R_2}$

$v_{R_1} = -v_{ab} = 96.0$ V; $v_{R_2} = i_2R_2 = (2.40\text{ A})(25.0\ \Omega) = 60.0$ V

Then $v_L = v_{R_1} + v_{R_2} = 96.0\text{ V} + 60.0\text{ V} = 156$ V.

As you travel counterclockwise around the circuit in the direction of the current, the voltage drops across each resistor, so it must rise across the inductor and point d is at higher potential than point c. The current is decreasing, so the induced emf in the inductor is directed in the direction of the current. Thus, $v_{cd} = -156$ V.

(h) Point d is at higher potential.

EVALUATE: The voltage across R_1 is constant once the switch is closed. In the branch containing R_2, just after S is closed the voltage drop is all across L and after a long time it is all across R_2. Just after S is opened the same current flows in the single loop as had been flowing through the inductor and the sum of the voltage across the resistors equals the voltage across the inductor. This voltage dies away, as the energy stored in the inductor is dissipated in the resistors.

30.68. **IDENTIFY:** Apply the loop rule to the two loops. The current through the inductor doesn't change abruptly.

SET UP: For the inductor $|\mathcal{E}| = L\left|\dfrac{di}{dt}\right|$ and $\mathcal{E}$ is directed to oppose the change in current.

EXECUTE: **(a)** Switch is closed, then at some later time

$$\frac{di}{dt} = 50.0\ \text{A/s} \Rightarrow v_{cd} = L\frac{di}{dt} = (0.300\ \text{H})\ (50.0\ \text{A/s}) = 15.0\ \text{V}.$$

The top circuit loop: $60.0\ \text{V} = i_1 R_1 \Rightarrow i_1 = \dfrac{60.0\ \text{V}}{40.0\ \Omega} = 1.50\ \text{A}.$

The bottom loop: $60\ \text{V} - i_2 R_2 - 15.0\ \text{V} = 0 \Rightarrow i_2 = \dfrac{45.0\ \text{V}}{25.0\ \Omega} = 1.80\ \text{A}.$

(b) After a long time: $i_2 = \dfrac{60.0\ \text{V}}{25.0\ \Omega} = 2.40\ \text{A},$ and immediately when the switch is opened, the inductor maintains this current, so $i_1 = i_2 = 2.40\ \text{A}.$

EVALUATE: The current through R_1 changes abruptly when the switch is closed.

30.69. **IDENTIFY** and **SET UP:** The circuit is sketched in Figure 30.69a. Apply the loop rule. Just after S_1 is closed, $i = 0$. After a long time i has reached its final value and $di/dt = 0$. The voltage across a resistor depends on i and the voltage across an inductor depends on di/dt.

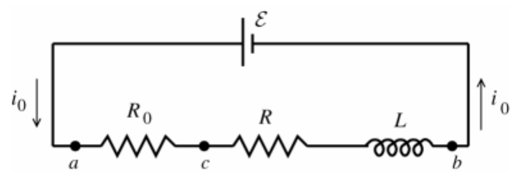


Figure 30.69a

EXECUTE: **(a)** At time $t = 0$, $i_0 = 0$ so $v_{ac} = i_0 R_0 = 0$. By the loop rule $\mathcal{E} - v_{ac} - v_{cb} = 0$, so $v_{cb} = \mathcal{E} - v_{ac} = \mathcal{E} = 36.0\ \text{V}.$ ($i_0 R = 0$ so this potential difference of 36.0 V is across the inductor and is an induced emf produced by the changing current.)

(b) After a long time $\dfrac{di_0}{dt} \to 0$ so the potential $-L\dfrac{di_0}{dt}$ across the inductor becomes zero. The loop rule gives $\mathcal{E} - i_0(R_0 + R) = 0.$

$$i_0 = \frac{\mathcal{E}}{R_0 + R} = \frac{36.0\ \text{V}}{50.0\ \Omega + 150\ \Omega} = 0.180\ \text{A}$$

$$v_{ac} = i_0 R_0 = (0.180\ \text{A})(50.0\ \Omega) = 9.0\ \text{V}$$

Thus $v_{cb} = i_0 R + L\dfrac{di_0}{dt} = (0.180\ \text{A})(150\ \Omega) + 0 = 27.0\ \text{V}$ (Note that $v_{ac} + v_{cb} = \mathcal{E}$.)

(c) $\mathcal{E} - v_{ac} - v_{cb} = 0$

$$\mathcal{E} - iR_0 - iR - L\frac{di}{dt} = 0$$

$$L\frac{di}{dt} = \mathcal{E} - i(R_0 + R) \text{ and } \left(\frac{L}{R + R_0}\right)\frac{di}{dt} = -i + \frac{\mathcal{E}}{R + R_0}$$

$$\frac{di}{-i + \mathcal{E}/(R + R_0)} = \left(\frac{R + R_0}{L}\right)dt$$

Integrate from $t = 0$, when $i = 0$, to t, when $i = i_0$: $\displaystyle\int_0^{i_0} \frac{di}{-i + \mathcal{E}/(R + R_0)} = \frac{R + R_0}{L}\int_0^t dt = -\ln\left[-i + \frac{\mathcal{E}}{R + R_0}\right]_0^{i_0} = \left(\frac{R + R_0}{L}\right)t,$

so $\ln\left(-i_0 + \dfrac{\mathcal{E}}{R + R_0}\right) - \ln\left(\dfrac{\mathcal{E}}{R + R_0}\right) = -\left(\dfrac{R + R_0}{L}\right)t$

$$\ln\left(\frac{-i_0 + \mathcal{E}/(R + R_0)}{\mathcal{E}/(R + R_0)}\right) = -\left(\frac{R + R_0}{L}\right)t$$

Taking exponentials of both sides gives $\dfrac{-i_0 + \mathcal{E}/(R + R_0)}{\mathcal{E}/(R + R_0)} = e^{-(R + R_0)t/L}$ and $i_0 = \dfrac{\mathcal{E}}{R + R_0}\left(1 - e^{-(R + R_0)t/L}\right)$

Substituting in the numerical values gives $i_0 = \dfrac{36.0\ \text{V}}{50\ \Omega + 150\ \Omega}\left(1 - e^{-(200\ \Omega/4.00\ \text{H})t}\right) = (0.180\ \text{A})\left(1 - e^{-t/0.020\ \text{s}}\right)$

At $t \to 0$, $i_0 = (0.180 \text{ A})(1-1) = 0$ (agrees with part (a)). At $t \to \infty$, $i_0 = (0.180 \text{ A})(1-0) = 0.180 \text{ A}$ (agrees with part (b)).

$v_{ac} = i_0 R_0 = \dfrac{\mathcal{E}R_0}{R+R_0}\left(1-e^{-(R+R_0)t/L}\right) = 9.0 \text{ V}\left(1-e^{-t/0.020 \text{ s}}\right)$

$v_{cb} = \mathcal{E} - v_{ac} = 36.0 \text{ V} - 9.0 \text{ V}(1-e^{-t/0.020 \text{ s}}) = 9.0 \text{ V}(3.00 + e^{-t/0.020 \text{ s}})$

At $t \to 0$, $v_{ac} = 0$, $v_{cb} = 36.0$ V (agrees with part (a)). At $t \to \infty$, $v_{ac} = 9.0$ V, $v_{cb} = 27.0$ V (agrees with part (b)).
The graphs are given in Figure 30.69b.

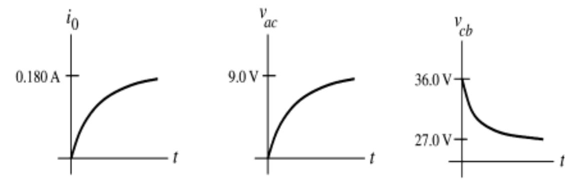


Figure 30.69b

EVALUATE: The expression for $i(t)$ we derived becomes Eq.(30.14) if the two resistors R_0 and R in series are replaced by a single equivalent resistance $R_0 + R$.

30.70. **IDENTIFY:** Apply the loop rule. The current through the inductor doesn't change abruptly.
SET UP: With S_2 closed, v_{cb} must be zero.
EXECUTE: **(a)** Immediately after S_2 is closed, the inductor maintains the current $i = 0.180$ A through R. The loop rule around the outside of the circuit yields

$\mathcal{E} + \mathcal{E}_L - iR - i_0R_0 = 36.0 \text{ V} + (0.18 \text{ A})(150 \ \Omega) - (0.18 \text{ A})(150 \ \Omega) - i_0(50 \ \Omega) = 0$. $i_0 = \dfrac{36 \text{ V}}{50 \ \Omega} = 0.720 \text{ A}$.

$v_{ac} = (0.72 \text{ A})(50 \text{ V}) = 36.0 \text{ V}$ and $v_{cb} = 0$.

(b) After a long time, $v_{ac} = 36.0$ V, and $v_{cb} = 0$. Thus $i_0 = \dfrac{\mathcal{E}}{R_0} = \dfrac{36.0 \text{ V}}{50 \ \Omega} = 0.720 \text{ A}$, $i_R = 0$ and $i_{s2} = 0.720 \text{ A}$.

(c) $i_0 = 0.720 \text{ A}$, $i_R(t) = \dfrac{\mathcal{E}}{R_{\text{total}}} e^{-(R/L)t}$ and $i_R(t) = (0.180 \text{ A})e^{-(12.5 \text{ s}^{-1})t}$.

$i_{s2}(t) = (0.720 \text{ A}) - (0.180 \text{ A})e^{-(12.5 \text{ s}^{-1})t} = (0.180 \text{ A})\left(4 - e^{-(12.5 \text{ s}^{-1})t}\right)$. The graphs of the currents are given in Figure 30.70.

EVALUATE: R_0 is in a loop that contains just $\mathcal{E}$ and R_0, so the current through R_0 is constant. After a long time the current through the inductor isn't changing and the voltage across the inductor is zero. Since v_{cb} is zero, the voltage across R must be zero and i_R becomes zero.

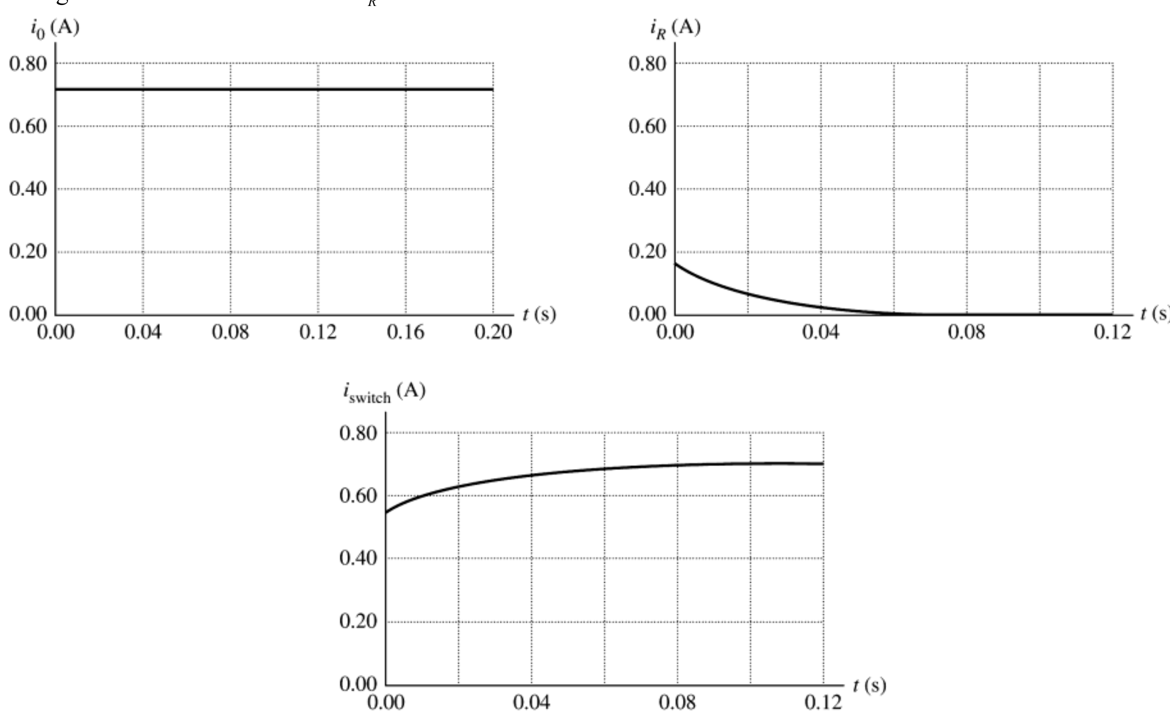


Figure 30.70

30.71. **IDENTIFY:** The current through an inductor doesn't change abruptly. After a long time the current isn't changing and the voltage across each inductor is zero.

SET UP: Problem 30.47 shows how to find the equivalent inductance of inductors in series and parallel.

EXECUTE: **(a)** Just after the switch is closed there is no current in the inductors. There is no current in the resistors so there is no voltage drop across either resistor. A reads zero and V reads 20.0 V.

(b) After a long time the currents are no longer changing, there is no voltage across the inductors, and the inductors can be replaced by short-circuits. The circuit becomes equivalent to the circuit shown in Figure 30.71a. $I = (20.0\text{ V})/(75.0\ \Omega) = 0.267\text{ A}$. The voltage between points a and b is zero, so the voltmeter reads zero.

(c) Use the results of Problem 30.49 to combine the inductor network into its equivalent, as shown in Figure 30.71b. $R = 75.0\ \Omega$ is the equivalent resistance. Eq.(30.14) says $i = (\mathcal{E}/R)(1-e^{-t/\tau})$ with

$\tau = L/R = (10.8\text{ mH})/(75.0\ \Omega) = 0.144\text{ ms}$. $\mathcal{E} = 20.0\text{ V}$, $R = 75.0\ \Omega$, $t = 0.115\text{ ms}$ so $i = 0.147\text{ A}$.

$V_R = iR = (0.147\text{ A})(75.0\ \Omega) = 11.0\text{ V}$. $20.0\text{ V} - V_R - V_L = 0$ and $V_L = 20.0\text{ V} - V_R = 9.0\text{ V}$. The ammeter reads 0.147 A and the voltmeter reads 9.0 V.

EVALUATE: The current through the battery increases from zero to a final value of 0.267 A. The voltage across the inductor network drops from 20.0 V to zero.

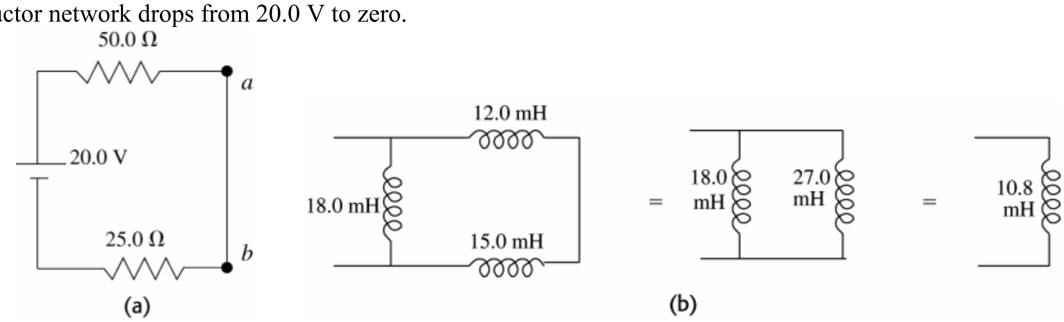


Figure 30.71

30.72. **IDENTIFY:** At steady state with the switch in position 1, no current flows to the capacitors and the inductors can be replaced by wires. Apply conservation of energy to the circuit with the switch in position 2.

SET UP: Replace the series combinations of inductors and capacitors by their equivalents. For the inductors use the results of Problem 30.47.

EXECUTE: **(a)** At steady state $i = \dfrac{\mathcal{E}}{R} = \dfrac{75.0\text{ V}}{125\ \Omega} = 0.600\text{ A}$.

(b) The equivalent circuit capacitance of the two capacitors is given by $\dfrac{1}{C_s} = \dfrac{1}{25\ \mu\text{F}} + \dfrac{1}{35\ \mu\text{F}}$ and $C_s = 14.6\ \mu\text{F}$.

$L_s = 15.0\text{ mH} + 5.0\text{ mH} = 20.0\text{ mH}$. The equivalent circuit is sketched in Figure 30.72a.

Energy conservation: $\dfrac{q^2}{2C} = \dfrac{1}{2}Li_0^2$. $q = i_0\sqrt{LC} = (0.600\text{ A})\sqrt{(20\times10^{-3}\text{ H})(14.6\times10^{-6}\text{ F})} = 3.24\times10^{-4}\text{ C}$. As shown in Figure 30.72b, the capacitors have their maximum charge at $t = T/4$.

$t = \tfrac{1}{4}T = \tfrac{1}{4}(2\pi\sqrt{LC}) = \dfrac{\pi}{2}\sqrt{LC} = \dfrac{\pi}{2}\sqrt{(20\times10^{-3}\text{ H})(14.6\times10^{-6}\text{ F})} = 8.49\times10^{-4}\text{ s}$

EVALUATE: With the switch closed the battery stores energy in the inductors. This then is the energy in the L-C circuit when the switch is in position 2.

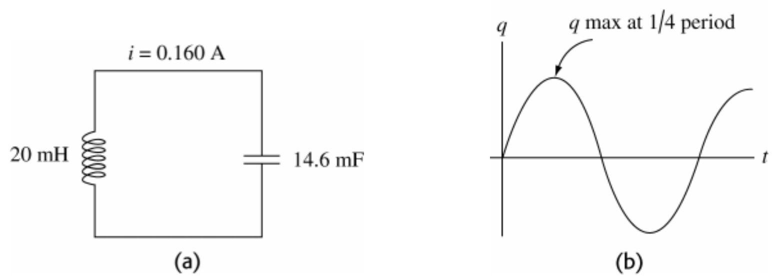


Figure 30.72

30.73. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: Find the flux through a ring of height h, radius r and thickness dr. Example 28.19 shows that $B = \dfrac{\mu_0 Ni}{2\pi r}$ inside the toroid.

EXECUTE: **(a)** $\Phi_B = \int_a^b B(h\,dr) = \int_a^b \left(\frac{\mu_0 Ni}{2\pi r}\right)(h\,dr) = \frac{\mu_0 Nih}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 Nih}{2\pi}\ln(b/a).$

(b) $L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 h}{2\pi}\ln(b/a).$

(c) $\ln(b/a) = \ln(1-(b-a)/a) \approx \frac{b-a}{a} + \frac{(b-a)^2}{2a^2} + \cdots \Rightarrow L \approx \frac{\mu_0 N^2 h}{2\pi}\left(\frac{b-a}{a}\right).$

EVALUATE: $h(b-a)$ is the cross-sectional area A of the toroid and a is approximately the radius r, so this result is approximately the same as the result derived in Example 30.3.

30.74. **IDENTIFY:** The direction of the current induced in circuit A is given by Lenz's law.

SET UP: When the switch is closed current flows counterclockwise in the circuit on the left, from the positive plate of the capacitor. The current decreases as a function of time, as the charge and voltage of the capacitor decrease.

EXECUTE: At loop A the magnetic field from the wire of the other circuit adjacent to A is into the page. The magnetic field of this current is decreasing, as the current decreases. Therefore, the magnetic field of the induced current in A is directed into the page inside A and to produce a magnetic field in this direction the induced current is clockwise.

EVALUATE: The magnitude of the emf induced in circuit A decreases with time after the switch is closed, because the rate of change of the current in the other circuit decreases.

30.75. **(a) IDENTIFY** and **SET UP:** With switch S closed the circuit is shown in Figure 30.75a.

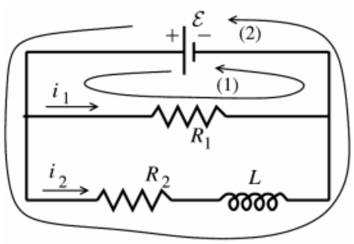


Figure 30.75a

Apply the loop rule to loops 1 and 2.

EXECUTE:

loop 1

$\mathcal{E} - i_1 R_1 = 0$

$i_1 = \frac{\mathcal{E}}{R_1}$ (independent of t)

loop (2)

$\mathcal{E} - i_2 R_2 - L\frac{di_2}{dt} = 0$

This is in the form of equation (30.12), so the solution is analogous to Eq.(30.14): $i_2 = \frac{\mathcal{E}}{R_2}\left(1 - e^{-R_2 t/L}\right)$

(b) EVALUATE: The expressions derived in part (a) give that as $t \to \infty$, $i_1 = \frac{\mathcal{E}}{R_1}$ and $i_2 = \frac{\mathcal{E}}{R_2}$. Since $\frac{di_2}{dt} \to 0$ at steady-state, the inductance then has no effect on the circuit. The current in R_1 is constant; the current in R_2 starts at zero and rises to $\mathcal{E}/R_2$.

(c) IDENTIFY and **SET UP:** The circuit now is as shown in Figure 30.75b.

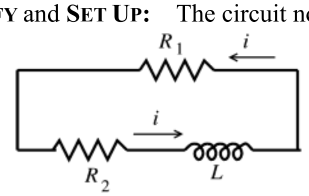


Figure 30.75b

Let $t = 0$ now be when S is opened.

At $t = 0$, $i = \frac{\mathcal{E}}{R_2}$.

Apply the loop rule to the single current loop.

EXECUTE: $-i(R_1 + R_2) - L\frac{di}{dt} = 0.$ (Now $\frac{di}{dt}$ is negative.)

$L\frac{di}{dt} = -i(R_1 + R_2)$ gives $\frac{di}{i} = -\left(\frac{R_1 + R_2}{L}\right)dt$

Integrate from $t = 0$, when $i = I_0 = \mathcal{E}/R_2$, to t.

$\int_{I_0}^{i}\frac{di}{i} = -\left(\frac{R_1 + R_2}{L}\right)\int_0^t dt$ and $\ln\left(\frac{i}{I_0}\right) = -\left(\frac{R_1 + R_2}{L}\right)t$

Taking exponentials of both sides of this equation gives $i = I_0 e^{-(R_1+R_2)t/L} = \frac{\mathcal{E}}{R_2}e^{-(R_1+R_2)t/L}$

(d) IDENTIFY and SET UP: Use the equation derived in part (c) and solve for R_2 and $\mathcal{E}$.

EXECUTE: $L = 22.0\text{ H}$

$R_{R_1} = \dfrac{V^2}{R_1} = 40.0\text{ W}$ gives $R_1 = \dfrac{V^2}{P_{R_1}} = \dfrac{(120\text{ V})^2}{40.0\text{ W}} = 360\ \Omega.$

We are asked to find R_2 and $\mathcal{E}$. Use the expression derived in part (c).

$I_0 = 0.600\text{ A}$ so $\mathcal{E}/R_2 = 0.600\text{ A}$

$i = 0.150\text{ A}$ when $t = 0.080\text{ s}$, so $i = \dfrac{\mathcal{E}}{R_2}e^{-(R_1+R_2)t/L}$ gives $0.150\text{ A} = (0.600\text{ A})e^{-(R_1+R_2)t/L}$

$\frac{1}{4} = e^{-(R_1+R_2)t/L}$ so $\ln 4 = (R_1+R_2)t/L$

$R_2 = \dfrac{L\ln 4}{t} - R_1 = \dfrac{(22.0\text{ H})\ln 4}{0.080\text{ s}} - 360\ \Omega = 381.2\ \Omega - 360\ \Omega = 21.2\ \Omega$

Then $\mathcal{E} = (0.600\text{ A})R_2 = (0.600\text{ A})(21.2\ \Omega) = 12.7\text{ V}.$

(e) IDENTIFY and SET UP: Use the expressions derived in part (a).

EXECUTE: The current through the light bulb before the switch is opened is $i_1 = \dfrac{\mathcal{E}}{R_1} = \dfrac{12.7\text{ V}}{360\ \Omega} = 0.0353\text{ A}$

EVALUATE: When the switch is opened the current through the light bulb jumps from 0.0353 A to 0.600 A. Since the electrical power dissipated in the bulb (brightness) depend on i^2, the bulb suddenly becomes much brighter.

30.76. IDENTIFY: Follow the steps specified in the problem.

SET UP: The current in an inductor does not change abruptly.

EXECUTE: **(a)** Using Kirchhoff's loop rule on the left and right branches:

Left: $\mathcal{E} - (i_1+i_2)R - L\dfrac{di_1}{dt} = 0 \Rightarrow R(i_1+i_2) + L\dfrac{di_1}{dt} = \mathcal{E}.$

Right: $\mathcal{E} - (i_1+i_2)R - \dfrac{q_2}{C} = 0 \Rightarrow R(i_1+i_2) + \dfrac{q_2}{C} = \mathcal{E}.$

(b) Initially, with the switch just closed, $i_1 = 0,\ i_2 = \dfrac{\mathcal{E}}{R}$ and $q_2 = 0.$

(c) The substitution of the solutions into the circuit equations to show that they satisfy the equations is a somewhat tedious exercise but straightforward exercise. We will show that the initial conditions are satisfied:

At $t = 0,\ q_2 = \dfrac{\mathcal{E}}{\omega R}e^{-\beta t}\sin(\omega t) = \dfrac{\mathcal{E}}{\omega R}\sin(0) = 0.$

$i_1(t) = \dfrac{\mathcal{E}}{R}(1 - e^{-\beta t}[(2\omega RC)^{-1}\sin(\omega t) + \cos(\omega t)] \Rightarrow i_1(0) = \dfrac{\mathcal{E}}{R}(1 - [\cos(0)]) = 0.$

(d) When does i_2 first equal zero? $\omega = \sqrt{\dfrac{1}{LC} - \dfrac{1}{(2RC)^2}} = 625\text{ rad/s}.$

$i_2(t) = 0 = \dfrac{\mathcal{E}}{R}e^{-\beta t}[-(2\omega RC)^{-1}\sin(\omega t) + \cos(\omega t)] \Rightarrow -(2\omega RC)^{-1}\tan(\omega t) + 1 = 0$ and

$\tan(\omega t) = +2\omega RC = +2(625\text{ rad/s})(400\ \Omega)(2.00\times 10^{-6}\text{ F}) = +1.00.$

$\omega t = \arctan(+1.00) = +0.785 \Rightarrow t = \dfrac{0.785}{625\text{ rad/s}} = 1.256\times 10^{-3}\text{s}.$

EVALUATE: As $t \to \infty,\ i_1 \to \mathcal{E}/R,\ q_2 \to 0$ and $i_2 \to 0$.

30.77. IDENTIFY: Apply $L = \dfrac{N\Phi_B}{i}$ to calculate L.

SET UP: In the air the magnetic field is $B_{\text{Air}} = \dfrac{\mu_0 Ni}{W}$. In the liquid, $B_{\text{L}} = \dfrac{\mu Ni}{W}$

EXECUTE: **(a)** $\Phi_B = BA = B_{\text{L}}A_{\text{L}} + B_{\text{Air}}A_{\text{Air}} = \dfrac{\mu_0 Ni}{W}((D-d)W) + \dfrac{K\mu_0 Ni}{W}(dW) = \mu_0 Ni[(D-d)+Kd].$

$L = \dfrac{N\Phi_B}{i} = \mu_0 N^2[(D-d)+Kd] = L_0 - L_0\dfrac{d}{D} + L_{\text{f}}\dfrac{d}{D} = L_0 + \left(\dfrac{L_{\text{f}} - L_0}{D}\right)d.$

$d = \left(\dfrac{L - L_0}{L_{\text{f}} - L_0}\right)D$, where $L_0 = \mu_0 N^2 D$, and $L_f = K\mu_0 N^2 D.$

(b) Using $K = \chi_m + 1$ we can find the inductance for any height $L = L_0\left(1 + \chi_m \dfrac{d}{D}\right)$.

Height of Fluid	Inductance of Liquid Oxygen	Inductance of Mercury
$d = D/4$	0.63024 H	0.63000 H
$d = D/2$	0.63048 H	0.62999 H
$d = 3D/4$	0.63072 H	0.62999 H
$d = D$	0.63096 H	0.62998 H

The values $\chi_m(O_2) = 1.52\times10^{-3}$ and $\chi_m(\text{Hg}) = -2.9\times10^{-5}$ have been used.

EVALUATE: **(d)** The volume gauge is much better for the liquid oxygen than the mercury because there is an easily detectable spread of values for the liquid oxygen, but not for the mercury.

30.78. **IDENTIFY:** The induced emf across the two coils is due to both the self-inductance of each and the mutual inductance of the pair of coils.

SET UP: The equivalent inductance is defined by $\mathcal{E} = L_{eq}\dfrac{di}{dt}$, where $\mathcal{E}$ and i are the total emf and current across the combination.

EXECUTE: Series: $L_1\dfrac{di_1}{dt} + L_2\dfrac{di_2}{dt} + M_{21}\dfrac{di_1}{dt} + M_{12}\dfrac{di_2}{dt} \equiv L_{eq}\dfrac{di}{dt}$.

But $i = i_1 + i_2 \Rightarrow \dfrac{di}{dt} = \dfrac{di_1}{dt} + \dfrac{di_2}{dt}$ and $M_{12} = M_{21} \equiv M$, so $(L_1 + L_2 + 2M)\dfrac{di}{dt} = L_{eq}\dfrac{di}{dt}$ and $L_{eq} = L_1 + L_2 + 2M$.

Parallel: We have $L_1\dfrac{di_1}{dt} + M_{12}\dfrac{di_2}{dt} = L_{eq}\dfrac{di}{dt}$ and $L_2\dfrac{di_2}{dt} + M_{21}\dfrac{di_1}{dt} = L_{eq}\dfrac{di}{dt}$, with $\dfrac{di_1}{dt} + \dfrac{di_2}{dt} = \dfrac{di}{dt}$ and $M_{12} = M_{21} \equiv M$.

To simplify the algebra let $A = \dfrac{di_1}{dt}, B = \dfrac{di_2}{dt}$, and $C = \dfrac{di}{dt}$. So $L_1A + MB = L_{eq}C$, $L_2B + MA = L_{eq}C$, $A + B = C$. Now solve for A and B in terms of C. $(L_1 - M)A + (M - L_2)B = 0$ using $A = C - B$. $(L_1 - M)(C - B) + (M - L_2)B = 0$.

$(L_1 - M)C - (L_1 - M)B + (M - L_2)B = 0$. $(2M - L_1 - L_2)B = (M - L_1)C$ and $B = \dfrac{(M - L_1)}{(2M - L_1 - L_2)}C$.

But $A = C - B = C - \dfrac{(M - L_1)C}{(2M - L_1 - L_2)} = \dfrac{(2M - L_1 - L_2) - M + L_1}{(2M - L_1 - L_2)}C$, or $A = \dfrac{M - L_2}{2M - L_1 - L_2}C$. Substitute A in B back into original equation:

$\dfrac{L_1(M - L_2)C}{2M - L_1 - L_2} + \dfrac{M(M - L_1)}{(2M - L_1 - L_2)}C = L_{eq}C$ and $\dfrac{M^2 - L_1L_2}{2M - L_1 - L_2}C = L_{eq}C$. Finally, $L_{eq} = \dfrac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$.

EVALUATE: If the flux of one coil doesn't pass through the other coil, so $M = 0$, then the results reduce to those of problem 30.47.

30.79. **IDENTIFY:** Apply Kirchhoff's loop rule to the top and bottom branches of the circuit.

SET UP: Just after the switch is closed the current through the inductor is zero and the charge on the capacitor is zero.

EXECUTE: $\mathcal{E} - i_1R_1 - L\dfrac{di_1}{dt} = 0 \Rightarrow i_1 = \dfrac{\mathcal{E}}{R_1}(1 - e^{-(R_1/L)t})$. $\mathcal{E} - i_2R_2 - \dfrac{q_2}{C} = 0 \Rightarrow -\dfrac{di_2}{dt}R_2 - \dfrac{i_2}{C} = 0 \Rightarrow i_2 = \dfrac{\mathcal{E}}{R_2}e^{-(1/R_2C)t}$).

$q_2 = \int_0^t i_2dt' = -\dfrac{\mathcal{E}}{R_2}R_2Ce^{-(1/R_2C)t'}\Big|_0^t = \mathcal{E}C(1 - e^{-(1/R_2C)t})$.

(b) $i_1(0) = \dfrac{\mathcal{E}}{R_1}(1 - e^0) = 0$, $i_2 = \dfrac{\mathcal{E}}{R_2}e^0 = \dfrac{48.0\text{ V}}{5000\ \Omega} = 9.60\times10^{-3}$ A.

(c) As $t \to \infty$: $i_1(\infty) = \dfrac{\mathcal{E}}{R_1}(1 - e^{-\infty}) = \dfrac{\mathcal{E}}{R_1} = \dfrac{48.0\text{ V}}{25.0\ \Omega} = 1.92$ A, $i_2 = \dfrac{\mathcal{E}}{R_2}e^{-\infty} = 0$. A good definition of a "long time" is many time constants later.

(d) $i_1 = i_2 \Rightarrow \frac{\mathcal{E}}{R_1}(1 - e^{-(R_1/L)t}) = \frac{\mathcal{E}}{R_2} e^{-(1/R_2C)t} \Rightarrow (1 - e^{-(R_1/L)t}) = \frac{R_1}{R_2} e^{-(1/R_2C)t}$. Expanding the exponentials like $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots$, we find: $\frac{R_1}{L}t - \frac{1}{2}\left(\frac{R_1}{L}\right)^2 t^2 + \cdots = \frac{R_1}{R_2}\left(1 - \frac{t}{RC} + \frac{t^2}{2R^2C^2} - \cdots\right)$ and $t\left(\frac{R_1}{L} + \frac{R_1}{R_2{}^2C}\right) + O(t^2) + \cdots = \frac{R_1}{R_2}$, if we have assumed that $t << 1$. Therefore:

$$t \approx \frac{1}{R_2}\left(\frac{1}{(1/L) + (1/R_2{}^2C)}\right) = \left(\frac{LR_2C}{L + R_2{}^2C}\right) = \left(\frac{(8.0\text{ H})(5000\ \Omega)(2.0\times10^{-5}\text{ F})}{8.0\text{ H} + (5000\ \Omega)^2(2.0\times10^{-5}\text{ F})}\right) = 1.6\times10^{-3}\text{s.}$$

(e) At $t = 1.57\times10^{-3}\text{ s}$: $i_1 = \frac{\mathcal{E}}{R_1}(1 - e^{-(R_1/L)t}) = \frac{48\text{ V}}{25\ \Omega}(1 - e^{-(25/8)t}) = 9.4\times10^{-3}\text{ A}$.

(f) We want to know when the current is half its final value. We note that the current i_2 is very small to begin with, and just gets smaller, so we ignore it and find:

$i_{1/2} = 0.960\text{ A} = i_1 = \frac{\mathcal{E}}{R_1}(1 - e^{-(R_1/L)t}) = (1.92\text{ A})(1 - e^{-(R_1/L)t})$. $e^{-(R_1/L)t} = 0.500 \Rightarrow t = \frac{L}{R_1}\ln(0.5) = \frac{8.0\text{ H}}{25\ \Omega}\ln(0.5) = 0.22\text{ s}$.

EVALUATE: i_1 is initially zero and rises to a final value of 1.92 A. i_2 is initially 9.60 mA and falls to zero, q_2 is initially zero and rises to $q_2 = \mathcal{E}C = 960\ \mu\text{C}$.

ALTERNATING CURRENT

31

31.1. **IDENTIFY:** $i = I\cos\omega t$ and $I_{\text{rms}} = I/\sqrt{2}$.

SET UP: The specified value is the root-mean-square current; $I_{\text{rms}} = 0.34$ A.

EXECUTE: **(a)** $I_{\text{rms}} = 0.34$ A

(b) $I = \sqrt{2}I_{\text{rms}} = \sqrt{2}(0.34 \text{ A}) = 0.48$ A.

(c) Since the current is positive half of the time and negative half of the time, its average value is zero.

(d) Since I_{rms} is the square root of the average of i^2, the average square of the current is $I_{\text{rms}}^2 = (0.34 \text{ A})^2 = 0.12 \text{ A}^2$.

EVALUATE: The current amplitude is larger than its rms value.

31.2. **IDENTIFY** and **SET UP:** Apply Eqs.(31.3) and (31.4)

EXECUTE: **(a)** $I = \sqrt{2}I_{\text{rms}} = \sqrt{2}(2.10 \text{ A}) = 2.97$ A.

(b) $I_{\text{rav}} = \dfrac{2}{\pi}I = \dfrac{2}{\pi}(2.97 \text{ A}) = 1.89$ A.

EVALUATE: **(c)** The root-mean-square voltage is always greater than the rectified average, because squaring the current before averaging, and then taking the square root to get the root-mean-square value will always give a larger value than just averaging.

31.3. **IDENTIFY** and **SET UP:** Apply Eq.(31.5).

EXECUTE: **(a)** $V_{\text{rms}} = \dfrac{V}{\sqrt{2}} = \dfrac{45.0 \text{ V}}{\sqrt{2}} = 31.8$ V.

(b) Since the voltage is sinusoidal, the average is zero.

EVALUATE: The voltage amplitude is larger than V_{rms}.

31.4. **IDENTIFY:** $V = IX_C$ with $X_C = \dfrac{1}{\omega C}$.

SET UP: ω is the angular frequency, in rad/s.

EXECUTE: **(a)** $V = IX_C = \dfrac{I}{\omega C}$ so $I = V\omega C = (60.0 \text{ V})(100 \text{ rad/s})(2.20\times10^{-6} \text{ F}) = 0.0132$ A.

(b) $I = V\omega C = (60.0 \text{ V})(1000 \text{ rad/s})(2.20\times10^{-6} \text{ F}) = 0.132$ A.

(c) $I = V\omega C = (60.0 \text{ V})(10{,}000 \text{ rad/s})(2.20\times10^{-6} \text{ F}) = 1.32$ A.

(d) The plot of $\log I$ versus $\log\omega$ is given in Figure 31.4.

EVALUATE: $I = \omega VC$ so $\log I = \log(VC) + \log\omega$. A graph of $\log I$ versus $\log\omega$ should be a straight line with slope +1, and that is what Figure 31.4 shows.

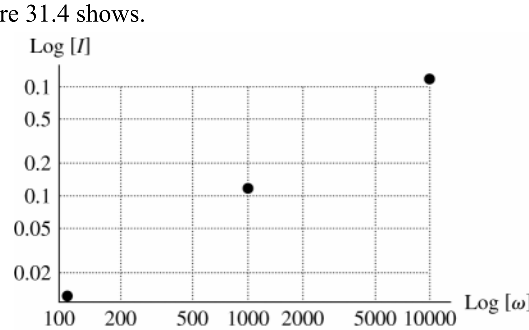


Figure 31.4

31.5. **IDENTIFY:** $V = IX_L$ with $X_L = \omega L$.

SET UP: ω is the angular frequency, in rad/s.

EXECUTE: **(a)** $V = IX_L = I\omega L$ and $I = \dfrac{V}{\omega L} = \dfrac{60.0\text{ V}}{(100\text{ rad/s})(5.00\text{ H})} = 0.120\text{ A}$.

(b) $I = \dfrac{V}{\omega L} = \dfrac{60.0\text{ V}}{(1000\text{ rad/s})(5.00\text{ H})} = 0.0120\text{ A}$.

(c) $I = \dfrac{V}{\omega L} = \dfrac{60.0\text{ V}}{(10{,}000\text{ rad/s})(5.00\text{ H})} = 0.00120\text{ A}$.

(d) The plot of $\log I$ versus $\log \omega$ is given in Figure 31.5.

EVALUATE: $I = \dfrac{V}{\omega L}$ so $\log I = \log(V/L) - \log \omega$. A graph of $\log I$ versus $\log \omega$ should be a straight line with slope -1, and that is what Figure 31.5 shows.

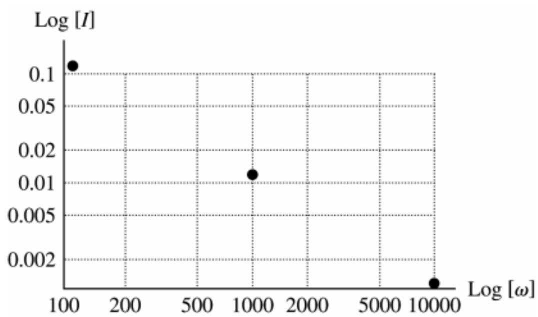


Figure 31.5

31.6. **IDENTIFY:** The reactance of capacitors and inductors depends on the angular frequency at which they are operated, as well as their capacitance or inductance.

SET UP: The reactances are $X_C = 1/\omega C$ and $X_L = \omega L$.

EXECUTE: **(a)** Equating the reactances gives $\omega L = \dfrac{1}{\omega C} \Rightarrow \omega = \dfrac{1}{\sqrt{LC}}$

(b) Using the numerical values we get $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(5.00\text{ mH})(3.50\ \mu\text{F})}} = 7560\text{ rad/s}$

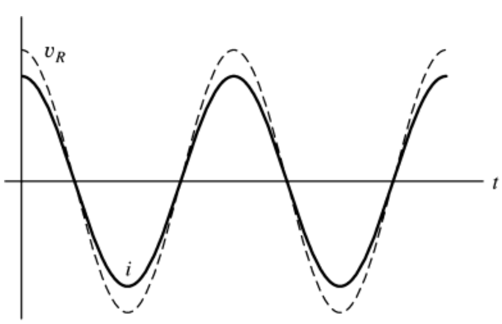
$$X_C = X_L = \omega L = (7560\text{ rad/s})(5.00\text{ mH}) = 37.8\ \Omega$$

EVALUATE: At other angular frequencies, the two reactances could be very different.

31.7. **IDENTIFY** and **SET UP:** For a resistor $v_R = iR$. For an inductor, $v_L = V\cos(\omega t + 90°)$. For a capacitor, $v_C = V\cos(\omega t - 90°)$.

EXECUTE: The graphs are sketched in Figures 31.7a-c. The phasor diagrams are given in Figure 31.7d.

EVALUATE: For a resistor only in the circuit, the current and voltage in phase. For an inductor only, the voltage leads the current by $90°$. For a capacitor only, the voltage lags the current by $90°$.



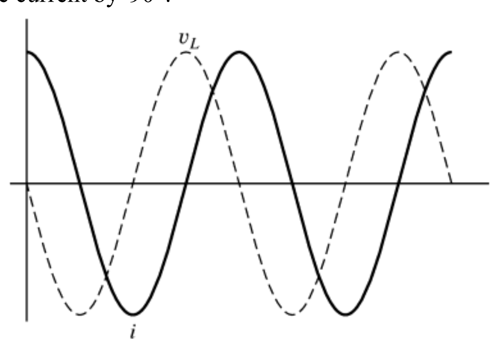


Figure 31.7a and b

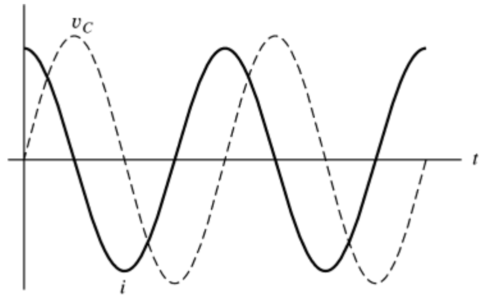


Figure 31.7c

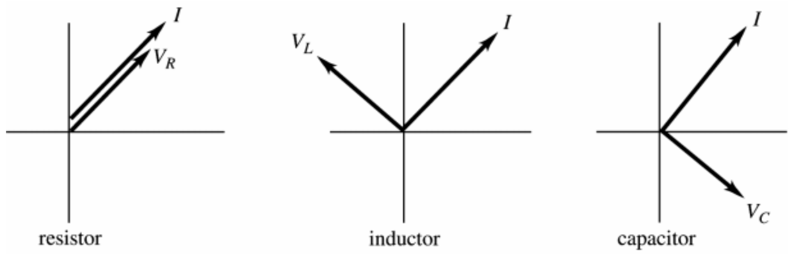


Figure 31.7d

31.8. **IDENTIFY:** The reactance of an inductor is $X_L = \omega L = 2\pi fL$. The reactance of a capacitor is $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC}$.

SET UP: The frequency *f* is in Hz.

EXECUTE: **(a)** At 60.0 Hz, $X_L = 2\pi(60.0\text{ Hz})(0.450\text{ H}) = 170\ \Omega$. X_L is proportional to *f* so at 600 Hz, $X_L = 1700\ \Omega$.

(b) At 60.0 Hz, $X_C = \dfrac{1}{2\pi(60.0\text{ Hz})(2.50\times10^{-6}\text{ F})} = 1.06\times10^{3}\ \Omega$. X_C is proportional to $1/f$, so at 600 Hz, $X_C = 106\ \Omega$.

(c) $X_L = X_C$ says $2\pi fL = \dfrac{1}{2\pi fC}$ and $f = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(0.450\text{ H})(2.50\times10^{-6}\text{ F})}} = 150\text{ Hz}$.

EVALUATE: X_L increases when *f* increases. X_C increases when *f* increases.

31.9. **IDENTIFY** and **SET UP:** Use Eqs.(31.12) and (31.18).

EXECUTE: **(a)** $X_L = \omega L = 2\pi fL = 2\pi(80.0\text{ Hz})(3.00\text{ H}) = 1510\ \Omega$

(b) $X_L = 2\pi fL$ gives $L = \dfrac{X_L}{2\pi f} = \dfrac{120\ \Omega}{2\pi(80.0\text{ Hz})} = 0.239\text{ H}$

(c) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(80.0\text{ Hz})(4.00\times10^{-6}\text{ F})} = 497\ \Omega$

(d) $X_C = \dfrac{1}{2\pi fC}$ gives $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi(80.0\text{ Hz})(120\ \Omega)} = 1.66\times10^{-5}\text{ F}$

EVALUATE: X_L increases when *L* increases; X_C decreases when *C* increases.

31.10. **IDENTIFY:** $V_L = I\omega L$

SET UP: ω is the angular frequency, in rad/s. $f = \dfrac{\omega}{2\pi}$ is the frequency in Hz.

EXECUTE: $V_L = I\omega L$ so $f = \dfrac{V_L}{2\omega IL} = \dfrac{(12.0\text{ V})}{2\pi(2.60\times10^{-3}\text{ A})(4.50\times10^{-4}\text{ H})} = 1.63\times10^{6}\text{ Hz}$.

EVALUATE: When *f* is increased, *I* decreases.

31.11. **IDENTIFY** and **SET UP:** Apply Eqs.(31.18) and (31.19).

EXECUTE: $V = IX_C$ so $X_C = \dfrac{V}{I} = \dfrac{170\text{ V}}{0.850\text{ A}} = 200\ \Omega$

$X_C = \dfrac{1}{\omega C}$ gives $C = \dfrac{1}{2\pi f X_C} = \dfrac{1}{2\pi(60.0\text{ Hz})(200\ \Omega)} = 1.33\times10^{-5}\text{ F} = 13.3\ \mu\text{F}$

EVALUATE: The reactance relates the voltage amplitude to the current amplitude and is similar to Ohm's law.

31.12. **IDENTIFY:** Compare v_C that is given in the problem to the general form $v_C = \frac{I}{\omega C}\sin\omega t$ and determine ω.

SET UP: $X_C = \frac{1}{\omega C}$. $v_R = iR$ and $i = I\cos\omega$.

EXECUTE: **(a)** $X_C = \frac{1}{\omega C} = \frac{1}{(120 \text{ rad/s})(4.80\times10^{-6} \text{ F})} = 1736\ \Omega$

(b) $I = \frac{V_C}{X_C} = \frac{7.60 \text{ V}}{1736\ \Omega} = 4.378\times10^{-3}$ A and $i = I\cos\omega t = (4.378\times10^{-3} \text{ A})\cos[(120 \text{ rad/s})t]$. Then

$v_R = iR = (4.38\times10^{-3} \text{ A})(250\ \Omega)\cos((120 \text{ rad/s})t) = (1.10 \text{ V})\cos((120 \text{ rad/s})t)$.

EVALUATE: The voltage across the resistor has a different phase than the voltage across the capacitor.

31.13. **IDENTIFY** and **SET UP:** The voltage and current for a resistor are related by $v_R = iR$. Deduce the frequency of the voltage and use this in Eq.(31.12) to calculate the inductive reactance. Eq.(31.10) gives the voltage across the inductor.

EXECUTE: **(a)** $v_R = (3.80 \text{ V})\cos[(720 \text{ rad/s})t]$

$v_R = iR$, so $i = \frac{v_R}{R} = \left(\frac{3.80 \text{ V}}{150\ \Omega}\right)\cos[(720 \text{ rad/s})t] = (0.0253 \text{ A})\cos[(720 \text{ rad/s})t]$

(b) $X_L = \omega L$

$\omega = 720$ rad/s, $L = 0.250$ H, so $X_L = \omega L = (720 \text{ rad/s})(0.250 \text{ H}) = 180\ \Omega$

(c) If $i = I\cos\omega t$ then $v_L = V_L\cos(\omega t + 90°)$ (from Eq.31.10). $V_L = I\omega L = IX_L = (0.02533 \text{ A})(180\ \Omega) = 4.56$ V

$v_L = (4.56 \text{ V})\cos[(720 \text{ rad/s})t + 90°]$

But $\cos(a + 90°) = -\sin a$ (Appendix B), so $v_L = -(4.56 \text{ V})\sin[(720 \text{ rad/s})t]$.

EVALUATE: The current is the same in the resistor and inductor and the voltages are $90°$ out of phase, with the voltage across the inductor leading.

31.14. **IDENTIFY:** Calculate the reactance of the inductor and of the capacitor. Calculate the impedance and use that result to calculate the current amplitude.

SET UP: With no capacitor, $Z = \sqrt{R^2 + X_L^2}$ and $\tan\phi = \frac{X_L}{R}$. $X_L = \omega L$. $I = \frac{V}{Z}$. $V_L = IX_L$ and $V_R = IR$. For an inductor, the voltage leads the current.

EXECUTE: **(a)** $X_L = \omega L = (250 \text{ rad/s})(0.400 \text{ H}) = 100\ \Omega$. $Z = \sqrt{(200\ \Omega)^2 + (100\ \Omega)^2} = 224\ \Omega$.

(b) $I = \frac{V}{Z} = \frac{30.0 \text{ V}}{224\ \Omega} = 0.134$ A

(c) $V_R = IR = (0.134 \text{ A})(200\ \Omega) = 26.8$ V. $V_L = IX_L = (0.134 \text{ A})(100\ \Omega) = 13.4$ V.

(d) $\tan\phi = \frac{X_L}{R} = \frac{100\ \Omega}{200\ \Omega}$ and $\phi = +26.6°$. Since ϕ is positive, the source voltage leads the current.

(e) The phasor diagram is sketched in Figure 31.14.

EVALUATE: Note that $V_R + V_L$ is greater than V. The loop rule is satisfied at each instance of time but the voltages across R and L reach their maxima at different times.

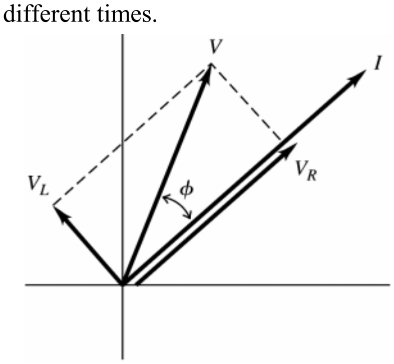


Figure 31.14

31.15. **IDENTIFY:** $v_R(t)$ is given by Eq.(31.8). $v_L(t)$ is given by Eq.(31.10).

SET UP: From Exercise 31.14, $V = 30.0$ V, $V_R = 26.8$ V, $V_L = 13.4$ V and $\phi = 26.6°$.

EXECUTE: **(a)** The graph is given in Figure 31.15.

(b) The different voltages are $v = (30.0 \text{ V})\cos(250t + 26.6°)$, $v_R = (26.8 \text{ V})\cos(250t)$, $v_L = (13.4 \text{ V})\cos(250t + 90°)$. At $t = 20$ ms: $v = 20.5$ V, $v_R = 7.60$ V, $v_L = 12.85$ V. Note that $v_R + v_L = v$.

(c) At $t = 40$ ms: $v = -15.2$ V, $v_R = -22.49$ V, $v_L = 7.29$ V. Note that $v_R + v_L = v$.

EVALUATE: It is important to be careful with radians versus degrees in above expressions!

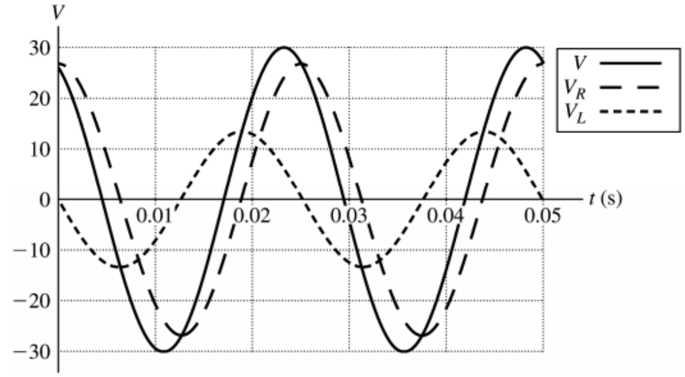


Figure 31.15

31.16. IDENTIFY: Calculate the reactance of the inductor and of the capacitor. Calculate the impedance and use that result to calculate the current amplitude.

SET UP: With no resistor, $Z = \sqrt{(X_L - X_C)^2} = |X_L - X_C|$. $\tan\phi = \dfrac{X_L - X_C}{\text{zero}}$. $X_C = \dfrac{1}{\omega C}$. $X_L = \omega L$. For an inductor, the voltage leads the current. For a capacitor, the voltage lags the current.

EXECUTE: **(a)** $X_L = \omega L = (250 \text{ rad/s})(0.400 \text{ H}) = 100\ \Omega$. $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(250 \text{ rad/s})(6.00\times 10^{-6} \text{ F})} = 667\ \Omega$.

$Z = |X_L - X_C| = |100\ \Omega - 667\ \Omega| = 567\ \Omega$.

(b) $I = \dfrac{V}{Z} = \dfrac{30.0 \text{ V}}{567\ \Omega} = 0.0529$ A

(c) $V_C = IX_C = (0.0529 \text{ A})(667\ \Omega) = 35.3$ V. $V_L = IX_L = (0.0529 \text{ A})(100\ \Omega) = 5.29$ V.

(d) $\tan\phi = \dfrac{X_L - X_C}{\text{zero}} = \dfrac{100\ \Omega - 667\ \Omega}{\text{zero}} = -\infty$ and $\phi = -90°$. The phase angle is negative and the source voltage lags the current.

(e) The phasor diagram is sketched in Figure 31.16.

EVALUATE: When $X_C > X_L$ the phase angle is negative and the source voltage lags the current.

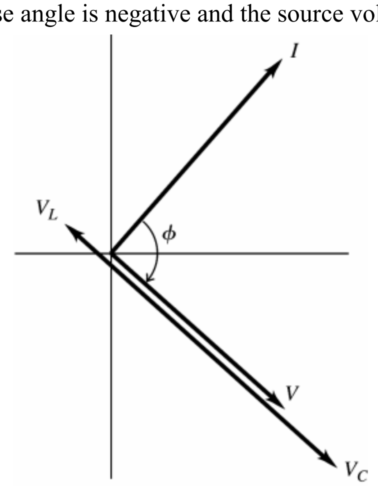


Figure 31.16

31.17. IDENTIFY and SET UP: Calculate the impendance of the circuit and use Eq.(31.22) to find the current amplitude. The voltage amplitudes across each circuit element are given by Eqs.(31.7), (31.13), and (31.19). The phase angle is calculated using Eq.(31.24). The circuit is shown in Figure 31.17a.

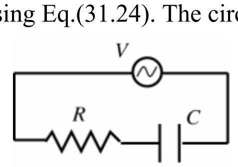


Figure 31.17a

No inductor means $X_L = 0$

$R = 200\ \Omega$, $C = 6.00\times 10^{-6}$ F,

$V = 30.0$ V, $\omega = 250$ rad/s

EXECUTE: **(a)** $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(250\text{ rad/s})(6.00\times10^{-6}\text{ F})} = 666.7\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\ \Omega)^2 + (666.7\ \Omega)^2} = 696\ \Omega$

(b) $I = \dfrac{V}{Z} = \dfrac{30.0\text{ V}}{696\ \Omega} = 0.0431\text{ A} = 43.1\text{ mA}$

(c) Voltage amplitude across the resistor: $V_R = IR = (0.0431\text{ A})(200\ \Omega) = 8.62\text{ V}$

Voltage amplitude across the capacitor: $V_C = IX_C = (0.0431\text{ A})(666.7\ \Omega) = 28.7\text{ V}$

(d) $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{0 - 666.7\ \Omega}{200\ \Omega} = -3.333$ so $\phi = -73.3°$

The phase angle is negative, so the source voltage lags behind the current.

(e) The phasor diagram is sketched qualitatively in Figure 31.17b.

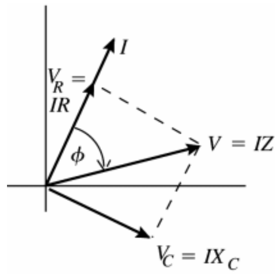


Figure 31.17b

EVALUATE: The voltage across the resistor is in phase with the current and the capacitor voltage lags the current by 90°. The presence of the capacitor causes the source voltage to lag behind the current. Note that $V_R + V_C > V$. The instantaneous voltages in the circuit obey the loop rule at all times but because of the phase differences the voltage amplitudes do not.

31.18. **IDENTIFY:** $v_R(t)$ is given by Eq.(31.8). $v_C(t)$ is given by Eq.(31.16).

SET UP: From Exercise 31.17, $V = 30.0$ V, $V_R = 8.62$ V, $V_C = 28.7$ V and $\phi = -73.3°$.

EXECUTE: **(a)** The graph is given in Figure 31.18.

(b) The different voltage are:

$v = (30.0\text{ V})\cos(250t - 73.3°)$, $v_R = (8.62\text{ V})\cos(250t)$, $v_C = (28.7\text{ V})\cos(250t - 90°)$. At $t = 20$ ms:

$v = -25.1$ V, $v_R = 2.45$ V, $v_C = -27.5$ V. Note that $v_R + v_C = v$.

(c) At $t = 40$ ms: $v = -22.9$ V, $v_R = -7.23$ V, $v_C = -15.6$ V. Note that $v_R + v_C = v$.

EVALUATE: It is important to be careful with radians vs. degrees!

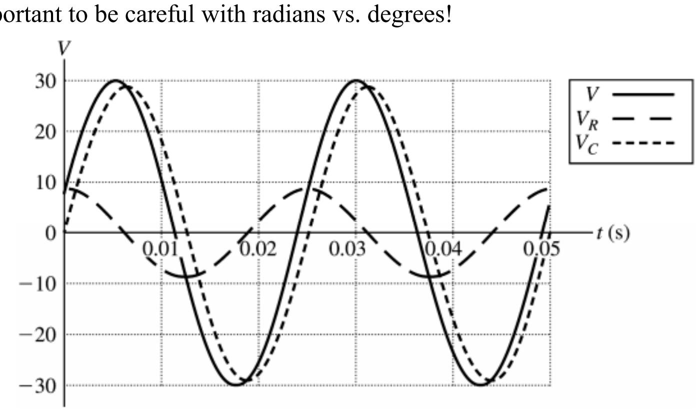


Figure 31.18

31.19. **IDENTIFY:** Apply the equations in Section 31.3.

SET UP: $\omega = 250$ rad/s, $R = 200\ \Omega$, $L = 0.400$ H, $C = 6.00\ \mu$F and $V = 30.0$ V.

EXECUTE: **(a)** $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$.

$Z = \sqrt{(200\ \Omega)^2 + ((250\text{ rad/s})(0.0400\text{ H}) - 1/((250\text{ rad/s})(6.00\times10^{-6}\text{ F})))^2} = 601\ \Omega$

(b) $I = \dfrac{V}{Z} = \dfrac{30\text{ V}}{601\ \Omega} = 0.0499$ A.

(c) $\phi = \arctan\left(\dfrac{\omega L - 1/\omega C}{R}\right) = \arctan\left(\dfrac{100\ \Omega - 667\ \Omega}{200\ \Omega}\right) = -70.6°$, and the voltage lags the current.

(d) $V_R = IR = (0.0499\text{ A})(200\ \Omega) = 9.98\text{ V}$;

$V_L = I\omega L = (0.0499\text{ A})(250\text{ rad/s})(0.400\text{ H}) = 4.99\text{ V}$; $V_C = \dfrac{I}{\omega C} = \dfrac{(0.0499\text{ A})}{(250\text{ rad/s})(6.00\times 10^{-6}\text{ F})} = 33.3\text{ V}$.

EVALUATE: **(e)** At any instant, $v = v_R + v_C + v_L$. But v_C and v_L are 180° out of phase, so v_C can be larger than v at a value of t, if $v_L + v_R$ is negative at that t.

31.20. **IDENTIFY:** $v_R(t)$ is given by Eq.(31.8). $v_C(t)$ is given by Eq.(31.16). $v_L(t)$ is given by Eq.(31.10).

SET UP: From Exercise 31.19, $V = 30.0\text{ V}$, $V_L = 4.99\text{ V}$, $V_R = 9.98\text{ V}$, $V_C = 33.3\text{ V}$ and $\phi = -70.6°$.

EXECUTE: **(a)** The graph is sketched in Figure 31.20. The different voltages plotted in the graph are:
$v = (30\text{ V})\cos(250t - 70.6°)$, $v_R = (9.98\text{ V})\cos(250t)$, $v_L = (4.99\text{ V})\cos(250t + 90°)$ and $v_C = (33.3\text{ V})\cos(250t - 90°)$.

(b) At $t = 20$ ms: $v = -24.3\text{ V}$, $v_R = 2.83\text{ V}$, $v_L = 4.79\text{ V}$, $v_C = -31.9\text{ V}$.

(c) At $t = 40$ ms: $v = -23.8\text{ V}$, $v_R = -8.37\text{ V}$, $v_L = 2.71\text{ V}$, $v_C = -18.1\text{ V}$.

EVALUATE: In both parts (b) and (c), note that the source voltage equals the sum of the other voltages at the given instant. Be careful with degrees versus radians!

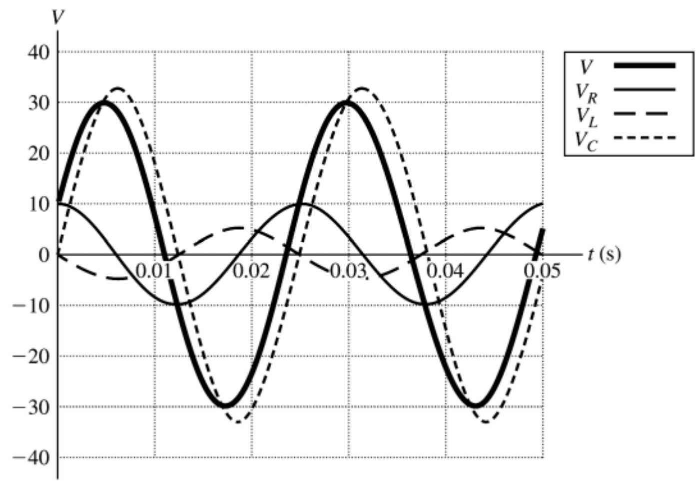


Figure 31.20

31.21. **IDENTIFY and SET UP:** The current is largest at the resonance frequency. At resonance, $X_L = X_C$ and $Z = R$. For part (b), calculate Z and use $I = V/Z$.

EXECUTE: **(a)** $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 113\text{ Hz}$. $I = V/R = 15.0\text{ mA}$.

(b) $X_C = 1/\omega C = 500\ \Omega$. $X_L = \omega L = 160\ \Omega$. $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\ \Omega)^2 + (160\ \Omega - 500\ \Omega)^2} = 394.5\ \Omega$.
$I = V/Z = 7.61\text{ mA}$. $X_C > X_L$ so the source voltage lags the current.

EVALUATE: $\omega_0 = 2\pi f_0 = 710\text{ rad/s}$. $\omega = 400\text{ rad/s}$ and is less than ω_0. When $\omega < \omega_0$, $X_C > X_L$. Note that I in part (b) is less than I in part (a).

31.22. **IDENTIFY:** The impedance and individual reactances depend on the angular frequency at which the circuit is driven.

SET UP: The impedance is $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$, the current amplitude is $I = V/Z$, and the instantaneous values of the potential and current are $v = V\cos(\omega t + \phi)$, where $\tan\phi = (X_L - X_C)/R$, and $i = I\cos\omega t$.

EXECUTE: **(a)** Z is a minimum when $\omega L = \dfrac{1}{\omega C}$, which gives $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(8.00\text{ mH})(12.5\ \mu\text{F})}}$ = 3162 rad/s = 3160 rad/s and $Z = R = 175\ \Omega$.

(b) $I = V/Z = (25.0\text{ V})/(175\ \Omega) = 0.143\text{ A}$

(c) $i = I\cos\omega t = I/2$, so $\cos\omega t = \frac{1}{2}$, which gives $\omega t = 60° = \pi/3$ rad. $v = V\cos(\omega t + \phi)$, where $\tan\phi = (X_L - X_C)/R = 0/R = 0$. So, $v = (25.0\text{ V})\cos\omega t = (25.0\text{ V})(1/2) = 12.5\text{ V}$.
$v_R = Ri = (175\ \Omega)(1/2)(0.143\text{ A}) = 12.5\text{ V}$.
$v_C = V_C\cos(\omega t - 90°) = IX_C\cos(\omega t - 90°) = \dfrac{0.143\text{ A}}{(3162\text{ rad/s})(12.5\ \mu\text{F})}\cos(60° - 90°) = +3.13\text{ V}$.

$v_L = V_L\cos(\omega t + 90°) = IX_L\cos(\omega t + 90°) = (0.143\text{ A})(3162\text{ rad/s})(8.00\text{ mH})\cos(60° + 90°)$.
$v_L = -3.13$ V.
(d) $v_R + v_L + v_C = 12.5\text{ V} + (-3.13\text{ V}) + 3.13\text{ V} = 12.5\text{ V} = v_{source}$

EVALUATE: The instantaneous potential differences across all the circuit elements always add up to the value of the source voltage at that instant. In this case (resonance), the potentials across the inductor and capacitor have the same magnitude but are 180° out of phase, so they add to zero, leaving all the potential difference across the resistor.

31.23. IDENTIFY and SET UP: Use the equation that preceeds Eq.(31.20): $V^2 = V_R^2 + (V_L - V_C)^2$

EXECUTE: $V = \sqrt{(30.0\text{ V})^2 + (50.0\text{ V} - 90.0\text{ V})^2} = 50.0\text{ V}$

EVALUATE: The equation follows directly from the phasor diagrams of Fig.31.13 (b or c). Note that the voltage amplitudes do not simply add to give 170.0 V for the source voltage.

31.24. IDENTIFY and SET UP: $X_L = \omega L$and $X_C = \dfrac{1}{\omega C}$.

EXECUTE: **(a)** If $\omega = \omega_0 = \dfrac{1}{\sqrt{LC}}$, then $X = \omega L - \dfrac{1}{\omega C}$and $X = \dfrac{L}{\sqrt{LC}} - \dfrac{1}{C/\sqrt{LC}} = 0$.

(b) When $\omega > \omega_0,\ X > 0$

(c) When $\omega > \omega_0,\ X < 0$

(d) The graph of X versus ω is given in Figure 31.24.

EVALUATE: $Z = \sqrt{R^2 + X^2}$ and $\tan\phi = X/R$.

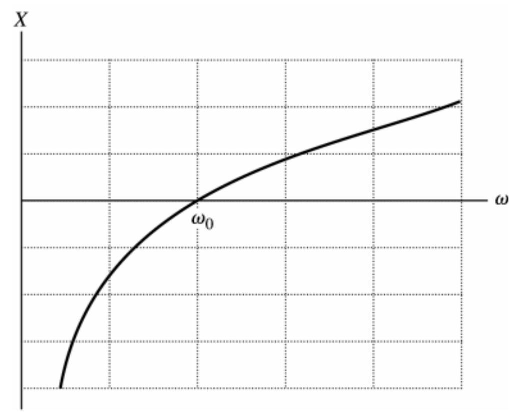


Figure 31.24

31.25. IDENTIFY: For a pure resistance, $P_{av} = V_{rms}I_{rms} = I_{rms}^2 R$.

SET UP: 20.0 W is the average power P_{av}.

EXECUTE: **(a)** The average power is one-half the maximum power, so the maximum instantaneous power is 40.0 W.

(b) $I_{rms} = \dfrac{P_{av}}{V_{rms}} = \dfrac{20.0\text{ W}}{120\text{ V}} = 0.167\text{ A}$

(c) $R = \dfrac{P_{av}}{I_{rms}^2} = \dfrac{20.0\text{ W}}{(0.167\text{ A})^2} = 720\ \Omega$

EVALUATE: We can also calculate the average power as $P_{av} = \dfrac{V_{R,rms}^2}{R} = \dfrac{V_{rms}^2}{R} = \dfrac{(120\text{ V})^2}{750\ \Omega} = 20.0\text{ W}$.

31.26. IDENTIFY: The average power supplied by the source is $P = V_{rms}I_{rms}\cos\phi$. The power consumed in the resistance is $P = I_{rms}^2 R$.

SET UP: $\omega = 2\pi f = 2\pi(1.25\times10^3\text{ Hz}) = 7.854\times10^3\text{ rad/s}$. $X_L = \omega L = 157\ \Omega$. $X_C = \dfrac{1}{\omega C} = 909\ \Omega$.

EXECUTE: **(a)** First, let us find the phase angle between the voltage and the current:
$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{157\ \Omega - 909\ \Omega}{350\ \Omega}$ and $\phi = -65.04°$. The impedance of the circuit is
$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(350\ \Omega)^2 + (-752\ \Omega)^2} = 830\ \Omega$. The average power provided by the generator is then
$P = V_{rms}I_{rms}\cos(\phi) = \dfrac{V_{rms}^2}{Z}\cos(\phi) = \dfrac{(120\text{ V})^2}{830\ \Omega}\cos(-65.04°) = 7.32\text{ W}$

(b) The average power dissipated by the resistor is $P_R = I_{\text{rms}}^2 R = \left(\dfrac{120\ \text{V}}{830\ \Omega}\right)^2 (350\ \Omega) = 7.32\ \text{W}.$

EVALUATE: Conservation of energy requires that the answers to parts (a) and (b) are equal.

31.27. **IDENTIFY:** The power factor is $\cos\phi$, where ϕ is the phase angle in Fig.31.13. The average power is given by Eq.(31.31). Use the result of part (a) to rewrite this expression.

(a) SET UP: The phasor diagram is sketched in Figure 31.27.

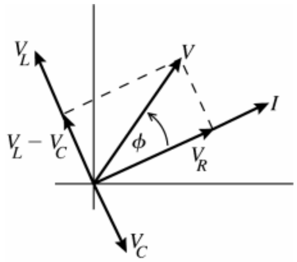


Figure 31.27

EXECUTE: From the diagram $\cos\phi = \dfrac{V_R}{V} = \dfrac{IR}{IZ} = \dfrac{R}{Z}$, as was to be shown.

(b) $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi = V_{\text{rms}} I_{\text{rms}} \left(\dfrac{R}{Z}\right) = \left(\dfrac{V_{\text{rms}}}{Z}\right) I_{\text{rms}} R.$ But $\dfrac{V_{\text{rms}}}{Z} = I_{\text{rms}}$, so $P_{\text{av}} = I_{\text{rms}}^2 R.$

EVALUATE: In an *L-R-C* circuit, electrical energy is stored and released in the inductor and capacitor but none is dissipated in either of these circuit elements. The power delivered by the source equals the power dissipated in the resistor.

31.28. **IDENTIFY** and **SET UP:** $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi.$ $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z}.$ $\cos\phi = \dfrac{R}{Z}.$

EXECUTE: $I_{\text{rms}} = \dfrac{80.0\ \text{V}}{105\ \Omega} = 0.762\ \text{A}.$ $\cos\phi = \dfrac{75.0\ \Omega}{105\ \Omega} = 0.714.$ $P_{\text{av}} = (80.0\ \text{V})(0.762\ \text{A})(0.714) = 43.5\ \text{W}.$

EVALUATE: Since the average power consumed by the inductor and by the capacitor is zero, we can also calculate the average power as $P_{\text{av}} = I_{\text{rms}}^2 R = (0.762\ \text{A})^2 (75.0\ \Omega) = 43.5\ \text{W}.$

31.29. **IDENTIFY** and **SET UP:** Use the equations of Section 31.3 to calculate ϕ, Z and V_{rms}. The average power delivered by the source is given by Eq.(31.31) and the average power dissipated in the resistor is $I_{\text{rms}}^2 R$

EXECUTE: **(a)** $X_L = \omega L = 2\pi f L = 2\pi(400\ \text{Hz})(0.120\ \text{H}) = 301.6\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi(400\ \text{Hz})(7.3\times10^{-6}\ \text{Hz})} = 54.51\ \Omega$$

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{301.6\ \Omega - 54.41\ \Omega}{240\ \Omega}$, so $\phi = +45.8°$. The power factor is $\cos\phi = +0.697$.

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(240\ \Omega)^2 + (301.6\ \Omega - 54.51\ \Omega)^2} = 344\ \Omega$

(c) $V_{\text{rms}} = I_{\text{rms}} Z = (0.450\ \text{A})(344\ \Omega) = 155\ \text{V}$

(d) $P_{\text{av}} = I_{\text{rms}} V_{\text{rms}} \cos\phi = (0.450\ \text{A})(155\ \text{V})(0.697) = 48.6\ \text{W}$

(e) $P_{\text{av}} = I_{\text{rms}}^2 R = (0.450\ \text{A})^2 (240\ \Omega) = 48.6\ \text{W}$

EVALUATE: The average electrical power delivered by the source equals the average electrical power consumed in the resistor.

(f) All the energy stored in the capacitor during one cycle of the current is released back to the circuit in another part of the cycle. There is no net dissipation of energy in the capacitor.

(g) The answer is the same as for the capacitor. Energy is repeatedly being stored and released in the inductor, but no net energy is dissipated there.

31.30. **IDENTIFY:** The angular frequency and the capacitance can be used to calculate the reactance X_C of the capacitor. The angular frequency and the inductance can be used to calculate the reactance X_L of the inductor. Calculate the phase angle ϕ and then the power factor is $\cos\phi$. Calculate the impedance of the circuit and then the rms current in the circuit. The average power is $P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$. On the average no power is consumed in the capacitor or the inductor, it is all consumed in the resistor.

SET UP: The source has rms voltage $V_{\text{rms}} = \dfrac{V}{\sqrt{2}} = \dfrac{45\ \text{V}}{\sqrt{2}} = 31.8\ \text{V}.$

EXECUTE: $X_L = \omega L = (360\ \text{rad/s})(15\times 10^{-3}\ \text{H}) = 5.4\ \Omega$. $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(360\ \text{rad/s})(3.5\times 10^{-6}\ \text{F})} = 794\ \Omega$.

$\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{5.4\ \Omega - 794\ \Omega}{250\ \Omega}$ and $\phi = -72.4°$. The power factor is $\cos\phi = 0.302$.

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(250\ \Omega)^2 + (5.4\ \Omega - 794\ \Omega)^2} = 827\ \Omega$. $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{31.8\ \text{V}}{827\ \Omega} = 0.0385\ \text{A}$.

$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi = (31.8\ \text{V})(0.0385\ \text{A})(0.302) = 0.370\ \text{W}$.

(c) The average power delivered to the resistor is $P_{\text{av}} = I_{\text{rms}}^2 R = (0.0385\ \text{A})^2(250\ \Omega) = 0.370\ \text{W}$. The average power delivered to the capacitor and to the inductor is zero.

EVALUATE: On average the power delivered to the circuit equals the power consumed in the resistor. The capacitor and inductor store electrical energy during part of the current oscillation but each return the energy to the circuit during another part of the current cycle.

31.31. IDENTIFY and SET UP: At the resonance frequency, $Z = R$. Use that $V = IZ$, $V_R = IR$, $V_L = IX_L$ and $V_C = IX_C$. P_{av} is given by Eq.(31.31).

(a) EXECUTE: $V = IZ = IR = (0.500\ \text{A})(300\ \Omega) = 150\ \text{V}$

(b) $V_R = IR = 150\ \text{V}$

$X_L = \omega L = L(1/\sqrt{LC}) = \sqrt{L/C} = 2582\ \Omega$; $V_L = IX_L = 1290\ \text{V}$

$X_C = 1/(\omega C) = \sqrt{L/C} = 2582\ \Omega$; $V_C = IX_C = 1290\ \text{V}$

(c) $P_{\text{av}} = \frac{1}{2}VI\cos\phi = \frac{1}{2}I^2R$, since $V = IR$ and $\cos\phi = 1$ at resonance.

$P_{\text{av}} = \frac{1}{2}(0.500\ \text{A})^2(300\ \Omega) = 37.5\ \text{W}$

EVALUATE: At resonance $V_L = V_C$. Note that $V_L + V_C > V$. However, at any instant $v_L + v_C = 0$.

31.32. IDENTIFY: The current is maximum at the resonance frequency, so choose C such that $\omega = 50.0\ \text{rad/s}$ is the resonance frequency. At the resonance frequency $Z = R$.

SET UP: $V_L = I\omega L$

EXECUTE: **(a)** The amplitude of the current is given by $I = \dfrac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$. Thus, the current will have a maximum amplitude when $\omega L = \dfrac{1}{\omega C}$. Therefore, $C = \dfrac{1}{\omega^2 L} = \dfrac{1}{(50.0\ \text{rad/s})^2(9.00\ \text{H})} = 44.4\ \mu\text{F}$.

(b) With the capacitance calculated above we find that $Z = R$, and the amplitude of the current is $I = \dfrac{V}{R} = \dfrac{120\ \text{V}}{400\ \Omega} = 0.300\ \text{A}$. Thus, the amplitude of the voltage across the inductor is

$V_L = I(\omega L) = (0.300\ \text{A})(50.0\ \text{rad/s})(9.00\ \text{H}) = 135\ \text{V}$.

EVALUATE: Note that V_L is greater than the source voltage amplitude.

31.33. IDENTIFY and SET UP: At resonance $X_L = X_C$, $\phi = 0$ and $Z = R$. $R = 150\ \Omega$, $L = 0.750\ \text{H}$, $C = 0.0180\ \mu\text{F}$, $V = 150\ \text{V}$

EXECUTE: **(a)** At the resonance frequency $X_L = X_C$ and from $\tan\phi = \dfrac{X_L - X_C}{R}$ we have that $\phi = 0°$ and the power factor is $\cos\phi = 1.00$.

(b) $P_{\text{av}} = \frac{1}{2}VI\cos\phi$ (Eq.31.31)

At the resonance frequency $Z = R$, so $I = \dfrac{V}{Z} = \dfrac{V}{R}$

$P_{\text{av}} = \frac{1}{2}V\left(\dfrac{V}{R}\right)\cos\phi = \frac{1}{2}\dfrac{V^2}{R} = \frac{1}{2}\dfrac{(150\ \text{V})^2}{150\ \Omega} = 75.0\ \text{W}$

(c) EVALUATE: When C and f are changed but the circuit is kept on resonance, nothing changes in $P_{\text{av}} = V^2/(2R)$, so the average power is unchanged: $P_{\text{av}} = 75.0\ \text{W}$. The resonance frequency changes but since $Z = R$ at resonance the current doesn't change.

31.34. IDENTIFY: $\omega_0 = \dfrac{1}{\sqrt{LC}}$. $V_C = IX_C$. $V = IZ$.

SET UP: At resonance, $Z = R$.

EXECUTE: **(a)** $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.350\ \text{H})(0.0120\times10^{-6}\ \text{F})}} = 1.54\times10^4\ \text{rad/s}$

(b) $V = IZ = \left(\dfrac{V_C}{X_C}\right)Z = \left(\dfrac{V_C}{X_C}\right)R.\ \ X_C = \dfrac{1}{\omega C} = \dfrac{1}{(1.54\times10^4\ \text{rad/s})(0.0120\times10^{-6}\ \text{F})} = 5.41\times10^3\ \Omega.$

$V = \left(\dfrac{550\ \text{V}}{5.41\times10^3\ \Omega}\right)(400\ \Omega) = 40.7\ \text{V}.$

EVALUATE: The voltage amplitude for the capacitor is more than a factor of 10 times greater than the voltage amplitude of the source.

31.35. **IDENTIFY** and **SET UP:** The resonance angular frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$. $X_L = \omega L$. $X_C = \dfrac{1}{\omega C}$ and $Z = \sqrt{R^2 + (X_L - X_C)^2}$. At the resonance frequency $X_L = X_C$ and $Z = R$.

EXECUTE: **(a)** $Z = R = 115\ \Omega$

(b) $\omega_0 = \dfrac{1}{\sqrt{(4.50\times10^{-3}\ \text{H})(1.26\times10^{-6}\ \text{F})}} = 1.33\times10^4\ \text{rad/s}$. $\omega = 2\omega_0 = 2.66\times10^4\ \text{rad/s}.$

$X_L = \omega L = (2.66\times10^4\ \text{rad/s})(4.50\times10^{-3}\ \text{H}) = 120\ \Omega.\ \ X_C = \dfrac{1}{\omega C} = \dfrac{1}{(2.66\times10^4\ \text{rad/s})(1.25\times10^{-6}\ \text{F})} = 30\ \Omega$

$Z = \sqrt{(115\ \Omega)^2 + (120\ \Omega - 30\ \Omega)^2} = 146\ \Omega$

(c) $\omega = \omega_0/2 = 6.65\times10^3\ \text{rad/s}.\ \ X_L = 30\ \Omega.\ \ X_C = \dfrac{1}{\omega C} = 120\ \Omega.\ \ Z = \sqrt{(115\ \Omega)^2 + (30\ \Omega - 120\ \Omega)^2} = 146\ \Omega,$ the same value as in part (b).

EVALUATE: For $\omega = 2\omega_0$, $X_L > X_C$. For $\omega = \omega_0/2$, $X_L < X_C$. But $(X_L - X_C)^2$ has the same value at these two frequencies, so Z is the same.

31.36. **IDENTIFY:** At resonance $Z = R$ and $X_L = X_C$.

SET UP: $\omega_0 = \dfrac{1}{\sqrt{LC}}$. $V = IZ$. $V_R = IR$, $V_L = IX_L$ and $V_C = V_L$.

EXECUTE: **(a)** $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.280\ \text{H})(4.00\times10^{-6}\ \text{F})}} = 945\ \text{rad/s}.$

(b) I = 1.20 A at resonance, so $R = Z = \dfrac{V}{I} = \dfrac{120\ \text{V}}{1.70\ \text{A}} = 70.6\ \Omega$

(c) At resonance, $V_R = 120\ \text{V}$, $V_L = V_C = I\omega L = (1.70\ \text{A})(945\ \text{rad/s})(0.280\ \text{H}) = 450\ \text{V}.$

EVALUATE: At resonance, $V_R = V$ and $V_L - V_C = 0$.

31.37. **IDENTIFY** and **SET UP:** Eq.(31.35) relates the primary and secondary voltages to the number of turns in each. $I = V/R$ and the power consumed in the resistive load is $I_{\text{rms}}^2 = V_{\text{rms}}^2/R$.

EXECUTE: **(a)** $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$ so $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2} = \dfrac{120\ \text{V}}{12.0\ \text{V}} = 10$

(b) $I_2 = \dfrac{V_2}{R} = \dfrac{12.0\ \text{V}}{5.00\ \Omega} = 2.40\ \text{A}$

(c) $P_{\text{av}} = I_2^2 R = (2.40\ \text{A})^2(5.00\ \Omega) = 28.8\ \text{W}$

(d) The power drawn from the line by the transformer is the 28.8 W that is delivered by the load.

$$P_{\text{av}} = \frac{V^2}{R} \text{ so } R = \frac{V^2}{P_{\text{av}}} = \frac{(120\ \text{V})^2}{28.8\ \text{W}} = 500\ \Omega$$

And $\left(\dfrac{N_1}{N_2}\right)^2(5.00\ \Omega) = (10)^2(5.00\ \Omega) = 500\ \Omega,$ as was to be shown.

EVALUATE: The resistance is “transformed”. A load of resistance R connected to the secondary draws the same power as a resistance $(N_1/N_2)^2 R$ connected directly to the supply line, without using the transformer.

31.38. IDENTIFY: $P_{av} = VI$ and $P_{av,1} = P_{av,2}$. $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2}$.

SET UP: $V_1 = 120$ V. $V_2 = 13{,}000$ V.

EXECUTE: (a) $\dfrac{N_2}{N_1} = \dfrac{V_2}{V_1} = \dfrac{13{,}000 \text{ V}}{120 \text{ V}} = 108$

(b) $P_{av} = V_2 I_2 = (13{,}000 \text{ V})(8.50 \times 10^{-3} \text{ A}) = 110$ W

(c) $I_1 = \dfrac{P_{av}}{V_1} = \dfrac{110 \text{ W}}{120 \text{ V}} = 0.917$ A

EVALUATE: Since the power supplied to the primary must equal the power delivered by the secondary, in a step-up transformer the current in the primary is greater than the current in the secondary.

31.39. IDENTIFY: A transformer transforms voltages according to $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$. The effective resistance of a secondary circuit of resistance R is $R_{eff} = \dfrac{R}{(N_2/N_1)^2}$. Resistance R is related to P_{av} and V by $P_{av} = \dfrac{V^2}{R}$. Conservation of energy requires $P_{av,1} = P_{av,2}$ so $V_1 I_1 = V_2 I_2$.

SET UP: Let $V_1 = 240$ V and $V_2 = 120$ V, so $P_{2,av} = 1600$ W. These voltages are rms.

EXECUTE: (a) $V_1 = 240$ V and we want $V_2 = 120$ V, so use a step-down transformer with $N_2/N_1 = \frac{1}{2}$.

(b) $P_{av} = VI$, so $I = \dfrac{P_{av}}{V} = \dfrac{1600 \text{ W}}{240 \text{ V}} = 6.67$ A.

(c) The resistance R of the blower is $R = \dfrac{V^2}{P} = \dfrac{(120 \text{ V})^2}{1600 \text{ W}} = 9.00\ \Omega$. The effective resistance of the blower is $R_{eff} = \dfrac{9.00\ \Omega}{(1/2)^2} = 36.0\ \Omega$.

EVALUATE: $I_2 V_2 = (13.3 \text{ A})(120 \text{ V}) = 1600$ W. Energy is provided to the primary at the same rate that it is consumed in the secondary. Step-down transformers step up resistance and the current in the primary is less than the current in the secondary.

31.40. IDENTIFY: $Z = \sqrt{R^2 + (X_L - X_C)^2}$, with $X_L = \omega L$ and $X_C = \dfrac{1}{\omega C}$.

SET UP: The woofer has a R and L in series and the tweeter has a R and C in series.

EXECUTE: (a) $Z_{tweeter} = \sqrt{R^2 + (1/\omega C)^2}$

(b) $Z_{woofer} = \sqrt{R^2 + (\omega L)^2}$

(c) If $Z_{tweeter} = Z_{woofer}$, then the current splits evenly through each branch.

(d) At the crossover point, where currents are equal, $R^2 + \left(1/\omega C^2\right) = R^2 + \left(\omega L\right)^2$. $\omega = \dfrac{1}{\sqrt{LC}}$ and $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi\sqrt{LC}}$.

EVALUATE: The crossover frequency corresponds to the resonance frequency of a *R*-*C*-*L* circuit, since the crossover frequency is where $X_L = X_C$.

31.41. IDENTIFY and SET UP: Use Eq.(31.24) to relate L and R to ϕ. The voltage across the coil leads the current in it by $52.3°$, so $\phi = +52.3°$.

EXECUTE: $\tan\phi = \dfrac{X_L - X_C}{R}$. But there is no capacitance in the circuit so $X_C = 0$. Thus $\tan\phi = \dfrac{X_L}{R}$ and $X_L = R\tan\phi = (48.0\ \Omega)\tan 52.3° = 62.1\ \Omega$. $X_L = \omega L = 2\pi f L$ so $L = \dfrac{X_L}{2\pi f} = \dfrac{62.1\ \Omega}{2\pi(80.0 \text{ Hz})} = 0.124$ H.

EVALUATE: $\phi > 45°$ when $(X_L - X_C) > R$, which is the case here.

31.42. IDENTIFY: $Z = \sqrt{R^2 + (X_L - X_C)^2}$. $I_{rms} = \dfrac{V_{rms}}{Z}$. $V_{rms} = I_{rms}R$. $V_{C,rms} = I_{rms}X_C$. $V_{L,rms} = I_{rms}X_L$.

SET UP: $V_{rms} = \dfrac{V}{\sqrt{2}} = \dfrac{30.0 \text{ V}}{\sqrt{2}} = 21.2$ V.

EXECUTE: **(a)** $\omega = 200\text{ rad/s}$, so $X_L = \omega L = (200\text{ rad/s})(0.400\text{ H}) = 80.0\ \Omega$ and

$X_C = \dfrac{1}{\omega C} = \dfrac{1}{(200\text{ rad/s})(6.00\times 10^{-6}\text{ F})} = 833\ \Omega.$ $Z = \sqrt{(200\ \Omega)^2 + (80.0\ \Omega - 833\ \Omega)^2} = 779\ \Omega.$

$I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{21.2\text{ V}}{779\ \Omega} = 0.0272\text{ A}.$ V_1 reads $V_{R,\text{rms}} = I_{\text{rms}}R = (0.0272\text{ A})(200\ \Omega) = 5.44\text{ V}.$ V_2 reads

$V_{L,\text{rms}} = I_{\text{rms}}X_L = (0.0272\text{ A})(80.0\ \Omega) = 2.18\text{ V}.$ V_3 reads $V_{C,\text{rms}} = I_{\text{rms}}X_C = (0.0272\text{ A})(833\ \Omega) = 22.7\text{ V}.$ V_4 reads

$\left|\dfrac{V_L - V_C}{\sqrt{2}}\right| = |V_{L,\text{rms}} - V_{C,\text{rms}}| = |2.18\text{ V} - 22.7\text{ V}| = 20.5\text{ V}.$ V_5 reads $V_{\text{rms}} = 21.2\text{ V}.$

(b) $\omega = 1000\text{ rad/s}$ so $X_L = \omega L = (5)(80.0\ \Omega) = 400\ \Omega$ and $X_C = \dfrac{1}{\omega C} = \dfrac{833\ \Omega}{5} = 167\ \Omega.$

$Z = \sqrt{(200\ \Omega)^2 + (400\ \Omega - 167\ \Omega)^2} = 307\ \Omega.$ $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{21.2\text{ V}}{307\ \Omega} = 0.0691\text{ A}.$ V_1 reads $V_{R,\text{rms}} = 13.8\text{ V}.$ V_2 reads

$V_{L,\text{rms}} = 27.6\text{ V}.$ V_3 reads $V_{C,\text{rms}} = 11.5\text{ V}.$ V_4 reads $|V_{L,\text{rms}} - V_{C,\text{rms}}| = |27.6\text{ V} - 11.5\text{ V}| = 16.1\text{ V}.$ V_5 reads

$V_{\text{rms}} = 21.2\text{ V}.$

EVALUATE: The resonance frequency for this circuit is $\omega_0 = \dfrac{1}{\sqrt{LC}} = 645\text{ rad/s}.$ 200 rad/s is less than the resonance frequency and $X_C > X_L$. 1000 rad/s is greater than the resonance frequency and $X_L > X_C$.

31.43. **IDENTIFY** and **SET UP:** The rectified current equals the absolute value of the current *i*. Evaluate the integral as specified in the problem.

EXECUTE: **(a)** From Fig.31.3b, the rectified current is zero at the same values of *t* for which the sinusoidal current is zero. At these *t*, $\cos\omega t = 0$ and $\omega t = \pm\pi/2,\ \pm 3\pi/2, \ldots$. The two smallest positive times are

$t_1 = \pi/2\omega,\ t_2 = 3\pi/2\omega.$

(b) $A = \left|\int_{t_1}^{t_2} i\,dt\right| = -\int_{t_1}^{t_2} I\cos\omega t\,dt = -I\left[\dfrac{1}{\omega}\sin\omega t\right]_{t_1}^{t_2} = -\dfrac{I}{\omega}(\sin\omega t_2 - \sin\omega t_1)$

$\sin\omega t_1 = \sin[\omega(\pi/2\omega)] = \sin(\pi/2) = 1$

$\sin\omega t_2 = \sin[\omega(3\pi/2\omega)] = \sin(3\pi/2) = -1$

$A = \left(\dfrac{I}{\omega}\right)(1-(-1)) = \dfrac{2I}{\omega}$

(c) $I_{\text{rav}}(t_2 - t_1) = 2I/\omega$

$I_{\text{rav}} = \dfrac{2I}{\omega(t_2 - t_1)} = \dfrac{2I}{\omega(3\pi/2\omega - \pi/2\omega)} = \dfrac{2I}{\pi}$, which is Eq.(31.3).

EVALUATE: We have shown that Eq.(31.3) is correct. The average rectified current is less than the current amplitude *I*, since the rectified current varies between 0 and *I*. The average of the current is zero, since it has both positive and negative values.

31.44. **IDENTIFY:** $X_L = \omega L.$ $P_{\text{av}} = V_{\text{rms}}I_{\text{rms}}\cos\phi$

SET UP: $f = 120\text{ Hz};$ $\omega = 2\pi f.$

EXECUTE: **(a)** $X_L = \omega L \Rightarrow L = \dfrac{X_L}{\omega} = \dfrac{250\ \Omega}{2\pi(120\text{ Hz})} = 0.332\ \Omega$

(b) $Z = \sqrt{R^2 + X_L{}^2} = \sqrt{(400\ \Omega)^2 + (250\ \Omega)^2} = 472\ \Omega.$ $\cos\phi = \dfrac{R}{Z}$ and $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z}.$ $P_{\text{av}} = \dfrac{V_{\text{rms}}^2}{Z}\dfrac{R}{Z}$, so

$V_{\text{rms}} = Z\sqrt{\dfrac{P_{\text{av}}}{R}} = (472\ \Omega)\sqrt{\dfrac{800\text{ W}}{400\ \Omega}} = 668\text{ V}.$

EVALUATE: $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{668\text{ V}}{472\ \Omega} = 1.415\text{ A}.$ We can calculate P_{av} as $I_{\text{rms}}^2 R = (1.415\text{ A})^2(400\ \Omega) = 800\text{ W},$ which checks.

31.45. **(a) IDENTIFY** and **SET UP:** Source voltage lags current so it must be that $X_C > X_L$ and we must add an inductor in series with the circuit. When $X_C = X_L$ the power factor has its maximum value of unity, so calculate the additional *L* needed to raise X_L to equal X_C.

(b) EXECUTE: power factor $\cos\phi$ equals 1 so $\phi = 0$ and $X_C = X_L$. Calculate the present value of $X_C - X_L$ to see how much more X_L is needed: $R = Z\cos\phi = (60.0\ \Omega)(0.720) = 43.2\ \Omega$

$\tan\phi = \dfrac{X_L - X_C}{R}$ so $X_L - X_C = R\tan\phi$

$\cos\phi = 0.720$ gives $\phi = -43.95°$ (ϕ is negative since the voltage lags the current)

Then $X_L - X_C = R\tan\phi = (43.2\ \Omega)\tan(-43.95°) = -41.64\ \Omega$.

Therefore need to add $41.64\ \Omega$ of X_L.

$X_L = \omega L = 2\pi f L$ and $L = \dfrac{X_L}{2\pi f} = \dfrac{41.64\ \Omega}{2\pi(50.0\ \text{Hz})} = 0.133\ \text{H}$, amount of inductance to add.

EVALUATE: From the information given we can't calculate the original value of L in the circuit, just how much to add. When this L is added the current in the circuit will increase.

31.46. IDENTIFY: Use $V_{rms} = I_{rms}Z$ to calculate Z and then find R. $P_{av} = I_{rms}^2 R$

SET UP: $X_C = 50.0\ \Omega$

EXECUTE: $Z = \dfrac{V_{rms}}{I_{rms}} = \dfrac{240\ \text{V}}{3.00\ \text{A}} = 80.0\ \Omega = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (50.0\ \Omega)^2}$. Thus,

$R = \sqrt{(80.0\ \Omega)^2 - (50.0\ \Omega)^2} = 62.4\ \Omega$. The average power supplied to this circuit is equal to the power dissipated by the resistor, which is $P = I_{rms}^2 R = (3.00\ \text{A})^2(62.4\ \Omega) = 562\ \text{W}$.

EVALUATE: $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{-50.0\ \Omega}{62.4\ \Omega}$ and $\phi = -38.7°$.

$P_{av} = V_{rms}I_{rms}\cos\phi = (240\ \text{V})(3.00\ \text{A})\cos(-38.7°) = 562\ \text{W}$, which checks.

31.47. IDENTIFY: The voltage and current amplitudes are the maximum values of these quantities, not necessarily the instantaneous values.

SET UP: The voltage amplitudes are $V_R = RI$, $V_L = X_L I$, and $V_C = X_C I$, where $I = V/Z$ and

$Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$.

EXECUTE: **(a)** $\omega = 2\pi f = 2\pi(1250\ \text{Hz}) = 7854$ rad/s. Carrying extra figures in the calculator gives $X_L = \omega L = (7854\ \text{rad/s})(3.50\ \text{mH}) = 27.5\ \Omega$; $X_C = 1/\omega C = 1/[(7854\ \text{rad/s})(10.0\ \mu\text{F})] = 12.7\ \Omega$;

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50.0\ \Omega)^2 + (27.5\ \Omega - 12.7\ \Omega)^2} = 52.1\ \Omega$;

$I = V/Z = (60.0\ \text{V})/(52.1\ \Omega) = 1.15$ A; $V_R = RI = (50.0\ \Omega)(1.15\ \text{A}) = 57.5$ V;

$V_L = X_L I = (27.5\ \Omega)(1.15\ \text{A}) = 31.6$ V; $V_C = X_C I = (12.7\ \Omega)(1.15\ \text{A}) = 14.7$ V.

The voltage amplitudes can add to more than 60.0 V because these voltages do not all occur at the same instant of time. At any instant, the instantaneous voltages all add to 60.0 V.

(b) All of them will change because they all depend on ω. $X_L = \omega L$ will double to 55.0 Ω, and $X_C = 1/\omega C$ will decrease by half to 6.35 Ω. Therefore $Z = \sqrt{(50.0\ \Omega)^2 + (55.0\ \Omega - 6.35\ \Omega)^2} = 69.8\ \Omega$; $I = V/Z = (60.0\ \text{V})/(69.8\ \Omega) = 0.860$ A; $V_R = IR = (0.860\ \text{A})(50.0\ \Omega) = 43.0$ V;

$V_L = IX_L = (0.860\ \text{A})(55.0\ \Omega) = 47.3$ V; $V_C = IX_C = (0.860\ \text{A})(6.35\ \Omega) = 5.47$ V.

EVALUATE: The new amplitudes in part (b) are not simple multiples of the values in part (a) because the impedance and reactances are not all the same simple multiple of the angular frequency.

31.48. IDENTIFY and SET UP: $X_C = \dfrac{1}{\omega C}$. $X_L = \omega L$.

EXECUTE: **(a)** $\dfrac{1}{\omega_1 C} = \omega_1 L$ and $LC = \dfrac{1}{\omega_1^2}$. At angular frequency ω_2, $\dfrac{X_L}{X_C} = \dfrac{\omega_2 L}{1/\omega_2 C} = \omega_2^2 LC = (2\omega_1)^2 \dfrac{1}{\omega_1^2} = 4$.

$X_L > X_C$.

(b) At angular frequency ω_3, $\dfrac{X_L}{X_C} = \omega_3^2 LC = \left(\dfrac{\omega_1}{3}\right)^2\left(\dfrac{1}{\omega_1^2}\right) = \dfrac{1}{9}$. $X_C > X_L$.

EVALUATE: When ω increases, X_L increases and X_C decreases. When ω decreases, X_L decreases and X_C increases.

(c) The resonance angular frequency ω_0 is the value of ω for which $X_C = X_L$, so $\omega_0 = \omega_1$.

31.49. **IDENTIFY** and **SET UP:** Express Z and I in terms of ω, L, C and R. The voltages across the resistor and the inductor are $90°$ out of phase, so $V_{\text{out}} = \sqrt{V_R^2 + V_L^2}$.

EXECUTE: The circuit is sketched in Figure 31.49.

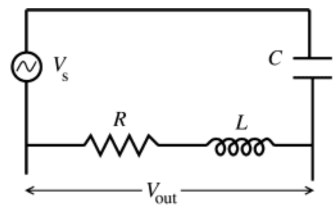


Figure 31.49

$$X_L = \omega L,\ X_C = \frac{1}{\omega C}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$I = \frac{V_s}{Z} = \frac{V_s}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$V_{\text{out}} = I\sqrt{R^2 + X_L^2} = I\sqrt{R^2 + \omega^2 L^2} = V_s\sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\frac{V_{\text{out}}}{V_s} = \sqrt{\frac{R^2 + \omega^2 L^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

ω small

As ω gets small, $R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \to \frac{1}{\omega^2 C^2}, R^2 + \omega^2 L^2 \to R^2$

Therefore $\frac{V_{\text{out}}}{V_s} \to \sqrt{\frac{R^2}{(1/\omega^2 C^2)}} = \omega RC$ as ω becomes small.

ω large

As ω gets large, $R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 \to R^2 + \omega^2 L^2 \to \omega^2 L^2,\ R^2 + \omega^2 L^2 \to \omega^2 L^2$

Therefore, $\frac{V_{\text{out}}}{V_s} \to \sqrt{\frac{\omega^2 L^2}{\omega^2 L^2}} = 1$ as ω becomes large.

EVALUATE: $V_{\text{out}}/V_s \to 0$ as ω becomes small, so there is V_{out} only when the frequency ω of V_s is large. If the source voltage contains a number of frequency components, only the high frequency ones are passed by this filter.

31.50. **IDENTIFY:** $V = V_C = IX_C$. $I = V/Z$.

SET UP: $X_L = \omega L$, $X_C = \frac{1}{\omega C}$.

EXECUTE: $V_{\text{out}} = V_C = \frac{I}{\omega C} \Rightarrow \frac{V_{\text{out}}}{V_s} = \frac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

If ω is large: $\frac{V_{\text{out}}}{V_s} = \frac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \approx \frac{1}{\omega C\sqrt{(\omega L)^2}} = \frac{1}{(LC)\omega^2}$.

If ω is small: $\frac{V_{\text{out}}}{V_s} \approx \frac{1}{\omega C\sqrt{(1/\omega C)^2}} = \frac{\omega C}{\omega C} = 1$.

EVALUATE: When ω is large, X_C is small and X_L is large so Z is large and the current is small. Both factors in $V_C = IX_C$ are small. When ω is small, X_C is large and the voltage amplitude across the capacitor is much larger than the voltage amplitudes across the resistor and the inductor.

31.51. **IDENTIFY:** $I = V/Z$ and $P_{\text{av}} = \frac{1}{2}I^2 R$.

SET UP: $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

EXECUTE: **(a)** $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$.

(b) $P_{av} = \frac{1}{2}I^2R = \frac{1}{2}\left(\frac{V}{Z}\right)^2 R = \frac{V^2R/2}{R^2 + (\omega L - 1/\omega C)^2}.$

(c) The average power and the current amplitude are both greatest when the denominator is smallest, which occurs for $\omega_0 L = \frac{1}{\omega_0 C}$, so $\omega_0 = \frac{1}{\sqrt{LC}}$.

(d) $P_{av} = \frac{(100\text{ V})^2(200\,\Omega)/2}{(200\,\Omega)^2 + \left(\omega(2.00\text{ H}) - 1/[\omega(0.500\times10^{-6}\text{ F})]\right)^2} = \frac{25\omega^2}{40{,}000\omega^2 + (2\omega^2 - 2{,}000{,}000)^2}.$

The graph of P_{av} versus ω is sketched in Figure 31.51.

EVALUATE: Note that as the angular frequency goes to zero, the power and current are zero, just as they are when the angular frequency goes to infinity. This graph exhibits the same strongly peaked nature as the light purple curve in Figure 31.19 in the textbook.

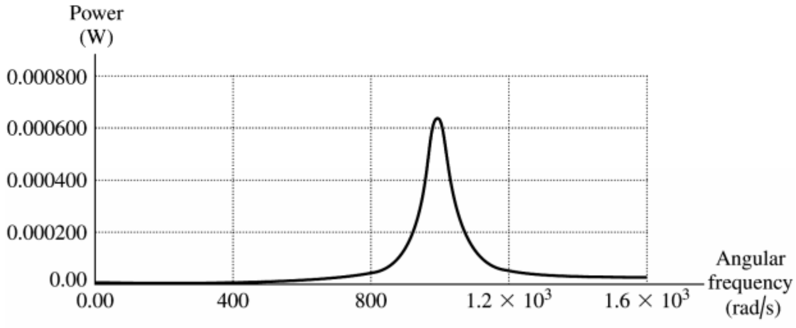


Figure 31.51

31.52. IDENTIFY: $V_L = I\omega L$ and $V_C = \frac{I}{\omega C}$.

SET UP: Problem 31.51 shows that $I = \frac{V}{\sqrt{R^2 + (\omega L - 1/[\omega C])^2}}$.

EXECUTE: **(a)** $V_L = I\omega L = \frac{V\omega L}{Z} = \frac{V\omega L}{\sqrt{R^2 + (\omega L - 1/[\omega C])^2}}$.

(b) $V_C = \frac{I}{\omega C} = \frac{I}{\omega CZ} = \frac{1}{\omega C\sqrt{R^2 + (\omega L - 1/[\omega C])^2}}$.

(c) The graphs are given in Figure 31.52.

EVALUATE: **(d)** When the angular frequency is zero, the inductor has zero voltage while the capacitor has voltage of 100 V (equal to the total source voltage). At very high frequencies, the capacitor voltage goes to zero, while the inductor's voltage goes to 100 V. At resonance, $\omega_0 = \frac{1}{\sqrt{LC}} = 1000\text{ rad/s}$, the two voltages are equal, and are a maximum, 1000 V.

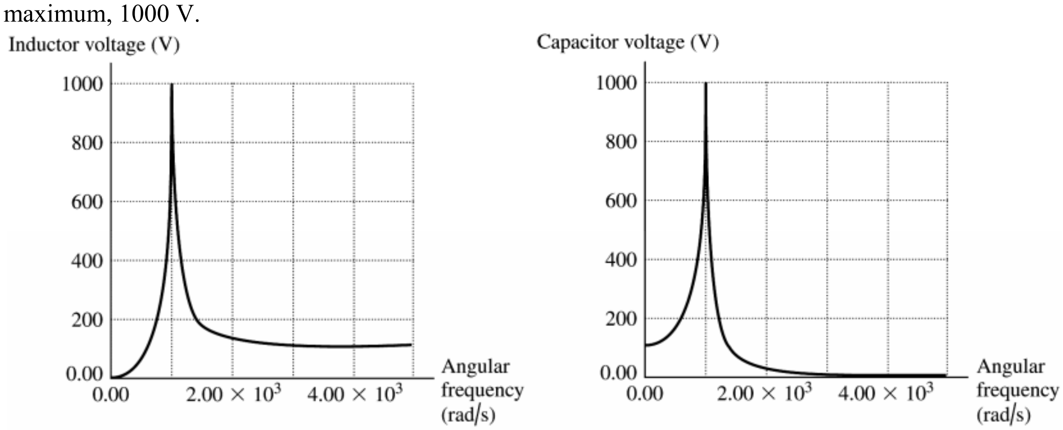


Figure 31.52

31.53. **IDENTIFY:** $U_B = \frac{1}{2}Li^2$. $U_E = \frac{1}{2}Cv^2$.

SET UP: Let $\langle x \rangle$ denote the average value of the quantity x. $\langle i^2 \rangle = \frac{1}{2}I^2$ and $\langle v_C^2 \rangle = \frac{1}{2}V_C^2$. Problem 31.51 shows that $I = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/[\omega C])^2}}$. Problem 31.52 shows that $V_C = \dfrac{V}{\omega C\sqrt{R^2 + (\omega L - 1/[\omega C])^2}}$.

EXECUTE: **(a)** $U_B = \frac{1}{2}Li^2 \Rightarrow \langle U_B \rangle = \frac{1}{2}L\langle i^2 \rangle = \frac{1}{2}LI_{\text{rms}}^2 = \frac{1}{2}L\left(\dfrac{I}{\sqrt{2}}\right)^2 = \frac{1}{4}LI^2$.

$U_E = \frac{1}{2}Cv_C^2 \Rightarrow \langle U_E \rangle = \dfrac{1}{2}C\langle v_C^2 \rangle = \frac{1}{2}CV_{C,\text{rms}}^2 = \frac{1}{2}C\left(\dfrac{V_C}{\sqrt{2}}\right)^2 = \frac{1}{4}CV_C^2$

(b) Using Problem 31.51a

$$\langle U_B \rangle = \frac{1}{4}LI^2 = \frac{1}{4}L\left(\frac{V^2}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right)^2 = \frac{LV^2}{4\left(R^2 + (\omega L - 1/\omega C)^2\right)}.$$

Using Problem (31.47b): $\langle U_E \rangle = \dfrac{1}{4}CV_C{}^2 = \dfrac{1}{4}C\dfrac{V^2}{\omega^2 C^2\left(R^2 + (\omega L - 1/\omega C)^2\right)} = \dfrac{V^2}{4\omega^2 C\left(R^2 + (\omega L - 1/\omega C)^2\right)}$.

(c) The graphs of the magnetic and electric energies are given in Figure 31.53.

EVALUATE: **(d)** When the angular frequency is zero, the magnetic energy stored in the inductor is zero, while the electric energy in the capacitor is $U_E = CV^2/4$. As the frequency goes to infinity, the energy noted in both inductor and capacitor go to zero. The energies equal each other at the resonant frequency where $\omega_0 = \dfrac{1}{\sqrt{LC}}$ and $U_B = U_E = \dfrac{LV^2}{4R^2}$.

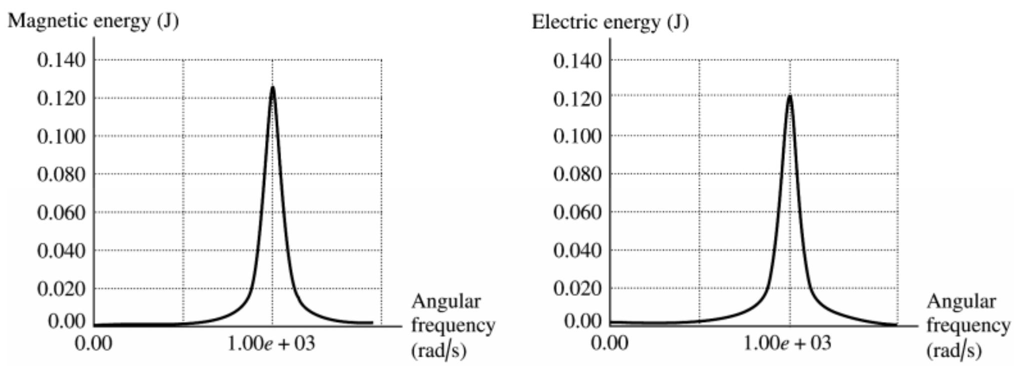


Figure 31.53

31.54. **IDENTIFY:** At any instant of time the same rules apply to the parallel ac circuit as to parallel dc circuit: the voltages are the same and the currents add.

SET UP: For a resistor the current and voltage in phase. For an inductor the voltage leads the current by $90°$ and for a capacitor the voltage lags the current by $90°$.

EXECUTE: **(a)** The parallel *L-R-C* circuit must have equal potential drops over the capacitor, inductor and resistor, so $v_R = v_L = v_C = v$. Also, the sum of currents entering any junction must equal the current leaving the junction. Therefore, the sum of the currents in the branches must equal the current through the source: $i = i_R + i_L + i_C$.

(b) $i_R = \dfrac{v}{R}$ is always in phase with the voltage. $i_L = \dfrac{v}{\omega L}$ lags the voltage by $90°$, and $i_C = v\omega C$ leads the voltage by $90°$. The phase diagram is sketched in Figure 31.54.

(c) From the diagram, $I^2 = I_R{}^2 + (I_C - I_L)^2 = \left(\dfrac{V}{R}\right)^2 + \left(V\omega C - \dfrac{V}{\omega L}\right)^2$.

(d) From part (c): $I = V\sqrt{\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)^2}$. But $I = \dfrac{V}{Z}$, so $\dfrac{1}{Z} = \sqrt{\dfrac{1}{R^2} + \left(\omega C - \dfrac{1}{\omega L}\right)^2}$.

EVALUATE: For large ω, $Z \to \frac{1}{\omega C}$. The current in the capacitor branch is much larger than the current in the other branches. For small ω, $Z \to \omega L$. The current in the inductive branch is much larger than the current in the other branches.

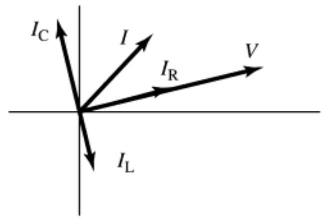


Figure 31.54

31.55. **IDENTIFY:** Apply the expression for I from problem 31.54 when $\omega_0 = 1/\sqrt{LC}$.

SET UP: From Problem 31.54, $I = V\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$

EXECUTE: **(a)** At resonance, $\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0 C = \frac{1}{\omega_0 L} \Rightarrow I_C = V\omega_0 C = \frac{V}{\omega_0 L} = I_L$ so $I = I_R$ and I is a minimum.

(b) $P_{\text{av}} = \frac{V_{\text{rms}}^2}{Z}\cos\phi = \frac{V^2}{R}$ at resonance where $R < Z$ so power is a maximum.

(c) At $\omega = \omega_0$, I and V are in phase, so the phase angle is zero, which is the same as a series resonance.

EVALUATE: **(d)** The parallel circuit is sketched in Figure 31.55. At resonance, $|i_C| = |i_L|$ and at any instant of time these two currents are in opposite directions. Therefore, the net current between a and b is always zero.
(e) If the inductor and capacitor each have some resistance, and these resistances aren't the same, then it is no longer true that $i_C + i_L = 0$ and the statement in part (d) isn't valid.

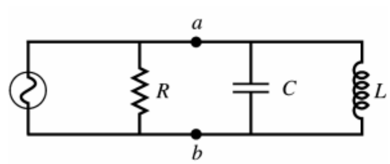


Figure 31.55

31.56. **IDENTIFY:** Refer to the results and the phasor diagram in Problem 31.54. The source voltage is applied across each parallel branch.

SET UP: $V = \sqrt{2}V_{\text{rms}} = 311\text{ V}$

EXECUTE: **(a)** $I_R = \frac{V}{R} = \frac{311\text{ V}}{400\ \Omega} = 0.778\text{ A}$.

(b) $I_C = V\omega C = (311\text{ V})(360\text{ rad/s})(6.00\times10^{-6}\text{ F}) = 0.672\text{ A}$.

(c) $\phi = \arctan\left(\frac{I_C}{I_R}\right) = \arctan\left(\frac{0.672\text{ A}}{0.778\text{ A}}\right) = 40.8°$.

(d) $I = \sqrt{I_R^2 + I_C^2} = \sqrt{(0.778\text{ A})^2 + (0.672\text{ A})^2} = 1.03\text{ A}$.

(e) Leads since $\phi > 0$.

EVALUATE: The phasor diagram shows that the current in the capacitor always leads the source voltage.

31.57. **IDENTIFY** and **SET UP:** Refer to the results and the phasor diagram in Problem 31.54. The source voltage is applied across each parallel branch.

EXECUTE: **(a)** $I_R = \frac{V}{R};\ I_C = V\omega C;\ I_L = \frac{V}{\omega L}$.

(b) The graph of each current versus ω is given in Figure 31.57a.

(c) $\omega \to 0: I_C \to 0;\ I_L \to \infty.\quad \omega \to \infty: I_C \to \infty;\ I_L \to 0.$

At low frequencies, the current is not changing much so the inductor's back-emf doesn't "resist." This allows the current to pass fairly freely. However, the current in the capacitor goes to zero because it tends to "fill up" over the slow period, making it less effective at passing charge. At high frequency, the induced emf in the inductor resists the violent changes and passes little current. The capacitor never gets a chance to fill up so passes charge freely.

(d) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(2.0\text{ H})(0.50\times 10^{-6}\text{ F})}} = 1000\text{ rad/sec}$ and $f = 159\text{ Hz}$. The phasor diagram is sketched in Figure 31.57b.

(e) $I = \sqrt{\left(\dfrac{V}{R}\right)^2 + \left(V\omega C - \dfrac{V}{\omega L}\right)^2}$.

$$I = \sqrt{\left(\frac{100\text{ V}}{200\ \Omega}\right)^2 + \left((100\text{ V})(1000\text{ s}^{-1})(0.50\times 10^{-6}\text{ F}) - \frac{100\text{ V}}{(1000\text{ s}^{-1})(2.0\text{ H})}\right)^2} = 0.50\text{ A}$$

(f) At resonance $I_L = I_C = V\omega C = (100\text{ V})(1000\text{ s}^{-1})(0.50\times 10^{-6}\text{ F}) = 0.0500\text{ A}$ and $I_R = \dfrac{V}{R} = \dfrac{100\text{ V}}{200\ \Omega} = 0.50\text{ A}$.

EVALUATE: At resonance $i_C = i_L = 0$ at all times and the current through the source equals the current through the resistor.

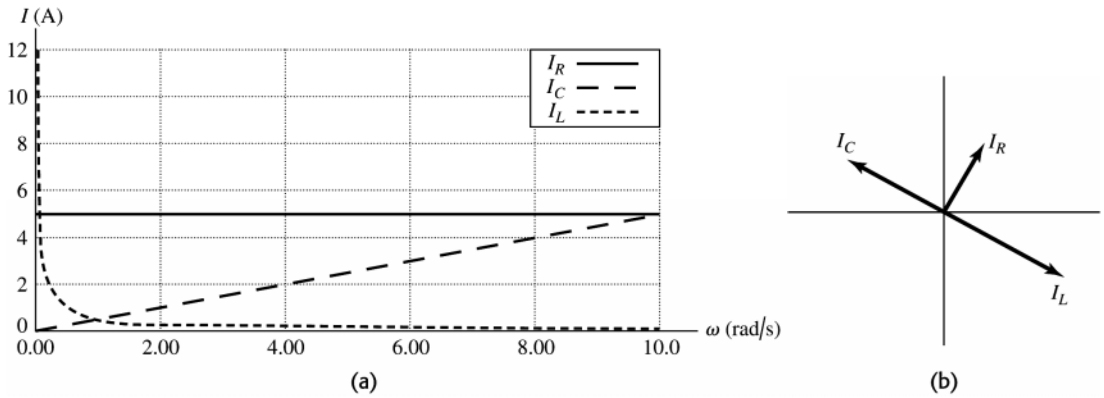


Figure 31.57

31.58. **IDENTIFY:** The average power depends on the phase angle ϕ.

SET UP: The average power is $P_{av} = V_{rms}I_{rms}\cos\phi$, and the impedance is $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$.

EXECUTE: **(a)** $P_{av} = V_{rms}I_{rms}\cos\phi = \frac{1}{2}(V_{rms}I_{rms})$, which gives $\cos\phi = \frac{1}{2}$, so $\phi = \pi/3 = 60°$. $\tan\phi = (X_L - X_C)/R$, which gives $\tan 60° = (\omega L - 1/\omega C)/R$. Using $R = 75.0\ \Omega$, $L = 5.00$ mH, and $C = 2.50\ \mu$F and solving for ω we get $\omega = 28760$ rad/s $= 28{,}800$ rad/s.

(b) $Z = \sqrt{R^2 + (X_L - X_C)^2}$, where $X_L = \omega L = (28{,}760\text{ rad/s})(5.00\text{ mH}) = 144\ \Omega$ and $X_C = 1/\omega C = 1/[(28{,}760\text{ rad/s})(2.50\ \mu\text{F})] = 13.9\ \Omega$, giving $Z = \sqrt{(75\ \Omega)^2 + (144\ \Omega - 13.9\ \Omega)^2} = 150\ \Omega$;
$I = V/Z = (15.0\text{ V})/(150\ \Omega) = 0.100$ A and $P_{av} = \frac{1}{2}VI\cos\phi = \frac{1}{2}(15.0\text{ V})(0.100\text{ A})(1/2) = 0.375$ W.

EVALUATE: All this power is dissipated in the resistor because the average power delivered to the inductor and capacitor is zero.

31.59. **IDENTIFY:** We know R, X_C and ϕ so Eq.(31.24) tells us X_L. Use $P_{av} = I_{rms}^2 R$ from Exercise 31.27 to calculate I_{rms}. Then calculate Z and use Eq.(31.26) to calculate V_{rms} for the source.

SET UP: Source voltage lags current so $\phi = -54.0°$. $X_C = 350\ \Omega$, $R = 180\ \Omega$, $P_{av} = 140\text{ W}$

EXECUTE: **(a)** $\tan\phi = \dfrac{X_L - X_C}{R}$

$X_L = R\tan\phi + X_C = (180\ \Omega)\tan(-54.0°) + 350\ \Omega = -248\ \Omega + 350\ \Omega = 102\ \Omega$

(b) $P_{av} = V_{rms}I_{rms}\cos\phi = I_{rms}^2 R$ (Exercise 31.27). $I_{rms} = \sqrt{\dfrac{P_{av}}{R}} = \sqrt{\dfrac{140\text{ W}}{180\ \Omega}} = 0.882\text{ A}$

(c) $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(180\ \Omega)^2 + (102\ \Omega - 350\ \Omega)^2} = 306\ \Omega$

$V_{rms} = I_{rms}Z = (0.882\text{ A})(306\ \Omega) = 270\text{ V}$.

EVALUATE: We could also use Eq.(31.31): $P_{av} = V_{rms}I_{rms}\cos\phi$

$V_{rms} = \dfrac{P_{av}}{I_{rms}\cos\phi} = \dfrac{140\text{ W}}{(0.882\text{ A})\cos(-54.0°)} = 270\text{ V}$, which agrees. The source voltage lags the current when $X_C > X_L$, and this agrees with what we found.

31.60. **IDENTIFY** and **SET UP:** Calculate Z and $I = V/Z$.

EXECUTE: **(a)** For $\omega = 800$ rad/s:

$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} = \sqrt{(500\ \Omega)^2 + ((800\ \text{rad/s})(2.0\ \text{H}) - 1/((800\ \text{rad/s})(5.0\times10^{-7}\ \text{F})))^2}$. $Z = 1030\ \Omega$.

$I = \dfrac{V}{Z} = \dfrac{100\ \text{V}}{1030\ \Omega} = 0.0971\ \text{A}$. $V_R = IR = (0.0971\ \text{A})(500\ \Omega) = 48.6\ \text{V}$, $V_C = \dfrac{1}{\omega C} = \dfrac{0.0971\ \text{A}}{(800\ \text{rad/s})(5.0\times10^{-7}\ \text{F})} = 243\ \text{V}$

and $V_L = I\omega L = (0.0971\ \text{A})(800\ \text{rad/s})(2.00\ \text{H}) = 155\ \text{V}$. $\phi = \arctan\left(\dfrac{\omega L - 1/(\omega C)}{R}\right) = -60.9°$. The graph of each voltage versus time is given in Figure 31.60a.

(b) Repeating exactly the same calculations as above for $\omega = 1000$ rad/s:

$Z = R = 500\ \Omega$; $\phi = 0$; $I = 0.200$ A; $V_R = V = 100$ V; $V_C = V_L = 400$ V. The graph of each voltage versus time is given in Figure 31.60b.

(c) Repeating exactly the same calculations as part (a) for $\omega = 1250$ rad/s:

$Z = R = 1030\ \Omega$; $\phi = +60.9°$; $I = 0.0971$ A; $V_R = 48.6$ V; $V_C = 155$ V; $V_L = 243$ V. The graph of each voltage versus time is given in Figure 31.60c.

EVALUATE: The resonance frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(2.00\ \text{H})(0.500\ \mu\text{F})}} = 1000$ rad/s. For $\omega < \omega_0$ the phase angle is negative and for $\omega > \omega_0$ the phase angle is positive.

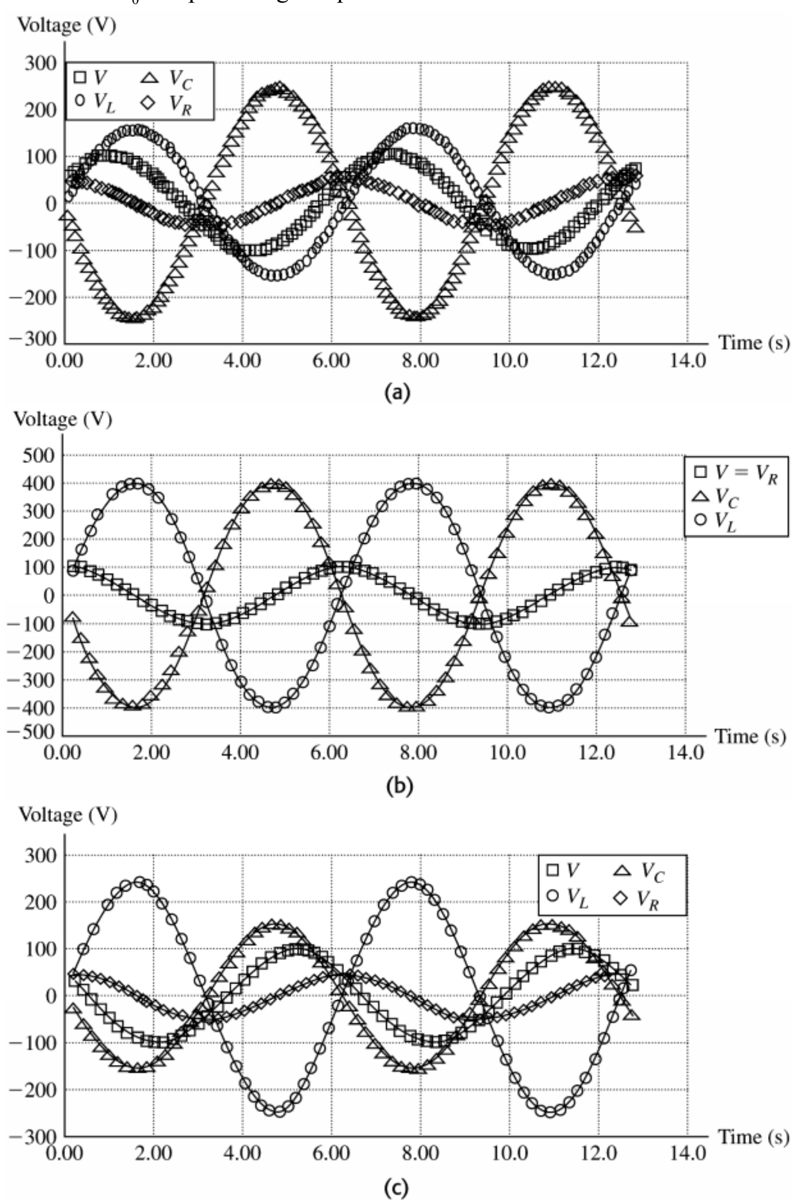


Figure 31.60

31.61. **IDENTIFY** and **SET UP:** Eq.(31.19) allows us to calculate I and then Eq.(31.22) gives Z. Solve Eq.(31.21) for L.

EXECUTE: **(a)** $V_C = IX_C$ so $I = \dfrac{V_C}{X_C} = \dfrac{360\text{ V}}{480\ \Omega} = 0.750\text{ A}$

(b) $V = IZ$ so $Z = \dfrac{V}{I} = \dfrac{120\text{ V}}{0.750\text{ A}} = 160\ \Omega$

(c) $Z^2 = R^2 + (X_L - X_C)^2$

$X_L - X_C = \pm\sqrt{Z^2 - R^2}$, so

$X_L = X_C \pm \sqrt{Z^2 - R^2} = 480\ \Omega \pm \sqrt{(160\ \Omega)^2 - (80.0\ \Omega)^2} = 480\ \Omega \pm 139\ \Omega$

$X_L = 619\ \Omega$ or $341\ \Omega$

(d) EVALUATE: $X_C = \dfrac{1}{\omega C}$ and $X_L = \omega L$. At resonance, $X_C = X_L$. As the frequency is lowered below the resonance frequency X_C increases and X_L decreases. Therefore, for $\omega < \omega_0, X_L < X_C$. So for $X_L = 341\ \Omega$ the angular frequency is less than the resonance angular frequency. ω is greater than ω_0 when $X_L = 619\ \Omega$. But at these two values of X_L, the magnitude of $X_L - X_C$ is the same so Z and I are the same. In one case $(X_L = 691\ \Omega)$ the source voltage leads the current and in the other $(X_L = 341\ \Omega)$ the source voltage lags the current.

31.62. **IDENTIFY** and **SET UP:** The maximum possible current amplitude occurs at the resonance angular frequency because the impedance is then smallest.

EXECUTE: **(a)** At the resonance angular frequency $\omega_0 = 1/\sqrt{LC}$, the current is a maximum and $Z = R$, giving $I_{max} = V/R$. At the required frequency, $I = I_{max}/3$. $I = V/Z = I_{max}/3 = (V/R)/3$, which means that $Z = 3R$. Squaring gives $R^2 + (\omega L - 1/\omega C)^2 = 9R^2$. Solving for ω gives $\omega = 3.192 \times 10^5$ rad/s and $\omega = 8.35 \times 10^4$ rad/s.

(b) $V = \sqrt{2}V_{rms} = \sqrt{2}(35.0\text{ V}) = 49.5\text{ V}$. $I = \dfrac{I_{max}}{3} = \dfrac{V}{3R} = \dfrac{49.5\text{ V}}{3(125\ \Omega)} = 0.132\text{ A}$.

For $\omega = 8.35\times10^4$ rad/s: $R = 125\ \Omega$ and $V_R = IR = 16.5\ \Omega$; $X_L = \omega L = 125\ \Omega$ and $V_L = 16.5$ V;

$X_C = \dfrac{1}{\omega C} = 479\ \Omega$ and $V_C = 63.2$ V.

For $\omega = 3.192\times10^5$ rad/s: $R = 125\ \Omega$ and $V_R = IR = 16.5\ \Omega$; $X_L = \omega L = 479\ \Omega$ and $V_L = 63.2$ V;

$X_C = \dfrac{1}{\omega C} = 125\ \Omega$ and $V_C = 16.5$ V.

EVALUATE: For the lower frequency, $X_C > X_L$ and $V_C > V_L$. For the higher frequency, $X_L > X_C$ and $V_L > V_C$.

31.63. **IDENTIFY** and **SET UP:** Consider the cycle of the repeating current that lies between $t_1 = \tau/2$ and $t_2 = 3\tau/2$. In this interval $i = \dfrac{2I_0}{\tau}(t-\tau)$. $I_{av} = \dfrac{1}{t_2 - t_1}\displaystyle\int_{t_1}^{t_2} i\,dt$ and $I_{rms}^2 = \dfrac{1}{t_2 - t_1}\displaystyle\int_{t_1}^{t_2} i^2 dt$

EXECUTE: $I_{av} = \dfrac{1}{t_2 - t_1}\displaystyle\int_{t_1}^{t_2} i\,dt = \dfrac{1}{\tau}\int_{\tau/2}^{3\tau/2}\dfrac{2I_0}{\tau}(t-\tau)\,dt = \dfrac{2I_0}{\tau^2}\left[\dfrac{1}{2}t^2 - \tau t\right]_{\tau/2}^{3\tau/2}$

$$I_{av} = \left(\frac{2I_0}{\tau^2}\right)\left(\frac{9\tau^2}{8} - \frac{3\tau 2}{2} - \frac{\tau^2}{8} + \frac{\tau^2}{2}\right) = (2I_0)\tfrac{1}{8}(9 - 12 - 1 + 4) = \frac{I_0}{4}(13 - 13) = 0.$$

$$I_{rms}^2 = (I^2)_{av} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} i^2 dt = \frac{1}{\tau}\int_{\tau/2}^{3\tau/2}\frac{4I_0^2}{\tau^2}(t-\tau)^2\,dt$$

$$I_{rms}^2 = \frac{4I_0^2}{\tau^3}\int_{\tau/2}^{3\tau/2}(t-\tau)^2 dt = \frac{4I_0^2}{\tau^3}\left[\tfrac{1}{3}(t-\tau)^3\right]_{\tau/2}^{3\tau/2} = \frac{4I_0^2}{3\tau^3}\left[\left(\frac{\tau}{2}\right)^3 - \left(-\frac{\tau}{2}\right)^3\right]$$

$$I_{rms}^2 = \frac{I_0^2}{6}[1+1] = \tfrac{1}{3}I_0^2$$

$$I_{rms} = \sqrt{I_{rms}^2} = \frac{I_0}{\sqrt{3}}.$$

EVALUATE: In each cycle the current has as much negative value as positive value and its average is zero. i^2 is always positive and its average is not zero. The relation between I_{rms} and the current amplitude for this current is different from that for a sinusoidal current (Eq.31.4).

31.64. IDENTIFY: Apply $V_{\text{rms}} = I_{\text{rms}}Z$

SET UP: $\omega_0 = \dfrac{1}{\sqrt{LC}}$ and $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

EXECUTE: **(a)** $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(1.80\text{ H})(9.00\times10^{-7}\text{ F})}} = 786\text{ rad/s}$.

(b) $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$. $Z = \sqrt{(300\ \Omega)^2 + ((786\text{ rad/s})(1.80\text{ H}) - 1/((786\text{ rad/s})(9.00\times10^{-7}\text{ F})))^2} = 300\ \Omega$.

$I_{\text{rms-0}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{60\text{ V}}{300\ \Omega} = 0.200\text{ A}$.

(c) We want $I = \dfrac{1}{2}I_{\text{rms-0}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{V_{\text{rms}}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$. $R^2 + (\omega L - 1/\omega C)^2 = \dfrac{4V_{\text{rms}}^2}{I_{\text{rms-0}}^2}$.

$\omega^2 L^2 + \dfrac{1}{\omega^2 C^2} - \dfrac{2L}{C} + R^2 - \dfrac{4V_{\text{rms}}^2}{I_{\text{rms-0}}^2} = 0$ and $(\omega^2)^2 L^2 + \omega^2\left(R^2 - \dfrac{2L}{C} - \dfrac{4V_{\text{rms}}^2}{I_{\text{rms-0}}^2}\right) + \dfrac{1}{C^2} = 0.$

Substituting in the values for this problem, the equation becomes $(\omega^2)^2(3.24) + \omega^2(-4.27\times10^6) + 1.23\times10^{12} = 0$.

Solving this quadratic equation in ω^2 we find $\omega^2 = 8.90\times10^5\text{ rad}^2/\text{s}^2$ or $4.28\times10^5\text{ rad}^2/\text{s}^2$ and $\omega = 943\text{ rad/s}$ or 654 rad/s.

(d) (i) $R = 300\ \Omega$, $I_{\text{rms-0}} = 0.200\text{ A}$, $|\omega_1 - \omega_2| = 289\text{ rad/s}$. (ii) $R = 30\ \Omega$, $I_{\text{rms-0}} = 2\text{A}$, $|\omega_1 - \omega_2| = 28\text{ rad/s}$.

(iii) $R = 3\ \Omega$, $I_{\text{rms-0}} = 20\text{ A}$, $|\omega_1 - \omega_2| = 2.88\text{ rad/s}$.

EVALUATE: The width gets smaller as R gets smaller; $I_{\text{rms-0}}$ gets larger as R gets smaller.

31.65. IDENTIFY: The resonance frequency, the reactances, and the impedance all depend on the values of the circuit elements.

SET UP: The resonance frequency is $\omega_0 = 1/\sqrt{LC}$, the reactances are $X_L = \omega L$ and $X_C = 1/\omega C$, and the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

EXECUTE: **(a)** $\omega_0 = 1/\sqrt{LC}$ becomes $\dfrac{1}{\sqrt{2L}\sqrt{2C}} \to 1/2$, so ω_0 decreases by $\frac{1}{2}$.

(b) Since $X_L = \omega L$, if L is doubled, X_L increases by a factor of 2.

(c) Since $X_C = 1/\omega C$, doubling C decreases X_C by a factor of $\frac{1}{2}$.

(d) $Z = \sqrt{R^2 + (X_L - X_C)^2} \to Z = \sqrt{(2R)^2 + (2X_L - \frac{1}{2}X_C)^2}$, so Z does not change by a simple factor of 2 or $\frac{1}{2}$.

EVALUATE: The impedance does not change by a simple factor, even though the other quantities do.

31.66. IDENTIFY: A transformer transforms voltages according to $\dfrac{V_2}{V_1} = \dfrac{N_2}{N_1}$. The effective resistance of a secondary circuit of resistance R is $R_{\text{eff}} = \dfrac{R}{(N_2/N_1)^2}$.

SET UP: $N_2 = 275$ and $V_1 = 25.0\text{ V}$.

EXECUTE: **(a)** $V_2 = V_1(N_2/N_1) = (25.0\text{ V})(834/275) = 75.8\text{ V}$

(b) $R_{\text{eff}} = \dfrac{R}{(N_2/N_1)^2} = \dfrac{125\ \Omega}{(834/275)^2} = 13.6\ \Omega$

EVALUATE: The voltage across the secondary is greater than the voltage across the primary since $N_2 > N_1$. The effective load resistance of the secondary is less than the resistance R connected across the secondary.

31.67. IDENTIFY: The resonance angular frequency is $\omega_0 = \dfrac{1}{\sqrt{LC}}$ and the resonance frequency is $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$.

SET UP: ω_0 is independent of R.

EXECUTE: **(a)** ω_0(or f_0) depends only on L and C so change these quantities.

(b) To double ω_0, decrease L and C by multiplying each of them by $\frac{1}{2}$.

EVALUATE: Increasing L and C decreases the resonance frequency; decreasing L and C increases the resonance frequency.

31.68. IDENTIFY: At resonance, $Z = R$. $I = V/R$. $V_R = IR$, $V_C = IX_C$ and $V_L = IX_L$. $U_E = \frac{1}{2}CV_C^2$ and $U_L = \frac{1}{2}LI^2$.

SET UP: The amplitudes of each time dependent quantity correspond to the maximum values of those quantities.

EXECUTE: **(a)** $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$. At resonance $\omega L = \frac{1}{\omega C}$ and $I_{\max} = \frac{V}{R}$.

(b) $V_C = IX_C = \frac{V}{R\omega_0 C} = \frac{V}{R}\sqrt{\frac{L}{C}}$.

(c) $V_L = IX_L = \frac{V}{R}\omega_0 L = \frac{V}{R}\sqrt{\frac{L}{C}}$.

(d) $U_C = \frac{1}{2}CV_C^2 = \frac{1}{2}C\frac{V^2}{R^2}\frac{L}{C} = \frac{1}{2}L\frac{V^2}{R^2}$.

(e) $U_L = \frac{1}{2}LI^2 = \frac{1}{2}L\frac{V^2}{R^2}$.

EVALUATE: At resonance $V_C = V_L$ and the maximum energy stored in the inductor equals the maximum energy stored in the capacitor.

31.69. **IDENTIFY:** $I = V/R$. $V_R = IR$, $V_C = IX_C$ and $V_L = IX_L$. $U_E = \frac{1}{2}CV_C^2$ and $U_L = \frac{1}{2}LI^2$.

SET UP: The amplitudes of each time dependent quantity correspond to the maximum values of those quantities.

EXECUTE: $\omega = \frac{\omega_0}{2}$.

(a) $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\frac{\omega_0 L}{2} - 2/\omega_0 C\right)^2}} = \frac{V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}$.

(b) $V_C = IX_C = \frac{2}{\omega_0 C}\frac{V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}} = \sqrt{\frac{L}{C}}\frac{2V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}$.

(c) $V_L = IX_L = \frac{\omega_0 L}{2}\frac{V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}} = \sqrt{\frac{L}{C}}\frac{V/2}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}$.

(d) $U_C = \frac{1}{2}CV_C^2 = \frac{2LV^2}{R^2 + \frac{9}{4}\frac{L}{C}}$.

(e) $U_L = \frac{1}{2}LI^2 = \frac{1}{2}\frac{LV^2}{R^2 + \frac{9}{4}\frac{L}{C}}$.

EVALUATE: For $\omega < \omega_0$, $V_C > V_L$ and the maximum energy stored in the capacitor is greater than the maximum energy stored in the inductor.

31.70. **IDENTIFY:** $I = V/R$. $V_R = IR$, $V_C = IX_C$ and $V_L = IX_L$. $U_E = \frac{1}{2}CV_C^2$ and $U_L = \frac{1}{2}LI^2$.

SET UP: The amplitudes of each time dependent quantity correspond to the maximum values of those quantities.

EXECUTE: $\omega = 2\omega_0$.

(a) $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (2\omega_0 L - 1/2\omega_0 C)^2}} = \frac{V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}$.

(b) $V_C = IX_C = \frac{1}{2\omega_0 C}\frac{V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}} = \sqrt{\frac{L}{C}}\frac{V/2}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}$.

(c) $V_L = IX_L = 2\omega_0 L\frac{V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}} = \sqrt{\frac{L}{C}}\frac{2V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}$.

(d) $U_C = \frac{1}{2}CV_C^2 = \frac{LV^2}{8\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}$.

(e) $U_L = \frac{1}{2}LI^2 = \dfrac{LV^2}{2\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}.$

EVALUATE: For $\omega > \omega_0$, $V_L > V_C$ and the maximum energy stored in the inductor is greater than the maximum energy stored in the capacitor.

31.71. IDENTIFY and **SET UP:** Assume the angular frequency ω of the source and the resistance R of the resistor are known.

EXECUTE: Connect the source, capacitor, resistor, and inductor in series. Measure V_R and V_L. $\dfrac{V_L}{V_R} = \dfrac{I\omega L}{IR} = \dfrac{\omega L}{R}$ and L can be calculated.

EVALUATE: There are a number of other approaches. The frequency could be varied until $V_C = V_L$, and then this frequency is equal to $1/\sqrt{LC}$. If C is known, then L can be calculated.

31.72. IDENTIFY: $P_{av} = V_{rms}I_{rms}\cos\phi$ and $I_{rms} = \dfrac{V_{rms}}{Z}$. Calculate Z. $R = Z\cos\phi$.

SET UP: $f = 50.0$ Hz and $\omega = 2\pi f$. The power factor is $\cos\phi$.

EXECUTE: **(a)** $P_{av} = \dfrac{V_{rms}^2}{Z}\cos\phi$. $Z = \dfrac{V_{rms}^2\cos\phi}{P_{av}} = \dfrac{(120\text{ V})^2(0.560)}{(220\text{ W})} = 36.7\ \Omega.$

$R = Z\cos\phi = (36.7\ \Omega)(0.560) = 20.6\ \Omega.$

(b) $Z = \sqrt{R^2 + X_L{}^2} \cdot X_L = \sqrt{Z^2 - R^2} = \sqrt{(36.7\ \Omega)^2 - (20.6\ \Omega)^2} = 30.4\ \Omega$. But $\phi = 0$ is at resonance, so the inductive and capacitive reactances equal each other. Therefore we need to add $X_C = 30.4\ \Omega$. $X_C = \dfrac{1}{\omega C}$ therefore gives

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50.0\text{ Hz})(30.4\ \Omega)} = 1.05\times10^{-4}\text{ F}.$$

(c) At resonance, $P_{av} = \dfrac{V^2}{R} = \dfrac{(120\text{ V})^2}{20.6\ \Omega} = 699$ W.

EVALUATE: $P_{av} = I_{rms}^2 R$ and I_{rms} is maximum at resonance, so the power drawn from the line is maximum at resonance.

31.73. IDENTIFY: $p_R = i^2R$. $p_L = iL\dfrac{di}{dt}$. $p_C = \dfrac{q}{C}i$.

SET UP: $i = I\cos\omega t$

EXECUTE: **(a)** $p_R = i^2R = I^2\cos^2(\omega t)R = V_R I\cos^2(\omega t) = \frac{1}{2}V_R I(1+\cos(2\omega t)).$

$$P_{av}(R) = \frac{1}{T}\int_0^T p_R dt = \frac{V_R I}{2T}\int_0^T (1+\cos(2\omega t))dt = \frac{V_R I}{2T}[t]_0^T = \tfrac{1}{2}V_R I.$$

(b) $p_L = Li\dfrac{di}{dt} = -\omega LI^2\cos(\omega t)\sin(\omega t) = -\frac{1}{2}V_L I\sin(2\omega t)$. But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{av}(L) = 0.$

(c) $p_C = \dfrac{q}{C}i = v_C i = V_C I\sin(\omega t)\cos(\omega t) = \frac{1}{2}V_C I\sin(2\omega t)$. But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{av}(C) = 0.$

(d) $p = p_R + p_L + p_c = V_R I\cos^2(\omega t) - \frac{1}{2}V_L I\sin(2\omega t) + \frac{1}{2}V_C I\sin(2\omega t)$ and

$p = I\cos(\omega t)(V_R\cos(\omega t) - V_L\sin(\omega t) + V_C\sin(\omega t))$. But $\cos\phi = \dfrac{V_R}{V}$ and $\sin\phi = \dfrac{V_L - V_C}{V}$, so

$p = VI\cos(\omega t)(\cos\phi\cos(\omega t) - \sin\phi\sin(\omega t))$, at any instant of time.

EVALUATE: At an instant of time the energy stored in the capacitor and inductor can be changing, but there is no net consumption of electrical energy in these components.

31.74. IDENTIFY: $V_L = IX_L$. $\dfrac{dV_L}{d\omega} = 0$ at the ω where V_L is a maximum. $V_C = IX_C$. $\dfrac{dV_C}{d\omega} = 0$ at the ω where V_C is a maximum.

SET UP: Problem 31.51 shows that $I = \dfrac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$

EXECUTE: **(a)** V_R =maximum when $V_C = V_L \Rightarrow \omega = \omega_0 = \dfrac{1}{\sqrt{LC}}.$

(b) V_L =maximum when $\frac{dV_L}{d\omega}=0$.Therefore: $\frac{dV_L}{d\omega}=0=\frac{d}{d\omega}\left(\frac{V\omega L}{\sqrt{R^2+(\omega L-1/\omega C)^2}}\right)$.

$0=\frac{VL}{\sqrt{R^2+(\omega L-1/\omega C)^2}}-\frac{V\omega^2 L(L-1/\omega^2 C)(L+1/\omega^2 C)}{(R^2+(\omega L-1/\omega C)^2)^{3/2}}$. $R^2+(\omega L-1/\omega C)^2=\omega^2(L^2-1/\omega^4C^2)$.

$R^2+\frac{1}{\omega^2C^2}-\frac{2L}{C}=-\frac{1}{\omega^2C^2}$. $\frac{1}{\omega^2}=LC-\frac{R^2C^2}{2}$ and $\omega=\frac{1}{\sqrt{LC-R^2C^2/2}}$.

(c) V_C =maximum when $\frac{dV_C}{d\omega}=0$. Therefore: $\frac{dV_C}{d\omega}=0=\frac{d}{d\omega}\left(\frac{V}{\omega C\sqrt{R^2+(\omega L-1/\omega C)^2}}\right)$.

$0=-\frac{V}{\omega^2C\sqrt{R^2+(\omega L-1/\omega C)^2}}-\frac{V(L-1/\omega^2C)(L+1/\omega^2C)}{C(R^2+(\omega L-1/\omega C)^2)^{3/2}}$. $R^2+(\omega L-1/\omega C)^2=-\omega^2(L^2-1/\omega^4C^2)$.

$R^2+\omega^2L^2-\frac{2L}{C}=-\omega^2L^2$ and $\omega=\sqrt{\frac{1}{LC}-\frac{R^2}{2L^2}}$.

$R^2+\omega^2L^2-\frac{2L}{C}=-\omega^2L^2$.

EVALUATE: V_L is maximum at a frequency greater than the resonance frequency and X_C is a maximum at a frequency less than the resonance frequency. These frequencies depend on R , as well as on L and on C.

31.75. IDENTIFY: Follow the steps specified in the problem.

SET UP: In part (a) use Eq.(31.23) to calculate Z and then $I=V/Z$. ϕ is given by Eq.(31.24). In part (b) let $Z=R+iX$.

EXECUTE: **(a)** From the current phasors we know that $Z=\sqrt{R^2+(\omega L-1/\omega C)^2}$.

$$Z=\sqrt{(400\ \Omega)^2+\left((1000\text{ rad/s})(0.50\text{ H})-\frac{1}{(1000\text{ rad/s})(1.25\times10^{-6}\text{ F})}\right)^2}=500\ \Omega.$$

$I=\frac{V}{Z}=\frac{200\text{ V}}{500\ \Omega}=0.400$ A.

(b) $\phi=\arctan\left(\frac{\omega L-1/(\omega C)}{R}\right)$. $\phi=\arctan\left(\frac{(1000\text{ rad/s})(0.500\text{ H})-1/(1000\text{ rad/s})(1.25\times10^{-6}\text{ F})}{400\ \Omega}\right)=+36.9°$

(c) $Z_{\text{cpx}}=R+i\left(\omega L-\frac{1}{\omega C}\right)$. $Z_{\text{cpx}}=400\ \Omega-i\left((1000\text{ rad/s})(0.50\text{ H})-\frac{1}{(1000\text{ rad/s})(1.25\times10^{-6}\text{ F})}\right)=400\ \Omega-300\ \Omega i.$

$Z=\sqrt{(400\ \Omega)^2+(-300\ \Omega)^2}=500\ \Omega.$

(d) $I_{\text{cpx}}=\frac{V}{Z_{\text{cpx}}}=\frac{200\text{ V}}{(400-300i)\ \Omega}=\left(\frac{8+6i}{25}\right)\text{A}=(0.320\text{ A})+(0.240\text{ A})i.$ $I=\sqrt{\left(\frac{8+6i}{25}\right)\left(\frac{8-6i}{25}\right)}=0.400$ A.

(e) $\tan\phi=\frac{\text{Im}(I_{\text{cpx}})}{\text{Re}(I_{\text{cpx}})}=\frac{6/25}{8/25}=0.75\Rightarrow\phi=+36.9°.$

(f) $V_{R\text{cpx}}=I_{\text{cpx}}R=\left(\frac{8+6i}{25}\right)(400\ \Omega)=(128+96i)\text{V}.$

$V_{L\text{cpx}}=iI_{\text{cpx}}\omega L=i\left(\frac{8+6i}{25}\right)(1000\text{ rad/s})(0.500\text{ H})=(-120+160i)\text{ V}.$

$V_{C\text{cpx}}=i\frac{I_{\text{cpx}}}{\omega C}=i\left(\frac{8+6i}{25}\right)\frac{1}{(1000\text{ rad/s})(1.25\times10^{-6}\text{ F})}=(+192-256i)\text{ V}.$

(g) $V_{\text{cpx}}=V_{R\text{cpx}}+V_{L\text{cpx}}+V_{C\text{cpx}}=(128+96i)\text{ V}+(-120+160i)\text{V}+(192-256i)\text{ V}=200\text{ V}.$

EVALUATE: Both approaches yield the same value for I and for ϕ.

ELECTROMAGNETIC WAVES

32

32.1. **IDENTIFY:** Since the speed is constant, distance $x = ct$.

SET UP: The speed of light is $c = 3.00\times10^8$ m/s . 1 yr $= 3.156\times10^7$ s.

EXECUTE: **(a)** $t = \dfrac{x}{c} = \dfrac{3.84\times10^8 \text{ m}}{3.00\times10^8 \text{ m/s}} = 1.28 \text{ s}$

(b) $x = ct = (3.00\times10^8 \text{ m/s})(8.61 \text{ yr})(3.156\times10^7 \text{ s/yr}) = 8.15\times10^{16} \text{ m} = 8.15\times10^{13} \text{ km}$

EVALUATE: The speed of light is very great. The distance between stars is very large compared to terrestrial distances.

32.2. **IDENTIFY:** Since the speed is constant the difference in distance is $c\Delta t$.

SET UP: The speed of electromagnetic waves in air is $c = 3.00\times10^8$ m/s.

EXECUTE: A total time difference of $0.60\ \mu$s corresponds to a difference in distance of

$c\Delta t = (3.00\times10^8 \text{ m/s})(0.60\times10^{-6} \text{ s}) = 180 \text{ m}$.

EVALUATE: The time delay doesn't depend on the distance from the transmitter to the receiver, it just depends on the difference in the length of the two paths.

32.3. **IDENTIFY:** Apply $c = f\lambda$.

SET UP: $c = 3.00\times10^8$ m/s

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.0\times10^8 \text{ m/s}}{5000 \text{ m}} = 6.0\times10^4 \text{ Hz}$.

(b) $f = \dfrac{c}{\lambda} = \dfrac{3.0\times10^8 \text{ m/s}}{5.0 \text{ m}} = 6.0\times10^7 \text{ Hz}$.

(c) $f = \dfrac{c}{\lambda} = \dfrac{3.0\times10^8 \text{ m/s}}{5.0\times10^{-6} \text{ m}} = 6.0\times10^{13} \text{ Hz}$.

(d) $f = \dfrac{c}{\lambda} = \dfrac{3.0\times10^8 \text{ m/s}}{5.0\times10^{-9} \text{ m}} = 6.0\times10^{16} \text{ Hz}$.

EVALUATE: f increases when λ decreases.

32.4. **IDENTIFY:** $c = f\lambda$ and $k = \dfrac{2\pi}{\lambda}$.

SET UP: $c = 3.00\times10^8$ m/s .

EXECUTE: **(a)** $f = \dfrac{c}{\lambda}$. UVA: 7.50×10^{14} Hz to 9.38×10^{14} Hz . UVB: 9.38×10^{14} Hz to 1.07×10^{15} Hz .

(b) $k = \dfrac{2\pi}{\lambda}$. UVA: 1.57×10^7 rad/m to 1.96×10^7 rad/m . UVB: 1.96×10^7 rad/m to 2.24×10^7 rad/m .

EVALUATE: Larger λ corresponds to smaller f and k.

32.5. **IDENTIFY:** $c = f\lambda$. $E_{max} = cB_{max}$. $k = 2\pi/\lambda$. $\omega = 2\pi f$.

SET UP: Since the wave is traveling in empty space, its wave speed is $c = 3.00\times10^8$ m/s .

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8 \text{ m/s}}{432\times10^{-9} \text{ m}} = 6.94\times10^{14} \text{ Hz}$

(b) $E_{max} = cB_{max} = (3.00\times10^8 \text{ m/s})(1.25\times10^{-6} \text{ T}) = 375 \text{ V/m}$

(c) $k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{432\times10^{-9} \text{ m}} = 1.45\times10^{7} \text{ rad/m}$. $\omega = (2\pi \text{ rad})(6.94\times10^{14} \text{ Hz}) = 4.36\times10^{15} \text{ rad/s}$.

$E = E_{\max}\cos(kx-\omega t) = (375 \text{ V/m})\cos([1.45\times10^{7} \text{ rad/m}]x - [4.36\times10^{15} \text{ rad/s}]t)$

$B = B_{\max}\cos(kx-\omega t) = (1.25\times10^{-6} \text{ T})\cos([1.45\times10^{7} \text{ rad/m}]x - [4.36\times10^{15} \text{ rad/s}]t)$

EVALUATE: The $\cos(kx-\omega t)$ factor is common to both the electric and magnetic field expressions, since these two fields are in phase.

32.6. **IDENTIFY:** $c = f\lambda$. $E_{\max} = cB_{\max}$. Apply Eqs.(32.17) and (32.19).

SET UP: The speed of the wave is $c = 3.00\times10^{8}$ m/s.

EXECUTE: **(a)** $f = \frac{c}{\lambda} = \frac{3.00\times10^{8} \text{ m/s}}{435\times10^{-9} \text{ m}} = 6.90\times10^{14} \text{ Hz}$

(b) $B_{\max} = \frac{E_{\max}}{c} = \frac{2.70\times10^{-3} \text{ V/m}}{3.00\times10^{8} \text{ m/s}} = 9.00\times10^{-12} \text{ T}$

(c) $k = \frac{2\pi}{\lambda} = 1.44\times10^{7} \text{ rad/m}$. $\omega = 2\pi f = 4.34\times10^{15} \text{ rad/s}$. If $\vec{E}(z,\ t) = \hat{i}E_{\max}\cos(kz+\omega t)$, then

$\vec{B}(z,\ t) = -\hat{j}B_{\max}\cos(kz+\omega t)$, so that $\vec{E}\times\vec{B}$ will be in the $-\hat{k}$ direction.

$\vec{E}(z,\ t) = \hat{i}(2.70\times10^{-3} \text{ V/m})\cos([1.44\times10^{7} \text{ rad/s})z + [4.34\times10^{15} \text{ rad/s}]t)$ and

$\vec{B}(z,\ t) = -\hat{j}(9.00\times10^{-12} \text{ T})\cos([1.44\times10^{7} \text{ rad/s})z + [4.34\times10^{15} \text{ rad/s}]t)$.

EVALUATE: The directions of $\vec{E}$ and $\vec{B}$ and of the propagation of the wave are all mutually perpendicular. The argument of the cosine is $kz+\omega t$ since the wave is traveling in the $-z$-direction . Waves for visible light have very high frequencies.

32.7. **IDENTIFY** and **SET UP:** The equations are of the form of Eqs.(32.17), with x replaced by z. $\vec{B}$ is along the y-axis; deduce the direction of $\vec{E}$.

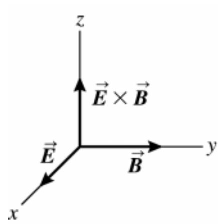
EXECUTE: $\omega = 2\pi f = 2\pi(6.10\times10^{14} \text{ Hz}) = 3.83\times10^{15} \text{ rad/s}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{\omega}{c} = \frac{3.83\times10^{15} \text{ rad/s}}{3.00\times10^{8} \text{ m/s}} = 1.28\times10^{7} \text{ rad/m}$

$B_{\max} = 5.80\times10^{-4} \text{ T}$

$E_{\max} = cB_{\max} = (3.00\times10^{8} \text{ m/s})(5.80\times10^{-4} \text{ T}) = 1.74\times10^{5} \text{ V/m}$

$\vec{B}$ is along the y-axis. $\vec{E}\times\vec{B}$ is in the direction of propagation (the $+z$-direction). From this we can deduce the direction of $\vec{E}$, as shown in Figure 32.7.



$\vec{E}$ is along the x-axis.

Figure 32.7

$\vec{E} = E_{\max}\hat{i}\cos(kz-\omega t) = (1.74\times10^{5} \text{ V/m})\hat{i}\cos[(1.28\times10^{7} \text{ rad/m})z - (3.83\times10^{15} \text{ rad/s})t]$

$\vec{B} = B_{\max}\hat{j}\cos(kz-\omega t) = \left(5.80\times10^{-4} \text{ T}\right)\hat{j}\cos[(1.28\times10^{7} \text{ rad/m})z - (3.83\times10^{15} \text{ rad/s})t]$

EVALUATE: $\vec{E}$ and $\vec{B}$ are perpendicular and oscillate in phase.

32.8. **IDENTIFY:** For an electromagnetic wave propagating in the negative x direction, $E = E_{\max}\cos(kx+\omega t)$. $\omega = 2\pi f$ and $k = \frac{2\pi}{\lambda}$. $T = \frac{1}{f}$. $E_{\max} = cB_{\max}$.

SET UP: The wave specified in the problem has a different phase, so $E = -E_{\max}\sin(kx+\omega t)$. $E_{\max} = 375 \text{ V/m}$, $k = 1.99\times10^{7}$ rad/m and $\omega = 5.97\times10^{15}$ rad/s .

EXECUTE: **(a)** $B_{\max} = \frac{E_{\max}}{c} = 1.25\ \mu\text{T}$.

(b) $f = \dfrac{\omega}{2\pi} = 9.50\times10^{14}$ Hz. $\lambda = \dfrac{2\pi}{k} = 3.16\times10^{-7}$ m $= 316$ nm. $T = \dfrac{1}{f} = 1.05\times10^{-15}$ s. This wavelength is too short to be visible.

(c) $c = f\lambda = (9.50\times10^{14}\text{ Hz})(3.16\times10^{-7}\text{ m}) = 3.00\times10^{8}$ m/s. This is what the wave speed should be for an electromagnetic wave propagating in vacuum.

EVALUATE: $c = f\lambda = \left(\dfrac{\omega}{2\pi}\right)\left(\dfrac{2\pi}{k}\right) = \dfrac{\omega}{k}$ is an alternative expression for the wave speed.

32.9. **IDENTIFY** and **SET UP:** Compare the $\vec{E}(y,\ t)$ given in the problem to the general form given by Eq.(32.17). Use the direction of propagation and of $\vec{E}$ to find the direction of $\vec{B}$.

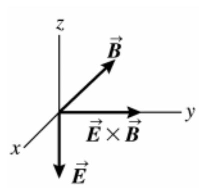
(a) EXECUTE: The equation for the electric field contains the factor $\sin(ky-\omega t)$ so the wave is traveling in the $+y$-direction. The equation for $\vec{E}(y,\ t)$ is in terms of $\sin(ky-\omega t)$ rather than $\cos(ky-\omega t)$; the wave is shifted in phase by $90°$ relative to one with a $\cos(ky-\omega t)$ factor.

(b) $\vec{E}(y,t) = -(3.10\times10^{5}\text{ V/m})\hat{k}\sin[ky-(2.65\times10^{12}\text{ rad/s})t]$

Comparing to Eq.(32.17) gives $\omega = 2.65\times10^{12}$ rad/s

$\omega = 2\pi f = \dfrac{2\pi c}{\lambda}$ so $\lambda = \dfrac{2\pi c}{\omega} = \dfrac{2\pi(2.998\times10^{8}\text{ m/s})}{(2.65\times10^{12}\text{ rad/s})} = 7.11\times10^{-4}$ m

(c)



$\vec{E}\times\vec{B}$ must be in the $+y$-direction (the direction in which the wave is traveling). When $\vec{E}$ is in the $-z$-direction then $\vec{B}$ must be in the $-x$-direction, as shown in Figure 32.9.

Figure 32.9

$k = \dfrac{2\pi}{\lambda} = \dfrac{\omega}{c} = \dfrac{2.65\times10^{12}\text{ rad/s}}{2.998\times10^{8}\text{ m/s}} = 8.84\times10^{3}$ rad/m

$E_{max} = 3.10\times10^{5}$ V/m

Then $B_{max} = \dfrac{E_{max}}{c} = \dfrac{3.10\times10^{5}\text{ V/m}}{2.998\times10^{8}\text{ m/s}} = 1.03\times10^{-3}$ T

Using Eq.(32.17) and the fact that $\vec{B}$ is in the $-\hat{i}$ direction when $\vec{E}$ is in the $-\hat{k}$ direction,

$\vec{B} = -(1.03\times10^{-3}\text{ T})\hat{i}\sin[(8.84\times10^{3}\text{ rad/m})y-(2.65\times10^{12}\text{ rad/s})t]$

EVALUATE: $\vec{E}$ and $\vec{B}$ are perpendicular and oscillate in phase.

32.10. **IDENTIFY:** Apply Eqs.(32.17) and (32.19). $f = c/\lambda$ and $k = 2\pi/\lambda$.

SET UP: The wave in this problem has a different phase, so $B_y(z,\ t) = B_{max}\sin(kx+\omega t)$.

EXECUTE: **(a)** The phase of the wave is given by $kx+\omega t$, so the wave is traveling in the $-x$ direction.

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{c}$. $f = \dfrac{kc}{2\pi} = \dfrac{(1.38\times10^{4}\text{ rad/m})(3.0\times10^{8}\text{ m/s})}{2\pi} = 6.59\times10^{11}$ Hz.

(c) Since the magnetic field is in the $+y$-direction, and the wave is propagating in the $-x$-direction, then the electric field is in the $+z$-direction so that $\vec{E}\times\vec{B}$ will be in the $-x$-direction.

$\vec{E}(x,\ t) = +cB(x,\ t)\hat{k} = cB_{max}\sin(kx+\omega t)\hat{k}$.

$\vec{E}(x,\ t) = (c(3.25\times10^{-9}\text{ T}))\sin\left((1.38\times10^{4}\text{ rad/m})x+(4.14\times10^{12}\text{ rad/s})t\right)\hat{k}$.

$\vec{E}(x,\ t) = +(2.48\text{ V/m})\sin\left((1.38\times10^{4}\text{ rad/m})x+(4.14\times10^{12}\text{ rad/s})t\right)\hat{k}$.

EVALUATE: $\vec{E}$ and $\vec{B}$ have the same phase and are in perpendicular directions.

32.11. **IDENTIFY** and **SET UP:** $c = f\lambda$ allows calculation of λ. $k = 2\pi/\lambda$ and $\omega = 2\pi f$. Eq.(32.18) relates the electric and magnetic field amplitudes.

EXECUTE: **(a)** $c = f\lambda$ so $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^{8}\text{ m/s}}{830\times10^{3}\text{ Hz}} = 361$ m

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi \text{ rad}}{361 \text{ m}} = 0.0174 \text{ rad/m}$

(c) $\omega = 2\pi f = (2\pi)(830\times10^3 \text{ Hz}) = 5.22\times10^6 \text{ rad/s}$

(d) Eq.(32.18): $E_{\max} = cB_{\max} = (2.998\times10^8 \text{ m/s})(4.82\times10^{-11} \text{ T}) = 0.0144 \text{ V/m}$

EVALUATE: This wave has a very long wavelength; its frequency is in the AM radio braodcast band. The electric and magnetic fields in the wave are very weak.

32.12. **IDENTIFY:** $E_{\max} = cB_{\max}$.

SET UP: The magnetic field of the earth is about 10^{-4} T.

EXECUTE: $B = \dfrac{E}{c} = \dfrac{3.85\times10^{-3} \text{ V/m}}{3.00\times10^8 \text{ m/s}} = 1.28\times10^{-11} \text{ T}$.

EVALUATE: The field is much smaller than the earth's field.

32.13. **IDENTIFY** and **SET UP:** $v = f\lambda$ relates frequency and wavelength to the speed of the wave. Use Eq.(32.22) to calculate n and K.

EXECUTE: **(a)** $\lambda = \dfrac{v}{f} = \dfrac{2.17\times10^8 \text{ m/s}}{5.70\times10^{14} \text{ Hz}} = 3.81\times10^{-7} \text{ m}$

(b) $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^8 \text{ m/s}}{5.70\times10^{14} \text{ Hz}} = 5.26\times10^{-7} \text{ m}$

(c) $n = \dfrac{c}{v} = \dfrac{2.998\times10^8 \text{ m/s}}{2.17\times10^8 \text{ m/s}} = 1.38$

(d) $n = \sqrt{KK_{\text{m}}} \approx \sqrt{K}$ so $K = n^2 = (1.38)^2 = 1.90$

EVALUATE: In the material $v < c$ and f is the same, so λ is less in the material than in air. $v < c$ always, so n is always greater than unity.

32.14. **IDENTIFY:** Apply Eq.(32.21). $E_{\max} = cB_{\max}$. $v = f\lambda$. Apply Eq.(32.29) with $\mu = K_{\text{m}}\mu_0$ in place of μ_0.

SET UP: $K = 3.64$. $K_{\text{m}} = 5.18$

EXECUTE: **(a)** $v = \dfrac{c}{\sqrt{KK_{\text{m}}}} = \dfrac{(3.00\times10^8 \text{ m/s})}{\sqrt{(3.64)(5.18)}} = 6.91\times10^7 \text{ m/s}$.

(b) $\lambda = \dfrac{v}{f} = \dfrac{6.91\times10^7 \text{ m/s}}{65.0 \text{ Hz}} = 1.06\times10^6 \text{ m}$.

(c) $B_{\max} = \dfrac{E_{\max}}{v} = \dfrac{7.20\times10^{-3} \text{ V/m}}{6.91\times10^7 \text{ m/s}} = 1.04\times10^{-10} \text{ T}$.

(d) $I = \dfrac{E_{\max}B_{\max}}{2K_{\text{m}}\mu_0} = \dfrac{(7.20\times10^{-3} \text{ V/m})(1.04\times10^{-10} \text{ T})}{2(5.18)\mu_0} = 5.75\times10^{-8} \text{ W/m}^2$.

EVALUATE: The wave travels slower in this material than in air.

32.15. **IDENTIFY:** $I = P/A$. $I = \tfrac{1}{2}\epsilon_0 cE_{\max}^2$. $E_{\max} = cB_{\max}$.

SET UP: The surface area of a sphere of radius r is $A = 4\pi r^2$. $\epsilon_0 = 8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2$.

EXECUTE: **(a)** $I = \dfrac{P}{A} = \dfrac{(0.05)(75 \text{ W})}{4\pi(3.0\times10^{-2} \text{ m})^2} = 330 \text{ W/m}^2$.

(b) $E_{\max} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(330 \text{ W/m}^2)}{(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8 \text{ m/s})}} = 500 \text{ V/m}$. $B_{\max} = \dfrac{E_{\max}}{c} = 1.7\times10^{-6} \text{ T} = 1.7 \ \mu\text{T}$.

EVALUATE: At the surface of the bulb the power radiated by the filament is spread over the surface of the bulb. Our calculation approximates the filament as a point source that radiates uniformly in all directions.

32.16. **IDENTIFY** and **SET UP:** The direction of propagation is given by $\vec{E}\times\vec{B}$.

EXECUTE: **(a)** $\hat{S} = \hat{i}\times(-\hat{j}) = -\hat{k}$.

(b) $\hat{S} = \hat{j}\times\hat{i} = -\hat{k}$.

(c) $\hat{S} = (-\hat{k})\times(-\hat{i}) = \hat{j}$.

(d) $\hat{S} = \hat{i}\times(-\hat{k}) = \hat{j}$.

EVALUATE: In each case the directions of $\vec{E}$, $\vec{B}$ and the direction of propagation are all mutually perpendicular.

32.17. **IDENTIFY:** $E_{max} = cB_{max}$. $\vec{E}\times\vec{B}$ is in the direction of propagation.

SET UP: $c = 3.00\times10^8$ m/s. $E_{max} = 4.00$ V/m.

EXECUTE: $B_{max} = E_{max}/c = 1.33\times10^{-8}$ T. For $\vec{E}$ in the +*x*-direction, $\vec{E}\times\vec{B}$ is in the +*z*-direction when $\vec{B}$ is in the +*y*-direction.

EVALUATE: $\vec{E}$, $\vec{B}$ and the direction of propagation are all mutually perpendicular.

32.18. **IDENTIFY:** The intensity of the electromagnetic wave is given by Eq.(32.29): $I = \frac{1}{2}\epsilon_0 cE_{max}^2 = \epsilon_0 cE_{rms}^2$. The total energy passing through a window of area A during a time t is IAt.

SET UP: $\epsilon_0 = 8.85\times10^{-12}$ F/m

EXECUTE: $\text{Energy} = \epsilon_0 cE_{rms}^2 At = (8.85\times10^{-12}\ \text{F/m})(3.00\times10^8\ \text{m/s})(0.0200\ \text{V/m})^2(0.500\ \text{m}^2)(30.0\ \text{s}) = 15.9\ \mu\text{J}$

EVALUATE: The intensity is proportional to the square of the electric field amplitude.

32.19. **IDENTIFY and SET UP:** Use Eq.(32.29) to calculate *I*, Eq.(32.18) to calculate B_{max}, and use $I = P_{av}/4\pi r^2$ to calculate P_{av}.

(a) EXECUTE: $I = \frac{1}{2}\epsilon_0 E_{max}^2$; $E_{max} = 0.090$ V/m, so $I = 1.1\times10^{-5}\ \text{W/m}^2$

(b) $E_{max} = cB_{max}$ so $B_{max} = E_{max}/c = 3.0\times10^{-10}$ T

(c) $P_{av} = I(4\pi r^2) = (1.075\times10^{-5}\ \text{W/m}^2)(4\pi)(2.5\times10^3\ \text{m})^2 = 840$ W

(d) EVALUATE: The calculation in part (c) assumes that the transmitter emits uniformly in all directions.

32.20. **IDENTIFY and SET UP:** $I = P_{av}/A$ and $I = \epsilon_0 cE_{rms}^2$.

EXECUTE: **(a)** The average power from the beam is $P_{av} = IA = (0.800\ \text{W/m}^2)(3.0\times10^{-4}\ \text{m}^2) = 2.4\times10^{-4}$ W.

(b) $$E_{rms} = \sqrt{\frac{I}{\epsilon_0 c}} = \sqrt{\frac{0.800\ \text{W/m}^2}{(8.85\times10^{-12}\ \text{F/m})(3.00\times10^8\ \text{m/s})}} = 17.4\ \text{V/m}$$

EVALUATE: The laser emits radiation only in the direction of the beam.

32.21: **IDENTIFY:** $I = P_{av}/A$

SET UP: At a distance *r* from the star, the radiation from the star is spread over a spherical surface of area $A = 4\pi r^2$.

EXECUTE: $P_{av} = I(4\pi r^2) = (5.0\times10^3\ \text{W/m}^2)(4\pi)(2.0\times10^{10}\ \text{m})^2 = 2.5\times10^{25}$ J

EVALUATE: The intensity decreases with distance from the star as $1/r^2$.

32.22. **IDENTIFY and SET UP:** $c = f\lambda$, $E_{max} = cB_{max}$ and $I = E_{max}B_{max}/2\mu_0$

EXECUTE: **(a)** $$f = \frac{c}{\lambda} = \frac{3.00\times10^8\ \text{m/s}}{0.354\ \text{m}} = 8.47\times10^8\ \text{Hz}.$$

(b) $$B_{max} = \frac{E_{max}}{c} = \frac{0.0540\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 1.80\times10^{-10}\ \text{T}.$$

(c) $$I = S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{(0.0540\ \text{V/m})(1.80\times10^{-10}\ \text{T})}{2\mu_0} = 3.87\times10^{-6}\ \text{W/m}^2.$$

EVALUATE: Alternatively, $I = \frac{1}{2}\epsilon_0 cE_{max}^2$.

32.23. **IDENTIFY:** $P_{av} = IA$ and $I = \frac{1}{2}\epsilon_0 cE_{max}^2$

SET UP: The surface area of a sphere is $A = 4\pi r^2$.

EXECUTE: $$P_{av} = S_{av}A = \left(\frac{E_{max}^2}{2c\mu_0}\right)(4\pi r^2).\quad E_{max} = \sqrt{\frac{P_{av}c\mu_0}{2\pi r^2}} = \sqrt{\frac{(60.0\ \text{W})(3.00\times10^8\ \text{m/s})\mu_0}{2\pi(5.00\ \text{m})^2}} = 12.0\ \text{V/m}.$$

$$B_{max} = \frac{E_{max}}{c} = \frac{12.0\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 4.00\times10^{-8}\ \text{T}.$$

EVALUATE: E_{max} and B_{max} are both inversely proportional to the distance from the source.

32.24. **IDENTIFY:** The Poynting vector is $\vec{S} = \vec{E}\times\vec{B}$.

SET UP: The electric field is in the +*y*-direction, and the magnetic field is in the +*z*-direction. $\cos^2\phi = \frac{1}{2}(1+\cos 2\phi)$

EXECUTE: **(a)** $\hat{S} = \hat{E}\times\hat{B} = (-\hat{j})\times\hat{k} = -\hat{i}$. The Poynting vector is in the –*x*-direction, which is the direction of propagation of the wave.

(b) $S(x,\,t)=\dfrac{E(x,\,t)B(x,\,t)}{\mu_0}=\dfrac{E_{\max}B_{\max}}{\mu_0}\cos^2(kx+\omega t)=\dfrac{E_{\max}B_{\max}}{2\mu_0}\big(1+\cos(2(\omega t+kx))\big).$ But over one period, the cosine function averages to zero, so we have $S_{\text{av}}=\dfrac{E_{\max}B_{\max}}{2\mu_0}$. This is Eq.(32.29).

EVALUATE: We can also show that these two results also apply to the wave represented by Eq.(32.17).

32.25. **IDENTIFY:** Use the radiation pressure to find the intensity, and then $P_{\text{av}}=I(4\pi r^2)$.

SET UP: For a perfectly absorbing surface, $p_{\text{rad}}=\dfrac{I}{c}$

EXECUTE: $p_{\text{rad}}=I/c$ so $I=cp_{\text{rad}}=2.70\times10^3\ \text{W/m}^2$. Then

$P_{\text{av}}=I(4\pi r^2)=(2.70\times10^3\ \text{W/m}^2)(4\pi)(5.0\ \text{m})^2=8.5\times10^5\ \text{W}.$

EVALUATE: Even though the source is very intense the radiation pressure 5.0 m from the surface is very small.

32.26. **IDENTIFY:** The intensity and the energy density of an electromagnetic wave depends on the amplitudes of the electric and magnetic fields.

SET UP: Intensity is $I=P_{\text{av}}/A$, and the average power is $P_{\text{av}}=2I/c$, where $I=\tfrac{1}{2}\epsilon_0 cE_{\max}^2$. The energy density is $u=\epsilon_0E^2$.

EXECUTE: **(a)** $I=P_{\text{av}}/A=\dfrac{316{,}000\ \text{W}}{2\pi(5000\ \text{m})^2}=0.00201\ \text{W/m}^2$. $P_{\text{av}}=2I/c=\dfrac{2(0.00201\ \text{W/m}^2)}{3.00\times10^8\ \text{m/s}}=1.34\times10^{-11}\ \text{Pa}$

(b) $I=\tfrac{1}{2}\epsilon_0cE_{\max}^2$ gives

$$E_{\max}=\sqrt{\frac{2I}{\epsilon_0c}}=\sqrt{\frac{2(0.00201\,\text{W/m}^2)}{(8.85\times10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\ \text{m/s})}}=1.23\ \text{N/C}$$

$$B_{\max}=E_{\max}/c=(1.23\ \text{N/C})/(3.00\times10^8\ \text{m/s})=4.10\times10^{-9}\ \text{T}$$

(c) $u=\epsilon_0E^2$, so $u_{\text{av}}=\epsilon_0(E_{\text{av}})^2$ and $E_{\text{av}}=\dfrac{E_{\max}}{\sqrt{2}}$, so

$$u_{\text{av}}=\frac{\epsilon_0E_{\max}^2}{2}=\frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(1.23\ \text{N/C})^2}{2}=6.69\times10^{-12}\ \text{J/m}^3$$

(d) As was shown in Section 32.4, the energy density is the same for the electric and magnetic fields, so each one has 50% of the energy density.

EVALUATE: Compared to most laboratory fields, the electric and magnetic fields in ordinary radiowaves are extremely weak and carry very little energy.

32.27. **IDENTIFY and SET UP:** Use Eqs.(32.30) and (32.31).

EXECUTE: **(a)** By Eq.(32.30) the average momentum density is $\dfrac{dp}{dV}=\dfrac{S_{\text{av}}}{c^2}=\dfrac{I}{c^2}$

$\dfrac{dp}{dV}=\dfrac{0.78\times10^3\ \text{W/m}^2}{(2.998\times10^8\ \text{m/s})^2}=8.7\times10^{-15}\ \text{kg/m}^2\cdot\text{s}$

(b) By Eq.(32.31) the average momentum flow rate per unit area is $\dfrac{S_{\text{av}}}{c}=\dfrac{I}{c}=\dfrac{0.78\times10^3\ \text{W/m}^2}{2.998\times10^8\ \text{m/s}}=2.6\times10^{-6}\ \text{Pa}$

EVALUATE: The radiation pressure that the sunlight would exert on an absorbing or reflecting surface is very small.

32.28. **IDENTIFY:** Apply Eqs.(32.32) and (32.33). The average momentum density is given by Eq.(32.30), with S replaced by $S_{\text{av}}=I$.

SET UP: $1\ \text{atm}=1.013\times10^5\ \text{Pa}$

EXECUTE: **(a)** Absorbed light: $p_{\text{rad}}=\dfrac{I}{c}=\dfrac{2500\ \text{W/m}^2}{3.0\times10^8\ \text{m/s}}=8.33\times10^{-6}\ \text{Pa}$. Then

$p_{\text{rad}}=\dfrac{8.33\times10^{-6}\ \text{Pa}}{1.013\times10^5\ \text{Pa/atm}}=8.23\times10^{-11}\ \text{atm}.$

(b) Reflecting light: $p_{\text{rad}}=\dfrac{2I}{c}=\dfrac{2(2500\ \text{W/m}^2)}{3.0\times10^8\ \text{m/s}}=1.67\times10^{-5}\ \text{Pa}$. Then

$p_{\text{rad}}=\dfrac{1.67\times10^{-5}\ \text{Pa}}{1.013\times10^5\ \text{Pa/atm}}=1.65\times10^{-10}\ \text{atm}.$

(c) The momentum density is $\dfrac{dp}{dV}=\dfrac{S_{av}}{c^2}=\dfrac{2500\ \text{W/m}^2}{(3.0\times10^8\ \text{m/s})^2}=2.78\times10^{-14}\ \text{kg/m}^2\cdot\text{s}.$

EVALUATE: The factor of 2 in p_{rad} for the reflecting surface arises because the momentum vector totally reverses direction upon reflection. Thus the *change* in momentum is twice the original momentum.

32.29. **IDENTIFY:** Apply Eq.(32.4) and (32.9).

SET UP: Eq.(32.26) is $S=\epsilon_0 cE^2$.

EXECUTE: $S=\dfrac{\epsilon_0}{\sqrt{\epsilon_0\mu_0}}E^2=\sqrt{\dfrac{\epsilon_0}{\mu_0}}E^2=\sqrt{\dfrac{\epsilon_0}{\mu_0}}Ec\dfrac{E}{c}=c\sqrt{\dfrac{\epsilon_0}{\mu_0}}EB=\dfrac{1}{\sqrt{\epsilon_0\mu_0}}\sqrt{\dfrac{\epsilon_0}{\mu_0}}EB=\dfrac{EB}{\mu_0}=\dfrac{E^2}{\mu_0 c}=\epsilon_0 cE^2$

EVALUATE: We can also write $S=\epsilon_0 c(cB)^2=\epsilon_0 c^3B^2$. S can be written solely in terms of E or solely in terms of B.

32.30. **IDENTIFY:** The electric field at the nodes is zero, so there is no force on a point charge placed at a node.
SET UP: The location of the nodes is given by Eq.(32.36), where x is the distance from one of the planes. $\lambda=c/f$.

EXECUTE: $\Delta x_{\text{nodes}}=\dfrac{\lambda}{2}=\dfrac{c}{2f}=\dfrac{3.00\times10^8\ \text{m/s}}{2(7.50\times10^8\ \text{Hz})}=0.200\ \text{m}=20.0\ \text{cm}$. There must be nodes at the planes, which are 80.0 cm apart, and there are two nodes between the planes, each 20.0 cm from a plane. It is at 20 cm, 40 cm, and 60 cm from one plane that a point charge will remain at rest, since the electric fields there are zero.
EVALUATE: The magnetic field amplitude at these points isn't zero, but the magnetic field doesn't exert a force on a stationary charge.

32.31. **IDENTIFY** and **SET UP:** Apply Eqs.(32.36) and (32.37).

EXECUTE: **(a)** By Eq.(32.37) we see that the nodal planes of the $\vec{B}$ field are a distance $\lambda/2$ apart, so $\lambda/2=3.55$ mm and $\lambda=7.10$ mm.

(b) By Eq.(32.36) we see that the nodal planes of the $\vec{E}$ field are also a distance $\lambda/2=3.55$ mm apart.

(c) $v=f\lambda=(2.20\times10^{10}\ \text{Hz})(7.10\times10^{-3}\ \text{m})=1.56\times10^8\ \text{m/s}.$

EVALUATE: The spacing between the nodes of $\vec{E}$ is the same as the spacing between the nodes of $\vec{B}$. Note that $v<c$, as it must.

32.32. **IDENTIFY:** The nodal planes of $\vec{E}$ and $\vec{B}$ are located by Eqs.(32.26) and (32.27).

SET UP: $\lambda=\dfrac{c}{f}=\dfrac{3.00\times10^8\ \text{m/s}}{75.0\times10^6\ \text{Hz}}=4.00\ \text{m}$

EXECUTE: **(a)** $\Delta x=\dfrac{\lambda}{2}=2.00\ \text{m}.$

(b) The distance between the electric and magnetic nodal planes is one-quarter of a wavelength, so is $\dfrac{\lambda}{4}=\dfrac{\Delta x}{2}=\dfrac{2.00\ \text{m}}{2}=1.00\ \text{m}.$

EVALUATE: The nodal planes of $\vec{B}$ are separated by a distance $\lambda/2$ and are midway between the nodal planes of $\vec{E}$.

32.33. **(a) IDENTIFY** and **SET UP:** The distance between adjacent nodal planes of $\vec{B}$ is $\lambda/2$. There is an antinodal plane of $\vec{B}$ midway between any two adjacent nodal planes, so the distance between a nodal plane and an adjacent antinodal plane is $\lambda/4$. Use $v=f\lambda$ to calculate λ.

EXECUTE: $\lambda=\dfrac{v}{f}=\dfrac{2.10\times10^8\ \text{m/s}}{1.20\times10^{10}\ \text{Hz}}=0.0175\ \text{m}$

$\dfrac{\lambda}{4}=\dfrac{0.0175\ \text{m}}{4}=4.38\times10^{-3}\ \text{m}=4.38\ \text{mm}$

(b) IDENTIFY and **SET UP:** The nodal planes of $\vec{E}$ are at $x=0,\ \lambda/2,\ \lambda,\ 3\lambda/2,\ \ldots,$ so the antinodal planes of $\vec{E}$ are at $x=\lambda/4,\ 3\lambda/4,\ 5\lambda/4,\ \ldots.$ The nodal planes of $\vec{B}$ are at $x=\lambda/4,\ 3\lambda/4,\ 5\lambda/4,\ \ldots,$ so the antinodal planes of $\vec{B}$ are at $\lambda/2,\ \lambda,\ 3\lambda/2,\ \ldots.$

EXECUTE: The distance between adjacent antinodal planes of $\vec{E}$ and antinodal planes of $\vec{B}$ is therefore $\lambda/4=4.38$ mm.

(c) From Eqs.(32.36) and (32.37) the distance between adjacent nodal planes of $\vec{E}$ and $\vec{B}$ is $\lambda/4=4.38$ mm.

EVALUATE: The nodes of $\vec{E}$ coincide with the antinodes of $\vec{B}$, and conversely. The nodes of $\vec{B}$ and the nodes of $\vec{E}$ are equally spaced.

32.34. **IDENTIFY:** Evaluate the derivatives of the expressions for $E_y(x,\ t)$ and $B_z(x,\ t)$ that are given in Eqs.(32.34) and (32.35).

SET UP: $\frac{\partial}{\partial x}\sin kx = k\cos kx$, $\frac{\partial}{\partial t}\sin\omega t = \omega\cos\omega t$. $\frac{\partial}{\partial x}\cos kx = -k\sin kx$, $\frac{\partial}{\partial t}\cos\omega t = -\omega\sin\omega t$.

EXECUTE: **(a)** $\frac{\partial^2 E_y(x,\ t)}{\partial x^2} = \frac{\partial^2}{\partial x^2}(-2E_{\max}\sin kx\sin\omega t) = \frac{\partial}{\partial x}(-2kE_{\max}\cos kx\sin\omega t)$ and

$$\frac{\partial^2 E_y(x,\ t)}{\partial x^2} = 2k^2E_{\max}\sin kx\sin\omega t = \frac{\omega^2}{c^2}2E_{\max}\sin kx\sin\omega t = \epsilon_0\mu_0\frac{\partial^2 E_y(x,\ t)}{\partial t^2}.$$

Similarly: $\frac{\partial^2 B_z(x,\ t)}{\partial x^2} = \frac{\partial^2}{\partial x^2}(-2B_{\max}\cos kx\cos\omega t) = \frac{\partial}{\partial x}(+2kB_{\max}\sin kx\cos\omega t)$ and

$$\frac{\partial^2 B_z(x,\ t)}{\partial x^2} = 2k^2B_{\max}\cos kx\cos\omega t = \frac{\omega^2}{c^2}2B_{\max}\cos kx\cos\omega t = \epsilon_0\mu_0\frac{\partial^2 B_z(x,\ t)}{\partial t^2}.$$

(b) $\frac{\partial E_y(x,\ t)}{\partial x} = \frac{\partial}{\partial x}(-2E_{\max}\sin kx\sin\omega t) = -2kE_{\max}\cos kx\sin\omega t$.

$$\frac{\partial E_y(x,\ t)}{\partial x} = -\frac{\omega}{c}2E_{\max}\cos kx\sin\omega t = -\omega 2\frac{E_{\max}}{c}\cos kx\sin\omega t = -\omega 2B_{\max}\cos kx\sin\omega t\ .$$

$$\frac{\partial E_y(x,\ t)}{\partial x} = +\frac{\partial}{\partial t}(2B_{\max}\cos kx\cos\omega t) = -\frac{\partial B_z(x,\ t)}{\partial t}.$$

Similarly: $-\frac{\partial B_z(x,\ t)}{\partial x} = \frac{\partial}{\partial x}(+2B_{\max}\cos kx\cos\omega t) = -2kB_{\max}\sin kx\cos\omega t$.

$$-\frac{\partial B_z(x,\ t)}{\partial x} = -\frac{\omega}{c}2B_{\max}\sin kx\cos\omega t = -\frac{\omega}{c^2}2cB_{\max}\sin kx\cos\omega t\ .$$

$$-\frac{\partial B_z(x,\ t)}{\partial x} = -\epsilon_0\mu_0\omega 2E_{\max}\sin kx\cos\omega t = \epsilon_0\mu_0\frac{\partial}{\partial t}(-2E_{\max}\sin kx\sin\omega t) = \epsilon_0\mu_0\frac{\partial E_y(x,\ t)}{\partial t}.$$

EVALUATE: The standing waves are linear superpositions of two traveling waves of the same k and ω .

32.35. **IDENTIFY:** The nodal and antinodal planes are each spaced one-half wavelength apart.

SET UP: $2\frac{1}{2}$ wavelengths fit in the oven, so $\left(2\frac{1}{2}\right)\lambda = L$, and the frequency of these waves obeys the equation $f\lambda = c$.

EXECUTE: **(a)** Since $\left(2\frac{1}{2}\right)\lambda = L,$ we have L = (5/2)(12.2 cm) = 30.5 cm.

(b) Solving for the frequency gives $f = c/\lambda$ = (3.00 × 10^8 m/s)/(0.122 m) = 2.46 × 10^9 Hz.

(c) L = 35.5 cm in this case. $\left(2\frac{1}{2}\right)\lambda = L,$ so $\lambda = 2L/5$ = 2(35.5 cm)/5 = 14.2 cm.

$$f = c/\lambda = (3.00 \times 10^8\ \text{m/s})/(0.142\ \text{m}) = 2.11 \times 10^9\ \text{Hz}$$

EVALUATE: Since microwaves have a reasonably large wavelength, microwave ovens can have a convenient size for household kitchens. Ovens using radiowaves would need to be far too large, while ovens using visible light would have to be microscopic.

32.36. **IDENTIFY:** Evaluate the partial derivatives of the expressions for $E_y(x,\ t)$ and $B_z(x,\ t)$.

SET UP: $\frac{\partial}{\partial x}\sin(kx-\omega t) = k\cos(kx-\omega t)$, $\frac{\partial}{\partial t}\sin(kx-\omega t) = -\omega\cos(kx-\omega t)$. $\frac{\partial}{\partial x}\cos(kx-\omega t) = -k\sin(kx-\omega t)$,

$\frac{\partial}{\partial t}\cos(kx-\omega t) = \omega\sin(kx-\omega t)$

EXECUTE: Assume $\vec{E} = E_{\max}\hat{j}\sin(kx-\omega t)$ and $\vec{B} = B_{\max}\hat{k}\sin(kx-\omega t+\phi)$, with $-\pi < \phi < \pi$. Eq. (32.12) is $\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$. This gives $kE_{\max}\cos(kx-\omega t) = +\omega B_{\max}\cos(kx-\omega t+\phi)$, so $\phi = 0$, and $kE_{\max} = \omega B_{\max}$, so $E_{\max} = \frac{\omega}{k}B_{\max} = \frac{2\pi f}{2\pi/\omega}B_{\max} = f\omega B_{\max} = cB_{\max}$. Similarly for Eq.(32.14), $-\frac{\partial B_z}{\partial x} = \epsilon_0\mu_0\frac{\partial E_y}{\partial t}$ gives $-kB_{\max}\cos(kx-\omega t+\phi) = -\epsilon_0\mu_0\omega E_{\max}\cos(kx-\omega t)$, so $\phi = 0$ and $kB_{\max} = \epsilon_0\mu_0\omega E_{\max}$, so

$$B_{\max} = \frac{\epsilon_0\mu_0\omega}{k}E_{\max} = \frac{2\pi f}{c^2 2\pi/\omega}E_{\max} = \frac{f\omega}{c^2}E_{\max} = \frac{1}{c}E_{\max}.$$

EVALUATE: The $\vec{E}$ and $\vec{B}$ fields must oscillate in phase.

32.37. **IDENTIFY** and **SET UP:** Take partial derivatives of Eqs.(32.12) and (32.14), as specified in the problem.

EXECUTE: Eq.(32.12): $\dfrac{\partial E_y}{\partial x} = -\dfrac{\partial B_z}{\partial t}$

Taking $\dfrac{\partial}{\partial t}$ of both sides of this equation gives $\dfrac{\partial^2 E_y}{\partial x \partial t} = -\dfrac{\partial^2 B_z}{\partial t^2}$. Eq.(32.14) says $-\dfrac{\partial B_z}{\partial x} = \epsilon_0 \mu_0 \dfrac{\partial E_y}{\partial t}$. Taking $\dfrac{\partial}{\partial x}$ of both sides of this equation gives $-\dfrac{\partial^2 B_z}{\partial x^2} = \epsilon_0 \mu_0 \dfrac{\partial^2 E_y}{\partial t \partial x}$, so $\dfrac{\partial^2 E_y}{\partial t \partial x} = -\dfrac{1}{\epsilon_0 \mu_0}\dfrac{\partial^2 B_z}{\partial x^2}$. But $\dfrac{\partial^2 E_y}{\partial x \partial t} = \dfrac{\partial^2 E_y}{\partial t \partial x}$ (The order in which the partial derivatives are taken doesn't change the result.) So $-\dfrac{\partial^2 B_z}{\partial t^2} = -\dfrac{1}{\epsilon_0 \mu_0}\dfrac{\partial^2 B_z}{\partial x^2}$ and $\dfrac{\partial^2 B_z}{\partial x^2} = \epsilon_0 \mu_0 \dfrac{\partial^2 B_z}{\partial t^2}$, as was to be shown.

EVALUATE: Both fields, electric and magnetic, satisfy the wave equation, Eq.(32.10). We have also shown that both fields propagate with the same speed $v = 1/\sqrt{\epsilon_0 \mu_0}$.

32.38. **IDENTIFY:** The average energy density in the electric field is $u_{E,\text{av}} = \frac{1}{2}\epsilon_0 (E^2)_{\text{av}}$ and the average energy density in the magnetic field is $u_{B,\text{av}} = \dfrac{1}{2\mu_0}(B^2)_{\text{av}}$.

SET UP: $\left(\cos^2(kx - \omega t)\right)_{\text{av}} = \frac{1}{2}$.

EXECUTE: $E_y(x,\, t) = E_{\max}\cos(kx - \omega t)$. $u_E = \frac{1}{2}\epsilon_0 E_y^2 = \frac{1}{2}\epsilon_0 E_{\max}^2 \cos^2(kx - \omega t)$ and $u_{E,\text{av}} = \frac{1}{4}\epsilon_0 E_{\max}^2$.

$B_z(x,\, t) = B_{\max}\cos(kx - \omega t)$, so $u_B = \dfrac{1}{2\mu_0}B_z^2 = \dfrac{1}{2\mu_0}B_{\max}^2\cos^2(kx - \omega t)$ and $u_{B,\text{av}} = \dfrac{1}{4\mu_0}B_{\max}^2$. $E_{\max} = cB_{\max}$, so $u_{E,\text{av}} = \frac{1}{4}\epsilon_0 c^2 B_{\max}^2$. $c = \dfrac{1}{\sqrt{\epsilon_0 \mu_0}}$, so $u_{E,\text{av}} = \dfrac{1}{2\mu_0}B_{\max}^2$, which equals $u_{B,\text{av}}$.

EVALUATE: Our result allows us to write $u_{\text{av}} = 2u_{E,\text{av}} = \frac{1}{2}\epsilon_0 E_{\max}^2$ and $u_{\text{av}} = 2u_{B,\text{av}} = \dfrac{1}{2\mu_0}B_{\max}^2$.

32.39. **IDENTIFY:** The intensity of an electromagnetic wave depends on the amplitude of the electric and magnetic fields. Such a wave exerts a force because it carries energy.

SET UP: The intensity of the wave is $I = P_{\text{av}}/A = \frac{1}{2}\epsilon_0 c E_{\max}^2$, and the force is $F = P_{\text{av}}A$ where $P_{\text{av}} = I/c$.

EXECUTE: **(a)** $I = P_{\text{av}}/A = (25{,}000\text{ W})/[4\pi(575\text{ m})^2] = 0.00602\text{ W/m}^2$

(b) $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$, so $E_{\max} = \sqrt{\dfrac{2I}{\epsilon_0 c}} = \sqrt{\dfrac{2(0.00602\text{ W/m}^2)}{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\text{ m/s})}} = 2.13\text{ N/C}$.

$$B_{\max} = E_{\max}/c = (2.13\text{ N/C})/(3.00\times10^8\text{ m/s}) = 7.10\times10^{-9}\text{ T}$$

(c) $F = P_{\text{av}}A = (I/c)A = (0.00602\text{ W/m}^2)(0.150\text{ m})(0.400\text{ m})/(3.00\times10^8\text{ m/s}) = 1.20\times10^{-12}\text{ N}$

EVALUATE: The fields are very weak compared to ordinary laboratory fields, and the force is hardly worth worrying about!

32.40. **IDENTIFY:** $c = f\lambda$. $E_{\max} = cB_{\max}$. $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$. For a totally absorbing surface the radiation pressure is $\dfrac{I}{c}$.

SET UP: The wave speed in air is $c = 3.00\times10^8\text{ m/s}$.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\text{ m/s}}{3.84\times10^{-2}\text{ m}} = 7.81\times10^9\text{ Hz}$

(b) $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{1.35\text{ V/m}}{3.00\times10^8\text{ m/s}} = 4.50\times10^{-9}\text{ T}$

(c) $I = \frac{1}{2}\epsilon_0 c E_{\max}^2 = \frac{1}{2}(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3.00\times10^8\text{ m/s})(1.35\text{ V/m})^2 = 2.42\times10^{-3}\text{ W/m}^2$

(d) $F = (\text{pressure})A = \dfrac{IA}{c} = \dfrac{(2.42\times10^{-3}\text{ W/m}^2)(0.240\text{ m}^2)}{3.00\times10^8\text{ m/s}} = 1.94\times10^{-12}\text{ N}$

EVALUATE: The intensity depends only on the amplitudes of the electric and magnetic fields and is independent of the wavelength of the light.

32.41. (a) IDENTIFY and SET UP: Calculate I and then use Eq.(32.29) to calculate $E_{\max}$ and Eq.(32.18) to calculate $B_{\max}$.

EXECUTE: The intensity is power per unit area: $I = \dfrac{P}{A} = \dfrac{3.20\times10^{-3}\text{ W}}{\pi(1.25\times10^{-3}\text{ m})^2} = 652\text{ W/m}^2$.

$I = \dfrac{E_{\max}^2}{2\mu_0 c}$, so $E_{\max} = \sqrt{2\mu_0 cI}$

$$E_{\max} = \sqrt{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(2.998\times10^8\text{ m/s})(652\text{ W/m}^2)} = 701\text{ V/m}$$

$$B_{\max} = \frac{E_{\max}}{c} = \frac{701\text{ V/m}}{2.998\times10^8\text{ m/s}} = 2.34\times10^{-6}\text{ T}$$

EVALUATE: The magnetic field amplitude is quite small.

(b) IDENTIFY and SET UP: Eqs.(24.11) and (30.10) give the energy density in terms of the electric and magnetic field values at any time. For sinusoidal fields average over E^2 and B^2 to get the average energy densities.

EXECUTE: The energy density in the electric field is $u_E = \frac{1}{2}\epsilon_0 E^2$. $E = E_{\max}\cos(kx-\omega t)$ and the average value of $\cos^2(kx-\omega t)$ is $\frac{1}{2}$. The average energy density in the electric field then is $u_{E,\text{av}} = \frac{1}{4}\epsilon_0 E_{\max}^2 = \frac{1}{4}(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(701\text{ V/m})^2 = 1.09\times10^{-6}\text{ J/m}^3$. The energy density in the magnetic field is $u_B = \dfrac{B^2}{2\mu_0}$. The average value is $u_{B,\text{av}} = \dfrac{B_{\max}^2}{4\mu_0} = \dfrac{(2.34\times10^{-6}\text{ T})^2}{4(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})} = 1.09\times10^{-6}\text{ J/m}^3$.

EVALUATE: Our result agrees with the statement in Section 32.4 that the average energy density for the electric field is the same as the average energy density for the magnetic field.

(c) IDENTIFY and SET UP: The total energy in this length of beam is the total energy density $u_{\text{av}} = u_{E,\text{av}} + u_{B,\text{av}} = 2.18\times10^{-6}\text{ J/m}^3$ times the volume of this part of the beam.

EXECUTE: $U = u_{\text{av}}LA = (2.18\times10^{-6}\text{ J/m}^3)(1.00\text{ m})\pi(1.25\times10^{-3}\text{ m})^2 = 1.07\times10^{-11}\text{ J}$.

EVALUATE: This quantity can also be calculated as the power output times the time it takes the light to travel L = 1.00 m: $U = P\left(\dfrac{L}{c}\right) = (3.20\times10^{-3}\text{ W})\left(\dfrac{1.00\text{ m}}{2.998\times10^8\text{ m/s}}\right) = 1.07\times10^{-11}\text{ J}$, which checks.

32.42. IDENTIFY: Use the gaussian surface specified in the hint.

SET UP: The wave is in free space, so in Gauss's law for the electric field, $Q_{\text{encl}} = 0$ and $\oint \vec{E}\cdot d\vec{A} = 0$. Gauss's law for the magnetic field says $\oint \vec{B}\cdot d\vec{A} = 0$

EXECUTE: Use a gaussian surface such that the front surface is ahead of the wave front (no electric or magnetic fields) and the back face is behind the wave front, as shown in Figure 32.42. $\oint \vec{E}\cdot d\vec{A} = E_x A = \dfrac{Q_{\text{encl}}}{\varepsilon_0} = 0$, so $E_x = 0$.

$\oint \vec{B}\cdot d\vec{A} = B_x A = 0$ and $B_x = 0$.

EVALUATE: The wave must be transverse, since there are no components of the electric or magnetic field in the direction of propagation.

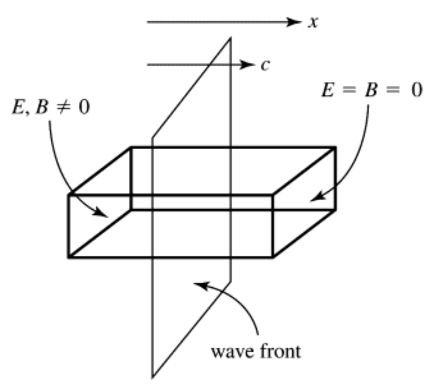


Figure 32.42

32.43. IDENTIFY: $I = P_{\text{av}}/A$. For an absorbing surface, the radiation pressure is $p_{\text{rad}} = \dfrac{I}{c}$

SET UP: Assume the electromagnetic waves are formed at the center of the sun, so at a distance r from the center of the sun $I = P_{\text{av}}/(4\pi r^2)$.

EXECUTE: **(a)** At the sun's surface: $I = \dfrac{P_{av}}{4\pi R^2} = \dfrac{3.9\times 10^{26}\ \text{W}}{4\pi(6.96\times 10^8\ \text{m})^2} = 6.4\times 10^7\ \text{W/m}^2$ and

$p_{rad} = \dfrac{I}{c} = \dfrac{6.4\times 10^7\ \text{W/m}^2}{3.00\times 10^8\ \text{m/s}} = 0.21\ \text{Pa}.$

(b) Halfway out from the sun's center, the intensity is 4 times more intense, and so is the radiation pressure: $I = 2.6\times 10^8\ \text{W/m}^2$ and $p_{rad} = 0.85\ \text{Pa}$. At the top of the earth's atmosphere, the measured sunlight intensity is $1400\ \text{W/m}^2 = 5\times 10^{-6}\ \text{Pa}$, which is about 100,000 times less than the values above.

EVALUATE: **(b)** The gas pressure at the sun's surface is 50,000 times greater than the radiation pressure, and halfway out of the sun the gas pressure is believed to be about 6×10^{13} times greater than the radiation pressure. Therefore it is reasonable to ignore radiation pressure when modeling the sun's interior structure.

32.44. **IDENTIFY:** $I = \dfrac{P}{A}$. $I = \frac{1}{2}\epsilon_0 c E_{max}^2$.

SET UP: $3.00\times 10^8\ \text{m/s}$

EXECUTE: $I = \dfrac{P}{A} = \dfrac{2.80\times 10^3\ \text{W}}{36.0\ \text{m}^2} = 77.8\ \text{W/m}^2$.

$$E_{max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(77.8\ \text{W/m}^2)}{(8.854\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times 10^8\ \text{m/s})}} = 242\ \text{N/C}.$$

EVALUATE: This value of E_{max} is similar to the electric field amplitude in ordinary light sources.

32.45. **IDENTIFY:** The same intensity light falls on both reflectors, but the force on the reflecting surface will be twice as great as the force on the absorbing surface. Therefore there will be a net torque about the rotation axis.

SET UP: For a totally absorbing surface, $F = P_{av}A = (I/c)A$, while for a totally reflecting surface the force will be twice as great. The intensity of the wave is $I = \frac{1}{2}\epsilon_0 c E_{max}^2$. Once we have the torque, we can use the rotational form of Newton's second law, $\tau_{net} = I\alpha$, to find the angular acceleration.

EXECUTE: The force on the absorbing reflector is $F_{Abs} = p_{av}A = (I/c)A = \dfrac{\frac{1}{2}\epsilon_0 c E_{max}^2 A}{c} = \frac{1}{2}\epsilon_0 A E_{max}^2$

For a totally reflecting surface, the force will be twice as great, which is $\epsilon_0 c E_{max}^2$. The net torque is therefore

$\tau_{net} = F_{Refl}(L/2) - F_{Abs}(L/2) = \epsilon_0 A E_{max}^2 L/4$

Newton's 2nd law for rotation gives $\tau_{net} = I\alpha$. $\epsilon_0 A E_{max}^2 L/4 = 2m(L/2)^2\alpha$

Solving for α gives $\alpha = \epsilon_0 A E_{max}^2/(2mL) = \dfrac{\left(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(0.0150\ \text{m})^2(1.25\ \text{N/C})^2}{(2)(0.00400\ \text{kg})(1.00\ \text{m})} = 3.89\times 10^{-13}\ \text{rad/s}^2$

EVALUATE: This is an extremely small angular acceleration. To achieve a larger value, we would have to greatly increase the intensity of the light wave or decrease the mass of the reflectors.

32.46. **IDENTIFY:** For light of intensity I_{abs} incident on a totally absorbing surface, the radiation pressure is $p_{rad,abs} = \dfrac{I_{abs}}{c}$. For light of intensity I_{refl} incident on a totally reflecting surface, $p_{rad,refl} = \dfrac{2I_{refl}}{c}$.

SET UP: The total radiation pressure is $p_{rad} = p_{rad,abs} + p_{rad,refl}$. $I_{abs} = wI$ and $I_{refl} = (1-w)I$

EXECUTE: **(a)** $p_{rad} = p_{rad,abs} + p_{rad,refl} = \dfrac{I_{abs}}{c} + \dfrac{2I_{refl}}{c} = \dfrac{wI}{c} + \dfrac{2(1-w)I}{c} = \dfrac{(2-w)I}{c}$.

(b) (i) For totally absorbing $w = 1$ so $p_{rad} = \dfrac{I}{c}$. (ii) For totally reflecting $w = 0$ so $p_{rad} = \dfrac{2I}{c}$. These are just equations 32.32 and 32.33.

(c) For $w = 0.9$ and $I = 1.40\times 10^2\ \text{W/m}^2$, $p_{rad} = \dfrac{(2-0.9)(1.40\times 10^3\ \text{W/m}^2)}{3.00\times 10^8\ \text{m/s}} = 5.13\times 10^{-6}\ \text{Pa}$. For $w = 0.1$ and $I = 1.40\times 10^3\ \text{W/m}^2$, $p_{rad} = \dfrac{(2-0.1)(1.40\times 10^2\ \text{W/m}^2)}{3.00\times 10^8\ \text{m/s}} = 8.87\times 10^{-6}\ \text{Pa}$.

EVALUATE: The radiation pressure is greater when a larger fraction is reflected.

32.47. **IDENTIFY** and **SET UP:** In the wire the electric field is related to the current density by Eq.(25.7). Use Ampere's law to calculate $\vec{B}$. The Poynting vector is given by Eq.(32.28) and the equation that follows it relates the energy flow through a surface to $\vec{S}$.

EXECUTE: **(a)** The direction of $\vec{E}$ is parallel to the axis of the cylinder, in the direction of the current. From Eq.(25.7), $E = \rho J = \rho I/\pi a^2$. ($E$ is uniform across the cross section of the conductor.)

(b) A cross-sectional view of the conductor is given in Figure 32.47a; take the current to be coming out of the page.

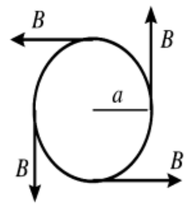


Figure 32.47a

Apply Ampere's law to a circle of radius a.

$\oint \vec{B} \cdot d\vec{l} = B(2\pi a)$

$I_{\text{encl}} = I$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$ gives $B(2\pi a) = \mu_0 I$ and $B = \dfrac{\mu_0 I}{2\pi a}$

The direction of $\vec{B}$ is counterclockwise around the circle.

(c) The directions of $\vec{E}$ and $\vec{B}$ are shown in Figure 32.47b.

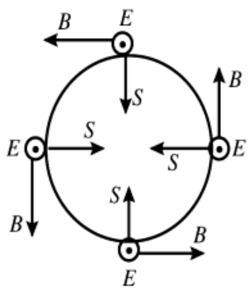


Figure 32.47b

The direction of $\vec{S} = \dfrac{1}{\mu_0}\vec{E} \times \vec{B}$ is radially inward.

$S = \dfrac{1}{\mu_0}EB = \dfrac{1}{\mu_0}\left(\dfrac{\rho I}{\pi a^2}\right)\left(\dfrac{\mu_0 I}{2\pi a}\right)$

$S = \dfrac{\rho I^2}{2\pi^2 a^3}$

(d) EVALUATE: Since S is constant over the surface of the conductor, the rate of energy flow P is given by S times the surface of a length l of the conductor: $P = SA = S(2\pi a l) = \dfrac{\rho I^2}{2\pi^2 a^3}(2\pi a l) = \dfrac{\rho l I^2}{\pi a^2}$. But $R = \dfrac{\rho l}{\pi a^2}$, so the result from the Poynting vector is $P = RI^2$. This agrees with $P_R = I^2 R$, the rate at which electrical energy is being dissipated by the resistance of the wire. Since $\vec{S}$ is radially inward at the surface of the wire and has magnitude equal to the rate at which electrical energy is being dissipated in the wire, this energy can be thought of as entering through the cylindrical sides of the conductor.

32.48. **IDENTIFY:** The intensity of the wave, not the electric field strength, obeys an inverse-square distance law.

SET UP: The intensity is inversely proportional to the distance from the source, and it depends on the amplitude of the electric field by $I = S_{\text{av}} = \frac{1}{2}\epsilon_0 c E_{\text{max}}{}^2$.

EXECUTE: Since $I = \frac{1}{2}\epsilon_0 c E_{\text{max}}{}^2$, $E_{\text{max}} \propto \sqrt{I}$. A point at 20.0 cm (0.200 m) from the source is 50 times closer to the source than a point that is 10.0 m from it. Since $I \propto 1/r^2$ and (0.200 m)/(10.0 m) = 1/50, we have $I_{0.20} = 50^2\, I_{10}$. Since $E_{\text{max}} \propto \sqrt{I}$, we have $E_{0.20} = 50E_{10}$ = (50)(1.50 N/C) = 75.0 N/C.

EVALUATE: While the intensity increases by a factor of $50^2 = 2500$, the amplitude of the wave only increases by a factor of 50. Recall that the intensity of *any* wave is proportional to the *square* of its amplitude.

32.49. **IDENTIFY** and **SET UP:** The magnitude of the induced emf is given by Faraday's law: $|\mathcal{E}| = \left|\dfrac{d\Phi_B}{dt}\right|$. To calculate $d\Phi_B/dt$ we need dB/dt at the antenna. Use the total power output to calculate I and then combine Eq.(32.29) and (32.18) to calculate B_{max}. The time dependence of B is given by Eq.(32.17).

EXECUTE: $\Phi_B = B\pi R^2$, where R = 0.0900 m is the radius of the loop. (This assumes that the magnetic field is uniform across the loop, an excellent approximation.) $|\mathcal{E}| = \pi R^2 \left|\dfrac{dB}{dt}\right|$

$B = B_{\text{max}}\cos(kx - \omega t)$ so $\left|\dfrac{dB}{dt}\right| = B_{\text{max}}\omega \sin(kx - \omega t)$

The maximum value of $\left|\dfrac{dB}{dt}\right|$ is $B_{\text{max}}\omega$, so $|\mathcal{E}|_{\text{max}} = \pi R^2 B_{\text{max}}\omega$.

$R = 0.0900\text{ m}$, $\omega = 2\pi f = 2\pi(95.0\times10^6\text{ Hz}) = 5.97\times10^8\text{ rad/s}$

Calculate the intensity I at this distance from the source, and from that the magnetic field amplitude $B_{\max}$:

$$I = \frac{P}{4\pi r^2} = \frac{55.0\times 10^3\ \text{W}}{4\pi(2.50\times 10^3\ \text{m})^2} = 7.00\times 10^{-4}\ \text{W/m}^2.\quad I = \frac{E_{\max}^2}{2\mu_0 c} = \frac{(cB_{\max})^2}{2\mu_0 c} = \frac{c}{2\mu_0}B_{\max}^2$$

Thus $B_{\max} = \sqrt{\frac{2\mu_0 I}{c}} = \sqrt{\frac{2(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(7.00\times 10^{-4}\ \text{W/m}^2)}{2.998\times 10^8\ \text{m/s}}} = 2.42\times 10^{-9}\ \text{T}$. Then

$|\mathcal{E}|_{\max} = \pi R^2 B_{\max}\omega = \pi(0.0900\ \text{m})^2(2.42\times 10^{-9}\ \text{T})(5.97\times 10^8\ \text{rad/s}) = 0.0368\ \text{V}$.

EVALUATE: An induced emf of this magnitude is easily detected.

32.50. IDENTIFY: The nodal planes are one-half wavelength apart.

SET UP: The nodal planes of B are at $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots$, which are $\lambda/2$ apart.

EXECUTE: **(a)** The wavelength is $\lambda = c/f = (3.000\times 10^8\ \text{m/s})/(110.0\times 10^6\ \text{Hz}) = 2.727\ \text{m}$. So the nodal planes are at $(2.727\ \text{m})/2 = 1.364\ \text{m}$ apart.

(b) For the nodal planes of E, we have $\lambda_n = 2L/n$, so $L = n\lambda/2 = (8)(2.727\ \text{m})/2 = 10.91\ \text{m}$

EVALUATE: Because radiowaves have long wavelengths, the distances involved are easily measurable using ordinary metersticks.

32.51. IDENTIFY and **SET UP:** Find the force on you due to the momentum carried off by the light. Express this force in terms of the radiated power of the flashlight. Use this force to calculate your acceleration and use a constant acceleration equation to find the time.

(a) EXECUTE: $p_{\text{rad}} = I/c$ and $F = p_{\text{rad}}A$ gives $F = IA/c = P_{\text{av}}/c$

$a_x = F/m = P_{\text{av}}/(mc) = (200\ \text{W})/[(150\ \text{kg})(3.00\times 10^8\ \text{m/s})] = 4.44\times 10^{-9}\ \text{m/s}^2$

Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \sqrt{2(x-x_0)/a_x} = \sqrt{2(16.0\ \text{m})/(4.44\times 10^{-9}\ \text{m/s}^2)} = 8.49\times 10^4\ \text{s} = 23.6\ \text{h}$

EVALUATE: The radiation force is very small. In the calculation we have ignored any other forces on you.

(b) You could throw the flashlight in the direction away from the ship. By conservation of linear momentum you would move toward the ship with the same magnitude of momentum as you gave the flashlight.

32.52. IDENTIFY: $P_{\text{av}} = IA$ and $I = \frac{1}{2}\epsilon_0 cE_{\max}^2$. $E_{\max} = cB_{\max}$

SET UP: The power carried by the current i is $P = Vi$.

EXECUTE: $I = \frac{P_{\text{av}}}{A} = \frac{1}{2}\epsilon_0 cE^2$ and $E_{\max} = \sqrt{\frac{2P_{\text{av}}}{A\epsilon_0 c}} = \sqrt{\frac{2Vi}{A\epsilon_0 c}} = \sqrt{\frac{2(5.00\times 10^5\ \text{V})(1000\ \text{A})}{(100\ \text{m}^2)\epsilon_0(3.00\times 10^8\ \text{m/s})}} = 6.14\times 10^4\ \text{V/m}$.

$$B_{\max} = \frac{E_{\max}}{c} = \frac{6.14\times 10^4\ \text{V/m}}{3.00\times 10^8\ \text{m/s}} = 2.05\times 10^{-4}\ \text{T}.$$

EVALUATE: $I = Vi/A = \frac{(5.00\times 10^5\ \text{V})(1000\ \text{A})}{100\ \text{m}^2} = 5.00\times 10^6\ \text{W/m}^2$. This is a very intense beam spread over a large area.

32.53. IDENTIFY: The orbiting satellite obeys Newton's second law of motion. The intensity of the electromagnetic waves it transmits obeys the inverse-square distance law, and the intensity of the waves depends on the amplitude of the electric and magnetic fields.

SET UP: Newton's second law applied to the satellite gives $mv^2/R = GmM/r^2$, where M is the mass of the Earth and m is the mass of the satellite. The intensity I of the wave is $I = S_{\text{av}} = \frac{1}{2}\epsilon_0 cE_{\max}{}^2$, and by definition, $I = P_{\text{av}}/A$.

EXECUTE: **(a)** The period of the orbit is 12 hr. Applying Newton's 2nd law to the satellite gives $mv^2/R = GmM/r^2$, which gives $\frac{m(2\pi r/T)^2}{r} = \frac{GmM}{r^2}$. Solving for r, we get

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left[\frac{(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.97\times 10^{24}\ \text{kg})(12\times 3600\ \text{s})^2}{4\pi^2}\right]^{1/3} = 2.66\times 10^7\ \text{m}$$

The height above the surface is $h = 2.66\times 10^7\ \text{m} - 6.38\times 10^6\ \text{m} = 2.02\times 10^7\ \text{m}$. The satellite only radiates its energy to the lower hemisphere, so the area is 1/2 that of a sphere. Thus, from the definition of intensity, the intensity at the ground is

$$I = P_{\text{av}}/A = P_{\text{av}}/(2\pi h^2) = (25.0\ \text{W})/[2\pi(2.02\times 10^7\ \text{m})^2] = 9.75\times 10^{-15}\ \text{W/m}^2$$

(b) $I = S_{\text{av}} = \frac{1}{2}\epsilon_0 cE_{\max}{}^2$, so $E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(9.75\times 10^{-15}\ \text{W/m}^2)}{(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times 10^8\ \text{m/s})}} = 2.71\times 10^{-6}\ \text{N/C}$

$$B_{\max} = E_{\max}/c = (2.71\times 10^{-6}\ \text{N/C})/(3.00\times 10^8\ \text{m/s}) = 9.03\times 10^{-15}\ \text{T}$$

$$t = d/c = (2.02\times 10^7\ \text{m})/(3.00\times 10^8\ \text{m/s}) = 0.0673\ \text{s}$$

(c) $P_{av} = I/c = (9.75\times10^{-15}\ \text{W/m}^2)/(3.00\times10^8\ \text{m/s}) = 3.25\times10^{-23}$ Pa

(d) $\lambda = c/f = (3.00\times10^8\ \text{m/s})/(1575.42\times10^6\ \text{Hz}) = 0.190$ m

EVALUATE: The fields and pressures due to these waves are very small compared to typical laboratory quantities.

32.54. **IDENTIFY:** For a totally reflective surface the radiation pressure is $\dfrac{2I}{c}$. Find the force due to this pressure and express the force in terms of the power output P of the sun. The gravitational force of the sun is $F_g = G\dfrac{mM_{sun}}{r^2}$.

SET UP: The mass of the sum is $M_{sun} = 1.99\times10^{30}$ kg. $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

EXECUTE: **(a)** The sail should be reflective, to produce the maximum radiation pressure.

(b) $F_{rad} = \left(\dfrac{2I}{c}\right)A$, where A is the area of the sail. $I = \dfrac{P}{4\pi r^2}$, where r is the distance of the sail from the sun.

$$F_{rad} = \left(\frac{2A}{c}\right)\left(\frac{P}{4\pi r^2}\right) = \frac{PA}{2\pi r^2 c}.\quad F_{rad} = F_g \text{ so } \frac{PA}{2\pi r^2 c} = G\frac{mM_{sun}}{r^2}.$$

$$A = \frac{2\pi cGmM_{sun}}{P} = \frac{2\pi(3.00\times10^8\ \text{m/s})(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(10{,}000\ \text{kg})(1.99\times10^{30}\ \text{kg})}{3.9\times10^{26}\ \text{W}}.$$

$$A = 6.42\times10^6\ \text{m}^2 = 6.42\ \text{km}^2.$$

(c) Both the gravitational force and the radiation pressure are inversely proportional to the square of the distance from the sun, so this distance divides out when we set $F_{rad} = F_g$.

EVALUATE: A very large sail is needed, just to overcome the gravitational pull of the sun.

32.55. **IDENTIFY** and **SET UP:** The gravitational force is given by Eq.(12.2). Express the mass of the particle in terms of its density and volume. The radiation pressure is given by Eq.(32.32); relate the power output L of the sun to the intensity at a distance r. The radiation force is the pressure times the cross sectional area of the particle.

EXECUTE: **(a)** The gravitational force is $F_g = G\dfrac{mM}{r^2}$. The mass of the dust particle is $m = \rho V = \rho\frac{4}{3}\pi R^3$. Thus

$$F_g = \frac{4\rho G\pi MR^3}{3r^2}.$$

(b) For a totally absorbing surface $p_{rad} = \dfrac{I}{c}$. If L is the power output of the sun, the intensity of the solar radiation a distance r from the sun is $I = \dfrac{L}{4\pi r^2}$. Thus $p_{rad} = \dfrac{L}{4\pi cr^2}$. The force F_{rad} that corresponds to p_{rad} is in the direction of propagation of the radiation, so $F_{rad} = p_{rad}A_\perp$, where $A_\perp = \pi R^2$ is the component of area of the particle perpendicular to the radiation direction. Thus $F_{rad} = \left(\dfrac{L}{4\pi cr^2}\right)(\pi R^2) = \dfrac{LR^2}{4cr^2}$.

(c) $F_g = F_{rad}$

$$\frac{4\rho G\pi MR^3}{3r^2} = \frac{LR^2}{4cr^2}$$

$$\left(\frac{4\rho G\pi M}{3}\right)R = \frac{L}{4c} \text{ and } R = \frac{3L}{16c\rho G\pi M}$$

$$R = \frac{3(3.9\times10^{26}\ \text{W})}{16(2.998\times10^8\ \text{m/s})(3000\ \text{kg/m}^3)(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\ \text{kg}^2)\pi(1.99\times10^{30}\ \text{kg})}$$

$$R = 1.9\times10^{-7}\ \text{m} = 0.19\ \mu\text{m}.$$

EVALUATE: The gravitational force and the radiation force both have a r^{-2} dependence on the distance from the sun, so this distance divides out in the calculation of R.

(d) $\dfrac{F_{rad}}{F_g} = \left(\dfrac{LR^2}{4cr^2}\right)\left(\dfrac{3r^2}{4\rho G\pi mR^3}\right) = \dfrac{3L}{16c\rho G\pi MR}$. F_{rad} is proportional to R^2 and F_g is proportional to R^3, so this ratio is proportional to $1/R$. If $R < 0.20\ \mu\text{m}$ then $F_{rad} > F_g$ and the radiation force will drive the particles out of the solar system.

32.56. **IDENTIFY:** The electron has acceleration $a = \dfrac{v^2}{R}$.

SET UP: $1\ \text{eV} = 1.60\times10^{-19}$ C. An electron has $|q| = e = 1.60\times10^{-19}$ C.

EXECUTE: For the electron in the classical hydrogen atom, its acceleration is
$a = \frac{v^2}{R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \frac{2(13.6 \text{ eV})(1.60\times10^{-19} \text{ J/eV})}{(9.11\times10^{-31} \text{ kg})(5.29\times10^{-11} \text{ m})} = 9.03\times10^{22} \text{ m/s}^2$. Then using the formula for the rate of energy emission given in Problem 32.57:
$\frac{dE}{dt} = \frac{q^2a^2}{6\pi\epsilon_0 c^3} = \frac{(1.60\times10^{-19} \text{ C})^2(9.03\times10^{22} \text{ m/s}^2)^2}{6\pi\epsilon_0(3.00\times10^8 \text{ m/s})^3} = 4.64\times10^{-8} \text{ J/s} = 2.89\times10^{11} \text{ eV/s}$. This large value of $\frac{dE}{dt}$ would mean that the electron would almost immediately lose all its energy!

EVALUATE: The classical physics result in Problem 32.57 must not apply to electrons in atoms.

32.57. **IDENTIFY:** The orbiting particle has acceleration $a = \frac{v^2}{R}$.

SET UP: $K = \frac{1}{2}mv^2$. An electron has mass $m_e = 9.11\times10^{-31}$ kg and a proton has mass $m_p = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $\left[\frac{q^2a^2}{6\pi\epsilon_0 c^3}\right] = \frac{\text{C}^2(\text{m/s}^2)^2}{(\text{C}^2/\text{N}\cdot\text{m}^2)(\text{m/s})^3} = \frac{\text{N}\cdot\text{m}}{\text{s}} = \frac{\text{J}}{\text{s}} = \text{W} = \left[\frac{dE}{dt}\right]$.

(b) For a proton moving in a circle, the acceleration is
$a = \frac{v^2}{R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \frac{2(6.00\times10^6 \text{ eV})(1.6\times10^{-19} \text{ J/eV})}{(1.67\times10^{-27} \text{ kg})(0.75 \text{ m})} = 1.53\times10^{15} \text{ m/s}^2$. The rate at which it emits energy because of its acceleration is $\frac{dE}{dt} = \frac{q^2a^2}{6\pi\epsilon_0 c^3} = \frac{(1.6\times10^{-19} \text{ C})^2(1.53\times10^{15} \text{ m/s}^2)^2}{6\pi\epsilon_0(3.0\times10^8 \text{ m/s})^3} = 1.33\times10^{-23} \text{ J/s} = 8.32\times10^{-5} \text{ eV/s}$.

Therefore, the fraction of its energy that it radiates every second is $\frac{(dE/dt)(1 \text{ s})}{E} = \frac{8.32\times10^{-5} \text{ eV}}{6.00\times10^6 \text{ eV}} = 1.39\times10^{-11}$.

(c) Carry out the same calculations as in part (b), but now for an electron at the same speed and radius. That means the electron's acceleration is the same as the proton, and thus so is the rate at which it emits energy, since they also have the same charge. However, the electron's initial energy differs from the proton's by the ratio of their masses:
$E_e = E_p\frac{m_e}{m_p} = (6.00\times10^6 \text{ eV})\frac{(9.11\times10^{-31} \text{ kg})}{(1.67\times10^{-27} \text{ kg})} = 3273 \text{ eV}$. Therefore, the fraction of its energy that it radiates every second is $\frac{(dE/dt)(1 \text{ s})}{E} = \frac{8.32\times10^{-5} \text{ eV}}{3273 \text{ eV}} = 2.54\times10^{-8}$.

EVALUATE: The proton has speed $v = \sqrt{\frac{2E}{m_p}} = \sqrt{\frac{2(6.0\times10^6 \text{ eV})(1.60\times10^{-19} \text{ J/eV})}{1.67\times10^{-27} \text{ kg}}} = 3.39\times10^7 \text{ m/s}$. The electron has the same speed and kinetic energy 3.27 keV. The particles in the accelerator radiate at a much smaller rate than the electron in Problem 32.56 does, because in the accelerator the orbit radius is very much larger than in the atom, so the acceleration is much less.

32.58. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** $E_y(x,t) = E_{\max}e^{-k_Cx}\sin(k_Cx - \omega t)$.

$$\frac{\partial E_y}{\partial x} = E_{\max}(-k_C)e^{-k_Cx}\sin(k_Cx-\omega t) + E_{\max}(+k_C)e^{-k_Cx}\cos(k_Cx-\omega t)$$

$$\frac{\partial^2 E_y}{\partial x^2} = E_{\max}(+k_C^2)e^{-k_Cx}\sin(k_Cx-\omega t) + E_{\max}(-k_C^2)e^{-k_Cx}\cos(k_Cx-\omega t)$$
$$+E_{\max}(-k_C^2)e^{-k_Cx}\cos(k_Cx-\omega t) + E_{\max}(-k_C^2)e^{-k_Cx}\sin(k_Cx-\omega t).$$

$\frac{\partial^2 E_y}{\partial x^2} = -2E_{\max}k_C^2e^{-k_Cx}\cos(k_Cx-\omega t)$. $\frac{\partial E_y}{\partial t} = E_{\max}e^{-k_Cx}\,\omega\cos(k_Cx-\omega t)$.

Setting $\frac{\partial^2 E_y}{\partial x^2} = \frac{\mu\partial E_y}{\rho\partial t}$ gives $2E_{\max}k_C^2e^{-k_Cx}\cos(k_Cx-\omega t) = E_{\max}e^{-k_Cx}\,\omega\cos(k_Cx-\omega t)$. This will only be true if $\frac{2k_C^2}{\omega} = \frac{\mu}{\rho}$, or $k_C = \sqrt{\frac{\omega\mu}{2\rho}}$.

(b) The energy in the wave is dissipated by the i^2R heating of the conductor.

(c) $E_y = \frac{E_{y0}}{e} \Rightarrow k_Cx = 1,\ x = \frac{1}{k_C} = \sqrt{\frac{2\rho}{\omega\mu}} = \sqrt{\frac{2(1.72\times10^{-8}\ \Omega\cdot\text{m})}{2\pi(1.0\times10^6 \text{ Hz})\mu_0}} = 6.60\times10^{-5}\text{ m}$.

EVALUATE: The lower the frequency of the waves, the greater is the distance they can penetrate into a conductor. A dielectric (insulator) has a much larger resistivity and these waves can penetrate a greater distance in these materials.

33

THE NATURE AND PROPAGATION OF LIGHT

33.1. **IDENTIFY:** For reflection, $\theta_r = \theta_a$.

SET UP: The desired path of the ray is sketched in Figure 33.1.

EXECUTE: $\tan\phi = \dfrac{14.0\text{ cm}}{11.5\text{ cm}}$, so $\phi = 50.6°$. $\theta_r = 90° - \phi = 39.4°$ and $\theta_r = \theta_a = 39.4°$.

EVALUATE: The angle of incidence is measured from the normal to the surface.

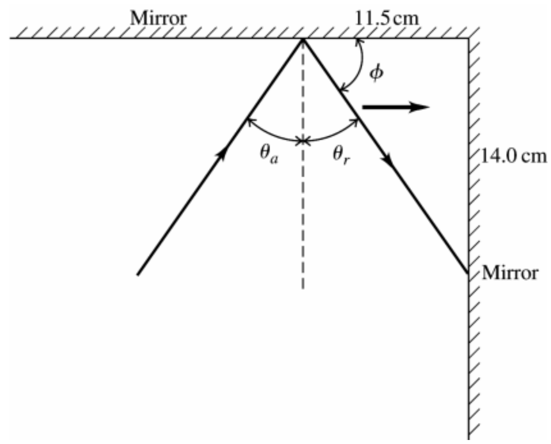


Figure 33.1

33.2 **IDENTIFY:** For reflection, $\theta_r = \theta_a$.

SET UP: The angles of incidence and reflection at each reflection are shown in Figure 33.2. For the rays to be perpendicular when they cross, $\alpha = 90°$.

EXECUTE: **(a)** $\theta + \phi = 90°$ and $\beta + \phi = 90°$, so $\beta = \theta$. $\dfrac{\alpha}{2} + \beta = 90°$ and $\alpha = 180° - 2\theta$.

(b) $\theta = \frac{1}{2}(180° - \alpha) = \frac{1}{2}(180° - 90°) = 45°$.

EVALUATE: As $\theta \to 0°$, $\alpha \to 180°$. This corresponds to the incident and reflected rays traveling in nearly the same direction. As $\theta \to 90°$, $\alpha \to 0°$. This corresponds to the incident and reflected rays traveling in nearly opposite directions.

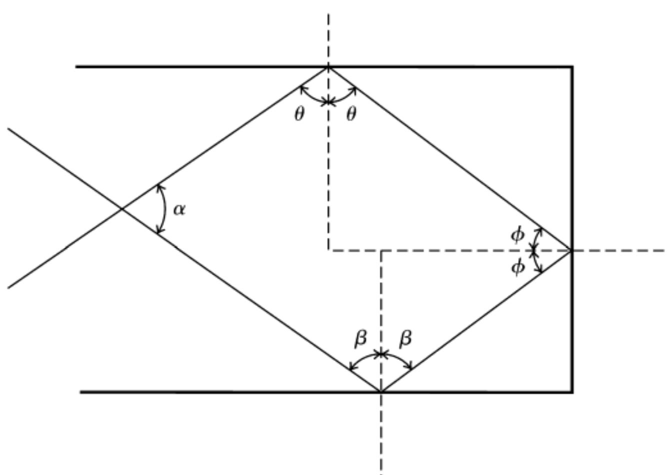


Figure 33.2

33.3. **IDENTIFY** and **SET UP:** Use Eqs.(33.1) and (33.5) to calculate v and λ.

EXECUTE: **(a)** $n = \frac{c}{v}$ so $v = \frac{c}{n} = \frac{2.998 \times 10^8 \text{ m/s}}{1.47} = 2.04 \times 10^8 \text{ m/s}$

(b) $\lambda = \frac{\lambda_0}{n} = \frac{650 \text{ nm}}{1.47} = 442 \text{ nm}$

EVALUATE: Light is slower in the liquid than in vacuum. By $v = f\lambda$, when v is smaller, λ is smaller.

33.4. **IDENTIFY:** In air, $c = f\lambda_0$. In glass, $\lambda = \frac{\lambda_0}{n}$.

SET UP: $c = 3.00 \times 10^8 \text{ m/s}$

EXECUTE: **(a)** $\lambda_0 = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.80 \times 10^{14} \text{ Hz}} = 517 \text{ nm}$

(b) $\lambda = \frac{\lambda_0}{n} = \frac{517 \text{ nm}}{1.52} = 340 \text{ nm}$

EVALUATE: In glass the light travels slower than in vacuum and the wavelength is smaller.

33.5. **IDENTIFY:** $n = \frac{c}{v}$. $\lambda = \frac{\lambda_0}{n}$, where λ_0 is the wavelength in vacuum.

SET UP: $c = 3.00 \times 10^8 \text{ m/s}$. n for air is only slightly larger than unity.

EXECUTE: **(a)** $n = \frac{c}{v} = \frac{3.00 \times 10^8 \text{ m/s}}{1.94 \times 10^8 \text{ m/s}} = 1.54.$

(b) $\lambda_0 = n\lambda = (1.54)(3.55 \times 10^{-7} \text{ m}) = 5.47 \times 10^{-7} \text{ m}.$

EVALUATE: In quartz the speed is lower and the wavelength is smaller than in air.

33.6. **IDENTIFY:** $\lambda = \frac{\lambda_0}{n}$.

SET UP: From Table 33.1, $n_{\text{water}} = 1.333$ and $n_{\text{benzene}} = 1.501$.

EXECUTE: **(a)** $\lambda_{\text{water}} n_{\text{water}} = \lambda_{\text{benzene}} n_{\text{benzene}} = \lambda_0$. $\lambda_{\text{benzene}} = \lambda_{\text{water}} \left(\frac{n_{\text{water}}}{n_{\text{benzene}}}\right) = (438 \text{ nm})\left(\frac{1.333}{1.501}\right) = 389 \text{ nm}$.

(b) $\lambda_0 = \lambda_{\text{water}} n_{\text{water}} = (438 \text{ nm})(1.333) = 584 \text{ nm}$

EVALUATE: λ is smallest in benzene, since n is largest for benzene.

33.7. **IDENTIFY:** Apply Eqs.(33.2) and (33.4) to calculate θ_r and θ_b. The angles in these equations are measured with respect to the normal, not the surface.

(a) SET UP: The incident, reflected and refracted rays are shown in Figure 33.7.

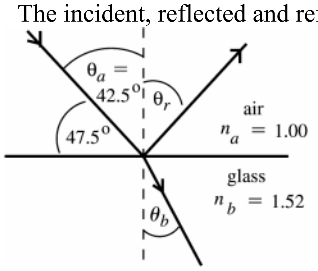


Figure 33.7

EXECUTE: $\theta_r = \theta_a = 42.5°$

The reflected ray makes an angle of $90.0° - \theta_r = 47.5°$ with the surface of the glass.

(b) $n_a \sin\theta_a = n_b \sin\theta_b$, where the angles are measured from the normal to the interface.

$$\sin\theta_b = \frac{n_a \sin\theta_a}{n_b} = \frac{(1.00)(\sin 42.5°)}{1.66} = 0.4070$$

$\theta_b = 24.0°$

The refracted ray makes an angle of $90.0° - \theta_b = 66.0°$ with the surface of the glass.

EVALUATE: The light is bent toward the normal when the light enters the material of larger refractive index.

33.8. **IDENTIFY:** Use the distance and time to find the speed of light in the plastic. $n = \frac{c}{v}$.

SET UP: $c = 3.00 \times 10^8 \text{ m/s}$

EXECUTE: $v = \frac{d}{t} = \frac{2.50 \text{ m}}{11.5 \times 10^{-9} \text{s}} = 2.17 \times 10^8 \text{ m/s}$. $n = \frac{c}{v} = \frac{3.00 \times 10^8 \text{ m/s}}{2.17 \times 10^8 \text{ m/s}} = 1.38$.

EVALUATE: In air light travels this same distance in $\frac{2.50 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 8.3 \text{ ns}$.

33.9. **IDENTIFY** and **SET UP:** Use Snell's law to find the index of refraction of the plastic and then use Eq.(33.1) to calculate the speed v of light in the plastic.
EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$

$$n_b = n_a\left(\frac{\sin\theta_a}{\sin\theta_b}\right) = 1.00\left(\frac{\sin 62.7°}{\sin 48.1°}\right) = 1.194$$

$$n = \frac{c}{v} \text{ so } v = \frac{c}{n} = (3.00\times10^8 \text{ m/s})/1.194 = 2.51\times10^8 \text{ m/s}$$

EVALUATE: Light is slower in plastic than in air. When the light goes from air into the plastic it is bent toward the normal.

33.10. **IDENTIFY:** Apply Snell's law at both interfaces.
SET UP: The path of the ray is sketched in Figure 33.10. Table 33.1 gives $n = 1.329$ for the methanol.
EXECUTE: **(a)** At the air-glass interface $(1.00)\sin 41.3° = n_{\text{glass}}\sin\alpha$. At the glass-methanol interface $n_{\text{glass}}\sin\alpha = (1.329)\sin\theta$. Combining these two equations gives $\sin 41.3° = 1.329\sin\theta$ and $\theta = 29.8°$.
(b) The same figures applies as for part (a), except $\theta = 20.2°$. $(1.00)\sin 41.3° = n\sin 20.2°$ and $n = 1.91$.
EVALUATE: The angle α is $25.2°$. The index of refraction of methanol is less than that of the glass and the ray is bent away from the normal at the glass $\to$ methanol interface. The unknown liquid has an index of refraction greater than that of the glass, so the ray is bent toward the normal at the glass $\to$ liquid interface.

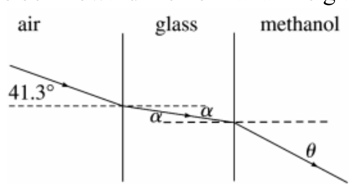


Figure 33.10

33.11. **IDENTIFY:** Apply Snell's law to each refraction.
SET UP: Let the light initially be in the material with refractive index n_a and let the final slab have refractive index n_b. In part (a) let the middle slab have refractive index n_1.
EXECUTE: **(a)** 1st interface: $n_a\sin\theta_a = n_1\sin\theta_1$. 2nd interface: $n_1\sin\theta_1 = n_b\sin\theta_b$. Combining the two equations gives $n_a\sin\theta_a = n_b\sin\theta_b$. This is the equation that would apply if the middle slab were absent.
(b) For N slabs, $n_a\sin\theta_a = n_1\sin\theta_1$, $n_1\sin\theta_1 = n_2\sin\theta_2$, …, $n_{N-2}\sin\theta_{N-2} = n_b\sin\theta_b$. Combining all these equations gives $n_a\sin\theta_a = n_b\sin\theta_b$.
EVALUATE: The final direction of travel depends on the angle of incidence in the first slab and the refractive indices of the first and last slabs.

33.12. **IDENTIFY:** Apply Snell's law to the refraction at each interface.
SET UP: $n_{\text{air}} = 1.00$. $n_{\text{water}} = 1.333$.
EXECUTE: **(a)** $\theta_{\text{water}} = \arcsin\left(\frac{n_{\text{air}}}{n_{\text{water}}}\sin\theta_{\text{air}}\right) = \arcsin\left(\frac{1.00}{1.333}\sin 35.0°\right) = 25.5°.$
EVALUATE: **(b)** This calculation has no dependence on the glass because we can omit that step in the chain: $n_{\text{air}}\sin\theta_{\text{air}} = n_{\text{glass}}\sin\theta_{\text{glass}} = n_{\text{water}}\sin\theta_{\text{water}}$.

33.13. **IDENTIFY:** When a wave passes from one material into another, the number of waves per second that cross the boundary is the same on both sides of the boundary, so the frequency does not change. The wavelength and speed of the wave, however, do change.
SET UP: In a material having index of refraction n, the wavelength is $\lambda = \frac{\lambda_0}{n}$, where λ_0 is the wavelength in vacuum, and the speed is $\frac{c}{n}$.
EXECUTE: **(a)** The frequency is the same, so it is still f. The wavelength becomes $\lambda = \frac{\lambda_0}{n}$, so $\lambda_0 = n\lambda$. The speed is $v = \frac{c}{n}$, so $c = nv$.
(b) The frequency is still f. The wavelength becomes $\lambda' = \frac{\lambda_0}{n'} = \frac{n\lambda}{n'} = \left(\frac{n}{n'}\right)\lambda$ and the speed becomes

$$v' = \frac{c}{n'} = \frac{nv}{n'} = \left(\frac{n}{n'}\right)v$$

EVALUATE: These results give the speed and wavelength in a new medium in terms of the original medium without referring them to the values in vacuum (or air).

33.14. **IDENTIFY:** Apply the law of reflection.

SET UP: The mirror in its original position and after being rotated by an angle θ are shown in Figure 33.14. α is the angle through which the reflected ray rotates when the mirror rotates. The two angles labeled ϕ are equal and the two angles labeled ϕ' are equal because of the law of reflection. The two angles labeled θ are equal because the lines forming one angle are perpendicular to the lines forming the other angle.

EXECUTE: From the diagram, $\alpha = 2\phi' - 2\phi = 2(\phi' - \phi)$ and $\theta = \phi' - \phi$. $\alpha = 2\theta$, as was to be shown.

EVALUATE: This result is independent of the initial angle of incidence.

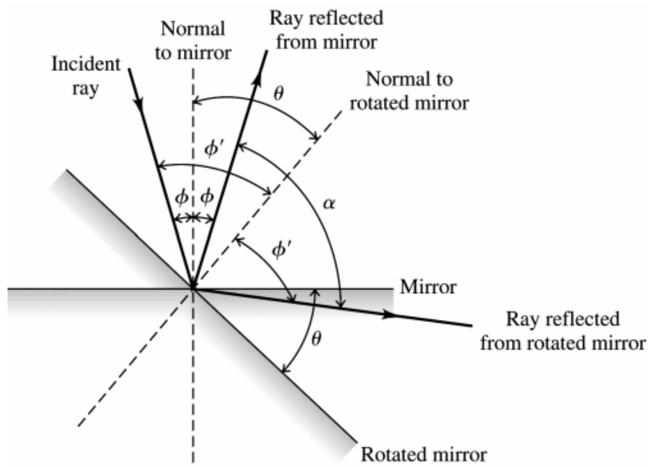


Figure 33.14

33.15. **IDENTIFY:** Apply $n_a \sin\theta_a = n_b \sin\theta_b$.

SET UP: The light refracts from the liquid into the glass, so $n_a = 1.70$, $\theta_a = 62.0°$. $n_b = 1.58$.

EXECUTE: $\sin\theta_b = \left(\dfrac{n_a}{n_b}\right)\sin\theta_a = \left(\dfrac{1.70}{1.58}\right)\sin 62.0° = 0.950$ and $\theta_b = 71.8°$.

EVALUATE: The ray refracts into a material of smaller *n*, so it is bent away from the normal.

33.16. **IDENTIFY:** Apply Snell's law.

SET UP: θ_a and θ_b are measured relative to the normal to the surface of the interface. $\theta_a = 60.0° - 15.0° = 45.0°$.

EXECUTE: $\theta_b = \arcsin\left(\dfrac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\dfrac{1.33}{1.52}\sin 45.0°\right) = 38.2°$. But this is the angle from the normal to the surface, so the angle from the vertical is an additional 15° because of the tilt of the surface. Therefore, the angle is 53.2°.

EVALUATE: Compared to Example 33.1, θ_a is shifted by 15° but the shift in θ_b is only $53.2° - 49.3° = 3.9°$.

33.17. **IDENTIFY:** The critical angle for total internal reflection is θ_a that gives $\theta_b = 90°$ in Snell's law.

SET UP: In Figure 33.17 the angle of incidence θ_a is related to angle θ by $\theta_a + \theta = 90°$.

EXECUTE: **(a)** Calculate θ_a that gives $\theta_b = 90°$. $n_a = 1.60$, $n_b = 1.00$ so $n_a \sin\theta_a = n_b \sin\theta_b$ gives $(1.60)\sin\theta_a = (1.00)\sin 90°$. $\sin\theta_a = \dfrac{1.00}{1.60}$ and $\theta_a = 38.7°$. $\theta = 90° - \theta_a = 51.3°$.

(b) $n_a = 1.60$, $n_b = 1.333$. $(1.60)\sin\theta_a = (1.333)\sin 90°$. $\sin\theta_a = \dfrac{1.333}{1.60}$ and $\theta_a = 56.4°$. $\theta = 90° - \theta_a = 33.6°$.

EVALUATE: The critical angle increases when the ratio $\dfrac{n_a}{n_b}$ increases.

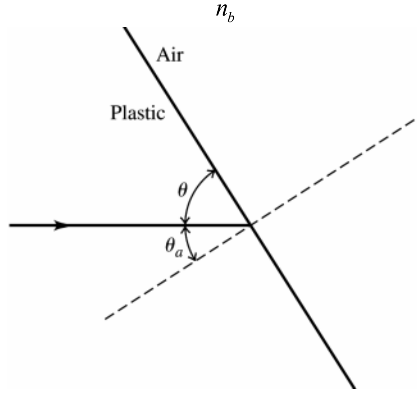


Figure 33.17

33.18. **IDENTIFY:** Since the refractive index of the glass is greater than that of air or water, total internal reflection will occur at the cube surface if the angle of incidence is greater than or equal to the critical angle.

SET UP: At the critical angle θ_c, Snell's law gives $n_{\text{glass}} \sin \theta_c = n_{\text{air}} \sin 90°$ and likewise for water.

EXECUTE: **(a)** At the critical angle θ_c, $n_{\text{glass}} \sin \theta_c = n_{\text{air}} \sin 90°$. $1.53 \sin \theta_c = (1.00)(1)$ and $\theta_c = 40.8°$.

(b) Using the same procedure as in part (a), we have $1.53 \sin \theta_c = 1.333 \sin 90°$ and $\theta_c = 60.6°$.

EVALUATE: Since the refractive index of water is closer to the refractive index of glass than the refractive index of air is, the critical angle for glass-to-water is greater than for glass-to-air.

33.19. **IDENTIFY:** Use the critical angle to find the index of refraction of the liquid.

SET UP: Total internal reflection requires that the light be incident on the material with the larger n, in this case the liquid. Apply $n_a \sin\theta_a = n_b \sin\theta_b$ with a = liquid and b = air, so $n_a = n_{\text{liq}}$ and $n_b = 1.0$.

EXECUTE: $\theta_a = \theta_{\text{crit}}$ when $\theta_b = 90°$, so $n_{\text{liq}} \sin\theta_{\text{crit}} = (1.0)\sin 90°$

$$n_{\text{liq}} = \frac{1}{\sin\theta_{\text{crit}}} = \frac{1}{\sin 42.5°} = 1.48.$$

(a) $n_a \sin\theta_a = n_b \sin\theta_b$ (a = liquid, b = air)

$$\sin\theta_b = \frac{n_a \sin\theta_a}{n_b} = \frac{(1.48)\sin 35.0°}{1.0} = 0.8489 \text{ and } \theta_b = 58.1°$$

(b) Now $n_a \sin\theta_a = n_b \sin\theta_b$ with a = air, b = liquid

$$\sin\theta_b = \frac{n_a \sin\theta_a}{n_b} = \frac{(1.0)\sin 35.0°}{1.48} = 0.3876 \text{ and } \theta_b = 22.8°$$

EVALUATE: For light traveling liquid $\to$ air the light is bent away from the normal. For light traveling air $\to$ liquid the light is bent toward the normal.

33.20. **IDENTIFY:** The largest angle of incidence for which any light refracts into the air is the critical angle for water $\to$ air.

SET UP: Figure 33.20 shows a ray incident at the critical angle and therefore at the edge of the ring of light. The radius of this circle is r and $d = 10.0$ m is the distance from the ring to the surface of the water.

EXECUTE: From the figure, $r = d\tan\theta_{\text{crit}}$. θ_{crit} is calculated from $n_a \sin\theta_a = n_b \sin\theta_b$ with $n_a = 1.333$, $\theta_a = \theta_{\text{crit}}$, $n_b = 1.00$ and $\theta_b = 90°$. $\sin\theta_{\text{crit}} = \frac{(1.00)\sin 90°}{1.333}$ and $\theta_{\text{crit}} = 48.6°$. $r = (10.0 \text{ m})\tan 48.6° = 11.3$ m.

$A = \pi r^2 = \pi(11.3 \text{ m})^2 = 401 \text{ m}^2$.

EVALUATE: When the incident angle in the water is larger than the critical angle, no light refracts into the air.

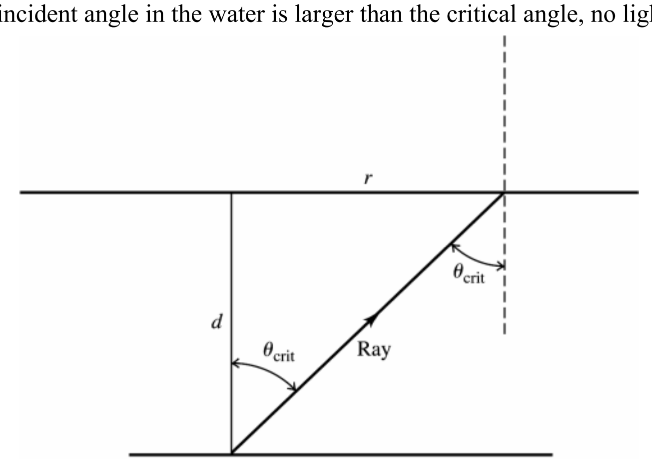


Figure 33.20

33.21. **IDENTIFY** and **SET UP:** For glass $\to$ water, $\theta_{\text{crit}} = 48.7°$. Apply Snell's law with $\theta_a = \theta_{\text{crit}}$ to calculate the index of refraction n_a of the glass.

EXECUTE: $n_a \sin\theta_{\text{crit}} = n_b \sin 90°$, so $n_a = \frac{n_b}{\sin\theta_{\text{crit}}} = \frac{1.333}{\sin 48.7°} = 1.77$

EVALUATE: For total internal reflection to occur the light must be incident in the material of larger refractive index. Our results give $n_{\text{glass}} > n_{\text{water}}$, in agreement with this.

33.22. IDENTIFY: If no light refracts out of the glass at the glass to air interface, then the incident angle at that interface is θ_{crit}.

SET UP: The ray has an angle of incidence of $0°$ at the first surface of the glass, so enters the glass without being bent, as shown in Figure 33.22. The figure shows that $\alpha + \theta_{\text{crit}} = 90°$.

EXECUTE: **(a)** For the glass-air interface $\theta_a = \theta_{\text{crit}}$, $n_a = 1.52$, $n_b = 1.00$ and $\theta_b = 90°$. $n_a \sin\theta_a = n_b \sin\theta_b$ gives $\sin\theta_{\text{crit}} = \dfrac{(1.00)(\sin 90°)}{1.52}$ and $\theta_{\text{crit}} = 41.1°$. $\alpha = 90° - \theta_{\text{crit}} = 48.9°$.

(b) Now the second interface is glass $\rightarrow$ water and $n_b = 1.333$. $n_a \sin\theta_a = n_b \sin\theta_b$ gives $\sin\theta_{\text{crit}} = \dfrac{(1.333)(\sin 90°)}{1.52}$ and $\theta_{\text{crit}} = 61.3°$. $\alpha = 90° - \theta_{\text{crit}} = 28.7°$.

EVALUATE: The critical angle increases when the air is replaced by water and rays are bent as they refract out of the glass.

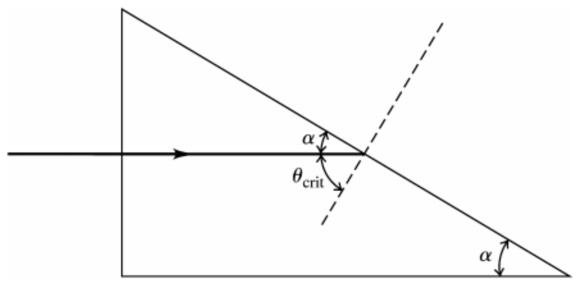


Figure 33.22

33.23. IDENTIFY: Apply $n_a \sin\theta_a = n_b \sin\theta_b$.

SET UP: The light is in diamond and encounters an interface with air, so $n_a = 2.42$ and $n_b = 1.00$. The largest θ_a is when $\theta_b = 90°$.

EXECUTE: $(2.42)\sin\theta_a = (1.00)\sin 90°$. $\sin\theta_a = \dfrac{1}{2.42}$ and $\theta_a = 24.4°$.

EVALUATE: Diamond has an usually large refractive index, and this results in a small critical angle.

33.24. IDENTIFY: Snell's law is $n_a \sin\theta_a = n_b \sin\theta_b$. $v = \dfrac{c}{n}$.

SET UP: $a = \text{air}$, $b = \text{glass}$.

EXECUTE: **(a)** red: $n_b = \dfrac{n_a \sin\theta_a}{\sin\theta_b} = \dfrac{(1.00)\sin 57.0°}{\sin 38.1°} = 1.36$. violet: $n_b = \dfrac{(1.00)\sin 57.0°}{\sin 36.7°} = 1.40$.

(b) red: $v = \dfrac{c}{n} = \dfrac{3.00\times 10^8 \text{ m/s}}{1.36} = 2.21\times 10^8 \text{ m/s}$; violet: $v = \dfrac{c}{n} = \dfrac{3.00\times 10^8 \text{ m/s}}{1.40} = 2.14\times 10^8 \text{ m/s}$.

EVALUATE: n is larger for the violet light and therefore this light is bent more toward the normal, and the violet light has a smaller speed in the glass than the red light.

33.25. IDENTIFY: When unpolarized light passes through a polarizer the intensity is reduced by a factor of $\frac{1}{2}$ and the transmitted light is polarized along the axis of the polarizer. When polarized light of intensity $I_{\max}$ is incident on a polarizer, the transmitted intensity is $I = I_{\max}\cos^2\phi$, where ϕ is the angle between the polarization direction of the incident light and the axis of the filter.

SET UP: For the second polarizer $\phi = 60°$. For the third polarizer, $\phi = 90° - 60° = 30°$.

EXECUTE: **(a)** At point A the intensity is $I_0/2$ and the light is polarized along the vertical direction. At point B the intensity is $(I_0/2)(\cos 60°)^2 = 0.125I_0$, and the light is polarized along the axis of the second polarizer. At point C the intensity is $(0.125I_0)(\cos 30°)^2 = 0.0938I_0$.

(b) Now for the last filter $\phi = 90°$ and $I = 0$.

EVALUATE: Adding the middle filter increases the transmitted intensity.

33.26. IDENTIFY: Apply Snell's law.

SET UP: The incident, reflected and refracted rays are shown in Figure 33.26.

EXECUTE: From the figure, $\theta_b = 37.0°$ and $n_b = n_a \dfrac{\sin\theta_a}{\sin\theta_b} = 1.33\dfrac{\sin 53°}{\sin 37°} = 1.77.$

EVALUATE: The refractive index of *b* is greater than that of *a,* and the ray is bent toward the normal when it refracts.

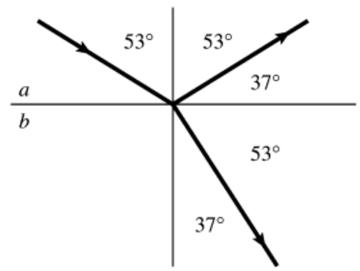


Figure 33.26

33.27. IDENTIFY and **SET UP:** Reflected beam completely linearly polarized implies that the angle of incidence equals the polarizing angle, so $\theta_p = 54.5°$. Use Eq.(33.8) to calculate the refractive index of the glass. Then use Snell's law to calculate the angle of refraction.

EXECUTE: **(a)** $\tan\theta_p = \dfrac{n_b}{n_a}$ gives $n_{glass} = n_{air}\tan\theta_p = (1.00)\tan 54.5° = 1.40.$

(b) $n_a \sin\theta_a = n_b \sin\theta_b$

$\sin\theta_b = \dfrac{n_a \sin\theta_a}{n_b} = \dfrac{(1.00)\sin 54.5°}{1.40} = 0.5815$ and $\theta_b = 35.5°$

EVALUATE:

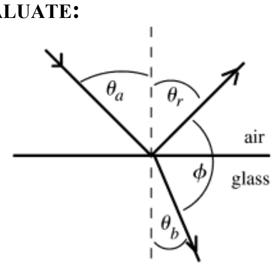


Figure 33.27

Note: $\phi = 180.0° - \theta_r - \theta_b$ and $\theta_r = \theta_a$. Thus $\phi = 180.0° - 54.5° - 35.5° = 90.0°$; the reflected ray and the refracted ray are perpendicular to each other. This agrees with Fig.33.28.

33.28. IDENTIFY: Set $I = I_0/10$, where I is the intensity of light passed by the second polarizer.

SET UP: When unpolarized light passes through a polarizer the intensity is reduced by a factor of $\frac{1}{2}$ and the transmitted light is polarized along the axis of the polarizer. When polarized light of intensity I_{max} is incident on a polarizer, the transmitted intensity is $I = I_{max}\cos^2\phi$, where ϕ is the angle between the polarization direction of the incident light and the axis of the filter.

EXECUTE: **(a)** After the first filter $I = \dfrac{I_0}{2}$ and the light is polarized along the vertical direction. After the second filter we want $I = \dfrac{I_0}{10}$, so $\dfrac{I_0}{10} = \left(\dfrac{I_0}{2}\right)(\cos\phi)^2$. $\cos\phi = \sqrt{2/10}$ and $\phi = 63.4°$.

(b) Now the first filter passes the full intensity I_0 of the incident light. For the second filter $\dfrac{I_0}{10} = I_0(\cos\phi)^2$.

$\cos\phi = \sqrt{1/10}$ and $\phi = 71.6°$.

EVALUATE: When the incident light is polarized along the axis of the first filter, ϕ must be larger to achieve the same overall reduction in intensity than when the incident light is unpolarized.

33.29. IDENTIFY: From Malus's law, the intensity of the emerging light is proportional to the *square* of the cosine of the angle between the polarizing axes of the two filters.

SET UP: If the angle between the two axes is θ, the intensity of the emerging light is $I = I_{max}\cos^2\theta$.

EXECUTE: At angle θ, $I = I_{max}\cos^2\theta$, and at the new angle α, $\dfrac{1}{2}I = I_{max}\cos^2\alpha$. Taking the ratio of the intensities gives $\dfrac{I_{max}\cos^2\alpha}{I_{max}\cos^2\theta} = \dfrac{\frac{1}{2}I}{I}$, which gives us $\cos\alpha = \dfrac{\cos\theta}{\sqrt{2}}$. Solving for α yields $\alpha = \arccos\left(\dfrac{\cos\theta}{\sqrt{2}}\right)$.

EVALUATE: Careful! This result is not $\cos^2\theta$.

33.30. **IDENTIFY:** The reflected light is completely polarized when the angle of incidence equals the polarizing angle θ_p, where $\tan\theta_p = \frac{n_b}{n_a}$.

SET UP: $n_b = 1.66$.

EXECUTE: **(a)** $n_a = 1.00$. $\tan\theta_p = \frac{1.66}{1.00}$ and $\theta_p = 58.9°$.

(b) $n_a = 1.333$. $\tan\theta_p = \frac{1.66}{1.333}$ and $\theta_p = 51.2°$.

EVALUATE: The polarizing angle depends on the refractive indicies of both materials at the interface.

33.31. **IDENTIFY:** When unpolarized light of intensity I_0 is incident on a polarizing filter, the transmitted light has intensity $\frac{1}{2}I_0$ and is polarized along the filter axis. When polarized light of intensity I_0 is incident on a polarizing filter the transmitted light has intensity $I_0 \cos^2\phi$.

SET UP: For the second filter, $\phi = 62.0° - 25.0° = 37.0°$.

EXECUTE: After the first filter the intensity is $\frac{1}{2}I_0 = 10.0\ \text{W/m}^2$ and the light is polarized along the axis of the first filter. The intensity after the second filter is $I = I_0\cos^2\phi$, where $I_0 = 10.0\ \text{W/m}^2$ and $\phi = 37.0°$. This gives $I = 6.38\ \text{W/m}^2$.

EVALUATE: The transmitted intensity depends on the angle between the axes of the two filters.

33.32. **IDENTIFY:** After passing through the first filter the light is linearly polarized along the filter axis. After the second filter, $I = I_{max}(\cos\phi)^2$, where ϕ is the angle between the axes of the two filters.

SET UP: The maximum amount of light is transmitted when $\phi = 0$.

EXECUTE: **(a)** $I = I_0(\cos 22.5°)^2 = 0.854 I_0$

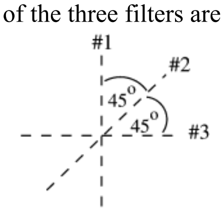
(b) $I = I_0(\cos 45.0°)^2 = 0.500 I_0$

(c) $I = I_0(\cos 67.5°)^2 = 0.146 I_0$

EVALUATE: As ϕ increases toward $90°$ the axes of the two filters are closer to being perpendicular to each other and the transmitted intensity decreases.

33.33. **IDENTIFY** and **SET UP:** Apply Eq.(33.7) to polarizers #2 and #3. The light incident on the first polarizer is unpolarized, so the transmitted light has half the intensity of the incident light, and the transmitted light is polarized.

(a) EXECUTE: The axes of the three filters are shown in Figure 33.33a.

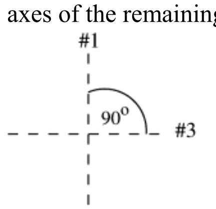


$$I = I_{max}\cos^2\phi$$

Figure 33.33a

After the first filter the intensity is $I_1 = \frac{1}{2}I_0$ and the light is linearly polarized along the axis of the first polarizer. After the second filter the intensity is $I_2 = I_1\cos^2\phi = (\frac{1}{2}I_0)(\cos 45.0°)^2 = 0.250 I_0$ and the light is linearly polarized along the axis of the second polarizer. After the third filter the intensity is $I_3 = I_2\cos^2\phi = 0.250 I_0(\cos 45.0°)^2 = 0.125 I_0$ and the light is linearly polarized along the axis of the third polarizer.

(b) The axes of the remaining two filters are shown in Figure 33.33b.



After the first filter the intensity is $I_1 = \frac{1}{2}I_0$ and the light is linearly polarized along the axis of the first polarizer.

Figure 33.33b

After the next filter the intensity is $I_3 = I_1\cos^2\phi = (\frac{1}{2}I_0)(\cos 90.0°)^2 = 0$. No light is passed.

EVALUATE: Light is transmitted through all three filters, but no light is transmitted if the middle polarizer is removed.

33.34. **IDENTIFY:** Use the transmitted intensity when all three polairzers are present to solve for the incident intensity I_0. Then repeat the calculation with only the first and third polarizers.

SET UP: For unpolarized light incident on a filter, $I = \frac{1}{2}I_0$ and the light is linearly polarized along the filter axis. For polarized light incident on a filter, $I = I_{max}(\cos\phi)^2$, where I_{max} is the intensity of the incident light, and the emerging light is linearly polarized along the filter axis.

EXECUTE: With all three polarizers, if the incident intensity is I_0 the transmitted intensity is $I = (\frac{1}{2}I_0)(\cos 23.0°)^2(\cos[62.0° - 23.0°])^2 = 0.256I_0$. $I_0 = \dfrac{I}{0.256} = \dfrac{75.0\text{ W/cm}^2}{0.256} = 293\text{ W/cm}^2$. With only the first and third polarizers, $I = (\frac{1}{2}I_0)(\cos 62.0°)^2 = 0.110I_0 = (0.110)(293\text{ W/cm}^2) = 32.2\text{ W/cm}^2$.

EVALUATE: The transmitted intensity is greater when all three filters are present.

33.35. **IDENTIFY:** The shorter the wavelength of light, the more it is scattered. The intensity is inversely proportional to the fourth power of the wavelength.

SET UP: The intensity of the scattered light is proportional to $1/\lambda^4$, we can write it as $I = (\text{constant})/\lambda^4$.

EXECUTE: **(a)** Since I is proportional to $1/\lambda^4$, we have $I = (\text{constant})/\lambda^4$. Taking the ratio of the intensity of the red light to that of the green light gives $\dfrac{I_R}{I} = \dfrac{(\text{constant})/\lambda_R^4}{(\text{constant})/\lambda_G^4} = \left(\dfrac{\lambda_G}{\lambda_R}\right)^4 = \left(\dfrac{520\text{ nm}}{665\text{ nm}}\right)^4 = 0.374$, so $I_R = 0.374I$.

(b) Following the same procedure as in part (a) gives $\dfrac{I_V}{I} = \left(\dfrac{\lambda_G}{\lambda_V}\right)^4 = \left(\dfrac{520\text{ nm}}{420\text{ nm}}\right)^4 = 2.35$, so $I_V = 2.35I$.

EVALUATE: In the scattered light, the intensity of the short-wavelength violet light is about 7 times as great as that of the red light, so this scattered light will have a blue-violet color.

33.36. **IDENTIFY:** As the wave front reaches the sharp object, every point on the front will act as a source of secondary wavelets.

SET UP: Consider a wave front that is just about to go past the corner. Follow it along and draw the successive wave fronts.

EXECUTE: The path of the wavefront is drawn in Figure 33.36.

EVALUATE: The wave fronts clearly bend around the sharp point, just as water waves bend around a rock and light waves bend around the edge of a slit.

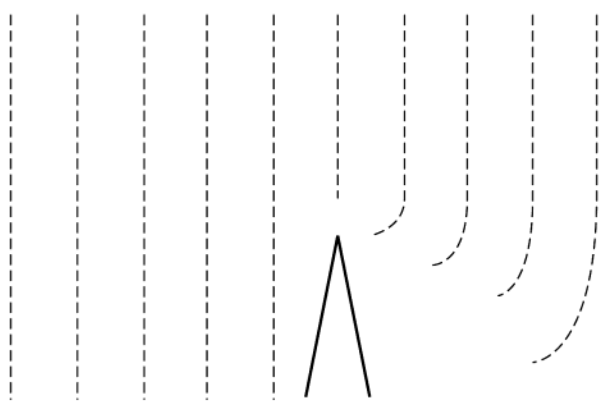


Figure 33.36

33.37. **IDENTIFY:** Reflection reverses the sign of the component of light velocity perpendicular to the reflecting surface but leaves the other components unchanged.

SET UP: Consider three mirrors, M_1 in the (x,y)-plane, M_2 in the (y,z)-plane, and M_3 in the (x,z)-plane.

EXECUTE: A light ray reflecting from M_1 changes the sign of the z-component of the velocity, reflecting from M_2 changes the x-component, and from M_3 changes the y-component. Thus the velocity, and hence also the path, of the light beam flips by 180°

EVALUATE: Example 33.3 discusses some uses of corner reflectors.

33.38. **IDENTIFY:** The light travels slower in the jelly than in the air and hence will take longer to travel the length of the tube when it is filled with jelly than when it contains just air.

SET UP: The definition of the index of refraction is $n = c/v$, where v is the speed of light in the jelly.

EXECUTE: First get the length L of the tube using air. In the air, we have $L = ct = (3.00 \times 10^8\text{ m/s})(8.72\text{ ns}) = 2.616\text{ m}$. The speed in the jelly is $v = \dfrac{L}{t} = (2.616\text{ m})/(8.72\text{ ns} + 2.04\text{ ns}) = 2.431 \times 10^8\text{ m/s}$. $n = \dfrac{c}{v} =$ $(3.00 \times 10^8\text{ m/s})/(2.431 \times 10^8\text{ m/s}) = 1.23$

EVALUATE: A high-speed timer would be needed to measure times as short as a few nanoseconds.

33.39. **IDENTIFY** and **SET UP:** Apply Snell's law at each interface.

EXECUTE: **(a)** $n_1 \sin\theta_1 = n_2 \sin\theta_2$ and $n_2 \sin\theta_2 = n_3 \sin\theta_3$, so $n_1 \sin\theta_1 = n_3 \sin\theta_3$ and $\sin\theta_3 = (n_1 \sin\theta_1)/n_3$.

(b) $n_3 \sin\theta_3 = n_2 \sin\theta_2$ and $n_2 \sin\theta_2 = n_1 \sin\theta_1$, so $n_1 \sin\theta_1 = n_3 \sin\theta_3$ and the light makes the same angle with respect to the normal in the material that has refractive index n_1 as it did in part (a).

(c) For reflection, $\theta_r = \theta_a$. These angles are still equal if θ_r becomes the incident angle; reflected rays are also reversible.

EVALUATE: Both the refracted and reflected rays are reversible, in the sense that if the direction of the light is reversed then each of these rays follow the path of the incident ray.

33.40. **IDENTIFY:** Use the change in transit time to find the speed v of light in the slab, and then apply $n = \dfrac{c}{v}$ and $\lambda = \dfrac{\lambda_0}{n}$.

SET UP: It takes the light an additional 4.2 ns to travel 0.840 m after the glass slab is inserted into the beam.

EXECUTE: $\dfrac{0.840\text{ m}}{c/n} - \dfrac{0.840\text{ m}}{c} = (n-1)\dfrac{0.840\text{ m}}{c} = 4.2\text{ ns}$. We can now solve for the index of refraction:

$n = \dfrac{(4.2\times10^{-9}\text{s})\,(3.00\times10^{8}\text{ m/s})}{0.840\text{ m}} + 1 = 2.50$. The wavelength inside of the glass is $\lambda = \dfrac{490\text{ nm}}{2.50} = 196\text{ nm}$.

EVALUATE: Light travels slower in the slab than in air and the wavelength is shorter.

33.41. **IDENTIFY:** The angle of incidence at *A* is to be the critical angle. Apply Snell's law at the air to glass refraction at the top of the block.

SET UP: The ray is sketched in Figure 33.41.

EXECUTE: For glass $\to$ air at point *A*, Snell's law gives $(1.38)\sin\theta_{\text{crit}} = (1.00)\sin 90°$ and $\theta_{\text{crit}} = 46.4°$.

$\theta_b = 90° - \theta_{\text{crit}} = 43.6°$. Snell's law applied to the refraction from air to glass at the top of the block gives $(1.00)\sin\theta_a = (1.38)\sin(43.6°)$ and $\theta_a = 72.1°$.

EVALUATE: If θ_a is larger than $72.1°$ then the angle of incidence at point *A* is less than the initial critical angle and total internal reflection doesn't occur.

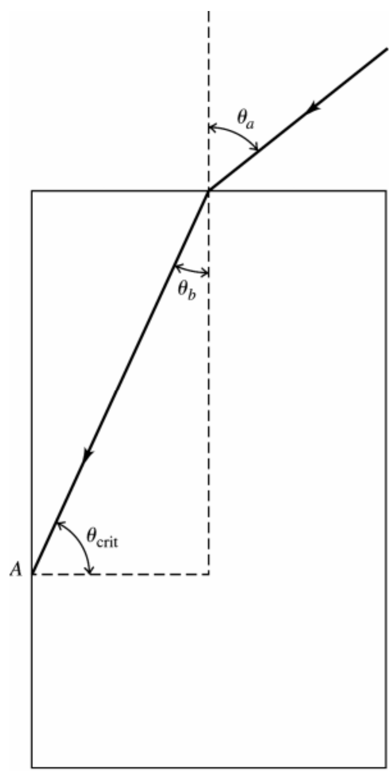


Figure 33.41

33.42. **IDENTIFY:** As the light crosses the glass-air interface along *AB*, it is refracted and obeys Snell's law.

SET UP: Snell's law is $n_a \sin\theta_a = n_b \sin\theta_b$ and $n = 1.000$ for air. At point *B* the angle of the prism is $30.0°$.

EXECUTE: Apply Snell's law at *AB*. The prism angle at *A* is 60.0°, so for the upper ray, the angle of incidence at *AB* is 60.0° + 12.0° = 72.0°. Using this value gives $n_1 \sin 60.0° = \sin 72.0°$ and $n_1 = 1.10$. For the lower ray, the angle of incidence at *AB* is 60.0° + 12.0° + 8.50° = 80.5°, giving $n_2 \sin 60.0° = \sin 80.5°$ and $n_2 = 1.14$.

EVALUATE: The lower ray is deflected more than the upper ray because that wavelength has a slightly greater index of refraction than the upper ray.

33.43. **IDENTIFY:** Circularly polarized light consists of the superposition of light polarized in two perpendicular directions, with a quarter-cycle ($90°$) phase difference between the two polarization components.

SET UP: A quarter-wave plate shifts the relative phase of the two perpendicular polarization components by $90°$.

EXECUTE: In the circularly polarized light the two perpendicular polarization components are $90°$ out of phase. The quarter-wave plate shifts the relative phase by $\pm 90°$ and then the two components are either in phase or $180°$ out of phase. Either corresponds to linearly polarized light.

EVALUATE: Either left circularly polarized light or right circularly polarized light is converted to linearly polarized light by the quarter-wave plate.

33.44. **IDENTIFY:** Apply $\lambda = \dfrac{\lambda_0}{n}$. The number of wavelengths in a distance d of a material is $\dfrac{d}{\lambda}$ where λ is the wavelength in the material.

SET UP: The distance in glass is $d_{\text{glass}} = 0.00250\text{ m}$. The distance in air is

$d_{\text{air}} = 0.0180\text{ m} - 0.00250\text{ m} = 0.0155\text{ m}$

EXECUTE: number of wavelengths = number in air + number in glass.

$$\text{number of wavelengths} = \frac{d_{\text{air}}}{\lambda} + \frac{d_{\text{glass}}}{\lambda}n = \frac{0.0155\text{ m}}{5.40\times10^{-7}\text{ m}} + \frac{0.00250\text{ m}}{5.40\times10^{-7}\text{ m}}(1.40) = 3.52\times10^{4}.$$

EVALUATE: Without the glass plate the number of wavelengths between the source and screen is $\dfrac{0.0180\text{ m}}{5.40\times10^{-3}\text{ m}} = 3.33\times10^{4}$. The wavelength is shorter in the glass so there are more wavelengths in a distance in glass than there are in the same distance in air.

33.45. **IDENTIFY:** Find the critical angle for glass $\rightarrow$ air. Light incident at this critical angle is reflected back to the edge of the halo.

SET UP: The ray incident at the critical angle is sketched in Figure 33.45.

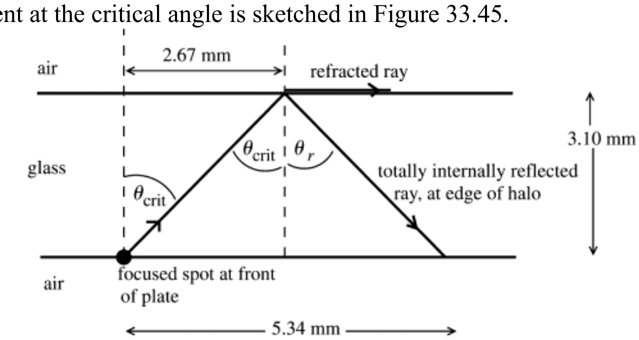


Figure 33.45

EXECUTE: From the distances given in the sketch, $\tan\theta_{\text{crit}} = \dfrac{2.67\text{ mm}}{3.10\text{ mm}} = 0.8613;\ \theta_{\text{crit}} = 40.7°.$

Apply Snell's law to the total internal reflection to find the refractive index of the glass: $n_a \sin\theta_a = n_b \sin\theta_b$

$n_{\text{glass}} \sin\theta_{\text{crit}} = 1.00 \sin 90°$

$$n_{\text{glass}} = \frac{1}{\sin\theta_{\text{crit}}} = \frac{1}{\sin 40.7°} = 1.53$$

EVALUATE: Light incident on the back surface is also totally reflected if it is incident at angles greater than θ_{crit}. If it is incident at less than θ_{crit} it refracts into the air and does not reflect back to the emulsion.

33.46. **IDENTIFY:** Apply Snell's law to the refraction of the light as it passes from water into air.

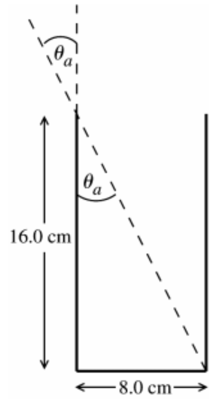
SET UP: $\theta_a = \arctan\left(\dfrac{1.5\text{ m}}{1.2\text{ m}}\right) = 51°$. $n_a = 1.00$. $n_b = 1.333$.

EXECUTE: $\theta_b = \arcsin\left(\dfrac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\dfrac{1.00}{1.333}\sin 51°\right) = 36°$. Therefore, the distance along the bottom of the pool from directly below where the light enters to where it hits the bottom is $x = (4.0\text{ m})\tan\theta_b = (4.0\text{ m})\tan 36° = 2.9\text{ m}$. $x_{\text{total}} = 1.5\text{ m} + x = 1.5\text{ m} + 2.9\text{ m} = 4.4\text{ m}$.

EVALUATE: The light ray from the flashlight is bent toward the normal when it refracts into the water.

33.47. **IDENTIFY:** Use Snell's law to determine the effect of the liquid on the direction of travel of the light as it enters the liquid.

SET UP: Use geometry to find the angles of incidence and refraction. Before the liquid is poured in the ray along your line of sight has the path shown in Figure 33.47a.

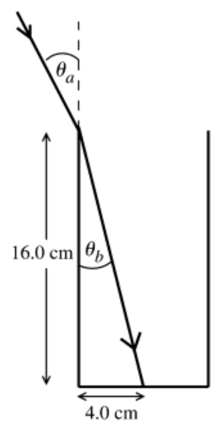


$$\tan\theta_a = \frac{8.0\text{ cm}}{16.0\text{ cm}} = 0.500$$

$$\theta_a = 26.57°$$

Figure 33.47a

After the liquid is poured in, θ_a is the same and the refracted ray passes through the center of the bottom of the glass, as shown in Figure 33.47b.



$$\tan\theta_b = \frac{4.0\text{ cm}}{16.0\text{ cm}} = 0.250$$

$$\theta_b = 14.04°$$

Figure 33.47b

EXECUTE: Use Snell's law to find n_b, the refractive index of the liquid:

$$n_a \sin\theta_a = n_b \sin\theta_b$$

$$n_b = \frac{n_a \sin\theta_a}{\sin\theta_b} = \frac{(1.00)(\sin 26.57°)}{\sin 14.04°} = 1.84$$

EVALUATE: When the light goes from air to liquid (larger refractive index) it is bent toward the normal.

33.48. **IDENTIFY:** Apply Snell's law to each refraction and apply the law of reflection at the mirrored bottom.

SET UP: The path of the ray is sketched in Figure 33.48. The problem asks us to calculate θ_b'.

EXECUTE: Apply Snell's law to the air $\rightarrow$ liquid refraction. $(1.00)\sin(42.5°) = (1.63)\sin\theta_b$ and $\theta_b = 24.5°$. $\theta_b = \phi$ and $\phi = \theta_a'$, so $\theta_a' = \theta_b = 24.5°$. Snell's law applied to the liquid $\rightarrow$ air refraction gives $(1.63)\sin(24.5°) = (1.00)\sin\theta_b'$ and $\theta_b' = 42.5°$.

EVALUATE: The light emerges from the liquid at the same angle from the normal as it entered the liquid.

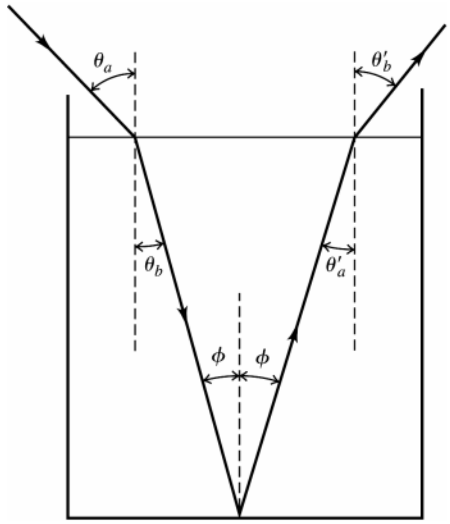


Figure 33.48

33.49. IDENTIFY: Apply Snell's law to the water $\rightarrow$ ice and ice $\rightarrow$ air interfaces.

(a) SET UP: Consider the ray shown in Figure 33.49.

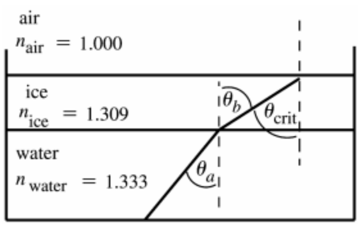


Figure 33.49

We want to find the incident angle θ_a at the water-ice interface that causes the incident angle at the ice-air interface to be the critical angle.

EXECUTE: ice-air interface: $n_{ice}\sin\theta_{crit} = 1.0\sin 90°$

$n_{ice}\sin\theta_{crit} = 1.0$ so $\sin\theta_{crit} = \dfrac{1}{n_{ice}}$

But from the diagram we see that $\theta_b = \theta_{crit}$, so $\sin\theta_b = \dfrac{1}{n_{ice}}$.

water-ice interface: $n_w \sin\theta_a = n_{ice}\sin\theta_b$

But $\sin\theta_b = \dfrac{1}{n_{ice}}$ so $n_w \sin\theta_a = 1.0$. $\sin\theta_a = \dfrac{1}{n_w} = \dfrac{1}{1.333} = 0.7502$ and $\theta_a = 48.6°$.

(b) EVALUATE: The angle calculated in part (a) is the critical angle for a water-air interface; the answer would be the same if the ice layer wasn't there!

33.50. IDENTIFY: The incident angle at the prism $\rightarrow$ water interface is to be the critical angle.

SET UP: The path of the ray is sketched in Figure 33.50. The ray enters the prism at normal incidence so is not bent. For water, $n_{water} = 1.333$.

EXECUTE: From the figure, $\theta_{crit} = 45°$. $n_a\sin\theta_a = n_b\sin\theta_b$ gives $n_{glass}\sin 45° = (1.333)\sin 90°$.

$$n_{glass} = \frac{1.333}{\sin 45°} = 1.89 .$$

EVALUATE: For total internal reflection the ray must be incident in the material of greater refractive index. $n_{\text{glass}} > n_{\text{water}}$, so that is the case here.

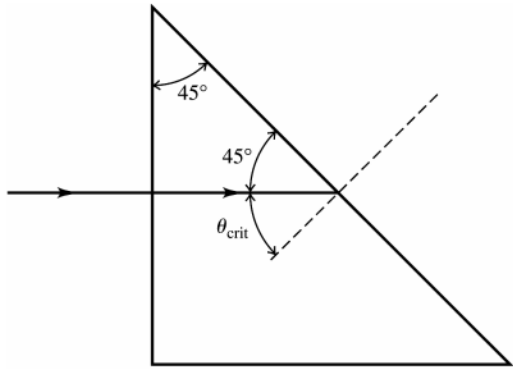


Figure 33.50

33.51. **IDENTIFY:** Apply Snell's law to the refraction of each ray as it emerges from the glass. The angle of incidence equals the angle $A = 25.0°$.

SET UP: The paths of the two rays are sketched in Figure 33.51.

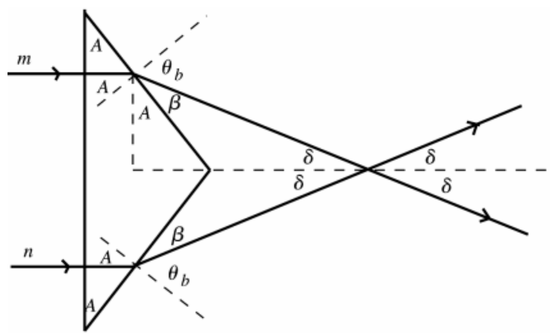


Figure 33.51

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$

$n_{\text{glas}} \sin 25.0° = 1.00 \sin\theta_b$

$\sin\theta_b = n_{\text{glass}} \sin 25.0°$

$\sin\theta_b = 1.66 \sin 25.0° = 0.7015$

$\theta_b = 44.55°$

$\beta = 90.0° - \theta_b = 45.45°$

Then $\delta = 90.0° - A - \beta = 90.0° - 25.0° - 45.45° = 19.55°$. The angle between the two rays is $2\delta = 39.1°$.

EVALUATE: The light is incident normally on the front face of the prism so the light is not bent as it enters the prism.

33.52. **IDENTIFY:** The ray shown in the figure that accompanies the problem is to be incident at the critical angle.

SET UP: $\theta_b = 90°$. The incident angle for the ray in the figure is $60°$.

EXECUTE: $n_a \sin\theta_a = n_b \sin\theta_b$ gives $n_b = \left(\dfrac{n_a \sin\theta_a}{\sin\theta_b}\right) = \left(\dfrac{1.62 \sin 60°}{\sin 90°}\right) = 1.40.$

EVALUATE: Total internal reflection occurs only when the light is incident in the material of the greater refractive index.

33.53. **IDENTIFY:** No light enters the gas because total internal reflection must have occurred at the water-gas interface.

SET UP: At the minimum value of S, the light strikes the water-gas interface at the critical angle. We apply Snell's law, $n_a \sin\theta_a = n_b \sin\theta_b$, at that surface.

EXECUTE: **(a)** In the water, $\theta = \dfrac{S}{R}$ = (1.09 m)/(1.10 m) = 0.991 rad = 56.77°. This is the critical angle. So, using the refractive index for water from Table 33.1, we get n = (1.333) sin 56.77° = 1.12

(b) (i) The laser beam stays in the water all the time, so

$$t = 2R/v = 2R/\left(\frac{c}{n_{\text{water}}}\right) = \frac{Dn_{\text{water}}}{c} = (2.20\ \text{m})(1.333)/(3.00 \times 10^8\ \text{m/s}) = 9.78\ \text{ns}$$

(ii) The beam is in the water half the time and in the gas the other half of the time.

$$t_{\text{gas}} = \frac{Rn_{\text{gas}}}{c} = (1.10 \text{ m})(1.12)/(3.00 \times 10^8 \text{ m/s}) = 4.09 \text{ ns}$$

The total time is 4.09 ns + (9.78 ns)/2 = 8.98 ns

EVALUATE: The gas must be under considerable pressure to have a refractive index as high as 1.12.

33.54. **IDENTIFY:** No light enters the water because total internal reflection must have occurred at the glass-water surface.

SET UP: A little geometry tells us that θ is the angle of incidence at the glass-water face in the water. Also, θ = 59.2° must be the critical angle at that surface, so the angle of refraction is 90.0°. Snell's law, $n_a \sin\theta_a = n_b \sin\theta_b$, applies at that glass-water surface, and the index of refraction is defined as $n = \frac{c}{v}$.

EXECUTE: Snell's law at the glass-water surface gives $n \sin 59.2° = (1.333)(1.00)$, which gives $n = 1.55$. $v = \frac{c}{n} = (3.00 \times 10^8 \text{ m/s})/1.55 = 1.93 \times 10^8$ m/s.

EVALUATE: Notice that θ is *not* the angle of incidence at the reflector, but it is the angle of incidence at the glass-water surface.

33.55. **(a) IDENTIFY:** Apply Snell's law to the refraction of the light as it enters the atmosphere.

SET UP: The path of a ray from the sun is sketched in Figure 33.55.

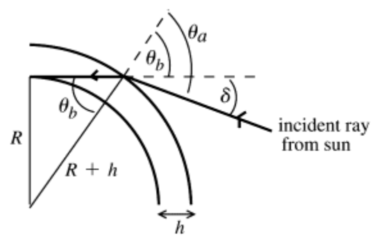


Figure 33.55

$$\delta = \theta_a - \theta_b$$

From the diagram $\sin\theta_b = \dfrac{R}{R+h}$

$$\theta_b = \arcsin\left(\frac{R}{R+h}\right)$$

EXECUTE: Apply Snell's law to the refraction that occurs at the top of the atmosphere: $n_a \sin\theta_a = n_b \sin\theta_b$

(a = vacuum of space, refractive, index 1.0; b = atmosphere, refractive index n)

$$\sin\theta_a = n\sin\theta_b = n\left(\frac{R}{R+h}\right) \text{ so } \theta_a = \arcsin\left(\frac{nR}{R+h}\right)$$

$$\delta = \theta_a - \theta_b = \arcsin\left(\frac{nR}{R+h}\right) - \arcsin\left(\frac{R}{R+h}\right)$$

(b) $\dfrac{R}{R+h} = \dfrac{6.38\times10^6 \text{ m}}{6.38\times10^6 \text{ m} + 20\times10^3 \text{ m}} = 0.99688$

$$\frac{nR}{R+h} = 1.0003(0.99688) = 0.99718$$

$$\theta_b = \arcsin\left(\frac{R}{R+h}\right) = 85.47°$$

$$\theta_b = \arcsin\left(\frac{nR}{R+h}\right) 85.70°$$

$$\delta = \theta_a - \theta_b = 85.70° - 85.47° = 0.23°$$

EVALUATE: The calculated δ is about the same as the angular radius of the sun.

33.56. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** The distance traveled by the light ray is the sum of the two diagonal segments: $d = (x^2 + y_1^2)^{1/2} + ((l-x)^2 + y_2^2)^{1/2}$. Then the time taken to travel that distance is $t = \dfrac{d}{c} = \dfrac{(x^2 + y_1^2)^{1/2} + ((l-x)^2 + y_2^2)^{1/2}}{c}$.

(b) Taking the derivative with respect to x of the time and setting it to zero yields

$$\frac{dt}{dx} = \frac{1}{c}\frac{d}{dt}\left[(x^2+y_1^2)^{1/2} + \left((l-x)^2 + y_2^2\right)^{1/2}\right]\frac{dt}{dx} = \frac{1}{c}\left[x(x^2+y_1^2)^{-1/2} - (l-x)\left((l-x)^2 + y_2^2\right)^{-1/2}\right] = 0$$. This gives

$$\frac{x}{\sqrt{x^2+y_1^2}} = \frac{(l-x)}{\sqrt{(l-x)^2+y_2^2}}, \ \sin\theta_1 = \sin\theta_2 \text{ and } \theta_1 = \theta_2.$$

EVALUATE: For any other path between points 1 and 2, that includes a point on the reflective surface, the distance traveled and therefore the travel time is greater than for this path.

33.57. **IDENTIFY** and **SET UP:** Find the distance that the ray travels in each medium. The travel time in each medium is the distance divided by the speed in that medium.

(a) EXECUTE: The light travels a distance $\sqrt{h_1^2 + x^2}$ in traveling from point A to the interface. Along this path the speed of the light is v_1, so the time it takes to travel this distance is $t_1 = \dfrac{\sqrt{h_1^2 + x^2}}{v_1}$. The light travels a distance $\sqrt{h_2^2 + (l-x)^2}$ in traveling from the interface to point B. Along this path the speed of the light is v_2, so the time it takes to travel this distance is $t_2 = \dfrac{\sqrt{h_2^2 + (l-x)^2}}{v_2}$. The total time to go from A to B is

$$t = t_1 + t_2 = \frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (l+x)^2}}{v_2}.$$

(b) $$\frac{dt}{dx} = \frac{1}{v_1}\left(\frac{1}{2}\right)(h_1^2 + x^2)^{-1/2}(2x) + \frac{1}{v_2}\left(\frac{1}{2}\right)(h_2^2 + (l-x)^2)^{-1/2}2(l-x)(-1) = 0$$

$$\frac{x}{v_1\sqrt{h_1^2 + x^2}} = \frac{l-x}{v_2\sqrt{h_2^2 + (l-x)^2}}$$

Multiplying both sides by c gives $\dfrac{c}{v_1}\dfrac{x}{\sqrt{h_1^2 + x^2}} = \dfrac{c}{v_2}\dfrac{l-x}{\sqrt{h_2^2 + (l-x)^2}}$

$\dfrac{c}{v_1} = n_1$ and $\dfrac{c}{v_2} = n_2$ (Eq.33.1)

From Fig.33.55 in the textbook, $\sin\theta_1 = \dfrac{x}{\sqrt{h_1^2 + x^2}}$ and $\sin\theta_2 = \dfrac{l-x}{\sqrt{h_2^2 + (l-x)^2}}$.

So $n_1 \sin\theta_1 = n_2 \sin\theta_2$, which is Snell's law.

EVALUATE: Snell's law is a result of a change in speed when light goes from one material to another.

33.58. **IDENTIFY:** Apply Snell's law to each refraction.

SET UP: Refer to the angles and distances defined in the figure that accompanies the problem.

EXECUTE: **(a)** For light in air incident on a parallel-faced plate, Snell's Law yields:

$n\sin\theta_a = n'\sin\theta'_b = n'\sin\theta_b = n\sin\theta'_a \Rightarrow \sin\theta_a = \sin\theta'_a \Rightarrow \theta_a = \theta'_a$.

(b) Adding more plates just adds extra steps in the middle of the above equation that always cancel out. The requirement of parallel faces ensures that the angle $\theta'_n = \theta_n$ and the chain of equations can continue.

(c) The lateral displacement of the beam can be calculated using geometry:

$d = L\sin(\theta_a - \theta'_b)$ and $L = \dfrac{t}{\cos\theta'_b} \Rightarrow d = \dfrac{t\sin(\theta_a - \theta'_b)}{\cos\theta'_b}$.

(d) $\theta'_b = \arcsin\left(\dfrac{n\sin\theta_a}{n'}\right) = \arcsin\left(\dfrac{\sin 66.0^\circ}{1.80}\right) = 30.5^\circ$ and $d = \dfrac{(2.40\text{ cm})\sin(66.0^\circ - 30.5^\circ)}{\cos 30.5^\circ} = 1.62\text{ cm}$.

EVALUATE: The lateral displacement in part (d) is large, of the same order as the thickness of the plate.

33.59. **IDENTIFY:** Apply Snell's law to each refraction and apply the law of reflection to each reflection.

SET UP: The paths of rays A and B are sketched in Figure 33.59. Let θ be the angle of incidence for the combined ray.

EXECUTE: For ray A its final direction of travel is at an angle θ with respect to the normal, by the law of reflection. Let the final direction of travel for ray B be at angle ϕ with respect to the normal. At the upper surface, Snell's law gives $n_1\sin\theta = n_2\sin\alpha$. The lower surface reflects ray B at angle α. Ray B returns to the upper surface of the film at an angle of incidence α. Snell's law applied to the refraction as ray B leaves the film gives $n_2\sin\alpha = n_1\sin\phi$. Combining the two equations gives $n_1\sin\theta = n_1\sin\phi$ and $\theta = \phi$; the two rays are parallel after they emerge from the film.

EVALUATE: Ray B is bent toward the normal as it enters the film and away from the normal as it refracts out of the film.

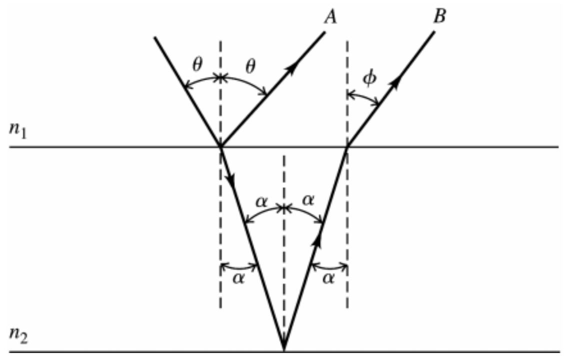


Figure 33.59

33.60. IDENTIFY: Apply Snell's law and the results of Problem 33.58.

SET UP: From Figure 33.58 in the textbook, $n_r = 1.61$ for red light and $n_v = 1.66$ for violet. In the notation of Problem 33.58, t is the thickness of the glass plate and the lateral displacement is d. We want the difference in d for the two colors of light to be 1.0 mm. $\theta_a = 70.0°$. For red light, $n_a \sin\theta_a = n_b \sin\theta_b'$ gives $\sin\theta_b' = \dfrac{(1.00)\sin 70.0°}{1.61}$ and $\theta_b' = 35.71°$. For violet light, $\sin\theta_b' = \dfrac{(1.00)\sin 70.0°}{1.66}$ and $\theta_b' = 34.48°$.

EXECUTE: **(a)** n decreases with increasing λ, so n is smaller for red than for blue. So beam a is the red one.

(b) Problem 33.58 says $d = t\dfrac{\sin(\theta_a - \theta_b')}{\cos\theta_b'}$. For red light, $d_r = t\dfrac{\sin(70° - 35.71°)}{\cos 35.71°} = 0.6938t$ and for violet light,

$d_v = t\dfrac{\sin(70° - 35.48°)}{\cos 35.48°} = 0.7048t$. $d_v - d_r = 1.0$ mm gives $t = \dfrac{0.10 \text{ cm}}{0.7048 - 0.6958} = 9.1$ cm.

EVALUATE: Our calculation shown that the violet light has greater lateral displacement and this is ray b.

33.61. IDENTIFY: Apply Snell's law to the two refractions of the ray.

SET UP: Refer to the figure that accompanies the problem.

EXECUTE: **(a)** $n_a \sin\theta_a = n_b \sin\theta_b$ gives $\sin\theta_a = n_b \sin\dfrac{A}{2}$. But $\theta_a = \dfrac{A}{2} + \alpha$, so $\sin\left(\dfrac{A}{2} + \alpha\right) = \sin\dfrac{A + 2\alpha}{2} = n\sin\dfrac{A}{2}$.

At each face of the prism the deviation is α, so $2\alpha = \delta$ and $\sin\dfrac{A+\delta}{2} = n\sin\dfrac{A}{2}$.

(b) From part (a), $\delta = 2\arcsin\left(n\sin\dfrac{A}{2}\right) - A$. $\delta = 2\arcsin\left((1.52)\sin\dfrac{60.0°}{2}\right) - 60.0° = 38.9°$.

(c) If two colors have different indices of refraction for the glass, then the deflection angles for them will differ:

$$\delta_{\text{red}} = 2\arcsin\left((1.61)\sin\frac{60.0°}{2}\right) - 60.0° = 47.2°$$

$$\delta_{\text{violet}} = 2\arcsin\left((1.66)\sin\frac{60.0°}{2}\right) - 60.0° = 52.2° \Rightarrow \Delta\delta = 52.2° - 47.2° = 5.0°$$

EVALUATE: The violet light has a greater refractive index and therefore the angle of deviation is greater for the violet light.

33.62. IDENTIFY: The reflected light is totally polarized when light strikes a surface at Brewster's angle.

SET UP: At the plastic wall, Brewster's angle obeys the equation $\tan\theta_p = n_b/n_a$, and Snell's law, $n_a \sin\theta_a = n_b \sin\theta_b$, applies at the air-water surface.

EXECUTE: To be totally polarized, the reflected sunlight must have struck the wall at Brewster's angle. $\tan\theta_p = n_b/n_a = (1.61)/(1.00)$ and $\theta_p = 58.15°$

This is the angle of incidence at the wall. A little geometry tells us that the angle of incidence at the water surface is $90.00° - 58.15° = 31.85°$. Applying Snell's law at the water surface gives

$$(1.00)\sin 31.85° = 1.333\sin\theta \text{ and } \theta = 23.3°$$

EVALUATE: We have two different principles involved here: Reflection at Brewster's angle at the wall and Snell's law at the water surface.

33.63. **IDENTIFY** and **SET UP:** The polarizer passes $\frac{1}{2}$ of the intensity of the unpolarized component, independent of ϕ. Out of the intensity I_p of the polarized component the polarizer passes intensity $I_p \cos^2(\phi-\theta)$, where $\phi-\theta$ is the angle between the plane of polarization and the axis of the polarizer.

(a) Use the angle where the transmitted intensity is maximum or minimum to find θ. See Figure 33.63.

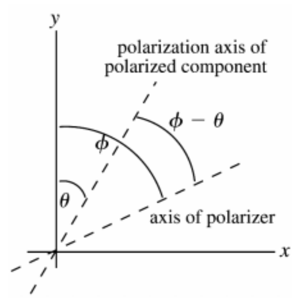


Figure 33.63

EXECUTE: The total transmitted intensity is $I = \frac{1}{2}I_0 + I_p \cos^2(\phi-\theta)$. This is maximum when $\theta = \phi$ and from the table of data this occurs for ϕ between $30°$ and $40°$, say at $35°$ and $\theta = 35°$. Alternatively, the total transmitted intensity is minimum when $\phi - \theta = 90°$ and from the data this occurs for $\phi = 125°$. Thus, $\theta = \phi - 90° = 125° - 90° = 35°$, in agreement with the above.

(b) IDENTIFY and **SET UP:** $I = \frac{1}{2}I_0 + I_p \cos^2(\phi-\theta)$

Use data at two values of ϕ to determine the two constants I_0 and I_p. Use data where the I_p term is large $(\phi = 30°)$ and where it is small $(\phi = 130°)$ to have the greatest sensitivity to both I_0 and I_p:

EXECUTE: $\phi = 30°$ gives $24.8\ \text{W/m}^2 = \frac{1}{2}I_0 + I_p \cos^2(30° - 35°)$

$24.8\ \text{W/m}^2 = 0.500 I_0 + 0.9924 I_p$

$\phi = 130°$ gives $5.2\ \text{W/m}^2 = \frac{1}{2}I_0 + I_p \cos^2(130° - 35°)$

$5.2\ \text{W/m}^2 = 0.500 I_0 + 0.0076 I_p$

Subtracting the second equation from the first gives $19.6\ \text{W/m}^2 = 0.9848 I_p$ and $I_p = 19.9\ \text{W/m}^2$. And then $I_0 = 2(5.2\ \text{W/m}^2 - 0.0076(19.9\ \text{W/m}^2)) = 10.1\ \text{W/m}^2$.

EVALUATE: Now that we have I_0, I_p and θ we can verify that $I = \frac{1}{2}I_0 + I_p \cos^2(\phi-\theta)$ describes that data in the table.

33.64. **IDENTIFY:** The number of wavelengths in a distance D of material is D/λ, where λ is the wavelength of the light in the material.

SET UP: The condition for a quarter-wave plate is $\dfrac{D}{\lambda_1} = \dfrac{D}{\lambda_2} + \dfrac{1}{4}$, where we have assumed $n_1 > n_2$ so $\lambda_2 > \lambda_1$.

EXECUTE: **(a)** $\dfrac{n_1 D}{\lambda_0} = \dfrac{n_2 D}{\lambda_0} + \dfrac{1}{4}$ and $D = \dfrac{\lambda_0}{4(n_1 - n_2)}$.

(b) $D = \dfrac{\lambda_0}{4(n_1 - n_2)} = \dfrac{5.89 \times 10^{-7}\ \text{m}}{4(1.875 - 1.635)} = 6.14 \times 10^{-7}\text{m}.$

EVALUATE: The thickness of the quarter-wave plate in part (b) is 614 nm, which is of the same order as the wavelength in vacuum of the light.

33.65. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: $\cos(\alpha - \beta) = \sin\alpha\sin\beta + \cos\alpha\cos\beta$. $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

EXECUTE: **(a)** Multiplying Eq.(1) by $\sin\beta$ and Eq.(2) by $\sin\alpha$ yields:

(1): $\dfrac{x}{a}\sin\beta = \sin\omega t\cos\alpha\sin\beta - \cos\omega t\sin\alpha\sin\beta$ and (2): $\dfrac{y}{a}\sin\alpha = \sin\omega t\cos\beta\sin\alpha - \cos\omega t\sin\beta\sin\alpha$.

Subtracting yields: $\dfrac{x\sin\beta - y\sin\alpha}{a} = \sin\omega t(\cos\alpha\sin\beta - \cos\beta\sin\alpha).$

(b) Multiplying Eq. (1) by $\cos\beta$ and Eq. (2) by $\cos\alpha$ yields:

$(1): \dfrac{x}{a}\cos\beta = \sin\omega t\cos\alpha\cos\beta - \cos\omega t\sin\alpha\cos\beta$ and $(2): \dfrac{y}{a}\cos\alpha = \sin\omega t\cos\beta\cos\alpha - \cos\omega t\sin\beta\cos\alpha$.

Subtracting yields: $\dfrac{x\cos\beta - y\cos\alpha}{a} = -\cos\omega t(\sin\alpha\cos\beta - \sin\beta\cos\alpha).$

(c) Squaring and adding the results of parts (a) and (b) yields:

$$(x\sin\beta - y\sin\alpha)^2 + (x\cos\beta - y\cos\alpha)^2 = a^2(\sin\alpha\cos\beta - \sin\beta\cos\alpha)^2$$

(d) Expanding the left-hand side, we have:

$$x^2(\sin^2\beta + \cos^2\beta) + y^2(\sin^2\alpha + \cos^2\alpha) - 2xy(\sin\alpha\sin\beta + \cos\alpha\cos\beta)$$
$$= x^2 + y^2 - 2xy(\sin\alpha\sin\beta + \cos\alpha\cos\beta) = x^2 + y^2 - 2xy\cos(\alpha - \beta).$$

The right-hand side can be rewritten: $a^2(\sin\alpha\cos\beta - \sin\beta\cos\alpha)^2 = a^2\sin^2(\alpha - \beta)$. Therefore, $x^2 + y^2 - 2xy\cos(\alpha - \beta) = a^2\sin^2(\alpha - \beta)$. Or, $x^2 + y^2 - 2xy\cos\delta = a^2\sin^2\delta$, where $\delta = \alpha - \beta$.

EVALUATE: **(e)** $\delta = 0: x^2 + y^2 - 2xy = (x - y)^2 = 0 \Rightarrow x = y$, which is a straight diagonal line

$\delta = \dfrac{\pi}{4}: x^2 + y^2 - \sqrt{2}xy = \dfrac{a^2}{2}$, which is an ellipse

$\delta = \dfrac{\pi}{2}: x^2 + y^2 = a^2$, which is a circle. This pattern repeats for the remaining phase differences.

33.66. **IDENTIFY:** Apply Snell's law to each refraction.

SET UP: Refer to the figure that accompanies the problem.

EXECUTE: **(a)** By the symmetry of the triangles, $\theta_b^A = \theta_a^B$, and $\theta_a^C = \theta_r^B = \theta_a^B = \theta_b^A$. Therefore, $\sin\theta_b^C = n\sin\theta_a^C = n\sin\theta_b^A = \sin\theta_a^A = \theta_b^C = \theta_a^A$.

(b) The total angular deflection of the ray is $\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_a^B + \theta_b^C - \theta_a^C = 2\theta_a^A - 4\theta_b^A + \pi$.

(c) From Snell's Law, $\sin\theta_a^A = n\sin\theta_b^A \Rightarrow \theta_b^A = \arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right)$.

$\Delta = 2\theta_a^A - 4\theta_b^A + \pi = 2\theta_a^A - 4\arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right) + \pi.$

(d) $\dfrac{d\Delta}{d\theta_a^A} = 0 = 2 - 4\dfrac{d}{d\theta_a^A}\left(\arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right)\right) \Rightarrow 0 = 2 - \dfrac{4}{\sqrt{1 - \dfrac{\sin^2\theta_1}{n^2}}}\cdot\left(\dfrac{\cos\theta_1}{n}\right)$. $4\left(1 - \dfrac{\sin^2\theta_1}{n^2}\right) = \left(\dfrac{16\cos^2\theta_1}{n^2}\right)$.

$4\cos^2\theta_1 = n^2 - 1 + \cos^2\theta_1$. $3\cos^2\theta_1 = n^2 - 1$. $\cos^2\theta_1 = \dfrac{1}{3}(n^2 - 1)$.

(e) For violet: $\theta_1 = \arccos\left(\sqrt{\dfrac{1}{3}(n^2 - 1)}\right) = \arccos\left(\sqrt{\dfrac{1}{3}(1.342^2 - 1)}\right) = 58.89°$.

$\Delta_{\text{violet}} = 139.2° \Rightarrow \theta_{\text{violet}} = 40.8°$.

For red: $\theta_1 = \arccos\left(\sqrt{\dfrac{1}{3}(n^2 - 1)}\right) = \arccos\left(\sqrt{\dfrac{1}{3}(1.330^2 - 1)}\right) = 59.58°$. $\Delta_{\text{red}} = 137.5° \Rightarrow \theta_{\text{red}} = 42.5°$.

EVALUATE: The angles we have calculated agree with the values given in Figure 37.20d in the textbook. θ_1 is larger for red than for violet, so red in the rainbow is higher above the horizon.

33.67. **IDENTIFY:** Follow similar steps to Challenge Problem 33.66.

SET UP: Refer to Figure 33.20e in the textbook.

EXECUTE: The total angular deflection of the ray is $\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_b^A + \pi - 2\theta_b^A + \theta_a^A - \theta_b^A = 2\theta_a^A - 6\theta_b^A + 2\pi$, where we have used the fact from the previous problem that all the internal angles are equal and the two external equals are equal. Also using the Snell's Law relationship, we have: $\theta_b^A = \arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right)$. $\Delta = 2\theta_a^A - 6\theta_b^A + 2\pi = 2\theta_a^A - 6\arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right) + 2\pi$.

(b) $\dfrac{d\Delta}{d\theta_a^A} = 0 = 2 - 6\dfrac{d}{d\theta_a^A}\left(\arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right)\right) \Rightarrow 0 = 2 - \dfrac{6}{\sqrt{1-\dfrac{\sin\theta_2}{n^2}}}\cdot\left(\dfrac{\cos\theta_2}{n}\right)$.

$n^2\left(1-\dfrac{\sin^2\theta_2}{n^2}\right) = (n^2 - 1 + \cos^2\theta_2) = 9\cos^2\theta_2$. $\cos^2\theta_2 = \dfrac{1}{8}(n^2-1)$.

(c) For violet, $\theta_2 = \arccos\left(\sqrt{\dfrac{1}{8}(n^2-1)}\right) = \arccos\left(\sqrt{\dfrac{1}{8}(1.342^2-1)}\right) = 71.55°$. $\Delta_{\text{violet}} = 233.2°$ and $\theta_{\text{violet}} = 53.2°$.

For red, $\theta_2 = \arccos\left(\sqrt{\dfrac{1}{8}(n^2-1)}\right) = \arccos\left(\sqrt{\dfrac{1}{8}(1.330^2-1)}\right) = 71.94°$. $\Delta_{\text{red}} = 230.1°$ and $\theta_{\text{red}} = 50.1°$.

EVALUATE: The angles we calculated agree with those given in Figure 37.20e in the textbook. The color that appears higher above the horizon is violet. The colors appear in reverse order in a secondary rainbow compared to a primary rainbow.

34

GEOMETRIC OPTICS

34.1. **IDENTIFY** and **SET UP:** Plane mirror: $s = -s'$ (Eq.34.1) and $m = y'/y = -s'/s = +1$ (Eq.34.2). We are given s and y and are asked to find s' and y'.

EXECUTE: The object and image are shown in Figure 34.1.

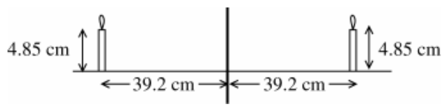


Figure 34.1

$$s' = -s = -39.2 \text{ cm}$$
$$|y'| = |m||y| = (+1)(4.85 \text{ cm})$$
$$|y'| = 4.85 \text{ cm}$$

The image is 39.2 cm to the right of the mirror and is 4.85 cm tall.

EVALUATE: For a plane mirror the image is always the same distance behind the mirror as the object is in front of the mirror. The image always has the same height as the object.

34.2. **IDENTIFY:** Similar triangles say $\dfrac{h_{\text{tree}}}{h_{\text{mirror}}} = \dfrac{d_{\text{tree}}}{d_{\text{mirror}}}$.

SET UP: $d_{\text{mirror}} = 0.350$ m, $h_{\text{mirror}} = 0.0400$ m and $d_{\text{tree}} = 28.0 \text{ m} + 0.350$ m.

EXECUTE: $h_{\text{tree}} = h_{\text{mirror}} \dfrac{d_{\text{tree}}}{d_{\text{mirror}}} = 0.040 \text{ m} \dfrac{28.0 \text{ m} + 0.350 \text{ m}}{0.350 \text{ m}} = 3.24$ m.

EVALUATE: The image of the tree formed by the mirror is 28.0 m behind the mirror and is 3.24 m tall.

34.3. **IDENTIFY:** Apply the law of reflection.

SET UP: If up is the $+y$-direction and right is the $+x$-direction, then the object is at $(-x_0, -y_0)$ and P_2' is at $(x_0, -y_0)$.

EXECUTE: Mirror 1 flips the y-values, so the image is at (x_0, y_0) which is P_3'.

EVALUATE: Mirror 2 uses P_1' as an object and forms an image at P_3'.

34.4. **IDENTIFY:** $f = R/2$

SET UP: For a concave mirror $R > 0$.

EXECUTE: **(a)** $f = \dfrac{R}{2} = \dfrac{34.0 \text{ cm}}{2} = 17.0$ cm

EVALUATE: **(b)** The image formation by the mirror is determined by the law of reflection and that is unaffected by the medium in which the light is traveling. The focal length remains 17.0 cm.

34.5. **IDENTIFY** and **SET UP:** Use Eq.(34.6) to calculate s' and use Eq.(34.7) to calculate y'. The image is real if s' is positive and is erect if $m > 0$. Concave means R and f are positive, $R = +22.0$ cm; $f = R/2 = +11.0$ cm.

EXECUTE: **(a)**

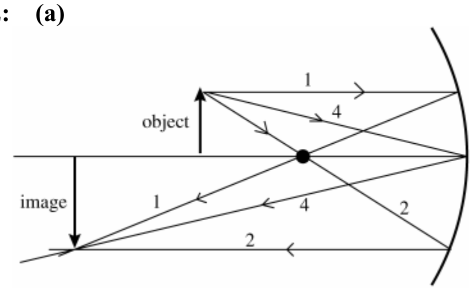


Figure 34.5

Three principal rays, numbered as in Sect. 34.2, are shown in Figure 34.5. The principal ray diagram shows that the image is real, inverted, and enlarged.

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$

$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{sf}$ so $s'=\frac{sf}{s-f}=\frac{(16.5\text{ cm})(11.0\text{ cm})}{16.5\text{ cm}-11.0\text{ cm}}=+33.0\text{ cm}$

$s'>0$ so real image, 33.0 cm to left of mirror vertex

$m=-\frac{s'}{s}=-\frac{33.0\text{ cm}}{16.5\text{ cm}}=-2.00$ ($m<0$ means inverted image) $|y'|=|m||y|=2.00(0.600\text{ cm})=1.20\text{ cm}$

EVALUATE: The image is 33.0 cm to the left of the mirror vertex. It is real, inverted, and is 1.20 cm tall (enlarged). The calculation agrees with the image characterization from the principal ray diagram. A concave mirror used alone always forms a real, inverted image if $s>f$ and the image is enlarged if $f<s<2f$.

34.6. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=-\frac{s'}{s}$.

SET UP: For a convex mirror, $R<0$. $R=-22.0\text{ cm}$ and $f=\frac{R}{2}=-11.0\text{ cm}$.

EXECUTE: **(a)** The principal-ray diagram is sketched in Figure 34.6.

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $s'=\frac{sf}{s-f}=\frac{(16.5\text{ cm})(-11.0\text{ cm})}{16.5\text{ cm}-(-11.0\text{ cm})}=-6.6\text{ cm}$. $m=-\frac{s'}{s}=-\frac{-6.6\text{ cm}}{16.5\text{ cm}}=+0.400$.

$|y'|=|m|y=(0.400)(0.600\text{ cm})=0.240\text{ cm}$. The image is 6.6 cm to the right of the mirror. It is 0.240 cm tall.

$s'<0$, so the image is virtual. $m>0$, so the image is erect.

EVALUATE: The calculated image properties agree with the image characterization from the principal-ray diagram.

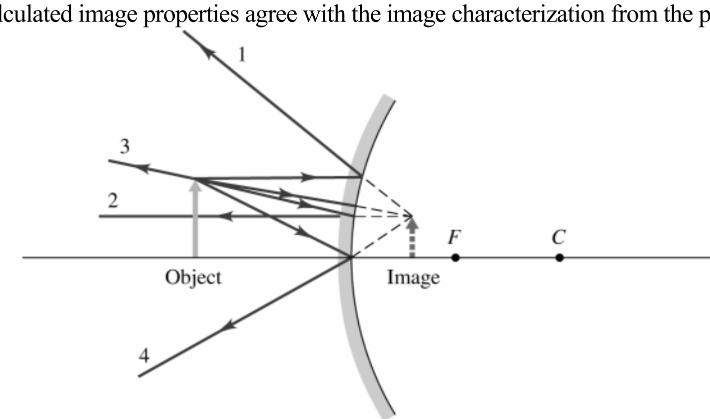


Figure 34.6

34.7. IDENTIFY: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $m=-\frac{s'}{s}$. $|m|=\frac{|y'|}{y}$. Find m and calculate y'.

SET UP: $f=+1.75\text{ m}$.

EXECUTE: $s\gg f$ so $s'=f=1.75\text{ m}$.

$m=-\frac{s'}{s}=-\frac{1.75\text{ m}}{5.58\times10^{10}\text{ m}}=-3.14\times10^{-11}$. $|y'|=|m|y=(3.14\times10^{-11})(6.794\times10^{6}\text{ m})=2.13\times10^{-4}\text{ m}=0.213\text{ mm}$.

EVALUATE: The image is real and is 1.75 m in front of the mirror.

34.8. IDENTIFY: Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=-\frac{s'}{s}$.

SET UP: The mirror surface is convex so $R=-3.00\text{ cm}$. $s=24.0\text{ cm}-3.00\text{ cm}=21.0\text{ cm}$.

EXECUTE: $f=\frac{R}{2}=-1.50\text{ cm}$. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $s'=\frac{sf}{s-f}=\frac{(21.0\text{ cm})(-1.50\text{ cm})}{21.0\text{ cm}-(-1.50\text{ cm})}=-1.40\text{ cm}$. The image is 1.40 cm behind the surface so it is $3.00\text{ cm}-1.40\text{ cm}=1.60\text{ cm}$ from the center of the ornament, on the same side as the object. $m=-\frac{s'}{s}=-\frac{-1.40\text{ cm}}{21.0\text{ cm}}=+0.0667$. $|y'|=|m|y=(0.0667)(3.80\text{ mm})=0.253\text{ mm}$.

EVALUATE: The image is virtual, upright and smaller than the object.

34.9. IDENTIFY: The shell behaves as a spherical mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and its magnification is given by $m=-\frac{s'}{s}$.

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s}=\frac{2}{-18.0\text{ cm}}-\frac{1}{-6.00\text{ cm}}\Rightarrow s=18.0\text{ cm}$ from the vertex.

$m=-\frac{s'}{s}=-\frac{-6.00\text{ cm}}{18.0\text{ cm}}=\frac{1}{3}\Rightarrow y'=\frac{1}{3}(1.5\text{ cm})=0.50\text{ cm}$. The image is 0.50 cm tall, erect, and virtual.

EVALUATE: Since the magnification is less than one, the image is smaller than the object.

34.10. **IDENTIFY:** The bottom surface of the bowl behaves as a spherical convex mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and its magnification is given by $m=-\frac{s'}{s}$.

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=\frac{-2}{35\text{ cm}}-\frac{1}{90\text{ cm}}\Rightarrow s'=-15\text{ cm}$ behind bowl.

$m=-\frac{s'}{s}=\frac{15\text{ cm}}{90\text{ cm}}=0.167\Rightarrow y'=(0.167)(2.0\text{cm})=0.33\text{ cm}$. The image is 0.33 cm tall, erect, and virtual.

EVALUATE: Since the magnification is less than one, the image is smaller than the object.

34.11. **IDENTIFY:** We are dealing with a spherical mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and its magnification is given by $m=-\frac{s'}{s}$.

EXECUTE: (a) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{fs}\Rightarrow s'=\frac{sf}{s-f}$. Also $m=-\frac{s'}{s}=\frac{f}{f-s}$.

(b) The graph is given in Figure 34.11a.

(c) $s'>0$ for $s>f$, $s<0$.

(d) $s'<0$ for $0<s<f$.

(e) The image is at negative infinity, "behind" the mirror.

(f) At the focal point, $s=f$.

(g) The image is at the mirror, $s'=0$.

(h) The graph is given in Figure 34.11b.

(i) Erect and larger if $0<s<f$.

(j) Inverted if $s>f$.

(k) The image is smaller if $s>2f$ or $s<0$.

(l) As the object is moved closer and closer to the focal point, the magnification increases to infinite values.

EVALUATE: As the object crosses the focal point, both the image distance and the magnification undergo discontinuities.

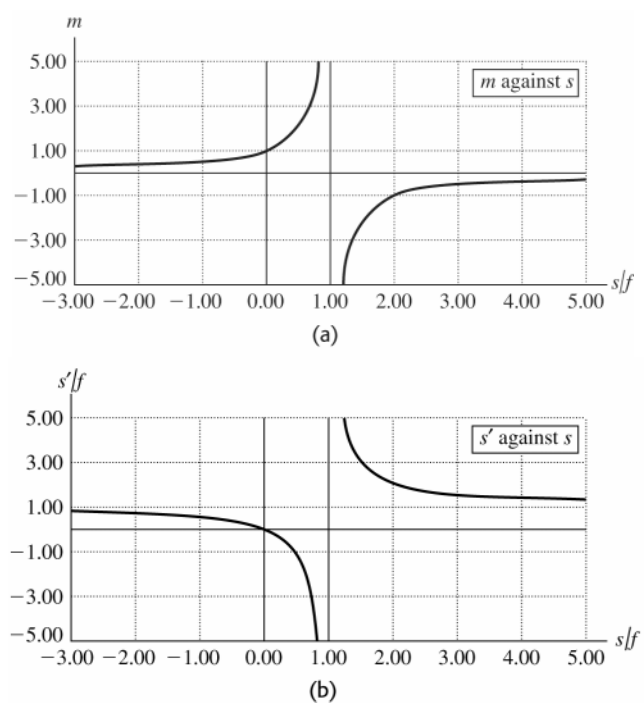


Figure 34.11

34.12. IDENTIFY: $s' = \dfrac{sf}{s-f}$ and $m = \dfrac{f}{f-s}$.

SET UP: With $f = -|f|$, $s' = -\dfrac{s|f|}{s+|f|}$ and $m = \dfrac{|f|}{s+|f|}$.

EXECUTE: The graphs are given in Figure 34.12.

(a) $s' > 0$ for $-|f| < s < 0$.

(b) $s' < 0$ for $s < -|f|$ and $s < 0$.

(c) If the object is at infinity, the image is at the outward going focal point.

(d) If the object is next to the mirror, then the image is also at the mirror

(e) The image is erect (magnification greater than zero) for $s > -|f|$.

(f) The image is inverted (magnification less than zero) for $s < -|f|$.

(g) The image is larger than the object (magnification greater than one) for $-2|f| < s < 0$.

(h) The image is smaller than the object (magnification less than one) for $s > 0$ and $s < -2|f|$.

EVALUATE: For a real image $(s > 0)$, the image formed by a convex mirror is always virtual and smaller than the object.

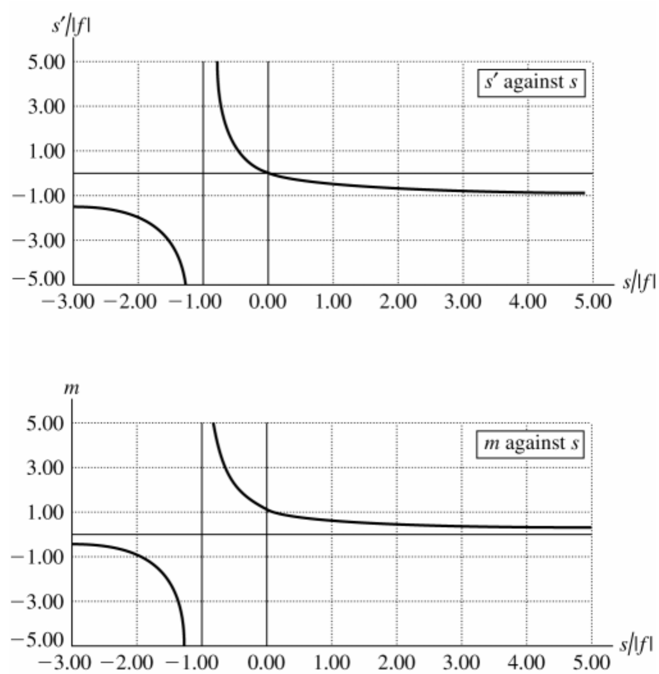


Figure 34.12

34.13. IDENTIFY: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$.

SET UP: $m = +2.00$ and $s = 1.25$ cm. An erect image must be virtual.

EXECUTE: (a) $s' = \dfrac{sf}{s-f}$ and $m = -\dfrac{f}{s-f}$. For a concave mirror, m can be larger than 1.00. For a convex mirror, $|f| = -f$ so $m = +\dfrac{|f|}{s+|f|}$ and m is always less than 1.00. The mirror must be concave ($f > 0$).

(b) $\dfrac{1}{f} = \dfrac{s'+s}{ss'}$. $f = \dfrac{ss'}{s+s'}$. $m = -\dfrac{s'}{s} = +2.00$ and $s' = -2.00s$. $f = \dfrac{s(-2.00s)}{s-2.00s} = +2.00s = +2.50\text{ cm}$.

$R = 2f = +5.00\text{ cm}$.

(c) The principal ray diagram is drawn in Figure 34.13.

EVALUATE: The principal-ray diagram agrees with the description from the equations.

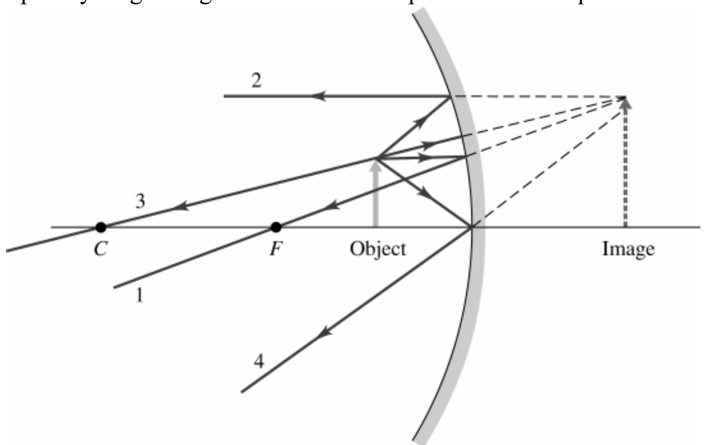


Figure 34.13

34.14. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=-\frac{s'}{s}$.

SET UP: For a concave mirror, $R>0$. $R=32.0\text{ cm}$ and $f=\frac{R}{2}=16.0\text{ cm}$.

EXECUTE: **(a)** $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. $s'=\frac{sf}{s-f}=\frac{(12.0\text{ cm})(16.0\text{ cm})}{12.0\text{ cm}-16.0\text{ cm}}=-48.0\text{ cm}$. $m=-\frac{s'}{s}=-\frac{-48.0\text{ cm}}{12.0\text{ cm}}=+4.00$.

(b) $s'=-48.0\text{ cm}$, so the image is 48.0 cm to the right of the mirror. $s'<0$ so the image is virtual.

(c) The principal-ray diagram is sketched in Figure 34.14. The rules for principal rays apply only to paraxial rays. Principal ray 2, that travels to the mirror along a line that passes through the focus, makes a large angle with the optic axis and is not described well by the paraxial approximation. Therefore, principal ray 2 is not included in the sketch.

EVALUATE: A concave mirror forms a virtual image whenever $s<f$.

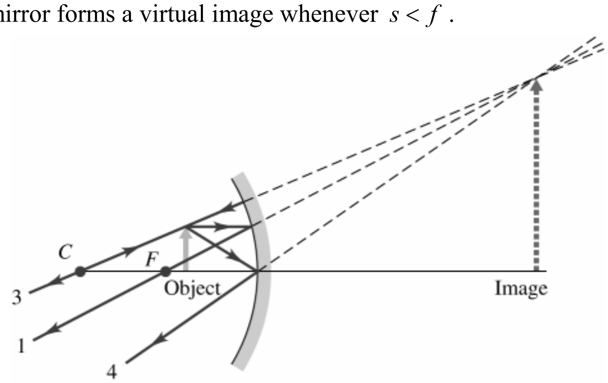


Figure 34.14

34.15. **IDENTIFY:** Apply Eq.(34.11), with $R\to\infty$. $|s'|$ is the apparent depth.

SET UP The image and object are shown in Figure 34.15.

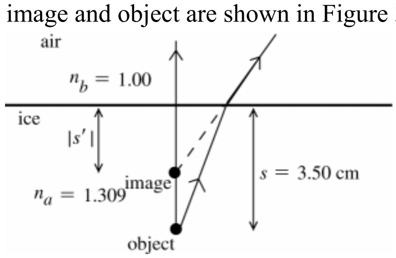


Figure 34.15

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R};$$

$R\to\infty$ (flat surface), so

$$\frac{n_a}{s}+\frac{n_b}{s'}=0$$

EXECUTE: $s'=-\frac{n_b s}{n_a}=-\frac{(1.00)(3.50\text{ cm})}{1.309}=-2.67\text{ cm}$

The apparent depth is 2.67 cm.

EVALUATE: When the light goes from ice to air (larger to smaller n), it is bent away from the normal and the virtual image is closer to the surface than the object is.

34.16. **IDENTIFY:** The surface is flat so $R \to \infty$ and $\frac{n_a}{s} + \frac{n_b}{s'} = 0$.

SET UP: The light travels from the fish to the eye, so $n_a = 1.333$ and $n_b = 1.00$. When the fish is viewed, $s = 7.0\text{ cm}$. The fish is $20.0\text{ cm} - 7.0\text{ cm} = 13.0\text{ cm}$ above the mirror, so the image of the fish is 13.0 cm below the mirror and $20.0\text{ cm} + 13.0\text{ cm} = 33.0\text{ cm}$ below the surface of the water. When the image is viewed, $s = 33.0\text{ cm}$.

EXECUTE: **(a)** $s' = -\left(\frac{n_b}{n_a}\right)s = -\left(\frac{1.00}{1.333}\right)(7.0\text{ cm}) = -5.25\text{ cm}$. The apparent depth is 5.25 cm.

(b) $s' = -\left(\frac{n_b}{n_a}\right)s = -\left(\frac{1.00}{1.333}\right)(33.0\text{ cm}) = -24.8\text{ cm}$. The apparent depth of the image of the fish in the mirror is 24.8 cm.

EVALUATE: In each case the apparent depth is less than the actual depth of what is being viewed.

34.17. **IDENTIFY:** $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$. $m = -\frac{n_a s'}{n_b s}$. Light comes from the fish to the person's eye.

SET UP: $R = -14.0\text{ cm}$. $s = +14.0\text{ cm}$. $n_a = 1.333$ (water). $n_b = 1.00$ (air). Figure 34.17 shows the object and the refracting surface.

EXECUTE: **(a)** $\frac{1.333}{14.0\text{ cm}} + \frac{1.00}{s'} = \frac{1.00 - 1.333}{-14.0\text{ cm}}$. $s' = -14.0\text{ cm}$. $m = -\frac{(1.333)(-14.0\text{ cm})}{(1.00)(14.0\text{ cm})} = +1.33$.

The fish's image is 14.0 cm to the left of the bowl surface so is at the center of the bowl and the magnification is 1.33.

(b) The focal point is at the image location when $s \to \infty$. $\frac{n_b}{s'} = \frac{n_b - n_a}{R}$. $n_a = 1.00$. $n_b = 1.333$. $R = +14.0\text{ cm}$.

$\frac{1.333}{s'} = \frac{1.333 - 1.00}{14.0\text{ cm}}$. $s' = +56.0\text{ cm}$. s' is greater than the diameter of the bowl, so the surface facing the sunlight does not focus the sunlight to a point inside the bowl. The focal point is outside the bowl and there is no danger to the fish.

EVALUATE: In part (b) the rays refract when they exit the bowl back into the air so the image we calculated is not the final image.

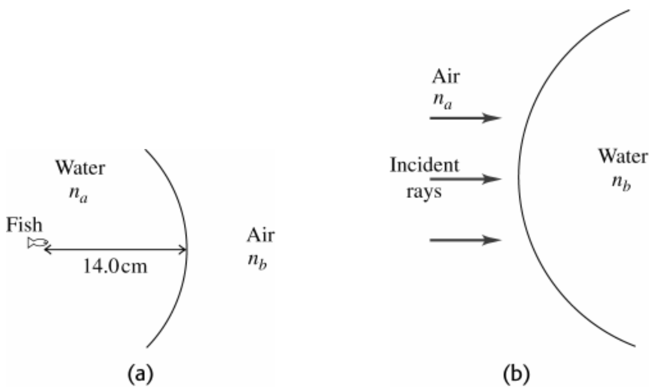


Figure 34.17

34.18. **IDENTIFY:** Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$.

SET UP: For a convex surface, $R > 0$. $R = +3.00\text{ cm}$. $n_a = 1.00$, $n_b = 1.60$.

EXECUTE: **(a)** $s \to \infty$. $\frac{n_b}{s'} = \frac{n_b - n_a}{R}$. $s' = \left(\frac{n_b}{n_b - n_a}\right)R = \left(\frac{1.60}{1.60 - 1.00}\right)(+3.00\text{ cm}) = +8.00\text{ cm}$. The image is 8.00 cm to the right of the vertex.

(b) $s = 12.0\text{ cm}$. $\frac{1.00}{12.0\text{ cm}} + \frac{1.60}{s'} = \frac{1.60 - 1.00}{3.00\text{ cm}}$. $s' = +13.7\text{ cm}$. The image is 13.7 cm to the right of the vertex.

(c) $s = 2.00\text{ cm}$. $\frac{1.00}{2.00\text{ cm}} + \frac{1.60}{s'} = \frac{1.60 - 1.00}{3.00\text{ cm}}$. $s' = -5.33\text{ cm}$. The image is 5.33 cm to the left of the vertex.

EVALUATE: The image can be either real ($s' > 0$) or virtual ($s' < 0$), depending on the distance of the object from the refracting surface.

34.19. **IDENTIFY:** The hemispherical glass surface forms an image by refraction. The location of this image depends on the curvature of the surface and the indices of refraction of the glass and oil.

SET UP: The image and object distances are related to the indices of refraction and the radius of curvature by the equation $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$.

EXECUTE: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}\Rightarrow\dfrac{1.45}{s}+\dfrac{1.60}{1.20\text{ m}}=\dfrac{0.15}{0.0300\text{ m}}\Rightarrow s=0.395\text{ cm}$

EVALUATE: The presence of the oil changes the location of the image.

34.20. **IDENTIFY:** $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}$. $m=-\dfrac{n_a s'}{n_b s}$.

SET UP: $R=+4.00$ cm. $n_a=1.00$. $n_b=1.60$. $s=24.0$ cm.

EXECUTE: $\dfrac{1}{24.0\text{ cm}}+\dfrac{1.60}{s'}=\dfrac{1.60-1.00}{4.00\text{ cm}}$. $s'=+14.8$ cm. $m=-\dfrac{(1.00)(14.8\text{ cm})}{(1.60)(24.0\text{ cm})}=-0.385$.

$|y'|=|m|y=(0.385)(1.50\text{ mm})=0.578\text{ mm}$. The image is 14.8 cm to the right of the vertex and is 0.578 mm tall. $m<0$, so the image is inverted.

EVALUATE: The image is real.

34.21. **IDENTIFY:** Apply Eqs.(34.11) and (34.12). Calculate s' and y'. The image is erect if $m>0$.

SET UP: The object and refracting surface are shown in Figure 34.21.

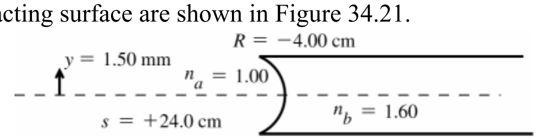


Figure 34.21

EXECUTE: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}$

$\dfrac{1.00}{24.0\text{ cm}}+\dfrac{1.60}{s'}=\dfrac{1.60-1.00}{-4.00\text{ cm}}$

Multiplying by 24.0 cm gives $1.00+\dfrac{38.4}{s'}=-3.60$

$\dfrac{38.4\text{ cm}}{s'}=-4.60$ and $s'=-\dfrac{38.4\text{ cm}}{4.60}=-8.35\text{ cm}$

Eq.(34.12): $m=-\dfrac{n_a s'}{n_b s}=-\dfrac{(1.00)(-8.35\text{ cm})}{(1.60)(+24.0\text{ cm})}=+0.217$

$|y'|=|m||y|=(0.217)(1.50\text{ mm})=0.326\text{ mm}$

EVALUATE: The image is virtual $(s'<0)$ and is 8.35 cm to the left of the vertex. The image is erect $(m>0)$ and is 0.326 mm tall. R is negative since the center of curvature of the surface is on the incoming side.

34.22. **IDENTIFY:** The hemispherical glass surface forms an image by refraction. The location of this image depends on the curvature of the surface and the indices of refraction of the glass and liquid.

SET UP: The image and object distances are related to the indices of refraction and the radius of curvature by the equation $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}$.

EXECUTE: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}\Rightarrow\dfrac{n_a}{14.0\text{ cm}}+\dfrac{1.60}{9.00\text{ cm}}=\dfrac{1.60-n_a}{4.00\text{ cm}}\Rightarrow n_a=1.24$.

EVALUATE: The result is a reasonable refractive index for liquids.

34.23. **IDENTIFY:** Use $\dfrac{1}{f}=(n-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$ to calculate *f*. The apply $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$ and $m=\dfrac{y'}{y}=-\dfrac{s'}{s}$.

SET UP: $R_1\to\infty$. $R_2=-13.0$ cm. If the lens is reversed, $R_1=+13.0$ cm and $R_2\to\infty$.

EXECUTE: **(a)** $\dfrac{1}{f}=(0.70)\left(\dfrac{1}{\infty}-\dfrac{1}{-13.0\text{ cm}}\right)=\dfrac{0.70}{13.0\text{ cm}}$ and $f=18.6$ cm. $\dfrac{1}{s'}=\dfrac{1}{f}-\dfrac{1}{s}=\dfrac{s-f}{sf}$.

$s'=\dfrac{sf}{s-f}=\dfrac{(22.5\text{ cm})(18.6\text{ cm})}{22.5\text{ cm}-18.6\text{ cm}}=107\text{ cm}$. $m=-\dfrac{s'}{s}=-\dfrac{107\text{ cm}}{22.5\text{ cm}}=-4.76$.

$y'=my=(-4.76)(3.75\text{ mm})=-17.8\text{ mm}$. The image is 107 cm to the right of the lens and is 17.8 mm tall. The image is real and inverted.

(b) $\dfrac{1}{f}=(n-1)\left(\dfrac{1}{13.0\text{ cm}}-\dfrac{1}{\infty}\right)$ and $f=18.6$ cm. The image is the same as in part (a).

EVALUATE: Reversing a lens does not change the focal length of the lens.

34.24. **IDENTIFY:** $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$. The sign of f determines whether the lens is converging or diverging.

SET UP: $s=16.0\text{ cm}$. $s'=-12.0\text{ cm}$.

EXECUTE: **(a)** $f=\frac{ss'}{s+s'}=\frac{(16.0\text{ cm})(-12.0\text{ cm})}{16.0\text{ cm}+(-12.0\text{ cm})}=-48.0\text{ cm}$. $f<0$ and the lens is diverging.

(b) $m=-\frac{s'}{s}=-\frac{-12.0\text{ cm}}{16.0\text{ cm}}=+0.750$. $|y'|=|m|y=(0.750)(8.50\text{ mm})=6.38\text{ mm}$. $m>0$ and the image is erect.

(c) The principal-ray diagram is sketched in Figure 34.24.

EVALUATE: A diverging lens always forms an image that is virtual, erect and reduced in size.

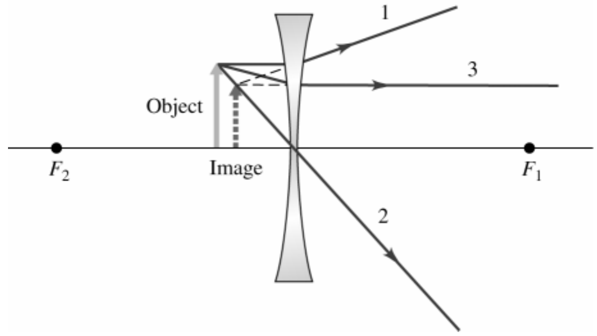


Figure 34.24

34.25. **IDENTIFY:** The liquid behaves like a lens, so the lensmaker's equation applies.

SET UP: The lensmaker's equation is $\frac{1}{s}+\frac{1}{s'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$, and the magnification of the lens is $m=-\frac{s'}{s}$.

EXECUTE: **(a)** $\frac{1}{s}+\frac{1}{s'}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\Rightarrow\frac{1}{24.0\text{ cm}}+\frac{1}{s'}=(1.52-1)\left(\frac{1}{-7.00\text{ cm}}-\frac{1}{-4.00\text{ cm}}\right)$

$$\Rightarrow s'=71.2\text{ cm}\text{, to the right of the lens.}$$

(b) $m=-\frac{s'}{s}=-\frac{71.2\text{ cm}}{24.0\text{ cm}}=-2.97$

EVALUATE: Since the magnification is negative, the image is inverted.

34.26. **IDENTIFY:** Apply $m=\frac{y'}{y}=-\frac{s'}{s}$ to relate s' and s and then use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$.

SET UP: Since the image is inverted, $y'<0$ and $m<0$.

EXECUTE: $m=\frac{y'}{y}=\frac{-4.50\text{ cm}}{3.20\text{ cm}}=-1.406$. $m=-\frac{s'}{s}$ gives $s'=+1.406s$. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $\frac{1}{s}+\frac{1}{1.406s}=\frac{1}{90.0\text{ cm}}$ and $s=154\text{ cm}$. $s'=(1.406)(154\text{ cm})=217\text{ cm}$. The object is 154 cm to the left of the lens. The image is 217 cm to the right of the lens and is real.

EVALUATE: For a single lens an inverted image is always real.

34.27. **IDENTIFY:** The thin-lens equation applies in this case.

SET UP: The thin-lens equation is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and the magnification is $m=-\frac{s'}{s}=\frac{y'}{y}$.

EXECUTE: $m=\frac{y'}{y}=\frac{34.0\text{ mm}}{8.00\text{ mm}}=4.25=-\frac{s'}{s}=-\frac{-12.0\text{ cm}}{s}\Rightarrow s=2.82\text{ cm}$. The thin-lens equation gives $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow f=3.69\text{ cm}$.

EVALUATE: Since the focal length is positive, this is a converging lens. The image distance is negative because the object is inside the focal point of the lens.

34.28. **IDENTIFY:** Apply $m=-\frac{s'}{s}$ to relate s and s'. Then use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$.

SET UP: Since the image is to the right of the lens, $s'>0$. $s'+s=6.00\text{ m}$.

EXECUTE: **(a)** $s'=80.0s$ and $s+s'=6.00\text{ m}$ gives $81.00s=6.00\text{ m}$ and $s=0.0741\text{ m}$. $s'=5.93\text{ m}$.

(b) The image is inverted since both the image and object are real $(s' > 0, s > 0)$.

(c) $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{0.0741\text{ m}} + \dfrac{1}{5.93\text{ m}} \Rightarrow f = 0.0732$ m, and the lens is converging.

EVALUATE: The object is close to the lens and the image is much farther from the lens. This is typical for slide projectors.

34.29. **IDENTIFY:** Apply $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$.

SET UP: For a distant object the image is at the focal point of the lens. Therefore, $f = 1.87$ cm. For the double-convex lens, $R_1 = +R$ and $R_2 = -R$, where $R = 2.50$ cm.

EXECUTE: $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R} - \dfrac{1}{-R}\right) = \dfrac{2(n-1)}{R}$. $n = \dfrac{R}{2f} + 1 = \dfrac{2.50\text{ cm}}{2(1.87\text{ cm})} + 1 = 1.67$.

EVALUATE: $f > 0$ and the lens is converging. A double-convex lens is always converging.

34.30. **IDENTIFY** and **SET UP:** Apply $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

EXECUTE: We have a converging lens if the focal length is positive, which requires

$\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) > 0 \Rightarrow \left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) > 0$. This can occur in one of three ways:

(i) R_1 and R_2 both positive and $R_1 < R_2$. (ii) $R_1 \geq 0, R_2 \leq 0$ (double convex and planoconvex).

(iii) R_1 and R_2 both negative and $|R_1| > |R_2|$ (meniscus). The three lenses in Figure 35.32a in the textbook fall into these categories.

We have a diverging lens if the focal length is negative, which requires

$\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) < 0 \Rightarrow \left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right) < 0$. This can occur in one of three ways:

(i) R_1 and R_2 both positive and $R_1 > R_2$ (meniscus). (ii) R_1 and R_2 both negative and $|R_2| > |R_1|$. (iii) $R_1 \leq 0, R_2 \geq 0$ (planoconcave and double concave). The three lenses in Figure 34.32b in the textbook fall into these categories.

EVALUATE: The converging lenses in Figure 34.32a are all thicker at the center than at the edges. The diverging lenses in Figure 34.32b are all thinner at the center than at the edges.

34.31. **IDENTIFY** and **SET UP:** The equations $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and $m = -\dfrac{s'}{s}$ apply to both thin lenses and spherical mirrors.

EXECUTE: **(a)** The derivation of the equations in Exercise 34.11 is identical and one gets:

$\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s'} = \dfrac{1}{f} - \dfrac{1}{s} = \dfrac{s-f}{fs} \Rightarrow s' = \dfrac{sf}{s-f}$, and also $m = -\dfrac{s'}{s} = \dfrac{f}{f-s}$.

(b) Again, one gets exactly the same equations for a converging lens rather than a concave mirror because the equations are identical. The difference lies in the interpretation of the results. For a lens, the outgoing side is *not* that on which the object lies, unlike for a mirror. So for an object on the left side of the lens, a positive image distance means that the image is on the right of the lens, and a negative image distance means that the image is on the left side of the lens.

(c) Again, for Exercise 34.12, the change from a convex mirror to a diverging lens changes nothing in the exercises, except for the interpretation of the location of the images, as explained in part (b) above.

EVALUATE: Concave mirrors and converging lenses both have $f > 0$. Convex mirrors and diverging lenses both have $f < 0$.

34.32. **IDENTIFY:** Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$.

SET UP: $f = +12.0$ cm and $s' = -17.0$ cm.

EXECUTE: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s} = \dfrac{1}{12.0\text{ cm}} - \dfrac{1}{-17.0\text{ cm}} \Rightarrow s = 7.0$ cm.

$m = -\dfrac{s'}{s} = -\dfrac{(-17.0)}{7.2} = +2.4 \Rightarrow y = \dfrac{y'}{m} = \dfrac{0.800\text{ cm}}{+2.4} = +0.34$ cm, so the object is 0.34 cm tall, erect, same side as the image. The principal-ray diagram is sketched in Figure 34.32.

EVALUATE: When the object is inside the focal point, a converging lens forms a virtual, enlarged image.

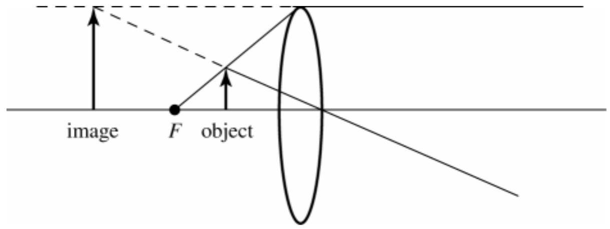


Figure 34.32

34.33. **IDENTIFY:** Use Eq.(34.16) to calculate the object distance s. m calculated from Eq.(34.17) determines the size and orientation of the image.

SET UP: $f = -48.0$ cm. Virtual image 17.0 cm from lens so $s' = -17.0$ cm.

EXECUTE: $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$, so $\dfrac{1}{s'}=\dfrac{1}{f}-\dfrac{1}{s}=\dfrac{s-f}{sf}$

$$s=\frac{s'f}{s'-f}=\frac{(-17.0\text{ cm})(-48.0\text{ cm})}{-17.0\text{ cm}-(-48.0\text{ cm})}=+26.3\text{ cm}$$

$$m=-\frac{s'}{s}=-\frac{-17.0\text{ cm}}{+26.3\text{ cm}}=+0.646$$

$$m=\frac{y'}{y}\text{ so }|y|=\frac{|y'|}{|m|}=\frac{8.00\text{ mm}}{0.646}=12.4\text{ mm}$$

The principal-ray diagram is sketched in Figure 34.33.

EVALUATE: Virtual image, real object ($s > 0$) so image and object are on same side of lens. $m > 0$ so image is erect with respect to the object. The height of the object is 12.4 mm.

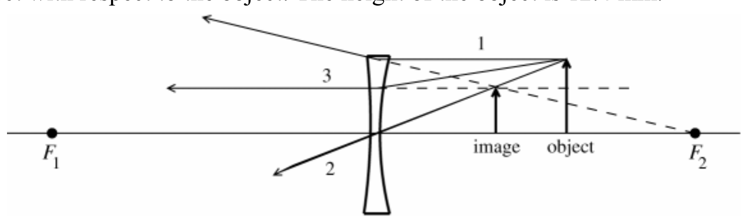


Figure 34.33

34.34. **IDENTIFY:** Apply $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$.

SET UP: The sign of f determines whether the lens is converging or diverging. $s = 16.0$ cm. $s' = +36.0$ cm. Use $m=-\dfrac{s'}{s}$ to find the size and orientation of the image.

EXECUTE: **(a)** $f=\dfrac{ss'}{s+s'}=\dfrac{(16.0\text{ cm})(36.0\text{ cm})}{16.0\text{ cm}+36.0\text{ cm}}=11.1\text{ cm}$. $f > 0$ and the lens is converging.

(b) $m=-\dfrac{s'}{s}=-\dfrac{36.0\text{ cm}}{16.0\text{ cm}}=-2.25$. $|y'|=|m|y=(2.25)(8.00\text{ mm})=18.0\text{ mm}$. $m < 0$ so the image is inverted.

(c) The principal-ray diagram is sketched in Figure 34.34.

EVALUATE: The image is real so the lens must be converging.

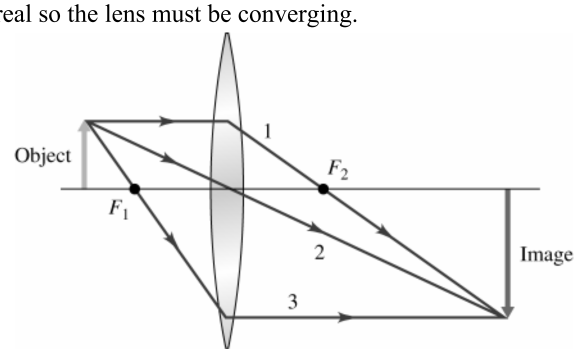


Figure 34.34

34.35. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$.

SET UP: The image is to be formed on the film, so $s'=+20.4\text{ cm}$.

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s}+\frac{1}{20.4\text{ cm}}=\frac{1}{20.0\text{ cm}}\Rightarrow s=1020\text{ cm}=10.2\text{ m}.$

EVALUATE: The object distance is much greater than *f*, so the image is just outside the focal point of the lens.

34.36. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $m=\frac{y'}{y}=-\frac{s'}{s}$.

SET UP: $s=3.90\text{ m}$. $f=0.085\text{ m}$.

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{3.90\text{ m}}+\frac{1}{s'}=\frac{1}{0.085\text{ m}}\Rightarrow s'=0.0869\text{ m}.$ $y'=-\frac{s'}{s}y=-\frac{0.0869}{3.90}1750\text{ mm}=-39.0\text{ mm}$, so it will not fit on the $24\text{-mm}\times 36\text{-mm}$ film.

EVALUATE: The image is just outside the focal point and $s'\approx f$. To have $y'=36\text{ mm}$, so that the image will fit on the film, $s=-\frac{s'y}{y'}\approx-\frac{(0.085\text{ m})(1.75\text{ m})}{-0.036\text{ m}}=4.1\text{ m}$. The person would need to stand about 4.1 m from the lens.

34.37. **IDENTIFY:** $|m|=\left|\frac{s'}{s}\right|$.

SET UP: $s\gg f$, so $s'\approx f$.

EXECUTE: **(a)** $|m|=\frac{s'}{s}\approx\frac{f}{s}\Rightarrow|m|=\frac{28\text{ mm}}{200{,}000\text{ mm}}=1.4\times10^{-4}.$

(b) $|m|=\frac{s'}{s}\approx\frac{f}{s}\Rightarrow|m|=\frac{105\text{ mm}}{200{,}000\text{ mm}}=5.3\times10^{-4}.$

(c) $|m|=\frac{s'}{s}\approx\frac{f}{s}\Rightarrow|m|=\frac{300\text{ mm}}{200{,}000\text{ mm}}=1.5\times10^{-3}.$

EVALUATE: The magnitude of the magnification increases when *f* increases.

34.38. **IDENTIFY:** $|m|=\left|\frac{s'}{s}\right|=\frac{|y'|}{y}$

SET UP: $s\gg f$, so $s'\approx f$.

EXECUTE: $|y'|=\frac{s'}{s}y\approx\frac{f}{s}y=\frac{5.00\text{ m}}{9.50\times10^{3}\text{ m}}(70.7\text{ m})=0.0372\text{ m}=37.2\text{ mm}.$

EVALUATE: A very long focal length lens is needed to photograph a distant object.

34.39. **IDENTIFY** and **SET UP:** Find the lateral magnification that results in this desired image size. Use Eq.(34.17) to relate *m* and s' and Eq.(34.16) to relate *s* and s' to *f*.

EXECUTE: **(a)** We need $m=-\frac{24\times10^{-3}\text{ m}}{160\text{ m}}=-1.5\times10^{-4}$. Alternatively, $m=-\frac{36\times10^{-3}\text{ m}}{240\text{ m}}=-1.5\times10^{-4}$.

$s\gg f$ so $s'\approx f$

Then $m=-\frac{s'}{s}=-\frac{f}{s}=-1.5\times10^{-4}$ and $f=(1.5\times10^{-4})(600\text{ m})=0.090\text{ m}=90\text{ mm}.$

A smaller *f* means a smaller s' and a smaller *m*, so with *f* = 85 mm the object's image nearly fills the picture area.

(b) We need $m=-\frac{36\times10^{-3}\text{ m}}{9.6\text{ m}}=-3.75\times10^{-3}$. Then, as in part (a), $\frac{f}{s}=3.75\times10^{-3}$ and

$f=(40.0\text{ m})(3.75\times10^{-3})=0.15\text{ m}=150\text{ mm}.$ Therefore use the 135 mm lens.

EVALUATE: When $s\gg f$ and $s'\approx f$, $y'=-f(y/s)$. For the mobile home *y*/*s* is smaller so a larger *f* is needed. Note that *m* is very small; the image is much smaller than the object.

34.40. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to each lens. The image of the first lens serves as the object for the second lens.

SET UP: For a distant object, $s\to\infty$

EXECUTE: **(a)** $s_1=\infty\Rightarrow s_1'=f_1=12\text{ cm}.$

(b) $s_2=4.0\text{ cm}-12\text{ cm}=-8\text{ cm}.$

(c) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{-8\text{ cm}}+\frac{1}{s_2'}=\frac{1}{-12\text{ cm}}\Rightarrow s_2'=24\text{ cm}$, to the right.

(d) $s_1=\infty\Rightarrow s_1'=f_1=12\text{ cm}.\ \ s_2=8.0\text{ cm}-12\text{ cm}=-4\text{ cm}.\ \ \frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{-4\text{ cm}}+\frac{1}{s_2'}=\frac{1}{-12\text{ cm}}\Rightarrow s_2'=6\text{ cm}.$

EVALUATE: In each case the image of the first lens serves as a virtual object for the second lens, and $s_2<0$.

34.41. **IDENTIFY:** The *f*-number of a lens is the ratio of its focal length to its diameter. To maintain the same exposure, the amount of light passing through the lens during the exposure must remain the same.
SET UP: The *f*-number is *f/D*.

EXECUTE: **(a)** $f\text{-number}=\frac{f}{D}\Rightarrow f\text{-number}=\frac{180.0\text{ mm}}{16.36\text{ mm}}\Rightarrow f\text{-number}=f/11$. (The *f*-number is an integer.)

(b) *f*/11 to *f*/2.8 is four steps of 2 in intensity, so one needs 1/16th the exposure. The exposure should be 1/480 s = 2.1×10^{-3} s = 2.1 ms.

EVALUATE: When opening the lens from *f*/11 to *f*/2.8, the area increases by a factor of 16, so 16 times as much light is allowed in. Therefore the exposure time must be decreased by a factor of 1/16 to maintain the same exposure on the film or light receptors of a digital camera.

34.42. **IDENTIFY** and **SET UP:** The square of the aperture diameter is proportional to the length of the exposure time required.

EXECUTE: $\left(\frac{1}{30}\text{ s}\right)\left(\frac{8\text{ mm}}{23.1\text{ mm}}\right)^2\approx\left(\frac{1}{250}\text{ s}\right)$

EVALUATE: An increase in the aperture diameter decreases the exposure time.

34.43. **IDENTIFY** and **SET UP:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to calculate s'.

EXECUTE: **(a)** A real image is formed at the film, so the lens must be convex.

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ so $\frac{1}{s'}=\frac{s-f}{sf}$ and $s'=\frac{sf}{s-f}$, with $f=+50.0.0\text{ mm}$. For $s=45\text{ cm}=450\text{ mm}, s'=56\text{ mm}$. For $s=\infty, s'=f=50\text{ mm}$. The range of distances between the lens and film is 50 mm to 56 mm.

EVALUATE: The lens is closer to the film when photographing more distant objects.

34.44. **IDENTIFY:** The projector lens can be modeled as a thin lens.

SET UP: The thin-lens equation is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, and the magnification of the lens is $m=-\frac{s'}{s}$.

EXECUTE: **(a)** $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{f}=\frac{1}{0.150\text{ m}}+\frac{1}{9.00\text{ m}}\Rightarrow f=147.5\text{ mm}$, so use a *f* = 148 mm lens.

(b) $m=-\frac{s'}{s}\Rightarrow|m|=60\Rightarrow$ Area $=1.44\text{ m}\times2.16\text{ m}$.

EVALUATE: The lens must produce a real image to be viewed on the screen. Since the magnification comes out negative, the slides to be viewed must be placed upside down in the tray.

34.45. **(a) IDENTIFY:** The purpose of the corrective lens is to take an object 25 cm from the eye and form a virtual image at the eye's near point. Use Eq.(34.16) to solve for the image distance when the object distance is 25 cm.

SET UP: $\frac{1}{f}=+2.75$ diopters means $f=+\frac{1}{2.75}\text{ m}=+0.3636\text{ m}$ (converging lens)

$f=36.36\text{ cm}; s=25\text{ cm}; s'=?$

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ so

$$s'=\frac{sf}{s-f}=\frac{(25\text{ cm})(36.36\text{ cm})}{25\text{ cm}-36.36\text{ cm}}=-80.0\text{ cm}$$

The eye's near point is 80.0 cm from the eye.

(b) IDENTIFY: The purpose of the corrective lens is to take an object at infinity and form a virtual image of it at the eye's far point. Use Eq.(34.16) to solve for the image distance when the object is at infinity.

SET UP: $\frac{1}{f}=-1.30$ diopters means $f=-\frac{1}{1.30}\text{ m}=-0.7692\text{ m}$ (diverging lens)

$f=-76.92\text{ cm}; s=\infty; s'=?$

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and $s=\infty$ says $\frac{1}{s'}=\frac{1}{f}$ and $s'=f=-76.9\text{ cm}$. The eye's far point is 76.9 cm from the eye.

EVALUATE: In each case a virtual image is formed by the lens. The eye views this virtual image instead of the object. The object is at a distance where the eye can't focus on it, but the virtual image is at a distance where the eye can focus.

34.46. **IDENTIFY:** $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$

SET UP: $n_a=1.00$, $n_b=1.40$. $s=40.0\text{ cm}$, $s'=2.60\text{ cm}$.

EXECUTE: $\frac{1}{40.0\text{ cm}}+\frac{1.40}{2.60\text{ cm}}=\frac{0.40}{R}$ and $R=0.710\text{ cm}$.

EVALUATE: The cornea presents a convex surface to the object, so $R>0$.

34.47. **IDENTIFY:** In each case the lens forms a virtual image at a distance where the eye can focus. Power in diopters equals $1/f$, where f is in meters.

SET UP: In part (a), $s=25$ cm and in part (b), $s\to\infty$.

EXECUTE: **(a)** $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{0.25\text{ m}}+\frac{1}{-0.600\text{ m}}\Rightarrow\text{power}=\frac{1}{f}=+2.33$ diopters.

(b) $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{\infty}+\frac{1}{-0.600\text{ m}}\Rightarrow\text{power}=\frac{1}{f}=-1.67$ diopters.

EVALUATE: A converging lens corrects the near vision and a diverging lens corrects the far vision.

34.48. **IDENTIFY:** When the object is at the focal point, $M=\frac{25.0\text{ cm}}{f}$. In part (b), apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to calculate s for $s'=-25.0\text{ cm}$.

SET UP: Our calculation assumes the near point is 25.0 cm from the eye.

EXECUTE: **(a)** Angular magnification $M=\frac{25.0\text{ cm}}{f}=\frac{25.0\text{ cm}}{6.00\text{ cm}}=4.17.$

(b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s}+\frac{1}{-25.0\text{ cm}}=\frac{1}{6.00\text{ cm}}\Rightarrow s=4.84\text{ cm}.$

EVALUATE: In part (b), $\theta'=\frac{y}{s}$, $\theta=\frac{y}{25.0\text{ cm}}$ and $M=\frac{25.0\text{ cm}}{s}=\frac{25.0\text{ cm}}{4.84\text{ cm}}=5.17$. M is greater when the image is at the near point than when the image is at infinity.

34.49. **IDENTIFY:** Use Eqs.(34.16) and (34.17) to calculate s and y'.

(a) SET UP: $f=8.00$ cm; $s'=-25.0$ cm; $s=?$

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, so $\frac{1}{s}=\frac{1}{f}-\frac{1}{s'}=\frac{s'-f}{s'f}$

EXECUTE: $s=\frac{s'f}{s'-f}=\frac{(-25.0\text{ cm})(+8.00\text{ cm})}{-25.0\text{ cm}-8.00\text{ cm}}=+6.06\text{ cm}$

(b) $m=-\frac{s'}{s}=-\frac{-25.0\text{ cm}}{6.06\text{ cm}}=+4.125$

$|m|=\frac{|y'|}{|y|}$ so $|y'|=|m||y|=(4.125)(1.00\text{ mm})=4.12\text{ mm}$

EVALUATE: The lens allows the object to be much closer to the eye than the near point. The lense allows the eye to view an image at the near point rather than the object.

34.50. **IDENTIFY:** For a thin lens, $-\frac{s'}{s}=\frac{y'}{y}$, so $\left|\frac{y'}{s'}\right|=\left|\frac{y}{s}\right|$, and the angular size of the image equals the angular size of the object.

SET UP: The object has angular size $\theta=\frac{y}{f}$, with θ in radians.

EXECUTE: $\theta=\frac{y}{f}\Rightarrow f=\frac{y}{\theta}=\frac{2.00\text{ mm}}{0.025\text{ rad}}=80.0\text{ mm}=8.00\text{ cm}.$

EVALUATE: If the insect is at the near point of a normal eye, its angular size is $\frac{2.00\text{ mm}}{250\text{ mm}}=0.0080\text{ rad}$.

34.51. **IDENTIFY:** The thin-lens equation applies to the magnifying lens.

SET UP: The thin-lens equation is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$.

EXECUTE: The image is behind the lens, so $s' < 0$. The thin-lens equation gives

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s}=\frac{1}{5.00\text{ cm}}-\frac{1}{-25.0\text{ cm}}\Rightarrow s=4.17\text{ cm, on the same side of the lens as the ant.}$$

EVALUATE: Since $s' < 0$, the image will be erect.

34.52. **IDENTIFY:** Apply Eq.(34.24).

SET UP: $s_1' = 160\text{ mm} + 5.0\text{ mm} = 165\text{ mm}$

EXECUTE: **(a)** $M = \dfrac{(250\text{ mm})s_1'}{f_1 f_2} = \dfrac{(250\text{ mm})(165\text{ mm})}{(5.00\text{ mm})(26.0\text{ mm})} = 317.$

(b) The minimum separation is $\dfrac{0.10\text{ mm}}{M} = \dfrac{0.10\text{ mm}}{317} = 3.15\times10^{-4}\text{ mm}.$

EVALUATE: The angular size of the image viewed by the eye when looking through the microscope is 317 times larger than if the object is viewed at the near-point of the unaided eye.

34.53. **(a) IDENTIFY** and **SET UP:**

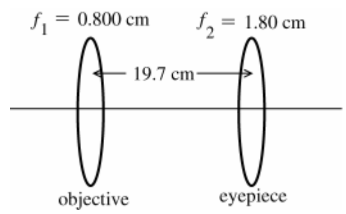


Figure 34.53

Final image is at ∞ so the object for the eyepiece is at its focal point. But the object for the eyepiece is the image of the objective so the image formed by the objective is 19.7 cm – 1.80 cm = 17.9 cm to the right of the lens. Apply Eq.(34.16) to the image formation by the objective, solve for the object distance *s*.

$f = 0.800$ cm; $s' = 17.9$ cm; $s = ?$

$\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}$, so $\dfrac{1}{s}=\dfrac{1}{f}-\dfrac{1}{s'}=\dfrac{s'-f}{s'f}$

EXECUTE: $s = \dfrac{s'f}{s'-f} = \dfrac{(17.9\text{ cm})(+0.800\text{ cm})}{17.9\text{ cm}-0.800\text{ cm}} = +8.37\text{ mm}$

(b) SET UP: Use Eq.(34.17).

EXECUTE: $m_1 = -\dfrac{s'}{s} = -\dfrac{17.9\text{ cm}}{0.837\text{ cm}} = -21.4$

The linear magnification of the objective is 21.4.

(c) SET UP: Use Eq.(34.23): $M = m_1 M_2$

EXECUTE: $M_2 = \dfrac{25\text{ cm}}{f_2} = \dfrac{25\text{ cm}}{1.80\text{ cm}} = 13.9$

$M = m_1 M_2 = (-21.4)(13.9) = -297$

EVALUATE: M is not accurately given by $(25\text{ cm})s_1'/f_1 f_2 = 311$, because the object is not quite at the focal point of the objective ($s_1 = 0.837$ cm and $f_1 = 0.800$ cm).

34.54. **IDENTIFY:** Eq.(34.24) can be written $M = |m_1| M_2 = \left|\dfrac{s_1'}{f_1}\right| M_2.$

SET UP: $s_1' = f_1 + 120\text{ mm}$

EXECUTE: $f = 16\text{ mm}: s' = 120\text{ mm} + 16\text{ mm} = 136\text{ mm}; s = 16\text{ mm}.$ $|m_1| = \dfrac{s'}{s} = \dfrac{136\text{ mm}}{16\text{ mm}} = 8.5.$

$f = 4\text{ mm}: s' = 120\text{ mm} + 4\text{ mm} = 124\text{ mm}; s = 4\text{ mm} \Rightarrow |m_1| = \dfrac{s'}{s} = \dfrac{124\text{ mm}}{4\text{ mm}} = 31.$

$f = 1.9\text{ mm}: s' = 120\text{ mm} + 1.9\text{ mm} = 122\text{ mm}; s = 1.9\text{ mm} \Rightarrow |m_1| = \dfrac{s'}{s} = \dfrac{122\text{ mm}}{1.9\text{ mm}} = 64.$

The eyepiece magnifies by either 5 or 10, so:

(a) The maximum magnification occurs for the 1.9-mm objective and 10x eyepiece:

$M = |m_1| M_e = (64)(10) = 640.$

(b) The minimum magnification occurs for the 16-mm objective and 5x eyepiece:

$M = |m_1| M_e = (8.5)(5) = 43.$

EVALUATE: The smaller the focal length of the objective, the greater the overall magnification.

34.55. **IDENTIFY:** f-number $= f/D$

SET UP: $D = 1.02$ m

EXECUTE: $\frac{f}{D} = 19.0 \Rightarrow f = (19.0)D = (19.0)(1.02\text{ m}) = 19.4\text{ m}.$

EVALUATE: Camera lenses can also have an *f*-number of 19.0. For a camera lens, both the focal length and lens diameter are much smaller, but the *f*-number is a measure of their ratio.

34.56. **IDENTIFY:** For a telescope, $M = -\frac{f_1}{f_2}$.

SET UP: $f_2 = 9.0$ cm. The distance between the two lenses equals $f_1 + f_2$.

EXECUTE: $f_1 + f_2 = 1.80\text{ m} \Rightarrow f_1 = 1.80\text{ m} - 0.0900\text{ m} = 1.71\text{ m}$. $M = -\frac{f_1}{f_2} = -\frac{171}{9.00} = -19.0.$

EVALUATE: For a telescope, $f_1 \gg f_2$.

34.57. **(a) IDENTIFY** and **SET UP:** Use Eq.(34.24), with $f_1 = 95.0$ cm (objective) and $f_2 = 15.0$ cm (eyepiece).

EXECUTE: $M = -\frac{f_1}{f_2} = -\frac{95.0\text{ cm}}{15.0\text{ cm}} = -6.33$

(b) IDENTIFY and **SET UP:** Use Eq.(34.17) to calculate y'.

SET UP: $s = 3.00 \times 10^3$ m

$s' = f_1 = 95.0$ cm (since s is very large, $s' \approx f$)

EXECUTE: $m = -\frac{s'}{s} = -\frac{0.950\text{ m}}{3.00 \times 10^3\text{ m}} = -3.167 \times 10^{-4}$

$|y'| = |m||y| = (3.167 \times 10^{-4})(60.0\text{ m}) = 0.0190\text{ m} = 1.90\text{ cm}$

(c) IDENTIFY: Use Eq.(34.21) and the angular magnification *M* obtained in part (a) to calculate θ'. The angular size θ of the image formed by the objective (object for the eyepiece) is its height divided by its distance from the objective.

EXECUTE: The angular size of the object for the eyepiece is $\theta = \frac{0.0190\text{ m}}{0.950\text{ m}} = 0.0200$ rad.

(Note that this is also the angular size of the object for the objective: $\theta = \frac{60.0\text{ m}}{3.00 \times 10^3\text{ m}} = 0.0200$ rad. For a thin lens the object and image have the same angular size and the image of the objective is the object for the eyepiece.)

$M = \frac{\theta'}{\theta}$ (Eq.34.21) so the angular size of the image is $\theta' = M\theta = -(6.33)(0.0200\text{ rad}) = -0.127$ rad (The minus sign shows that the final image is inverted.)

EVALUATE: The lateral magnification of the objective is small; the image it forms is much smaller than the object. But the total angular magnification is larger than 1.00; the angular size of the final image viewed by the eye is 6.33 times larger than the angular size of the original object, as viewed by the unaided eye.

34.58. **IDENTIFY:** The angle subtended by Saturn with the naked eye is the same as the angle subtended by the image of Saturn formed by the objective lens (see Fig. 34.53 in the textbook).

SET UP: The angle subtended by Saturn is $\theta = \frac{\text{diameter of Saturn}}{\text{distance to Saturn}} = \frac{y'}{f_1}$.

EXECUTE: Putting in the numbers gives $\theta = \frac{y'}{f_1} = \frac{1.7\text{ mm}}{18\text{ m}} = \frac{0.0017\text{ m}}{18\text{ m}} = 9.4 \times 10^{-5}\text{ rad} = 0.0054°$

EVALUATE: The angle subtended by the final image, formed by the eyepiece, would be much larger than 0.0054°.

34.59. **IDENTIFY:** $f = R/2$ and $M = -\frac{f_1}{f_2}$.

SET UP: For object and image both at infinity, $f_1 + f_2$ equals the distance *d* between the two mirrors.

$f_2 = 1.10$ cm. $R_1 = 1.30$ m.

EXECUTE: **(a)** $f_1 = \frac{R_1}{2} = 0.650\text{ m} \Rightarrow d = f_1 + f_2 = 0.661\text{ m}.$

(b) $|M| = \frac{f_1}{f_2} = \frac{0.650\text{ m}}{0.011\text{ m}} = 59.1.$

EVALUATE: For a telescope, $f_1 \gg f_2$.

34.60. **IDENTIFY:** The primary mirror forms an image which then acts as the object for the secondary mirror.
SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$.

EXECUTE: For the first image (formed by the primary mirror):

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=\frac{1}{2.5\text{ m}}-\frac{1}{\infty}\Rightarrow s'=2.5\text{ m}.$$

For the second image (formed by the secondary mirror), the distance between the two vertices is x. Assuming that the image formed by the primary mirror is to the right of the secondary mirror, the object distance is $s = x - 2.5$ m and the image distance is $s' = x + 0.15$ m. Therefore we have

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{x-2.5\text{ m}}+\frac{1}{x+0.15\text{ m}}=\frac{1}{-1.5\text{ m}}$$

The positive root of the quadratic equation gives $x = 1.7$ m, which is the distance between the vertices.
EVALUATE: Some light is blocked by the secondary mirror, but usually not enough to make much difference.

34.61. **IDENTIFY** and **SET UP:** For a plane mirror $s'=-s$. $v=\frac{ds}{dt}$ and $v'=\frac{ds'}{dt}$, so $v'=-v$.

EXECUTE: The velocities of the object and image relative to the mirror are equal in magnitude and opposite in direction. Thus both you and your image are receding from the mirror surface at 2.40 m/s, in opposite directions. Your image is therefore moving at 4.80 m/s relative to you.
EVALUATE: The result derives from the fact that for a plane mirror the image is the same distance behind the mirror as the object is in front of the mirror.

34.62. **IDENTIFY:** Apply the law of reflection.
SET UP: The image of one mirror can serve as the object for the other mirror.
EXECUTE: **(a)** There are three images formed, as shown in Figure 34.62a.
(b) The paths of rays for each image are sketched in Figure 34.62b.
EVALUATE: Our results agree with Figure 34.9 in the textbook.

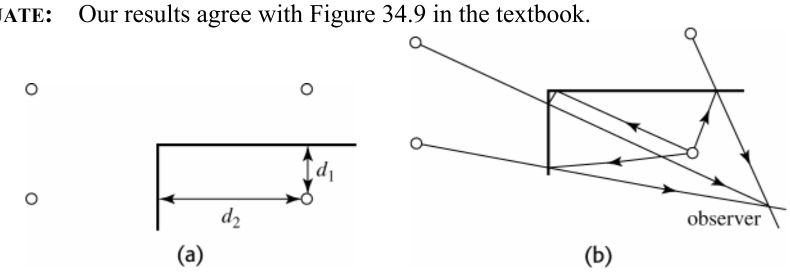


Figure 34.62

34.63. **IDENTIFY:** Apply the law of reflection for rays from the feet to the eyes and from the top of the head to the eyes.
SET UP: In Figure 34.63, ray 1 travels from the feet of the woman to her eyes and ray 2 travels from the top of her head to her eyes. The total height of the woman is h.
EXECUTE: The two angles labeled θ_1 are equal because of the law of reflection, as are the two angles labeled θ_2. Since these angles are equal, the two distances labeled y_1 are equal and the two distances labeled y_2 are equal. The height of the woman is $h_w = 2y_1 + 2y_2$. As the drawing shows, the height of the mirror is $h_m = y_1 + y_2$. Comparing, we find that $h_m = h_w/2$. The minimum height required is half the height of the woman.

EVALUATE: The height of the image is the same as the height of the woman, so the height of the image is twice the height of the mirror.

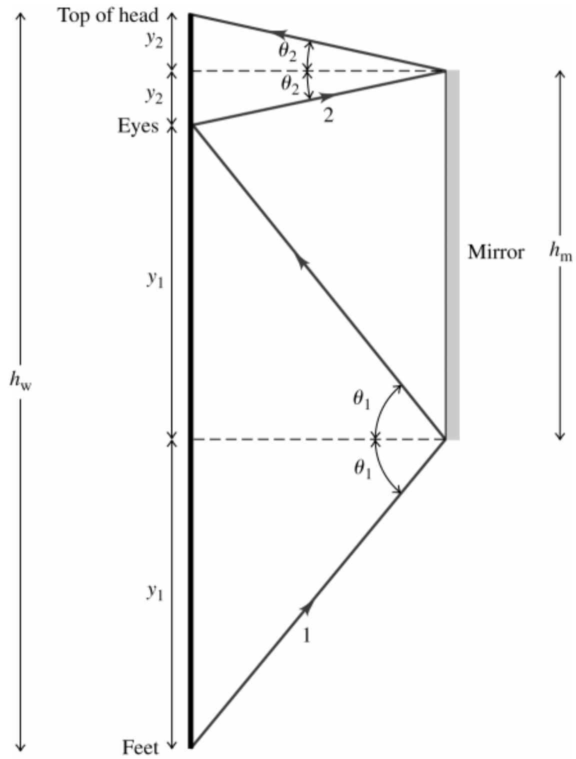


Figure 34.63

34.64. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}$ and $m=-\frac{s'}{s}$.

SET UP: Since the image is projected onto the wall it is real and $s'>0$. $m=-\frac{s'}{s}$ so m is negative and $m=-2.25$. The object, mirror and wall are sketched in Figure 34.64. The sketch shows that $s'-s=400\text{ cm}$.

EXECUTE: $m=-2.25=-\frac{s'}{s}$ and $s'=2.25s$. $s'-s=2.25s-s=400\text{ cm}$ and $s=320\text{ cm}$.

$s'=400\text{ cm}+\ 320\text{ cm}=720\text{ cm}$. The mirror should be 7.20 m from the wall. $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}$. $\frac{1}{320\text{ cm}}+\frac{1}{720\text{ cm}}=\frac{2}{R}$.

$R=4.43\text{ m}$.

EVALUATE: The focal length of the mirror is $f=R/2=222\text{ cm}$. $s>f$, as it must if the image is to be real.

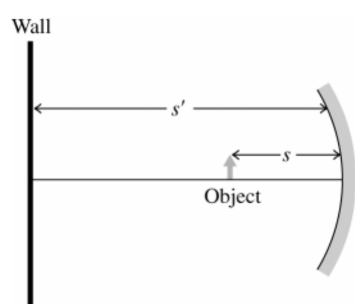


Figure 34.64

34.65. **IDENTIFY:** We are given the image distance, the image height, and the object height. Use Eq.(34.7) to calculate the object distance s. Then use Eq.(34.4) to calculate R.

(a) SET UP: Image is to be formed on screen so is real image; $s'>0$. Mirror to screen distance is 8.00 m, so $s'=+800\text{ cm}$. $m=-\frac{s'}{s}<0$ since both s and s' are positive.

EXECUTE: $|m| = \frac{|y'|}{|y|} = \frac{36.0 \text{ m}}{0.600 \text{ cm}} = 60.0$ and $m = -60.0$. Then $m = -\frac{s'}{s}$ gives $s = -\frac{s'}{m} = -\frac{800 \text{ cm}}{-60.0} = +13.3$ cm.

(b) $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$, so $\frac{2}{R} = \frac{s+s'}{ss'}$

$R = 2\left(\frac{ss'}{s+s'}\right) = 2\left(\frac{(13.3 \text{ cm})(800 \text{ cm})}{800 \text{ cm} + 13.3 \text{ cm}}\right) = 26.2 \text{ cm}$

EVALUATE: R is calculated to be positive, which is correct for a concave mirror. Also, in part (a) s is calculated to be positive, as it should be for a real object.

34.66. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ to calculate s' and then use $m = -\frac{s'}{s} = \frac{y'}{y}$ to find the height of the image.

SET UP: For a convex mirror, $R < 0$, so $R = -18.0$ cm and $f = \frac{R}{2} = -9.00$ cm.

EXECUTE: **(a)** $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$. $s' = \frac{sf}{s-f} = \frac{(1300 \text{ cm})(-9.00 \text{ cm})}{1300 \text{ cm} - (-9.00 \text{ cm})} = -8.94 \text{ cm}$. $m = -\frac{s'}{s} = -\frac{-8.94 \text{ cm}}{1300 \text{ cm}} = 6.88 \times 10^{-3}$.

$|y'| = |m| y = (6.88 \times 10^{-3})(1.5 \text{ m}) = 0.0103 \text{ m} = 1.03 \text{ cm}$.

(b) The height of the image is much less than the height of the car, so the car appears to be farther away than its actual distance.

EVALUATE: The image formed by a convex mirror is always virtual and smaller than the object.

34.67. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$ and $m = -\frac{s'}{s}$.

SET UP: $R = +19.4$ cm.

EXECUTE: **(a)** $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \Rightarrow \frac{1}{8.0 \text{ cm}} + \frac{1}{s'} = \frac{2}{19.4 \text{ cm}} \Rightarrow s' = -46 \text{ cm}$, so the image is virtual.

(b) $m = -\frac{s'}{s} = -\frac{-46}{8.0} = 5.8$, so the image is erect, and its height is $y' = (5.8)y = (5.8)(5.0 \text{ mm}) = 29 \text{ mm}$.

EVALUATE: **(c)** When the filament is 8 cm from the mirror, the image is virtual and cannot be projected onto a wall.

34.68. IDENTIFY: Combine $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$ and $m = -\frac{s'}{s}$.

SET UP: $m = +2.50$. $R > 0$.

EXECUTE: $m = -\frac{s'}{s} = +2.50$. $s' = -2.50s$. $\frac{1}{s} + \frac{1}{-2.50s} = \frac{2}{R}$. $\frac{0.600}{s} = \frac{2}{R}$ and $s = 0.300R$.

$s' = -2.50s = (-2.50)(0.300R) = -0.750R$. The object is a distance of $0.300R$ in front of the mirror and the image is a distance of $0.750R$ behind the mirror.

EVALUATE: For a single mirror an erect image is always virtual.

34.69. IDENTIFY and SET UP: Apply Eqs.(34.6) and (34.7). For a virtual object $s < 0$. The image is real if $s' > 0$.

EXECUTE: **(a)** Convex implies $R < 0$; $R = -24.0$ cm; $f = R/2 = -12.0$ cm

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, so $\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s-f}{sf}$

$s' = \frac{sf}{s-f} = \frac{(-12.0 \text{ cm})s}{s + 12.0 \text{ cm}}$

s is negative, so write as $s = -|s|$; $s' = +\frac{(12.0 \text{ cm})|s|}{12.0 \text{ cm} - |s|}$. Thus $s' > 0$ (real image) for $|s| < 12.0$ cm. Since s is negative this means $-12.0 \text{ cm} < s < 0$. A real image is formed if the virtual object is closer to the mirror than the focus.

(b) $m = -\frac{s'}{s}$; real image implies $s' > 0$; virtual object implies $s < 0$. Thus $m > 0$ and the image is erect.

(c) The principal-ray diagram is given in Figure 34.69.

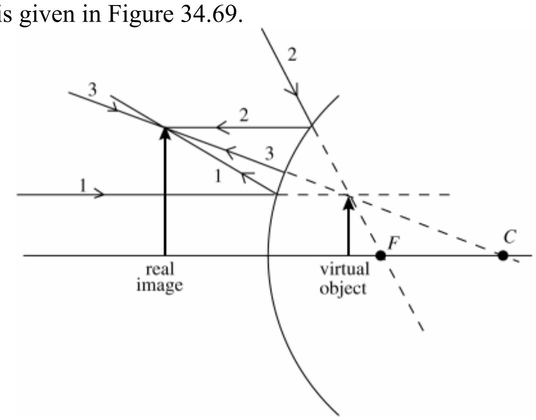


Figure 34.69

EVALUATE: For a real object, only virtual images are formed by a convex mirror. The virtual object considered in this problem must have been produced by some other optical element, by another lens or mirror in addition to the convex one we considered.

34.70. **IDENTIFY:** Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$, with $R\to\infty$ since the surfaces are flat.

SET UP: The image formed by the first interface serves as the object for the second interface.

EXECUTE: For the water-benzene interface to get the apparent water depth:

$$\frac{n_a}{s}+\frac{n_b}{s'}=0\Rightarrow\frac{1.33}{6.50\text{ cm}}+\frac{1.50}{s'}=0\Rightarrow s'=-7.33\text{ cm.}$$

For the benzene-air interface, to get the total apparent distance to the bottom:

$$\frac{n_a}{s}+\frac{n_b}{s'}=0\Rightarrow\frac{1.50}{(7.33\text{ cm}+2.60\text{ cm})}+\frac{1}{s'}=0\Rightarrow s'=-6.62\text{ cm.}$$

EVALUATE: At the water-benzene interface the light refracts into material of greater refractive index and the overall effect is that the apparent depth is greater than the actual depth.

34.71. **IDENTIFY:** The focal length is given by $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$.

SET UP: $R_1=\pm4.0$ cm or ±8.0 cm. $R_2=\pm8.0$ cm or ±4.0 cm. The signs are determined by the location of the center of curvature for each surface.

EXECUTE: $\frac{1}{f}=(0.60)\left(\frac{1}{\pm4.00\text{ cm}}-\frac{1}{\pm8.00\text{ cm}}\right)$, so $f=\pm4.44$ cm, ±13.3 cm. The possible lens shapes are sketched in Figure 34.71.

$f_1=+13.3$ cm; $f_2=+4.44$ cm; $f_3=4.44$ cm; $f_4=-13.3$ cm; $f_5=-13.3$ cm; $f_6=+13.3$ cm;
$f_7=-4.44$ cm; $f_8=-4.44$ cm.

EVALUATE: f is the same whether the light travels through the lens from right to left or left to right, so for the pairs (1,6), (4,5) and (7,8) the focal lengths are the same.

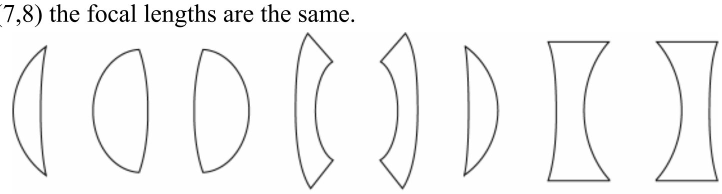


Figure 34.71

34.72. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and the concept of principal rays.

SET UP: $s=10.0$ cm. If extended backwards the ray comes from a point on the optic axis 18.0 cm from the lens and the ray is parallel to the optic axis after it passes through the lens.

EXECUTE: **(a)** The ray is bent toward the optic axis by the lens so the lens is converging.

(b) The ray is parallel to the optic axis after it passes through the lens so it comes from the focal point; $f=18.0$ cm.

(c) The principal ray diagram is drawn in Figure 34.72. The diagram shows that the image is 22.5 cm to the left of the lens.

(d) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives $s'=\frac{sf}{s-f}=\frac{(10.0\text{ cm})(18.0\text{ cm})}{10.0\text{ cm}-18.0\text{ cm}}=-22.5\text{ cm}$. The calculated image position agrees with the principal ray diagram.

EVALUATE: The image is virtual. A converging lens produces a virtual image when the object is inside the focal point.

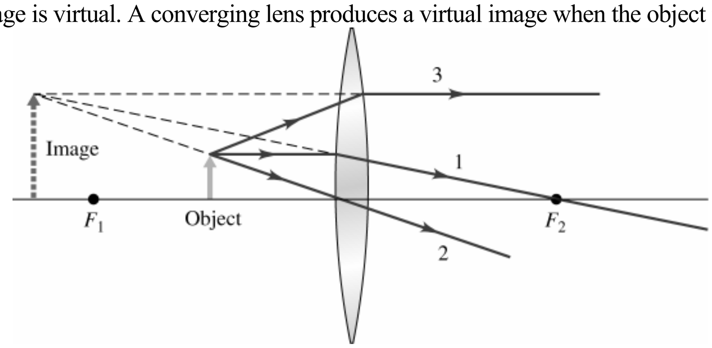


Figure 34.72

34.73. **IDENTIFY:** Since the truck is moving toward the mirror, its image will also be moving toward the mirror.

SET UP: The equation relating the object and image distances to the focal length of a spherical mirror is $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$, where $f = R/2$.

EXECUTE: Since the mirror is convex, $f = R/2 = (-1.50\text{ m})/2 = -0.75$ m. Applying the equation for a spherical mirror gives $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow s'=\frac{fs}{s-f}$.

Using the chain rule from calculus and the fact that $v = ds/dt$, we have $v'=\frac{ds'}{dt}=\frac{ds'}{ds}\frac{ds}{dt}=v\frac{f^2}{(s-f)^2}$

Solving for v gives $v=v'\left(\frac{s-f}{f}\right)^2=(1.5\text{ m/s})\left[\frac{2.0\text{ m}-(-0.75\text{ m})}{-0.75\text{ m}}\right]^2=20.2\text{ m/s}$.

This is the velocity of the truck relative to the mirror, so the truck is approaching the mirror at 20.2 m/s. You are traveling at 25 m/s, so the truck must be traveling at 25 m/s + 20.2 m/s = 45 m/s relative to the highway.

EVALUATE: Even though the truck and car are moving at constant speed, the image of the truck is *not* moving at constant speed because its location depends on the distance from the mirror to the truck.

34.74. **IDENTIFY:** In this context, the microscope just looks at an image or object. Apply $\frac{n_a}{s}+\frac{n_b}{s'}=0$ to the image formed by refraction at the top surface of the second plate. In this calculation the object is the bottom surface of the second plate.

SET UP: The thickness of the second plate is $2.50\text{ mm}+0.78\text{ mm}$, and this is s. The image is 2.50 mm below the top surface, so $s'=-2.50\text{ mm}$.

EXECUTE: $\frac{n_a}{s}+\frac{n_b}{s'}=0\Rightarrow\frac{n}{s}+\frac{1}{s'}=0\Rightarrow n=-\frac{s}{s'}=-\frac{2.50\text{ mm}+0.780\text{ mm}}{-2.50\text{ mm}}=1.31.$

EVALUATE: The object and image distances are measured from the front surface of the second plate, and the image is virtual.

34.75. **IDENTIFY** and **SET UP:** In part (a) use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to evaluate ds'/ds. Compare to $m=-\frac{s'}{s}$. In part (b) use $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}$ to find the location of the image of each face of the cube.

EXECUTE: **(a)** $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ and taking its derivative with respect to s we have $0=\frac{d}{ds}\left(\frac{1}{s}+\frac{1}{s'}-\frac{1}{f}\right)=-\frac{1}{s^2}-\frac{1}{s'^2}\frac{ds'}{ds}$

and $\frac{ds'}{ds}=-\frac{s'^2}{s^2}=-m^2$. But $\frac{ds'}{ds}=m'$, so $m'=-m^2$. Images are always inverted longitudinally.

(b) (i) Front face: $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}\Rightarrow\frac{1}{200.000\text{ cm}}+\frac{1}{s'}=\frac{2}{150.000\text{ cm}}\Rightarrow s'=120.00\text{ cm}.$

Rear face: $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R} \Rightarrow \frac{1}{200.100 \text{ cm}}+\frac{1}{s'}=\frac{2}{150.000 \text{ cm}} \Rightarrow s'=119.96 \text{ cm}.$

(ii) $m=-\frac{s'}{s}=-\frac{120.000}{200.000}=-0.600$. $m'=-m^2=-(-0.600)^2=-0.360.$

(iii) The two faces perpendicular to the axis (the front and rear faces): squares with side length 0.600 mm. The four faces parallel to the axis (the side faces): rectangles with sides of length 0.360 mm parallel to the axis and 0.600 mm perpendicular to the axis.

EVALUATE: Since the lateral and longitudinal magnifications have different values the image of the cube is not a cube.

34.76. IDENTIFY: $m'=ds'/ds$ and $m=-\frac{n_a s'}{n_b s}$.

SET UP: Use $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ to evaluate ds'/ds.

EXECUTE: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ and taking its derivative with respect to s we have

$$0=\frac{d}{ds}\left(\frac{n_a}{s}+\frac{n_b}{s'}-\frac{n_b-n_a}{R}\right)=-\frac{n_a}{s^2}-\frac{n_b}{s'^2}\frac{ds'}{ds} \text{ and } \frac{ds'}{ds}=-\frac{s'^2}{s^2}\frac{n_a}{n_b}=-\left(\frac{s'^2}{s^2}\frac{n_a^2}{n_b^2}\right)\frac{n_b}{n_a}=-m^2\frac{n_b}{n_a}.$$

But $\frac{ds'}{ds}=m'$, so $m'=-m^2\frac{n_b}{n_a}$.

EVALUATE: m' is always negative. This means that images are always inverted longitudinally.

34.77. IDENTIFY and **SET UP:** Rays that pass through the hole are undeflected. All other rays are blocked. $m=-\frac{s'}{s}$.

EXECUTE: **(a)** The ray diagram is drawn in Figure 34.77. The ray shown is the only ray from the top of the object that reaches the film, so this ray passes through the top of the image. An inverted image is formed on the far side of the box, no matter how far this side is from the pinhole and no matter how far the object is from the pinhole.

(b) $s=1.5$ m. $s'=20.0$ cm. $m=-\frac{s'}{s}=-\frac{20.0 \text{ cm}}{150 \text{ cm}}=-0.133$. $y'=my=(-0.133)(18 \text{ cm})=-2.4$ cm. The image is 2.4 cm tall.

EVALUATE: A defect of this camera is that not much light energy passes through the small hole each second, so long exposure times are required.

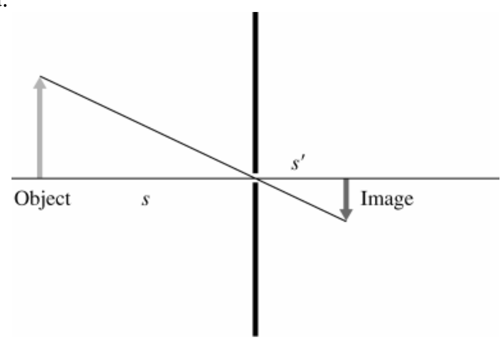


Figure 34.77

34.78. IDENTIFY: Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ and $m=-\frac{n_a s'}{n_b s}$ to each refraction. The overall magnification is $m=m_1 m_2$.

SET UP: For the first refraction, $R=+6.0$ cm, $n_a=1.00$ and $n_b=1.60$. For the second refraction, $R=-12.0$ cm, $n_a=1.60$ and $n_b=1.00$.

EXECUTE: **(a)** The image from the left end acts as the object for the right end of the rod.

(b) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1}{23.0 \text{ cm}}+\frac{1.60}{s'}=\frac{0.60}{6.0 \text{ cm}} \Rightarrow s'=28.3 \text{ cm}.$

So the second object distance is $s_2=40.0 \text{ cm}-28.3 \text{ cm}=11.7$ cm. $m_1=-\frac{n_a s'}{n_b s}=-\frac{28.3}{(1.60)(23.0)}=-0.769.$

(c) The object is real and inverted.

(d) $\frac{n_a}{s_2}+\frac{n_b}{s_2'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1.60}{11.7 \text{ cm}}+\frac{1}{s_2'}=\frac{-0.60}{-12.0 \text{ cm}} \Rightarrow s'=-11.5 \text{ cm}.$

$m_2=-\frac{n_a s'}{n_b s}=-\frac{(1.60)(-11.5)}{11.7}=1.57 \Rightarrow m=m_1 m_2=(-0.769)(1.57)=-1.21.$

(e) The final image is virtual, and inverted.

(f) $y' = (1.50\text{ mm})(-1.21) = -1.82\text{ mm}.$

EVALUATE: The first image is to the left of the second surface, so it serves as a real object for the second surface, with positive object distance.

34.79. **IDENTIFY:** Apply Eqs.(34.11) and (34.12) to the refraction as the light enters the rod and as it leaves the rod. The image formed by the first surface serves as the object for the second surface. The total magnification is $m_{tot} = m_1 m_2$, where m_1 and m_2 are the magnifications for each surface.

SET UP: The object and rod are shown in Figure 34.79.

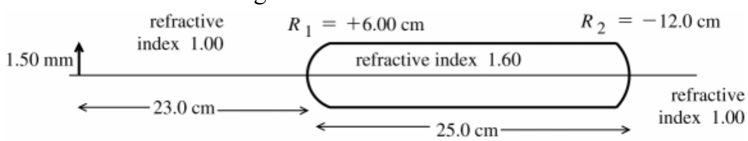


Figure 34.79

(a) image formed by refraction at first surface (left end of rod):

$s = +23.0\text{ cm};\ n_a = 1.00;\ n_b = 1.60;\ R = +6.00\text{ cm}$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

EXECUTE: $\dfrac{1}{23.0\text{ cm}} + \dfrac{1.60}{s'} = \dfrac{1.60 - 1.00}{6.00\text{ cm}}$

$$\frac{1.60}{s'} = \frac{1}{10.0\text{ cm}} - \frac{1}{23.0\text{ cm}} = \frac{23 - 10}{230\text{ cm}} = \frac{13}{230\text{ cm}}$$

$s' = 1.60\left(\dfrac{230\text{ cm}}{13}\right) = +28.3\text{ cm};$ image is 28.3 cm to right of first vertex.

This image serves as the object for the refraction at the second surface (right-hand end of rod). It is $28.3\text{ cm} - 25.0\text{ cm} = 3.3\text{ cm}$ to the right of the second vertex. For the second surface $s = -3.3\text{ cm}$ (virtual object).

(b) EVALUATE: Object is on side of outgoing light, so is a virtual object.

(c) SET UP: Image formed by refraction at second surface (right end of rod):

$s = -3.3\text{ cm};\ n_a = 1.60;\ n_b = 1.00;\ R = -12.0\text{ cm}$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

EXECUTE: $\dfrac{1.60}{-3.3\text{ cm}} + \dfrac{1.00}{s'} = \dfrac{1.00 - 1.60}{-12.0\text{ cm}}$

$s' = +1.9\text{ cm};\ s' > 0$ so image is 1.9 cm to right of vertex at right-hand end of rod.

(d) $s' > 0$ so final image is real.

Magnification for first surface:

$$m = -\frac{n_a s'}{n_b s} = -\frac{(1.60)(+28.3\text{ cm})}{(1.00)(+23.0\text{ cm})} = -0.769$$

Magnification for second surface:

$$m = -\frac{n_a s'}{n_b s} = -\frac{(1.60)(+1.9\text{ cm})}{(1.00)(-3.3\text{ cm})} = +0.92$$

The overall magnification is $m_{tot} = m_1 m_2 = (-0.769)(+0.92) = -0.71$ $m_{tot} < 0$ so final image is inverted with respect to the original object.

(e) $y' = m_{tot} y = (-0.71)(1.50\text{ mm}) = -1.06\text{ mm}$

The final image has a height of 1.06 mm.

EVALUATE: The two refracting surfaces are not close together and Eq.(34.18) does not apply.

34.80. **IDENTIFY:** Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$. The type of lens determines the sign of *f*. The sign of s' determines whether the image is real or virtual.

SET UP: $s = +8.00\text{ cm}$. $s' = -3.00\text{ cm}$. s' is negative because the image is on the same side of the lens as the object.

EXECUTE: **(a)** $\dfrac{1}{f} = \dfrac{s + s'}{ss'}$ and $f = \dfrac{ss'}{s + s'} = \dfrac{(8.00\text{ cm})(-3.00\text{ cm})}{8.00\text{ cm} - 3.00\text{ cm}} = -4.80\text{ cm}$. f is negative so the lens is diverging.

(b) $m = -\dfrac{s'}{s} = -\dfrac{-3.00\text{ cm}}{8.00\text{ cm}} = +0.375$. $y' = my = (0.375)(6.50\text{ mm}) = 2.44\text{ mm}$. $s' < 0$ and the image is virtual.

EVALUATE: A converging lens can also form a virtual image, if the object distance is less than the focal length. But in that case $|s'| > s$ and the image would be farther from the lens than the object is.

34.81. **IDENTIFY:** $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$. The type of lens determines the sign of *f*. $m = \dfrac{y'}{y} = -\dfrac{s'}{s}$. The sign of s' depends on whether the image is real or virtual. $s = 16.0\text{ cm}$.

SET UP: $s' = -22.0\text{ cm}$; s' is negative because the image is on the same side of the lens as the object.

EXECUTE: **(a)** $\dfrac{1}{f} = \dfrac{s+s'}{ss'}$ and $f = \dfrac{ss'}{s+s'} = \dfrac{(16.0\text{ cm})(-22.0\text{ cm})}{16.0\text{ cm} - 22.0\text{ cm}} = +58.7\text{ cm}$. *f* is positive so the lens is converging.

(b) $m = -\dfrac{s'}{s} = -\dfrac{-22.0\text{ cm}}{16.0\text{ cm}} = 1.38$. $y' = my = (1.38)(3.25\text{ mm}) = 4.48\text{ mm}$. $s' < 0$ and the image is virtual.

EVALUATE: A converging lens forms a virtual image when the object is closer to the lens than the focal point.

34.82. **IDENTIFY:** Apply $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}$. Use the image distance when viewed from the flat end to determine the refractive index *n* of the rod.

SET UP: When viewing from the flat end, $n_a = n$, $n_b = 1.00$ and $R \to \infty$. When viewing from the curved end, $n_a = n$, $n_b = 1.00$ and $R = -10.0\text{ cm}$.

EXECUTE: $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = 0 \Rightarrow \dfrac{n}{15.0\text{ cm}} + \dfrac{1}{-9.50\text{ cm}} = 0 \Rightarrow n = \dfrac{15.0}{9.50} = 1.58$. When viewed from the curved end of the rod $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R} \Rightarrow \dfrac{n}{s} + \dfrac{1}{s'} = \dfrac{1-n}{R} \Rightarrow \dfrac{1.58}{15.0\text{ cm}} + \dfrac{1}{s'} = \dfrac{-0.58}{-10.0\text{ cm}}$, and $s' = -21.1\text{ cm}$. The image is 21.1 cm within the rod from the curved end.

EVALUATE: In each case the image is virtual and on the same side of the surface as the object.

34.83. **(a) IDENTIFY:** Apply Snell's law to the refraction of a ray at each side of the beam to find where these rays strike the table.

SET UP: The path of a ray is sketched in Figure 34.83.

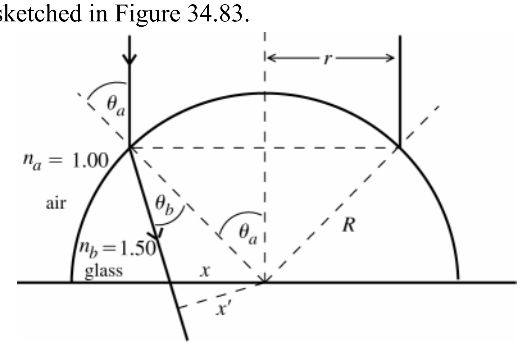


Figure 34.83

The width of the incident beam is exaggerated in the sketch, to make it easier to draw. Since the diameter of the beam is much less than the radius of the hemisphere, angles θ_a and θ_b are small. The diameter of the circle of light formed on the table is $2x$. Note the two right triangles containing the angles θ_a and θ_b.

$r = 0.190\text{ cm}$ is the radius of the incident beam.

$R = 12.0\text{ cm}$ is the radius of the glass hemisphere.

EXECUTE: θ_a and θ_b small imply $x \approx x'$; $\sin\theta_a = \dfrac{r}{R}$, $\sin\theta_b = \dfrac{x'}{R} \approx \dfrac{x}{R}$

Snell's law: $n_a \sin\theta_a = n_b \sin\theta_b$

Using the above expressions for $\sin\theta_a$ and $\sin\theta_b$ gives $n_a \dfrac{r}{R} = n_b \dfrac{x}{R}$

$n_a r = n_b x$ so $x = \dfrac{n_a r}{n_b} = \dfrac{1.00(0.190\text{ cm})}{1.50} = 0.1267\text{ cm}$

The diameter of the circle on the table is $2x = 2(0.1267\text{ cm}) = 0.253\text{ cm}$.

(b) EVALUATE: *R* divides out of the expression; the result for the diameter of the spot is independent of the radius *R* of the hemisphere. It depends only on the diameter of the incident beam and the index of refraction of the glass.

34.84. IDENTIFY and SET UP: Treating each of the goblet surfaces as spherical surfaces, we have to pass, from left to right, through four interfaces. Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ to each surface. The image formed by one surface serves as the object for the next surface.

EXECUTE: **(a)** For the empty goblet:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{\infty}+\frac{1.50}{s_1'}=\frac{0.50}{4.00\text{ cm}}\Rightarrow s_1'=12\text{ cm}.$$

$$s_2=0.60\text{ cm}-12\text{ cm}=-11.4\text{ cm}\Rightarrow\frac{1.50}{-11.4\text{ cm}}+\frac{1}{s_2'}=\frac{-0.50}{3.40\text{ cm}}\Rightarrow s_2'=-64.6\text{ cm}.$$

$$s_3=64.6\text{ cm}+6.80\text{ cm}=71.4\text{ cm}\Rightarrow\frac{1}{71.4\text{ cm}}+\frac{1.50}{s_3'}=\frac{0.50}{-3.40\text{ cm}}\Rightarrow s_3'=-9.31\text{ cm}.$$

$$s_4=9.31\text{ cm}+0.60\text{ cm}=9.91\text{ cm}\Rightarrow\frac{1.50}{9.91\text{ cm}}+\frac{1}{s_4'}=\frac{-0.50}{-4.00\text{ cm}}\Rightarrow s_4'=-37.9\text{ cm}.$$ The final image is $37.9\text{ cm}-2(4.0\text{ cm})=29.9\text{ cm}$ to the left of the goblet.

(b) For the wine-filled goblet:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{\infty}+\frac{1.50}{s_1'}=\frac{0.50}{4.00\text{ cm}}\Rightarrow s_1'=12\text{ cm}.$$

$$s_2=0.60\text{ cm}-12\text{ cm}=-11.4\text{ cm}\Rightarrow\frac{1.50}{-11.4\text{ cm}}+\frac{1.37}{s_2'}=\frac{-0.13}{3.40\text{ cm}}\Rightarrow s_2'=14.7\text{ cm}.$$

$$s_3=6.80\text{ cm}-14.7\text{ cm}=-7.9\text{ cm}\Rightarrow\frac{1.37}{-7.9\text{ cm}}+\frac{1.50}{s_3'}=\frac{0.13}{-3.40\text{ cm}}\Rightarrow s_3'=11.1\text{ cm}.$$

$$s_4=0.60\text{ cm}-11.1\text{ cm}=-10.5\text{ cm}\Rightarrow\frac{1.50}{-10.5\text{ cm}}+\frac{1}{s_4'}=\frac{-0.50}{-4.00\text{ cm}}\Rightarrow s_4'=3.73\text{ cm}.$$ The final image is 3.73 cm to the right of the goblet.

EVALUATE: If the object for a surface is on the outgoing side of the light, then the object is virtual and the object distance is negative.

34.85. IDENTIFY: The image formed by refraction at the surface of the eye is located by $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$.

SET UP: $n_a=1.00$, $n_b=1.35$. $R>0$. For a distant object, $s\approx\infty$ and $\frac{1}{s}\approx 0$.

EXECUTE: **(a)** $s\approx\infty$ and $s'=2.5$ cm: $\frac{1.35}{2.5\text{ cm}}=\frac{1.35-1.00}{R}$ and $R=0.648\text{ cm}=6.48\text{ mm}$.

(b) $R=0.648$ cm and $s=25$ cm: $\frac{1.00}{25\text{ cm}}+\frac{1.35}{s'}=\frac{1.35-1.00}{0.648}$. $\frac{1.35}{s'}=0.500$ and $s'=2.70\text{ cm}=27.0\text{ mm}$. The image is formed behind the retina.

(c) Calculate s' for $s\approx\infty$ and $R=0.50$ cm: $\frac{1.35}{s'}=\frac{1.35-1.00}{0.50\text{ cm}}$. $s'=1.93\text{ cm}=19.3\text{ mm}$. The image is formed in front of the retina.

EVALUATE: The cornea alone cannot achieve focus of both close and distant objects.

34.86. IDENTIFY: Apply $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$ and $m=-\frac{n_a s'}{n_b s}$ to each surface. The overall magnification is $m=m_1m_2$. The image formed by the first surface is the object for the second surface.

SET UP: For the first surface, $n_a=1.00$, $n_b=1.60$ and $R=+15.0$ cm. For the second surface, $n_a=1.60$, $n_b=1.00$ and $R=\rightarrow\infty$.

EXECUTE: **(a)** $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{12.0\text{ cm}}+\frac{1.60}{s'}=\frac{0.60}{15.0\text{ cm}}\Rightarrow s'=-36.9$ cm. The object distance for the far end of the rod is $50.0\text{ cm}-(-36.9\text{ cm})=86.9$ cm. The final image is 4.3 cm to the left of the vertex of the hemispherical surface. $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.60}{86.9\text{ cm}}+\frac{1}{s'}=0\Rightarrow s'=-54.3$ cm.

(b) The magnification is the product of the two magnifications:

$$m_1=-\frac{n_a s'}{n_b s}=-\frac{-36.9}{(1.60)(12.0)}=1.92,\ m_2=1.00\Rightarrow m=m_1m_2=1.92.$$

EVALUATE: The final image is virtual, erect and larger than the object.

34.87. **IDENTIFY:** Apply Eq.(34.11) to the image formed by refraction at the front surface of the sphere.

SET UP: Let n_g be the index of refraction of the glass. The image formation is shown in Figure 34.87.

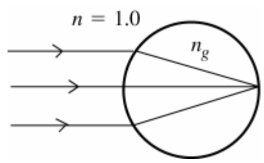


Figure 34.87

$s = \infty$

$s' = +2r$, where r is the radius of the sphere

$n_a = 1.00$, $n_b = n_g$, $R = +r$

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$$

EXECUTE: $\dfrac{1}{\infty} + \dfrac{n_g}{2r} = \dfrac{n_g - 1.00}{r}$

$\dfrac{n_g}{2r} = \dfrac{n_g}{r} - \dfrac{1}{r}$; $\dfrac{n_g}{2r} = \dfrac{1}{r}$ and $n_g = 2.00$

EVALUATE: The required refractive index of the glass does not depend on the radius of the sphere.

34.88. **IDENTIFY:** Apply $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}$ to each surface. The image of the first surface is the object for the second surface. The relation between s_1' and s_2 involves the length d of the rod.

SET UP: For the first surface, $n_a = 1.00$, $n_b = 1.55$ and $R = +6.00\text{ cm}$. For the second surface, $n_a = 1.55$, $n_b = 1.00$ and $R = -6.00\text{ cm}$.

EXECUTE: We have images formed from both ends. From the first surface:

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{25.0\text{ cm}} + \frac{1.55}{s'} = \frac{0.55}{6.00\text{ cm}} \Rightarrow s' = 30.0\text{ cm}.$$

This image becomes the object for the second end: $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R} \Rightarrow \dfrac{1.55}{d - 30.0\text{ cm}} + \dfrac{1}{65.0\text{ cm}} = \dfrac{-0.55}{-6.00\text{ cm}}$.

$d - 30.0\text{ cm} = 20.3\text{ cm} \Rightarrow d = 50.3\text{ cm}.$

EVALUATE: The final image is real. The first image is 20.3 cm to the right of the second surface and serves as a real object.

34.89. **IDENTIFY:** The first lens forms an image which then acts as the object for the second lens.

SET UP: The thin-lens equation is $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and the magnification is $m = -\dfrac{s'}{s}$.

EXECUTE: **(a)** For the first lens: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{5.00\text{ cm}} + \dfrac{1}{s'} = \dfrac{1}{-15.0\text{ cm}} \Rightarrow s' = -3.75\text{ cm}$, to the left of the lens (virtual image).

(b) For the second lens, s = 12.0 cm + 3.75 cm = 15.75 cm.

$\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{15.75\text{ cm}} + \dfrac{1}{s'} = \dfrac{1}{15.0\text{ cm}} \Rightarrow s' = 315\text{ cm}$, or 332 cm from the object.

(c) The final image is real.

(d) $m = -\dfrac{s'}{s}$, $m_1 = 0.750$, $m_2 = -20.0$, $m_{\text{total}} = -15.0 \Rightarrow y' = -6.00\text{ cm}$, inverted.

EVALUATE: Note that the total magnification is the product of the individual magnifications.

34.90. **IDENTIFY** and **SET UP:** Use $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$ to calculate the focal length of the lenses. The image formed by the first lens serves at the object for the second lens. $m_{\text{tot}} = m_1 m_2$. $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ gives $s' = \dfrac{sf}{s-f}$.

EXECUTE: **(a)** $\dfrac{1}{f} = (0.60)\left(\dfrac{1}{12.0\text{ cm}} - \dfrac{1}{28.0\text{ cm}}\right)$ and $f = +35.0\text{ cm}$.

<u>Lens 1</u>: $f_1 = +35.0\text{ cm}$. $s_1 = +45.0\text{ cm}$. $s_1' = \dfrac{s_1 f_1}{s_1 - f_1} = \dfrac{(45.0\text{ cm})(35.0\text{ cm})}{45.0\text{ cm} - 35.0\text{ cm}} = +158\text{ cm}$.

$m_1 = -\dfrac{s_1'}{s_1} = -\dfrac{158\text{ cm}}{45.0\text{ cm}} = -3.51$. $|y_1'| = |m_1| y_1 = (3.51)(5.00\text{ mm}) = 17.6\text{ mm}$. The image of the first lens is 158 cm to the right of lens 1 and is 17.6 mm tall.

(b) The image of lens 1 is $315\text{ cm} - 158\text{ cm} = 157\text{ cm}$ to the left of lens 2. $f_2 = +35.0\text{ cm}$. $s_2 = +157\text{ cm}$.

$$s_2' = \frac{s_2 f_2}{s_2 - f_2} = \frac{(157\text{ cm})(35.0\text{ cm})}{157\text{ cm} - 35.0\text{ cm}} = +45.0\text{ cm}.\quad m_2 = -\frac{s_2'}{s_2} = -\frac{45.0\text{ cm}}{157\text{ cm}} = -0.287.$$

$m_{\text{tot}} = m_1 m_2 = (-3.51)(-0.287) = +1.00$. The final image is 45.0 cm to the right of lens 2. The final image is 5.00 mm tall. $m_{\text{tot}} > 0$. So the final image is erect.

EVALUATE: The final image is real. It is erect because each lens produces an inversion of the image, and two inversions return the image to the orientation of the object.

34.91. **IDENTIFY** and **SET UP:** Apply Eq.(34.16) for each lens position. The lens to screen distance in each case is the image distance. There are two unknowns, the original object distance *x* and the focal length *f* of the lens. But each lens position gives an equation, so there are two equations for these two unknowns. The object, lens and screen before and after the lens is moved are shown in Figure 34.91.

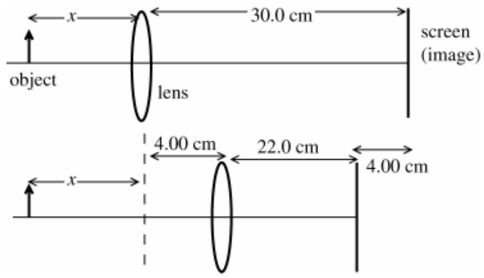


Figure 34.91

$s = x$; $s' = 30.0\text{ cm}$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{x} + \frac{1}{30.0\text{ cm}} = \frac{1}{f}$$

$s = x + 4.00\text{ cm}$; $s' = 22.0\text{ cm}$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \text{ gives } \frac{1}{x + 4.00\text{ cm}} + \frac{1}{22.0\text{ cm}} = \frac{1}{f}$$

EXECUTE: Equate these two expressions for $1/f$:

$$\frac{1}{x} + \frac{1}{30.0\text{ cm}} = \frac{1}{x + 4.00\text{ cm}} + \frac{1}{22.0\text{ cm}}$$

$$\frac{1}{x} - \frac{1}{x + 4.00\text{ cm}} = \frac{1}{22.0\text{ cm}} - \frac{1}{30.0\text{ cm}}$$

$$\frac{x + 4.00\text{ cm} - x}{x(x + 4.00\text{ cm})} = \frac{30.0 - 22.0}{660\text{ cm}} \text{ and } \frac{4.00\text{ cm}}{x(x + 4.00\text{ cm})} = \frac{8}{660\text{ cm}}$$

$$x^2 + (4.00\text{ cm})x - 330\text{ cm}^2 = 0 \text{ and } x = \frac{1}{2}(-4.00 \pm \sqrt{16.0 + 4(330)})\text{ cm}$$

x must be positive so $x = \frac{1}{2}(-4.00 + 36.55)\text{ cm} = 16.28\text{ cm}$

Then $\dfrac{1}{x} + \dfrac{1}{30.0\text{ cm}} = \dfrac{1}{f}$ and $\dfrac{1}{f} = \dfrac{1}{16.28\text{ cm}} + \dfrac{1}{30.0\text{ cm}}$

$f = +10.55\text{ cm}$, which rounds to 10.6 cm. $f > 0$; the lens is converging.

EVALUATE: We can check that $s = 16.28\text{ cm}$ and $f = 10.55\text{ cm}$ gives $s' = 30.0\text{ cm}$ and that $s = (16.28 + 4.0)\text{ cm} = 20.28\text{ cm}$ and $f = 10.55\text{ cm}$ gives $s' = 22.0\text{ cm}$.

34.92. **IDENTIFY** and **SET UP:** Apply $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}$.

EXECUTE: **(a)** $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R} \Rightarrow \dfrac{n_a}{f} + \dfrac{n_b}{\infty} = \dfrac{n_b - n_a}{R}$ and $\dfrac{n_a}{\infty} + \dfrac{n_b}{f'} = \dfrac{n_b - n_a}{R}$.

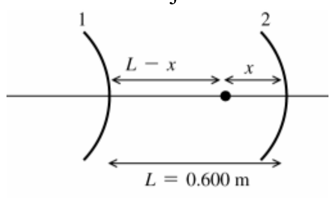
$\dfrac{n_a}{f} = \dfrac{n_b - n_a}{R}$ and $\dfrac{n_b}{f'} = \dfrac{n_b - n_a}{R}$. Therefore, $\dfrac{n_a}{f} = \dfrac{n_b}{f'}$ and $n_a / n_b = \dfrac{f}{f'}$.

(b) $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R} \Rightarrow \dfrac{n_b f}{s f'} + \dfrac{n_b}{s'} = \dfrac{n_b(1 - f/f')}{R}$. Therefore, $\dfrac{f}{s} + \dfrac{f'}{s'} = \dfrac{f'(1 - f/f')}{R} = \dfrac{f' - f}{R} = 1$.

EVALUATE: For a thin lens the first and second focal lengths are equal.

34.93. **(a) IDENTIFY:** Use Eq.(34.6) to locate the image formed by each mirror. The image formed by the first mirror serves as the object for the 2nd mirror.

SET UP: The positions of the object and the two mirrors are shown in Figure 34.93a.



$|R| = 0.360$ m

$|f| = |R|/2 = 0.180$ m

Figure 34.93a

EXECUTE: Image formed by convex mirror (mirror #1):

convex means $f_1 = -0.180$ m; $s_1 = L - x$

$$s_1' = \frac{s_1 f_1}{s_1 - f_1} = \frac{(L-x)(-0.180\text{ m})}{L - x + 0.180\text{ m}} = -(0.180\text{ m})\left(\frac{0.600\text{ m} - x}{0.780\text{ m} - x}\right) < 0$$

The image is $(0.180\text{ m})\left(\frac{0.600\text{ m} - x}{0.780\text{ m} - x}\right)$ to the left of mirror #1 so is

$$0.600\text{ m} + (0.180\text{ m})\left(\frac{0.600\text{ m} - x}{0.780\text{ m} - x}\right) = \frac{0.576\text{ m}^2 - (0.780\text{ m})x}{0.780\text{ m} - x}$$ to the left of mirror #2.

Image formed by concave mirror (mirror #2);

concave implies $f_2 = +0.180$ m

$$s_2 = \frac{0.576\text{ m}^2 - (0.780\text{ m})x}{0.780\text{ m} - x}$$

Rays return to the source implies $s_2' = x$. Using these expressions in $s_2 = \frac{s_2' f_2}{s_2' - f_2}$ gives

$$\frac{0.576\text{ m}^2 - (0.780\text{ m})x}{0.780\text{ m} - x} = \frac{(0.180\text{ m})x}{x - 0.180\text{ m}}$$

$$0.600x^2 - (0.576\text{ m})x + 0.10368\text{ m}^2 = 0$$

$$x = \frac{1}{1.20}\left(0.576 \pm \sqrt{(0.576)^2 - 4(0.600)(0.10368)}\right)\text{ m} = \frac{1}{1.20}(0.576 \pm 0.288)\text{ m}$$

$x = 0.72$ m (impossible; can't have $x > L = 0.600$ m) or $x = 0.24$ m.

(b) SET UP: Which mirror is #1 and which is #2 is now reversed form part (a). This is shown in Figure 34.93b.

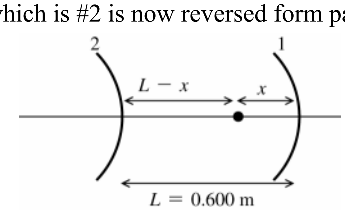


Figure 34.93b

EXECUTE: Image formed by concave mirror (mirror #1):

concave means $f_1 = +0.180$ m; $s_1 = x$

$$s_1' = \frac{s_1 f_1}{s_1 - f_1} = \frac{(0.180\text{ m})x}{x - 0.180\text{ m}}$$

The image is $\frac{(0.180\text{ m})x}{x - 0.180\text{ m}}$ to the left of mirror #1, so $s_2 = 0.600\text{ m} - \frac{(0.180\text{ m})x}{x - 0.180\text{ m}} = \frac{(0.420\text{ m})x - 0.180\text{ m}^2}{x - 0.180\text{ m}}$

Image formed by convex mirror (mirror #2):

convex means $f_2 = -0.180$ m

rays return to the source means $s_2' = L - x = 0.600\text{ m} - x$

$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ gives

$$\frac{x - 0.180\text{ m}}{(0.420\text{ m})x - 0.180\text{ m}^2} + \frac{1}{0.600\text{ m} - x} = -\frac{1}{0.180\text{ m}}$$

$$\frac{x - 0.180\text{ m}}{(0.420\text{ m})x - 0.180\text{ m}^2} = -\left(\frac{0.780\text{ m} - x}{0.180\text{ m}^2 - (0.180\text{ m})x}\right)$$

$$0.600x^2 - (0.576\text{ m})x + 0.1036\text{ m}^2 = 0$$

This is the same quadratic equation as obtained in part (a), so again $x = 0.24$ m.

EVALUATE: For $x = 0.24$ m the image is at the location of the source, both for rays that initially travel from the source toward the left and for rays that travel from the source toward the right.

34.94. IDENTIFY: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ gives $s' = \frac{sf}{s-f}$, for both the mirror and the lens.

SET UP: For the second image, the image formed by the mirror serves as the object for the lens. For the mirror, $f_{\text{m}} = +10.0$ cm. For the lens, $f = 32.0$ cm. The center of curvature of the mirror is $R = 2f_{\text{m}} = 20.0$ cmto the right of the mirror vertex.

EXECUTE: **(a)** The principal-ray diagrams from the two images are sketched in Figures 34.94a-b. In Figure 34.94b, only the image formed by the mirror is shown. This image is at the location of the candle so the principal ray diagram that shows the image formation when the image of the mirror serves as the object for the lens is analogous to that in Figure 34.94a and is not drawn.

(b) Image formed by the light that passes directly through the lens: The candle is 85.0 cm to the left of the lens. $s' = \frac{sf}{s-f} = \frac{(85.0\text{ cm})(32.0\text{ cm})}{85.0\text{ cm} - 32.0\text{ cm}} = +51.3\text{ cm}$. $m = -\frac{s'}{s} = -\frac{51.3\text{ cm}}{85.0\text{ cm}} = -0.604$. This image is 51.3 cm to the right of the lens. $s' > 0$ so the image is real. $m < 0$ so the image is inverted. Image formed by the light that first reflects off the mirror: First consider the image formed by the mirror. The candle is 20.0 cm to the right of the mirror, so $s = +20.0$ cm. $s' = \frac{sf}{s-f} = \frac{(20.0\text{ cm})(10.0\text{ cm})}{20.0\text{ cm} - 10.0\text{ cm}} = 20.0\text{ cm}$. $m_1 = -\frac{s_1'}{s_1} = -\frac{20.0\text{ cm}}{20.0\text{ cm}} = -1.00$. The image formed by the mirror is at the location of the candle, so $s_2 = +85.0$ cmand $s_2' = 51.3$ cm. $m_2 = -0.604$. $m_{\text{tot}} = m_1 m_2 =$ $(-1.00)(-0.604) = 0.604$. The second image is 51.3 cm to the right of the lens. $s_2' > 0$, so the final image is real. $m_{\text{tot}} > 0$, so the final image is erect.

EVALUATE: The two images are at the same place. They are the same size. One is erect and one is inverted.

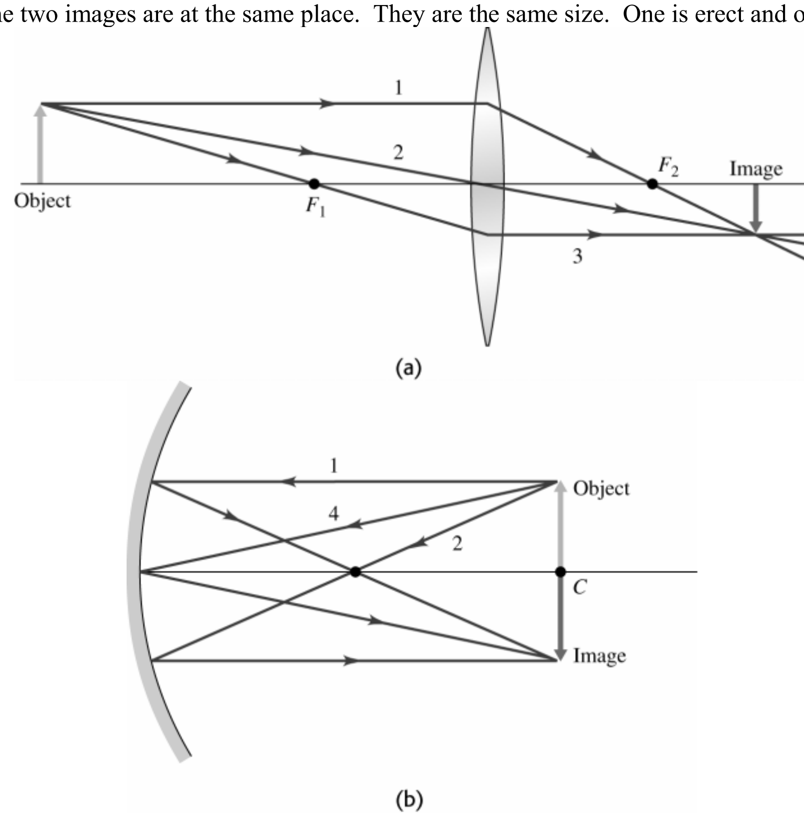


Figure 34.94

34.95. IDENTIFY: Apply $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ to each case.

SET UP: $s = 20.0$ cm. $R > 0$. Use $s' = +9.12$ cm to find *R*. For this calculation, $n_a = 1.00$ and $n_b = 1.55$. Then repeat the calculation with $n_a = 1.33$.

EXECUTE: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ gives $\frac{1.00}{20.0\text{ cm}} + \frac{1.55}{9.12\text{ cm}} = \frac{1.55 - 1.00}{R}$. $R = 2.50$ cm.

Then $\frac{1.33}{20.0\text{ cm}}+\frac{1.55}{s'}=\frac{1.55-1.33}{2.50\text{ cm}}$ gives $s'=-72.1\text{ cm}$. The image is 72.1 cm to the left of the surface vertex.

EVALUATE: With the rod in air the image is real and with the rod in water the image is virtual.

34.96. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to each lens. The image formed by the first lens serves as the object for the second lens. The focal length of the lens combination is defined by $\frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{f}$. In part (b) use $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ to calculate *f* for the meniscus lens and for the CCl_4, treated as a thin lens.

SET UP: With two lenses of different focal length in contact, the image distance from the first lens becomes exactly minus the object distance for the second lens.

EXECUTE: **(a)** $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}\Rightarrow\frac{1}{s_1'}=\frac{1}{f_1}-\frac{1}{s_1}$ and $\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{-s_1'}+\frac{1}{s_2'}=\left(\frac{1}{s_1}-\frac{1}{f_1}\right)+\frac{1}{s_2'}=\frac{1}{f_2}$. But overall for the lens system, $\frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{f}\Rightarrow\frac{1}{f}=\frac{1}{f_2}+\frac{1}{f_1}$.

(b) With carbon tetrachloride sitting in a meniscus lens, we have two lenses in contact. All we need in order to calculate the system's focal length is calculate the individual focal lengths, and then use the formula from part (a).

For the meniscus lens $\frac{1}{f_\text{m}}=(n_b-n_a)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(0.55)\left(\frac{1}{4.50\text{ cm}}-\frac{1}{9.00\text{ cm}}\right)=0.061\text{ cm}^{-1}$ and $f_\text{m}=16.4\text{ cm}$.

For the CCl_4: $\frac{1}{f_\text{w}}=(n_b-n_a)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(0.46)\left(\frac{1}{9.00\text{ cm}}-\frac{1}{\infty}\right)=0.051\text{ cm}^{-1}$ and $f_\text{w}=19.6\text{ cm}$.

$\frac{1}{f}=\frac{1}{f_\text{w}}+\frac{1}{f_\text{m}}=0.112\text{ cm}^{-1}$ and $f=8.93\text{ cm}$.

EVALUATE: $f=\frac{f_1f_2}{f_1+f_2}$, so *f* for the combination is less than either f_1 or f_2.

34.97. **IDENTIFY:** Apply Eq.(34.11) with $R\to\infty$ to the refraction at each surface. For refraction at the first surface the point *P* serves as a virtual object. The image formed by the first refraction serves as the object for the second refraction.

SET UP: The glass plate and the two points are shown in Figure 37.97.

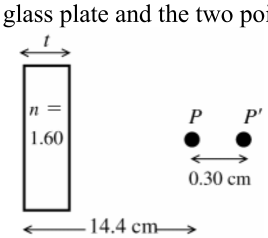


Figure 34.97

plane faces means $R\to\infty$ and

$$\frac{n_a}{s}+\frac{n_b}{s'}=0$$

$$s'=-\frac{n_b}{n_a}s$$

EXECUTE: <u>refraction at the first (left-hand) surface of the piece of glass:</u>

The rays converging toward point *P* constitute a virtual object for this surface, so $s=-14.4$ cm.

$n_a=1.00,\ n_b=1.60.$

$$s'=-\frac{1.60}{1.00}(-14.4\text{ cm})=+23.0\text{ cm}$$

This image is 23.0 cm to the right of the first surface so is a distance $23.0\text{ cm}-t$ to the right of the second surface. This image serves as a virtual object for the second surface.

<u>refraction at the second (right-hand) surface of the piece of glass:</u>

The image is at P' so $s'=14.4\text{ cm}+0.30\text{ cm}-t=14.7\text{ cm}-t$. $s=-(23.0\text{ cm}-t)$; $n_a=1.60$; $n_b=1.00$ $s'=-\frac{n_b}{n_a}s$ gives $14.7\text{ cm}-t=-\left(\frac{1.00}{1.60}\right)(-[23.0\text{ cm}-t])$. $14.7\text{ cm}-t=+14.4\text{ cm}-0.625t$.

$0.375t=0.30\text{ cm}$ and $t=0.80\text{ cm}$

EVALUATE: The overall effect of the piece of glass is to diverge the rays and move their convergence point to the right. For a real object, refraction at a plane surface always produces a virtual image, but with a virtual object the image can be real.

34.98. **IDENTIFY:** Apply the two equations $\frac{n_a}{s_1}+\frac{n_b}{s_1'}=\frac{n_b-n_a}{R_1}$ and $\frac{n_b}{s_2}+\frac{n_c}{s_2'}=\frac{n_c-n_b}{R_2}$.

SET UP: $n_a = n_{\text{liq}} = n_c$, $n_b = n$, and $s_1' = -s_2$.

EXECUTE: **(a)** $\frac{n_{\text{liq}}}{s_1}+\frac{n}{s_1'}=\frac{n-n_{\text{liq}}}{R_1}$ and $\frac{n}{-s_1'}+\frac{n_{\text{liq}}}{s_2'}=\frac{n_{\text{liq}}-n}{R_2}$. $\frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{f'}=(n/n_{\text{liq}}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$.

(b) Comparing the equations for focal length in and out of air we have:

$$f(n-1)=f'(n/n_{\text{liq}}-1)=f'\left(\frac{n-n_{\text{liq}}}{n_{\text{liq}}}\right)\Rightarrow f'=\left[\frac{n_{\text{liq}}(n-1)}{n-n_{\text{liq}}}\right]f.$$

EVALUATE: When $n_{\text{liq}}=1$, $f'=f$, as it should.

34.99. **IDENTIFY:** Apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$.

SET UP: The image formed by the converging lens is 30.0 cm from the converging lens, and becomes a virtual object for the diverging lens at a position 15.0 cm to the right of the diverging lens. The final image is projected $15\text{ cm}+19.2\text{ cm}=34.2\text{ cm}$ from the diverging lens.

EXECUTE: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{-15.0\text{ cm}}+\frac{1}{34.2\text{ cm}}=\frac{1}{f}\Rightarrow f=-26.7\text{ cm}.$

EVALUATE: Our calculation yields a negative value of *f*, which should be the case for a diverging lens.

34.100. **IDENTIFY:** The spherical mirror forms an image of the object. It forms another image when the image of the plane mirror serves as an object.

SET UP: For the convex mirror $f=-24.0\text{ cm}$. The image formed by the plane mirror is 10.0 cm to the right of the plane mirror, so is $20.0\text{ cm}+10.0\text{ cm}=30.0\text{ cm}$ from the vertex of the spherical mirror.

EXECUTE: The first image formed by the spherical mirror is the one where the light immediately strikes its surface, without bouncing from the plane mirror.

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{10.0\text{ cm}}+\frac{1}{s'}=\frac{1}{-24.0\text{ cm}}\Rightarrow s'=-7.06\text{ cm},$ and the image height

is $y'=-\frac{s'}{s}y=-\frac{-7.06}{10.0}(0.250\text{ cm})=0.177\text{ cm}.$

The second image is of the plane mirror image is located 30.0 cm from the vertex of the spherical mirror.

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{30.0\text{ cm}}+\frac{1}{s'}=\frac{1}{-24.0\text{ cm}}\Rightarrow s'=-13.3\text{ cm}$ and the image height is

$y'=-\frac{s'}{s}y=-\frac{-13.3}{30.0}(0.250\text{ cm})=0.111\text{ cm}.$

EVALUATE: Other images are formed by additional reflections from the two mirrors.

34.101. **IDENTIFY:** In the sketch in Figure 34.101 the light travels upward from the object. Apply Eq.(34.11) with $R\to\infty$ to the refraction at each surface. The image formed by the first surface serves as the object for the second surface.

SET UP: The locations of the object and the glass plate are shown in Figure 34.101.

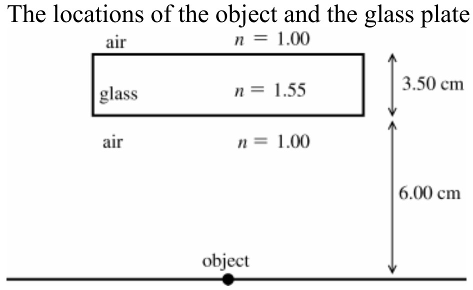


Figure 34.101

For a plane (flat) surface

$R\to\infty$ so $\frac{n_a}{s}+\frac{n_b}{s'}=0$

$s'=-\frac{n_b}{n_a}s$

EXECUTE: First refraction (air $\to$ glass):

$n_a=1.00$; $n_b=1.55$; $s=6.00\text{ cm}$

$s'=-\frac{n_b}{n_a}s=-\frac{1.55}{1.00}(6.00\text{ cm})=-9.30\text{ cm}$

The image is 9.30 cm below the lower surface of the glass, so is $9.30\text{ cm}+3.50\text{ cm}=12.8\text{ cm}$ below the upper surface.

Second refraction (glass $\rightarrow$ air):

$n_a = 1.55$; $n_b = 1.00$; $s = +12.8$ cm

$$s' = -\frac{n_b}{n_a}s = -\frac{1.00}{1.55}(12.8\text{ cm}) = -8.26\text{ cm}$$

The image of the page is 8.26 cm below the top surface of the glass plate and therefore $9.50\text{ cm} - 8.26\text{ cm} = 1.24\text{ cm}$ above the page.

EVALUATE: The image is virtual. If you view the object by looking down from above the plate, the image of the page that you see is closer to your eye than the page is.

34.102. IDENTIFY: Light refracts at the front surface of the lens, refracts at the glass-water interface, reflects from the plane mirror and passes through the two interfaces again, now traveling in the opposite direction.

SET UP: Use the focal length in air to find the radius of curvature R of the lens surfaces.

EXECUTE: **(a)** $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{40\text{ cm}} = 0.52\left(\frac{2}{R}\right) \Rightarrow R = 41.6\text{ cm}.$

At the air–lens interface: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{70.0\text{ cm}} + \frac{1.52}{s_1'} = \frac{0.52}{41.6\text{ cm}}$ and $s_1' = -851$ cm and $s_2 = 851$ cm.

At the lens–water interface: $\Rightarrow \frac{1.52}{851\text{ cm}} + \frac{1.33}{s_2'} = \frac{-0.187}{-41.6\text{ cm}}$ and $s_2' = 491$ cm.

The mirror reflects the image back (since there is just 90 cm between the lens and mirror.) So, the position of the image is 401 cm to the left of the mirror, or 311 cm to the left of the lens.

At the water–lens interface: $\Rightarrow \frac{1.33}{-311\text{ cm}} + \frac{1.52}{s_3'} = \frac{0.187}{41.6\text{ cm}}$ and $s_3' = +173$ cm.

At the lens–air interface: $\Rightarrow \frac{1.52}{-173\text{ cm}} + \frac{1}{s_4'} = \frac{-0.52}{-41.6\text{ cm}}$ and $s_4' = +47.0$ cm, to the left of lens.

$$m = m_1 m_2 m_3 m_4 = \left(\frac{n_{a1}s_1'}{n_{b1}s_1}\right)\left(\frac{n_{a2}s_2'}{n_{b2}s_2}\right)\left(\frac{n_{a3}s_3'}{n_{b3}s_3}\right)\left(\frac{n_{a4}s_4'}{n_{b4}s_4}\right) = \left(\frac{-851}{70}\right)\left(\frac{491}{-851}\right)\left(\frac{+173}{-311}\right)\left(\frac{+47.0}{-173}\right) = -1.06.$$

(Note all the indices of refraction cancel out.)

(b) The image is real.

(c) The image is inverted.

(d) The final height is $y' = my = (1.06)(4.00\text{ mm}) = 4.24$ mm.

EVALUATE: The final image is real even though it is on the same side of the lens as the object!

34.103. IDENTIFY: The camera lens can be modeled as a thin lens that forms an image on the film.

SET UP: The thin-lens equation is $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$, and the magnification of the lens is $m = -\frac{s'}{s}$.

EXECUTE: **(a)** $m = -\frac{s'}{s} = \frac{y'}{y} = \frac{1}{4}\frac{(0.0360\text{ m})}{(12.0\text{ m})} \Rightarrow s' = (7.50\times10^{-4})\,s$,

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{s} + \frac{1}{(7.50\times10^{-4})s} = \frac{1}{s}\left(1 + \frac{1}{7.50\times10^{-4}}\right) = \frac{1}{f} = \frac{1}{0.0350\text{ m}} \Rightarrow s = 46.7\text{ m}.$$

(b) To just fill the frame, the magnification must be 3.00×10^{-3} so:

$$\frac{1}{s}\left(1 + \frac{1}{3.00\times10^{-3}}\right) = \frac{1}{f} = \frac{1}{0.0350\text{ m}} \Rightarrow s = 11.7\text{ m}.$$

Since the boat is originally 46.7 m away, the distance you must move closer to the boat is 46.7 m – 11.7 m = 35.0 m.

EVALUATE: This result seems to imply that if you are 4 times as far, the image is ¼ as large on the film. However this result is only an approximation, and would not be true for very close distances. It is a better approximation for large distances.

34.104. IDENTIFY: Apply $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and $m = -\frac{s'}{s}$.

SET UP: $s + s' = 18.0$ cm

EXECUTE: **(a)** $\frac{1}{18.0\text{ cm} - s'} + \frac{1}{s'} = \frac{1}{3.00\text{ cm}}$. $(s')^2 - (18.0\text{ cm})s' + 54.0\text{ cm}^2 = 0$ so $s' = 14.2$ cm or 3.80 cm.

$s = 3.80$ cm or 14.2 cm, so the screen must either be 3.80 cm or 14.2 cm from the object.

(b) $s = 3.80\text{ cm}: m = -\dfrac{s'}{s} = -\dfrac{3.80}{14.2} = -0.268.$ $s = 14.2\text{ cm}: m = -\dfrac{s'}{s} = -\dfrac{14.2}{3.80} = -3.74.$

EVALUATE: Since the image is projected onto the screen, the image is real and s' is positive. We assumed this when we wrote the condition $s + s' = 18.0\text{ cm}$.

34.105. IDENTIFY: Apply Eq.(34.16) to calculate the image distance for each lens. The image formed by the 1st lens serves as the object for the 2nd lens, and the image formed by the 2nd lens serves as the object for the 3rd lens.

SET UP: The positions of the object and lenses are shown in Figure 34.105.

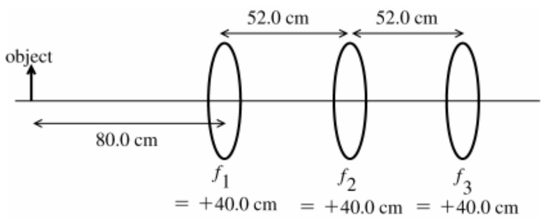


Figure 34.105

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{s - f}{sf}$$

$$s' = \frac{sf}{s - f}$$

EXECUTE: <u>lens #1</u>

$s = +80.0\text{ cm}; f = +40.0\text{ cm}$

$$s' = \frac{sf}{s - f} = \frac{(+80.0\text{ cm})(+40.0\text{ cm})}{+80.0\text{ cm} - 40.0\text{ cm}} = +80.0\text{ cm}$$

The image formed by the first lens is 80.0 cm to the right of the first lens, so it is $80.0\text{ cm} - 52.0\text{ cm} = 28.0\text{ cm}$ to the right of the second lens.

<u>lens #2</u>

$s = -28.0\text{ cm}; f = +40.0\text{ cm}$

$$s' = \frac{sf}{s - f} = \frac{(-28.0\text{ cm})(+40.0\text{ cm})}{-28.0\text{ cm} - 40.0\text{ cm}} = +16.47\text{ cm}$$

The image formed by the second lens is 16.47 cm to the right of the second lens, so it is $52.0\text{ cm} - 16.47\text{ cm} = 35.53\text{ cm}$ to the left of the third lens.

<u>lens #3</u>

$s = +35.53\text{ cm}; f = +40.0\text{ cm}$

$$s' = \frac{sf}{s - f} = \frac{(+35.53\text{ cm})(+40.0\text{ cm})}{+35.53\text{ cm} - 40.0\text{ cm}} = -318\text{ cm}$$

The final image is 318 cm to the left of the third lens, so it is $318\text{ cm} - 52\text{ cm} - 52\text{ cm} - 80\text{ cm} = 134\text{ cm}$ to the left of the object.

EVALUATE: We used the separation between the lenses and the sign conventions for s and s' to determine the object distances for the 2nd and 3rd lenses. The final image is virtual since the final s' is negative.

34.106. IDENTIFY: Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ and calculate s' for each s.

SET UP: $f = 90\text{ mm}$

EXECUTE: $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{1300\text{ mm}} + \dfrac{1}{s'} = \dfrac{1}{90\text{ mm}} \Rightarrow s' = 96.7\text{ mm}.$

$\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{6500\text{ mm}} + \dfrac{1}{s'} = \dfrac{1}{90\text{ mm}} \Rightarrow s' = 91.3\text{ mm}.$

$\Rightarrow \Delta s' = 96.7\text{ mm} - 91.3\text{ mm} = 5.4\text{ mm}$ toward the film

EVALUATE: $s' = \dfrac{sf}{s - f}$. For $f > 0$ and $s > f$, s' decreases as s increases.

34.107. IDENTIFY and SET UP: The generalization of Eq.(34.22) is $M = \dfrac{\text{near point}}{f}$, so $f = \dfrac{\text{near point}}{M}$.

EXECUTE: **(a)** age 10, near point = 7 cm

$$f = \frac{7\text{ cm}}{2.0} = 3.5\text{ cm}$$

(b) age 30, near point = 14 cm

$$f = \frac{14\text{ cm}}{2.0} = 7.0\text{ cm}$$

(c) age 60, near point = 200 cm

$f = \dfrac{200\text{ cm}}{2.0} = 100\text{ cm}$

(d) $f = 3.5\text{ cm}$ (from part (a)) and near point = 200 cm (for 60-year-old)

$M = \dfrac{200\text{ cm}}{3.5\text{ cm}} = 57$

(e) EVALUATE: No. The reason $f = 3.5\text{ cm}$ gives a larger M for a 60-year-old than for a 10-year-old is that the eye of the older person can't focus on as close an object as the younger person can. The unaided eye of the 60-year-old must view a much smaller angular size, and that is why the same f gives a much larger M. The angular size of the image depends only on f and is the same for the two ages.

34.108. IDENTIFY: Use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to calculate s that gives $s' = -25\text{ cm}$. $M = \dfrac{\theta'}{\theta}$.

SET UP: Let the height of the object be y, so $\theta' = \dfrac{y}{s}$ and $\theta = \dfrac{y}{25\text{ cm}}$.

EXECUTE: **(a)** $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s} + \dfrac{1}{-25\text{ cm}} = \dfrac{1}{f} \Rightarrow s = \dfrac{f(25\text{ cm})}{f + 25\text{ cm}}$.

(b) $\theta' = \arctan\left(\dfrac{y}{s}\right) = \arctan\left(\dfrac{y(f + 25\text{ cm})}{f(25\text{ cm})}\right) \approx \dfrac{y(f + 25\text{ cm})}{f(25\text{ cm})}$.

(c) $M = \dfrac{\theta'}{\theta} = \dfrac{y(f + 25\text{ cm})}{f(25\text{ cm})}\dfrac{1}{y/25\text{ cm}} = \dfrac{f + 25\text{ cm}}{f}$.

(d) If $f = 10\text{ cm} \Rightarrow M = \dfrac{10\text{ cm} + 25\text{ cm}}{10\text{ cm}} = 3.5$. This is 1.4 times greater than the magnification obtained if the image if formed at infinity $(M_\infty = \dfrac{25\text{ cm}}{f} = 2.5)$.

EVALUATE: **(e)** Having the first image form just within the focal length puts one in the situation described above, where it acts as a source that yields an enlarged virtual image. If the first image fell just outside the second focal point, then the image would be real and diminished.

34.109. IDENTIFY: Apply $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$. The near point is at infinity, so that is where the image must be formed for any objects that are close.

SET UP: The power in diopters equals $\dfrac{1}{f}$, with f in meters.

EXECUTE: $\dfrac{1}{f} = \dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{24\text{ cm}} + \dfrac{1}{-\infty} = \dfrac{1}{0.24\text{ m}} = 4.17$ diopters.

EVALUATE: To focus on closer objects, the power must be increased.

34.110. IDENTIFY: Apply $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}$.

SET UP: $n_a = 1.00$, $n_b = 1.40$.

EXECUTE: $\dfrac{1}{36.0\text{ cm}} + \dfrac{1.40}{s'} = \dfrac{0.40}{0.75\text{ cm}} \Rightarrow s' = 2.77\text{ cm}$.

EVALUATE: This distance is greater than the normal eye, which has a cornea vertex to retina distance of about 2.6 cm.

34.111. IDENTIFY: Use similar triangles in Figure 34.63 in the textbook and Eq.(34.16) to derive the expressions called for in the problem.

(a) SET UP: The effect of the converging lens on the ray bundle is sketched in Figure 34.111.

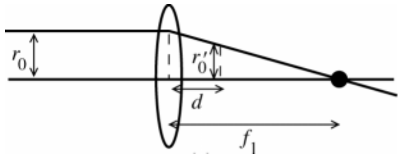


Figure 34.111a

EXECUTE: From similar triangles in Figure 34.111a,

$\dfrac{r_0}{f_1} = \dfrac{r_0'}{f_1 - d}$

Thus $r_0' = \left(\dfrac{f_1 - d}{f_1}\right) r_0$, as was to be shown.

(b) SET UP: The image at the focal point of the first lens, a distance f_1 to the right of the first lens, serves as the object for the second lens. The image is a distance $f_1 - d$ to the right of the second lens, so $s_2 = -(f_1 - d) = d - f_1$.

EXECUTE: $s_2' = \dfrac{s_2 f_2}{s_2 - f_2} = \dfrac{(d - f_1) f_2}{d - f_1 - f_2}$

$f_2 < 0$ so $|f_2| = -f_2$ and $s_2' = \dfrac{(f_1 - d)|f_2|}{|f_2| - f_1 + d}$, as was to be shown.

(c) SET UP: The effect of the diverging lens on the ray bundle is sketched in Figure 34.111b.

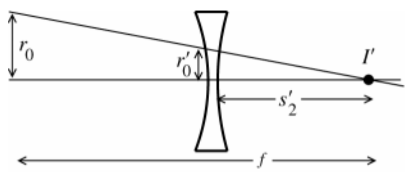


Figure 34.111b

EXECUTE: From similar triangles in the sketch, $\dfrac{r_0}{f} = \dfrac{r_0'}{s_2'}$

Thus $\dfrac{r_0}{r_0'} = \dfrac{f}{s_2'}$

From the results of part (a), $\dfrac{r_0}{r_0'} = \dfrac{f_1}{f_1 - d}$. Combining the two results gives $\dfrac{f_1}{f_1 - d} = \dfrac{f}{s_2'}$

$f = s_2'\left(\dfrac{f_1}{f_1 - d}\right) = \dfrac{(f_1 - d)|f_2| f_1}{(|f_2| - f_1 + d)(f_1 - d)} = \dfrac{f_1|f_2|}{|f_2| - f_1 + d}$, as was to be shown.

(d) SET UP: Put the numerical values into the expression derived in part (c).

EXECUTE: $f = \dfrac{f_1|f_2|}{|f_2| - f_1 + d}$

$f_1 = 12.0$ cm, $|f_2| = 18.0$ cm, so $f = \dfrac{216 \text{ cm}^2}{6.0 \text{ cm} + d}$

$d = 0$ gives $f = 36.0$ cm; maximum f

$d = 4.0$ cm gives $f = 21.6$ cm; minimum f

$f = 30.0$ cm says $30.0 \text{ cm} = \dfrac{216 \text{ cm}^2}{6.0 \text{ cm} + d}$

$6.0 \text{ cm} + d = 7.2$ cm and $d = 1.2$ cm

EVALUATE: Changing d produces a range of effective focal lengths. The effective focal length can be both smaller and larger than $f_1 + |f_2|$.

34.112. IDENTIFY: $|M| = \dfrac{\theta'}{\theta}$. $\theta = \left|\dfrac{y_1'}{f_1}\right|$, and $\theta' = \left|\dfrac{y_2'}{s_2'}\right|$. This gives $|M| = \left|\dfrac{y_2'}{s_2'} \cdot \dfrac{f_1}{y_1'}\right|$.

SET UP: Since the image formed by the objective is used as the object for the eyepiece, $y_1' = y_2$.

EXECUTE: $|M| = \left|\dfrac{y_2'}{s_2'} \cdot \dfrac{f_1}{y_2}\right| = \left|\dfrac{y_2'}{y_2} \cdot \dfrac{f_1}{s_2'}\right| = \left|\dfrac{s_2'}{s_2} \cdot \dfrac{f_1}{s_2'}\right| = \left|\dfrac{f_1}{s_2}\right|$. Therefore, $s_2 = \dfrac{f_1}{|M|} = \dfrac{48.0 \text{ cm}}{36} = 1.33$ cm, and this is just outside the eyepiece focal point.

Now the distance from the mirror vertex to the lens is $f_1 + s_2 = 49.3$ cm, and so

$\dfrac{1}{s_2} + \dfrac{1}{s_2'} = \dfrac{1}{f_2} \Rightarrow s_2' = \left(\dfrac{1}{1.20 \text{ cm}} - \dfrac{1}{1.33 \text{ cm}}\right)^{-1} = 12.3$ cm. Thus we have a final image which is real and 12.3 cm from the eyepiece. (Take care to carry plenty of figures in the calculation because two close numbers are subtracted.)

EVALUATE: Eq.(34.25) gives $|M| = 40$, somewhat larger than $|M|$ for this telescope.

34.113. IDENTIFY and SET UP: The image formed by the objective is the object for the eyepiece. The total lateral magnification is $m_{tot} = m_1 m_2$. $f_1 = 8.00$ mm (objective); $f_2 = 7.50$ cm (eyepiece)

(a) The locations of the object, lenses and screen are shown in Figure 34.113.

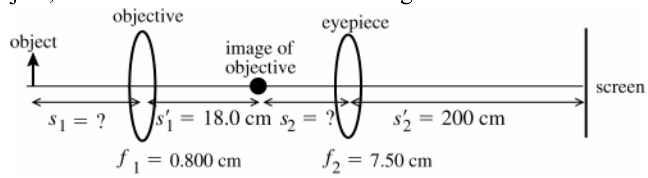


Figure 34.113

EXECUTE: Find the object distance s_1 for the objective:

$s'_1 = +18.0 \text{ cm}, f_1 = 0.800 \text{ cm}, s_1 = ?$

$\frac{1}{s_1}+\frac{1}{s'_1}=\frac{1}{f_1}$, so $\frac{1}{s_1}=\frac{1}{f_1}-\frac{1}{s'_1}=\frac{s'_1-f_1}{s'_1 f_1}$

$s_1=\frac{s'_1 f_1}{s'_1-f_1}=\frac{(18.0\text{ cm})(0.800\text{ cm})}{18.0\text{ cm}-0.800\text{ cm}}=0.8372\text{ cm}$

Find the object distance s_2 for the eyepiece:

$s'_2 = +200 \text{ cm}, f_2 = 7.50 \text{ cm}, s_2 = ?$

$\frac{1}{s_2}+\frac{1}{s'_2}=\frac{1}{f_2}$

$s_2=\frac{s'_2 f_2}{s'_2-f_2}=\frac{(200\text{ cm})(7.50\text{ cm})}{200\text{ cm}-7.50\text{ cm}}=7.792\text{ cm}$

Now we calculate the magnification for each lens:

$m_1=-\frac{s'_1}{s_1}=-\frac{18.0\text{ cm}}{0.8372\text{ cm}}=-21.50$

$m_2=-\frac{s'_2}{s_2}=-\frac{200\text{ cm}}{7.792\text{ cm}}=-25.67$

$m_{\text{tot}}=m_1 m_2=(-21.50)(-25.67)=552.$

(b) From the sketch we can see that the distance between the two lenses is $s'_1+s_2=18.0\text{ cm}+7.792\text{ cm}=25.8\text{ cm}$.

EVALUATE: The microscope is not being used in the conventional way; it merely serves as a two-lens system. In particular, the final image formed by the eyepiece in the problem is real, not virtual as is the case normally for a microscope. Eq.(34.23) does not apply here, and in any event gives the angular not the lateral magnification.

34.114. IDENTIFY: For u and u' as defined in Figure 34.64 in the textbook, $M=\frac{u'}{u}$.

SET UP: f_2 is negative. From Figure 34.64, the length of the telescope is f_1+f_2.

EXECUTE: **(a)** From the figure, $u=\frac{y}{f_1}$ and $u'=\frac{y}{|f_2|}=-\frac{y}{f_2}$. The angular magnification is $M=\frac{u'}{u}=-\frac{f_1}{f_2}$.

(b) $M=-\frac{f_1}{f_2}\Rightarrow f_2=-\frac{f_1}{M}=-\frac{95.0\text{ cm}}{6.33}=-15.0\text{ cm}$.

(c) The length of the telescope is $95.0\text{ cm}-15.0\text{ cm}=80.0\text{ cm}$, compared to the length of 110 cm for the telescope in Exercise 34.57.

EVALUATE: An advantage of this construction is that the telescope is somewhat shorter.

34.115. IDENTIFY: Use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to calculate s' (the distance of each point from the lens), for points A, B and C.

SET UP: The object and lens are shown in Figure 34.115a.

EXECUTE: **(a)** For point C: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{45.0\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}}\Rightarrow s'=36.0\text{ cm}$.

$y'=-\frac{s'}{s}y=-\frac{36.0}{45.0}(15.0\text{ cm})=-12.0\text{ cm}$, so the image of point C is 36.0 cm to the right of the lens, and 12.0 cm below the axis.

For point A: $s=45.0\text{ cm}+8.00\text{ cm}(\cos 45°)=50.7\text{ cm}$.

$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{50.7\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}}\Rightarrow s'=33.0\text{ cm}$. $y'=-\frac{s'}{s}y=-\frac{33.0}{45.0}(15.0\text{ cm}-8.00\text{ cm}(\sin 45°))=-6.10\text{ cm}$,

so the image of point A is 33.0 cm to the right of the lens, and 6.10 cm below the axis.

For point B: $s = 45.0\text{ cm} - 8.00\text{ cm}(\cos 45°) = 39.3\text{ cm}$. $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{39.3\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}} \Rightarrow s' = 40.7\text{ cm}$.

$y' = -\frac{s'}{s}y = -\frac{40.7}{39.3}(15.0\text{ cm} + 8.00\text{ cm}(\sin 45°)) = -21.4\text{ cm}$, so the image of point B is 40.7 cm to the right of the lens, and 21.4 cm below the axis. The image is shown in Figure 34.115b.

(b) The length of the pencil is the distance from point A to B:

$L = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} = \sqrt{(33.0\text{ cm} - 40.7\text{ cm})^2 + (6.10\text{ cm} - 21.4\text{ cm})^2} = 17.1\text{ cm}$

EVALUATE: The image is below the optic axis and is larger than the object.

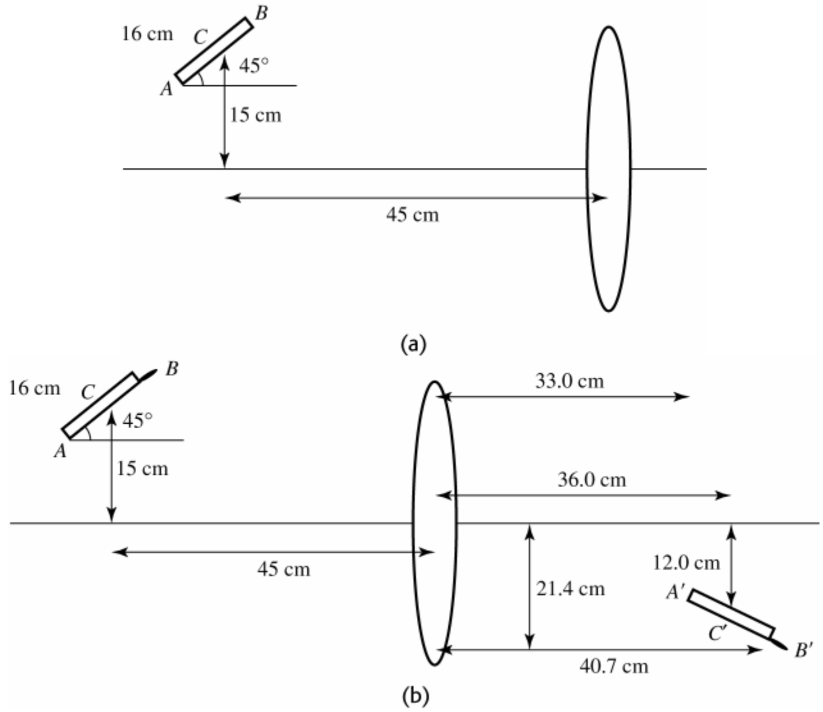


Figure 34.115

34.116. IDENTIFY and SET UP: Consider the ray diagram drawn in Figure 34.116.

EXECUTE: **(a)** Using the diagram and law of sines, $\frac{\sin\theta}{(R-f)} = \frac{\sin\alpha}{g}$ but $\sin\theta = \frac{h}{R} = \sin\alpha$ (law of reflection), $g = (R-f)$. Bisecting the triangle: $\cos\theta = \frac{R/2}{(R-f)} \Rightarrow R\cos\theta - f\cos\theta = \frac{R}{2}$.

$f = \frac{R}{2}\left[2 - \frac{1}{\cos\theta}\right] = f_0\left[2 - \frac{1}{\cos\theta}\right]$. $f_0 = \frac{R}{2}$ is the value of f for θ near zero (incident ray near the axis). When θ increases, $(2 - 1/\cos\theta)$ decreases and f decreases.

(b) $\frac{f - f_0}{f_0} = -0.02 \Rightarrow \frac{f}{f_0} = 0.98$ so $2 - \frac{1}{\cos\theta} = 0.98$. $\cos\theta = \frac{1}{2 - 0.98} = 0.98$ and $\theta = 11.4°$.

EVALUATE: For $\theta = 45°$, $f = 0.586 f_0$, and f approaches zero as θ approaches $60°$.

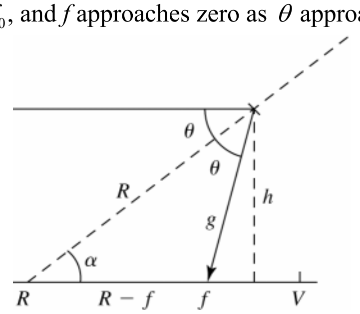


Figure 34.116

34.117. IDENTIFY: The distance between image and object can be calculated by taking the derivative of the separation distance and minimizing it.

SET UP: For a real image $s' > 0$ and the distance between the object and the image is $D = s + s'$. For a real image must have $s > f$.

EXECUTE: $D = s + s'$ but $s' = \dfrac{sf}{s-f} \Rightarrow D = s + \dfrac{sf}{s-f} = \dfrac{s^2}{s-f}$.

$\dfrac{dD}{ds} = \dfrac{d}{ds}\left(\dfrac{s^2}{s-f}\right) = \dfrac{2s}{s-f} - \dfrac{s^2}{(s-f)^2} = \dfrac{s^2 - 2sf}{(s-f)^2} = 0$. $s^2 - 2sf = 0$. $s(s-2f) = 0$. $s = 2f$ is the solution for which $s > f$. For $s = 2f$, $s' = 2f$. Therefore, the minimum separation is $2f + 2f = 4f$.

(b) A graph of D/f versus s/f is sketched in Figure 34.117. Note that the minimum does occur for $D = 4f$.

EVALUATE: If, for example, $s = 3f/2$, then $s' = 3f$ and $D = s + s' = 4.5f$, greater than the minimum value.

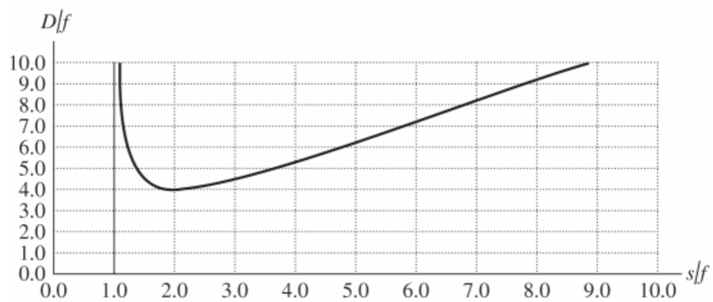


Figure 34.117

34.118. IDENTIFY and SET UP: For a plane mirror, $s' = -s$.

EXECUTE: (a) By the symmetry of image production, any image must be the same distance *D* as the object from the mirror intersection point. But if the images and the object are equal distances from the mirror intersection, they lie on a circle with radius equal to *D*.

(b) The center of the circle lies at the mirror intersection as discussed above.

(c) The diagram is sketched in Figure 34.118.

EVALUATE: To see the image, light from the object must be able to reflect from each mirror and reach the person's eyes.

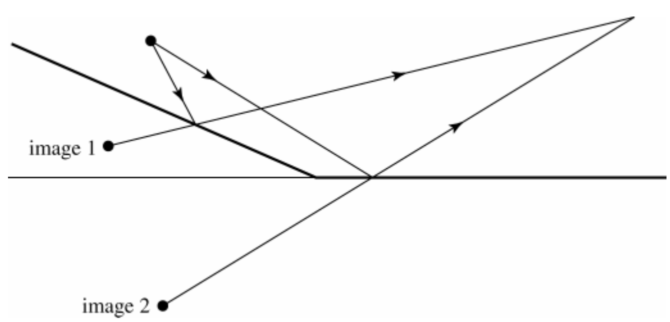


Figure 34.118

34.119. IDENTIFY: Apply $\dfrac{n_a}{s} + \dfrac{n_b}{s'} = \dfrac{n_b - n_a}{R}$ to refraction at the cornea to find where the object for the cornea must be in order for the image to be at the retina. Then use $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ to calculate *f* so that the lens produces an image of a distant object at this point.

SET UP: For refraction at the cornea, $n_a = 1.33$ and $n_b = 1.40$. The distance from the cornea to the retina in this model of the eye is 2.60 cm. From Problem 34.46, $R = 0.71\text{ cm}$.

EXECUTE: (a) People with normal vision cannot focus on distant objects under water because the image is unable to be focused in a short enough distance to form on the retina. Equivalently, the radius of curvature of the normal eye is about five or six times too great for focusing at the retina to occur.

(b) When introducing glasses, let's first consider what happens at the eye:

$\dfrac{n_a}{s_2} + \dfrac{n_b}{s_2'} = \dfrac{n_b - n_a}{R} \Rightarrow \dfrac{1.33}{s_2} + \dfrac{1.40}{2.6\text{ cm}} = \dfrac{0.07}{0.71\text{ cm}} \Rightarrow s_2 = -3.02$ cm. That is, the object for the cornea must be 3.02 cm behind the cornea. Now, assume the glasses are 2.00 cm in front of the eye, so $s_1' = 2.00\text{ cm} + s_2 = 5.02\text{ cm}$.

$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1'}$ gives $\frac{1}{\infty}+\frac{1}{5.02\text{ cm}}=\frac{1}{f_1'}$ and $f_1'=5.02$ cm. This is the focal length in water, but to get it in air, we use the formula from Problem 34.98: $f_1=f_1'\left[\frac{n-n_{\text{liq}}}{n_{\text{liq}}(n-1)}\right]=(5.02\text{ cm})\left[\frac{1.52-1.333}{1.333(1.52-1)}\right]=1.35\text{ cm}$.

EVALUATE: A converging lens is needed.

35

INTERFERENCE

35.1. **IDENTIFY:** Compare the path difference to the wavelength.
SET UP: The separation between sources is 5.00 m, so for points between the sources the largest possible path difference is 5.00 m.
EXECUTE: **(a)** For constructive interference the path difference is $m\lambda, m = 0, \pm 1, \pm 2, \ldots$ Thus only the path difference of zero is possible. This occurs midway between the two sources, 2.50 m from *A*.
(b) For destructive interference the path difference is $(m + \frac{1}{2})\lambda,\ m = 0, \pm 1, \pm 2, \ldots$
A path difference of $\pm\lambda/2 = 3.00$ m is possible but a path difference as large as $3\lambda/2 = 9.00$ m is not possible. For a point a distance *x* from *A* and $5.00 - x$ from *B* the path difference is
$x - (5.00\text{ m} - x)$. $x - (5.00\text{ m} - x) = +3.00$ m gives $x = 4.00$ m. $x - (5.00\text{ m} - x) = -3.00$ m gives $x = 1.00$ m .
EVALUATE: The point of constructive interference is midway between the points of destructive interference.

35.2. **IDENTIFY:** For destructive interference the path difference is $(m + \frac{1}{2})\lambda, m = 0, \pm 1, \pm 2, \ldots$. The longest wavelength is for $m = 0$. For constructive interference the path difference is $m\lambda,\ m = 0, \pm 1, \pm 2, \ldots$ The longest wavelength is for $m = 1$.
SET UP: The path difference is 120 m.
EXECUTE: **(a)** For destructive interference $\dfrac{\lambda}{2} = 120\text{ m} \Rightarrow \lambda = 240$ m.
(b) The longest wavelength for constructive interference is $\lambda = 120$ m.
EVALUATE: The path difference doesn't depend on the distance of point *Q* from *B*.

35.3. **IDENTIFY:** Use $c = f\lambda$ to calculate the wavelength of the transmitted waves. Compare the difference in the distance from *A* to *P* and from *B* to *P*. For constructive interference this path difference is an integer multiple of the wavelength.
SET UP: Consider Figure 35.3

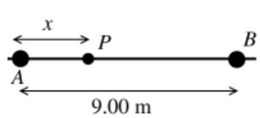


Figure 35.3

The distance of point *P* from each coherent source is $r_A = x$ and $r_B = 9.00\text{ m} - x$.

EXECUTE: The path difference is $r_B - r_A = 9.00\text{ m} - 2x$.
$r_B - r_A = m\lambda,\ m = 0,\ \pm 1,\ \pm 2,\ \ldots$

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8\text{ m/s}}{120 \times 10^6\text{ Hz}} = 2.50\text{ m}$$

Thus $9.00\text{ m} - 2x = m(2.50\text{ m})$ and $x = \dfrac{9.00\text{ m} - m(2.50\text{ m})}{2} = 4.50\text{ m} - (1.25\text{ m})m.$ *x* must lie in the range 0 to 9.00 m since *P* is said to be between the two antennas.
$m = 0$ gives $x = 4.50$ m
$m = +1$ gives $x = 4.50\text{ m} - 1.25\text{ m} = 3.25$ m
$m = +2$ gives $x = 4.50\text{ m} - 2.50\text{ m} = 2.00$ m
$m = +3$ gives $x = 4.50\text{ m} - 3.75\text{ m} = 0.75$ m
$m = -1$ gives $x = 4.50\text{ m} + 1.25\text{ m} = 5.75$ m
$m = -2$ gives $x = 4.50\text{ m} + 2.50\text{ m} = 7.00$ m
$m = -3$ gives $x = 4.50\text{ m} + 3.75\text{ m} = 8.25$ m

All other values of m give values of x out of the allowed range. Constructive interference will occur for $x = 0.75$ m, 2.00 m, 3.25 m, 4.50 m, 5.75 m, 7.00 m, and 8.25 m.

EVALUATE: Constructive interference occurs at the midpoint between the two sources since that point is the same distance from each source. The other points of constructive interference are symmetrically placed relative to this point.

35.4. **IDENTIFY:** For constructive interference the path difference d is related to λ by $d = m\lambda,\ m = 0,1,2,\ldots$ For destructive interference $d = (m+\tfrac{1}{2})\lambda,\ m = 0,1,2,\ldots$

SET UP: $d = 2040$ nm

EXECUTE: **(a)** The brightest wavelengths are when constructive interference occurs:

$d = m\lambda_m \Rightarrow \lambda_m = \dfrac{d}{m} \Rightarrow \lambda_3 = \dfrac{2040\text{ nm}}{3} = 680\text{ nm},\ \lambda_4 = \dfrac{2040\text{ nm}}{4} = 510\text{ nm}$ and

$\lambda_5 = \dfrac{2040\text{ nm}}{5} = 408\text{ nm}.$

(b) The path-length difference is the same, so the wavelengths are the same as part (a).

(c) $d = (m+\tfrac{1}{2})\lambda_m$ so $\lambda_m = \dfrac{d}{m+\frac{1}{2}} = \dfrac{2040\text{ nm}}{m+\frac{1}{2}}$. The visible wavelengths are $\lambda_3 = 583$ nm and $\lambda_4 = 453$ nm.

EVALUATE: The wavelengths for constructive interference are between those for destructive interference.

35.5. **IDENTIFY:** If the path difference between the two waves is equal to a whole number of wavelengths, constructive interference occurs, but if it is an odd number of half-wavelengths, destructive interference occurs.

SET UP: We calculate the distance traveled by both waves and subtract them to find the path difference.

EXECUTE: Call P_1 the distance from the right speaker to the observer and P_2 the distance from the left speaker to the observer.

(a) $P_1 = 8.0$ m and $P_2 = \sqrt{(6.0\text{ m})^2 + (8.0\text{ m})^2} = 10.0$ m. The path distance is

$$\Delta P = P_2 - P_1 = 10.0\text{ m} - 8.0\text{ m} = 2.0\text{ m}$$

(b) The path distance is one wavelength, so constructive interference occurs.

(c) $P_1 = 17.0$ m and $P_2 = \sqrt{(6.0\text{m})^2 + (17.0\text{m})^2} = 18.0$m. The path difference is 18.0 m – 17.0 m = 1.0 m, which is one-half wavelength, so destructive interference occurs.

EVALUATE: Constructive interference also occurs if the path difference 2λ, 3λ, 4λ, etc., and destructive interference occurs if it is $\lambda/2$, $3\lambda/2$, $5\lambda/2$, etc.

35.6. **IDENTIFY:** At an antinode the interference is constructive and the path difference is an integer number of wavelengths; path difference $= m\lambda,\ m = 0,\pm1,\pm2,\ldots$ at an antinode.

SET UP: The maximum magnitude of the path difference is the separation d between the two sources.

EXECUTE: **(a)** At $S_1, r_2 - r_1 = 4\lambda$, and this path difference stays the same all along the y-axis, so $m = +4$. At $S_2, r_2 - r_1 = -4\lambda$, and the path difference below this point, along the negative y-axis, stays the same, so $m = -4$.

(b) The wave pattern is sketched in Figure 35.6.

(c) The maximum and minimum m-values are determined by the largest integer less than or equal to $\dfrac{d}{\lambda}$.

(d) If $d = 7\dfrac{1}{2}\lambda \Rightarrow -7 \le m \le +7$, so there will be a total of 15 antinodes between the sources.

EVALUATE: We are considering points close to the two sources and the antinodal curves are not straight lines.

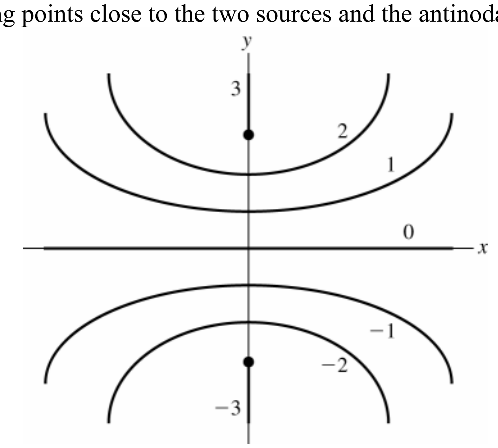


Figure 35.6

35.7. **IDENTIFY:** At an antinodal point the path difference is equal to an integer number of wavelengths.
SET UP: For $m = 3$, the path difference is 3λ.
EXECUTE: Measuring with a ruler from both S_1 and S_2 to the different points in the antinodal line labeled $m = 3$, we find that the difference in path length is three times the wavelength of the wave, as measured from one crest to the next on the diagram.
EVALUATE: There is a whole curve of points where the path difference is 3λ.

35.8. **IDENTIFY:** The value of y_{20} is much smaller than R and the approximate expression $y_m = R\dfrac{m\lambda}{d}$ is accurate.

SET UP: $y_{20} = 10.6\times10^{-3}$ m.

EXECUTE: $d = \dfrac{20R\lambda}{y_{20}} = \dfrac{(20)(1.20\text{ m})(502\times10^{-9}\text{ m})}{10.6\times10^{-3}\text{ m}} = 1.14\times10^{-3}\text{ m} = 1.14\text{ mm}$

EVALUATE: $\tan\theta_{20} = \dfrac{y_{20}}{R}$ so $\theta_{20} = 0.51°$ and the approximation $\sin\theta_{20} \approx \tan\theta_{20}$ is very accurate.

35.9. **IDENTIFY** and **SET UP:** The dark lines correspond to destructive interference and hence are located by Eq.(35.5):

$$d\sin\theta = \left(m+\frac{1}{2}\right)\lambda \text{ so } \sin\theta = \frac{\left(m+\frac{1}{2}\right)\lambda}{d},\ m = 0, \pm1, \pm2, \ldots$$

Solve for θ that locates the second and third dark lines. Use $y = R\tan\theta$ to find the distance of each of the dark lines from the center of the screen.
EXECUTE: 1st dark line is for $m = 0$

2nd dark line is for $m = 1$ and $\sin\theta_1 = \dfrac{3\lambda}{2d} = \dfrac{3(500\times10^{-9}\text{ m})}{2(0.450\times10^{-3}\text{m})} = 1.667\times10^{-3}$ and $\theta_1 = 1.667\times10^{-3}$ rad

3rd dark line is for $m = 2$ and $\sin\theta_2 = \dfrac{5\lambda}{2d} = \dfrac{5(500\times10^{-9}\text{ m})}{2(0.450\times10^{-3}\text{m})} = 2.778\times10^{-3}$ and $\theta_2 = 2.778\times10^{-3}$ rad

(Note that θ_1 and θ_2 are small so that the approximation $\theta \approx \sin\theta \approx \tan\theta$ is valid.) The distance of each dark line from the center of the central bright band is given by $y_m = R\tan\theta$, where $R = 0.850$ m is the distance to the screen.

$\tan\theta \approx \theta$ so $y_m = R\theta_m$

$y_1 = R\theta_1 = (0.750\text{ m})(1.667\times10^{-3}\text{ rad}) = 1.25\times10^{-3}\text{ m}$

$y_2 = R\theta_2 = (0.750\text{ m})(2.778\times10^{-3}\text{ rad}) = 2.08\times10^{-3}\text{ m}$

$\Delta y = y_2 - y_1 = 2.08\times10^{-3}\text{ m} - 1.25\times10^{-3}\text{ m} = 0.83\text{ mm}$

EVALUATE: Since θ_1 and θ_2 are very small we could have used Eq.(35.6), generalized to destructive interference: $y_m = R\left(m+\dfrac{1}{2}\right)\lambda/d$.

35.10. **IDENTIFY:** Since the dark fringes are eqully spaced, $R \gg y_m$, the angles are small and the dark bands are located by $y_{m+\frac{1}{2}} = R\dfrac{(m+\frac{1}{2})\lambda}{d}$.

SET UP: The separation between adjacent dark bands is $\Delta y = \dfrac{R\lambda}{d}$.

EXECUTE: $\Delta y = \dfrac{R\lambda}{d} \Rightarrow d = \dfrac{R\lambda}{\Delta y} = \dfrac{(1.80\text{ m})(4.50\times10^{-7}\text{ m})}{4.20\times10^{-3}\text{ m}} = 1.93\times10^{-4}\text{ m} = 0.193\text{ m}.$

EVALUATE: When the separation between the slits decreases, the separation between dark fringes increases.

35.11. **IDENTIFY** and **SET UP:** The positions of the bright fringes are given by Eq.(35.6): $y_m = R(m\lambda/d)$. For each fringe the adjacent fringe is located at $y_{m+1} = R(m+1)\lambda/d$. Solve for λ.
EXECUTE: The separation between adjacent fringes is $\Delta y = y_{m+1} - y_m = R\lambda/d$.

$$\lambda = \frac{d\Delta y}{R} = \frac{(0.460\times10^{-3}\text{ m})(2.82\times10^{-3}\text{ m})}{2.20\text{ m}} = 5.90\times10^{-7}\text{ m} = 590\text{ nm}$$

EVALUATE: Eq.(35.6) requires that the angular position on the screen be small. The angular position of bright fringes is given by $\sin\theta = m\lambda/d$. The slit separation is much larger than the wavelength $(\lambda/d = 1.3\times10^{-3})$, so θ is small so long as m is not extremely large.

35.12. IDENTIFY: The width of a bright fringe can be defined to be the distance between its two adjacent destructive minima. Assuming the small angle formula for destructive interference $y_m = R\dfrac{(m+\frac{1}{2})\lambda}{d}$.

SET UP: $d = 0.200\times 10^{-3}$ m. $R = 4.00$ m.

EXECUTE: The distance between any two successive minima is $y_{m+1} - y_m = R\dfrac{\lambda}{d} = (4.00 \text{ m})\dfrac{(400\times 10^{-9}\text{m})}{(0.200\times 10^{-3}\text{ m})} = 8.00$ mm. Thus, the answer to both part (a) and part (b) is that the width is 8.00 mm.

EVALUATE: For small angles, when $y_m \ll R$, the interference minima are equally spaced.

35.13. IDENTIFY and SET UP: The dark lines are located by $d\sin\theta = \left(m + \dfrac{1}{2}\right)\lambda.$ The distance of each line from the center of the screen is given by $y = R\tan\theta.$

EXECUTE: First dark line is for $m = 0$ and $d\sin\theta_1 = \lambda/2.$

$\sin\theta_1 = \dfrac{\lambda}{2d} = \dfrac{550\times 10^{-9}\text{ m}}{2(1.80\times 10^{-6}\text{ m})} = 0.1528$ and $\theta_1 = 8.789°.$ Second dark line is for $m = 1$ and $d\sin\theta_2 = 3\lambda/2.$

$\sin\theta_2 = \dfrac{3\lambda}{2d} = 3\left(\dfrac{550\times 10^{-9}\text{ m}}{2(1.80\times 10^{-6}\text{ m})}\right) = 0.4583$ and $\theta_2 = 27.28°.$

$y_1 = R\tan\theta_1 = (0.350\text{ m})\tan 8.789° = 0.0541$ m

$y_2 = R\tan\theta_2 = (0.350\text{ m})\tan 27.28° = 0.1805$ m

The distance between the lines is $\Delta y = y_2 - y_1 = 0.1805\text{ m} - 0.0541\text{ m} = 0.126\text{ m} = 12.6$ cm.

EVALUATE: $\sin\theta_1 = 0.1528$ and $\tan\theta_1 = 0.1546.$ $\sin\theta_2 = 0.4583$ and $\tan\theta_2 = 0.5157.$ As the angle increases, $\sin\theta \approx \tan\theta$ becomes a poorer approximation.

35.14. IDENTIFY: Using Eq.(35.6) for small angles: $y_m = R\dfrac{m\lambda}{d}$.

SET UP: First-order means $m = 1$.

EXECUTE: The distance between corresponding bright fringes is

$$\Delta y = \frac{Rm}{d}\Delta\lambda = \frac{(5.00\text{ m})(1)}{(0.300\times 10^{-3}\text{ m})}(660 - 470)\times(10^{-9}\text{ m}) = 3.17\text{ mm}.$$

EVALUATE: The separation between these fringes for different wavelengths increases when the slit separation decreases.

35.15. IDENTIFY and SET UP: Use the information given about the bright fringe to find the distance *d* between the two slits. Then use Eq.(35.5) and $y = R\tan\theta$ to calculate λ for which there is a first-order dark fringe at this same place on the screen.

EXECUTE: $y_1 = \dfrac{R\lambda_1}{d}$, so $d = \dfrac{R\lambda_1}{y_1} = \dfrac{(3.00\text{ m})(600\times 10^{-9}\text{ m})}{4.84\times 10^{-3}\text{ m}} = 3.72\times 10^{-4}$ m. (*R* is much greater than *d,* so Eq.35.6 is valid.) The dark fringes are located by $d\sin\theta = \left(m + \dfrac{1}{2}\right)\lambda,$ $m = 0, \pm 1, \pm 2, \ldots$ The first order dark fringe is located by $\sin\theta = \lambda_2/2d,$ where λ_2 is the wavelength we are seeking.

$y = R\tan\theta \approx R\sin\theta = \dfrac{\lambda_2 R}{2d}$

We want λ_2 such that $y = y_1.$ This gives $\dfrac{R\lambda_1}{d} = \dfrac{R\lambda_2}{2d}$ and $\lambda_2 = 2\lambda_1 = 1200$ nm.

EVALUATE: For $\lambda = 600$ nm the path difference from the two slits to this point on the screen is 600 nm. For this same path difference (point on the screen) the path difference is $\lambda/2$ when $\lambda = 1200$ nm.

35.16. IDENTIFY: Bright fringes are located at $y_m = R\dfrac{m\lambda}{d}$, when $y_m \ll R$. Dark fringes are at $d\sin\theta = (m + \frac{1}{2})\lambda$ and $y = R\tan\theta$.

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times 10^{8}\text{ m/s}}{6.32\times 10^{14}\text{ Hz}} = 4.75\times 10^{-7}$ m. For the third bright fringe (not counting the central bright spot), $m = 3$. For the third dark fringe, $m = 2$.

EXECUTE: **(a)** $d = \dfrac{m\lambda R}{y_m} = \dfrac{3(4.75\times10^{-7}\ \text{m})(0.850\ \text{m})}{0.0311\ \text{m}} = 3.89\times10^{-5}\ \text{m} = 0.0389\ \text{mm}$

(b) $\sin\theta = (2+\tfrac{1}{2})\dfrac{\lambda}{d} = (2.5)\left(\dfrac{4.75\times10^{-7}\ \text{m}}{3.89\times10^{-5}\ \text{m}}\right) = 0.0305$ and $\theta = 1.75°$. $y = R\tan\theta = (85.0\ \text{cm})\tan1.75° = 2.60\ \text{cm}$.

EVALUATE: The third dark fringe is closer to the center of the screen than the third bright fringe on one side of the central bright fringe.

35.17. **IDENTIFY:** Bright fringes are located at angles θ given by $d\sin\theta = m\lambda$.

SET UP: The largest value $\sin\theta$ can have is 1.00.

EXECUTE: **(a)** $m = \dfrac{d\sin\theta}{\lambda}$. For $\sin\theta = 1$, $m = \dfrac{d}{\lambda} = \dfrac{0.0116\times10^{-3}\ \text{m}}{5.85\times10^{-7}\ \text{m}} = 19.8$. Therefore, the largest m for fringes on the screen is $m = 19$. There are $2(19)+1 = 39$ bright fringes, the central one and 19 above and 19 below it.

(b) The most distant fringe has $m = \pm19$. $\sin\theta = m\dfrac{\lambda}{d} = \pm19\left(\dfrac{5.85\times10^{-7}\ \text{m}}{0.0116\times10^{-3}\ \text{m}}\right) = \pm0.958$ and $\theta = \pm73.3°$.

EVALUATE: For small θ the spacing Δy between adjacent fringes is constant but this is no longer the case for larger angles.

35.18. **IDENTIFY:** At large distances from the antennas the equation $d\sin\theta = m\lambda,\ m = 0,\pm1,\pm2,\ldots$ gives the angles where maximum intensity is observed and $d\sin\theta = (m+\tfrac{1}{2})\lambda,\ m = 0,\pm1,\pm2,\ldots$ gives the angles where minimum intensity is observed.

SET UP: $d = 12.0\ \text{m}$. $\lambda = \dfrac{c}{f}$.

EXECUTE: **(a)** $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\ \text{m/s}}{107.9\times10^{6}\ \text{Hz}} = 2.78\ \text{m}$. $\sin\theta = \dfrac{m\lambda}{d} = m\left(\dfrac{2.78\ \text{m}}{12.0\ \text{m}}\right) = m(0.232)$.

$\theta = \pm13.4°,\ \pm27.6°,\ \pm44.1°,\ \pm68.1°$.

(b) $\sin\theta = (m+\tfrac{1}{2})\dfrac{\lambda}{d} = (m+\tfrac{1}{2})(0.232)$. $\theta = \pm6.66°,\ \pm20.4°,\ \pm35.5°,\ \pm54.3°$.

EVALUATE: The angles for zero intensity are approximately midway between those for maximum intensity.

35.19. **IDENTIFY:** Eq.(35.10): $I = I_0\cos^2(\phi/2)$. Eq.(35.11): $\phi = (2\pi/\lambda)(r_2 - r_1)$.

SET UP: ϕ is the phase difference and $(r_2 - r_1)$ is the path difference.

EXECUTE: **(a)** $I = I_0(\cos 30.0°)^2 = 0.750I_0$

(b) $60.0° = (\pi/3)\ \text{rad}$. $(r_2 - r_1) = (\phi/2\pi)\lambda = [(\pi/3)/2\pi]\lambda = \lambda/6 = 80\ \text{nm}$.

EVALUATE: $\phi = 360°/6$ and $(r_2 - r_1) = \lambda/6$.

35.20. **IDENTIFY:** $\dfrac{\Delta\phi}{2\pi} = \dfrac{\text{path difference}}{\lambda}$ relates the path difference to the phase difference $\Delta\phi$.

SET UP: The sources and point P are shown in Figure 35.20.

EXECUTE: $\Delta\phi = 2\pi\left(\dfrac{524\ \text{cm} - 486\ \text{cm}}{2\ \text{cm}}\right) = 119\ \text{radians}$

EVALUATE: The distances from B to P and A to P aren't important, only the difference in these distances.

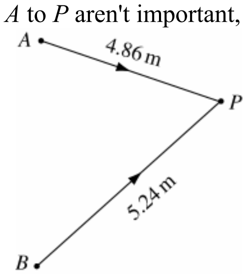


Figure 35.20

35.21. **IDENTIFY** and **SET UP:** The phase difference ϕ is given by $\phi = (2\pi d/\lambda)\sin\theta$ (Eq.35.13.)

EXECUTE: $\phi = [2\pi(0.340\times10^{-3}\ \text{m})/(500\times10^{-9}\ \text{m})]\sin 23.0° = 1670\ \text{rad}$

EVALUATE: The mth bright fringe occurs when $\phi = 2\pi m$, so there are a large number of bright fringes within $23.0°$ from the centerline. Note that Eq.(35.13) gives ϕ in radians.

35.22. IDENTIFY: The maximum intensity occurs at all the points of constructive interference. At these points, the path difference between waves from the two transmitters is an integral number of wavelengths.

SET UP: For constructive interference, $\sin\theta = m\lambda/d$.

EXECUTE: **(a)** First find the wavelength of the UHF waves:

$$\lambda = c/f = (3.00\times10^8 \text{ m/s})/(1575.42 \text{ MHz}) = 0.1904 \text{ m}$$

For maximum intensity $(\pi d \sin\theta)/\lambda = m\pi$, so

$$\sin\theta = m\lambda/d = m[(0.1904 \text{ m})/(5.18 \text{ m})] = 0.03676m$$

The maximum possible m would be for $\theta = 90°$, or $\sin\theta = 1$, so

$$m_{\text{max}} = d/\lambda = (5.18 \text{ m})/(0.1904 \text{ m}) = 27.2$$

which must be ±27 since m is an integer. The total number of maxima is 27 on either side of the central fringe, plus the central fringe, for a total of 27 + 27 + 1 = 55 bright fringes.

(b) Using $\sin\theta = m\lambda/d$, where $m = 0, \pm1, \pm2$, and ±3, we have

$$\sin\theta = m\lambda/d = m[(0.1904 \text{ m})/(5.18 \text{ m})] = 0.03676m$$

$m = 0$: $\sin\theta = 0$, which gives $\theta = 0°$

$m = \pm1$: $\sin\theta = \pm(0.03676)(1)$, which gives $\theta = \pm2.11°$

$m = \pm2$: $\sin\theta = \pm(0.03676)(2)$, which gives $\theta = \pm4.22°$

$m = \pm3$: $\sin\theta = \pm(0.03676)(3)$, which gives $\theta = \pm6.33°$

(c) $I = I_0\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right) = \left(2.00 \text{ W/m}^2\right)\cos^2\left[\dfrac{\pi(5.18 \text{ m})\sin(4.65°)}{0.1904 \text{ m}}\right] = 1.28 \text{ W/m}^2.$

EVALUATE: Notice that $\sin\theta$ increases in integer steps, but θ only increases in integer steps for small θ.

35.23. (a) IDENTIFY and SET UP: The minima are located at angles θ given by $d\sin\theta = \left(m+\frac{1}{2}\right)\lambda$. The first minimum corresponds to $m = 0$. Solve for θ. Then the distance on the screen is $y = R\tan\theta$.

EXECUTE: $\sin\theta = \dfrac{\lambda}{2d} = \dfrac{660\times10^{-9} \text{ m}}{2(0.260\times10^{-3} \text{ m})} = 1.27\times10^{-3}$ and $\theta = 1.27\times10^{-3}$ rad

$y = (0.700 \text{ m})\tan(1.27\times10^{-3} \text{ rad}) = 0.889 \text{ mm}.$

(b) IDENTIFY and SET UP: Eq.(35.15) given the intensity I as a function of the position y on the screen: $I = I_0\cos^2\left(\dfrac{\pi dy}{\lambda R}\right)$. Set $I = I_0/2$ and solve for y.

EXECUTE: $I = \frac{1}{2}I_0$ says $\cos^2\left(\dfrac{\pi dy}{\lambda R}\right) = \dfrac{1}{2}$

$\cos\left(\dfrac{\pi dy}{\lambda R}\right) = \dfrac{1}{\sqrt{2}}$ so $\dfrac{\pi dy}{\lambda R} = \dfrac{\pi}{4}$ rad

$y = \dfrac{\lambda R}{4d} = \dfrac{(660\times10^{-9} \text{ m})(0.700 \text{ m})}{4(0.260\times10^{-3} \text{ m})} = 0.444 \text{ mm}$

EVALUATE: $I = I_0/2$ at a point on the screen midway between where $I = I_0$ and $I = 0$.

35.24. IDENTIFY: Eq. (35.14): $I = I_0\cos^2\left(\dfrac{\pi d}{\lambda}\sin\theta\right)$.

SET UP: The intensity goes to zero when the cosine's argument becomes an odd integer multiple of $\dfrac{\pi}{2}$

EXECUTE: $\dfrac{\pi d}{\lambda}\sin\theta = (m+1/2)\pi$ gives $d\sin\theta = \lambda(m+1/2)$, which is Eq. (35.5).

EVALUATE: Section 35.3 shows that the maximum-intensity directions from Eq.(35.14) agree with Eq.(35.4).

35.25. IDENTIFY: The intensity decreases as we move away from the central maximum.

SET UP: The intensity is given by $I = I_0\cos^2\left(\dfrac{\pi dy}{\lambda R}\right)$.

EXECUTE: First find the wavelength: $\lambda = c/f = (3.00\times10^8 \text{ m/s})/(12.5 \text{ MHz}) = 24.00$ m

At the farthest the receiver can be placed, $I = I_0/4$, which gives

$$\frac{I_0}{4} = I_0\cos^2\left(\frac{\pi dy}{\lambda R}\right) \Rightarrow \cos^2\left(\frac{\pi dy}{\lambda R}\right) = \frac{1}{4} \Rightarrow \cos\left(\frac{\pi dy}{\lambda R}\right) = \pm\frac{1}{2}$$

The solutions are $\pi dy/\lambda R = \pi/3$ and $2\pi/3$. Using $\pi/3$, we get

$$y = \lambda R/3d = (24.00\text{ m})(500\text{ m})/[3(56.0\text{ m})] = 71.4\text{ m}$$

It must remain within 71.4 m of point *C*.

EVALUATE: Using $\pi dy/\lambda R = 2\pi/3$ gives $y = 142.8$ m. But to reach this point, the receiver would have to go beyond 71.4 m from *C*, where the signal would be too weak, so this second point is not possible.

35.26. **IDENTIFY:** The phase difference ϕ and the path difference $r_1 - r_2$ are related by $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2)$. The intensity is given by $I = I_0\cos^2\left(\dfrac{\phi}{2}\right)$.

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8\text{ m/s}}{1.20\times10^8\text{ Hz}} = 2.50\text{ m}$. When the receiver measures zero intensity I_0, $\phi = 0$.

EXECUTE: **(a)** $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2) = \dfrac{2\pi}{2.50\text{ m}}(1.8\text{ m}) = 4.52\text{ rad}$.

(b) $I = I_0\cos^2\left(\dfrac{\phi}{2}\right) = I_0\cos^2\left(\dfrac{4.52\text{ rad}}{2}\right) = 0.404I_0$.

EVALUATE: $(r_1 - r_2)$ is greater than $\lambda/2$, so one minimum has been passed as the receiver is moved.

35.27. **IDENTIFY:** Consider interference between rays reflected at the upper and lower surfaces of the film. Consider phase difference due to the path difference of $2t$ and any phase differences due to phase changes upon reflection.

SET UP: Consider Figure 35.27.

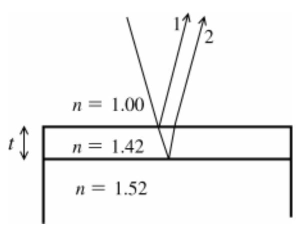


Figure 35.27

Both rays (1) and (2) undergo a 180° phase change on reflection, so these is no net phase difference introduced and the condition for destructive interference is $2t = \left(m + \dfrac{1}{2}\right)\lambda.$

EXECUTE: $t = \dfrac{\left(m + \frac{1}{2}\right)\lambda}{2}$; thinnest film says $m = 0$ so $t = \dfrac{\lambda}{4}$

$\lambda = \dfrac{\lambda_0}{1.42}$ and $t = \dfrac{\lambda_0}{4(1.42)} = \dfrac{650\times10^{-9}\text{ m}}{4(1.42)} = 1.14\times10^{-7}\text{ m} = 114\text{ nm}$

EVALUATE: We compared the path difference to the wavelength in the film, since that is where the path difference occurs.

35.28. **IDENTIFY:** Require destructive interference for light reflected at the front and rear surfaces of the film.

SET UP: At the front surface of the film, light in air ($n = 1.00$) reflects from the film ($n = 2.62$) and there is a 180° phase shift due to the reflection. At the back surface of the film, light in the film ($n = 2.62$) reflects from glass ($n = 1.62$) and there is no phase shift due to reflection. Therefore, there is a net 180° phase difference produced by the reflections. The path difference for these two rays is $2t$, where t is the thickness of the film. The wavelength in the film is $\lambda = \dfrac{505\text{ nm}}{2.62}$.

EXECUTE: **(a)** Since the reflection produces a net 180° phase difference, destructive interference of the reflected light occurs when $2t = m\lambda$. $t = m\left(\dfrac{505\text{ nm}}{2[2.62]}\right) = (96.4\text{ nm})m$. The minimum thickness is 96.4 nm.

(b) The next three thicknesses are for $m = 2$, 3 and 4: 192 nm, 289 nm and 386 nm.

EVALUATE: The minimum thickness is for $t = \lambda/2n$. Compare this to Problem 35.27, where the minimum thickness for destructive interference is $t = \lambda/4n$.

35.29. **IDENTIFY:** The fringes are produced by interference between light reflected from the top and bottom surfaces of the air wedge. The refractive index of glass is greater than that of air, so the waves reflected from the top surface of the air wedge have no reflection phase shift and the waves reflected from the bottom surface of the air wedge do

have a half-cycle reflection phase shift. The condition for constructive interference (bright fringes) is therefore $2t = (m + \frac{1}{2})\lambda$.

SET UP: The geometry of the air wedge is sketched in Figure 35.29. At a distance x from the point of contact of the two plates, the thickness of the air wedge is t.

EXECUTE: $\tan\theta = \frac{t}{x}$ so $t = x\tan\theta$. $t_m = (m + \frac{1}{2})\frac{\lambda}{2}$. $x_m = (m + \frac{1}{2})\frac{\lambda}{2\tan\theta}$ and $x_{m+1} = (m + \frac{3}{2})\frac{\lambda}{2\tan\theta}$. The distance along the plate between adjacent fringes is $\Delta x = x_{m+1} - x_m = \frac{\lambda}{2\tan\theta}$. 15.0 fringes/cm $= \frac{1.00}{\Delta x}$ and $\Delta x = \frac{1.00}{15.0 \text{ fringes/cm}} = 0.0667 \text{ cm}$. $\tan\theta = \frac{\lambda}{2\Delta x} = \frac{546\times10^{-9} \text{ m}}{2(0.0667\times10^{-2} \text{ m})} = 4.09\times10^{-4}$. The angle of the wedge is $4.09\times10^{-4} \text{ rad} = 0.0234°$.

EVALUATE: The fringes are equally spaced; Δx is independent of m.

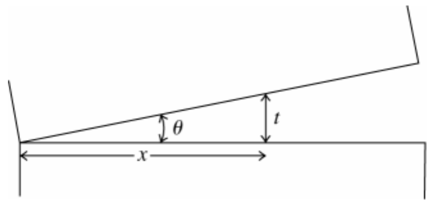


Figure 35.29

35.30. **IDENTIFY:** The fringes are produced by interference between light reflected from the top and from the bottom surfaces of the air wedge. The refractive index of glass is greater than that of air, so the waves reflected from the top surface of the air wedge have no reflection phase shift and the waves reflected from the bottom surface of the air wedge do have a half-cycle reflection phase shift. The condition for constructive interference (bright fringes) therefore is $2t = (m + \frac{1}{2})\lambda$.

SET UP: The geometry of the air wedge is sketched in Figure 35.30.

EXECUTE: $\tan\theta = \frac{0.0800 \text{ mm}}{90.0 \text{ mm}} = 8.89\times10^{-4}$. $\tan\theta = \frac{t}{x}$ so $t = (8.89\times10^{-4})x$. $t_m = (m + \frac{1}{2})\frac{\lambda}{2}$.

$x_m = (m + \frac{1}{2})\frac{\lambda}{2(8.89\times10^{-4})}$ and $x_{m+1} = (m + \frac{3}{2})\frac{\lambda}{2(8.89\times10^{-4})}$. The distance along the plate between adjacent fringes is $\Delta x = x_{m+1} - x_m = \frac{\lambda}{2(8.89\times10^{-4})} = \frac{656\times10^{-9} \text{ m}}{2(8.89\times10^{-4})} = 3.69\times10^{-4} \text{ m} = 0.369 \text{ mm}$. The number of fringes per cm is $\frac{1.00}{\Delta x} = \frac{1.00}{0.0369 \text{ cm}} = 27.1 \text{ fringes/cm}$.

EVALUATE: As $t \to 0$ the interference is destructive and there is a dark fringe at the line of contact between the two plates.

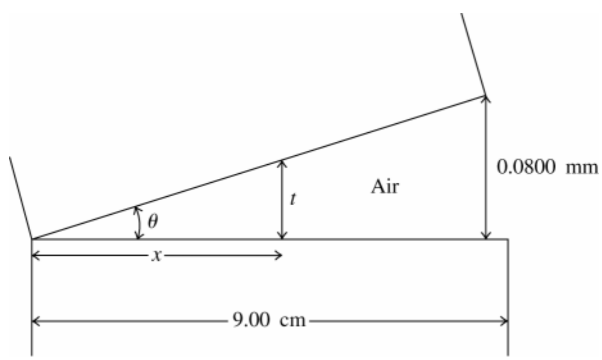


Figure 35.30

35.31. **IDENTIFY:** The light reflected from the top of the TiO_2 film interferes with the light reflected from the top of the glass surface. These waves are out of phase due to the path difference in the film and the phase differences caused by reflection.

SET UP: There is a π phase change at the TiO_2 surface but none at the glass surface, so for destructive interference the path difference must be $m\lambda$ in the film.

EXECUTE: **(a)** Calling T the thickness of the film gives $2T = m\lambda_0/n$, which yields $T = m\lambda_0/(2n)$. Substituting the numbers gives

$$T = m \text{ (520.0 nm)}/[2(2.62)] = 99.237m$$

T must be greater than 1036 nm, so $m = 11$, which gives $T = 1091.6$ nm, since we want to know the minimum thickness to add.

$$\Delta T = 1091.6 \text{ nm} - 1036 \text{ nm} = 55.6 \text{ nm}$$

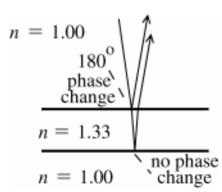
(b) (i) Path difference = $2T = 2(1092 \text{ nm}) = 2184 \text{ nm} = 2180$ nm.
(ii) The wavelength in the film is $\lambda = \lambda_0/n = (520.0 \text{ nm})/2.62 = 198.5$ nm.

$$\text{Path difference} = (2180 \text{ nm})/[(198.5 \text{ nm})/\text{wavelength}] = 11.0 \text{ wavelengths}$$

EVALUATE: Because the path difference in the film is 11.0 wavelengths, the light reflected off the top of the film will be 180° out of phase with the light that traveled through the film and was reflected off the glass due to the phase change at reflection off the top of the film.

35.32. **IDENTIFY:** Consider the phase difference produced by the path difference and by the reflections. For destructive interference the total phase difference is an integer number of half cycles.
SET UP: The reflection at the top surface of the film produces a half-cycle phase shift. There is no phase shift at the reflection at the bottom surface.
EXECUTE: **(a)** Since there is a half-cycle phase shift at just one of the interfaces, the minimum thickness for constructive interference is $t = \dfrac{\lambda}{4} = \dfrac{\lambda_0}{4n} = \dfrac{550 \text{ nm}}{4(1.85)} = 74.3$ nm.
(b) The next smallest thickness for constructive interference is with another half wavelength thickness added:
$t = \dfrac{3\lambda}{4} = \dfrac{3\lambda_0}{4n} = \dfrac{3(550 \text{ nm})}{4(1.85)} = 223$ nm.
EVALUATE: Note that we must compare the path difference to the wavelength in the film.

35.33. **IDENTIFY:** Consider the interference between rays reflected from the two surfaces of the soap film. Strongly reflected means constructive interference. Consider phase difference due to the path difference of $2t$ and any phase difference due to phase changes upon reflection.
(a) SET UP: Consider Figure 35.33.



There is a 180° phase change when the light is reflected from the outside surface of the bubble and no phase change when the light is reflected from the inside surface.

Figure 35.33

EXECUTE: The reflections produce a net 180° phase difference and for there to be constructive interference the path difference $2t$ must correspond to a half-integer number of wavelengths to compensate for the $\lambda/2$ shift due to the reflections. Hence the condition for constructive interference is $2t = \left(m + \frac{1}{2}\right)(\lambda_0/n)$, $m = 0, 1, 2, \ldots$ Here λ_0 is the wavelength in air and (λ_0/n) is the wavelength in the bubble, where the path difference occurs.

$$\lambda_0 = \frac{2tn}{m + \frac{1}{2}} = \frac{2(290 \text{ nm})(1.33)}{m + \frac{1}{2}} = \frac{771.4 \text{ nm}}{m + \frac{1}{2}}$$

for $m = 0$, $\lambda = 1543$ nm; for $m = 1$, $\lambda = 514$ nm; for $m = 2$, $\lambda = 308$ nm;… Only 514 nm is in the visible region; the color for this wavelength is green.

(b) $\lambda_0 = \dfrac{2tn}{m + \frac{1}{2}} = \dfrac{2(340 \text{ nm})(1.33)}{m + \frac{1}{2}} = \dfrac{904.4 \text{ nm}}{m + \frac{1}{2}}$

for $m = 0$, $\lambda = 1809$ nm; for $m = 1$, $\lambda = 603$ nm; for $m = 2$, $\lambda = 362$ nm;… Only 603 nm is in the visible region; the color for this wavelength is orange.
EVALUATE: The dominant color of the reflected light depends on the thickness of the film. If the bubble has varying thickness at different points, these points will appear to be different colors when the light reflected from the bubble is viewed.

35.34. **IDENTIFY:** The number of waves along the path is the path length divided by the wavelength. The path difference and the reflections determine the phase difference.
SET UP: The path length is $2t = 17.52 \times 10^{-6}$ m. The wavelength in the film is $\lambda = \dfrac{\lambda_0}{n}$.

EXECUTE: **(a)** $\lambda = \dfrac{648 \text{ nm}}{1.35} = 480 \text{ nm}$. The number of waves is $\dfrac{2t}{\lambda} = \dfrac{17.52\times 10^{-6} \text{ m}}{480\times 10^{-9} \text{ m}} = 36.5$.

(b) The path difference introduces a $\lambda/2$, or $180°$, phase difference. The ray reflected at the top surface of the film undergoes a $180°$ phase shift upon reflection. The reflection at the lower surface introduces no phase shift. Both rays undergo a $180°$ phase shift, one due to reflection and one due to reflection. The two effects cancel and the two rays are in phase as they leave the film.

EVALUATE: Note that we must use the wavelength in the film to determine the number of waves in the film.

35.35. **IDENTIFY:** Require destructive interference between light reflected from the two points on the disc.

SET UP: Both reflections occur for waves in the plastic substrate reflecting from the reflective coating, so they both have the same phase shift upon reflection and the condition for destructive interference (cancellation) is $2t = (m+\frac{1}{2})\lambda$, where t is the depth of the pit. $\lambda = \dfrac{\lambda_0}{n}$. The minimum pit depth is for $m = 0$.

EXECUTE: $2t = \dfrac{\lambda}{2}$. $t = \dfrac{\lambda}{4} = \dfrac{\lambda_0}{4n} = \dfrac{790 \text{ nm}}{4(1.8)} = 110 \text{ nm} = 0.11\ \mu\text{m}$.

EVALUATE: The path difference occurs in the plastic substrate and we must compare the wavelength in the substrate to the path difference.

35.36. **IDENTIFY:** Consider light reflected at the front and rear surfaces of the film.

SET UP: At the front surface of the film, light in air ($n = 1.00$) reflects from the film ($n = 2.62$) and there is a $180°$ phase shift due to the reflection. At the back surface of the film, light in the film ($n = 2.62$) reflects from glass ($n = 1.62$) and there is no phase shift due to reflection. Therefore, there is a net $180°$ phase difference produced by the reflections. The path difference for these two rays is $2t$, where t is the thickness of the film. The wavelength in the film is $\lambda = \dfrac{505 \text{ nm}}{2.62}$.

EXECUTE: **(a)** Since the reflection produces a net $180°$ phase difference, destructive interference of the reflected light occurs when $2t = m\lambda$. $t = m\left(\dfrac{505 \text{ nm}}{2[2.62]}\right) = (96.4 \text{ nm})m$. The minimum thickness is 96.4 nm.

(b) The next three thicknesses are for $m = 2$, 3 and 4: 192 nm, 289 nm and 386 nm.

EVALUATE: The minimum thickness is for $t = \lambda/2n$. Compare this to Problem 34.27, where the minimum thickness for destructive interference is $t = \lambda/4n$.

35.37. **IDENTIFY** and **SET UP:** Apply Eq.(35.19) and calculate y for $m = 1800$.

EXECUTE: Eq.(35.19): $y = m(\lambda/2) = 1800(633\times 10^{-9} \text{ m})/2 = 5.70\times 10^{-4} \text{ m} = 0.570 \text{ mm}$

EVALUATE: A small displacement of the mirror corresponds to many wavelengths and a large number of fringes cross the line.

35.38. **IDENTIFY:** Apply Eq.(35.19).

SET UP: $m = 818$. Since the fringes move in opposite directions, the two people move the mirror in opposite directions.

EXECUTE: **(a)** For Jan, the total shift was $y_1 = \dfrac{m\lambda_1}{2} = \dfrac{818(6.06\times 10^{-7} \text{ m})}{2} = 2.48\times 10^{-4}$ m. For Linda, the total shift was $y_2 = \dfrac{m\lambda_2}{2} = \dfrac{818(5.02\times 10^{-7} \text{ m})}{2} = 2.05\times 10^{-4}$ m.

(b) The net displacement of the mirror is the difference of the above values:

$$\Delta y = y_1 - y_2 = 0.248 \text{ mm} - 0.205 \text{ mm} = 0.043 \text{ mm}.$$

EVALUATE: The person using the larger wavelength moves the mirror the greater distance.

35.39. **IDENTIFY:** Consider the interference between light reflected from the top and bottom surfaces of the air film between the lens and the glass plate.

SET UP: For maximum intensity, with a net half-cycle phase shift due to reflections, $2t = \left(m + \dfrac{1}{2}\right)\lambda$. $t = R - \sqrt{R^2 - r^2}$.

EXECUTE: $\dfrac{(2m+1)\lambda}{4} = R - \sqrt{R^2 - r^2} \Rightarrow \sqrt{R^2 - r^2} = R - \dfrac{(2m+1)\lambda}{4}$

$$\Rightarrow R^2 - r^2 = R^2 + \left[\frac{(2m+1)\lambda}{4}\right]^2 - \frac{(2m+1)\lambda R}{2} \Rightarrow r = \sqrt{\frac{(2m+1)\lambda R}{2} - \left[\frac{(2m+1)\lambda}{4}\right]^2}$$

$$\Rightarrow r \approx \sqrt{\frac{(2m+1)\lambda R}{2}}, \text{ for } R \gg \lambda.$$

The second bright ring is when $m = 1$:

$$r \approx \sqrt{\frac{(2(1)+1)\,(5.80\times10^{-7}\,\text{m})\,(0.952\ \text{m})}{2}} = 9.10\times10^{-4}\,\text{m} = 0.910\ \text{mm}.$$

So the diameter of the second bright ring is 1.82 mm.

EVALUATE: The diameter of the mth ring is proportional to $\sqrt{2m+1}$, so the rings get closer together as m increases. This agrees with Figure 35.17b in the textbook.

35.40. IDENTIFY: As found in Problem 35.39, the radius of the mth bright ring is $r \approx \sqrt{\frac{(2m+1)\lambda R}{2}}$, for $R \gg \lambda$.

SET UP: Introducing a liquid between the lens and the plate just changes the wavelength from λ to $\frac{\lambda}{n}$, where n is the refractive index of the liquid.

EXECUTE: $r(n) \approx \sqrt{\frac{(2m+1)\lambda R}{2n}} = \frac{r}{\sqrt{n}} = \frac{0.850\ \text{mm}}{\sqrt{1.33}} = 0.737\ \text{mm}.$

EVALUATE: The refractive index of the water is less than that of the glass plate, so the phase changes on reflection are the same as when air is in the space.

35.41. IDENTIFY: The liquid alters the wavelength of the light and that affects the locations of the interference minima.

SET UP: The interference minima are located by $d\sin\theta = (m+\frac{1}{2})\lambda$. For a liquid with refractive index n, $\lambda_{\text{liq}} = \frac{\lambda_{\text{air}}}{n}$.

EXECUTE: $\frac{\sin\theta}{\lambda} = \frac{(m+\frac{1}{2})}{d} =$ constant, so $\frac{\sin\theta_{\text{air}}}{\lambda_{\text{air}}} = \frac{\sin\theta_{\text{liq}}}{\lambda_{\text{liq}}}$. $\frac{\sin\theta_{\text{air}}}{\lambda_{\text{air}}} = \frac{\sin\theta_{\text{liq}}}{\lambda_{\text{air}}/n}$ and $n = \frac{\sin\theta_{\text{air}}}{\sin\theta_{\text{liq}}} = \frac{\sin 35.20^\circ}{\sin 19.46^\circ} = 1.730$.

EVALUATE: In the liquid the wavelength is shorter and $\sin\theta = (m+\frac{1}{2})\frac{\lambda}{d}$ gives a smaller θ than in air, for the same m.

35.42. IDENTIFY: As the brass is heated, thermal expansion will cause the two slits to move farther apart.

SET UP: For destructive interference, $d\sin\theta = \lambda/2$. The change in separation due to thermal expansion is $dw = \alpha w_0\,dT$, where w is the distance between the slits.

EXECUTE: The first dark fringe is at $d\sin\theta = \lambda/2 \Rightarrow \sin\theta = \lambda/2d$.

Call $d \equiv w$ for these calculations to avoid confusion with the differential. $\sin\theta = \lambda/2w$

Taking differentials gives $d(\sin\theta) = d(\lambda/2w)$ and $\cos\theta\,d\theta = -\lambda/2\ dw/w^2$.

For thermal expansion, $dw = \alpha w_0\,dT$, which gives $\cos\theta\,d\theta = -\frac{\lambda}{2}\frac{\alpha w_0 dT}{w_0^2} = -\frac{\lambda\alpha dT}{2w_0}$. Solving for $d\theta$ gives

$d\theta = -\frac{\lambda\alpha dT}{2w_0\cos\theta_0}$. Get λ: $w_0\sin\theta_0 = \lambda/2 \rightarrow \lambda = 2w_0\sin\theta_0$. Substituting this quantity into the equation for $d\theta$ gives

$$d\theta = -\frac{2w_0\sin\theta_0\alpha dT}{2w_0\cos\theta_0} = -\tan\theta_0\,\alpha dT.$$

$$d\theta = -\tan(32.5^\circ)(2.0\times10^{-5}\ \text{K}^{-1})(115\ \text{K}) = -0.001465\ \text{rad} = -0.084^\circ$$

The minus sign tells us that the dark fringes move closer together.

EVALUATE: We can also see that the dark fringes move closer together because $\sin\theta$ is proportional to $1/d$, so as d increases due to expansion, θ decreases.

35.43. IDENTIFY: *Both* frequencies will interfere constructively when the path difference from both of them is an integral number of wavelengths.

SET UP: Constructive interference occurs when $\sin\theta = m\lambda/d$.

EXECUTE: First find the two wavelengths.

$$\lambda_1 = v/f_1 = (344\ \text{m/s})/(900\ \text{Hz}) = 0.3822\ \text{m}$$

$$\lambda_2 = v/f_2 = (344\ \text{m/s})/(1200\ \text{Hz}) = 0.2867\ \text{m}$$

To interfere constructively at the same angle, the angles must be the same, and hence the sines of the angles must be equal. Each sine is of the form $\sin\theta = m\lambda/d$, so we can equate the sines to get

$$m_1\lambda_1/d = m_2\lambda 2/d$$

$$m_1(0.3822\ \text{m}) = m_2(0.2867\ \text{m})$$

$$m_2 = 4/3\ m_1$$

Since both m_1 and m_2 must be integers, the allowed pairs of values of m_1 and m_2 are

$$m_1 = m_2 = 0$$
$$m_1 = 3,\ m_2 = 4$$
$$m_1 = 6,\ m_2 = 8$$
$$m_1 = 9,\ m_2 = 12$$
etc.

For $m_1 = m_2 = 0$, we have $\theta = 0$.
For $m_1 = 3$, $m_2 = 4$, we have $\sin\theta_1 = (3)(0.3822\text{ m})/(2.50\text{ m})$, giving $\theta_1 = 27.3°$
For $m_1 = 6$, $m_2 = 8$, we have $\sin\theta_1 = (6)(0.3822\text{ m})/(2.50\text{ m})$, giving $\theta_1 = 66.5°$
For $m_1 = 9$, $m_2 = 12$, we have $\sin\theta_1 = (9)(0.3822\text{ m})/(2.50\text{ m}) = 1.38 > 1$, so no angle is possible.
EVALUATE: At certain other angles, one frequency will interfere constructively, but the other will not.

35.44. **IDENTIFY:** For destructive interference, $d = r_2 - r_1 = \left(m + \frac{1}{2}\right)\lambda$.

SET UP: $r_2 - r_1 = \sqrt{(200\text{ m})^2 + x^2} - x$

EXECUTE: $(200\text{ m})^2 + x^2 = x^2 + \left[\left(m + \frac{1}{2}\right)\lambda\right]^2 + 2x\left(m + \frac{1}{2}\right)\lambda$.

$x = \dfrac{20{,}000\text{ m}^2}{\left(m + \frac{1}{2}\right)\lambda} - \dfrac{1}{2}\left(m + \frac{1}{2}\right)\lambda$. The wavelength is calculated by $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8\text{ m/s}}{5.80\times10^6\text{ Hz}} = 51.7\text{ m}$.

$m = 0: x = 761\text{ m};\ m = 1: x = 219\text{ m};\ m = 2: x = 90.1\text{ m};\ m = 3; x = 20.0\text{ m}$.

EVALUATE: For $m = 3$, $d = 3.5\lambda = 181\text{ m}$. The maximum possible path difference is the separation of 200 m between the sources.

35.45. **IDENTIFY:** The two scratches are parallel slits, so the light that passes through them produces an interference pattern. However the light is traveling through a medium (plastic) that is different from air.
SET UP: The central bright fringe is bordered by a dark fringe on each side of it. At these dark fringes, $d\sin\theta = ½\ \lambda/n$, where n is the refractive index of the plastic.
EXECUTE: First use geometry to find the angles at which the two dark fringes occur. At the first dark fringe $\tan\theta = [(5.82\text{ mm})/2]/(3250\text{ mm})$, giving $\theta = \pm 0.0513°$
For destructive interference, we have $d\sin\theta = ½\ \lambda/n$ and

$$n = \lambda/(2d\sin\theta) = (632.8\text{ nm})/[2(0.000225\text{ m})(\sin 0.0513°)] = 1.57$$

EVALUATE: The wavelength of the light in the plastic is reduced compared to what it would be in air.

35.46. **IDENTIFY:** Interference occurs due to the path difference of light in the thin film.
SET UP: Originally the path difference was an odd number of half-wavelengths for cancellation to occur. If the path difference decreases by ½ wavelength, it will be a multiple of the wavelength, so constructive interference will occur.
EXECUTE: Calling ΔT the thickness that must be removed, we have

path difference = $2\Delta T = ½\ \lambda/n$ and $\Delta T = \lambda/4n = (525\text{ nm})/[4(1.40)] = 93.75\text{ nm}$,

At 4.20 nm/yr, we have $(4.20\text{ nm/yr})t = 93.75$ nm and $t = 22.3$ yr.
EVALUATE: If you were giving a warranty on this film, you certainly could not give it a "lifetime guarantee"!

35.47. **IDENTIFY** and **SET UP:** If the total phase difference is an integer number of cycles the interference is constructive and if it is a half-integer number of cycles it is destructive.
EXECUTE: **(a)** If the two sources are out of phase by one half-cycle, we must add an extra half a wavelength to the path difference equations Eq.(35.1) and Eq.(35.2). This exactly changes one for the other, for $m \to m + \frac{1}{2}$ and $m + \frac{1}{2} \to m$, since m in any integer.

(b) If one source leads the other by a phase angle ϕ, the fraction of a cycle difference is $\dfrac{\phi}{2\pi}$. Thus the path length difference for the two sources must be adjusted for both destructive and constructive interference, by this amount. So for constructive inference: $r_1 - r_2 = (m + \phi/2\pi)\lambda$, and for destructive interference, $r_1 - r_2 = (m + 1/2 + \phi/2\pi)\lambda$, where in each case $m = 0, \pm1, \pm2, \ldots$

EVALUATE: If $\phi = 0$ these results reduce to Eqs.(35.1) and (35.2).

35.48. **IDENTIFY:** Follow the steps specified in the problem.
SET UP: Use $\cos(\omega t + \phi/2) = \cos(\omega t)\cos(\phi/2) - \sin(\omega t)\sin(\phi/2)$. Then

$2\cos(\phi/2)\cos(\omega t + \phi/2) = 2\cos(\omega t)\cos^2(\phi/2) - 2\sin(\omega t)\sin(\phi/2)\cos(\phi/2)$. Then use $\cos^2(\phi/2) = \dfrac{1 + \cos(\phi)}{2}$ and

$2\sin(\phi/2)\cos(\phi/2)=\sin\phi$. This gives $\cos(\omega t)+(\cos(\omega t)\cos(\phi)-\sin(\omega t)\sin(\phi))=\cos(\omega t)+\cos(\omega t+\phi)$, using again the trig identity for the cosine of the sum of two angles.

EXECUTE: **(a)** The electric field is the sum of the two fields and can be written as $E_P(t)=E_2(t)+E_1(t)=E\cos(\omega t)+E\cos(\omega t+\phi)$. $E_P(t)=2E\cos(\phi/2)\cos(\omega t+\phi/2)$.

(b) $E_p(t)=A\cos(\omega t+\phi/2)$, so comparing with part (a), we see that the amplitude of the wave (which is always positive) must be $A=2E\,|\cos(\phi/2)|$.

(c) To have an interference maximum, $\dfrac{\phi}{2}=2\pi m$. So, for example, using $m=1$, the relative phases are

E_2: 0; E_1: $\phi=4\pi$; E_p: $\dfrac{\phi}{2}=2\pi$, and all waves are in phase.

(d) To have an interference minimum, $\dfrac{\phi}{2}=\pi\left(m+\dfrac{1}{2}\right)$. So, for example using $m=0$, relative phases are

E_2: 0; E_1: $\phi=\pi$; E_p: $\phi/2=\pi/2$, and the resulting wave is out of phase by a quarter of a cycle from both of the original waves.

(e) The instantaneous magnitude of the Poynting vector is

$$|\vec{S}|=\varepsilon_0 cE_p^2(t)=\varepsilon_0 c(4E^2\cos^2(\phi/2)\cos^2(\omega t+\phi/2)).$$

For a time average, $\cos^2(\omega t+\phi/2)=\dfrac{1}{2}$, so $|S_{av}|=2\varepsilon_0 cE^2\cos^2(\phi/2)$.

EVALUATE: The result of part (e) shows that the intensity at a point depends on the phase difference ϕ at that point for the waves from each source.

35.49. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: The definition of hyperbola is the locus of points such that the difference between P to S_2 and P to S_1 is a constant.

EXECUTE: **(a)** $\Delta r=m\lambda$. $r_1=\sqrt{x^2+(y-d)^2}$ and $r_2=\sqrt{x^2+(y+d)^2}$.

$\Delta r=\sqrt{x^2+(y+d)^2}-\sqrt{x^2+(y-d)^2}=m\lambda$.

(b) For a given m and λ, Δr is a constant and we get a hyperbola. Or, in the case of all m for a given λ, a family of hyperbolas.

(c) $\sqrt{x^2+(y+d)^2}-\sqrt{x^2+(y-d)^2}=(m+\tfrac{1}{2})\lambda.$

EVALUATE: The hyperbolas approach straight lines at large distances from the source.

35.50. **IDENTIFY:** Follow the derivation of Eq.(35.7), but with different amplitudes for the two waves.

SET UP: $\cos(\pi-\phi)=-\cos\phi$

EXECUTE: **(a)** $E_p^2=E_1^2+E_2^2-2E_1E_2\cos(\pi-\phi)=E^2+4E^2+4E^2\cos\phi=5E^2+4E^2\cos\phi$

$I=\dfrac{1}{2}\varepsilon_0 cE_p{}^2=\varepsilon_0 c\left[\left(\dfrac{5}{2}E^2\right)+\left(\dfrac{4}{2}E^2\right)\cos\phi\right]$. $\phi=0\Rightarrow I_0=\dfrac{9}{2}\varepsilon_0 cE^2$. Therefore, $I=I_0\left[\dfrac{5}{9}+\dfrac{4}{9}\cos\phi\right]$.

(b) The graph is shown in Figure 35.50. $I_{min}=\dfrac{1}{9}I_0$ which occurs when $\phi=n\pi(n\text{ odd})$.

EVALUATE: The maxima and minima occur at the same points on the screen as when the two sources have the same amplitude, but when the amplitudes are different the intensity is no longer zero at the minima.

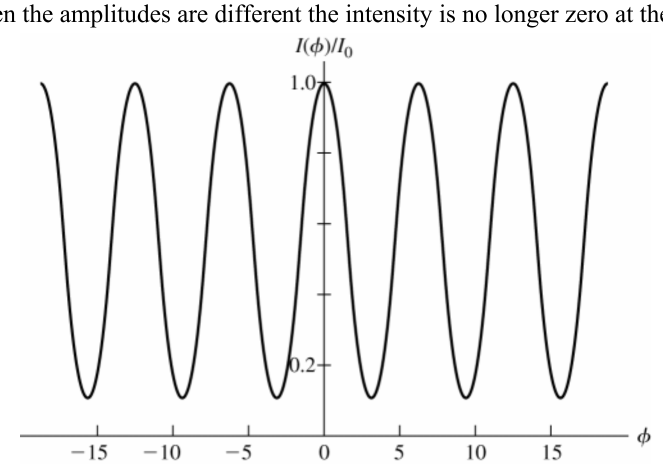


Figure 35.50

35.51. **IDENTIFY** and **SET UP:** Consider interference between rays reflected from the upper and lower surfaces of the film to relate the thickness of the film to the wavelengths for which there is destructive interference. The thermal expansion of the film changes the thickness of the film when the temperature changes.

EXECUTE: For this film on this glass, there is a net $\lambda/2$ phase change due to reflection and the condition for destructive interference is $2t = m(\lambda/n)$, where $n = 1.750$.

Smallest nonzero thickness is given by $t = \lambda/2n$.

At $20.0°\text{C}$, $t_0 = (582.4\ \text{nm})/[(2)(1.750)] = 166.4\ \text{nm}$.

At $170°\text{C}$, $t_0 = (588.5\ \text{nm})/[(2)(1.750)] = 168.1\ \text{nm}$.

$t = t_0(1 + \alpha\Delta T)$ so

$\alpha = (t - t_0)/(t_0\Delta T) = (1.7\ \text{nm})/[(166.4\ \text{nm})(150°\text{C})] = 6.8\times10^{-5}(\text{C}°)^{-1}$

EVALUATE: When the film is heated its thickness increases, and it takes a larger wavelength in the film to equal $2t$. The value we calculated for α is the same order of magnitude as those given in Table 17.1.

35.52. **IDENTIFY** and **SET UP:** At the $m = 3$ bright fringe for the red light there must be destructive interference at this same θ for the other wavelength.

EXECUTE: For constructive interference: $d\sin\theta = m\lambda_1 \Rightarrow d\sin\theta = 3(700\ \text{nm}) = 2100\ \text{nm}$. For destructive interference: $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda_2 \Rightarrow \lambda_2 = \frac{d\sin\theta}{m+\frac{1}{2}} = \frac{2100\ \text{nm}}{m+\frac{1}{2}}$. So the possible wavelengths are

$\lambda_2 = 600\ \text{nm}$, for $m = 3$, and $\lambda_2 = 467\ \text{nm}$, for $m = 4$.

EVALUATE: Both d and θ drop out of the calculation since their combination is just the path difference, which is the same for both types of light.

35.53. **IDENTIFY:** Apply $I = I_0\cos\left(\frac{\pi d}{\lambda}\sin\theta\right)$.

SET UP: $I = I_0/2$ when $\frac{\pi d}{\lambda}\sin\theta$ is $\frac{\pi}{4}$ rad, $\frac{3\pi}{4}$ rad, ...

EXECUTE: First we need to find the angles at which the intensity drops by one-half from the value of the mth bright fringe. $I = I_0\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) = \frac{I_0}{2} \Rightarrow \frac{\pi d}{\lambda}\sin\theta \approx \frac{\pi d\theta_m}{\lambda} = (m+1/2)\frac{\pi}{2}$.

$m = 0: \theta = \theta_m^- = \frac{\lambda}{4d}; m = 1: \theta = \theta_m^+ = \frac{3\lambda}{4d} \Rightarrow \Delta\theta_m = \frac{\lambda}{2d}$.

EVALUATE: There is no dependence on the m-value of the fringe, so all fringes at small angles have the same half-width.

34.54. **IDENTIFY:** Consider the phase difference produced by the path difference and by the reflections.

SET UP: There is just one half-cycle phase change upon reflection, so for constructive interference $2t = (m_1 + \frac{1}{2})\lambda_1 = (m_2 + \frac{1}{2})\lambda_2$, where these wavelengths are in the glass. The two different wavelengths differ by just one m-value, $m_2 = m_1 - 1$.

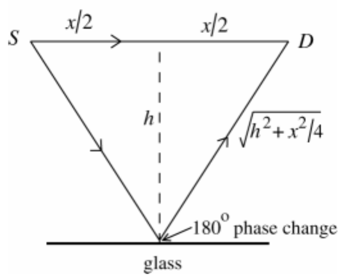
EXECUTE: $\left(m_1 + \frac{1}{2}\right)\lambda_1 = \left(m_1 - \frac{1}{2}\right)\lambda_2 \Rightarrow m_1(\lambda_2 - \lambda_1) = \frac{\lambda_1 + \lambda_2}{2} \Rightarrow m_1 = \frac{\lambda_1 + \lambda_2}{2(\lambda_2 - \lambda_1)}$.

$m_1 = \frac{477.0\ \text{nm} + 540.6\ \text{nm}}{2(540.6\ \text{nm} - 477.0\ \text{nm})} = 8$. $2t = \left(8 + \frac{1}{2}\right)\frac{\lambda_{01}}{n} \Rightarrow t = \frac{17(477.0\ \text{nm})}{4(1.52)} = 1334\ \text{nm}$.

EVALUATE: Now that we have t we can calculate all the other wavelengths for which there is constructive interference.

35.55. **IDENTIFY:** Consider the phase difference due to the path difference and due to the reflection of one ray from the glass surface.

(a) SET UP: Consider Figure 35.55



path difference = $2\sqrt{h^2 + x^2/4} - x = \sqrt{4h^2 + x^2} - x$

Figure 35.55

Since there is a $180°$ phase change for the reflected ray, the condition for constructive interference is path difference $=\left(m+\frac{1}{2}\right)\lambda$ and the condition for destructive interference is path difference $=m\lambda$.

(b) EXECUTE: Constructive interference: $\left(m+\frac{1}{2}\right)\lambda=\sqrt{4h^2+x^2}-x$ and $\lambda=\dfrac{\sqrt{4h^2+x^2}-x}{m+\frac{1}{2}}$. Longest λ is for $m=0$ and then $\lambda=2\left(\sqrt{4h^2+x^2}-x\right)=2\left(\sqrt{4(0.24\text{ m})^2+(0.14\text{ m})^2}-0.14\text{ m}\right)=0.72\text{ m}$

EVALUATE: For $\lambda=0.72\text{ m}$ the path difference is $\lambda/2$.

35.56. **IDENTIFY:** Require constructive interference for the reflection from the top and bottom surfaces of each cytoplasm layer and each guanine layer.

SET UP: At the water (or cytoplasm) to guanine interface, there is a half-cycle phase shift for the reflected light, but there is not one at the guanine to cytoplasm interface. Therefore there will always be one half-cycle phase difference between two neighboring reflected beams, just due to the reflections.

EXECUTE: For the guanine layers:

$$2t_{\text{g}}=(m+\frac{1}{2})\frac{\lambda}{n_{\text{g}}}\Rightarrow\lambda=\frac{2t_{\text{g}}n_{\text{g}}}{(m+\frac{1}{2})}=\frac{2(74\text{ nm})\,(1.80)}{(m+\frac{1}{2})}=\frac{266\text{ nm}}{(m+\frac{1}{2})}\Rightarrow\lambda=533\text{ nm }(m=0).$$

For the cytoplasm layers:

$$2t_{\text{c}}=\left(m+\frac{1}{2}\right)\frac{\lambda}{n_{\text{c}}}\Rightarrow\lambda=\frac{2t_{\text{c}}n_{\text{c}}}{(m+\frac{1}{2})}=\frac{2(100\text{ nm})\,(1.333)}{(m+\frac{1}{2})}=\frac{267\text{ nm}}{(m+\frac{1}{2})}\Rightarrow\lambda=533\text{ nm }(m=0).$$

(b) By having many layers the reflection is strengthened, because at each interface some more of the transmitted light gets reflected back, increasing the total percentage reflected.

(c) At different angles, the path length in the layers changes (always to a larger value than the normal incidence case). If the path length changes, then so do the wavelengths that will interfere constructively upon reflection.

EVALUATE: The thickness of the guanine and cytoplasm layers are inversely proportional to their refractive indices $\left(\frac{100}{74}=\frac{1.80}{1.333}\right)$, so both kinds of layers produce constructive interference for the same wavelength in air.

35.57. **IDENTIFY:** The slits will produce an interference pattern, but in the liquid, the wavelength of the light will be less than it was in air.

SET UP: The first bright fringe occurs when $d\sin\theta=\lambda/n$.

EXECUTE: In air: $d\sin 18.0°=\lambda$. In the liquid: $d\sin 12.6°=\lambda/n$. Dividing the equations gives

$$n=(\sin 18.0°)/(\sin 12.6°)=1.42$$

EVALUATE: It was not necessary to know the spacing of the slits, since it was the same in both air and the liquid.

35.58. **IDENTIFY:** Consider light reflected at the top and bottom surfaces of the film. Wavelengths that are predominant in the transmitted light are those for which there is destructive interference in the reflected light.

SET UP: For the waves reflected at the top surface of the oil film there is a half-cycle reflection phase shift. For the waves reflected at the bottom surface of the oil film there is no reflection phase shift. The condition for constructive interference is $2t=(m+\frac{1}{2})\lambda$. The condition for destructive interference is $2t=m\lambda$. The range of visible wavelengths is approximately 400 nm to 700 nm. In the oil film, $\lambda=\dfrac{\lambda_0}{n}$.

EXECUTE: **(a)** $2t=(m+\frac{1}{2})\lambda=(m+\frac{1}{2})\dfrac{\lambda_0}{n}$. $\lambda_0=\dfrac{2tn}{m+\frac{1}{2}}=\dfrac{2(380\text{ nm})(1.45)}{m+\frac{1}{2}}=\dfrac{1102\text{ nm}}{m+\frac{1}{2}}$.

$m=0$: $\lambda_0=2200\text{ nm}$. $m=1$: $\lambda_0=735\text{ nm}$. $m=2$: $\lambda_0=441\text{ nm}$. $m=3$: $\lambda_0=315\text{ nm}$. The visible wavelength for which there is constructive interference in the reflected light is 441 nm.

(b) $2t=m\lambda=m\dfrac{\lambda_0}{n}$. $\lambda_0=\dfrac{2tn}{m}=\dfrac{1102\text{ nm}}{m}$. $m=1$: $\lambda_0=1102\text{ nm}$. $m=2$: $\lambda_0=551\text{ nm}$. $m=3$: $\lambda_0=367\text{ nm}$.

The visible wavelength for which there is destructive interference in the reflected light is 551 nm. This is the visible wavelength predominant in the transmitted light.

EVALUATE: At a particular wavelength the sum of the intensities of the reflected and transmitted light equals the intensity of the incident light.

35.59. **(a) IDENTIFY:** The wavelength in the glass is decreased by a factor of $1/n$, so for light through the upper slit a shorter path is needed to produce the same phase at the screen. Therefore, the interference pattern is shifted downward on the screen.

(b) SET UP: Consider the total phase difference produced by the path length difference and also by the different wavelength in the glass.

EXECUTE: At a point on the screen located by the angle θ the difference in path length is $d\sin\theta$. This introduces a phase difference of $\phi=\left(\frac{2\pi}{\lambda_0}\right)(d\sin\theta)$, where λ_0 is the wavelength of the light in air or vacuum.

In the thickness L of glass the number of wavelengths is $\frac{L}{\lambda}=\frac{nL}{\lambda_0}$. A corresponding length L of the path of the ray through the lower slit, in air, contains L/λ_0 wavelengths. The phase difference this introduces is

$\phi=2\pi\left(\frac{nL}{\lambda_0}-\frac{L}{\lambda_0}\right)$ and $\phi=2\pi(n-1)(L/\lambda_0)$. The total phase difference is the sum of these two,

$\left(\frac{2\pi}{\lambda_0}\right)(d\sin\theta)+2\pi(n-1)(L/\lambda_0)=(2\pi/\lambda_0)(d\sin\theta+L(n-1))$. Eq.(35.10) then gives

$$I=I_0\cos^2\left[\left(\frac{\pi}{\lambda_0}\right)(d\sin\theta+L(n-1))\right].$$

(c) Maxima means $\cos\phi/2=\pm1$ and $\phi/2=m\pi$, $m=0,\ \pm1,\ \pm2,\ldots$ $(\pi/\lambda_0)(d\sin\theta+L(n-1))=m\pi$

$$d\sin\theta+L(n-1)=m\lambda_0$$

$$\sin\theta=\frac{m\lambda_0-L(n-1)}{d}$$

EVALUATE: When $L\to0$ or $n\to1$ the effect of the plate goes away and the maxima are located by Eq.(35.4).

35.60. IDENTIFY: Dark fringes occur because the path difference is one-half of a wavelength.

SET UP: At the first dark fringe, $d\sin\theta=\lambda/2$. The intensity at any angle θ is given by $I=I_0\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)$.

(a) At the first dark fringe, we have

$$d\sin\theta=\lambda/2$$

$$d/\lambda=2/(2\sin15.0°)=1.93$$

(b) $I=I_0\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)=\frac{I_0}{10}\ \Rightarrow\ \cos\left(\frac{\pi d\sin\theta}{\lambda}\right)=\frac{1}{\sqrt{10}}$

$$\frac{\pi d\sin\theta}{\lambda}=\arccos\left(\frac{1}{\sqrt{10}}\right)=71.57°=1.249\text{ rad}$$

Using the result from part (a), that $d/\lambda=1.93$, we have $\pi(1.93)\sin\theta=1.249$. $\sin\theta=0.2060$ and

$$\theta=\pm11.9°$$

EVALUATE: Since the first dark fringes occur at ±15.0°, it is reasonable that at ≈12° the intensity is reduced to only 1/10 of its maximum central value.

35.61. IDENTIFY: There are two effects to be considered: first, the expansion of the rod, and second, the change in the rod's refractive index.

SET UP: $\lambda=\frac{\lambda_0}{n}$ and $\Delta n=n_0(2.50\times10^{-5}\ (\text{C}°)^{-1})\Delta T$. $\Delta L=L_0(5.00\times10^{-6}\ (\text{C}°)^{-1})\Delta T$.

EXECUTE: The extra length of rod replaces a little of the air so that the change in the number of wavelengths due to this is given by: $\Delta N_1=\frac{2n_{\text{glass}}\Delta L}{\lambda_0}-\frac{2n_{\text{air}}\Delta L}{\lambda_0}=\frac{2(n_{\text{glass}}-1)L_0\alpha\Delta T}{\lambda_0}$ and

$$\Delta N_1=\frac{2(1.48-1)(0.030\text{ m})(5.00\times10^{-6}/\text{C}°)(5.00\text{ C}°)}{5.89\times10^{-7}\text{ m}}=1.22.$$

The change in the number of wavelengths due to the change in refractive index of the rod is:

$$\Delta N_2=\frac{2\Delta n_{\text{glass}}L_0}{\lambda_0}=\frac{2(2.50\times10^{-5}/\text{C}°)(5.00\text{ C}°/\text{min})(1.00\text{ min})(0.0300\text{ m})}{5.89\times10^{-7}\text{m}}=12.73.$$

So, the total change in the number of wavelengths as the rod expands is $\Delta N=12.73+1.22=14.0$ fringes/minute.

EVALUATE: Both effects increase the number of wavelengths along the length of the rod. Both ΔL and Δn_{glass} are very small and the two effects can be considered separately.

35.62. IDENTIFY: Apply Snell's law to the refraction at the two surfaces of the prism. S_1 and S_2 serve as coherent sources so the fringe spacing is $\Delta y=\frac{R\lambda}{d}$, where d is the distance between S_1 and S_2.

SET UP: For small angles, $\sin\theta \approx \theta$, with θ expressed in radians.

EXECUTE: **(a)** Since we can approximate the angles of incidence on the prism as being small, Snell's Law tells us that an incident angle of θ on the flat side of the prism enters the prism at an angle of θ/n, where n is the index of refraction of the prism. Similarly on leaving the prism, the in-going angle is $\theta/n - A$ from the normal, and the outgoing angle, relative to the prism, is $n(\lambda/n - A)$. So the beam leaving the prism is at an angle of $\theta' = n(\theta/n - A) + A$ from the optical axis. So $\theta - \theta' = (n-1)A$. At the plane of the source S_0, we can calculate the height of one image above the source: $\dfrac{d}{2} = \tan(\theta - \theta')a \approx (\theta - \theta')a = (n-1)Aa \Rightarrow d = 2aA(n-1)$.

(b) To find the spacing of fringes on a screen, we use

$$\Delta y = \frac{R\lambda}{d} = \frac{R\lambda}{2aA(n-1)} = \frac{(2.00\text{ m} + 0.200\text{ m})\,(5.00\times10^{-7}\text{ m})}{2(0.200\text{ m})\,(3.50\times10^{-3}\text{ rad})\,(1.50-1.00)} = 1.57\times10^{-3}\text{ m}.$$

EVALUATE: The fringe spacing is proportional to the wavelength of the light. The biprism serves as an alternative to two closely spaced narrow slits.

DIFFRACTION

36

36.1. **IDENTIFY:** Use $y = x\tan\theta$ to calculate the angular position θ of the first minimum. The minima are located by Eq.(36.2): $\sin\theta = \dfrac{m\lambda}{a}$, $m = \pm1, \pm2, \ldots$ First minimum means $m = 1$ and $\sin\theta_1 = \lambda / a$ and $\lambda = a\sin\theta_1$. Use this equation to calculate λ.

SET UP: The central maximum is sketched in Figure 36.1.

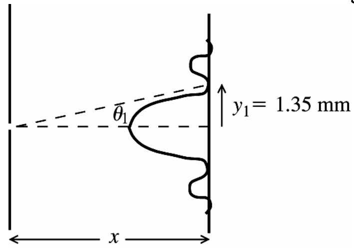


Figure 36.1

EXECUTE: $y_1 = x\tan\theta_1$

$\tan\theta_1 = \dfrac{y_1}{x} =$

$\dfrac{1.35\times10^{-3}\text{ m}}{2.00\text{ m}} = 0.675\times10^{-3}$

$\theta_1 = 0.675\times10^{-3}$ rad

$\lambda = a\sin\theta_1 = (0.750\times10^{-3}\text{ m})\sin(0.675\times10^{-3}\text{ rad}) = 506\text{ nm}$

EVALUATE: θ_1 is small so the approximation used to obtain Eq.(36.3) is valid and this equation could have been used.

36.2. **IDENTIFY:** The angle is small, so $y_m = x\dfrac{m\lambda}{a}$.

SET UP: $y_1 = 10.2$ mm

EXECUTE: $y_1 = \dfrac{x\lambda}{a} \Rightarrow a\dfrac{x\lambda}{y_1} = \dfrac{(0.600\text{ m})(5.46\times10^{-7}\text{ m})}{10.2\times10^{-3}\text{ m}} = 3.21\times10^{-5}\text{ m}.$

EVALUATE: The diffraction pattern is observed at a distance of 60.0 cm from the slit.

36.3. **IDENTIFY:** The dark fringes are located at angles θ that satisfy $\sin\theta = \dfrac{m\lambda}{a}$, $m = \pm1, \pm2, \ldots.$

SET UP: The largest value of $|\sin\theta|$ is 1.00.

EXECUTE: **(a)** Solve for *m* that corresponds to $\sin\theta = 1$: $m = \dfrac{a}{\lambda} = \dfrac{0.0666\times10^{-3}\text{ m}}{585\times10^{-9}\text{ m}} = 113.8$. The largest value *m* can have is 113. $m = \pm1, \pm2, \ldots, \pm113$ gives 226 dark fringes.

(b) For $m = \pm113$, $\sin\theta = \pm113\left(\dfrac{585\times10^{-9}\text{ m}}{0.0666\times10^{-3}\text{ m}}\right) = \pm0.9926$ and $\theta = \pm83.0°$.

EVALUATE: When the slit width *a* is decreased, there are fewer dark fringes. When $a < \lambda$ there are no dark fringes and the central maximum completely fills the screen.

36.4. **IDENTIFY** and **SET UP:** λ / a is very small, so the approximate expression $y_m = R\dfrac{m\lambda}{a}$ is accurate. The distance between the two dark fringes on either side of the central maximum is $2y_1$.

EXECUTE: $y_1 = \dfrac{\lambda R}{a} = \dfrac{(633\times10^{-9}\text{ m})(3.50\text{ m})}{0.750\times10^{-3}\text{ m}} = 2.95\times10^{-3}\text{ m} = 2.95\text{ mm}$. $2y_1 = 5.90$ mm.

EVALUATE: When *a* is decreased, the width $2y_1$ of the central maximum increases.

36.5. **IDENTIFY:** The minima are located by $\sin\theta = \dfrac{m\lambda}{a}$

SET UP: $a = 12.0$ cm. $x = 40.0$ cm.

EXECUTE: The angle to the first minimum is $\theta = \arcsin\left(\frac{\lambda}{a}\right) = \arcsin\left(\frac{9.00\text{ cm}}{12.00\text{ cm}}\right) = 48.6°$.

So the distance from the central maximum to the first minimum is just $y_1 = x\tan\theta =$ $(40.0\text{ cm})\tan(48.6°) = \pm 45.4\text{ cm}$.

EVALUATE: $2\lambda/a$ is greater than 1, so only the $m = 1$ minimum is seen.

36.6. **IDENTIFY:** The angle that locates the first diffraction minimum on one side of the central maximum is given by $\sin\theta = \frac{\lambda}{a}$. The time between crests is the period *T*. $f = \frac{1}{T}$ and $\lambda = \frac{v}{f}$.

SET UP: The time between crests is the period, so $T = 1.0\text{ h}$.

EXECUTE: **(a)** $f = \frac{1}{T} = \frac{1}{1.0\text{ h}} = 1.0\text{ h}^{-1}$. $\lambda = \frac{v}{f} = \frac{800\text{ km/h}}{1.0\text{ h}^{-1}} = 800\text{ km}$.

(b) Africa-Antarctica: $\sin\theta = \frac{800\text{ km}}{4500\text{ km}}$ and $\theta = 10.2°$.

Australia-Antarctica: $\sin\theta = \frac{800\text{ km}}{3700\text{ km}}$ and $\theta = 12.5°$.

EVALUATE: Diffraction effects are observed when the wavelength is about the same order of magnitude as the dimensions of the opening through which the wave passes.

36.7. **IDENTIFY:** We can model the hole in the concrete barrier as a single slit that will produce a single-slit diffraction pattern of the water waves on the shore.

SET UP: For single-slit diffraction, the angles at which destructive interference occurs are given by $\sin\theta_m = m\lambda/a$, where $m = 1, 2, 3, \ldots$.

EXECUTE: **(a)** The frequency of the water waves is $f = 75.0\text{ min}^{-1} = 1.25\text{ s}^{-1} = 1.25\text{ Hz}$, so their wavelength is $\lambda = v/f =$ (15.0 cm/s)/(1.25 Hz) = 12.0 cm.

At the first point for which destructive interference occurs, we have $\tan\theta = (0.613\text{ m})/(3.20\text{ m}) \Rightarrow \theta = 10.84°$. $a\sin\theta = \lambda$ and

$$a = \lambda/\sin\theta = (12.0\text{ cm})/(\sin 10.84°) = 63.8\text{ cm}.$$

(b) First find the angles at which destructive interference occurs.

$$\sin\theta_2 = 2\lambda/a = 2(12.0\text{ cm})/(63.8\text{ cm}) \rightarrow \theta_2 = \pm 22.1°$$

$$\sin\theta_3 = 3\lambda/a = 3(12.0\text{ cm})/(63.8\text{ cm}) \rightarrow \theta_3 = \pm 34.3°$$

$$\sin\theta_4 = 4\lambda/a = 4(12.0\text{ cm})/(63.8\text{ cm}) \rightarrow \theta_4 = \pm 48.8°$$

$$\sin\theta_5 = 5\lambda/a = 5(12.0\text{ cm})/(63.8\text{ cm}) \rightarrow \theta_5 = \pm 70.1°$$

EVALUATE: These are large angles, so we cannot use the approximation that $\theta_m \approx m\lambda/a$.

36.8. **IDENTIFY:** The minima are located by $\sin\theta = \frac{m\lambda}{a}$. For part (b) apply Eq.(36.7).

SET UP: For the first minimum, $m = 1$. The intensity at $\theta = 0$ is I_0.

EXECUTE: **(a)** $\sin\theta = \frac{m\lambda}{a} = \sin 90.0° = 1 = \frac{m\lambda}{a} = \frac{\lambda}{a}$. Thus $a = \lambda = 580\text{ nm} = 5.80\times10^{-4}\text{ mm}$.

(b) According to Eq.(36.7),

$$\frac{I}{I_0} = \left\{\frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda}\right\}^2 = \left\{\frac{\sin[\pi(\sin\pi/4)]}{\pi(\sin\pi/4)}\right\}^2 = 0.128.$$

EVALUATE: If $a = \lambda/2$, for example, then at $\theta = 45°$, $\frac{I}{I_0} = \left\{\frac{\sin[(\pi/2)(\sin\pi/4)]}{(\pi/2)(\sin\pi/4)}\right\}^2 = 0.81$. As a/λ decreases, the screen becomes more uniformly illuminated.

36.9. **IDENTIFY and SET UP:** $v = f\lambda$ gives λ. The person hears no sound at angles corresponding to diffraction minima. The diffraction minima are located by $\sin\theta = m\lambda/a$, $m = \pm1, \pm2, \ldots$ Solve for θ.

EXECUTE: $\lambda = v/f = (344\text{ m/s})/(1250\text{ Hz}) = 0.2752\text{ m}$; $a = 1.00\text{ m}$. $m = \pm1$, $\theta = \pm16.0°$; $m = \pm2$, $\theta = \pm33.4°$; $m = \pm3$, $\theta = \pm55.6°$; no solution for larger m

EVALUATE: $\lambda/a = 0.28$ so for the large wavelength sound waves diffraction by the doorway is a large effect. Diffraction would not be observable for visible light because its wavelength is much smaller and $\lambda/a \ll 1$.

36.10. **IDENTIFY:** Compare E_y to the expression $E_y = E_{\max}\sin(kx - \omega t)$ and determine k, and from that calculate λ. $f = c/\lambda$. The dark bands are located by $\sin\theta = \dfrac{m\lambda}{a}$.

SET UP: $c = 3.00\times10^8$ m/s. The first dark band corresponds to $m = 1$.

EXECUTE: **(a)** $E = E_{\max}\sin(kx - \omega t)$. $k = \dfrac{2\pi}{\lambda} \Rightarrow \lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{1.20\times10^7\ \text{m}^{-1}} = 5.24\times10^{-7}\text{m}$.

$f\lambda = c \Rightarrow f = \dfrac{c}{\lambda} = \dfrac{3.0\times10^8\ \text{m/s}}{5.24\times10^{-7}\text{m}} = 5.73\times10^{14}$ Hz.

(b) $a\sin\theta = \lambda$. $a = \dfrac{\lambda}{\sin\theta} = \dfrac{5.24\times10^{-7}\ \text{m}}{\sin 28.6^\circ} = 1.09\times10^{-6}$ m.

(c) $a\sin\theta = m\lambda\ (m = 1,\ 2,\ 3,\ \ldots)$. $\sin\theta_2 = \pm2\dfrac{\lambda}{a} = \pm2\dfrac{5.24\times10^{-7}\text{m}}{1.09\times10^{-6}\text{m}}$ and $\theta_2 = \pm74^\circ$.

EVALUATE: For $m = 3$, $\dfrac{m\lambda}{a}$ is greater than 1 so only the first and second dark bands appear.

36.11. **IDENTIFY** and **SET UP:** $\sin\theta = \lambda/a$ locates the first minimum. $y = x\tan\theta$.

EXECUTE: $\tan\theta = y/x = (36.5\ \text{cm})/(40.0\ \text{cm})$ and $\theta = 42.38^\circ$.

$a = \lambda/\sin\theta = (620\times10^{-9}\ \text{m})/(\sin 42.38^\circ) = 0.920\ \mu\text{m}$

EVALUATE: $\theta = 0.74$ rad and $\sin\theta = 0.67$, so the approximation $\sin\theta \approx \theta$ would not be accurate.

36.12. **IDENTIFY:** The angle is small, so $y_m = x\dfrac{m\lambda}{a}$ applies.

SET UP: The width of the central maximum is $2y_1$, so $y_1 = 3.00$ mm.

EXECUTE: **(a)** $y_1 = \dfrac{x\lambda}{a} \Rightarrow a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50\ \text{m})(5.00\times10^{-7}\text{m})}{3.00\times10^{-3}\text{m}} = 4.17\times10^{-4}$ m.

(b) $a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50\ \text{m})(5.00\times10^{-5}\ \text{m})}{3.00\times10^{-3}\ \text{m}} = 4.17\times10^{-2}\ \text{m} = 4.2$ cm.

(c) $a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50\ \text{m})(5.00\times10^{-10}\ \text{m})}{3.00\times10^{-3}\ \text{m}} = 4.17\times10^{-7}$ m.

EVALUATE: The ratio a/λ stays constant, so a is smaller when λ is smaller.

36.13. **IDENTIFY:** Calculate the angular positions of the minima and use $y = x\tan\theta$ to calculate the distance on the screen between them.

(a) SET UP: The central bright fringe is shown in Figure 36.13a.

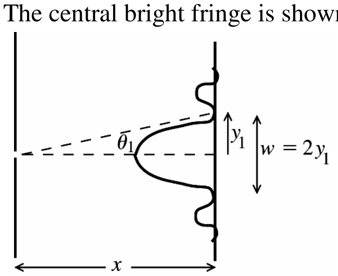


Figure 36.13a

EXECUTE: The first minimum is located by

$\sin\theta_1 = \dfrac{\lambda}{a} = \dfrac{633\times10^{-9}\ \text{m}}{0.350\times10^{-3}\ \text{m}} = 1.809\times10^{-3}$

$\theta_1 = 1.809\times10^{-3}$ rad

$y_1 = x\tan\theta_1 = (3.00\ \text{m})\tan(1.809\times10^{-3}\ \text{rad}) = 5.427\times10^{-3}$ m

$w = 2y_1 = 2(5.427\times10^{-3}\ \text{m}) = 1.09\times10^{-2}\ \text{m} = 10.9$ mm

(b) SET UP: The first bright fringe on one side of the central maximum is shown in Figure 36.13b.

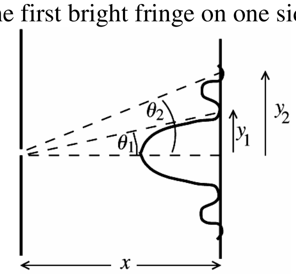


Figure 36.13b

EXECUTE: $w = y_2 - y_1$

$y_1 = 5.427\times10^{-3}$ m (part (a))

$\sin\theta_2 = \dfrac{2\lambda}{a} = 3.618\times10^{-3}$

$\theta_2 = 3.618\times10^{-3}$ rad

$y_2 = x\tan\theta_2 = 1.085\times10^{-2}$ m

$w = y_2 - y_1 = 1.085\times10^{-2}\ \text{m} - 5.427\times10^{-3}\ \text{m} = 5.4$ mm

EVALUATE: The central bright fringe is twice as wide as the other bright fringes.

36.14. **IDENTIFY:** $I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$. $\beta = \frac{2\pi}{\lambda}a\sin\theta$.

SET UP: The angle θ is small, so $\sin\theta \approx \tan\theta \approx y/x$.

EXECUTE: $\beta = \frac{2\pi a}{\lambda}\sin\theta \approx \frac{2\pi a}{\lambda}\frac{y}{x} = \frac{2\pi(4.50\times10^{-4}\text{ m})}{(6.20\times10^{-7}\text{m})(3.00\text{ m})}y = (1520\text{ m}^{-1})y.$

(a) $y = 1.00\times10^{-3}\text{m}: \frac{\beta}{2} = \frac{(1520\text{ m}^{-1})(1.00\times10^{-3}\text{m})}{2} = 0.760.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(0.760)}{0.760}\right)^2 = 0.822I_0$$

(b) $y = 3.00\times10^{-3}\text{m}: \frac{\beta}{2} = \frac{(1520\text{ m}^{-1})(3.00\times10^{-3}\text{m})}{2} = 2.28.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(2.28)}{2.28}\right)^2 = 0.111I_0.$$

(c) $y = 5.00\times10^{-3}\text{m}: \frac{\beta}{2} = \frac{(1520\text{ m}^{-1})(5.00\times10^{-3}\text{m})}{2} = 3.80.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(3.80)}{3.80}\right)^2 = 0.0259I_0.$$

EVALUATE: The first minimum occurs at $y_1 = \frac{\lambda x}{a} = \frac{(6.20\times10^{-7}\text{ m})(3.00\text{ m})}{4.50\times10^{-4}\text{ m}} = 4.1\text{ mm}$. The distances in parts (a) and (b) are within the central maximum. $y = 5.00\text{ mm}$ is within the first secondary maximum.

36.15. **(a) IDENTIFY:** Use Eq.(36.2) with $m = 1$ to locate the angular position of the first minimum and then use $y = x\tan\theta$ to find its distance from the center of the screen.

SET UP: The diffraction pattern is sketched in Figure 36.15.

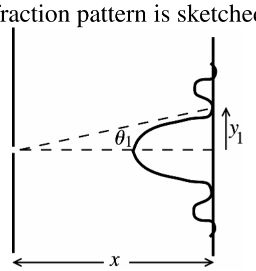


Figure 36.15

$\sin\theta_1 = \frac{\lambda}{a} =$

$\frac{540\times10^{-9}\text{ m}}{0.240\times10^{-3}\text{ m}} = 2.25\times10^{-3}$

$\theta_1 = 2.25\times10^{-3}\text{ rad}$

$y_1 = x\tan\theta_1 = (3.00\text{ m})\tan(2.25\times10^{-3}\text{ rad}) = 6.75\times10^{-3}\text{ m} = 6.75\text{ mm}$

(b) IDENTIFY and **SET UP:** Use Eqs.(36.5) and (36.6) to calculate the intensity at this point.

EXECUTE: Midway between the center of the central maximum and the first minimum implies

$y = \frac{1}{2}(6.75\text{ mm}) = 3.375\times10^{-3}\text{ m}.$

$\tan\theta = \frac{y}{x} = \frac{3.375\times10^{-3}\text{ m}}{3.00\text{ m}} = 1.125\times10^{-3};\ \theta = 1.125\times10^{-3}\text{ rad}$

The phase angle β at this point on the screen is

$\beta = \left(\frac{2\pi}{\lambda}\right)a\sin\theta = \frac{2\pi}{540\times10^{-9}\text{ m}}(0.240\times10^{-3}\text{ m})\sin(1.125\times10^{-3}\text{ rad}) = \pi.$

Then $I = I_0\left(\frac{\sin\beta/2}{\beta/2}\right)^2 = (6.00\times10^{-6}\text{ W/m}^2)\left(\frac{\sin\pi/2}{\pi/2}\right)^2$

$I = \left(\frac{4}{\pi^2}\right)(6.00\times10^{-6}\text{ W/m}^2) = 2.43\times10^{-6}\text{ W/m}^2.$

EVALUATE: The intensity at this point midway between the center of the central maximum and the first minimum is less than half the maximum intensity. Compare this result to the corresponding one for the two-slit pattern, Exercise 35.23.

36.16. **IDENTIFY:** In the single-slit diffraction pattern, the intensity is a maximum at the center and zero at the dark spots. At other points, it depends on the angle at which one is observing the light.

SET UP: Dark fringes occur when $\sin\theta_m = m\lambda/a$, where m = 1, 2, 3, …, and the intensity is given by $I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2 = \dfrac{\pi a\sin\theta}{\lambda}$.

EXECUTE: **(a)** At the maximum possible angle, $\theta = 90°$, so

$$m_{\max} = (a\sin 90°)/\lambda = (0.0250\text{ mm})/(632.8\text{ nm}) = 39.5$$

Since m must be an integer and $\sin\theta$ must be ≤ 1, $m_{\max} = 39$. The total number of dark fringes is 39 on each side of the central maximum for a total of 78.

(b) The farthest dark fringe is for $m = 39$, giving

$$\sin\theta_{39} = (39)(632.8\text{ nm})/(0.0250\text{ mm}) \Rightarrow \theta_{39} = \pm 80.8°$$

(c) The next closer dark fringe occurs at $\sin\theta_{38} = (38)(632.8\text{ nm})/(0.0250\text{ mm}) \Rightarrow \theta_{38} = 74.1°$.

The angle midway these two extreme fringes is $(80.8° + 74.1°)/2 = 77.45°$, and the intensity at this angle is $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta/2 = \dfrac{\pi a\sin\theta}{\lambda} = \dfrac{\pi(0.0250\,\text{mm})\sin(77.45°)}{632.8\,\text{nm}}$ = 121.15 rad, which gives $I = \left(8.50\text{ W/m}^2\right)\left[\dfrac{\sin(121.15\,\text{rad})}{121.15\,\text{rad}}\right]^2 = 5.55\times 10^{-4}\text{ W/m}^2$

EVALUATE: At the angle in part (c), the intensity is so low that the light would be barely perceptible.

36.17. **IDENTIFY** and **SET UP:** Use Eq.(36.6) to calculate λ and use Eq.(36.5) to calculate I. $\theta = 3.25°$, $\beta = 56.0$ rad, $a = 0.105\times 10^{-3}$ m.

(a) EXECUTE: $\beta = \left(\dfrac{2\pi}{\lambda}\right)a\sin\theta$ so

$$\lambda = \frac{2\pi a\sin\theta}{\beta} = \frac{2\pi(0.105\times 10^{-3}\text{ m})\sin 3.25°}{56.0\text{ rad}} = 668\text{ nm}$$

(b) $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2 = I_0\left(\dfrac{4}{\beta^2}\right)(\sin(\beta/2))^2 = I_0\dfrac{4}{(56.0\text{ rad})^2}[\sin(28.0\text{ rad})]^2 = 9.36\times 10^{-5} I_0$

EVALUATE: At the first minimum $\beta = 2\pi$ rad and at the point considered in the problem $\beta = 17.8\pi$ rad, so the point is well outside the central maximum. Since β is close to $m\pi$ with $m = 18$, this point is near one of the minima. The intensity here is much less than I_0.

36.18. **IDENTIFY:** Use $\beta = \dfrac{2\pi a}{\lambda}\sin\theta$ to calculate β.

SET UP: The total intensity is given by drawing an arc of a circle that has length E_0 and finding the length of the chord which connects the starting and ending points of the curve.

EXECUTE: **(a)** $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi a}{\lambda}\dfrac{\lambda}{2a} = \pi$. From Figure 36.18a, $\pi\dfrac{E_p}{2} = E_0 \Rightarrow E_p = \dfrac{2}{\pi}E_0$.

The intensity is $I = \left(\dfrac{2}{\pi}\right)^2 I_0 = \dfrac{4I_0}{\pi^2} = 0.405 I_0$. This agrees with Eq.(36.5).

(b) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi a}{\lambda}\dfrac{\lambda}{a} = 2\pi$. From Figure 36.18b, it is clear that the total amplitude is zero, as is the intensity. This also agrees with Eq.(36.5).

(c) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi a}{\lambda}\dfrac{3\lambda}{2a} = 3\pi$. From Figure 36.18c, $3\pi\dfrac{E_p}{2} = E_0 \Rightarrow E_p = \dfrac{2}{3\pi}E_0$. The intensity is $I = \left(\dfrac{2}{3\pi}\right)^2 I_0 = \dfrac{4}{9\pi^2}I_0$. This agrees with Eq.(36.5).

EVALUATE: In part (a) the point is midway between the center of the central maximum and the first minimum. In part (b) the point is at the first maximum and in (c) the point is approximately at the location of the first secondary maximum. The phasor diagrams help illustrate the rapid decrease in intensity at successive maxima.

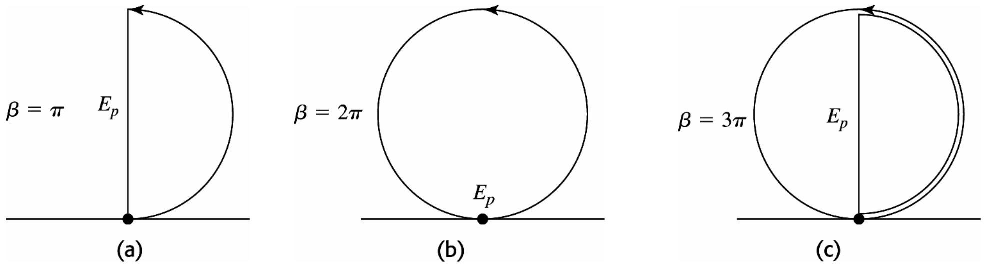


Figure 36.18

36.19. **IDENTIFY:** The space between the skyscrapers behaves like a single slit and diffracts the radio waves.
SET UP: Cancellation of the waves occurs when $a \sin \theta = m\lambda$, $m = 1, 2, 3, \ldots$, and the intensity of the waves is given by $I_0\left(\frac{\sin \beta/2}{\beta/2}\right)^2$, where $\beta/2 = \frac{\pi a \sin\theta}{\lambda}$.

EXECUTE: **(a)** First find the wavelength of the waves:
$\lambda = c/f = (3.00 \times 10^8 \text{ m/s})/(88.9 \text{ MHz}) = 3.375 \text{ m}$
For no signal, $a \sin \theta = m\lambda$.
$m = 1$: $\sin \theta_1 = (1)(3.375 \text{ m})/(15.0 \text{ m}) \Rightarrow \theta_1 = \pm 13.0°$
$m = 2$: $\sin \theta_2 = (2)(3.375 \text{ m})/(15.0 \text{ m}) \Rightarrow \theta_2 = \pm 26.7°$
$m = 3$: $\sin \theta_3 = (3)(3.375 \text{ m})/(15.0 \text{ m}) \Rightarrow \theta_3 = \pm 42.4°$
$m = 4$: $\sin \theta_4 = (4)(3.375 \text{ m})/(15.0 \text{ m}) \Rightarrow \theta_4 = \pm 64.1°$

(b) $I_0\left(\frac{\sin \beta/2}{\beta/2}\right)^2$, where $\beta/2 = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi(15.0\,\text{m})\sin(5.00°)}{3.375\,\text{m}} = 1.217 \text{ rad}$

$$I = \left(3.50\,\text{W/m}^2\right)\left[\frac{\sin(1.217 \text{ rad})}{1.217 \text{ rad}}\right]^2 = 2.08 \text{ W/m}^2$$

EVALUATE: The wavelength of the radio waves is very long compared to that of visible light, but it is still considerably shorter than the distance between the buildings.

36.20. **IDENTIFY:** The net intensity is the *product* of the factor due to single-slit diffraction and the factor due to double slit interference.

SET UP: The double-slit factor is $I_{\text{DS}} = I_0\left(\cos^2\frac{\phi}{2}\right)$ and the single-slit factor is $I_{\text{SS}} = \left(\frac{\sin\beta/2}{\beta/2}\right)^2$.

EXECUTE: **(a)** $d \sin\theta = m\lambda \Rightarrow \sin\theta = m\lambda/d$.
$\sin\theta_1 = \lambda/d$, $\sin\theta_2 = 2\lambda/d$, $\sin\theta_3 = 3\lambda/d$, $\sin\theta_4 = 4\lambda/d$

(b) At the interference bright fringes, $\cos^2\phi/2 = 1$ and $\beta/2 = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi(d/3)\sin\theta}{\lambda}$.

At θ_1, $\sin \theta_1 = \lambda/d$, so $\beta/2 = \frac{\pi(d/3)(\lambda/d)}{\lambda} = \pi/3$. The intensity is therefore

$$I_1 = I_0\left(\cos^2\frac{\phi}{2}\right)\left(\frac{\sin\beta/2}{\beta/2}\right)^2 = I_0(1)\left(\frac{\sin\pi/3}{\pi/3}\right)^2 = 0.684\, I_0$$

At θ_2, $\sin \theta_2 = 2\lambda/d$, so $\beta/2 = \frac{\pi(d/3)(2\lambda/d)}{\lambda} = 2\pi/3$. Using the same procedure as for θ_1, we have $I_2 = I_0(1)\left(\frac{\sin 2\pi/3}{2\pi/3}\right)^2 = 0.171\, I_0$

At θ_3, we get $\beta/2 = \pi$, which gives $I_3 = 0$ since $\sin \pi = 0$.

At θ_4, $\sin \theta_4 = 4\lambda/d$, so $\beta/2 = 4\pi/3$, which gives $I_4 = I_0\left(\frac{\sin 4\pi/3}{4\pi/3}\right)^2 = 0.0427\, I_0$

(c) Since $d = 3a$, every third interference maximum is missing.
(d) In Figure 36.12c in the textbook, every fourth interference maximum at the sides is missing because $d = 4a$.

EVALUATE: The result in this problem is different from that in Figure 36.12c because in this case $d = 3a$, so every third interference maximum at the sides is missing. Also the "envelope" of the intensity function decreases more rapidly here than in Figure 36.12c because the first diffraction minimum is reached sooner, and the decrease in intensity from one interference maximum to the next is faster for $a = d/3$ than for $a = d/4$.

36.21. **(a) IDENTIFY and SET UP:** The interference fringes (maxima) are located by $d\sin\theta = m\lambda$, with $m = 0, \pm 1, \pm 2, \ldots$. The intensity I in the diffraction pattern is given by $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, with $\beta = \left(\dfrac{2\pi}{\lambda}\right)a\sin\theta$.

We want $m = \pm 3$ in the first equation to give θ that makes $I = 0$ in the second equation.

EXECUTE: $d\sin\theta = m\lambda$ gives $\beta = \left(\dfrac{2\pi}{\lambda}\right)a\left(\dfrac{3\lambda}{d}\right) = 2\pi(3a/d)$.

$I = 0$ says $\dfrac{\sin\beta/2}{\beta/2} = 0$ so $\beta = 2\pi$ and then $2\pi = 2\pi(3a/d)$ and $(d/a) = 3$.

(b) IDENTIFY and SET UP: Fringes $m = 0, \pm 1, \pm 2$ are within the central diffraction maximum and the $m = \pm 3$ fringes coincide with the first diffraction minimum. Find the value of m for the fringes that coincide with the second diffraction minimum.

EXECUTE: Second minimum implies $\beta = 4\pi$.

$$\beta = \left(\frac{2\pi}{\lambda}\right)a\sin\theta = \left(\frac{2\pi}{\lambda}\right)a\left(\frac{m\lambda}{d}\right) = 2\pi m(a/d) = 2\pi(m/3)$$

Then $\beta = 4\pi$ says $4\pi = 2\pi(m/3)$ and $m = 6$. Therefore the $m = \pm 4$ and $m = +5$ fringes are contained within the first diffraction maximum on one side of the central maximum; two fringes.

EVALUATE: The central maximum is twice as wide as the other maxima so it contains more fringes.

36.22. **IDENTIFY and SET UP:** Use Figure 36.14b in the textbook. There is totally destructive interference between slits whose phasors are in opposite directions.

EXECUTE: By examining the diagram, we see that every fourth slit cancels each other.

EVALUATE: The total electric field is zero so the phasor diagram corresponds to a point of zero intensity. The first two maxima are at $\phi = 0$ and $\phi = \pi$, so this point is not midway between two maxima.

36.23. **(a) IDENTIFY and SET UP:** If the slits are very narrow then the central maximum of the diffraction pattern for each slit completely fills the screen and the intensity distribution is given solely by the two-slit interference. The maxima are given by
$d\sin\theta = m\lambda$ so $\sin\theta = m\lambda/d$. Solve for θ.

EXECUTE: 1st order maximum: $m = 1$, so $\sin\theta = \dfrac{\lambda}{d} = \dfrac{580\times10^{-9}\text{ m}}{0.530\times10^{-3}\text{ m}} = 1.094\times10^{-3}$; $\theta = 0.0627°$

2nd order maximum: $m = 2$, so $\sin\theta = \dfrac{2\lambda}{d} = 2.188\times10^{-3}$; $\theta = 0.125°$

(b) IDENTIFY and SET UP: The intensity is given by Eq.(36.12): $I = I_0\cos^2(\phi/2)\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$. Calculate ϕ and β at each θ from part (a).

EXECUTE: $\phi = \left(\dfrac{2\pi d}{\lambda}\right)\sin\theta = \left(\dfrac{2\pi d}{\lambda}\right)\left(\dfrac{m\lambda}{d}\right) = 2\pi m$, so $\cos^2(\phi/2) = \cos^2(m\pi) = 1$

(Since the angular positions in part (a) correspond to interference maxima.)

$$\beta = \left(\frac{2\pi a}{\lambda}\right)\sin\theta = \left(\frac{2\pi a}{\lambda}\right)\left(\frac{m\lambda}{d}\right) = 2\pi m(a/d) = m2\pi\left(\frac{0.320\text{ mm}}{0.530\text{ mm}}\right) = m(3.794\text{ rad})$$

1st order maximum: $m = 1$, so $I = I_0(1)\left(\dfrac{\sin(3.794/2)\text{ rad}}{(3.794/2)\text{ rad}}\right)^2 = 0.249I_0$

2nd order maximum: $m = 2$, so $I = I_0(1)\left(\dfrac{\sin 3.794\text{ rad}}{3.794\text{ rad}}\right)^2 = 0.0256I_0$

EVALUATE: The first diffraction minimum is at an angle θ given by $\sin\theta = \lambda/a$ so $\theta = 0.104°$. The first order fringe is within the central maximum and the second order fringe is inside the first diffraction maximum on one side of the central maximum. The intensity here at this second fringe is much less than I_0.

36.24. **IDENTIFY:** A double-slit bright fringe is missing when it occurs at the same angle as a double-slit dark fringe.

SET UP: Single-slit diffraction dark fringes occur when $a\sin\theta = m\lambda$, and double-slit interference bright fringes occur when $d\sin\theta = m'\lambda$.

EXECUTE: (a) The angles are the same for cancellation, so dividing the equations gives

$$d/a = m'/m \Rightarrow m'/m = 7 \Rightarrow m' = 7m$$

When $m = 1$, $m' = 7$; when $m = 2$, $m' = 14$, and so forth, so every 7th bright fringe is missing from the double-slit interference pattern.

EVALUATE: (b) The result is independent of the wavelength, so every 7th fringe will be cancelled for all wavelengths. But the bright interference fringes occur when $d \sin\theta = m\lambda$, so the *location* of the cancelled fringes *does* depend on the wavelength.

36.25. **IDENTIFY** and **SET UP:** The phasor diagrams are similar to those in Fig.36.14. An interference minimum occurs when the phasors add to zero.

EXECUTE: **(a)** The phasor diagram is given in Figure 36.25a

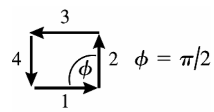


Figure 36.25a

There is destructive interference between the light through slits 1 and 3 and between 2 and 4.

(b) The phasor diagram is given in Figure 36.25b.

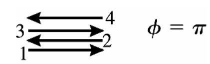


Figure 36.25b

There is destructive interference between the light through slits 1 and 2 and between 3 and 4.

(c) The phasor diagram is given in Figure 36.25c.

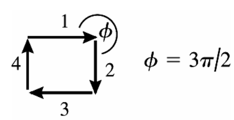


Figure 36.25c

There is destructive interference between light through slits 1 and 3 and between 2 and 4.

EVALUATE: Maxima occur when $\phi = 0,\ 2\pi,\ 4\pi$, etc. Our diagrams show that there are three minima between the maxima at $\phi = 0$ and $\phi = 2\pi$. This agrees with the general result that for N slits there are $N-1$ minima between each pair of principal maxima.

36.26. **IDENTIFY:** A double-slit bright fringe is missing when it occurs at the same angle as a double-slit dark fringe.

SET UP: Single-slit diffraction dark fringes occur when $a \sin\theta = m\lambda$, and double-slit interference bright fringes occur when $d \sin\theta = m'\lambda$.

EXECUTE: **(a)** The angle at which the first bright fringe occurs is given by
$\tan\theta_1 = (1.53\text{ mm})/(2500\text{ mm}) \Rightarrow \theta_1 = 0.03507°$. $d \sin\theta_1 = \lambda$ and

$$d = \lambda/(\sin\theta_1) = (632.8\text{ nm})/\sin(0.03507°) = 0.00103\text{ m} = 1.03\text{ mm}$$

(b) The 7th double-slit interference bright fringe is just cancelled by the 1st diffraction dark fringe, so $\sin\theta_{\text{diff}} = \lambda/a$ and $\sin\theta_{\text{interf}} = 7\lambda/d$

The angles are equal, so $\lambda/a = 7\lambda/d \rightarrow a = d/7 = (1.03\text{ mm})/7 = 0.148\text{ mm}$.

EVALUATE: We can generalize that if $d = na$, where n is a positive integer, then every n^{th} double-slit bright fringe will be missing in the pattern.

36.27. **IDENTIFY:** The diffraction minima are located by $\sin\theta = \dfrac{m_d\lambda}{a}$ and the two-slit interference maxima are located by $\sin\theta = \dfrac{m_i\lambda}{d}$. The third bright band is missing because the first order single slit minimum occurs at the same angle as the third order double slit maximum.

SET UP: The pattern is sketched in Figure 36.27. $\tan\theta = \dfrac{3\text{ cm}}{90\text{ cm}}$, so $\theta = 1.91°$.

EXECUTE: Single-slit dark spot: $a\sin\theta = \lambda$ and $a = \dfrac{\lambda}{\sin\theta} = \dfrac{500\text{ nm}}{\sin 1.91°} = 1.50\times10^4\text{ nm} = 15.0\ \mu\text{m}$ (width)

Double-slit bright fringe: $d\sin\theta = 3\lambda$ and $d = \dfrac{3\lambda}{\sin\theta} = \dfrac{3(500\text{ nm})}{\sin 1.91°} = 4.50\times10^4\text{ nm} = 45.0\ \mu\text{m}$ (separation).

EVALUATE: Note that $d/a = 3.0$.

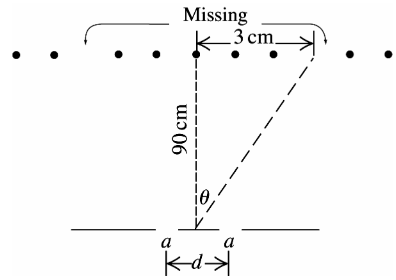


Figure 36.27

36.28. **IDENTIFY:** The maxima are located by $d\sin\theta = m\lambda$.

SET UP: The order corresponds to the values of *m*.

EXECUTE: First-order: $d\sin\theta_1 = \lambda$. Fourth-order: $d\sin\theta_4 = 4\lambda$.

$\dfrac{d\sin\theta_4}{d\sin\theta_1} = \dfrac{4\lambda}{\lambda}$, $\sin\theta_4 = 4\sin\theta_1 = 4\sin 8.94°$ and $\theta_4 = 38.4°$.

EVALUATE: We did not have to solve for *d*.

36.29. **IDENTIFY** and **SET UP:** The bright bands are at angles θ given by $d\sin\theta = m\lambda$. Solve for *d* and then solve for θ for the specified order.

EXECUTE: **(a)** $\theta = 78.4°$ for $m = 3$ and $\lambda = 681$ nm, so $d = m\lambda/\sin\theta = 2.086\times10^{-4}$ cm

The number of slits per cm is $1/d = 4790$ slits/cm

(b) 1st order: $m = 1$, so $\sin\theta = \lambda/d = (681\times10^{-9}\text{ m})/(2.086\times10^{-6}\text{ m})$ and $\theta = 19.1°$

2nd order: $m = 2$, so $\sin\theta = 2\lambda/d$ and $\theta = 40.8°$

(c) For $m = 4$, $\sin\theta = 4\lambda/d$ is greater than 1.00, so there is no 4th-order bright band.

EVALUATE: The angular position of the bright bands for a particular wavelength increases as the order increases.

36.30. **IDENTIFY:** The bright spots are located by $d\sin\theta = m\lambda$.

SET UP: Third-order means $m = 3$ and second-order means $m = 2$.

EXECUTE: $\dfrac{m\lambda}{\sin\theta} = d = \text{constant}$, so $\dfrac{m_r\lambda_r}{\sin\theta_r} = \dfrac{m_v\lambda_v}{\sin\theta_v}$.

$$\sin\theta_v = \sin\theta_r\left(\frac{m_v}{m_r}\right)\left(\frac{\lambda_v}{\lambda_r}\right) = (\sin 65.0°)\left(\frac{2}{3}\right)\left(\frac{400\text{ nm}}{700\text{ nm}}\right) = 0.345 \text{ and } \theta_v = 20.2°.$$

EVALUATE: The third-order line for a particular λ occurs at a larger angle than the second-order line. In a given order, the line for violet light (400 nm) occurs at a smaller angle than the line for red light (700 nm).

36.31. **IDENTIFY** and **SET UP:** Calculate *d* for the grating. Use Eq.(36.13) to calculate θ for the longest wavelength in the visible spectrum and verify that θ is small. Then use Eq.(36.3) to relate the linear separation of lines on the screen to the difference in wavelength.

EXECUTE: **(a)** $d = \left(\dfrac{1}{900}\right)\text{ cm} = 1.111\times10^{-5}\text{ m}$

For $\lambda = 700$ nm, $\lambda/d = 6.3\times10^{-2}$. The first-order lines are located at $\sin\theta = \lambda/d$; $\sin\theta$ is small enough for $\sin\theta \approx \theta$ to be an excellent approximation.

(b) $y = x\lambda/d$, where $x = 2.50$ m.

The distance on the screen between 1st order bright bands for two different wavelengths is $\Delta y = x(\Delta\lambda)/d$, so

$\Delta\lambda = d(\Delta y)/x = (1.111\times10^{-5}\text{ m})(3.00\times10^{-3}\text{ m})/(2.50\text{ m}) = 13.3\text{ nm}$

EVALUATE: The smaller *d* is (greater number of lines per cm) the smaller the $\Delta\lambda$ that can be measured.

36.32. **IDENTIFY:** The maxima are located by $d\sin\theta = m\lambda$.

SET UP: $350\text{ slits/mm} \Rightarrow d = \dfrac{1}{3.50\times10^5\text{ m}^{-1}} = 2.86\times10^{-6}\text{ m}$

EXECUTE: $m=1$: $\theta_{400}=\arcsin\left(\frac{\lambda}{d}\right)=\arcsin\left(\frac{4.00\times10^{-7}\text{ m}}{2.86\times10^{-6}\text{ m}}\right)=8.05^\circ$.

$\theta_{700}=\arcsin\left(\frac{\lambda}{d}\right)=\arcsin\left(\frac{7.00\times10^{-7}\text{ m}}{2.86\times10^{-6}\text{ m}}\right)=14.18^\circ$. $\Delta\theta_1=14.18^\circ-8.05^\circ=6.13^\circ$.

$m=3$: $\theta_{400}=\arcsin\left(\frac{3\lambda}{d}\right)=\arcsin\left(\frac{3(4.00\times10^{-7}\text{ m})}{2.86\times10^{-6}\text{ m}}\right)=24.8^\circ$.

$\theta_{700}=\arcsin\left(\frac{3\lambda}{d}\right)=\arcsin\left(\frac{3(7.00\times10^{-7}\text{ m})}{2.86\times10^{-6}\text{ m}}\right)=47.3^\circ$. $\Delta\theta_1=47.3^\circ-24.8^\circ=22.5^\circ$.

EVALUATE: $\Delta\theta$ is larger in third order.

36.33. **IDENTIFY:** The maxima are located by $d\sin\theta=m\lambda$.

SET UP: $d=1.60\times10^{-6}$ m

EXECUTE: $\theta=\arcsin\left(\frac{m\lambda}{d}\right)=\arcsin\left(\frac{m[6.328\times10^{-7}\text{ m}]}{1.60\times10^{-6}\text{ m}}\right)=\arcsin([0.396]m)$. For $m=1$, $\theta_1=23.3^\circ$. For $m=2$, $\theta_2=52.3^\circ$. There are no other maxima.

EVALUATE: The reflective surface produces the same interference pattern as a grating with slit separation *d*.

36.34. **IDENTIFY:** The maxima are located by $d\sin\theta=m\lambda$.

SET UP: $5000\text{ slits/cm}\Rightarrow d=\frac{1}{5.00\times10^{5}\text{ m}^{-1}}=2.00\times10^{-6}\text{ m}.$

EXECUTE: **(a)** $\lambda=\frac{d\sin\theta}{m}=\frac{(2.00\times10^{-6}\text{ m})\sin13.5^\circ}{1}=4.67\times10^{-7}\text{ m}.$

(b) $m=2$: $\theta=\arcsin\left(\frac{m\lambda}{d}\right)=\arcsin\left(\frac{2(4.67\times10^{-7}\text{ m})}{2.00\times10^{-6}\text{ m}}\right)=27.8^\circ.$

EVALUATE: Since the angles are fairly small, the second-order deviation is approximately twice the first-order deviation.

36.35. **IDENTIFY:** The maxima are located by $d\sin\theta=m\lambda$.

SET UP: $350\text{ slits/mm}\Rightarrow d=\frac{1}{3.50\times10^{5}\text{ m}^{-1}}=2.86\times10^{-6}\text{ m}$

EXECUTE: $\theta=\arcsin\left(\frac{m\lambda}{d}\right)=\arcsin\left(\frac{m(5.20\times10^{-7}\text{ m})}{2.86\times10^{-6}\text{ m}}\right)=\arcsin((0.182)m)$.

$m=1$: $\theta=10.5^\circ$; $m=2$: $\theta=21.3^\circ$; $m=3$: $\theta=33.1^\circ$.

EVALUATE: The angles are not precisely proportional to *m*, and deviate more from being proportional as the angles increase.

36.36. **IDENTIFY:** The resolution is described by $R=\frac{\lambda}{\Delta\lambda}=Nm$. Maxima are located by $d\sin\theta=m\lambda$.

SET UP: For 500 slits/mm, $d=(500\text{ slits/mm})^{-1}=(500{,}000\text{ slits/m})^{-1}$.

EXECUTE: **(a)** $N=\frac{\lambda}{m\Delta\lambda}=\frac{6.5645\times10^{-7}\text{m}}{2(6.5645\times10^{-7}\text{ m}-6.5627\times10^{-7}\text{ m})}=1820\text{ slits}.$

(b) $\theta=\sin^{-1}\left(\frac{m\lambda}{d}\right)\Rightarrow\theta_1=\sin^{-1}((2)(6.5645\times10^{-7}\text{ m})(500{,}000\text{ m}^{-1}))=41.0297^\circ$ and

$\theta_2=\sin^{-1}((2)(6.5627\times10^{-7}\text{ m})(500{,}000\text{ m}^{-1}))=41.0160^\circ$. $\Delta\theta=0.0137^\circ$

EVALUATE: $d\cos\theta\, d\theta=\lambda/N$, so for 1820 slits the angular interval $\Delta\theta$ between each of these maxima and the first adjacent minimum is $\Delta\theta=\frac{\lambda}{Nd\cos\theta}=\frac{6.56\times10^{-7}\text{ m}}{(1820)(2.0\times10^{-6}\text{ m})\cos41^\circ}=0.0137^\circ$. This is the same as the angular separation of the maxima for the two wavelengths and 1820 slits is just sufficient to resolve these two wavelengths in second order.

36.37. **IDENTIFY:** The resolving power depends on the line density and the width of the grating.

SET UP: The resolving power is given by $R = Nm = = \lambda/\Delta\lambda$.

EXECUTE: (a) $R = Nm$ = (5000 lines/cm)(3.50 cm)(1) = 17,500

(b) The resolving power needed to resolve the sodium doublet is

$$R = \lambda/\Delta\lambda = (589\text{ nm})/(589.59\text{ nm} - 589.00\text{ nm}) = 998$$

so this grating can easily resolve the doublet.

(c) (i) $R = \lambda/\Delta\lambda$. Since R = 17,500 when m = 1, $R = 2 \times 17{,}500 = 35{,}000$ for m = 2. Therefore

$$\Delta\lambda = \lambda/R = (587.8\text{ nm})/35{,}000 = 0.0168\text{ nm}$$

$$\lambda_{min} = \lambda + \Delta\lambda = 587.8002\text{ nm} + 0.0168\text{ nm} = 587.8170\text{ nm}$$

(ii) $\lambda_{max} = \lambda - \Delta\lambda = 587.8002\text{ nm} - 0.0168\text{ nm} = 587.7834\text{ nm}$

EVALUATE: (iii) Therefore the range of resolvable wavelengths is 587.7834 nm < λ < 587.8170 nm.

36.38. IDENTIFY and SET UP: $\dfrac{\lambda}{\Delta\lambda} = Nm$

EXECUTE: $N = \dfrac{\lambda}{m\Delta\lambda} = \dfrac{587.8002\text{ nm}}{(587.9782\text{ nm} - 587.8002\text{ nm})} = \dfrac{587.8002}{0.178} = 3302\text{ slits}$. $\dfrac{N}{1.20\text{ cm}} = \dfrac{3302}{1.20\text{ cm}} = 2752\dfrac{\text{slits}}{\text{cm}}$.

EVALUATE: A smaller number of slits would be needed to resolve these two lines in higher order.

36.39. IDENTIFY and SET UP: The maxima occur at angles θ given by Eq.(36.16), $2d\sin\theta = m\lambda$, where d is the spacing between adjacent atomic planes. Solve for d.

EXECUTE: second order says $m = 2$.

$$d = \frac{m\lambda}{2\sin\theta} = \frac{2(0.0850\times10^{-9}\text{ m})}{2\sin 21.5°} = 2.32\times10^{-10}\text{ m} = 0.232\text{ nm}$$

EVALUATE: Our result is similar to d calculated in Example 36.5.

36.40. IDENTIFY: The maxima are given by $2d\sin\theta = m\lambda$, $m = 1, 2, \ldots$

SET UP: $d = 3.50\times10^{-10}\text{ m}$.

EXECUTE: **(a)** $m = 1$ and $\lambda = \dfrac{2d\sin\theta}{m} = 2(3.50\times10^{-10}\text{ m})\sin 15.0° = 1.81\times10^{-10}\text{ m} = 0.181\text{ nm}$. This is an x ray.

(b) $\sin\theta = m\left(\dfrac{\lambda}{2d}\right) = m\left(\dfrac{1.81\times10^{-10}\text{ m}}{2[3.50\times10^{-10}\text{ m}]}\right) = m(0.2586)$. $m = 2$: $\theta = 31.1°$. $m = 3$: $\theta = 50.9°$. The equation doesn't have any solutions for $m > 3$.

EVALUATE: In this problem $\lambda/d = 0.52$.

36.41. IDENTIFY: Rayleigh's criterion says $\sin\theta = 1.22\dfrac{\lambda}{D}$

SET UP: The best resolution is 0.3 arcseconds, which is about $(8.33\times10^{-5})°$.

EXECUTE: **(a)** $D = \dfrac{1.22\lambda}{\sin\theta} = \dfrac{1.22(5.5\times10^{-7}\text{ m})}{\sin(8.33\times10^{-5}°)} = 0.46\text{ m}$

EVALUATE: **(b)** The Keck telescopes are able to gather more light than the Hale telescope, and hence they can detect fainter objects. However, their larger size does not allow them to have greater resolution—atmospheric conditions limit the resolution.

36.42. IDENTIFY: Apply $\sin\theta = 1.22\dfrac{\lambda}{D}$.

SET UP: $\theta = (1/60)°$

EXECUTE: $D = \dfrac{1.22\lambda}{\sin\theta} = \dfrac{1.22(5.5\times10^{-7}\text{ m})}{\sin(1/60)°} = 2.31\times10^{-3}\text{ m} = 2.3\text{ mm}$

EVALUATE: The larger the diameter the smaller the angle that can be resolved.

36.43. IDENTIFY: Apply $\sin\theta = 1.22\dfrac{\lambda}{D}$.

SET UP: $\theta = \dfrac{W}{h}$, where W = 28 km and $h = 1200$ km. θ is small, so $\sin\theta \approx \theta$.

EXECUTE: $D = \dfrac{1.22\lambda}{\sin\theta} = 1.22\lambda\dfrac{h}{W} = 1.22(0.036\text{ m})\dfrac{1.2\times10^{6}\text{ m}}{2.8\times10^{4}\text{ m}} = 1.88\text{ m}$

EVALUATE: D must be significantly larger than the wavelength, so a much larger diameter is needed for microwaves than for visible wavelengths.

36.44. IDENTIFY: Apply $\sin\theta = 1.22\dfrac{\lambda}{D}$.

SET UP: θ is small, so $\sin\theta \approx \theta = 1.00\times10^{-8}$ rad.

EXECUTE: $\lambda = \dfrac{D\sin\theta}{1.22} \approx \dfrac{D\theta}{1.22} = \dfrac{(8.00\times10^{6}\text{ m})(1.00\times10^{-8})}{1.22} = 0.0656\text{ m} = 6.56\text{ cm}$

EVALUATE: λ corresponds to microwaves.

36.45. **IDENTIFY** and **SET UP:** The angular size of the first dark ring is given by $\sin\theta_1 = 1.22\lambda/D$ (Eq.36.17). Calculate θ_1, and then the diameter of the ring on the screen is $2(4.5\text{ m})\tan\theta_1$.

EXECUTE: $\sin\theta_1 = 1.22\left(\dfrac{620\times10^{-9}\text{ m}}{7.4\times10^{-6}\text{ m}}\right) = 0.1022;\ \theta_1 = 0.1024\text{ rad}$

The radius of the Airy disk (central bright spot) is $r = (4.5\text{ m})\tan\theta_1 = 0.462\text{ m}$. The diameter is $2r = 0.92\text{ m} = 92\text{ cm}$.

EVALUATE: $\lambda/D = 0.084$. For this small D the central diffraction maximum is broad.

36.46. **IDENTIFY:** Rayleigh's criterion limits the angular resolution.

SET UP: Rayleigh's criterion is $\sin\theta \approx \theta = 1.22\ \lambda/D$.

EXECUTE: **(a)** Using Rayleigh's criterion

$$\sin\theta \approx \theta = 1.22\ \lambda/D = (1.22)(550\text{ nm})/(135/4\text{ mm}) = 1.99\times10^{-5}\text{ rad}$$

On the bear this angle subtends a distance x. $\theta = x/R$ and

$$x = R\theta = (11.5\text{ m})(1.99\times10^{-5}\text{ rad}) = 2.29\times10^{-4}\text{ m} = 0.23\text{ mm}$$

(b) At *f*/22, D is 4/22 times as large as at *f*/4. Since θ is proportional to $1/D$, and x is proportional to θ, x is 1/(4/22) = 22/4 times as large as it was at *f*/4. $x = (0.229\text{ mm})(22/4) = 1.3\text{ mm}$

EVALUATE: A wide-angle lens, such as one having a focal length of 28 mm, would have a much smaller opening at *f*/22 and hence would have an even less resolving ability.

36.47. **IDENTIFY** and **SET UP:** Resolved by Rayleigh's criterion means angular separation θ of the objects equals $1.22\lambda/D$. The angular separation θ of the objects is their linear separation divided by their distance from the telescope.

EXECUTE: $\theta = \dfrac{250\times10^3\text{ m}}{5.93\times10^{11}\text{ m}}$, where $5.93\times10^{11}\text{ m}$ is the distance from earth to Jupiter. Thus $\theta = 4.216\times10^{-7}$.

Then $\theta = 1.22\dfrac{\lambda}{D}$ and $D = \dfrac{1.22\lambda}{\theta} = \dfrac{1.22(500\times10^{-9}\text{ m})}{4.216\times10^{-7}} = 1.45\text{ m}$

EVALUATE: This is a very large telescope mirror. The greater the angular resolution the greater the diameter the lens or mirror must be.

36.48. **IDENTIFY:** Rayleigh's criterion says $\theta_{\text{res}} = 1.22\dfrac{\lambda}{D}$.

SET UP: $D = 7.20\text{ cm}$. $\theta_{\text{res}} = \dfrac{y}{s}$, where s is the distance of the object from the lens and $y = 4.00\text{ mm}$.

EXECUTE: $\dfrac{y}{s} = 1.22\dfrac{\lambda}{D}$. $s = \dfrac{yD}{1.22\lambda} = \dfrac{(4.00\times10^{-3}\text{ m})(7.20\times10^{-2}\text{ m})}{1.22(550\times10^{-9}\text{ m})} = 429\text{ m}$.

EVALUATE: The focal length of the lens doesn't enter into the calculation. In practice, it is difficult to achieve resolution that is at the diffraction limit.

36.49. **IDENTIFY** and **SET UP:** Let y be the separation between the two points being resolved and let s be their distance from the telescope. Then the limit of resolution corresponds to $1.22\dfrac{\lambda}{D} = \dfrac{y}{s}$.

EXECUTE: **(a)** Let the two points being resolved be the opposite edges of the crater, so y is the diameter of the crater. For the moon, $s = 3.8\times10^8$ m. $y = 1.22\lambda s/D$.

Hubble: $D = 2.4$ m and $\lambda = 400$ nm gives the maximum resolution, so $y = 77$ m

Arecibo: $D = 305$ m and $\lambda = 0.75$ m; $y = 1.1\times10^6$ m

(b) $s = \dfrac{yD}{1.22\lambda}$. Let $y \approx 0.30$ (the size of a license plate). $s = (0.30\text{ m})(2.4\text{ m})/[(1.22)(400\times10^{-9}\text{ m})] = 1500\text{ km}$.

EVALUATE: D/λ is much larger for the optical telescope and it has a much larger resolution even though the diameter of the radio telescope is much larger.

36.50. **IDENTIFY:** Apply $\sin\theta = 1.22\dfrac{\lambda}{D}$.

SET UP: θ is small, so $\sin\theta \approx \theta$. Smallest resolving angle is for short-wavelength light (400 nm).

EXECUTE: $\theta \approx 1.22\dfrac{\lambda}{D} = (1.22)\dfrac{400\times10^{-9}\text{ m}}{5.08\text{ m}} = 9.61\times10^{-8}\text{ rad}$. $\theta = \dfrac{10{,}000\text{ mi}}{R}$, where R is the distance to the star.

$R = \dfrac{10{,}000\text{ mi}}{\theta} = \dfrac{16{,}000\text{ km}}{9.6\times10^{-8}\text{rad}} = 1.7\times10^{11}\text{ km}$.

EVALUATE: This is less than a light year, so there are no stars this close.

36.51. **IDENTIFY:** Let y be the separation between the two points being resolved and let s be their distance from the telescope. The limit of resolution corresponds to $1.22\,\lambda/D = y/s$.

SET UP: $s = 4.28\text{ ly} = 4.05\times10^{16}\text{ m}$. Assume visible light, with $\lambda = 400\text{ m}$.

EXECUTE: $y = 1.22\,\lambda s/D = 1.22(400\times10^{-9}\text{ m})(4.05\times10^{16}\text{ m}/(10.0\text{ m}) = 2.0\times10^{9}\text{ m}$

EVALUATE: The diameter of Jupiter is 1.38×10^{8} m, so the resolution is insufficient, by about one order of magnitude.

36.52. **IDENTIFY:** If the apparatus of Exercise 36.4 is placed in water, then all that changes is the wavelength $\lambda \to \lambda' = \dfrac{\lambda}{n}$.

SET UP: For $y \ll x$, the distance between the two dark fringes on either side of the central maximum is $D' = 2y'$. Let $D = 2y$ be the separation of 5.91×10^{-3} m found in Exercise 36.4.

EXECUTE: $2y_1' = \dfrac{2x\lambda'}{a} = \dfrac{2x\lambda}{an} = \dfrac{D}{n} = \dfrac{5.91\times10^{-3}\text{ m}}{1.33} = 4.44\times10^{-3}\text{ m} = 4.44\text{ mm}.$

EVALUATE: The water shortens the wavelength and this decreases the width of the central maximum.

36.53. **(a) IDENTIFY** and **SET UP:** The intensity in the diffraction pattern is given by Eq.(36.5): $I = I_0\left(\dfrac{\sin\beta/2}{\beta/2}\right)^2$, where $\beta = \left(\dfrac{2\pi}{\lambda}\right)a\sin\theta$. Solve for θ that gives $I = \dfrac{1}{2}I_0$. The angles θ_+ and θ_- are shown in Figure 36.53.

EXECUTE: $I = \dfrac{1}{2}I_0$ so $\dfrac{\sin\beta/2}{\beta/2} = \dfrac{1}{\sqrt{2}}$

Let $x = \beta/2$; the equation for x is $\dfrac{\sin x}{x} = \dfrac{1}{\sqrt{2}} = 0.7071.$

Use trial and error to find the value of x that is a solution to this equation.

x	$(\sin x)/x$
1.0 rad	0.841
1.5 rad	0.665
1.2 rad	0.777
1.4 rad	0.7039
1.39 rad	0.7077;

thus $x = 1.39$ rad and $\beta = 2x = 2.78$ rad

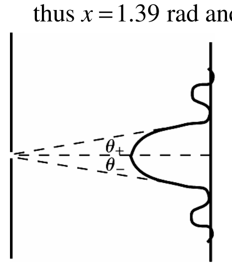


Figure 36.53

$\Delta\theta = |\theta_+ - \theta_-| = 2\theta_+$

$\sin\theta_+ = \dfrac{\lambda\beta}{2\pi a} =$

$\dfrac{\lambda}{a}\left(\dfrac{2.78\text{ rad}}{2\pi\text{ rad}}\right) = 0.4425\left(\dfrac{\lambda}{a}\right)$

(i) For $\dfrac{a}{\lambda} = 2$, $\sin\theta_+ = 0.4425\left(\dfrac{1}{2}\right) = 0.2212$; $\theta_+ = 12.78°$; $\Delta\theta = 2\theta_+ = 25.6°$

(ii) For $\dfrac{a}{\lambda} = 5$, $\sin\theta_+ = 0.4425\left(\dfrac{1}{5}\right) = 0.0885$; $\theta_+ = 5.077°$; $\Delta\theta = 2\theta_+ = 10.2°$

(iii) For $\dfrac{a}{\lambda} = 10$, $\sin\theta_+ = 0.4425\left(\dfrac{1}{10}\right) = 0.04425$; $\theta_+ = 2.536°$; $\Delta\theta = 2\theta_+ = 5.1°$

(b) IDENTIFY and **SET UP:** $\sin\theta_0 = \dfrac{\lambda}{a}$ locates the first minimum. Solve for θ_0.

EXECUTE: (i) For $\dfrac{a}{\lambda} = 2$, $\sin\theta_0 = \dfrac{1}{2}$; $\theta_0 = 30.0°$; $2\theta_0 = 60.0°$

(ii) For $\dfrac{a}{\lambda} = 5$, $\sin\theta_0 = \dfrac{1}{5}$; $\theta_0 = 11.54°$; $2\theta_0 = 23.1°$

(iii) For $\frac{a}{\lambda}=10,\ \sin\theta_0=\left(\frac{1}{10}\right);\theta_0=5.74°;\ 2\theta_0=11.5°$

EVALUATE: Either definition of the width shows that the central maximum gets narrower as the slit gets wider.

36.54. **IDENTIFY:** The two holes behave like double slits and cause the sound waves to interfere after they pass through the holes. The motion of the speakers causes a Doppler shift in the wavelength of the sound.

SET UP: The wavelength of the sound that strikes the wall is $\lambda = \lambda_0 - v_s T_s$, and destructive interference first occurs where $\sin\theta = \lambda/2$.

EXECUTE: **(a)** First find the wavelength of the sound that strikes the openings in the wall.

$$\lambda = \lambda_0 - v_s T_s = v/f_s - v_s/f_s = (v - v_s)/f_s = (344\ \text{m/s} - 80.0\ \text{m/s})/(1250\ \text{Hz}) = 0.211\ \text{m}$$

Destructive interference first occurs where $d \sin\theta = \lambda/2$, which gives

$$d = \lambda/(2\sin\theta) = (0.211\ \text{m})/(2\sin 12.7°) = 0.480\ \text{m}$$

(b) $\lambda = v/f = (344\ \text{m/s})/(1250\ \text{Hz}) = 0.275\ \text{m}$

$$\sin\theta = \lambda/2d = (0.275\ \text{m})/[2(0.480\ \text{m})] \rightarrow \theta = \pm 16.7°$$

EVALUATE: The moving source produces sound of shorter wavelength than the stationary source, so the angles at which destructive interference occurs are smaller for the moving source than for the stationary source.

36.55. **IDENTIFY** and **SET UP:** $\sin\theta = \lambda/a$ locates the first dark band. In the liquid the wavelength changes and this changes the angular position of the first diffraction minimum.

EXECUTE: $\sin\theta_{\text{air}} = \frac{\lambda_{\text{air}}}{a};\ \sin\theta_{\text{liquid}} = \frac{\lambda_{\text{liquid}}}{a}$

$\lambda_{\text{liquid}} = \lambda_{\text{air}}\left(\frac{\sin\theta_{\text{liquid}}}{\sin\theta_{\text{air}}}\right) = 0.4836$

$\lambda = \lambda_{\text{air}}/n$ (Eq.33.5), so $n = \lambda_{\text{air}}/\lambda_{\text{liquid}} = 1/0.4836 = 2.07$

EVALUATE: Light travels faster in air and n must be >1.00. The smaller λ in the liquid reduces θ that located the first dark band.

36.56. **IDENTIFY:** $d=\frac{1}{N}$, so the bright fringes are located by $\frac{1}{N}\sin\theta = \lambda$

SET UP: Red: $\frac{1}{N}\sin\lambda_R = 700\ \text{nm}$. Violet: $\frac{1}{N}\sin\lambda_V = 400\ \text{nm}$.

EXECUTE: $\frac{\sin\theta_R}{\sin\theta_V}=\frac{7}{4}$. $\theta_R-\theta_V=15° \rightarrow \theta_R=\theta_V+15°$. $\frac{\sin(\theta_V+15°)}{\sin\theta_V}=\frac{7}{4}$. Using a trig identify from Appendix B,

$\frac{\sin\theta_V\cos 15° + \cos\theta_V \sin 15°}{\sin\theta_V} = 7/4$. $\cos 15° + \cot\theta_V \sin 15° = 7/4$.

$\tan\theta_V = 0.330 \Rightarrow \theta_V = 18.3°$ and $\theta_R = \theta_V + 15° = 18.3° + 15° = 33.3°$. Then $\frac{1}{N}\sin\theta_R = 700$ nm gives

$N = \frac{\sin\theta_R}{700\ \text{nm}} = \frac{\sin 33.3°}{700\times10^{-9}\ \text{m}} = 7.84\times10^5\ \text{lines/m} = 7840\ \text{lines/cm}$. The spectrum begins at $18.3°$ and ends at $33.3°$.

EVALUATE: As N is increased, the angular range of the visible spectrum increases.

36.57. **(a) IDENTIFY** and **SET UP:** The angular position of the first minimum is given by $a\sin\theta = m\lambda$ (Eq.36.2), with $m=1$. The distance of the minimum from the center of the pattern is given by $y = x\tan\theta$.

$\sin\theta = \frac{\lambda}{a} = \frac{540\times10^{-9}\ \text{m}}{0.360\times10^{-3}\ \text{m}} = 1.50\times10^{-3};\ \theta = 1.50\times10^{-3}\ \text{rad}$

$y_1 = x\tan\theta = (1.20\ \text{m})\tan(1.50\times10^{-3}\ \text{rad}) = 1.80\times10^{-3}\ \text{m} = 1.80\ \text{mm}.$

(Note that θ is small enough for $\theta \approx \sin\theta \approx \tan\theta$, and Eq.(36.3) applies.)

(b) IDENTIFY and **SET UP:** Find the phase angle β where $I = I_0/2$. Then use Eq.(36.6) to solve for θ and $y = x\tan\theta$ to find the distance.

EXECUTE: From part (a) of Problem 36.53, $I = \frac{1}{2}I_0$ when $\beta = 2.78$ rad.

$\beta = \left(\frac{2\pi}{\lambda}\right)a\sin\theta$ (Eq.(36.6)), so $\sin\theta = \frac{\beta\lambda}{2\pi a}$.

$y = x\tan\theta \approx x\sin\theta \approx \frac{\beta\lambda x}{2\pi a} = \frac{(2.78\ \text{rad})(540\times10^{-9}\ \text{m})(1.20\ \text{m})}{2\pi(0.360\times10^{-3}\ \text{m})} = 7.96\times10^{-4}\ \text{m} = 0.796\ \text{mm}$

EVALUATE: The point where $I = I_0/2$ is not midway between the center of the central maximum and the first minimum; see Exercise 36.15.

36.58. **IDENTIFY:** $I = I_0\left(\dfrac{\sin\gamma}{\gamma}\right)^2$. The maximum intensity occurs when the derivative of the intensity function with respect to γ is zero.

SET UP: $\dfrac{d\sin\gamma}{d\gamma} = \cos\gamma$. $\dfrac{d}{d\gamma}\left(\dfrac{1}{\gamma}\right) = -\dfrac{1}{\gamma^2}$.

EXECUTE: $\dfrac{dI}{d\gamma} = I_0\dfrac{d}{d\gamma}\left(\dfrac{\sin\gamma}{\gamma}\right)^2 = 2\left(\dfrac{\sin\gamma}{\gamma}\right)\left(\dfrac{\cos\gamma}{\gamma} - \dfrac{\sin\gamma}{\gamma^2}\right) = 0$. $\dfrac{\cos\gamma}{\gamma} - \dfrac{\sin\gamma}{\gamma^2} \Rightarrow \gamma\cos\gamma = \sin\gamma \Rightarrow \gamma = \tan\gamma$.

(b) The graph in Figure 36.58 is a plot of $f(\gamma) = \gamma - \tan\gamma$. When $f(\gamma)$ equals zero, there is an intensity maximum. Getting estimates from the graph, and then using trial and error to narrow in on the value, we find that the three smallest γ-values are $\gamma = 4.49$ rad 7.73 rad, and 10.9 rad.

EVALUATE: $\gamma = 0$ is the central maximum. The three values of γ we found are the locations of the first three secondary maxima. The first four minima are at $\gamma = 3.14$ rad , 6.28 rad, 9.42 rad, and 12.6 rad. The maxima are between adjacent minima, but not precisely midway between them.

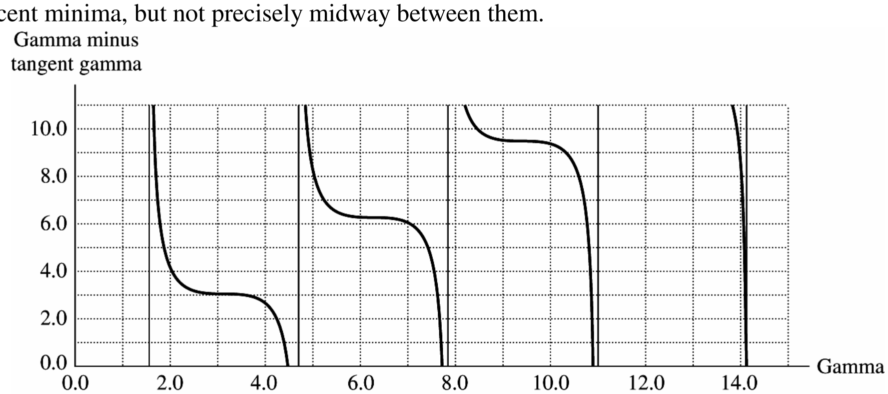


Figure 36.58

36.59. **IDENTIFY** and **SET UP:** Relate the phase difference between adjacent slits to the sum of the phasors for all slits. The phase difference between adjacent slits is $\phi = \dfrac{2\pi d}{\lambda}\sin\theta \approx \dfrac{2\pi d\theta}{\lambda}$ when θ is small and $\sin\theta \approx \theta$. Thus $\theta = \dfrac{\lambda\phi}{2\pi d}$.

EXECUTE: A principal maximum occurs when $\phi = \phi_{\max} = m2\pi$, where m is an integer, since then all the phasors add. The first minima on either side of the mth principal maximum occur when $\phi = \phi^{\pm}_{\min} = m2\pi \pm (2\pi/N)$ and the phasor diagram for N slits forms a closed loop and the resultant phasor is zero. The angular position of a principal maximum is $\theta = \left(\dfrac{\lambda}{2\pi d}\right)\phi_{\max}$. The angular position of the adjacent minimum is $\theta^{\pm}_{\min} = \left(\dfrac{\lambda}{2\pi d}\right)\phi^{\pm}_{\min}$.

$$\theta^{+}_{\min} = \left(\frac{\lambda}{2\pi d}\right)\left(\phi_{\max} + \frac{2\pi}{N}\right) = \theta + \left(\frac{\lambda}{2\pi d}\right)\left(\frac{2\pi}{N}\right) = \theta + \frac{\lambda}{Nd}$$

$$\theta^{-}_{\min} = \left(\frac{\lambda}{2\pi d}\right)\left(\phi_{\max} - \frac{2\pi}{N}\right) = \theta - \frac{\lambda}{Nd}$$

The angular width of the principal maximum is $\theta^{+}_{\min} - \theta^{-}_{\min} = \dfrac{2\lambda}{Nd}$, as was to be shown.

EVALUATE: The angular width of the principal maximum decreases like $1/N$ as N increases.

36.60. **IDENTIFY:** The change in wavelength of the H_α line is due to a Doppler shift in the wavelength due to the motion of the galaxy.

SET UP: From Equation 16.30, the Doppler effect formula for light is $f_R = \sqrt{\dfrac{c-v}{c+v}}f_S$.

EXECUTE: First find the wavelength of the light using the grating information.

$$\lambda = d\sin\theta_1 = [1/(575{,}800 \text{ lines/m})]\sin 23.41° = 6.900\times10^{-7}\text{ m} = 690.0\text{ nm}$$

Using Equation 16.30, we have $f_R = \sqrt{\dfrac{c-v}{c+v}}f_S$. In this case, f_R is the frequency of the 690.0-nm light that the cosmologist measures, and f_S is the frequency of the 656.3-nm light of the H_α line obtained in the laboratory.

Solving for v gives $v = \dfrac{1-(f_R/f_S)^2}{1+(f_R/f_S)^2}c$. Since $f\lambda = c$, $f = c/\lambda$, which gives $f_R/f_S = \lambda_S/\lambda_R$. Substituting this into the equation for v, we get

$$v = \frac{1-\left(\dfrac{\lambda_S}{\lambda_R}\right)^2}{1+\left(\dfrac{\lambda_S}{\lambda_R}\right)^2}c = \frac{1-\left(\dfrac{656.3\text{ nm}}{690.0\text{ nm}}\right)^2}{1+\left(\dfrac{656.3\text{ nm}}{690.0\text{ nm}}\right)^2}(3.00\times10^8\text{ m/s}) = 1.501\times10^7\text{ m/s},$$

which is 5.00% the speed of light.

EVALUATE: Since v is positive, the galaxy is moving *away from* us. We can also see this because the wavelength has increased due to the motion.

36.61. IDENTIFY and SET UP: Draw the specified phasor diagrams. There is totally destructive interference between two slits when their phasors are in opposite directions.

EXECUTE: **(a)** For eight slits, the phasor diagrams must have eight vectors. The diagrams for each specified value of ϕ are sketched in Figure 36.61a. In each case the phasors all sum to zero.

(b) The additional phasor diagrams for $\phi = 3\pi/2$ and $3\pi/4$ are sketched in Figure 36.61b.

For $\phi = \dfrac{3\pi}{4}$, $\phi = \dfrac{5\pi}{4}$, and $\phi = \dfrac{7\pi}{4}$, totally destructive interference occurs between slits four apart. For $\phi = \dfrac{3\pi}{2}$, totally destructive interference occurs with every second slit.

EVALUATE: At a minimum the phasors for all slits sum to zero.

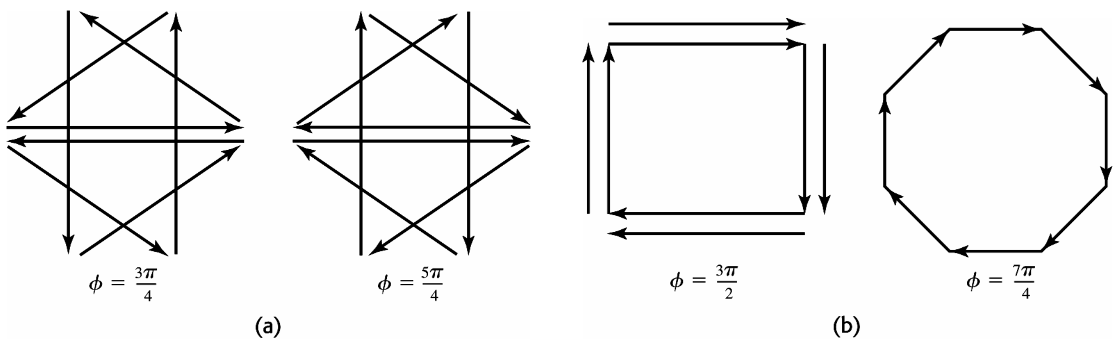


Figure 36.61

36.62. IDENTIFY: Maxima are given by $2d\sin\theta = m\lambda$.

SET UP: d is the separation between crystal planes.

EXECUTE: **(a)** $\theta = \arcsin\left(\dfrac{m\lambda}{2d}\right) = \arcsin\left(m\dfrac{0.125\text{ nm}}{2(0.282\text{ nm})}\right) = \arcsin(0.2216m)$.

For $m=1: \theta = 12.8°$, $m=2: \theta = 26.3°$, $m=3: \theta = 41.7°$, and $m=4: \theta = 62.4°$. No larger m values yield answers.

(b) If the separation $d = \dfrac{a}{\sqrt{2}}$, then $\theta = \arcsin\left(\dfrac{\sqrt{2}m\lambda}{2a}\right) = \arcsin(0.3134m)$.

So for $m=1$: $\theta = 18.3°$, $m=2$: $\theta = 38.8°$, and $m=3$: $\theta = 70.1°$. No larger m values yield answers.

EVALUATE: In part (b), where d is smaller, the maxima for each m are at larger θ

36.63. IDENTIFY and SET UP: In each case consider the relevant phasor diagram.

EXECUTE: **(a)** For the maxima to occur for N slits, the sum of all the phase differences between the slits must add to zero (the phasor diagram closes on itself). This requires that, adding up all the relative phase shifts, $N\phi = 2\pi m$, for some integer m. Therefore $\phi = \dfrac{2\pi m}{N}$, for m not an integer multiple of N, which would give a maximum.

(b) The sum of N phase shifts $\phi = \dfrac{2\pi m}{N}$ brings you full circle back to the maximum, so only the $N-1$ previous phases yield minima between each pair of principal maxima.

EVALUATE: The $N-1$ minima between each pair of principal maxima cause the maxima to become sharper as N increases.

36.64. IDENTIFY: Set $d = a$ in the expressions for ϕ and β and use the results in Eq.(36.12).

SET UP: Figure 36.64 shows a pair of slits whose width and separation are equal

EXECUTE: Figure 36.64 shows that the two slits are equivalent to a single slit of width $2a$.

$\phi = \dfrac{2\pi d}{\lambda}\sin\theta$, so $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \phi$. So then the intensity is

$$I = I_0\cos^2(\beta/2)\left(\frac{\sin^2(\beta/2)}{(\beta/2)^2}\right) = I_0\frac{(2\sin(\beta/2)\cos(\beta/2))^2}{\beta^2} = I_0\frac{\sin^2\beta}{\beta^2} = I_0\frac{\sin^2(\beta'/2)}{(\beta'/2)^2}, \text{ where } \beta' = \frac{2\pi(2a)}{\lambda}\sin\theta,$$

which is Eq. (35.5) with double the slit width.

EVALUATE: In Chapter 35 we considered the limit where $a << d$. $a > d$ is not possible.

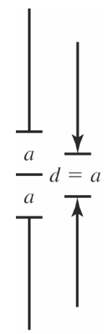


Figure 36.64

36.65. IDENTIFY and SET UP: The condition for an intensity maximum is $d\sin\theta = m\lambda,\ m = 0,\ \pm1,\ \pm2,\ldots$ Third order means $m = 3$. The longest observable wavelength is the one that gives $\theta = 90°$ and hence $\theta = 1$.

EXECUTE: 6500 lines/cm so 6.50×10^5 lines/m and $d = \dfrac{1}{6.50\times10^5}\text{ m} = 1.538\times10^{-6}\text{ m}$

$$\lambda = \frac{d\sin\theta}{m} = \frac{(1.538\times10^{-6}\text{ m})(1)}{3} = 5.13\times10^{-7}\text{ m} = 513\text{ nm}$$

EVALUATE: The longest wavelength that can be obtained decreases as the order increases.

36.66. IDENTIFY and SET UP: As the rays first reach the slits there is already a phase difference between adjacent slits of $\dfrac{2\pi d\sin\theta'}{\lambda}$. This, added to the usual phase difference introduced after passing through the slits, yields the condition for an intensity maximum. For a maximum the total phase difference must equal $2\pi m$.

EXECUTE: $\dfrac{2\pi d\sin\theta}{\lambda} + \dfrac{2\pi d\sin\theta'}{\lambda} = 2\pi m \Rightarrow d(\sin\theta + \sin\theta') = m\lambda$

(b) $600\text{ slits/mm} \Rightarrow d = \dfrac{1}{6.00\times10^5\text{ m}^{-1}} = 1.67\times10^{-6}\text{ m}.$

For $\theta' = 0°$,

$m = 0$: $\theta = \arcsin(0) = 0.$

$m = 1$: $\theta = \arcsin\left(\dfrac{\lambda}{d}\right) = \arcsin\left(\dfrac{6.50\times10^{-7}\text{ m}}{1.67\times10^{-6}\text{ m}}\right) = 22.9°.$

$m = -1$: $\theta = \arcsin\left(-\dfrac{\lambda}{d}\right) = \arcsin\left(-\dfrac{6.50\times10^{-7}\text{ m}}{1.67\times10^{-6}\text{ m}}\right) = -22.9°.$

For $\theta' = 20.0°$,

$m = 0$: $\theta = \arcsin(-\sin 20.0°) = -20.0°.$

$m = 1$: $\theta = \arcsin\left(\dfrac{6.50\times10^{-7}\text{ m}}{1.67\times10^{-6}\text{ m}} - \sin 20.0°\right) = 2.71°.$

$m = -1$: $\theta = \arcsin\left(-\dfrac{6.50\times10^{-7}\text{ m}}{1.67\times10^{-6}\text{ m}} - \sin 20.0°\right) = -47.0°.$

EVALUATE: When $\theta' > 0$, the maxima are shifted downward on the screen, toward more negative angles.

36.67. IDENTIFY: The maxima are given by $d\sin\theta = m\lambda$. We need $\sin\theta = \dfrac{m\lambda}{d} \le 1$in order for all the visible wavelengths are to be seen.

SET UP: For $650\text{ slits/mm} \Rightarrow d = \dfrac{1}{6.50\times10^5\text{ m}^{-1}} = 1.53\times10^{-6}\text{ m}.$

EXECUTE: $\lambda_1 = 4.00\times10^{-7}\text{m}: m=1: \frac{\lambda_1}{d} = 0.26; m=2: \frac{2\lambda_1}{d} = 0.52; m=3: \frac{3\lambda_1}{d} = 0.78.$

$\lambda_2 = 7.00\times10^{-7}\text{m}: m=1: \frac{\lambda_2}{d} = 0.46; m=2: \frac{2\lambda_2}{d} = 0.92; m=3: \frac{3\lambda_2}{d} = 1.37.$ So, the third order does not contain the violet end of the spectrum, and therefore only the first and second order diffraction patterns contain all colors of the spectrum.

EVALUATE: θ for each maximum is larger for longer wavelengths.

36.68. IDENTIFY: Apply $\sin\theta = 1.22\frac{\lambda}{D}$.

SET UP: θ is small, so $\sin\theta \approx \frac{\Delta x}{R}$, where Δx is the size of the detail and $R = 7.2\times10^8$ ly. 1 ly $= 9.41\times10^{12}$ km. $\lambda = c/f$

EXECUTE: $\sin\,\theta = 1.22\frac{\lambda}{D} \approx \frac{\Delta x}{R} \Rightarrow \Delta x = \frac{1.22\lambda R}{D} = \frac{(1.22)cR}{Df} = \frac{(1.22)(3.00\times10^5\text{ km/s})(7.2\times10^8\text{ ly})}{(77.000\times10^3\text{ km})(1.665\times10^9\text{ Hz})} = 2.06\text{ ly}.$

$(9.41\times10^{12}\text{ km/ly})(2.06\text{ ly}) = 1.94\times10^{13}\text{ km}.$

EVALUATE: $\lambda = 18$ cm. λ/D is very small, so $\frac{\Delta x}{R}$ is very small. Still, R is very large and Δx is many orders of magnitude larger than the diameter of the sun.

36.69. IDENTIFY and SET UP: Add the phases between adjacent sources.

EXECUTE: **(a)** $d\sin\theta = m\lambda$. Place 1st maximum at ∞ or $\theta = 90^\circ$. $d = \lambda$. If $d < \lambda$, this puts the first maximum "beyond ∞." Thus, if $d < \lambda$ there is only a single principal maximum.

(b) At a principal maximum when $\delta = 0$, the phase difference due to the path difference between adjacent slits is $\Phi_{\text{path}} = 2\pi\left(\frac{d\sin\theta}{\lambda}\right)$. This just scales 2π radians by the fraction the wavelength is of the path difference between adjacent sources. If we add a relative phase δ between sources, we still must maintain a total phase difference of zero to keep our principal maximum.

$$\Phi_{\text{path}} \pm \delta = 0 \Rightarrow \frac{2\pi d\sin\theta}{\lambda} = \pm\delta \text{ or } \theta = \sin^{-1}\left(\frac{\delta\lambda}{2\pi d}\right)$$

(c) $d = \frac{0.280\text{ m}}{14} = 0.0200$ m (count the number of spaces between 15 points). Let $\theta = 45^\circ$. Also recall $f\lambda = c$, so

$$\delta_{\max} = \pm\frac{2\pi(0.0200\text{ m})(8.800\times10^9\text{ Hz})\sin45^\circ}{(3.00\times10^8\text{ m/s})} = \pm2.61\text{ radians.}$$

EVALUATE: δ must vary over a wider range in order to sweep the beam through a greater angle.

36.70. IDENTIFY: The wavelength of the light is smaller under water than it is in air, which will affect the resolving power of the lens, by Rayleigh's criterion.

SET UP: The wavelength under water is $\lambda = \lambda_0/n$, and for small angles Rayleigh's criterion is $\theta = 1.22\lambda/D$.

EXECUTE: **(a)** In air the wavelength is $\lambda_0 = c/f = (3.00\times10^8\text{ m/s})/(6.00\times10^{14}\text{ Hz}) = 5.00\times10^{-7}$ m. In water the wavelength is $\lambda = \lambda_0/n = (5.00\times10^{-7}\text{ m})/1.33 = 3.76\times10^{-7}$ m. With the lens open all the way, we have $D = f/2.8 = (35.0\text{ mm})/2.80 = (0.0350\text{ m})/2.80$. In the water, we have

$$\sin\theta \approx \theta = 1.22\,\lambda/D = (1.22)(3.76\times10^{-7}\text{ m})/[(0.0350\text{ m})/2.80] = 3.67\times10^{-5}\text{ rad}$$

Calling w the width of the resolvable detail, we have

$$\theta = w/R \to w = R\theta = (2750\text{ mm})(3.67\times10^{-5}\text{ rad}) = 0.101\text{ mm}$$

(b) $\theta = 1.22\,\lambda/D = (1.22)(5.00\times10^{-7}\text{ m})/[(0.0350\text{ m})/2.80] = 4.88\times10^{-5}$ rad

$$w = R\theta = (2750\text{ mm})(4.88\times10^{-5}\text{ rad}) = 0.134\text{ mm}$$

EVALUATE: Due to the reduced wavelength underwater, the resolution of the lens is better under water than in air.

36.71. IDENTIFY and SET UP: Resolved by Rayleigh's criterion means the angular separation θ of the objects is given by $\theta = 1.22\lambda/D$. $\theta = y/s$, where $y = 75.0$ m is the distance between the two objects and s is their distance from the astronaut (her altitude).

EXECUTE: $\frac{y}{s} = 1.22\frac{\lambda}{D}$

$$s = \frac{yD}{1.22\lambda} = \frac{(75.0\text{ m})(4.00\times10^{-3}\text{ m})}{1.22(500\times10^{-9}\text{ m})} = 4.92\times10^5\text{ m} = 492\text{ km}$$

EVALUATE: In practice, this diffraction limit of resolution is not achieved. Defects of vision and distortion by the earth's atmosphere limit the resolution more than diffraction does.

36.72. **IDENTIFY:** Apply $\sin\theta = 1.22\dfrac{\lambda}{D}$.

SET UP: θ is small, so $\sin\theta \approx \dfrac{\Delta x}{R}$, where Δx is the size of the details and R is the distance to the earth.

$1 \text{ ly} = 9.41\times10^{15}$ m.

EXECUTE: **(a)** $R = \dfrac{D\Delta x}{1.22\lambda} = \dfrac{(6.00\times10^6 \text{ m})(2.50\times10^5 \text{ m})}{(1.22)(1.0\times10^{-5} \text{ m})} = 1.23\times10^{17} \text{ m} = 13.1 \text{ ly}$

(b) $\Delta x = \dfrac{1.22\lambda R}{D} = \dfrac{(1.22)(1.0\times10^{-5} \text{ m})(4.22 \text{ ly})(9.41\times10^{15} \text{ m/ly})}{1.0 \text{ m}} = 4.84\times10^8 \text{ km}$. This is about 10,000 times the diameter of the earth! Not enough resolution to see an earth-like planet! Δx is about 3 times the distance from the earth to the sun.

(c) $\Delta x = \dfrac{(1.22)(1.0\times10^{-5} \text{ m})(59 \text{ ly})(9.41\times10^{15} \text{ m/ly})}{6.00\times10^6 \text{ m}} = 1.13\times10^6 \text{ m} = 1130 \text{ km}.$

$\dfrac{\Delta x}{D_{\text{planet}}} = \dfrac{1130 \text{ km}}{1.38\times10^5 \text{ km}} = 8.19\times10^{-3}$; Δx is small compared to the size of the planet.

EVALUATE: The very large diameter of *Planet Imager* allows it to resolve planet-sized detail at great distances.

36.73. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** From the segment dy', the fraction of the amplitude of E_0 that gets through is

$E_0\left(\dfrac{dy'}{a}\right) \Rightarrow dE = E_0\left(\dfrac{dy'}{a}\right)\sin(kx-\omega t).$

(b) The path difference between each little piece is

$y'\sin\theta \Rightarrow kx = k(D - y'\sin\theta) \Rightarrow dE = \dfrac{E_0 dy'}{a}\sin(k(D-y'\sin\theta)-\omega t)$. This can be rewritten as

$dE = \dfrac{E_0 dy'}{a}(\sin(kD-\omega t)\cos(ky'\sin\theta) + \sin(ky'\sin\theta)\cos(kD-\omega t)).$

(c) So the total amplitude is given by the integral over the slit of the above.

$\Rightarrow E = \int_{-a/2}^{a/2} dE = \dfrac{E_0}{a}\int_{-a/2}^{a/2} dy'\,(\sin(kD-\omega t)\cos(ky'\sin\theta) + \sin(ky'\sin\theta)\cos(kD-\omega t)).$

But the second term integrates to zero, so we have:

$$E = \frac{E_0}{a}\sin(kD-\omega t)\int_{-a/2}^{a/2} dy'(\cos(ky'\sin\theta)) = E_0\sin(kD-\omega t)\left[\left(\frac{\sin(ky'\sin\theta)}{ka\sin\theta/2}\right)\right]_{-a/2}^{a/2}$$

$$\Rightarrow E = E_0\sin(kD-\omega t)\left(\frac{\sin(ka(\sin\theta)/2)}{ka(\sin\theta)/2}\right) = E_0\sin(kD-\omega t)\left(\frac{\sin(\pi a(\sin\theta)/\lambda)}{\pi a(\sin\theta)/\lambda}\right).$$

At $\theta = 0$, $\dfrac{\sin[\ldots]}{[\ldots]} = 1 \Rightarrow E = E_0\sin(kD-\omega t).$

(d) Since $I \propto E^2 \Rightarrow I = I_0\left(\dfrac{\sin(ka(\sin\theta)/2)}{ka(\sin\theta)/2}\right)^2 = I_0\left(\dfrac{\sin(\beta/2)}{\beta/2}\right)^2$, where we have used $I_0 = E_0^2\sin^2(kx-\omega t).$

EVALUATE: The same result for $I(\theta)$ is obtained as was obtained using phasors.

36.74. **IDENTIFY** and **SET UP:** Follow the steps specified in the problem.

EXECUTE: **(a)** Each source can be thought of as a traveling wave evaluated at $x = R$ with a maximum amplitude of E_0. However, each successive source will pick up an extra phase from its respective pathlength to point P. $\phi = 2\pi\left(\dfrac{d\sin\theta}{\lambda}\right)$ which is just 2π, the maximum phase, scaled by whatever fraction the path difference, $d\sin\theta$, is of the wavelength, λ. By adding up the contributions from each source (including the accumulating phase difference) this gives the expression provided.

(b) $e^{i(kR-\omega t+n\phi)} = \cos(kR-\omega t+n\phi) + i\sin(kR-\omega t+n\phi)$. The real part is just $\cos(kR-\omega t+n\phi)$. So,

$\text{Re}\left[\sum_{n=0}^{N-1} E_0 e^{i(kR-\omega t+n\phi)}\right] = \sum_{n=0}^{N-1} E_0\cos(kR-\omega t+n\phi)$. (Note: Re means "the real part of . . . ."). But this is just

$E_0\cos(kR-\omega t) + E_0\cos(kR-\omega t+\phi) + E_0\cos(kR-\omega t+2\phi) + \cdots + E_0\cos(kR-\omega t+(N-1)\phi)$

(c) $\sum_{n=0}^{N-1} E_0\, e^{i(kR-\omega t+n\phi)} = E_0 \sum_{n=0}^{N-1} e^{-i\omega t}\, e^{+ikR} e^{in\phi} = E_0 e^{i(kR-\omega t)} \sum_{n=0}^{N-1} e^{in\phi}$. $\sum_{n=0}^{\infty} e^{in\phi} = \sum_{n=0}^{N-1} (e^{i\phi})^n$. But recall $\sum_{n=0}^{N-1} x^n = \dfrac{x^N - 1}{x-1}$.

Let $x = e^{i\phi}$ so $\sum_{n=0}^{N-1} (e^{i\phi})^n = \dfrac{e^{iN\phi}-1}{e^{i\phi}-1}$(nice trick!). But $\dfrac{e^{iN\phi}-1}{e^{i\phi}-1} = \dfrac{e^{iN\phi/2}(e^{iN\phi/2}-e^{-iN\phi/2})}{e^{i\phi/2}(e^{i\phi/2}-e^{-i\phi/2})} = e^{i(N-1)\phi/2}\dfrac{(e^{iN\phi/2}-e^{-iN\phi/2})}{(e^{i\phi/2}-e^{-i\phi/2})}$.

Putting everything together:

$$\sum_{n=0}^{N-1} E_0 e^{i(kR-\omega t+n\phi)} = E_0 e^{i(kR-\omega t+(N-1)\phi/2)}\,\frac{(e^{iN\phi/2}-e^{-iN\phi/2})}{(e^{i\phi/2}-e^{-i\phi/2})}$$

$$= E_0\left[\cos(kR-\omega t+(N-1)\phi/2) + i\sin(kR-\omega t+(N-1)\phi/2)\right]\left[\frac{\cos N\phi/2 + i\sin N\phi/2 - \cos N\phi/2 + i\sin N\phi/2}{\cos\phi/2 + i\sin\phi/2 - \cos\phi/2 + i\sin\phi/2}\right]$$

Taking only the real part gives $\Rightarrow E_0\cos(kR-\omega t+(N-1)\phi/2)\dfrac{\sin(N\phi/2)}{\sin\phi/2} = E.$

(d) $I = |E|^2_{\text{av}} = I_0\dfrac{\sin^2(N\phi/2)}{\sin^2(\phi/2)}$. (The $\cos^2$ term goes to $\frac{1}{2}$ in the time average and is included in the definition of I_0.)

$I_0 \propto \dfrac{E_0^2}{2}$.

EVALUATE: **(e)** $N = 2$. $I = I_0\dfrac{\sin^2(2\phi/2)}{\sin^2\phi/2} = \dfrac{I_0(2\sin\phi/2\cos\phi/2)^2}{\sin^2\phi/2} = 4I_0\cos^2\dfrac{\phi}{2}$. Looking at Eq.(35.9),

$I_0' \propto 2E_0^2$ but for us $I_0 \propto \dfrac{E_0^2}{2} = \dfrac{I_0'}{4}$.

36.75. **IDENTIFY** and **SET UP:** From Problem 36.74, $I = I_0\dfrac{\sin^2(N\phi/2)}{\sin^2\phi/2}$. Use this result to obtain each result specified in the problem.

EXECUTE: **(a)** $\lim_{\phi\to 0} I \to \dfrac{0}{0}$. Use l'Hôpital's rule: $\lim_{\phi\to 0}\dfrac{\sin(N\phi/2)}{\sin\phi/2} = \lim_{\phi\to 0}\left(\dfrac{N/2}{1/2}\right)\dfrac{\cos(N\phi/2)}{\cos(\phi/2)} = N$. So $\lim_{\phi\to 0} I = N^2 I_0$.

(b) The location of the first minimum is when the numerator first goes to zero at $\dfrac{N}{2}\phi_{\text{min}} = \pi$ or $\phi_{\text{min}} = \dfrac{2\pi}{N}$. The width of the central maximum goes like $2\phi_{\text{min}}$, so it is proportional to $\dfrac{1}{N}$.

(c) Whenever $\dfrac{N\phi}{2} = n\pi$ where n is an integer, the numerator goes to zero, giving a minimum in intensity. That is, I is a minimum wherever $\phi = \dfrac{2n\pi}{N}$. This is true assuming that the denominator doesn't go to zero as well, which occurs when $\dfrac{\phi}{2} = m\pi$, where m is an integer. When both go to zero, using the result from part(a), there is a maximum. That is, if $\dfrac{n}{N}$ is an integer, there will be a maximum.

(d) From part (c), if $\dfrac{n}{N}$ is an integer we get a maximum. Thus, there will be $N-1$ minima. (Places where $\dfrac{n}{N}$ is not an integer for fixed N and integer n.) For example, $n = 0$ will be a maximum, but $n = 1, 2\ldots, N-1$ will be minima with another maximum at $n = N$.

(e) Between maxima $\dfrac{\phi}{2}$ is a half-integer multiple of $\pi\left(\text{i.e. } \dfrac{\pi}{2}, \dfrac{3\pi}{2}, \text{etc.}\right)$ and if N is odd then

$\dfrac{\sin^2(N\phi/2)}{\sin^2\phi/2} \to 1$, so $I \to I_0$.

EVALUATE: These results show that the principal maxima become sharper as the number of slits is increased.

37

RELATIVITY

37.1. **IDENTIFY** and **SET UP:** Consider the distance A to O' and B to O' as observed by an observer on the ground (Figure 37.1).

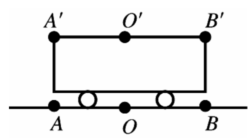


Figure 37.1

EXECUTE: Simultaneous to observer on train means light pulses from A' and B' arrive at O' at the same time. To observer at O light from A' has a longer distance to travel than light from B' so O will conclude that the pulse from $A(A')$ started before the pulse at $B(B')$. To observer at O bolt A appeared to strike first.

EVALUATE: Section 37.2 shows that if they are simultaneous to the observer on the ground then an observer on the train measures that the bolt at B' struck first.

37.2. **(a)** $\gamma = \dfrac{1}{\sqrt{1-(0.9)^2}} = 2.29.\quad t = \gamma\tau = (2.29)\,(2.20\times10^{-6}\text{s}) = 5.05\times10^{-6}\,\text{s}.$

(b) $d = vt = (0.900)\,(3.00\times10^{8}\text{ m/s})\,(5.05\times10^{-6}\text{s}) = 1.36\times10^{3}\text{ m} = 1.36\text{ km}.$

37.3. **IDENTIFY** and **SET UP:** The problem asks for u such that $\Delta t_0/\Delta t = \dfrac{1}{2}.$

EXECUTE: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ gives $u = c\sqrt{1-\left(\Delta t_0/\Delta t\right)^2} = (3.00\times10^{8}\text{ m/s})\sqrt{1-\left(\dfrac{1}{2}\right)^2} = 2.60\times10^{8}\text{ m/s}$; $\dfrac{u}{c} = 0.867$

Jet planes fly at less than ten times the speed of sound, less than about 3000 m/s. Jet planes fly at much lower speeds than we calculated for u.

37.4. **IDENTIFY:** Time dilation occurs because the rocket is moving relative to Mars.

SET UP: The time dilation equation is $\Delta t = \gamma\Delta t_0$, where t_0 is the proper time.

EXECUTE: **(a)** The two time measurements are made at the same place on Mars by an observer at rest there, so the observer on Mars measures the proper time.

(b) $\Delta t = \gamma\Delta t_0 = \dfrac{1}{\sqrt{1-(0.985)^2}}(75.0\ \mu\text{s}) = 435\ \mu\text{s}$

EVALUATE: The pulse lasts for a shorter time relative to the rocket than it does relative to the Mars observer.

37.5. **(a) IDENTIFY** and **SET UP:** $\Delta t_0 = 2.60\times10^{-8}$ s; $\Delta t = 4.20\times10^{-7}$ s. In the lab frame the pion is created and decays at different points, so this time is not the proper time.

EXECUTE: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ says $1-\dfrac{u^2}{c^2} = \left(\dfrac{\Delta t_0}{\Delta t}\right)^2$

$$\frac{u}{c} = \sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = \sqrt{1-\left(\frac{2.60\times10^{-8}\text{ s}}{4.20\times10^{-7}\text{ s}}\right)^2} = 0.998;\ u = 0.998c$$

EVALUATE: $u < c$, as it must be, but u/c is close to unity and the time dilation effects are large.

(b) IDENTIFY and **SET UP:** The speed in the laboratory frame is $u = 0.998c$; the time measured in this frame is Δt, so the distance as measured in this frame is $d = u\Delta t$

EXECUTE: $d = (0.998)(2.998\times10^{8}\text{ m/s})(4.20\times10^{-7}\text{ s}) = 126\text{ m}$

EVALUATE: The distance measured in the pion's frame will be different because the time measured in the pion's frame is different (shorter).

37.6. $\gamma = 1.667$

(a) $\Delta t_0 = \dfrac{\Delta t}{\gamma} = \dfrac{1.20\times10^8\ \text{m}}{\gamma(0.800c)} = 0.300\ \text{s}.$

(b) $(0.300\ \text{s})\,(0.800c) = 7.20\times10^7\text{m}.$

(c) $\Delta t_0 = 0.300\ \text{s}/\gamma = 0.180\ \text{s}.$ (This is what the *racer* measures *your* clock to read at that instant.) At *your* origin you read the original $\dfrac{1.20\times10^8\text{m}}{(0.800)\,(3\times10^8\ \text{m/s})} = 0.5\ \text{s}.$ Clearly the observers (you and the racer) will not agree on the order of events!

37.7. **IDENTIFY** and **SET UP:** A clock moving with respect to an observer appears to run more slowly than a clock at rest in the observer's frame. The clock in the spacecraft measurers the proper time Δt_0. $\Delta t = 365\ \text{days} = 8760\ \text{hours}.$

EXECUTE: The clock on the moving spacecraft runs slow and shows the smaller elapsed time.
$\Delta t_0 = \Delta t\sqrt{1-u^2/c^2} = (8760\ \text{h})\sqrt{1-(4.80\times10^6/3.00\times10^8)^2} = 8758.88\ \text{h}$. The difference in elapsed times is $8760\ \text{h} - 8758.88\ \text{h} = 1.12\ \text{h}$.

37.8. **IDENTIFY** and **SET UP:** The proper time is measured in the frame where the two events occur at the same point.
EXECUTE: **(a)** The time of 12.0 ms measured by the first officer on the craft is the proper time.

(b) $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ gives $u = c\sqrt{1-(\Delta t_0/\Delta t)^2} = c\sqrt{1-(12.0\times10^{-3}/0.190)^2} = 0.998c.$

EVALUATE: The observer at rest with respect to the searchlight measures a much shorter duration for the event.

37.9. **IDENTIFY** and **SET UP:** $l = l_0\sqrt{1-u^2/c^2}$. The length measured when the spacecraft is moving is $l = 74.0$ m; l_0 is the length measured in a frame at rest relative to the spacecraft.

EXECUTE: $l_0 = \dfrac{l}{\sqrt{1-u^2/c^2}} = \dfrac{74.0\ \text{m}}{\sqrt{1-(0.600c/c)^2}} = 92.5\ \text{m}.$

EVALUATE: $l_0 > l$. The moving spacecraft appears to an observer on the planet to be shortened along the direction of motion.

37.10. **IDENTIFY** and **SET UP:** When the meterstick is at rest with respect to you, you measure its length to be 1.000 m, and that is its proper length, l_0. $l = 0.3048$ m.

EXECUTE: $l = l_0\sqrt{1-u^2/c^2}$ gives $u = c\sqrt{1-(l/l_0)^2} = c\sqrt{1-(0.3048/1.00)^2} = 0.9524c = 2.86\times10^8\ \text{m/s}.$

37.11. **IDENTIFY** and **SET UP:** The 2.2 μs lifetime is Δt_0 and the observer on earth measures Δt. The atmosphere is moving relative to the muon so in its frame the height of the atmosphere is l and l_0 is 10 km.

EXECUTE: **(a)** The greatest speed the muon can have is c, so the greatest distance it can travel in 2.2×10^{-6} s is $d = vt = (3.00\times10^8\ \text{m/s})(2.2\times10^{-6}\ \text{s}) = 660\ \text{m} = 0.66\ \text{km}.$

(b) $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}} = \dfrac{2.2\times10^{-6}\ \text{s}}{\sqrt{1-(0.999)^2}} = 4.9\times10^{-5}\ \text{s}$

$d = vt = (0.999)(3.00\times10^8\ \text{m/s})(4.9\times10^{-5}\ \text{s}) = 15\ \text{km}$

In the frame of the earth the muon can travel 15 km in the atmosphere during its lifetime.

(c) $l = l_0\sqrt{1-u^2/c^2} = (10\ \text{km})\sqrt{1-(0.999)^2} = 0.45\ \text{km}$

In the frame of the muon the height of the atmosphere is less than the distance it moves during its lifetime.

37.12. **IDENTIFY** and **SET UP:** The scientist at rest on the earth's surface measures the proper length of the separation between the point where the particle is created and the surface of the earth, so $l_0 = 45.0$ km. The transit time measured in the particle's frame is the proper time, Δt_0.

EXECUTE: **(a)** $t = \dfrac{l_0}{v} = \dfrac{45.0\times10^3\ \text{m}}{(0.99540)(3.00\times10^8\ \text{m/s})} = 1.51\times10^{-4}\ \text{s}$

(b) $l = l_0\sqrt{1-u^2/c^2} = (45.0\ \text{km})\sqrt{1-(0.99540)^2} = 4.31\ \text{km}$

(c) time dilation formula: $\Delta t_0 = \Delta t\sqrt{1-u^2/c^2} = (1.51\times10^{-4}\ \text{s})\sqrt{1-(0.99540)^2} = 1.44\times10^{-5}\ \text{s}$

from Δl: $t = \dfrac{l}{v} = \dfrac{4.31\times10^3\ \text{m}}{(0.99540)(3.00\times10^8\ \text{m/s})} = 1.44\times10^{-5}\ \text{s}$

The two results agree.

37.13. **(a)** $l_0 = 3600\ \text{m}$.

$$l = l_0\sqrt{1-\frac{u^2}{c^2}} = l_0(3600\ \text{m})\sqrt{1-\frac{(4.00\times10^7\ \text{m/s})^2}{(3.00\times10^8\ \text{m/s})^2}} = (3600\ \text{m})(0.991) = 3568\ \text{m}.$$

(b) $\Delta t_0 = \dfrac{l_0}{u} = \dfrac{3600 \text{ m}}{4.00\times 10^7 \text{ m/s}} = 9.00\times 10^{-5} \text{ s}.$

(c) $\Delta t = \dfrac{l}{u} = \dfrac{3568 \text{ m}}{4.00\times 10^7 \text{ m/s}} = 8.92\times 10^{-5} \text{ s}.$

37.14. Multiplying the last equation of (37.21) by u and adding to the first to eliminate t gives

$$x' + ut' = \gamma x\left(1 - \frac{u^2}{c^2}\right) = \frac{1}{\gamma}x,$$

and multiplying the first by $\dfrac{u}{c^2}$ and adding to the last to eliminate x gives

$$t' + \frac{u}{c^2}x' = \gamma t\left(1 - \frac{u^2}{c^2}\right) = \frac{1}{\gamma}t,$$

so $x = \gamma(x' + ut')$ and $t = \gamma(t' + ux'/c^2)$, which is indeed the same as Eq. (37.21) with the primed coordinates replacing the unprimed, and a change of sign of u.

37.15. **(a)** $v = \dfrac{v' + u}{1 + uv'/c^2} = \dfrac{0.400c + 0.600c}{1 + (0.400)\,(0.600)} = 0.806c$

(b) $v = \dfrac{v' + u}{1 + uv'/c^2} = \dfrac{0.900c + 0.600c}{1 + (0.900)(0.600)} = 0.974c$

(c) $v = \dfrac{v' + u}{1 + uv'/c^2} = \dfrac{0.990c + 0.600c}{1 + (0.990)(0.600)} = 0.997c.$

37.16. $\gamma = 1.667 (\gamma = 5/3 \text{ if } u = (4/5)c).$

(a) In Mavis's frame the event "light on" has space-time coordinates $x' = 0$ and $t' = 5.00$ s, so from the result of Exercise 37.14 or Example 37.7, $x = \gamma(x' + ut')$ and $t = \gamma\left(t' + \dfrac{ux'}{c^2}\right) \Rightarrow x = \gamma ut' = 2.00\times 10^9 \text{ m}, t = \gamma t' = 8.33 \text{ s}$.

(b) The 5.00-s interval in Mavis's frame is the proper time Δt_0 in Eq.(37.6), so $\Delta t = \gamma \Delta t_0 = 8.33$ s, as in part (a).

(c) $(8.33 \text{ s})\,(0.800c) = 2.00\times 10^9$ m, which is the distance x found in part (a).

37.17. **IDENTIFY:** The relativistic velocity addition formulas apply since the speeds are close to that of light.

SET UP: The relativistic velocity addition formula is $v_x' = \dfrac{v_x - u}{1 - \dfrac{uv_x}{c^2}}$.

EXECUTE: **(a)** For the pursuit ship to catch the cruiser, the distance between them must be decreasing, so the velocity of the cruiser relative to the pursuit ship must be directed toward the pursuit ship.

(b) Let the unprimed frame be Tatooine and let the primed frame be the pursuit ship. We want the velocity v' of the cruiser knowing the velocity of the primed frame u and the velocity of the cruiser v in the unprimed frame (Tatooine).

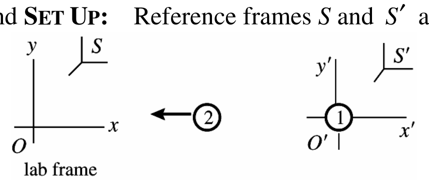
$$v_x' = \frac{v_x - u}{1 - \dfrac{uv_x}{c^2}} = \frac{0.600c - 0.800c}{1 - (0.600)\,(0.800)} = -0.385c$$

The result implies that the cruiser is moving toward the pursuit ship at $0.385c$.

EVALUATE: The nonrelativistic formula would have given $-0.200c$, which is considerably different from the correct result.

37.18. Let u_y be the y-component of the velocity of S' relative to S. Following the steps used in the derivation of Eq.(37.23) we get $v_y = \dfrac{v_y' + u_y}{1 + u_y v_y'/c^2}$.

37.19. **IDENTIFY** and **SET UP:** Reference frames S and S' are shown in Figure 37.19.



Frame S is at rest in the laboratory. Frame S' is attached to particle 1.

Figure 37.19

u is the speed of S' relative to S; this is the speed of particle 1 as measured in the laboratory. Thus $u = +0.650c$. The speed of particle 2 in S' is $0.950c$. Also, since the two particles move in opposite directions, 2 moves in the $-x'$ direction and $v_x' = -0.950c$. We want to calculate v_x, the speed of particle 2 in frame S; use Eq.(37.23).

EXECUTE: $v_x = \dfrac{v_x' + u}{1 + uv_x'/c^2} = \dfrac{-0.950c + 0.650c}{1 + (0.950c)(-0.650c)/c^2} = \dfrac{-0.300c}{1 - 0.6175} = -0.784c$. The speed of the second particle, as measured in the laboratory, is $0.784c$.

EVALUATE: The incorrect Galilean expression for the relative velocity gives that the speed of the second particle in the lab frame is $0.300c$. The correct relativistic calculation gives a result more than twice this.

37.20. **IDENTIFY** and **SET UP:** Let S be the laboratory frame and let S' be the frame of one of the particles, as shown in Figure 37.20. Let the positive x direction for both frames be from particle 1 to particle 2. In the lab frame particle 1 is moving in the $+x$ direction and particle 2 is moving in the $-x$ direction. Then $u = 0.9520c$ and $v = -0.9520c$. v' is the velocity of particle 2 relative to particle 1.

EXECUTE: $v' = \dfrac{v - u}{1 - uv/c^2} = \dfrac{-0.9520c - 0.9520c}{1 - (0.9520c)(-0.9520c)/c^2} = -0.9988c$. The speed of particle 2 relative to particle 1 is $0.9988c$. $v' < 0$ shows particle 2 is moving toward particle 1.

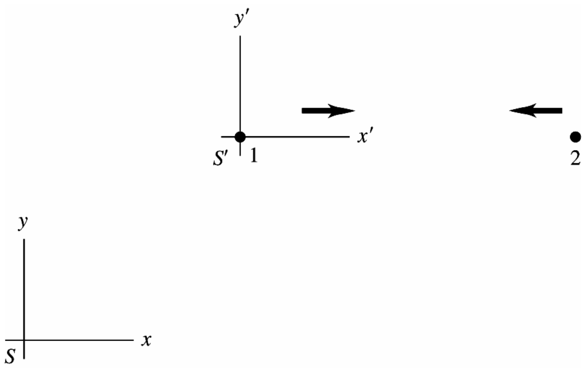


Figure 37.20

37.21. **IDENTIFY:** The relativistic velocity addition formulas apply since the speeds are close to that of light.

SET UP: The relativistic velocity addition formula is $v_x' = \dfrac{v_x - u}{1 - \dfrac{uv_x}{c^2}}$.

EXECUTE: In the relativistic velocity addition formula for this case, v_x' is the relative speed of particle 1 with respect to particle 2, v is the speed of particle 2 measured in the laboratory, and u is the speed of particle 1 measured in the laboratory, $u = -v$.

$v_x' = \dfrac{v - (-v)}{1 - (-v)v/c^2} = \dfrac{2v}{1 + v^2/c^2}$. $\dfrac{v_x'}{c^2}v^2 - 2v + v_x' = 0$ and $(0.890c)v^2 - 2c^2v + (0.890c^3) = 0$.

This is a quadratic equation with solution $v = 0.611c$ (v must be less than c).

EVALUATE: The nonrelativistic result would be $0.445c$, which is considerably different from this result.

37.22. **IDENTIFY** and **SET UP:** Let the starfighter's frame be S and let the enemy spaceship's frame be S'. Let the positive x direction for both frames be from the enemy spaceship toward the starfighter. Then $u = +0.400c$. $v' = +0.700c$. v is the velocity of the missile relative to you.

EXECUTE: **(a)** $v = \dfrac{v' + u}{1 + uv'/c^2} = \dfrac{0.700c + 0.400c}{1 + (0.400)(0.700)} = 0.859c$

(b) Use the distance it moves as measured in your frame and the speed it has in your frame to calculate the time it takes in your frame. $t = \dfrac{8.00 \times 10^9\ \text{m}}{(0.859)(3.00 \times 10^8\ \text{m/s})} = 31.0\ \text{s}$.

37.23. **IDENTIFY** and **SET UP:** The reference frames are shown in Figure 37.23.

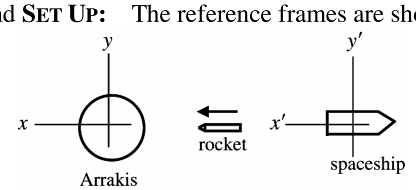


Figure 37.23

S = Arrakis frame
S' = spaceship frame
The object is the rocket.

u is the velocity of the spaceship relative to Arrakis.

$v_x = +0.360c$; $v_x' = +0.920c$

(In each frame the rocket is moving in the positive coordinate direction.)

Use the Lorentz velocity transformation equation, Eq.(37.22): $v'_x = \dfrac{v_x - u}{1 - uv_x/c^2}$.

EXECUTE: $v'_x = \dfrac{v_x - u}{1 - uv_x/c^2}$ so $v'_x - u\left(\dfrac{v_x v'_x}{c^2}\right) = v_x - u$ and $u\left(1 - \dfrac{v_x v'_x}{c^2}\right) = v_x - v'_x$

$$u = \frac{v_x - v'_x}{1 - v_x v'_x/c^2} = \frac{0.360c - 0.920c}{1 - (0.360c)(0.920c)/c^2} = -\frac{0.560c}{0.6688} = -0.837c$$

The speed of the spacecraft relative to Arrakis is $0.837c = 2.51\times10^8$ m/s. The minus sign in our result for u means that the spacecraft is moving in the $-x$-direction, so it is moving away from Arrakis.

EVALUATE: The incorrect Galilean expression also says that the spacecraft is moving away from Arrakis, but with speed $0.920c - 0.360c = 0.560c$.

37.24. **IDENTIFY:** We need to use the relativistic Doppler shift formula.

SET UP: The relativistic Doppler shift formula, Eq.(37.25), is $f = \sqrt{\dfrac{c+u}{c-u}} f_0$.

EXECUTE: $f^2 = \dfrac{c+u}{c-u} f_0^2$. $(c-u)f^2 = (c+u)f_0^2$. $cf^2 - uf^2 = cf_0^2 + uf_0^2$. $cf^2 - cf_0^2 = uf^2 + uf_0^2$ and

$$u = \frac{c(f^2 - f_0^2)}{f^2 + f_0^2} = \frac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1} c.$$

(a) For $f/f_0 = 0.95$, $u = -0.051c$ moving away from the source.

(b) For $f/f_0 = 5.0$, $u = 0.923c$ moving towards the source.

EVALUATE: Note that the speed required to achieve a 10 times greater Doppler shift is not 10 times the original speed.

37.25. **IDENTIFY** and **SET UP:** Source and observer are approaching, so use Eq.(37.25): $f = \sqrt{\dfrac{c+u}{c-u}} f_0$. Solve for u, the speed of the light source relative to the observer.

(a) EXECUTE: $f^2 = \left(\dfrac{c+u}{c-u}\right) f_0^2$

$(c-u)f^2 = (c+u)f_0^2$ and $u = \dfrac{c(f^2 - f_0^2)}{f^2 + f_0^2} = c\left(\dfrac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1}\right)$

$\lambda_0 = 675$ nm, $\lambda = 575$ nm

$u = \left(\dfrac{(675\text{ nm}/575\text{ nm})^2 - 1}{(675\text{ nm}/575\text{ nm})^2 + 1}\right) c = 0.159c = (0.159)(2.998\times10^8\text{ m/s}) = 4.77\times10^7$ m/s; definitely speeding

(b) $4.77\times10^7\text{ m/s} = (4.77\times10^7\text{ m/s})(1\text{ km}/1000\text{ m})(3600\text{ s}/1\text{ h}) = 1.72\times10^8$ km/h. Your fine would be $\$1.72\times10^8$ (172 million dollars).

EVALUATE: The source and observer are approaching, so $f > f_0$ and $\lambda < \lambda_0$. Our result gives $u < c$, as it must.

37.26. Using $u = -0.600c = -(3/5)c$ in Eq.(37.25) gives

$$f = \sqrt{\frac{1-(3/5)}{1+(3/5)}} f_0 = \sqrt{\frac{2/5}{8/5}} f_0 = f_0/2.$$

37.27. **IDENTIFY** and **SET UP:** If $\vec{F}$ is parallel to $\vec{v}$ then $\vec{F}$ changes the magnitude of $\vec{v}$ and not its direction.

$$F = \frac{dp}{dt} = \frac{d}{dt}\left(\frac{mv}{\sqrt{1 - v^2/c^2}}\right)$$

Use the chain rule to evaluate the derivative: $\dfrac{d}{dt} f(v(t)) = \dfrac{df}{dv}\dfrac{dv}{dt}$.

EXECUTE: **(a)** $F = \dfrac{m}{(1 - v^2/c^2)^{1/2}}\left(\dfrac{dv}{dt}\right) + \dfrac{mv}{(1 - v^2/c^2)^{3/2}}\left(-\dfrac{1}{2}\right)\left(-\dfrac{2v}{c^2}\right)\left(\dfrac{dv}{dt}\right)$

$$F = \frac{dv}{dt}\frac{m}{(1 - v^2/c^2)^{3/2}}\left(1 - \frac{v^2}{c^2} + \frac{v^2}{c^2}\right) = \frac{dv}{dt}\frac{m}{(1 - v^2/c^2)^{3/2}}$$

But $\dfrac{dv}{dt} = a$, so $a = (F/m)(1 - v^2/c^2)^{3/2}$.

EVALUATE: Our result agrees with Eq.(37.30).

(b) IDENTIFY and SET UP: If $\vec{F}$ is perpendicular to $\vec{v}$ then $\vec{F}$ changes the direction of $\vec{v}$ and not its magnitude.

$$\vec{F} = \frac{d}{dt}\left(\frac{m\vec{v}}{\sqrt{1-v^2/c^2}}\right).$$

$\vec{a} = d\vec{v}/dt$ but the magnitude of v in the denominator of Eq.(37.29) is constant.

EXECUTE: $F = \dfrac{ma}{\sqrt{1-v^2/c^2}}$ and $a = (F/m)(1-v^2/c^2)^{1/2}$.

EVALUATE: This result agrees with Eq.(37.33).

37.28. **IDENTIFY and SET UP:** $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$. If γ is 1.0% greater than 1 then $\gamma = 1.010$, if γ is 10% greater than 1 then $\gamma = 1.10$ and if γ is 100% greater than 1 then $\gamma = 2.00$.

EXECUTE: $v = c\sqrt{1-1/\gamma^2}$

(a) $v = c\sqrt{1-1/(1.010)^2} = 0.140c$

(b) $v = c\sqrt{1-1/(1.10)^2} = 0.417c$

(c) $v = c\sqrt{1-1/(2.00)^2} = 0.866c$

37.29. **(a)** $p = \dfrac{mv}{\sqrt{1-v^2/c^2}} = 2mv$.

$$\Rightarrow 1 = 2\sqrt{1-v^2/c^2} \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow v^2 = \frac{3}{4}c^2 \Rightarrow v = \frac{\sqrt{3}}{2}c = 0.866c.$$

(b) $F = \gamma^3 ma = 2ma \Rightarrow \gamma^3 = 2 \Rightarrow \gamma = (2)^{1/3}$ so $\dfrac{1}{1-\dfrac{v^2}{c^2}} = 2^{2/3} \Rightarrow \dfrac{v}{c} = \sqrt{1-2^{-2/3}} = 0.608$

37.30. The force is found from Eq.(37.32) or Eq.(37.33).

(a) Indistinguishable from $F = ma = 0.145$ N.

(b) $\gamma^3 ma = 1.75$ N.

(c) $\gamma^3 ma = 51.7$ N.

(d) $\gamma ma = 0.145$ N, 0.333 N, 1.03 N.

37.31. **(a)** $K = \dfrac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = mc^2$

$$\Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 2 \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow v = \sqrt{\frac{3}{4}}c = 0.866c.$$

(b) $K = 5mc^2 \Rightarrow \dfrac{1}{\sqrt{1-v^2/c^2}} = 6 \Rightarrow \dfrac{1}{36} = 1 - \dfrac{v^2}{c^2} \Rightarrow v = \sqrt{\dfrac{35}{36}}c = 0.986c.$

37.32. $E = 2mc^2 = 2(1.67\times10^{-27}\,\text{kg})(3.00\times10^8\,\text{m/s})^2 = 3.01\times10^{-10}\,\text{J} = 1.88\times10^9\,\text{eV}$.

37.33. **IDENTIFY and SET UP:** Use Eqs.(37.38) and (37.39).

EXECUTE: (a) $E = mc^2 + K$, so $E = 4.00mc^2$ means $K = 3.00mc^2 = 4.50\times10^{-10}$ J

(b) $E^2 = (mc^2)^2 + (pc)^2$; $E = 4.00mc^2$, so $15.0(mc^2)^2 = (pc)^2$

$p = \sqrt{15}mc = 1.94\times10^{-18}$ kg · m/s

(c) $E = mc^2/\sqrt{1-v^2/c^2}$

$E = 4.00mc^2$ gives $1 - v^2/c^2 = 1/16$ and $v = \sqrt{15/16}c = 0.968c$

EVALUATE: The speed is close to c since the kinetic energy is greater than the rest energy. Nonrelativistic expressions relating E, K, p and v will be very inaccurate.

37.34. **(a)** $W = \Delta K = (\gamma_f - 1)mc^2 = (4.07\times10^{-3})\,mc^2$.

(b) $(\gamma_f - \gamma_i)mc^2 = 4.79mc^2$.

(c) The result of part (b) is far larger than that of part (a).

37.35. **IDENTIFY:** Use $E = mc^2$ to relate the mass increase to the energy increase.

(a) SET UP: Your total energy E increases because your gravitational potential energy mgy increases.

EXECUTE: $\Delta E = mg\Delta y$

$\Delta E = (\Delta m)c^2$ so $\Delta m = \Delta E / c^2 = mg(\Delta y)/c^2$

$\Delta m / m = (g\Delta y)/c^2 = (9.80\text{ m/s}^2)(30\text{ m})/(2.998\times 10^8\text{ m/s})^2 = 3.3\times 10^{-13}\%$

This increase is much, much too small to be noticed.

(b) SET UP: The energy increases because potential energy is stored in the compressed spring.

EXECUTE: $\Delta E = \Delta U = \frac{1}{2}kx^2 = \frac{1}{2}(2.00\times 10^4\text{ N/m})(0.060\text{ m})^2 = 36.0\text{ J}$

$\Delta m = (\Delta E)/c^2 = 4.0\times 10^{-16}\text{ kg}$

Energy increases so mass increases. The mass increase is much, much too small to be noticed.

EVALUATE: In both cases the energy increase corresponds to a mass increase. But since c^2 is a very large number the mass increase is very small.

37.36. **(a)** $E_0 = m_0c^2$. $2E = mc^2 = 2m_0c^2$. Therefore, $m = 2m_0 \Rightarrow \dfrac{m_0}{\sqrt{1-v^2/c^2}} = 2m_0$.

$$\frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow \frac{v^2}{c^2} = \frac{3}{4} \Rightarrow v = c\sqrt{3/4} = 0.866c = 2.60\times 10^8\text{ m/s}$$

(b) $10\, m_0c^2 = mc^2 = \dfrac{m_0}{\sqrt{1-v^2/c^2}}c^2$.

$$1 - \frac{v^2}{c^2} = \frac{1}{100} \Rightarrow \frac{v^2}{c^2} = \frac{99}{100}.\quad v = c\sqrt{\frac{99}{100}} = 0.995c = 2.98\times 10^8\text{ m/s}.$$

37.37. **IDENTIFY** and **SET UP:** The energy equivalent of mass is $E = mc^2$. $\rho = 7.86\text{ g/cm}^3 = 7.86\times 10^3\text{ kg/m}^3$. For a cube, $V = L^3$.

EXECUTE: **(a)** $m = \dfrac{E}{c^2} = \dfrac{1.0\times 10^{20}\text{ J}}{(3.00\times 10^8\text{ m/s})^2} = 1.11\times 10^3\text{ kg}$

(b) $\rho = \dfrac{m}{V}$ so $V = \dfrac{m}{\rho} = \dfrac{1.11\times 10^3\text{ kg}}{7.86\times 10^3\text{ kg/m}^3} = 0.141\text{ m}^3$. $L = V^{1/3} = 0.521\text{ m} = 52.1\text{ cm}$

EVALUATE: Particle/antiparticle annihilation has been observed in the laboratory, but only with small quantities of antimatter.

37.38. $(5.52\times 10^{-27}\text{ kg})(3.00\times 10^8\text{ m/s})^2 = 4.97\times 10^{-10}\text{ J} = 3105\text{ MeV}$.

37.39. **IDENTIFY** and **SET UP:** The total energy is given in terms of the momentum by Eq.(37.39). In terms of the total energy *E*, the kinetic energy *K* is $K = E - mc^2$ (from Eq.37.38). The rest energy is mc^2.

EXECUTE: **(a)** $E = \sqrt{(mc^2)^2 + (pc)^2} =$

$$\sqrt{[(6.64\times 10^{-27})(2.998\times 10^8)^2]^2 + [(2.10\times 10^{-18})(2.998\times 10^8)]^2}\text{ J}$$

$E = 8.67\times 10^{-10}\text{ J}$

(b) $mc^2 = (6.64\times 10^{-27}\text{ kg})(2.998\times 10^8\text{ m/s})^2 = 5.97\times 10^{-10}\text{ J}$

$K = E - mc^2 = 8.67\times 10^{-10}\text{ J} - 5.97\times 10^{-10}\text{ J} = 2.70\times 10^{-10}\text{ J}$

(c) $\dfrac{K}{mc^2} = \dfrac{2.70\times 10^{-10}\text{ J}}{5.97\times 10^{-10}\text{ J}} = 0.452$

EVALUATE: The incorrect nonrelativistic expressions for *K* and *p* give $K = p^2/2m = 3.3\times 10^{-10}$ J; the correct relativistic value is less than this.

37.40. $E = (m^2c^4 + p^2c^2)^{1/2} = mc^2\left(1 + \left(\dfrac{p}{mc}\right)^2\right)^{1/2}$

$E \approx mc^2\left(1 + \dfrac{1}{2}\dfrac{p^2}{m^2c^2}\right) = mc^2 + \dfrac{p^2}{2m} = mc^2 + \dfrac{1}{2}mv^2$**, the sum of the rest mass energy and the classical kinetic energy.**

37.41. **(a)** $v = 8\times 10^7\text{ m/s} \Rightarrow \gamma = \dfrac{1}{\sqrt{1-v^2/c^2}} = 1.0376$. For $m = m_p$, $K_{\text{nonrel}} = \dfrac{1}{2}mv^2 = 5.34\times 10^{-12}$ J.

$K_{\text{rel}} = (\gamma - 1)mc^2 = 5.65\times 10^{-12}$ J. $\dfrac{K_{\text{rel}}}{K_{\text{nonrel}}} = 1.06$.

(b) $v = 2.85\times 10^8\text{ m/s};\ \gamma = 3.203$.

$K_{\text{rel}} = \dfrac{1}{2}mv^2 = 6.78\times 10^{-11}\text{ J};\ K_{\text{rel}} = (\gamma - 1)mc^2 = 3.31\times 10^{-10}\text{ J};\ \ K_{\text{rel}}/K_{\text{nonrel}} = 4.88.$

37.42. IDENTIFY: Since the speeds involved are close to that of light, we must use the relativistic formula for kinetic energy.

SET UP: The relativistic kinetic energy is $K = (\gamma - 1)mc^2 = \left(\dfrac{1}{\sqrt{1 - v^2/c^2}} - 1\right)mc^2$.

(a) $K = (\gamma - 1)mc^2 = \left(\dfrac{1}{\sqrt{1 - v^2/c^2}} - 1\right)mc^2 = (1.67\times10^{-27}\text{ kg})(3.00\times10^{8}\text{ m/s})^2\left(\dfrac{1}{\sqrt{1-(0.100c/c)^2}} - 1\right)$

$K = (1.50\times10^{-10}\text{ J})\left(\dfrac{1}{\sqrt{1-0.0100}} - 1\right) = 7.56\times10^{-13}\text{ J} = 4.73\text{ MeV}$

(b) $K = (1.50\times10^{-10}\text{J})\left(\dfrac{1}{\sqrt{1-(0.500)^2}} - 1\right) = 2.32\times10^{-11}\text{ J} = 145\text{ MeV}$

(c) $K = (1.50\times10^{-10}\text{ J})\left(\dfrac{1}{\sqrt{1-(0.900)^2}} - 1\right) = 1.94\times10^{-10}\text{ J} = 1210\text{ MeV}$

(d) $\Delta E = 2.32\times10^{-11}\text{ J} - 7.56\times10^{-13}\text{ J} = 2.24\times10^{-11}\text{ J} = 140\text{ MeV}$

(e) $\Delta E = 1.94\times10^{-10}\text{ J} - 2.32\times10^{-11}\text{ J} = 1.71\times10^{-10}\text{ J} = 1070\text{ MeV}$

(f) Without relativity, $K = \frac{1}{2}mv^2$. The work done in accelerating a proton from $0.100c$ to $0.500c$ in the nonrelativistic limit is $\Delta E = \frac{1}{2}m(0.500c)^2 - \frac{1}{2}m(0.100c)^2 = 1.81\times10^{-11}\text{ J} = 113\text{ MeV}$.

The work done in accelerating a proton from $0.500c$ to $0.900c$ in the nonrelativistic limit is

$$\Delta E = \frac{1}{2}m(0.900c)^2 - \frac{1}{2}m(0.500c)^2 = 4.21\times10^{-11}\text{ J} = 263\text{ MeV}.$$

EVALUATE: We see in the first case the nonrelativistic result is within 20% of the relativistic result. In the second case, the nonrelativistic result is very different from the relativistic result since the velocities are closer to c.

37.43. IDENTIFY and SET UP: Use Eq.(23.12) and conservation of energy to relate the potential difference to the kinetic energy gained by the electron. Use Eq.(37.36) to calculate the kinetic energy from the speed.

EXECUTE: (a) $K = q\Delta V = e\Delta V$

$K = mc^2\left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right) = 4.025mc^2 = 3.295\times10^{-13}\text{ J} = 2.06\text{ MeV}$

$\Delta V = K/e = 2.06\times10^{6}\text{ V}$

(b) From part (a), $K = 3.30\times10^{-13}\text{ J} = 2.06\text{ MeV}$

EVALUATE: The speed is close to c and the kinetic energy is four times the rest mass.

37.44. (a) According to Eq.(37.38) and conservation of mass-energy

$$2Mc^2 + mc^2 = \gamma 2Mc^2 \Rightarrow \gamma = 1 + \frac{m}{2M} = 1 + \frac{9.75}{2(16.7)} = 1.292.$$

Note that since $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$, we have that $\dfrac{v}{c} = \sqrt{1 - \dfrac{1}{\gamma^2}} = \sqrt{1 - \dfrac{1}{(1.292)^2}} = 0.6331$.

(b) According to Eq.(37.36), the kinetic energy of each proton is

$K = (\gamma - 1)Mc^2 = (1.292 - 1)(1.67\times10^{-27}\text{ kg})(3.00\times10^{8}\text{ m/s})^2\left(\dfrac{1.00\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) = 274\text{ MeV}.$

(c) The rest energy of η^0 is $mc^2 = (9.75\times10^{-28}\text{ kg})(3.00\times10^{8}\text{ m/s})^2\left(\dfrac{1.00\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) = 548\text{ MeV}.$

(d) The kinetic energy lost by the protons is the energy that produces the η^0,

$$548\text{ MeV} = 2(274\text{ MeV}).$$

37.45. IDENTIFY: The relativistic expression for the kinetic energy is $K = (\gamma - 1)mc^2$, where $\gamma = \dfrac{1}{\sqrt{1-x}}$ and $x = v^2/c^2$.

The Newtonian expression for the kinetic energy is $K_N = \frac{1}{2}mv^2$.

SET UP: Solve for v such that $K = \frac{3}{2}K_N$.

EXECUTE: $(\gamma-1)mc^2=\frac{3}{4}mv^2$. $\frac{1}{\sqrt{1-x}}-1=\frac{3}{4}x$. $\frac{1}{1-x}=\left(1+\frac{3}{4}x\right)^2$. After a little algebra this becomes

$9x^2+15x-8=0$. $x=\frac{1}{18}\left(-15\pm\sqrt{(15)^2+4(9)(8)}\right)$. The positive root is $x=0.425$. $x=v^2/c^2$, so

$v=\sqrt{x}\ c=0.652c$.

EVALUATE: The fractional increase of the relativistic expression above the nonrelativistic one increases as v increases.

37.46. The fraction of the initial mass (a) that becomes energy is $1-\frac{(4.0015\text{ u})}{2(2.0136\text{ u})}=6.382\times10^{-3}$, and so the energy released per kilogram is $(6.382\times10^{-3})(1.00\text{ kg})(3.00\times10^{8}\text{ m/s})^2=5.74\times10^{14}$ J.

(b) $\frac{1.0\times10^{19}\text{ J}}{5.74\times10^{14}\text{ J/kg}}=1.7\times10^{4}$ kg.

37.47. **(a)** $E=mc^2, m=E/c^2=(3.8\times10^{26}\text{ J})/(2.998\times10^{8}\text{ m/s})^2=4.2\times10^{9}\text{ kg}$.

1 kg is equivalent to 2.2 lbs, so $m=4.6\times10^{6}$ tons

(b) The current mass of the sun is 1.99×10^{30} kg, so it would take it

$(1.99\times10^{30}\text{ kg})/(4.2\times10^{9}\text{ kg/s})=4.7\times10^{20}\text{s}=1.5\times10^{13}$ years to use up all its mass.

37.48. **IDENTIFY:** Since the final speed is close to the speed of light, there will be a considerable difference between the relativistic and nonrelativistic results.

SET UP: The nonrelativistic work-energy theorem is $F\Delta x=\frac{1}{2}mv^2-\frac{1}{2}mv_0^2$, and the relativistic formula for a constant force is $F\Delta x=(\gamma-1)mc^2$.

(a) Using the classical work-energy theorem and solving for Δx, we obtain

$$\Delta x=\frac{m(v^2-v_0^2)}{2F}=\frac{(0.100\times10^{-9}\text{ kg})[(0.900)(3.00\times10^{8}\text{ m/s})]^2}{2(1.00\times10^{6}\text{ N})}=3.65\text{ m}.$$

(b) Using the relativistic work-energy theorem for a constant force, we obtain

$$\Delta x=\frac{(\gamma-1)mc^2}{F}.$$

For the given speed, $\gamma=\frac{1}{\sqrt{1-0.900^2}}=2.29$, thus

$$\Delta x=\frac{(2.29-1)(0.100\times10^{-9}\text{ kg})(3.00\times10^{8}\text{ m/s})^2}{(1.00\times10^{6}\text{N})}=11.6\text{ m}.$$

EVALUATE: **(c)** The distance obtained from the relativistic treatment is greater. As we have seen, more energy is required to accelerate an object to speeds close to c, so that force must act over a greater distance.

37.49. **(a) IDENTIFY** and **SET UP:** $\Delta t_0=2.60\times10^{-8}$ s is the proper time, measured in the pion's frame. The time measured in the lab must satisfy $d=c\Delta t$, where $u\approx c$. Calculate Δt and then use Eq.(37.6) to calculate u.

EXECUTE: $\Delta t=\frac{d}{c}=\frac{1.20\times10^{3}\text{ m}}{2.998\times10^{8}\text{ m/s}}=4.003\times10^{-6}$ s

$\Delta t=\frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ so $(1-u^2/c^2)^{1/2}=\frac{\Delta t_0}{\Delta t}$ and $(1-u^2/c^2)=\left(\frac{\Delta t_0}{\Delta t}\right)^2$

Write $u=(1-\Delta)c$ so that $(u/c)^2=(1-\Delta)^2=1-2\Delta+\Delta^2\approx1-2\Delta$ since Δ is small.

Using this in the above gives $1-(1-2\Delta)=\left(\frac{\Delta t_0}{\Delta t}\right)^2$

$\Delta=\frac{1}{2}\left(\frac{\Delta t_0}{\Delta t}\right)^2=\frac{1}{2}\left(\frac{2.60\times10^{-8}\text{ s}}{4.003\times10^{-6}\text{ s}}\right)^2=2.11\times10^{-5}$

EVALUATE: An alternative calculation is to say that the length of the tube must contract relative to the moving pion so that the pion travels that length before decaying. The contracted length must be

$l=c\Delta t_0=(2.998\times10^{8}\text{ m/s})(2.60\times10^{-8}\text{ s})=7.79$ m.

$l=l_0\sqrt{1-u^2/c^2}$ so $1-u^2/c^2=\left(\frac{l}{l_0}\right)^2$

Then $u=(1-\Delta)c$ gives $\Delta=\frac{1}{2}\left(\frac{l}{l_0}\right)^2=\frac{1}{2}\left(\frac{7.79\text{ m}}{1.20\times10^3\text{ m}}\right)^2=2.11\times10^{-5}$, which checks.

(b) IDENTIFY and SET UP: $E=\gamma mc^2$ (Eq.(37.38).

EXECUTE: $\gamma=\frac{1}{\sqrt{1-u^2/c^2}}=\frac{1}{\sqrt{2\Delta}}=\frac{1}{\sqrt{2(2.11\times10^{-5})}}=154$

$E=154(139.6\text{ MeV})=2.15\times10^4\text{ MeV}=21.5\text{ GeV}$

EVALUATE: The total energy is 154 times the rest energy.

37.50. **IDENTIFY and SET UP:** The proper length of a side is $l_0=a$. The side along the direction of motion is shortened to $l=l_0\sqrt{1-v^2/c^2}$. The sides in the two directions perpendicular to the motion are unaffected by the motion and still have a length a.

EXECUTE: $V=a^2l=a^3\sqrt{1-v^2/c^2}$

37.51. **IDENTIFY and SET UP:** There must be a length contraction such that the length a becomes the same as b; $l_0=a,\ l=b.\ l_0$ is the distance measured by an observer at rest relative to the spacecraft. Use Eq.(37.16) and solve for u.

EXECUTE: $\frac{l}{l_0}=\sqrt{1-u^2/c^2}$ so $\frac{b}{a}=\sqrt{1-u^2/c^2}$;

$a=1.40b$ gives $b/1.40b=\sqrt{1-u^2/c^2}$ and thus $1-u^2/c^2=1/(1.40)^2$

$u=\sqrt{1-1/(1.40)^2}\,c=0.700c=2.10\times10^8\text{ m/s}$

EVALUATE: A length on the spacecraft in the direction of the motion is shortened. A length perpendicular to the motion is unchanged.

37.52. **IDENTIFY and SET UP:** The proper time $\Delta t_0$is the time that elapses in the frame of the space probe. Δt is the time that elapses in the frame of the earth. The distance traveled is 42.2 light years, as measured in the earth frame.

EXECUTE: **(a)** Light travels 42.2 light years in 42.2 yr, so $\Delta t=\left(\frac{c}{0.9910c}\right)(42.2\text{ yr})=42.6\text{ yr}$.

$\Delta t_0=\Delta t\sqrt{1-u^2/c^2}=(42.6\text{ yr})\sqrt{1-(0.9910)^2}=5.7\text{ yr}$. She measures her biological age to be $19\text{ yr}+5.7\text{ yr}=24.7\text{ yr}$.

(b) Her age measured by someone on earth is $19\text{ yr}+42.6\text{ yr}=61.6\text{ yr}$.

37.53. **(a)** $E=\gamma mc^2$ and $\gamma=10=\frac{1}{\sqrt{1-(v/c)^2}}\Rightarrow\frac{v}{c}=\sqrt{\frac{\gamma^2-1}{\gamma^2}}\Rightarrow\frac{v}{c}=\sqrt{\frac{99}{100}}=0.995.$

(b) $(pc)^2=m^2v^2\gamma^2c^2,\ E^2=m^2c^4\left(\left(\frac{v}{c}\right)^2\gamma^2+1\right)$

$$\Rightarrow\frac{E^2-(pc)^2}{E^2}=\frac{1}{1+\gamma^2\left(\frac{v}{c}\right)^2}=\frac{1}{1+(10/(0.995))^2}=0.01=1\%.$$

37.54. **IDENTIFY and SET UP:** The clock on the plane measures the proper time Δt_0.

$\Delta t=4.00\text{ h}=4.00\text{ h }(3600\text{ s/1 h})=1.44\times10^4\text{ s}.$

$\Delta t=\frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ and $\Delta t_0=\Delta t\sqrt{1-u^2/c^2}$

EXECUTE: $\frac{u}{c}$ small so $\sqrt{1-u^2/c^2}=(1-u^2/c^2)^{1/2}\approx1-\frac{1}{2}\frac{u^2}{c^2}$; thus $\Delta t_0=\Delta t\left(1-\frac{1}{2}\frac{u^2}{c^2}\right)$

The difference in the clock readings is $\Delta t-\Delta t_0=\frac{1}{2}\frac{u^2}{c^2}\Delta t=\frac{1}{2}\left(\frac{250\text{ m/s}}{2.998\times10^8\text{ m/s}}\right)^2(1.44\times10^4\text{ s})=5.01\times10^{-9}\text{ s}$. The clock on the plane has the shorter elapsed time.

EVALUATE: Δt_0 is always less than Δt; our results agree with this. The speed of the plane is much less than the speed of light, so the difference in the reading of the two clocks is very small.

37.55. **IDENTIFY:** Since the speed is very close to the speed of light, we must use the relativistic formula for kinetic energy.

SET UP: The relativistic formula for kinetic energy is $K = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)$ and the relativistic mass is $m_{\text{rel}} = \frac{m}{\sqrt{1-v^2/c^2}}$.

EXECUTE: (a) $K = 7\times10^{12}\text{ eV} = 1.12\times10^{-6}\text{ J}$. Using this value in the relativistic kinetic energy formula and substituting the mass of the proton for *m*, we get $K = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}}-1\right)$

which gives $\frac{1}{\sqrt{1-v^2/c^2}} = 7.45\times10^3$ and $1-\frac{v^2}{c^2} = \frac{1}{(7.45\times10^3)^2}$. Solving for *v* gives

$1-\frac{v^2}{c^2} = \frac{(c+v)(c-v)}{c^2} = \frac{2(c-v)}{c}$, since $c + v \approx 2c$. Substituting $v = (1-\Delta)c$, we have.

$1-\frac{v^2}{c^2} = \frac{2(c-v)}{c} = \frac{2[c-(1-\Delta)c]}{c} = 2\Delta$. Solving for Δ gives $\Delta = \frac{1-v^2/c^2}{2} = \frac{\frac{1}{(7.45\times10^3)^2}}{2} = 9\times10^{-9}$, to one significant digit.

(b) Using the relativistic mass formula and the result that $\frac{1}{\sqrt{1-v^2/c^2}} = 7.45\times10^3$, we have

$m_{\text{rel}} = \frac{m}{\sqrt{1-v^2/c^2}} = m\left(\frac{1}{\sqrt{1-v^2/c^2}}\right) = (7\times10^3)m$, to one significant digit.

EVALUATE: At such high speeds, the proton's mass is over 7000 times as great as its rest mass.

37.56. **IDENTIFY** and **SET UP:** The energy released is $E = (\Delta m)c^2$. $\Delta m = \left(\frac{1}{10^4}\right)(8.00\text{ kg})$. $P_{\text{av}} = \frac{E}{t}$. The change in gravitational potential energy is $mg\Delta y$.

EXECUTE: **(a)** $E = (\Delta m)c^2 = \left(\frac{1}{10^4}\right)(8.00\text{ kg})(3.00\times10^8\text{ m/s})^2 = 7.20\times10^{13}\text{ J}$

(b) $P_{\text{av}} = \frac{E}{t} = \frac{7.20\times10^{13}\text{ J}}{4.00\times10^{-6}\text{ s}} = 1.80\times10^{19}\text{ W}$

(c) $E = \Delta U = mg\Delta y$. $m = \frac{E}{g\Delta y} = \frac{7.20\times10^{13}\text{ J}}{(9.80\text{ m/s}^2)(1.00\times10^3\text{ m})} = 7.35\times10^9\text{ kg}$

37.57. **IDENTIFY** and **SET UP:** In crown glass the speed of light is $v = \frac{c}{n}$. Calculate the kinetic energy of an electron that has this speed.

EXECUTE: $v = \frac{2.998\times10^8\text{ m/s}}{1.52} = 1.972\times10^8\text{ m/s}.$

$K = mc^2(\gamma-1)$

$mc^2 = (9.109\times10^{-31}\text{ kg})(2.998\times10^8\text{ m/s})^2 = 8.187\times10^{-14}\text{ J}(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 0.5111\text{ MeV}$

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-((1.972\times10^8\text{ m/s})/(2.998\times10^8\text{ m/s}))^2}} = 1.328$

$K = mc^2(\gamma-1) = (0.5111\text{ MeV})(1.328-1) = 0.168\text{ MeV}$

EVALUATE: No object can travel faster than the speed of light in vacuum but there is nothing that prohibits an object from traveling faster than the speed of light in some material.

37.58. **(a)** $v = \frac{p}{m} = \frac{(E/c)}{m} = \frac{E}{mc}$, where the atom and the photon have the same magnitude of momentum, E/c.

(b) $v = \frac{E}{mc} \ll c$, so $E \ll mc^2$.

37.59. **IDENTIFY** and **SET UP:** Let *S* be the lab frame and S' be the frame of the proton that is moving in the +*x* direction, so $u = +c/2$. The reference frames and moving particles are shown in Figure 37.59. The other proton moves in

the $-x$ direction in the lab frame, so $v = -c/2$. A proton has rest mass $m_p = 1.67\times10^{-27}$ kg and rest energy $m_p c^2 = 938$ MeV.

EXECUTE: **(a)** $v' = \dfrac{v-u}{1-uv/c^2} = \dfrac{-c/2 - c/2}{1-(c/2)(-c/2)/c^2} = -\dfrac{4c}{5}$

The speed of each proton relative to the other is $\dfrac{4}{5}c$.

(b) In nonrelativistic mechanics the speeds just add and the speed of each relative to the other is c.

(c) $K = \dfrac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2$

(i) Relative to the lab frame each proton has speed $v = c/2$. The total kinetic energy of each proton is

$$K = \frac{938 \text{ MeV}}{\sqrt{1-\left(\frac{1}{2}\right)^2}} - (938 \text{ MeV}) = 145 \text{ MeV}.$$

(ii) In its rest frame one proton has zero speed and zero kinetic energy and the other has speed $\dfrac{4}{5}c$. In this frame the kinetic energy of the moving proton is $K = \dfrac{938 \text{ MeV}}{\sqrt{1-\left(\frac{4}{5}\right)^2}} - (938 \text{ MeV}) = 625 \text{ MeV}$

(d) (i) Each proton has speed $v = c/2$ and kinetic energy

$$K = \frac{1}{2}mv^2 = \left(\frac{1}{2}m\right)(c/2)^2 = \frac{mc^2}{8} = \frac{938 \text{ MeV}}{8} = 117 \text{ MeV}$$

(ii) One proton has speed $v = 0$ and the other has speed c. The kinetic energy of the moving proton is $K = \dfrac{1}{2}mc^2 = \dfrac{938 \text{ MeV}}{2} = 469 \text{ MeV}$

EVALUATE: The relativistic expression for K gives a larger value than the nonrelativistic expression. The kinetic energy of the system is different in different frames.

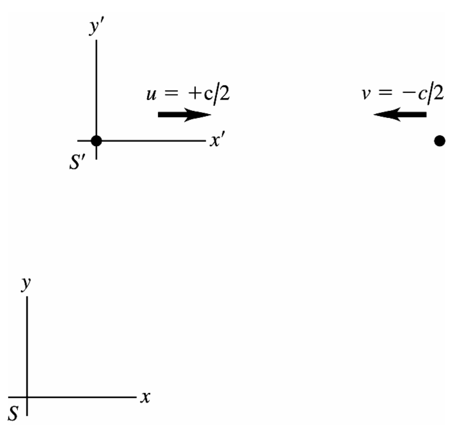


Figure 37.59

37.60. **IDENTIFY** and **SET UP:** Let S be the lab frame and let S' the frame of the proton that is moving in the $+x$ direction in the lab frame, as shown in Figure 37.60. In S' the other proton moves in the $-x'$ direction with speed $c/2$, so $v' = -c/2$. In the lab frame each proton has speed αc, where α is a constant that we need to solve for.

EXECUTE: **(a)** $v = \dfrac{v'+u}{1+uv'/c^2}$ with $v = -\alpha c$, $u = +\alpha c$ and $v' = -0.50c$ gives $-\alpha c = \dfrac{-0.50c + \alpha c}{1+(\alpha c)(-0.50c)/c^2}$ and $-\alpha = \dfrac{-0.50+\alpha}{1-0.50\alpha}$. $\alpha^2 - 4\alpha + 1 = 0$ and $\alpha = 0.268$ or $\alpha = 3.73$. Can't have $v > c$, so only $\alpha = 0.268$ is physically allowed. The speed measured by the observer in the lab is $0.268c$.

(b) (i) $v = 0.269c$. $\gamma = 1.0380$. $K = (\gamma - 1)mc^2 = 35.6$ MeV.

(ii) $v = 0.500c$. $\gamma = 1.1547$. $K = (\gamma - 1)mc^2 = 145$ MeV.

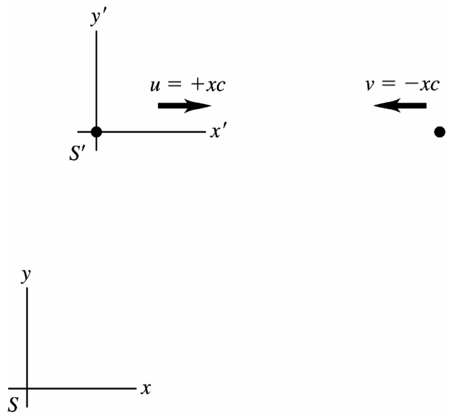


Figure 37.60

37.61. $x'^2 = c^2t'^2 \Rightarrow (x-ut)^2\gamma^2 = c^2\gamma^2\left(t - ux/c^2\right)^2$

$\Rightarrow x - ut = c(t - ux/c^2) \Rightarrow x\left(1 + \dfrac{u}{c}\right) = \dfrac{1}{c}x(u+c) = t(u+c) \Rightarrow x = ct \Rightarrow x^2 = c^2t^2.$

37.62. **IDENTIFY** and **SET UP:** Let S be the lab frame and let S' be the frame of the nucleus. Let the +x direction be the direction the nucleus is moving. $u = 0.7500c$.

EXECUTE: **(a)** $v' = +0.9995c$. $v = \dfrac{v'+u}{1+uv'/c^2} = \dfrac{0.9995c + 0.7500c}{1+(0.7500)(0.9995)} = 0.999929c$

(b) $v' = -0.9995c$. $v = \dfrac{-0.9995c + 0.7500c}{1+(0.7500)(-0.9995)} = -0.9965c$

(c) emitted in same direction:

(i) $K = \left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right)mc^2 = (0.511\text{ MeV})\left(\dfrac{1}{\sqrt{1-(0.999929)^2}} - 1\right) = 42.4\text{ MeV}$

(ii) $K' = \left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right)mc^2 = (0.511\text{ MeV})\left(\dfrac{1}{\sqrt{1-(0.9995)^2}} - 1\right) = 15.7\text{ MeV}$

(d) emitted in opposite direction:

(i) $K = \left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right)mc^2 = (0.511\text{ MeV})\left(\dfrac{1}{\sqrt{1-(0.9965)^2}} - 1\right) = 5.60\text{ MeV}$

(ii) $K' = \left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right)mc^2 = (0.511\text{ MeV})\left(\dfrac{1}{\sqrt{1-(0.9995)^2}} - 1\right) = 15.7\text{ MeV}$

37.63. **IDENTIFY** and **SET UP:** Use Eq.(37.30), with $a = dv/dt$, to obtain an expression for dv/dt. Separate the variables v and t and integrate to obtain an expression for $v(t)$. In this expression, let $t \to \infty$.

EXECUTE: $a = \dfrac{dv}{dt} = \dfrac{F}{m}(1 - v^2/c^2)^{3/2}$. (One-dimensional motion is assumed, and all the F, v, and a refer to x-components.)

$$\frac{dv}{(1-v^2/c^2)^{3/2}} = \left(\frac{F}{m}\right)dt$$

Integrate from $t = 0$, when $v = 0$, to time t, when the velocity is v.

$$\int_0^v \frac{dv}{(1-v^2/c^2)^{3/2}} = \int_0^t \left(\frac{F}{m}\right)dt$$

Since F is constant, $\int_0^t \left(\dfrac{F}{m}\right)dt = \dfrac{Ft}{m}$. In the velocity integral make the change of variable $y = v/c$; then $dy = dv/c$.

$$\int_0^v \frac{dv}{(1-v^2/c^2)^{3/2}} = c\int_0^{v/c} \frac{dy}{(1-y^2)^{3/2}} = c\left[\frac{y}{(1-y^2)^{1/2}}\right]_0^{v/c} = \frac{v}{\sqrt{1-v^2/c^2}}$$

Thus $\dfrac{v}{\sqrt{1-v^2/c^2}} = \dfrac{Ft}{m}$.

Solve this equation for v:

$$\frac{v^2}{1-v^2/c^2}=\left(\frac{Ft}{m}\right)^2 \text{ and } v^2=\left(\frac{Ft}{m}\right)^2(1-v^2/c^2)$$

$$v^2\left(1+\left(\frac{Ft}{mc}\right)^2\right)=\left(\frac{Ft}{m}\right)^2 \text{ so } v=\frac{(Ft/m)}{\sqrt{1+(Ft/mc)^2}}=c\frac{Ft}{\sqrt{m^2c^2+F^2t^2}}$$

As $t\to\infty$, $\dfrac{Ft}{\sqrt{m^2c^2+F^2t^2}}\to\dfrac{Ft}{\sqrt{F^2t^2}}\to 1$, so $v\to c$.

EVALUATE: Note that $\dfrac{Ft}{\sqrt{m^2c^2+F^2t^2}}$ is always less than 1, so $v<c$ always and v approaches c only when $t\to\infty$.

37.64. Setting $x=0$ in Eq.(37.21), the first equation becomes $x'=-\gamma ut$ and the last, upon multiplication by c, becomes $ct'=\gamma ct$. Squaring and subtracting gives $c^2t'^2-x'^2=\gamma^2(c^2t^2-u^2t^2)=c^2t^2$, or $x'=c\sqrt{t'^2-t^2}=4.53\times10^8$ m.

37.65. **(a) IDENTIFY** and **SET UP:** Use the Lorentz coordinate transformation (Eq.37.21) for (x_1,t_1) and (x_2,t_2):

$$x_1'=\frac{x_1-ut_1}{\sqrt{1-u^2/c^2}},\quad x_2'=\frac{x_2-ut_2}{\sqrt{1-u^2/c^2}}$$

$$t_1'=\frac{t_1-ux_1/c^2}{\sqrt{1-u^2/c^2}},\quad t_2'=\frac{t_2-ux_2/c^2}{\sqrt{1-u^2/c^2}}$$

Same point in S' implies $x_1'=x_2'$. What then is $\Delta t'=t_2'-t_1'$?

EXECUTE: $x_1'=x_2'$ implies $x_1-ut_1=x_2-ut_2$

$$u(t_2-t_1)=x_2-x_1 \text{ and } u=\frac{x_2-x_1}{t_2-t_1}=\frac{\Delta x}{\Delta t}$$

From the time transformation equations,

$$\Delta t'=t_2'-t_1'=\frac{1}{\sqrt{1-u^2/c^2}}(\Delta t-u\Delta x/c^2)$$

Using the result that $u=\dfrac{\Delta x}{\Delta t}$ gives

$$\Delta t'=\frac{1}{\sqrt{1-(\Delta x)^2/((\Delta t)^2c^2)}}(\Delta t-(\Delta x)^2/((\Delta t)c^2))$$

$$\Delta t'=\frac{\Delta t}{\sqrt{(\Delta t)^2-(\Delta x)^2/c^2}}(\Delta t-(\Delta x)^2/((\Delta t)c^2))$$

$$\Delta t'=\frac{(\Delta t)^2-(\Delta x)^2/c^2}{\sqrt{(\Delta t)^2-(\Delta x)^2/c^2}}=\sqrt{(\Delta t)^2-(\Delta x/c)^2}\text{, as was to be shown.}$$

This equation doesn't have a physical solution (because of a negative square root) if $(\Delta x/c)^2>(\Delta t)^2$ or $\Delta x\geq c\Delta t$.

(b) IDENTIFY and **SET UP:** Now require that $t_2'=t_1'$ (the two events are simultaneous in S') and use the Lorentz coordinate transformation equations.

EXECUTE: $t_2'=t_1'$ implies $t_1-ux_1/c^2=t_2-ux_2/c^2$

$$t_2-t_1=\left(\frac{x_2-x_1}{c^2}\right)u \text{ so } \Delta t=\left(\frac{\Delta x}{c^2}\right)u \text{ and } u=\frac{c^2\Delta t}{\Delta x}$$

From the Lorentz transformation equations,

$$\Delta x'=x_2'-x_1'=\left(\frac{1}{\sqrt{1-u^2/c^2}}\right)(\Delta x-u\Delta t).$$

Using the result that $u=c^2\Delta t/\Delta x$ gives

$$\Delta x'=\frac{1}{\sqrt{1-c^2(\Delta t)^2/(\Delta x)^2}}(\Delta x-c^2(\Delta t)^2/\Delta x)$$

$$\Delta x'=\frac{\Delta x}{\sqrt{(\Delta x)^2-c^2(\Delta t)^2}}(\Delta x-c^2(\Delta t)^2/\Delta x)$$

$$\Delta x'=\frac{(\Delta x)^2-c^2(\Delta t)^2}{\sqrt{(\Delta x)^2-c^2(\Delta t)^2}}=\sqrt{(\Delta x)^2-c^2(\Delta t)^2}$$

(c) IDENTIFY and **SET UP:** The result from part (b) is $\Delta x'=\sqrt{(\Delta x)^2-c^2(\Delta t)^2}$

Solve for Δt: $(\Delta x')^2 = (\Delta x)^2 - c^2(\Delta t)^2$

EXECUTE: $\Delta t = \dfrac{\sqrt{(\Delta x)^2 - (\Delta x')^2}}{c} = \dfrac{\sqrt{(5.00\text{ m})^2 - (2.50\text{ m})^2}}{2.998\times10^8\text{ m/s}} = 1.44\times10^{-8}\text{ s}$

EVALUATE: This provides another illustration of the concept of simultaneity (Section 37.2): events observed to be simultaneous in one frame are not simultaneous in another frame that is moving relative to the first.

37.66. **(a)** 80.0 m/s is non-relativistic, and $K = \frac{1}{2}mv^2 = 186\text{ J}$.

(b) $(\gamma - 1)mc^2 = 1.31\times10^{15}\text{ J}$.

(c) In Eq. (37.23), c) $v' = 2.20\times10^8\text{ m/s}$, $u = -1.80\times10^8\text{ m/s}$, and so $v = 7.14\times10^7\text{m/s}$.

(d) $\dfrac{20.0\text{ m}}{\gamma} = 13.6\text{ m}$.

(e) $\dfrac{20.0\text{ m}}{2.20\times10^8\text{ m/s}} = 9.09\times10^{-8}\text{ s}$.

(f) $t' = \dfrac{t}{\gamma} = 6.18\times10^{-8}\text{ s}$, or $t' = \dfrac{13.6\text{ m}}{2.20\times10^8\text{ m/s}} = 6.18\times10^{-8}\text{ s}$.

37.67. **IDENTIFY** and **SET UP:** An increase in wavelength corresponds to a decrease in frequency $(f = c/\lambda)$, so the atoms are moving away from the earth. Receding, so use Eq.(37.26): $f = \sqrt{\dfrac{c-u}{c+u}}f_0$

EXECUTE: Solve for u: $(f/f_0)^2(c+u) = c-u$ and $u = c\left(\dfrac{1-(f/f_0)^2}{1+(f/f_0)^2}\right)$

$f = c/\lambda,\ f_0 = c/\lambda_0$ so $f/f_0 = \lambda_0/\lambda$

$$u = c\left(\frac{1-(\lambda_0/\lambda)^2}{1+(\lambda_0/\lambda)^2}\right) = c\left(\frac{1-(656.3/953.4)^2}{1+(656.3/953.4)^2}\right) = 0.357c = 1.07\times10^8\text{ m/s}$$

EVALUATE: The relative speed is large, 36% of c. The cosmological implication of such observations will be discussed in Section 44.6.

37.68. The baseball had better be moving non-relativistically, so the Doppler shift formula (Eq.(37.25)) becomes $f \cong f_0(1-(u/c))$. In the baseball's frame, this is the frequency with which the radar waves strike the baseball, and the baseball reradiates at *f*. But in the coach's frame, the reflected waves are Doppler shifted again, so the detected frequency is $f(1-(u/c)) = f_0(1-(u/c))^2 \approx f_0(1-2(u/c))$, so $\Delta f = 2f_0(u/c)$ and the fractional frequency shift is $\dfrac{\Delta f}{f_0} = 2(u/c)$. In this case,

$$u = \frac{\Delta f}{2f_0}c = \frac{(2.86\times10^{-7})}{2}(3.00\times10^8\text{ m}) = 42.9\text{ m/s} = 154\text{ km/h} = 92.5\text{ mi/h}.$$

37.69. **IDENTIFY** and **SET UP:** 500 light years $= 4.73\times10^{18}$ m. The proper distance l_0 to the star is 500 light years. The energy needed is the kinetic energy of the rocket at its final speed.

EXECUTE: **(a)** $u = 0.50c$. $\Delta t = \dfrac{d}{u} = \dfrac{4.73\times10^{18}\text{ m}}{(0.50)(3.00\times10^8\text{ m/s})} = 3.2\times10^{10}\text{ s} = 1000\text{ yr}$

The proper time is measured by the astronauts. $\Delta t_0 = \Delta t\sqrt{1-u^2/c^2} = 866\text{ yr}$

$$K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = (1000\text{ kg})(3.00\times10^8\text{ m/s})^2\left(\frac{1}{\sqrt{1-(0.500)^2}} - 1\right) = 1.4\times10^{19}\text{ J}$$

This is 140% of the U.S. yearly use of energy.

(b) $u = 0.99c$. $\Delta t = \dfrac{d}{u} = \dfrac{4.73\times10^{18}\text{ m}}{(0.99)(3.00\times10^8\text{ m/s})} = 1.6\times10^{10}\text{ s} = 505\text{ yr}$, $\Delta t_0 = 71\text{ yr}$

$$K = (9.00\times10^{19}\text{ J})\left(\frac{1}{\sqrt{1-(0.99)^2}} - 1\right) = 5.5\times10^{20}\text{ J}$$

This is 55 times the U.S. yearly use.

(c) $u = 0.9999c$. $\Delta t = \frac{d}{u} = \frac{4.73\times10^{18}\text{ m}}{(0.9999)(3.00\times10^{8}\text{ m/s})} = 1.58\times10^{10}\text{ s} = 501\text{ yr}$, $\Delta t_0 = 7.1\text{ yr}$

$$K = (9.00\times10^{19}\text{ J})\left(\frac{1}{\sqrt{1-(0.9999)^2}} - 1\right) = 6.3\times10^{21}\text{ J}$$

This is 630 times the U.S. yearly use.
The energy cost of accelerating a rocket to these speeds is immense.

37.70. **(a)** As in the hint, both the sender and the receiver measure the same distance. However, in our frame, the ship has moved between emission of successive wavefronts, and we can use the time $T = 1/f$ as the proper time, with the result that $f = \gamma f_0 > f_0$.

(b) Toward: $f = f_0\sqrt{\frac{c+u}{c-u}} = 345\text{ MHz}\left(\frac{1+0.758}{1-0.758}\right)^{1/2} = 930\text{ MHz}$

$$f - f_0 = 930\text{ MHz} - 345\text{ MHz} = 585\text{ MHz}.$$

Away: $f = f_0\sqrt{\frac{c-u}{c+u}} = 345\text{ MHz}\left(\frac{1-0.758}{1+0.758}\right)^{1/2} = 128\text{ MHz}$ and $f - f_0 = -217\text{ MHz}$.

(c) $\gamma f_0 = 1.53 f_0 = 528\text{ MHz}$, $f - f_0 = 183\text{ MHz}$. The shift is still bigger than f_0 , but not as large as the approaching frequency.

37.71. The crux of this problem is the question of simultaneity. To be "in the barn at one time" for the runner is different than for a stationary observer in the barn. The diagram in Figure 37.71a shows the rod fitting into the barn at time $t = 0$, according to the stationary observer. The diagram in Figure 37.71b is in the runner's frame of reference. The front of the rod enters the barn at time t_1 and leaves the back of the barn at time t_2. However, the back of the rod does not enter the front of the barn until the later time t_3.

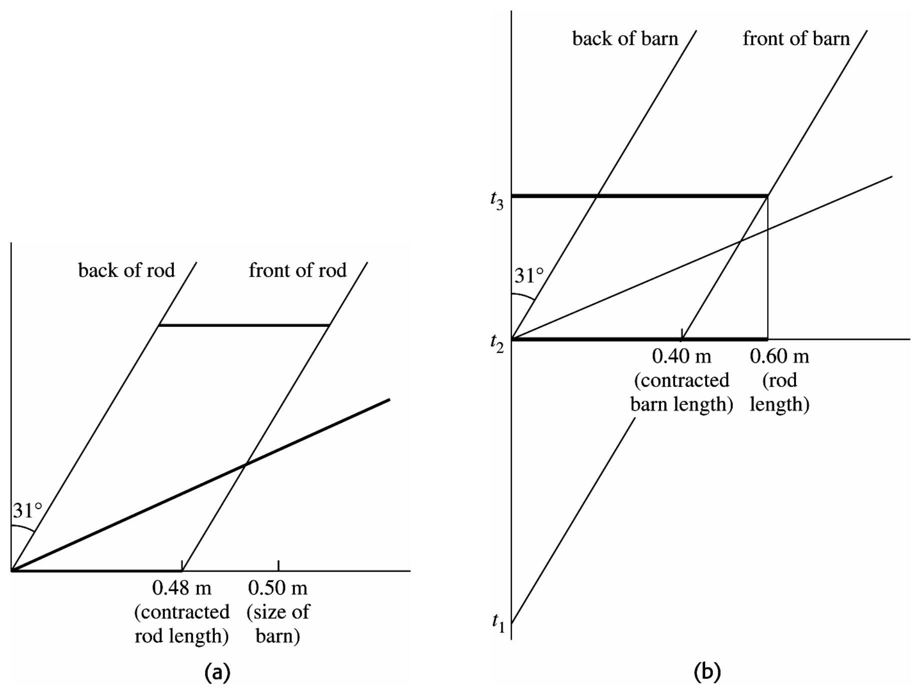


Figure 37.71

37.72. In Eq.(37.23), $u = V$, $v' = (c/n)$, and so $v = \frac{(c/n)+V}{1+\frac{cV}{nc^2}} = \frac{(c/n)+V}{1+(V/nc)}$. For V non-relativistic, this is

$v \approx ((cn)+V)(1-(V/nc)) = (nc/n) + V - (V/n^2) - (V^2/nc) \approx \frac{c}{n} + \left(1-\frac{1}{n^2}\right)V$, so $k = \left(1-\frac{1}{n^2}\right)$. For water, $n = 1.333$ and $k = 0.437$.

37.73. **(a)** $a' = \dfrac{dv}{dt'}$. $dt' = \gamma(dt - u\,dx/c^2)$. $dv' = \dfrac{dv}{(1-uv/c^2)} + \dfrac{v-u}{(1-uv/c^2)^2}\dfrac{u}{c^2}dv$

$\dfrac{dv'}{dv} = \dfrac{1}{1-uv/c^2} + \dfrac{v-u}{(1-uv/c^2)^2}\left(\dfrac{u}{c^2}\right)$.

$$dv' = dv\left(\frac{1}{1-uv/c^2} + \frac{(v-u)u/c^2}{(1-uv/c^2)^2}\right) = dv\left(\frac{1-u^2/c^2}{(1-uv/c^2)^2}\right)$$

$$a' = \frac{dv\dfrac{(1-u^2/c^2)}{(1-uv/c^2)^2}}{\gamma dt - u\gamma dx/c^2} = \frac{dv}{dt}\frac{(1-u^2/c^2)}{(1-uv/c^2)^2}\frac{1}{\gamma(1-uv/c^2)}$$

$$= a(1-u^2/c^2)^{3/2}\,(1-uv/c^2)^{-3}.$$

(b) Changing frames from $S' \to S$ just involves changing $a \to a'$, $v \to -v' \Rightarrow a = a'(1-u^2/c^2)^{3/2}\left(1+\dfrac{uv'}{c^2}\right)^{-3}$.

37.74. **(a)** The speed v' is measured relative to the rocket, and so for the rocket and its occupant, $v' = 0$. The acceleration as seen in the rocket is given to be $a' = g$, and so the acceleration as measured on the earth is

$a = \dfrac{du}{dt} = g\left(1-\dfrac{u^2}{c^2}\right)^{3/2}$.

(b) With $v_1 = 0$ when $t = 0$,

$dt = \dfrac{1}{g}\dfrac{du}{(1-u^2/c^2)^{3/2}}$. $\int_0^{t_1} dt = \dfrac{1}{g}\int_0^{v_1}\dfrac{du}{(1-u^2/c^2)^{3/2}}$. $t_1 = \dfrac{v_1}{g\sqrt{1-v_1^2/c^2}}$.

(c) $dt' = \gamma dt = dt/\sqrt{1-u^2/c^2}$, so the relation in part (b) between *dt* and *du,* expressed in terms of dt' and *du,* is

$dt' = \gamma dt = \dfrac{1}{\sqrt{1-u^2/c^2}}\dfrac{du}{g(1-u^2/c^2)^{3/2}} = \dfrac{1}{g}\dfrac{du}{(1-u^2/c^2)^2}$.

Integrating as above (perhaps using the substitution $z = u/c$) gives $t_1' = \dfrac{c}{g}\,\text{arctanh}\left(\dfrac{v_1}{c}\right)$. For those who wish to avoid inverse hyperbolic functions, the above integral may be done by the method of partial fractions;

$g\,dt' = \dfrac{du}{(1+u/c)(1-u/c)} = \dfrac{1}{2}\left[\dfrac{du}{1+u/c} + \dfrac{du}{1-uc}\right]$, which integrates to $t_1' = \dfrac{c}{2g}\ln\left(\dfrac{c+v_1}{c-v_1}\right)$.

(d) Solving the expression from part (c) for v_1 in terms of t_1, $(v_1/c) = \tanh(gt_1'/c)$, so that $\sqrt{1-(v_1/c)^2} = 1/\cosh(gt_1'/c)$, using the appropriate indentities for hyperbolic functions. Using this in the expression found in part (b), $t_1 = \dfrac{c}{g}\dfrac{\tanh(gt_1'/c)}{1/\cosh(gt_1'/c)} = \dfrac{c}{g}\sinh(gt_1'/c)$, which may be rearranged slightly as $\dfrac{gt_1}{c} = \sinh\left(\dfrac{gt_1'}{c}\right)$. If hyperbolic functions are not used, v_1 in terms of t_1' is found to be $\dfrac{v_1}{c} = \dfrac{e^{gt_1'/c} - e^{-gt_1'/c}}{e^{gt_1'/c} + e^{-gt_1'/c}}$ which is the same as $\tanh(gt_1'/c)$. Inserting this expression into the result of part (b) gives, after much algebra, $t_1 = \dfrac{c}{2g}(e^{gt_1'/c} - e^{-gt_1'/c})$, which is equivalent to the expression found using hyperbolic functions.

(e) After the first acceleration period (of 5 years by Stella's clock), the elapsed time on earth is

$$t_1' = \frac{c}{g}\sinh(gt_1'/c) = 2.65\times10^9\ \text{s} = 84.0\ \text{yr}.$$

The elapsed time will be the same for each of the four parts of the voyage, so when Stella has returned, Terra has aged 336 yr and the year is 2436. (Keeping more precision than is given in the problem gives February 7 of that year.)

37.75. **(a)** $f_0 = 4.568110\times10^{14}$ Hz; $f_+ = 4.568910\times10^{14}$ Hz; $f_- = 4.567710\times10^{14}$ Hz

$$\left.\begin{aligned} f_+ &= \sqrt{\frac{c+(u+v)}{c-(u+v)}}\,f_0 \\ f_- &= \sqrt{\frac{c+(u-v)}{c-(u-v)}}\,f_0 \end{aligned}\right\} \Rightarrow \begin{aligned} f_+^2(c-(u+v)) &= f_0^2\,(c+(u+v)) \\ f_-^2(c-(u-v)) &= f_0^2(c+(u-v)) \end{aligned}$$

where u is the velocity of the center of mass and v is the orbital velocity.

$$\Rightarrow (u+v) = \frac{(f_+/f_0)^2 - 1}{(f_+/f_0)^2 + 1}c \text{ and } (u-v) = \frac{(f_-^2/f_0^2) - 1}{(f_-^2/f_0^2) + 1}c$$

$$\Rightarrow u + v = 5.25\times10^4 \text{ m/s and } u - v = -2.63\times10^4 \text{ m/s}.$$

This gives $u = +1.31\times10^4$ m/s (moving toward at 13.1 km/s) and $v = 3.94\times10^4$ m/s.

(b) $v = 3.94\times10^4$ m/s; $T = 11.0$ days. $2\pi R = vt \Rightarrow$

$R = \dfrac{(3.94\times10^4 \text{ m/s})(11.0 \text{ days})(24 \text{ hrs/day})(3600 \text{ sec/hr})}{2\pi} = 5.96\times10^9$ m. This is about 0.040 times the earth-sun distance.

Also the gravitational force between them (a distance of $2R$) must equal the centripetal force from the center of mass:

$$\frac{(Gm^2)}{(2R)^2} = \frac{mv^2}{R} \Rightarrow m = \frac{4Rv^2}{G} = \frac{4(5.96\times10^9 \text{ m})(3.94\times10^4 \text{ m/s})^2}{6.672\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2} = 5.55\times10^{29} \text{ kg} = 0.279 \text{ m}_{\text{sun}}.$$

37.76. For any function $f = f(x,t)$ and $x = x(x',t'), t = t(x',t')$, let $F(x',t') = f(x(x',t'), t(x',t'))$ and use the standard (but mathematically improper) notation $F(x',t') = f(x',t')$. The chain rule is then

$$\frac{\partial f(x',t')}{\partial x} = \frac{\partial f(x,t)}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial f(x',t')}{\partial t'}\frac{\partial t'}{\partial x},$$
$$\frac{\partial f(x',t')}{\partial t} = \frac{\partial f(x,t)}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial f(x',t')}{\partial t'}\frac{\partial t'}{\partial t}.$$

In this solution, the explicit dependence of the functions on the sets of dependent variables is suppressed, and the above relations are then $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial f}{\partial t'}\frac{\partial t'}{\partial x}$, $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial f}{\partial t'}\frac{\partial t'}{\partial t}$.

(a) $\frac{\partial x'}{\partial x} = 1, \frac{\partial x'}{\partial t} = -v, \frac{\partial t'}{\partial x} = 0$ and $\frac{\partial t'}{\partial t} = 1$. Then, $\frac{\partial E}{\partial x} = \frac{\partial E}{\partial x'}$, and $\frac{\partial^2 E}{\partial x^2} = \frac{\partial^2 E}{\partial x'^2}$. For the time derivative, $\frac{\partial E}{\partial t} = -v\frac{\partial E}{\partial x'} + \frac{\partial E}{\partial t'}$. To find the second time derivative, the chain rule must be applied to both terms; that is,

$$\frac{\partial}{\partial t}\frac{\partial E}{\partial x'} = -v\frac{\partial^2 E}{\partial x'^2} + \frac{\partial^2 E}{\partial t'\partial x'},$$
$$\frac{\partial}{\partial t}\frac{\partial E}{\partial t'} = -v\frac{\partial^2 E}{\partial x'\partial t'} + \frac{\partial^2 E}{\partial t'^2}.$$

Using these in $\frac{\partial^2 E}{\partial t^2}$, collecting terms and equating the mixed partial derivatives gives

$\frac{\partial^2 E}{\partial t^2} = v^2\frac{\partial^2 E}{\partial x'^2} - 2v\frac{\partial^2 E}{\partial x'\partial t'} + \frac{\partial^2 E}{\partial t'^2}$, and using this and the above expression for $\frac{\partial^2 E}{\partial x'^2}$ gives the result.

(b) For the Lorentz transformation, $\frac{\partial x'}{\partial x} = \gamma, \frac{\partial x'}{\partial t} = \gamma v, \frac{\partial t'}{\partial x} = \gamma v/c^2$ and $\frac{\partial t'}{\partial t} = \gamma$.

The first partials are then

$$\frac{\partial E}{\partial x} = \gamma\frac{\partial E}{\partial x'} - \gamma\frac{v}{c^2}\frac{\partial E}{\partial t'}, \frac{\partial E}{\partial t} = -\gamma v\frac{\partial E}{\partial x'} + \gamma\frac{\partial E}{\partial t'}$$

and the second partials are (again equating the mixed partials)

$$\frac{\partial^2 E}{\partial x^2} = \gamma^2\frac{\partial^2 E}{\partial x'^2} + \gamma^2\frac{v^2}{c^4}\frac{\partial^2 E}{\partial t'^2} - 2\gamma^2\frac{v}{c^2}\frac{\partial^2 E}{\partial x'\partial t'}$$
$$\frac{\partial^2 E}{\partial t^2} = \gamma^2 v^2\frac{\partial^2 E}{\partial x'^2} + \gamma^2\frac{\partial^2 E}{\partial t'^2} - 2\gamma^2 v\frac{\partial^2 E}{\partial x'\partial t'}.$$

Substituting into the wave equation and combining terms (note that the mixed partials cancel),

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \gamma^2\left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2 E}{\partial x'^2} + \gamma^2\left(\frac{v^2}{c^4} - \frac{1}{c^2}\right)\frac{\partial^2 E}{\partial t'^2} = \frac{\partial^2 E}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t'^2} = 0.$$

37.77. **(a)** In the center of momentum frame, the two protons approach each other with equal velocities (since the protons have the same mass). After the collision, the two protons are at rest—but now there are kaons as well. In this situation the kinetic energy of the protons must equal the total rest energy of the two kaons $\Rightarrow 2(\gamma_{\text{cm}}-1)m_{\text{p}}c^2 = 2m_{\text{k}}c^2 \Rightarrow \gamma_{\text{cm}} = 1+\dfrac{m_{\text{k}}}{m_{\text{p}}} = 1.526.$ The velocity of a proton in the center of momentum frame is then

$$v_{\text{cm}} = c\sqrt{\frac{\gamma_{\text{cm}}^2-1}{\gamma_{\text{cm}}^2}} = 0.7554c.$$

To get the velocity of this proton in the lab frame, we must use the Lorentz velocity transformations. This is the same as "hopping" into the proton that will be our target and asking what the velocity of the projectile proton is. Taking the lab frame to be the unprimed frame moving to the left, $u = v_{\text{cm}}$ and $v' = v_{\text{cm}}$ (the velocity of the projectile proton in the center of momentum frame).

$$v_{\text{lab}} = \frac{v'+u}{1+\dfrac{uv'}{c^2}} = \frac{2v_{\text{cm}}}{1+\dfrac{v_{\text{cm}}^2}{c^2}} = 0.9619c \Rightarrow \gamma_{\text{lab}} = \frac{1}{\sqrt{1-\dfrac{v_{\text{lab}}^2}{c^2}}} = 3.658 \Rightarrow K_{\text{lab}} = (\gamma_{\text{lab}}-1)m_{\text{p}}c^2 = 2494\text{ MeV}.$$

(b) $\dfrac{K_{\text{lab}}}{2m_{\text{k}}} = \dfrac{2494\text{ MeV}}{2(493.7\text{ MeV})} = 2.526.$

(c) The center of momentum case considered in part (a) is the same as this situation. Thus, the kinetic energy required *is* just twice the rest mass energy of the kaons. $K_{\text{cm}} = 2(493.7\text{ MeV}) = 987.4\text{ MeV}.$ This offers a substantial advantage over the fixed target experiment in part (b). It takes less energy to create two kaons in the proton center of momentum frame.

PHOTONS, ELECTRONS, AND ATOMS

38

38.1. **IDENTIFY** and **SET UP:** The stopping potential V_0 is related to the frequency of the light by $V_0 = \frac{h}{e} f - \frac{\phi}{e}$. The slope of V_0 versus f is h/e. The value f_{th} of f when $V_0 = 0$ is related to ϕ by $\phi = hf_{th}$.

EXECUTE: **(a)** From the graph, $f_{th} = 1.25\times10^{15}$ Hz. Therefore, with the value of h from part (b), $\phi = hf_{th} = 4.8$ eV.

(b) From the graph, the slope is 3.8×10^{-15} V·s. $h = (e)(\text{slope}) = (1.60\times10^{-16}\ \text{C})(3.8\times10^{-15}\ \text{V}\cdot\text{s}) = 6.1\times10^{-34}\ \text{J}\cdot\text{s}$

(c) No photoelectrons are produced for $f < f_{th}$.

(d) For a different metal f_{th} and ϕ are different. The slope is h/e so would be the same, but the graph would be shifted right or left so it has a different intercept with the horizontal axis.

EVALUATE: As the frequency f of the light is increased above f_{th} the energy of the photons in the light increases and more energetic photons are produced. The work function we calculated is similar to that for gold or nickel.

38.2. **IDENTIFY** and **SET UP:** $c = f\lambda$ relates frequency and wavelength and $E = hf$ relates energy and frequency for a photon. $c = 3.00\times10^8$ m/s. $1\ \text{eV} = 1.60\times10^{-16}$ J.

EXECUTE: **(a)** $f = \frac{c}{\lambda} = \frac{3.00\times10^8\ \text{m/s}}{505\times10^{-9}\ \text{m}} = 5.94\times10^{14}\ \text{Hz}$

(b) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(5.94\times10^{14}\ \text{Hz}) = 3.94\times10^{-19}\ \text{J} = 2.46\ \text{eV}$

(c) $K = \frac{1}{2}mv^2$ so $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(3.94\times10^{-19}\ \text{J})}{9.5\times10^{-15}\ \text{kg}}} = 9.1\ \text{mm/s}$

38.3. $f = \frac{c}{\lambda} = \frac{3.00\times10^8\ \text{m/s}}{5.20\times10^{-7}\ \text{m}} = 5.77\times10^{14}\ \text{Hz}$

$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{5.20\times10^{-7}\ \text{m}} = 1.28\times10^{-27}\ \text{kg}\cdot\text{m/s}$

$E = pc = (1.28\times10^{-27}\ \text{kg}\cdot\text{m/s})(3.00\times10^8\ \text{m/s}) = 3.84\times10^{-19}\ \text{J} = 2.40\ \text{eV}.$

38.4. **IDENTIFY** and **SET UP:** $P_{av} = \frac{\text{energy}}{t}$. $1\ \text{eV} = 1.60\times10^{-19}$ J. For a photon, $E = hf = \frac{hc}{\lambda}$. $h = 6.63\times10^{-34}$ J·s.

EXECUTE: **(a)** $\text{energy} = P_{av}t = (0.600\ \text{W})(20.0\times10^{-3}\ \text{s}) = 1.20\times10^{-2}\ \text{J} = 7.5\times10^{16}\ \text{eV}$

(b) $E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{652\times10^{-9}\ \text{m}} = 3.05\times10^{-19}\ \text{J} = 1.91\ \text{eV}$

(c) The number of photons is the total energy in a pulse divided by the energy of one photon:

$\frac{1.20\times10^{-2}\ \text{J}}{3.05\times10^{-19}\ \text{J/photon}} = 3.93\times10^{16}$ photons.

EVALUATE: The number of photons in each pulse is very large.

38.5. **IDENTIFY** and **SET UP:** Eq.(38.2) relates the photon energy and wavelength. $c = f\lambda$ relates speed, frequency and wavelength for an electromagnetic wave.

EXECUTE: **(a)** $E = hf$ so $f = \frac{E}{h} = \frac{(2.45\times10^6\ \text{eV})(1.602\times10^{-19}\ \text{J/1 eV})}{6.626\times10^{-34}\ \text{J}\cdot\text{s}} = 5.92\times10^{20}\ \text{Hz}$

(b) $c = f\lambda$ so $\lambda = \frac{c}{f} = \frac{2.998\times10^8\ \text{m/s}}{5.92\times10^{20}\ \text{Hz}} = 5.06\times10^{-13}\ \text{m}$

(c) **EVALUATE:** λ is comparable to a nuclear radius. Note that in doing the calculation the energy in MeV was converted to the SI unit of Joules.

38.6. **IDENTIFY** and **SET UP:** $\lambda_{th} = 272\text{ nm}$. $c = f\lambda$. $\frac{1}{2}mv_{max}^2 = hf - \phi$. At the threshold frequency, f_{th}, $v_{max} \to 0$. $h = 4.136\times10^{-15}\text{ eV}\cdot\text{s}$.

EXECUTE: **(a)** $f_{th} = \dfrac{c}{\lambda_{th}} = \dfrac{3.00\times10^8\text{ m/s}}{272\times10^{-9}\text{ m}} = 1.10\times10^{15}\text{ Hz}$.

(b) $\phi = hf_{th} = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(1.10\times10^{15}\text{ Hz}) = 4.55\text{ eV}$.

(c) $\frac{1}{2}mv_{max}^2 = hf - \phi = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(1.45\times10^{15}\text{ Hz}) - 4.55\text{ eV} = 6.00\text{ eV} - 4.55\text{ eV} = 1.45\text{ eV}$

EVALUATE: The threshold wavelength depends on the work function for the surface.

38.7. **IDENTIFY** and **SET UP:** Eq.(38.3): $\frac{1}{2}mv_{max}^2 = hf - \phi = \dfrac{hc}{\lambda} - \phi$. Take the work function ϕ from Table 38.1. Solve for v_{max}. Note that we wrote *f* as c/λ.

EXECUTE: $\frac{1}{2}mv_{max}^2 = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{235\times10^{-9}\text{ m}} - (5.1\text{ eV})(1.602\times10^{-19}\text{ J/1 eV})$

$\frac{1}{2}mv_{max}^2 = 8.453\times10^{-19}\text{ J} - 8.170\times10^{-19}\text{ J} = 2.83\times10^{-20}\text{ J}$

$v_{max} = \sqrt{\dfrac{2(2.83\times10^{-20}\text{ J})}{9.109\times10^{-31}\text{ kg}}} = 2.49\times10^5\text{ m/s}$

EVALUATE: The work function in eV was converted to joules for use in Eq.(38.3). A photon with $\lambda = 235\text{ nm}$ has energy greater then the work function for the surface.

38.8. **IDENTIFY** and **SET UP:** $\phi = hf_{th} = \dfrac{hc}{\lambda_{th}}$. The minimum ϕ corresponds to the minimum λ.

EXECUTE: $\phi = \dfrac{hc}{\lambda_{th}} = \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^8\text{ m/s})}{700\times10^{-9}\text{ m}} = 1.77\text{ eV}$

38.9. **IDENTIFY** and **SET UP:** $c = f\lambda$. The source emits $(0.05)(75\text{ J}) = 3.75\text{ J}$ of energy as visible light each second. $E = hf$, with $h = 6.63\times10^{-34}\text{ J}\cdot\text{s}$.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\text{ m/s}}{600\times10^{-9}\text{ m}} = 5.00\times10^{14}\text{ Hz}$

(b) $E = hf = (6.63\times10^{-34}\text{ J}\cdot\text{s})(5.00\times10^{14}\text{ Hz}) = 3.32\times10^{-19}\text{ J}$. The number of photons emitted per second is $\dfrac{3.75\text{ J}}{3.32\times10^{-19}\text{ J/photon}} = 1.13\times10^{19}$ photons.

(c) No. The frequency of the light depends on the energy of each photon. The number of photons emitted per second is proportional to the power output of the source.

38.10. **IDENTIFY:** In the photoelectric effect, the energy of the photon is used to eject an electron from the surface, and any excess energy goes into kinetic energy of the electron.

SET UP: The energy of a photon is $E = hf$, and the work function is given by $\phi = hf_0$, where f_0 is the threshold frequency.

EXECUTE: (a) From the graph, we see that $K_{max} = 0$ when $\lambda = 250$ nm, so the threshold wavelength is 250 nm. Calling f_0 the threshold frequency, we have

$$f_0 = c/\lambda_0 = (3.00 \times 10^8\text{ m/s})/(250\text{ nm}) = 1.2 \times 10^{15}\text{ Hz}.$$

(b) $\phi = hf_0 = (4.136 \times 10^{-15}\text{ eV}\cdot\text{s})(1.2 \times 10^{15}\text{ Hz}) = 4.96\text{ eV} = 5.0\text{ eV}$

(c) The graph (see Figure 38.10) is linear for $\lambda < \lambda_0$ $(1/\lambda > 1/\lambda_0)$, and linear graphs are easier to interpret than curves.

EVALUATE: If the wavelength of the light is longer than the threshold wavelength (that is, if $1/\lambda < 1/\lambda_0$), the kinetic energy of the electrons is really not defined since no photoelectrons are ejected from the metal.

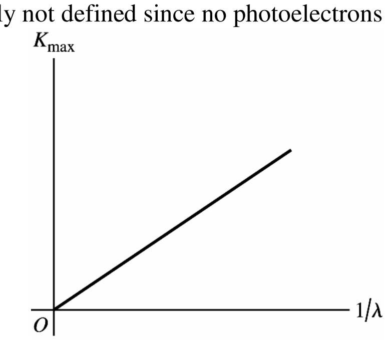


Figure 38.10

38.11. **IDENTIFY:** Protons have mass and photons are massless.

(a) SET UP: For a particle with mass, $K = p^2/2m$.

EXECUTE: $p_2 = 2p_1$ means $K_2 = 4K_1$.

(b) SET UP: For a photon, $E = pc$.

EXECUTE: $p_2 = 2p_1$ means $E_2 = 2E_1$.

EVALUATE: The relation between E and p is different for particles with mass and particles without mass.

38.12. **IDENTIFY** and **SET UP:** $eV_0 = \frac{1}{2}mv_{\text{max}}^2$, where V_0 is the stopping potential. The stopping potential in volts equals eV_0 in electron volts. $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi$.

EXECUTE: **(a)** $eV_0 = \frac{1}{2}mv_{\text{max}}^2$ so

$eV_0 = hf - \phi = \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{250\times10^{-9}\text{ m}} - 2.3\text{ eV} = 4.96\text{ eV} - 2.3\text{ eV} = 2.7\text{ eV}$. The stopping potential is 2.7 electron volts.

(b) $\frac{1}{2}mv_{\text{max}}^2 = 2.7\text{ eV}$

(c) $v_{\text{max}} = \sqrt{\dfrac{2(2.7\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}} = 9.7\times10^{5}\text{ m/s}$

38.13. **(a) IDENTIFY:** First use Eq.(38.4) to find the work function ϕ.

SET UP: $eV_0 = hf - \phi$ so $\phi = hf - eV_0 = \dfrac{hc}{\lambda} - eV_0$

EXECUTE: $\phi = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{254\times10^{-9}\text{ m}} - (1.602\times10^{-19}\text{ C})(0.181\text{ V})$

$\phi = 7.821\times10^{-19}\text{ J} - 2.900\times10^{-20}\text{ J} = 7.531\times10^{-19}\text{ J}(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 4.70\text{ eV}$

IDENTIFY and **SET UP:** The threshold frequency f_{th} is the smallest frequency that still produces photoelectrons. It corresponds to $K_{\text{max}} = 0$ in Eq.(38.3), so $hf_{\text{th}} = \phi$.

EXECUTE: $f = \dfrac{c}{\lambda}$ says $\dfrac{hc}{\lambda_{\text{th}}} = \phi$

$\lambda_{\text{th}} = \dfrac{hc}{\phi} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{7.531\times10^{-19}\text{ J}} = 2.64\times10^{-7}\text{ m} = 264\text{ nm}$

(b) EVALUATE: As calculated in part (a), $\phi = 4.70$ eV. This is the value given in Table 38.1 for copper.

38.14. **IDENTIFY** and **SET UP:** A photon has zero rest mass, so its energy and momentum are related by Eq.(37.40). Eq.(38.5) then relates its momentum and wavelength.

EXECUTE: **(a)** $E = pc = (8.24\times10^{-28}\text{ kg}\cdot\text{m/s})(2.998\times10^{8}\text{ m/s}) = 2.47\times10^{-19}\text{ J} =$ $(2.47\times10^{-19}\text{ J})(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 1.54\text{ eV}$

(b) $p = \dfrac{h}{\lambda}$ so $\lambda = \dfrac{h}{p} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{8.24\times10^{-28}\text{ kg}\cdot\text{m/s}} = 8.04\times10^{-7}\text{ m} = 804\text{ nm}$

EVALUATE: This wavelength is longer than visible wavelengths; it is in the infrared region of the electromagnetic spectrum. To check our result we could verify that the same E is given by Eq.(38.2), using the λ we have calculated.

38.15. **IDENTIFY** and **SET UP:** Balmer's formula is $\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right)$. For the H_γ spectral line $n = 5$. Once we have λ, calculate f from $f = c/\lambda$ and E from Eq.(38.2).

EXECUTE: **(a)** $\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{5^2}\right) = R\left(\dfrac{25-4}{100}\right) = R\left(\dfrac{21}{100}\right)$.

Thus $\lambda = \dfrac{100}{21R} = \dfrac{100}{21(1.097\times10^{7})}\text{ m} = 4.341\times10^{-7}\text{ m} = 434.1\text{ nm}$.

(b) $f = \dfrac{c}{\lambda} = \dfrac{2.998\times10^{8}\text{ m/s}}{4.341\times10^{-7}\text{ m}} = 6.906\times10^{14}\text{ Hz}$

(c) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(6.906\times10^{14}\ \text{Hz}) = 4.576\times10^{-19}\ \text{J} = 2.856\ \text{eV}$

EVALUATE: Section 38.3 shows that the longest wavelength in the Balmer series (H_α) is 656 nm and the shortest is 365 nm. Our result for H_γ falls within this range. The photon energies for hydrogen atom transitions are in the eV range, and our result is of this order.

38.16. **IDENTIFY** and **SET UP:** For the Lyman series the final state is $n=1$ and the wavelengths are given by $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$, $n = 2,3,....$ For the Paschen series the final state is $n=3$ and the wavelengths are given by $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, $n = 4,5,....$ $R = 1.097\times10^{7}\ \text{m}^{-1}$. The longest wavelength is for the smallest *n* and the shortest wavelength is for $n \to \infty$.

EXECUTE: <u>Lyman</u> Longest: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$. $\lambda = \frac{4}{3(1.097\times10^{7}\ \text{m}^{-1})} = 121.5\ \text{nm}$.

Shortest: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R$. $\lambda = \frac{1}{1.097\times10^{7}\ \text{m}^{-1}} = 91.16\ \text{nm}$

<u>Paschen</u> Longest: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7R}{144}$. $\lambda = \frac{144}{7(1.097\times10^{7}\ \text{m}^{-1})} = 1875\ \text{nm}$.

Shortest: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) = \frac{R}{9}$.

38.17. **(a)** $E_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{8.60\times10^{-7}\ \text{m}} = 2.31\times10^{-19}\ \text{J} = 1.44\ \text{eV}.$

So the internal energy of the atom increases by 1.44 eV to $E = -6.52\ \text{eV} + 1.44\ \text{eV} = -5.08\ \text{eV}$.

(b) $E_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{4.20\times10^{-7}\ \text{m}} = 4.74\times10^{-19}\ \text{J} = 2.96\ \text{eV}.$

So the final internal energy of the atom decreases to $E = -2.68\ \text{eV} - 2.96\ \text{eV} = -5.64\ \text{eV}$.

38.18. **IDENTIFY** and **SET UP:** The ionization threshold is at $E = 0$. The energy of an absorbed photon equals the energy gained by the atom and the energy of an emitted photon equals the energy lost by the atom.
EXECUTE: **(a)** $\Delta E = 0 - (-20\ \text{eV}) = 20\ \text{eV}$

(b) When the atom in the $n=1$ level absorbs a 18 eV photon, the final level of the atom is $n=4$. The possible transitions from $n=4$ and corresponding photon energies are $n=4 \to n=3,\ 3\ \text{eV}$; $n=4 \to n=2,\ 8\ \text{eV}$; $n=4 \to n=1,\ 18\ \text{eV}$. Once the atom has gone to the $n=3$ level, the following transitions can occur: $n=3 \to n=2,\ 5\ \text{eV}$; $n=3 \to n=1,\ 15\ \text{eV}$. Once the atom has gone to the $n=2$ level, the following transition can occur: $n=2 \to n=1,\ 10\ \text{eV}$. The possible energies of emitted photons are: 3 eV, 5 eV, 8 eV, 10 eV, 15 eV, and 18 eV.
(c) There is no energy level 8 eV higher in energy than the ground state, so the photon cannot be absorbed.
(d) The photon energies for $n=3 \to n=2$ and for $n=3 \to n=1$ are 5 eV and 15 eV. The photon energy for $n=4 \to n=3$ is 3 eV. The work function must have a value between 3 eV and 5 eV.

38.19. **IDENTIFY** and **SET UP:** The wavelength of the photon is related to the transition energy $E_\text{i} - E_\text{f}$ of the atom by $E_\text{i} - E_\text{f} = \frac{hc}{\lambda}$ where $hc = 1.240\times10^{-6}\ \text{eV}\cdot\text{m}$.

EXECUTE: **(a)** The minimum energy to ionize an atom is when the upper state in the transition has $E = 0$, so $E_1 = -17.50\ \text{eV}$. For $n=5 \to n=1$, $\lambda = 73.86\ \text{nm}$ and $E_5 - E_1 = \frac{1.240\times10^{-6}\ \text{eV}\cdot\text{m}}{73.86\times10^{-9}\ \text{m}} = 16.79\ \text{eV}$.
$E_5 = -17.50\ \text{eV} + 16.79\ \text{eV} = -0.71\ \text{eV}$. For $n=4 \to n=1$, $\lambda = 75.63\ \text{nm}$ and $E_4 = -1.10\ \text{eV}$. For $n=3 \to n=1$, $\lambda = 79.76\ \text{nm}$ and $E_3 = -1.95\ \text{eV}$. For $n=2 \to n=1$, $\lambda = 94.54\ \text{nm}$ and $E_2 = -4.38\ \text{eV}$.

(b) $E_\text{i} - E_\text{f} = E_4 - E_2 = -1.10\ \text{eV} - (-4.38\ \text{eV}) = 3.28\ \text{eV}$ and $\lambda = \frac{hc}{E_\text{i} - E_\text{f}} = \frac{1.240\times10^{-6}\ \text{eV}\cdot\text{m}}{3.28\ \text{eV}} = 378\ \text{nm}$

EVALUATE: The $n=4 \to n=2$ transition energy is smaller than the $n=4 \to n=1$ transition energy so the wavelength is longer. In fact, this wavelength is longer than for any transition that ends in the $n=1$ state.

38.20. **(a)** Equating initial kinetic energy and final potential energy and solving for the separation radius r,

$$r = \frac{1}{4\pi\epsilon_0}\frac{(92e)\,(2e)}{K} = \frac{1}{4\pi\epsilon_0}\frac{(184)\,(1.60\times10^{-19}\ \text{C})}{(4.78\times10^{6}\ \text{J/C})} = 5.54\times10^{-14}\ \text{m}.$$

(b) The above result may be substituted into Coulomb's law, or, the relation between the magnitude of the force and the magnitude of the potential energy in a Coulombic field is

$$F = \frac{K}{r} = \frac{(4.78\times10^{6}\ \text{eV})\,(1.6\times10^{-19}\ \text{J/ev})}{(5.54\times10^{-14}\ \text{m})} = 13.8\ \text{N}.$$

38.21. **(a) IDENTIFY:** If the particles are treated as point charges, $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$.

SET UP: $q_1 = 2e$ (alpha particle); $q_2 = 82e$ (gold nucleus); r is given so we can solve for U.

EXECUTE: $U = (8.987\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(2)(82)(1.602\times10^{-19}\ \text{C})^2}{6.50\times10^{-14}\ \text{m}} = 5.82\times10^{-13}\ \text{J}$

$U = 5.82\times10^{-13}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 3.63\times10^{6}\ \text{eV} = 3.63\ \text{MeV}$

(b) IDENTIFY: Apply conservation of energy: $K_1 + U_1 = K_2 + U_2$.

SET UP: Let point 1 be the initial position of the alpha particle and point 2 be where the alpha particle momentarily comes to rest. Alpha particle is initially far from the lead nucleus implies $r_1 \approx \infty$ and $U_1 = 0$. Alpha particle stops implies $K_2 = 0$.

EXECUTE: Conservation of energy thus says $K_1 = U_2 = 5.82\times10^{-13}\ \text{J} = 3.63\ \text{MeV}$.

(c) $K = \frac{1}{2}mv^2$ so $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(5.82\times10^{-13}\ \text{J})}{6.64\times10^{-27}\ \text{kg}}} = 1.32\times10^{7}\ \text{m/s}$

EVALUATE: $v/c = 0.044$, so it is ok to use the nonrelativistic expression to relate K and v. When the alpha particle stops, all its initial kinetic energy has been converted to electrostatic potential energy.

38.22. **(a)**, **(b)** For either atom, the magnitude of the angular momentum is $\dfrac{h}{2\pi} = 1.05\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}$.

38.23. **IDENTIFY** and **SET UP:** Use the energy to calculate n for this state. Then use the Bohr equation, Eq.(38.10), to calculate L.

EXECUTE: $E_n = -(13.6\ \text{eV})/n^2$, so this state has $n = \sqrt{13.6/1.51} = 3$. In the Bohr model. $L = n\hbar$ so for this state $L = 3\hbar = 3.16\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}$.

EVALUATE: We will find in Section 41.1 that the modern quantum mechanical description gives a different result.

38.24. **IDENTIFY** and **SET UP:** For a hydrogen atom $E_n = -\dfrac{13.6\ \text{eV}}{n^2}$. $\Delta E = \dfrac{hc}{\lambda}$, where ΔE is the magnitude of the energy change for the atom and λ is the wavelength of the photon that is absorbed or emitted.

EXECUTE: $\Delta E = E_4 - E_1 = -(13.6\ \text{eV})\left(\dfrac{1}{4^2} - \dfrac{1}{1^2}\right) = +12.75\ \text{eV}$.

$\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{12.75\ \text{eV}} = 97.3\ \text{nm}$. $f = \dfrac{c}{\lambda} = 3.08\times10^{15}\ \text{Hz}$.

38.25. **IDENTIFY:** The force between the electron and the nucleus in Be^{3+} is $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{Ze^2}{r^2}$, where $Z = 4$ is the nuclear charge. All the equations for the hydrogen atom apply to Be^{3+} if we replace e^2 by Ze^2.

(a) SET UP: Modify Eq.(38.18).

EXECUTE: $E_n = -\dfrac{1}{\epsilon_0}\dfrac{me^4}{8n^2h^2}$ (hydrogen) becomes

$$E_n = -\frac{1}{\epsilon_0}\frac{m(Ze^2)^2}{8n^2h^2} = Z^2\left(-\frac{1}{\epsilon_0}\frac{me^4}{8n^2h^2}\right) = Z^2\left(-\frac{13.60\ \text{eV}}{n^2}\right)\ (\text{for Be}^{3+})$$

The ground-level energy of Be^{3+} is $E_1 = 16\left(-\dfrac{13.60\ \text{eV}}{1^2}\right) = -218\ \text{eV}$.

EVALUATE: The ground-level energy of Be^{3+} is $Z^2 = 16$ times the ground-level energy of H.

(b) SET UP: The ionization energy is the energy difference between the $n \to \infty$ level energy and the $n = 1$ level energy.

EXECUTE: The $n \to \infty$ level energy is zero, so the ionization energy of Be^{3+} is 218 eV.
EVALUATE: This is 16 times the ionization energy of hydrogen.

(c) SET UP: $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ just as for hydrogen but now R has a different value.

EXECUTE: $R_{\text{H}} = \frac{me^4}{8\epsilon_0 h^3 c} = 1.097 \times 10^7\ \text{m}^{-1}$ for hydrogen becomes

$R_{\text{Be}} = Z^2 \frac{me^4}{8\epsilon_0 h^3 c} = 16(1.097 \times 10^7\ \text{m}^{-1}) = 1.755 \times 10^8\ \text{m}^{-1}$ for Be^{3+}.

For $n = 2$ to $n = 1$, $\frac{1}{\lambda} = R_{\text{Be}}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 3R/4.$

$\lambda = 4/(3R) = 4/(3(1.755 \times 10^8\ \text{m}^{-1})) = 7.60 \times 10^{-9}\ \text{m} = 7.60\ \text{nm}.$

EVALUATE: This wavelength is smaller by a factor of 16 compared to the wavelength for the corresponding transition in the hydrogen atom.

(d) SET UP: Modify Eq.(38.12): $r_n = \epsilon_0 \frac{n^2 h^2}{\pi m e^2}$ (hydrogen).

EXECUTE: $r_n = \epsilon_0 \frac{n^2 h^2}{\pi m (Ze^2)}$ (Be^{3+}).

EVALUATE: For a given n the orbit radius for Be^{3+} is smaller by a factor of $Z = 4$ compared to the corresponding radius for hydrogen.

38.26. **(a)** We can find the photon's energy from Eq. 38.8

$E = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = (6.63 \times 10^{-34}\ \text{J} \cdot \text{s})\,(3.00 \times 10^8\ \text{m/s})\,(1.097 \times 10^7 \text{m}^{-1})\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = 4.58 \times 10^{-19}\ \text{J}$. The corresponding wavelength is $\lambda = \frac{E}{hc} = 434\ \text{nm}.$

(b) In the Bohr model, the angular momentum of an electron with principal quantum number n is given by Eq. 38.10: $L = n\frac{h}{2\pi}$. Thus, when an electron makes a transition from $n = 5$ to $n = 2$ orbital, there is the following loss in angular momentum (which we would assume is transferred to the photon):

$$\Delta L = (2 - 5)\frac{h}{2\pi} = -\frac{3(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})}{2\pi} = -3.17 \times 10^{-34}\ \text{J} \cdot \text{s}.$$

However, this prediction of the Bohr model is wrong (as shown in Chapter 41).

38.27. **(a)** $v_n = \frac{1}{\epsilon_0} \frac{e^2}{2nh} : n = 1 \Rightarrow v_1 = \frac{(1.60 \times 10^{-19}\ \text{C})^2}{\epsilon_0 2\,(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})} = 2.18 \times 10^6\ \text{m/s}$

$$h = 2 \Rightarrow v_2 = \frac{v_1}{2} = 1.09 \times 10^6\ \text{m/s}.\quad h = 3 \Rightarrow v_3 = \frac{v_1}{3} = 7.27 \times 10^5\ \text{m/s}.$$

(b) Orbital period $= \frac{2\pi r_n}{v_n} = \frac{2\epsilon_0 n^2 h^2 / me^2}{1/\epsilon_0 \cdot e^2 / 2nh} = \frac{4\epsilon_0^2 n^3 h^3}{me^4}$

$$n = 1 \Rightarrow T_1 = \frac{4\epsilon_0^2 (6.63 \times 10^{-34}\ \text{J} \cdot \text{s})^3}{(9.11 \times 10^{-31}\ \text{kg})\,(1.60 \times 10^{-19}\ \text{C})^4} = 1.53 \times 10^{-16}\ \text{s}$$

$$n = 2:\ T_2 = T_1(2)^3 = 1.22 \times 10^{-15}\ \text{s}.\quad n = 3:\ T_3 = T_1(3)^3 = 4.13 \times 10^{-15}\ \text{s}.$$

(c) number of orbits $= \frac{1.0 \times 10^{-8} \text{s}}{1.22 \times 10^{-15}\ \text{s}} = 8.2 \times 10^6.$

38.28. **IDENTIFY** and **SET UP:** $E_n = -\frac{13.6\ \text{eV}}{n^2}$

EXECUTE: **(a)** $E_n = -\frac{13.6\ \text{eV}}{n^2}$ and $E_{n+1} = -\frac{13.6\ \text{eV}}{(n+1)^2}$

$\Delta E = E_{n+1} - E_n = (-13.6\ \text{eV})\left[\frac{1}{(n+1)^2} - \frac{1}{n^2}\right] = -(13.6\ \text{eV})\frac{n^2 - (n+1)^2}{(n^2)(n+1)^2}$

$\Delta E = (13.6\ \text{eV})\frac{2n+1}{(n^2)(n+1)^2}$ As n becomes large, $\Delta E \to (13.6\ \text{eV})\frac{2n}{n^4} = (13.6\ \text{eV})\frac{2}{n^3}$

Thus ΔE becomes small as n becomes large.

(b) $r_n = n^2 r_1$ so the orbits get farther apart in space as n increases.

38.29. **IDENTIFY** and **SET UP:** The number of photons emitted each second is the total energy emitted divided by the energy of one photon. The energy of one photon is given by Eq.(38.2). $E = Pt$ gives the energy emitted by the laser in time t.

EXECUTE: In 1.00 s the energy emitted by the laser is $(7.50\times10^{-3}\text{ W})(1.00\text{ s}) = 7.50\times10^{-3}\text{ J}$.

The energy of each photon is $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{10.6\times10^{-6}\text{ m}} = 1.874\times10^{-20}\text{ J}$.

Therefore $\dfrac{7.50\times10^{-3}\text{ J/s}}{1.874\times10^{-20}\text{ J/photon}} = 4.00\times10^{17}$ photons/s

EVALUATE: The number of photons emitted per second is extremely large.

38.30. **IDENTIFY** and **SET UP:** Visible light has wavelengths from about 400 nm to about 700 nm. The energy of each photon is $E = hf = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\text{ J}\cdot\text{m}}{\lambda}$. The power is the total energy per second and the total energy E_{tot} is the number of photons N times the energy E of each photon.

EXECUTE: **(a)** 193 nm is shorter than visible light so is in the ultraviolet.

(b) $E = \dfrac{hc}{\lambda} = 1.03\times10^{-18}\text{ J} = 6.44\text{ eV}$

(c) $P = \dfrac{E_{\text{tot}}}{t} = \dfrac{NE}{t}$ so $N = \dfrac{Pt}{E} = \dfrac{(1.50\times10^{-3}\text{ W})(12.0\times10^{-9}\text{ s})}{1.03\times10^{-18}\text{ J}} = 1.75\times10^{7}$ photons

EVALUATE: A very small amount of energy is delivered to the lens in each pulse, but this still corresponds to a large number of photons.

38.31. **IDENTIFY:** Apply Eq.(38.21): $\dfrac{n_{5s}}{n_{3p}} = e^{-(E_{5s}-E_{3p})/kT}$

SET UP: From Fig.38.24a in the textbook, $E_{5s} = 20.66\text{ eV}$ and $E_{3p} = 18.70\text{ eV}$

EXECUTE: $E_{5s} - E_{3p} = 20.66\text{ eV} - 18.70\text{ eV} = 1.96\text{ eV}(1.602\times10^{-19}\text{ J/1 eV}) = 3.140\times10^{-19}\text{ J}$

(a) $\dfrac{n_{5s}}{n_{3p}} = e^{-(3.140\times10^{-19}\text{ J})/[(1.38\times10^{-23}\text{ J/K})(300\text{ K})]} = e^{-75.79} = 1.2\times10^{-33}$

(b) $\dfrac{n_{5s}}{n_{3p}} = e^{-(3.140\times10^{-19}\text{ J})/[(1.38\times10^{-23}\text{ J/K})(600\text{ K})]} = e^{-37.90} = 3.5\times10^{-17}$

(c) $\dfrac{n_{5s}}{n_{3p}} = e^{-(3.140\times10^{-19}\text{ J})/[(1.38\times10^{-23}\text{ J/K})(1200\text{ K})]} = e^{-18.95} = 5.9\times10^{-9}$

(d) EVALUATE: At each of these temperatures the number of atoms in the 5*s* excited state, the initial state for the transition that emits 632.8 nm radiation, is quite small. The ratio increases as the temperature increases.

38.32. $\dfrac{n_{2P_{3/2}}}{n_{2P_{1/2}}} = e^{-(E_{2P3/2} - E_{2P1/2})/KT}$.

From the diagram $\Delta E_{3/2-\text{g}} = \dfrac{hc}{\lambda_1} = \dfrac{(6.626\times10^{-34}\text{ J})(3.000\times10^{8}\text{ m/s})}{5.890\times10^{-7}\text{ m}} = 3.375\times10^{-19}\text{ J}$.

$\Delta E_{1/2-\text{g}} = \dfrac{hc}{\lambda_2} = \dfrac{(6.626\times10^{-34}\text{ J})(3.000\times10^{8}\text{ m/s})}{5.896\times10^{-7}\text{ m}} = 3.371\times10^{-19}\text{ J}$. so $\Delta E_{3/2-1/2} = 3.375\times10^{-19}\text{ J} - 3.371\times10^{-19}\text{ J} =$

4.00×10^{-22} J. $\dfrac{n_{2P_{3/2}}}{n_{2P_{1/2}}} = e^{-(4.00\times10^{-22}\text{ J})/(1.38\times10^{-23}\text{ J/K}\cdot500\text{ K}).} = 0.944$. So more atoms are in the $2p_{1/2}$ state.

38.33. $eV_{\text{AC}} = hf_{\max} = \dfrac{hc}{\lambda_{\min}}$

$$\Rightarrow \lambda_{\min} = \frac{hc}{eV_{\text{AC}}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(4000\text{ V})} = 3.11\times10^{-10}\text{ m}$$

This is the same answer as would be obtained if electrons of this energy were used. Electron beams are much more easily produced and accelerated than proton beams.

38.34. **IDENTIFY** and **SET UP:** $\dfrac{hc}{\lambda} = eV$, where λ is the wavelength of the x ray and V is the accelerating voltage.

EXECUTE: **(a)** $V = \dfrac{hc}{e\lambda} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.150\times10^{-9}\ \text{m})} = 8.29\ \text{kV}$

(b) $\lambda = \dfrac{hc}{eV} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(30.0\times10^{3}\ \text{V})} = 4.14\times10^{-11}\ \text{m} = 0.0414\ \text{nm}$

(c) No. A proton has the same magnitude of charge as an electron and therefore gains the same amount of kinetic energy when accelerated by the same magnitude of potential difference.

38.35. **IDENTIFY:** The initial electrical potential energy of the accelerated electrons is converted to kinetic energy which is then given to a photon.

SET UP: The electrical potential energy of an electron is eV_{AC}, where V_{AC} is the accelerating potential, and the energy of a photon is hf. Since the energy of the electron is all given to a photon, we have $eV_{AC} = hf$. For any wave, $f\lambda = v$.

EXECUTE: **(a)** $eV_{AC} = hf_{min}$ gives

$$f_{min} = eV_{AC}/h = (1.60\times10^{-19}\ \text{C})(25{,}000\ \text{V})/(6.626\times10^{-34}\ \text{J}\cdot\text{s}) = 6.037\times10^{18}\ \text{Hz}$$

$$= 6.04\times10^{18}\ \text{Hz, rounded to three digits}$$

(b) $\lambda_{min} = c/f_{max} = (3.00\times10^{8}\ \text{m/s})/(6.037\times10^{18}\ \text{Hz}) = 4.97\times10^{-11}\ \text{m} = 0.0497\ \text{nm}$

(c) We assume that all the energy of the electron produces only one photon on impact with the screen.

EVALUATE: These photons are in the x-ray and γ-ray part of the electromagnetic spectrum (see Figure 32.4 in the textbook) and would be harmful to the eyes without protective glass on the screen to absorb them.

38.36. **IDENTIFY** and **SET UP:** The wavelength of the x rays produced by the tube is give by $\dfrac{hc}{\lambda} = eV$.

$\lambda' = \lambda + \dfrac{h}{mc}(1-\cos\phi)$. $\dfrac{h}{mc} = 2.426\times10^{-12}\ \text{m}$. The energy of the scattered x ray is $\dfrac{hc}{\lambda'}$.

EXECUTE: **(a)** $\lambda = \dfrac{hc}{eV} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(18.0\times10^{3}\ \text{V})} = 6.91\times10^{-11}\ \text{m} = 0.0691\ \text{nm}$

(b) $\lambda' = \lambda + \dfrac{h}{mc}(1-\cos\phi) = 6.91\times10^{-11}\ \text{m} + (2.426\times10^{-12}\ \text{m})(1-\cos45.0°)$.

$\lambda' = 6.98\times10^{-11}\ \text{m} = 0.0698\ \text{nm}$.

(c) $E = \dfrac{hc}{\lambda'} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{6.98\times10^{-11}\ \text{m}} = 17.8\ \text{keV}$

EVALUATE: The incident x ray has energy 18.0 keV. In the scattering event, the photon loses energy and its wavelength increases.

38.37. **IDENTIFY:** Apply Eq.(38.23): $\lambda' - \lambda = \dfrac{h}{mc}(1-\cos\phi) = \lambda_C(1-\cos\phi)$

SET UP: Solve for λ': $\lambda' = \lambda + \lambda_C(1-\cos\phi)$

The largest λ' corresponds to $\phi = 180°$, so $\cos\phi = -1$.

EXECUTE: $\lambda' = \lambda + 2\lambda_C = 0.0665\times10^{-9}\ \text{m} + 2(2.426\times10^{-12}\ \text{m}) = 7.135\times10^{-11}\ \text{m} = 0.0714\ \text{nm}$. This wavelength occurs at a scattering angle of $\phi = 180°$.

EVALUATE: The incident photon transfers some of its energy and momentum to the electron from which it scatters. Since the photon loses energy its wavelength increases, $\lambda' > \lambda$.

38.38. **(a)** From Eq. (38.23), $\cos\phi = 1 - \dfrac{\Delta\lambda}{(h/mc)}$, and so $\Delta\lambda = 0.0542\ \text{nm} - 0.0500\ \text{nm}$,

$\cos\phi = 1 - \dfrac{0.0042\ \text{nm}}{0.002426\ \text{nm}} = -0.731$, and $\phi = 137°$.

(b) $\Delta\lambda = 0.0521\ \text{nm} - 0.0500\ \text{nm}$. $\cos\phi = 1 - \dfrac{0.0021\ \text{nm}}{0.002426\ \text{nm}} = 0.134$. $\phi = 82.3°$.

(c) $\Delta\lambda = 0$, the photon is undeflected, $\cos\phi = 1$ and $\phi = 0$.

38.39. **IDENTIFY** and **SET UP:** The shift in wavelength of the photon is $\lambda' - \lambda = \dfrac{h}{mc}(1-\cos\phi)$ where λ' is the wavelength after the scattering and $\dfrac{h}{mc} = \lambda_c = 2.426\times10^{-12}\ \text{m}$. The energy of a photon of wavelength λ is

$E = \frac{hc}{\lambda} = \frac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{\lambda}$. Conservation of energy applies to the collision, so the energy lost by the photon equals the energy gained by the electron.

EXECUTE: **(a)** $\lambda' - \lambda = \lambda_c(1-\cos\phi) = (2.426\times10^{-12}\ \text{m})(1-\cos35.0°) = 4.39\times10^{-13}\ \text{m} = 4.39\times10^{-4}\ \text{nm}$

(b) $\lambda' = \lambda + 4.39\times10^{-4}\ \text{nm} = 0.04250\ \text{nm} + 4.39\times10^{-4}\ \text{nm} = 0.04294\ \text{nm}$

(c) $E_\lambda = \frac{hc}{\lambda} = 2.918\times10^4\ \text{eV}$ and $E_{\lambda'} = \frac{hc}{\lambda'} = 2.888\times10^4\ \text{eV}$ so the photon loses 300 eV of energy.

(d) Energy conservation says the electron gains 300 eV of energy.

38.40. The change in wavelength of the scattered photon is given by Eq. 38.23

$$\frac{\Delta\lambda}{\lambda} = \frac{h}{mc\lambda}(1-\cos\phi) \Rightarrow \lambda = \frac{h}{mc\left(\frac{\Delta\lambda}{\lambda}\right)}(1-\cos\phi).$$

Thus, $\lambda = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(1.67\times10^{-27}\ \text{kg})(3.00\times10^8\ \text{m/s})(0.100)}(1+1) = 2.65\times10^{-14}\ \text{m}.$

38.41. The derivation of Eq.(38.23) is explicitly shown in Equations (38.24) through (38.27) with the final substitution of $p' = h/\lambda'$ and $p = h/\lambda$ yielding $\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$.

38.42. From Eq. (38.30), (a) $\lambda_m = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{3.00\ \text{K}} = 0.966\ \text{mm}$, and $f = \frac{c}{\lambda_m} = 3.10\times10^{11}$ Hz. Note that a more precise value of the Wien displacement law constant has been used.

(b) A factor of 100 increase in the temperature lowers λ_m by a factor of 100 to 9.66 μm and raises the frequency by the same factor, to 3.10×10^{13} Hz.

(c) Similarly, $\lambda_m = 966\ \text{nm}$ and $f = 3.10\times10^{14}$ Hz.

38.43. **(a)** $H = Ae\sigma T^4$; $A = \pi r^2 l$

$$T = \left(\frac{H}{Ae\sigma}\right)^{1/4} = \left(\frac{100\ \text{W}}{2\pi(0.20\times10^{-3}\ \text{m})(0.30\ \text{m})(0.26)(5.671\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}\right)^{1/4}$$

$T = 2.06\times10^3$ K

(b) $\lambda_m T = 2.90\times10^{-3}\ \text{m}\cdot\text{K}$; $\lambda_m = 1410\ \text{nm}$

Much of the emitted radiation is in the infrared.

38.44. $T = \frac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{\lambda_m} = \frac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{400\times10^{-9}\ \text{m}} = 7.25\times10^3\ \text{K}.$

38.45. **IDENTIFY** and **SET UP:** The wavelength λ_m where the Planck distribution peaks is given by Eq.(38.30).

EXECUTE: $\lambda_m = \frac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{2.728\ \text{K}} = 1.06\times10^{-3}\ \text{m} = 1.06\ \text{mm}.$

EVALUATE: This wavelength is in the microwave portion of the electromagnetic spectrum. This radiation is often referred to as the "microwave background" (Section 44.7). Note that in Eq.(38.30), T must be in kelvins.

38.46. **IDENTIFY:** Since the stars radiate as blackbodies, they obey the Stefan-Boltzmann law and Wien's displacement law.

SET UP: The Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma AT^4$. Wien's displacement law tells us that the peak-intensity wavelength is λ_m = (constant)/T.

EXECUTE: **(a)** The hot and cool stars radiate the same total power, so the Stefan-Boltzmann law gives $\sigma A_h T_h^4 = \sigma A_c T_c^4 \Rightarrow 4\pi R_h^2 T_h^4 = 4\pi R_c^2 T_c^4 = 4\pi(3R_h)^2 T_c^4 \Rightarrow T_h^4 = 9T^4 \Rightarrow T_h = T\sqrt{3} = 1.7T$, rounded to two significant digits.

(b) Using Wien's law, we take the ratio of the wavelengths, giving

$$\frac{\lambda_m(\text{hot})}{\lambda_m(\text{cool})} = \frac{T_c}{T_h} = \frac{T}{T\sqrt{3}} = \frac{1}{\sqrt{3}} = 0.58,\ \text{rounded to two significant digits.}$$

EVALUATE: Although the hot star has only 1/9 the surface area of the cool star, its absolute temperature has to be only 1.7 times as great to radiate the same amount of energy.

38.47. **(a)** Let $\alpha = hc/kT$. To find the maximum in the Planck distribution:

$$\frac{dI}{d\lambda} = \frac{d}{d\lambda}\left(\frac{2\pi hc^2}{\lambda^5(e^{\alpha/\lambda}-1)}\right) = 0 = -5\frac{(2\pi hc^2)}{\lambda^5(e^{\alpha/\lambda}-1)} - \frac{2\pi hc^2(-\alpha/\lambda^2)}{\lambda^5(e^{\alpha/\lambda}-1)^2}$$

$\Rightarrow -5(e^{\alpha/\lambda}-1)\lambda = \alpha \Rightarrow -5e^{\alpha/\lambda} + 5 = \alpha/\lambda \Rightarrow$ Solve $5 - x = 5e^x$ where $x = \frac{\alpha}{\lambda} = \frac{hc}{\lambda kT}$.

Its root is 4.965, so $\frac{\alpha}{\lambda} = 4.965 \Rightarrow \lambda = \frac{hc}{(4.965)kT}$.

(b) $\lambda_{\text{m}}T = \frac{hc}{(4.965)k} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(4.965)(1.38\times10^{-23}\ \text{J/K})} = 2.90\times10^{-3}\ \text{m}\cdot\text{K}.$

38.48. **IDENTIFY:** Since the stars radiate as blackbodies, they obey the Stefan-Boltzmann law.
SET UP: The Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$.
EXECUTE: **(a)** $I = \sigma T^4 = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(24{,}000\ \text{K})^4 = 1.9\times10^{10}\ \text{W/m}^2$
(b) Wien's law gives $\lambda_{\text{m}} = (0.00290\ \text{m}\cdot\text{K})/(24{,}000\ \text{K}) = 1.2\times10^{-7}\ \text{m} = 120\ \text{nm}$
This is not visible since the wavelength is less than 400 nm.
(c) $P = AI \Rightarrow 4\pi R^2 = P/I = (1.00\times10^{25}\ \text{W})/(1.9\times10^{10}\ \text{W/m}^2)$
which gives $R_{\text{Sirius}} = 6.51\times10^6\ \text{m} = 6510\ \text{km}$.
$R_{\text{Sirius}}/R_{\text{sun}} = (6.51\times10^6\ \text{m})/(6.96\times10^9\ \text{m}) = 0.0093$, which gives

$$R_{\text{Sirius}} = 0.0093\ R_{\text{sun}} \approx 1\%\ R_{\text{sun}}$$

(d) Using the Stefan-Boltzmann law, we have

$$\frac{P_{\text{sun}}}{P_{\text{Sirius}}} = \frac{\sigma A_{\text{sun}}T_{\text{sun}}^4}{\sigma A_{\text{Sirius}}T_{\text{Sirius}}^4} = \frac{4\pi R_{\text{sun}}^2 T_{\text{sun}}^4}{4\pi R_{\text{Sirius}}^2 T_{\text{Sirius}}^4} = \left(\frac{R_{\text{sun}}}{R_{\text{Sirius}}}\right)^2\left(\frac{T_{\text{sun}}}{T_{\text{Sirius}}}\right)^4 \cdot \frac{P_{\text{sun}}}{P_{\text{Sirius}}} = \left(\frac{R_{\text{sun}}}{0.00935R_{\text{sun}}}\right)^2\left(\frac{5800\ \text{K}}{24{,}000\ \text{K}}\right)^4 = 39$$

EVALUATE: Even though the absolute surface temperature of Sirius B is about 4 times that of our sun, it radiates only 1/39 times as much energy per second as our sun because it is so small.

38.49. Eq. (38.32): $I(\lambda) = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)}$ but $e^x = 1 + x + \frac{x^2}{2} + \cdots \approx 1 + x$ for

$x \ll 1 \Rightarrow I(\lambda) \approx \frac{2\pi hc^2}{\lambda^5(hc/\lambda kT)} = \frac{2\pi ckT}{\lambda^4} =$ Eq. (38.31), which is Rayleigh's distribution.

38.50. **(a)** Wien's law: $\lambda_{\text{m}} = \frac{k}{T}$. $\lambda_{\text{m}} = \frac{2.90\times10^{-3}\ \text{K}\cdot\text{m}}{30{,}000\ \text{K}} = 9.7\times10^{-8}\text{m} = 97\ \text{nm}$

This peak is in the ultraviolet region, which is *not* visible. The star is blue because the largest part of the visible light radiated is in the blue/violet part of the visible spectrum

(b) $P = \sigma A T^4$ (Stefan-Boltzmann law)

$$(100{,}000)(3.86\times10^{26}\ \text{W}) = \left(5.67\times10^{-8}\frac{\text{W}}{\text{m}^2\text{K}^4}\right)(4\pi R^2)(30{,}000\ \text{K})^4$$

$$R = 8.2\times10^9\ \text{m}$$

$$R_{\text{star}}/R_{\text{sun}} = \frac{8.2\times10^9\ \text{m}}{6.96\times10^8\ \text{m}} = 12$$

(c) The visual luminosity is proportional to the power radiated at visible wavelengths. Much of the power is radiated nonvisible wavelengths, which does not contribute to the visible luminosity.

38.51. **IDENTIFY** and **SET UP:** Use $c = f\lambda$ to relate frequency and wavelength and use $E = hf$ to relate photon energy and frequency.
EXECUTE: **(a)** One photon dissociates one AgBr molecule, so we need to find the energy required to dissociate a single molecule. The problem states that it requires 1.00×10^5 J to dissociate one mole of AgBr, and one mole contains Avogadro's number (6.02×10^{23}) of molecules, so the energy required to dissociate one AgBr is

$$\frac{1.00\times10^5\ \text{J/mol}}{6.02\times10^{23}\ \text{molecules/mol}} = 1.66\times10^{-19}\ \text{J/molecule}.$$

The photon is to have this energy, so $E = 1.66\times10^{-19}\ \text{J}(1\text{eV}/1.602\times10^{-19}\ \text{J}) = 1.04\ \text{eV}$.

(b) $E = \frac{hc}{\lambda}$ so $\lambda = \frac{hc}{E} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{1.66\times10^{-19}\ \text{J}} = 1.20\times10^{-6}\ \text{m} = 1200\ \text{nm}$

(c) $c = f\lambda$ so $f = \frac{c}{\lambda} = \frac{2.998\times10^8\ \text{m/s}}{1.20\times10^{-6}\ \text{m}} = 2.50\times10^{14}\ \text{Hz}$

(d) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(100\times10^6\ \text{Hz}) = 6.63\times10^{-26}\ \text{J}$

$E = 6.63\times10^{-26}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 4.14\times10^{-7}\ \text{eV}$

(e) EVALUATE: A photon with frequency $f = 100$ MHz has too little energy, by a large factor, to dissociate a AgBr molecule. The photons in the visible light from a firefly do individually have enough energy to dissociate AgBr. The huge number of 100 MHz photons can't compensate for the fact that individually they have too little energy.

38.52. **(a)** Assume a non-relativistic velocity and conserve momentum $\Rightarrow mv = \dfrac{h}{\lambda} \Rightarrow v = \dfrac{h}{m\lambda}$.

(b) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\left(\dfrac{h}{m\lambda}\right)^2 = \dfrac{h^2}{2m\lambda^2}$.

(c) $\dfrac{K}{E} = \dfrac{h^2}{2m\lambda^2}\cdot\dfrac{\lambda}{hc} = \dfrac{h}{2mc\lambda}$. Recoil becomes an important concern for small m and small λ since this ratio becomes large in those limits.

(d) $E = 10.2\text{ eV} \Rightarrow \lambda = \dfrac{hc}{E} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(10.2\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 1.22\times10^{-7}\text{ m} = 122\text{ nm}.$

$$K = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{2(1.67\times10^{-27}\text{ kg})(1.22\times10^{-7}\text{ m})^2} = 8.84\times10^{-27}\text{ J} = 5.53\times10^{-8}\text{ eV}.$$

$\dfrac{K}{E} = \dfrac{5.53\times10^{-8}\text{ eV}}{10.2\text{ eV}} = 5.42\times10^{-9}$. This is quite small so recoil can be neglected.

38.53. **IDENTIFY** and **SET UP:** $f = \dfrac{c}{\lambda}$. The (f, V_0) values are: $(8.20\times10^{14}\text{ Hz, }1.48\text{ V})$, $(7.41\times10^{14}\text{ Hz, }1.15\text{ V})$, $(6.88\times10^{14}\text{ Hz, }0.93\text{ V})$, $(6.10\times10^{14}\text{ Hz, }0.62\text{ V})$, $(5.49\times10^{14}\text{ Hz, }0.36\text{ V})$, $(5.18\times10^{14}\text{ Hz, }0.24\text{ V})$. The graph of V_0 versus f is given in Figure 38.53.

EXECUTE: **(a)** The threshold frequency, f_{th}, is f where $V_0 = 0$. From the graph this is $f_{\text{th}} = 4.56\times10^{14}$ Hz.

(b) $\lambda_{\text{th}} = \dfrac{c}{f_{\text{th}}} = \dfrac{3.00\times10^{8}\text{ m/s}}{4.56\times10^{14}\text{ Hz}} = 658\text{ nm}$

(c) $\phi = hf_{\text{th}} = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(4.56\times10^{14}\text{ Hz}) = 1.89\text{ eV}$

(d) $eV_0 = hf - \phi$ so $V_0 = \left(\dfrac{h}{e}\right)f - \phi$. The slope of the graph is $\dfrac{h}{e}$.

$\dfrac{h}{e} = \left(\dfrac{1.48\text{ V} - 0.24\text{ V}}{8.20\times10^{14}\text{ Hz} - 5.18\times10^{14}\text{ Hz}}\right) = 4.11\times10^{-15}$ V/Hz and

$h = (4.11\times10^{-15}\text{ V/Hz})(1.60\times10^{-19}\text{ C}) = 6.58\times10^{-34}\text{ J}\cdot\text{s}$.

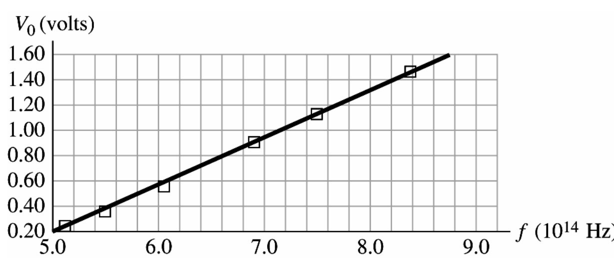


Figure 38.53

38.54. **(a)** $\dfrac{dN}{dt} = \dfrac{(dE/dt)}{(dE/dN)} = \dfrac{P}{hf} = \dfrac{(200\text{ W})(0.10)}{h(5.00\times10^{14}\text{ Hz})} = 6.03\times10^{19}$ photons/sec.

(b) Demand $\dfrac{(dN/dt)}{4\pi r^2} = 1.00\times10^{11}$ photons/sec $\cdot$ cm^2.

Therefore, $r = \left(\dfrac{6.03\times10^{19}\text{ photons/sec}}{4\pi(1.00\times10^{11}\text{ photons/sec}\cdot\text{cm}^2)}\right)^{1/2} = 6930\text{ cm} = 69.3\text{ m}.$

38.55. **(a) IDENTIFY:** Apply the photoelectric effect equation, Eq.(38.4).

SET UP: $eV_0 = hf - \phi = (hc/\lambda) - \phi$. Call the stopping potential V_{01} for λ_1 and V_{02} for λ_2. Thus $eV_{01} = (hc/\lambda_1) - \phi$ and $eV_{02} = (hc/\lambda_2) - \phi$. Note that the work function ϕ is a property of the material and is independent of the wavelength of the light.

EXECUTE: Subtracting one equation from the other gives $e(V_{02} - V_{01}) = hc\left(\dfrac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right)$.

(b) $\Delta V_0 = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{1.602\times10^{-19}\ \text{C}}\left(\dfrac{295\times10^{-9}\ \text{m}-265\times10^{-9}\ \text{m}}{(295\times10^{-9}\ \text{m})(265\times10^{-9}\ \text{m})}\right)=0.476\ \text{V}.$

EVALUATE: $e\Delta V_0$, which is 0.476 eV, is the increase in photon energy from 295 nm to 265 nm. The stopping potential increases when λ deceases because the photon energy increases when the wavelength decreases.

38.56. **IDENTIFY:** The photoelectric effect occurs, so the energy of the photon is used to eject an electron, with any excess energy going into kinetic energy of the electron.

SET UP: Conservation of energy gives $hf = hc/\lambda = K_{max} + \phi$.

EXECUTE: **(a)** Using $hc/\lambda = K_{max} + \phi$, we solve for the work function:

$$\phi = hc/\lambda - K_{max} = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})/(124\ \text{nm}) - 4.16\ \text{eV} = 5.85\ \text{eV}$$

(b) The number N of photoelectrons per second is equal to the number of photons per second that strike the metal per second. $N\times$ (energy of a photon) = 2.50 W. $N(hc/\lambda)$ = 2.50 W.

$N = (2.50\ \text{W})(124\ \text{nm})/[(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})] = 1.56\times10^{18}$ electrons/s

(c) N is proportional to the power, so if the power is cut in half, so is N, which gives

$$N = (1.56\times10^{18}\ \text{el/s})/2 = 7.80\times10^{17}\ \text{el/s}$$

(d) If we cut the wavelength by half, the energy of each photon is doubled since $E = hc/\lambda$. To maintain the same power, the number of photons must be half of what they were in part (b), so N is cut in half to 7.80×10^{17} el/s. We could also see this from part (b), where N is proportional to λ. So if the wavelength is cut in half, so is N.

EVALUATE: In part (c), reducing the power does not reduce the maximum kinetic energy of the photons; it only reduces the number of ejected electrons. In part (d), reducing the wavelength *does* change the maximum kinetic energy of the photoelectrons because we have increased the energy of each photon.

38.57. **IDENTIFY** and **SET UP:** The energy added to mass m of the blood to heat it to $T_f = 100\ °\text{C}$ and to vaporize it is $Q = mc(T_f - T_i) + mL_v$, with $c = 4190\ \text{J/kg}\cdot\text{K}$ and $L_v = 2.256\times10^{6}\ \text{J/kg}$. The energy of one photon is $E = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$.

EXECUTE: **(a)** $Q = (2.0\times10^{-9}\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(100°\text{C} - 33°\text{C}) + (2.0\times10^{-9}\ \text{kg})(2.256\times10^{6}\ \text{J/kg}) = 5.07\times10^{-3}\ \text{J}$

The pulse must deliver 5.07 mJ of energy.

(b) $P = \dfrac{\text{energy}}{t} = \dfrac{5.07\times10^{-3}\ \text{J}}{450\times10^{-6}\ \text{s}} = 11.3\ \text{W}$

(c) One photon has energy $E = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{585\times10^{-9}\ \text{m}} = 3.40\times10^{-19}\ \text{J}$. The number N of photons per pulse is the energy per pulse divided by the energy of one photon: $N = \dfrac{5.07\times10^{-3}\ \text{J}}{3.40\times10^{-19}\ \text{J/photon}} = 1.49\times10^{16}$ photons

38.58. **(a)** $\lambda_0 = \dfrac{hc}{E}$, and the wavelengths are: cesium: 590 nm, copper: 264 nm, potassium: 539 nm, zinc: 288 nm.

b) The wavelengths of copper and zinc are in the ultraviolet, and visible light is not energetic enough to overcome the threshold energy of these metals.

38.59. **(a) IDENTIFY** and **SET UP:** Apply Eq.(38.20): $m_r = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{207 m_e m_p}{207 m_e + m_p}$

EXECUTE: $m_r = \dfrac{207(9.109\times10^{-31}\ \text{kg})(1.673\times10^{-27}\ \text{kg})}{207(9.109\times10^{-31}\ \text{kg}) + 1.673\times10^{-27}\ \text{kg}} = 1.69\times10^{-28}\ \text{kg}$

We have used m_e to denote the electron mass.

(b) IDENTIFY: In Eq.(38.18) replace $m = m_e$ by m_r: $E_n = -\dfrac{1}{\epsilon_0^2}\dfrac{m_r e^4}{8n^2h^2}$.

SET UP: Write as $E_n = \left(\dfrac{m_r}{m_H}\right)\left(-\dfrac{1}{\epsilon_0^2}\dfrac{m_H e^4}{8n^2h^2}\right)$, since we know that $\dfrac{1}{\epsilon_0^2}\dfrac{m_H e^4}{8h^2} = 13.60\ \text{eV}$. Here m_H denotes the reduced mass for the hydrogen atom; $m_H = 0.99946(9.109\times10^{-31}\ \text{kg}) = 9.104\times10^{-31}\ \text{kg}$.

EXECUTE: $E_n = \left(\dfrac{m_r}{m_H}\right)\left(-\dfrac{13.60\ \text{eV}}{n^2}\right)$

$$E_1 = \frac{1.69\times10^{-28}\ \text{kg}}{9.104\times10^{-31}\ \text{kg}}(-13.60\ \text{eV}) = 186(-13.60\ \text{eV}) = -2.53\ \text{keV}$$

(c) SET UP: From part (b), $E_n = \left(\frac{m_r}{m_H}\right)\left(-\frac{R_H ch}{n^2}\right)$, where $R_H = 1.097\times10^7\ \text{m}^{-1}$ is the Rydberg constant for the hydrogen atom. Use this result in $\frac{hc}{\lambda} = E_i - E_f$ to find an expression for $1/\lambda$. The initial level for the transition is the $n_i = 2$ level and the final level is the $n_f = 1$ level.

EXECUTE: $\frac{hc}{\lambda} = \frac{m_r}{m_H}\left(-\frac{R_H ch}{n_i^2} - \left(-\frac{R_H ch}{n_f^2}\right)\right)$

$$\frac{1}{\lambda} = \frac{m_r}{m_H} R_H \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\frac{1}{\lambda} = \frac{1.69\times10^{-28}\ \text{kg}}{9.104\times10^{-31}\ \text{kg}}(1.097\times10^7\ \text{m}^{-1})\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 1.527\times10^9\ \text{m}^{-1}$$

$\lambda = 0.655$ nm

EVALUATE: From Example 38.6 the wavelength of the radiation emitted in this transition in hydrogen is 122 nm. The wavelength for muonium is $\frac{m_H}{m_r} = 5.39\times10^{-3}$ times this. The reduced mass for hydrogen is very close to the electron mass because the electron mass is much less then the proton mass: $m_p/m_e = 1836$. The muon mass is $207m_e = 1.886\times10^{-28}$ kg. The proton is only about 10 times more massive than the muon, so the reduced mass is somewhat smaller than the muon mass. The muon-proton atom has much more strongly bound energy levels and much shorter wavelengths in its spectrum than for hydrogen.

38.60. **(a)** The change in wavelength of the scattered photon is given by Eq. 38.23

$$\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi) \Rightarrow \lambda = \lambda' - \frac{h}{mc}(1-\cos\phi) =$$

$$(0.0830\times10^{-9}\ \text{m}) - \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(9.11\times10^{-31}\ \text{kg})(3.00\times10^8\ \text{m/s})}(1+1) = 0.0781\ \text{nm}.$$

(b) Since the collision is one-dimensional, the magnitude of the electron's momentum must be equal to the magnitude of the change in the photon's momentum. Thus,

$$p_e = h\left(\frac{1}{\lambda} - \frac{-1}{\lambda'}\right) = (6.63\times10^{-34}\ \text{J}\cdot\text{s})\left(\frac{1}{0.0781} + \frac{1}{0.0830}\right)(10^9\ \text{m}^{-1})$$
$$= 1.65\times10^{-23}\ \text{kg}\cdot\text{m/s} \approx 2\times10^{-23}\ \text{kg}\cdot\text{m/s}.$$

(c) Since the electron is non relativistic $(\beta = 0.06)$, $K_e = \frac{p_e^2}{2m} = 1.49\times10^{-16}\ \text{J} \approx 10^{-16}$ J.

38.61. **IDENTIFY** and **SET UP:** $\lambda' = \lambda + \frac{h}{mc}(1-\cos\phi)$

$\phi = 180°$ so $\lambda' = \lambda + \frac{2h}{mc} = 0.09485$ m. Use Eq.(38.5) to calculate the momentum of the scattered photon. Apply conservation of energy to the collision to calculate the kinetic energy of the electron after the scattering. The energy of the photon is given by Eq.(38.2),

EXECUTE: **(a)** $p' = h/\lambda' = 6.99\times10^{-24}\ \text{kg}\cdot\text{m/s}$.

(b) $E = E' + E_e$; $hc/\lambda = hc/\lambda' + E_e$

$$E_e = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = (hc)\frac{\lambda' - \lambda}{\lambda\lambda'} = 1.129\times10^{-16}\ \text{J} = 705\ \text{eV}$$

EVALUATE: The energy of the incident photon is 13.8 keV, so only about 5% of its energy is transferred to the electron. This corresponds to a fractional shift in the photon's wavelength that is also 5%.

38.62. **(a)** $\phi = 180°$ so $(1-\cos\phi) = 2 \Rightarrow \Delta\lambda = \frac{2h}{mc} = 0.0049$ nm, so $\lambda' = 0.1849$ nm.

(b) $\Delta E = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = 2.93\times10^{-17}\ \text{J} = 183\ \text{eV}$. This will be the kinetic energy of the electron.

(c) The kinetic energy is far less than the rest mass energy, so a non-relativistic calculation is adequate; $v = \sqrt{2K/m} = 8.02\times10^6$ m/s.

38.63. **IDENTIFY** and **SET UP:** The H_α line in the Balmer series corresponds to the $n=3$ to $n=2$ transition.

$E_n = -\dfrac{13.6\text{ eV}}{n^2}$. $\dfrac{hc}{\lambda} = \Delta E$.

EXECUTE: **(a)** The atom must be given an amount of energy $E_3 - E_1 = -(13.6\text{ eV})\left(\dfrac{1}{3^2} - \dfrac{1}{1^2}\right) = 12.1\text{ eV}$.

(b) There are three possible transitions. $n=3 \to n=1$: $\Delta E = 12.1\text{ eV}$ and $\lambda = \dfrac{hc}{\Delta E} = 103\text{ nm}$;

$n=3 \to n=2$: $\Delta E = -(13.6\text{ eV})\left(\dfrac{1}{3^2} - \dfrac{1}{2^2}\right) = 1.89\text{ eV}$ and $\lambda = 657\text{ nm}$; $n=2 \to n=1$:

$\Delta E = -(13.6\text{ eV})\left(\dfrac{1}{2^2} - \dfrac{1}{1^2}\right) = 10.2\text{ eV}$ and $\lambda = 122\text{ nm}$.

38.64. $\dfrac{n_2}{n_1} = e^{-(E_{\text{ex}} - E_{\text{g}})/kT} \Rightarrow T = \dfrac{-(E_{\text{ex}} - E_{\text{g}})}{k\ln(n_2/n_1)}$.

$$E_{\text{ex}} = E_2 = \frac{-13.6\text{ eV}}{4} = -3.4\text{ eV}.\ E_{\text{g}} = -13.6\text{ eV}.\ E_{\text{ex}} - E_{\text{g}} = 10.2\text{ eV} = 1.63\times10^{-18}\text{ J}.$$

(a) $\dfrac{n_2}{n_1} = 10^{-12}$. $T = \dfrac{-(1.63\times10^{-18}\text{ J})}{(1.38\times10^{-23}\text{ J/K})\ln(10^{-12})} = 4275\text{ K}$.

(b) $\dfrac{n_2}{n_1} = 10^{-8}$. $T = \dfrac{-(1.63\times10^{-18}\text{ J})}{(1.38\times10^{-23}\text{ J/K})\ln(10^{-8})} = 6412\text{ K}$.

(c) $\dfrac{n_2}{n_1} = 10^{-4}$. $T = \dfrac{-(1.63\times10^{-18}\text{ J})}{(1.38\times10^{-23}\text{ J/K})\ln(10^{-4})} = 12824\text{ K}$.

(d) For absorption to take place in the Balmer series, hydrogen must *start* in the $n=2$ state. From part (a), colder stars have fewer atoms in this state leading to weaker absorption lines.

38.65. **(a) IDENTIFY** and **SET UP:** The photon energy is given to the electron in the atom. Some of this energy overcomes the binding energy of the atom and what is left appears as kinetic energy of the free electron. Apply $hf = E_{\text{f}} - E_{\text{i}}$, the energy given to the electron in the atom when a photon is absorbed.

EXECUTE: The energy of one photon is $\dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{85.5\times10^{-9}\text{ m}}$

$\dfrac{hc}{\lambda} = 2.323\times10^{-18}\text{ J}(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 14.50\text{ eV}$.

The final energy of the electron is $E_{\text{f}} = E_{\text{i}} + hf$. In the ground state of the hydrogen atom the energy of the electron is $E_{\text{i}} = -13.60\text{ eV}$. Thus $E_{\text{f}} = -13.60\text{ eV} + 14.50\text{ eV} = 0.90\text{ eV}$.

(b) EVALUATE: At thermal equilibrium a few atoms will be in the $n=2$ excited levels, which have an energy of $-13.6\text{ eV}/4 = -3.40\text{ eV}$, 10.2 eV greater than the energy of the ground state. If an electron with $E = -3.40\text{ eV}$ gains 14.5 eV from the absorbed photon, it will end up with $14.5\text{ eV} - 3.4\text{ eV} = 11.1\text{ eV}$ of kinetic energy.

38.66. **IDENTIFY:** The diffraction grating allows us to determine the peak-intensity wavelength of the light. Then Wien's displacement law allows us to calculate the temperature of the blackbody, and the Stefan-Boltzmann law allows us to calculate the rate at which it radiates energy.

SET UP: The bright spots for a diffraction grating occur when $d \sin\theta = m\lambda$. Wien's displacement law is $\lambda_{\text{peak}} = \dfrac{2.90\times10^{-3}\text{ m}\cdot\text{K}}{T}$, and the Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$.

EXECUTE: **(a)** First find the wavelength of the light:

$$\lambda = d\sin\theta = [1/(385{,}000\text{ lines/m})]\sin(11.6°) = 5.22\times10^{-7}\text{ m}$$

Now use Wien's law to find the temperature: $T = (2.90\times10^{-3}\text{ m}\cdot\text{K})/(5.22\times10^{-7}\text{ m}) = 5550\text{ K}$.

(b) The energy radiated by the blackbody is equal to the power times the time, giving $U = Pt = IAt = \sigma A T^4 t$, which gives

$t = U/(\sigma A T^4) = (12.0\times10^{6}\text{ J})/[(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(4\pi)(0.0750\text{ m})^2(5550\text{ K})^4] = 3.16\text{ s}$.

EVALUATE: By ordinary standards, this blackbody is very hot, so it does not take long to radiate 12.0 MJ of energy.

38.67. **IDENTIFY:** Assuming that Betelgeuse radiates like a perfect blackbody, Wien's displacement and the Stefan-Boltzmann law apply to its radiation.

SET UP: Wien's displacement law is $\lambda_{\text{peak}} = \dfrac{2.90\times10^{-3}\,\text{m}\cdot\text{K}}{T}$, and the Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$.

EXECUTE: **(a)** First use Wien's law to find the peak wavelength:

$$\lambda_\text{m} = (2.90\times10^{-3}\ \text{m}\cdot\text{K})/(3000\ \text{K}) = 9.667\times10^{-7}\ \text{m}$$

Call N the number osf photons/second radiated. $N\times$ (energy per photon) $= IA = \sigma AT^4$.

$N(hc/\lambda_\text{m}) = \sigma AT^4$. $N = \dfrac{\lambda_\text{m}\sigma AT^4}{hc}$.

$$N = \frac{(9.667\times10^{-7}\ \text{m})(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(4\pi)(600\times6.96\times10^8\ \text{m})^2(3000\,\text{K})^4}{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}.$$

$N = 5\times10^{49}$ photons/s.

(b) $\dfrac{I_\text{B}A_\text{B}}{I_\text{S}A_\text{S}} = \dfrac{\sigma A_\text{B}T_\text{B}^4}{\sigma A_\text{S}T_\text{S}^4} = \dfrac{4\pi R_\text{B}^2T_\text{B}^4}{4\pi R_\text{S}^2T_\text{S}^4} = \left(\dfrac{600R_\text{S}}{R_\text{S}}\right)^2\left(\dfrac{3000\ \text{K}}{5800\ \text{K}}\right)^4 = 3\times10^4$

EVALUATE: Betelgeuse radiates 30,000 times as much energy per second as does our sun!

38.68. **IDENTIFY:** The blackbody radiates heat into the water, but the water also radiates heat back into the blackbody. The net heat entering the water causes evaporation. Wien's law tells us the peak wavelength radiated, but a thermophile in the water measures the wavelength and frequency of the light in the water.

SET UP: By the Stefan-Boltzman law, the net power radiated by the blackbody is $\dfrac{dQ}{dt} = \sigma A\left(T_\text{sphere}^4 - T_\text{water}^4\right)$. Since this heat evaporates water, the rate at which water evaporates is $\dfrac{dQ}{dt} = L_v\dfrac{dm}{dt}$. Wien's displacement law is $\lambda_\text{m} = \dfrac{2.90\times10^{-3}\,\text{m}\cdot\text{K}}{T}$, and the wavelength in the water is $\lambda_\text{w} = \lambda_0/n$.

EXECUTE: (a) The net radiated heat is $\dfrac{dQ}{dt} = \sigma A\left(T_\text{sphere}^4 - T_\text{water}^4\right)$ and the evaporation rate is $\dfrac{dQ}{dt} = L\text{v}\dfrac{dm}{dt}$, where dm is the mass of water that evaporates in time dt. Equating these two rates gives $L_\text{v}\dfrac{dm}{dt} = \sigma A\left(T_\text{sphere}^4 - T_\text{water}^4\right)$.

$$\frac{dm}{dt} = \frac{\sigma\left(4\pi R^2\right)\left(T_\text{sphere}^4 - T_\text{water}^4\right)}{L_\text{v}}.$$

$$\frac{dm}{dt} = \frac{\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)(4\pi)(0.120\,\text{m})^2\left[(498\,\text{K})^4 - (373\,\text{K})^4\right]}{2256\times10^3\ \text{J/Kg}} = 1.92\times10^{-4}\ \text{kg/s} = 0.193\ \text{g/s}$$

(b) (i) Wien's law gives $\lambda_\text{m} = (0.00290\ \text{m}\cdot\text{K})/(498\ \text{K}) = 5.82\times10^{-6}$ m

But this would be the wavelength in vacuum. In the water the thermophile organism would measure $\lambda_\text{w} = \lambda_0/n = (5.82\times10^{-6}\ \text{m})/1.333 = 4.37\times10^{-6}\ \text{m} = 4.37\ \mu\text{m}$

(ii) The frequency is the same as if the wave were in air, so

$$f = c/\lambda_0 = (3.00\times10^8\ \text{m/s})/(5.82\times10^{-6}\ \text{m}) = 5.15\times10^{13}\ \text{Hz}$$

EVALUATE: An alternative way is to use the quantities in the water: $f = \dfrac{c/n}{\lambda_0/n} = c/\lambda_0$, which gives the same answer for the frequency. An organism in the water would measure the light coming to it through the water, so the wavelength it would measure would be reduced by a factor of $1/n$.

38.69. **IDENTIFY:** The energy of the peak-intensity photons must be equal to the energy difference between the $n = 1$ and the $n = 4$ states. Wien's law allows us to calculate what the temperature of the blackbody must be for it to radiate with its peak intensity at this wavelength.

SET UP: In the Bohr model, the energy of an electron in shell n is $E_n = -\dfrac{13.6\ \text{eV}}{n^2}$, and Wien's displacement law is $\lambda_\text{m} = \dfrac{2.90\times10^{-3}\,\text{m}\cdot\text{K}}{T}$. The energy of a photon is $E = hf = hc/\lambda$.

EXECUTE: First find the energy (ΔE) that a photon would need to excite the atom. The ground state of the atom is $n = 1$ and the third excited state is $n = 4$. This energy is the *difference* between the two energy levels. Therefore

$\Delta E = (-13.6\ \text{eV})\left(\frac{1}{4^2}-\frac{1}{1^2}\right) = 12.8$ eV. Now find the wavelength of the photon having this amount of energy.

$hc/\lambda = 12.8$ eV and

$$\lambda = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})/(12.8\ \text{eV}) = 9.73\times10^{-8}\ \text{m}$$

Now use Wien's law to find the temperature. $T = (0.00290\ \text{m}\cdot\text{K})/(9.73\times10^{-8}\ \text{m}) = 2.98\times10^{4}$ K.

EVALUATE: This temperature is well above ordinary room temperatures, which is why hydrogen atoms are not in excited states during everyday conditions.

38.70. IDENTIFY and **SET UP:** Electrical power is *VI*. $Q = mc\Delta T$.

EXECUTE: **(a)** $(0.010)VI = (0.010)(18.0\times10^{3}\ \text{V})(60.0\times10^{-3}\ \text{A}) = 10.8\ \text{W} = 10.8\ \text{J/s}$

(b) The energy in the electron beam that isn't converted to x rays stays in the target and appears as thermal energy. For $t = 1.00$ s, $Q = (0.990)VI(1.00\ \text{s}) = 1.07\times10^{3}$ J and $\Delta T = \frac{Q}{mc} = \frac{1.07\times10^{3}\ \text{J}}{(0.250\ \text{kg})(130\ \text{J/kg}\cdot\text{K})} = 32.9\ \text{K}$. The temperature rises at a rate of 32.9 K/s.

EVALUATE: The target must be made of a material that has a high melting point.

38.71. IDENTIFY: Apply conservation of energy and conservation of linear momentum to the system of atom plus photon.

(a) SET UP: Let E_{tr} be the transition energy, E_{ph} be the energy of the photon with wavelength λ', and E_{r} be the kinetic energy of the recoiling atom. Conservation of energy gives $E_{\text{ph}} + E_{\text{r}} = E_{\text{tr}}$.

$E_{\text{ph}} = \frac{hc}{\lambda'}$ so $\frac{hc}{\lambda'} = E_{\text{tr}} - E_{\text{r}}$ and $\lambda' = \frac{hc}{E_{\text{tr}} - E_{\text{r}}}$.

EXECUTE: If the recoil energy is neglected then the photon wavelength is $\lambda = hc/E_{\text{tr}}$.

$$\Delta\lambda = \lambda' - \lambda = hc\left(\frac{1}{E_{\text{tr}} - E_{\text{r}}} - \frac{1}{E_{\text{tr}}}\right) = \left(\frac{hc}{E_{\text{tr}}}\right)\left(\frac{1}{1 - E_{\text{r}}/E_{\text{tr}}} - 1\right)$$

$$\frac{1}{1 - E_{\text{r}}/E_{\text{tr}}} = \left(1 - \frac{E_{\text{r}}}{\text{E}_{\text{tr}}}\right)^{-1} \approx 1 + \frac{E_{\text{r}}}{E_{\text{tr}}} \text{ since } \frac{E_{\text{r}}}{E_{\text{tr}}} \ll 1$$

(We have used the binomial theorem, Appendix B.)

Thus $\Delta\lambda = \frac{hc}{E_{\text{tr}}}\left(\frac{E_{\text{r}}}{E_{\text{tr}}}\right)$, or since $E_{\text{tr}} = hc/\lambda$, $\Delta\lambda = \left(\frac{E_{\text{r}}}{hc}\right)\lambda^2$.

SET UP: Use conservation of linear momentum to find E_{r}: Assuming that the atom is initially at rest, the momentum p_{r} of the recoiling atom must be equal in magnitude and opposite in direction to the momentum $p_{\text{ph}} = h/\lambda$ of the emitted photon: $h/\lambda = p_{\text{r}}$.

EXECUTE: $E_{\text{r}} = \frac{p_{\text{r}}^2}{2m}$, where *m* is the mass of the atom, so $E_{\text{r}} = \frac{h^2}{2m\lambda^2}$.

Use this result in the above equation: $\Delta\lambda = \left(\frac{E_{\text{r}}}{hc}\right)\lambda^2 = \left(\frac{h^2}{2m\lambda^2}\right)\left(\frac{\lambda^2}{hc}\right) = \frac{h}{2mc}$;

note that this result for $\Delta\lambda$ is independent of the atomic transition energy.

(b) For a hydrogen atom $m = m_{\text{p}}$ and $\Delta\lambda = \frac{h}{2m_{\text{p}}c} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{2(1.673\times10^{-27}\ \text{kg})(2.998\times10^{8}\ \text{m/s})} = 6.61\times10^{-16}\ \text{m}$

EVALUATE: The correction is independent of *n*. The wavelengths of photons emitted in hydrogen atom transitions are on the order of $100\ \text{nm} = 10^{-7}$ m, so the recoil correction is exceedingly small.

38.72. (a) $\Delta\lambda_1 = (h/mc)(1-\cos\theta_1)$, $\Delta\lambda_2 = (h/mc)(1-\cos\theta_2)$, and so the overall wavelength shift is $\Delta\lambda = (h/mc)(2 - \cos\theta_1 - \cos\theta_2)$.

(b) For a single scattering through angle θ, $\Delta\lambda_{\text{s}} = (h/mc)(1-\cos\theta)$. For two successive scatterings through an angle of $\theta/2$ for each scattering,

$$\Delta\lambda_{\text{t}} = 2(h/mc)(1-\cos\theta/2).$$

$$1-\cos\theta = 2(1-\cos^2(\theta/2)) \text{ and } \Delta\lambda_{\text{s}} = (h/mc)2(1-\cos^2(\theta/2))$$

$$\cos(\theta/2) \le 1 \text{ so } 1-\cos^2(\theta/2) \ge (1-\cos(\theta/2)) \text{ and } \Delta\lambda_{\text{s}} \ge \Delta\lambda_{\text{t}}$$

Equality holds only when $\theta = 180°$.

(c) $(h/mc)2(1-\cos 30.0°) = 0.268(h/mc)$.

(d) $(h/mc)(1-\cos 60°) = 0.500(h/mc)$, which is indeed greater than the shift found in part (c).

38.73. **IDENTIFY** and **SET UP:** Find the average change in wavelength for one scattering and use that in $\Delta\lambda$ in Eq.(38.23) to calculate the average scattering angle ϕ.

EXECUTE: **(a)** The wavelength of a 1 MeV photon is

$$\lambda = \frac{hc}{E} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{1\times10^{6}\text{ eV}} = 1\times10^{-12}\text{ m}$$

The total change in wavelength therefore is $500\times10^{-9}\text{ m} - 1\times10^{-12}\text{ m} = 500\times10^{-9}\text{ m}$.

If this shift is produced in 10^{26} Compton scattering events, the wavelength shift in each scattering event is

$$\Delta\lambda = \frac{500\times10^{-9}\text{ m}}{1\times10^{26}} = 5\times10^{-33}\text{ m}.$$

(b) Use this $\Delta\lambda$ in $\Delta\lambda = \dfrac{h}{mc}(1-\cos\phi)$ and solve for ϕ. We anticipate that ϕ will be very small, since $\Delta\lambda$ is much less than h/mc, so we can use $\cos\phi \approx 1-\phi^2/2$.

$$\Delta\lambda = \frac{h}{mc}(1-(1-\phi^2/2)) = \frac{h}{2mc}\phi^2$$

$$\phi = \sqrt{\frac{2\Delta\lambda}{(h/mc)}} = \sqrt{\frac{2(5\times10^{-33}\text{ m})}{2.426\times10^{-12}\text{ m}}} = 6.4\times10^{-11}\text{ rad} = (4\times10^{-9})°$$

ϕ in radians is much less than 1 so the approximation we used is valid.

(c) IDENTIFY and **SET UP:** We know the total transit time and the total number of scatterings, so we can calculate the average time between scatterings.

EXECUTE: The total time to travel from the core to the surface is $(10^6\text{ y})(3.156\times10^7\text{ s/y}) = 3.2\times10^{13}\text{ s}$. There are 10^{26} scatterings during this time, so the average time between scatterings is $t = \dfrac{3.2\times10^{13}\text{ s}}{10^{26}} = 3.2\times10^{-13}\text{ s}$.

The distance light travels in this time is $d = ct = (3.0\times10^8\text{ m/s})(3.2\times10^{-13}\text{ s}) = 0.1\text{ mm}$

EVALUATE: The photons are on the average scattered through a very small angle in each scattering event. The average distance a photon travels between scatterings is very small.

38.74. **(a)** The final energy of the photon is $E' = \dfrac{hc}{\lambda'}$, and $E = E' + K$, where K is the kinetic energy of the electron after the collision. Then,

$$\lambda = \frac{hc}{E'+K} = \frac{hc}{(hc/\lambda')+K} = \frac{hc}{(hc/\lambda')+(\gamma-1)mc^2} = \frac{\lambda'}{1+\dfrac{\lambda' mc}{h}\left[\dfrac{1}{(1-v^2/c^2)^{1/2}}-1\right]}.$$

($K = mc^2(\gamma-1)$ since the relativistic expression must be used for three-figure accuracy).

(b) $\phi = \arccos(1-\Delta\lambda/(h/mc))$.

(c) $\gamma - 1 = \dfrac{1}{\left(1-\left(\frac{1.80}{3.00}\right)^2\right)^{1/2}} - 1 = 1.25 - 1 = 0.250,\ \dfrac{h}{mc} = 2.43\times10^{-12}\text{ m}$

$$\Rightarrow \lambda = \frac{5.10\times10^{-3}\text{ mm}}{1+\dfrac{(5.10\times10^{-12}\text{ m})(9.11\times10^{-31}\text{ kg})(3.00\times10^{8}\text{ m/s})(0.250)}{(6.63\times10^{-34}\text{ J}\cdot\text{s})}} = 3.34\times10^{-3}\text{ nm}.$$

$$\phi = \arccos\left(1-\frac{(5.10\times10^{-12}\text{ m}-3.34\times10^{-12}\text{ m})}{2.43\times10^{-12}\text{ m}}\right) = 74.0°.$$

38.75. **(a) IDENTIFY** and **SET UP:** Conservation of energy applied to the collision gives $E_\lambda = E_{\lambda'} + E_e$, where E_e is the kinetic energy of the electron after the collision and E_λ and $E_{\lambda'}$ are the energies of the photon before and after the collision. The energy of a photon is related to its wavelength according to Eq.(38.2).

EXECUTE: $E_e = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = hc\left(\frac{\lambda' - \lambda}{\lambda\lambda'}\right)$

$$E_e = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})\left(\frac{0.0032\times10^{-9}\ \text{m}}{(0.1100\times10^{-9}\ \text{m})(0.1132\times10^{-9}\ \text{m})}\right)$$

$$E_e = 5.105\times10^{-17}\ \text{J} = 319\ \text{eV}$$

$$E_e = \frac{1}{2}mv^2 \text{ so } v = \sqrt{\frac{2E_e}{m}} = \sqrt{\frac{2(5.105\times10^{-17}\ \text{J})}{9.109\times10^{-31}\ \text{kg}}} = 1.06\times10^{7}\ \text{m/s}$$

(b) The wavelength λ of a photon with energy E_e is given by $E_e = hc/\lambda$ so

$$\lambda = \frac{hc}{E_e} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{5.105\times10^{-17}\ \text{J}} = 3.89\ \text{nm}$$

EVALUATE: Only a small portion of the incident photon's energy is transferred to the struck electron; this is why the wavelength calculated in part (b) is much larger than the wavelength of the incident photon in the Compton scattering.

38.76. **IDENTIFY:** Apply the Compton scattering formula $\lambda' - \lambda = \Delta\lambda = \frac{h}{mc}(1-\cos\phi) = \lambda_c(1-\cos\phi)$

(a) SET UP: Largest $\Delta\lambda$ is for $\phi = 180°$.

EXECUTE: For $\phi = 180°$, $\Delta\lambda = 2\lambda_c = 2(2.426\ \text{pm}) = 4.85\ \text{pm}$.

(b) SET UP: $\lambda' - \lambda = \lambda_c(1-\cos\phi)$

Wavelength doubles implies $\lambda' = 2\lambda$ so $\lambda' - \lambda = \lambda$. Thus $\lambda = \lambda_C(1-\cos\phi)$. λ is related to E by Eq.(38.2).

EXECUTE: $E = hc/\lambda$, so smallest energy photon means largest wavelength photon, so $\phi = 180°$ and

$\lambda = 2\lambda_c = 4.85\ \text{pm}$. Then $E = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{4.85\times10^{-12}\ \text{m}} = 4.096\times10^{-14}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) =$ 0.256 MeV.

EVALUATE: Any photon Compton scattered at $\phi = 180°$ has a wavelength increase of $2\lambda_c = 4.85$ pm. 4.85 pm is near the short-wavelength end of the range of x-ray wavelengths.

38.77. **(a)** $I(\lambda) = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)}$ but $\lambda = \frac{c}{f}$

$$\Rightarrow I(f) = \frac{2\pi hc^2}{(c/f)^5(e^{hf/kT}-1)} = \frac{2\pi hf^5}{c^3(e^{hf/kT}-1)}$$

(b) $\int_0^\infty I(\lambda)\,d\lambda = \int_\infty^0 I(f)\,df\left(\frac{-c}{f^2}\right)$

$$= \int_0^\infty \frac{2\pi hf^3\,df}{c^2(e^{hf/kT}-1)} = \frac{2\pi(kT)^4}{c^2h^3}\int_0^\infty \frac{x^3}{e^x-1}\,dx = \frac{2\pi(kT)^4}{c^2h^3}\frac{1}{240}(2\pi)^4 = \frac{(2\pi)^5(kT)^4}{240h^3c^2} = \frac{2\pi^5k^4T^4}{15c^2h^3}$$

(c) The expression $\frac{2\pi^5k^4T^4}{15h^3c^2} = \sigma$ as shown in Eq. (38.36). Plugging in the values for the constants we get $\sigma = 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4$.

38.78. $I = \sigma T^4$, $P = IA$, and $\Delta E = Pt$; combining,

$$t = \frac{\Delta E}{A\sigma T^4} = \frac{(100\ \text{J})}{(4.00\times10^{-6}\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(473\ \text{K})^4} = 8.81\times10^{3}\ \text{s} = 2.45\ \text{hrs}.$$

38.79. **(a)** The period was found in Exercise 38.27b: $T = \frac{4\epsilon_0^2n^3h^3}{me^4}$ and frequency is just $f = \frac{1}{T} = \frac{me^4}{4\epsilon_0^2n^3h^3}$.

(b) Eq. (38.6) tells us that $f = \frac{1}{h}(E_2 - E_1)$. So $f = \frac{me^4}{8\epsilon_0^2h^3}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$ (from Eq. (38.18)).

If $n_2 = n$ and $n_1 = n+1$, then $\frac{1}{n_2^2} - \frac{1}{n_1^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$

$$= \frac{1}{n^2}\left(1 - \frac{1}{(1+1/n)^2}\right) \approx \frac{1}{n^2}\left(1 - \left(1 - \frac{2}{n} + \cdots\right)\right) = \frac{2}{n^3} \text{ for large } n \Rightarrow f \approx \frac{me^4}{4\epsilon_0^2n^3h^3}.$$

38.80. Each photon has momentum $p = \dfrac{h}{\lambda}$, and if the rate at which the photons strike the surface is (dN/dt), the force on the surface is $(h/\lambda)(dN/dt)$, and the pressure is $(h/\lambda)(dN/dt)/A$. The intensity is $I = (dN/dt)(E)/A = (dN/dt)(hc/\lambda)/A$, and comparison of the two expressions gives the pressure as (I/c).

38.81. Momentum: $\vec{\boldsymbol{p}} + \vec{\boldsymbol{P}} = \vec{\boldsymbol{p}}' + \vec{\boldsymbol{P}}' \Rightarrow p - P = -p' - P' \;\Rightarrow\; p' = P - (p + P')$

energy: $pc + E = p'c + E' = p'c + \sqrt{(P'c)^2 + (mc^2)^2}$

$$\Rightarrow (pc - p'c + E)^2 = (P'c)^2 + (mc^2)^2 = (Pc)^2 + ((p + p')c)^2 - 2P(p + p')c^2 + (mc^2)^2$$

$$(pc - p'c)^2 + E^2 = E^2 + (pc + p'c)^2 - 2(Pc^2)(p + p') + 2Ec(p - p') - 4pp'c^2 + 2Ec(p - p')$$

$$+2(Pc^2)(p + p') = 0$$

$$\Rightarrow p'(Pc^2 - 2pc^2 - Ec) = p(-Ec - Pc^2)$$

$$\Rightarrow p' = p\frac{Ec + Pc^2}{2pc^2 + Ec - Pc^2} = p\frac{E + Pc}{2pc + (E - Pc)}$$

$$\Rightarrow \lambda' = \lambda\left(\frac{2hc/\lambda + (E - Pc)}{E + Pc}\right) = \lambda\left(\frac{E - Pc}{E + Pc}\right) + \frac{2hc}{E + Pc}$$

$$\Rightarrow \lambda' = \frac{(\lambda(E - Pc) + 2hc)}{E + Pc}$$

If $E \gg mc^2, Pc = \sqrt{E^2 - (mc^2)^2} = E\sqrt{1 - \left(\dfrac{mc^2}{E}\right)^2} \approx E\left(1 - \dfrac{1}{2}\left(\dfrac{mc^2}{E}\right)^2 + \cdots\right)$

$$\Rightarrow E - Pc \approx \frac{1}{2}\frac{(mc^2)^2}{E} \;\Rightarrow \lambda_1 \approx \frac{\lambda(mc^2)^2}{2E(2E)} + \frac{hc}{E} = \frac{hc}{E}\left(1 + \frac{m^2c^4\lambda}{4hcE}\right)$$

(b) If $\lambda = 10.6 \times 10^{-6}\ \text{m}, E = 1.00 \times 10^{10}\ \text{eV} = 1.60 \times 10^{-9}\ \text{J}$

$$\Rightarrow \lambda' \approx \frac{hc}{1.60 \times 10^{-9}\ \text{J}}\left(1 + \frac{(9.11 \times 10^{-31}\ \text{kg})^2 c^4 (10.6 \times 10^{-6}\ \text{m})}{4hc\,(1.6 \times 10^{-9}\ \text{J})}\right)$$

$$= (1.24 \times 10^{-16}\ \text{m})(1 + 56.0) = 7.08 \times 10^{-15}\ \text{m}.$$

(c) These photons are gamma rays. We have taken infrared radiation and converted it into gamma rays! Perhaps useful in nuclear medicine, nuclear spectroscopy, or high energy physics: wherever controlled gamma ray sources might be useful.

39

THE WAVE NATURE OF PARTICLES

39.1. **IDENTIFY** and **SET UP:** $\lambda = \frac{h}{p} = \frac{h}{mv}$. For an electron, $m = 9.11\times10^{-31}$ kg. For a proton, $m = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(4.70\times10^{6}\ \text{m/s})} = 1.55\times10^{-10}\ \text{m} = 0.155\ \text{nm}$

(b) λ is proportional to $\frac{1}{m}$, so $\lambda_\text{p} = \lambda_\text{e}\left(\frac{m_\text{e}}{m_\text{p}}\right) = (1.55\times10^{-10}\ \text{m})\left(\frac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}\right) = 8.46\times10^{-14}\ \text{m}$.

39.2. **IDENTIFY** and **SET UP:** For a photon, $E = \frac{hc}{\lambda}$. For an electron or proton, $p = \frac{h}{\lambda}$ and $E = \frac{p^2}{2m}$, so $E = \frac{h^2}{2m\lambda^2}$.

EXECUTE: **(a)** $E = \frac{hc}{\lambda} = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{0.20\times10^{-9}\ \text{m}} = 6.2\ \text{keV}$

(b) $E = \frac{h^2}{2m\lambda^2} = \left(\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{0.20\times10^{-9}\ \text{m}}\right)^2 \frac{1}{2(9.11\times10^{-31}\ \text{kg})} = 6.03\times10^{-18}\ \text{J} = 38\ \text{eV}$

(c) $E_\text{p} = E_\text{e}\left(\frac{m_\text{e}}{m_\text{p}}\right) = (38\ \text{eV})\left(\frac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}\right) = 0.021\ \text{eV}$

EVALUATE: For a given wavelength a photon has much more energy than an electron, which in turn has more energy than a proton.

39.3. **(a)** $\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(2.80\times10^{-10}\ \text{m})} = 2.37\times10^{-24}\ \text{kg}\cdot\text{m/s}.$

(b) $K = \frac{p^2}{2m} = \frac{(2.37\times10^{-24}\ \text{kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\ \text{kg})} = 3.08\times10^{-18}\ \text{J} = 19.3\ \text{eV}.$

39.4. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

$$= \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(6.64\times10^{-27}\ \text{kg})\,(4.20\times10^{6}\ \text{eV})\,(1.60\times10^{-19}\ \text{J/eV})}} = 7.02\times10^{-15}\ \text{m}.$$

39.5. **IDENTIFY** and **SET UP:** The de Broglie wavelength is $\lambda = \frac{h}{p} = \frac{h}{mv}$. In the Bohr model, $mvr_n = n(h/2\pi)$, so $mv = nh/(2\pi r_n)$. Combine these two expressions and obtain an equation for λ in terms of n. Then

$\lambda = h\left(\frac{2\pi r_n}{nh}\right) = \frac{2\pi r_n}{n}.$

EXECUTE: **(a)** For $n = 1$, $\lambda = 2\pi r_1$ with $r_1 = a_0 = 0.529\times10^{-10}$ m, so $\lambda = 2\pi(0.529\times10^{-10}\ \text{m}) = 3.32\times10^{-10}$ m

$\lambda = 2\pi r_1$; the de Broglie wavelength equals the circumference of the orbit.

(b) For $n = 4$, $\lambda = 2\pi r_4/4$.

$r_n = n^2 a_0$ so $r_4 = 16a_0$.

$\lambda = 2\pi(16a_0)/4 = 4(2\pi a_0) = 4(3.32\times10^{-10}\ \text{m}) = 1.33\times10^{-9}$ m

$\lambda = 2\pi r_4/4$; the de Broglie wavelength is $\frac{1}{n} = \frac{1}{4}$ times the circumference of the orbit.

EVALUATE: As n increases the momentum of the electron increases and its de Broglie wavelength decreases. For any n, the circumference of the orbits equals an integer number of de Broglie wavelengths.

39.6. (**a**) For a nonrelativistic particle, $K = \dfrac{p^2}{2m}$, so $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2Km}}$.

(**b**) $(6.63\times10^{-34}\ \text{J}\cdot\text{s})\big/\sqrt{2(800\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})(9.11\times10^{-31}\ \text{kg})} = 4.34\times10^{-11}\ \text{m}.$

39.7. **IDENTIFY:** A person walking through a door is like a particle going through a slit and hence should exhibit wave properties.

SET UP: The de Broglie wavelength of the person is $\lambda = h/mv$.

EXECUTE: (**a**) Assume m = 75 kg and v = 1.0 m/s.

$$\lambda = h/mv = (6.626 \times 10^{-34}\ \text{J}\cdot\text{s})/[(75\ \text{kg})(1.0\ \text{m/s})] = 8.8 \times 10^{-36}\ \text{m}$$

EVALUATE: (**b**) A typical doorway is about 1 m wide, so the person's de Broglie wavelength is much too small to show wave behavior through a "slit" that is about 10^{35} times as wide as the wavelength. Hence ordinary objects do not show wave behavior in everyday life.

39.8. Combining Equations 37.38 and 37.39 gives $p = mc\sqrt{\gamma^2 - 1}$.

(**a**) $\lambda = \dfrac{h}{p} = (h/mc)\big/\sqrt{\gamma^2-1} = 4.43\times10^{-12}$ m. (The incorrect nonrelativistic calculation gives 5.05×10^{-12} m.)

(**b**) $(h/mc)\big/\sqrt{\gamma^2-1} = 7.07\times10^{-13}$ m.

39.9. **IDENTIFY** and **SET UP:** A photon has zero mass and its energy and wavelength are related by Eq.(38.2). An electron has mass. Its energy is related to its momentum by $E = p^2/2m$ and its wavelength is related to its momentum by Eq.(39.1).

EXECUTE: (**a**) photon

$$E = \frac{hc}{\lambda} \text{ so } \lambda = \frac{hc}{E} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{(20.0\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})} = 62.0\ \text{nm}$$

electron

$$E = p^2/(2m) \text{ so } p = \sqrt{2mE} = \sqrt{2(9.109\times10^{-31}\ \text{kg})(20.0\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})} = 2.416\times10^{-24}\ \text{kg}\cdot\text{m/s}$$

$$\lambda = h/p = 0.274\ \text{nm}$$

(**b**) photon $E = hc/\lambda = 7.946\times10^{-19}\ \text{J} = 4.96\ \text{eV}$

electron $\lambda = h/p$ so $p = h/\lambda = 2.650\times10^{-27}\ \text{kg}\cdot\text{m/s}$

$$E = p^2/(2m) = 3.856\times10^{-24}\ \text{J} = 2.41\times10^{-5}\ \text{eV}$$

(**c**) **EVALUATE:** You should use a probe of wavelength approximately 250 nm. An electron with $\lambda = 250$ nm has much less energy than a photon with $\lambda = 250$ nm, so is less likely to damage the molecule. Note that $\lambda = h/p$ applies to all particles, those with mass and those with zero mass. $E = hf = hc/\lambda$ applies only to photons and $E = p^2/2m$ applies only to particles with mass.

39.10. **IDENTIFY:** Any moving particle has a de Broglie wavelength. The speed of a molecule, and hence its de Broglie wavelength, depends on the temperature of the gas.

SET UP: The average kinetic energy of the molecule is $K_{av} = 3/2\ kT$, and the de Broglie wavelength is $\lambda = h/mv = h/p$.

EXECUTE: (**a**) Combining $K_{av} = 3/2\ kT$ and $K = p^2/2m$ gives $3/2\ kT = p_{av}{}^2/2m$ and $p_{av} = \sqrt{3mkT}$. The de Broglie wavelength is $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{3mkT}} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{3(2\times1.67\times10^{-27}\ \text{kg})(1.38\times10^{-23}\ \text{J/K})(273\ \text{K})}} = 1.08\times10^{-10}\ \text{m}$.

(**b**) For an electron, $\lambda = h/p = h/mv$ gives

$$v = \frac{h}{m\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(1.08\times10^{-10}\ \text{m})} = 6.75 \times 10^{6}\ \text{m/s}$$

This is about 2% the speed of light, so we do *not* need to use relativity.

(**c**) For photon:

$$E = hc/\lambda = (6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})/(1.08 \times 10^{-10}\ \text{m}) = 1.84 \times 10^{-15}\ \text{J}$$

For the H_2 molecule: $K_{av} = (3/2)kT = 3/2\ (1.38 \times 10^{-23}\ \text{J/K})(273\ \text{K}) = 5.65 \times 10^{-21}$ J

For the electron: $K = ½\ mv^2 = ½\ (9.11 \times 10^{-31}\ \text{kg})(6.73 \times 10^{6}\ \text{m/s})^2 = 2.06 \times 10^{-17}$ J

EVALUATE: The photon has about 100 times more energy than the electron and 300,000 times more energy than the H_2 molecule. This shows that photons of a given wavelength will have much more energy than particles of the same wavelength.

39.11. **IDENTIFY** and **SET UP:** Use Eq.(39.1).

EXECUTE: $\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(5.00\times10^{-3}\ \text{kg})(340\ \text{m/s})} = 3.90\times10^{-34}\ \text{m}$

EVALUATE: This wavelength is extremely short; the bullet will not exhibit wavelike properties.

39.12. **(a)** $\lambda = h/mv \rightarrow v = h/m\lambda$

Energy conservation: $e\Delta V = \frac{1}{2}mv^2$

$$\Delta V = \frac{mv^2}{2e} = \frac{m\left(\frac{h}{m\lambda}\right)^2}{2e} = \frac{h^2}{2em\lambda^2} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.60\times10^{-19}\ \text{C})\,(9.11\times10^{-31}\ \text{kg})\,(0.15\times10^{-9}\ \text{m})^2} = 66.9\ \text{V}$$

(b) $E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})\,(3.0\times10^{8}\ \text{m/s})}{0.15\times10^{-9}\ \text{m}} = 1.33\times10^{-15}\ \text{J}$

$e\Delta V = K = E_{\text{photon}}$ and $\Delta V = \frac{E_{\text{photon}}}{e} = \frac{1.33\times10^{-15}\ \text{J}}{1.6\times10^{-19}\ \text{C}} = 8310\ \text{V}$

39.13. **(a)** $\lambda = 0.10\ \text{nm}$. $p = mv = h/\lambda$ so $v = h/(m\lambda) = 7.3\times10^{6}\ \text{m/s}$.

(b) $E = \frac{1}{2}mv^2 = 150\ \text{eV}$

(c) $\text{E} = hc/\lambda = 12\ \text{KeV}$

(d) The electron is a better probe because for the same λ it has less energy and is less damaging to the structure being probed.

39.14. **IDENTIFY:** The electrons behave like waves and are diffracted by the slit.

SET UP: We use conservation of energy to find the speed of the electrons, and then use this speed to find their de Broglie wavelength, which is $\lambda = h/mv$. Finally we know that the first dark fringe for single-slit diffraction occurs when $a \sin\theta = \lambda$.

EXECUTE: **(a)** Use energy conservation to find the speed of the electron: $\frac{1}{2}mv^2 = eV$.

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\left(1.60\times10^{-19}\ \text{C}\right)(100\ \text{V})}{9.11\times10^{-31}\ \text{kg}}} = 5.93\times10^{6}\ \text{m/s}$$

which is about 2% the speed of light, so we can ignore relativity.

(b) First find the de Broglie wavelength:

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(5.93\times10^{6}\ \text{m/s}\right)} = 1.23\times10^{-10}\ \text{m} = 0.123\ \text{nm}$$

For the first single slit dark fringe, we have $a \sin\theta = \lambda$, which gives

$$a = \frac{\lambda}{\sin\theta} = \frac{1.23\times10^{-10}\ \text{m}}{\sin(11.5°)} = 6.16\times10^{-10}\ \text{m} = 0.616\ \text{nm}$$

EVALUATE: The slit width is around 5 times the de Broglie wavelength of the electron, and both are much smaller than the wavelength of visible light.

39.15. For $m = 1$, $\lambda = d\sin\theta = \frac{h}{\sqrt{2mE}}$.

$$E = \frac{h^2}{2md^2\sin^2\theta} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.675\times10^{-27}\ \text{kg})\,(9.10\times10^{-11}\ \text{m})^2\sin^2(28.6°)} = 6.91\times10^{-20}\ \text{J} = 0.432\ \text{eV}.$$

39.16. Intensity maxima occur when $d\sin\theta = m\lambda$. $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2ME}}$ so $d\sin\theta = \frac{mh}{\sqrt{2ME}}$. (Careful! Here, m is the order of the maxima, whereas M is the mass of the incoming particle.)

(a) $d = \frac{mh}{\sqrt{2ME}\sin\theta} = \frac{(2)(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(188\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}\sin(60.6°)}$

$= 2.06\times10^{-10}\ \text{m} = 0.206\ \text{nm}.$

(b) $m = 1$ also gives a maximum.

$$\theta = \arcsin\left(\frac{(1)\,(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\ \text{kg})\,(188\ \text{eV})\,(1.60\times10^{-19}\ \text{J/eV})}\,(2.06\times10^{-10}\ \text{m})}\right) = 25.8°.$$

This is the only other one. If we let $m \ge 3$, then there are no more maxima.

(c) $E = \dfrac{m^2h^2}{2Md^2\sin^2\theta} = \dfrac{(1)^2(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{2(9.11\times10^{-31}\text{ kg})(2.60\times10^{-10}\text{ m})^2\sin^2(60.6°)}$

$= 7.49\times10^{-18}\text{ J} = 46.8\text{ eV}.$

Using this energy, if we let $m = 2$, then $\sin\theta > 1$. Thus, there is no $m = 2$ maximum in this case.

39.17. The condition for a maximum is $d\sin\theta = m\lambda$. $\lambda = \dfrac{h}{p} = \dfrac{h}{Mv}$, so $\theta = \arcsin\left(\dfrac{mh}{dMv}\right)$. (Careful! Here, *m* is the order of the maximum, whereas *M* is the incoming particle mass.)

(a) $m = 1 \Rightarrow \theta_1 = \arcsin\left(\dfrac{h}{dMv}\right)$

$$= \arcsin\left(\frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{(1.60\times10^{-6}\text{ m})(9.11\times10^{-31}\text{ kg})(1.26\times10^{4}\text{ m/s})}\right) = 2.07°.$$

$$m = 2 \Rightarrow \theta_2 = \arcsin\left(\frac{(2)(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(1.60\times10^{-6}\text{ m})(9.11\times10^{-31}\text{ kg})(1.26\times10^{4}\text{ m/s})}\right) = 4.14°.$$

(b) For small angles (in radians!) $y \cong D\theta$, so $y_1 \approx (50.0\text{ cm})(2.07°)\left(\dfrac{\pi\text{ radians}}{180°}\right) = 1.81\text{ cm}$,

$y_2 \approx (50.0\text{ cm})(4.14°)\left(\dfrac{\pi\text{ radians}}{180°}\right) = 3.61\text{ cm}$ and $y_2 - y_1 = 3.61\text{ cm} - 1.81\text{ cm} = 1.81\text{ cm}.$

39.18. **IDENTIFY:** Since we know only that the mosquito is somewhere in the room, there is an uncertainty in its position. The Heisenberg uncertainty principle tells us that there is an uncertainty in its momentum.

SET UP: The uncertainty principle is $\Delta x\Delta p_x \ge \hbar$.

EXECUTE: **(a)** You know the mosquito is somewhere in the room, so the maximum uncertainty in its horizontal position is $\Delta x = 5.0$ m.

(b) The uncertainty principle gives $\Delta x\Delta p_x \ge \hbar$, and $\Delta p_x = m\Delta v_x$ since we know the mosquito's mass. This gives $\Delta x\, m\Delta v_x \ge \hbar$, which we can solve for Δv_x to get the minimum uncertainty in v_x.

$$\Delta v_x = \frac{\hbar}{m\Delta x} = \frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{(1.5\times10^{-6}\text{ kg})(5.0\text{ m})} = 1.4\times10^{-29}\text{ m/s}$$

which is hardly a serious impediment!

EVALUATE: For something as "large" as a mosquito, the uncertainty principle places a negligible limitation on our ability to measure its speed.

39.19. **(a) IDENTIFY** and **SET UP:** Use $\Delta x\Delta p_x \ge h/2\pi$ to calculate Δx and obtain Δv_x from this.

EXECUTE: $\Delta p_x \ge \dfrac{h}{2\pi\Delta x} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2\pi(1.00\times10^{-6}\text{ m})} = 1.055\times10^{-28}\text{ kg}\cdot\text{m/s}$

$$\Delta v_x = \frac{\Delta p_x}{m} = \frac{1.055\times10^{-28}\text{ kg}\cdot\text{m/s}}{1200\text{ kg}} = 8.79\times10^{-32}\text{ m/s}$$

(b) EVALUATE: Even for this very small Δx the minimum Δv_x required by the Heisenberg uncertainty principle is very small. The uncertainty principle does not impose any practical limit on the simultaneous measurements of the positions and velocities of ordinary objects.

39.20. **IDENTIFY:** Since we know that the marble is somewhere on the table, there is an uncertainty in its position. The Heisenberg uncertainty principle tells us that there is therefore an uncertainty in its momentum.

SET UP: The uncertainty principle is $\Delta x\Delta p_x \ge \hbar$.

EXECUTE: **(a)** Since the marble is somewhere on the table, the maximum uncertainty in its horizontal position is $\Delta x = 1.75$ m.

(b) Following the same procedure as in part (b) of problem 39.18, the minimum uncertainty in the horizontal velocity of the marble is

$$\Delta v_x = \frac{\hbar}{m\Delta x} = \frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{(0.0100\text{ kg})(1.75\text{ m})} = 6.03\times10^{-33}\text{ m/s}$$

(c) The uncertainty principle tells us that we cannot know that the marble's horizontal velocity is *exactly* zero, so the smallest we could measure it to be is 6.03×10^{-33} m/s, from part (b). The longest time it could remain on the

table is the time to travel the full width of the table (1.75 m), so $t = x/v_x$ = (1.75 m)/(6.03×10^{-33} m/s) = 2.90×10^{32} s = 9.20×10^{24} years

Since the universe is about 14×10^9 years old, this time is about

$$\frac{9.0\times10^{24}\ \text{yr}}{14\times10^{9}\ \text{yr}} \approx 6\times10^{14} \text{ times the age of the universe! Don't hold your breath!}$$

EVALUATE: For household objects, the uncertainty principle places a negligible limitation on our ability to measure their speed.

39.21. Heisenberg's Uncertainty Principles tells us that $\Delta x \Delta p_x \geq \frac{h}{2\pi}$. We can treat the standard deviation as a direct measure of uncertainty. Here $\Delta x \Delta p_x = (1.2\times10^{-10}\ \text{m})\,(3.0\times10^{-25}\ \text{kg}\cdot\text{m/s}) = 3.6\times10^{-35}\ \text{J}\cdot\text{s}$ but $\frac{h}{2\pi} = 1.05\times10^{-34}\ \text{J}\cdot\text{s}$

Therefore $\Delta x \Delta p_x < \frac{h}{2\pi}$ so the claim is *not valid*.

39.22. **(a)** $(\Delta x)\,(m\Delta v_x) \geq h/2\pi$, and setting $\Delta v_x = (0.010)v_x$ and the product of the uncertainties equal to $h/2\pi$ (for the minimum uncertainty) gives $v_x = h/(2\pi m(0.010)\Delta x) = 57.9\ \text{m/s}$.

(b) Repeating with the proton mass gives 31.6 mm/s.

39.23. $\Delta E > \frac{h}{2\pi\Delta t} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi(5.2\times10^{-3}\ \text{s})} = 2.03\times10^{-32}\ \text{J} = 1.27\times10^{-13}\ \text{eV}.$

39.24. **IDENTIFY** and **SET UP:** The Heisenberg Uncertainty Principle says $\Delta x \Delta p_x \geq \frac{h}{2\pi}$. The minimum allowed $\Delta x \Delta p_x$ is $h/2\pi$. $\Delta p_x = m\Delta v_x$.

EXECUTE: **(a)** $m\Delta x \Delta v_x = \frac{h}{2\pi}$. $\Delta v_x = \frac{h}{2\pi m \Delta x} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(1.67\times10^{-27}\ \text{kg})(2.0\times10^{-12}\ \text{m})} = 3.2\times10^{4}\ \text{m/s}$

(b) $\Delta x = \frac{h}{2\pi m \Delta v_x} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(9.11\times10^{-31}\ \text{kg})(0.250\ \text{m/s})} = 4.6\times10^{-4}\ \text{m}$

39.25. $\Delta E \Delta t = \frac{h}{2\pi}$. $\Delta E = \frac{h}{2\pi\Delta t} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi(7.6\times10^{-21}\ \text{s})} = 1.39\times10^{-14}\ \text{J} = 8.69\times10^{4}\ \text{eV} = 0.0869\ \text{MeV}.$

$$\frac{\Delta E}{E} = \frac{0.0869\ \text{MeV}/c^2}{3097\ \text{MeV}/c^2} = 2.81\times10^{-5}.$$

39.26. $\Delta E \Delta t = \frac{h}{2\pi}$. $\Delta E = \Delta m c^2$. $\Delta m = 2.06\times10^{9}\ \text{eV}/c^2 = 3.30\times10^{-10}\ \text{J}/c^2$.

$\Delta t = \frac{h}{2\pi\Delta m c^2} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(3.30\times10^{-10}\ \text{J})} = 3.20\times10^{-25}\ \text{s}.$

39.27. **IDENTIFY** and **SET UP:** For a photon $E_{\text{ph}} = \frac{hc}{\lambda} = \frac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$. For an electron $E_{\text{e}} = \frac{p^2}{2m} = \frac{1}{2m}\left(\frac{h}{\lambda}\right)^2 = \frac{h^2}{2m\lambda^2}$.

EXECUTE: **(a)** photon $E_{\text{ph}} = \frac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{10.0\times10^{-9}\ \text{m}} = 1.99\times10^{-17}\ \text{J}$

electron $E_{\text{e}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\ \text{kg})(10.0\times10^{-9}\ \text{m})^2} = 2.41\times10^{-21}\ \text{J}$

$\frac{E_{\text{ph}}}{E_{\text{e}}} = \frac{1.99\times10^{-17}\ \text{J}}{2.41\times10^{-21}\ \text{J}} = 8.26\times10^{3}$

(b) The electron has much less energy so would be less damaging.

EVALUATE: For a particle with mass, such as an electron, $E \sim \lambda^{-2}$. For a massless photon $E \sim \lambda^{-1}$.

39.28. **(a)** $eV = K = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m}$, so $V = \frac{(h/\lambda)^2}{2me} = 419\ \text{V}$.

(b) The voltage is reduced by the ratio of the particle masses, $(419\ \text{V})\frac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}} = 0.229\ \text{V}$.

39.29. **IDENTIFY** and **SET UP:** $\psi(x) = A\sin kx$. The position probability density is given by $|\psi(x)|^2 = A^2\sin^2 kx$.

EXECUTE: **(a)** The probability is highest where $\sin kx = 1$ so $kx = 2\pi x/\lambda = n\pi/2$, $n = 1, 3, 5, \ldots$

$x = n\lambda/4$, $n = 1, 3, 5, \ldots$ so $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots$

(b) The probability of finding the particle is zero where $|\psi|^2 = 0$, which occurs where $\sin kx = 0$ and
$kx = 2\pi x/\lambda = n\pi,\ n = 0,\ 1,\ 2,\ldots$
$x = n\lambda/2,\ n = 0,\ 1,\ 2,\ldots$ so $x = 0,\ \lambda/2,\ \lambda,\ 3\lambda/2,\ldots$

EVALUATE: The situation is analogous to a standing wave, with the probability analogous to the square of the amplitude of the standing wave.

39.30. $\Psi^* = \psi^* \sin\omega t$, so $|\Psi|^2 = |\Psi^*\Psi| = \psi^*\psi\sin^2\omega t = |\psi|^2\sin^2\omega t$. $|\Psi|^2$ is not time-independent, so Ψ is not the wavefunction for a stationary state.

39.31. **IDENTIFY:** To describe a real situation, a wave function must be normalizable.

SET UP: $|\psi|^2\,dV$ is the probability that the particle is found in volume dV. Since the particle must be *somewhere*, ψ must have the property that $\int|\psi|^2\,dV = 1$ when the integral is taken over all space.

EXECUTE: **(a)** In one dimension, as we have here, the integral discussed above is of the form $\int_{-\infty}^{\infty}|\psi(x)|^2\,dx = 1$.

(b) Using the result from part (a), we have $\int_{-\infty}^{\infty}\left(e^{ax}\right)^2 dx = \int_{-\infty}^{\infty}e^{2ax}dx = \left.\dfrac{e^{2ax}}{2a}\right|_{-\infty}^{\infty} = \infty$. Hence this wave function cannot be normalized and therefore cannot be a valid wave function.

(c) We only need to integrate this wave function of 0 to ∞ because it is zero for $x < 0$. For normalization we have

$1 = \int_{-\infty}^{\infty}|\psi|^2\,dx = \int_0^{\infty}\left(Ae^{-bx}\right)^2 dx = \int_0^{\infty}A^2e^{-2bx}dx = \left.\dfrac{A^2e^{-2bx}}{-2b}\right|_0^{\infty} = \dfrac{A^2}{2b}$, which gives $\dfrac{A^2}{2b} = 1$, so $A = \sqrt{2b}$.

EVALUATE: If b were positive, the given wave function could not be normalized, so it would not be allowable.

39.32. **(a)** The uncertainty in the particle position is proportional to the width of $\psi(x)$, and is inversely proportional to $\sqrt{\alpha}$. This can be seen by either plotting the function for different values of α, finding the expectation value $\langle x^2\rangle = \int\psi^2x^2dx$ for the normalized wave function or by finding the full width at half-maximum. The particle's uncertainty in position decreases with increasing α. The dependence of the expectation value $\langle x^2\rangle$ on α may be found by considering

$$\langle x^2\rangle = \frac{\int_{-\infty}^{\infty}x^2e^{-2\alpha x^2}dx}{\int_{-\infty}^{\infty}e^{-2\alpha x^2}dx} = -\frac{1}{2}\frac{\partial}{\partial\alpha}\ln\left[\int_{-\infty}^{\infty}e^{-2\alpha x^2}dx\right] = -\frac{1}{2}\frac{\partial}{\partial\alpha}\ln\left[\frac{1}{\sqrt{2\alpha}}\int_{-\infty}^{\infty}e^{-u^2}du\right] = \frac{1}{4\alpha},$$

where the substitution $u = \sqrt{\alpha}x$ has been made.

(b) Since the uncertainty in position decreases, the uncertainty in momentum must increase.

39.33. $f(x,y) = \left(\dfrac{x-iy}{x+iy}\right)$ and $f^*(x,y) = \left(\dfrac{x+iy}{x-iy}\right) \Rightarrow |f|^2 = f\,f^* = \left(\dfrac{x-iy}{x+iy}\right)\cdot\left(\dfrac{x+iy}{x-iy}\right) = 1.$

39.34. The same. $|\psi(x,y,z)|^2 = \psi^*(x,y,z)\psi(x,y,z)$

$|\psi(x,y,z)e^{i\phi}|^2 = (\psi^*(x,y,z)e^{-i\phi})(\psi(x,y,z)e^{+i\phi}) = \psi^*(x,y,z)\psi(x,y,z).$

The complex conjugate means convert all i's to–i's and vice-versa. $e^{i\phi}\cdot e^{-i\phi} = 1.$

39.35. **IDENTIFY:** To describe a real situation, a wave function must be normalizable.

SET UP: $|\psi|^2\,dV$ is the probability that the particle is found in volume dV. Since the particle must be *somewhere*, ψ must have the property that $\int|\psi|^2\,dV = 1$ when the integral is taken over all space.

EXECUTE: **(a)** For normalization of the one-dimensional wave function, we have

$1 = \int_{-\infty}^{\infty}|\psi|^2\,dx = \int_{-\infty}^{0}\left(Ae^{bx}\right)^2 dx + \int_0^{\infty}\left(Ae^{-bx}\right)^2 dx = \int_{-\infty}^{0}A^2e^{2bx}dx + \int_0^{\infty}A^2e^{-2bx}dx.$

$1 = A^2\left\{\left.\dfrac{e^{2bx}}{2b}\right|_{-\infty}^{0} + \left.\dfrac{e^{-2bx}}{-2b}\right|_0^{\infty}\right\} = \dfrac{A^2}{b}$, which gives $A = \sqrt{b} = \sqrt{2.00\ \text{m}^{-1}} = 1.41\ \text{m}^{-1/2}$

(b) The graph of the wavefunction versus x is given in Figure 39.35.

(c) (i) $P = \int_{-0.500\ \text{m}}^{+5.00\ \text{m}}|\psi|^2\,dx = 2\int_0^{+5.00\ \text{m}}A^2e^{-2bx}dx$, where we have used the fact that the wave function is an even function of x. Evaluating the integral gives

$$P = \frac{-A^2}{b}\left(e^{-2b(0.500\ \text{m})} - 1\right) = \frac{-(2.00\,\text{m}^{-1})}{2.00\,\text{m}^{-1}}\left(e^{-2.00} - 1\right) = 0.865$$

There is a little more than an 86% probability that the particle will be found within 50 cm of the origin.

(ii) $P = \int_{-\infty}^{0} \left(Ae^{bx}\right)^2 dx = \int_{-\infty}^{0} A^2 e^{2bx} dx = \dfrac{A^2}{2b} = \dfrac{2.00\ \text{m}^{-1}}{2(2.00\ \text{m}^{-1})} = \dfrac{1}{2} = 0.500$

There is a 50-50 chance that the particle will be found to the left of the origin, which agrees with the fact that the wave function is symmetric about the y-axis.

(iii) $P = \int_{0.500\ \text{m}}^{1.00\ \text{m}} A^2 e^{-2bx} dx$

$$= \frac{A^2}{-2b}\left(e^{-2(2.00\ \text{m}^{-1})(1.00\ \text{m})} - e^{-2(2.00\ \text{m}^{-1})(0.500\ \text{m})}\right) = -\frac{1}{2}\left(e^{-4} - e^{-2}\right) = 0.0585$$

EVALUATE: There is little chance of finding the particle in regions where the wave function is small.

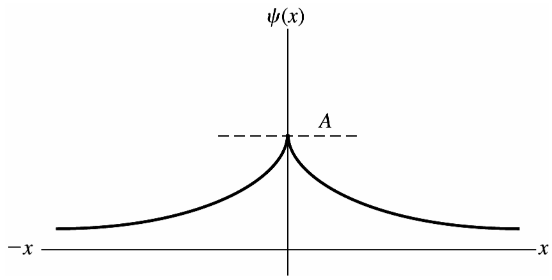


Figure 39.35

39.36. Eq. (39.18): $\dfrac{-\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} + U\psi = E\psi$. Let $\psi = A\psi_1 + B\psi_2$

$\Rightarrow \dfrac{-\hbar^2}{2m}\dfrac{d^2}{dx^2}(A\psi_1 + B\psi_2) + U(A\psi_1 + B\psi_2) = E(A\psi_1 + B\psi_2)$

$\Rightarrow A\left(-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi_1}{dx^2} + U\psi_1 - E\psi_1\right) + B\left(-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi_2}{dx^2} + U\psi_2 - E\psi_2\right) = 0.$

But each of ψ_1 and ψ_2 satisfy Schrödinger's equation separately so the equation still holds true, for any A or B.

39.37. $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} + U\psi = BE_1\psi_1 + CE_2\psi_2$. If ψ were a solution with energy E, then $BE_1\psi_1 + CE_2\psi_2 = BE\psi_1 + CE\psi_2$ or $B(E_1 - E)\psi_1 = C(E - E_2)\psi_2$. This would mean that ψ_1 is a constant multiple of ψ_2, and ψ_1 and ψ_2 would be wave functions with the same energy. However, $E_1 \neq E_2$, so this is not possible, and ψ cannot be a solution to Eq. (39.18).

39.38. **(a)** $\lambda = \dfrac{h}{\sqrt{2mK}} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(40\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}} = 1.94\times10^{-10}\ \text{m}.$

(b) $\dfrac{R}{v} = \dfrac{R}{\sqrt{2E/m}} = \dfrac{(2.5\ \text{m})(9.11\times10^{-31}\ \text{kg})^{1/2}}{\sqrt{2(40\ \text{eV})(1.6\times10^{-19}\ \text{J/eV})}} = 6.67\times10^{-7}\ \text{s}.$

(c) The width w is $w = 2R\dfrac{\lambda}{a}$' and $w = \Delta v_y t = \Delta p_y t/m$, where t is the time found in part (b) and a is the slit width. Combining the expressions for w, $\Delta p_y = \dfrac{2m\lambda R}{at} = 2.65\times10^{-28}\ \text{kg}\cdot\text{m/s}.$

(d) $\Delta y = \dfrac{h}{2\pi\Delta p_y} = 0.40\ \mu\text{m},$ which is the same order of magnitude.

39.39. **(a)** $E = hc/\lambda = 12\ \text{eV}$

(b) Find E for an electron with $\lambda = 0.10\times10^{-6}$ m. $\lambda = h/p$ so $p = h/\lambda = 6.626\times10^{-27}\ \text{kg}\cdot\text{m/s}$.

$E = p^2/(2m) = 1.5\times10^{-4}\ \text{eV}$. $E = q\Delta V$ so $\Delta V = 1.5\times10^{-4}\ \text{V}$

$v = p/m = (6.626\times10^{-27}\ \text{kg}\cdot\text{m/s})/(9.109\times10^{-31}\ \text{kg}) = 7.3\times10^{3}\ \text{m/s}$

(c) Same λ so same p. $E = p^2/(2m)$ but now $m = 1.673\times10^{-27}$ kg so $E = 8.2\times10^{-8}$ eV and $\Delta V = 8.2\times10^{-8}$ V.

$v = p/m = (6.626\times10^{-27}\ \text{kg}\cdot\text{m/s})/(1.673\times10^{-27}\ \text{kg}) = 4.0\ \text{m/s}$

39.40. **(a)** Single slit diffraction: $a\sin\theta = m\lambda$. $\lambda = a\sin\theta = (150\times10^{-9}\ \text{m})\sin20° = 5.13\times10^{-8}\ \text{m}$

$$\lambda = h/mv \rightarrow v = h/m\lambda.\ \ v = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(5.13\times10^{-8}\ \text{m})} = 1.42\times10^{4}\ \text{m/s}$$

(b) $a\sin\theta_2 = 2\lambda$. $\sin\theta_2 = \pm2\dfrac{\lambda}{a} = \pm2\left(\dfrac{5.13\times10^{-8}\ \text{m}}{150\times10^{-9}\ \text{m}}\right) = \pm0.684$. $\theta_2 = \pm43.2°$

39.41. **IDENTIFY:** The electrons behave like waves and produce a double-slit interference pattern after passing through the slits.

SET UP: The first angle at which destructive interference occurs is given by $d \sin \theta = \lambda/2$. The de Broglie wavelength of each of the electrons is $\lambda = h/mv$.

EXECUTE: **(a)** First find the wavelength of the electrons. For the first dark fringe, we have $d \sin \theta = \lambda/2$, which gives (1.25 nm)(sin 18.0°) = $\lambda/2$, and λ = 0.7725 nm. Now solve the de Broglie wavelength equation for the speed of the electron:

$$v = \frac{h}{m\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(0.7725\times10^{-9}\ \text{m})} = 9.42\times10^{5}\ \text{m/s}$$

which is about 0.3% the speed of light, so they are *nonrelativistic*.

(b) Energy conservation gives $eV = ½\, mv^2$ and

$$V = mv^2/2e = (9.11\times10^{-31}\ \text{kg})(9.42\times10^{5}\ \text{m})^2/[2(1.60\times10^{-19}\ \text{C})] = 2.52\ \text{V}$$

EVALUATE: The hole must be much smaller than the wavelength of visible light for the electrons to show diffraction.

39.42. **IDENTIFY:** The alpha particles and protons behave as waves and exhibit circular-aperture diffraction after passing through the hole.

SET UP: For a round hole, the first dark ring occurs at the angle θ for which $\sin\theta = 1.22\lambda/D$, where D is the diameter of the hole. The de Broglie wavelength for a particle is $\lambda = h/p = h/mv$.

EXECUTE: Taking the ratio of the sines for the alpha particle and proton gives

$$\frac{\sin\theta_\alpha}{\sin\theta_\text{p}} = \frac{1.22\lambda_\alpha}{1.22\lambda_\text{p}} = \frac{\lambda_\alpha}{\lambda_\text{p}}$$

The de Broglie wavelength gives $\lambda_\text{p} = h/p_\text{p}$ and $\lambda_\alpha = h/p_\alpha$, so $\frac{\sin\theta_\alpha}{\sin\theta_\text{p}} = \frac{h/p_\alpha}{h/p_\text{p}} = \frac{p_\text{p}}{p_\alpha}$. Using $K = p^2/2m$, we have $p = \sqrt{2mK}$. Since the alpha particle has twice the charge of the proton and both are accelerated through the same potential difference, $K_\alpha = 2K_\text{p}$. Therefore $p_\text{p} = \sqrt{2m_\text{p}K_\text{p}}$ and $p_\alpha = \sqrt{2m_\alpha K_\alpha} = \sqrt{2m_\alpha(2K_\text{p})} = \sqrt{4m_\alpha K_\text{p}}$. Substituting these quantities into the ratio of the sines gives

$$\frac{\sin\theta_\alpha}{\sin\theta_\text{p}} = \frac{p_\text{p}}{p_\alpha} = \frac{\sqrt{2m_\text{p}K_\text{p}}}{\sqrt{4m_\alpha K_\text{p}}} = \sqrt{\frac{m_\text{p}}{2m_\alpha}}$$

Solving for $\sin\theta_\alpha$ gives $\sin\theta_\alpha = \sqrt{\frac{1.67\times10^{-27}\ \text{kg}}{2(6.64\times10^{-27}\ \text{kg})}}\ \sin15.0°$ and $\theta_\alpha = 5.3°$.

EVALUATE: Since $\sin\theta$ is inversely proportional to the mass of the particle, the larger-mass alpha particles form their first dark ring at a smaller angle than the ring for the lighter protons.

39.43. **IDENTIFY:** Both the electrons and photons behave like waves and exhibit single-slit diffraction after passing through their respective slits.

SET UP: The energy of the photon is $E = hc/\lambda$ and the de Broglie wavelength of the electron is $\lambda = h/mv = h/p$. Destructive interference for a single slit first occurs when $a \sin\theta = \lambda$.

EXECUTE: **(a)** For the photon: $\lambda = hc/E$ and $a \sin\theta = \lambda$. Since the a and θ are the same for the photons and electrons, they must both have the same wavelength. Equating these two expressions for λ gives $a \sin\theta = hc/E$. For the electron, $\lambda = h/p = \frac{h}{\sqrt{2mK}}$ and $a \sin\theta = \lambda$. Equating these two expressions for λ gives $a \sin\theta = \frac{h}{\sqrt{2mK}}$.

Equating the two expressions for $a\sin\theta$ gives $hc/E = \frac{h}{\sqrt{2mK}}$, which gives $E = c\sqrt{2mK} = (4.05\times10^{-7}\ \text{J}^{1/2})\sqrt{K}$

(b) $\frac{E}{K} = \frac{c\sqrt{2mK}}{K} = \sqrt{\frac{2mc^2}{K}}$. Since $v \ll c$, $mc^2 > K$, so the square root is >1. Therefore $E/K > 1$, meaning that the photon has more energy than the electron.

EVALUATE: As we have seen in Problem 39.10, when a photon and a particle have the same wavelength, the photon has more energy than the particle.

39.44. According to Eq.(35.4) $\lambda = \frac{d\sin\theta}{m} = \frac{(40.0\times10^{-6}\ \text{m})\sin(0.0300\ \text{rad})}{2} = 600\ \text{nm}$. The velocity of an electron with this wavelength is given by Eq.(39.1)

$$v = \frac{p}{m} = \frac{h}{m\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(9.11\times10^{-31}\ \text{kg})(600\times10^{-9}\ \text{m})} = 1.21\times10^{3}\ \text{m/s}.$$

Since this velocity is much smaller than c we can calculate the energy of the electron classically

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31}\ \text{kg})(1.21\times10^3\ \text{m/s})^2 = 6.70\times10^{-25}\ \text{J} = 4.19\ \mu\text{eV}.$$

39.45. The de Broglie wavelength of the blood cell is

$$\lambda = \frac{h}{mv} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(1.00\times10^{-14}\ \text{kg})(4.00\times10^{-3}\ \text{m/s})} = 1.66\times10^{-17}\text{m}.$$

We need not be concerned about wave behavior.

39.46. **(a)** $\lambda = \dfrac{h}{p} = \dfrac{h\left(1-\dfrac{v^2}{c^2}\right)^{1/2}}{mv} \Rightarrow \lambda^2 m^2 v^2 = h^2\left(1-\dfrac{v^2}{c^2}\right) = h^2 - \dfrac{h^2v^2}{c^2} \Rightarrow \lambda^2m^2v^2 + h^2\dfrac{v^2}{c^2} = h^2$

$$\Rightarrow v^2 = \frac{h^2}{\left(\lambda^2m^2 + \dfrac{h^2}{c^2}\right)} = \frac{c^2}{\left(\dfrac{\lambda^2m^2c^2}{h^2}+1\right)} \Rightarrow v = \frac{c}{\left(1+\left(\dfrac{mc\lambda}{h}\right)^2\right)^{1/2}}.$$

(b) $v = \dfrac{c}{\left(1+\left(\dfrac{\lambda}{(h/mc)}\right)^2\right)^{1/2}} \approx c\left(1-\dfrac{1}{2}\left(\dfrac{mc\lambda}{h}\right)^2\right) = (1-\Delta)c.\quad \Delta = \dfrac{m^2c^2\lambda^2}{2h^2}.$

(c) $\lambda = 1.00\times10^{-15}\,\text{m} << \dfrac{h}{mc}.$

$$\Delta = \frac{(9.11\times10^{-31}\ \text{kg})^2(3.00\times10^8\ \text{m/s})^2(1.00\times10^{-15}\text{m})^2}{2(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2} = 8.50\times10^{-8}$$

$\Rightarrow v = (1-\Delta)c = (1-8.50\times10^{-8})c.$

39.47. **(a)** Recall $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}} = \dfrac{h}{\sqrt{2mq\Delta V}}$. So for an electron:

$$\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(1.60\times10^{-19}\ \text{C})(125\ \text{V})}} \Rightarrow \lambda = 1.10\times10^{-10}\ \text{m}.$$

(b) For an alpha particle: $\lambda = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(6.64\times10^{-27}\ \text{kg})2(1.60\times10^{-19}\ \text{C})(125\ \text{V})}} = 9.10\times10^{-13}\ \text{m}.$

39.48. **IDENTIFY** and **SET UP:** The minimum uncertainty product is $\Delta x\Delta p_x = \dfrac{h}{2\pi}$. $\Delta x = r_1$, where r_1 is the radius of the $n=1$ Bohr orbit. In the $n=1$ Bohr orbit, $mv_1r_1 = \dfrac{h}{2\pi}$ and $p_1 = mv_1 = \dfrac{h}{2\pi r_1}$.

EXECUTE: $\Delta p_x = \dfrac{h}{2\pi\Delta x} = \dfrac{h}{2\pi r_1} = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(0.529\times10^{-10}\ \text{m})} = 2.0\times10^{-24}\ \text{kg}\cdot\text{m/s}$. This is the same as the magnitude of the momentum of the electron in the $n=1$ Bohr orbit.

EVALUATE: Since the momentum is the same order of magnitude as the uncertainty in the momentum, the uncertainty principle plays a large role in the structure of atoms.

39.49. **IDENTIFY** and **SET UP:** Combining the two equations in the hint gives $PC = \sqrt{K(K+2mc^2)}$ and $\lambda = \dfrac{hc}{\sqrt{K(K+2mc^2)}}$.

EXECUTE: **(a)** With $K = 3mc^2$ this becomes $\lambda = \dfrac{hc}{\sqrt{3mc^2(3mc^2+2mc^2)}} = \dfrac{h}{\sqrt{15}mc}$.

(b) (i) $K = 3mc^2 = 3(9.109\times10^{-31}\ \text{kg})(2.998\times10^8\ \text{m/s})^2 = 2.456\times10^{-13}\ \text{J} = 1.53\ \text{MeV}$

$$\lambda = \frac{h}{\sqrt{15}mc} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{15}(9.109\times10^{-31}\ \text{kg})(2.998\times10^8\ \text{m/s})} = 6.26\times10^{-13}\ \text{m}$$

(ii) K is proportional to m, so for a proton $K = (m_p/m_e)(1.53\ \text{MeV}) = 1836(1.53\ \text{MeV}) = 2810\ \text{MeV}$

λ is proportional to $1/m$, so for a proton $\lambda = (m_e/m_p)(6.26\times10^{-13}\ \text{m}) = (1/1836)(6.626\times10^{-13}\ \text{m}) = 3.41\times10^{-16}\ \text{m}$

EVALUATE: The proton has a larger rest mass energy so its kinetic energy is larger when $K = 3mc^2$. The proton also has larger momentum so has a smaller λ.

39.50. **(a)** $\dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi(5.0\times10^{-15}\ \text{m})}=2.1\times10^{-20}\ \text{kg}\cdot\text{m/s}.$

(b) $K=\sqrt{(pc)^2+(mc^2)^2}-mc^2=1.3\times10^{-13}\ \text{J}=0.82\ \text{MeV}.$

(c) The result of part (b), about $1\ \text{MeV}=1\times10^6\ \text{eV}$, is many orders of magnitude larger than the potential energy of an electron in a hydrogen atom.

39.51. **(a) IDENTIFY** and **SET UP:** $\Delta x\Delta p_x\ge h/2\pi$

Estimate Δx as $\Delta x\approx5.0\times10^{-15}$ m.

EXECUTE: Then the minimum allowed Δp_x is $\Delta p_x\approx\dfrac{h}{2\pi\Delta x}=\dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(5.0\times10^{-15}\ \text{m})}=2.1\times10^{-20}\ \text{kg}\cdot\text{m/s}$

(b) IDENTIFY and **SET UP:** Assume $p\approx2.1\times10^{-20}\ \text{kg}\cdot\text{m/s}$. Use Eq.(37.39) to calculate *E*, and then $K=E-mc^2$.

EXECUTE: $E=\sqrt{(mc^2)^2+(pc)^2}$

$mc^2=(9.109\times10^{-31}\ \text{kg})(2.998\times10^8\ \text{m/s})^2=8.187\times10^{-14}\ \text{J}$

$pc=(2.1\times10^{-20}\ \text{kg}\cdot\text{m/s})(2.998\times10^8\ \text{m/s})=6.296\times10^{-12}\ \text{J}$

$E=\sqrt{(8.187\times10^{-14}\ \text{J})^2+(6.296\times10^{-12}\ \text{J})^2}=6.297\times10^{-12}\ \text{J}$

$K=E-mc^2=6.297\times10^{-12}\ \text{J}-8.187\times10^{-14}\ \text{J}=6.215\times10^{-12}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J})=39\ \text{MeV}$

(c) IDENTIFY and **SET UP:** The Coulomb potential energy for a pair of point charges is given by Eq.(23.9). The proton has charge +*e* and the electron has charge –*e*.

EXECUTE: $U=-\dfrac{ke^2}{r}=-\dfrac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.602\times10^{-19}\ \text{C})^2}{5.0\times10^{-15}\ \text{m}}=-4.6\times10^{-14}\ \text{J}=-0.29\ \text{MeV}$

EVALUATE: The kinetic energy of the electron required by the uncertainty principle would be much larger than the magnitude of the negative Coulomb potential energy. The total energy of the electron would be large and positive and the electron could not be bound within the nucleus.

39.52. **(a)** Take the direction of the electron beam to be the *x*-direction and the direction of motion perpendicular to the beam to be the *y*-direction. $\Delta v_y=\dfrac{\Delta p_y}{m}=\dfrac{h}{2\pi m\Delta y}=\dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(9.11\times10^{-31}\ \text{kg})(0.50\times10^{-3}\ \text{m})}=0.23\ \text{m/s}$

(b) The uncertainty Δr in the position of the point where the electrons strike the screen is

$$\Delta r=\Delta v_y t=\frac{\Delta p_y}{m}\frac{x}{v_x}=\frac{h}{2\pi m\Delta y}\frac{x}{\sqrt{2K/m}}=9.56\times10^{-10}\ \text{m},$$

(c) This is far too small to affect the clarity of the picture.

39.53. **IDENTIFY** and **SET UP:** $\Delta E\Delta t\ge\dfrac{h}{2\pi}$. Take the minimum uncertainty product, so $\Delta E=\dfrac{h}{2\pi\Delta t}$, with $\Delta t=8.4\times10^{-17}\ \text{s}$. $m=264m_e$. $\Delta m=\dfrac{\Delta E}{c^2}$.

EXECUTE: $\Delta E=\dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(8.4\times10^{-17}\ \text{s})}=1.26\times10^{-18}\ \text{J}$. $\Delta m=\dfrac{1.26\times10^{-18}\ \text{J}}{(3.00\times10^8\ \text{m/s})^2}=1.4\times10^{-35}\ \text{kg}$.

$\dfrac{\Delta m}{m}=\dfrac{1.4\times10^{-35}\ \text{kg}}{(264)(9.11\times10^{-31}\ \text{kg})}=5.8\times10^{-8}$

39.54. **IDENTIFY:** The insect behaves like a wave as it passes through the hole in the screen.

SET UP: **(a)** For wave behavior to show up, the wavelength of the insect must be of the order of the diameter of the hole. The de Broglie wavelength is $\lambda=h/mv$.

EXECUTE: The de Broglie wavelength of the insect must be of the order of the diameter of the hole in the screen, so $\lambda\approx5.00$ mm. The de Broglie wavelength gives

$$v=\frac{h}{m\lambda}=\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(1.25\times10^{-6}\ \text{kg}\right)\left(0.00400\ \text{m}\right)}=1.33\times10^{-25}\ \text{m/s}$$

(b) $t=x/v=(0.000500\ \text{m})/(1.33\times10^{-25}\ \text{m/s})=3.77\times10^{21}\ \text{s}=1.4\times10^{10}\ \text{yr}$

The universe is about 14 billion years old (1.4×10^{10} yr), so this time would be about 85,000 times the age of the universe.

EVALUATE: Don't expect to see a diffracting insect! Wave behavior of particles occurs only at the very small scale.

39.55. **IDENTIFY** and **SET UP:** Use Eq.(39.1) to relate your wavelength and speed.

EXECUTE: **(a)** $\lambda=\dfrac{h}{mv}$, so $v=\dfrac{h}{m\lambda}=\dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(60.0\ \text{kg})(1.0\ \text{m})}=1.1\times10^{-35}\ \text{m/s}$

(b) $t = \dfrac{\text{distance}}{\text{velocity}} = \dfrac{0.80 \text{ m}}{1.1\times10^{-35} \text{ m/s}} = 7.3\times10^{34} \text{ s}(1 \text{ y}/3.156\times10^{7} \text{ s}) = 2.3\times10^{27} \text{ y}$

Since you walk through doorways much more quickly than this, you will not experience diffraction effects.

EVALUATE: A 1 kg object moving at 1 m/s has a de Broglie wavelength $\lambda = 6.6\times10^{-34}$ m, which is exceedingly small. An object like you has a very, very small λ at ordinary speeds and does not exhibit wavelike properties.

39.56. **(a)** $E = 2.58 \text{ eV} = 4.13\times10^{-19}$ J, with a wavelength of $\lambda = \dfrac{hc}{E} = 4.82\times10^{-7} \text{ m} = 482 \text{ nm}$

(b) $\Delta E = \dfrac{h}{2\pi\Delta t} = \dfrac{(6.63\times10^{-34} \text{ J}\cdot\text{s})}{2\pi(1.64\times10^{-7} \text{ s})} = 6.43\times10^{-28} \text{ J} = 4.02\times10^{-9} \text{ eV}.$

(c) $\lambda E = hc$, so $(\Delta\lambda)E + \lambda\Delta E = 0$, and $|\Delta E/E| = |\Delta\lambda/\lambda|$, so

$$\Delta\lambda = \lambda|\Delta E/E| = (4.82\times10^{-7} \text{ m})\left(\frac{6.43\times10^{-28} \text{ J}}{4.13\times10^{-19} \text{ J}}\right) = 7.50\times10^{-16} \text{ m} = 7.50\times10^{-7} \text{ nm}.$$

39.57. **IDENTIFY:** The electrons behave as waves whose wavelength is equal to the de Broglie wavelength.

SET UP: The de Broglie wavelength is $\lambda = h/mv$, and the energy of a photon is $E = hf = hc/\lambda$.

EXECUTE: **(a)** Use the de Broglie wavelength to find the speed of the electron.

$$v = \frac{h}{m\lambda} = \frac{6.626\times10^{-34} \text{ J}\cdot\text{s}}{(9.11\times10^{-31} \text{ kg})(1.00\times10^{-9} \text{ m})} = 7.27 \times 10^{5} \text{ m/s}$$

which is much less than the speed of light, so it is nonrelativistic.

(b) Energy conservation gives $eV = ½ mv^2$.

$$V = mv^2/2e = (9.11 \times 10^{-31} \text{ kg})(7.27 \times 10^{5} \text{ m/s})^2/[2(1.60 \times 10^{-19} \text{ C})] = 1.51 \text{ V}$$

(c) $K = eV = e(1.51 \text{ V}) = 1.51$ eV, which is about ¼ the potential energy of the NaCl crystal, so the electron would not be too damaging.

(d) $E = hc/\lambda = (4.136 \times 10^{-15} \text{ eV s})(3.00 \times 10^{8} \text{ m/s})/(1.00 \times 10^{-9} \text{ m}) = 1240 \text{ eV}$

which would certainly destroy the molecules under study.

EVALUATE: As we have seen in Problems 39.10 and 39.43, when a particle and a photon have the same wavelength, the photon has much more energy.

39.58. $\sin\theta' = \dfrac{\lambda'}{\lambda}\sin\theta$, and $\lambda' = (h/p') = (h/\sqrt{2mE'})$, and so $\theta' = \arcsin\left(\dfrac{h}{\lambda\sqrt{2mE'}}\sin\theta\right)$.

$$\theta' = \arcsin\left(\frac{(6.63\times10^{-34} \text{ J}\cdot\text{s})\sin 35.8°}{(3.00\times10^{-11} \text{ m})\sqrt{2(9.11\times10^{-31} \text{ kg})(4.50\times10^{+3})(1.60\times10^{-19} \text{ J/eV})}}\right) = 20.9°$$

39.59. **(a)** The maxima occur when $2d\sin\theta = m\lambda$ as described in Section 38.7.

(b) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$. $\lambda = \dfrac{(6.63\times10^{-34} \text{ J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-37} \text{ kg})(71.0 \text{ eV})(1.60\times10^{-19} \text{ J/eV})}} = 1.46\times10^{-10}\text{m} = 0.146 \text{ nm}$.

$\theta = \sin^{-1}\left(\dfrac{m\lambda}{2d}\right)$ (Note: This m is the order of the maximum, not the mass.)

$$\Rightarrow \sin^{-1}\left(\frac{(1)(1.46\times10^{-10} \text{ m})}{2(9.10\times10^{-11} \text{ m})}\right) = 53.3°.$$

(c) The work function of the metal acts like an attractive potential increasing the kinetic energy of incoming electrons by $e\phi$. An increase in kinetic energy is an increase in momentum that leads to a smaller wavelength. A smaller wavelength gives a smaller angle θ (see part (b)).

39.60. **(a)** Using the given approximation, $E = \dfrac{1}{2}\left((h/x)^2/m + kx^2\right)$, $(dE/dx) = kx - (h^2/mx^3)$, and the minimum energy occurs when $kx = (h^2/mx^3)$, or $x^2 = \dfrac{h}{\sqrt{mk}}$. The minimum energy is then $h\sqrt{k/m}$.

(b) They are the same.

39.61. **(a) IDENTIFY** and **SET UP:** $U = A|x|$. Eq.(7.17) relates force and potential. The slope of the function $A|x|$ is not continuous at $x = 0$ so we must consider the regions $x > 0$ and $x < 0$ separately.

EXECUTE: For $x > 0$, $|x| = x$ so $U = Ax$ and $F = -\dfrac{d(Ax)}{dx} = -A$. For $x < 0$, $|x| = -x$ so $U = -Ax$ and $F = -\dfrac{d(-Ax)}{dx} = +A$. We can write this result as $F = -A|x|/x$, valid for all x except for $x = 0$.

(b) IDENTIFY and SET UP: Use the uncertainty principle, expressed as $\Delta p \Delta x \approx h,$ and as in Problem 39.50 estimate Δp by p and Δx by x. Use this to write the energy E of the particle as a function of x. Find the value of x that gives the minimum E and then find the minimum E.

EXECUTE: $E = K + U = \dfrac{p^2}{2m} + A|x|$

$px \approx h,$ so $p \approx h/x$

Then $E \approx \dfrac{h^2}{2mx^2} + A|x|.$

For $x > 0, E = \dfrac{h^2}{2mx^2} + Ax.$

To find the value of x that gives minimum E set $\dfrac{dE}{dx} = 0.$

$$0 = \frac{-2h^2}{2mx^3} + A$$

$$x^3 = \frac{h^2}{mA} \text{ and } x = \left(\frac{h^2}{mA}\right)^{1/3}$$

With this x the minimum E is

$$E = \frac{h^2}{2m}\left(\frac{mA}{h^2}\right)^{2/3} + A\left(\frac{h^2}{mA}\right)^{1/3} = \frac{1}{2}h^{2/3}m^{-1/3}A^{2/3} + h^{2/3}m^{-1/3}A^{2/3}$$

$$E = \tfrac{3}{2}\left(\frac{h^2A^2}{m}\right)^{1/3}$$

EVALUATE: The potential well is shaped like a V. The larger A is the steeper the slope of U and the smaller the region to which the particle is confined and the greater is its energy. Note that for the x that minimizes E, $2K = U$.

39.62. For this wave function, $\Psi^* = \psi_1^* e^{i\omega_1 t} + \psi_2^* e^{i\omega_2 t}$, so

$$\Psi^2 = \Psi^*\Psi = (\psi_1^* e^{i\omega_1 t} + \psi_2^* e^{i\omega_2 t})(\psi_1 e^{-i\omega_1 t} + \psi_2 e^{-i\omega_2 t}) = \psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_1^*\psi_2 e^{i(\omega_1 - \omega_2)t} + \psi_2^*\psi_1 e^{i(\omega_2 - \omega_1)t}.$$

The frequencies ω_1 and ω_2 are given as not being the same, so $|\Psi|^2$ is not time-independent, and Ψ is not the wave function for a stationary state.

39.63. The time-dependent equation, with the separated form for $\Psi(x, t)$ as given becomes

$$i\hbar\psi(-i\omega) = \left(-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi\right).$$

Since ψ is a solution of the time-independent solution with energy $E,$ the term in parenthesis is $E\psi,$ and so $\omega\hbar = E,$ and $\omega = (E/\hbar).$

39.64. **(a)** $\omega = 2\pi f = \dfrac{2\pi E}{h} = \dfrac{E}{\hbar}.$ $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{h}p = \dfrac{p}{\hbar}.$ $\hbar\omega = E = K = \dfrac{p^2}{2m} = \dfrac{(\hbar k)^2}{2m} \Rightarrow \omega = \dfrac{\hbar k^2}{2m}.$

(b) From Problem 39.63 the time-dependent Schrödinger's equation is $-\dfrac{\hbar^2}{2m}\dfrac{\partial^2\psi(x,t)}{\partial x^2} +$ $U(x)\psi(x,t) = i\hbar\dfrac{\partial\psi(x,t)}{\partial t}$. $U(x) = 0$ for a free particle, so $\dfrac{\partial^2\psi(x,t)}{\partial x^2} = -\dfrac{2mi}{\hbar}\dfrac{\partial\psi(x,t)}{\partial t}.$

Try $\psi(x,t) = \cos(kx - \omega t)$:

$$\frac{\partial\psi}{\partial t}(x,t) = A\omega\sin(kx - \omega t)$$

$$\frac{\partial\psi(x,t)}{\partial x} = -Ak\sin(kx - \omega t) \text{ and } \frac{\partial^2\psi}{\partial x^2} = Ak^2\cos(kx - \omega t).$$

Putting this into the Schrödinger's equation, $Ak^2\cos(kx - \omega t) = -\left(\dfrac{2mi}{\hbar}\right)A\omega\sin(kx - \omega t).$

This is not generally true for all x and t so is not a solution.

(c) Try $\psi(x,t) = A\sin(kx - \omega t)$:

$$\frac{\partial\psi(x,t)}{\partial t} = -A\omega\cos(kx - \omega t)$$

$$\frac{\partial\psi(x,t)}{\partial x} = Ak\cos(kx - \omega t) \text{ and } \frac{\partial^2\psi(x,t)}{\partial x^2} = -Ak^2\sin(kx - \omega t).$$

Again, $-Ak^2\sin(kx - \omega t) = -\left(\frac{2mi}{\hbar}\right)A\omega\cos(kx - \omega t)$ is not generally true for *all* x and t so is not a solution.

(d) Try $\psi(x,t) = A\cos(kx - \omega t) + B\sin(kx - \omega t)$:

$$\frac{\partial\psi(x,t)}{\partial t} = +A\omega\sin(kx - \omega t) - B\omega\cos(kx - \omega t)$$

$$\frac{\partial\psi(x,t)}{\partial x} = -Ak\sin(kx - \omega t) + Bk\cos(kx - \omega t) \text{ and } \frac{\partial^2\psi(x,t)}{\partial x^2} = -Ak^2\cos((kx - \omega t) - Bk^2\sin(kx - \omega t).$$

Putting this into the Schrödinger's equation,

$$-Ak^2\cos(kx - \omega t) - Bk^2\sin(kx - \omega t) = -\frac{2mi}{\hbar}(+A\omega\sin(kx - \omega t) - B\omega\cos(kx - \omega t)).$$

Recall that $\omega = \frac{\hbar k^2}{2m}$. Collect sin and cos terms.

$(A + iB)k^2\cos(kx - \omega t) + (iA - B)k^2\sin(kx - \omega t) = 0.$ This is only true if $B = iA$.

39.65. **(a) IDENTIFY** and **SET UP:** Let the *y*-direction be from the thrower to the catcher, and let the *x*-direction be horizontal and perpendicular to the *y*-direction. A cube with volume $V = 125\text{ cm}^3 = 0.125\times10^{-3}\text{ m}^3$ has side length $l = V^{1/3} = (0.125\times10^{-3}\text{ m}^3)^{1/3} = 0.050\text{ m}$. Thus estimate Δx as $\Delta x \approx 0.050\text{ m}$. Use the uncertainty principle to estimate Δp_x.

EXECUTE: $\Delta x\Delta p_x \geq h/2\pi$ then gives $\Delta p_x \approx \frac{h}{2\pi\Delta x} = \frac{0.0663\text{ J}\cdot\text{s}}{2\pi(0.050\text{ m})} = 0.21\text{ kg}\cdot\text{m/s}$

(The value of *h* in this other universe has been used.)

(b) IDENTIFY and **SET UP:** $\Delta x = (\Delta v_x)t$ is the uncertainty in the *x*-coordinate of the ball when it reaches the catcher, where *t* is the time it takes the ball to reach the second student. Obtain Δv_x from Δp_x.

EXECUTE: The uncertainty in the ball's horizontal velocity is $\Delta v_x = \frac{\Delta p_x}{m} = \frac{0.21\text{ kg}\cdot\text{m/s}}{0.25\text{ kg}} = 0.84\text{ m/s}$

The time it takes the ball to travel to the second student is $t = \frac{12\text{ m}}{6.0\text{ m/s}} = 2.0\text{ s}$. The uncertainty in the *x*-coordinate of the ball when it reaches the second student that is introduced by Δv_x is $\Delta x = (\Delta v_x)t = (0.84\text{ m/s})(2.0\text{ s}) = 1.7\text{ m}$. The ball could miss the second student by about 1.7 m.

EVALUATE: A game of catch would be very different in this universe. We don't notice the effects of the uncertainty principle in everyday life because *h* is so small.

39.66. **(a)** $|\psi^2| = A^2x^2e^{-2(\alpha x^2 + \beta y^2 + \gamma z^2)}$. To save some algebra, let $u = x^2$, so that $|\psi|^2 = ue^{-2\alpha u}f(y,z)$.

$\frac{\partial}{\partial u}|\psi|^2 = (1 - 2\alpha u)|\psi|^2$; the maximum occurs at $u_0 = \frac{1}{2\alpha}$, $x_0 = \pm\frac{1}{\sqrt{2\alpha}}$.

(b) ψ vanishes at $x = 0$, so the probability of finding the particle in the $x = 0$ plane is zero. The wave function vanishes for $x = \pm\infty$.

39.67. **(a) IDENTIFY** and **SET UP:** The probability is $P = |\psi|^2\,dV$ with $dV = 4\pi r^2 dr$

EXECUTE: $|\psi|^2 = A^2e^{-2\alpha r^2}$ so $P = 4\pi A^2r^2e^{-2\alpha r^2}\,dr$

(b) IDENTIFY and **SET UP:** *P* is maximum where $\frac{dP}{dr} = 0$

EXECUTE: $\frac{d}{dr}(r^2e^{-2\alpha r^2}) = 0$

$2re^{-2\alpha r^2} - 4\alpha r^3e^{-2\alpha r^2} = 0$ and this reduces to $2r - 4\alpha r^3 = 0$

$r = 0$ is a solution of the equation but corresponds to a minimum not a maximum. Seek *r* not equal to 0 so divide by *r* and get $2 - 4\alpha r^2 = 0$

This gives $r = \frac{1}{\sqrt{2\alpha}}$ (We took the positive square root since r must be positive.)

EVALUATE: This is different from the value of r, $r = 0$, where $|\psi|^2$ is a maximum. At $r = 0$, $|\psi|^2$ has a maximum but the volume element $dV = 4\pi r^2 dr$ is zero here so P does not have a maximum at $r = 0$.

39.68. **(a)** $B(k) = e^{-\alpha^2 k^2}$ $\quad B(0) = B_{\max} = 1$

$$B(k_{\text{h}}) = \frac{1}{2} = e^{-\alpha^2 k_{\text{h}}^2} \Rightarrow \ln(1/2) = -\alpha^2 k_{\text{h}}^2 \;\Rightarrow k_{\text{h}} = \frac{1}{\alpha}\sqrt{\ln(2)} = \omega_k.$$

(b) Using integral tables: $\psi(x) = \int_0^\infty e^{-\alpha^2 k^2} \cos kx dk = \frac{\sqrt{\pi}}{2\alpha}(e^{-x^2/4\alpha^2})$. $\psi(x)$ is a maximum when $x = 0$.

(c) $\psi(x_{\text{h}}) = \frac{\sqrt{\pi}}{4\alpha}$ when $e^{-x_{\text{h}}^2/4\alpha^2} = \frac{1}{2} \Rightarrow \frac{-x_{\text{h}}^2}{4\alpha^2} = \ln(1/2) \;\Rightarrow x_{\text{h}} = 2\alpha\sqrt{\ln 2} = \omega_{\text{x}}$

(d) $\omega_p \omega_x = \left(\frac{h\omega_k}{2\pi}\right)\omega_x = \frac{h}{2\pi}\left(\frac{1}{\alpha}\sqrt{\ln 2}\right)\left(2\alpha\sqrt{\ln 2}\right) = \frac{h}{2\pi}(2\ln 2) = \frac{h \ln 2}{\pi}$.

39.69. **(a)** $\psi(x) = \int_0^\infty B(k)\cos kx dk = \int_0^{k_0}\left(\frac{1}{k_0}\right)\cos kx dk = \left.\frac{\sin kx}{k_0 x}\right|_0^{k_0} = \frac{\sin k_0 x}{k_0 x}$

(b) $\psi(x)$ has a maximum value at the origin $x = 0$. $\psi(x_0) = 0$ when $k_0 x_0 = \pi$ so $x_0 = \frac{\pi}{k_0}$. Thus the width of this function $w_x = 2x_0 = \frac{2\pi}{k_0}$. If $k_0 = \frac{2\pi}{L}, w_x = L$. $B(k)$ versus k is graphed in Figure 39.69a. The graph of $\psi(x)$ versus x is in Figure 39.69b.

(c) If $k_0 = \frac{\pi}{L} w_x = 2L$.

(d) $w_p w_x = \left(\frac{hw_k}{2\pi}\right)\left(\frac{2\pi}{k_0}\right) = \frac{hw_k}{k_0} = \frac{hk_0}{k_0} = h$. The uncertainty principle states that $w_p w_x \ge \frac{h}{2\pi}$. For us, no matter what k_0 is, $w_p w_x = h$, which is greater than $\frac{h}{2\pi}$.

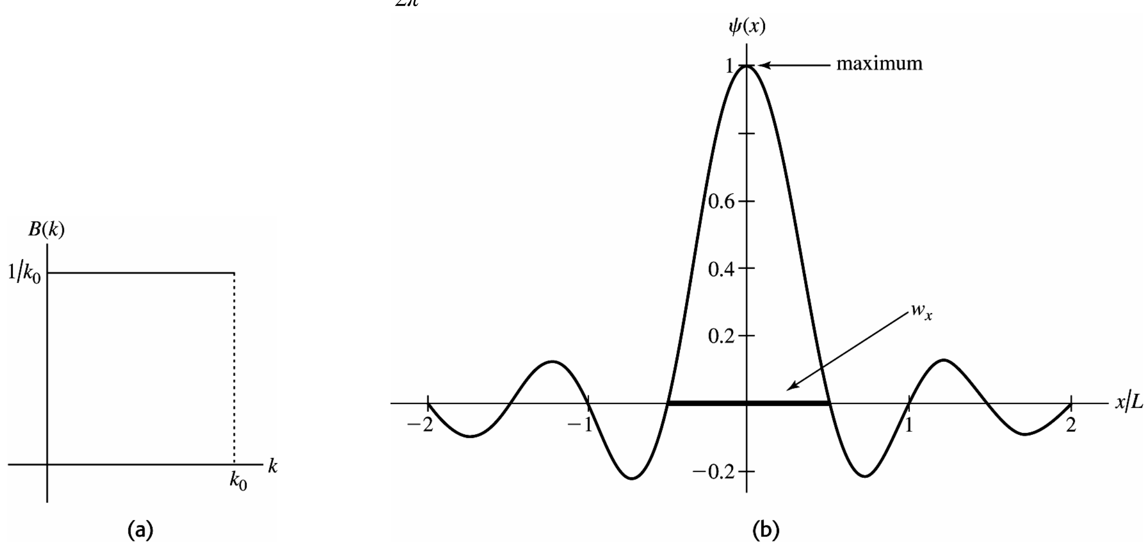


Figure 39.69

39.70. **(a)** For a standing wave, $n\lambda = 2L$, and $E_n = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{n^2 h^2}{8mL^2}$.

(b) With $L = a_0 = 0.5292 \times 10^{-10}$ m, $E_1 = 2.15 \times 10^{-17}$ J $= 134$ eV.

39.71. Time of flight of the marble, from a free-fall kinematic equation is just $t=\sqrt{\frac{2y}{g}}=\sqrt{\frac{2(25.0\text{ m})}{9.81\text{ m/s}^2}}=2.26\text{ s}$.

$$\Delta x_f=\Delta x_i+(\Delta v_x)t=\Delta x_i+\left(\frac{\Delta p_x}{m}\right)t=\frac{ht}{2\pi\Delta x_i m}+\Delta x_i$$

To minimize Δx_f with respect to Δx_i, $\frac{d(\Delta x_f)}{d(\Delta x_i)}=0=\frac{-ht}{2\pi m(\Delta x_i)^2}+1$

$$\Rightarrow \Delta x_i(\text{min})=\sqrt{\left(\frac{ht}{2\pi m}\right)}$$

$$\Rightarrow \Delta x_f(\text{min})=\sqrt{\frac{ht}{2\pi m}}+\sqrt{\frac{ht}{2\pi m}}=\sqrt{\frac{2ht}{\pi m}}=\sqrt{\frac{2(6.63\times10^{-34}\text{ J}\cdot\text{s})(2.26\text{ s})}{\pi(0.0200\text{ kg})}}=2.18\times10^{-16}\text{ m}=2.18\times10^{-7}\text{ nm.}$$

40

QUANTUM MECHANICS

40.1. **IDENTIFY** and **SET UP:** The energy levels for a particle in a box are given by $E_n = \dfrac{n^2h^2}{8mL^2}$.

EXECUTE: **(a)** The lowest level is for $n=1$, and $E_1 = \dfrac{(1)(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{8(0.20\ \text{kg})(1.5\ \text{m})^2} = 1.2\times10^{-67}\ \text{J}.$

(b) $E = \dfrac{1}{2}mv^2$ so $v = \sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2(1.2\times10^{-67}\ \text{J})}{0.20\ \text{kg}}} = 1.1\times10^{-33}\ \text{m/s}.$ If the ball has this speed the time it would take it to travel from one side of the table to the other is $t = \dfrac{1.5\ \text{m}}{1.1\times10^{-33}\ \text{m/s}} = 1.4\times10^{33}\ \text{s}.$

(c) $E_1 = \dfrac{h^2}{8mL^2},\ E_2 = 4E_1,$ so $\Delta E = E_2 - E_1 = 3E_1 = 3(1.2\times10^{-67}\ \text{J}) = 3.6\times10^{-67}\ \text{J}$

(d) EVALUATE: No, quantum mechanical effects are not important for the game of billiards. The discrete, quantized nature of the energy levels is completely unobservable.

40.2. $L = \dfrac{h}{\sqrt{8mE_1}}$

$$L = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{8(1.673\times10^{-27}\ \text{kg})(5.0\times10^{6}\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}} = 6.4\times10^{-15}\ \text{m}.$$

40.3. **IDENTIFY:** An electron in the lowest energy state in this box must have the same energy as it would in the ground state of hydrogen.

SET UP: The energy of the nth level of an electron in a box is $E_n = \dfrac{nh^2}{8mL^2}$.

EXECUTE: An electron in the ground state of hydrogen has an energy of $-13.6\ \text{eV}$, so find the width corresponding to an energy of $E_1 = 13.6\ \text{eV}$. Solving for L gives

$$L = \frac{h}{\sqrt{8mE_1}} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{8(9.11\times10^{-31}\ \text{kg})(13.6\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}} = 1.66\times10^{-10}\ \text{m}.$$

EVALUATE: This width is of the same order of magnitude as the diameter of a Bohr atom with the electron in the K shell.

40.4. **(a)** The energy of the given photon is

$$E = hf = h\frac{c}{\lambda} = (6.63\times10^{-34}\ \text{J}\cdot\text{s})\frac{(3.00\times10^{3}\ \text{m/s})}{(122\times10^{-9}\ \text{m})} = 1.63\times10^{-18}\ \text{J}.$$

The energy levels of a particle in a box are given by Eq.40.9

$$\Delta E = \frac{h^2}{8mL^2}(n^2 - n_2).\quad L = \sqrt{\frac{h^2(n_1^2 - n_2)}{8m\Delta E}} = \sqrt{\frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2(2^2-1^2)}{8(9.11\times10^{-31}\ \text{kg})(1.63\times10^{-20}\ \text{J})}} = 3.33\times10^{-10}\ \text{m}.$$

(b) The ground state energy for an electron in a box of the calculated dimensions is

$$E = \frac{h^2}{8mL^2} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{8(9.11\times10^{-31}\ \text{kg})(3.33\times10^{-10}\ \text{m})^2} = 5.43\times10^{-19}\ \text{J} = 3.40\ \text{eV}\ \text{(one-third of the original photon energy),}$$

which does not correspond to the $-13.6\ \text{eV}$ ground state energy of the hydrogen atom. Note that the energy levels for a particle in a box are proportional to n^2, whereas the energy levels for the hydrogen atom are proportional to $-\dfrac{1}{n^2}$.

40.5. **IDENTIFY** and **SET UP:** Eq.(40.9) gives the energy levels. Use this to obtain an expression for $E_2 - E_1$ and use the value given for this energy difference to solve for *L*.

EXECUTE: Ground state energy is $E_1 = \dfrac{h^2}{8mL^2}$; first excited state energy is $E_2 = \dfrac{4h^2}{8mL^2}$. The energy separation between these two levels is $\Delta E = E_2 - E_1 = \dfrac{3h^2}{8mL^2}$. This gives $L = h\sqrt{\dfrac{3}{8m\Delta E}} =$

$$L = 6.626\times10^{-34}\ \text{J}\cdot\text{s}\sqrt{\frac{3}{8(9.109\times10^{-31}\ \text{kg})(3.0\ \text{eV})(1.602\times10^{-19}\ \text{J/1 eV})}} = 6.1\times10^{-10}\ \text{m} = 0.61\ \text{nm}.$$

EVALUATE: This energy difference is typical for an atom and *L* is comparable to the size of an atom.

40.6. **(a)** The wave function for $n = 1$ vanishes only at $x = 0$ and $x = L$ in the range $0 \le x \le L$.
(b) In the range for x, the sine term is a maximum only at the middle of the box, $x = L/2$.
(c) The answers to parts (a) and (b) are consistent with the figure.

40.7. **IDENTIFY** and **SET UP:** For the $n = 2$ first excited state the normalized wave function is given by Eq.(40.13). $\psi_2(x) = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{2\pi x}{L}\right)$. $|\psi_2(x)|^2\,dx = \dfrac{2}{L}\sin^2\left(\dfrac{2\pi x}{L}\right)dx$. Examine $|\psi_2(x)|^2\,dx$ and find where it is zero and where it is maximum.

EXECUTE: **(a)** $|\psi_2|^2\,dx = 0$ implies $\sin\left(\dfrac{2\pi x}{L}\right) = 0$

$\dfrac{2\pi x}{L} = m\pi,\ m = 0,\ 1,\ 2,\ \ldots;\ x = m(L/2)$

For $m = 0,\ x = 0$; for $m = 1,\ x = L/2$; for $m = 2,\ x = L$

The probability of finding the particle is zero at $x = 0,\ L/2$, and L.

(b) $|\psi_2|^2\,dx$ is maximum when $\sin\left(\dfrac{2\pi x}{L}\right) = \pm1$

$\dfrac{2\pi x}{L} = m(\pi/2),\ m = 1,\ 3,\ 5,\ \ldots;\ x = m(L/4)$

For $m = 1,\ x = L/4$; for $m = 3,\ x = 3L/4$

The probability of finding the particle is largest at $x = L/4$ and $3L/4$.

(c) EVALUATE: The answers to part (a) correspond to the zeros of $|\psi|^2$ shown in Fig.40.5 in the textbook and the answers to part (b) correspond to the two values of *x* where $|\psi|^2$ in the figure is maximum.

40.8. $\dfrac{d^2\psi}{dx^2} = -k^2\psi$, and for ψ to be a solution of Eq.(40.3), $k^2 = E\dfrac{8\pi^2 m}{h^2} = E\dfrac{2m}{\hbar^2}$.
(b) The wave function must vanish at the rigid walls; the given function will vanish at $x = 0$ for any k, but to vanish at $x = L,\ kL = n\pi$ for integer n.

40.9. **(a) IDENTIFY** and **SET UP:** $\psi = A\cos kx$. Calculate $d\psi^2/dx^2$ and substitute into Eq.(40.3) to see if this equation is satisfied.

EXECUTE: Eq.(40.3): $-\dfrac{h^2}{8\pi^2 m}\dfrac{d^2\psi}{dx^2} = E\psi$

$\dfrac{d\psi}{dx} = A(-k\sin kx) = -Ak\sin kx$

$\dfrac{d^2\psi}{dx^2} = -Ak(k\cos kx) = -Ak^2\cos kx$

Thus Eq.(40.3) requires $-\dfrac{h^2}{8\pi^2 m}(-Ak^2\cos kx) = E(A\cos kx)$.

This says $-\dfrac{h^2k^2}{8\pi^2 m} = E;\ k = \dfrac{\sqrt{2mE}}{(h/2\pi)} = \dfrac{\sqrt{2mE}}{\hbar}$

$\psi = A\cos kx$ is a solution to Eq.(40.3) if $k = \dfrac{\sqrt{2mE}}{\hbar}$.

(b) EVALUATE: The wave function for a particle in a box with rigid walls at $x = 0$ and $x = L$ must satisfy the boundary conditions $\psi = 0$ at $x = 0$ and $\psi = 0$ at $x = L$. $\psi(0) = A\cos 0 = A$, since $\cos 0 = 1$. Thus ψ is not 0 at $x = 0$ and this wave function isn't acceptable because it doesn't satisfy the required boundary condition, even though it is a solution to the Schrödinger equation.

40.10. **(a)** The third excited state is $n = 4$, so

$$\Delta E = (4^2 - 1)\frac{h^2}{8mL^2} = \frac{15(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{8(9.11\times10^{-31}\ \text{kg})(0.125\times10^{-9}\ \text{m})^2} = 5.78\times10^{-17}\ \text{J} = 361\ \text{eV}.$$

(b) $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.0\times10^{8}\ \text{m/s})}{5.78\times10^{-17}\ \text{J}} = 3.44\ \text{nm}$

40.11. Recall $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}.$

(a) $E_1 = \dfrac{h^2}{8mL^2} \Rightarrow \lambda_1 = \dfrac{h}{\sqrt{2mh^2/8mL^2}} = 2L = 2(3.0\times10^{-10}\ \text{m}) = 6.0\times10^{-10}\ \text{m}.$ The wavelength is twice the width of the box. $p_1 = \dfrac{h}{\lambda_1} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{6.0\times10^{-10}\ \text{m}} = 1.1\times10^{-24}\ \text{kg}\cdot\text{m/s}$

(b) $E_2 = \dfrac{4h^2}{8mL^2} \Rightarrow \lambda_2 = L = 3.0\times10^{-10}\ \text{m}.$ The wavelength is the same as the width of the box.

$p_2 = \dfrac{h}{\lambda_2} = 2p_1 = 2.2\times10^{-24}\ \text{kg}\cdot\text{m/s}.$

(c) $E_3 = \dfrac{9h^2}{8mL^2} \Rightarrow \lambda_3 = \dfrac{2}{3}L = 2.0\times10^{-10}\ \text{m}.$ The wavelength is two-thirds the width of the box.

$p_3 = 3p_1 = 3.3\times10^{-24}\ \text{kg}\cdot\text{m/s}.$

40.12. **IDENTIFY:** If the given wave function is a solution to the Schrödinger equation, we will get an identity when we substitute that wave function into the Schrödinger equation.

SET UP: We must substitute the equation $\Psi(x,t) = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{n\pi x}{L}\right)e^{-iE_n t/\hbar}$ into the one-dimensional Schrödinger equation $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x).$

EXECUTE: Taking the second derivative of $\Psi(x,t)$ with respect to x gives $\dfrac{d^2\Psi(x,\ t)}{dx^2} = -\left(\dfrac{n\pi}{L}\right)^2\Psi(x,t)$

Substituting this result into $-\dfrac{\hbar^2}{2m}\dfrac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x),$ we get $\dfrac{\hbar^2}{2m}\left(\dfrac{n\pi}{L}\right)^2\Psi(x,t) = E\Psi(x,t)$ which gives $E_n = \dfrac{\hbar^2}{2m}\left(\dfrac{n\pi}{L}\right)^2,$ the energies of a particle in a box.

EVALUATE: Since this process gives us the energies of a particle in a box, the given wave function is a solution to the Schrödinger equation.

40.13. **(a)** Eq.(40.1): $\dfrac{-\hbar^2}{2m}\dfrac{d^2\psi}{dx^2} + U\psi = E\psi.$

Left-hand side: $\dfrac{-\hbar^2}{2m}\dfrac{d^2}{dx^2}(A\sin kx) + U_0 A\sin kx = \dfrac{\hbar^2k^2}{2m}A\sin kx + U_0 A\sin kx = \left(\dfrac{\hbar^2k^2}{2m} + U_0\right)\psi.$

But $\dfrac{\hbar^2k^2}{2m} + U_0 > U_0 > E$ for constant k. But $\dfrac{\hbar^2k^2}{2m} + U_0$ should equal $E \Rightarrow$ no solution.

(b) If $E > U_0$, then $\dfrac{\hbar^2k^2}{2m} + U_0 = E$ is consistent and so $\psi = A\sin kx$ is a solution of Eq.(40.1) for this case.

40.14. According to Eq.(40.17), the wavelength of the electron inside of the square well is given by $k = \dfrac{\sqrt{2mE}}{\hbar} \Rightarrow \lambda_{\text{in}} = \dfrac{h}{\sqrt{2m(3U_0)}}.$ By an analysis similar to that used to derive Eq.40.17, we can show that outside the box

$$\lambda_{\text{out}} = \frac{h}{\sqrt{2m(E-U_0)}} = \frac{h}{\sqrt{2m(2U_0)}}.$$

Thus, the ratio of the wavelengths is $\dfrac{\lambda_{\text{out}}}{\lambda_{\text{in}}} = \dfrac{\sqrt{2m(3U_0)}}{\sqrt{2m(2U_0)}} = \sqrt{\dfrac{3}{2}}.$

40.15. $E_1 = 0.625E_\infty = 0.625\dfrac{\pi^2\hbar^2}{2mL^2}$; $E_1 = 2.00\text{ eV} = 3.20\times10^{-19}\text{ J}$

$$L = \pi\hbar\left(\frac{0.625}{2(9.109\times10^{-31}\text{ kg})(3.20\times10^{-19}\text{ J})}\right)^{1/2} = 3.43\times10^{-10}\text{ m}$$

40.16. Since $U_0 = 6E_\infty$ we can use the result $E_1 = 0.625E_\infty$ from Section 40.3, so $U_0 - E_1 = 5.375E_\infty$ and the maximum wavelength of the photon would be

$$\lambda = \frac{hc}{U_0 - E_1} = \frac{hc}{(5.375)(h^2/8mL^2)} = \frac{8mL^2c}{(5.375)h}$$

$$\lambda = \frac{8(9.11\times10^{-31}\text{ kg})(1.50\times10^{-9}\text{ m})^2(3.00\times10^8\text{ m/s})}{(5.375)(6.63\times10^{-34}\text{ J}\cdot\text{s})} = 1.38\times10^{-6}\text{ m.}$$

40.17. Eq.(40.16): $\psi = A\sin\dfrac{\sqrt{2mE}}{\hbar}x + B\cos\dfrac{\sqrt{2mE}}{\hbar}x$

$$\frac{d^2\psi}{dx^2} = -A\left(\frac{2mE}{\hbar^2}\right)\sin\frac{\sqrt{2mE}}{\hbar}x - B\left(\frac{2mE}{\hbar^2}\right)\cos\frac{\sqrt{2mE}}{\hbar}x = \frac{-2mE}{\hbar^2}(\psi) = \text{Eq.(40.15).}$$

40.18. $\dfrac{d\psi}{dx} = \kappa(Ce^{\kappa x} - De^{-\kappa x})$, $\dfrac{d^2\psi}{dx^2} = \kappa^2(Ce^{\kappa x} + De^{-\kappa x}) = \kappa^2\psi$ for all constants C and D. Hence ψ is a solution to Eq.(40.1) for $-\dfrac{\hbar^2}{2m}\kappa^2 + U_0 = E$, or $\kappa = [2m(U_0 - E)]^{1/2}/\hbar$, and κ is real for $E < U_0$.

40.19. **IDENTIFY:** Find the transition energy ΔE and set it equal to the energy of the absorbed photon. Use $E = hc/\lambda$ to find the wavelength of the photon.

SET UP: $U_0 = 6E_\infty$, as in Fig.40.8 in the textbook, so $E_1 = 0.625E_\infty$ and $E_3 = 5.09E_\infty$ with $E_\infty = \dfrac{\pi^2\hbar^2}{2mL^2}$. In this problem the particle bound in the well is a proton, so $m = 1.673\times10^{-27}\text{ kg}$.

EXECUTE: $E_\infty = \dfrac{\pi^2\hbar^2}{2mL^2} = \dfrac{\pi^2(1.055\times10^{-34}\text{ J}\cdot\text{s})^2}{2(1.673\times10^{-27}\text{ kg})(4.0\times10^{-15}\text{ m})^2} = 2.052\times10^{-12}\text{ J}$. The transition energy is

$\Delta E = E_3 - E_1 = (5.09 - 0.625)E_\infty = 4.465E_\infty$. $\Delta E = 4.465(2.052\times10^{-12}\text{ J}) = 9.162\times10^{-12}\text{ J}$

The wavelength of the photon that is absorbed is related to the transition energy by $\Delta E = hc/\lambda$, so

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{9.162\times10^{-12}\text{ J}} = 2.2\times10^{-14}\text{ m} = 22\text{ fm.}$$

EVALUATE: The wavelength of the photon is comparable to the size of the box.

40.20. **IDENTIFY:** The longest wavelength corresponds to the smallest energy change.

SET UP: The ground level energy level of the infinite well is $E_\infty = \dfrac{h^2}{8mL^2}$, and the energy of the photon must be equal to the energy difference between the two shells.

EXECUTE: The 400.0 nm photon must correspond to the $n = 1$ to $n = 2$ transition. Since $U_0 = 6E_\infty$, we have $E_2 = 2.43E_\infty$ and $E_1 = 0.625E_\infty$. The energy of the photon is equal to the energy difference between the two levels, and $E_\infty = \dfrac{h^2}{8mL^2}$, which gives $E_\gamma = E_2 - E_1 \Rightarrow \dfrac{hc}{\lambda} = (2.43 - 0.625)E_\infty = \dfrac{1.805\,h^2}{8mL^2}$

Solving for L gives $L = \sqrt{\dfrac{(1.805)h\lambda}{8mc}} = \sqrt{\dfrac{(1.805)(6.626\times10^{-34}\text{ J}\cdot\text{s})(4.00\times10^{-7}\text{ m})}{8(9.11\times10^{-31}\text{ kg})(3.00\times10^8\text{ m/s})}} = 4.68\times10^{-10}\text{ m} = 0.468\text{ nm.}$

EVALUATE: This width is approximately half that of a Bohr hydrogen atom.

40.21. $T = 16\dfrac{E}{U_0}\left(1 - \dfrac{E}{U_0}\right)e^{-2L\sqrt{2m(U_0-E)}/\hbar}$. $\dfrac{E}{U_0} = \dfrac{6.0\text{ eV}}{11.0\text{ eV}}$ and $E - U_0 = 5\text{ eV} = 8.0\times10^{-19}\text{ J}$.

(a) $L = 0.80\times10^{-9}\text{ m}$: $T = 16\left(\dfrac{6.0\text{ eV}}{11.0\text{ eV}}\right)\left(1 - \dfrac{6.0\text{ ev}}{11.0\text{ eV}}\right)e^{-2(0.80\times10^{-9}\text{ m})\sqrt{2(9.11\times10^{-31}\text{ kg})(8.0\times10^{-19}\text{ J})}/1.055\times10^{-34}\text{ J}\cdot\text{s}} = 4.4\times10^{-8}$

(b) $L = 0.40\times10^{-9}\text{ m}$: $T = 4.2\times10^{-4}$.

40.22. The transmission coefficient is $T = 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)e^{-2\sqrt{2m(U_0-E)}L/\hbar}$, with $E = 5.0$ eV, $L = 0.60\times10^{-9}$ m, and $m = 9.11\times10^{-31}$ kg

(a) $U_0 = 7.0\text{ eV} \Rightarrow T = 5.5\times10^{-4}$.

(b) $U_0 = 9.0\text{ eV} \Rightarrow T = 1.8\times10^{-5}$

(c) $U_0 = 13.0\text{ eV} \Rightarrow T = 1.1\times10^{-7}$.

40.23. **IDENTIFY** and **SET UP:** Use Eq.(39.1), where $K = p^2/2m$ and $E = K + U$.

EXECUTE: $\lambda = h/p = h/\sqrt{2mK}$, so $\lambda\sqrt{K}$ is constant

$\lambda_1\sqrt{K_1} = \lambda_2\sqrt{K_2}$; λ_1 and K_1 are for $x > L$ where $K_1 = 2U_0$ and λ_2 and K_2 are for $0 < x < L$ where $K_2 = E - U_0 = U_0$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}} = \sqrt{\frac{U_0}{2U_0}} = \frac{1}{\sqrt{2}}$$

EVALUATE: When the particle is passing over the barrier its kinetic energy is less and its wavelength is larger.

40.24. **IDENTIFY:** The probability of tunneling depends on the energy of the particle and the width of the barrier.

SET UP: The probability of tunneling is approximately $T = Ge^{-2\kappa L}$, where $G = 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)$ and $\kappa = \frac{\sqrt{2m(U_0-E)}}{\hbar}$.

EXECUTE: $G = 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right) = 16\frac{50.0\text{ eV}}{70.0\text{ eV}}\left(1-\frac{50.0\text{ eV}}{70.0\text{ eV}}\right) = 3.27.$

$$\kappa = \frac{\sqrt{2m(U_0-E)}}{\hbar} = \frac{\sqrt{2(1.67\times10^{-27}\text{ kg})(70.0\text{ eV} - 50.0\text{ eV})(1.60\times10^{-19}\text{ J/eV})}}{(6.63\times10^{-34}\text{ J}\cdot\text{s})/2\pi} = 9.8\times10^{11}\text{ m}^{-1}$$

Solving $T = Ge^{-2\kappa L}$ for L gives $L = \frac{1}{2\kappa}\ln(G/T) = \frac{1}{2(9.8\times10^{11}\text{ m}^{-1})}\ln\left(\frac{3.27}{0.0030}\right) = 3.6\times10^{-12}\text{ m} = 3.6\text{ pm}$

If the proton were replaced with an electron, the electron's mass is much smaller so L would be larger.

EVALUATE: An electron can tunnel through a much wider barrier than a proton of the same energy.

40.25. **IDENTIFY** and **SET UP:** The probability is $T = Ae^{-2\kappa L}$, with $A = 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)$ and $\kappa = \frac{\sqrt{2m(U_0-E)}}{\hbar}$.

$E = 32$ eV, $U_0 = 41$ eV, $L = 0.25\times10^{-9}$ m. Calculate T.

EXECUTE: **(a)** $A = 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right) = 16\frac{32}{41}\left(1-\frac{32}{41}\right) = 2.741.$

$$\kappa = \frac{\sqrt{2m(U_0-E)}}{\hbar}$$

$$\kappa = \frac{\sqrt{2(9.109\times10^{-31}\text{ kg})(41\text{ eV} - 32\text{ eV})(1.602\times10^{-19}\text{ J/eV})}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = 1.536\times10^{10}\text{ m}^{-1}$$

$T = Ae^{-2\kappa L} = (2.741)e^{-2(1.536\times10^{10}\text{ m}^{-1})(0.25\times10^{-9}\text{ m})} = 2.741e^{-7.68} = 0.0013$

(b) The only change in the mass m, which appears in κ.

$$\kappa = \frac{\sqrt{2m(U_0-E)}}{\hbar}$$

$$\kappa = \frac{\sqrt{2(1.673\times10^{-27}\text{ kg})(41\text{ eV} - 32\text{ eV})(1.602\times10^{-19}\text{ J/eV})}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = 6.584\times10^{11}\text{ m}^{-1}$$

Then $T = Ae^{-2\kappa L} = (2.741)e^{-2(6.584\times10^{11}\text{ m}^{-1})(0.25\times10^{-9}\text{ m})} = 2.741e^{-392.2} = 10^{-143}$

EVALUATE: The more massive proton has a much smaller probability of tunneling than the electron does.

40.26. $T = Ge^{-2\kappa L}$ with $G = 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)$ and $\kappa = \frac{\sqrt{2m(U_0-E)}}{\hbar}$, so $T = 16\frac{E}{U_0}\left(1-\frac{E}{U_0}\right)e^{\frac{-2\sqrt{2m(U_0-E)}}{\hbar}L}$.

(a) If $U_0 = 30.0\times10^6$ eV, $L = 2.0\times10^{-15}$ m, $m = 6.64\times10^{-27}$ kg and $U_0 - E = 1.0\times10^6$ eV ($E = 29.0\times10^6$ eV), $T = 0.090$.

(b) If $U_0 - E = 10.0\times10^6$ eV ($E = 20.0\times10^6$ eV), $T = 0.014$.

40.27. **IDENTIFY** and **SET UP:** The energy levels are given by Eq.(40.26), where $\omega = \sqrt{\frac{k'}{m}}$.

EXECUTE: $\omega = \sqrt{\frac{k'}{m}} = \sqrt{\frac{110 \text{ N/m}}{0.250 \text{ kg}}} = 21.0$ rad/s

The ground state energy is given by Eq.(40.26):

$E_0 = \frac{1}{2}\hbar\omega = \frac{1}{2}(1.055\times10^{-34}\text{ J}\cdot\text{s})(21.0\text{ rad/s}) = 1.11\times10^{-33}\text{ J}(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 6.93\times10^{-15}$ eV

$E_n = \left(n+\frac{1}{2}\right)\hbar\omega;\; E_{(n+1)} = \left(n+1+\frac{1}{2}\right)\hbar\omega$

The energy separation between these adjacent levels is

$\Delta E = E_{n+1} - E_n = \hbar\omega = 2E_0 = 2(1.11\times10^{-33}\text{ J}) = 2.22\times10^{-33}\text{ J} = 1.39\times10^{-14}$ eV

EVALUATE: These energies are extremely small; quantum effects are not important for this oscillator.

40.28. Let $\sqrt{mk'}/2\hbar = \delta$, and so $\frac{d\psi}{dx} = -2x\delta\psi$ and $\frac{d^2\psi}{dx^2} = (4x^2\delta^2 - 2\delta)\psi$, and ψ is a solution of Eq.(40.21) if

$E = \frac{\hbar^2}{m}\delta = \frac{1}{2}\hbar\sqrt{k'/m} = \frac{1}{2}\hbar\omega.$

40.29. **IDENTIFY:** We can model the molecule as a harmonic oscillator. The energy of the photon is equal to the energy difference between the two levels of the oscillator.

SET UP: The energy of a photon is $E_\gamma = hf = hc/\lambda$, and the energy levels of a harmonic oscillator are given by

$E_n = \left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}} = \left(n+\frac{1}{2}\right)\hbar\omega.$

EXECUTE: **(a)** The photon's energy is $E_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{5.8\times10^{-6}\text{ m}} = 0.21$ eV

(b) The transition energy is $\Delta E = E_{n+1} - E_n = \hbar\omega = \hbar\sqrt{\frac{k'}{m}}$, which gives $\frac{2\pi\hbar c}{\lambda} = \hbar\sqrt{\frac{k'}{m}}$. Solving for k', we get

$k' = \frac{4\pi^2c^2m}{\lambda^2} = \frac{4\pi^2(3.00\times10^8\text{ m/s})^2(5.6\times10^{-26}\text{ kg})}{(5.8\times10^{-6}\text{ m})^2} = 5{,}900$ N/m.

EVALUATE: This would be a rather strong spring in the physics lab.

40.30. According to Eq.(40.26), the energy released during the transition between two adjacent levels is twice the ground state energy $E_3 - E_2 = \hbar\omega = 2E_0 = 11.2$ eV.

For a photon of energy E

$$E = hf \Rightarrow \lambda = \frac{c}{f} = \frac{hc}{E} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{(11.2\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 111\text{ nm}.$$

40.31. **IDENTIFY** and **SET UP:** Use the energies given in Eq.(40.26) to solve for the amplitude A and maximum speed $v_{\max}$ of the oscillator. Use these to estimate Δx and Δp_x and compute the uncertainty product $\Delta x\Delta p_x$.

EXECUTE: The total energy of a Newtonian oscillator is given by $E = \frac{1}{2}k'A^2$ where k' is the force constant and A is the amplitude of the oscillator. Set this equal to the energy $E = (n+\frac{1}{2})\hbar\omega$ of an excited level that has quantum number n, where $\omega = \sqrt{\frac{k'}{m}}$, and solve for A: $\frac{1}{2}k'A^2 = (n+\frac{1}{2})\hbar\omega$

$A = \sqrt{\frac{(2n+1)\hbar\omega}{k'}}$

The total energy of the Newtonian oscillator can also be written as $E = \frac{1}{2}mv_{\max}^2$. Set this equal to $E = (n+\frac{1}{2})\hbar\omega$ and solve for $v_{\max}$: $\frac{1}{2}mv_{\max}^2 = (n+\frac{1}{2})\hbar\omega$

$v_{\max} = \sqrt{\frac{(2n+1)\hbar\omega}{m}}$

Thus the maximum linear momentum of the oscillator is $p_{\max} = mv_{\max} = \sqrt{(2n+1)\hbar m\omega}$. Assume that A represents the uncertainty Δx in position and that $p_{\max}$ is the corresponding uncertainty Δp_x in momentum. Then the uncertainty product is $\Delta x \Delta p_x = \sqrt{\frac{(2n+1)\hbar\omega}{k'}}\sqrt{(2n+1)\hbar m\omega} = (2n+1)\hbar\omega\sqrt{\frac{m}{k'}} = (2n+1)\hbar\omega\left(\frac{1}{\omega}\right) = (2n+1)\hbar.$

EVALUATE: For $n = 1$ this gives $\Delta x \Delta p_x = 3\hbar$, in agreement with the result derived in Section 40.4. The uncertainty product $\Delta x \Delta p_x$ increases with n.

40.32. **(a)** $\frac{|\psi(A)|^2}{|\psi(0)|^2} = \exp\left(-\frac{\sqrt{mk'}}{\hbar}A^2\right) = \exp\left(-\sqrt{mk'}\frac{\omega}{k'}\right) = e^{-1} = 0.368.$

This is consistent with what is shown in Figure 40.20 in the textbook.

(b) $\frac{|\psi(2A)^2|}{|\psi(0)|^2} = \exp\left(-\frac{\sqrt{mk'}}{\hbar}(2A)^2\right) = \exp\left(-\sqrt{mk'}4\frac{\omega}{k'}\right) = e^{-4} = 1.83\times10^{-2}.$

This figure cannot be read this precisely, but the qualitative decrease in amplitude with distance is clear.

40.33. **IDENTIFY:** We model the atomic vibration in the crystal as a harmonic oscillator.

SET UP: The energy levels of a harmonic oscillator are given by $E_n = \left(n + \frac{1}{2}\right)\hbar\sqrt{\frac{k'}{m}} = \left(n + \frac{1}{2}\right)\hbar\omega.$

EXECUTE: **(a)** The ground state energy of a simple harmonic oscillator is

$$E_0 = \frac{1}{2}\hbar\omega = \frac{1}{2}\hbar\sqrt{\frac{k'}{m}} = \frac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{2}\sqrt{\frac{12.2\ \text{N/m}}{3.82\times10^{-26}\ \text{kg}}} = 9.43\times10^{-22}\ \text{J} = 5.89\times10^{-3}\ \text{eV}$$

(b) $E_4 - E_3 = \hbar\omega = 2E_0 = 0.0118\ \text{eV}$, so $\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{1.88\times10^{-21}\ \text{J}} = 106\ \mu\text{m}$

(c) $E_{n+1} - E_n = \hbar\omega = 2E_0 = 0.0118\ \text{eV}$

EVALUATE: These energy differences are much smaller than those due to electron transitions in the hydrogen atom.

40.34. **IDENTIFY:** If the given wave function is a solution to the Schrödinger equation, we will get an identity when we substitute that wave function into the Schrödinger equation.

SET UP: The given function is $\psi(x) = Ae^{ikx}$, and the one-dimensional Schrödinger equation is

$$-\frac{\hbar}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x).$$

EXECUTE: Start with the given function and take the indicated derivatives: $\psi(x) = Ae^{ikx}$. $\frac{d\psi(x)}{dx} = Aike^{ikx}$.

$\frac{d^2\psi(x)}{dx^2} = Ai^2k^2e^{ikx} = -Ak^2e^{ikx}$. $\frac{d^2\psi(x)}{dx^2} = -k^2\psi(x)$. $-\frac{\hbar}{2m}\frac{d^2\psi(x)}{dx^2} = \frac{\hbar^2}{2m}\ k^2\psi(x)$. Substituting these results into the one-dimensional Schrödinger equation gives $\frac{\hbar^2k^2}{2m}\ \psi(x) + U_0\psi(x) = E\ \psi(x)$.

EVALUATE: $\psi(x) = A\ e^{ikx}$ is a solution to the one-dimensional Schrödinger equation if $E - U_0 = \frac{\hbar^2k^2}{2m}$ or $k = \sqrt{\frac{2m(E-U_0)}{\hbar^2}}$. (Since $U_0 < E$ was given, k is the square root of a positive quantity.) In terms of the particle's momentum p: $k = p/\hbar$, and in terms of the particle's de Broglie wavelength λ: $k = 2\pi/\lambda$.

40.35. **IDENTIFY:** Let I refer to the region $x < 0$ and let II refer to the region $x > 0$, so $\psi_I(x) = Ae^{ik_1x} + Be^{-ik_1x}$ and $\psi_{II}(x) = Ce^{ik_2x}$. Set $\psi_I(0) = \psi_{II}(0)$ and $\frac{d\psi_I}{dx} = \frac{d\psi_{II}}{dx}$ at $x = 0$.

SET UP: $\frac{d}{dx}(e^{ikx}) = ike^{ikx}$.

EXECUTE: $\psi_I(0) = \psi_{II}(0)$ gives $A + B = C$. $\frac{d\psi_I}{dx} = \frac{d\psi_{II}}{dx}$ at $x = 0$ gives $ik_1A - ik_1B = ik_2C$. Solving this pair of equations for B and C gives $B = \left(\frac{k_1 - k_2}{k_1 + k_2}\right)A$ and $C = \left(\frac{2k_2}{k_1 + k_2}\right)A.$

EVALUATE: The probability of reflection is $R = \frac{B^2}{A^2} = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}$. The probability of transmission is $T = \frac{C^2}{A^2} = \frac{4k_1^2}{(k_1 + k_2)^2}$. Note that $R + T = 1$.

40.36. **(a)** $R_n = \frac{(n+1)^2 - n^2}{n^2} = \frac{2n+1}{n^2} = \frac{2}{n} + \frac{1}{n^2}$. This is never larger than it is for $n = 1$, and $R_1 = 3$.

(b) R approaches zero; in the classical limit, there is no quantization, and the spacing of successive levels is vanishingly small compared to the energy levels.

40.37. **IDENTIFY** and **SET UP:** The energy levels are given by Eq.(40.9): $E_n = \frac{n^2h^2}{8mL^2}$. Calculate ΔE for the transition and set $\Delta E = hc/\lambda$, the energy of the photon.

EXECUTE: **(a)** Ground level, $n = 1$, $E_1 = \frac{h^2}{8mL^2}$

First excited level, $n = 2$, $E_2 = \frac{4h^2}{8mL^2}$

The transition energy is $\Delta E = E_2 - E_1 = \frac{3h^2}{8mL^2}$. Set the transition energy equal to the energy hc/λ of the emitted photon. This gives $\frac{hc}{\lambda} = \frac{3h^2}{8mL^2}$.

$$\lambda = \frac{8mcL^2}{3h} = \frac{8(9.109\times10^{-31}\text{ kg})(2.998\times10^{8}\text{ m/s})(4.18\times10^{-9}\text{ m})^2}{3(6.626\times10^{-34}\text{ J}\cdot\text{s})}$$

$\lambda = 1.92\times10^{-5}\text{ m} = 19.2\ \mu\text{m}$.

(b) Second excited level has $n = 3$ and $E_3 = \frac{9h^2}{8mL^2}$. The transition energy is $\Delta E = E_3 - E_2 = \frac{9h^2}{8mL^2} - \frac{4h^2}{8mL^2} = \frac{5h^2}{8mL^2}$.

$\frac{hc}{\lambda} = \frac{5h^2}{8mL^2}$ so $\lambda = \frac{8mcL^2}{5h} = \frac{3}{5}(19.2\ \mu\text{m}) = 11.5\ \mu\text{m}$.

EVALUATE: The energy spacing between adjacent levels increases with n, and this corresponds to a shorter wavelength and more energetic photon in part (b) than in part (a).

40.38. **(a)** $\frac{2}{L}\int_0^{L/4}\sin^2\frac{\pi x}{L}dx = \frac{2}{L}\int_0^{L/4}\frac{1}{2}\left(1-\cos\frac{2\pi x}{L}\right)dx = \frac{1}{L}\left(x - \frac{L}{2\pi}\sin\frac{2\pi x}{L}\right)_0^{L/4} = \frac{1}{4} - \frac{1}{2\pi}$, which is about 0.0908.

(b) Repeating with limits of $L/4$ and $L/2$ gives $\frac{1}{L}\left(x - \frac{L}{2\pi}\sin\frac{2\pi x}{L}\right)_{L/4}^{L/2} = \frac{1}{4} + \frac{1}{2\pi}$, about 0.409.

(c) The particle is much likely to be nearer the middle of the box than the edge.

(d) The results sum to exactly 1/2, which means that the particle is as likely to be between $x = 0$ and $L/2$ as it is to be between $x = L/2$ and $x = L$.

(e) These results are represented in Figure 40.5b in the textbook.

40.39. **IDENTIFY:** The probability of the particle being between x_1 and x_2 is $\int_{x_1}^{x_2}|\psi|^2 dx$, where ψ is the normalized wave function for the particle.

(a) SET UP: The normalized wave function for the ground state is $\psi_1 = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right)$.

EXECUTE: The probability P of the particle being between $x = L/4$ and $x = 3L/4$ is $P = \int_{L/4}^{3L/4}|\psi_1|^2 dx = \frac{2}{L}\int_{L/4}^{3L/4}\sin^2\left(\frac{\pi x}{L}\right)dx$. Let $y = \pi x/L$; $dx = (L/\pi)dy$ and the integration limits become $\pi/4$ and $3\pi/4$.

$$P = \frac{2}{L}\left(\frac{L}{\pi}\right)\int_{\pi/4}^{3\pi/4}\sin^2 y\,dy = \frac{2}{\pi}\left[\frac{1}{2}y - \frac{1}{4}\sin 2y\right]_{\pi/4}^{3\pi/4}$$

$$P = \frac{2}{\pi}\left[\frac{3\pi}{8} - \frac{\pi}{8} - \frac{1}{4}\sin\left(\frac{3\pi}{2}\right) + \frac{1}{4}\sin\left(\frac{\pi}{2}\right)\right]$$

$P = \frac{2}{\pi}\left(\frac{\pi}{4} - \frac{1}{4}(-1) + \frac{1}{4}(1)\right) = \frac{1}{2} + \frac{1}{\pi} = 0.818$. (Note: The integral formula $\int\sin^2 y\,dy = \frac{1}{2}y - \frac{1}{4}\sin 2y$ was used.)

(b) SET UP: The normalized wave function for the first excited state is $\psi_2 = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$

EXECUTE: $P = \int_{L/4}^{3L/4} |\psi_2|^2 dx = \frac{2}{L}\int_{L/4}^{3L/4} \sin^2\left(\frac{2\pi x}{L}\right) dx.$ Let $y = 2\pi x/L$; $dx = (L/2\pi)dy$ and the integration limits become $\pi/2$ and $3\pi/2$.

$$P = \frac{2}{L}\left(\frac{L}{2\pi}\right)\int_{\pi/2}^{3\pi/2} \sin^2 y\, dy = \frac{1}{\pi}\left[\frac{1}{2}y - \frac{1}{4}\sin 2y\right]_{\pi/2}^{3\pi/2} = \frac{1}{\pi}\left(\frac{3\pi}{4} - \frac{\pi}{4}\right) = 0.500$$

(c) EVALUATE: These results are consistent with Fig.40.4b in the textbook. That figure shows that $|\psi|^2$ is more concentrated near the center of the box for the ground state than for the first excited state; this is consistent with the answer to part (a) being larger than the answer to part (b). Also, this figure shows that for the first excited state half the area under $|\psi|^2$ curve lies between L/4 and 3L/4, consistent with our answer to part (b).

40.40. Using the normalized wave function $\psi_1 = \sqrt{2/L}\,\sin(\pi x/L)$, the probabilities $|\psi|^2\, dx$ are

(a) $(2/L)\sin^2(\pi/4)dx = dx/L$

(b) $(2/L)\sin^2(\pi/2)dx = 2dx/L$

(c) $(2/L)\sin^2(3\pi/4) = dx/L.$

40.41. **IDENTIFY** and **SET UP:** The normalized wave function for the $n = 2$ first excited level is $\psi_2 = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$.

$P = |\psi(x)|^2\, dx$ is the probability that the particle will be found in the interval x to $x + dx$.

EXECUTE: **(a)** $x = L/4$

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\left(\frac{2\pi}{L}\right)\left(\frac{L}{4}\right)\right) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi}{2}\right) = \sqrt{\frac{2}{L}}.$$

$P = (2/L)dx$

(b) $x = L/2$

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\left(\frac{2\pi}{L}\right)\left(\frac{L}{2}\right)\right) = \sqrt{\frac{2}{L}}\sin(\pi) = 0$$

$P = 0$

(c) $x = 3L/4$

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\left(\left(\frac{2\pi}{L}\right)\left(\frac{3L}{4}\right)\right) = \sqrt{\frac{2}{L}}\sin\left(\frac{3\pi}{2}\right) = -\sqrt{\frac{2}{L}}.$$

$P = (2/L)dx$

EVALUATE: Our results are consistent with the $n = 2$ part of Fig.40.5 in the textbook. $|\psi|^2$ is zero at the center of the box and is symmetric about this point.

40.42. $\Delta\vec{p} = \vec{p}_{\text{final}} - \vec{p}_{\text{initial}}$. $|\vec{p}| = \hbar k = \frac{\hbar n\pi}{L} = \frac{hn}{2L}$. At $x = 0$ the initial momentum at the wall is $\vec{p}_{\text{initial}} = -\frac{hn}{2L}\hat{i}$ and the final momentum, after turning around, is $\vec{p}_{\text{final}} = +\frac{hn}{2L}\hat{i}$. So, $\Delta\vec{p} = +\frac{hn}{2L}\hat{i} - \left(-\frac{hn}{2L}\hat{i}\right) = +\frac{hn}{L}\hat{i}$. At $x = L$ the initial momentum is $\vec{p}_{\text{initial}} = +\frac{hn}{2L}\hat{i}$ and the final momentum, after turning around, is $\vec{p}_{\text{final}} = -\frac{hn}{2L}\hat{i}$. So,

$$\Delta\vec{p} = -\frac{hn}{2L}\hat{i} - \frac{hn}{2L}\hat{i} = -\frac{hn}{L}\hat{i}$$

40.43. **(a)** For a free particle, $U(x) = 0$ so Schrödinger's equation becomes $\frac{d^2\psi(x)}{dx^2} = -\frac{2m}{h^2}E\psi(x)$. The graph is given in Figure 40.43.

(b) For $x < 0$: $\psi(x) = e^{+\kappa x}$. $\frac{d\psi(x)}{dx} = \kappa e^{+\kappa x}$. $\frac{d^2\psi(x)}{dx} = \kappa^2 e^{+\kappa x}$. So $\kappa^2 = -\frac{2m}{\hbar^2}E \Rightarrow E = -\frac{\hbar^2\kappa^2}{2m}$.

(c) For $x > 0$: $\psi(x) = e^{-\kappa x}$. $\frac{d\psi(x)}{dx} = -ke^{-\kappa x}$. $\frac{d^2\psi(x)}{dx} = \kappa^2 e^{-\kappa x}$

So again $\kappa^2 = -\frac{2m}{\hbar^2}E \Rightarrow E = \frac{-\hbar^2\kappa^2}{2m}$. Parts (c) and (d) show $\psi(x)$ satisfies the Schrödinger's equation, provided $E = \frac{-\hbar^2\kappa^2}{2m}$.

(d) Note $\frac{d\psi(x)}{dx}$ is discontinuous at $x = 0$. (That is, negative for $x > 0$ and positive for $x < 0$.)

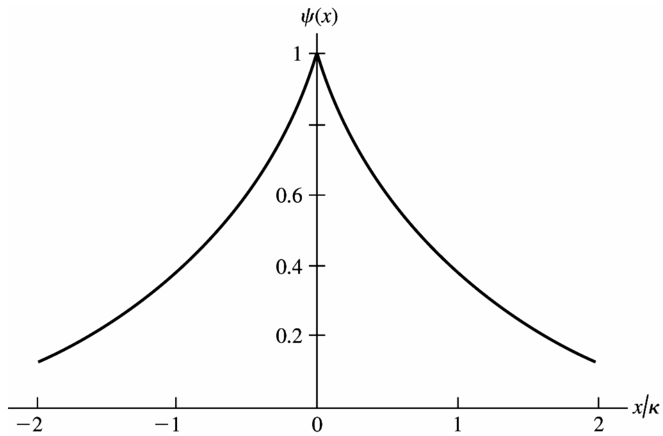


Figure 40.43

40.44. IDENTIFY: We start with the penetration distance formula given in the problem.

SET UP: The given formula is $\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}}$.

EXECUTE: **(a)** Substitute the given numbers into the formula:

$$\eta = \frac{\hbar}{\sqrt{2m(U_0 - E)}} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(20\ \text{eV} - 13\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}} = 7.4\times10^{-11}\ \text{m}$$

(b) $\eta = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(1.67\times10^{-27}\ \text{kg})(30\ \text{MeV} - 20\ \text{MeV})(1.602\times10^{-13}\ \text{J/MeV})}} = 1.44\times10^{-15}\ \text{m}$

EVALUATE: The penetration depth varies widely depending on the mass and energy of the particle.

40.45. **(a)** We set the solutions for inside and outside the well equal to each other at the well boundaries, $x = 0$ and L.

$x = 0: A\sin(0) + B = C \Rightarrow B = C$, since we must have $D = 0$ for $x < 0$.

$x = L$: $A\sin\frac{\sqrt{2mE}L}{\hbar} + B\cos\frac{\sqrt{2mE}L}{\hbar} = +De^{-\kappa L}$ since $C = 0$ for $x > L$.

This gives $A\sin kL + B\cos kL = De^{-\kappa L}$, where $k = \frac{\sqrt{2mE}}{\hbar}$.

(b) Requiring continuous derivatives at the boundaries yields

$x = 0$: $\frac{d\psi}{dx} = kA\cos(k\cdot 0) - kB\sin(k\cdot 0) = kA = \kappa Ce^{k\cdot 0} \Rightarrow kA = \kappa C$

$x = L$: $kA\cos kL - kB\sin kL = -\kappa De^{-\kappa L}$.

40.46. $T = Ge^{-2\kappa L}$ with $G = 16\frac{E}{U_0}\left(1 - \frac{E}{U_0}\right)$ and $\kappa = \frac{\sqrt{2m(U_0 - E)}}{\hbar} \Rightarrow L = -\frac{1}{2\kappa}\ln\left(\frac{T}{G}\right)$.

If $E = 5.5\ \text{eV}, U_0 = 10.0\ \text{eV}, m = 9.11\times10^{-31}\ \text{kg}$, and $T = 0.0010$. Then

$$\kappa = \frac{\sqrt{2(9.11\times10^{-31}\ \text{kg})(4.5\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}}{(1.054\times10^{-34}\ \text{J}\cdot\text{s})} = 1.09\times10^{10}\ \text{m}^{-1} \text{ and } G = 16\frac{5.5\ \text{eV}}{10.0\ \text{eV}}\left(1 - \frac{5.5\ \text{eV}}{10.0\ \text{eV}}\right) = 3.96$$

so $L = -\frac{1}{2(1.09\times10^{10}\ \text{m}^{-1})}\ln\left(\frac{0.0010}{3.96}\right) = 3.8\times10^{-10}\ \text{m} = 0.38\ \text{nm}$.

40.47. IDENTIFY and SET UP: When κL is large, then $e^{\kappa L}$ is large and $e^{-\kappa L}$ is small. When κL is small, $\sinh\kappa L \to \kappa L$. Consider both κL large and κL small limits.

EXECUTE: **(a)** $T = \left[1 + \frac{(U_0\sinh\kappa L)^2}{4E(U_0 - E)}\right]^{-1}$

$\sinh \kappa L = \dfrac{e^{\kappa L} - e^{-\kappa L}}{2}$

For $\kappa L \gg 1$, $\sinh \kappa L \to \dfrac{e^{\kappa L}}{2}$ and $T \to \left[1 + \dfrac{U_0^2 e^{2\kappa L}}{16E(U_0 - E)}\right]^{-1} = \dfrac{16E(U_0 - E)}{16E(U_0 - E) + U_0^2 e^{2\kappa L}}$

For $\kappa L \gg 1$, $16E(U_0 - E) + U_0^2 e^{2\kappa L} \to U_0^2 e^{2\kappa L}$

$T \to \dfrac{16E(U_0 - E)}{U_0^2 e^{2\kappa L}} = 16\left(\dfrac{E}{U_0}\right)\left(1 - \dfrac{E}{U_0}\right) e^{-2\kappa L}$, which is Eq.(40.21).

(b) $\kappa L = \dfrac{L\sqrt{2m(U_0 - E)}}{\hbar}$. So $\kappa L \gg 1$ when L is large (barrier is wide) or $U_0 - E$ is large. (E is small compared to U_0.)

(c) $\kappa = \dfrac{\sqrt{2m(U_0 - E)}}{\hbar}$; κ becomes small as E approaches U_0. For κ small, $\sinh \kappa L \to \kappa L$ and

$T \to \left[1 + \dfrac{U_0^2 \kappa^2 L^2}{4E(U_0 - E)}\right]^{-1} = \left[1 + \dfrac{U_0^2 2m(U_0 - E)L^2}{\hbar^2 4E(U_0 - E)}\right]^{-1}$ (using the definition of κ)

Thus $T \to \left[1 + \dfrac{2U_0^2 L^2 m}{4E\hbar^2}\right]^{-1}$

$U_0 \to E$ so $\dfrac{U_0^2}{E} \to E$ and $T \to \left[1 + \dfrac{2EL^2 m}{4\hbar^2}\right]^{-1}$

But $k^2 = \dfrac{2mE}{\hbar^2}$, so $T \to \left[1 + \left(\dfrac{kL}{2}\right)^2\right]^{-1}$, as was to be shown.

EVALUATE: When κL is large Eq.(40.20) applies and T is small. When $E \to U_0$, T does not approach unity.

40.48. **(a)** $E = \frac{1}{2}mv^2 = (n + (1/2))\hbar\omega = (n + (1/2))hf$, and solving for n,

$$n = \frac{\frac{1}{2}mv^2}{hf} - \frac{1}{2} = \frac{(1/2)(0.020\text{ kg})(0.360\text{ m/s})^2}{(6.63\times10^{-34}\text{ J}\cdot\text{s})(1.50\text{ Hz})} - \frac{1}{2} = 1.3\times10^{30}.$$

(b) The difference between energies is $\hbar\omega = hf = (6.63\times10^{-34}\text{ J}\cdot\text{s})(1.50\text{ Hz}) = 9.95\times10^{-34}$ J. This energy is too small to be detected with current technology

40.49. **IDENTIFY** and **SET UP:** Calculate the angular frequency ω of the pendulum and apply Eq.(40.26) for the energy levels.

EXECUTE: $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.500\text{ s}} = 4\pi\text{ s}^{-1}$

The ground-state energy is $E_0 = \frac{1}{2}\hbar\omega = \frac{1}{2}(1.055\times10^{-34}\text{ J}\cdot\text{s})(4\pi\text{ s}^{-1}) = 6.63\times10^{-34}$ J.

$E_0 = 6.63\times10^{-34}\text{ J}(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 4.14\times10^{-15}\text{ eV}$

$E_n = \left(n + \dfrac{1}{2}\right)\hbar\omega$

$E_{n+1} = \left(n + 1 + \dfrac{1}{2}\right)\hbar\omega$

The energy difference between the adjacent energy levels is

$\Delta E = E_{n+1} - E_n = \hbar\omega = 2E_0 = 1.33\times10^{-33}\text{ J} = 8.30\times10^{-15}\text{ eV}$

EVALUATE: These energies are much too small to detect. Quantum effects are not important for ordinary size objects.

40.50. **IDENTIFY:** We model the electrons in the lattice as a particle in a box. The energy of the photon is equal to the energy difference between the two energy states in the box.

SET UP: The energy of an electron in the nth level is $E_n = \dfrac{n^2h^2}{8mL^2}$. We do not know the initial or final levels, but we do know they differ by 1. The energy of the photon, hc/λ, is equal to the energy difference between the two states.

EXECUTE: The energy difference between the levels is $\Delta E = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})}{1.649\times10^{-7}\text{ m}} =$ 1.206×10^{-18} J. Using the formula for the energy levels in a box, this energy difference is equal to

$\Delta E = \left[n^2 - (n-1)^2\right]\dfrac{h^2}{8mL^2} = (2n-1)\dfrac{h^2}{8mL^2}$.

Solving for n gives $n=\left(\dfrac{\Delta E 8mL^2}{h^2}+1\right)=\dfrac{1}{2}\left(\dfrac{(1.206\times10^{-18}\ \text{J})8(9.11\times10^{-31}\ \text{kg})(0.500\times10^{-9}\ \text{m})^2}{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}+1\right)=3.$

The transition is from $n=3$ to $n=2$.

EVALUATE: We know the transition is not from the $n=4$ to the $n=3$ state because we let n be the higher state and $n-1$ the lower state.

40.51. IDENTIFY: If the given wave function is a solution to the Schrödinger equation, we will get an identity when we substitute that wave function into the Schrödinger equation.

SET UP: The given wave function is $\psi_0(x)=A_0e^{-\alpha^2x^2/2}$ and the Schrödinger equation is

$$-\frac{\hbar}{2m}\frac{d^2\psi(x)}{dx^2}+\frac{k'x^2}{2}\psi(x)=E\ \psi(x).$$

EXECUTE: **(a)** Start by taking the derivatives: $\psi_0(x)=A_0e^{-\alpha^2x^2/2}$. $\dfrac{d\psi_0(x)}{dx}=-\alpha^2xA_0e^{-\alpha^2x^2/2}$.

$\dfrac{d^2\psi_0(x)}{dx^2}=-A_0\alpha^2e^{-\alpha^2x^2/2}+(\alpha^2)^2x^2A_0e^{-\alpha^2x^2/2}$. $\dfrac{d^2\psi_0(x)}{dx^2}=[-\alpha^2+(\alpha^2)^2x^2]\ \psi_0(x)$.

$-\dfrac{\hbar}{2m}\dfrac{d^2\psi_0(x)}{dx^2}=-\dfrac{\hbar^2}{2m}[-\alpha^2+(\alpha^2)^2x^2]\ \psi_0(x)$. Equation (40.22) is $-\dfrac{\hbar}{2m}\dfrac{d^2\psi(x)}{dx^2}+\dfrac{k'x^2}{2}\psi(x)=E\ \psi(x)$. Substituting the above result into that equation gives $-\dfrac{\hbar^2}{2m}[-\alpha^2+(\alpha^2)^2x^2]\ \psi_0(x)+\dfrac{k'x^2}{2}\psi_0(x)=E\ \psi_0(x)$. Since $\alpha^2=\dfrac{m\omega}{\hbar}$ and $\omega=\sqrt{\dfrac{k'}{m}}$, the coefficient of x^2 is $-\dfrac{\hbar^2}{2m}(\alpha^2)^2+\dfrac{k'}{2}=-\dfrac{\hbar^2}{2m}\left(\dfrac{m\omega}{\hbar}\right)^2+\dfrac{m\omega^2}{2}=0.$

(b) $A_0=\left(\dfrac{m\omega}{\hbar\pi}\right)^{1/4}$

(c) The classical turning points are at $A=\pm\sqrt{\dfrac{\hbar}{\omega\, m}}$. The probability density function $|\psi|^2$ is

$|\psi_0(x)|^2=A_0^2e^{-\alpha^2x^2}=\left(\dfrac{m\omega}{\hbar\pi}\right)^{1/2}e^{-m\omega x^2/\hbar}$. At $x=0$, $|\psi_0|^2=\left(\dfrac{m\omega}{\hbar\pi}\right)^{1/2}$.

$\dfrac{d|\psi_0(x)|^2}{dx}=\left(\dfrac{m\omega}{\hbar\pi}\right)^{1/2}(-\alpha^2 2x)e^{-\alpha^2x^2}=-2\dfrac{m\omega}{\hbar}\left(\dfrac{m\omega}{\hbar\pi}\right)^{1/2}xe^{-\alpha^2x^2}$. At $x=0$, $\dfrac{d|\psi_0(x)|^2}{dx}=0$.

$\dfrac{d^2|\psi_0(x)|^2}{dx^2}=-2\dfrac{m\omega}{\hbar}\left(\dfrac{m\omega}{\hbar\pi}\right)^{1/2}[1-2\alpha^2x^2]e^{-\alpha^2x^2}$. At $x=0$, $\dfrac{d^2|\psi_0(x)|^2}{dx^2}<0$. Therefore, at $x=0$, the first derivative is zero and the second derivative is negative. Therefore, the probability density function has a maximum at $x=0$.

EVALUATE: $\psi_0(x)=A_0e^{-\alpha^2x^2/2}$ is a solution to equation (40.22) if $-\dfrac{\hbar^2}{2m}(-\alpha^2)\psi_0(x)=E\ \psi_0(x)$ or $E=\dfrac{\hbar^2\alpha^2}{2m}=\dfrac{\hbar\omega}{2}$. $E_0=\dfrac{\hbar\omega}{2}$ corresponds to $n=0$ in Equation (40.26).

40.52. IDENTIFY: If the given wave function is a solution to the Schrödinger equation, we will get an identity when we substitute that wave function into the Schrödinger equation.

SET UP: The given wave function is $\psi_1(x)=A_1 2xe^{-\alpha^2x^2/2}$ and the Schrödinger equation is

$$-\frac{\hbar}{2m}\frac{d^2\psi(x)}{dx^2}+\frac{k'x^2}{2}\psi(x)=E\ \psi(x).$$

EXECUTE: **(a)** Start by taking the indicated derivatives: $\psi_1(x)=A_1 2xe^{-\alpha^2x^2/2}$.

$\dfrac{d\psi_1(x)}{dx}=-2\alpha^2x^2A_1e^{-\alpha^2x^2/2}+2A_1e^{-\alpha^2x^2/2}$. $\dfrac{d^2\psi_1(x)}{dx^2}=-2A_1\alpha^2 2xe^{-\alpha^2x^2/2}-2A_1\alpha^2x^2(-\alpha^2x)e^{-\alpha^2x^2/2}+2A_1(-\alpha^2x)e^{-\alpha^2x^2/2}$.

$$\frac{d^2\psi_1(x)}{dx^2}=[-2\alpha^2+(\alpha^2)^2x^2-\alpha^2]\ \psi_1(x)=[-3\alpha^2+(\alpha^2)^2x^2]\ \psi_1(x)$$

$$-\frac{\hbar}{2m}\frac{d^2\psi_1(x)}{dx^2}=-\frac{\hbar^2}{2m}[-3\alpha^2+(\alpha^2)^2x^2]\ \psi_1(x)$$

Equation (40.22) is $-\frac{\hbar}{2m}\frac{d^2\psi(x)}{dx^2}+\frac{k'x^2}{2}\psi(x)=E\ \psi(x)$. Substituting the above result into that equation gives $-\frac{\hbar^2}{2m}[-3\alpha^2+(\alpha^2)^2x^2]\ \psi_1(x)+\frac{k'x^2}{2}\psi_1(x)=E\ \psi_1(x)$. Since $\alpha^2=\frac{m\omega}{\hbar}$ and $\omega=\sqrt{\frac{k'}{m}}$, the coefficient of x^2 is $-\frac{\hbar^2}{2m}(\alpha^2)^2+\frac{k'}{2}=-\frac{\hbar^2}{2m}\left(\frac{m\omega}{\hbar}\right)^2+\frac{m\omega^2}{2}=0$

(b) $A_1=\frac{1}{\sqrt{2}}\left(\frac{m\omega}{\hbar\pi}\right)^{1/4}$

(c) The probability density function $|\psi|^2$ is $|\psi_1(x)|^2=A_1^2 4x^2e^{-\alpha^2x^2}=\frac{1}{2}\left(\frac{m\omega}{\hbar\pi}\right)^{1/2}4x^2e^{-\frac{m\omega x^2}{\hbar}}$

At $x=0,\ |\psi_1|^2=0.\ \frac{d|\psi_1(x)|^2}{dx}=A_1^2 8xe^{-\alpha^2x^2}+A_1^2 4x^2(-\alpha^2 2x)e^{-\alpha^2x^2}=A_1^2 8xe^{-\alpha^2x^2}-A_1^2 8x^3\alpha^2e^{-\alpha^2x^2}$

At $x=0,\ \frac{d|\psi_1(x)|^2}{dx}=0.$ At $x=\pm\frac{1}{\alpha},\ \frac{d|\psi_1(x)|^2}{dx}=0.$

$\frac{d^2|\psi_1(x)|^2}{dx^2}=A_1^2 8e^{-\alpha^2x^2}+A_1^2 8x(-\alpha^2 2x)e^{-\alpha^2x^2}-A_1^2 8(3x^2)\alpha^2e^{-\alpha^2x^2}-A_1^2 8x^3\alpha^2(-\alpha^2 2x)e^{-\alpha^2x^2}.$

$\frac{d^2|\psi_1(x)|^2}{dx^2}=A_1^2 8e^{-\alpha^2x^2}-A_1^2 16x^2\alpha^2e^{-\alpha^2x^2}-A_1^2 24x^2\alpha^2e^{-\alpha^2x^2}+A_1^2 16x^4(\alpha^2)^2e^{-\alpha^2x^2}$. At $x=0,\ \frac{d^2|\psi_1(x)|^2}{dx^2}>0.$ So at $x=0,$ the first derivative is zero and the second derivative is positive. Therefore, the probability density function has a minimum at $x=0.$ At $x=\pm\frac{1}{\alpha},\ \frac{d^2|\psi_1(x)|^2}{dx^2}<0.$ So at $x=\pm\frac{1}{\alpha}$, the first derivative is zero and the second derivative is negative. Therefore, the probability density function has maxima at $x=\pm\frac{1}{\alpha}$, corresponding to the classical turning points for $n=0$ as found in the previous question.

EVALUATE: $\psi_1(x)=A_1 2xe^{-\alpha^2x^2/2}$ is a solution to equation (40.22) if $-\frac{\hbar^2}{2m}(-3\alpha^2)\psi_1(x)=E\ \psi_1(x)$ or $E=\frac{3\hbar^2\alpha^2}{2m}=\frac{3\hbar\omega}{2}.\ E_1=\frac{3\hbar\omega}{2}$ corresponds to $n=1$ in Equation (40.26).

40.53. **IDENTIFY** and **SET UP:** Evaluate $\partial^2\psi/\partial x^2$, $\partial^2\psi/\partial y^2$, and $\partial^2\psi/\partial z^2$ for the proposed ψ and put Eq.(40.29). Use that ψ_{n_x}, ψ_{n_y}, and ψ_{n_z} are each solutions to Eq.(40.22).

EXECUTE: **(a)** $-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+U\psi=E\psi$

ψ_{n_x}, ψ_{n_y}, ψ_{n_z} are each solutions of Eq.(40.22), so $-\frac{\hbar^2}{2m}\frac{d^2\psi_{n_x}}{dx^2}+\frac{1}{2}k'x^2\psi_{n_x}=E_{n_x}\psi_{n_x}$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{n_y}}{dy^2}+\frac{1}{2}k'y^2\psi_{n_y}=E_{n_y}\psi_{n_y}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{n_z}}{dz^2}+\frac{1}{2}k'z^2\psi_{n_z}=E_{n_z}\psi_{n_z}$$

$$\psi=\psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z),\ U=\frac{1}{2}k'x^2+\frac{1}{2}k'y^2+\frac{1}{2}k'z^2$$

$$\frac{\partial^2\psi}{\partial x^2}=\left(\frac{d^2\psi_{n_x}}{dx^2}\right)\psi_{n_y}\psi_{n_z},\frac{\partial^2\psi}{\partial y^2}=\left(\frac{d^2\psi_{n_y}}{dy^2}\right)\psi_{n_x}\psi_{n_z},\frac{\partial^2\psi}{\partial z^2}=\left(\frac{d^2\psi_{n_z}}{dz^2}\right)\psi_{n_x}\psi_{n_y}.$$

So $-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+U\psi=\left(-\frac{\hbar^2}{2m}\frac{d^2\psi_{n_x}}{dx^2}+\frac{1}{2}k'x^2\psi_{n_x}\right)\psi_{n_y}\psi_{n_z}$

$$+\left(-\frac{\hbar^2}{2m}\frac{d^2\psi_{n_y}}{dy^2}+\frac{1}{2}k'y^2\psi_{n_y}\right)\psi_{n_x}\psi_{n_z}+\left(-\frac{\hbar^2}{2m}\frac{d^2\psi_{n_z}}{dz^2}+\frac{1}{2}k'z^2\psi_{n_z}\right)\psi_{n_x}\psi_{n_y}$$

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+U\psi=(E_{n_x}+E_{n_y}+E_{n_z})\psi$$

Therefore, we have shown that this ψ is a solution to Eq.(40.29), with energy

$$E_{n_x n_y n_z} = E_{n_x} + E_{n_y} + E_{n_z} = \left(n_x + n_y + n_z + \frac{3}{2}\right)\hbar\omega$$

(b) and **(c)** The ground state has $n_x = n_y = n_z = 0,$ so the energy is $E_{000} = \frac{3}{2}\hbar\omega.$ There is only one set of n_x, n_y and n_z that give this energy.

First-excited state: $n_x = 1,\ n_y = n_z = 0$ or $n_y = 1,\ n_x = n_z = 0$ or $n_z = 1,\ n_x = n_y = 0$ and $E_{100} = E_{010} = E_{001} = \frac{5}{2}\hbar\omega$

There are three different sets of $n_x,\ n_y,\ n_z$ quantum numbers that give this energy, so there are three different quantum states that have this same energy.

EVALUATE: For the three-dimensional isotropic harmonic oscillator, the wave function is a product of one-dimensional harmonic oscillator wavefunctions for each dimension. The energy is a sum of energies for three one-dimensional oscillators. All the excited states are degenerate, with more than one state having the same energy.

40.54. $\omega_1 = \sqrt{k_1'/m},\ \omega_2 = \sqrt{k_2'/m}.$ Let $\psi_{n_x}(x)$ be a solution of Eq.(40.22) with $E_{n_x} = \left(n_x + \frac{1}{2}\right)\hbar\omega_1, \psi_{n_x}(y)$ be a similar solution, $\psi_{n_z}(z)$ be a solution of Eq.(40.22) but with *z* as the independent variable instead of *x*, and energy $E_{n_z} = \left(n_z + \frac{1}{2}\right)\omega_2.$

(a) As in Problem 40.53, look for a solution of the form $\psi(x,y,z) = \psi_{n_x}(x)\psi_{n_y}(y)\psi_{n_z}(z).$ Then,

$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = \left(E_{n_x} - \frac{1}{2}k_1'x^2\right)\psi$ with similar relations for $\frac{\partial^2\psi}{\partial y^2}$ and $\frac{\partial^2\psi}{\partial z^2}$. Adding,

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) = \left(E_{n_x} + E_{n_y} + E_{n_z} - \frac{1}{2}k_1'x^2 - \frac{1}{2}k_1'y^2 - \frac{1}{2}k_2'z^2\right)\psi$$

$$= (E_{n_x} + E_{n_y} + E_{n_z} - U)\psi = (E - U)\psi$$

where the energy *E* is $E = E_{n_x} + E_{n_y} + E_{n_z} = \hbar\left[(n_x + n_y + 1)\omega_1^2 + \left(n_z + \frac{1}{2}\right)\omega_2^2\right]$,with n_x, n_y and n_z all nonnegative integers.

(b) The ground level corresponds to $n_x = n_y = n_z = 0,$ and $E = \hbar\left(\omega_1^2 + \frac{1}{2}\omega_2^2\right)$. The first excited level corresponds to $n_x = n_y = 0$and $n_z = 1,$ since $\omega_1^2 > \omega_2^2,$ and $E = \hbar\omega\left(\omega_1^2 + \frac{3}{2}\omega_2^2\right)$. There is only one set of quantum numbers for both the ground state and the first excited state.

40.55. **(a)** $\psi(x) = A\sin kx$ and $\psi(-L/2) = 0 = \psi(+L/2)$

$$\Rightarrow 0 = A\sin\left(\frac{+kL}{2}\right) \Rightarrow \frac{+kL}{2} = n\pi \Rightarrow k = \frac{2n\pi}{L} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = \frac{L}{n} \Rightarrow p_n = \frac{h}{\lambda n} = \frac{nh}{L} \Rightarrow E_n = \frac{p_n^2}{2m} = \frac{n^2h^2}{2mL^2} = \frac{(2n)^2h^2}{8mL^2},\ \text{where } n = 1, 2...$$

(b) $\psi(x) = A\cos kx$ and $\psi(-L/2) = 0 = \psi(+L/2)$

$$\Rightarrow 0 = A\cos\left(\frac{kL}{2}\right) \Rightarrow \frac{kL}{2} = (2n+1)\frac{\pi}{2} \Rightarrow k = \frac{(2n+1)\pi}{L} = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = \frac{2L}{(2n+1)} \Rightarrow p_n = \frac{(2n+1)h}{2L}$$

$$\Rightarrow E_n = \frac{(2n+1)^2h^2}{8mL^2}\ n = 0, 1, 2...$$

(c) The combination of all the energies in parts (a) and (b) is the same energy levels as given in Eq.(40.9), where $E_n = \frac{n^2h^2}{8mL^2}.$

(d) Part (a)'s wave functions are odd, and part (b)'s are even.

40.56. **(a)** As with the particle in a box, $\psi(x) = A\sin kx,$ where *A* is a constant and $k^2 = 2mE/\hbar^2.$ Unlike the particle in a box, however, k and hence E do not have simple forms.

(b) For $x > L$, the wave function must have the form of Eq.(40.18). For the wave function to remain finite as $x \to \infty, C = 0.$ The constant $\kappa^2 = 2m(U_0 - E)/\hbar$, as in Eq.(14.17) and Eq.(40.18).

(c) At $x = L, A\sin kL = De^{-\kappa L}$ and $kA\cos kL = -\kappa De^{-\kappa L}$. Dividing the second of these by the first gives $k\cot kL = -\kappa$, a transcendental equation that must be solved numerically for different values of the length L and the ratio E/U_0.

40.57. **(a)** $E = K + U(x) = \dfrac{p^2}{2m} + U(x) \Rightarrow p = \sqrt{2m(E - U(x))}.$ $\lambda = \dfrac{h}{p} \Rightarrow \lambda(x) = \dfrac{h}{\sqrt{2m(E - U(x))}}.$

(b) As $U(x)$ gets larger (i.e., $U(x)$ approaches E from below—recall $k \ge 0$), $E - U(x)$ gets smaller, so $\lambda(x)$ gets larger.

(c) When $E = U(x)$, $E - U(x) = 0$, so $\lambda(x) \to \infty$.

(d) $\displaystyle\int_a^b \frac{dx}{\lambda(x)} = \int_a^b \frac{dx}{h/\sqrt{2m(E-U(x))}} = \frac{1}{h}\int_a^b \sqrt{2m(E-U(x))}\, dx = \frac{n}{2} \Rightarrow \int_a^b \sqrt{2m(E-U(x))}\, dx = \frac{hn}{2}.$

(e) $U(x) = 0$ for $0 < x < L$ with classical turning points at $x = 0$ and $x = L$. So,
$\displaystyle\int_a^b \sqrt{2m(E-U(x))}\, dx = \int_0^L \sqrt{2mE}dx = \sqrt{2mE}\int_0^L dx = \sqrt{2mE}L.$ So, from part (d),

$$\sqrt{2mE}L = \frac{hn}{2} \Rightarrow E = \frac{1}{2m}\left(\frac{hn}{2L}\right)^2 = \frac{h^2n^2}{8mL^2}.$$

(f) Since $U(x) = 0$ in the region between the turning points at $x = 0$ and $x = L$, the results is the *same* as part (e). The height U_0 never enters the calculation. WKB is best used with *smoothly* varying potentials $U(x)$.

40.58. **(a)** At the turning points $E = \dfrac{1}{2}k'x_{\text{TP}}^2 \Rightarrow x_{\text{TP}} = \pm\sqrt{\dfrac{2E}{k'}}.$

(b) $\displaystyle\int_{-\sqrt{2E/k'}}^{+\sqrt{2E/k'}} \sqrt{2m\left(E - \frac{1}{2}k'x^2\right)}dx = \frac{nh}{2}.$ To evaluate the integral, we want to get it into a form that matches the standard integral given. $\sqrt{2m\left(E - \dfrac{1}{2}k'x^2\right)} = \sqrt{2mE - mk'x^2} = \sqrt{mk'}\sqrt{\dfrac{2mE}{mk'} - x^2} = \sqrt{mk'}\sqrt{\dfrac{2E}{k'} - x^2}.$

Letting $A^2 = \dfrac{2E}{k'}$, $a = -\sqrt{\dfrac{2E}{k'}}$, and $b = +\sqrt{\dfrac{2E}{k'}}$

$$\Rightarrow \sqrt{mk'}\int_a^b \sqrt{A^2 - x^2}\, dx = 2\frac{\sqrt{mk'}}{2}\left[x\sqrt{A^2 - x^2} + A^2 \arcsin\left(\frac{x}{|A|}\right)\right]\Bigg|_0^b$$

$$= \sqrt{mk'}\left[\sqrt{\frac{2E}{k'}}\sqrt{\frac{2E}{k'} - \frac{2E}{k'}} + \frac{2E}{k'}\arcsin\left(\frac{\sqrt{2E/k'}}{\sqrt{2E/k'}}\right)\right] = \sqrt{mk'}\,\frac{2E}{k'}\arcsin(1) = 2E\sqrt{\frac{m}{k'}}\left(\frac{\pi}{2}\right).$$

Using WKB, this is equal to $\dfrac{hn}{2}$, so $E\sqrt{\dfrac{m}{k'}}\pi = \dfrac{hn}{2}$. Recall $\omega = \sqrt{\dfrac{k'}{m}}$, so $E = \dfrac{h}{2\pi}\omega n = \hbar\omega n.$

(c) We are missing the zero-point-energy offset of $\dfrac{\hbar\omega}{2}\left(\text{recall } E = \hbar\omega\left(n + \dfrac{1}{2}\right)\right)$. However, our approximation isn't bad at all!

40.59. **(a)** At the turning points $E = A|x_{\text{TP}}| \Rightarrow x_{\text{TP}} = \pm\dfrac{E}{A}.$

(b) $\displaystyle\int_{-E/A}^{+E/A} \sqrt{2m(E - A|x|)}dx = 2\int_0^{E/A} \sqrt{2m(E - Ax)}\, dx.$ Let $y = 2m(E - Ax) \Rightarrow$

$dy = -2mA\, dx$ when $x = \dfrac{E}{A}$, $y = 0$, and when $x = 0$, $y = 2mE$. So

$\displaystyle 2\int_0^{E/A} \sqrt{2m(E - Ax)}dx = -\frac{1}{mA}\int_{2mE}^0 y^{1/2}dy = -\frac{2}{3mA}y^{3/2}\Big|_{2mE}^0 = \frac{2}{3mA}(2mE)^{3/2}.$ Using WKB, this is equal to $\dfrac{hn}{2}$. So,

$$\frac{2}{3mA}(2mE)^{3/2} = \frac{hn}{2} \Rightarrow E = \frac{1}{2m}\left(\frac{3mAh}{4}\right)^{2/3} n^{2/3}.$$

(c) The difference in energy decreases between successive levels. For example:

$$1^{2/3} - 0^{2/3} = 1,\ 2^{2/3} - 1^{2/3} = 0.59,\ 3^{3/2} - 2^{3/2} = 0.49,...$$

- A sharp ∞ step gave ever-increasing level differences $(\sim n^2)$.
- A parabola $(\sim x^2)$ gave evenly spaced levels ($\sim n$).
- Now, a linear potential $(\sim x)$ gives ever-decreasing level differences ($\sim n^{2/3}$).

Roughly speaking, if the curvature of the potential (~ second derivative) is bigger than that of a parabola, then the level differences will increase. If the curvature is less than a parabola, the differences will decrease.

41

ATOMIC STRUCTURE

41.1. **IDENTIFY** and **SET UP:** $L=\sqrt{l(l+1)}\hbar$. $L_z=m_l\hbar$. $l=0,\ 1,\ 2,...,\ n-1$. $m_l=0,\ \pm1,\ \pm2,...,\ \pm l$. $\cos\theta=L_z/L$.

EXECUTE: **(a)** $\underline{l=0}$: $L=0$, $L_z=0$. $\underline{l=1}$: $L=\sqrt{2}\hbar$, $L_z=\hbar,0,-\hbar$. $\underline{l=2}$: $L=\sqrt{6}\hbar$, $L_z=2\hbar,\hbar,0,-\hbar,-2\hbar$.

(b) In each case $\cos\theta=L_z/L$. $L=0$: θ not defined. $L=\sqrt{2}\hbar$: $45.0°,\ 90.0°,\ 135.0°$. $L=\sqrt{6}\hbar$: $35.3°,\ 65.9°,\ 90.0°,\ 114.1°,\ 144.7°$.

EVALUATE: There is no state where $\vec{L}$ is totally aligned along the z axis.

41.2. **IDENTIFY** and **SET UP:** $L=\sqrt{l(l+1)}\hbar$. $L_z=m_l\hbar$. $l=0,1,2,...,n-1$. $m_l=0,\pm1,\pm2,...,\pm l$. $\cos\theta=L_z/L$.

EXECUTE: **(a)** $\underline{l=0}$: $L=0$, $L_z=0$. $\underline{l=1}$: $L=\sqrt{2}\hbar$, $L_z=\hbar,0,-\hbar$. $\underline{l=2}$: $L=\sqrt{6}\hbar$, $L_z=2\hbar,\hbar,0,-\hbar,-2\hbar$. $\underline{l=3}$: $L=2\sqrt{3}\hbar$, $L_z=3\hbar,2\hbar,\hbar,0,-\hbar,-2\hbar,-3\hbar$. $\underline{l=4}$: $L=2\sqrt{5}\hbar$, $L_z=4\hbar,3\hbar,2\hbar,\hbar,0,-\hbar,-2\hbar,-3\hbar,-4\hbar$.

(b) $L=0$: θ not defined. $L=\sqrt{2}\hbar$: $45.0°,90.0°,135.0°$. $L=\sqrt{6}\hbar$: $35.3°,65.9°,90.0°,114.1°,144.7°$. $L=2\sqrt{3}\hbar$: $54.7°,73.2°,90.0°,106.8°,125.3°,150.0°$. $L=2\sqrt{5}\hbar$: $26.6°,47.9°,63.4°,77.1°,90.0°,102.9°,116.6°,132.1°,153.4°$.

(c) The minimum angle is $26.6°$ and occurs for $l=4$, $m_l=+4$. The maximum angle is $153.4°$ and occurs for $l=4$, $m_l=-4$.

41.3. **IDENTIFY** and **SET UP:** The magnitude of the orbital angular momentum L is related to the quantum number l by Eq.(41.4): $L=\sqrt{l(l+1)}\hbar,\ 1=0,\ 1,\ 2,\ldots$

EXECUTE: $l(l+1)=\left(\dfrac{L}{\hbar}\right)^2=\left(\dfrac{4.716\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}\right)=20$

And then $l(l+1)=20$ gives that $l=4$.

EVALUATE: l must be integer.

41.4. **(a)** $(m_l)_{\text{max}}=2$, so $(L_z)_{\text{max}}=2\hbar$.

(b) $\sqrt{l(l+1)}\hbar=\sqrt{6}\hbar=2.45\hbar$.

(c) The angle is arccos $\left(\dfrac{L_z}{L}\right)=\arccos\left(\dfrac{m_l}{\sqrt{6}}\right)$, and the angles are, for $m_l=-2$ to $m_l=2$, $144.7°$, $114.1°,\ 90.0°,\ 65.9°, 35.3°$. The angle corresponding to $m_l=l$ will always be larger for larger l.

41.5. **IDENTIFY** and **SET UP:** The angular momentum L is related to the quantum number l by Eq.(41.4), $L=\sqrt{l(l+1)}\hbar$. The maximum l, l_{max}, for a given n is $l_{\text{max}}=n-1$.

EXECUTE: For $n=2$, $l_{\text{max}}=1$ and $L=\sqrt{2}\hbar=1.414\hbar$.

For $n=20$, $l_{\text{max}}=19$ and $L=\sqrt{(19)(20)}\hbar=19.49\hbar$.

For $n=200$, $l_{\text{max}}=199$ and $L=\sqrt{(199)(200)}\hbar=199.5\hbar$.

EVALUATE: As n increases, the maximum L gets closer to the value $n\hbar$ postulated in the Bohr model.

41.6. The (l, m_l) combinations are (0, 0), (1, 0), (1, ±1), (2, 0), (2, ±1), (2, ±2), (3, 0), (3, ±1), (3, ±2), (3, ±3), (4, 0), (4, ±1), (4, ±2), (4, ±3), and (4, ±4), a total of 25.

(b) Each state has the same energy (n is the same), $-\dfrac{13.60\ \text{eV}}{25}=-0.544\ \text{eV}$.

41.7. $U=\dfrac{1}{4\pi\epsilon_0}\dfrac{q_1q_2}{r}=\dfrac{-1}{4\pi\epsilon_0}\dfrac{(1.60\times10^{-19}\ \text{C})^2}{1.0\times10^{-10}\ \text{m}}=-2.3\times10^{-18}\ \text{J}$

$$U=\frac{-2.3\times10^{-18}\ \text{J}}{1.60\times10^{-19}\ \text{J/eV}}=-14.4\ \text{eV}.$$

41.8. **(a)** As in Example 41.3, the probability is

$$P=\int_0^{a/2}|\psi_{1s}|^2\,4\pi r^2dr=\frac{4}{a^3}\left[\left(-\frac{ar^2}{2}-\frac{a^2r}{2}-\frac{a^3}{4}\right)e^{-2r/a}\right]_0^{a/2}=1-\frac{5e^{-1}}{2}=0.0803\,.$$

(b) The difference in the probabilities is $(1-5e^{-2})-(1-(5/2)e^{-1})=(5/2)(e^{-1}-2e^{-2})=0.243.$

41.9. **(a)** $|\psi|^2=\psi^*\psi=|R(r)|^2|\Theta(\theta)|^2\,(Ae^{-im_l\phi})(Ae^{+im_l\phi})=A^2|R(r)|^2|\Theta(\theta)|^2,$ which is independent of ϕ .

(b) $\int_0^{2\pi}|\Phi(\phi)|^2\,d\phi=A^2\int_0^{2\pi}d\phi=2\pi A^2=1\Rightarrow A=\dfrac{1}{\sqrt{2\pi}}.$

41.10. $E_n=-\dfrac{1}{(4\pi\epsilon_0)^2}\dfrac{m_re^4}{2n^2\hbar^2}\Delta E_{12}=E_2-E_1=\dfrac{E_1}{2^2}-E_1=-(0.75)E_1.$

(a) If $m_\text{r}=m=9.11\times10^{-31}$ kg

$$\frac{m_\text{r}e^4}{(4\pi\epsilon_0)^2\hbar^2}=\frac{(9.109\times10^{-31}\text{ kg})(1.602\times10^{-19}\text{ C})^4}{2(1.055\times10^{-34}\text{ J}\cdot\text{s})^2}\left(8.988\times10^9\text{ N}\cdot\text{m}^2/\text{C}\right)^2=2.177\times10^{-18}\text{ J}=13.59\text{ eV}$$

For $2\rightarrow1$ transition, the coefficient is $(0.75)(13.59\text{ eV})=10.19$ eV.

(b) If $m_\text{r}=\dfrac{m}{2},$ using the result from part (a),

$$\frac{m_\text{r}e^4}{(4\pi\epsilon_0)^2\hbar^2}=(13.59\text{ eV})\left(\frac{m/2}{m}\right)=\left(\frac{13.59\text{ eV}}{2}\right)=6.795\text{ eV}.$$

Similarly, the $2\rightarrow1$ transition, $\Rightarrow\left(\dfrac{10.19\text{ eV}}{2}\right)=5.095$ eV.

(c) If $m_\text{r}=185.8m,$ using the result from part (a),

$$\frac{m_re^4}{(4\pi\epsilon_0)^2\hbar^2}=(13.59\text{ eV})\left(\frac{185.8m}{m}\right)=2525\text{ eV},$$

and the $2\rightarrow1$ transition gives $\Rightarrow$ (10.19 eV)(185.8) = 1893 eV.

41.11. **IDENTIFY** and **SET UP:** Eq.(41.8) gives $a=\dfrac{4\pi\epsilon_0\hbar^2}{m_\text{r}e^2}=\dfrac{\epsilon_0h^2}{\pi m_\text{r}e^2}.$

EXECUTE: **(a)** $m_\text{r}=m$

$$a=\frac{\epsilon_0h^2}{\pi m_\text{r}e^2}=\frac{(8.854\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{\pi(9.109\times10^{-31}\text{ kg})(1.602\times10^{-19}\text{ C})^2}=0.5293\times10^{-10}\text{ m}$$

(b) $m_\text{r}=m/2$

$$a=2\left(\frac{\epsilon_0h^2}{\pi m_\text{r}e^2}\right)=1.059\times10^{-10}\text{ m}$$

(c) $m_\text{r}=185.8m$

$$a=\frac{1}{185.8}\left(\frac{\epsilon_0h^2}{\pi m_\text{r}e^2}\right)=2.849\times10^{-13}\text{ m}$$

EVALUATE: a is the radius for the $n=1$ level in the Bohr model. When the reduced mass m_r increases, a decreases. For positronium and muonium the reduced mass effect is large.

41.12. $e^{im_l\phi}=\cos(m_l\phi)+i\sin(m_l\phi),$ and to be periodic with period $2\pi,$ $m_l2\pi$ must be an integer multiple of $2\pi,$ so m_l must be an integer.

41.13. $P(a)=\int_0^a|\psi_{1s}|2V=\int_0^a\dfrac{1}{\pi a^3}e^{-2r/a}(4\pi r^2dr)\,.$

$$P(a)=\frac{4}{a^3}\int_o^a r^2e^{-2r/a}dr=\frac{4}{a^3}\left[\left(\frac{-ar^2}{2}-\frac{a^2r}{2}-\frac{a^2}{4}\right)e^{-2r/a}\right]_0^a=\frac{4}{a^3}\left[\left(\frac{-a^3}{2}-\frac{a^3}{2}-\frac{a^3}{4}\right)e^{-2}+\frac{a^3}{4}e^0\right]$$

$\Rightarrow P(a)=1-5e^{-2}.$

41.14. **(a)** $\Delta E=\mu_\text{B}B=(5.79\times10^{-5}\text{ eV/T})(0.400\text{ T})=2.32\times10^{-5}$ eV

(b) $m_l=-2$ the lowest possible value of m_l.

(c) The energy level diagram is sketched in Figure 41.14.

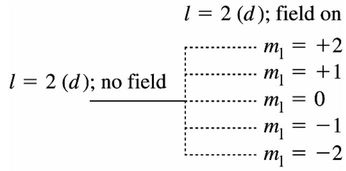


Figure 41.14

41.15. **IDENTIFY** and **SET UP:** The interaction energy between an external magnetic field and the orbital angular momentum of the atom is given by Eq.(41.18). The energy depends on m_l with the most negative m_l value having the lowest energy.

EXECUTE: **(a)** For the 5*g* level, $l = 4$ and there are $2l + 1 = 9$ different m_l states. The 5*g* level is split into 9 levels by the magnetic field.

(b) Each m_l level is shifted in energy an amount given by $U = m_l \mu_B B$. Adjacent levels differ in m_l by one, so $\Delta U = \mu_B B$.

$$\mu_B = \frac{e\hbar}{2m} = \frac{(1.602\times10^{-19}\ \text{C})(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{2(9.109\times10^{-31}\ \text{kg})} = 9.277\times10^{-24}\ \text{A}\cdot\text{m}^2$$

$$\Delta U = \mu_B B = (9.277\times10^{-24}\ \text{A/m}^2)(0.600\ \text{T}) = 5.566\times10^{-24}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 3.47\times10^{-5}\ \text{eV}$$

(c) The level of highest energy is for the largest m_l, which is $m_l = l = 4$; $U_4 = 4\mu_B B$. The level of lowest energy is for the smallest m_l, which is $m_l = -l = -4$; $U_{-4} = -4\mu_B B$. The separation between these two levels is $U_4 - U_{-4} = 8\mu_B B = 8(3.47\times10^{-5}\ \text{eV}) = 2.78\times10^{-4}\ \text{eV}$.

EVALUATE: The energy separations are proportional to the magnetic field. The energy of the $n = 5$ level in the absence of the external magnetic field is $(-13.6\ \text{eV})/5^2 = -0.544\ \text{eV}$, so the interaction energy with the magnetic field is much less than the binding energy of the state.

41.16. **(a)** According to Figure 41.11 in the textbook there are three different transitions that are consistent with the selection rules. The initial m_l values are 0, ±1; and the final m_l value is 0.

(b) The transition from $m_l = 0$ to $m_l = 0$ produces the same wavelength (122 nm) that was seen without the magnetic field.

(c) The larger wavelength (smaller energy) is produced from the $m_l = -1$ to $m_l = 0$ transition.

(d) The shorter wavelength (greater energy) is produced from the $m_l = +1$ to $m_l = 0$ transition.

41.17. $3p \Rightarrow n = 3, l = 1, \Delta U = \mu_B B \Rightarrow B = \dfrac{U}{\mu_B} = \dfrac{(2.71\times10^{-5}\ \text{eV})}{(5.79\times10^{-5}\ \text{eV/T})} = 0.468\ \text{T}$

(b) Three: $m_l = 0, \pm1$.

41.18. **(a)** $U = +(2.00232)\left(\dfrac{e}{2m}\right)\left(\dfrac{-\hbar}{2}\right)B = -\dfrac{(2.00232)}{2}\mu_B B$

$$U = -\frac{(2.00232)}{2}(5.788\times10^{-5}\ \text{eV/T})(0.480\ \text{T}) = -2.78\times10^{-5}\ \text{eV}.$$

(b) Since *n* = 1, *l* = 0 so there is no orbital magnetic dipole interaction. But if $n \neq 0$ there could be since $l < n$ allows for $l \neq 0$.

41.19. **IDENTIFY** and **SET UP:** The interaction energy is $U = -\vec{\mu}\cdot\vec{B}$, with μ_z given by Eq.(41.22).

EXECUTE: $U = -\vec{\mu}\cdot\vec{B} = +\mu_z B$, since the magnetic field is in the negative *z*-direction.

$\mu_z = -(2.00232)\left(\dfrac{e}{2m}\right)S_z$, so $U = -(2.00232)\left(\dfrac{e}{2m}\right)S_z B$

$S_z = m_s\hbar$, so $U = -2.00232\left(\dfrac{e\hbar}{2m}\right)m_s B$

$\dfrac{e\hbar}{2m} = \mu_B = 5.788\times10^{-5}\ \text{eV/T}$

$U = -2.00232\mu_B m_s B$

The $m_s = +\dfrac{1}{2}$ level has lower energy.

$$\Delta U = U\left(m_s = -\frac{1}{2}\right) - U\left(m_s = +\frac{1}{2}\right) = -2.00232\mu_B B\left(-\frac{1}{2} - \left(+\frac{1}{2}\right)\right) = +2.00232\mu_B B$$

$$\Delta U = +2.00232(5.788\times10^{-5}\ \text{eV/T})(1.45\ \text{T}) = 1.68\times10^{-4}\ \text{eV}$$

EVALUATE: The interaction energy with the electron spin is the same order of magnitude as the interaction energy with the orbital angular momentum for states with $m_l \neq 0$. But a 1*s* state has $l = 0$ and $m_l = 0$, so there is no orbital magnetic interaction.

41.20. The allowed (l, j) combinations are $\left(0, \frac{1}{2}\right), \left(1, \frac{1}{2}\right), \left(1, \frac{3}{2}\right), \left(2, \frac{3}{2}\right)$ and $\left(2, \frac{5}{2}\right)$.

41.21. **IDENTIFY** and **SET UP:** *j* can have the values $l + 1/2$ and $l - 1/2$.
EXECUTE: If *j* takes the values 7/2 and 9/2 it must be that $l - 1/2 = 7/2$ and $l = 8/2 = 4$. The letter that labels this *l* is *g*.
EVALUATE: *l* must be an integer.

41.22. **(a)** $\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(300\times10^{8}\ \text{m/s})}{(5.9\times10^{-6}\ \text{eV})} = 21\ \text{cm}$, $f = \frac{c}{\lambda} = \frac{(3.00\times10^{8}\ \text{m/s})}{0.21\ \text{m}} = 1.4\times10^{9}\ \text{Hz}$, a short radio wave.
(b) As in Example 41.6, the effective field is $B \cong \Delta E/2\mu_\text{B} = 5.1\times10^{-2}\ \text{T}$, for smaller than that found in the example.

41.23. **IDENTIFY** and **SET UP:** For a classical particle $L = I\omega$. For a uniform sphere with mass *m* and radius *R*, $I = \frac{2}{5}mR^2$, so $L = \left(\frac{2}{5}mR^2\right)\omega$. Solve for ω and then use $v = r\omega$ to solve for *v*.

EXECUTE: **(a)** $L = \sqrt{\frac{3}{4}}\hbar$ so $\frac{2}{5}mR^2\omega = \sqrt{\frac{3}{4}}\hbar$

$$\omega = \frac{5\sqrt{3/4}\hbar}{2mR^2} = \frac{5\sqrt{3/4}(1.055\times10^{-34}\ \text{J}\cdot\text{s})}{2(9.109\times10^{-31}\ \text{kg})(1.0\times10^{-17}\ \text{m})^2} = 2.5\times10^{30}\ \text{rad/s}$$

(b) $v = r\omega = (1.0\times10^{-17}\ \text{m})(2.5\times10^{30}\ \text{rad/s}) = 2.5\times10^{13}\ \text{m/s}$.

EVALUATE: This is much greater than the speed of light *c*, so the model cannot be valid.

41.24. However the number of electrons is obtained, the results must be consistent with Table (41.3); adding two more electrons to the zinc configuration gives $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^2$.

41.25. The ten lowest energy levels for electrons are in the *n* = 1 and *n* = 2 shells.

$$n = 1, l = 0, m_l = 0, \qquad m_s = \pm\frac{1}{2}: 2\ \text{states.}$$

$$n = 2, l = 0, m_l = 0, \qquad m_s = \pm\frac{1}{2}: 2\ \text{states.}$$

$$n = 2, l = 1, m_l = 0, \pm1, \quad m_s = \pm\frac{1}{2}: 6\ \text{states.}$$

41.26. For the outer electrons, there are more inner electrons to screen the nucleus.

41.27. **IDENTIFY** and **SET UP:** The energy of an atomic level is given in terms of *n* and Z_eff by Eq.(41.27), $E_n = -\left(\frac{Z_\text{eff}^2}{n^2}\right)(13.6\ \text{eV})$. The ionization energy for a level with energy $-E_n$ is $+E_n$.

EXECUTE: $n = 5$ and $Z_\text{eff} = 2.771$ gives $E_5 = -\frac{(2.771)^2}{5^2}(13.6\ \text{eV}) = -4.18\ \text{eV}$

The ionization energy is 4.18 eV.
EVALUATE: The energy of an atomic state is proportional to Z_eff^2.

41.28. For the $4s$ state, $E = -4.339\ \text{eV}$ and $Z_\text{eff} = 4\sqrt{(-4.339)/(-13.6)} = 2.26$. Similarly, $Z_\text{eff} = 1.79$ for the 4*p* state and 1.05 for the 4*d* state. The electrons in the states with higher *l* tend to be further away from the filled subshells and the screening is more complete.

41.29. **IDENTIFY** and **SET UP:** Use the exclusion principle to determine the ground-state electron configuration, as in Table 41.3. Estimate the energy by estimating Z_eff, taking into account the electron screening of the nucleus.

EXECUTE: **(a)** $Z = 7$ for nitrogen so a nitrogen atom has 7 electrons. N^{2+} has 5 electrons: $1s^2 2s^2 2p$.

(b) $Z_\text{eff} = 7 - 4 = 3$ for the 2*p* level.

$$E_n = -\left(\frac{Z_\text{eff}^2}{n^2}\right)(13.6\ \text{eV}) = -\frac{3^2}{2^2}(13.6\ \text{eV}) = -30.6\ \text{eV}$$

(c) $Z = 15$ for phosphorus so a phosphorus atom has 15 electrons.
P^{2+} has 13 electrons: $1s^2 2s^2 2p^6 3s^2 3p$

(d) $Z_{\text{eff}} = 15 - 12 = 3$ for the $3p$ level.

$$E_n = -\left(\frac{Z_{\text{eff}}^2}{n^2}\right)(13.6\text{ eV}) = -\frac{3^2}{3^2}(13.6\text{ eV}) = -13.6\text{ eV}$$

EVALUATE: In these ions there is one electron outside filled subshells, so it is a reasonable approximation to assume full screening by these inner-subshell electrons.

41.30. **(a)** $E_2 = -\frac{13.6\text{ eV}}{4}Z_{\text{eff}}^2$, so $Z_{\text{eff}} = 1.26$.

(b) Similarly, $Z_{\text{eff}} = 2.26$.

(c) Z_{eff} becomes larger going down the columns in the periodic table.

41.31. **IDENTIFY** and **SET UP:** Estimate Z_{eff} by considering electron screening and use Eq.(41.27) to calculate the energy. Z_{eff} is calculated as in Example 41.8.

EXECUTE: **(a)** The element Be has nuclear charge $Z = 4$. The ion Be^+ has 3 electrons. The outermost electron sees the nuclear charge screened by the other two electrons so $Z_{\text{eff}} = 4 - 2 = 2$.

$$E_n = -\left(\frac{Z_{\text{eff}}^2}{n^2}\right)(13.6\text{ eV}) \text{ so } E_2 = -\frac{2^2}{2^2}(13.6\text{ eV}) = -13.6\text{ eV}$$

(b) The outermost electron in Ca^+ sees a $Z_{\text{eff}} = 2$. $E_4 = -\frac{2^2}{4^2}(13.6\text{ eV}) = -3.4\text{ eV}$

EVALUATE: For the electron in the highest l-state it is reasonable to assume full screening by the other electrons, as in Example 41.8. The highest l-states of Be^+, Mg^+, Ca^+, etc. all have a $Z_{\text{eff}} = 2$. But the energies are different because for each ion the outermost sublevel has a different n quantum number.

41.32. $E_{kx} \cong (Z-1)^2(10.2\text{ eV})$. $Z \approx 1 + \sqrt{\frac{7.46\times10^3\text{ eV}}{10.2\text{ eV}}} = 28.0$, which corresponds to the element Nickel (Ni).

41.33. **(a)** $Z = 20: f = (2.48\times10^{15}\text{ Hz})(20-1)^2 = 8.95\times10^{17}\text{ Hz}$.

$E = hf = (4.14\times10^{-15}\text{ eV}\cdot\text{s})(8.95\times10^{17}\text{ Hz}) = 3.71\text{ keV}$. $\lambda = \frac{c}{f} = \frac{3.00\times10^8\text{ m/s}}{8.95\times10^{17}\text{ Hz}} = 3.35\times10^{-10}\text{ m}$.

(b) $Z = 27$: $f = 1.68\times10^{18}$ Hz. $E = 6.96$ keV. $\lambda = 1.79\times10^{-10}$ m.

(c) $Z = 48$: $f = 5.48\times10^{18}$ Hz, $E = 22.7$ keV, $\lambda = 5.47\times10^{-11}$ m.

41.34. **IDENTIFY:** The orbital angular momentum is limited by the shell the electron is in.

SET UP: For an electron in the n shell, its orbital angular momentum quantum number l is limited by $0 \le l < n$, and its orbital angular momentum is given by $L = \sqrt{l(l+1)}\,\hbar$. The z-component of its angular momentum is $L_z = m_l\hbar$, where $m_l = 0, \pm1, \ldots, \pm l$, and its spin angular momentum is $S = \sqrt{3/4}\,\hbar$ for all electrons. Its energy in the nth shell is $E_n = -(13.6\text{eV})/n^2$.

EXECUTE: **(a)** $L = \sqrt{l(l+1)}\,\hbar = 12\hbar \Rightarrow l = 3$. Therefore the smallest that n can be is 4, so $E_n = -(13.6\text{ eV})/n^2 = -(13.6\text{ eV})/4^2 = -0.8500$ eV.

(b) For $l = 3$, $m_l = \pm3, \pm2, \pm1, 0$. Since $L_z = m_l\hbar$, the largest L_z can be is $3\hbar$ and the smallest it can be is $-3\hbar$.

(c) $S = \sqrt{3/4}\,\hbar$ for *all* electrons.

(d) In this case, $n = 3$, so $l = 2, 1, 0$. Therefore the maximum that L can be is $L_{\max} = \sqrt{2(2+1)}\,\hbar = \sqrt{6}\,\hbar$. The minimum L can be is zero when $l = 0$.

EVALUATE: At the quantum level, electrons in atoms can have only certain allowed values of their angular momentum.

41.35. **IDENTIFY:** The total energy determines what shell the electron is in, which limits its angular momentum.

SET UP: The electron's orbital angular momentum is given by $L = \sqrt{l(l+1)}\,\hbar$, and its total energy in the nth shell is $E_n = -(13.6\text{ eV})/n^2$.

EXECUTE: **(a)** First find n: $E_n = -(13.6\text{eV})/n^2 = -0.5440$ eV which gives $n = 5$, so $l = 4, 3, 2, 1, 0$. Therefore the possible values of L are given by $L = \sqrt{l(l+1)}\,\hbar$, giving $L = 0, \sqrt{2}\hbar, \sqrt{6}\hbar, \sqrt{12}\hbar, \sqrt{20}\hbar$.

(b) $E_6 = -(13.6\text{ eV})/6^2 = -0.3778$ eV. $\Delta E = E_6 - E_5 = -0.3778\text{ eV} - (-0.5440\text{ eV}) = +0.1662$ eV

This must be the energy of the photon, so $\Delta E = hc/\lambda$, which gives

$\lambda = hc/\Delta E = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^8\text{ m/s})/(0.1662\text{ eV}) = 7.47\times10^{-6}\text{ m} = 7470$ nm, which is in the infrared and hence not visible.

EVALUATE: The electron can have any of the five possible values for its angular momentum, but it cannot have any others.

41.36. **IDENTIFY:** For the N shell, $n = 4$, which limits the values of the other quantum numbers.
SET UP: In the nth shell, $0 \le l < n$, $m_l = 0, \pm 1, \ldots, \pm l$, and $m_s = \pm 1/2$. The orbital angular momentum of the electron is $L = \sqrt{l(l+1)}\,\hbar$ and its spin angular momentum is $S = \sqrt{3/4}\,\hbar$.
EXECUTE: **(a)** For $l = 3$ we can have $m_l = \pm 3, \pm 2\pm, \pm 1, 0$ and $m_s = \pm½$; for $l = 2$ we can have $m_l = \pm 2, \pm 1, 0$ and $m_s = \pm½$; for $l = 1$, we can have $m_l = \pm 1, 0$ and $m_s = \pm 1/2$; for $l = 0$, we can have $m_l = 0$ and $m_s = \pm 1/2$.
(b) For the N shell, $n = 4$, and for an f-electron, $l = 3$, giving $L = \sqrt{l(l+1)}\,\hbar = \sqrt{3(3+1)}\,\hbar = \sqrt{12}\hbar$. $L_z = m_l\hbar = \pm 3\hbar,\ \pm 2\hbar,\ \pm\hbar,\ 0,$ so the maximum value is $3\hbar$. $S = \sqrt{3/4}\,\hbar$ for all electrons.
(c) For a d-state electron, $l = 2$, giving $L = \sqrt{2(2+1)}\,\hbar = \sqrt{6}\hbar$. $L_z = m_l\hbar,$ and the maximum value of m_l is 2, so the maximum value of L_z is $2\,\hbar$. The smallest angle occurs when L_z is most closely aligned along the angular momentum vector, which is when L_z is greatest. Therefore $\cos\theta_{\text{min}} = \dfrac{L_z}{L} = \dfrac{2\hbar}{\sqrt{6}\hbar} = \dfrac{2}{\sqrt{6}}$ and $\theta_{\text{min}} = 35.3°$. The largest angle occurs when L_z is as far as possible from the L-vector, which is when L_z is most negative. Therefore $\cos\theta_{\text{max}} = \dfrac{-2\hbar}{\sqrt{6}\hbar} = -\dfrac{2}{\sqrt{6}}$ and $\theta_{\text{max}} = 144.7°$.
(d) This is not possible since $l = 3$ for an f-electron, but in the M shell the maximum value of l is 2.
EVALUATE: The fact that the angle in part (c) cannot be zero tells us that the orbital angular momentum of the electron cannot be totally aligned along any specified direction.

41.37. **IDENTIFY:** The inner electrons shield part of the nuclear charge from the outer electron.
SET UP: The electron's energy in the nth shell, due to shielding, is $E_n = -\dfrac{Z_{\text{eff}}^2}{n^2}(13.6\text{eV})$, where $Z_{\text{eff}}e$ is the effective charge that the electron "sees" for the nucleus.
EXECUTE: **(a)** $E_n = -\dfrac{Z_{\text{eff}}^2}{n^2}(13.6\text{ eV})$ and $n = 4$ for the 4s state. Solving for Z_{eff} gives $Z_{\text{eff}} = \sqrt{-\dfrac{(4^2)(-1.947\text{ eV})}{13.6\text{ eV}}}$ $= 1.51$. The nucleus contains a charge of $+11e$, so the average number of electrons that screen this nucleus must be $11 - 1.51 = 9.49$ electrons
(b) (i) The charge of the nucleus is $+19e$, but $17.2e$ is screened by the electrons, so the outer electron "sees" $19e - 17.2e = 1.8e$ and $Z_{\text{eff}} = 1.8$.
(ii) $E_n = -\dfrac{Z_{\text{eff}}^2}{n^2}(13.6\text{ eV}) = -\dfrac{(1.8)^2}{4^2}(13.6\text{ eV}) = -2.75\text{ eV}$
EVALUATE: Sodium has 11 protons, so the inner 10 electrons shield a large portion of this charge from the outer electron. But they don't shield 10 of the protons, since the inner electrons are not totally equivalent to a uniform spherical shell. (They are lumpy.)

41.38. See Example 41.3; $r^2|\psi|^2 = Cr^2e^{-2r/a}$, $\dfrac{d(r^2|\psi|^2)}{dr} = Ce^{-2r/a}(2r-(2r^2/a)),$ and for a maximum, $r = a,$ the distance of the electron from the nucleus in the Bohr model.

41.39. **(a) IDENTIFY** and **SET UP:** The energy is given by Eq.(38.18), which is identical to Eq.(41.3). The potential energy is given by Eq.(23.9), with $q = +Ze$ and $q_0 = -e$.

EXECUTE: $E_{1s} = -\dfrac{1}{(4\pi\epsilon_0)^2}\dfrac{me^4}{2\hbar^2};\ U(r) = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{r}$

$E_{1s} = U(r)$ gives $-\dfrac{1}{(4\pi\epsilon_0)^2}\dfrac{me^4}{2\hbar^2} = -\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{r}$

$$r = \frac{(4\pi\epsilon_0)2\hbar^2}{me^2} = 2a$$

EVALUATE: The turning point is twice the Bohr radius.
(b) IDENTIFY and **SET UP:** For the 1s state the probability that the electron is in the classically forbidden region is $P(r > 2a) = \int_{2a}^{\infty}|\psi_{1s}|^2 dV = 4\pi\int_{2a}^{\infty}|\psi_{1s}|^2 r^2 dr.$ The normalized wave function of the 1s state of hydrogen is given in Example 41.3: $\psi_{1s}(r) = \dfrac{1}{\sqrt{\pi a^3}}e^{-r/a}$. Evaluate the integral; the integrand is the same as in Example 41.3.

EXECUTE: $P(r > 2a) = 4\pi\left(\dfrac{1}{\pi a^3}\right)\int_{2a}^{\infty} r^2 e^{-2r/a}dr$

Use the integral formula $\int r^2 e^{-\alpha r} dr = -e^{-\alpha r}\left(\frac{r^2}{\alpha} + \frac{2r}{\alpha^2} + \frac{2}{\alpha^3}\right)$, with $\alpha = 2/a$.

$$P(r > 2a) = -\frac{4}{a^3}\left[e^{-2r/a}\left(\frac{ar^2}{2} + \frac{a^2 r}{2} + \frac{a^3}{4}\right)\right]_{2a}^{\infty} = +\frac{4}{a^3}e^{-4}(2a^3 + a^3 + a^3/4)$$

$P(r > 2a) = 4e^{-4}(13/4) = 13e^{-4} = 0.238.$

EVALUATE: These is a 23.8% probability of the electron being found in the classically forbidden region, where classically its kinetic energy would be negative.

41.40. **(a)** For large values of n, the inner electrons will completely shield the nucleus, so $Z_{\text{eff}} = 1$ and the ionization energy would be $\frac{13.60\text{ eV}}{n^2}$.

(b) $\frac{13.60\text{ eV}}{350^2} = 1.11\times10^{-4}$ eV, $r_{350} = (350)^2 a_0 = (350)^2(0.529\times10^{-10}\text{ m}) = 6.48\times10^{-6}$ m .

(c) Similarly for $n = 650$, $\frac{13.60\text{ eV}}{(650)^2} = 3.22\times10^{-5}$ eV, $r_{650} = (650)^2(0.529\times10^{-10}\text{ m}) = 2.24\times10^{-5}$ m.

41.41. $\psi_{2s}(r) = \frac{1}{\sqrt{32\pi a^3}}\left(2 - \frac{r}{a}\right)e^{-r/2a}$

(a) IDENTIFY and **SET UP:** Let $I = \int_0^\infty |\psi_{2s}|^2 dV = 4\pi\int_0^\infty |\psi_{2s}|^2 r^2 dr$. If ψ_{2s} is normalized then we will find that $I = 1$.

EXECUTE: $I = 4\pi\left(\frac{1}{32\pi a^3}\right)\int_0^\infty \left(2 - \frac{r}{a}\right)^2 e^{-r/a} r^2 dr = \frac{1}{8a^3}\int_0^\infty \left(4r^2 - \frac{4r^3}{a} + \frac{r^4}{a^2}\right)e^{-r/a}dr$

Use the integral formula $\int_0^\infty x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}$, with $\alpha = 1/a$

$I = \frac{1}{8a^3}\left(4(2!)(a^3) - \frac{4}{a}(3!)(a)^4 + \frac{1}{a^2}(4!)(a)^5\right) = \frac{1}{8}(8 - 24 + 24) = 1$; this ψ_{2s} is normalized.

(b) SET UP: For a spherically symmetric state such as the 2s, the probability that the electron will be found at $r < 4a$ is $P(r < 4a) = \int_0^{4a} |\psi_{2s}|^2 dV = 4\pi\int_0^{4a} |\psi_{2s}|^2 r^2 dr$.

EXECUTE: $P(r < 4a) = \frac{1}{8a^3}\int_0^{4a}\left(4r^2 - \frac{4r^3}{a} + \frac{r^4}{a^2}\right)e^{-r/a}dr$

Let $P(r < 4a) = \frac{1}{8a^3}(I_1 + I_2 + I_3)$.

$I_1 = 4\int_0^{4a} r^2 e^{-r/a} dr$

Use the integral formula $\int r^2 e^{-\alpha r} dr = -e^{-\alpha r}\left(\frac{r^2}{\alpha} + \frac{2r}{\alpha^2} + \frac{2}{\alpha^3}\right)$ with $\alpha = 1/a$.

$I_1 = -4[e^{-r/a}(r^2 a + 2ra^2 + 2a^3)]_0^{4a} = (-104e^{-4} + 8)a^3$.

$I_2 = -\frac{4}{a}\int_0^{4a} r^3 e^{-r/a} dr$

Use the integral formula $\int r^3 e^{-\alpha r} dr = -e^{-\alpha r}\left(\frac{r^3}{\alpha} + \frac{3r^2}{\alpha^2} + \frac{6r}{a^3} + \frac{6}{\alpha^4}\right)$ with $\alpha = 1/a$.

$I_2 = \frac{4}{a}[e^{-r/a}(r^3 a + 3r^2 a^2 + 6ra^3 + 6a^4)]_0^{4a} = (568e^{-4} - 24)a^3$.

$I_3 = \frac{1}{a^2}\int_0^{4a} r^4 e^{-r/a} dr$

Use the integral formula $\int r^4 e^{-\alpha r} dr = -e^{-\alpha r}\left(\frac{r^4}{\alpha} + \frac{4r^3}{\alpha^2} + \frac{12r^2}{\alpha^3} + \frac{24r}{a^4} + \frac{24}{a^5}\right)$ with $\alpha = 1/a$.

$I_3 = -\frac{1}{a^2}[e^{-r/a}(r^4 a + 4r^3 a^2 + 12r^2 a^3 + 24ra^4 + 24a^5)]_0^{4a} = (-824e^{-4} + 24)a^3$.

Thus $P(r<4a)=\frac{1}{8a^3}(I_1+I_2+I_3)=\frac{1}{8a^3}a^3([8-24+24]+e^{-4}[-104+568-824])$

$P(r<4a)=\frac{1}{8}(8-360e^{-4})=1-45e^{-4}=0.176.$

EVALUATE: There is an 82.4% probability that the electron will be found at $r>4a$. In the Bohr model the electron is for certain at $r=4a$; this is a poor description of the radial probability distribution for this state.

41.42. **(a)** Since the given $\psi(r)$ is real, $r^2|\psi|^2=r^2\psi^2$. The probability density will be an extreme when $\frac{d}{dr}(r^2\psi^2)=2\left(r\psi^2+r^2\psi\frac{d\psi}{dr}\right)=2r\psi\left(\psi+r\frac{d\psi}{dr}\right)=0$. This occurs at $r=0$, a minimum, and when $\psi=0$, also a minimum. A maximum must correspond to $\psi+r\frac{d\psi}{dr}=0$. Within a multiplicative constant, $\psi(r)=(2-r/a)e^{-r/2a}$, $\frac{d\psi}{dr}=-\frac{1}{a}(2-r/2a)e^{-r/2a}$, and the condition for a maximum is $(2-r/a)=(r/a)(2-r/2a)$, or $r^2-6ra+4a^2=0$. The solutions to the quadratic are $r=a(3\pm\sqrt{5})$. The ratio of the probability densities at these radii is 3.68, with the larger density at $r=a(3+\sqrt{5})$.

(b) $\psi=0$ at $r=2a$

Parts (a) and (b) are consistent with Figure 41.5 in the textbook; note the two relative maxima, one on each side of the minimum of zero at $r=2a$.

41.43. **IDENTIFY:** Use Figure 41.2 in the textbook to relate θ_L to L_z and L: $\cos\theta_L=\frac{L_z}{L}$ so $\theta_L=\arccos\left(\frac{L_z}{L}\right)$

(a) SET UP: The smallest angle $(\theta_L)_{\min}$ is for the state with the largest L and the largest L_z. This is the state with $l=n-1$ and $m_l=l=n-1$.

EXECUTE: $L_z=m_l\hbar=(n-1)\hbar$

$L=\sqrt{l(l+1)}\hbar=\sqrt{(n-1)n}\hbar$

$(\theta_L)_{\min}=\arccos\left(\frac{(n-1)\hbar}{\sqrt{(n-1)n}\hbar}\right)=\arccos\left(\frac{(n-1)}{\sqrt{(n-1)n}}\right)=\arccos\left(\sqrt{\frac{n-1}{n}}\right)=\arccos(\sqrt{1-1/n}).$

EVALUATE: Note that $(\theta_L)_{\min}$ approaches $0°$ as $n\to\infty$.

(b) SET UP: The largest angle $(\theta_L)_{\max}$ is for $l=n-1$ and $m_l=-l=-(n-1)$.

EXECUTE: A similar calculation to part (a) yields $(\theta_L)_{\max}=\arccos(-\sqrt{1-1/n})$

EVALUATE: Note that $(\theta_L)_{\max}$ approaches $180°$ as $n\to\infty$.

41.44. **(a)** $L_x^2+L_y^2=L^2-L_z^2=l(l+1)\hbar^2-m_l^2\hbar^2$ so $\sqrt{L_x^2+L_y^2}=\sqrt{l(l+1)-m_l^2}\hbar.$

(b) This is the magnitude of the component of angular momentum perpendicular to the z-axis.

(c) The maximum value is $\sqrt{l(l+1)}\hbar=L$, when $m_l=0$. That is, if the electron is known to have no z-component of angular momentum, the angular momentum must be perpendicular to the z-axis. The minimum is $\sqrt{l}\hbar$ when $m_l=\pm l$.

41.45. $P(r)=\left(\frac{1}{24a^5}\right)r^4e^{-r/2a}$. $\frac{dP}{dr}=\left(\frac{1}{24a^5}\right)\left(4r^3-\frac{r^4}{a}\right)e^{-r/2a}$. $\frac{dP}{dr}=0$ when $4r^3-\frac{r^4}{a}=0$; $r=4a$. In the Bohr model, $r_n=n^2a$ so $r_2=4a$, which agrees.

41.46. The time required to transit the horizontal 50 cm region is $t=\frac{\Delta x}{v_x}=\frac{0.500\text{ m}}{525\text{ m/s}}=0.952$ ms. The force required to deflect each spin component by 0.50 mm is $F_z=ma_z=\pm m\frac{2\Delta z}{t^2}=\pm\left(\frac{0.1079\text{ kg/mol}}{6.022\times10^{23}\text{ atoms/mol}}\right)\frac{2(0.50\times10^{-3}\text{ m})}{(0.952\times10^{-3}\text{ s})^2}=$ $\pm1.98\times10^{-22}$ N. According to Eq.(41.22), the value of μ_z is $|\mu_z|=9.28\times10^{-24}\text{ A}\cdot\text{m}^2$. Thus, the required magnetic-field gradient is $\left|\frac{dB_z}{dz}\right|=\left|\frac{F_z}{\mu_z}\right|=\frac{1.98\times10^{-22}\text{ N}}{9.28\times10^{-24}\text{ J/T}}=21.3$ T/m.

41.47. Decay from a $3d$ to $2p$ state in hydrogen means that $n=3 \to n=2$ and $m_l = \pm 2, \pm 1, 0 \to m_l = \pm 1, 0.$ However selection rules limit the possibilities for decay. The emitted photon carries off one unit of angular momentum so l must change by 1 and hence m_l must change by 0 or $\pm 1.$ The shift in the transition energy from the zero field value is just $U = (m_{l_3} - m_{l_2})\mu_B B = \frac{e\hbar B}{2m}(m_{l_3} - m_{l_2}),$ where m_{l_3} is the $3d$ m_l value and m_{l_2} is the $2p$ m_l value. Thus there are only three different energy shifts. They and the transitions that have them, labeled by the m_l names, are:

$$\frac{e\hbar B}{2m}: 2 \to 1, \quad 1 \to 0, \quad 0 \to -1$$
$$0: 1 \to 1, \quad 0 \to 0, \quad -1 \to -1$$
$$-\frac{e\hbar B}{2m}: 0 \to 1, \quad -1 \to 0, \quad -2 \to -1$$

41.48. **IDENTIFY:** The presence of an external magnetic field shifts the energy levels up or down, depending upon the value of m_l.

SET UP: The selection rules tell us that for allowed transitions, $\Delta l = 1$ and $\Delta m_l = 0$ or ± 1.

EXECUTE: **(a)** $E = hc/\lambda = (4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})/(475.082\ \text{nm}) = 2.612\ \text{eV}$.

(b) For allowed transitions, $\Delta l = 1$ and $\Delta m_l = 0$ or ± 1. For the $3d$ state, $n = 3$, $l = 2$, and m_l can have the values 2, 1, 0, –1, –2. In the $2p$ state, $n = 2$, $l = 1$, and m_l can be 1, 0, –1. Therefore the 9 allowed transitions from the $3d$ state in the presence of a magnetic field are:

$l = 2$, $m_l = 2 \to l = 1$, $m_l = 1$;
$l = 2$, $m_l = 1 \to l = 1$, $m_l = 0$
$l = 2$, $m_l = 1 \to l = 1$, $m_l = 1$
$l = 2$, $m_l = 0 \to l = 1$, $m_l = 0$
$l = 2$, $m_l = 0 \to l = 1$, $m_l = 1$
$l = 2$, $m_l = 0 \to l = 1$, $m_l = -1$
$l = 2$, $m_l = -1 \to l = 1$, $m_l = 0$
$l = 2$, $m_l = -1 \to l = 1$, $m_l = -1$
$l = 2$, $m_l = -2 \to l = 1$, $m_l = -1$

(c) $\Delta E = \mu_B B = (5.788 \times 10^{-5}\ \text{eV/T})(3.500\ \text{T}) = 0.000203\ \text{eV}$

So the energies of the new states are –8.50000 eV + 0 and –8.50000 eV ± 0.000203 eV, giving energies of: –8.50020 eV, –8.50000 eV, and –8.49980 eV

(d) The energy differences of the allowed transitions are equal to the energy differences if no magnetic field were present (2.61176 eV, from part (a)), and that value $\pm\Delta E$ (0.000203 eV, from part (c)). Therefore we get the following.

For $E = 2.61176$ eV: $\lambda = 475.082$ nm (which was given)

For $E = 2.61176\ \text{eV} + 0.000203\ \text{eV} = 2.611963\ \text{eV}$:

$$\lambda = hc/E = (4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})/(2.611963\ \text{eV}) = 475.045\ \text{nm}$$

For $E = 2.61176\ \text{eV} - 0.000203\ \text{eV} = 2.61156\ \text{eV}$:

$$\lambda = hc/E = (4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})/(2.61156\ \text{eV}) = 475.119\ \text{nm}$$

EVALUATE: Even a strong magnetic field produces small changes in the energy levels, and hence in the wavelengths of the emitted light.

41.49. **IDENTIFY:** The presence of an external magnetic field shifts the energy levels up or down, depending upon the value of m_l.

SET UP: The energy difference due to the magnetic field is $\Delta E = \mu_B B$ and the energy of a photon is $E = hc/\lambda$.

EXECUTE: For the p state, $m_l = 0$ or ± 1, and for the s state $m_l = 0$. Between any two adjacent lines, $\Delta E = \mu_B B$. Since the change in the wavelength ($\Delta\lambda$) is very small, the energy change (ΔE) is also very small, so we can use differentials. $E = hc/\lambda$. $|dE| = \frac{hc}{\lambda^2}d\lambda$ and $\Delta E = \frac{hc\Delta\lambda}{\lambda^2}$. Since $\Delta E = \mu_B B$, we get $\mu_B B = \frac{hc\Delta\lambda}{\lambda^2}$ and $B = \frac{hc\Delta\lambda}{\mu_B \lambda^2}$.

$B = (4.136 \times 10^{-15}\ \text{eV}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})(0.0462\ \text{nm})/(5.788 \times 10^{-5}\ \text{eV/T})(575.050\ \text{nm})^2 = 3.00\ \text{T}$

EVALUATE: Even a strong magnetic field produces small changes in the energy levels, and hence in the wavelengths of the emitted light.

41.50. **(a)** The energy shift from zero field is $\Delta U_0 = m_l \mu_B B.$

For $m_l = 2, \Delta U_0 = (2)\,(5.79 \times 10^{-5}\ \text{eV/T})\,(1.40\ \text{T}) = 1.62 \times 10^{-4}\ \text{eV}.$

For $m_l = 1, \Delta U_0 = (1)(5.79 \times 10^{-5}\ \text{eV/T})\,(1.40\ \text{T}) = 8.11 \times 10^{-5}\ \text{eV}.$

(b) $|\Delta\lambda| = \lambda_0 \frac{|\Delta E|}{E_0},$ where $E_0 = (13.6\ \text{eV})((1/4) - (1/9)),$ $\lambda_0 = \left(\frac{36}{5}\right)\frac{1}{R} = 6.563 \times 10^{-7}\ \text{m}$

and $\Delta E = 1.62\times10^{-4}$ eV $- 8.11\times10^{-5}$ eV $= 8.09\times10^{-5}$ eV from part (a). Then, $|\Delta\lambda| = 2.81\times10^{-11}$ m $= 0.0281$ nm . The wavelength corresponds to a larger energy change, and so the wavelength is smaller.

41.51. **IDENTIFY:** The ratio according to the Boltzmann distribution is given by Eq.(38.21): $\dfrac{n_1}{n_0} = e^{-(E_1-E_0)/kT}$, where 1 is the higher energy state and 0 is the lower energy state.

SET UP: The interaction energy with the magnetic field is $U = -\mu_z B = 2.00232\left(\dfrac{e\hbar}{2m}\right)m_s B$ (Example 41.5.). The energy of the $m_s = +\dfrac{1}{2}$ level is increased and the energy of the $m_s = -\dfrac{1}{2}$ level is decreased.

$$\frac{n_{1/2}}{n_{-1/2}} = e^{-(U_{1/2}-U_{-1/2})/kT}$$

EXECUTE: $U_{1/2} - U_{-1/2} = 2.00232\left(\dfrac{e\hbar}{2m}\right)B\left(\dfrac{1}{2}-\left(-\dfrac{1}{2}\right)\right) = 2.00232\left(\dfrac{e\hbar}{2m}\right)B = 2.00232\mu_B B$

$$\frac{n_{1/2}}{n_{-1/2}} = e^{-(2.00232)\mu_B B/kT}$$

(a) $B = 5.00\times10^{-5}$ T

$$\frac{n_{1/2}}{n_{-1/2}} = e^{-2.00232(9.274\times10^{-24}\ \text{A/m}^2)(5.00\times10^{-5}\ \text{T})/([1.381\times10^{-23}\ \text{J/K}][300\ \text{K}])}$$

$$\frac{n_{1/2}}{n_{-1/2}} = e^{-2.24\times10^{-7}} = 0.99999978 = 1 - 2.2\times10^{-7}$$

(b) $B = 5.00\times10^{-5}$ T, $\dfrac{n_{1/2}}{n_{-1/2}} = e^{-2.24\times10^{-3}} = 0.9978$

(c) $B = 5.00\times10^{-5}$ T, $\dfrac{n_{1/2}}{n_{-1/2}} = e^{-2.24\times10^{-2}} = 0.978$

EVALUATE: For small fields the energy separation between the two spin states is much less than kT for $T = 300$ K and the states are equally populated. For $B = 5.00$ T the energy spacing is large enough for there to be a small excess of atoms in the lower state.

41.52. Using Eq.(41.4), $L = mvr = \sqrt{l(l+1)}\hbar$, and the Bohr radius from Eq.(38.15), we obtain the following value for v:

$v = \dfrac{\sqrt{l(l+1)}\hbar}{m(n^2 a_0)} = \dfrac{\sqrt{2}(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi(9.11\times10^{-31}\ \text{kg})\,(4)\,(5.29\times10^{-11}\ \text{m})} = 7.74\times10^5$ m/s. The magnetic field generated by the "moving" proton at the electrons position can be calculated from Eq.(28.1):

$$B = \frac{\mu_0}{4\pi}\frac{|q|v\sin\phi}{r^2} = (10^{-7}\ \text{T}\cdot\text{m/A})\frac{(1.60\times10^{-19}\text{C})\,(7.74\times10^5\ \text{m/s})\sin(90°)}{(4)^2(5.29\times10^{-11}\ \text{m})^2} = 0.277\ \text{T}.$$

41.53. **IDENTIFY** and **SET UP:** m_s can take on 4 different values: $m_s = -\dfrac{3}{2},\ -\dfrac{1}{2},\ +\dfrac{1}{2},\ +\dfrac{3}{2}$. Each nlm_l state can have 4 electrons, each with one of the four different m_s values. Apply the exclusion principle to determine the electron configurations.

EXECUTE: **(a)** For a filled $n=1$ shell, the electron configuration would be $1s^4$; four electrons and $Z=4$. For a filled $n=2$ shell, the electron configuration would be $1s^4 2s^4 2p^{12}$; twenty electrons and $Z=20$.

(b) Sodium has $Z=11$; 11 electrons. The ground-state electron configuration would be $1s^4 2s^4 2p^3$.

EVALUATE: The chemical properties of each element would be very different.

41.54. **(a)** $Z^2\,(-13.6\ \text{eV}) = (7)^2\,(-13.6\ \text{eV}) = -666$ eV.

(b) The negative of the result of part (a), 666 eV.

(c) The radius of the ground state orbit is inversely proportional to the nuclear charge, and $\dfrac{a}{Z} = (0.529\times10^{-10}\ \text{m})/7 = 7.56\times10^{-12}$ m.

(d) $\lambda = \dfrac{hc}{\Delta E} = \dfrac{hc}{E_0\left(\frac{1}{1^2}-\frac{1}{2^2}\right)}$, where E_0 is the energy found in part (b), and $\lambda = 2.49$ nm.

41.55. **(a) IDENTIFY and SET UP:** The energy of the photon equals the transition energy of the atom: $\Delta E = hc/\lambda$. The energies of the states are given by Eq.(41.3).

EXECUTE: $E_n = -\dfrac{13.60 \text{ eV}}{n^2}$ so $E_2 = -\dfrac{13.60 \text{ eV}}{4}$ and $E_1 = -\dfrac{13.60 \text{ eV}}{1}$

$$\Delta E = E_2 - E_1 = 13.60 \text{ eV}\left(-\frac{1}{4}+1\right) = \frac{3}{4}(13.60 \text{ eV}) = 10.20 \text{ eV} = (10.20 \text{ eV})(1.602\times10^{-19} \text{ J/eV}) = 1.634\times10^{-18} \text{ J}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34} \text{ J}\cdot\text{s})(2.998\times10^{8} \text{ m/s})}{1.634\times10^{-18} \text{ J}} = 1.22\times10^{-7} \text{ m} = 122 \text{ nm}$$

(b) IDENTIFY and SET UP: Calculate the change in ΔE due to the orbital magnetic interaction energy, Eq.(41.17), and relate this to the shift $\Delta\lambda$ in the photon wavelength.

EXECUTE: The shift of a level due to the energy of interaction with the magnetic field in the *z*-direction is $U = m_l\mu_B B$. The ground state has $m_l = 0$ so is unaffected by the magnetic field. The $n = 2$ initial state has $m_l = -1$ so its energy is shifted downward an amount $U = m_l\mu_B B = (-1)(9.274\times10^{-24} \text{ A/m}^2)(2.20 \text{ T}) =$

$(-2.040\times10^{-23} \text{ J})(1 \text{ eV}/1.602\times10^{-19} \text{ J}) = 1.273\times10^{-4} \text{ eV}$

Note that the shift in energy due to the magnetic field is a very small fraction of the 10.2 eV transition energy. Problem 39.56c shows that in this situation $|\Delta\lambda/\lambda| = |\Delta E/E|$. This gives

$$|\Delta\lambda| = \lambda|\Delta E/E| = 122 \text{ nm}\left(\frac{1.273\times10^{-4} \text{ eV}}{10.2 \text{ eV}}\right) = 1.52\times10^{-3} \text{ nm} = 1.52 \text{ pm.}$$

EVALUATE: The upper level in the transition is lowered in energy so the transition energy is decreased. A smaller ΔE means a larger λ; the magnetic field increases the wavelength. The fractional shift in wavelength, $\Delta\lambda/\lambda$ is small, only 1.2×10^{-5}.

41.56. The effective field is that which gives rise to the observed difference in the energy level transition,

$B = \dfrac{\Delta E}{\mu_B} = \dfrac{hc}{\mu_B}\left(\dfrac{\lambda_1-\lambda_2}{\lambda_1\lambda_2}\right) = \dfrac{2\pi mc}{e}\left(\dfrac{\lambda_1-\lambda_2}{\lambda_1\lambda_2}\right)$. Substitution of numerical values gives $B = 3.64\times10^{-3}$ T, much smaller than that for sodium.

41.57. **IDENTIFY:** Estimate the atomic transition energy and use Eq.(38.6) to relate this to the photon wavelength.

(a) SET UP: vanadium, $Z = 23$

<u>minimum wavelength</u>; corresponds to largest transition energy

EXECUTE: The highest occupied shell is the *N* shell $(n = 4)$. The highest energy transition is $N \to K$, with transition energy $\Delta E = E_N - E_K$. Since the shell energies scale like $1/n^2$ neglect E_N relative to E_K, so $\Delta E = E_K = (Z-1)^2(13.6 \text{ eV}) = (23-1)^2(13.6 \text{ eV}) = 6.582\times10^{3} \text{ eV} = 1.055\times10^{-15}$ J. The energy of the emitted photon equals this transition energy, so the photon's wavelength is given by $\Delta E = hc/\lambda$ so $\lambda = hc/\Delta E$.

$$\lambda = \frac{(6.626\times10^{-34} \text{ J}\cdot\text{s})(2.998\times10^{8} \text{ m/s})}{1.055\times10^{-15} \text{ J}} = 1.88\times10^{-10} \text{ m} = 0.188 \text{ nm.}$$

SET UP: <u>maximum wavelength</u>; corresponds to smallest transition energy, so for the K_α transition

EXECUTE: The frequency of the photon emitted in this transition is given by Moseley's law (Eq.41.29):

$f = (2.48\times10^{15} \text{ Hz})(Z-1)^2 = (2.48\times10^{15} \text{ Hz})(23-1)^2 = 1.200\times10^{18}$ Hz

$$\lambda = \frac{c}{f} = \frac{2.998\times10^{8} \text{ m/s}}{1.200\times10^{18} \text{ Hz}} = 2.50\times10^{-10} \text{ m} = 0.250 \text{ nm}$$

(b) rhenium, $Z = 45$

Apply the analysis of part (a), just with this different value of *Z*.

<u>minimum wavelength</u>

$\Delta E = E_K = (Z-1)^2(13.6 \text{ eV}) = (45-1)^2(13.6 \text{ eV}) = 2.633\times10^{4} \text{ eV} = 4.218\times10^{-15}$ J.

$$\lambda = hc/\Delta E = \frac{(6.626\times10^{-34} \text{ J}\cdot\text{s})(2.998\times10^{8} \text{ m/s})}{4.218\times10^{-15} \text{ J}} = 4.71\times10^{-11} \text{ m} = 0.0471 \text{ nm.}$$

<u>maximum wavelength</u>

$f = (2.48\times10^{15} \text{ Hz})(Z-1)^2 = (2.48\times10^{15} \text{ Hz})(45-1)^2 = 4.801\times10^{18}$ Hz

$$\lambda = \frac{c}{f} = \frac{2.998\times10^{8} \text{ m/s}}{4.801\times10^{18} \text{ Hz}} = 6.24\times10^{-11} \text{ m} = 0.0624 \text{ nm}$$

EVALUATE: Our calculated wavelengths have values corresponding to x rays. The transition energies increase when *Z* increases and the photon wavelengths decrease.

41.58. **(a)** $\Delta E = (2.00232)\dfrac{e}{2m}B\Delta S_z \approx \dfrac{e\hbar}{m}B = \dfrac{hc}{\lambda} \Rightarrow B = \dfrac{2\pi mc}{\lambda e}.$

(b) $B = \dfrac{2\pi(9.11\times10^{-31}\text{ kg})(3.00\times10^{8}\text{ m/s})}{(0.0350\text{ m})(1.60\times10^{-19}\text{ C})} = 0.307\text{ T}.$

41.59. **(a)** To calculate the total number of states for the nth principal quantum number shell we must multiply all the possibilities. The spin states multiply everything by 2. The maximum l value is (n –1), and each l value has $(2l+1)m_l$ values. So the total number of states is

$$N = 2\sum_{l=0}^{n-1}(2l+1) = 2\sum_{l=0}^{n-1}1 + 4\sum_{l=0}^{n-1}l = 2n + \frac{4(n-1)(n)}{2} = 2n + 2n^2 - 2n = 2n^2.$$

(b) The n = 5 shell (O-shell) has 50 states.

41.60. **IDENTIFY:** We treat the Earth as an electron.

SET UP: The intrinsic spin angular momentum of an electron is $S = \sqrt{\frac{3}{4}}\hbar$, and the angular momentum of the spinning Earth is $S = I\omega$, where $I = 2/5\, mR^2$.

EXECUTE: **(a)** Using $S = I\omega = \sqrt{\frac{3}{4}}\hbar$ and solving for ω gives

$$\omega = \frac{\sqrt{\frac{3}{4}}\hbar}{\frac{2}{5}mR^2} = \frac{\sqrt{\frac{3}{4}}\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)}{\frac{2}{5}\left(5.97\times10^{24}\text{ kg}\right)\left(6.38\times10^{6}\text{ m}\right)^2} = 9.40\times10^{-73}\text{ rad/s}$$

(b) We could not use this approach on the electron because in quantum physics we do not view it in the classical sense as a spinning ball.

EVALUATE: The angular velocity we have just calculated for the Earth would certainly be masked by its present angular spin of one revolution per day.

41.61. The potential $U(x) = \frac{1}{2}k'x^2$ is that of a simple harmonic oscillator. Treated quantum mechanically (see Section 40.4) each *energy* state has energy $E_n = \hbar\omega\,(n+\frac{1}{2})$. Since electrons obey the exclusion principle, this allows us to put *two* electrons (one for each $m_s = \pm\frac{1}{2}$) for every value of n—each quantum state is then defined by the ordered pair of quantum numbers (n, m_s). By placing two electrons in each energy level the lowest energy is then

$$2\left(\sum_{n=0}^{N-1}E_n\right) = 2\left(\sum_{n=0}^{N-1}\hbar\omega\left(n+\frac{1}{2}\right)\right) = 2\hbar\omega\left[\sum_{n=0}^{N-1}n + \sum_{n=0}^{N-1}\frac{1}{2}\right] = 2\hbar\omega\left[\frac{(N-1)(N)}{2} + \frac{N}{2}\right] =$$

$$\hbar\omega[N^2 - N + N] = \hbar\omega N^2 = \hbar N^2\sqrt{\frac{k'}{m}}.$$

Here we used the hint from Problem 41.59 to do the first sum, realizing that the first value of n is zero and the last value of n is N – 1, giving us a total of N energy levels filled.

41.62. **(a)** Apply Coulomb's law to the orbiting electron and set it equal to the centripetal force. There is an attractive force with charge +2e a distance r away and a repulsive force a distance 2r away. So, $\dfrac{(+2e)(-e)}{4\pi\epsilon_0 r^2} + \dfrac{(-e)(-e)}{4\pi\epsilon_0 (2r)^2} = \dfrac{-mv^2}{r}$. But, from the quantization of angular momentum in the first Bohr orbit, $L = mvr = \hbar \Rightarrow v = \dfrac{\hbar}{mr}$.

So $\dfrac{-2e^2}{4\pi\epsilon_0 r^2} + \dfrac{e^2}{4\pi\epsilon_0 (4r)^2} = \dfrac{-mv^2}{r} = \dfrac{-m\left(\dfrac{\hbar}{mr}\right)^2}{r} = -\dfrac{\hbar^2}{mr^3} \Rightarrow \dfrac{-7}{4}\dfrac{e^2}{r^2} = -\dfrac{4\pi\epsilon_0\hbar^2}{mr^3}$.

$$r = \frac{4}{7}\left(\frac{4\pi\epsilon_0\hbar^2}{me^2}\right) = \frac{4}{7}a_0 = \frac{4}{7}(0.529\times10^{-10}\text{ m}) = 3.02\times10^{-11}\text{ m}.$$

And $v = \dfrac{-\hbar}{mr} = \dfrac{7}{4}\dfrac{\hbar}{ma_0} = \dfrac{7}{4}\dfrac{(1.054\times10^{-34}\text{ J}\cdot\text{s})}{(9.11\times10^{-31}\text{ kg})(0.529\times10^{-10}\text{ m})} = 3.83\times10^{6}\text{ m/s}.$

(b) $K = 2\left(\dfrac{1}{2}mv^2\right) = 9.11\times10^{-31}\text{ kg }(3.83\times10^{6}\text{ m/s})^2 = 1.34\times10^{-17}\text{ J} = 83.5\text{ eV}.$

(c) $U = 2\left(\dfrac{-2e^2}{4\pi\epsilon_0 r}\right) + \dfrac{e^2}{4\pi\epsilon_0(2r)} = \dfrac{-4e^2}{4\pi\epsilon_0 r} + \dfrac{e^2}{4\pi E_0(2r)} = \dfrac{-7}{2}\left(\dfrac{e^2}{4\pi\epsilon_0 r}\right) = -2.67\times10^{-17}\text{ J} = -166.9\text{ eV}$

(d) $E_\infty = -[-166.9\text{ eV} + 83.5\text{ eV}] = 83.4\text{ eV},$ which is only off by about 5% from the real value of 79.0 eV.

41.63. **(a)** The radius is inversely proportional to *Z*, so the classical turning radius is $2a/Z$.

(b) The normalized wave function is $\psi_{1s}(r) = \dfrac{1}{\sqrt{\pi a^3/Z^3}} e^{-Zr/a}$ and the probability of the electron being found outside the classical turning point is $P = \int_{2a/Z}^{\infty} |\psi_{1s}|^2 4\pi r^2 dr = \dfrac{4}{a^3/Z^3} \int_{2a/Z}^{\infty} e^{-2Zr/a} r^2 dr.$ Making the change of variable $u = Zr/a$, $dr = (a/Z)du$ changes the integral to $P = 4\int_2^{\infty} e^{-2u} u^2 du$, which is independent of *Z*. The probability is that found in Problem 41.39, 0.238, independent of *Z*.

MOLECULES AND CONDENSED MATTER

42

42.1. **(a)** $K = \frac{3}{2}kT \Rightarrow T = \frac{2K}{3k} = \frac{2(7.9\times10^{-4}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{3(1.38\times10^{-23}\ \text{J/K})} = 6.1\ \text{K}$

(b) $T = \frac{2(4.48\ \text{eV})\ (1.60\times10^{-19}\ \text{J/eV})}{3(1.38\times10^{-23}\ \text{J/K})} = 34{,}600\ \text{K}.$

(c) The thermal energy associated with room temperature (300 K) is much greater than the bond energy of He_2 (calculated in part (a)), so the typical collision at room temperature will be more than enough to break up He_2. However, the thermal energy at 300 K is much less than the bond energy of H_2, so we would expect it to remain intact at room temperature.

42.2. **(a)** $U = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} = -5.0\ \text{eV}.$

(b) $-5.0\ \text{eV} + (4.3\ \text{eV} - 3.5\ \text{eV}) = -4.2\ \text{eV}.$

42.3. **IDENTIFY:** The energy given to the photon comes from a transition between rotational states.

SET UP: The rotational energy of a molecule is $E = l(l+1)\frac{\hbar^2}{2I}$ and the energy of the photon is $E = hc/\lambda$.

EXECUTE: Use the energy formula, the energy difference between the $l = 3$ and $l = 1$ rotational levels of the molecule is $\Delta E = \frac{\hbar^2}{2I}\left[3(3+1) - 1(1+1)\right] = \frac{5\hbar^2}{I}$. Since $\Delta E = hc/\lambda$, we get $hc/\lambda = 5\hbar^2/I$. Solving for I gives

$$I = \frac{5\hbar\lambda}{2\pi c} = \frac{5\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)(1.780\ \text{nm})}{2\pi\left(3.00\times10^{8}\ \text{m/s}\right)} = 4.981\times10^{-52}\ \text{kg}\cdot\text{m}^2.$$

Using $I = m_r r_0^2$, we can solve for r_0: $r_0 = \sqrt{\frac{I\left(m_N + m_H\right)}{m_N m_H}} = \sqrt{\frac{\left(4.981\times10^{-52}\ \text{kg}\cdot\text{m}^2\right)\left(2.33\times10^{-26}\ \text{kg} + 1.67\times10^{-27}\ \text{kg}\right)}{\left(2.33\times10^{-26}\ \text{kg}\right)\left(1.67\times10^{-27}\ \text{kg}\right)}}$

$r_0 = 5.65\times10^{-13}$ m

EVALUATE: This separation is much smaller than the diameter of a typical atom and is not very realistic. But we are treating a *hypothetical* NH molecule.

42.4. The energy of the emitted photon is 1.01×10^{-5} eV, and so its frequency and wavelength are $f = \frac{E}{h} = \frac{(1.01\times10^{-5}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})} = 2.44\ \text{GHz}$ and $\lambda = \frac{c}{f} = \frac{(3.00\times10^{8}\ \text{m/s})}{(2.44\times10^{9}\ \text{Hz})} = 0.123\ \text{m}$. This frequency corresponds to that given for a microwave oven.

42.5. Let 1 refer to C and 2 to O. $m_1 = 1.993\times10^{-26}$ kg, $m_2 = 2.656\times10^{-26}$ kg, $r_0 = 0.1128$ nm.

$r_1 = \left(\frac{m_2}{m_1 + m_2}\right)r_0 = 0.0644\ \text{nm (carbon)}$; $r_2 = \left(\frac{m_1}{m_1 + m_2}\right)r_0 = 0.0484\ \text{nm (oxygen)}$

(b) $I = m_1 r_1^2 + m_2 r_2^2 = 1.45\times10^{-46}\ \text{kg}\cdot\text{m}^2$; yes, this agrees with Example 42.2.

42.6. Each atom has a mass m and is at a distance $L/2$ from the center, so the moment of inertia is $2(m)(L/2)^2 = mL^2/2 = 2.21\times10^{-44}\ \text{kg}\cdot\text{m}^2$.

42.7. **IDENTIFY** and **SET UP:** Set $K = E_1$ from Example 42.2. Use $K = \frac{1}{2}I\omega^2$ to solve for ω and $v = r\omega$ to solve for v.

EXECUTE: **(a)** From Example 42.2, $E_1 = 0.479\ \text{meV} = 7.674\times10^{-23}$ J and $I = 1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2$

$K = \frac{1}{2}I\omega^2$ and $K = E$ gives $\omega = \sqrt{2E_1/I} = 1.03\times10^{12}$ rad/s

(b) $v_1 = r_1\omega_1 = (0.0644\times10^{-9}\text{ m})(1.03\times10^{12}\text{ rad/s}) = 66.3\text{ m/s}$ (carbon)

$v_2 = r_2\omega_2 = (0.0484\times10^{-9}\text{ m})(1.03\times10^{12}\text{ rad/s}) = 49.8\text{ m/s}$ (oxygen)

(c) $T = 2\pi/\omega = 6.10\times10^{-12}\text{ s}$

EVALUATE: From the information in Example 42.3 we can calculate the vibrational period to be $T = 2\pi/\omega = 2\pi\sqrt{m_r/k'} = 1.5\times10^{-14}\text{ s}$. The rotational motion is over an order of magnitude slower than the vibrational motion.

42.8. $\Delta E = \dfrac{hc}{\lambda} = \hbar\sqrt{k'/m_r}$, and solving for k', $k' = \left(\dfrac{2\pi c}{\lambda}\right)^2 m_r = 205\text{ N/m}$.

42.9. **IDENTIFY** and **SET UP:** The energy of a rotational level with quantum number l is $E_l = l(l+1)\hbar^2/2I$ (Eq.(42.3)). $I = m_r r^2$, with the reduced mass m_r given by Eq.(42.4). Calculate I and ΔE and then use $\Delta E = hc/\lambda$ to find λ.

EXECUTE: **(a)** $m_r = \dfrac{m_1 m_2}{m_1+m_2} = \dfrac{m_{Li}m_H}{m_{Li}+m_H} = \dfrac{(1.17\times10^{-26}\text{ kg})(1.67\times10^{-27}\text{ kg})}{1.17\times10^{-26}\text{ kg}+1.67\times10^{-27}\text{ kg}} = 1.461\times10^{-27}\text{ kg}$

$I = m_r r^2 = (1.461\times10^{-27}\text{ kg})(0.159\times10^{-9}\text{ m})^2 = 3.694\times10^{-47}\text{ kg}\cdot\text{m}^2$

$l = 3:\ E = 3(4)\left(\dfrac{\hbar^2}{2I}\right) = 6\left(\dfrac{\hbar^2}{I}\right)$

$l = 4:\ E = 4(5)\left(\dfrac{\hbar^2}{2I}\right) = 10\left(\dfrac{\hbar^2}{I}\right)$

$\Delta E = E_4 - E_3 = 4\left(\dfrac{\hbar^2}{I}\right) = 4\left(\dfrac{(1.055\times10^{-34}\text{ J}\cdot\text{s})^2}{3.694\times10^{-47}\text{ kg}\cdot\text{m}^2}\right) = 1.20\times10^{-21}\text{ J} = 7.49\times10^{-3}\text{ eV}$

(b) $\Delta E = hc/\lambda$ so $\lambda = \dfrac{hc}{\Delta E} = \dfrac{(4.136\times10^{-15}\text{ eV})(2.998\times10^8\text{ m/s})}{7.49\times10^{-3}\text{ eV}} = 166\,\mu\text{m}$

EVALUATE: LiH has a smaller reduced mass than CO and λ is somewhat smaller here than the λ calculated for CO in Example 42.2

42.10. **IDENTIFY:** The vibrational energy of the molecule is related to its force constant and reduced mass, while the rotational energy depends on its moment of inertia, which in turn depends on the reduced mass.

SET UP: The vibrational energy is $E_n = \left(n+\dfrac{1}{2}\right)\hbar\omega = \left(n+\dfrac{1}{2}\right)\hbar\sqrt{\dfrac{k'}{m_r}}$ and the rotational energy is $E_l = l(l+1)\dfrac{\hbar^2}{2I}$.

EXECUTE: For a vibrational transition, we have $\Delta E_v = \hbar\sqrt{\dfrac{k'}{m_r}}$, so we first need to find m_r. The energy for a rotational transition is $\Delta E_R = \dfrac{\hbar^2}{2I}\left[2(2+1)-1(1+1)\right] = \dfrac{2\hbar^2}{I}$. Solving for I and using the fact that $I = m_r r_0^2$, we have $m_r r_0^2 = \dfrac{2\hbar^2}{\Delta E_R}$, which gives

$$m_r = \frac{2\hbar^2}{r_0^2\Delta E_R} = \frac{2\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)\left(6.583\times10^{-16}\text{ eV}\cdot\text{s}\right)}{\left(0.8860\times10^{-9}\text{ m}\right)^2\left(8.841\times10^{-4}\text{ eV}\right)} = 2.0014\times10^{-28}\text{ kg}$$

Now look at the vibrational transition to find the force constant.

$$\Delta E_v = \hbar\sqrt{\frac{k'}{m_r}} \quad\Rightarrow\quad k' = \frac{m_r(\Delta E_v)^2}{\hbar^2} = \frac{\left(2.0014\times10^{-28}\text{ kg}\right)(0.2560\text{ eV})^2}{\left(6.583\times10^{-16}\text{ eV}\cdot\text{s}\right)^2} = 30.27\text{ N/m}$$

EVALUATE: This would be a rather weak spring in the laboratory.

42.11. **(a)** $E_l = \dfrac{l(l+1)\hbar^2}{2I}, E_{l-1} = \dfrac{l(l-1)\hbar^2}{2I} \Rightarrow \Delta E = \dfrac{\hbar^2}{2I}(l^2+l-l^2+l) = \dfrac{l\hbar^2}{I}$

(b) $f = \dfrac{\Delta E}{h} = \dfrac{\Delta E}{2\pi\hbar} = \dfrac{l\hbar}{2\pi I}$.

42.12. **IDENTIFY:** Find ΔE for the transition and compute λ from $\Delta E = hc/\lambda$.

SET UP: From Example 42.2, $E_l = l(l+1)\dfrac{\hbar^2}{2I}$, with $\dfrac{\hbar^2}{2I} = 0.2395\times10^{-3}$ eV. From Example 42.3, $\Delta E = 0.2690$ eV is the spacing between vibrational levels. Thus $E_n = (n+\tfrac{1}{2})\hbar\omega$, with $\hbar\omega = 0.2690$ eV. By Eq.(42.9),

$$E = E_n + E_l = (n+\tfrac{1}{2})\hbar\omega + l(l+1)\frac{\hbar^2}{2I}.$$

EXECUTE: **(a)** $n=0 \to n=1$ and $l=1 \to l=2$

For $n=0,\ l=1,\ E_i = \tfrac{1}{2}\hbar\omega + 2\left(\dfrac{\hbar^2}{2I}\right)$.

For $n=1,\ l=2,\ E_f = \tfrac{3}{2}\hbar\omega + 6\left(\dfrac{\hbar^2}{2I}\right)$.

$$\Delta E = E_f - E_i = \hbar\omega + 4\left(\frac{\hbar^2}{2I}\right) = 0.2690\text{ eV} + 4(0.2395\times10^{-3}\text{ eV}) = 0.2700\text{ eV}$$

$$\frac{hc}{\lambda} = \Delta E \text{ so } \lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{0.2700\text{ eV}} = 4.592\times10^{-6}\text{ m} = 4.592\,\mu\text{m}$$

(b) $n=0 \to n=1$ and $l=2 \to l=1$

For $n=0,\ l=2,\ E_i = \tfrac{1}{2}\hbar\omega + 6\left(\dfrac{\hbar^2}{2I}\right)$.

For $n=1,\ l=1,\ E_f = \tfrac{3}{2}\hbar\omega + 2\left(\dfrac{\hbar^2}{2I}\right)$.

$$\Delta E = E_f - E_i = \hbar\omega - 4\left(\frac{\hbar^2}{2I}\right) = 0.2690\text{ eV} - 4(0.2395\times10^{-3}\text{ eV}) = 0.2680\text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{0.2680\text{ eV}} = 4.627\times10^{-6}\text{ m} = 4.627\,\mu\text{m}$$

(c) $n=0 \to n=1$ and $l=3 \to l=2$

For $n=0,\ l=3,\ E_i = \tfrac{1}{2}\hbar\omega + 12\left(\dfrac{\hbar^2}{2I}\right)$.

For $n=1,\ l=2,\ E_f = \tfrac{3}{2}\hbar\omega + 6\left(\dfrac{\hbar^2}{2I}\right)$.

$$\Delta E = E_f - E_i = \hbar\omega - 6\left(\frac{\hbar^2}{2I}\right) = 0.2690\text{ eV} - 6(0.2395\times10^{-3}\text{ eV}) = 0.2676\text{ eV}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{0.2676\text{ eV}} = 4.634\times10^{-6}\text{ m} = 4.634\,\mu\text{m}$$

EVALUATE: All three transitions are for $n=0 \to n=1$. The spacing between vibrational levels is larger than the spacing between rotational levels, so the difference in λ for the various rotational transitions is small. When the transition is to a larger l, $\Delta E > \hbar\omega$ and when the transition is to a smaller l, $\Delta E < \hbar\omega$.

42.13. **(a) IDENTIFY** and **SET UP:** Use $\omega = \sqrt{k'/m_r}$ and $\omega = 2\pi f$ to calculate k'. The atomic masses are used in Eq.(42.4) to calculate m_r.

EXECUTE: $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k'}{m_r}}$, so $k' = m_r(2\pi f)^2$

$$m_r = \frac{m_1 m_2}{m_1+m_2} = \frac{m_H m_F}{m_H+m_F} = \frac{(1.67\times10^{-27}\text{ kg})(3.15\times10^{-26}\text{ kg})}{1.67\times10^{-27}\text{ kg}+3.15\times10^{-26}\text{ kg}} = 1.586\times10^{-27}\text{ kg}$$

$$k' = m_r(2\pi f)^2 = (1.586\times10^{-27}\text{ kg})(2\pi[1.24\times10^{14}\text{ Hz}])^2 = 963\text{ N/m}$$

(b) IDENTIFY and **SET UP:** The energy levels are given by Eq.(42.7). $E_n = (n+\tfrac{1}{2})\hbar\omega = (n+\tfrac{1}{2})hf$, since $\hbar\omega = (h/2\pi)\omega$ and $(\omega/2\pi) = f$. The energy spacing between adjacent levels is $\Delta E = E_{n+1} - E_n = (n+1+\tfrac{1}{2}-n-\tfrac{1}{2})hf = hf$, independent of n.

EXECUTE: $\Delta E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(1.24\times10^{14}\ \text{Hz}) = 8.22\times10^{-20}\ \text{J} = 0.513\ \text{eV}$

(c) IDENTIFY and SET UP: The photon energy equals the transition energy so $\Delta E = hc/\lambda$.

EXECUTE: $hf = hc/\lambda$ so $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^{8}\ \text{m/s}}{1.24\times10^{14}\ \text{Hz}} = 2.42\times10^{-6}\ \text{m} = 2.42\ \mu\text{m}$

EVALUATE: This photon is infrared, which is typical for vibrational transitions.

42.14. For an average spacing a, the density is $\rho = m/a^3$, where m is the average of the ionic masses, and so

$$a^3 = \frac{m}{\rho} = \frac{\left(6.49\times10^{-26}\ \text{kg} + 1.33\times10^{-25}\ \text{kg}\right)/2}{(2.75\times10^{3}\ \text{kg/m}^3)} = 3.60\times10^{-29}\ \text{m}^3,$$

and $a = 3.30\times10^{-10}\ \text{m} = 0.330\ \text{nm}$.

(b) The larger (higher atomic number) atoms have the larger spacing.

42.15. **IDENTIFY and SET UP:** Find the volume occupied by each atom. The density is the average mass of Na and Cl divided by this volume.

EXECUTE: Each atom occupies a cube with side length 0.282 nm. Therefore, the volume occupied by each atom is $V = (0.282\times10^{-9}\ \text{m})^3 = 2.24\times10^{-29}\ \text{m}^3$. In NaCl there are equal numbers of Na and Cl atoms, so the average mass of the atoms in the crystal is $m = \tfrac{1}{2}(m_{\text{Na}} + m_{\text{Cl}}) = \tfrac{1}{2}(3.82\times10^{-26}\ \text{kg} + 5.89\times10^{-26}\ \text{kg}) = 4.855\times10^{-26}\ \text{kg}$

The density then is $\rho = \dfrac{m}{V} = \dfrac{4.855\times10^{-26}\ \text{kg}}{2.24\times10^{-29}\ \text{m}^3} = 2.17\times10^{3}\ \text{kg/m}^3$.

EVALUATE: The density of water is $1.00\times10^{3}\ \text{kg/m}^3$, so our result is reasonable.

42.16. **(a)** As a photon, $\lambda = \dfrac{hc}{E} = \dfrac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(6.20\times10^{3}\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)} = 0.200\ \text{nm}.$

(b) As a matter wave,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)}{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)(37.6\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)}} = 0.200\ \text{nm}$$

(c) As a matter wave,

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)}{\sqrt{2\left(1.67\times10^{-27}\ \text{kg}\right)(0.0205\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)}} = 0.200\ \text{nm}.$$

42.17. **IDENTIFY:** The energy gap is the energy of the maximum-wavelength photon.

SET UP: The energy difference is equal to the energy of the photon, so $\Delta E = hc/\lambda$.

EXECUTE: **(a)** Using the photon wavelength to find the energy difference gives

$$\Delta E = hc/\lambda = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})/(1.11\times10^{-6}\ \text{m}) = 1.12\ \text{eV}$$

(b) A wavelength of 1.11 μm = 1110 nm is in the infrared, shorter than that of visible light.

EVALUATE: Since visible photons have more than enough energy to excite electrons from the valence to the conduction band, visible light will be absorbed, which makes silicon opaque.

42.18. **(a)** $\dfrac{hc}{\Delta E} = 2.27\times10^{-7}\ \text{m} = 227\ \text{nm}$, in the ultraviolet.

(b) Visible light lacks enough energy to excite the electrons into the conduction band, so visible light passes through the diamond unabsorbed.

(c) Impurities can lower the gap energy making it easier for the material to absorb shorter wavelength visible light. This allows longer wavelength visible light to pass through, giving the diamond color.

42.19. $\Delta E = \dfrac{hc}{\lambda} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{9.31\times10^{-13}\ \text{m}} = 2.14\times10^{-13}\ \text{J} = 1.34\times10^{6}\ \text{eV}$. So the number of electrons that can be excited to the conduction band is $n = \dfrac{1.34\times10^{6}\ \text{eV}}{1.12\ \text{eV}} = 1.20\times10^{6}$ electrons

42.20. $1 = \int |\psi|^2\, dV$

$$= A^2\left(\int_0^L \sin^2\left(\frac{n_x\pi x}{L}\right)dx\right)\left(\int_0^L \sin^2\left(\frac{n_y\pi y}{L}\right)dy\right)\left(\int_0^L \sin^2\left(\frac{n_z\pi z}{L}\right)dz\right) = A^2\left(\frac{L}{2}\right)^3$$

so $A = (2/L)^{3/2}$ (assuming A to be real positive).

42.21. Density of states:

$$g(E)=\frac{(2m)^{3/2}V}{2\pi^2\hbar^3}E^{1/2}=\frac{(2(9.11\times10^{-31}\ \text{kg}))^{3/2}(1.0\times10^{-6}\ \text{m}^3)(5.0\ \text{eV})^{1/2}(1.60\times10^{-19}\ \text{J/eV})^{1/2}}{2\pi^2(1.054\times10^{-34}\ \text{J}\cdot\text{s})^3}$$

$$g(E)=(9.5\times10^{40}\ \text{states/J})(1.60\times10^{-19}\ \text{J/eV})=1.5\times10^{22}\ \text{states/eV}.$$

42.22. $v_{\text{rms}}=\sqrt{3kT/m}=1.17\times10^5\ \text{m/s}$, as found in Example 42.9. The equipartition theorem does not hold for the electrons at the Fermi energy. Although these electrons are very energetic, they cannot lose energy, unlike electrons in a free electron gas.

42.23. **(a) IDENTIFY** and **SET UP:** The three-dimensional Schrödinger equation is $-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)+U\psi=E\psi$ (Eq.40.29). For free electrons, $U=0$. Evaluate $\partial^2\psi/\partial x^2$, $\partial^2\psi/\partial y^2$, and $\partial^2\psi/\partial z^2$ for ψ as given by Eq.(42.10). Put the results into Eq.(40.20) and see if the equation is satisfied.

EXECUTE: $\frac{\partial\psi}{\partial x}=\frac{n_x\pi}{L}A\cos\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)\sin\left(\frac{n_z\pi z}{L}\right)$

$$\frac{\partial^2\psi}{\partial x^2}=-\left(\frac{n_x\pi}{L}\right)^2A\sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)\sin\left(\frac{n_z\pi z}{L}\right)=-\left(\frac{n_x\pi}{L}\right)^2\psi$$

Similarly $\frac{\partial^2\psi}{\partial y^2}=-\left(\frac{n_y\pi}{L}\right)^2\psi$ and $\frac{\partial^2\psi}{\partial z^2}=-\left(\frac{n_z\pi}{L}\right)^2\psi.$

Therefore, $-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)=\frac{\hbar^2}{2m}\left(\frac{\pi^2}{L^2}\right)(n_x^2+n_y^2+n_z^2)\psi=\frac{(n_x^2+n_y^2+n_z^2)\pi^2\hbar^2}{2mL^2}\psi$

This equals $E\psi$, with $E=\frac{(n_x^2+n_y^2+n_z^2)\pi^2\hbar^2}{2mL^2}$, which is Eq.(42.11).

EVALUATE: ψ given by Eq.(42.10) is a solution to Eq.(40.29), with *E* as given by Eq.(42.11).

(b) IDENTIFY and **SET UP:** Find the set of quantum numbers n_x, n_y, and n_z that give the lowest three values of *E*. The degeneracy is the number of sets n_x, n_y, n_z and m_s that give the same *E*.

EXECUTE: <u>Ground level</u>: lowest *E* so $n_x=n_y=n_z=1$ and $E=\frac{3\pi^2\hbar^2}{2mL^2}$. No other combination of n_x, n_y, and n_z gives this same *E*, so the only degeneracy is the degeneracy of two due to spin.

<u>First excited level</u>: next lower *E* so one *n* equals 2 and the others equal 1. $E=(2^2+1^2+1^2)\frac{\pi^2\hbar^2}{2mL^2}=\frac{6\pi^2\hbar^2}{2mL^2}$

There are three different sets of n_x, n_y, n_z values that give this *E*:

$n_x=2,\ n_y=1,\ n_z=1;\ n_x=1,\ n_y=2,\ n_z=1;\ n_x=1,\ n_y=1,\ n_z=2$

This gives a degeneracy of 3 so the total degeneracy, with the factor of 2 from spin, is 6.

<u>Second excited level</u>: next lower *E* so two of n_x, n_y, n_z equal 2 and the other equals 1.

$$E=(2^2+2^2+1^2)\frac{\pi^2\hbar^2}{2mL^2}=\frac{9\pi^2\hbar^2}{2mL^2}$$

There are different sets of n_x, n_y, n_z values that give this *E*:

$n_x=2,\ n_y=2,\ n_z=1;\ n_x=2,\ n_y=1,\ n_z=2;\ n_x=1,\ n_y=2,\ n_z=2.$

Thus, as for the first excited level, the total degeneracy, including spin, is 6.

EVALUATE: The wavefunction for the 3-dimensional box is a product of the wavefunctions for a 1-dimensional box in the *x*, *y*, and *z* coordinates and the energy is the sum of energies for three 1-dimensional boxes. All levels except for the ground level have a degeneracy greater than two. Compare to the 3-dimensional isotropic harmonic oscillator treated in Problem 40.53.

42.24. Eq.(42.13) may be solved for $n_{\text{rs}}=(2mE)^{1/2}(L/\hbar\pi)$, and substituting this into Eq. (42.12), using $L^3=V$, gives Eq.(42.14).

42.25. **(a) IDENTIFY** and **SET UP:** The electron contribution to the molar heat capacity at constant volume of a metal is $C_V=\left(\frac{\pi^2KT}{2E_{\text{F}}}\right)R.$

EXECUTE: $C_V=\frac{\pi^2(1.381\times10^{-23}\ \text{J/K})(300\ \text{K})}{2(5.48\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}R=0.0233R.$

(b) EVALUATE: The electron contribution found in part (a) is $0.0233R = 0.194\ \text{J/mol}\cdot\text{K}$. This is $0.194/25.3 = 7.67\times10^{-3} = 0.767\%$ of the total C_V.

(c) Only a small fraction of C_V is due to the electrons. Most of C_V is due to the vibrational motion of the ions.

42.26. **(a)** From Eq. (42.22), $E_{\text{av}} = \dfrac{3}{5}E_{\text{F}} = 1.94\ \text{eV}$.

(b) $\sqrt{2E/m} = \sqrt{\dfrac{2(1.94\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\ \text{kg}}} = 8.25\times10^{5}\ \text{m/s}$.

(c) $\dfrac{E_{\text{F}}}{k} = \dfrac{(3.23\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{(1.38\times10^{-23}\ \text{J/K})} = 3.74\times10^{4}\ \text{K}$.

42.27. **IDENTIFY:** The probability is given by the Fermi-Dirac distribution.

SET UP: The Fermi-Dirac distribution is $f(E) = \dfrac{1}{e^{(E-E_{\text{F}})/kT}+1}$.

EXECUTE: We calculate the value of $f(E)$, where $E = 8.520$ eV, $E_{\text{F}} = 8.500$ eV, $k = 1.38\times10^{-23}$ J/K $= 8.625\times10^{-5}$ eV/K, and $T = 20°\text{C} = 293$ K. The result is $f(E) = 0.312 = 31.2\%$.

EVALUATE: Since the energy is close to the Fermi energy, the probability is quite high that the state is occupied by an electron.

42.28. **(a)** See Example 42.10: The probabilities are 1.78×10^{-7}, 2.37×10^{-6}, and 1.51×10^{-5}.

(b) The Fermi distribution, Eq.(42.17), has the property that $f(E_{\text{F}} - E) = 1 - f(E)$ (see Problem (42.48)), and so the probability that a state at the top of the valence band is occupied is the same as the probability that a state of the bottom of the conduction band is filled (this result depends on having the Fermi energy in the middle of the gap).

42.29. **IDENTIFY:** Use Eq.(42.17), $f(E) = \dfrac{1}{e^{(E-E_{\text{F}})/kT}+1}$. Solve for $E - E_{\text{F}}$.

SET UP: $e^{(E-E_{\text{F}})/kT} = \dfrac{1}{f(E)} - 1$

The problem states that $f(E) = 4.4\times10^{-4}$ for E at the bottom of the conduction band.

EXECUTE: $e^{(E-E_{\text{F}})/kT} = \dfrac{1}{4.4\times10^{-4}} - 1 = 2.272\times10^{3}$.

$E - E_{\text{F}} = kT\ln(2.272\times10^{3}) = (1.3807\times10^{-23}\ \text{J/T})(300\ \text{K})\ln(2.272\times10^{3}) = 3.201\times10^{-20}\ \text{J} = 0.20\ \text{eV}$

$E_{\text{F}} = E - 0.20$ eV; the Fermi level is 0.20 eV below the bottom of the conduction band.

EVALUATE: The energy gap between the Fermi level and bottom of the conduction band is large compared to kT at $T = 300$ K and as a result $f(E)$ is small.

42.30. **IDENTIFY:** The current depends on the voltage across the diode and its temperature, so the resistance also depends on these quantities.

SET UP: The current is $I = I_{\text{S}}(e^{eV/kT} - 1)$ and the resistance is $R = V/I$.

EXECUTE: **(a)** The resistance is $R = \dfrac{V}{I} = \dfrac{V}{I_{\text{S}}(e^{eV/kT}-1)}$. The exponent is $\dfrac{eV}{kT} = \dfrac{e(0.0850\ \text{V})}{(8.625\times10^{-5}\ \text{eV/K})(293\ \text{K})} =$ 3.3635, giving $R = \dfrac{85.0\ \text{mV}}{(0.750\ \text{mA})(e^{3.3635}-1)} = 4.06\ \Omega$.

(b) In this case, the exponent is $\dfrac{eV}{kT} = \dfrac{e(-0.050\ \text{V})}{(8.625\times10^{-5}\ \text{eV/K})(293\ \text{K})} = -1.979$

which gives $R = \dfrac{-50.0\ \text{mV}}{(0.750\ \text{mA})(e^{-1.979}-1)} = 77.4\ \Omega$

EVALUATE: Reversing the voltage can make a considerable change in the resistance of a diode.

42.31. **IDENTIFY** and **SET UP:** The voltage-current relation is given by Eq.(42.23): $I = I_{\text{s}}(e^{eV/kT} - 1)$. Use the current for $V = +15.0$ mV to solve for the constant I_{s}.

EXECUTE: **(a)** Find I_{s}: $V = +15.0\times10^{-3}$ V gives $I = 9.25\times10^{-3}$ A

$$\frac{eV}{kT} = \frac{(1.602\times10^{-19}\ \text{C})(15.0\times10^{-3}\ \text{V})}{(1.381\times10^{-23}\ \text{J/K})(300\ \text{K})} = 0.5800$$

$$I_{\text{s}} = \frac{I}{e^{eV/kT}-1} = \frac{9.25\times10^{-3}\ \text{A}}{e^{0.5800}-1} = 1.177\times10^{-2} = 11.77\ \text{mA}$$

Then can calculate I for $V = 10.0$ mV: $\dfrac{eV}{kT} = \dfrac{(1.602\times10^{-19}\text{ C})(10.0\times10^{-3}\text{ V})}{(1.381\times10^{-23}\text{ J/K})(300\text{ K})} = 0.3867$

$I = I_s(e^{eV/kT} - 1) = (11.77\text{ mA})(e^{0.3867} - 1) = 5.56\text{ mA}$

(b) $\dfrac{eV}{kT}$ has the same magnitude as in part (a) but not V is negative so $\dfrac{eV}{kT}$ is negative.

$\underline{V = -15.0\text{ mV}}: \dfrac{eV}{kT} = -0.5800$ and $I = I_s(e^{eV/kT} - 1) = (11.77\text{ mA})(e^{-0.5800} - 1) = -5.18\text{ mA}$

$\underline{V = -10.0\text{ mV}}: \dfrac{eV}{kT} = -0.3867$ and $I = I_s(e^{eV/kT} - 1) = (11.77\text{ mA})(e^{-0.3867} - 1) = -3.77\text{ mA}$

EVALUATE: There is a directional asymmetry in the current, with a forward-bias voltage producing more current than a reverse-bias voltage of the same magnitude, but the voltage is small enough for the asymmetry not be pronounced. Compare to Example 42.11, where more extreme voltages are considered.

42.32. **(a)** Solving Eq.(42.23) for the voltage as a function of current,

$$V = \frac{kT}{e}\ln\left(\frac{I}{I_S} + 1\right) = \frac{kT}{e}\ln\left(\frac{40.0\text{ mA}}{3.60\text{ mA}} + 1\right) = 0.0645\text{ V}.$$

(b) From part (a), the quantity $e^{eV/kT} = 12.11$, so far a reverse-bias voltage of the same magnitude,

$I = I_S\left(e^{-eV/kT} - 1\right) = I_S\left(\dfrac{1}{12.11} - 1\right) = -3.30\text{ mA}.$

42.33. **IDENTIFY:** During the transition, the molecule emits a photon of light having energy equal to the energy difference between the two vibrational states of the molecule.

SET UP: The vibrational energy is $E_n = \left(n + \dfrac{1}{2}\right)\hbar\omega = \left(n + \dfrac{1}{2}\right)\hbar\sqrt{\dfrac{k'}{m_r}}$.

EXECUTE: **(a)** The energy difference between two adjacent energy states is $\Delta E = \hbar\sqrt{\dfrac{k'}{m_r}}$, and this is the energy of the photon, so $\Delta E = hc/\lambda$. Equating these two expressions for ΔE and solving for k', we have $k' = m_r\left(\dfrac{\Delta E}{\hbar}\right)^2 =$ $\dfrac{m_H m_O}{m_H + m_O}\left(\dfrac{\Delta E}{\hbar}\right)^2$, and using $\dfrac{\Delta E}{\hbar} = \dfrac{hc/\lambda}{\hbar} = \dfrac{2\pi c}{\lambda}$ with the appropriate numbers gives us

$$k' = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(2.656\times10^{-26}\text{ kg}\right)}{1.67\times10^{-27}\text{ kg} + 2.656\times10^{-26}\text{ kg}}\left[\frac{2\pi\left(3.00\times10^{8}\text{ m/s}\right)}{2.39\times10^{-6}\text{ m}}\right]^2 = 977\text{ N/m}$$

(b) $f = \dfrac{\omega}{2\pi} = \dfrac{1}{2\pi}\sqrt{\dfrac{k'}{m_r}} = \dfrac{1}{2\pi}\sqrt{\dfrac{\dfrac{m_H m_O}{m_H + m_O}}{k'}}$. Substituting the appropriate numbers gives us

$$f = \frac{1}{2\pi}\sqrt{\frac{\dfrac{\left(1.67\times10^{-27}\text{ kg}\right)\left(2.656\times10^{-26}\text{ kg}\right)}{1.67\times10^{-27}\text{ kg} + 2.656\times10^{-26}\text{ kg}}}{977\text{ N/m}}} = 1.25\times10^{14}\text{ Hz}$$

EVALUATE: The frequency is close to, but not quite in, the visible range.

42.34. $I = \dfrac{2\hbar^2}{\Delta E} = \dfrac{h\lambda}{2\pi^2 c} = 7.14\times10^{-48}\text{ kg}\cdot\text{m}^2.$

42.35. **IDENTIFY** and **SET UP:** Eq.(21.14) gives the electric dipole moment as $p = qd$, where the dipole consists of charges $\pm q$ separated by distance d.

EXECUTE: **(a)** Point charges $+e$ and $-e$ separated by distance d, so

$p = ed = (1.602\times10^{-19}\text{ C})(0.24\times10^{-9}\text{ m}) = 3.8\times10^{-29}\text{ C}\cdot\text{m}$

(b) $p = qd$ so $q = \dfrac{p}{d} = \dfrac{3.0\times10^{-29}\text{ C}\cdot\text{m}}{0.24\times10^{-9}\text{ m}} = 1.3\times10^{-19}\text{ C}$

(c) $\dfrac{q}{e} = \dfrac{1.3\times10^{-19}\text{ C}}{1.602\times10^{-19}\text{ C}} = 0.81$

(d) $q = \frac{p}{d} = \frac{1.5\times10^{-30}\ \text{C}\cdot\text{m}}{0.16\times10^{-9}\ \text{m}} = 9.37\times10^{-21}\ \text{C}$

$\frac{q}{e} = \frac{9.37\times10^{-21}\ \text{C}}{1.602\times10^{-19}\ \text{C}} = 0.058$

EVALUATE: The fractional ionic character for the bond in HI is much less than the fractional ionic character for the bond in NaCl. The bond in HI is mostly covalent and not very ionic.

42.36. The electrical potential energy is $U = -5.13\ \text{eV}$, and $r = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{U} = 2.8\times10^{-10}\ \text{m}$.

42.37. **(a) IDENTIFY:** $E(\text{Na}) + E(\text{Cl}) = E(\text{Na}^+) + E(\text{Cl}^-) + U(r)$. Solving for $U(r)$ gives $U(r) = -[E(\text{Na}^+) - E(\text{Na})] + [E(\text{Cl}) - E(\text{Cl}^-)]$.

SET UP: $[E(\text{Na}^+) - E(\text{Na})]$ is the ionization energy of Na, the energy required to remove one electron, and is equal to 5.1 eV. $[E(\text{Cl}) - E(\text{Cl}^-)]$ is the electron affinity of Cl, the magnitude of the decrease in energy when an electron is attached to a neutral Cl atom, and is equal to 3.6 eV.

EXECUTE: $U = -5.1\ \text{eV} + 3.6\ \text{eV} = -1.5\ \text{eV} = -2.4\times10^{-19}\ \text{J}$, and $-\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = -2.4\times10^{-19}\ \text{J}$

$$r = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{e^2}{2.4\times10^{-19}\ \text{J}} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.602\times10^{-19}\ \text{C})^2}{2.4\times10^{-19}\ \text{J}}$$

$r = 9.6\times10^{-10}\ \text{m} = 0.96\ \text{nm}$

(b) ionization energy of $\text{K} = 4.3\ \text{eV}$; electron affinity of $\text{Br} = 3.5\ \text{eV}$

Thus $U = -4.3\ \text{eV} + 3.5\ \text{eV} = -0.8\ \text{eV} = -1.28\times10^{-19}\ \text{J}$, and $-\frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = -1.28\times10^{-19}\ \text{J}$

$$r = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{e^2}{1.28\times10^{-19}\ \text{J}} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.602\times10^{-19}\ \text{C})^2}{1.28\times10^{-19}\ \text{J}}$$

$r = 1.8\times10^{-9}\ \text{m} = 1.8\ \text{nm}$

EVALUATE: K has a smaller ionization energy than Na and the electron affinities of Cl and Br are very similar, so it takes less energy to make $\text{K}^+ + \text{Br}^-$ from $\text{K} + \text{Br}$ than to make $\text{Na}^+ + \text{Cl}^-$ from $\text{Na} + \text{Cl}$. Thus, the stabilization distance is larger for KBr than for NaCl.

42.38. The energies corresponding to the observed wavelengths are 3.29×10^{-21} J, 2.87×10^{-21} J, 2.47×10^{-21} J, 2.06×10^{-21} J and 1.65×10^{-21} J. The average spacing of these energies is 0.410×10^{-21} J and these are seen to correspond to transition from levels 8, 7, 6, 5 and 4 to the respective next lower levels. Then, $\frac{\hbar^2}{I} = 0.410\times10^{-21}\ \text{J}$, from which $I = 2.71\times10^{-47}\ \text{kg}\cdot\text{m}^2$.

42.39. **(a) IDENTIFY:** The rotational energies of a molecule depend on its moment of inertia, which in turn depends on the separation between the atoms in the molecule.

SET UP: Problem 42.38 gives $I = 2.71\times10^{-47}\ \text{kg}\cdot\text{m}^2$. $I = m_r r^2$. Calculate m_r and solve for *r*.

EXECUTE: $m_r = \frac{m_H m_{Cl}}{m_H + m_{Cl}} = \frac{(1.67\times10^{-27}\ \text{kg})(5.81\times10^{-26}\ \text{kg})}{1.67\times10^{-27}\ \text{kg} + 5.81\times10^{-26}\ \text{kg}} = 1.623\times10^{-27}\ \text{kg}$

$$r = \sqrt{\frac{I}{m_r}} = \sqrt{\frac{2.71\times10^{-47}\ \text{kg}\cdot\text{m}^2}{1.623\times10^{-27}\ \text{kg}}} = 1.29\times10^{-10}\ \text{m} = 0.129\ \text{nm}$$

EVALUATE: This is a typical atomic separation for a diatomic molecule; see Example 42.2 for the corresponding distance for CO.

(b) IDENTIFY: Each transition is from the level *l* to the level $l-1$. The rotational energies are given by Eq.(42.3). The transition energy is related to the photon wavelength by $\Delta E = hc/\lambda$.

SET UP: $E_l = l(l+1)\hbar^2/2I$, so $\Delta E = E_l - E_{l-1} = [l(l+1) - l(l-1)]\left(\frac{\hbar^2}{2I}\right) = l\left(\frac{\hbar^2}{I}\right)$.

EXECUTE: $l\left(\frac{\hbar^2}{I}\right) = \frac{hc}{\lambda}$

$$l = \frac{2\pi cI}{\hbar\lambda} = \frac{2\pi(2.998\times10^8\ \text{m/s})(2.71\times10^{-47}\ \text{kg}\cdot\text{m}^2)}{(1.055\times10^{-34}\ \text{J}\cdot\text{s})\lambda} = \frac{4.843\times10^{-4}\ \text{m}}{\lambda}$$

For $\lambda = 60.4\ \mu\text{m},\ l = \dfrac{4.843\times10^{-4}\ \text{m}}{60.4\times10^{-6}\ \text{m}} = 8.$

For $\lambda = 69.0\ \mu\text{m},\ l = \dfrac{4.843\times10^{-4}\ \text{m}}{69.0\times10^{-6}\ \text{m}} = 7.$

For $\lambda = 80.4\ \mu\text{m},\ l = \dfrac{4.843\times10^{-4}\ \text{m}}{80.4\times10^{-6}\ \text{m}} = 6.$

For $\lambda = 96.4\ \mu\text{m},\ l = \dfrac{4.843\times10^{-4}\ \text{m}}{96.4\times10^{-6}\ \text{m}} = 5.$

For $\lambda = 120.4\ \mu\text{m},\ l = \dfrac{4.843\times10^{-4}\ \text{m}}{120.4\times10^{-6}\ \text{m}} = 4.$

EVALUATE: In each case l is an integer, as it must be.

(c) IDENTIFY and **SET UP:** Longest λ implies smallest ΔE, and this is for the transition from $l = 1$ to $l = 0$.

EXECUTE: $\Delta E = l\left(\dfrac{\hbar^2}{I}\right) = (1)\dfrac{(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}{2.71\times10^{-47}\ \text{kg}\cdot\text{m}^2} = 4.099\times10^{-22}\ \text{J}$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{4.099\times10^{-22}\ \text{J}} = 4.85\times10^{-4}\ \text{m} = 485\,\mu\text{m}.$$

EVALUATE: This is longer than any wavelengths in part (b).

(d) IDENTIFY: What changes is m_r, the reduced mass of the molecule.

SET UP: The transition energy is $\Delta E = l\left(\dfrac{\hbar^2}{I}\right)$ and $\Delta E = \dfrac{hc}{\lambda}$, so $\lambda = \dfrac{2\pi cI}{l\hbar}$ (part (b)). $I = m_\text{r}r^2$, so λ is directly proportional to m_r. $\dfrac{\lambda(\text{HCl})}{m_\text{r}(\text{HCl})} = \dfrac{\lambda(\text{DCl})}{m_\text{r}(\text{DCl})}$ so $\lambda(\text{DCl}) = \lambda(\text{HCl})\dfrac{m_\text{r}(\text{DCl})}{m_\text{r}(\text{HCl})}$

EXECUTE: The mass of a deuterium atom is approximately twice the mass of a hydrogen atom, so $m_\text{D} = 3.34\times10^{-27}\ \text{kg}$.

$$m_\text{r}(\text{DCl}) = \frac{\text{m}_\text{D}\text{m}_\text{Cl}}{\text{m}_\text{D}+\text{m}_\text{Cl}} = \frac{(3.34\times10^{-27}\ \text{kg})(5.81\times10^{-27}\ \text{kg})}{3.34\times10^{-27}\ \text{kg}+5.81\times10^{-26}\ \text{kg}} = 3.158\times10^{-27}\ \text{kg}$$

$$\lambda(\text{DCl}) = \lambda(\text{HCl})\left(\frac{3.158\times10^{-27}\ \text{kg}}{1.623\times10^{-27}\ \text{kg}}\right) = (1.946)\lambda(\text{HCl})$$

$l = 8 \rightarrow l = 7; \lambda = (60.4\ \mu\text{m})(1.946) = 118\ \mu\text{m}$

$l = 7 \rightarrow l = 6; \lambda = (69.0\ \mu\text{m})(1.946) = 134\ \mu\text{m}$

$l = 6 \rightarrow l = 5; \lambda = (80.4\ \mu\text{m})(1.946) = 156\ \mu\text{m}$

$l = 5 \rightarrow l = 4; \lambda = (96.4\ \mu\text{m})(1.946) = 188\ \mu\text{m}$

$l = 4 \rightarrow l = 3; \lambda = (120.4\ \mu\text{m})(1.946) = 234\ \mu\text{m}$

EVALUATE: The moment of inertia increases when H is replaced by D, so the transition energies decrease and the wavelengths increase. The larger the rotational inertia the smaller the rotational energy for a given l (Eq.42.3).

42.40. From the result of Problem 42.11, the moment inertia of the molecule is $I = \dfrac{\hbar^2 l}{\Delta E} = \dfrac{hl\lambda}{4\pi^2 c} = 6.43\times10^{-46}\ \text{kg}\cdot\text{m}^2$ and from Eq.(42.6) the separation is $r_0 = \sqrt{\dfrac{I}{m_\text{r}}} = 0.193\ \text{nm}.$

42.41. **(a)** $E_\text{ex} = \dfrac{L^2}{2I} = \dfrac{\hbar^2 l(l+1)}{2I}$. $E_g = 0$ $(l = 0)$, and there is an additional multiplicative factor of $2l + 1$ because for each l state there are really $(2l+1)$ m_l -states with the same energy. So $\dfrac{n_l}{n_0} = (2l+1)e^{-\hbar^2 l(l+1)/(2IkT)}$.

(b) $T = 300\ \text{K}, I = 1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2$.

(i) $E_{l=1} = \dfrac{\hbar^2(1)\,(1+1)}{2(1.449\times10^{-46}\ \text{kg}\cdot\text{m}^2)} = 7.67\times10^{-23}\ \text{J}.$ $\dfrac{E_{l=1}}{kT} = \dfrac{7.67\times10^{-23}\ \text{J}}{(1.38\times10^{-23}\ \text{J/K})\,(300\ \text{K})} = 0.0185.$

$(2l+1) = 3$, so $\dfrac{n_{l=1}}{n_0} = (3)e^{-0.0185} = 2.95.$

(ii) $\dfrac{E_{l=2}}{kT} = \dfrac{\hbar^2(2)\,(2+1)}{2(1.449\times10^{-46}\text{ kg}\cdot\text{m}^2)\,(1.38\times10^{-23}\text{ J/K})\,(300\text{ K})} = 0.0556.$

$(2l+1) = 5$, so $\dfrac{n_{l=1}}{n_0} = (5)(e^{-0.0556}) = 4.73.$

(iii) $\dfrac{E_{l=10}}{kT} = \dfrac{\hbar^2(10)\,(10+1)}{2(1.449\times10^{-46}\text{ kg}\cdot\text{m}^2)\,(1.38\times10^{-23}\text{ J/K})\,(300\text{ K})} = 1.02.$

$(2l+1) = 21$, so $\dfrac{n_{l=10}}{n_0} = (21)\,(e^{-1.02}) = 7.57.$

(iv) $\dfrac{E_{l=20}}{kT} = \dfrac{\hbar^2(20)\,(20+1)}{2(1.449\times10^{-46}\text{ kg}\cdot\text{m}^2)\,(1.38\times10^{-23}\text{ J/K})\,(300\text{ K})} = 3.89.$

$(2l+1) = 41$, so $\dfrac{n_{l=20}}{n_0} = (41)e^{-3.89} = 0.838.$

(v) $\dfrac{E_{l=50}}{kT} = \dfrac{\hbar^2(50)\,(50+1)}{2(1.449\times10^{-46}\text{ kg}\cdot\text{m}^2)\,(1.38\times10^{-23}\text{ J/K})\,(300\text{ K})} = 23.6.$

$(2l+1) = 101$, so $\dfrac{n_{l=50}}{n_0} = (101)e^{-23.6} = 5.69\times10^{-9}.$

(c) There is a competing effect between the $(2l + 1)$ term and the decaying exponential. The $2l + 1$ term dominates for small l, while the exponential term dominates for large l.

42.42. **(a)** $I_{CO} = 1.449\times10^{-46}\text{ kg}\cdot\text{m}^2.$

$$E_{l=1} = \frac{\hbar^2 l(l+1)}{2I} = \frac{(1.054\times10^{-34}\text{ J}\cdot\text{s})^2(1)\,(1+1)}{2(1.449\times10^{-46}\text{ kg}\cdot\text{m}^2)} = 7.67\times10^{-23}\text{ J}.\quad E_{l=0} = 0.$$

$\Delta E = 7.67\times10^{-23}\text{ J} = 4.79\times10^{-4}\text{ eV}.$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})\,(3.00\times10^8\text{ m/s})}{(7.67\times10^{-23}\text{ J})} = 2.59\times10^{-3}\text{ m} = 2.59\text{ mm}.$$

(b) Let's compare the value of kT when $T = 20$ K to that of ΔE for the $l=1 \to l=0$ rotational transition:

$$kT = (1.38\times10^{-23}\text{ J/K})\,(20\text{ K}) = 2.76\times10^{-22}\text{ J}.$$

$$\Delta E = 7.67\times10^{-23}\text{ J (from part (a)). So } \frac{kT}{\Delta E} = 3.60.$$

Therefore, although T is quite small, there is still plenty of energy to excite CO molecules into the first rotational level. This allows astronomers to detect the 2.59 mm wavelength radiation from such molecular clouds.

42.43. **IDENTIFY** and **SET UP:** $E_l = l(l+1)\hbar^2/2I$, so E_l and the transition energy ΔE depend on I. Different isotopic molecules have different I.

EXECUTE: **(a)** Calculate I for $Na^{35}Cl$: $m_r = \dfrac{m_{Na}m_{Cl}}{m_{Na}+m_{Cl}} = \dfrac{(3.8176\times10^{-26}\text{ kg})(5.8068\times10^{-26}\text{ kg})}{3.8176\times10^{-26}\text{ kg} + 5.8068\times10^{-26}\text{ kg}} = 2.303\times10^{-26}\text{ kg}$

$I = m_r r^2 = (2.303\times10^{-26}\text{ kg})(0.2361\times10^{-9}\text{ m})^2 = 1.284\times10^{-45}\text{ kg}\cdot\text{m}^2$

<u>$l=2 \to l=1$ transition</u>

$$\Delta E = E_2 - E_1 = (6-2)\left(\frac{\hbar^2}{2I}\right) = \frac{2\hbar^2}{I} = \frac{2(1.055\times10^{-34}\text{ J}\cdot\text{s})^2}{1.284\times10^{-45}\text{ kg}\cdot\text{m}^2} = 1.734\times10^{-23}\text{ J}$$

$$\Delta E = \frac{hc}{\lambda}\text{ so } \lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{1.734\times10^{-23}\text{ J}} = 1.146\times10^{-2}\text{ m} = 1.146\text{ cm}$$

<u>$l=1 \to l=0$ transition</u>

$$\Delta E = E_1 - E_0 = (2-0)\left(\frac{\hbar^2}{2I}\right) = \frac{\hbar^2}{I} = \frac{1}{2}(1.734\times10^{-23}\text{ J}) = 8.67\times10^{-24}\text{ J}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{8.67\times10^{-24}\text{ J}} = 2.291\text{ cm}$$

(b) Calculate I for $Na^{37}Cl$: $m_r = \dfrac{m_{Na}m_{Cl}}{m_{Na}+m_{Cl}} = \dfrac{(3.8176\times10^{-26}\text{ kg})(6.1384\times10^{-26}\text{ kg})}{3.8176\times10^{-26}\text{ kg} + 6.1384\times10^{-26}\text{ kg}} = 2.354\times10^{-26}\text{ kg}$

$I = m_r r^2 = (2.354\times10^{-26}\text{ kg})(0.2361\times10^{-9}\text{ m})^2 = 1.312\times10^{-45}\text{ kg}\cdot\text{m}^2$

$\underline{l=2 \to l=1 \text{ transition}}$

$$\Delta E = \frac{2\hbar^2}{I} = \frac{2(1.055\times10^{-34}\ \text{J}\cdot\text{s})^2}{1.312\times10^{-45}\ \text{kg}\cdot\text{m}^2} = 1.697\times10^{-23}\ \text{J}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{1.697\times10^{-23}\ \text{J}} = 1.171\times10^{-2}\ \text{m} = 1.171\ \text{cm}$$

$\underline{l=1 \to l=0 \text{ transition}}$

$$\Delta E = \frac{\hbar^2}{I} = \frac{1}{2}(1.697\times10^{-23}\ \text{J}) = 8.485\times10^{-24}\ \text{J}$$

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{8.485\times10^{-24}\ \text{J}} = 2.341\ \text{cm}$$

The differences in the wavelengths for the two isotopes are:
$l=2 \to l=1$ transition: $1.171\ \text{cm} - 1.146\ \text{cm} = 0.025\ \text{cm}$
$l=1 \to l=0$ transition: $2.341\ \text{cm} - 2.291\ \text{cm} = 0.050\ \text{cm}$

EVALUATE: Replacing ^{35}Cl by ^{37}Cl increases I, decreases ΔE and increases λ. The effect on λ is small but measurable.

42.44. The vibration frequency is, from Eq.(42.8), $f = \dfrac{\Delta E}{h} = 1.12\times10^{14}$ Hz. The force constant is $k' = (2\pi f)^2 m_r = 777\ \text{N/m}$.

42.45. $E_n = \left(n+\frac{1}{2}\right)\hbar\sqrt{\dfrac{k'}{m_r}} \Rightarrow E_0 = \dfrac{1}{2}\hbar\sqrt{\dfrac{2k'}{m_H}}$

$$\Rightarrow E_0 = \frac{1}{2}(1.054\times10^{-34}\ \text{J}\cdot\text{s})\sqrt{\frac{2(576\ \text{N/m})}{1.67\times10^{-27}\ \text{kg}}} = 4.38\times10^{-20}\ \text{J} = 0.274\ \text{eV}.$$

This is much less than the H_2 bond energy.

42.46. **(a)** The frequency is proportional to the reciprocal of the square root of the reduced mass, and in terms of the atomic masses, the frequency of the isotope with the deuterium atom is

$$f = f_0\left(\frac{m_F m_H/(m_H+m_F)}{m_F m_D/(m_D+m_F)}\right)^{1/2} = f_0\left(\frac{1+(m_F/m_D)}{1+(m_F/m_H)}\right)^{1/2}.$$

Using f_0 from Exercise 42.13 and the given masses, $f = 8.99\times10^{13}$ Hz.

42.47. **IDENTIFY** and **SET UP:** Use Eq.(42.6) to calculate I. The energy levels are given by Eq.(42.9). The transition energy ΔE is related to the photon wavelength by $\Delta E = hc/\lambda$.

EXECUTE: **(a)** $m_r = \dfrac{m_H m_I}{m_H + m_I} = \dfrac{(1.67\times10^{-27}\ \text{kg})(2.11\times10^{-25}\ \text{kg})}{1.67\times10^{-27}\ \text{kg} + 2.11\times10^{-25}\ \text{kg}} = 1.657\times10^{-27}\ \text{kg}$

$I = m_r r^2 = (1.657\times10^{-27}\ \text{kg})(0.160\times10^{-9}\ \text{m})^2 = 4.24\times10^{-47}\ \text{kg}\cdot\text{m}^2$

(b) The energy levels are $E_{nl} = l(l+1)\left(\dfrac{\hbar^2}{2I}\right) + (n+\frac{1}{2})\hbar\sqrt{\dfrac{k'}{m_r}}$ (Eq.(42.9))

$\sqrt{\dfrac{k'}{m}} = \omega = 2\pi f$ so $E_{nl} = l(l+1)\left(\dfrac{\hbar^2}{2I}\right) + (n+\frac{1}{2})hf$

(i) transition $n=1 \to n=0,\ l=1 \to l=0$

$$\Delta E = (2-0)\left(\frac{\hbar^2}{2I}\right) + (1+\tfrac{1}{2}-\tfrac{1}{2})hf = \frac{\hbar^2}{I} + hf$$

$$\Delta E = \frac{hc}{\lambda} \text{ so } \lambda = \frac{hc}{\Delta E} = \frac{hc}{(\hbar^2/I)+hf} = \frac{c}{(\hbar/2\pi I)+f}$$

$$\frac{\hbar}{2\pi I} = \frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(4.24\times10^{-47}\ \text{kg}\cdot\text{m}^2)} = 3.960\times10^{11}\ \text{Hz}$$

$$\lambda = \frac{c}{(\hbar/2\pi I)+f} = \frac{2.998\times10^{8}\ \text{m/s}}{3.960\times10^{11}\ \text{Hz} + 6.93\times10^{13}\ \text{Hz}} = 4.30\ \mu\text{m}$$

(ii) transition $n=1 \to n=0,\ l=2 \to l=1$

$$\Delta E = (6-2)\left(\frac{\hbar^2}{2I}\right) + hf = \frac{2\hbar^2}{I} + hf$$

$$\lambda = \frac{c}{2(\hbar/2\pi I) + f} = \frac{2.998\times10^8\ \text{m/s}}{2(3.960\times10^{11}\ \text{Hz}) + 6.93\times10^{13}\ \text{Hz}} = 4.28\ \mu\text{m}$$

(iii) transition $n=2 \to n=1,\ l=2 \to l=3$

$$\Delta E = (6-12)\left(\frac{\hbar^2}{2I}\right) + hf = -\frac{3\hbar^2}{I} + hf$$

$$\lambda = \frac{c}{-3(\hbar/2\pi I) + f} = \frac{2.998\times10^8\ \text{m/s}}{-3(3.960\times10^{11}\ \text{Hz}) + 6.93\times10^{13}\ \text{Hz}} = 4.40\,\mu\text{m}$$

EVALUATE: The vibrational energy change for the $n=1 \to n=0$ transition is the same as for the $n=2 \to n=1$ transition. The rotational energies are much smaller than the vibrational energies, so the wavelengths for all three transitions don't differ much.

42.48. The sum of the probabilities is $f(E_{\text{F}} + \Delta E) + f(E_{\text{F}} - \Delta E) = \dfrac{1}{e^{-\Delta E/kT}+1} + \dfrac{1}{e^{\Delta E/kT}+1} = \dfrac{1}{e^{-\Delta E/kT}+1} + \dfrac{e^{-\Delta E/kT}}{1+e^{-\Delta E/kT}} = 1.$

42.49. Since potassium is a metal we approximate $E_{\text{F}} = E_{\text{F0}}$. $\Rightarrow E_{\text{F}} = \dfrac{3^{2/3}\pi^{4/3}\hbar^2 n^{2/3}}{2m}$.

But the electron concentration $n = \dfrac{\rho}{m} \Rightarrow n = \dfrac{851\ \text{kg/m}^3}{6.49\times10^{-26}\ \text{kg}} = 1.31\times10^{28}\ \text{electron/m}^3$

$$\Rightarrow E_{\text{F}} = \frac{3^{2/3}\pi^{4/3}(1.054\times10^{-34}\ \text{J}\cdot\text{s})^2(1.31\times10^{28}/\text{m}^3)^{2/3}}{2(9.11\times10^{-31}\ \text{kg})} = 3.24\times10^{-19}\ \text{J} = 2.03\ \text{eV}.$$

42.50. **IDENTIFY:** The only difference between the two isotopes is their mass, which will affect their reduced mass and hence their moment of inertia.

SET UP: The rotational energy states are given by $E = l(l+1)\dfrac{\hbar^2}{2I}$ and the reduced mass is given by $m_{\text{r}} = m_1m_2/(m_1 + m_2)$.

EXECUTE: **(a)** If we call m the mass of the H-atom, the mass of the deuterium atom is $2m$ and the reduced masses of the molecules are

H_2 (hydrogen): $m_{\text{r}}(\text{H}) = mm/(m + m) = m/2$

D_2 (deuterium): $m_{\text{r}}(\text{D}) = (2m)(2m)/(2m + 2m) = m$

Using $I = m_r r_0^2$, the moments of inertia are $I_{\text{H}} = mr_0^2/2$ and $I_{\text{D}} = mr_0^2$. The ratio of the rotational energies is then

$$\frac{E_{\text{H}}}{E_{\text{D}}} = \frac{l(l+1)\left(\hbar^2/2I_{\text{H}}\right)}{l(l+1)\left(\hbar^2/2I_{\text{D}}\right)} = \frac{I_{\text{D}}}{I_{\text{H}}} = \frac{mr_0^2}{\frac{m}{2}r_0^2} = 2.$$

(b) The ratio of the vibrational energies is $\dfrac{E_{\text{H}}}{E_{\text{D}}} = \dfrac{\left(n+\frac{1}{2}\right)\hbar\sqrt{\dfrac{k'}{m_{\text{r}}(\text{H})}}}{\left(n+\frac{1}{2}\right)\hbar\sqrt{\dfrac{k'}{m_{\text{r}}(\text{D})}}} = \sqrt{\dfrac{m_{\text{r}}(\text{D})}{m_{\text{r}}(\text{H})}} = \sqrt{\dfrac{m}{m/2}} = \sqrt{2}$.

EVALUATE: The electrical force is the same for both molecules since both H and D have the same charge, so it is reasonable that the force constant would be the same for both of them.

41.51. **IDENTIFY** and **SET UP:** Use the description of the bcc lattice in Fig.42.12c in the textbook to calculate the number of atoms per unit cell and then the number of atoms per unit volume.

EXECUTE: **(a)** Each unit cell has one atom at its center and 8 atoms at its corners that are each shared by 8 other unit cells. So there are $1+8/8=2$ atoms per unit cell.

$$\frac{n}{V} = \frac{2}{(0.35\times10^{-9}\ \text{m})^3} = 4.66\times10^{28}\ \text{atoms/m}^3$$

(b) $E_{\text{F0}} = \dfrac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\left(\dfrac{N}{V}\right)^{2/3}$

In this equation N/V is the number of free electrons per m^3. But the problem says to assume one free electron per atom, so this is the same as n/V calculated in part (a).

$m = 9.109\times10^{-31}$ kg (the electron mass), so $E_{\text{F0}} = 7.563\times10^{-19}\ \text{J} = 4.7\ \text{eV}$

EVALUATE: Our result for metallic lithium is similar to that calculated for copper in Example 42.8.

42.52. **(a)** $\frac{d}{dr}U_{\text{tot}} = \frac{\alpha e^2}{4\pi\epsilon_0}\frac{1}{r^2} - 8A\frac{1}{r^9}$. Setting this equal to zero when $r = r_0$ gives $r_0^7 = \frac{8A4\pi\epsilon_0}{\alpha e^2}$ and so $U_{\text{tot}} = \frac{\alpha e^2}{4\pi\epsilon_0}\left(-\frac{1}{r} + \frac{r_0^7}{8r^8}\right)$. At $r = r_0$, $U_{\text{tot}} = -\frac{7\alpha e^2}{32\pi\epsilon_0 r_0} = -1.26\times10^{-18}\text{ J} = -7.85\text{ eV}$.

(b) To remove a Na^+Cl^- ion pair from the crystal requires 7.85 eV. When neutral Na and Cl atoms are formed from the Na^+ and Cl^- atoms there is a net release of energy $-5.14\text{ eV} + 3.61\text{ eV} = -1.53\text{ eV}$, so the net energy required to remove a neutral Na, Cl pair from the crystal is $7.85\text{ eV} - 1.53\text{ eV} = 6.32\text{ eV}$.

42.53. **(a) IDENTIFY** and **SET UP:** $p = -\frac{dE_{\text{tot}}}{dV}$. Relate E_{tot} to E_{F0} and evaluate the derivative.

EXECUTE: $E_{\text{tot}} = NE_{\text{av}} = \frac{3N}{5}E_{\text{F0}} = \frac{3}{5}\left(\frac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\right)N^{5/3}V^{-2/3}$

$\frac{dE_{\text{tot}}}{dV} = \frac{3}{5}\left(\frac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\right)N^{5/3}\left(-\frac{2}{3}V^{-5/3}\right)$ so $p = \left(\frac{3^{2/3}\pi^{4/3}\hbar^2}{5m}\right)\left(\frac{N}{V}\right)^{5/3}$, as was to be shown.

(b) $N/V = 8.45\times10^{28}\text{ m}^{-3}$

$$p = \left(\frac{3^{2/3}\pi^{4/3}(1.055\times10^{-34}\text{ J}\cdot\text{s})^2}{5(9.109\times10^{-31}\text{ kg})}\right)(8.45\times10^{28}\text{ m}^{-3})^{5/3} = 3.81\times10^{10}\text{ Pa} = 3.76\times10^5\text{ atm}.$$

(c) EVALUATE: Normal atmospheric pressure is about 10^5 Pa, so these pressures are extremely large. The electrons are held in the metal by the attractive force exerted on them by the copper ions.

42.54. **(a)** From Problem 42.53, $p = \frac{3^{2/3}\pi^{4/3}\hbar^2}{5m}\left(\frac{N}{V}\right)^{5/3}$. $B = -V\frac{dp}{dV} = -V\left[\frac{5}{3}\cdot\frac{3^{2/3}\pi^{4/3}\hbar^2}{5m}\cdot\left(\frac{N}{V}\right)^{2/3}\left(\frac{-N}{V^2}\right)\right] = \frac{5}{3}p.$

(b) $\frac{N}{V} = 8.45\times10^{28}\text{ m}^{-3}$. $B = \frac{5}{3}\cdot\frac{3^{2/3}\pi^{4/3}\hbar^2}{5m}(8.45\times10^{28}\text{m}^{-3})^{5/3} = 6.33\times10^{10}\text{ Pa}.$

(c) $\frac{6.33\times10^{10}\text{ Pa}}{1.4\times10^{11}\text{ Pa}} = 0.45.$ The copper ions themselves make up the remaining fraction.

42.55. **(a)** $E_{\text{F0}} = \frac{3^{2/3}\pi^{4/3}\hbar^2}{2m}\left(\frac{N}{V}\right)^{2/3}$. Let $E_{\text{F0}} = \frac{1}{100}mc^2$.

$$\left(\frac{N}{V}\right) = \left[\frac{2m^2c^2}{(100)3^{2/3}\pi^{4/3}\hbar^2}\right]^{3/2} = \frac{2^{3/2}m^3c^3}{100^{3/2}3\pi^2\hbar^3} = \frac{2^{3/2}m^3c^3}{3000\pi^2\hbar^3} = 1.67\times10^{33}\text{ m}^{-3}.$$

(b) $\frac{8.45\times10^{28}\text{ m}^{-3}}{1.67\times10^{33}\text{ m}^{-3}} = 5.06\times10^{-5}$. Since the real concentration of electrons in copper is less than one part in 10^{-4} of the concentration where relativistic effects are important, it is safe to ignore relativistic effects for most applications.

(c) The number of electrons is $N_e = \frac{6(2\times10^{30}\text{ kg})}{1.99\times10^{-26}\text{ kg}} = 6.03\times10^{56}$. The concentration is

$$\frac{N_e}{V} = \frac{6.03\times10^{56}}{\frac{4}{3}\pi(6.00\times10^6\text{ m})^3} = 6.66\times10^{35}\text{ m}^{-3}.$$

(d) Comparing this to the result from part (a) $\frac{6.66\times10^{35}\text{ m}^{-3}}{1.67\times10^{33}\text{ m}^{-3}} \cong 400$ so relativistic effects will be very important.

42.56. **IDENTIFY:** The current through the diode is related to the voltage across it.
SET UP: The current through the diode is given by $I = I_S(e^{eV/kT} - 1)$.
EXECUTE: **(a)** The current through the resistor is (35.0 V)/(125 Ω) = 0.280 A = 280 mA, which is also the current through the diode. This current is given by $I = I_S(e^{eV/kT} - 1)$, giving 280 mA = 0.625 mA$(e^{eV/kT} - 1)$ and 1 + (280/0.625) = 449 = $e^{eV/kT}$. Solving for V at T = 293 K gives $V = \frac{kT\ln 449}{e} = \frac{(1.38\times10^{-23}\text{ J/K})(293\text{ K})\ln 449}{1.60\times10^{-19}\text{ C}}$ = 0.154 V
(b) $R = V/I$ = (0.154 V)/(0.280 A) = 0.551 Ω
EVALUATE: At a different voltage, the diode would have different resistance.

42.57. **(a)** $U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}} = \frac{q^2}{4\pi\epsilon_0}\left(\frac{-1}{d}+\frac{1}{r}-\frac{1}{r+d}-\frac{1}{r-d}+\frac{1}{r}-\frac{1}{d}\right) = \frac{q^2}{4\pi\epsilon_0}\left(\frac{2}{r}-\frac{2}{d}-\frac{1}{r+d}-\frac{1}{r-d}\right).$

But $\frac{1}{r+d}+\frac{1}{r-d} = \frac{1}{r}\left(\frac{1}{1+\frac{d}{r}}+\frac{1}{1-\frac{d}{r}}\right) \approx \frac{1}{r}\left(1-\frac{d}{r}+\frac{d^2}{r^2}+\cdots+1+\frac{d}{r}+\frac{d^2}{r^2}\right) \approx \frac{2}{r}+\frac{2d^2}{r^3}$

$\Rightarrow U = \frac{-2q^2}{4\pi\epsilon_0}\left(\frac{1}{d}+\frac{d^2}{r^3}\right) = \frac{-2p^2}{4\pi\epsilon_0 r^3}-\frac{2p^2}{4\pi\epsilon_0 d^3}.$

(b) $U = \frac{1}{4\pi\epsilon_0}\sum_{i<j}\frac{q_i q_j}{r_{ij}} = \frac{q^2}{4\pi\epsilon_0}\left(\frac{-1}{d}-\frac{1}{r}+\frac{1}{r+d}+\frac{1}{r-d}-\frac{1}{r}-\frac{1}{d}\right) = \frac{q^2}{4\pi\epsilon_0}\left(\frac{-2}{d}-\frac{2}{r}+\frac{2}{r}+\frac{2d^2}{r^3}\right) =$

$\frac{-2q^2}{4\pi\epsilon_0}\left(\frac{1}{d}-\frac{d^2}{r^3}\right) \Rightarrow U = \frac{-2p^2}{4\pi\epsilon_0 d^3}+\frac{2p^2}{4\pi\epsilon_0 r^3}.$

If we ignore the potential energy involved in forming each individual molecule, which just involves a different choice for the zero of potential energy, then the answers are:

(a) $U = \frac{-2p^2}{4\pi\epsilon_0 r^3}$. The interaction is attractive.

(b) $U = \frac{+2p^2}{4\pi\epsilon_0 r^3}$. The interaction is repulsive.

42.58. **(a)** Following the hint, $k'dr = -d\left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}\right)_{r=r_0} = \frac{1}{2\pi\epsilon_0}\frac{e^2}{r_0^3}dr$ and $\hbar\omega = \hbar\sqrt{2k'/m} = \hbar\sqrt{\frac{1}{\pi\epsilon_0}\frac{e^2}{mr_0^3}} =$

$1.23\times10^{-19}\ \text{J} = 0.77\ \text{eV}$, where $(m/2)$ has been used for the reduced mass.

(b) The reduced mass is doubled, and the energy is reduced by a factor of $\sqrt{2}$ to 0.54 eV.

43

NUCLEAR PHYSICS

43.1. **(a)** ${}^{28}_{14}\text{Si}$ has 14 protons and 14 neutrons.

(b) ${}^{85}_{37}\text{Rb}$ has 37 protons and 48 neutrons.

(c) ${}^{205}_{81}\text{Tl}$ has 81 protons and 124 neutrons.

43.2. **(a)** Using $R = (1.2\text{ fm})A^{1/3}$, the radii are roughly 3.6 fm, 5.3 fm, and 7.1 fm.

(b) Using $4\pi R^2$ for each of the radii in part (a), the areas are $163\ \text{fm}^2$, $353\ \text{fm}^2$ and $633\ \text{fm}^2$.

(c) $\frac{4}{3}\pi R^3$ gives $195\ \text{fm}^3$, $624\ \text{fm}^3$ and $1499\ \text{fm}^3$.

(d) The density is the same, since the volume and the mass are both proportional to A: $2.3\times10^{17}\ \text{kg/m}^3$ (see Example 43.1).

(e) Dividing the result of part (d) by the mass of a nucleon, the number density is $0.14/\text{fm}^3 = 1.40\times10^{44}/\text{m}^3$.

43.3. **IDENTIFY:** Calculate the spin magnetic energy shift for each spin state of the 1*s* level. Calculate the energy splitting between these states and relate this to the frequency of the photons.

SET UP: When the spin component is parallel to the field the interaction energy is $U = -\mu_z B.$ When the spin component is antiparallel to the field the interaction energy is $U = +\mu_z B.$ The transition energy for a transition between these two states is $\Delta E = 2\mu_z B,$ where $\mu_z = 2.7928\mu_\text{n}$. The transition energy is related to the photon frequency by $\Delta E = hf$, so $2\mu_z B = hf$.

EXECUTE: $B = \dfrac{hf}{2\mu_z} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(22.7\times10^6\ \text{Hz})}{2(2.7928)(5.051\times10^{-27}\ \text{J/T})} = 0.533\ \text{T}$

EVALUATE: This magnetic field is easily achievable. Photons of this frequency have wavelength $\lambda = c/f = 13.2$ m. These are radio waves.

43.4. **(a)** As in Example 43.2, $\Delta E = 2(1.9130)(3.15245\times10^{-8}\ \text{eV/T})(2.30\ \text{T}) = 2.77\times10^{-7}\ \text{eV}.$ Since $\vec{\mu}$ and $\vec{S}$ are in opposite directions for a neutron, the antiparallel configuration is lower energy. This result is smaller than but comparable to that found in the example for protons.

(b) $f = \dfrac{\Delta E}{h} = 66.9\ \text{MHz},\ \lambda = \dfrac{c}{f} = 4.48\ \text{m}.$

43.5. **IDENTIFY:** Calculate the spin magnetic energy shift for each spin component. Calculate the energy splitting between these states and relate this to the frequency of the photons.

(a) SET UP: From Example 43.2, when the *z*-component of $\vec{S}$ (and $\vec{\mu}$) is parallel to $\vec{B}$, $U = -|\mu_z|B = -2.7928\mu_\text{n}B.$ When the *z*-component of $\vec{S}$ (and $\vec{\mu}$) is antiparallel to $\vec{B}$, $U = -|\mu_z|B = +2.7928\mu_\text{n}B.$ The state with the proton spin component parallel to the field lies lower in energy. The energy difference between these two states is $\Delta E = 2(2.7928\mu_\text{n}B).$

EXECUTE: $\Delta E = hf$ so $f = \dfrac{\Delta E}{h} = \dfrac{2(2.7928\mu_\text{n})B}{h} = \dfrac{2(2.7928)(5.051\times10^{-27}\ \text{J/T})(1.65\ \text{T})}{6.626\times10^{-34}\ \text{J}\cdot\text{s}}$

$f = 7.03\times10^7\ \text{Hz} = 7.03\ \text{MHz}$

And then $\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^8\ \text{m/s}}{7.03\times10^7\ \text{Hz}} = 4.26\ \text{m}$

EVALUATE: From Figure 32.4 in the textbook, these are radio waves.

(b) SET UP: From Eqs. (27.27) and (41.22) and Fig.41.14 in the textbook, the state with the z-component of $\vec{\mu}$ parallel to $\vec{B}$ has lower energy. But, since the charge of the electron is negative, this is the state with the electron spin component antiparallel to $\vec{B}$. That is, for the $m_s = -\frac{1}{2}$ state lies lower in energy.

EXECUTE: For the $m_s = +\frac{1}{2}$ state, $U = +(2.00232)\left(\frac{e}{2m}\right)\left(+\frac{\hbar}{2}\right)B = +\frac{1}{2}(2.00232)\left(\frac{e\hbar}{2m}\right)B = +\frac{1}{2}(2.00232)\mu_B B.$

For the $m_s = -\frac{1}{2}$ state, $U = -\frac{1}{2}(2.00232)\mu_B B$. The energy difference between these two states is $\Delta E = (2.00232)\mu_B B.$

$\Delta E = hf$ so $f = \frac{\Delta E}{h} = \frac{2.00232\mu_B B}{h} = \frac{(2.00232)(9.274\times10^{-24}\ \text{J/T})(1.65\ \text{T})}{6.626\times10^{-34}\ \text{J}\cdot\text{s}} = 4.62\times10^{10}\ \text{Hz} = 46.2\ \text{GHz}$. And

$\lambda = \frac{c}{f} = \frac{2.998\times10^8\ \text{m/s}}{4.62\times10^{10}\ \text{Hz}} = 6.49\times10^{-3}\ \text{m} = 6.49\ \text{mm}.$

EVALUATE: From Figure 32.4 in the textbook, these are microwaves. The interaction energy with the magnetic field is inversely proportional to the mass of the particle, so it is less for the proton than for the electron. The smaller transition energy for the proton produces a larger wavelength.

43.6. **(a)** $146m_n + 92m_H - m_U = 1.93\ \text{u}$

(b) 1.80×10^3 MeV

(c) 7.56 MeV per nucleon (using 931.5 MeV/u and 238 nucleons).

43.7. **IDENTIFY** and **SET UP:** The text calculates that the binding energy of the deuteron is 2.224 MeV. A photon that breaks the deuteron up into a proton and a neutron must have at least this much energy.

$E = \frac{hc}{\lambda}$ so $\lambda = \frac{hc}{E}$

EXECUTE: $\lambda = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{2.224\times10^6\ \text{eV}} = 5.575\times10^{-13}\ \text{m} = 0.5575\ \text{pm}.$

EVALUATE: This photon has gamma-ray wavelength.

43.8. **IDENTIFY:** The binding energy of the nucleus is the energy of its constituent particles minus the energy of the carbon-12 nucleus.

SET UP: In terms of the masses of the particles involved, the binding energy is

$$E_B = (6m_H + 6m_n - m_{C\text{-}12})c^2.$$

EXECUTE: **(a)** Using the values from Table 43.2, we get

$$E_B = [6(1.007825\ \text{u}) + 6(1.008665\ \text{u}) - 12.000000\ \text{u})](931.5\ \text{MeV/u}) = 92.16\ \text{MeV}$$

(b) The binding energy per nucleon is (92.16 MeV)/(12 nucleons) = 7.680 MeV/nucleon

(c) The energy of the C-12 nucleus is (12.0000 u)(931.5 MeV/u) = 11178 MeV. Therefore the percent of the mass that is binding energy is $\frac{92.16\ \text{MeV}}{11178\ \text{MeV}} = 0.8245\%$.

EVALUATE: The binding energy of 92.16 MeV binds 12 nucleons. The binding energy per nucleon, rather than just the total binding energy, is a better indicator of the strength with which a nucleus is bound.

43.9. **IDENTIFY:** Conservation of energy tells us that the initial energy (photon plus deuteron) is equal to the energy after the split (kinetic energy plus energy of the proton and neutron). Therefore the kinetic energy released is equal to the energy of the photon minus the binding energy of the deuteron.

SET UP: The binding energy of a deuteron is 2.224 MeV and the energy of the photon is $E = hc/\lambda$. Kinetic energy is $K = \frac{1}{2}mv^2$.

EXECUTE: **(a)** The energy of the photon is

$$E_{ph} = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{3.50\times10^{-13}\ \text{m}} = 5.68\times10^{-13}\ \text{J}.$$

The binding of the deuteron is $E_B = 2.224\ \text{MeV} = 3.56\times10^{-13}\ \text{J}$. Therefore the kinetic energy is $K = (5.68 - 3.56)\times10^{-13}\ \text{J} = 2.12\times10^{-13}\ \text{J} = 1.32\ \text{MeV}$.

(b) The particles share the energy equally, so each gets half. Solving the kinetic energy for v gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.06\times10^{-13}\ \text{J})}{1.6605\times10^{-27}\ \text{kg}}} = 1.13\times10^7\ \text{m/s}$$

EVALUATE: Considerable energy has been released, because the particle speeds are in the vicinity of the speed of light.

43.10. **(a)** $7(m_n + m_H) - m_N = 0.112\ \text{u}$, which is 105 MeV, or 7.48 MeV per nucleon.

(b) Similarly, $2(m_H + m_n) - m_{He} = 0.03038\ \text{u} = 28.3\ \text{MeV}$, or 7.07 MeV per nucleon, slightly lower (compare to Figure 43.2 in the textbook).

43.11. **(a) IDENTIFY:** Find the energy equivalent of the mass defect.

SET UP: A $^{11}_{5}\text{B}$ atom has 5 protons, $11-5=6$ neutrons, and 5 electrons. The mass defect therefore is $\Delta M = 5m_p + 6m_n + 5m_e - M(^{11}_{5}\text{B})$.

EXECUTE: $\Delta M = 5(1.0072765\text{ u}) + 6(1.0086649\text{ u}) + 5(0.0005485799\text{ u}) - 11.009305\text{ u} = 0.08181\text{ u}$. The energy equivalent is $E_B = (0.08181\text{ u})(931.5\text{ MeV/u}) = 76.21\text{ MeV}$.

(b) IDENTIFY and **SET UP:** Eq.(43.11): $E_B = C_1A - C_2A^{2/3} - C_3Z(Z-1)/A^{1/3} - C_4(A-2Z)^2/A$

The fifth term is zero since Z is odd but N is even. $A = 11$ and $Z = 5$.

EXECUTE: $E_B = (15.75\text{ MeV})(11) - (17.80\text{ MeV})(11)^{2/3} - (0.7100\text{ MeV})5(4)/11^{1/3} - (23.69\text{ MeV})(11-10)^2/11$.

$E_B = +173.25\text{ MeV} - 88.04\text{ MeV} - 6.38\text{ MeV} - 2.15\text{ MeV} = 76.68\text{ MeV}$

The percentage difference between the calculated and measured E_B is $\dfrac{76.68\text{ MeV} - 76.21\text{ MeV}}{76.21\text{ MeV}} = 0.6\%$

EVALUATE: Eq.(43.11) has a greater percentage accuracy for ^{62}Ni. The semi-empirical mass formula is more accurate for heavier nuclei.

43.12. **(a)** $34m_n + 29m_H - m_{Cu} = 34(1.008665)\text{ u} + 29(1.007825)\text{ u} - 62.929601\text{ u} = 0.592\text{ u}$, which is 551 MeV, or 8.75 MeV per nucleon (using 931.5 MeV/u and 63 nucleons).

(b) In Eq.(43.11), $Z = 29$ and $N = 34$, so the fifth term is zero. The predicted binding energy is

$$E_B = (15.75\text{ MeV})(63) - (17.80\text{ MeV})(63)^{2/3} - (0.7100\text{ MeV})\frac{(29)(28)}{(63)^{1/3}} - (23.69\text{ MeV})\frac{(5)^2}{(63)}.$$

$E_B = 556\text{ MeV}$. The fifth term is zero since the number of neutrons is even while the number of protons is odd, making the pairing term zero. This result differs from the binding energy found from the mass deficit by 0.86%, a very good agreement comparable to that found in Example 43.4.

43.13. **IDENTIFY** In each case determine how the decay changes A and Z of the nucleus. The β^+ and β^- particles have charge but their nucleon number is $A = 0$.

(a) SET UP: α-decay: Z increases by 2, $A = N + Z$ decreases by 4 (an α particle is a $^{4}_{2}\text{He}$ nucleus)

EXECUTE: $^{239}_{94}\text{Pu} \to {}^{4}_{2}\text{He} + {}^{235}_{92}\text{U}$

(b) SET UP: β^- decay: Z increases by 1, $A = N + Z$ remains the same (a β^- particle is an electron, $^{0}_{-1}\text{e}$)

EXECUTE: $^{24}_{11}\text{Na} \to {}^{0}_{-1}\text{e} + {}^{24}_{12}\text{Mg}$

(c) SET UP β^+ decay: Z decreases by 1, $A = N + Z$ remains the same (a β^+ particle is a positron, $^{0}_{+1}\text{e}$)

EXECUTE: $^{15}_{8}\text{O} \to {}^{0}_{+1}\text{e} + {}^{15}_{7}\text{N}$

EVALUATE: In each case the total charge and total number of nucleons for the decay products equals the charge and number of nucleons for the parent nucleus; these two quantities are conserved in the decay.

43.14. **(a)** The energy released is the energy equivalent of $m_n - m_p - m_e = 8.40\times10^{-4}\text{ u}$, or 783 keV.

(b) $m_n > m_p$, and the decay is not possible.

43.15. **IDENTIFY:** The energy of the photon must be equal to the difference in energy of the two nuclear energy levels.

SET UP: The energy difference is $\Delta E = hc/\lambda$.

EXECUTE: $\Delta E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{0.0248\times10^{-9}\text{ J}} = 8.015\times10^{-15}\text{ J} = 0.0501\text{ MeV}$

EVALUATE: Since the wavelength of this photon is *much* shorter than the wavelengths of visible light, its energy is much greater than visible-light photons which are frequently emitted during electron transitions in atoms. This tells us that the energy difference between the nuclear shells is *much* greater than the energy difference between electron shells in atoms, meaning that nuclear energies are *much* greater than the energies of orbiting electrons.

43.16. **IDENTIFY:** The energy released is equal to the mass defect of the initial and final nuclei.

SET UP: The mass defect is equal to the difference between the initial and final masses of the constituent particles.

EXECUTE: **(a)** The mass defect is 238.050788 u – 234.043601 u – 4.002603 u = 0.004584 u. The energy released is (0.004584 u)(931.5 MeV/u) = 4.270 MeV.

(b) Take the ratio of the two kinetic energies, using the fact that $K = p^2/2m$:

$$\frac{K_{Th}}{K_\alpha} = \frac{\dfrac{p_{Th}^2}{2m_{Th}}}{\dfrac{p_\alpha^2}{2m_\alpha}} = \frac{m_\alpha}{m_{Th}} = \frac{4}{234}.$$

The kinetic energy of the Th is

$$K_{\text{Th}} = \frac{4}{234+4} K_{\text{Total}} = \frac{4}{238}(4.270\,\text{MeV}) = 0.07176\ \text{MeV} = 1.148\ \times 10^{-14}\ \text{J}$$

Solving for v in the kinetic energy gives

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(1.148\times 10^{-14}\ \text{J})}{(234.043601)\left(1.6605\times 10^{-27}\ \text{kg}\right)}} = 2.431\times 10^{5}\ \text{m/s}$$

EVALUATE: As we can see by the ratio of kinetic energies in part (b) , the alpha particle will have a much higher kinetic energy than the thorium.

43.17. If β^- decay of ^{14}C is possible, then we are considering the decay $^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + \beta^-$.

$\Delta m = M(^{14}_{6}\text{C}) - M(^{14}_{7}\text{N}) - m_{\text{e}}$

$\Delta m = (14.003242\ \text{u} - 6(0.000549\ \text{u})) - (14.003074\ \text{u} - 7(0.000549\ \text{u})) - 0.0005491\ \text{u}$

$\Delta m = +1.68\times 10^{-4}$ u. So $E = (1.68\times 10^{-4}\ \text{u})(931.5\ \text{MeV/u}) = 0.156\ \text{MeV} = 156\ \text{keV}$

43.18. **(a)** A proton changes to a neutron, so the emitted particle is a positron (β^+).

(b) The number of nucleons in the nucleus decreases by 4 and the number of protons by 2, so the emitted particle is an alpha-particle.

(c) A neutron changes to a proton, so the emitted particle is an electron (β^-).

43.19. **(a)** As in the example, $(0.000898\ \text{u})(931.5\ \text{MeV/u}) = 0.836\ \text{MeV}$.

(b) $0.836\ \text{MeV} - 0.122\ \text{MeV} - 0.014\ \text{MeV} = 0.700\ \text{MeV}$.

43.20. **(a)** $^{90}_{39}\text{Sr} \rightarrow \beta^- + {}^{90}_{39}\text{X}$. X has 39 protons and 90 protons plus neutrons, so it must be ^{90}Y.

(b) Use base 2 because we know the half life. $A = A_0 2^{-t/T_{1/2}}$ and $0.01A_0 = A_0 2^{-t/T_{1/2}}$.

$$t = -\frac{T_{1/2}\log 0.01}{\log 2} = -\frac{(28\ \text{yr})\log 0.01}{\log 2} = 190\,\text{yr}\ .$$

43.21. **IDENTIFY** and **SET UP:** $T_{1/2} = \dfrac{\ln 2}{\lambda}$ The mass of a single nucleus is $124m_{\text{p}} = 2.07\times 10^{-25}$ kg .

$|\Delta N/\Delta t| = 0.350\ \text{Ci} = 1.30\times 10^{10}\ \text{Bq}$; $|\Delta N/\Delta t| = \lambda N$

EXECUTE: $N = \dfrac{6.13\times 10^{-3}\ \text{kg}}{2.07\times 10^{-25}\ \text{kg}} = 2.96\times 10^{22}$; $\lambda = \dfrac{\Delta N/\Delta t}{N} = \dfrac{1.30\times 10^{10}\ \text{Bq}}{2.96\times 10^{22}} = 4.39\times 10^{-13}\ \text{s}^{-1}$

$T_{1/2} = \dfrac{\ln 2}{\lambda} = 1.58\times 10^{12}\ \text{s} = 5.01\times 10^{4}\ \text{yr}$

43.22. Note that Eq.(43.17) can be written as follows: $N = N_0 2^{-t/T_{1/2}}$. The amount of elapsed time since the source was created is roughly 2.5 years. Thus, we expect the current activity to be $N = (5000\ \text{Ci})2^{-(2.5\ \text{yr})/(5.271\ \text{yr})} = 3600\,\text{Ci}$. The source is barely usable. Alternatively, we could calculate $\lambda = \dfrac{\ln(2)}{T_{1/2}} = 0.132(\text{years})^{-1}$ and use the Eq. 43.17 directly to obtain the same answer.

43.23. **IDENTIFY** and **SET UP:** As discussed in Section 43.4, the activity $A = |dN/dt|$ obeys the same decay equation as Eq. (43.17): $A = A_0 e^{-\lambda t}$. For ^{14}C, $T_{1/2} = 5730$ y and $\lambda = \ln 2/T_{1/2}$ so $A = A_0 e^{-(\ln 2)t/T_{1/2}}$; Calculate A at each t; $A_0 = 180.0$ decays/min.

EXECUTE: **(a)** $t = 1000$ y, $A = 159$ decays/min

(b) $t = 50{,}000$ y, $A = 0.43$ decays/min

EVALUATE: The time in part (b) is 8.73 half-lives, so the decay rate has decreased by a factor or $(\frac{1}{2})^{8.73}$.

43.24. **IDENTIFY and SET UP:** The decay rate decreases by a factor of 2 in a time of one half-life.

EXECUTE: **(a)** 24 d is $3T_{1/2}$ so the activity is $(375\ \text{Bq})/(2^3) = 46.9\ \text{Bq}$

(b) The activity is proportional to the number of radioactive nuclei, so the percent is $\dfrac{17.0\ \text{Bq}}{46.9\ \text{Bq}} = 36.2\%$

(c) $^{131}_{53}\text{I} \rightarrow {}^{0}_{-1}\text{e} + {}^{131}_{54}\text{Xe}$ The nucleus $^{131}_{54}\text{Xe}$ is produced.

EVALUATE: Both the activity and the number of radioactive nuclei present decrease by a factor of 2 in one half-life.

43.25. **(a)** ${}^{3}_{1}\text{H} \rightarrow {}^{0}_{-1}\text{e} + {}^{3}_{2}\text{He}$

(b) $N = N_0 e^{-\lambda t}, N = 0.100N_0$ and $\lambda = (\ln 2)/T_{1/2}$

$$0.100 = e^{-t(\ln 2)/T_{1/2}};\quad -t(\ln 2)/T_{1/2} = \ln(0.100);\quad t = \frac{-\ln(0.100)T_{1/2}}{\ln 2} = 40.9 \text{ y}$$

43.26. **(a)** $\dfrac{dN}{dt} = 500\ \mu\text{Ci} = (500\times10^{-6})(3.70\times10^{10}\ \text{s}^{-1}) = 1.85\times10^{7}$ decays/s

$T_{1/2} = \dfrac{\ln 2}{\lambda} \rightarrow \lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{12\ \text{d}(86{,}400\text{s/d})} = 6.69\times10^{-7}\ \text{s}$.

$\dfrac{dN}{dt} = \lambda N \Rightarrow N = \dfrac{dN/dt}{\lambda} = \dfrac{1.85\times10^{7}\ \text{decays / s}}{6.69\times10^{-7}\text{s}^{-1}} = 2.77\times10^{13}$ nuclei . The mass of this many ^{131}Ba nuclei is

$m = 2.77\times10^{13}\,\text{nuclei}\times(131\times1.66\times10^{-27}\ \text{kg/nucleus}) = 6.0\times10^{-12}\,\text{kg} = 6.0\times10^{-9}\ \text{g} = 6.0\ \text{ng}$

(b) $A = A_0 e^{-\lambda t}$. $1\ \mu\text{Ci} = (500\ \mu\text{Ci})\, e^{-\lambda t}$. $\ln(1/500) = -\lambda t$.

$t = -\dfrac{\ln(1/500)}{\lambda} = -\dfrac{\ln(1/500)}{6.69\times10^{-7}\text{s}^{-1}} = 9.29\times10^{6}\ \text{s}\left(\dfrac{1\ \text{d}}{86{,}400\ \text{s}}\right) = 108$ days

43.27. $A = A_0 e^{-\lambda t} = A_0 e^{-t(\ln 2)/T_{1/2}}$. $-\dfrac{(\ln 2)t}{T_{½}} = \ln(A/A_0)$.

$T_{½} = -\dfrac{(\ln 2)t}{\ln(A/A_0)} = -\dfrac{(\ln 2)(4.00\ \text{days})}{\ln(3091/8318)} = 2.80$ days

43.28. $\dfrac{dN}{dt} = \lambda N$. $\lambda = \dfrac{\ln 2}{T_{½}} = \dfrac{\ln 2}{1620\ \text{yr}\left(3.15\times10^{7}\ \text{s/yr}\right)} = 1.36\times10^{-11}\ \text{s}^{-1}$.

$N = 1\ \text{g}\left(\dfrac{6.022\times10^{23}\text{atoms}}{226\ \text{g}}\right) = 2.665\times10^{25}$ atoms .

$$\frac{dN}{dt} = \lambda N = (2.665\times10^{25})(1.36\times10^{-11}\text{s}^{-1}) = 3.62\times10^{10}\ \text{decays/s} = 3.62\times10^{10}\,\text{Bq}$$

Convert to Ci: $3.62\times10^{10}\ \text{Bq}\left(\dfrac{1\,\text{Ci}}{3.70\times10^{10}\ \text{Bq}}\right) = 0.98\ \text{Ci}$

43.29. **IDENTIFY** and **SET UP:** Calculate the number N of ^{14}C atoms in the sample and then use Eq. (43.17) to find the decay constant λ. Eq. (43.18) then gives $T_{1/2}$.

EXECUTE: Find the total number of carbon atoms in the sample.

$n = m/M$;

$N_{\text{tot}} = nN_{\text{A}} = mN_{\text{A}}/M = (12.0\times10^{-3}\ \text{kg})(6.022\times10^{23}\ \text{atoms/mol})/(12.011\times10^{-3}\ \text{kg/mol})$

$\text{N}_{\text{tot}} = 6.016\times10^{23}$ atoms, so $(1.3\times10^{-12})(6.016\times10^{23}) = 7.82\times10^{11}$ carbon-14 atoms

$\Delta N/\Delta t = -180$ decays/min $= -3.00$ decays/s

$\Delta N/\Delta t = -\lambda N;\quad \lambda = \dfrac{-\Delta N/\Delta t}{N} = 3.836\times10^{-12}\ \text{s}^{-1}$

$T_{1/2} = (\ln 2)/\lambda = 1.807\times10^{11}\ \text{s} = 5730\ \text{y}$

EVALUATE: The value we calculated agrees with the value given in Section 43.4.

43.30. $\dfrac{360\times10^{6}\text{decays}}{86{,}400\ \text{s}} = 4.17\times10^{3}\ \text{Bq} = 1.13\times10^{-7}\ \text{Ci} = 0.113\ \mu\text{Ci}.$

43.31. **(a)** $\left|\dfrac{dN}{dt}\right| = 7.56\times10^{11}\ \text{Bq} = 7.56\times10^{11}$ decays/s . $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{(30.8\ \text{min})(60\ \text{s/min})} = 3.75\times10^{-4}\ \text{s}^{-1}$.

$N_0 = \dfrac{1}{\lambda}\left|\dfrac{dN}{dt}\right| = \dfrac{7.56\times10^{11}\ \text{decays/s}}{3.75\times10^{-4}\ \text{s}^{-1}} = 2.02\times10^{15}$ nuclei.

(b) The number of nuclei left after one half-life is $\dfrac{N_0}{2} = 1.01\times10^{15}$ nuclei, and the activity is half:

$\left|\dfrac{dN}{dt}\right| = 3.78\times10^{11}\,\text{decays/s}.$

(c) After three half lives (92.4 minutes) there is an eighth of the original amount, so $N = 2.53\times10^{14}$ nuclei, and an eighth of the activity: $\left(\dfrac{dN}{dt}\right) = 9.45\times10^{10}\,\text{decays/s}.$

43.32. The activity of the sample is $\dfrac{3070\text{ decays/min}}{(60\text{ sec/min})\,(0.500\text{ kg})} = 102\text{ Bq/kg}$, while the activity of atmospheric carbon is $255\ \text{Bq/kg}$ (see Example 43.9). The age of the sample is then $t = -\dfrac{\ln(102/255)}{\lambda} = -\dfrac{\ln(102/255)}{1.21\times10^{-4}/\text{y}} = 7573\text{ y}.$

43.33. **IDENTIFY** and **SET UP:** Find λ from the half-life and the number N of nuclei from the mass of one nucleus and the mass of the sample. Then use Eq.(43.16) to calculate $|dN/dt|$, the number of decays per second.

EXECUTE: **(a)** $|dN/dt| = \lambda N$

$\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{(1.28\times10^{9}\text{ y})(3.156\times10^{7}\text{ s/1 y})} = 1.715\times10^{-17}\text{ s}^{-1}$

The mass of ^{40}K atom is approximately 40 u, so the number of ^{40}K nuclei in the sample is

$N = \dfrac{1.63\times10^{-9}\text{ kg}}{40\text{ u}} = \dfrac{1.63\times10^{-9}\text{ kg}}{40(1.66054\times10^{-27}\text{ kg})} = 2.454\times10^{16}.$

Then $|dN/dt| = \lambda N = (1.715\times10^{-17}\text{ s}^{-1})(2.454\times10^{16}) = 0.421\text{ decays/s}$

(b) $|dN/dt| = (0.421\text{ decays/s})(1\text{ Ci}/(3.70\times10^{10}\text{ decays/s})) = 1.14\times10^{-11}\text{ Ci}$

EVALUATE: The very small sample still contains a very large number of nuclei. But the half life is very large, so the decay rate is small.

43.34. **(a)** rem = rad $\times$ RBE. 200 = x(10) and x = 20 rad.

(b) 1 rad deposits $0.010\ \text{J/kg}$, so 20 rad deposit $0.20\ \text{J/kg}$. This radiation affects 25 g (0.025 kg) of tissue, so the total energy is $(0.025\text{ kg})(0.20\text{ J/kg}) = 5.0\times10^{-3}\text{ J} = 5.0\text{ mJ}$

(c) Since RBE = 1 for β-rays, so rem = rad. Therefore 20 rad = 20 rem.

43.35. 1 rad = 10^{-2} Gy, so 1 Gy = 100 rad and the dose was 500 rad.

rem = (rad)(RBE) = (500 rad)(4.0) = 2000 rem. $1\text{ Gy} = 1\text{ J/kg}$, so 5.0 J/kg.

43.36. **IDENTIFY** and **SET UP:** For x rays $\text{RBE} = 1$ so the equivalent dose in Sv is the same as the absorbed dose in J/kg.

EXECUTE: One whole-body scan delivers $(75\text{ kg})(12\times10^{-3}\text{ J/kg}) = 0.90\text{ J}$. One chest x ray delivers $(5.0\text{ kg})(0.20\times10^{-3}\text{ J/kg}) = 1.0\times10^{-3}\text{ J}$. It takes $\dfrac{0.90\text{ J}}{1.0\times10^{-3}\text{ J}} = 900$ chest x rays to deliver the same total energy.

43.37. **IDENTIFY** and **SET UP:** For x rays $\text{RBE} = 1$ and the equivalent dose equals the absorbed dose.

EXECUTE: **(a)** $175\text{ krad} = 175\text{ krem} = 1.75\text{ kGy} = 1.75\text{ kSv}$

$(1.75\times10^{3}\text{ J/kg})(0.150\text{ kg}) = 2.62\times10^{2}\text{ J}$

(b) $175\text{ krad} = 1.75\text{ kGy}$; $(1.50)(175\text{ krad}) = 262\text{ krem} = 2.62\text{ kSv}$

The energy deposited would be $2.62\times10^{2}\text{ J}$, the same as in (a).

EVALUATE: The energy required to raise the temperature of 0.150 kg of water $1\ \text{C}^\circ$ is 628 J, and $2.62\times10^{2}\text{ J}$ is less than this. The energy deposited corresponds to a very small amount of heating.

43.38. **(a)** $5.4\text{ Sv}\,(100\text{ rem/Sv}) = 540\text{ rem}.$

(b) The RBE of 1 gives an absorbed dose of 540 rad.

(c) The absorbed dose is 5.4 Gy, so the total energy absorbed is $(5.4\text{ Gy})\,(65\text{ kg}) = 351\text{ J}$. The energy required to raise the temperature of 65 kg by 0.010° C is $(65\text{ kg})\,(4190\text{ J/kg}\cdot\text{K})\,(0.01\text{C}^\circ) = 3\text{ kJ}.$

43.39. **(a)** We need to know how many decays per second occur.

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(12.3\text{ y})\,(3.156\times10^{7}\text{ s/y})} = 1.79\times10^{-9}\text{s}^{-1}.$$

The number of tritium atoms is $N_0 = \dfrac{1}{\lambda}\left|\dfrac{dN}{dt}\right| = \dfrac{(0.35\text{ Ci})\,(3.70\times10^{10}\text{ Bq/Ci})}{1.79\times10^{-9}\text{ s}^{-1}} = 7.2540\times10^{18}$ nuclei.

The number of remaining nuclei after one week is

$N = N_0 e^{-\lambda t} = (7.25\times10^{18}) e^{-(1.79\times10^{-9}\,\text{s}^{-1})(7)(24)(3600\text{s})} = 7.2462\times10^{18}$ nuclei. $\Delta N = N_0 - N = 7.8\times10^{15}$ decays. So the energy absorbed is $E_{\text{total}} = \Delta N\, E_\gamma = (7.8\times10^{15})\,(5000\text{ eV})\,(1.60\times10^{-19}\text{ J/eV}) = 6.24\text{ J}$. The absorbed dose is $\dfrac{(6.24\text{ J})}{(50\text{ kg})} = 0.125\text{ J/kg} = 12.5\text{ rad}$. Since RBE = 1, then the equivalent dose is 12.5 rem.

(b) In the decay, antinetrinos are also emitted. These are not absorbed by the body, and so some of the energy of the decay is lost (about 12 keV).

43.40. $(0.72\times10^{-6}\text{ Ci})\,(3.7\times10^{10}\text{ Bq/Ci})\,(3.156\times10^{7}\text{ s}) = 8.41\times10^{11}\ \alpha$ particles. The absorbed dose is

$\dfrac{(8.41\times10^{11})\,(4.0\times10^{6}\text{ eV})\,(1.602\times10^{-19}\text{ J/eV})}{(0.50\text{ kg})} = 1.08\text{ Gy} = 108\text{ rad}$. The equivalent dose is $(20)\,(108\text{ rad}) = 2160\text{ rem}$.

43.41. **(a) IDENTIFY** and **SET UP:** Determine X by balancing the charge and nucleon number on the two sides of the reaction equation.

EXECUTE: X must have $A = 2 + 14 - 10 = 6$ and $Z = 1 + 7 - 5 = 3$. Thus X is ${}^{6}_{3}\text{Li}$ and the reaction is

${}^{2}_{1}\text{H} + {}^{14}_{7}\text{N} \rightarrow {}^{6}_{3}\text{Li} + {}^{10}_{5}\text{B}$

(b) IDENTIFY and **SET UP:** Calculate the mass decrease and find its energy equivalent.

EXECUTE: The neutral atoms on each side of the reaction equation have a total of 8 electrons, so the electron masses cancel when neutral atom masses are used. The neutral atom masses are found in Table 43.2.

mass of ${}^{2}_{1}\text{H} + {}^{14}_{7}\text{N}$ is $2.014102\text{ u} + 14.003074\text{ u} = 16.017176\text{ u}$

mass of ${}^{6}_{3}\text{Li} + {}^{10}_{5}\text{B}$ is $6.015121\text{ u} + 10.012937\text{ u} = 16.028058\text{ u}$

The mass increases, so energy is absorbed by the reaction. The Q value is

$(16.017176\text{ u} - 16.028058\text{ u})(931.5\text{ MeV/u}) = -10.14\text{ MeV}$

(c) IDENTIFY and **SET UP:** The available energy in the collision, the kinetic energy K_{cm} in the center of mass reference frame, is related to the kinetic energy K of the bombarding particle by Eq. (43.24).

EXECUTE: The kinetic energy that must be available to cause the reaction is 10.14 MeV. Thus $K_{\text{cm}} = 10.14$ MeV. The mass M of the stationary target $({}^{14}_{7}\text{N})$ is $M = 14$ u. The mass m of the colliding particle $({}^{2}_{1}\text{H})$ is 2 u. Then by Eq. (43.24) the minimum kinetic energy K that the ${}^{2}_{1}\text{H}$ must have is

$$K = \left(\frac{M + m}{M}\right) K_{\text{cm}} = \left(\frac{14\text{ u} + 2\text{ u}}{14\text{ u}}\right)(10.14\text{ MeV}) = 11.59\text{ MeV}$$

EVALUATE: The projectile $\left({}^{2}_{1}\text{H}\right)$ is much lighter than the target $\left({}^{14}_{7}\text{N}\right)$ so K is not much larger than K_{cm}. The K we have calculated is what is required to allow the mass increase. We would also need to check to see if at this energy the projectile can overcome the Coulomb repulsion to get sufficiently close to the target nucleus for the reaction to occur.

43.42. $m_{{}^{3}_{2}\text{He}} + m_{{}^{2}_{1}\text{H}} - m_{{}^{4}_{2}\text{He}} - m_{{}^{1}_{1}\text{H}} = 1.97\times10^{-2}\text{u}$, so the energy released is 18.4 MeV.

43.43. **IDENTIFY** and **SET UP:** Determine X by balancing the charge and the nucleon number on the two sides of the reaction equation.

EXECUTE: X must have $A = +2 + 9 - 4 = 7$ and $Z = +1 + 4 - 2 = 3$. Thus X is ${}^{7}_{3}\text{Li}$ and the reaction is

${}^{2}_{1}\text{H} + {}^{9}_{4}\text{Be} = {}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He}$

(b) IDENTIFY and **SET UP:** Calculate the mass decrease and find its energy equivalent.

EXECUTE: If we use the neutral atom masses then there are the same number of electrons (five) in the reactants as in the products. Their masses cancel, so we get the same mass defect whether we use nuclear masses or neutral atom masses. The neutral atoms masses are given in Table 43.2.

${}^{2}_{1}\text{H} + {}^{9}_{4}\text{Be}$ has mass $2.014102\text{ u} + 9.012182\text{ u} = 11.26284\text{ u}$

${}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He}$ has mass $7.016003\text{ u} + 4.002603\text{ u} = 11.018606\text{ u}$

The mass decrease is $11.026284\text{ u} - 11.018606\text{ u} = 0.007678\text{ u}$.

This corresponds to an energy release of $0.007678\text{ u}(931.5\text{ MeV/1 u}) = 7.152\text{ MeV}$.

(c) IDENTIFY and **SET UP:** Estimate the threshold energy by calculating the Coulomb potential energy when the ${}^{2}_{1}\text{H}$ and ${}^{9}_{4}\text{Be}$ nuclei just touch. Obtain the nuclear radii from Eq. (43.1).

EXECUTE: The radius R_{Be} of the ${}^{9}_{4}\text{Be}$ nucleus is $R_{\text{Be}} = (1.2\times10^{-15}\text{ m})(9)^{1/3} = 2.5\times10^{-15}\text{ m}$.

The radius R_{H} of the ${}^{2}_{1}\text{H}$ nucleus is $R_{\text{H}} = (1.2\times10^{-15}\text{ m})(2)^{1/3} = 1.5\times10^{-15}\text{ m}$.

The nuclei touch when their center-to-center separation is

$R = R_{\text{Be}} + R_{\text{H}} = 4.0\times10^{-15}\text{ m}$.

The Coulomb potential energy of the two reactant nuclei at this separation is

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r} = \frac{1}{4\pi\epsilon_0}\frac{e(4e)}{r}$$

$$U = (8.988\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{4(1.602\times10^{-19}\ \text{C})^2}{(4.0\times10^{-15}\ \text{m})(1.602\times10^{-19}\ \text{J/eV})} = 1.4\ \text{MeV}$$

This is an estimate of the threshold energy for this reaction.

EVALUATE: The reaction releases energy but the total initial kinetic energy of the reactants must be 1.4 MeV in order for the reacting nuclei to get close enough to each other for the reaction to occur. The nuclear force is strong but is very short-range.

43.44. **IDENTIFY and SET UP:** 0.7% of naturally occurring uranium is the isotope ^{235}U. The mass of one ^{235}U nucleus is about $235m_p$.

EXECUTE: **(a)** The number of fissions needed is $\frac{1.0\times10^{19}\ \text{J}}{(200\times10^6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = 3.13\times10^{29}$. The mass of ^{235}U required is $(3.13\times10^{29})(235m_p) = 1.23\times10^5\ \text{kg}$.

(b) $\frac{1.23\times10^5\ \text{kg}}{0.7\times10^{-2}} = 1.76\times10^7\ \text{kg}$

EVALUATE: The calculation assumes 100% conversion of fission energy to electrical energy.

43.45. **IDENTIFY** and **SET UP:** The energy released is the energy equivalent of the mass decrease. 1 u is equivalent to 931.5 MeV. The mass of one ^{235}U nucleus is $235m_p$.

EXECUTE: **(a)** $^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{144}_{56}\text{Ba} + ^{89}_{36}\text{Kr} + 3^{1}_{0}\text{n}$

We can use atomic masses since the same number of electrons are included on each side of the reaction equation and the electron masses cancel. The mass decrease is

$$\Delta M = m\left(^{235}_{92}\text{U}\right) + m\left(^{1}_{0}\text{n}\right) - \left[m\left(^{144}_{56}\text{Ba}\right) + m\left(^{89}_{36}\text{Kr}\right) + 3m\left(^{1}_{0}\text{n}\right)\right]$$

$$\Delta M = 235.043930\ \text{u} + 1.0086649\ \text{u} - 143.922953\ \text{u} - 88.917630\ \text{u} - 3(1.0086649\ \text{u})$$

$\Delta M = 0.1860\ \text{u}$. The energy released is $(0.1860\ \text{u})(931.5\ \text{MeV/u}) = 173.3\ \text{MeV}$.

(b) The number of ^{235}U nuclei in 1.00 g is $\frac{1.00\times10^{-3}\ \text{kg}}{235m_p} = 2.55\times10^{21}$. The energy released per gram is

$(173.3\ \text{MeV/nucleus})(2.55\times10^{21}\ \text{nuclei/g}) = 4.42\times10^{23}\ \text{MeV/g}$.

43.46. **(a)** $^{28}_{14}\text{Si} + \gamma \Rightarrow ^{24}_{12}\text{Mg} + ^{A}_{Z}\text{X}$. $A + 24 = 28$ so $A = 4$. $Z + 12 = 14$ so $Z = 2$. X is an α particle.

(b) $E_\gamma = -\Delta mc^2 = (23.985042\ \text{u} + 4.002603\ \text{u} - 27.976927\ \text{u})\ (931.5\ \text{MeV/u}) = 9.984\ \text{MeV}$

43.47. The energy liberated will be

$M(^{3}_{2}\text{He}) + M(^{4}_{2}\text{He}) - M(^{7}_{4}\text{Be}) = (3.016029\ \text{u} + 4.002603\ \text{u} - 7.016929\ \text{u})(931.5\ \text{MeV/u}) = 1.586\ \text{MeV}$.

43.48. **(a)** $Z = 3 + 2 - 0 = 5$ and $A = 4 + 7 - 1 = 10$.

(b) The nuclide is a boron nucleus, and $m_{\text{He}} + m_{\text{Li}} - m_{\text{n}} - m_{\text{B}} = -3.00\times10^{-3}\ \text{u}$, and so 2.79 MeV of energy is absorbed.

43.49. Nuclei: $^{A}_{Z}\text{X}^{Z+} \rightarrow ^{A-4}_{Z-2}\text{Y}^{(Z-2)+} + ^{4}_{2}\text{He}^{2+}$. Add the mass of Z electrons to each side and we find:

$\Delta m = M(^{A}_{Z}\text{X}) - M(^{A-4}_{Z-2}\text{Y}) - M(^{4}_{2}\text{He})$, where now we have the mass of the neutral atoms. So as long as the mass of the original neutral atom is greater than the sum of the neutral products masses, the decay can happen.

43.50. Denote the reaction as $^{A}_{Z}\text{X} \rightarrow {}_{Z+1}{}^{A}\text{Y} + \text{e}^-$. The mass defect is related to the change in the neutral atomic masses by

$$[m_{\text{X}} - Zm_{\text{e}}] - [m_{\text{Y}} - (Z+1)m_{\text{e}}] - m_{\text{e}} = (m_{\text{X}} - m_{\text{Y}}),$$

where m_{X} and m_{Y} are the masses as tabulated in, for instance, Table (43.2).

43.51. $^{A}_{Z}\text{X}^{Z+} \rightarrow {}_{Z-1}{}^{A}\text{Y}^{(Z-1)+} + \beta^+$. Adding $(Z-1)$ electrons to both sides yields $^{A}_{Z}\text{X}^{+} \rightarrow {}_{Z-1}{}^{A}\text{Y} + \beta^+$. So in terms of masses:

$\Delta m = M\left(^{A}_{Z}\text{X}^{+}\right) - M\left({}_{Z-1}{}^{A}\text{Y}\right) - m_{\text{e}} = \left(M\left(^{A}_{Z}\text{X}\right) - m_{\text{e}}\right) - M\left({}_{Z-1}{}^{A}\text{Y}\right) - m_{\text{e}} = M\left(^{A}_{Z}\text{X}\right) - M\left({}_{Z-1}{}^{A}\text{Y}\right) - 2m_{\text{e}}$. So the decay will occur as long as the original neutral mass is greater than the sum of the neutral product mass and two electron masses.

43.52. **IDENTIFY and SET UP:** $m = \rho V$. $1\ \text{gal} = 3.788\ \text{L} = 3.788\times10^{-3}\ \text{m}^3$. The mass of a ^{235}U nucleus is $235m_p$.

$1\ \text{MeV} = 1.60\times10^{-13}\ \text{J}$

EXECUTE: **(a)** For 1 gallon, $m = \rho V = (737\ \text{kg/m}^3)(3.788\times10^{-3}\ \text{m}^3) = 2.79\ \text{kg} = 2.79\times10^3\ \text{g}$

$$\frac{1.3\times10^8\ \text{J/gal}}{2.79\times10^3\ \text{g/gal}} = 4.7\times10^4\ \text{J/g}$$

(b) 1 g contains $\dfrac{1.00\times10^{-3}\ \text{kg}}{235m_\text{p}}=2.55\times10^{21}$ nuclei

$(200\ \text{MeV/nucleus})(1.60\times10^{-13}\ \text{J/MeV})(2.55\times10^{21}\ \text{nuclei})=8.2\times10^{10}\ \text{J/g}$

(c) A mass of $6m_\text{p}$ produces 26.7 MeV.

$$\frac{(26.7\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV})}{6m_\text{p}}=4.26\times10^{14}\ \text{J/kg}=4.26\times10^{11}\ \text{J/g}$$

(d) The total energy available would be $(1.99\times10^{30}\ \text{kg})(4.7\times10^{7}\ \text{J/kg})=9.4\times10^{37}\ \text{J}$

$$\text{power}=\frac{\text{energy}}{t}\ \text{ so }\ t=\frac{\text{energy}}{\text{power}}=\frac{9.4\times10^{37}\,\text{J}}{3.86\times10^{26}\ \text{W}}=2.4\times10^{11}\ \text{s}=7600\ \text{yr}$$

EVALUATE: If the mass of the sun were all proton fuel, it would contain enough fuel to last $(7600\ \text{yr})\left(\dfrac{4.3\times10^{11}\ \text{J/g}}{4.7\times10^{4}\ \text{J/g}}\right)=7.0\times10^{10}\ \text{yr}$.

43.53. Using Eq: (43.12): ${}^{A}_{Z}M=ZM_\text{H}+Nm_\text{n}-E_\text{B}/c^2\Rightarrow M({}^{24}_{11}\text{Na})=11M_\text{H}+13m_\text{n}-E_\text{B}/c^2$.

But $E_\text{B}=(15.75\ \text{MeV})(24)-(17.80\ \text{MeV})(24)^{2/3}-(0.7100\ \text{MeV})\dfrac{(11)(10)}{(24)^{1/3}}-$

$(23.69\ \text{MeV})\dfrac{(24-2(11))^2}{24}-(39\ \text{MeV})(24)^{-4/3}=198.31\ \text{MeV}.$

$$\Rightarrow M({}^{24}_{11}\text{Na})=11(1.007825\ \text{u})+13(1\ .008665\ \text{u})-\frac{(198.31\ \text{MeV})}{931.5\ \text{MeV/u}}=23.9858\ \text{u}$$

$$\%\ \text{error}=\frac{23.990963-23.9858}{23.990963}\times100=0.022\%.$$

If the binding energy term is neglected, $M({}^{24}_{11}\text{Na})=24.1987$ u and the percentage error would be

$$\frac{24.1987-23.990963}{23.990963}\times100=0.87\%.$$

43.54. The α-particle will have $\dfrac{226}{230}$ of the released energy (see Example 43.5). $\dfrac{226}{230}(m_\text{Th}-m_\text{Ra}-m_\alpha)=$ 5.032×10^{-3} u or 4.69 MeV.

43.55. **(a) IDENTIFY and SET UP:** The heavier nucleus will decay into the lighter one.

EXECUTE: ${}^{25}_{13}\text{Al}$ will decay into ${}^{25}_{12}\text{Mg}$.

(b) IDENTIFY and SET UP: Determine the emitted particle by balancing A and Z in the decay reaction.

EXECUTE: This gives ${}^{25}_{13}\text{Al}\rightarrow{}^{25}_{12}\text{Mg}+{}^{0}_{+1}\text{e}$. The emitted particle must have charge $+e$ and its nucleon number must be zero. Therefore, it is a β^+ particle, a positron.

(c) IDENTIFY and SET UP: Calculate the energy defect ΔM for the reaction and find the energy equivalent of ΔM. Use the nuclear masses for ${}^{25}_{13}\text{Al}$ and ${}^{25}_{12}\text{Mg}$, to avoid confusion in including the correct number of electrons if neutral atom masses are used.

EXECUTE: The nuclear mass for ${}^{25}_{13}\text{Al}$ is $M_\text{nuc}({}^{25}_{13}\text{Al})=24.990429\ \text{u}-13(0.000548580\ \text{u})=24.983297\ \text{u}$.

The nuclear mass for ${}^{25}_{12}\text{Mg}$ is $M_\text{nuc}({}^{25}_{12}\text{Mg})=24.985837\ \text{u}-12(0.000548580\ \text{u})=24.979254\ \text{u}$.

The mass defect for the reaction is

$$\Delta M=M_\text{nuc}({}^{25}_{13}\text{Al})-M_\text{nuc}({}^{25}_{12}\text{Mg})-M({}^{0}_{+1}\text{e})=24.983297\ \text{u}-24.979254\ \text{u}-0.00054858\ \text{u}=0.003494\ \text{u}$$

$$Q=(\Delta M)c^2=0.003494\ \text{u}(931.5\ \text{MeV/1 u})=3.255\ \text{MeV}$$

EVALUATE: The mass decreases in the decay and energy is released. Note: ${}^{25}_{13}\text{Al}$ can also decay into ${}^{25}_{12}\text{Mg}$ by the electron capture.

$${}^{25}_{13}\text{Al}+{}^{0}_{-1}\text{e}\rightarrow{}^{25}_{12}\text{Mg}$$

The ${}^{0}_{-1}$ electron in the reaction is an orbital electron in the neutral ${}^{25}_{13}\text{Al}$ atom. The mass defect can be calculated using the nuclear masses:

$$\Delta M=M_\text{nuc}\left({}^{25}_{13}\text{Al}\right)+M({}^{0}_{-1}\text{e})-M_\text{nuc}\left({}^{25}_{12}\text{Mg}\right)=24.983287\ \text{u}+0.00054858\ \text{u}-24.979254\ \text{u}=0.004592\ \text{u}.$$

$$Q=\left(\Delta M\right)^2\ c^2=(0.004592\ \text{u})(931.5\ \text{MeV/1 u})=4.277\ \text{MeV}$$

The mass decreases in the decay and energy is released.

43.56. **(a)** $m_{^{210}_{84}\text{Po}} - m_{^{206}_{82}\text{Pb}} - m_{^{4}_{2}\text{He}} = 5.81\times10^{-3}$ u, or $Q = 5.41$ MeV. The energy of the alpha particle is $(206/210)$ times this, or 5.30 MeV (see Example 43.5).

(b) $m_{^{210}_{84}\text{Po}} - m_{^{209}_{83}\text{Bi}} - m_{^{1}_{1}\text{H}} = -5.35\times10^{-3}\,\text{u} < 0$, so the decay is not possible.

(c) $m_{^{210}_{84}\text{Po}} - m_{^{209}_{84}\text{Po}} - m_{\text{n}} = -8.22\times10^{-3}\,\text{u} < 0$, so the decay is not possible.

(d) $m_{^{210}_{85}\text{At}} > m_{^{210}_{84}\text{Po}}$, so the decay is not possible (see Problem (43.50)).

(e) $m_{^{210}_{83}\text{Bi}} + 2m_{\text{e}} > m_{^{210}_{84}\text{Po}}$, so the decay is not possible (see Problem (43.51)).

43.57. **IDENTIFY and SET UP:** The amount of kinetic energy released is the energy equivalent of the mass change in the decay. $m_{\text{e}} = 0.0005486$ u and the atomic mass of $^{14}_{7}\text{N}$ is 14.003074 u. The energy equivalent of 1 u is 931.5 MeV. ^{14}C has a half-life of $T_{1/2} = 5730\ \text{yr} = 1.81\times10^{11}\ \text{s}$. The RBE for an electron is 1.0.

EXECUTE: **(a)** $^{14}_{6}\text{C} \rightarrow \text{e}^{-} + {}^{14}_{7}\text{N} + \overline{\nu}_{\text{e}}$

(b) The mass decrease is $\Delta M = m\left(^{14}_{6}\text{C}\right) - \left[m_{\text{e}} + m\left(^{14}_{7}\text{N}\right)\right]$. Use nuclear masses, to avoid difficulty in accounting for atomic electrons. The nuclear mass of $^{14}_{6}\text{C}$ is $14.003242\ \text{u} - 6m_{\text{e}} = 13.999950\ \text{u}$.

The nuclear mass of $^{14}_{7}\text{N}$ is $14.003074\ \text{u} - 7m_{\text{e}} = 13.999234\ \text{u}$.

$\Delta M = 13.999950\ \text{u} - 13.999234\ \text{u} - 0.000549\ \text{u} = 1.67\times10^{-4}\ \text{u}$. The energy equivalent of ΔM is 0.156 MeV.

(c) The mass of carbon is $(0.18)(75\ \text{kg}) = 13.5\ \text{kg}$. From Example 43.9, the activity due to 1 g of carbon in a living organism is 0.255 Bq. The number of decay/s due to 13.5 kg of carbon is $(13.5\times10^{3})(0.255\ \text{Bq/g}) =$ 3.4×10^{3} decays/s.

(d) Each decay releases 0.156 MeV so 3.4×10^{3} decays/s releases $530\ \text{MeV/s} = 8.5\times10^{-11}\ \text{J/s}$.

(e) The total energy absorbed in 1 yr is $(8.5\times10^{-11}\ \text{J/s})(3.156\times10^{7}\ \text{s}) = 2.7\times10^{-3}\ \text{J}$. The absorbed dose is $\dfrac{2.7\times10^{-3}\ \text{J}}{75\ \text{kg}} = 3.6\times10^{-5}\ \text{J/kg} = 36\ \mu\text{Gy} = 3.6\ \text{mrad}$. With $\text{RBE} = 1.0$, the equivalent dose is $36\ \mu\text{Sv} = 3.6\ \text{mrem}$.

43.58. **IDENTIFY** and **SET UP:** $m_{\pi} = 264m_{\text{e}} = 2.40\times10^{-28}$ kg. The total energy of the two photons equals the rest mass energy $m_{\pi}c^2$ of the pion.

EXECUTE: **(a)** $E_{\text{ph}} = \frac{1}{2}m_{\pi}c^2 = \frac{1}{2}(2.40\times10^{-28}\ \text{kg})(3.00\times10^{8}\ \text{m/s})^2 = 1.08\times10^{-11}\ \text{J} = 67.5\ \text{MeV}$

$$E_{\text{ph}} = \frac{hc}{\lambda} \text{ so } \lambda = \frac{hc}{E_{\text{ph}}} = \frac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{67.5\times10^{6}\ \text{eV}} = 1.84\times10^{-14}\ \text{m} = 18.4\ \text{fm}$$

These are gamma ray photons, so they have $\text{RBE} = 1.0$.

(b) Each pion delivers $2(1.08\times10^{-11}\ \text{J}) = 2.16\times10^{-11}\ \text{J}$.

The absorbed dose is $200\ \text{rad} = 2.00\ \text{Gy} = 2.00\ \text{J/kg}$.

The energy deposited is $(25\times10^{-3}\ \text{kg})(2.00\ \text{J/kg}) = 0.050\ \text{J}$.

The number of π^0 mesons needed is $\dfrac{0.050\ \text{J}}{2.16\times10^{-11}\ \text{J/meson}} = 2.3\times10^{9}$ mesons.

EVALUATE: Note that charge is conserved in the decay since the pion is neutral. If the pion is initially at rest the photons must have equal momenta in opposite directions so the two photons have the same λ and are emitted in opposite directions. The photons also have equal energies since they have the same momentum and $E = pc$.

43.59. **IDENTIFY** and **SET UP:** Find the energy equivalent of the mass decrease. Part of the released energy appears as the emitted photon and the rest as kinetic energy of the electron.

EXECUTE: $^{198}_{79}\text{Au} \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e}$

The mass change is $197.968225\ \text{u} - 197.966752\ \text{u} = 1.473\times10^{-3}\ \text{u}$

(The neutral atom masses include 79 electrons before the decay and 80 electrons after the decay. This one additional electron in the products accounts correctly for the electron emitted by the nucleus.) The total energy released in the decay is $(1.473\times10^{-3}\ \text{u})(931.5\ \text{MeV/u}) = 1.372\ \text{MeV}$. This energy is divided between the energy of the emitted photon and the kinetic energy of the β^- particle. Thus the β^- particle has kinetic energy equal to $1.372\ \text{MeV} - 0.412\ \text{MeV} = 0.960\ \text{MeV}$.

EVALUATE: The emitted electron is much lighter than the $^{198}_{80}\text{Hg}$ nucleus, so the electron has almost all the final kinetic energy. The final kinetic energy of the ^{198}Hg nucleus is very small.

43.60. (See Problem (43.51)) $m_{{}^{11}_{6}\text{C}} - m_{{}^{11}_{5}\text{B}} - 2m_{\text{e}} = 1.03\times10^{-3}$ u. Decay is energetically possible.

43.61. **IDENTIFY** and **SET UP:** The decay is energetically possible if the total mass decreases. Determine the nucleus produced by the decay by balancing *A* and *Z* on both sides of the equation. ${}^{13}_{7}\text{N} \rightarrow {}^{0}_{+1}\text{e} + {}^{13}_{6}\text{C}$. To avoid confusion in including the correct number of electrons with neutral atom masses, use nuclear masses, obtained by subtracting the mass of the atomic electrons from the neutral atom masses.

EXECUTE: The nuclear mass for ${}^{13}_{7}\text{N}$ is $M_{\text{nuc}}\left({}^{13}_{7}\text{N}\right) = 13.005739\ \text{u} - 7(0.00054858\ \text{u}) = 13.001899\ \text{u}$.

The nuclear mass for ${}^{13}_{6}\text{C}$ is $M_{\text{nuc}}\left({}^{13}_{6}\text{C}\right) = 13.003355\ \text{u} - 6(0.00054858\ \text{u}) = 13.000064\ \text{u}$.

The mass defect for the reaction is

$\Delta M = M_{\text{nuc}}\left({}^{13}_{7}\text{N}\right) - M_{\text{nuc}}\left({}^{13}_{6}\text{C}\right) - M\left({}^{0}_{+1}\text{e}\right)$. $\Delta M = 13.001899\ \text{u} - 13.000064\ \text{u} - 0.00054858\ \text{u} = 0.001286\ \text{u}$.

EVALUATE: The mass decreases in the decay, so energy is released. This decay is energetically possible.

43.62. **(a)** A least-squares fit to log of the activity *vs.* time gives a slope of $\lambda = 0.5995\ \text{h}^{-1}$, for a half-life of $\dfrac{\ln 2}{\lambda} = 1.16\ \text{h}$.

(b) The initial activity is $N_0\lambda$, and this gives $N_0 = \dfrac{(2.00\times10^4\ \text{Bq})}{(0.5995\ \text{hr}^{-1})(1\ \text{hr}/3600\ \text{s})} = 1.20\times10^8$.

(c) $N_0 e^{-\lambda t} = 1.81\times10^6$.

43.63. The activity $A(t) \equiv \dfrac{dN(t)}{dt}$ but $\dfrac{dN(t)}{dt} = -\lambda N(t)$ so $-\lambda N_0 = A_0$. Taking the derivative of

$N(t) = N_0 e^{-\lambda t} \Rightarrow \dfrac{dN(t)}{dt} = -\lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t}$, or $A(t) = A_0 e^{-\lambda t}$.

43.64. From Eq.43.17 $N(t) = N_0 e^{-\lambda t}$ but $N_0 e^{-\lambda t} = N_0 e^{-(\ln 2)\left(t/T_{1/2}\right)}$

$= N_0\left[e^{-(\ln 2)}\right]^{\left(t/T_{1/2}\right)} = N_0\left[e^{\ln(1/2)}\right]^{\left(t/T_{1/2}\right)}$. So $N(t) = N_0\left(\dfrac{1}{2}\right)^n$ where $n = \dfrac{t}{T_{1/2}}$.

(We have used that $a\ln x = \ln(x^a)$, $e^{ax} = (e^x)^a$, and $e^{\ln x} = x$.)

43.65. **IDENTIFY and SET UP:** One-half of the sample decays in a time of $T_{1/2}$.

EXECUTE: **(a)** $\dfrac{10\times10^9\ \text{yr}}{200{,}000\ \text{yr}} = 5.0\times10^4$

(b) $(\frac{1}{2})^{5.0\times10^4}$. This exponent is too large for most hand-held calculators. But $(\frac{1}{2}) = 10^{-0.301}$ so

$(\frac{1}{2})^{5.0\times10^4} = (10^{-0.301})^{5.0\times10^4} = 10^{-15{,}000}$

43.66. **IDENTIFY and SET UP:** $T_{1/2} = \dfrac{\ln 2}{\lambda}$. The mass of a single nucleus is $149m_{\text{p}} = 2.49\times10^{-25}$ kg. $\Delta N/\Delta t = -\lambda N$.

EXECUTE: $N = \dfrac{12.0\times10^{-3}\ \text{kg}}{2.49\times10^{-25}\ \text{kg}} = 4.82\times10^{22}$. $\Delta N/\Delta t = -2.65$ decays/s

$\lambda = -\dfrac{\Delta N/\Delta t}{N} = \dfrac{2.65\ \text{decays/s}}{4.82\times10^{22}} = 5.50\times10^{-23}\ \text{s}^{-1}$; $T_{1/2} = \dfrac{\ln 2}{\lambda} = 1.26\times10^{22}\ \text{s} = 3.99\times10^{14}\ \text{yr}$

43.67. **IDENTIFY:** Use Eq. (43.17) to relate the initial number of radioactive nuclei, N_0, to the number , *N*, left after time *t*.

SET UP: We have to be careful; after ${}^{87}\text{Rb}$ has undergone radioactive decay it is no longer a rubidium atom. Let N_{85} be the number of ${}^{85}\text{Rb}$ atoms; this number doesn't change. Let N_0 be the number of ${}^{87}\text{Rb}$ atoms on earth when the solar system was formed. Let *N* be the present number of ${}^{87}\text{Rb}$ atoms.

EXECUTE: The present measurements say that $0.2783 = N/(N + N_{85})$.

$(N + N_{85})(0.2783) = N$, so $N = 0.3856N_{85}$. The percentage we are asked to calculate is $N_0/(N_0 + N_{85})$.

N and N_0 are related by $N = N_0 e^{-\lambda t}$ so $N_0 = e^{+\lambda t}N$.

Thus $\dfrac{N_0}{N_0 + N_{85}} = \dfrac{Ne^{\lambda t}}{Ne^{\lambda t} + N_{85}} = \dfrac{(0.3855e^{\lambda t})N_{85}}{(0.3856e^{\lambda t})N_{85} + N_{85}} = \dfrac{0.3856e^{\lambda t}}{0.3856e^{\lambda t} + 1}$.

$t = 4.6\times10^9\ \text{y}$; $\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{4.75\times10^{10}\ \text{y}} = 1.459\times10^{-11}\ \text{y}^{-1}$

$e^{\lambda t} = e^{(1.459\times10^{-11}\ \text{y}^{-1})(4.6\times10^9\ \text{y})} = e^{0.16711} = 1.0694$

Thus $\dfrac{N_0}{N_0 + N_{85}} = \dfrac{(0.3856)(1.0694)}{(0.3856)(1.0694)+1} = 29.2\%.$

EVALUATE: The half-life for ^{87}Rb is a factor of 10 larger than the age of the solar system, so only a small fraction of the ^{87}Rb nuclei initially present have decayed; the percentage of rubidium atoms that are radioactive is only a bit less now than it was when the solar system was formed.

43.68. **(a)** $(6.25\times10^{12})(4.77\times10^{6}\text{ MeV})(1.602\times10^{-19}\text{ J/eV})/(70.0\text{ kg}) = 0.0682\text{ Gy} = 0.682\text{ rad}$

(b) (20)(6.82 rad) = 136 rem

(c) $N\lambda = \dfrac{m}{Am_\text{p}}\dfrac{\ln(2)}{T_{1/2}} = 1.17\times10^{9}\text{ Bq} = 31.6\text{ mCi}$.

(d) $\dfrac{6.25\times10^{12}}{1.17\times10^{9}\text{ Bq}} = 5.34\times10^{3}$ s, about an hour and a half. Note that this time is so small in comparison with the half-life that the decrease in activity of the source may be neglected.

43.69. **IDENTIFY** and **SET UP:** Find the energy emitted and the energy absorbed each second. Convert the absorbed energy to absorbed dose and to equivalent dose.

EXECUTE: **(a)** First find the number of decays each second:

$$2.6\times10^{-4}\text{ Ci}\left(\frac{3.70\times10^{10}\text{ decays/s}}{1\text{ Ci}}\right) = 9.6\times10^{6}\text{ decays/s}$$

The average energy per decay is 1.25 MeV, and one-half of this energy is deposited in the tumor. The energy delivered to the tumor per second then is

$\frac{1}{2}(9.6\times10^{6}\text{ decays/s})(1.25\times10^{6}\text{ eV/decay})(1.602\times10^{-19}\text{ J/eV}) = 9.6\times10^{-7}\text{ J/s}.$

(b) The absorbed dose is the energy absorbed divided by the mass of the tissue:

$$\frac{9.6\times10^{-7}\text{ J/s}}{0.500\text{ kg}} = (1.9\times10^{-6}\text{ J/kg}\cdot\text{s})(1\text{ rad}/(0.01\text{ J/kg})) = 1.9\times10^{-4}\text{ rad/s}$$

(c) equivalent dose (REM) = RBE × absorbed dose (rad)

In one second the equivalent dose is $0.70(1.9\times10^{-4}\text{ rad}) = 1.3\times10^{-4}$ rem.

(d) $(200\text{ rem}/1.3\times10^{-4}\text{ rem/s}) = 1/5\times10^{6}\text{ s}(1\text{ h}/3600\text{ s}) = 420\text{ h} = 17$ days.

EVALUATE: The activity of the source is small so that absorbed energy per second is small and it takes several days for an equivalent dose of 200 rem to be absorbed by the tumor. A 200 rem dose equals 2.00 Sv and this is large enough to damage the tissue of the tumor.

43.70. **(a)** After 4.0 min = 240 s, the ratio of the number of nuclei is $\dfrac{2^{-240/122.2}}{2^{-240/26.9}} = 2^{(240)\left(\frac{1}{26.9}-\frac{1}{122.2}\right)} = 124.$

(b) After 15.0 min = 900 s, the ratio is 7.15×10^{7}.

43.71. **IDENTIFY** and **SET UP:** The number of radioactive nuclei left after time *t* is given by $N = N_0 e^{-\lambda t}$. The problem says $N/N_0 = 0.21$; solve for *t*.

EXECUTE: $0.21 = e^{-\lambda t}$ so $\ln(0.21) = -\lambda t$ and $t = -\ln(0.21)/\lambda$

Example 43.9 gives $\lambda = 1.209\times10^{-4}\text{ y}^{-1}$ for ^{14}C. Thus $t = \dfrac{-\ln(0.21)}{1.209\times10^{-4}\text{ y}} = 1.3\times10^{4}$ y.

EVALUATE: The half-life of ^{14}C is 5730 y, so our calculated *t* is more than two half-lives, so the fraction remaining is less than $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

43.72. **IDENTIFY:** The tritium (H-3) decays to He-3. The ratio of the number of He-3 atoms to H-3 atoms allows us to calculate the time since the decay began, which is when the H-3 was formed by the nuclear explosion. The H-3 decay is exponential.

SET UP: The number of tritium (H-3) nuclei decreases exponentially as $N_\text{H} = N_{0,\text{H}}e^{-\lambda t}$, with a half-life of 12.3 years. The amount of He-3 present after a time *t* is equal to the original amount of tritium minus the number of tritium nuclei that are still undecayed after time *t*.

EXECUTE: The number of He-3 nuclei after time *t* is

$$N_\text{He} = N_{0,\text{H}} - N_\text{H} = N_{0,\text{H}} - N_{0,\text{H}}e^{-\lambda t} = N_{0,\text{H}}\left(1-e^{-\lambda t}\right).$$

Taking the ratio of the number of He-3 atoms to the number of tritium (H-3) atoms gives

$$\frac{N_\text{He}}{N_\text{H}} = \frac{N_{0,\text{H}}\left(1-e^{-\lambda t}\right)}{N_{0,\text{H}}e^{-\lambda t}} = \frac{1-e^{-\lambda t}}{e^{-\lambda t}} = e^{\lambda t} - 1.$$

Solving for t gives $t = \dfrac{\ln(1+N_{\text{He}}/N_{\text{H}})}{\lambda}$. Using the given numbers and $T_{1/2} = \dfrac{\ln 2}{\lambda}$, we have

$\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{12.3\ \text{y}} = 0.0563/\text{y}$ and $t = \dfrac{\ln(1+4.3)}{0.0563/\text{y}}$ = 30 years.

EVALUATE: One limitation on this method would be that after many years the ratio of H to He would be too small to measure accurately.

43.73. **(a) IDENTIFY** and **SET UP:** Use Eq.(43.1) to calculate the radius R of a ${}^2_1\text{H}$ nucleus. Calculate the Coulomb potential energy (Eq.23.9) of the two nuclei when they just touch.

EXECUTE: The radius of ${}^2_1\text{H}$ is $R = (1.2\times10^{-15}\ \text{m})(2)^{1/3} = 1.51\times10^{-15}$ m. The barrier energy is the Coulomb potential energy of two ${}^2_1\text{H}$ nuclei with their centers separated by twice this distance:

$$U = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r} = (8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.602\times10^{-19}\ \text{C})^2}{2(1.51\times10^{-15}\ \text{m})} = 7.64\times10^{-14}\ \text{J} = 0.48\ \text{MeV}$$

(b) IDENTIFY and **SET UP:** Find the energy equivalent of the mass decrease.

EXECUTE: ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + {}^1_0\text{n}$

If we use neutral atom masses there are two electrons on each side of the reaction equation, so their masses cancel. The neutral atom masses are given in Table 43.2.

${}^2_1\text{H} + {}^2_1\text{H}$ has mass $2(2.014102\ \text{u}) = 4.028204\ \text{u}$

${}^3_2\text{He} + {}^1_0\text{n}$ has mass $3.016029\ \text{u} + 1.008665\ \text{u} = 4.024694\ \text{u}$

The mass decrease is $4.028204\ \text{u} - 4.024694\ \text{u} = 3.510\times10^{-3}\ \text{u}$. This corresponds to a liberated energy of $(3.510\times10^{-3}\ \text{u})(931.5\ \text{MeV/u}) = 3.270\ \text{MeV}$, or $(3.270\times10^6\ \text{eV})(1.602\times10^{-19}\ \text{J/eV}) = 5.239\times10^{-13}$ J.

(c) IDENTIFY and **SET UP:** We know the energy released when two ${}^2_1\text{H}$ nuclei fuse. Find the number of reactions obtained with one mole of ${}^2_1\text{H}$.

EXECUTE: Each reaction takes two ${}^2_1\text{H}$ nuclei. Each mole of D_2 has 6.022×10^{23} molecules, so 6.022×10^{23} pairs of atoms. The energy liberated when one mole of deuterium undergoes fusion is $(6.022\times10^{23})(5.239\times10^{-13}\ \text{J}) = 3.155\times10^{11}$ J/mol.

EVALUATE: The energy liberated per mole is more than a million times larger than from chemical combustion of one mole of hydrogen gas.

43.74. In terms of the number N of cesium atoms that decay in one week and the mass $m = 1.0$ kg, the equivalent dose is

$3.5\ \text{Sv} = \dfrac{N}{m}((\text{RBE})_\gamma \text{E}_\gamma + (\text{RBE})_\text{e}\text{E}_\text{e}) = \dfrac{N}{m}((1)(0.66\ \text{MeV}) + (1.5)(0.51\ \text{MeV})) = \dfrac{N}{m}(2.283\times10^{-13}\ \text{J})$, so

$N = \dfrac{(1.0\ \text{kg})(3.5\ \text{Sv})}{(2.283\times10^{-13}\ \text{J})} = 1.535\times10^{13}$. The number N_0 of atoms present is related to

N by $N_0 = Ne^{\lambda t}$. $\lambda = \dfrac{\ln 2}{Ty_2}\dfrac{0.693}{(30.07\ \text{yr})(3.156\times10^7\ \text{sec/yr})} = 7.30\times10^{-10}\ \text{sec}^{-1}$.

Then $N_0 = Ne^{\lambda t} = (1.535\times10^{13})e^{(7.30\times10^{-10}\ \text{s}^{-1})(7\ \text{days})(8.64\times10^4\ \text{s/day})} = 1.536\times10^{13}$.

43.75. **(a)** $v_{\text{cm}} = v\dfrac{m}{m+M}$. $v'_m = v - v\dfrac{m}{m+M} = \left(\dfrac{M}{m+M}\right)v$. $v'_M = \dfrac{vm}{m+M}$.

$$K' = \frac{1}{2}mv'^2_m + \frac{1}{2}Mv'^2_M = \frac{1}{2}\frac{mM^2}{(m+M)^2}v^2 + \frac{1}{2}\frac{Mm^2}{(m+M)^2}v^2 = \frac{1}{2}\frac{M}{(m+M)}\left(\frac{mM}{m+M} + \frac{m^2}{m+M}\right)v^2.$$

$$K' = \frac{M}{m+M}\left(\frac{1}{2}mv^2\right) \Rightarrow K' = \frac{M}{m+M}K \equiv K_{\text{cm}}.$$

(b) For an endoergic reaction $K_{\text{cm}} = -Q\,(Q<0)$ at threshold. Putting this into part (a) gives

$$-Q = \frac{M}{M+m}K_{\text{th}} \Rightarrow K_{\text{th}} = \frac{-(M+m)}{M}Q$$

43.76. $K=\frac{M_\alpha}{M_\alpha+m}K_\infty$, where K_∞ is the energy that the α-particle would have if the nucleus were infinitely massive.

Then, $M=M_{\text{Os}}-M_\alpha-K_\infty=M_{\text{Os}}-M_\alpha-\frac{186}{182}\left(2.76\ \text{MeV}/c^2\right)=181.94821\ \text{u}$.

43.77. $\Delta m=M\left({}^{235}_{92}\text{U}\right)-M\left({}^{140}_{54}\text{Xe}\right)-M\left({}^{94}_{38}\text{Sr}\right)-m_{\text{n}}$

$\Delta m=235.043923\ \text{u}-139.921636\ \text{u}-93.915360\ \text{u}-1.008665\ \text{u}=0.1983\ \text{u}$

$\Rightarrow E=(\Delta m)c^2=(0.1983\ \text{u})(931.5\ \text{MeV/u})=185\ \text{MeV}$.

43.78. **(a)** A least-squares fit of the log of the activity *vs.* time for the times later than 4.0 h gives a fit with correlation $-\left(1-2\times10^{-6}\right)$ and decay constant of $0.361\ \text{h}^{-1}$, corresponding to a half-life of 1.92 h. Extrapolating this back to time 0 gives a contribution to the rate of about 2500/s for this longer-lived species. A least-squares fit of the log of the activity *vs*. time for times earlier than 2.0 h gives a fit with correlation = 0.994, indicating the presence of only two species.

(b) By trial and error, the data is fit by a decay rate modeled by $R=(5000\ \text{Bq})e^{-t(1.733/\text{h})}+(2500\ \text{Bq})e^{-t(0.361/\text{h})}$. This would correspond to half-lives of 0.400 h and 1.92 h.

(c) In this model, there are 1.04×10^7 of the shorter-lived species and 2.49×10^7 of the longer-lived species.

(d) After 5.0 h, there would be 1.80×10^3 of the shorter-lived species and 4.10×10^6 of the longer-lived species.

43.79. **(a)** There are two processes occurring: the creation of ${}^{128}\text{I}$ by the neutron irradiation, and the decay of the newly produced ${}^{128}\text{I}$. So $\frac{dN}{dt}=K-\lambda N$ where K is the rate of production by the neutron irradiation. Then

$\int_0^N\frac{dN'}{K-\lambda N'}=\int_0^t dt$. $\left[\ln\left(K-\lambda N'\right)\right]_0^N=-\lambda t$. $\ln(K-\lambda N)=\ln K-\lambda t$. $N(t)=\frac{K\left(1-e^{-\lambda t}\right)}{\lambda}$. The graph is given in Figure 43.79.

(b) The activity of the sample is $\lambda N(t)=K\left(1-e^{-\lambda t}\right)=\left(1.5\times10^6\ \text{decays/s}\right)\times\left(1-e^{-\left(\frac{0.693}{25\ \text{min}}\right)t}\right)$. So the activity is

$\left(1.5\times10^6\ \text{decays/s}\right)\left(1-e^{-0.02772t}\right)$, with *t* in minutes. So the activity $\left(\frac{-dN'}{dt}\right)$ at various times is:

$$\frac{-dN'}{dt}(t=1\ \text{min})=4.1\times10^4\ \text{Bq};\ \frac{-dN'}{dt}(t=10\ \text{min})=3.6\times10^5\ \text{Bq};$$
$$\frac{-dN'}{dt}(t=25\ \text{min})=7.5\times10^5\ \text{Bq};\ \frac{-dN'}{dt}(t=50\ \text{min})=1.1\times10^6\ \text{Bq};$$
$$\frac{-dN'}{dt}(t=75\ \text{min})=1.3\times10^6\ \text{Bq};\ \frac{-dN'}{dt}(t=180\ \text{min})=1.5\times10^6\ \text{Bq};$$

(c) $N_{\max}=\frac{K}{\lambda}=\frac{(1.5\times10^6)(60)}{(0.02772)}=3.2\times10^9\ \text{atoms}$.

(d) The maximum activity is at saturation, when the rate being produced equals that decaying and so it equals $1.5\times10^6\ \text{decays/s}$.

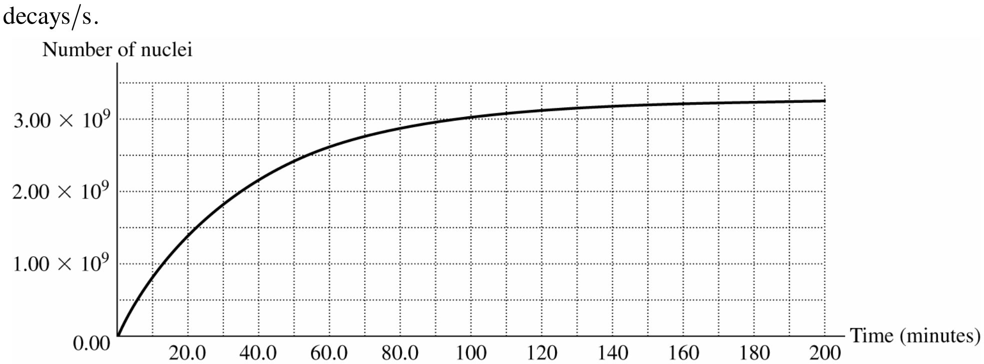


Figure 43.79

43.80. The activity of the original iron, after 1000 hours of operation, would be $(9.4\times10^{-6}\ \text{Ci})(3.7\times10^{10}\ \text{Bq/Ci})2^{-(1000\ \text{h})/(45\ \text{d}\times24\ \text{h/d})}=1.8306\times10^5\ \text{Bq}$. The activity of the oil is 84 Bq, or 4.5886×10^{-4} of the total iron activity, and this must be the fraction of the mass worn, or mass of $4.59\times10^{-2}\ \text{g}$. The rate at which the piston rings lost their mass is then $4.59\times10^{-5}\ \text{g/h}$.

44

PARTICLE PHYSICS AND COSMOLOGY

44.1. **(a) IDENTIFY and SET UP:** Use Eq.(37.36) to calculate the kinetic energy K.

EXECUTE: $K = mc^2\left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right) = 0.1547mc^2$

$m = 9.109\times10^{-31}$ kg, so $K = 1.27\times10^{-14}$ J

(b) IDENTIFY and SET UP: The total energy of the particles equals the sum of the energies of the two photons. Linear momentum must also be conserved.

EXECUTE: The total energy of each electron or positron is $E = K + mc^2 = 1.1547mc^2 = 9.46\times10^{-13}$ J. The total energy of the electron and positron is converted into the total energy of the two photons. The initial momentum of the system in the lab frame is zero (since the equal-mass particles have equal speeds in opposite directions), so the final momentum must also be zero. The photons must have equal wavelengths and must be traveling in opposite directions. Equal λ means equal energy, so each photon has energy 9.46×10^{-14} J.

(c) IDENTIFY and SET UP: Use Eq. (38.2) to relate the photon energy to the photon wavelength.

EXECUTE: $E = hc/\lambda$ so $\lambda = hc/E = hc/(9.46\times10^{-14}\text{ J}) = 2.10$ pm

EVALUATE: The wavelength calculated in Example 44.1 is 2.43 pm. When the particles also have kinetic energy, the energy of each photon is greater, so its wavelength is less.

44.2. The total energy of the positron is

$$E = K + mc^2 = 5.00\text{ MeV} + 0.511\text{ MeV} = 5.51\text{ MeV}.$$

We can calculate the speed of the positron from Eq.(37.38):

$$E = \frac{mc^2}{\sqrt{1-\dfrac{v^2}{c^2}}} \Rightarrow \frac{v}{c} = \sqrt{1-\left(\frac{mc^2}{E}\right)^2} = \sqrt{1-\left(\frac{0.511\text{ MeV}}{5.51\text{ MeV}}\right)^2} = 0.996.$$

44.3. **IDENTIFY and SET UP:** By momentum conservation the two photons must have equal and opposite momenta. Then $E = pc$ says the photons must have equal energies. Their total energy must equal the rest mass energy $E = mc^2$ of the pion. Once we have found the photon energy we can use $E = hf$ to calculate the photon frequency and use $\lambda = c/f$ to calculate the wavelength.

EXECUTE: The mass of the pion is $270m_e$, so the rest energy of the pion is $270(0.511\text{ MeV}) = 138$ MeV. Each photon has half this energy, or 69 MeV. $E = hf$ so $f = \dfrac{E}{h} = \dfrac{(69\times10^6\text{ eV})(1.602\times10^{-19}\text{ J/eV})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 1.7\times10^{22}$ Hz

$\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^8\text{ m/s}}{1.7\times10^{22}\text{ Hz}} = 1.8\times10^{-14}\text{ m} = 18$ fm.

EVALUATE: These photons are in the gamma ray part of the electromagnetic spectrum.

44.4. **(a)** The energy will be the proton rest energy, 938.3 MeV, corresponding to a frequency of 2.27×10^{23} Hz and a wavelength of 1.32×10^{-15} m.

(b) The energy of each photon will be $938.3\text{ MeV} + 830\text{ MeV} = 1768\text{ MeV}$, with frequency 42.8×10^{22} Hz and wavelength 7.02×10^{-16} m.

44.5. **(a)** $\Delta m = m_{\pi+} - m_{\mu+} = 270\ m_e - 207\ m_e = 63\ m_e \Rightarrow E = 63(0.511\text{ MeV}) = 32$ MeV.

(b) A positive muon has less mass than a positive pion, so if the decay from muon to pion was to happen, you could always find a frame where energy was not conserved. This cannot occur.

44.6. **(a)** $\lambda = \frac{hc}{E} = \frac{hc}{m_\mu c^2} = \frac{h}{m_\mu c} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})}{(207)(9.11\times10^{-31}\text{ kg})(3.00\times10^{8}\text{ m/s})} = 1.17\times10^{-14}\text{ m} = 0.0117\text{ pm}$

In this case, the muons are created at rest (no kinetic energy).

(b) Shorter wavelengths would mean higher photon energy, and the muons would be created with non-zero kinetic energy.

44.7. **IDENTIFY:** The energy released comes from the mass difference.

SET UP: The mass difference is the initial mass minus the final mass.

$$\Delta m = m_{\mu^-} - m_{e^-} - m_{e^+}$$

EXECUTE: Using the masses from Table 44.2, we have

$\Delta m = m_{\mu^-} - m_{e^-} - m_{e^+} = (105.7\text{ MeV}/c^2) - (0.511\text{ MeV}/c^2) - (0.511\text{ MeV}/c^2) = 105\text{ MeV}/c^2$

Multiplying these masses by c^2 gives E = 105 MeV.

EVALUATE: This energy is observed as kinetic energy of the electron and positron.

44.8. **IDENTIFY** and **SET UP:** Calculate the mass change in each reaction, using the atomic masses in Table 44.2. A mass change of 1 u is equivalent to an energy of 931.5 MeV.

EXECUTE: **(a)** and **(b)** Eq.(44.1): ${}^{4}_{2}\text{He} + {}^{9}_{4}\text{Be} \rightarrow {}^{12}_{6}\text{C} + {}^{1}_{0}\text{n}$

$\Delta M = m({}^{4}\text{He}) + m({}^{9}\text{Be}) - \left[m({}^{12}\text{C}) + m({}^{1}\text{n})\right]$

$\Delta M = 4.00260\text{ u} + 9.01218\text{ u} - 12.00000\text{ u} - 1.00866\text{ u} = 0.00612\text{ u}$

The mass decreases and the energy liberated is 5.70 MeV. The reaction is exoergic.

Eq.(44.2): ${}^{1}_{0}\text{n} + {}^{10}_{5}\text{B} \rightarrow {}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He}$

$\Delta M = m({}^{1}\text{n}) + m({}^{10}\text{B}) - \left[m({}^{7}\text{Li}) + m({}^{4}\text{He})\right]$

$\Delta M = 1.00866\text{ u} + 10.01294\text{ u} - 7.01600\text{ u} - 4.00260\text{ u} = 0.00300\text{ u}$

The mass decreases and the energy liberated is 2.79 MeV. The reaction is exoergic.

(c) The reactants in the reactions of Eq.(44.1) have positive nuclear charges and a threshold kinetic energy is required for the reactants to overcome their Coulomb repulsion and get close enough for the reaction to occur. The neutron in Eq.(44.2) is neutral so there is no Coulomb repulsion and no threshold energy for this reaction.

44.9. **IDENTIFY:** The antimatter annihilates with an equal amount of matter.

SET UP: The energy of the matter is $E = (\Delta m)c^2$.

EXECUTE: Putting in the numbers gives

$$E = (\Delta m)c^2 = (400\text{ kg} + 400\text{ kg})(3.00\times10^{8}\text{ m/s})^2 = 7.2\times10^{19}\text{ J}.$$

This is about 70% of the annual energy use in the U.S.

EVALUATE: If this huge amount of energy were released suddenly, it would blow up the *Enterprise*! Getting useable energy from matter-antimatter annihiliation is not so easy to do!

44.10. **IDENTIFY:** With a stationary target, only part of the initial kinetic energy of the moving electron is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be left over kinetic energy. Therefore not all of the initial energy is available.

SET UP: The available energy is given by $E_a^2 = 2mc^2\left(E_m + mc^2\right)$ for two particles of equal mass when one is initially stationary. In this case, the initial kinetic energy (20.0 GeV = 20,000 MeV) is much more than the rest energy of the electron (0.511 MeV), so the formula for available energy reduces to $E_a = \sqrt{2mc^2E_m}$.

EXECUTE: **(a)** Using the formula for available energy gives

$$E_a = \sqrt{2mc^2E_m} = \sqrt{2(0.511\text{ MeV})(20.0\text{ GeV})} = 143\text{ MeV}$$

(b) For colliding beams of equal mass, each particle has half the available energy, so each has 71.5 MeV. The *total* energy is twice this, or 143 MeV.

EVALUATE: Colliding beams provide considerably more available energy to do experiments than do beams hitting a stationary target. With a stationary electron target in part (a), we had to give the moving electron 20,000 MeV of energy to get the same available energy that we got with only 143 MeV of energy with the colliding beams.

44.11. **(a) IDENTIFY** and **SET UP:** Eq. (44.7) says $\omega = |q|B/m$ so $B = m\omega/|q|$. And since $\omega = 2\pi f$, this becomes $B = 2\pi mf/|q|$.

EXECUTE: A deuteron is a deuterium nucleus $\left({}^{2}_{1}\text{H}\right)$. Its charge is $q = +e$. Its mass is the mass of the neutral ${}^{2}_{1}\text{H}$ atom (Table 43.2) minus the mass of the one atomic electron:

$m = 2.014102\text{ u} - 0.0005486\text{ u} = 2.013553\text{ u}(1.66054\times10^{-27}\text{ kg}/1\text{ u}) = 3.344\times10^{-27}\text{ kg}$

$B = \frac{2\pi mf}{|q|} = \frac{2\pi(3.344\times10^{-27}\text{ kg})(9.00\times10^{6}\text{ Hz})}{1.602\times10^{-19}\text{ C}} = 1.18\text{ T}$

(b) Eq.(44.8): $K = \frac{q^2B^2R^2}{2m} = \frac{[(1.602\times10^{-19}\text{ C})(1.18\text{ T})(0.320\text{ m})]^2}{2(3.344\times10^{-27}\text{ kg})}$.

$K = 5.471\times10^{-13}\text{ J} = (5.471\times10^{-13}\text{ J})(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 3.42\text{ MeV}$

$K = \frac{1}{2}mv^2$ so $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.471\times10^{-13}\text{ J})}{3.344\times10^{-27}\text{ kg}}} = 1.81\times10^7\text{ m/s}$

EVALUATE: $v/c = 0.06$, so it is ok to use the nonrelativistic expression for kinetic energy.

44.12. **(a)** $2f = \frac{\omega}{\pi} = \frac{eB}{m\pi} = 3.97\times10^7/\text{s}$.

(b) $\omega R = \frac{eBR}{m} = 3.12\times10^7\text{ m/s}$

(c) For three-figure precision, the relativistic form of the kinetic energy must be used,

$eV = (\gamma-1)mc^2$, so $eV = (\gamma-1)mc^2$, so $V = \frac{(\gamma-1)mc^2}{e} = 5.11\times10^6\text{ V}$.

44.13. **(a) IDENTIFY** and **SET UP:** The masses of the target and projectile particles are equal, so Eq. (44.10) can be used. $E_\text{a}^2 = 2mc^2(E_m + mc^2)$. E_a is specified; solve for the energy E_m of the beam particles.

EXECUTE: $E_m = \frac{E_\text{a}^2}{2mc^2} - mc^2$

The mass for the alpha particle can be calculated by subtracting two electron masses from the ${}^4_2\text{He}$ atomic mass:

$m = m_\alpha = 4.002603\text{ u} - 2(0.0005486\text{ u}) = 4.001506\text{ u}$

Then $mc^2 = (4.001506\text{ u})(931.5\text{ MeV/u}) = 3.727\text{ GeV}$.

$E_m = \frac{E_\text{a}^2}{2mc^2} - mc^2 = \frac{(16.0\text{ GeV})^2}{2(3.727\text{ GeV})} - 3.727\text{ GeV} = 30.6\text{ GeV}$.

(b) Each beam must have $\frac{1}{2}E_\text{a} = 8.0\text{ GeV}$.

EVALUATE: For a stationary target the beam energy is nearly twice the available energy. In a colliding beam experiment all the energy is available and each beam needs to have just half the required available energy.

44.14. **(a)** $\gamma = \frac{1000\times10^3\text{ MeV}}{938.3\text{ MeV}} = 1065.8$, so $v = 0.999999559c$.

(b) Nonrelativistic: $\omega = \frac{eB}{m} = 3.83\times10^8\text{ rad/s}$.

Relativistic: $\omega = \frac{eB}{m}\frac{1}{\gamma} = 3.59\times10^5\text{ rad/s}$.

44.15. **(a) IDENTIFY** and **SET UP:** For a proton beam on a stationary proton target and since E_a is much larger than the proton rest energy we can use Eq.(44.11): $E_\text{a}^2 = 2mc^2E_m$.

EXECUTE: $E_m = \frac{E_\text{a}^2}{2mc^2} = \frac{(77.4\text{ GeV})^2}{2(0.938\text{ GeV})} = 3200\text{ GeV}$

(b) IDENTIFY and **SET UP:** For colliding beams the total momentum is zero and the available energy E_a is the total energy for the two colliding particles.

EXECUTE: For proton-proton collisions the colliding beams each have the same energy, so the total energy of each beam is $\frac{1}{2}E_\text{a} = 38.7\text{ GeV}$.

EVALUATE: For a stationary target less than 3% of the beam energy is available for conversion into mass. The beam energy for a colliding beam experiment is a factor of (1/83) times smaller than the required energy for a stationary target experiment.

44.16. **IDENTIFY:** Only part of the initial kinetic energy of the moving electron is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be left over kinetic energy.

SET UP: To create the η^0, the minimum available energy must be equal to the rest mass energy of the products, which in this case is the η^0 plus two protons. In a collider, all of the initial energy is available, so the beam energy is the available energy.

EXECUTE: The minimum amount of available energy must be rest mass energy

$$E_\text{a} = 2m_\text{p} + m_\eta = 2(938.3\text{ MeV}) + 547.3\text{ MeV} = 2420\text{ MeV}$$

Each incident proton has half of the rest mass energy, or 1210 MeV = 1.21 GeV.

EVALUATE: As we saw in problem 44.10, we would need much more initial energy if one of the initial protons were stationary. The result here (1.21 GeV) is the *minimum* amount of energy needed; the original protons could have more energy and still trigger this reaction.

44.17. Section 44.3 says $m(\mathrm{Z}^0) = 91.2\ \mathrm{GeV}/c^2$.

$$E = 91.2\times10^9\ \mathrm{eV} = 1.461\times10^{-8}\ \mathrm{J};\ m = E/c^2 = 1.63\times10^{-25}\ \mathrm{kg};\ m(\mathrm{Z}^0)/m(\mathrm{p}) = 97.2$$

44.18. **(a)** We shall assume that the kinetic energy of the Λ^0 is negligible. In that case we can set the value of the photon's energy equal to Q:

$$Q = (1193 - 1116)\ \mathrm{MeV} = 77\ \mathrm{MeV} = E_{\mathrm{photon}}\,.$$

(b) The momentum of this photon is

$$p = \frac{E_{\mathrm{photon}}}{c} = \frac{(77\times10^6\ \mathrm{eV})(1.60\times10^{-18}\ \mathrm{J/eV})}{(3.00\times10^8\ \mathrm{m/s})} = 4.1\times10^{-20}\ \mathrm{kg\cdot m/s}$$

To justify our original assumption, we can calculate the kinetic energy of a Λ^0 that has this value of momentum

$$K_{\Lambda^0} = \frac{p^2}{2m} = \frac{E^2}{2mc^2} = \frac{(77\ \mathrm{MeV})^2}{2(1116\ \mathrm{MeV})} = 2.7\ \mathrm{MeV} << Q = 77\ \mathrm{MeV}.$$

Thus, we can ignore the momentum of the Λ^0 without introducing a large error.

44.19. **IDENTIFY** and **SET UP:** Find the energy equivalent of the mass decrease.

EXECUTE: The mass decrease is $m(\Sigma^+) - m(\mathrm{p}) - m(\pi^0)$ and the energy released is $mc^2(\Sigma^+) - mc^2(\mathrm{p}) - mc^2(\pi^0) = 1189\ \mathrm{MeV} - 938.3\ \mathrm{MeV} - 135.0\ \mathrm{MeV} = 116\ \mathrm{MeV}$. (The mc^2 values for each particle were taken from Table 44.3.)

EVALUATE: The mass of the decay products is less than the mass of the original particle, so the decay is energetically allowed and energy is released.

44.20. **IDENTIFY:** If the initial and final rest mass energies were equal, there would be no left over energy for kinetic energy. Therefore the kinetic energy of the products is the difference between the mass energy of the initial particles and the final particles.

SET UP: The difference in mass is $\Delta m = M_{\Omega^-} - m_{\Lambda^0} - m_{\mathrm{K}^-}$.

EXECUTE: Using Table 44.3, the energy difference is

$$E = (\Delta m)c^2 = 1672\ \mathrm{MeV} - 1116\ \mathrm{MeV} - 494\ \mathrm{MeV} = 62\ \mathrm{MeV}$$

EVALUATE: There is less rest mass energy after the reaction than before because 62 MeV of the initial energy was converted to kinetic energy of the products.

44.21. Conservation of lepton number.

(a) $\mu^- \to \mathrm{e}^- + \nu_{\mathrm{e}} + \overline{\nu}_\mu \Rightarrow L_\mu : +1 \neq -1,\ L_{\mathrm{e}} : 0 \neq +1 + 1$, so lepton numbers are not conserved.

(b) $\tau^- \to \mathrm{e}^- + \overline{\nu}_{\mathrm{e}} + \nu_\tau \Rightarrow L_{\mathrm{e}} : 0 = +1 - 1;\ L_\tau : +1 = +1$, so lepton numbers are conserved.

(c) $\pi^+ \to \mathrm{e}^+ + \gamma$. Lepton numbers are not conserved since just one lepton is produced from zero original leptons.

(d) $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \overline{\gamma}_{\mathrm{e}} \Rightarrow L_{\mathrm{e}} : 0 = +1 - 1,$ so the lepton numbers are conserved.

44.22. **IDENTIFY** and **SET UP:** p and n have baryon number $+1$ and $\overline{\mathrm{p}}$ has baryon number -1. e^+, e^-, $\overline{\upsilon}_{\mathrm{e}}$ and γ all have baryon number zero. Baryon number is conserved if the total baryon number of the products equals the total baryon number of the reactants.

EXECUTE: **(a)** reactants: $B = 1 + 1 = 2$. Products: $B = 1 + 0 = 1$. Not conserved.

(b) reactants: $B = 1 + 1 = 2$. Products: $B = 0 + 0 = 0$. Not conserved.

(c) reactants: $B = +1$. Products: $B = 1 + 0 + 0 = +1$. Conserved.

(d) reactants: $B = 1 - 1 = 0$. Products: $B = 0$. Conserved.

44.23. **IDENTIFY** and **SET UP:** Compare the sum of the strangeness quantum numbers for the particles on each side of the decay equation. The strangeness quantum numbers for each particle are given Table 44.3.

EXECUTE: **(a)** $\mathrm{K}^+ \to \mu^+ + \nu_\mu;\ S_{\mathrm{K}+} = +1,\ S_{\mu+} = 0,\ S_{\nu_\mu} = 0$

$S = 1$ initially; $S = 0$ for the products; S is <u>not conserved</u>

(b) $\mathrm{n} + \mathrm{K}^+ \to \mathrm{p} + \pi^0;\ S_{\mathrm{n}} = 0,\ S_{\mathrm{K}^+} = +1,\ S_{\mathrm{p}} = 0,\ S_{\pi^0} = 0$

$S = 1$ initially; $S = 0$ for the products; S is <u>not conserved</u>

(c) $\mathrm{K}^+ + \mathrm{K}^- \to \pi^0 + \pi^0;\ S_{\mathrm{K}^+} = +1;\ S_{\mathrm{K}^-} = -1;\ S_{\pi^0} = 0$

$S = +1 - 1 = 0$ initially; $S = 0$ for the products; S is <u>conserved</u>

(d) $p + K^- \to \Lambda^0 + \pi^0$; $S_p = 0, S_{K^-} = -1, S_{\Lambda^0} = -1, S_{\pi^0} = 0.$

$S = -1$ initially; $S = -1$ for the products; S is conserved

EVALUATE: Strangeness is not a conserved quantity in weak interactions and strangeness non-conserving reactions or decays can occur.

44.24. **(a)** Using the values of the constants from Appendix F,

$$\frac{e^2}{4\pi\epsilon_0 \hbar c} = 7.29660475 \times 10^{-3} = \frac{1}{137.050044}, \text{ or } 1/137 \text{ to three figures.}$$

(b) From Section 38.5, $v_1 = \frac{e^2}{2\epsilon_0 h}$. But notice this is just $\left(\frac{e^2}{4\pi\epsilon_0 \hbar c}\right)c$, as claimed.

44.25. $\left[\frac{f^2}{\hbar c}\right] = \frac{(\text{J}\cdot\text{m})}{(\text{J}\cdot\text{s})(\text{m}\cdot\text{s}^{-1})} = 1$ and thus $\frac{f^2}{\hbar c}$ is dimensionless. (Recall f^2 has units of energy times distance.)

44.26. **(a)** The diagram is given in Figure 44.26. The Ω^- particle has $Q = -1$ (as its label suggests) and $S = -3$. Its appears as a "hole"in an otherwise regular lattice in the $S - Q$ plane. The mass difference between each S row is around 145 MeV (or so). This puts the Ω^- mass at about the right spot. As it turns out, all the other particles on this lattice had been discovered already and it was this "hole" and mass regularity that led to an accurate prediction of the properties of the Ω !

(b) See diagram. Use quark charges $\boldsymbol{u} = +\frac{2}{3}, \boldsymbol{d} = \frac{-1}{3}$, and $\boldsymbol{s} = \frac{-1}{3}$ as a guide.

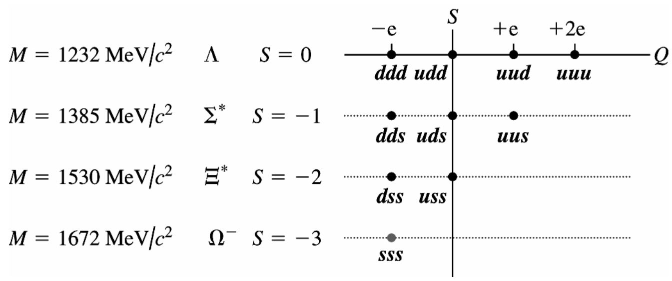


Figure 44.26

44.27. **IDENTIFY** and **SET UP:** Each value for the combination is the sum of the values for each quark. Use Table 44.4.

EXECUTE: **(a)** ***uds***

$Q = \frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0$

$B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

$S = 0 + 0 - 1 = -1$

$C = 0 + 0 + 0 = 0$

(b) $c\bar{u}$

The values for $\bar{u}$ are the negative for those for u.

$Q = \frac{2}{3}e - \frac{2}{3}e = 0$

$B = \frac{1}{3} - \frac{1}{3} = 0$

$S = 0 + 0 = 0$

$C = +1 + 0 = +1$

(c) ***ddd***

$Q = -\frac{1}{3}e - \frac{1}{3}e - \frac{1}{3}e = -e$

$B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = +1$

$S = 0 + 0 + 0 = 0$

$C = 0 + 0 + 0 = 0$

(d) $d\bar{c}$

$Q = -\frac{1}{3}e - \frac{2}{3}e = -e$

$B = \frac{1}{3} - \frac{1}{3} = 0$

$S = 0 + 0 = 0$

$C = 0 - 1 = -1$

EVALUATE: The charge, baryon number, strangeness and charm quantum numbers of a particle are determined by the particle's quark composition.

44.28. $(m_\gamma - 2m_\tau)c^2 = (9460\text{ MeV} - 2(1777\text{ MeV})) = 5906\text{ MeV}$ (see Sections 44.3 and 44.4 for masses).

44.29. **(a)** The antiparticle must consist of the antiquarks so $\overline{\text{n}} = \overline{\boldsymbol{u}}\overline{\boldsymbol{d}}\overline{\boldsymbol{d}}$.
(b) So $\text{n} = \boldsymbol{udd}$ is not its own antiparticle.
(c) $\psi = \boldsymbol{c\bar{c}}$ so $\overline{\psi} = \boldsymbol{\bar{c}c} = \psi$ so the ψ is its own antiparticle.

44.30. **(a)** $S = 1$ indicates the presence of one $\bar{\boldsymbol{s}}$ antiquark and no $\boldsymbol{s}$ quark. To have baryon number 0 there can be only one other quark, and to have net charge $+e$ that quark must be a $\boldsymbol{u}$, and the quark content is $\boldsymbol{u\bar{s}}$.
(b) The particle has an $\bar{\boldsymbol{s}}$ antiquark, and for a baryon number of –1 the particle must consist of three antiquarks. For a net charge of $-e$, the quark content must be $\overline{\boldsymbol{d}}\overline{\boldsymbol{d}}\,\overline{\boldsymbol{s}}$.
(c) $S = -2$ means that there are two $\boldsymbol{s}$ quarks, and for baryon number 1 there must be one more quark. For a charge of 0 the third quark must be a $\boldsymbol{u}$ quark and the quark content is $\boldsymbol{uss}$.

44.31. **IDENTIFY:** A proton is made up of ***uud*** quarks and a neutron consists of ***udd*** quarks.
SET UP: If a proton decays by β^+ decay, we have $\text{p} \rightarrow \text{e}^+ + \text{n} + \nu_e$ (both charge and lepton number are conserved).
EVALUATE: Since a proton consists of ***uud*** quarks and a neutron is ***udd*** quarks, it follows that in β^+ decay a ***u*** quark changes to a ***d*** quark.

44.32. **(a)** Using the definition of z from Example 44.9 we have that

$$1 + z = 1 + \frac{(\lambda_0 - \lambda_s)}{\lambda_0} = \frac{\lambda_0}{\lambda_s}.$$

Now we use Eq.(44.13) to obtain $1 + z = \sqrt{\dfrac{c+v}{c-v}} = \sqrt{\dfrac{1+v/c}{1-v/c}} = \sqrt{\dfrac{1+\beta}{1-\beta}}.$

(b) Solving the above equation for β we obtain $\beta = \dfrac{(1+z)^2 - 1}{(1+z)^2 + 1} = \dfrac{1.5^2 - 1}{1.5^2 + 1} = 0.3846.$

Thus, $v = 0.3846\ c = 1.15 \times 10^8\text{ m/s}.$

(c) We can use Eq.(44.15) to find the distance to the given galaxy,

$$r = \frac{v}{H_0} = \frac{(1.15 \times 10^8\text{ m/s})}{(7.1 \times 10^4\text{ (m/s)/Mpc)}} = 1.6 \times 10^3\text{ Mpc}$$

44.33. **(a) IDENTIFY and SET UP:** Use Eq.(44.14) to calculate v.

EXECUTE: $v = \left[\dfrac{(\lambda_0/\lambda_s)^2 - 1}{(\lambda_0/\lambda_s)^2 + 1}\right]c = \left[\dfrac{(658.5\text{ nm}/590\text{ nm})^2 - 1}{(658.5\text{ nm}/590\text{ nm})^2 + 1}\right]c = 0.1094c$

$v = (0.1094)(2.998 \times 10^8\text{ m/s}) = 3.28 \times 10^7\text{ m/s}$

(b) IDENTIFY and SET UP: Use Eq.(44.15) to calculate r.

EXECUTE: $r = \dfrac{v}{H_0} = \dfrac{3.28 \times 10^4\text{ km/s}}{(71\text{ (km/s)/Mpc})(1\text{ Mpc}/3.26\text{ Mly})} = 1510\text{ Mly}$

EVALUATE: The red shift $\lambda_0/\lambda_S - 1$ for this galaxy is 0.116. It is therefore about twice as far from earth as the galaxy in Examples 44.9 and 44.10, that had a red shift of 0.053.

44.34. From Eq.(44.15), $r = \dfrac{c}{H_0} = \dfrac{3.00 \times 10^8\text{ m/s}}{20\text{(km/s)/Mly}} = 1.5 \times 10^4\text{ Mly}.$

(b) This distance represents looking back in time so far that the light has not been able to reach us.

44.35. **(a) IDENTIFY and SET UP:** Hubble's law is Eq.(44.15), with $H_0 = 71$ (km/s)/(Mpc). 1 Mpc = 3.26 Mly.

EXECUTE: $r = 5210\text{ Mly}$ so $v = H_0 r = ((71\text{ km/s})/\text{Mpc})(1\text{ Mpc}/3.26\text{ Mly})(5210\text{ Mly}) = 1.1 \times 10^5\text{ km/s}$

(b) IDENTIFY and SET UP: Use v from part (a) in Eq. (44.13).

EXECUTE: $\dfrac{\lambda_0}{\lambda_S} = \sqrt{\dfrac{c+v}{c-v}} = \sqrt{\dfrac{1+v/c}{1-v/c}}$

$\dfrac{v}{c} = \dfrac{1.1 \times 10^8\text{ m/s}}{2.9980 \times 10^8\text{ m/s}} = 0.367$ so $\dfrac{\lambda_0}{\lambda_S} = \sqrt{\dfrac{1+0.367}{1-0.367}} = 1.5$

EVALUATE: The galaxy in Examples 44.9 and 44.10 is 710 Mly away so has a smaller recession speed and redshift than the galaxy in this problem.

44.36. **IDENTIFY and SET UP:** $m_\text{H} = 1.67 \times 10^{-27}\text{ kg}$. The ideal gas law says $pV = nRT$. Normal pressure is $1.013 \times 10^5\text{ Pa}$ and normal temperature is about $27\ ^\circ\text{C} = 300\text{ K}$. 1 mole is 6.02×10^{23} atoms.

EXECUTE: **(a)** $\dfrac{6.3\times10^{-27}\ \text{kg/m}^3}{1.67\times10^{-27}\ \text{kg/atom}} = 3.8\ \text{atoms/m}^3$

(b) $V = (4\ \text{m})(7\ \text{m})(3\ \text{m}) = 84\ \text{m}^3$ and $(3.8\ \text{atoms/m}^3)(84\ \text{m}^3) = 320$ atoms

(c) With $p = 1.013\times10^5$ Pa , $V = 84\ \text{m}^3$, $T = 300$ K the ideal gas law gives the number of moles to be

$$n = \frac{pV}{RT} = \frac{(1.013\times10^5\ \text{Pa})(84\ \text{m}^3)}{(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})} = 3.4\times10^3\ \text{moles}$$

$(3.4\times10^3\ \text{moles})(6.02\times10^{23}\ \text{atoms/mol}) = 2.0\times10^{27}$ atoms

EVALUATE: The average density of the universe is very small. Interstellar space contains a very small number of atoms per cubic meter, compared to the number of atoms per cubit meter in ordinary material on the earth, such as air.

44.37. **IDENTIFY** and **SET UP:** Find the energy equivalent of the mass decrease.

EXECUTE: **(a)** $\text{p} + {}^2_1\text{H} \rightarrow {}^3_2\text{He}$ or can write as ${}^1_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He}$

If neutral atom masses are used then the masses of the two atomic electrons on each side of the reaction will cancel.

Taking the atomic masses from Table 43.2, the mass decrease is $m\left({}^1_1\text{H}\right) + m\left({}^2_1\text{H}\right) - m\left({}^3_2\text{He}\right) =$ 1.007825 u + 2.014102 u − 3.016029 u = 0.005898 u. The energy released is the energy equivalent of this mass decrease: $(0.005898\ \text{u})(931.5\ \text{MeV/u}) = 5.494$ MeV

(b) ${}^1_0\text{n} + {}^3_2\text{He} \rightarrow {}^4_2\text{He}$

If neutral helium masses are used then the masses of the two atomic electrons on each side of the reaction equation will cancel. The mass decrease is $m\left({}^1_0\text{n}\right) + m\left({}^3_2\text{He}\right) - m\left({}^4_2\text{He}\right) = 1.008665$ u + 3.016029 u − 4.002603 u = 0.022091 u. The energy released is the energy equivalent of this mass decrease: $(0.022091\ \text{u})(931.l5\ \text{MeV/u}) = 20.58$ MeV

EVALUATE: These are important nucleosynthesis reactions, discussed in Section 44.7.

44.38. $3m({}^4\text{He}) - m({}^{12}\text{C}) = 7.80\times10^{-3}$ u, or 7.27 MeV.

44.39. $\Delta m = m_\text{e} + m_\text{p} - m_\text{n} - m_{\nu_\text{e}}$ so assuming $m_{\nu_\text{e}} \approx 0$,

$\Delta m = 0.0005486\ \text{u} + 1.007276\ \text{u} - 1.008665\ \text{u} = -8.40\times10^{-4}\ \text{u}$

$\Rightarrow E = (\Delta m)c^2 = (-8.40\times10^{-4}\ \text{u})(931.5\ \text{MeV/u}) = -0.783\ \text{MeV}$ and is endoergic.

44.40. $m_{{}^{12}_6\text{C}} + m_{{}^4_2\text{He}} - m_{{}^{16}_8\text{O}} = 7.69\times10^{-3}$ u, or 7.16 MeV, an exoergic reaction.

44.41. **IDENTIFY** and **SET UP:** The Wien displacement law (Eq.38.30) sys $\lambda_\text{m}T$ equals a constant. Use this to relate $\lambda_{\text{m},1}$ at T_1 to $\lambda_{\text{m},2}$ at T_2.

EXECUTE: $\lambda_{\text{m},1}T_1 = \lambda_{\text{m},2}T_2$

$$\lambda_{\text{m},1} = \lambda_{\text{m},2}\left(\frac{T_2}{T_1}\right) = 1.062\times10^{-3}\ \text{m}\left(\frac{2.728\ \text{K}}{3000\ \text{K}}\right) = 966\ \text{nm}$$

EVALUATE: The peak wavelength was much less when the temperature was much higher.

44.42. **(a)** The dimensions of $\hbar$ are energy times time, the dimensions of G are energy times time per mass squared, and so the dimensions of $\sqrt{\hbar G/c^3}$ are

$$\left[\frac{(\text{E}\cdot\text{T})(\text{E}\cdot\text{L}/\text{M}^2)}{(\text{L}/\text{T})^3}\right]^{1/2} = \left[\frac{\text{E}}{\text{M}}\right]\left[\frac{\text{T}^2}{\text{L}}\right] = \left[\frac{\text{L}}{\text{T}}\right]^2\left[\frac{\text{T}^2}{\text{L}}\right] = \text{L}.$$

(b) $$\left(\frac{\hbar G}{c^3}\right)^{1/2} = \left(\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)}{2\pi(3.00\times10^8\ \text{m/s})^3}\right)^{1/2} = 1.616\times10^{-35}\ \text{m}.$$

44.43. **IDENTIFY** and **SET UP:** For colliding beams the available energy is twice the beam energy. For a fixed-target experiment only a portion of the beam energy is available energy (Eqs.44.9 and 44.10).

EXECUTE: **(a)** $E_\text{a} = 2(7.0\ \text{TeV}) = 14.0$ TeV

(b) Need $E_\text{a} = 14.0\ \text{TeV} = 14.0\times10^6$ MeV. Since the target and projectile particles are both protons Eq. (44.10) can be used: $E_\text{a}^2 = 2mc^2(E_m + mc^2)$

$$E_m = \frac{E_\text{a}^2}{2mc^2} - mc^2 = \frac{(14.0\times10^6\ \text{MeV})^2}{2(938.3\ \text{MeV})} - 938.3\ \text{MeV} = 1.0\times10^{11}\ \text{MeV} = 1.0\times10^5\ \text{TeV}.$$

EVALUATE: This shows the great advantage of colliding beams at relativistic energies.

44.44. $K + m_p c^2 = \frac{hc}{\lambda}$, $K = \frac{hc}{\lambda} - m_p c^2 = 652\text{ MeV}$.

44.45. **IDENTIFY** and **SET UP:** Section 44.3 says the strong interaction is 100 times as strong as the electromagnetic interaction and that the weak interaction is 10^{-9} times as strong as the strong interaction. The Coulomb force is $F_e = \frac{kq_1q_2}{r^2}$ and the gravitational force is $F_g = G\frac{m_1m_2}{r^2}$.

EXECUTE: **(a)** $F_e = \frac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{(1\times10^{-15}\text{ m})^2} = 200\text{ N}$

$$F_g = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.67\times10^{-27}\text{ kg})^2}{(1\times10^{-15}\text{ m})^2} = 2\times10^{-34}\text{ N}$$

(b) $F_{str} \approx 100F_e \approx 2\times10^4\text{ N}$. $F_{weak} \approx 10^{-9}F_{str} \approx 2\times10^{-5}\text{ N}$

(c) $F_{str} > F_e > F_{weak} > F_g$

(d) $F_e \approx 1\times10^{36}F_g$. $F_{str} \approx 100F_e \approx 1\times10^{38}F_g$. $F_{weak} \approx 10^{-9}F_{str} \approx 1\times10^{29}F_g$

EVALUATE: The gravity force is much weaker than any of the other three forces. Gravity is important only when one very massive object is involved.

44.46. In Eq.(44.9), $E_a = (m_{\Sigma^0} + m_{K^0})c^2$, and with $M = m_p, m = m_{\pi^-}$ and $E_m = (m_{\pi^-})c^2 + K$,

$$K = \frac{E_a^2 - (m_\pi - c^2)^2 - (m_p c^2)^2}{2m_p c^2} - (m_{\pi^-})c^2$$

$$K = \frac{(1193\text{ MeV} + 497.7\text{ MeV})^2 - (139.6\text{ MeV})^2 - (938.3\text{ MeV})^2}{2(938.3\text{ MeV})} - 139.6\text{ MeV} = 904\text{ MeV}.$$

44.47. **IDENTIFY:** With a stationary target, only part of the initial kinetic energy of the moving proton is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be left over kinetic energy. Therefore not all of the initial energy is available.

SET UP: The available energy is given by $E_a^2 = 2mc^2\left(E_m + mc^2\right)$ for two particles of equal mass when one is initially stationary. The *minimum* available energy must be equal to the rest mass energies of the products, which in this case is two protons, a K^+ and a K^-. The available energy must be at least the sum of the final rest masses.

EXECUTE: The minimum amount of available energy must be

$E_a = 2m_p + m_{K^+} + m_{K^-} = 2(938.3\text{ MeV}) + 493.7\text{ MeV} + 493.7\text{ MeV} = 2864\text{ MeV} = 2.864\text{ GeV}$

Solving the available energy formula for E_m gives $E_a^2 = 2mc^2\left(E_m + mc^2\right)$ and

$$E_m = \frac{E_a^2}{2mc^2} - mc^2 = \frac{(2864\text{ MeV})^2}{2(938.3\text{ MeV})} - 938.3\text{ MeV} = 3432.6\text{ MeV}$$

Recalling that E_m is the *total* energy of the proton, including its rest mass energy (RME), we have

$$K = E_m - RME = 3432.6\text{ MeV} - 938.3\text{ MeV} = 2494\text{ MeV} = 2.494\text{ GeV}$$

Therefore the threshold kinetic energy is $K = 2494\text{ MeV} = 2.494\text{ GeV}$.

EVALUATE: Considerably less energy would be needed if the experiment were done using colliding beams of protons.

44.48. **(a)** The decay products must be neutral, so the only possible combinations are $\pi^0\pi^0\pi^0$ *or* $\pi^0\pi^+\pi^-$

(b) $m_{\eta_0} - 3m_{\pi^0} = 142.3\text{ MeV}/c^2$, so the kinetic energy of the π^0 mesons is 142.3 MeV. For the other reaction, $K = (m_{\eta_0} - m_{\pi^0} - m_{\pi^+} - m_{\pi^-})c^2 = 133.1\text{ MeV}$.

44.49. **IDENTIFY** and **SET UP:** Apply conservation of linear momentum to the collision. A photon has momentum $p = h/\lambda$, in the direction it is traveling. The energy of a photon is $E = pc = \frac{hc}{\lambda}$. All the mass of the electron and positron is converted to the total energy of the two photons, according to $E = mc^2$. The mass of an electron and of a positron is $m_e = 9.11\times10^{-31}\text{ kg}$

EXECUTE: **(a)** In the lab frame the initial momentum of the system is zero, since the electron and positron have equal speeds in opposite directions. According to momentum conservation, the final momentum of the system must also be zero. A photon has momentum, so the momentum of a single photon is not zero.

(b) For the two photons to have zero total momentum they must have the same magnitude of momentum and move in opposite directions. Since $E = pc$, equal p means equal E.

(c) $2E_{\text{ph}} = 2m_{\text{e}}c^2$ so $E_{\text{ph}} = m_{\text{e}}c^2$

$$E_{\text{ph}} = \frac{hc}{\lambda} \text{ so } \frac{hc}{\lambda} = m_{\text{e}}c^2 \text{ and } \lambda = \frac{h}{m_{\text{e}}c} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})} = 2.43\ \text{pm}$$

These are gamma ray photons.

EVALUATE: The total charge of the electron/positron system is zero and the photons have no charge, so charge is conserved in the particle-antiparticle annihilation.

44.50. **(a)** If the π^- decays, it must end in an electron and neutrinos. The rest energy of π^- (139.6 MeV) is shared between the electron rest energy (0.511 MeV) and kinetic energy (assuming the neutrino masses are negligible). So the energy released is 139.6 MeV – 0.511 MeV = 139.1 MeV.

(b) Conservation of momentum leads to the neutrinos carrying away most of the energy.

44.51. **(a)** The baryon number is 0, the charge is $+e$, the strangeness is 1, all lepton numbers are zero, and the particle is K^+.

(b) The baryon number is 0, the charge is $-e$, the strangeness is 0, all lepton numbers are zero, and the particle is π^-.

(c) The baryon numbers is –1, the charge is 0, the strangeness is zero, all lepton numbers are 0, and the particle is an antineutron.

(d) The baryon number is 0, the charge is $+e$, the strangeness is 0, the muonic lepton number is –1, all other lepton numbers are 0, and the particle is μ^+.

44.52. $$\Delta t = 7.6\times10^{-21}\ \text{s} \Rightarrow \Delta E = \frac{\hbar}{\Delta t} = \frac{1.054\times10^{-34}\ \text{J}\cdot\text{s}}{7.6\times10^{-21}\ \text{s}} = 1.39\times10^{-14}\ \text{J} = 87\ \text{keV}.$$

$$\frac{\Delta E}{m_\psi c^2} = \frac{0.087\ \text{MeV}}{3097\ \text{MeV}} = 2.8\times10^{-5}.$$

44.53. $$\frac{\hbar}{\Delta E} = \frac{(1.054\times10^{-34}\ \text{J}\cdot\text{s})}{(4.4\times10^{6}\ \text{eV})(1.6\times10^{-19}\ \text{J/eV})} = 1.5\times10^{-22}\ \text{s}.$$

44.54. **IDENTIFY** and **SET UP:** $\phi \to \text{K}^+ + \text{K}^-$. The total energy released is the energy equivalent of the mass decrease.

(a) EXECUTE: The mass decrease is $m(\phi) - m(\text{K}^+) - m(\text{K}^-)$. The energy equivalent of the mass decrease is $mc^2(\phi) - mc^2(\text{K}^+) - mc^2(\text{K}^-)$. The rest mass energy mc^2 for the ϕ meson is given in Problem 44.53, and the values for $\text{K}^+$and K^- are given in Table 44.3. The energy released then is $1019.4\ \text{MeV} - 2(493.7\ \text{MeV}) =$ $32.0\ \text{MeV}$. The K^+ gets half this, 16.0 MeV.

EVALUATE: **(b)** Does the decay $\phi \to \text{K}^+ + \text{K}^- + \pi^0$ occur? The energy equivalent of the $\text{K}^+ + \text{K}^- + \pi^0$ mass is 493.7 MeV + 493.7 MeV + 135.0 MeV = 1122 MeV. This is greater than the energy equivalent of the ϕ mass. The mass of the decay products would be greater than the mass of the parent particle; the decay is energetically forbidden.

(c) Does the decay $\phi \to \text{K}^+ + \pi^-$ occur? The reaction $\phi \to \text{K}^+ + \text{K}^-$ is observed. K^+ has strangeness $+1$ and K^- has strangeness -1, so the total strangeness of the decay products is zero. If strangeness must be conserved we deduce that the ϕ particle has strangeness zero. π^- has strangeness 0, so the product $\text{K}^+ + \pi^-$ has strangeness -1. The decay $\phi \to \text{K}^+ + \pi^-$ violates conservation of strangeness. Does the decay $\phi \to \text{K}^+ + \mu^-$ occur? μ^- has strangeness 0, so this decay would also violate conservation of strangeness.

44.55. **(a)** The number of protons in a kilogram is

$$(1.00\ \text{kg})\left(\frac{6.023\times10^{23}\ \text{molecules/mol}}{18.0\times10^{-3}\ \text{kg/mol}}\right)(2\ \text{protons/molecule}) = 6.7\times10^{25}.$$

Note that only the protons in the hydrogen atoms are considered as possible sources of proton decay. The energy per decay is $m_{\text{p}}c^2 = 938.3\ \text{MeV} = 1.503\times10^{-10}\ \text{J}$, and so the energy deposited in a year, per kilogram, is

$$(6.7\times10^{25})\left(\frac{\ln(2)}{1.0\times10^{18}\ \text{y}}\right)(1\ \text{y})\,(1.50\times10^{-10}\ \text{J}) = 7.0\times10^{-3}\ \text{Gy} = 0.70\ \text{rad}$$

(b) For an RBE of unity, the equivalent dose is (1) (0.70 rad) = 0.70 rem.

44.56. **IDENTIFY** and **SET UP:** The total released energy is the equivalent of the mass decrease. Use conservation of linear momentum to relate the kinetic energies of the decay particles.

EXECUTE: **(a)** The energy equivalent of the mass decrease is

$$mc^2(\Xi^-) - mc^2(\Lambda^0) - mc^2(\pi^-) = 1321\ \text{MeV} - 1116\ \text{MeV} - 139.6\ \text{MeV} = 65\ \text{MeV}$$

(**b**) The Ξ^- is at rest means that the linear momentum is zero. Conservation of linear momentum then says that the Λ^0 and π^- must have equal and opposite momenta:

$$m_{\Lambda^0} v_{\Lambda^0} = m_{\pi^-} v_{\pi^-}$$

$$v_{\pi^-} = \left(\frac{m_{\Lambda^0}}{m_{\pi^-}}\right) v_{\Lambda^0}$$

Also, the sum of the kinetic energies of the Λ^0 and π^- must equal the total kinetic energy $K_{\text{tot}} = 65$ MeV calculated in part (a):

$$K_{\text{tot}} = K_{\Lambda^0} + K_{\pi^-}$$

$$K_{\Lambda^0} + \tfrac{1}{2} m_{\pi^-} v_{\pi^-}^2 = K_{\text{tot}}$$

Use the momentum conservation result:

$$K_{\Lambda^0} + \tfrac{1}{2} m_{\pi^-} \left(\frac{m_{\Lambda^0}}{m_{\pi^-}}\right)^2 v_{\Lambda^0}^2 = K_{\text{tot}}$$

$$K_{\Lambda^0} + \left(\frac{m_{\Lambda^0}}{m_{\pi^-}}\right)\left(\tfrac{1}{2} m_{\Lambda^0} v_{\Lambda^0}^2\right) = K_{\text{tot}}$$

$$K_{\Lambda^0}\left(1 + \frac{m_{\Lambda^0}}{m_{\pi^-}}\right) = K_{\text{tot}}$$

$$K_{\Lambda^0} = \frac{K_{\text{tot}}}{1 + m_{\Lambda^0}/m_{\pi^-}} = \frac{65 \text{ MeV}}{1 + (1116 \text{ MeV})/(139.6 \text{ MeV})} = 7.2 \text{ MeV}$$

$K_{\Lambda^0} + K_{\pi^-} = K_{\text{tot}}$ so $K_{\pi^-} = K_{\text{tot}} - K_{\Lambda^0} = 65 \text{ MeV} - 7.2 \text{ MeV} = 57.8 \text{ MeV}$

The fraction for the Λ^0 is $\dfrac{7.2 \text{ MeV}}{65 \text{ MeV}} = 11\%.$

The fraction for the π^- is $\dfrac{57.8 \text{ MeV}}{65 \text{ MeV}} = 89\%.$

EVALUATE: The lighter particle carries off more of the kinetic energy that is released in the decay than the heavier particle does.

44.57. (**a**) For this model, $\dfrac{dR}{dt} = HR$, so $\dfrac{dR/dt}{R} = \dfrac{HR}{R} = H$, presumed to be the same for all points on the surface.

(**b**) For constant θ, $\dfrac{dr}{dt} = \dfrac{dR}{dt}\theta = HR\theta = Hr.$

(**c**) See part (a), $H_0 = \dfrac{dR/dt}{R}.$

(**d**) The equation $\dfrac{dR}{dt} = H_0 R$ is a differential equation, the solution to which, for constant H_0, is $R(t) = R_0 e^{H_0 t}$, where R_0 is the value of R at $t = 0$. This equation may be solved by separation of variables, as $\dfrac{dR/dt}{R} = \dfrac{d}{dt} \ln(R) = H_0$ and integrating both sides with respect to time.

(**e**) A constant H_0 would mean a constant critical density, which is inconsistent with uniform expansion.

44.58. From Problem 44.57, $r = R\theta \Rightarrow R = \dfrac{r}{\theta}$. So $\dfrac{dR}{dt} = \dfrac{1}{\theta}\dfrac{dr}{dt} - \dfrac{r}{\theta^2}\dfrac{d\theta}{dt} = \dfrac{1}{\theta}\dfrac{dr}{dt}$ since $\dfrac{d\theta}{dt} = 0.$

So $\dfrac{1}{R}\dfrac{dR}{dt} = \dfrac{1}{R\theta}\dfrac{dr}{dt} = \dfrac{1}{r}\dfrac{dr}{dt} \Rightarrow v = \dfrac{dr}{dt} = \left(\dfrac{1}{R}\dfrac{dR}{dt}\right) r = H_0 r.$ Now $\dfrac{dv}{d\theta} = 0 = \dfrac{d}{d\theta}\left(\dfrac{r}{R}\dfrac{dR}{dt}\right) = \dfrac{d}{d\theta}\left(\theta \dfrac{dR}{dt}\right)$

$\Rightarrow \theta \dfrac{dR}{dt} = K$ where K is a constant. $\Rightarrow \dfrac{dR}{dt} = \dfrac{K}{\theta} \Rightarrow R = \left(\dfrac{K}{\theta}\right) t$ since $\dfrac{d\theta}{dt} = 0 \Rightarrow H_0 = \dfrac{1}{R}\dfrac{dR}{dt} = \dfrac{\theta}{Kt}\dfrac{K}{\theta} = \dfrac{1}{t}.$ So the current value of the Hubble constant is $\dfrac{1}{T}$ where T is the present age of the universe.

44.59. (**a**) For mass m, in Eq. (37.23) $u = -v_{\text{cm}}$, $v' = v_0$, and so $v_m = \dfrac{v_0 - v_{\text{cm}}}{1 - v_0 v_{\text{cm}}/c^2}.$ For mass M, $u = -v_{\text{cm}}$, $v' = 0$, so $v_M = -v_{\text{cm}}.$

(b) The condition for no net momentum in the center of mass frame is $m\gamma_m v_m + M\gamma_M v_M = 0,$ where γ_m and γ_M correspond to the velocities found in part (a). The algebra reduces to $\beta_m\gamma_m = (\beta_0 - \beta')\gamma_0\gamma_M$, where $\beta_0 = \frac{v_0}{c},\ \beta' = \frac{v_{\text{cm}}}{c}$, and the condition for no net momentum becomes $m(\beta_0 - \beta')\gamma_0\gamma_M = M\beta'\gamma_M$, or

$$\beta' = \frac{\beta_0}{1+\dfrac{M}{m\gamma_0}} = \beta_0\frac{m}{m + M\sqrt{1-\beta_0^2}}.\quad v_{\text{cm}} = \frac{mv_0}{m + M\sqrt{1-(v_0/c)^2}}.$$

(c) Substitution of the above expression into the expressions for the velocities found in part (a) gives the relatively simple forms $v_m = v_0\gamma_0\frac{M}{m+M\gamma_0}, v_M = -v_0\gamma_0\frac{m}{m\gamma_0 + M}$. After some more algebra,

$\gamma_m = \frac{m + M\gamma_0}{\sqrt{m^2 + M^2 + 2mM\gamma_0}},\ \gamma_M = \frac{M + m\gamma_0}{\sqrt{m^2 + M^2 + 2mM\gamma_0}}$, from which $m\gamma_m + M\gamma_M = \sqrt{m^2 + M^2 + 2mM\gamma_0}$. This last expression, multiplied by c^2, is the available energy E_{a} in the center of mass frame, so that

$E_{\text{a}}^2 = (m^2 + M^2 + 2mM\gamma_0)c^4 = (mc^2)^2 + (Mc^2)^2 + (2Mc^2)(m\gamma_0 c^2) = (mc^2)^2 + (Mc^2)^2 + 2Mc^2E_m$, which is Eq.(44.9).

44.60. $\Lambda^0 \to \text{n} + \pi^0$

(a) $E = (\Delta m)c^2 = (m_{\Lambda^0})c^2 - (m_{\text{n}})c^2(m_{\pi^0})c^2 = 1116\ \text{MeV} - 939.6\ \text{MeV} - 135.0\ \text{MeV} = 41.4\ \text{MeV}$

(b) Using conservation of momentum and kinetic energy; we know that the momentum of the neutron and pion must have the same magnitude, $p_n = p_\pi$.

$K_{\text{n}} = E_{\text{n}} - m_{\text{n}}c^2 = \sqrt{(m_{\text{n}}c^2)^2 + (p_{\text{n}}c)^2} - m_{\text{n}}c^2 = \sqrt{(m_{\text{n}}c^2)^2 + (p_{\text{n}}c)^2} - m_{\text{n}}c^2$

$K_{\text{n}} = \sqrt{(m_{\text{n}}c^2)^2 + K_\pi^2 + 2m_\pi c^2 K_\pi} - m_{\text{n}}c^2 = K_\pi + K_{\text{n}} = K_\pi + \sqrt{(m_{\text{n}}c^2)^2 + K_\pi^2 + 2m_\pi c^2K_\pi} - m_{\text{n}}c^2 = E.$

$(m_{\text{n}}c^2)^2 + K_\pi^2 + 2m_\pi c^2K_\pi = E^2 + (m_{\text{n}}c^2)^2 + K_\pi^2 + 2Em_\pi c^2 - 2EK_\pi - 2m_{\text{n}}c^2K_\pi$. Collecting terms we find:

$K_\pi(2m_\pi c^2 + 2E + 2m_{\text{n}}c^2) = E^2 + 2Em_{\text{n}}c^2$

$$\Rightarrow K_\pi = \frac{(41.4\ \text{MeV})^2 + 2(41.4\ \text{MeV})(939.6\ \text{MeV})}{2(135.0\ \text{MeV}) + 2(41.4\ \text{MeV}) + 2(939.6\ \text{MeV})} = 35.62\ \text{MeV}.$$

So the fractional energy carried by the pion is $\frac{35.62}{41.4} = 0.86$, and that of the neutron is 0.14.